Uncertainty of Gauge Block Calibration by Mechanical Comparison: A Worked Example

Case 1: Gauges of Like Material*

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> *Although this Guide provides a framework for assessing uncertainty, it cannot substitute for critical thinking, intellectual honesty, and professional skill. The evaluation of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement. The quality and utility of the uncertainty quoted for the result of a measurement therefore ultimately depend on the understanding, critical analysis, and integrity of those who contribute to the assignment of its value.*

"Guide to the Expression of Uncertainty in Measurement", 1993, 1^{st} Edition (International Organization for Standardization, Switzerland), §3.4.8.

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List of Symbols

α Thermal dilatation coefficient of test gauge.

α_s Thermal dilatation coefficient of standard gauge.

$\delta\theta$ Relative temperature difference between standard and test gauge blocks.

θ Temperature offset of test gauge with respect to the reference temperature of 20°C.

θ_s Temperature offset of standard gauge with respect to the reference temperature of 20°C.

c_i, c_{ij}, c_{ijj} Sensitivity coefficients in the combined standard uncertainty. Each of c_i, c_{ij}, c_{ijj} represents partial derivatives of $f(x_i, x_j, \ldots)$, which are defined in §4 equation (10) below.

d Deviation from nominal length. Positive d indicates that the gauge is longer than the nominal length, negative d indicates that it is shorter.

$f(x_i, x_j, \ldots)$ Functional relationship describing the measured quantity in terms of influence factors x_i, x_j, etc. (§4.1.1, §4.1.2 *Guide*[1]).

k Coverage factor for the expanded uncertainty; typically $k = 2$.

l Length of test gauge, where $l = L + d$.

l_s Length of standard gauge.

L Nominal gauge length.

t Temperature in degrees Celcius.

t_{ref} Reference temperature $t_{\mathrm{ref}} = 20°\mathrm{C}$ (ISO 1 (1975)).

$u(x_i)$ Standard uncertainty attributed to the measured quantity x_i (§3.3.5, §4.1.5 *Guide*).

$u_c(x_i)$ Combined standard uncertainty (§3.3.6, §4.1.5 *Guide*); usually a quadrature sum of standard uncertainties of quantities influencing x_i.

$u_c(\theta_s)$ Combined standard uncertainty in measured gauge temperature.

$u(\theta_r)$ Standard uncertainty in gauge temperature measurement attributed to reading the thermometer indicator.

$u(\dot{\theta})$ Standard uncertainty in gauge temperature measurement attributed to the cyclic variation in room temperature.

[1] *Guide to the Expression of Uncertainty in Measurement*, 1993, 1^{st} Edition (International Organization for Standardization, Switzerland).

$u(\theta_{\text{cal}})$ Standard uncertainty in gauge temperature measurement attributed to the traceable calibration of the thermometer.

$u(\delta\theta)$ Standard uncertainty in temperature difference between standard and test gauges.

$u(\alpha_s)$ Standard uncertainty in the thermal dilatation coefficient of the standard.

$u(\alpha)$ Standard uncertainty in the thermal dilatation coefficient of the test gauge.

$u(\alpha_s)u(\theta_s)$ Cross term uncertainty representing the uncertainty attributed to the temperature deviation from the reference temperature of the standard.

$u(\alpha)u(\theta_s)$ Cross term uncertainty representing the uncertainty attributed to the temperature deviation from the reference temperature of the test gauge.

$u(\delta\theta)u(\alpha)$ Cross term uncertainty representing the uncertainty attributed to the temperature difference between standard and test gauges.

$u_c(d)$ Combined standard uncertainty attributed to the measured length difference.

$u(d_r)$ Standard uncertainty attributed to reading the indicator of the length difference between standard and test gauges.

$u(d_{\text{cal}})$ Standard uncertainty in the measured length difference attributed to the calibration of the mechanical comparator.

$u(d_g)$ Standard uncertainty in measured length difference attributed to the stability of the comparator gain electronics.

$u_c(l_s)$ Combined standard uncertainty in calibration of standard.

$u_{\text{cal}}(l_s)$ Calibration uncertainty of standard gauge, provided by the laboratory who calibrated the standards.

$u(\dot{l}_s)$ Uncertainty attributed to secular changes in gauge length.

$u_c(l_{ws})$ Combined standard uncertainty in the calibration of the working standard against the reference standard.

$u_c(l_{cl})$ Combined standard uncertainty in the calibration of a client gauge against the working standard in a direct chain beginning with the reference standard.

U The combined standard uncertainty $u_c(y)$ in a measured quantity y multiplied by a coverage factor k represents the expanded uncertainty.

U_{cl} Expanded uncertainty in the client gauge calibration, where the client gauge is calibrated against the working standard and reference standard in a direct chain.

1 Introduction

The Standards Council of Canada (SCC) and the National Research Council (NRC) Calibration Laboratory Assessment Service (CLAS) are aimed at aiding Canadian calibration laboratories in performing quality calibrations which are directly traceable to the definition of the metre (Pekelsky 1991, Quinn 1993/94). In the area of dimensional metrology, gauge block calibration is one of the key areas of CLAS accreditation.

This document provides a worked example of a typical evaluation of measurement uncertainty for the case of the calibration of gauge blocks by mechanical comparison. The comparison involves gauges of known and unknown length, herein referred to as the *standard gauge* and *test gauge* respectively. In laboratories that provide service to many clients, the lab's *reference standard gauge* is spared excessive handling and wear by calibrating a *working standard gauge*, which is used to calibrate the *client gauge*. Our two-gauge comparison model is general, in that it can apply first to the comparison of the reference standard, and working standard gauge, and then to the working standard and client gauges. At the end of the paper, it is shown how the uncertainties propagate along the traceability chain, from the reference standard, to the working standard, to the client gauge.

It is assumed that the reader is familiar with the gauge blocks, the mechanical comparator and the principles of their metrology. Our objective is to guide the reader through the evaluation of the overall calibration uncertainties.

The following evaluation of the measurement uncertainty is based on criteria expressed in the *ISO Guide to the Expression of Uncertainty in Measurement: 1993(E)*, cited hereafter simply as the *Guide*. The calculation of uncertainties can seem overwhelming at first glance at the *Guide*, but when broken down into a series of smaller tasks, or steps, the evaluation becomes more manageable. The gauge block example here is presented in the following steps:

Step 1: Analyse the measurement process and identify the influence quantities.

Step 2: List any simplifying assumptions and their impact or influence on the measurement.

Step 3: Form a mathematical model of the measurement in terms of the influence quantities (expressed in an optimal form).

Step 4: Evaluate the sensitivity coefficients of the influence quantities.

Step 5: Calculate the standard uncertainties of the influence quantities.

Step 6: Calculate the combined and expanded uncertainties for the overall process.

The remaining sections elaborate on these steps with enough generality that this document can be adapted to other cases, and with enough specific calculations using real-world values that a novice can follow the details to reach a final number for the uncertainty of gauge block calibration by mechanical comparison.[2]

2 Measurement Process and Assumptions

2.1 Identify the Influence Quantities

What are the influence factors involved in measuring length by mechanical comparison? Drawing a block diagram can often help elucidate these factors. In measuring length by mechanical comparison the length of the test gauge is compared to that of the standard gauge and the measured difference in their lengths determines the length of the test gauge. Referring to Figure 1, both the standard and test gauges are placed on the anvil of the comparator, and measured in turn between the styli. The gauge block comparator is of the type where there is a two-point contact, and the block is supported on the anvil. Firstly, the standard gauge is probed by the opposing styli and the length indication is adjusted to agree with its calibrated deviation from nominal length. Next, the test gauge is similarly probed, and the reading of the comparator is recorded. The deviation from nominal length of the test gauge is then reported as this reading.

Factors influencing this measurement are then: the length calibration of the standard, factors inherent in the comparator equipment used to measure the length difference such as scale linearity and reading capability, gauge geometry with respect to its effect on probing the length difference, the temperature of the environment as it influences the gauge temperatures, which is in turn influenced by the gauge materials, and so forth. In summary, they are:

- calibrated value of standard gauge
  - calibration value
  - secular changes in standard gauge
- measured length difference between standard and test gauges

[2] Specific product names or services are mentioned in this document only for the convenience of a worked example. NRC does not promote or endorse any of the products or services named in this article.

