Atlantic Canada
Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

CURRICULUM

Applications in Mathematics 113

(Implementation Edition)

2002

Additional copies of this document (Applications in Mathematics 113)
may be obtained from the Instructional Resources Branch.
Title Code (840910)

Acknowledgements

The departments of education of New Brunswick, Newfoundland and Labrador, Nova Scotia, and Prince Edward Island gratefully acknowledge the contributions of the following groups and individuals toward the development of this *Applications in Mathemetics 113* curriculum guide.

- The Regional Mathematics Curriculum Committee; current and past representatives include the following:

New Brunswick
Greta Gilmore, Mathematics Teacher,
Belleisle Regional High School

John Hildebrand, Mathematics Consultant,
Department of Education

Pierre Plourde, Mathematics Consultant,
Department of Education

Nova Scotia
Richard MacKinnon, Mathematics Consultant,
Department of Education & Culture

Lynn Evans Phillips, Mathematics Teacher,
Park View Education Centre

Newfoundland and Labrador
Sadie May, Distance Education Coordinator for Mathematics
Department of Education

Patricia Maxwell, Program Development Specialist,
Department of Education

Prince Edward Island
Elaine Somerville, Mathematics/Science Consultant,
Department of Education

- The Provincial Curriculum Working Group, comprising teachers and other educators in Nova Scotia, which served as lead province in drafting and revising the document.
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the development of the *Applications in Mathemetics 113* curriculum guide.

Table of Contents

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their learning experiences.

The *Foundation for the Atlantic Canada Mathematics Curriculum* (1996) firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* (1989) of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision, which embraces the principles of students learning to value and become active "doers" of mathematics and advocates a curriculum which focusses on the unifying ideas of mathematical problem solving, communication, reasoning, and connections. These principles and unifying ideas are reaffirmed with the publication of NCTM's *Principles and Standards for School Mathematics* (2000). The *Foundation for the Atlantic Canada Mathematics Curriculum* establishes a framework for the development of detailed grade-level documents describing mathematics curriculum and guiding instruction.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers with department of education officials to co-operatively plan and execute the development of curricula in mathematics, science, language arts, and other curricular areas. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the Essential Graduation Learnings (EGLs), also developed regionally. These EGLs and the contribution of the mathematics curriculum to their achievement are presented in the "Outcomes" section of the mathematics foundation document.

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. This curriculum guide is one of several which provide greater specificity and clarity for the classroom teacher. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—General Curriculum Outcomes (GCOs), which relate to subject strands, and Key-Stage Curriculum Outcomes (KSCOs), which articulate the GCOs further for the end of grades 3, 6, 9, and 12. This guide builds on the structure introduced in the foundation document, by relating Specific Curriculum Outcomes (SCOs) to KSCOs for *Applications in Mathematics 113*. Figure 1 further clarifies the outcome structure.

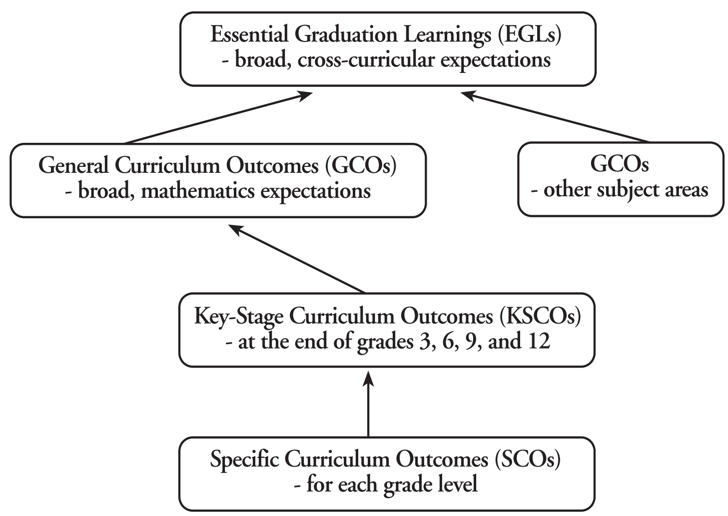


Figure 1: Outcome Framework

This mathematics guide is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice, including the following: (i) mathematics learning is an active and constructive process; (ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; (iii) learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort; (iv) learning is most effective when standards of expectation are made clear and assessment and feedback are ongoing; and (v) learners benefit, both socially and intellectually, from a variety of learning experiences, both independent and in collaboration with others.

II. Program Design and Components

A. Program Organization

As indicated previously, the mathematics curriculum is designed to support the Atlantic Canada Essential Graduation Learnings (EGLs). The curriculum is designed to significantly contribute to students meeting each of the six EGLs, with the communication and problem-solving EGLs relating particularly well with the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then goes on to present student outcomes at key stages of the student's school experience.

This curriculum guide presents specific curriculum outcomes for *Applications in Mathematics 113*. As illustrated in Figure 2, these outcomes represent the step-by-step means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and, ultimately, the essential graduation learnings.

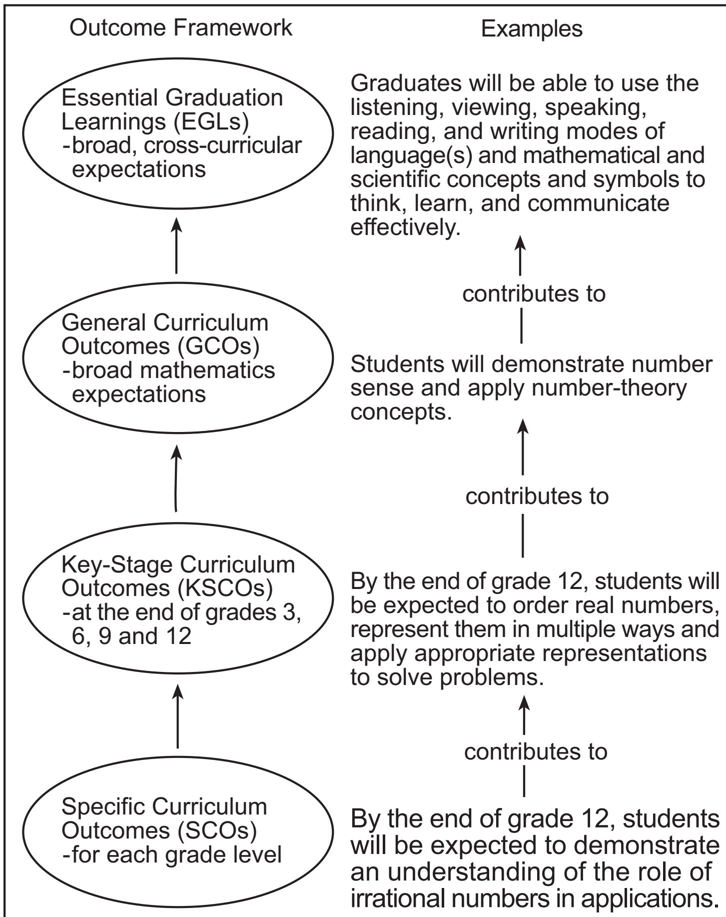


Figure 2: Examples of Outcomes

It is important to emphasize that the initial presentation of the specific curriculum outcomes for this course (pp. 17-32) follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a natural teaching sequence. In *Applications in Mathematics 113*, however, a suggested teaching order for specific curriculum outcomes has been given within a sequence of four topics or units (i.e., Statistics; Independent Study; Probability; and Decision Making in Consumer Situations). While the units are presented with a specific teaching sequence in mind, some flexibility exists as to the ordering of units within the course. It is expected that teachers will make individual decisions as to what sequence of topics will best suit their classes. In most instances, this will occur in consultation with fellow staff members, department heads, and/or district level personnel.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" topic for one group of students might be less effective in that role with a second group. Another consideration with respect to sequencing will be co-ordinating the mathematics program with other aspects of the students' school experience. An example of such co-ordination would be studying aspects of measurement in connection with appropriate topics in science. As well, sequencing could be influenced by events outside of the school, such as elections, special community celebrations, or natural occurrences.

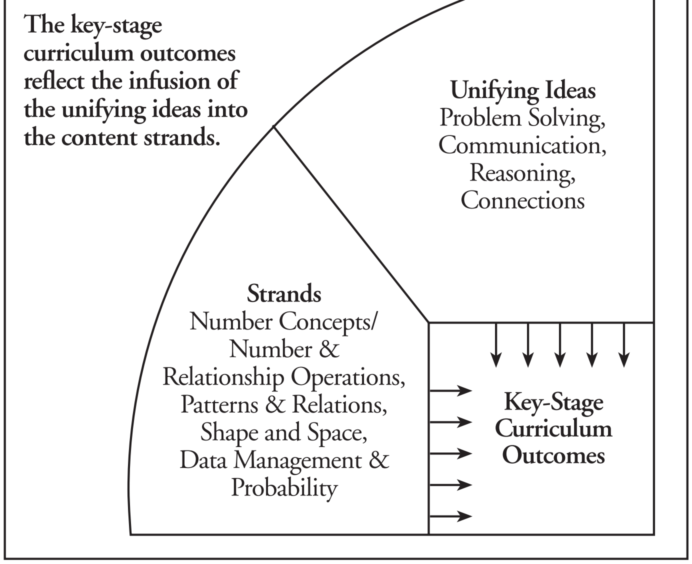
B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* (1989) and *Principles and Standards* (2000) establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. This is illustrated in Figure 3.

These unifying concepts serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode; classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically; via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning; and mathematics with which students are involved on a day-to-day basis must be connected to other mathematics, other disciplines, and/or the world around them.

Students will be expected to address routine and/or non-routine

Figure 3 - Integrating Strands and Unifying Ideas



mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling, whereas working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire in the later elementary years. During middle school and the 9/10 years, this repertoire will be extended to include such strategies as interpreting formulas, checking for hidden assumptions, examining systematic or critical cases, and solving algebraically. *Applications in Mathematics 113* will continue to develop students' problem-solving reportoires.

Opportunities should be created frequently to link mathematics and career opportunities. Students need to be aware of the importance of mathematics and the need for mathematics in so many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in higher-level mathematics programming in senior high mathematics and beyond.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline which lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, and actively participate in discourse and conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas in which reasoning and sense making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Meeting the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness, but they must also remain aware of avoiding gender and cultural biases in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

NCTM's *Principles and Standards* (2000) cites equity as a core element of its vision for mathematics education. "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students"(p. 12).

At grade 11 in New Brunswick, variations in student readiness, aptitude, and post-secondary intentions are addressed in significant part by the provision of courses at levels 1, 2 and 3. Students at all levels will work toward achievement of the same key-stage and general curriculum outcomes, and many of the course-specific curriculum outcomes will also be the same or similar. As well, the instructional environment and philosophy should be the same at all levels, with high expectations maintained for all students. The significant difference between levels will be the depth, breadth, and degree of sophistication and formalism expected with respect to each general outcome. Similarities between courses should allow some students to move from one course level to another.

By and large, Level 3 courses will be characterized by a greater focus on concrete activities, models, and applications, with less emphasis given to formalism, symbolism, computational or symbol-manipulating facility, and mathematical structure. Level 1 and 2 courses will involve greater attention to abstraction and more sophisticated generalizations, while Level 3 courses would see less time spent on complex exercises and connections with advanced mathematical ideas. Level 1 courses, which are designed for particularly talented students of mathematics, will be characterized by both more sophisticated engagement with mathematical concepts and techniques, and the extension of some topics beyond the scope provided at Level 2. These extensions will be included in Level 2 curriculum guides and identified with a symbol.

By way of a brief illustration, students at all levels should develop an understanding of exponential relationships. Students taking Level 3 courses have as much need as others to understand the nature of exponential relationships, given the central place of these relationships in universal, everyday issues such as investment, personal and government debt, and world population dynamics. The nature of exponential relationships can be developed through concrete, hands-on experiments and data analysis that does not require a lot of formalism or symbol manipulation. The more formal and symbolic operations on exponential relationships will be much more prevalent in Level 1 and 2 courses.

Finally, within any given course at any level, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Further, the practice of designing classroom activities to support a variety of learning styles must be extended to the assessment realm; such an extension implies the use of a wide variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This curriculum guide represents the central resource for the teacher of *Applications in Mathematics 113*. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and course-long planning, as well as a reference point to determine the extent to which the instructional outcomes should be met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent that they support the curriculum goals. Teachers will need professional resources as they seek to broaden their instructional and mathematical skills. Key among these are the NCTM publications, including the *Principles and Standards for School Mathematics, Assessment Standards for School Mathematics, Curriculum and Evaluation Standards for School Mathematics*, the *Addenda Series*, *Professional Standards for Teaching Mathematics,* and the various NCTM journals and yearbooks. As well, manipulative materials and appropriate access to technological resources (e.g., software, videos) should be available. Calculators will be an integral part of many learning activities.

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different than were those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their children's lives, assisting children with mathematical activities at home, and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

The concepts and skills developed in mathematics are applied in many other disciplines. These include science, social studies, music, technology education, art, physical education, and home economics. Efforts should be made to make connections and use examples which apply across a variety of discipline areas.

In science, for example, the concepts and skills of measurement are applied in the context of scientific investigations. Statistical concepts and skills are applied as students collect, present, and analyse data. Examples and applications of many mathematical relations and functions abound.

In social studies, knowledge of confidence intervals is valuable in intrepreting polling data, and an understanding of exponential growth is necessary to appreciate the significance of government debt and population growth. As well, students read, interpret, and construct tables, charts, and graphs in a variety of contexts such as demography.

Opportunities for mathematical connections are also plentiful in physical educaiton, many technological courses and the fine arts.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are critical, not only with respect to clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers make meaningful instructional decisions. (See "Assessing and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment should include the following: i) using a wide variety of assessment strategies and tools; ii) aligning assessment strategies and tools with the curriculum and instructional techniques; and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborate good assessment practice and serve as a guide with respect to student assessment for the mathematics foundation document. (See also, Appendix A, "Assessing and Evaluating Student Learning.")

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address such questions as the following: Are students meeting the curriculum outcomes? Is the curriculum being equitably applied across the region? Does the curriculum reflect a proper balance between procedural knowledge and conceptual understanding? Is technology fulfilling a proper role?

IV. Designing an Instructional Plan

It is important to develop an instructional plan for the duration of the course. Without such a plan, it is easy to run out of time before all aspects of the curriculum have been addressed. A plan for instruction that is comprehensive enough to cover all outcomes and topics will help to highlight the need for time management.

It is often advisable to use pre-testing to determine what students have retained from previous grades relative to a given topic or set of outcomes. In some cases, pre-testing may also identify students who have already acquired skills relevant to the current course. Pre-testing is often most useful when it occurs one to two weeks prior to the start of a a topic or set of outcomes. When the pre-test is done early enough and exposes deficiencies in prerequisite knowledge/ skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of adequate development or coverage in previous grades. This may imply that an adjustment is required to the starting point for instruction, as well as a meeting with other grade level teachers to address these concerns as necessary.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in an overall savings in time for both disciplines. The most obvious of these connections relate to the use of measurement in science and the use of a variety of data displays in social studies.

V. Curriculum Outcomes

The pages that follow provide details regarding both specific curriculum outcomes and the four topics/units that comprise *Applications in Mathematics 113*. The specific curriculum outcomes are presented initially, then the details of the units follow in a series of two-page spreads. (See Figure 4 on next page.)

This guide presents the curriculum for *Applications in Mathematics 113* so that a teacher may readily view the scope of the outcomes which students are expected to meet during the year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings in this course are part of a bigger picture of concept and skill development. (See Appendix B for a complete listing of the SCOs for grades 9 and 10.)

Within each unit, the specific curriculum outcomes are presented on two-page spreads. At the top of each page, the overarching topic is presented, with the appropriate SCO(s) displayed in the left-hand column. The second column of the layout is entitled "Elaboration-Instructional Strategies/Suggestions" and provides a clarification of the specific curriculum outcome(s), as well as some suggestions of possible strategies and/or activities which might be used to achieve the outcome(s). While the strategies and/or suggestions presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning, and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol ❒.

The third column of the two-page spread, "Worthwhile Tasks for Instruction and/or Assessment," might be used for assessment purposes or serve to further clarify the specific curriculum outcome(s). As well, those tasks regularly incorporate one or more of the four unifying ideas of the curriculum. These sample tasks are intended as examples only, and teachers will want to tailor them to meet the needs and interests of the students in their classrooms. The final column of each display is entitled "Suggested Resources" and will, over time, become a collection of useful references to resources which are particularly valuable with respect to achieving the outcome(s).