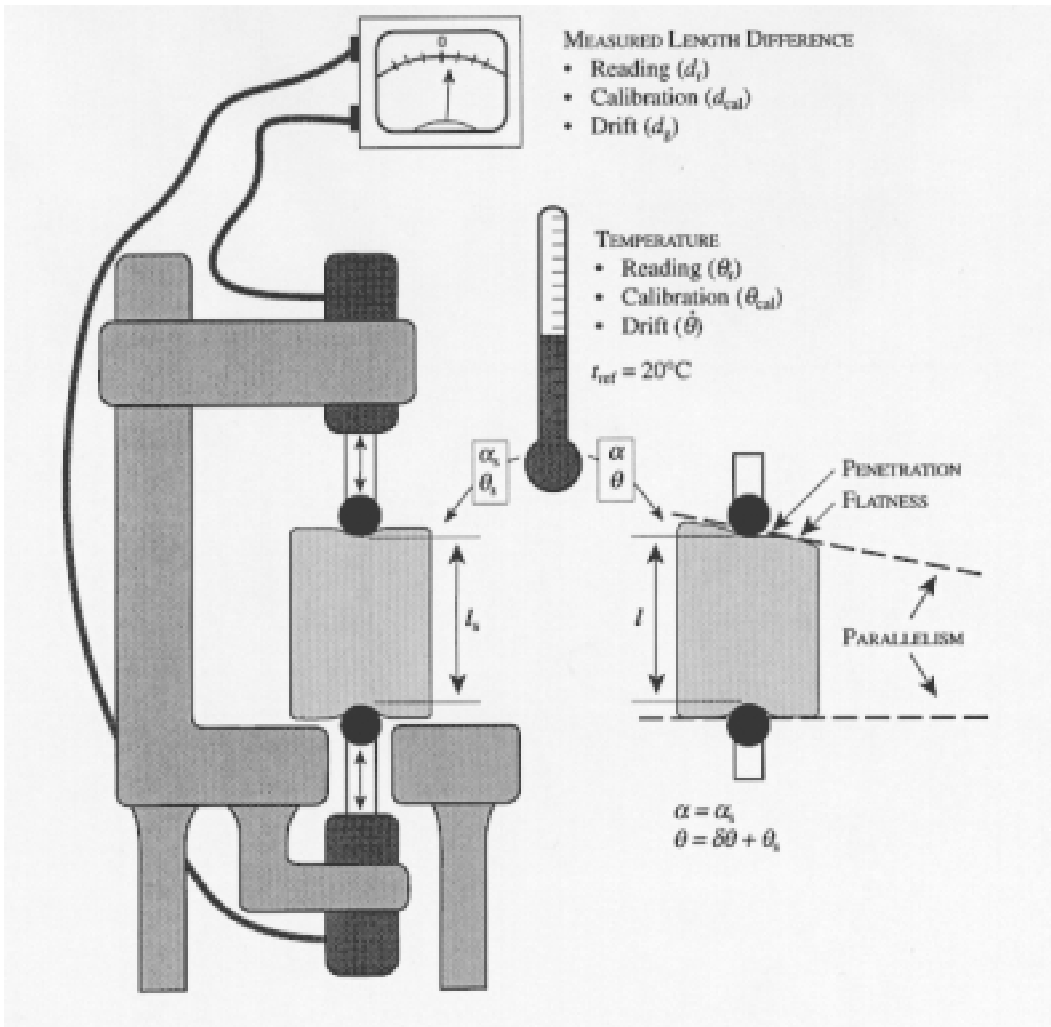


Figure 1: Schematic diagram of a gauge block comparator with opposing styli and digital readout. The diagram depicts the influence factors in the measurement, and the mathematical symbols which will represent them in the text.

  - comparator reading
  - linearity
  - drift
  - gauge positioning and geometry effects
  - gauge penetration variation
- temperature effects
  - thermometer reading
  - thermometer calibration
  - ambient room temperature variation
  - temperature difference between gauges
  - thermal expansion coefficient

The detailed calculation of the uncertainties attributed to the influence quantities is considered in §6. By setting some constraints on the measurement process, some of these factors can be combined, or made negligible.

2.2 The Assumptions

In general, an uncertainty is calculated for a very specific measurement scenario. The specifics of the calibration and the influence factors should be well defined before trying to consider what their uncertainties are. The acronym SWIPE (Standard, Workpiece, Instrument, Procedure/Personnel, Environment) is useful in recalling the key factors which influence a measurement, and can help in setting boundaries and limit cases for the measurement to be considered. The completely general case for gauge block calibration by mechanical comparator is beyond the scope of this paper. By making a few key assumptions, the model is greatly simplified. For this example, the following assumptions are made:

- best quality standard gauge (ISO 3650 grade 1 or better),
- good quality test gauge (ISO 3650 grade 2 or better),
- gauges of like-materials (same stylus penetration and thermal expansion coefficients),
- high-accuracy differential-mode mechanical comparator,
- best laboratory practice, free of blunders,
- environment meets or exceeds CLAS Type 1 lab (20 ± 1°C).

The first two assumptions address gauge geometry, quality and closeness to nominal length. If differences from nominal length are small, and gauge surfaces are flat, parallel and in new condition, then the operator's ability to centre each gauge under the stylus is a small factor that can be included in the repeatability of the comparator readings.

The calibration of like-material standard and test gauges is another key simplification made in this example. By limiting the discussion to similar materials, effects related to differences in thermal expansion simplify one of the largest sources of uncertainty in this kind of calibration, namely that of temperature. Penetration depth of the styli in different materials will be neglected. Variations in stylus penetration are considered to be included in the uncertainty attributed to repeated readings. For the purposes of this example, hardened steel has been selected as the material of both the standard and test gauges.

As for the instrument, it is assumed that the gauge block comparator is a high accuracy model of the opposing-styli type where only small length differences between the standard and test gauges are measured, i.e., both gauges have the same nominal length. Laboratory practices are assumed to be of a consistently high quality, especially in the thermal conditioning and handling of gauges, such that poor measurements as a result of blunders or bad practice are not included in the measurement uncertainty (§3.4.7 and §3.4.8 *Guide*). The laboratory environment is assumed to be maintained close to the standard reference temperature of $t_{\mathrm{ref}} = 20°\mathrm{C}$, for example meeting or exceeding the criteria of a CLAS Type 1 laboratory (Practices for Calibration Laboratories 1990). The reader is referred to the following references (and those mentioned therein) for the general subject of temperature measurement in the laboratory: Magison (1990), McGee (1988), Nicholas and White (1994).

3 Mathematical Model of the Measurement

The primary influence factors identified above can be expressed algebraically and combined to yield a mathematical model representing the measurement. Let the temperature offset θ be the difference between the gauge temperature t and the standard reference temperature $t_{\mathrm{ref}} = 20°\mathrm{C}$ (ISO 1 (1975))

$$\theta = t - t_{\mathrm{ref}}. \tag{1}$$

The length of a gauge at temperature offset θ is given by

$$l\{\theta\} = l(1 + \alpha\theta) \tag{2}$$

where l is the length at the reference temperature, and α is the thermal expansion coefficient for the gauge material. In our example, steel gauges are considered with

$$\alpha = 11.5 \times 10^{-6}/°\mathrm{C}. \tag{3}$$

The length difference d between a test gauge measured at temperature offset θ and a standard gauge at θ_s is

$$\begin{aligned} d &= l\{\theta\} - l_s\{\theta_s\} \\ &= l(1+\alpha\theta) - l_s(1+\alpha_s\theta_s), \end{aligned} \tag{4}$$

where the l and l_s are the lengths at the reference temperature of the test and standard gauges respectively, and α and α_s are their respective thermal expansion coefficients. Equation (4) can be re-arranged to isolate the length of the test gauge:

$$l = \frac{d + l_s(1+\alpha_s\theta_s)}{1+\alpha\theta}. \tag{5}$$

Equation (5) can be approximated as a polynomial by substituting $(1+\alpha\theta)^{-1}$ with its binomial expansion: $(1+x)^{-1} = 1-x+x^2-x^3+\ldots\ (x^2<1)$. Neglecting the quadratic and higher order terms in the polynomial, and retaining only significant first-order terms, it follows that

$$l \approx d + l_s(1+\alpha_s\theta_s - \alpha\theta). \tag{6}$$

For convenience we will define the following parameter which will represent the temperature difference between standard and test gauges[3]:

$$\delta\theta = \theta - \theta_s. \tag{7}$$

Rearranging (7) and applying it to equation (6), the expression for the calibration of a test gauge by mechanical comparison to be used in this evaluation of the measurement uncertainty is:

$$l = d + l_s(1+\alpha_s\theta_s - \alpha\delta\theta - \alpha\theta_s). \tag{8}$$

Recall that the case being considered is that where the standard and test gauges are of like material (steel standard to steel test gauges). Even though the thermal expansion coefficients for both standard and test gauges are nominally equal, and therefore $\alpha_s\theta_s$ and $\alpha\theta_s$ terms would cancel each other out in equation (8), these terms must be kept in the calculation to account for the non-cancelling uncertainties arising from each of these terms.