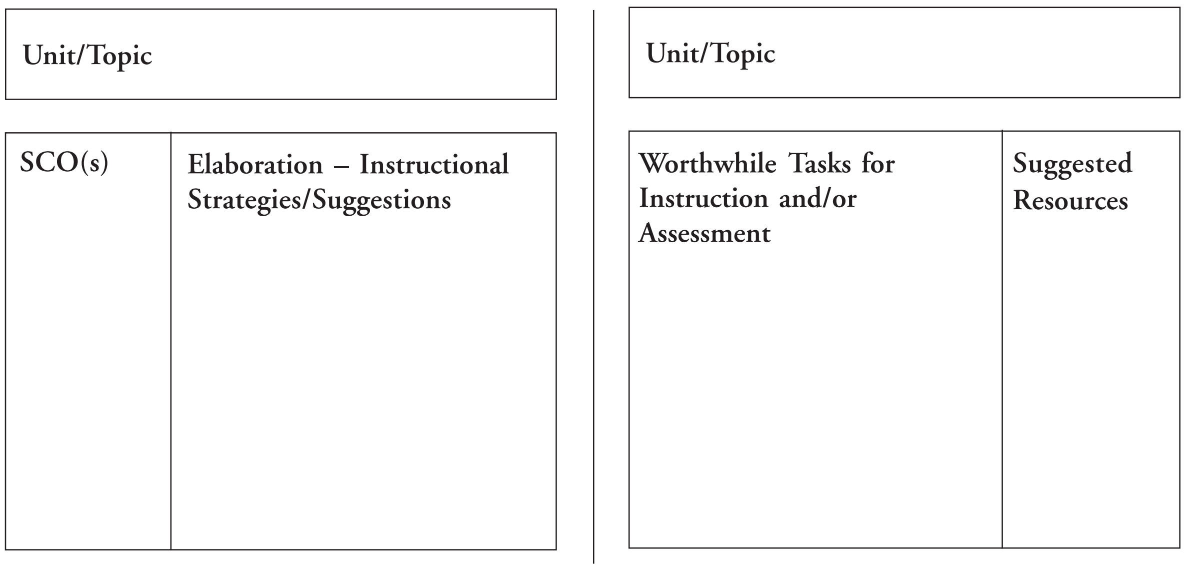


Figure 4: Layout of a 2-Page Spread

Specific Curriculum Outcomes

(by GCO)

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

ii) *order real numbers, represent them in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of *Applications in Mathematics 113*, students will be expected to

A3 **demonstrate an understanding of the application of random numbers to statistical sampling**

A6 **develop an understanding of factorial notation and apply it**

Elaboration

A3 Since it is often desirable to select a random sample when choosing a population sample, students should understand the importance of "randomizing" when conducting simulations. They will choose random samples in a number of ways, including the use of random number tables and random number generators on scientific/graphics calculators. Unit 1, pp. 36, 38

A6 Students will find it valuable to develop factorial notation in connection with tree diagrams and the fundamental counting principle, and will understand *n!* to represent the number of possible arrangements of *n* distinct objects. Factorial notation may also (but not necessarily) be applied when calculating permutations and combinations, in connection with SCOs G7 and $B8_3$.
Unit 3, pp. 78, 80

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism* SCO: By the end of *Applications in Mathematics 113*, students will be expected to **B4 use the calculator correctly and efficiently**	**B4** Students will need to use several calculator functions correctly, and make good choices with respect to their efficient use, in a number of situations. These include numerical and financial applications (see SCOs such as $B7_2$, $B8_2$, B9 and B10), probability applications (see $B7_3$) and graphing applications (see C8, F5 and F7). Unit 4, pp. 86, 88, 90
ii) derive, analyze and apply computational procedures in situations involving all representations of real numbers **$B7_3$ calculate probabilities to solve problems**	**$B7_3$** Students will solve problems in various probability situations. These will involve basic counting situations (see SCO G3), situations that may be aided with the use of tree or area diagrams (see G4) and situations involving simple permutations and combinations (see G7 and $B8_3$). Unit 3, pp. 70, 74, 82
$B8_3$ determine probabilities using permutations and combinations	**$B8_3$** Distinguishing between permutations and combinations is addressed in SCO G7. Students will apply these counting techniques in probability situations in which the number of permutations or combinations can be reasonably evaluated without the use of formulas (i.e., by itemizing cases or by using calculator functions). This outcome will be addressed in conjunction with $B7_3$. Unit 3, p. 82

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration

$B7_2$ estimate and calculate income and deductions

$B7_2$ As part of the overall personal budgeting process (see SCO $B8_2$), students will need to estimate and calculate (see B4) in many situations involving income (e.g., regular pay, overtime, commission, and piecework) and deductions (e.g., income tax, CPP, EI, pension plans, and professional dues). Unit 4, pp. 86, 88

$B8_2$ solve problems involving budgets

$B8_2$ Solving problems involving personal budgets is a principal focus of Unit 4. Students will need to apply decision-making techniques (see SCO C7) as they consider various factors (see, for example, $B7_2$, B9, B10 and C26) which impact on personal budgets. Unit 4, p. 90

B9 analyze situations and make decisions involving the financing of purchases

B9 Students will analyze situations involving the financing of purchases, with a view to making good decisions with respect to including such purchases within personal budgets. This outcome will be addressed in connection with SCOs C26, C7 and $B8_3$.
Unit 4, pp. 92, 94

B10 analyze situations and make decisions involving the cost of transportation

B10 Students will analyze situations involving the cost of transportation, with a view to making good decisions with respect to including such costs within personal budgets. This outcome will be addressed in connection with SCOs C26, C7 and $B8_3$.
Unit 4, p. 94

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) model real-world problems using functions, equations, inequalities and discrete structures* SCO: By the end of *Applications in Mathematics 113*, students will be expected to **C7 develop and apply informal decision-making flowcharts**	**C7** Students will use informal flowcharting techniques (e.g., mind maps, concept maps, and webs) to help make appropriate decisions with respect to budgets, particularly when considering the potential impact of changes in budget factors. This outcome will be addressed in conjunction with SCOs $B7_2$, $B8_2$, B9 and B10. Unit 4, pp. 90, 94
ii) represent functional relationships in multiple ways (e.g., written descriptions, tables, equations and graphs) and describe connections among these representations	
C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions	**C8** Students will need to translate between various representations of consumer data. For example, they might take a written presentation of a sales situation and represent data (e.g., sales, commission, and earnings) in tabular form or graphically (e.g., earnings vs. sales). In doing this they might need to interpret constraints (see SCO C11), draw inferences (F7) and/or interpolate or extrapolate (C18). Unit 4, pp. 86, 88
C11 express and interpret constraints	**C11** Students will express and interpret constraints with respect to such contexts as step commissions. This outcome will connect with SCOs such as $B7_2$ and C8. Unit 4, p. 86
iv) solve problems involving relationships, using graphing technology as well as paper-and-pencil techniques	
C18 interpolate and extrapolate to solve problems	**C18** Students will interpolate and extrapolate to answer questions (solve problems) with respect to many aspects of personal budget situations. See connections with outcomes such as $B8_2$, C8 and F7. Unit 4, pp. 86, 88

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration

v) *analyze and explain the behaviours, transformations and general properties of types of equations and relations*

C26 demonstrate an understanding of the difference between simple and compound interest

C26 Students will understand the difference between, and the long-term implications of, simple and compound interest. These implications will be important with respect to personal budgeting (see SCO $B8_2$), both in terms of investments and financing (see B9). Unit 4, p. 92

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

i) *understand sampling issues and their role with respect to statistical claims*

SCO: By the end of *Applications in Mathematics 113*, students will be expected to

F1 draw inferences about a population from a sample

F2 identify bias in data collection, interpretation and presentation

F3 demonstrate an understanding of what can be inferred about a population by examining sample means and dispersions

F4 demonstrate an understanding of how the size of a sample affects the variation in sample results

Elaboration

F1 Since it is often impractical to gather information about entire populations, sampling is a common statistical technique. Students will need to understand issues with respect to sampling strategies and sample size in order to properly draw inferences from sample data. These issues are addressed in SCOs F2, F4 and F3. Unit 1, pp. 32, 34, 38, 44

F2 Students should understand that some sampling methods (e.g., convenience, self-selected) may produce samples that are not representative of the population as a whole. They will need to identify the bias that can enter the interpretation of results based on such sampling, and learn how to create properly random samples. This outcome will be addressed in connection with SCOs A3, F1 and F7. Unit 1, pp. 32, 38, 40

F3 Students will understand the relationship between a population and a sample thereof, and how sample mean and dispersion relate to those characteristics of the population as a whole. This outcome is addressed in connection with SCOs F13, F4 and F1. Unit 1, pp. 42, 44

F4 By conducting experiments/simulations and examining the data collected, students should understand that larger sample sizes increase the likelihood that the statistical results will approximate expected values or population characteristics. This outcome will be addressed in connection with SCOs F15 and F3. Unit 1, pp. 34, 36, 44

GCO F: Students will solve problems involving the collection, display and analysis of data.

	Elaboration
ii) extend construction (both manually and via appropriate technology) of a wide variety of data displays	
F5 organize and display information in various ways with and without technology	**F5** Students will collect (see SCOs F15 and F17), organize and display data in various ways (e.g., box plots, bar graphs, and histograms), both by hand and using graphing technology when appropriate. This will take place in conjunction with outcomes such as F7 and F13. Unit 1, pp. 40, 42, 46
iv) determine, interpret and apply as appropriate a wide variety of statistical measures and distributions	
F7 draw inferences from graphs and tables	**F7** Students will draw inferences based on such information as data trends (e.g., linear vs. non-linear), data distribution (e.g., normal vs. skewed) and data variability (e.g., range). This outcome will be addressed in connection with SCOs such as F5, F13, C8 and C18. Unit 1, pp. 40, 46; Unit 4, pp. 86, 88
F12 interpret normal curves and standard deviation to express levels of confidence	**F12** In conjunction with SCO F17, students will use an understanding of the basic relationship between normal curves and standard deviation to express levels of confidence with respect to experimental/survey data. Unit 1, p. 50
F13 calculate, analyze and interpret various statistics	**F13** Students will determine such statistics as mean, median, range and standard deviation, using statistical technology as appropriate (see SCO F5). They will use such statistics to assist in making inferences concerning data (see F3 and F7). Unit 1, pp. 40, 42, 46
v) design and conduct relevant statistical experiments and analyze and communicate the results using a range of statistical arguments	
F15 design and conduct experiments/surveys to explore sampling variability	**F15** By designing and conducting experiments/simulations and examining the data collected, students will understand the impact of sampling issues on statistical results. This outcome will be addressed in connection with SCO F4. Unit 1, pp. 34, 42

GCO F: Students will solve problems involving the collection, display and analysis of data.

F17 design and conduct experiments/surveys and interpret and communicate level of confidence

Elaboration

F17 In conjunction with SCO F12, students will use an understanding of the basic relationship between normal curves and standard deviation to express levels of confidence with respect to data from experiments/surveys which they design and conduct. Unit 1, pp. 50, 52

GCO G: Students will represent and solve problems involving uncertainty.

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) design and conduct experiments a/o simulations to model and solve a wide variety of relevant probability problems, and interpret and judge the probabilistic arguments of others* SCO: By the end of *Applications in Mathematics 113*, students will be expected to **G1 develop and apply simulations to solve problems**	**G1** Students will design simulations to model problem situations and use them to solve the problems. Unit 3, p. 72
ii) build and apply formal concepts and techniques of theoretical probability	
G2 demonstrate an understanding that determining probability requires the quantifying of outcomes	**G2** Students will understand that determining probabilities requires counting both the number of possible outcomes and the number of successful outcomes. In this context, tree diagrams and area models (see SCO G4), the fundamental counting principle (see $G3_3$), and permutations and combinations (see G7) are all means of counting. Unit 3, p. 66
$G3_3$ demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events	**$G3_3$** Students will understand that, if an event A can occur in *n* ways and an unrelated event B in *m* ways, then A and then B can occur in *n* x *m* ways. They will also understand how to apply the principle when events are related to (dependent upon) one another. For further applications, see SCO $B7_3$. Unit 3, pp. 66, 68, 74, 76, 78, 80, 82
G4 apply area diagrams and tree diagrams to interpret and determine probabilities of independent and dependent events	**G4** Students will use diagrams to assist with quantifying outcomes and determining probabilities. This outcome will be addressed in connection with SCO $B7_3$. Unit 3, p. 74
G6 demonstrate an understanding of the difference between probability and odds	**G6** Students will understand that odds are a specific way to express probabilities and will be able to translate from one form of expression to the other. Unit 3, p. 70

GCO G: Students will represent and solve problems involving uncertainty.

	Elaboration
G7 **distinguish between situations that involve combinations and permutations**	**G7** Students will distinguish between situations involving unordered collections of objects (i.e., combinations) and those involving an ordering of objects (i.e., permutations), and will determine the number of permutations or combinations in situations in which they can be reasonably evaluated without the use of formulas (i.e., by itemizing cases or by using calculator functions). Making these distinctions will be critical with respect to determining probabilities involving permutations and combinations (see SCO $B8_3$). Unit 3, pp. 76, 78, 80
iv) relate probability and statistical situations	
$G3_2$ **graph sample distributions and interpret them using the language of probability**	**$G3_2$** In connection with SCOs F13, F5 and F7, students will graph and interpret histograms and normal curves. Unit 1, pp. 46, 48

Independent Study

Elaboration

Further to the topics that involve SCOs which contribute to students achieving the general curriculum outcomes, an Independent Study unit is included in *Applications in Mathematics 113*. This unit is intended to assist in the development of students' independent learning skills while allowing them to explore topics outside the prescribed curriculum. While the following SCOs fall outside the framework of GCOs A through G, they are to be considered integral to the curriculum.

SCO: By the end of *Applications in Mathematics 113*, students will be expected to

I1 demonstrate an understanding of a mathematical topic through independent research

I1 Students will research mathematical topics outside the prescribed curriculum and demonstrate an understanding of their topics by sharing the results of their research with their peers. (See also SCO I2.) Unit 2, pp. 56, 58, 60, 62

I2 communicate the results of the independent research

I2 Students will present the results of their research to their peers in one of a number of possible formats. Note that the presentation of results will be intimately connected to the achievement of SCO I3. Unit 2, pp. 56, 58, 60, 62

I3 demonstrate an understanding of the mathematical topics presented by other students

I3 Students will provide information in one or more of a number of formats to demonstrate that they have learned from the presentations of their peers. This outcome will be achieved in connection with SCO I2. Unit 2, pp. 60, 62

Unit 1
Statistics
(20 Hours)

Statistics

Outcomes

SCO: In this course, students will be expected to

F1 **draw inferences about a population from a sample**

F2 **identify bias in data collection, interpretation, and presentation**

Elaboration—Instructional Strategies/Suggestions

F1 Students might begin their study of statistics by reading and discussing some short articles that present results of statistical studies. They should discuss the different interpretations of the information that students might have and any terminology that might cause confusion. Students should expect to see terms like survey, census, sample, population, and sample/population proportions or percentages.

F1/F2 A television survey might have been conducted to find the fraction of the public that watches a particular television program on a particular night and if it is watched by more women than men. A quality engineer must estimate what percentage of bottles rolling off an assembly line are defective. In both these situations, information is gathered about a large group of people or things. The expense of contacting every person or inspecting every bottle, not to mention the time it would take, is formidable. So information is gathered about only part of the group (a sample) in order to draw conclusions about the whole group (the population). Choosing a representative sample from a large and varied population can be a complex task. It is important to be clear about what population is to be described and exactly what is to be measured.

Areas of concern

- How can a sample be chosen so that it is truly representative of the population?
- If a sample from a population differs from another sample from the same population, how confident can you be about predicting the true population percentage?
- Does the size of the sample make a difference?

F2 The process of sampling people at a mall is fast, cheap, and convenient. But convenience samples often produce unrepresentative data. Think about people in the mall who might be asked. Often it is those who are well dressed, respectable looking, and friendly types, because they look easier to approach. Often the sample from malls will over-represent the middle class and retired, and under-represent the poor. When an error occurs due to bad sampling, the difference between the results obtained and the truth about the whole population is called **bias**.

Students should take every opportunity to discuss bias with respect not only to data collection, but in its interpretation and presentation. These issues will be revisited on subsequent pages.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

Performance

F1

1) Some people say that statistics lie. State how confident you are on a scale of 1–5, 5 being very confident, with the sample results of the following situations:
   a) In March, the Gallup survey asked 1500 Nova Scotian adults this question: "Do you approve or disapprove of the way the Nova Scotia premier is handling his job?"
   b) Based on the survey, Gallup said that 56% of Nova Scotians approved of the way their premier is handling his job.
   c) After the same survey another newspaper reported that Gallup said 53% to 60% of Nova Scotians approved. They said they were 95% confident in their results.
   d) Shiniest toothpaste claims it's the best. Based on a survey of 10 dentists, 6 said Shiniest was tops.
   e) Based on the results of a survey of 100 music students at Swinging High, the school board will maintain the music program.
   f) Most Atlantic Canadians are opposed to nuclear power. This was the conclusion based on a survey conducted by phone in Lepreau, New Brunswick, site of a nuclear power plant.

F2

2) Discuss how bias may affect your confidence from each of the above situations.

Journal

F1

3) a) Describe the difference between a survey and a census.
   b) Describe the difference between a sample proportion and a population proportion.
4) List five ways that bias might affect the outcomes of a survey conducted in your town, Atlantic Canada.
5) You flip a coin 100 times and 62 heads turn up. Tell me what you think about my coin. How confident are you? Explain.

Suggested Resources

Garfunkel, Salomon, Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes,* New York, W. H. Freeman & Co., 1995.

Landwehr, James M. et. al. *Exploring Surveys and Information from Samples*, Quantitative Literacy Series, Palo, Alto, CA: Dale Seymour Publications, 1987.

Statistics

Outcomes

SCO: In this course, students will be expected to

F1 draw inferences about a population from a sample

F15 design and conduct experiments/ surveys to explore sampling variability

F4 demonstrate an understanding of how the size of a sample affects the variation in sample results

Elaboration—Instructional Strategies/Suggestions

F1/F15 Students should understand that sampling is conducted to get information about a population, and if chosen at random, the sampling is expected to represent the population. Your class may decide to ask a sample of your school population whether they watch a particular television program on a particular night. Gallup may decide to do the same survey by sampling the population of Atlantic Canada. The results of these samples from different populations are likely to vary. In fact, if repeated samples are taken from the same population, the results will vary from sample to sample. Can results from samples be trusted? How is trust affected by sample size? To explore this, students need to look more closely at sampling variability and consider how confident or certain they feel about the results. They can model this by taking samples from containers, flipping coins, rolling die, flipping cards, using random number generators, and simulating by using technology.

Students should be able to make the link between proportions and probability. If five of eight cubes in the bag are red, students should be able to infer that the probability of taking a red cube from the bag is 5/8 or 0.625, or that it will happen about 63% of the time.

Tossing coins is an easy way to start the exploration of sampling variability. Students would expect that since there are only two sides to a coin, heads would likely turn up one half of the time. Have students toss 10 coins and record the number of heads. Have them repeat this process many times. One thing that they should notice is how few times they will actually get 5 heads. Another would be that, after many trials, the results will cluster around the expected proportion (5 out of 10)(there will be more results of 4, 5, 6 than 1, 2, or 9, 10).

F15/F4 Increase the sample size to 40 (toss 40 coins many times) and revisit the proportions.

Students should notice that

- The probability of obtaining an exact particular result is greater for smaller samples. (You are more likely to get 5 heads when 10 coins are flipped than 50 heads when 100 coins are flipped.)
- The probability of obtaining an approximate particular expected result is greater for larger samples. (You are more likely to get about 50 heads (45–55) in 100 tosses than about 5 heads (4, 5, or 6) in 10 tosses.)

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F1/F15

Activity

1) a) It is assumed that the population proportion is 50% female. Sample the population. Select four persons at random and observe whether there are 0, 1, 2, 3, or 4 females. To simulate this

   i) Use a random device—four coins, four spinners, four two-colour counters, or a random number generator.
   ii) If coins are selected, toss four coins and count how many are heads (female) or use four spinners that have two colours, each representing the area.
   iii) Calculate the sample proportion. Will this same sample proportion be obtained each time?
   iv) Repeat the experiment 40 times and calculate the 40 sample proportions
   v) Collect the data in a table like the following:

Heads	Sample proportion	Tally	Frequency	Proportion of all trials
0	$\frac{0}{4} - 0.00$	$\vert\vert\vert$	3	$\frac{3}{40} - 0.075$
1	$\frac{1}{4} - 0.25$	$\cancel{\vert\vert\vert\vert}\ \vert\vert\vert$	8	$\frac{8}{40} - 0.200$
2	$\frac{2}{4} - 0.50$	$\cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}\ \vert$	16	$\frac{16}{40} - 0.400$
3	$\frac{3}{4} - 0.75$	$\cancel{\vert\vert\vert\vert}\ \cancel{\vert\vert\vert\vert}$	10	$\frac{10}{40} - 0.250$
4	$\frac{4}{4} = -1.00$	$\vert\vert\vert$	3	$\frac{3}{40} - 0.250$

   Are the outcomes equally likely?
   Make a chart or tree diagram to show how each toss of four coins, or spin on four spinners, results in a possible 16 outcomes (not 4) — FFFF, FFFM, FFMF, FMFF, ... to develop an understanding of equally likely.
   How many outcomes result in 0 females? 1 female? 2 females? 3 females? 4 females?
   How accurate was the simulation of 40 trials?

   b) Do the same thing now with eight coins or spinners to model eight persons being asked—spin or toss 10 times—complete a chart, 100 times—complete a chart. Then students should answer questions like (for tossing coins):

   i) Are you more likely to get a sample proportion of exactly 0.5 if you toss 4 or 8 coins? [Answer: 4 coins]
   ii) Are you more likely to get a sample proportion of between 0.25 and 0.75 if you toss 4 or 8 coins? [Answer: 8 coins]
   iii) Are you more likely to get exactly 10 heads from tossing 20 coins or exactly 50 heads from tossing 100 coins? [Answer: 10 from 20]
   iv) Are you more likely to get a sample proportion of exactly 0.25 and 0.75 from tossing 20 or 100 coins? [Answer: 100]

Suggested Resources

Garfunkel, Salomon, Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*, New York: W. H. Freeman & Co., 1995.

Landwehr, Swift, Watkins. *Exploring Surveys and Information from Samples*, Quantitative Literacy Series, Palo Alto, CA: Dale Seymour Publications, 1987.

Fathom Dynamic Statistics Software. Emeryville, CA: Key Curriculum Press.

Statistics

Outcomes

SCO: In this course, students will be expected to

F4 demonstrate an understanding of how the size of a sample affects the variation in sample results

A3 demonstrate an understanding of the application of random numbers to statistical sampling

Elaboration—Instructional Strategies/Suggestions

F4/A3 Students should approximate sampling distributions from populations for which the percentage of yeses is known, and study the variability that occurs through simulation. If, for example, students knew that 40% of the population of the town of Sackville answered yes when asked, "Would you agree to a 1% increase in town taxes to support the wild fowl park?" then they could take samples from the population to see how close their sample proportion of yeses would be to the 40%.

They might begin by taking small sample sizes of 5 or 10 and then trying some with large sample sizes of 40 or 80 to see the effect this would have on the distribution of the results. They should notice that as the sample size increases, the data would cluster more around the mean, decreasing the variation. If available, students could use technology to generate samples from known populations using various sample sizes and create histograms for each and note how the shape of the histograms change as the sample size increases. (For example, on the TI-83, they could use "randBin" found in the MATH PRB menu. They should use randBin because they are sampling from a population who can respond only "yes" or "no" (binary—only two choices.)

- ❐ randBin (10, 0.4, 100) would simulate 10n (the 10) spinners with 40% of the area marked "yes" (0.4) being spun 100 times (the 100). This would simulate the situation above, i.e., asking 10 people from the known population that is 40% in favour of increasing taxes, and repeating it 100 times.

As the sample size increases from 10 to 20, to 40, the distribution should cluster more and more closely to the mean. This would become clear on the histograms of the distribution, as they would grow taller and skinnier as the sample sizes increased.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F4

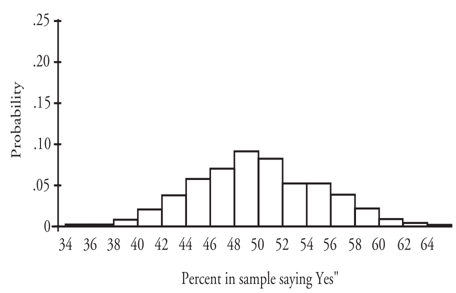
Activity

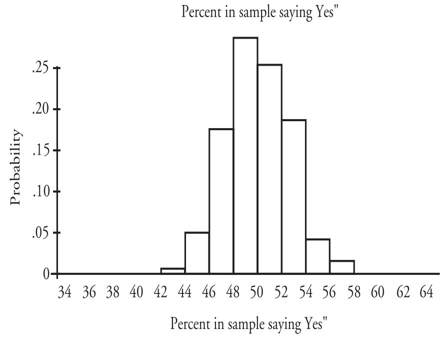
1) On a particular day in October, a census was taken of the entire grade 11 class at Your School. The results indicated that 62% of the 240 students answered yes to this question: "Should all grade 11 students take phys. ed. 3 hours/week? Yes or No?"
   The school staff couldn't believe the results. They decided to sample the population, asking the same question.
   a) Describe a process they might use to simulate a sample size 5, with 50 repetitions.
   b) Conduct the simulation and display the results on a histogram.
   c) Repeat the process described above, but for sample size 10, with 50 repetitions, then sample size 50 with 50 repetitions.
   d) Compare the three histograms to explain how they are different.
   e) Describe how your confidence about the initial census results as strengthened or weakened by the results of your sampling.

F4

Performance

2) Displayed are two histograms showing the probability that a student would answer yes to a question. The first graph represents a sample size 100 and 1000 trials, the second, 1500 and 1000 trials. Describe the differences and explain why the differences are occurring.





Suggested Resources

Garfunkel, Salomon, Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*, New York: W. H. Freeman & Co., 1995.

Landwehr, Swift, Watkins, *Exploring Surveys and Information from Samples*, Quantitative Literacy Series, Palo Alto, CA: Dale Seymour Publications, 1987.

Statistics

Outcomes

SCO: In this course, students will be expected to

F1 **draw inferences about a population from a sample**

F2 **identify bias in data collection, interpretation, and presentation**

A3 **demonstrate an understanding of the application of random numbers to statistical sampling**

Elaboration—Instructional Strategies/Suggestions

F1/F2 Students should become familiar with different kinds of sampling methods. The primary objective is for students to understand the differences in the sampling methods rather than memorizing the names by rote. Students should try to connect the names of the methods to the process used, in this way making sense of the name. The methods include

- simple random samples
- cluster samples
- systematic samples
- convenience samples
- self-selected samples

Students should understand that randomizing procedures are required to reduce bias in data collection. The bias that is most easily identified results from people's own misgivings about including all groups of people in the sampling process. Other sources of bias in data collection include poorly worded questions, leading questions, untruthful responses, and unresponsive participants.

A3 An experiment or survey must be replicable, that is, the results should be more or less the same after many repetitions. Much of this depends on how the sample data is selected. Students who need an unbiased selection method should use a random number table or generator. Computers and calculators should be used for this task. For example, scientific calculators have built-in programs that generate random numbers between zero and one, such as the list that may appear: .531, .574, .072, .924, .490, .751, .436, .099. Each three-digit decimal number between 0 and 1 has an equally likely chance of occurring. Random numbers can cluster, they just don't follow predictable or recurring patterns. If a student wants to interview 25 students from the grade 11 population in your school, she might multiply each of these random numbers by the total number of grade 11 students (212) in her school to find

$$.531x212 = 112\text{th person on the list}$$
$$.575x212 = 122\text{nd person on the list}$$
$$.072x212 = 15\text{th person on the list}$$

Students can use the menu MATH PRB on a graphing calculator to select various random number generators. For example, to simulate the tossing of a six-sided die 50 times, students could select 50 integers from 1 to 6 by using the command randInt (1, 6, 50). To simulate yes/no situations, students can randomly sample using randBin (10, .4, 50) ("Bin" for "binary," or "only two possible answers"). This command would produce data simular to that if a student selected 50 random samples each of size 10 from a population where 40% are known to answer yes to a particular question in a survey.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F1/F2/A3

Performance

1) A group of students wishes to take a survey of the 750 students in their school. Each student in the school has their own identification number, and each student in the school is registered in one English course. Identify the following methods as random sampling, convenience sampling, cluster samples, or systematic sampling.
   a) Select 100 identification numbers using the random number generator of the school computer and interview the students selected.
   b) One teacher has four English classes, with a total of 120 students in the four classes. Interview all the students in those classes.
   c) Place each of the students who are conducting the survey at a different entrance to the school and select 1 student per minute until each surveyor has interviewed 20 students.
   d) Select every seventh person on the list of students who attend the school.
   e) Interview 120 students who have study periods at the same times as the students doing the interview.
2) What does a random sample mean?
3) Review the following question from a statistics quiz. This question contains 132 letters. Count the e's. Based on your count, what is the percentage of e's used in the English language? How confident are you in your response (0–10, 10 being very confident)? Do you think that this is a biased sample of letters? Give a reason.
4) The following item appeared in a Saint John newspaper:
   "... is conducting a survey to determine the percentage of homes that have installed smoke-detecting devices. Questionnaires will be mailed to a sample of homes. If you would like to participate in this survey, please contact ..."
   Do you think that the results obtained from this survey will be biased? Give reasons.

Suggested Resources

Garfunkel, Salomon, Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*, New York: W.H. Freeman & Co., 1995.

Burrill, Gail, et al. *Data Analyses and Statistics, grades 9–12.* Reston, VA: NCTM, 1992.

Landwehr, Swift, Watkins, *Exploring Surveys and Information from Samples*, Quantitative Literacy Series. Palo Alto, CA: Dale Seymour Publications, 1987.

Statistics

Outcomes

SCO: In this course, students will be expected to

F5 **organize and display information in various ways, with and without technology**

F13 **calculate, analyse, and interpret various statistics**

F7 **draw inferences from graphs, tables, and reports**

F2 **identify bias in data collection, interpretation, and presentation**

Elaboration—Instructional Strategies/Suggestions

F5 Students will construct histograms, box plots, and other displays of data. They need to make decisions about which is the most appropriate display for the argument they are establishing or for the distribution they need to study.

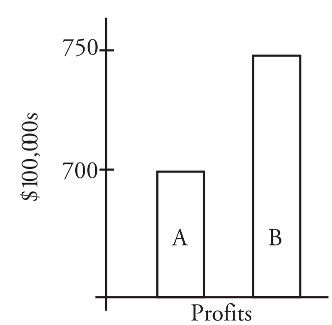
- ❐ A bar graph, circle graph, or a stem and leaf plot might be used to display test results in the grade 11 math class. Have students work in threes, each selecting one display method. Classmates should determine which graph displays the data in the most helpful or meaningful way.

Students should know that bar graphs are used to display discrete data, whereas histograms are intended for continuous data. Box plots are often used to compare variability in distributions. Students should note that the wider the box the more varied and widely distributed the data. Stem and leaf plots are like histograms, only they provide more detailed information.

F5/F13 Students should be encouraged to use appropriate statistical technology for calculating mean, median, and standard deviation (see page 44) and for creating displays (i.e., of histograms). Students will use histograms and/or frequency bar graphs to study distribution of data and to explore a distribution that is normal.

F7 Numerical facts are essential for making many kinds of decisions in our lives. Data is useful when we organize and present it so that it can be more easily interpreted. A few numbers computed from the data—averages, percentages and the like—can be very helpful. Some numbers computed from the data lead to statistical inference—drawing conclusions and stating our confidence about the conclusions.

Students should investigate graphs, tables, and displays of samples of data obtained in experiments and surveys. Students might come up with their own topics for a survey and survey the students in their class. They might then exchange displays of data or post them on the classroom walls from which whole-class discussions can take place. They should be encouraged to explain what they know based on the information on the display. From the display, they should be able to draw some conclusions about and describe the population the sample represents and attempt to make conclusions based on the sample.



F2 Students should analyse their displays for any bias that might be detected. Statisticians can alter the shape of a display in order to provide a picture that will help establish one particular side of an argument. Ask students how the graph to the right might mislead someone if they were working for company B and comparing their profits to those of company A.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F5/F13/F7/F2

Performance

1) The Golf Problem:
The very first annual district golf tournament is presently under way, and you are the coach of your school team. The prize up for grabs is a TI-83 graphing calculator for each math student in the winning school, to be paid for by the student council of the losing teams' schools. Each school selects five golfers and two alternates for their team. There will be two 18-hole games of golf twice a month beginning in May, and ending in September, with each team entering five players to play. The course to be played each half-month will be selected at random by the principal of the winning school in the previous game. At the end of September, the school team with the lowest accumulated score over the year will be declared the winner, and the other schools will supply them with their prize.

Ronnie:

83 82 101 92	75 72 99 93	77 90 87 95	82 88 89 88	95 85 76 85	93 81 72 91	91 95 82 93	101 97 90 99	103 105 101 105	92 91 78 101

Bobbie:

68 71 78 91	89 102 82 99	101 88 67 105	67 92 86 101	107 95 92 65	110 87 90 87	98 69 103 86	89 111 96 92	72 96 107 91	108 68 75 101

Situation 1: In early August the accumulated scores for each of the teams has been totalled and the results show that any of the teams could go on and be the winner. But wait, your school nurse has announced that your best golfer has developed a serious back problem and will not be able to play for the rest of this year. You must select one of your alternates to replace her for the remainder of the tournament. To assist you in your selection, the following tables represent the previous 20 golf scores recorded for each of your alternates (luckily they were members of the same golf course.)
Analyse this information and select the alternate you would choose to replace your star for the rest of the games this year. Present arguments and diagrams to support your decision.
Situation 2: There is only one game left in this year's tournament, and one of your players cannot make it due to having to write exams at his school. Again, you have the two individuals above to choose from as a replacement. Here is the situation: one game left; your accumulated team score is 4 points higher than the leading school; you really want those graphing calculators! Present arguments and diagrams to support your decision.

Suggested Resources

Garfunkel, Salomon, Consortium for Mathematics and Its Applications (COMAP). *For All Practical Purposes*, New York: W. H. Freeman & Co., 1995.

Burrill, Gail et al. *Data Analyses and Statistics, grades 9–12*, Reston, VA: NCTM, 1992.

Landwehr, Swift, Watkins, *Exploring Surveys and Information from Samples*, Quantitative Literacy Series, Palo Alto, CA: 1987.

Statistics

Outcomes

SCO: In this course, students will be expected to

F3 **demonstrate an understanding of what can be inferred about a population by examining sample means and dispersions**

F15 **design and conduct experiments/surveys to explore sampling variability**

F5 **organize and display information in various ways, with and without technology**

F13 **calculate, analyse, and interpret various statistics**

Elaboration—Instructional Strategies/Suggestions

F3 In studying the distribution of data (for more discussion on distribution, see page. 44), students should understand that experimental results about a variable often differ from what results are expected. ("Expected" here simply refers to that result one would expect to see. For example, most people would expect to see about 50% of all coin tosses to turn up heads.) Students must decide how much deviation from the expected is reasonable before it can be concluded that something "strange" is happening. For example, if 60% of the coin flips turn up heads, would that be considered a "strange" result? How about 57%? Students will deepen their understanding of distributions and will use mean and standard deviation to describe the dispersion of data.

F15/F5 They might begin by examining data with a mean and median that are the same, but with displays that look quite different. By examining these data displayed in box plots, students will see a longer box and perhaps longer whiskers for one set of data than for another, even though their means and medians are the same. Lower and flatter histograms indicate that there is more variation in the data than a narrow high histogram where the data seem to cluster around the mean.

F3/F13 Students should discuss how dispersion of data can be measured. They should consider the range of values in the data. They might talk about the fact that a certain range of values might be expected, but that if the range was overly large some data might be considered quite suspicious. In discussing acceptable range, students might talk about a way to measure the dispersion of data. They may decide that dispersion can be described by some value arrived at by subtracting data values from the mean value. In other words, finding some average amount by which each value point is away from the mean value point.

Students should develop an understanding of how standard deviation is measured using manual calculations at first, then using technology. They should understand each step in the calculation and that higher values for deviation mean more dispersion. They should think of standard deviation as representing the average dispersion from the mean. With technology, students should be able to access values for single-variable statistics such as mean, median, range, standard deviation, and quartile values. They should be expected to use technology, if available, to find these single-variable statistics.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F13/F15

Performance

1) The following data were gathered by a group of students performing an experiment to determine if amplitude has an effect on the period of a pendulum. The data represent the period of a pendulum, measured 20 times using a stop watch.

.58	.57	.51	.63	.61	.58	.59	.56	.55	.54
.54	.57	.59	.61	.53	.57	.56	.57	.58	.53

   a) Calculate the mean and standard deviation.
   b) Describe what you think is the period of the pendulum based on this experiment. Explain.
   c) Are you surprised by the different results that the students got within the same experiment? Explain.
   d) Describe how you think the experiment would be conducted.

F13/F3

2) This same experiment was conducted by 24 groups, each of which described the period as the mean of 20 trials. The following are the means.

.5624	.5705	.5680	.5581	.5713	.5615	.5496	.5564
.575	.5265	.5811	.5876	.5925	.5728	.5720	.5572
.5576	.5585	.5860	.5605	.5821	.5382	.6013	.5630

   a) Calculate the mean and standard deviation.
   b) Describe how this distribution is different than the one in 1) above.

F5

3) a) Construct histograms for the experimental results in 1) and 2) above.
   b) Construct 50% box plots for each as well.
   c) Describe how the box plots help you better understand the distributions.

F13

Journal

4) Explain why the differences from the mean are squared in the process of determining the standard deviation.
5) In the process of determining the standard deviation, each of the experimental results is subtracted from the mean. Why?
6) What does the standard deviation tell you?

Suggested Resources

Garfunkel, Salomon, *Consortium for Mathematics and Its Applications* (COMAP). *For All Practical Purposes*, New York: W. H. Freeman & Co., 1995.

Burrill, Gail et al. *Data Analyses and Statistics, grades 9–12*, Reston, VA: NCTM, 1992.