[3]The use of this difference parameter avoids having to deal with correlated uncertainty components (§F.1.2.4 *Guide*). One can assume that if θ and θ_s are measured with thermometers whose calibration traceability link at some point, then the temperature values for the two gauges will be correlated. Using $\delta\theta$ avoids having to calculate the uncertainty in the case of correlated components and thus simplifies the uncertainty calculation. Even if θ and θ_s are not correlated (i.e., independent), the result obtained by incorporating this substitution is only slightly more conservative. We will choose to leave α and α_s expressed explicitly for the reason that usually the manufacturer's stated values are used without further verification by measurement, and these stated values are assumed to be independent (uncorrelated). Having explicit α and α_s also readies this model for the case of different gauge materials. If the thermal expansion coefficients are indeed measured with the same system in the laboratory, the treatment of this variable could be altered to reduce the total uncertainty.

4 Uncertainty Equations

The uncertainty example calculation presented in this document follows internationally accepted general rules for the evaluation and expression of uncertainties as laid out in the *Guide*. The *Guide* contains detailed definitions of the statistical concepts and terminology, general equations for standard and expanded uncertainties, as well as recommendations for dealing with special uncertainty cases. Every attempt is made to reference the relevant section of the *Guide* where appropriate. The *Guide* also contains some specific worked examples in a variety of measurement domains.

4.1 Combined Standard Uncertainty

The combined standard uncertainty $u_c(l)$ is an estimate of the standard deviation of the distribution of possible values (or probability distribution) of the length of the test gauge l, here measured by mechanical comparison. The combined standard uncertainty, as its name implies, is just a sum of the uncertainties of all of the various influence factors $u(x_i)$, each weighted by a sensitivity coefficient c_i. As the *Guide* explains (§5.1.2), it is a sum of squared terms, given in general as

$$\begin{aligned} u_c^2(l) &= \sum_{i=1}^{N} c_i^2 u^2(x_i) \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left[\frac{1}{2}c_{ij}^2 + c_i \cdot c_{ijj}\right] u^2(x_i)u^2(x_j), \end{aligned} \tag{9}$$

where $u(x_i)$ are the standard uncertainties attributed to the influence quantities, with Type A or Type B evaluations (§4.2, 4.3 *Guide*), and where

$$c_i = \frac{\partial f}{\partial x_i}, \qquad c_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}, \qquad c_{ijj} = \frac{\partial^3 f}{\partial x_i \partial x_j^2}. \tag{10}$$

c_i, c_{ij}, c_{ijj} are the partial derivatives of the expression describing the length of the test gauge $l = f(d, l_s, \alpha_s, \theta_s, \alpha, \delta\theta)$, as developed in equation (8). c_i, c_{ij}, c_{ijj} are often referred to as sensitivity coefficients; their detailed calculation will be discussed below. To make this calculation a little more palatable, it is convenient to think of equation (9) as consisting of two parts: first-order terms containing $u^2(x_i)$, and higher order terms containing $u^2(x_i)u^2(x_j)$. In the subject area of dimensional metrology, the higher order terms should be evaluated with the first order terms.

4.2 Expanded Uncertainty

It is desirable to express measurement uncertainties in a form that encompasses a large portion of the distribution of possible values. The expanded uncertainty (§6.2 *Guide*)

$$U = ku_c(l), \tag{11}$$

is defined as the combined standard uncertainty multiplied by a coverage factor k. The value of the coverage factor is chosen depending on the approximate level of confidence that would facilitate the interpretation of the uncertainty. Most measurements are expressed with a value of k between 2 and 3. NRC has chosen to express the expanded uncertainty for $k = 2$, corresponding to an approximately 95% confidence level (§6.2.2 *Guide*). Multiplying by a coverage factor does not add any new information; it is a convention. The emphasis of this document is on the calculation of the combined standard uncertainty.

It is important that the reader distinguish between the *standard uncertainty* u, which is the $k = 1$ or 1σ value, and the *expanded uncertainty* $U = ku$, where $k > 1$. In §6, uncertainties with various k values will be adjusted to the 1σ ($k = 1$) level in order to work with the standard uncertainties throughout that section. The combined standard uncertainty is then expanded to U for $k = 2$ in §8.

4.3 Significant Digits for Calculated Uncertainty

Throughout the following calculations, three significant digits may be carried for purposes of resolution and to avoid rounding errors; however, for the scope of this uncertainty calculation, more than two significant digits is usually not warranted. Remember that uncertainties should always be rounded up (eg., 11.32 rounds to 11.4, which rounds to 12).

5 Evaluation of Sensitivity Coefficients

The general equation to be applied in the calculation of the combined standard uncertainty is equation (9). A model describing the measurement is represented mathematically by equation (8). It may seem complicated, but the calculation of the uncertainty begins by applying (9) to the mathematical expression (8). Simply making the substitution of our influence variables in place of the x_i in

(9), the combined standard uncertainty can be written as

$$
\begin{aligned}
u_c^2(l) = & \quad c_{l_s}^2 \; u^2(l_s) \\
+ & \quad c_d^2 \; u^2(d) \\
+ & \quad c_{\alpha_s}^2 \; u^2(\alpha_s) \\
+ & \quad c_\alpha^2 \; u^2(\alpha) \\
+ & \quad c_{\theta_s}^2 \; u^2(\theta_s) \\
+ & \quad c_{\delta\theta}^2 \; u^2(\delta\theta) \\
+ & \quad \text{higher order terms},
\end{aligned}
\tag{12}
$$

where the sensitivity coefficients for the first order terms are

$$
\begin{aligned}
c_{l_s} &= \frac{\partial l}{\partial l_s}, \\
c_d &= \frac{\partial l}{\partial d}, \\
c_{\alpha_s} &= \frac{\partial l}{\partial \alpha_s}, \\
c_\alpha &= \frac{\partial l}{\partial \alpha}, \\
c_{\theta_s} &= \frac{\partial l}{\partial \theta_s}, \\
c_{\delta\theta} &= \frac{\partial l}{\partial \delta\theta}.
\end{aligned}
\tag{13}
$$

The next step is to perform the partial derivatives for c_i, c_{ij}, c_{ijj} on equation (8) to determine the sensitivity coefficients. It is very helpful to use a table-style format in the book-keeping of the terms. Tables 1, 2 and 3 are constructed in sequence: the first partial derivatives of equation (8) listed in Table 1 are used in the calculation of the second partial derivatives c_{ij} in Table 2. Similarly, the second partial derivatives in Table 2 are used for the calculation of the third partial derivatives c_{ijj} in Table 3.

x_i	$c_i = \dfrac{\partial l}{\partial x_i}$
l_s	$1 + \alpha_s\theta_s - \alpha\delta\theta - \alpha\theta_s$
d	1
α_s	$l_s\theta_s$
α	$-l_s(\delta\theta + \theta_s)$
θ_s	$l_s(\alpha_s - \alpha)$
$\delta\theta$	$-l_s\alpha$

Table 1: Sensitivity coefficients for the first order standard uncertainty components where l is expressed by equation (8).

x_i	$c_{ij} = \dfrac{\partial^2 l}{\partial x_i \partial x_j}$					
	$x_j =$ l_s	d	α_s	α	θ_s	$\delta\theta$
l_s	$\cdot$	$\cdot$	θ_s	$-\delta\theta - \theta_s$	$\alpha_s - \alpha$	$-\alpha$
d	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
α_s	θ_s	$\cdot$	$\cdot$	$\cdot$	l_s	$\cdot$
α	$-\delta\theta - \theta_s$	$\cdot$	$\cdot$	$\cdot$	$-l_s$	$-l_s$
θ_s	$\alpha_s - \alpha$	$\cdot$	l_s	$-l_s$	$\cdot$	$\cdot$
$\delta\theta$	$-\alpha$	$\cdot$	$\cdot$	$-l_s$	$\cdot$	$\cdot$

Table 2: Sensitivity coefficients for the second order standard uncertainty components, where l is expressed by equation (8) and the first partial derivatives are given in Table 1. A dot $(\cdot)$ means the coefficient is zero.

x_i	$c_{ijj} = \dfrac{\partial^3 l}{\partial x_i \partial x_j^2}$					
	$x_j =$ l_s	d	α_s	α	θ_s	$\delta\theta$
l_s	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
d	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
α_s	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
α	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
θ_s	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$
$\delta\theta$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdot$

Table 3: Sensitivity coefficients for the second order standard uncertainty components, continued. A dot $(\cdot)$ means the coefficient is zero.