Statistics

Outcomes

SCO: In this course, students will be expected to

F1 **draw inferences about a population from a sample**

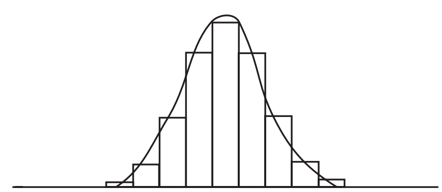
F3 **demonstrate an understanding of what can be inferred about a population by examining sample means and dispersions**

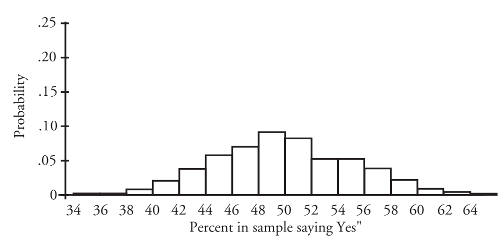
F4 **demonstrate an understanding of how the size of a sample affects the variation in sample results**

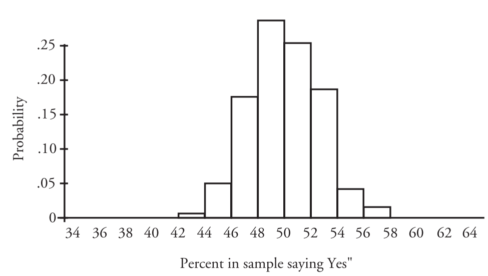
Elaboration—Instructional Strategies/Suggestions

F1/F3 In Year 10, students conducted multiple trials of certain experiments. They found that the measurement taken for each trial was not always the same. They explored this and came to an agreement that it is normal for this to happen. However, they wondered if some of the differences were normal or it they were caused by some error they were making. For example, when they attempted to measure the period of a pendulum (one complete swing), one time they might clock it at 1.80 seconds, the next time at 1.70 seconds, and the next at 1.85 seconds. In this course, students will formalize this notion that different measurements would be expected when performing the same experiment over and over again. What they need to formalize is how much deviation from the expected is reasonable before it can be concluded that the recorded measurement may have been affected by some other factor.

F1/F3/F4 When results vary, students need to question if the variation has occurred by chance or not. To do this, they should conduct many experiments of the same phenomena, pool all the results, and display them in histograms. For example, if a teacher had three or four math classes conducting these experiments and each class was made up of six–eight groups, each collecting their own data, the teacher could have each group describe their data by finding the mean. The teacher could then collect all the means and form a histogram. Students would see that as the number of samples get larger and larger, the histogram would take on a shape that becomes more and more like a normal distribution: that is, as the number of trials increased, the distribution of sample means would seem to cluster more and more around the mean of all the data. It would take on a more symmetrical, higher, narrower appearance rather than a low and flat shape. Students should understand that a similar change in shape of the histogram is connected to sample size and that the sample size affects dispersion. As the sample size increases, the deviation of the data from the mean decreases.







Statistics

Worthwhile Tasks for Instruction and/or Assessment

F1

Performance

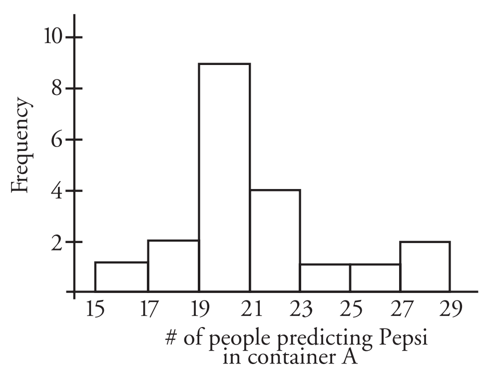
1) Barney and Martha set up a booth at their school's spring fair. They charged 25¢ for an opportunity to predict which cola was being tasted. Each participant would pay their 25¢ then taste two cups of liquid. They would then predict whether Container A or Container B was Pepsi. At the end of the day, Barney and Martha calculated their sample statistics. They had asked 125 people which container contained Pepsi. 42% of the people said container A was the Pepsi, with a standard deviation of 6.2%. Describe what you might hypothesize about the 1000 students at this school based on this sample.

F3

2) William wasn't satisfied with his hypothesis from question 1) above. He decided to conduct his own survey. He convinced 10 friends to help him. He asked each to conduct a similar survey at different locations around the school at noon time. Each was to collect 25 results and find the mean and repeat this one more time. William then examined a distribution of the 20 sample means. Describe what you think he would see. How might it be different than what he saw in 1). Explain.

F4

3) The histogram displayed shows the results of a survey similar to that in question 2), only this time the sample size was 50, not 25. There are 20 sample means, each representing a sample size 50. Explain how increasing the sample size to 50 from 25 has affected the distribution.



Suggested Resources

Garfunkel, Salomon, *Consortium for Mathematics and Its Applications* (COMAP). For All Practical Purposes, New York: W. H. Freeman & Co., 1995.

Burrill, Gail et al. *Data Analyses and Statistics, grades 9–12*, Reston, VA: NCTM, 1992.

Statistics

Outcomes

SCO: In this course, students will be expected to

$G3_2$ graph sample distributions and interpret them using the language of probability

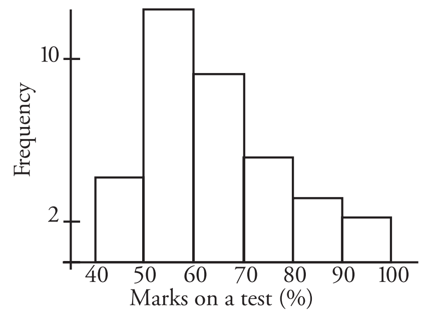
F13 calculate, analyse, and interpret various statistics

F5 organize and display information in various ways with and without technology

F7 draw inferences from graphs, tables, and reports

Elaboration—Instructional Strategies/Suggestions

$G3_2$ Students will begin to interpret the frequencies of events occurring within a population using probability. The concept being developed here is that the relative area under any portion of a histogram corresponds with the probability of randomly choosing a member of the population with that characteristic.



F13/F5/F7

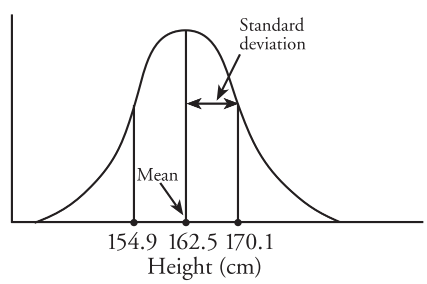
The probability of scoring between 40 and 50 on the test is determined by comparing in a ratio the area in the bar from 40–50, with the total area of all the bars. Class size is 35.

$$P(40-50)=\frac{4}{35} \text{ or } 0.114, \text{ or } 11\%$$

The probability that a particular student passed the test (a mark of 50% or better) is determined using the ratio $\frac{31}{35}$ or 0.89, or 89%.

F7 Given a histogram of a distribution that is fairly symmetrical and approaches the shape of a normal curve, students could be asked questions about what the probability is of a measurement being within one standard deviation of the mean. Using calculations like that above, students could calculate that area to include that measurement 68% of the time. Likewise, the 95% region and 99.7% region could be determined.

❒ Given data measuring heights of people with a mean height of 162.5 cm and standard deviation of 7.6 cm, a histogram could be constructed that approached the shape of a normal curve. Students could be asked what the probability is of having a height within one standard deviation of the mean. They would then approximate the area within the region below the curve between 154.9 and 170.1 cm and express it as a ratio of the total area and estimate that to be 0.68 or 68%.

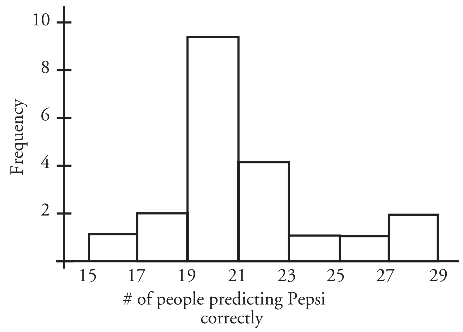


Statistics

Worthwhile Tasks for Instruction and/or Assessment

G3$_2$/F13/F5/F7
Performance

1) This frequency histogram represents the number of people who identified Pepsi in container A. It is based on 20 sample means from samples of size 50.



   a) Change this histogram to a probability bar graph representing the probability that a person would predict Pepsi in Container A.
   b) From the probability bar graph, what is the probability that from 19 to 21 people in every 50 would predict Pepsi in Container A? Explain.
   c) What is the probability that 30 of 50 would predict Pepsi in Container A? Explain.

F5/F13/G3$_2$/F7

2) Twelve of the 30 people in your math class are males. You have five tickets to a rock concert to give away to your classmates. Working as a team with other students, you are to draw 100 simple random samples of size 5 from this population (your class of 30). (Recommend that students use technology.)
   a) Complete the sample proportion of males in each sample.
   b) Make a probability bar graph of the situation.
   c) Describe the shape. In particular, does it look roughly normal?
   d) What is the average number of males in your 100 samples?
   e) You now give the five tickets to five students in the class. They were all females. Do you think there was any discrimination in giving the five tickets to the five females? Explain.

Suggested Resources

Statistics

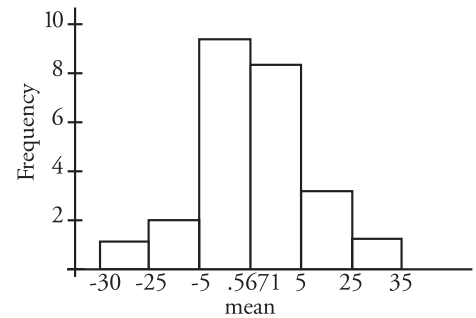
Outcomes

SCO: In this course, students will be expected to

G3$_2$ graph sample distributions and interpret them using the language of probability

Elaboration—Instructional Strategies/Suggestions

G3$_2$ On the previous two-page spread it was discussed how students would calculate probabilities based on the amount of area under the curve versus the total area.



Students should formalize this with an activity where they would calculate the areas between first, second, and third standard deviations. They could do this by superimposing a grid over a normal curve and estimating the area within the stated regions. They should understand that in a normal situation 95% of all the data should fall within two standard deviations of the mean. (Students should remember that a standard deviation represents an average dispersion from the mean.) Students should now use this information about mean and standard deviations in normal curves to describe samples and populations. For example, from a census taken as fans entered the ball park at a baseball game, it was known that 80% of the fans said they wanted the opportunity to buy peanuts while watching the game. The standard deviation was 3%. Students should be able to say that if random samples of fans were asked their opinion about peanuts they would expect that the sample proportions of those in favour would be in the range of 74% to 86%, 95% of the time.

In studying the distribution of data, students should understand that experimental results about a variable often differ from expected results. They decide how much deviation from the expected is reasonable before it can be concluded that some other factor is affecting the outcomes.

When 95% of the sample measurements are within two standard deviations of the mean, then there is only a 5% chance that a randomly chosen measurement will differ from the mean by more than this, or only a 5% chance that the difference is accidental. When a measurement falls within two standard deviations of the mean, it is not considered significantly different from the mean.

Students should calculate means and standard deviations from various contexts to determine whether values differ sufficiently from the mean to be considered “significantly different.”

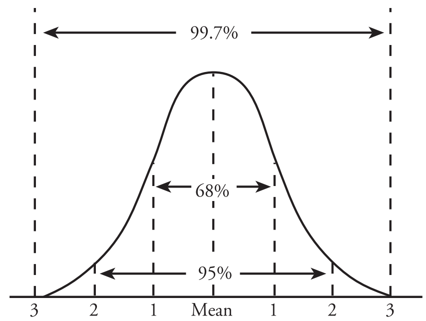
Statistics

Worthwhile Tasks for Instruction and/or Assessment

$G3_2$

Activity

1) On page 43, students were given problem 2) to address outcomes F13 and F3. Students calculated the mean and standard deviation for a distribution of 24 sample means. Each mean was the result of 20 trials. A histogram displays the distribution.



   a) Complete the labelling of the horizontal axis using standard deviation.
   b) Estimate the area on the histogram within the first standard deviation; within two standard deviations, within three standard deviations.
   c) How close to normal would you say this distribution is? Explain.
   d) If a pendulum's period was measured (as below) how confident (scale 0–5, 5 being very confident) would you be of each coming from this distribution? Explain.

i) .51	iv) .59
ii) .53	v) .60
iii) .56	vi) .62

2) The scores on a Canadian math contest were normally distributed with a mean of 105 and a standard deviation of 25.
   a) 68% of the students had scores between what two values?
   b) 95% of the students had scores between what values?
   c) A student scored 130. What percentage of students scored better than that?
   d) How likely is it that a student scored 50 points? Explain.

Journal

3) The following is a letter to the editor written by Bruce Wark, School of Journalism, University of King's College, taken from the March 1997 edition of the Halifax *Mail Star,* "Re: Cameron MacKeen's March 12 report Tories top Liberals, poll says. The article said the poll gave the Hamm Tories the support of 32% of decided voters, while the Savage Liberals lagged with only 25% ... A poll with a sample size of 400 has a standard deviation of $\pm 5\%$. That means, according to this poll, Tory support ranged from a high of 37% to a low of 27%. Liberal support ranged from 30% to 20%. It would be just as valid therefore, to report that the poll showed the Liberals ahead 30–27."
   a) Write a reaction to this letter. Do you agree with Mr. Wark's logic?
   b) Explain in your own words the logic in Mr. Wark's words in the last sentence.
   c) How else could the MacKeen report have announced the results for support?

Suggested Resources

Statistics

Outcomes

SCO: In this course, students will be expected to

F12 interpret normal curves and standard deviation to express levels of confidence

F17 design and conduct experiments/surveys and interpret and communicate level of confidence

Elaboration—Instructional Strategies/Suggestions

F12 When students conduct surveys and polls and collect data from experiments, they need to be able to interpret these results and to use them to predict, with confidence, behaviours of the larger population. By studying sampling distribution, students should come to understand that the distribution of any random phenomena tends to be normal if it is averaged over a large number of independent repetitions. Students must be able to decide how much deviation from the expected is reasonable before it can be concluded that another variable is affecting the outcome. In other words, how far out on the tail of the normal curve will a result have to be in order to be considered different or unlikely to happen. Since a given measurement has a 68% chance of being within one standard deviation of the mean and a 95% chance of being within two standard deviation, any value further away from the mean (beyond two standard deviations) may result in being a measurement that makes a difference to the outcome.

The following is an example of using normal curve and standard deviations to express results with some degree of confidence.

F17

- ❐ Sammy White owns a "fish for trout" business on a small lake in a river system in southern Nova Scotia. In mid-April, he advertises that if you fish in his lake, you will catch trout between 21cm and 30 cm. In fact, he guarantees that your catch will be in that size range or your money will be returned. How can he make that claim?
  Solution:
  During the month of April, Sammy closes his lake to fishing in order to conduct his survey of the trout population. Sammy has special trout traps set up in various parts of the lake. He spends most of his early mornings and late evenings canoeing from trap to trap to count and measure his trapped trout. By the middle of the month he tallies his results. He calculates the mean length for all the fish caught in each trap for each time that he checked the trap and produces a histogram of the distribution of "mean lengths" of trout. He notices that the histogram approaches the shape of a normal curve. He calculates the mean of the distribution to be 25.8 cm with a standard deviation of 2.1cm. From these results (and a little bit of rounding), he feels he can be 95% confident that any fish taken from his lake during the next few weeks will measure between 21cm and 30cm.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F12

Performance

1) At a local Cub rally, one event in which all the Cubs participated was the racing car event. Each Cub was given a block of wood, an axle, and four wheels and was expected to design a car for racing. The race consisted of releasing the car on a sloped track and timing the car as it passed a particular spot. At the end of the day the distribution of 100 times tended to be normal with a mean of 4.6 seconds and a standard deviation of 0.6.
   a) How many of the 100 times were within one standard deviation of the mean?
   b) How many seconds must a car be timed to be considered unusually fast?
   c) If it took 3.5 seconds for a car to pass the spot, do you think that car was likely to win the event?
   d) If a car took 5.1 seconds, how would you describe its chances of winning? Explain.
   e) Cars with times better than 5.3 made it to the final round. Estimate how many cars made it. Justify your estimate.

2) Suppose the following statement was made: 65% of high school students like music videos. Later, a random sample of 35 students finds that 18 like music videos. Is it fair to say that the statement is wrong? Why or why not?

F17

3) A committee at Yore School want to provide a late bus so that students can stay at the school longer for academic and recreational activities. They want you to design and conduct a survey that would help them with their arguments when they present this idea to the school board. You are to design and conduct a yes/no survey and present the results, including the predicted population mean, standard deviation, and your level of confidence, to the committee.

Suggested Resources

Statistics

Outcomes

SCO: In this course, students will be expected to

F17 **design and conduct experiments/surveys and interpret and communicate level of confidence**

Elaboration—Instructional Strategies/Suggestions

F17 Students should use their understandings of relationships between samples and populations to conduct a survey and interpret the results of it. They should be expected to demonstrate their understanding by presenting, both orally and in writing, the methods they used, their displays, and their findings.

Students should design and conduct their own survey or poll using a large sample size. When students collect and analyse their own data, their appreciation of the principles and practice of survey techniques grows, and they are able to pull together what they have learned in a practical way.

One suggested approach might be to split the class into groups of four. Each group plans a questionnaire, including the specific wording of the questions and instructions. Each student should choose one yes/no question, but the group as a whole should determine the wording of the question. The group should then field-test the questions by asking them to other students to make sure other students have no trouble answering yes or no. The group should then make any necessary revisions to the questions and decide on a sampling method. Each member of the group should collect data on all questions, but each students would prepare a report on his/her own question only.

Several evaluation schemes for survey projects are possible, but whatever the choice, make sure it is described to the students before they begin the project. One method is to give students a score from 0 to 4 in each of the following categories:

1. originality and independence
2. questionnaire design
3. sampling design
4. collection of data
5. report/presentation

❐ A group of students agreed to survey people randomly selected at Hector's Quay in Pictou (near the ferry) on a Saturday. It was the responsibility of each of the four people in the group to ask 10 people the question at three different time periods in the day. This was the question: "Should Northumberland Ferries Ltd. discontinue its ferry service from Nova Scotia to Prince Edward Island?" After pooling their results, the group said that based on their survey, they were 95% confident that that between 60% and 76% of the people in Pictou were against Northumberland Ferries Ltd. discontinuing their ferry service. Students should understand what this group did to be able to state those conclusions and whether, in fact, they were correct.
This is what they did: Students created a histogram with their pooled results that approximated a normal curve with a mean of 68% and a standard deviation of 4.1. They could be confident that if they repeated this survey 20 times, 19 of the sample means would be within two standard deviations of 68% saying no. In other words, roughly 60% to 76% of the people would say no, 19 times out of 20.

Statistics

Worthwhile Tasks for Instruction and/or Assessment

F17

Performance

1) A high school wants to survey adults in the community to find out what percentage would attend the "spring fling" event that the student council has in mind.
   a) Design and conduct a survey from which you can advise the student's council with 95% confidence.
   b) Describe in detail the design and how you conducted the survey.
   c) Give your conclusion to the student council as a statement of what you found using 95% confidence.

2) On page 52, a survey is discussed that deals with the ferry service between Nova Scotia and Prince Edward Island. Do you think the results from that survey reflect the feelings of all the people in Nova Scotia? Atlantic Canada? How might you design and conduct the survey so as to get a less biassed result?