Referring to Table 1 to make the substitutions for c_i, equation (12) becomes:

$$\begin{array}{rrl} u_c^2(l) = & (1+\alpha_s\theta_s-\alpha\delta\theta-\alpha\theta_s)^2 & u^2(l_s) \\ + & & u^2(d) \\ + & l_s^2\theta_s^2 & u^2(\alpha_s) \\ + & l_s^2(\delta\theta+\theta_s)^2 & u^2(\alpha) \\ + & l_s^2(\alpha_s-\alpha)^2 & u^2(\theta_s) \\ + & l_s^2\alpha^2 & u^2(\delta\theta) \\ + & & \text{higher order terms.} \end{array} \tag{14}$$

It is recognized that the first term involving $u^2(l_s)$ in equation (14) can be simplified a little at this point. It is known that regardless of the material, the expansion coefficient is in the neighborhood of $10^{-5}/^\circ$C. In the initial analysis of the measurement it has also been assumed that the comparator is placed in a temperature-controlled metrology laboratory, in which case ambient air temperature deviation, and therefore the standard gauge temperature deviation from the standard reference temperature, θ_s is small — less than 1°C. In addition, good metrological practices are assumed, therefore the temperature difference $\delta\theta$ between gauges is more than likely even smaller than that. Based on these reasonable assumptions, it is realized that the terms $\alpha_s\theta_s$, $\alpha\delta\theta$ and $\alpha\theta_s$ will have a small impact on $u^2(l_s)$ compared to unity, and can be neglected.

Now, consider the higher order terms in equation (9). Referring to Table 3, the sensitivity coefficients c_{ijj} are all exactly zero. This means that all the $c_i \cdot c_{ijj}$ terms disappear. The higher order terms remaining are those that are multiplied by c_{ij}, namely:

$$\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{2}c_{ij}^2u^2(x_i)u^2(x_j).$$

Reading c_{ij} from Table 2, the higher order contributions to the combined standard uncertainty are listed below (duplicate terms have been combined):

$$\begin{aligned} |c_{\alpha_s,l_s}|^2u^2(\alpha_s)u^2(l_s) &= \theta_s^2u^2(\alpha_s)u^2(l_s) \\ |c_{\alpha,l_s}|^2u^2(\alpha)u^2(l_s) &= (\delta\theta+\theta_s)^2u^2(\alpha)u^2(l_s) \\ |c_{\theta_s,l_s}|^2u^2(\theta_s)u^2(l_s) &= (\alpha_s-\alpha)^2u^2(\theta_s)u^2(l_s) \\ |c_{\delta\theta,l_s}|^2u^2(\delta\theta)u^2(l_s) &= \alpha^2u^2(\delta\theta)u^2(l_s) \\ |c_{\theta_s,\alpha_s}|^2u^2(\theta_s)u^2(\alpha_s) &= l_s^2u^2(\theta_s)u^2(\alpha_s) \\ |c_{\theta_s,\alpha}|^2u^2(\theta_s)u^2(\alpha) &= l_s^2u^2(\theta_s)u^2(\alpha) \\ |c_{\delta\theta,\alpha}|^2u^2(\delta\theta)u^2(\alpha) &= l_s^2u^2(\delta\theta)u^2(\alpha). \end{aligned} \tag{15}$$

The combined standard uncertainty in the mechanical comparison length measurement, according to the model described by equation (8), is thus:

$$
\begin{array}{rrll}
u_c^2(l) = & & u^2(l_s) & \\
+ & & u^2(d) & \\
+ & l_s^2\theta_s^2 & u^2(\alpha_s) & \\
+ & l_s^2(\delta\theta+\theta_s)^2 & u^2(\alpha) & \\
+ & l_s^2(\alpha_s-\alpha)^2 & u^2(\theta_s) & \\
+ & l_s^2\alpha^2 & u^2(\delta\theta) & \\
+ & \theta_s^2 & u^2(\alpha_s) & u^2(l_s) \\
+ & (\delta\theta+\theta_s)^2 & u^2(\alpha) & u^2(l_s) \\
+ & (\alpha_s-\alpha)^2 & u^2(\theta_s) & u^2(l_s) \\
+ & \alpha^2 & u^2(\delta\theta) & u^2(l_s) \\
+ & l_s^2 & u^2(\theta_s) & u^2(\alpha_s) \\
+ & l_s^2 & u^2(\theta_s) & u^2(\alpha) \\
+ & l_s^2 & u^2(\delta\theta) & u^2(\alpha).
\end{array}
\tag{16}
$$

Armed with the list of terms in this equation, we can now examine each term in detail. Table 4 is a preview of this process, listing the various uncertainty components, their physical source, an evaluated magnitude, and their combined influence.

6 Standard Uncertainties of Influence Quantities

6.1 Opening Remarks

The identification of the variables used in the evaluation of the standard uncertainties and the values of the uncertainty components are tabulated in Table 4. The following section provides a discussion of each case individually. *A specific example is presented here, and the detailed characterization of any system and its associated measurement uncertainties will be unique to a given set of conditions. By giving all the details at each step, this document is intended to be used as a guide for such a characterization.*

6.1.1 Nominal Length

The nominal length L is used for convenience in the calculation of length dependent coefficients in equation (16). The difference arising in the calculation from the substitution of L in the place of l_s is negligible.

6.1.2 Combined Standard Uncertainty Notation

The combined standard uncertainty is a quadrature sum (i.e., sum of squared terms) of the standard uncertainties attributed to the influence factors. It is convenient to sum the end effect uncertainties as a, and length dependent uncertainties as bL and express the combined uncertainty as $u_c = \sqrt{a^2 + b^2L^2}$ nm, where L is the nominal gauge length in millimeters. This notation allows one to estimate how the uncertainty will scale with the length of the gauge, and what the minimum uncertainty for short gauges having negligible length effects would be.

6.1.3 End and Length Dependence

Gauge block uncertainties can be grouped into those with length dependent effects and end effects. End effect uncertainties are those for which the uncertainty value is constant regardless of the length of the gauge. An example of an end effect uncertainty component is the indicator reading uncertainty, where the variation in reading the indicator is not related to the length of the gauge. Length dependent uncertainties are of the form of a coefficient multiplied by the nominal gauge length. The value changes with gauge length; for example, longer gauges display more dramatic changes in length with temperature variations than do shorter gauges, and therefore the uncertainties corresponding to temperature effects will scale as the length of the gauge.

6.1.4 Type A and Type B Uncertainty Evaluations

In the *Guide*, there is considerable concern about distinguishing between Type A and Type B uncertainty evaluations. Simply put: Type A evaluations are those for which repeated measurements are made and the 1σ standard deviation is calculated from the data, and used as the standard uncertainty. Type B evaluations are those for which repeated measurements cannot simply isolate the influence, and the uncertainty must be obtained by some other method based on the experience and expertise of the metrologist. The reader is referred to the *Guide* (§4.2, §4.3) for more background. In the sections that follow, the discussion gives worked examples of evaluating each type.

The rectangular distribution crops up frequently in Type B evaluation, and it is used in several instances in this document. As explained in §4.4.5 of the *Guide* (also see Figure 2, p. 17 *Guide*), if data to estimate the uncertainty distribution of an influence parameter is limited, often an adequate and useful approximation is to assume an upper $+a$ and lower $-a$ bound for a range of equally probable values. The standard uncertainty (§4.3.7 *Guide*) is then given by $a/\sqrt{3}$.

6.1.5 Interpreting Uncertainties to Identify Weaknesses

Uncertainty modeling and evaluation is an important tool for the metrologist as it provides a qualitative and quantitative insight in to the factors controlling the total uncertainty for a process. If any one uncertainty term is several times larger than the rest, when added in quadrature, it's magnitude dominates the combined uncertainty. As each component is investigated in the following sections of this example, the dominant terms will be noted. In a very practical sense, these dominant terms are the areas in which the most cost-effective improvements can be made to the calibration system.

6.2 Calibrated Value of Standard Gauge

Calibration Value The uncertainty of the calibration value of the standard gauge block $u_{\text{cal}}(l_s)$ is provided on the calibration certificate for the most recent calibration. This is a Type B evaluation, comprised of both length dependent and end effect components. The calibration uncertainty provided on the certificate indicates the uncertainty of the calibration at the time of measurement. Estimation of the long-term behaviour of the gauge length may or may not be included in this calculation; this matter should be discussed with whomever is responsible for the calibration. A calibration certificate from NRC will indicate the evaluated uncertainty for each gauge, as well as the parametric expanded uncertainty $U = k\sqrt{a^2 + b^2L^2}$ nm for a coverage factor of $k = 2$. For the example in this document, $U = 2\sqrt{10^2 + 0.21^2L^2}$ nm, where L is the nominal length of the gauge in millimeters. Therefore,

$$u_{\text{cal}}(l_s) = \sqrt{10^2 + 0.21^2L^2} \text{ nm}. \tag{17}$$

Secular Uncertainty One can estimate the effect of long-term drift of the standard gauge length by analyzing the trend of the gauge blocks over a period spanning several calibrations. With sufficient history, one can predict a value for the drift of the gauge length over the next calibration interval, as well as assign a value for the secular uncertainty $u(\dot{l}_s)$. This Type A evaluation of statistical data is the preferred method for determining this important length dependent uncertainty factor. It is the main reason that gauges with a long history of calibration are so valuable: secular characterization with a small uncertainty takes many years. A new set (as in this example) is an unknown performer. Depending on the variables of manufacturing the bulk material and processing it into blocks, the finished individual gauges may be growing or shrinking in a very uncertain fashion.