Suggested Resources

UNIT 2:
INDEPENDENT STUDY

Unit 2
Independent Study
(10–15 Hours)

Independent Study

Outcomes

SCO: In this course, students will be expected to

I1 demonstrate an understanding of a mathematical topic through independent research

I2 communicate the results of the independent research

Elaboration—Instructional Strategies/Suggestions

I1/I2 The purpose of this independent study is 1) to provide an opportunity for students to learn independently; and 2) to provide students with the opportunity to explore

- in more depth, mathematical content to which they have been exposed but would like to know more about
- new mathematical content areas
- mathematical topics of interest
- the history of mathematics and its connections to the mathematics we study today
- mathematics in our lives and related to careers
- mathematics through the Internet
- how people learn mathematics

Approximately 10–15 hours of class time should be devoted to this research project. Teachers should allow time for

- initial discussion and discovering ways to get information, what it means to learn mathematics independently, and why that is important
- discussing the expectations for the student presentations at the end of the unit and how they will be assessed
- brainstorming, topic-webbing, developing action plans and time lines, conferencing, etc.
- students to present the results of their explorations and learnings to the other students (short presentations, up to 10 minutes).

If students are working collaboratively on this project, it is expected that each would be responsible for gathering certain information and thus could be held responsible for the oral presentation that deals with that part of the project.

Independent Study

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

New Topics for Secondary School Mathematics, Matrices, NCTM, 1988

de Lange, Jan. *Meaningful Math: Matrices*, Pleasantvile, NY: Sunburst Inc., 1992.

de Lange, Jan. *Flying Through Math: Trig, Vectors, and Flying*, Pleasantville, NY: Sunburst Inc., 1991.

Froelich, Gary, et al. *Discrete Mathematics through Applications*. New York: W. H. Freeman and Company, 1994.

Jacobs, Harold R. *Mathematics, A Human Endeavor*, third edition, New York: W. H. Freeman and Company, 1994.

Serra, Michael. *Discovering Geometry*: An Inductive Approach, second edition, Eremyville, CA: Key Curriculum Press, 1997.

Charles, Randall et al. *How to Evaluate Progress in Problem Solving*, Reston, VA: NCTM, 1992.

Independent Study

Outcomes

SCO: In this course, students will be expected to

I1 demonstrate an understanding of a mathematical topic through independent research

I2 communicate the results of the independent research

Elaboration—Instructional Strategies/Suggestions

I1/I2

Managing the Project

Management of the project should be very teacher-directed:

- The nature of the topics will determine the appropriate group size.
- Class size might lend itself to the whole group doing the same project, but with small group or individual responsibility given to subtopics.
- Students will choose topics (perhaps from a teacher-prepared list) that are appropriate and of interest to them. Teachers should ensure the availability of reference resources (material and human) in or around the school or the community. Teachers should give final approval for any topic.
- Brainstorming or topic webbing should take place.
- Action plans should detail the tasks that have to be completed. It is assumed that each student will have responsibility for independent work within the structure of the project. Sample tasks include, for example:
  - writing letters to gather data or request materials
  - making phone calls for information
  - reading texts, newspapers, flyers, journals, and reports
  - completing a library search
  - interviewing resource people
  - reflecting on and sharing ideas with group members
  - preparing an oral presentation (see page 62, fourth bullet)
  - preparing a written submission
- Each student or group should detail his/her own time line to match that of the teacher. Deadlines should be determined for what has to be completed and for bringing completed work to class. Regular conferences regarding progress are crucial. This includes conferencing with group members and with the teacher.

Potential Topic and Content Areas

New Content

- vectors with respect to navigation and orientation—scale diagrams
- graph theory (e.g., four-colour problem and travelling salesman problem)
- geometry and its connections to art; fractals

Topics to Explore in for More Depth

- different measurement systems
- algebraic manipulation
- functions, combinations of functions—connection to art
- regression analysis
- the stories of infinity and zero
- periodic and sinusoidal situations, using technology

Mathematics in our Lives

- Fibonacci numbers—connecting to the world
- geometry in our lives (e.g., patterns, design, architecture)
- mathematics in jobs (e.g., interview a person about how he/she uses math in his/her job)
- Do statistics lie?
- consumer mathematics
- the role of mathematics in various career options
- leisure mathematics (e.g., non-routine problems, logic, math games, puzzles, games of chance)
- the Internet as a source of mathematics information
- Imperial Measurement System

Independent Study

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Independent Study

Outcomes

SCO: In this course, students will be expected to

I1 **demonstrate an understanding of a mathematical topic through independent research**

I2 **communicate the results of the independent research**

I3 **demonstrate an understanding of the mathematical topics presented by other students**

Elaboration—Instructional Strategies/Suggestions

I1/I2/I3 Teachers might facilitate this unit by

- focussing 10–15 hours on the project all at one time. (**Note**: Teachers need to remain aware of the time needed to gather and compile information.)
- spreading the project over a term, with some class periods being designated for the project introduction, as checkpoints (each with a particular expectation), and for finalizing, preparing, and performing presentations
- integrating it with a topic going on at the same time in the classroom (e.g., statistics, algebra, indirect measurement, trigonometry)

Expectations for assessment must be made clear to students. All students must be involved with the presentation of the mathematics they have researched. Presentations might take the form of

- oral presentations to the class
- oral presentations on video and played to the class
- conversations between students or among group members in front of the class or on video
- teacher/student interviews (private or in front of the class)
- some other variation

Executive summaries must be distributed to the class at the time of the presentations.

This means that students should summarize the new mathematics learned in such a way that other students can read over each summary, see a couple of examples and have some understanding of the new topics.

Assessment

- Criteria for the written submissions should be made clear.
- Criteria for presentations should be made clear. (**Note**: Students should not simply read their written submissions.)
- Criteria should be prepared by the teacher and discussed with the students at the outset of the independent study project.
- A rubric should be included (students could help design it) that allows for assessment of the written work as well as the presentation.
- Peer evaluation should be conducted during each presentation and by each group member in a group effort.

Independent Study

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Independent Study

Outcomes

SCO: In this course, students will be expected to

I1 **demonstrate an understanding of a mathematical topic through independent research**

I2 **communicate the results of the independent research**

I3 **demonstrate an understanding of the mathematical topics presented by other students**

Elaboration—Instructional Strategies/Suggestions

I1/I2 A rubric for the written component might look something like:

Top Level

- contains a complete response with clear, coherent, unambiguous, and elegant explanations
- includes clear and simple diagrams, charts, graphs, etc.
- communicates effectively to an identified audience
- shows understanding of the mathematical ideas and processes
- identifies all the important elements of the topic
- includes examples and counter-examples
- gives strong supporting arguments

Second Level

- contains a solid response with some of the characteristics above
- explains less elegantly, less completely
- does not go beyond the requirements of the project (or topic)

Third Level

- contains a complete response, but the explanation may be muddled
- presents incomplete arguments
- includes diagrams, but somewhat inappropriate or unclear
- indicates understanding of mathematical ideas, but not expressed clearly

Fourth Level

- omits significant parts
- has major errors
- uses inappropriate strategies

See the books in the Suggested Resources column for more examples of rubrics for evaluating projects and open-ended activities.

I3 Students should collect all executive summaries from students who are presenting, and they should ask questions for clarification at the end of presentations. Students might demonstrate what they have learned from the presentations of others by completing a questionnaire that focusses on the highlights of the presentations. On a test, teachers might ask students to discuss, showing examples, what was learned from any presentation. Another strategy for assessment might be through a conversation between the teacher and student about someone else's project.

Independent Study

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

Stenmark, Jean Kerr. *Assessment Alternatives in Mathematics,* Berkeley, CA: EQUALS Publishing/ Lawrence Hall of Science, University of California, 1989.

Stenmark, J.K. (ed). *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions,* Reston, VA:, NCTM, 1992.

Charles Randall, et al. *How to Evaluate Progress in Problem Solving*, NCTM, 1992.

Unit 3
Probability
(20 Hours)

Probability

Outcomes

SCO: In this course, students will be expected to

G2 **develop an understanding that determining probability requires the quantifying of outcomes**

$G3_3$ **demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events**

Elaboration—Instructional Strategies/Suggestions

G2 Every day students experience a variety of situations. Some involve making decisions based on their previous knowledge of similar situations.

- Should they do their math homework tonight or during their spare period before math class tomorrow?
- Should they challenge a friend to a game of racquetball or Blockers?
- Should they buy a ticket on a car raffle?
- Should they take their umbrella today?

Before making the decision, what they must ask themselves is "What is the chance of this decision working out in my favour?"

In probability, events are given numbers ranging from 0 to 1, where 0 refers to things that never happen and 1 refers to things that always happen.

In their previous studies (grades 7–9) students have created and solved problems using probabilities, including the use of tree and area diagrams, and simulations. They have compared theoretical and experimental probabilities of both single and complementary events, and dependent and independent events. Theoretical probabilities are those that result from theory (what should happen mathematically), while experimental probabilities are those that result from experiments or repeated trials of performing the event. Students also have examined how to calculate the probability of complementary events. The probability of an event happening and its complement add to make 1. They also study two independent events, A and B, where the probability of A and B is equal to $P(A)\times P(B)$.

G2/$G3_3$ Sometimes the task of listing and counting all the outcomes in a given situation is unrealistic because the sample space may contain hundreds or thousands of outcomes.

The fundamental counting principle enables students to find the number of outcomes without listing and counting each one. If the number of ways of choosing event A is $n(A)$ and the number of ways of choosing an independent event B is $n(B)$, then $n(A \text{ and then } B) = n(A)\times n(B)$, and $n(A \text{ or } B) = n(A) + n(B)$. The first is the multiplication principle, the second, the addition principle.

Sometimes events are not independent. For example, suppose a box contains three red marbles and two blue marbles, all the same size. A marble is drawn at random. The probability that it is red is $\frac{3}{5}$. If the marble is then replaced, the probability of picking a red marble again is $\frac{3}{5}$. However, if it is not replaced, then when another marble is picked the probability of it being red is now $\frac{2}{5}$. The second selection of a marble is dependent on the first selection not being returned to the box.

... continued

Probability

Worthwhile Tasks for Instruction and/or Assessment

G2/G3$_3$

Activity

1) Two students are playing "grab" with a deck of special "grab" cards. One student has a triangular shaped deck with 16 ones, 12 twos, 8 threes, and 4 fours. The other has a rectangular-shaped deck with 10 each of ones, twos, threes, and fours. The decks are well shuffled and each student plays the top card simultaneously. A "grab" is made when two cards match (a double).
   a) There are 40 cards in each deck. What is the total number of pairs of cards which could be played? [Answer: 40 pairs]
   b) How many of these pairs are "double ones"; that is, a one from the triangular deck and a one from the rectangular deck?
   c) How many are
      i) double twos?
      ii) double threes?
      iii) double fours?
   d) For equally likely outcomes, the probability of an event is "the number of outcomes that correspond to the event" divided by what?
   e) So, the probability of a double one is "what" divided by "the total number of pairs"?
   f) Use the fundamental counting principle and your answers to (c) to find the probability of
      i) a double one
      ii) a double two
      iii) a double
   g) A circular deck has 10 ones, 20 twos, 10 threes, and no fours. Calculate the probability of a "grab" if a triangular deck is played against a circular deck.

Performance

2) Telephone numbers are often used as random number generators. Assume that a computer randomly generates the last digit of a telephone number. What is the probability that the number is:
   a) an 8 or 9 ?
   b) odd or under 4 ?
   c) odd or greater than 2 ?

3) A airplane holds 176 passengers, 35 seats are reserved for business travellers, including 15 aisle seats, 40 of the remaining seats are aisle seats. If a late passenger is randomly assigned a seat, find the probability of getting an aisle seat or one in the business travellers' section.

4) Using the given table, which represents the number of people who died from accidents and their respective ages, and in each case assuming that one person is selected at random from this group
   a) Find the probability of selecting someone under 5 or over 74.
   b) Find the probability of selecting someone between 15and 64.
   c) Find the probability of selecting someone under 45 or between 25 and 74.

Age	Number
0–4	3843
5–14	4226
15–24	19 975
25–44	27 201
45–64	14 733
65–74	8 499
75+	16 800

Suggested Resources

Flewelling, Gary et at. *Mathematics 10: A Search for Meaning*, Toronto: Gage 1987.

Probability

Outcomes

SCO: In this course, students will be expected to

G3$_3$ demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

Elaboration—Instructional Strategies/Suggestions

G3$_3$ How is the fundamental counting principle related to probability? Consider the marble situation described at the bottom of the previous two-page spread.

The probability of selecting red is $\frac{3}{5}$, while the probability of selecting blue is $\frac{2}{4}$. The probability of selecting a red and a blue without replacement would be $P(r \text{ and } b) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20}$. Now, let us consider another situation:

Consider the experiment of a single toss of a standard die. There are six equally likely outcomes: 1, 2, 3, 4, 5, and 6. Define certain events as follows:

A: observe a 2
B: observe a 6
C: observe an even number
D: observe a number less than 5.

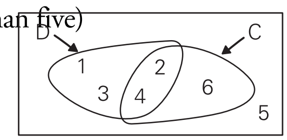
$P(A) = \frac{1}{6}$ (observe a 2), $P(B) = \frac{1}{6}$ (observe a 6). What about $P(A$ or $B)$ (observe a 2 or 6)? This can be shown two ways:

$\frac{n(A)+n(B)}{\text{total number of ways}} = \frac{1+1}{6} = \frac{2}{6}$ or $P(A \text{ or } B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$.

Will this be true for any two events? The events "observe a 2," and "observe a 6" are called mutually exclusive events, or disjoint because one can observe only a 2 or a 6, not both at the same time. On the other hand, events like C and D above have at least one element in common, and therefore are not mutually exclusive. Consider the events C and D.

The event (C or D) includes all the outcomes in C or D or both. That is,

$P(C \text{ or } D) = P$ (observe an even number of a number less than five)
$= $ (observe 2, 4, 6, or obserrve 1, 2, 3, 4)



Every outcome except five is included in (C or D).
Thus there are exactly 5 favourable outcomes. Thus $P(C \text{ or } D) = \frac{5}{6}$. But $P(C) + P(D) = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$, which cannot be possible since it exceeds 1.

The outcomes 2 and 4 are contained in both C and D and are being counted twice. They must be removed. There is an overlap.

$$P(C \text{ or } D) = P(C) + P(D) = \frac{3}{6} + \frac{4}{6} - \frac{2}{6} = \frac{5}{6}$$

Probability

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

G3$_3$

Performance

1) Discuss whether the following pairs of events are mutually exclusive and whether they are independent.
   a) The weather is fine; I walk to work.
   b) I cut a deck of cards and have a queen; you cut a 5.
   c) I cut the deck and have a red card; you cut a card with an odd number.
   d) I select a voter who registered Liberal; you select a voter who is registered Tory.
   e) I found a value for x to be greater than –2; you found x to have a value greater than 3.
   f) I selected two cards from the deck, the first was a face-card, the second was red.
2) If 366 different possible birthdays are each written on a different slip of paper and put in a hat and mixed,
   a) Find the probability of making one selection and getting a birthday in April or October.
   b) Find the probability of making one selection that is the first day of a month or a July date.
3) A store owner has three student part-time employees who are independent of each other. The store cannot open if all three are absent at the same time.
   a) If each of them averages an absenteeism rate of 5%, find the probability that the store cannot open on a particular day.
   b) If the absenteeism rates are 2.5%, 3%, and 6% respectively for three different employees, find the probability that the store cannot open on a particular day.
   c) Should the owner be concerned about opening in either situation a) or b)? Explain.
4) There are 6 defective bolts in a bin of 80 bolts. The entire bin is approved for shipping if no defects show up when 3 are randomly selected.
   a) Find the probability of approval if the selected bolts are replaced; are not replaced.
   b) Compare the results. Which procedure is more likely to reveal a defective bolt? Which procedure do you think is better? Explain.
5) Mary randomly selects a card from an ordinary deck of 52 playing cards. What is the probability that Mary will select either an ace or a diamond? Below is Fred's solution. Explain what Fred is thinking. Will his attempt lead to a correct answer? Explain.

$$P(\text{ace or diamond}) = \tfrac{4+13}{52} = \tfrac{17}{52}$$

Journal

6) Consider the table of experimental results. Comment on the following solution attempts.

	Seldane	Placebo	Control	Total
Drowsiness	70	54	113	237
No drowsiness	711	611	513	1835
	781	665	626	2072

   a) If one of the 2072 subjects is randomly selected, the probability of getting someone who took Seldane or a placebo is $\frac{781}{2072}+\frac{665}{2072}=\frac{1446}{4144}=0.3489$.
   b) If one of the 2072 subjects is randomly selected, the probability of getting someone who took Seldane or experienced drowsiness can be found by

$$\frac{781}{2072}+\frac{237}{2072}=\frac{1018}{2072}=0.491.$$

Probability

Outcomes

SCO: In this course, students will be expected to

G6 **demonstrate an understanding of the difference between probability and odds**

$B7_3$ **calculate probabilities to solve problems**

Elaboration—Instructional Strategies/Suggestions

G6 Expressions of likelihood are often given as odds. For example, 50:1, expressed "fifty to one," is an expression of odds for a situation where the event is not very likely to happen. The use of odds makes it easier to deal with money exchanges that result from gambling. The likelihood of an event can be expressed in terms of the odds against that event, or the odds in favour. For example, if

$P(A)=\frac{2}{5}$, then odds against $A=\frac{P(\overline{A})}{P(A)}=\frac{\frac{3}{5}}{\frac{2}{5}}=\frac{3}{2}$ where $\overline{A}$ is the complement of A. The answer is expressed as 3:2, or "three to two." The corresponding odds in favour are 2:3.

For bets, the odds against an event represent the ratio of net profit to the amount bet. odds against event A = (net profit) : (amount bet)

Suppose a bet pays 50:1. If the odds aren't specified as being in favour or against, they are probably the odds against the event occurring. If a person were to win a bet with 50:1 odds, that person would make a profit of $50 for each $1 bet. The person would collect $51.

G6/$B7_3$ Suppose an electrical circuit has 50:1 odds against failure. What are the odds against two such separate and independent circuits both failing? The best way to solve this problem is to first convert the 50:1 odds to the corresponding probability of failure $\left(\frac{1}{51}\right)$. Use the multiplication rule: $\frac{1}{51}\times\frac{1}{51}=\frac{1}{2601}$. This gives the probability of both circuits failing and is equivalent to odds of 2600:1.

When performing calculations involving likelihoods, use probability values between 0 and 1, not odds. This is why more time is spent on probability even though odds seem to be heard more often.

Probability

Worthwhile Tasks for Instruction and/or Assessment

G6

Pencil and Paper

1) Ask students to complete the following conversions:
   a) If P(A) = 2/7, find the odds against event A occurring. [Answer: 5:2]
   b) Find the probability of event A if the odds against it are 9:4. [Answer: 4/13]
   c) If the odds against an event are 7:3, what are the odds against this event occurring in all of three separate and independent trials? [Answer: 973:27]
   d) In a fair game, all of the money lost by some players is won by others. For one fair game, a $2 bet nets a profit of $16. Find the odds against winning and find the probability of winning. [Answer: 8:1, 1/9]
   e) A standard roulette wheel has 38 different slots numbered 1 through 36 and 0, and 00. If you bet on any individual number, the casino gives you odds of 35:1. What would be fair odds if the casino did not have an advantage? [Answer: 37:1]
   f) The actual odds against winning when you bet on "odds" at roulette are 10:9. What is the probability of winning? [Answer: 9/19]

G6/B7$_3$

Performance

2) A slot machine has three drums, each of which contains different symbols, often fruit and bars. When the machine is activated all three drums roll but each stops after the others. On this machine the first drum stops first, then the second drum, then the third. If the drums stop at a winning combination of symbols, then coins will fall into a metal tray. The chance of this happening depends on the distribution of the symbols on the drums. The table below shows a typical distribution:

Symbol	**drum1**	**drum2**	**drum3**
cherry	6	6	1
lemon	3	1	6
plum	1	5	7
orange	4	5	2
banana	3	1	4
bar	2	3	1
double bar	3	1	1

   a) Ask students to complete the following table:

Winning Ouctome	**average frequency per**	**probability**	**payoff odds**
dbl bar, dbl bar, dbl bar	$3 \times 3 \times 1 = 3$	3/7500=	500 to 1
bar, bar, bar			300 to 1
plum, plum, bar	$1 \times 5 \times 1 = 5$	5/7500=	400 to 1
orange, orange, banana		20 to 1	
cherry, cherry, plum		3 to 1	
lemon, lemon, lemon		75 to 1	
cherry, cherry, lemon		5 to 1	
plum, lemon, cherry		2 to 1	

   b) How do you think the pay-off odds were determined?
   c) For each of the winning combinations above, calculate the average total dollars paid out per 7500 plays.
   d) If these are the only winning combinations, about how many times, on average, can a player expect to win per 7500 plays?

Suggested Resources

Nova Scotia. Department of Health, *Drawing the Line: A Resource for the Prevention of Problem Gambling*. Halifax: Communications Nova Scotia, 1997.

Probability

Outcomes

SCO: In this course, students will be expected to

G1 develop and apply simulations to solve problems

Elaboration—Instructional Strategies/Suggestions

G1 Simulation is a procedure developed for answering questions about real problems by running experiments that closely resemble the real situation.

Suppose the students want to find the probability that a family with 3 children contains exactly one girl. If students cannot compute the theoretical probability and do not have the time to locate three-child families for observation, the best plan might be to simulate the outcomes for three-child families. One way to accomplish this is to toss three coins to represent the three births. A head could represent the birth of a girl. Then, observing exactly one head in a toss of three coins would be similar, in terms of probability, to observing exactly one girl in a three-child family. Students could easily toss the three coins many times to estimate the probability of seeing exactly one head. The result gives them an estimate of the probability of seeing exactly one girl in a three-child family. This is a simple problem to simulate, but the idea is very useful in complex problems for which theoretical probabilities may be nearly impossible to obtain.

Students need work on connecting simulation results to the original problem. When choosing a simple device to model the key components in the problem, they have to be careful to choose a model that generates outcomes with probabilities to match those of the real situation. Students could use devices such as coins, dice, spinners, objects in a bag, and random numbers.

Students need to understand that the experimental probability approaches the theoretical probability as the number of trials increases. They should also realize that knowing the probability of an event gives them no predicting power as to what the outcome of the next trial will be. However, after enough trials, they should be able to predict with some confidence what the overall results will be.

When conducting simulations students should follow a process like the one outlined below (see next page for an actual class activity):

Step 1: State the problem clearly.
Step 2: Define the key components.
Step 3: State the underlying assumptions.
Step 4: Select a model to generate the outcomes for a key component.
Step 5: Define and conduct a trial.
Step 6: Record the observation of interest.
Step 7: Repeat steps 5 and 6 until 50 trials are reached.
Step 8: Summarize the information and draw conclusions.

Probability

Worthwhile Tasks for Instruction and/or Assessment

G1

Activity

1) Consider the following problem:

   Marie has not studied for her history exam. She knows none of the answers on the seven-question true-and-false section of the test. She decides to guess at all seven. Estimate the probability that Marie will guess the correct answers to four or more of the seven questions. Ask students to complete the following:

   a) What is it that you are being asked to do?
   b) To perform a simulation, what assumptions should you make?
   c) Describe the model you would choose to perform the simulation.
   d) Pretend that you are watching the simulation. Describe what you observe for the entire simulation.
   e) What conclusion do you think would be made?

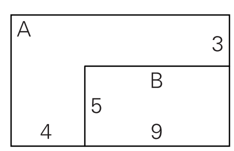
2) Suppose a stick, or a piece of raw spaghetti, has been broken at two random points. What is the probability that the three pieces will form a triangle? (pieces must touch end to end).

   a) Ask students to describe the process that might be used to estimate the answer using experimental probability.
   b) Instead, ask students to conduct a simulation. Assume the spaghetti is 100 units long. Generate two random numbers between 0 and 100 using each as a side of a triangle. Determine the length of the third side. Check to see if the numbers represent the lengths of the side of a triangle?
   c) Determine the answer.

Performance

3) Dale, a parachutist, jumps from an airplane and lands in a field. What are the chances that Dale will land in a particular numbered plot? Make a field grid using a normal sheet of graph paper divided into four equal areas.

   a) Model the situation by tossing a thumbtack onto the grid from a metre or more away. (If the tack bounces off the sheet, don't count it as a toss.) In your response consider several questions:
      i) Is there an equal chance to land in each plot?
      ii) How many times did Dale land in plot 1?
      iii) Compare what was found in the experiment with what you expected to find.
   b) Conduct the experiment again, but use a field divided into plots A and B to find the probability that Dale will land in Plot A.



   c) Perform a simulation to answer the same problem as in (b). Compare the results of the simulation with that of the experiment. Comment.

4) Perform simulations to solve the following problems:

   a) What is the probability that all five children in a family will be girls?

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

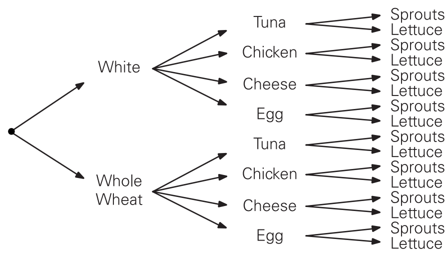
G4 **apply area diagrams and tree diagrams to interpret and determine probabilities of dependent and independent events**

$B7_3$ **calculate probabilities to solve problems**

$G3_3$ **demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events**

Elaboration—Instructional Strategies/Suggestions

G4/$G3_3$ A certain restaurant offers select-your-own sandwiches. That is, a person may select one item from each of the categories listed. It is important to be sure that all the possible outcomes are known and that they are equally likely. Only then can the theoretical probability of events be calculated. A tree diagram is one way to do this:

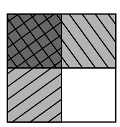


Bread	Filling	Extras
white	tuna	sprouts
whole-wheat	chicken	lettuce
	cheese	
	egg	

From the tree diagram students can really see that there are 16 equally likely outcomes They could use the fundamental counting principle to check their results $(2 \times 4 \times 2 = 16)$.

G4/$B7_3$/$G3_3$ Using an area model gives a pictorial representation of the analysis, which provides visual insights into the concepts of probability. Reliance on geometric skills allows the development of those concepts, which a lack of arithmetic skills would normally impede. Dividing a region in proportion to the appropriate probabilities appeals to students' intuitive understanding of probability. For example:

Rita has two dice, one red, one blue. Help her determine the probability of having the red die show an even number and the blue die an odd number. Using a square to represent one, Rita thinks she should shade $\frac{1}{2}$ of the square to represent the probability, the red die will show an even number. Students should be asked to explain why this makes sense. Have students complete the problem. They should then shade the upper half of the square to represent the blue die showing an odd number (three odd numbers of six possible numbers). The overlapped shaded region will indicate the probability of both events being true.



Ask the students using the same method, to find the probability that when Rita throws both dice, the red one shows a number less than five, and the blue one, a number greater than one.

The teacher might ask students to check their answers using tree diagrams and/or the fundamental counting principle $\left(\frac{4}{6} \times \frac{5}{6} = \frac{20}{36} = \frac{5}{9}\right)$.

Probability

Worthwhile Tasks for Instruction and/or Assessment

G4/B7$_3$/G3$_3$
Performance

1) Barb and Ann are having a contest to see who can hit a target first. Both Barb and Ann have a 50% chance of hitting their target on each shot. If Barb lets Ann go first each time, what is the probability that Ann wins?
2) A certain restaurant offers select-your-own desserts. That is, a person may select one item from each of the categories listed:

Ice Cream	**Sauce**	**Extras**
vanilla	chocolate	cherries
strawberry	caramel	peanuts
chocolate mint		

   a) Using a tree diagram, list all possible desserts that can be ordered.
   b) Would you expect the choices of a dessert to be equally likely for most customers?
   c) If the probability of selecting chocolate ice cream is 40%, and vanilla is 10%, chocolate sauce is 70%, and cherries 20%, describe the dessert with the highest probability of being selected.

3) A certain model car can be ordered with one of three engine sizes, with or without air conditioning, and with automatic or manual transmission.
   a) Show, by means of a tree diagram, all the possible ways this model car can be ordered.
   b) Suppose you want the car with the smallest engine, air conditioning, and manual transmission. A car agency tells you there is only one of the cars on hand. What is the probability that it has the features you want, if you assume the outcomes to be equally likely?

4) In a restaurant there are four kinds of soup, 12 entrees, six desserts, and three drinks. How many different four-course meals can a patron choose from? If 4 of the 12 entrees are chicken and two of the desserts involve cherries, what is the probability that someone will order wonton soup, a chicken dinner, a cherry dessert and milk?
5) Licence plates for cars often have three letters of the alphabet then three digits from 0 to 9. How many possible different licence plates can be produced? What is the probability of having the plate “CAR 000”?
6) A spinner is marked with an A or B as shown. Each round consists of either one or two spins. The player with the highest score wins. On each round player 1 spins first.
   If the spinner lands in the area marked A, player 1 scores a point, and this ends the round, and player 2 spins again to begin round 2. If on the first spin the spinner lands in the area marked B, then player 2 spins the spinner; player 2 scores 2 points if the spinner lands in B, and player 1 scores 1 point if it lands in A. Use the square grid to.

   a) Find P(A will score on a given round).
   b) Find P(B will score on a given round).

Suggested Resources

Newan, Claire et al. *Exploring Probability*. The Quantitative Literacy Series. Palo Alto, CA: Dale Seymour Publications, 1987.

Probability

Outcomes

SCO: In this course, students will be expected to

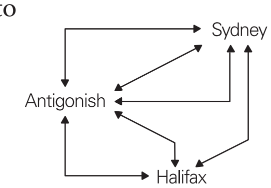
G7 **distinguish between situations that involve permutations and combinations**

G3$_3$ **demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events**

Elaboration—Instructional Strategies/Suggestions

G7/G3$_3$ Before describing different situations in terms of permutations and combinations, students need to have an opportunity to solve simple counting problems (see elaboration for G2/G3$_3$, p. 66). They may wish to organize their work into systematic lists and/or tree diagrams. As the number of choices increase, they will see the need for a way to count more efficiently. For example:

a) How many different routes can you take from Sydney to Halifax through Antigonish?
b) How many routes are there from Antigonish to either Halifax or Sydney?



Following this, the class might be split into two groups—one will do Problem A, the other Problem B. Students should present their solution to the class.

Problem A: Suppose there were three people, Adam, Marie, and Brian, standing in line at a banking machine. In how many different ways could they order themselves?

Problem B: The executive of the student council has five members. In how many ways can a committee of three people be formed?

Solutions might look like the following:

Problem A: Students might make a systematic list: A M B, A B M, M B A, M A B, B A M, B M A

Alternatively, using the fundamental counting principle, they might reason that there are initially 3 possibilities for the first person in line, then 2 possibilities for the second, and 1 possibility for the third. This would give $3 \times 2 \times 1 = 6$ ways for the line to be ordered.

Problem B: Making a systematic list, students might create committees such as ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, and CDE, a total of ten possible committees. Students will need to realize that within any committee the order of the three persons (say, A, B and C) is not important. Consequently, the kind of variations produced in the solution to Problem A do not provide different committees.

The essential difference between these two situations needs to be discussed and emphasized. Eventually, Problem A should be described as a permutation (order is important), Problem B as a combination (order not important).

Probability

Worthwhile Tasks for Instruction and/or Assessment

G7

Pencil and Paper

1) For each of the following, decide whether permutations or combinations are involved:
   a) the number of committees of 2 that can be formed from a group of 12 people
   b) the number of possible lineups for a baseball team that can be formed from 12 people (a baseball team consists of nine players, as follows: pitcher; catcher; first, second, and third basemen; shortstop; right, centre, and left-fielders)
   c) the number of five-letter licence plates that can be formed from 12 different letters
   d) the number of six subsets that can be formed from 12 different letters
   e) the number of five-man basketball teams that can be formed from 10 players
   f) the number of ordered triples that can be formed from 10 different numbers
   g) the number of ordered triples that can be formed from the numbers 1, 1, 1, 3, 3, 5, 5, 5, 5, and 4

2) As a promotion, a record store placed 12 tapes in one basket and 10 compact discs in another. Pierre was the one millionth customer and was allowed to select 4 tapes and 4 compact discs. To find how many selections that Pierre can make, does one use permutations or combinations? Explain.

G7/G3$_3$

3) The manager of a baseball team needs to decide the batting order for the season opener. In how many ways can the first four batters be arranged on the batting roster? Is this a permutation or combination question? Explain.

4) Three identical red balls (R) and two identical white balls (W) are placed in a box. How many ways are there of selecting the balls in the following order?

   RWRRW

5) Find the total number of arrangements of the letters of the word "SILK."

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

A6 **develop an understanding of factorial notation and apply it**

$G3_3$ **demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events**

G7 **distinguish between situations that involve combinations and permutations**

Elaboration—Instructional Strategies/Suggestions

A6/$G3_3$ As students refine their methods of counting, they are introduced to *n*! (*n* factorial) to represent the number of ways to arrange n distinct objects in a line. For example, the product rule can be used to find the number of possible arrangements for three people standing in a line. There are three people to choose from for the front of the line. For each of these choices, there are two people to choose for the second position in the line. For each of these choices, there is one person to choose from the end of the line. Therefore, there are six possible arrangements.

In another example, at a music festival, eight trumpet players competed in the Baroque class. After the judging, they were awarded 1st, 2nd, 3rd ... down to 8th place. In how many ways could their placements be awarded?

If all the trumpet players were given a standing, first, second, third, ... , eighth, then the total number of possible standings could be calculated by using reasoning like: There are eight people eligible for first, which leaves seven eligible for second, six people eligible for third ... leading to a calculation $8\times7\times6\times5\times4\times3\times2\times1$. This product can be written in a compact form as 8! and is read "eight factorial."

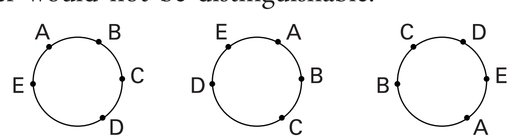
In general, $n! = n(n-1)(n-2)\ldots(3)(2)(1)$, where $n \in N$ and $0! = 1$.

$G3_3$/G7/A6 If there are only three prizes to be given, how many ways could placement be awarded?

Students should reason that eight people are eligible to come first, only seven are eligible to come second, and six are eligible to come third $\rightarrow 8\times7\times6 \rightarrow 336$. This could be worded "How many permutations are there of eight distinct objects taken three at a time?"

The symbol commonly used to represent this is ${}_8P_3$, or ${}_nP_r$ for the number of "*n*" objects taken "*r*" at a time. Students should notice that ${}_8P_3 = 8\cdot7\cdot6$. While some students may find it interesting to express ${}_8P_3$ as ${}_8P_3 = \frac{8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{5\cdot4\cdot3\cdot2\cdot1}$ or ${}_8P_3 = \frac{8!}{5!}$, students in this course are expected to evaluate permutations using lists, the fundamental counting principle, or the ${}_nP_r$ function on a calculator, rather than using a formula. Situations should generally be kept quite simple.

Students should note that when five people are to be arranged in a straight line there would be 5! or 120 ways to do this. However, if the same five people were to be arranged around a table in the order, say A, B, C, D, and E, their relative position to each other would not be distinguishable.



Thus, the total number of arrangements would be: $\frac{{}_5P_5}{5} = \frac{5!}{5} = 4! = 24$.

Probability

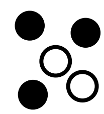
Worthwhile Tasks for Instruction and/or Assessment

$G3_3$/G7

Pencil and Paper

1) The town of Karsville, population 32 505, is designing its own licence plates for residents to place on the front of their automobiles.
   a) Ask students to use counting principles to determine the best of the following three options and to explain their choice:
      i) a licence made from using four single-digit numerals from 1 to 9
      ii) a licence made of three single-digit numerals from 1 to 9, and one letter from the alphabet
      iii) a licence made from three single-digit numerals from 1 to 9, and two letters from the alphabet.
   b) Ask students to select the best combination of single-digits from 1 to 9 and letters to suit the purposes of this town, and defend their selection.

2) The figure shows three black marbles and two white marbles. Suppose they are in a box. Without looking in the box, randomly choose two of the five marbles. How many ways are there to select two marbles that are the same colour? Each a different colour?



A6/G7

Pencil and Paper

3) There are five non-collinear points on a plane.
   a) How many segments can be formed using these five points as endpoints?
   b) If consecutive points are joined, a convex polygon is formed. How many diagonals does this polygon have?

4) A local pizza restaurant has a special on its 4-ingredient 20-cm pizza. If there are 15 ingredients from which to choose, how many different "specials" are possible?

5) a) Indicate which of the following are true (T) and which are false (F).
      i) $\frac{5!}{4!} = 5 \times 4$
      ii) $10 \times 9 \times 8 = \frac{10!}{7!}$
      iii) ${}_8P_2 = 56$
      iv) ${}_{100}P_4 = 100 \times 99 \times 98 \times 97$
   b) Create a story where each expression above would be used in the solution.