In the absence of statistical study of the specific members of the set, one must resort to a Type B best guess, based on the experience and research of others, and therefore potentially conservative numbers. For example, the stability of gauge block steel has been studied at the U. S. National Institute for Standards and Technology (NIST), where it was determined that steel gauges may shift in length by an amount $\dot{l} = 0.02l$ ppm/year,[4] with an uncertainty $u(\dot{l}) = 0.2l$ ppm/year (Doiron and Beers 1995). Assuming that the standards are calibrated annually

$$\begin{aligned} u(\dot{l}_s) &= \frac{0.2l_s \times 10^{-6}}{\text{year}} \times 1 \text{ year} \\ &= 0.2L \text{ nm, for 1 year,} \end{aligned} \tag{18}$$

for the gauge length L in millimeters. Notice that this is a length dependant uncertainty, and it can be quite large. If recalibration is delayed for several years, this term can grow to dominate. The price of a new set with no history is a costly secular uncertainty. For gauge sets with a long history, this term can be made negligible.

Combined Standard Uncertainty in Standard Gauge Calibration The above calibration and secular uncertainties are combined to give the uncertainty attributed to the calibration value of the standard as,

$$\begin{aligned} u_c(l_s) &= \sqrt{u^2_{\text{cal}}(l_s) + u^2(\dot{l}_s)} \\ &= \sqrt{10^2 + 0.21^2L^2 + 0.2L^2} \text{ nm} \\ &= \sqrt{10^2 + 0.290^2L^2} \text{ nm.} \end{aligned} \tag{19}$$

Recall from (16) that the sensitivity coefficient corresponding to this term is unity.

6.3 Measured Difference in Gauge Lengths

In this section, the uncertainty $u_c(d)$ in the value read from the comparator as the difference between the lengths of the standard and the test gauge is considered. This combined standard uncertainty is a combination of the following components.

Reading the Length Difference The uncertainty $u(d_r)$ in reading the length indicator due to the short-term perturbations can be evaluated by repeated difference readings of the same gauges (described below), which is a Type A evaluation. Somewhere in the uncertainty model, the operator(s) variance must also

[4]ppm: parts per million, 1 ppm = 10^{-6}

be accounted for. In this example, it is included in the reading uncertainty. One could also refer to the uncertainty evaluation for repeated readings provided by the manufacturer, which is a Type B uncertainty evaluation. Both methods evaluating this uncertainty are valid; however, the uncertainty attributed to repeated readings given by the manufacturer may be on the conservative side. In the example case presented here, one can get a more realistic value for this uncertainty by performing an experiment that isolates this influence factor.

In order to isolate the variation due to taking repeated readings, the readings of the gauge difference should be taken under constant conditions (recall SWIPE and the conditions laid out in §2.2):

- same gauges,
- same operator (repeat experiment for different operators),
- constant environment,
- constant instrument.

The experimental evaluation of this short-term uncertainty in the difference measurement simultaneously takes into account end effects such as: operator effects, electronic effects of the comparator, variations in the position of the stylus contact point with respect to the reference point, gauge geometry deviations in flatness and parallelism, and variations in probe penetration. These influence parameters will manifest themselves in the uncertainty of the repeated readings. Example data of the repeated readings of the length difference between standard and test gauges is shown in Table 5.

Length Difference Readings					
Example Gauge Block #1			Example Gauge Block #2		
Time	Temp. [°C]	Difference Reading [microinch]	Time	Temp. [°C]	Difference Reading [microinch]
4:19	19.9	+0.6	4:30	19.9	+5.5
4:19	19.9	+0.6	4:30	19.9	+5.4
4:20	19.9	+0.5	4:31	19.9	+5.4
4:20	19.9	+0.6	4:31	19.9	+5.5
4:21	19.9	+0.5	4:32	19.9	+5.4
standard deviation:		$1\sigma = 0.05$			$1\sigma = 0.05$

Table 5: Example of real data: repeated measurements of length difference between standard and test gauges. Experiment performed on Grade 1 steel rectangular gauge blocks.

Evaluating the standard deviation of the data in Table 5 yields a standard uncertainty in taking one difference measurement reading of (SI units as per CSA Standard 1989)

$$\begin{aligned} u(d_r) &= 0.05 \text{ microinch} \times \frac{25.4 \text{ nm}}{\text{microinch}} \\ &= 1.3 \text{ nm}. \end{aligned} \tag{20}$$

The uncertainty in taking the reading of the length difference is an end-effect uncertainty.

Comparator Linearity Calibration The length-measuring transducer for this example comparator is a linear variable differential transformer (LVDT) which has a long mechanical range (5 μm or 200 microinch) that is interpolated with a high resolution indicating device with a shorter range (2 μm). The indicator can be offset or stepped along the range of the LVDT by a selector switch. The linearity of readings within the step range is verified by using a calibrated set of 5–6 reference gauge blocks increasing in length by small steps within the test range. The indicator reading is compared with the calibrated length of each gauge. The shape of the plot of the indicator readings with respect to the calibration values is indicative of the nonlinearity within this measurement range. This nonlinearity correction could be applied to the comparator readings to reduce the variation of measurement made with the instrument. However, for best-quality comparators this nonlinearity is small and is more conveniently treated as a small uncertainty component. The gauge blocks used in this test are rather specialized (i.e., expensive) and although a Type A evaluation is desirable, it may be sufficient to assign an uncertainty based on a Type B evaluation of the manufacturer's claimed performance (assumed to be based on rigorous tests over the stated range of the instrument, see Figure 2). For our example, we consider the comparator specifications given in Figure 3, where the calibration accuracy is stated to be better than 0.2 microinch (2σ). Converting to 1σ and SI units:

$$\begin{aligned} u(d_{\text{cal}}) &= \frac{0.2 \text{ microinch}}{2} \times \frac{25.4 \text{ nm}}{\text{microinch}} \\ &= 2.5 \text{ nm}. \end{aligned} \tag{21}$$

In the case where the coverage factor is not supplied, one should first contact the manufacturer to clarify what the coverage factor is. Failing this, our policy at NRC is to assume a coverage factor of $k = 2$.

Comparator Drift The uncertainty attributed to scale-size drift in the readings due to the gain electronics can be determined from regular calibrations. Table 6 shows an example of the verification of gain stability on a regular basis

FEDERAL
AN ESTERLINE COMPANY

REPORT OF CALIBRATION

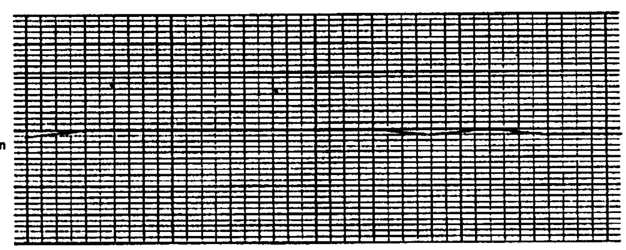
For: W. G. SLOAN INDUSTRIAL SUPPLIES LTD.
450 MATHESON BLVD. - UNIT 39
MISSISSAUGA L4Z 1R5
ONTARIO CANADA

P.O. No.: [redacted]
Dated: [redacted]
Item No.: 1
Model No.: 130B-24
Serial No.: [redacted]
REPORT NO. : [redacted]

Using the Federal Products Micro-Inch Calibrator which has been certified accurate to within 0.2μ" per .001" by the National Bureau of Standards, we have calibrated this Comparator and find its performance to be as shown below:

Specification: ☒ 1.0μ" in the central 100μ" and better than 1.0μ" in any 50μ"
☐ 0,03μm in the central 3 μm and better than 0,03μm in any 1,5 μm
☐

Deviation:
Each vertical chart graduation equals:
☒ .1μ" ☐ 0,0025μm
☐



Gage Head Position:
Each horizontal chart graduation equals:
☐ 10μ" ☒ 2.5μ" ☐ 0,25μm ☐ 0,05μm ☐

This calibration was performed with all auxiliary electronics functioning, as in actual operating conditions, with the Comparator's electrical zero control(s) in mean position and magnification adjusted, in accordance with the Comparator's Instruction Manual.

OUR CALIBRATION SYSTEM IS IN COMPLIANCE WITH MIL-STD-45662A.

Reference Temperature: 68°F/20°C

Reported by EJG

N.B.S. Test No.: 731/246660-90

Date 12/14/93

Dated: 8/2/90
PROCEDURE #PMC-EC-2

Edward J. Green
Supervisor, Precision Measurement Center
EDWARD J. GREEN

Federal Products Corporation, 1144 Eddy Street, P.O. Box 9400, Providence, R.I. 02940-9400 U.S.A.
Telephone: 401-781-9300 Telex-domestic: 927623 FEDGAGE PVD Telex-International: 6814119 FEDGAGE

Figure 2: Manufacturers calibration certificate demonstrating the linearity of the gauge block comparator. The graphed line has been re-drawn for illustration.