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

$G3_3$ demonstrate an understanding of the fundamental counting principle and apply it to calculate probabilities of dependent and independent events

G7 distinguish between situations that involve permutations and combinations

A6 develop an understanding of factorial notation and apply it

Elaboration—Instructional Strategies/Suggestions

$G3_3$/G7/A6 Refer back to the problem where there are five members on the executive of the students council. If these five were elected from a list of 10 candidates for executive position, the number of possible ways 10 people can be slotted into five positions could be found using permutations $\left({}_{10}P_5 = 30240\right)$ or the fundamental counting principle $(10\times 9\times 8\times 7\times 6 = 30240)$.

Now, from these five elected people a committee of three is struck. If the five people are represented by A, B, C, D, and E, then clearly a committee with A, B, and C is the same as a committee with C, A, and B. So, the order of the selection is not important, and the arrangement is called a combination. Therefore, since ABC, ACB, BAC, BCA, CAB, and CBA are all considered the same committee, they represent one combination. The number of permutations of A, B, and C is 3!. Thus, the number of committees from the original list of 10 candidates

$$= \frac{\text{number of ways the executive was chosen}}{3!}$$
$$= \frac{30240}{3!}$$
$$= 5040$$

That is ${}_{10}C_3 = \frac{{}_{10}P_3}{3!} = 5040$

and the number of committees from the five people on the executive selected would be ${}_5C_3 = \frac{{}_5P_3}{3!} = 10$.

Students should now apply combinations in a few simple problems where they would be expected to use basic counting techniques or technology that performs combination calculations, rather than memorize and apply a formula.

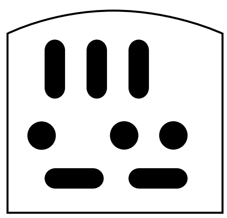
Probability

Worthwhile Tasks for Instruction and/or Assessment

G3$_3$/G7/A6

Performance

1) Mrs. Sandhurst has the following books on her reading list: *Great Expectations*, *Lord of the Flies*, *The Great Gatsby*, *Wuthering Heights*, *Fifth Business*, *The Stone Angel.*
   a) In how many ways can Mrs. Sandhurst arrange these books on her bookshelf?
   b) What is the probability that *Fifth Business* is next to *The Stone Angel* on her shelf?
   c) If a student borrows two of the books before she arranges them on the shelf, how many fewer arrangements does she have?
   d) If she arranges any four of the books on the shelf, how many fewer arrangements does she have?



2) On the pinball machine below, a ball falls from the top to the bottom. How many different paths can the ball follow assuming the ball falls without being pushed upwards?
3) In how many ways can
   a) a committee of four people be selected from eight people?
   b) a team of five players be selected from seven people?
   c) a study group of 4 people be selected from 10 students?

4) A local pizza restaurant has a special on its 5-ingredient 22-cm pizza. If there are 12 ingredients from which to choose, how many different "specials" are possible?

Suggested Resources

Probability

Outcomes

SCO: In this course, students will be expected to

$B8_3$ determine probabilities using permutations and combinations

$B7_3$ calculate probabilities to solve problems

$G3_3$ demonstrate an understanding of fundamental counting principle and apply it to calculate probabilities of dependent and independent events

Elaboration—Instructional Strategies/Suggestions

$B8_3/B7_3/G3_3$ One practical use of permutations and combinations is in the field of probability. For example, a deck of 52 cards is shuffled well. What is the probability that A, K, Q of spades will be dealt to you as the first three cards in that order?

The students would reason that since they want to see a particular three cards, in a fixed order, from 52 possible cards, they could use ${}_nP_r$ or ${}_{52}P_3$ to determine how many possible ways the first three cards could come up. (Alternately, they could use the fundamental counting principle.)

$${}_{52}P_3 = 132\ 600$$

Only one of those outcomes is favourable, so

$$P(A, K, Q \text{ in order}) = \frac{\#\ \text{favourable outcomes}}{\#\ \text{possible outcomes}} = \frac{1}{132600}$$

Combinations are sometimes used along with other counting techniques. For example, ask students to read the following problem and analyse Susan's solution and report their findings:

> Susan belongs to the school's seven member in-line skaters club. The club has been asked to select two girls and two boys to go to Toronto to take part in a skaters convention. What is the probability that Susan will be selected if there are three boys and four girls in the club? Ask students to respond to the solution given below:
>
> Susan's solution:
> – there are ${}_4C_2$ ways to select two girls
> – so, ${}_4C_2 = 6$ ways
> – there are ${}_3C_2$ ways to select two boys
> – so, ${}_3C_2 = 3$ ways
> – because there must be two girls and two boys, there are 6 + 3 = 9 ways of forming the group that is going
> – if the four people are selected at random, the probability that Susan is selected would be 1 in 9

(Comment: If there are 6 ways to chooose two girls and 3 ways to choose two boys, the fundamental counting principle gives $6 \times 3 = 18$ ways of making a group of four. Further, 9 of these potential groups contain Susan, so the probability of Susan being selected is 9 our of 18 or $\frac{1}{2}$. Note: A student might reason that the selection of the boys is irrelevant to Susan and that it is only the selection of the girls that matters. Since it is equally likely that Susan will be one of the two girls selected as one of the two girls left behind, the proability of her going is $\frac{1}{2}$.)

Probability

Worthwhile Tasks for Instruction and/or Assessment

$B8_3/G3_3$
Performance

1) There are 30 students in your mathematics class. Three students are selected to sit on a committee.

   a) How many committees can be formed if each member has equal status?
   b) How many committees can be formed if the first person chosen is the chairman, the second is the secretary, and the third is the treasurer?

2) Five identical red balls (R) and two identical white balls (W) are placed in a box.
   a) How many ways are there of selecting the balls in the following order?

RWRRWRR

$B8_3/B7_3/G3_3$
Performance

3) Nine people try out for nine positions on a baseball team. Each position is filled by selecting players at random. Assume all players are equally qualified for every position.
   a) In how many ways could the positions be filled?
   b) What is the probability that Duffy will be the pitcher?
   c) What is the probability that David, George or Duffy will be first baseman?
   d) What is the probability that David, George, or Duffy will be first baseman and Eleanor or Georgina will be pitcher?

4) The numbers on a raffle ticket contain three digits. The first digit cannot be zero.
   a) What is the probability of ticket number 917 winning the grand prize? What assumption did you make?
   b) What is the probability that a ticket with three as a second digit wins the grand prize?

5) Three black marbles and two white marbles are in a box. Without looking in the box, randomly choose two of the five marbles. If they are the same colour, player A wins, if they are a different colour, player B wins.
   a) What is the probability that player A wins? ___________
      What is the probability that player B wins? ___________
   b) Some combinations of black and white marbles will produce a fair game. Can you find a combination to make it a fair game? Can you find another?
   c) Create a simulation for this game.

Suggested Resources

Unit 4
Decision Making in Consumer Situations
(30 Hours)

Decision Making in Consumer Situations

Outcomes

SCO: In this course, students will be expected to

$B7_2$ estimate and calculate income and deductions

B4 use the calculator correctly and efficiently

C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions

C11 express and interpret constraints

F7 draw inferences from graphs, tables, and reports

C18 interpolate and extrapolate to solve problems

Elaboration—Instructional Strategies/Suggestions

$B7_2$/B4 Students should be able to estimate and calculate their weekly, bi-weekly, or monthly gross wages given the hourly rate of pay or the annual income. For example, Mary is earning \$12.50 an hour and works 37 hours a week. She might estimate her weekly income by multiplying 40 by 12, getting approximately \$480/week. Kate, on the other hand, is working part-time at a movie theatre at \$5.50 an hour. On the average, she works four four-hour shifts per week and her pay is deposited in her account every second Thursday. Kate should understand that her account balance will increase about \$160 every two weeks. Students should investigate various methods of calculating regular pay, including overtime (time-and-a-half, double time, etc.) salary, commissions, and piece work (e.g., textile industry, harvesting crops), and the approximate amounts that will be deducted.

❐ Suppose Nate is a car salesperson whose earnings are based on his sales. For example, Nate will earn 3% of his sales between \$0 and \$10 000. Once he sells more than \$10 000, his income will be \$300 + 2.5% of sales above \$10 000 and up to \$15 000. If he sells more than \$15 000, he makes \$425 + 1.5% of sales above \$15 000. Find his earnings if he sells cars amounting to \$18 500.

C8/C11/F7/B4 Students should analyse the situation described above using tables, graphs, and equations. Students might organize the above information into a table with the following headings: sales range, commission rate, earnings. They might complete the table to determine the possible weekly earnings (minima and maxima) for Nate. The amount of earning would be expressed as minimum and maximum earnings and should be recorded as inequalities, i.e., earning: $\$0 \leq E \leq \$300$.

Students should plot earnings versus sales using values calculated in the table. They should discuss dependent and independent variables and appropriate values for domains and ranges. Students should remember from Year 10 that when it is said that the graph represents earnings versus sales that earnings is the dependent variable and the independent variable is sales. From the table or graph, students should be able to describe in words, the relationship between earnings and sales. They should also be able to calculate and interpret a slope and come up with an equation that represents the relationship between commission earnings and sales.

C18 Students should use the graph and equation from the above example to interpolate and extrapolate.

Decision Making in Consumer Situations

Worthwhile Tasks for Instruction and/or Assessment

$B7_2$/C8/F7/C18

Performance

1) John and Mark both work as telemarketers for Sprint and ATT respectively. John is paid $20 for each new customer he signs up and Mark is paid $130.00 per week plus $12 for each new customer.

   a) State one advantage and one disadvantage for each of these ways of being paid.

   b) This graph gives a visual picture of their income.

      i) State the significance of points A (0, 0) and B (5, 190).

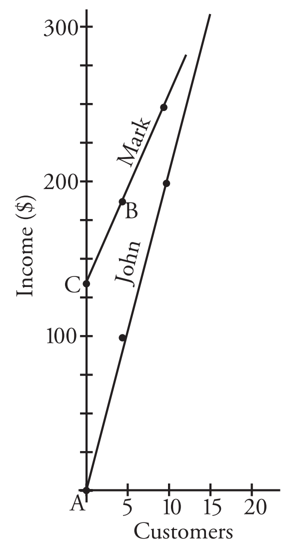
      ii) State the significance of point C. From the graph, give the coordinates of C.

      iii) From the graph, determine who earns more after each signs up 25 new customers in the first week.

      iv) Determine the equations that represent each person's total earnings per week. Use appropriate variables.

      v) What does the slope represent in each equation?

      vi) Use your equations to verify that your answer in part iii is correct.



C8/$B7_2$/F17/C18/C11/B4

2) a) Bobby-Lou rents bicycles for a non-refundable daily deposit plus a certain amount each hour. This equation represents her earnings *E*= 10+1.5h

   b) Hilda has a similar business. Her income is represented by this equation *E*=20+0.5h

   c) Graph each of these equations on the same axes. Label the axes.

   d) What is the significance of the point of intersection?

   e) Why might a person rent from Hilda, rather than Bobby-Lou?

   f) Select a point in the region above the first graph but below the second. What does this point represent?

   g) If I wanted to spend $300 on bicycling over several days, from which person should I rent?

Suggested Resources

Decision Making in Consumer Situations

Outcomes

SCO: In this course, students will be expected to

$B7_2$ estimate and calculate income and deductions

B4 use the calculator correctly and efficiently

C8 demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions

F7 draw inferences from graphs, tables and, reports

C18 interpolate and extrapolate to solve problems

Elaboration—Instructional Strategies/Suggestions

$B7_2$/B4 Students need to be aware of the deductions that their employer will make from their cheques and by how much this will reduce their gross income. Students should be aware of what is being deducted (income tax, CPP, EI, etc.) and approximately how much.

❒ Elaine earns $60 000 a year and needs to decide whether to be paid weekly, bi-weekly, twice a month, or monthly. What budgetary factors might she consider to help her make her decision (monthly payment-schedule bills, loans, mortgages, etc.)?

Elaine needs to examine how various deductions will be made from her earnings. Students should examine Employment Insurance (EI) payments. How are they calculated? at what rate? Often wage earners see only the deduction and wonder at what rate it is being deducted.

C8/F7/C18 Similarly, Canada Pension Plan (CPP) deductions are often presented in tables showing how much will be deducted for certain ranges of income. Students might examine such a table, sketch a graph to represent CPP deductions versus annual income, and use the graph to express the relationship as an equation. They might also examine the graphs for CPP deductions made based on weekly or bi-weekly earnings versus the equation obtained on deductions based on yearly earnings.

Students should examine federal and provincial tax deduction tables and note that provincial amounts are different and why that might be. Income tax deductions are a complex topic. Most students will be interested in actual deductions for different ranges of income, as well as different methods of payments. This type of information will help students make decisions when trying to budget their net incomes. Students should be able to determine the amount that will be deducted from Elaine's (above) earnings yearly, monthly, bi-weekly, etc., to see if there is a difference or not as to the method of payment. They should also examine situations where earnings are more modest.

Decision Making in Consumer Situations

Worthwhile Tasks for Instruction and/or Assessment

$B7_2$/B4

Performance

1) Sylvia earns $12.87 per hour and works 40 hours per week. She has two weeks paid vacation per year. What would be her gross pay if paid weekly? biweekly? monthly? Is the monthly payment four times the weekly payment? Why or why not?

$B7_2$/B4/F7

2) In 1999, Marcia paid the maximum contribution of $1329.90 to CPP. Will she also pay $936.00, the maximum contribution to EI? Explain.

$B7_2$/C8

3) The government is trying to decide if they should keep the present way of taxing people or switch to a flat tax rate. Find an occupation that would prefer to keep the present system and one that would prefer the flat tax rate and explain why each feels the way they do. Use examples to strengthen your position.

4) From the table of CPP contributions and federal and provincial tax deductions,

   a) determine the net pay for each of the following:

      i) $6500 annual income
      ii) $8500 annual income
      iii) $1600/month income
      iv) $950/month income
      v) $36 000 annual income

   b) Find the rate of deduction:
      i) CPP deduction for an annual income of $32 000
      ii) Tax deduction for monthly income of $1050/month; $3600/month

Suggested Resources

Decision Making in Consumer Situations

Outcomes

SCO: In this course, students will be expected to

$B8_2$ solve problems involving budgets

B4 use the calculator correctly and efficiently

C7 develop and apply informal decision-making flowcharts

Elaboration—Instructional Strategies/Suggestions

$B8_2$/B4/C7 This unit might be structured around a series of personal finance case studies (which could be examined on an individual student, small group, or whole class basis). The case studies characterize young people trying to make their way in the real world. Students will analyse their financial situations, draw up budgets, and make decisions about earnings and balancing budgets. For example, case study 1 might be a single parent with two children, earning minimum wage. The single parent is attending classes at a community college to become a massage therapist. Her two children are pre school-aged and require baby sitting. Case study 2 might be a single male, living at home, just finishing high school, working part time at a gas bar/service centre earning $7.25/hour. He's thinking of moving out on his own and purchasing a second-hand vehicle. Case study 3 might be a young married couple, living in a flat downtown. They have one pre schooler; the mother works as a mechanic, and the father is a receptionist. (The) student(s) would brainstorm to identify all elements of each case study, key process steps, decision points, etc. They might then develop an informal flowchart (or other type of map) that outlines steps in the process and what has to be considered at each step. Finally, they would apply the steps to arrive at appropriate possible decisions with respect to each case study. One of the first tasks to perform would be to determine a budget to ensure that they have enough money for the basic expenses of life and hopefully some that can be put away towards investments and emergencies.

In preparing budgets, groups will have to make decisions about the percentages of income to be allocated to categories like entertainment, clothing, food, savings, etc. To begin, they will need to determine and calculate their monthly fixed expenses (rent, telephone, electricity, etc.). To fully explore this may require the use of catalogues, advertising flyers, trips to grocery stores, and examination of telephone bills, electricity bills, etc. They should plan their budgets based on percentages allotted to (the above) various categories. These percentages can be decided upon by each group, perhaps with advice from parents and/or visits to the school by employees from financial institutions.

Students should brainstorm strategies for how they will organize the details and record their budgets. For example, organized lists or charts (and what would be the headings), spreadsheets, diagrams, graphs, etc. Students may wish, and should be encouraged, to use spreadsheets and appropriate software on which to record and keep track of their expenses and incomes, to update their budgets, and to display their budgets with bar and circle graphs. If using spreadsheets, they should plan their formula entries to simplify the calculations.

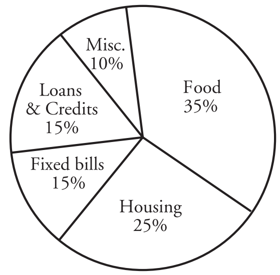
As the unit progresses, and changes in incomes and expenses are experienced, students will have to adjust their budgets and keep these current. For example, students will be asked to consider the purchase of a car (either new or used) and consider how the monthly payment will affect their budgets, and ultimately how their budgets may determine their purchases.

Decision Making in Consumer Situations

Worthwhile Tasks for Instruction and/or Assessment

$B8_2$/B4

Performance



1) The pie chart represents the budget of a family whose monthly income after deductions is $2850.00.
   a) What is the family's monthly mortgage payment?
   b) How much do they spend on food? Why do you think they spend this much? What advice might you give to reduce this amount?
   c) How much is this family paying for bills and loans?
   d) What categories might be included under miscellaneous?

C7

2) Outline the steps, using an informal flowchart, that you think are necessary if you have never created a budget, but now have to.

$B8_2$

3) When setting up a spreadsheet to keep track of your monthly income and expenses,
   a) What headings might you use at the top of the columns? Explain.
   b) What formulas might you build into your spreadsheet? Explain.
4) A young family with one child needs to buy a car. The family earns $1600 per month after deductions. Their rent and food takes $1000 per month, and they use some of the rest to pay for small credit card monthly payments and a monthly payment on an educational loan. What detailed advice might you give this family regarding the purchase of a car?
5) Emilie is saving for her graduation ring, which costs $255.00 plus 15% tax. She earns $86.00 per week at Subway. She must put $20.00 in her savings account each week. She decides to put one-third of the remainder towards her ring. How long will it take her to save for the ring?
6) Do you think that prices shown on goods for sale should have the tax included or should the tax be added at the checkout? Justify your answer.
7) Michelle was shopping in Montreal for shoes. She found a pair that cost $100.00. She mentally multiplied $100 by 1.145 and gave the sales clerk $114.50. The sales clerk said that the total price was $115.03. What was wrong with Michelle's reasoning?