Federal Products Co.
Instrumentation Group, Esterline Technologies
1144 Eddy Street
P.O. Box 9400
Providence, RI 02940 U.S.A
Phone: 401/781-9300
Fax: 401/941 5280

Esterline FEDERAL

PERFORMANCE SPECIFICATIONS
MODEL 130B-24 WITH DIGITAL READOUT

GENERAL DESCRIPTION

Bench Type, Electronic Millionth Class Gage Block Comparator. Measures length of square or rectangular gage blocks from 0-4" (0-100mm) at right angles to gaging surface without wringing.

PERFORMANCE SPECIFICATIONS

Repetition Accuracy: Standard deviation (sigma) of repeat, better than .2 micro inches (±3 sigma = ±1/2 micro inch using grade "AA" gage blocks.

Calibration Accuracy: Calibrated every 10 micro inches for at least 100 micro inches on equipment calibrated by NIST to an accuracy within 0.2 micro inches. Calibration chart (supplied) shows deviations in 0.1 micro inches.

Short Term Stability: Zero setting remains stable within 0.5% of full scale per hour (ambient temperature controlled to ±1.0°F).

Long Term Stability: Magnification remains stable within 1% of full scale in 6 month period (ambient temperature controlled to ±1.0°F).

DESIGN FEATURES

Amplifier: Solid-state, multi-stage. Power consumption 7 watts or less. Internal voltage regulation such that a ±10% line voltage variation produces a maximum of 0.75% of full scale meter variation at maximum voltage output.

Digital Display: Displays two ranges. In the inch mode, the range will be ±199.9 micro inches with .1 micro inch least digit. In the metric mode, the range will be ±5 micro meters with .01 micro meter least digit. Analog edge meter provided for quick set-up.

Gage Head: Reed spring suspension.

Gage Contacts: Diamond, with spherical radius. Positive contact throughout ranges. Will not cause permanent deformation of gage blocks. Constant spring rate.

Figure 3: Manufacturer's specification sheet listing the performance specifications. The Calibration Accuracy is used in the calculation of the measurement uncertainty.

and how, in this case, the uncertainty attributed to changes in gain can be estimated at a rectangularly distributed range (§6.1.4) of 0.1 microinch. Therefore

$$\begin{aligned} u(d_g) &= \frac{0.1 \text{ microinch}}{\sqrt{3}} \times \frac{25.4 \text{ nm}}{\text{microinch}} \\ &= 1.5 \text{ nm}, \end{aligned} \tag{22}$$

provided that the gain is calibrated on a regular basis.

Verification of Electronic Gain of Comparator						
	Comparator Reading [microinch]					
Date	0.1001 in	0.1002 in	0.1003 in	0.1004 in	0.1005 in	Adjust
1 Jan ‘96	-199.2	-104.4	-0.7	97.5	199.6	none
15 Jan ‘96	-199.2	-104.1	-0.7	97.5	199.7	none
22 Jan ‘96	-199.1	-104.0	-0.7	97.3	199.7	none
6 Feb ‘96	-199.1	-104.1	-0.7	97.4	199.6	none
19 Feb ‘96	-199.2	-104.1	-0.7	97.4	199.8	none
7 Mar ‘96	-199.0	-104.1	-0.7	97.5	199.6	none

Table 6: Real experimental data demonstrating the stability of the gauge block comparator electronic gain. In order to verify the electronic gain of the comparator, a series of known gauge blocks are used to probe the range within one step of the comparator. In this example, the upper and lower limits of the range are probed by the 0.1001 inch and 0.1005 inch gauge blocks, respectively. The 0.1002 inch and 0.1004 inch gauges verify the stability within the range. The comparator reading is set to that of the mid-range 0.1003 inch gauge block, and the values for the other blocks used in the test are recorded without adjustment. The data in this example demonstrates that there is a ±0.1 microinch performance band over the range.

Combined Standard Uncertainty in Comparator Readings The combined standard uncertainty $u_c(d)$ attributed to comparator effects in reading the length difference between the two gauges is the quadrature sum of the standard uncertainty attributed to repeated observations $u(d_r)$, the standard uncertainty attributed to comparator calibration $u(d_{\text{cal}})$, and the standard uncertainty attributed to comparator drift $u(d_g)$, namely:

$$\begin{aligned} u_c(d) &= \sqrt{u^2(d_r) + u^2(d_{\text{cal}}) + u^2(d_g)} \\ &= \sqrt{(1.3 \text{ nm})^2 + (2.5 \text{ nm})^2 + (1.5 \text{ nm})^2} \\ &= 3.19 \text{ nm}. \end{aligned} \tag{23}$$

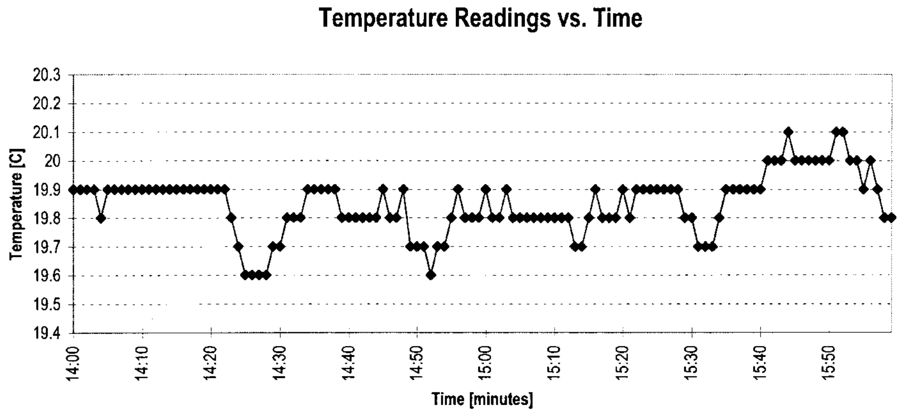


Figure 4: Temperature readings of the thermometer in the neighborhood of the gauge block comparator plotted for two hours. From this plot, the 0.1°C resolution and the cyclic variation in ambient room temperature can be observed.

6.4 Uncertainties in Measuring Temperature

Given the large ramifications of temperature on dimensional measurements, it is necessary to characterize the temperature behaviour in the location of the laboratory in which gauge blocks are calibrated, particularly when striving for lowest uncertainty calibrations.

Ambient Temperature Variation The cyclic variation of the temperature can be determined from an experimental plot of temperature vs. time, as shown in Figure 4. In any one hour period, the maximum offset of the room conditions from the mean temperature observed in the example is 0.3°C. This is a Type B evaluation, where the uncertainty is assumed to be rectangularly distributed across this range of ±0.3°C (§6.1.4), therefore

$$\begin{aligned} u(\dot{\theta}) &= \frac{0.3^{\circ}\mathrm{C}}{\sqrt{3}} \\ &= 0.17^{\circ}\mathrm{C}. \end{aligned} \tag{24}$$

This is a conservative evaluation of $u(\dot{\theta})$. The thermometer records the cycling of the air temperature, whereas the cyclic variation of the temperature of the gauge block has a large thermal inertia, and its temperature variation will be considerably damped compared to that of the air.

Reading Temperature By plotting temperature vs. time as in the example in Figure 4, one can observe that the readings are quantized by the digital resolution of the temperature indicator. In this case, the digital resolution is 0.1°C, therefore the uncertainty associated with the thermometer reading capability is

$$\begin{aligned} u(\theta_r) &= \frac{0.1°\mathrm{C}}{\sqrt{12}} \\ &= 0.03°\mathrm{C}, \end{aligned} \tag{25}$$

where the $\sqrt{12}$ factor arises from evaluating a digital resolution (§F.2.2.1 *Guide*). This is a Type B uncertainty evaluation.

Thermometer Calibration The calibration certificate should state the calibration uncertainty of the thermometer, and also the level of confidence or the coverage factor used. Using the uncertainty from a calibration report or certificate is a Type B evaluation. In the example in Figure 5, the uncertainty stated on the calibration certificate is 0.006°C. The calibration certificate also states that the coverage factor used is k=2. Therefore the standard uncertainty in the thermometer calibration is

$$\begin{aligned} u(\theta_{\mathrm{cal}}) &= \frac{0.006°\mathrm{C}}{2} \\ &= 0.003°\mathrm{C}. \end{aligned} \tag{26}$$

Combined Uncertainty in Temperature Measurement The combined standard uncertainty $u_c(\theta_s)$ in the temperature measurement of the standard gauge is then the quadrature sum of the above components:

$$\begin{aligned} u_c(\theta_s) &= \sqrt{u^2(\dot{\theta}) + u^2(\theta_r) + u^2(\theta_{\mathrm{cal}})} \\ &= \sqrt{(0.17)^2 + (0.03)^2 + (0.003)^2} \\ &= 0.173°\mathrm{C}. \end{aligned} \tag{27}$$

Notice that the room temperature drift contribution dominates the uncertainty compared to those for the reading and calibration of the thermometer.

The contribution to the combined uncertainty in the length measurement is $u_c(\theta_s)$ multiplied by the sensitivity coefficient determined for $u(\theta_s)$ in (16), namely $l_s(\alpha_s - \alpha)$. It is immediately noticed that the term $(\alpha_s - \alpha)$ is zero in the case of comparison of gauge blocks of like materials since the thermal expansion coefficients are nominally equal. It shall be shown below that the temperature effects make significant contributions to the uncertainty through the higher order terms.

[redacted] TEMPERATURE CALIBRATION CERTIFICATE

WORK ORDER NO:	[redacted]	CERTIFICATE #:	[redacted]
UUT TYPE:	THERMISTOR THERMOMETER	CLIENT:	[redacted]
UUT MODEL:	866	CAL PROCEDURE:	ES-273
UUT ASSET #:	NONE	CAL BY:	[redacted] [redacted]
UUT S/N:	T-55858	CAL DATE:	1995-09-05
MANUFACTURER:	TEGAM	LAB RH ± 5%:	55
		LAB TEMP ±1 °C:	22

TEMP STANDARD °C	UUT READING °C	UUT ERROR °C	CHECK STANDARD °C
18.993	19.0	0.0	19.005
19.500	19.5	-0.0	19.513
19.991	20.0	0.0	20.003
20.498	20.5	0.0	20.510
21.002	21.0	-0.0	21.014

Resolution of the UUT is 0.1 °C.

The total temperature calibration system uncertainty at the 95% (2S) confidence level is ±0.006 °C, which includes the uncertainty of the NRC to whom the standards are traceable.

EQUIPMENT USED:

C151014	ROSEMOUNT 162C SPRT THERMOMETER AS TEMP STANDARD
C151002	ROSEMOUNT 910AC CONSTANT TEMP FLUORINERT BATH
A081024	GUILDLINE 9975 RESISTANCE BRIDGE
A081025	GUILDLINE 9330 REFERENCE RESISTOR
C151024	GUILDLINE 9540 PRT THERMOMETER AS CHECK STANDARD
	PC FILE ES-263.WK1 TO CONVERT RESISTANCE TO TEMPERATURE

Figure 5: Calibration certificate for the digital thermometer used in this example.

6.5 Temperature Difference Between Gauges

For the laboratory in this example, the ability to resolve temperature is limited to 0.1°C, so it is difficult to make tests for a Type A evaluation demonstrating that gauge blocks are at the same temperature to a smaller uncertainty than this. Indeed, most metrologists rely on allowing blocks to stabilize to a common temperature over a period of time, rather than trying to measure gauge temperature with a thermometer. Given sufficient time to stabilize, a reasonable assumption is that the gauges have a temperature difference that is no larger than what can be resolved with the laboratory thermometer (0.1°C). This Type B uncertainty estimation is then based on a rectangular distribution (§6.1.4) spanning the range ±0.1°C. Therefore

$$\begin{aligned} u(\delta\theta) &= \frac{0.1^\circ\text{C}}{\sqrt{3}} \\ &= 0.06^\circ\text{C}. \end{aligned} \tag{28}$$

Recalling equation (16), the contribution to the combined standard uncertainty in the test gauge length measurement for steel gauge blocks is ($\alpha = 11.5 \times 10^{-6}/^\circ\text{C}$):

$$\begin{aligned} \alpha l_s u(\delta\theta) &= (11.5 \times 10^{-6}/^\circ\text{C})\ l_s\ (0.06^\circ\text{C}) \\ &= 0.690L \text{ nm}, \end{aligned} \tag{29}$$

where L is the nominal gauge length in millimeters.

6.6 Thermal Expansion Coefficient

The value for the thermal expansion coefficient used in most gauge calibration laboratories is that supplied by the manufacturer. Unless stated otherwise, an accepted practice is to assign a conservative uncertainty of about 10%. For this Type B evaluation we assume that this 10% is rectangularly distributed (§6.1.4). Thus, for the case of steel gauge blocks, the standard uncertainty in the thermal expansion coefficient is

$$\begin{aligned} u(\alpha) &= \frac{10\% \times 11.5 \times 10^{-6}/^\circ\text{C}}{\sqrt{3}} \\ &= 0.66 \times 10^{-6}/^\circ\text{C}. \end{aligned} \tag{30}$$

Recall from §2.2, the expansion coefficients for both standard and test gauges are assumed equal, so $u(\alpha_s) = u(\alpha)$. The standard uncertainties contributing to the combined standard uncertainty in the length of the test gauge includes multiplication by sensitivity coefficients determined earlier in equation (16). The sensitivity coefficients are $l_s\theta_s$ and $l_s(\delta\theta + \theta_s)$, respectively. The value for $\delta\theta$ has already been estimated in §6.5 to be 0.1°C. To estimate θ_s, the average value of the temperature deviation from 20°C shown in Figure 4 is used: $\theta_s = 0.15°\text{C}$. The contributions to $u_c(l)$ are then:

$$\begin{aligned} l_s \theta_s u(\alpha_s) &= l_s(0.15°\text{C})(0.66 \times 10^{-6}/°\text{C}) \\ &= 0.099L \text{ nm}, \end{aligned} \tag{31}$$

and

$$\begin{aligned} l_s(\delta\theta + \theta_s)u(\alpha) &= l_s(0.1°\text{C} + 0.15°\text{C})(0.66 \times 10^{-6}/°\text{C}) \\ &= 0.165L \text{ nm}, \end{aligned} \tag{32}$$

where L represents the nominal length of the gauge in millimeters. These are both length dependent uncertainty components.

6.7 Higher-Order Terms Associated with Temperature

Substituting values in each of the second order terms of (16), the following three terms are found to make significant contributions:

$$\begin{aligned} l_s u(\theta_s)u(\alpha_s) &= l_s(0.173°\text{C})(0.66 \times 10^{-6}/°\text{C}) \\ &= 0.114L \text{ nm} \end{aligned} \tag{33}$$

$$\begin{aligned} l_s u(\theta_s)u(\alpha) &= l_s(0.173°\text{C})(0.66 \times 10^{-6}/°\text{C}) \\ &= 0.114L \text{ nm} \end{aligned} \tag{34}$$

$$\begin{aligned} l_s u(\delta\theta)u(\alpha) &= l_s(0.06°\text{C})(0.66 \times 10^{-6}/°\text{C}) \\ &= 0.040L \text{ nm} \end{aligned} \tag{35}$$

where L is the nominal gauge length in millimeters. Notice that even if the lab had a mean temperature of 20°C (so $\theta = \theta_s = \delta\theta \equiv 0$), these parameters would still have non-zero uncertainty. Thus, the thermal expansion terms of §6.6 would vanish, but these higher order terms would remain.

Table 4: Hypothetical Values for Uncertainty in Gauge Block Calibration by Mechanical Comparison

Standard Uncertainty Component $u(x_i)$	Source	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient $\lvert c_i\rvert \equiv \frac{\partial l}{\partial x_i}$	Combined Standard Uncertainty Component $u_i(l) \equiv \lvert c_i\rvert u(x_i)$ [nm] for L [mm]
$u_c(l_s)$	Calibration of standard gauge	$\mathbf{\sqrt{10^2+0.290^2L^2}}$ **nm**	1	$\sqrt{10^2+0.290^2L^2}$
$u_{cal}(l_s)$	NRC calibration (Type B)	$\sqrt{10^2+0.21^2L^2}$ nm		
$u(\dot{l}_s)$	1-year secular change (Type B)	$0.2L$ nm		
$u_c(d)$	Measured length difference	**3.19 nm**	1	3.19
$u(d_r)$	Reading (Type A)	1.3 nm		
$u(d_{cal})$	Calibration (Type B)	2.5 nm		
$u(d_g)$	Gain electronics (Type B)	1.5 nm		
$u_c(\theta_s)$	Measured gauge temperature	**0.173°C**	$l_s(\alpha_s-\alpha)$	*nil*
$u(\theta_r)$	Reading (Type B)	0.03°C		
$u(\dot{\theta})$	Cyclic variation (Type B)	0.17°C		
$u(\theta_{cal})$	Calibration (Type B)	0.003°C		
$u(\delta\theta)$	Temperature difference between gauges (Type B)	0.06°C	$l_s\alpha$	$0.690L^\dagger$
$u(\alpha_s)$	Thermal dilatation of standard (Type B)	0.66 ppm/°C†	$l_s\theta_s$	$0.099L$
$u(\alpha)$	Thermal dilatation of test (Type B)	0.66 ppm/°C†	$l_s(\delta\theta+\theta_s)$	$0.165L$
	Cross terms:		$\lvert c_{ij}\rvert \equiv \frac{\partial^2 l}{\partial x_i \partial x_j}$	
$u(\alpha_s)u(\theta_s)$	Temperature effects on standard gauge	(0.66 ppm/°C)(0.173°C)	l_s	$0.114L$
$u(\alpha)u(\theta_s)$	Temperature effects on test gauge	(0.66 ppm/°C)(0.173°C)	l_s	$0.114L$
$u(\delta\theta)u(\alpha)$	Temperature difference between gauges	(0.06°C)(0.66 ppm/°C)	l_s	$0.040L$
	Combined Standard Uncertainty of Reference Standard to Working Standard Calibration:			$u_c(l_{ws}) = \sqrt{11^2+0.80^2L^2}$ nm
	Combined Standard Uncertainty of Working Standard to Client Gauge Calibration (see §7 text):			$u_c(l_{cl}) = \sqrt{12^2+1.1^2L^2}$ nm
	Expanded Uncertainty Attributed to Client Gauge Calibration for Coverage Factor $k=2$:			$U_{cl} = 2\sqrt{12^2+1.1^2L^2}$ nm

†Steel gauge: $\alpha = 11.5$ ppm/°C.

7 Combined Uncertainties

7.1 Combined Uncertainty Attributed to the Working Standard

The combined standard uncertainty is the quadrature sum of the standard uncertainties of the influence factors, written explicitly as equation (16). Values for the uncertainty components and their sensitivity coefficients have been calculated in §6 above for each of the identified influence quantities, and all that remains is to add them together in quadrature. This is facilitated with the summary format of Table 4. Adding the values in the right-most column of Table 4 in quadrature:

$$\begin{aligned} u_c(l_{ws}) &= \big(10^2 + 0.290^2L^2 + 3.19^2 + 0 + 0.690^2L^2 + 0.099^2L^2 \\ &\qquad + 0.165^2L^2 + 0.114^2L^2 + 0.114^2L^2 + 0.040^2L^2\big)^{1/2} \\ &= \sqrt{111 + 0.625L^2}\ \text{nm} \\ &= \sqrt{11^2 + 0.80^2L^2}\ \text{nm}, \end{aligned} \tag{36}$$

for L in millimeters. This is the uncertainty attributed to the single comparison measurement of the working standard against the reference standard, which has been calibrated at NRC.

7.2 Other Factors and Checking the Uncertainty Model Performance

The combined standard uncertainty in the gauge length calibration should be representative of the standard deviation of actual day-to-day measurements of gauge blocks. If the model is correct, and we have managed to include all of the significant influencing factors in a realistic manner, then the uncertainty should match the standard deviation of the measurements made with this system. This verification cannot be simply checked in-house by repeated readings. Only those few of the uncertainty components to do with repeatability will exhibit variation. The only way to verify the performance of the model is through intercomparison with other laboratories (as is done by the national labs), or by external audit, as is done for CLAS laboratories.

Referring to §4.1.2 of the *Guide*:

> *Thus, if data indicate that f does not model the measurement to the degree imposed by the required accuracy of the measurement result, additional input quantities must be included in f to eliminate*

the inadequacy (see 3.4.2). This may require introducing an input quantity to reflect incomplete knowledge of a phenomenon that affects the measurand.

7.3 Client Gauge Calibration Uncertainty

To calculate the uncertainty of the calibration of the client gauge from the working standard, the above evaluation can simply be replicated, where the combined standard uncertainty in the working standard $u_c(l_{ws}) = \sqrt{11^2 + 0.80^2 L^2}$ nm calculated in equation (36) is now substituted for the calibration uncertainty in the standard $u_{\mathrm{cal}}(l_s)$. Since the calibrations are performed in the same laboratory under similar conditions, values used for the other uncertainty components will be the same. The uncertainty in the client gauge block calibrated against the working standard is then:

$$\begin{aligned} u_c(l_{cl}) &= \big(11^2 + 0.80^2 L^2 + 0.1^2 L^2 + 3.91^2 + 0 + 0.690^2 L^2 + 0.099^2 L^2 \\ &\quad +0.165^2 L^2 + 0.114^2 L^2 + 0.114^2 L^2 + 0.040^2 L^2\big)^{1/2} \\ &= \sqrt{137 + 1.19 L^2} \text{ nm} \\ &= \sqrt{12^2 + 1.1^2 L^2} \text{ nm}, \end{aligned} \tag{37}$$

where the only difference from (36) is that the working standard has been assumed to be calibrated against the reference standard every 6 months, therefore the secular uncertainty attributed to the working standard gauge $u(\dot{l}_{ws}) = 0.1L$ nm.

In using this method of propagating the uncertainties from reference standard $\rightarrow$ working standard onto working standard $\rightarrow$ client gauge, there will be correlated components (§5.2 *Guide*). It turns out that the few uncorrelated uncertainty components are negligible compared to the other uncertainties that cannot be reduced by repeated measurements. The dominant sources of uncertainty are the calibration uncertainty in the reference standard $u_{\mathrm{cal}}(l_s)$ and temperature effects between gauges $u(\delta\theta)$. Reducing the uncertainty attributed to reading the comparator indicator through repeated readings, will be negligibly small in comparison.

8 Expanded Uncertainty

The expanded uncertainty (see §4.2) in the calibration of the working standard from the reference standard in this example is $U_{ws} = 2u_c(l_{ws}) = 2\sqrt{11^2 + 0.80^2 L^2}$ nm. The expanded uncertainty in the calibration of a client gauge from this working standard is $U_{cl} = 2u_c(l_{cl}) = 2\sqrt{12^2 + 1.1^2 L^2}$ nm.

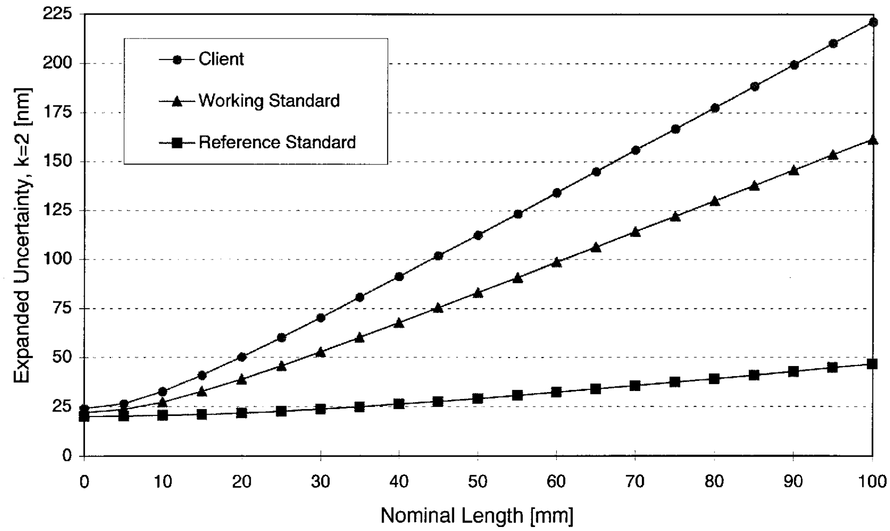


Figure 6: Expanded uncertainties plotted against nominal length for the cascade of calibrations beginning with an NRC calibrated reference standard → working standard → client gauge, assuming calibrations in a direct chain.

The calibration uncertainty for the cascade of calibrations beginning with the NRC calibrated reference standard, working standard and then client gauge are shown plotted against nominal gauge length in Figure 6. It is easily observed from this graph that the end effect uncertainties are dominant for shorter gauges, and beyond about 20-25 mm the length dependent uncertainties dominate. As mentioned above, the length dependent uncertainties predominantly stem from the reference standard calibration and temperature related effects. The main source of the end effect uncertainty is again the reference standard calibration. This is no mere coincidence. This example was designed for CLAS laboratories making a concerted effort to contribute as little uncertainty as possible while transferring the definition of length from NRC to their clients. One can never reduce the uncertainty to be below that of the reference gauge. The only avenue in that direction is to seek better calibration of one's reference standards. Herein lies the driving force of all progress in metrology.

9 Conclusion

Expressions describing the combined standard uncertainty for gauge block calibration by mechanical comparison were determined. Values for these components were evaluated, based on best practice in a CLAS laboratory. The combined standard uncertainty was formed from the quadrature sum of the standard uncertainties of the influence quantities, examples of which were considered separately and listed in Table 4.

Based on the quadrature sum of the components in Table 4, the expanded uncertainty in the calibration of the working standard against a calibrated reference standard is $U_{ws} = 2\sqrt{11^2 + 0.80^2 L^2}$ nm, for a coverage factor of $k = 2$, where L is in millimeters. Similarly, the expanded uncertainty in the calibration of the client gauge in a direct chain against the above working standard is $U_{cl} = 2\sqrt{12^2 + 1.1^2 L^2}$ nm.

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