Suggested Resources

Decision Making in Consumer Situations

Outcomes

SCO: In this course, students will be expected to

B9 analyse situations and make decisions involving the financing of purchases

C26 demonstrate an understanding of the difference between simple and compound interest

Elaboration—Instructional Strategies/Suggestions

B9 All of the case studies described in the previous page might experience an event that will require the replacement, or first purchase, of an appliance or TV and the use of credit to finance the purchase. Students should understand that different lending institutions offer availability to money at different costs to the consumer. Students should become aware that stores offer credit opportunities, some with their own credit card systems, others by having customers pay off the purchase price plus interest over a period of time. Usually store credit has high interest rates. Students should explore the effect of down payments, length of borrowing period, varying interest rates on monthly payments, and total cost over time.

Credit card companies are another option. Usually, the interest rate is lower with them. Students should study how they calculate the interest rate and monthly payments; when interest begins to accumulate on new additions to the card; and the overall cost compared to store credit.

B9/C26 Bank loans talk about principal amounts, amortization opportunities, interest rates (simple and compound). Students might want to work through a partial payment schedule to see the effect of partial payments on the principal and discover the true interest rate and total cost of the loan.

C26 Looking at investment opportunities such as guaranteed investment certificates (GICs) students can compare simple interest earnings ($I = Prt$) with earning obtained after compounding the interest. The formula for compound interest is not an outcome. Students should work on iterative process to see the difference between it and simple interest. More work and development of the formula will take place in a subsequent course.

Students should be able to explain the differences with respect to the math involved in the different types of credit and the costs to them, but need not have to remember the names associated with each type of credit.

Students should have to calculate the monthly interest and service charges on an unpaid credit card balance. They should compare credit card purchases with short-term loans. They should calculate the monthly payments for a loan, using formulas ($I = Prt$ and $A = P + I$), tables, and with appropriate technology.

Students should have to make a decision to use either a credit card or a short-term loan to make a purchase and the effect this has on their budgets and their life-styles.

Decision Making in Consumer Situations

Worthwhile Tasks for Instruction and/or Assessment

B9/C26

1) Mary and David bought their first dining room suite from McDonald's Used Furniture. Consider the following options and decide which method of payment would be the most economical.
   a) $100 down and 12 monthly payments of $125.00
   b) a bank loan for $1200 at 14% annual interest compounded monthly
   c) a credit card with a 19.9% rate and a minimum monthly payment of 4% of the unpaid balance

   Show all of your calculations to justify your decision.
2) The following has occurred on a credit card over a two-month period. Calculate the interest charge (1.5% of unpaid balance) and minimum monthly payment (3% of the unpaid balance) for the end of April.
   March: beginning balance of $37.05, credit purchases of $51.60 and 427.75; interest calculated for March and minimum payment made.
   April: purchases of $31.50, $10.60, and $17.25; interest calculated and minimum payment asked for.
3) President's Choice Financial is offering 4.4% interest, compounded monthly, on amounts in savings accounts over $10 000. Set up a spreadsheet or chart and determine how much interest you would have earned after 1 year for a $10 000 investment; After 10 years?
4) Ron and Ann have just won the $100 000 tag lottery. They have a young daughter who will be two in August. Ron suggests that they buy a GIC at eight and one-half percent with the $100 000 over a 16-year period. If they cash in the GIC after 16 years will they have enough interest to pay for their daughter's first year at university? First-year tuition is expected to grow 175% from the present $3500 in 16 years.

C26

Journal

5) Explain how the method of calculating the consumption tax in PEI is similar to the idea of compound interest. (The combination of sales tax and goods and services tax.)
6) Explain why some people put their savings in an account that pays simple interest of 2% per year when they could put it in a GIC that pays 4.6% per year. In this case, how much more interest would a $1500.00 earn after one year?

Suggested Resources

Decision Making in Consumer Situations

Outcomes

SCO: In this course, students will be expected to

B10 **analyse situations and make decisions involving the cost of transportation**

B9 **analyse situations and make decisions involving the financing of purchases**

C7 **develop and apply informal decision-making flowcharts**

Elaboration—Instructional Strategies/Suggestions

B10/B9 In their case study (described on p. 90), students will have to explore the costs of owning a car, including all the hidden costs (maintenance, depreciation, interest on loans, gasoline ...) to determine if their budgets will allow for a purchase. They need to consider the costs in other categories in their budget to help them decide if car ownership is feasible. They must think about ownership cost (financing, tax, registration, licence, depreciation, etc.) and operating costs (insurance, gas/oil, tires, wash, and maintenance). Students need to consider how other areas of budgeting may need to be adjusted to help with this new cost.

They should explore the cost differences between new and used car purchases and buying versus car leasing.

As alternative transportation, students should investigate the costs of bus, train, and air transportation, and road and bridge tolls. For example, if living in a city, would it be cheaper not to have a car, but rent a car when needed, take taxis or buses, etc.?

B9/C7 Teachers and students should work toward developing how to organize and lay out information (informal flow charting, mind maps, concept maps, webs) so that students can apply these strategies when making financial decisions.

Decision Making in Consumer Situations

Worthwhile Tasks for Instruction and/or Assessment

B10/B9/C7

Performance

1) Below are three options to lease a 2000 Neon. Determine the total cost of the lease in each case.

Time (months)	Monthly payment	Down payment
48	219	2840
48	250	1420
48	281	0

   Check a newspaper to determine what other charges might affect the total lease price.

2) A Suzuki wagon gets 8.3L/100 km gas mileage. Using a current price for gas, determine the cost of a return trip between Sydney and Halifax if the distance is 880 km. Mary can go by bus for $50.00. What do you think she should do?

3) You have decided to buy a car and travel from your home to the community college and take three paying customers. You know the following:
   - the weekly fixed costs total $55.00
   - the weekly variable costs total 8.7¢/km

   Set up an equation that represents the total weekly costs for the distance traveled. If you live 60 km from the community college, how much should you charge each person so that the four of you equally share the costs?

4) Samuel was discussing how a car depreciates in value. He knew that a car that was worth $12 000 now, will be worth $6 000 in three years. Do you agree or disagree with his idea that it will be worth $9 000 after 18 months?

Journal

5) Arthur has asked your advice on the pros and cons of investing in the purchase of a $1 200 half-ton truck or a $ 2 200 second-hand car. Write a response to Arthur. By the way, he has a part-time job that pays $7.50/hour. He works 30 hours a week and pays $35.00 a week to his mom and dad for board.

Suggested Resources

Appendix A: Assessing and Evaluating Student Learning

Assessing and Evaluating Student Learning

In recent years there have been calls for change in the practices used to assess and evaluate students' progress. Many factors have set the demands for change in motion, including the following:

- ***new expectations for mathematics education as outlined in*** **Curriculum and Evaluation Standards for School Mathematics** (NCTM 1989)

The *Curriculum Standards* provide educators with specific information about what students should be able to do in mathematics. These expectations go far beyond learning a list of mathematical facts; instead, they emphasize such competencies as creative and critical thinking, problem solving, working collaboratively, and the ability to manage one's own learning. Students are expected to be able to communicate mathematically, to solve and create problems, to use concepts to solve real-world applications, to integrate mathematics across disciplines, and to connect strands of mathematics. For the most part, assessments used in the past have not addressed these expectations. New approaches to assessment are needed if we are to address the expectations set out in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989).

- *understanding the bonds linking teaching, learning, and assessment*

Much of our understanding of learning has been based on a theory that viewed learning as the accumulation of discrete skills. Cognitive views of learning call for an active, constructive approach in which learners gain understanding by building their own knowledge and developing connections between facts and concepts. Problem solving and reasoning become the emphases rather than the acquisition of isolated facts. Conventional testing, which includes multiple choice or having students answer questions to determine if they can recall the type of question and the procedure to be used, provides a window into only one aspect of what a student has learned. Assessments that require students to solve problems, demonstrate skills, create products, and create portfolios of work reveal more about the student's understanding and reasoning of mathematics. If students are expected to develop reasoning and problem-solving competencies, then teaching must reflect such, and in turn, assessment must reflect what is valued in teaching and learning. Feedback from assessment directly affects learning. The development of problem-solving, and higher, order thinking skills will become a realization only if assessment practices are in alignment with these expectations.

- ***limitations of the traditional methods used to determine student achievement***

Do traditional methods of assessment provide the student with information on how to improve performance? We need to develop methods of assessment that provide us with accurate information about students' academic achievement and information to guide teachers in decision making to improve both learning and teaching.

What Is Assessment?

Assessment is the systematic process of gathering information on student learning. Assessment allows teachers to communicate to students what is really valued—what is worth learning, how it should be learned, what elements of quality are considered most important, and how well students are expected to perform. In order for teachers to assess student learning in a mathematics curriculum that emphasizes applications and problem solving, they need to employ strategies that recognize the reasoning involved in the process as well as in the product. *Assessment Standards for School Mathematics* (NCTM 1995, p. 3) describes assessment practices that enable teachers to gather evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes.

Assessment can be informal or formal. Informal assessment occurs while instruction is occurring. It is a mind-set, a daily activity that helps the teacher answer the question, Is what is taught being learned? Its primary purpose is to collect information about the instructional needs of students so that the teacher can make decisions to improve instructional strategies. For many teachers, the strategy of making annotated comments about a student's work is part of informal assessment. Assessment must do more than determine a score for the student. It should do more than portray a level of performance. It should direct teachers' communication and actions. Assessment must anticipate subsequent action.

Formal assessment requires the organization of an assessment event. In the past, mathematics teachers may have restricted these events to quizzes, tests, or exams. As the outcomes for mathematics education broaden, it becomes more obvious that these assessment methods become more limited. Some educators would argue that informal assessment provides better quality information because it is in a context that can be put to immediate use.

Why Should We Assess Student Learning?

We should assess student learning in order to

- improve instruction by identifying successful instructional strategies
- identify and address specific sources of the students' misunderstandings
- inform the students about their strengths in skills, knowledge, and learning strategies
- inform parents of their child's progress so that they can provide more effective support
- determine the level of achievement for each outcome

As an integral and ongoing part of the learning process, assessment must give each student optimal opportunity to demonstrate what he/

she knows and is able to do. It is essential, therefore, that teachers develop a repertoire of assessment strategies.

Assessment Strategies

Some assessment strategies that teachers may employ include the following.

Documenting classroom behaviours

In the past teachers have generally made observations of students' persistence, systematic working, organization, accuracy, conjecturing, modeling, creativity, and ability to communicate ideas, but often failed to document them. Certainly the ability to manage the documentation played a major part. Recording information signals to the student those behaviours that are truly valued. Teachers should focus on recording only significant events, which are those that represent a typical student's behaviour or a situation where the student demonstrates new understanding or a lack of understanding. Using a class list, teachers can expect to record comments on approximately four students per class. The use of an annotated class list allows the teacher to recognize where students are having difficulties and identify students who may be spectators in the classroom.

Using a portfolio or student journal

Having students assemble on a regular basis responses to various types of tasks is part of an effective assessment scheme. Responding to open-ended questions allows students to explore the bounds and the structure of mathematical categories. As an example, students are given a triangle in which they know two sides or an angle and a side and they are asked to find out everything they know about the triangle. This is preferable to asking students to find a particular side, because it is less prescriptive and allows students to explore the problem in many different ways and gives them the opportunity to use many different procedures and skills. Students should be monitoring their own learning by being asked to reflect and write about questions such as the following:

- What is the most interesting thing you learned in mathematics class this week?
- What do you find difficult to understand?
- How could the teacher improve mathematics instruction?
- Can you identify how the mathematics we are now studying is connected to the real world?

In the portfolio or in a journal, teachers can observe the development of the students' understanding and progress as a problem solver. Students should be doing problems that require varying lengths of time and represent both individual and group effort. What is most important is that teachers discuss with their peers what items are to be part of a meaningful portfolio, and that students also have some input into the assembling of a portfolio.

Projects and investigative reports

Students will have opportunities to do projects at various times throughout the year. For example, they may conduct a survey and do a statistical report, they may do a project by reporting on the contribution of a mathematician, or the project may involve building a complex three-dimensional shape or a set of three-dimensional shapes which relate to each other in some way. Students should also be given investigations in which they learn new mathematical concepts on their own. Excellent materials can be obtained from the National Council of Teachers of Mathematics, including the *Student Math Notes.* (These news bulletins can be downloaded from the Internet.)

Written tests, quizzes, and exams

Some critics allege that written tests are limited to assessing a student's ability to recall and replicate mathematical facts and procedures. Some educators would argue that asking students to solve contrived applications, usually within time limits, provides us with little knowledge of the students' understanding of mathematics.

How might we improve the use of written tests?

- Our challenge is to improve the nature of the questions being asked, so that we are gaining information about the students' understanding and comprehension.
- Tests must be designed so that questions being asked reflect the expectations of the outcomes being addressed.
- One way to do this is to have students construct assessment items for the test. Allowing students to contribute to the test permits them to reflect on what they were learning, and it is a most effective revision strategy.
- Teachers should reflect on the quality of the test being given to students. Are students being asked to evaluate, analyse, and synthesize information, or are they simply being asked to recall isolated facts from memory? Teachers should develop a table of specifications when planning their tests.
- In assessing students, teachers have a professional obligation to ensure that the assessment reflects those skills and behaviours that are truly valued. Good assessment goes hand-in-hand with good instruction and together they promote student achievement.

Appendix B:
SCOs for Grades 9 and 10

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers, and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, number system characteristics (e.g., density), etc.

By the end of grade 9, students will be expected to

A1 solve problems involving square root and principal square root

A2 graph, and write in symbols and in words, the solution set for equations and inequalities involving integers and other real numbers

A3 demonstrate an understanding of the meaning and uses of irrational numbers

A4 demonstrate an understanding of the inter-relationships of subsets of real numbers

A5 compare and order real numbers

A6 represent problem situations using matrices

By the end of grade 10, students will be expected to

A1 relate sets of numbers to solutions of inequalities

A2 analyse graphs or charts of situations to derive specific information

A3 demonstrate an understanding of the role of irrational numbers in applications

A4 approximate square roots

A5 demonstrate an understanding of the zero product property and its relationship to solving equations by factoring

A6 apply properties of numbers when operating upon expressions and equations

A7 demonstrate and apply an understanding of discrete and continuous number systems

A8 demonstrate an understanding of and apply properties to operations involving square roots

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 9, students will be expected to

B1 model, solve, and create problems involving real numbers

B2 add, subtract, multiply, and divide rational numbers in fractional and decimal forms, using the most appropriate method

B3 apply the order of operations in rational number computations

B4 demonstrate an understanding of, and apply the exponent laws, for integral exponents

B5 model, solve, and create problems involving numbers expressed in scientific notation

B6 judge the reasonableness of results in problem situations involving square roots, rational numbers, and numbers written in scientific notation

B7 model, solve, and create problems involving the matrix operations of addition, subtraction, and scalar multiplication

B8 add and subtract polynomial expressions symbolically to solve problems

B9 factor algebraic expressions with common monomial factors concretely, pictorially, and symbolically

B10 recognize that the dimensions of a rectangular model of a polynomial are its factors

B11 find products of two monomials, a monomial and a polynomial, and two binomials, concretely, pictorially, and symbolically

By the end of grade 10, students will be expected to

B1 model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations

B2 develop algorithms and perform operations on irrational numbers

B3 use concrete materials, pictorial representations, and algebraic symbolism to perform operations on polynomials

B4 identify and calculate the maximum and/or minimum values in a linear programming model

B5 develop, analyse, and apply procedures for matrix multiplication

B6 solve network problems using matrices

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 9, students will be expected to

B12 find quotients of polynomials with monomial divisors

B13 evaluate polynomial expressions

B14 demonstrate an understanding of the applicability of commutative, associative, distributive, identity, and inverse properties to operations involving algebraic expressions

B15 select and use appropriate strategies in problem situations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 9, students will be expected to

C1 represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

C2 interpret graphs that represent linear and non - linear data

C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

C4 determine the equations of lines by obtaining their slopes and y-intercepts from graphs, and sketch graphs of equations using y-intercepts and slopes

C5 explain the connections among different representations of patterns and relationships

C6 solve single-variable equations algebraically, and verify the solutions

C7 solve first-degree single-variable inequalities algebraically, verify the solutions, and display them on number lines

C8 solve, and create problems involving linear equations and inequalities

By the end of grade 10, students will be expected to

C1 express problems in terms of equations and vice versa

C2 model real-world phenomena with linear, quadratic, exponential, and power equations, and linear inequalities

C3 gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, and domain and range

C4 create and analyse scatter plots using appropriate technology

C5 sketch graphs from words, tables, and collected data

C6 apply linear programming to find optimal solutions to real-world problems

C7 model real-world situations with networks and matrices

C8 identify, generalize, and apply patterns

C9 construct and analyse graphs and tables relating two variables

C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

C11 write an inequality to describe its graph

C12 express and interpret constraints using inequalities

C13 determine the slope and y-intercept of a line from a table of values or a graph

C14 determine the equation of a line using the slope and y-intercept

C15 develop and apply strategies for solving problems

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally. (continued)

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 10, students will be expected to

C16 interpret solutions to equations based on context

C17 solve problems using graphing technology

C18 investigate and find the solution to a problem by graphing two linear equations with and without technology

C19 solve systems of linear equations using substitution and graphing methods

C20 evaluate and interpret non-linear equations using graphing technology

C21 explore and apply functional relationships notation, both formally and informally

C22 analyse and describe transformations of quadratic functions and apply them to absolute value functions

C23 express transformations algebraically and with mapping rules

C24 rearrange equations

C25 solve equations using graphs

C26 solve quadratic equations by factoring

C27 solve linear and simple radical, exponential, and absolute value equations and linear inequalities

C28 explore and describe the dynamics of change depicted in tables and graphs

C29 investigate, and make and test conjectures concerning, the steepness and direction of a line

C30 compare regression models of linear and non-linear functions

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally. (continued)

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 10, students will be expected to

C31 graph equations and inequalities and analyse graphs both with and without graphing technology

C32 determine if a graph is linear by plotting points in a given situation

C33 graph by constructing a table of values, by using graphing technology, and when appropriate, by the slope y-intercept method

C34 investigate and make and test conjectures about the solution to equations and inequalities using graphing technology

C35 expand and factor polynomial expressions using perimeter and area models

C36 explore, determine, and apply relationships between perimeter and area, surface area, and volume

C37 represent network problems using matrices and vice versa

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) and/or procedures (e.g., proportions) to determine measurements indirectly.

By the end of grade 9, students will be expected to

D1 solve indirect measurement problems by connecting rates and slopes

D2 solve measurement problems involving conversion among SI units

D3 relate the volumes of pyramids and cones to the volumes of corresponding prisms and cylinders

D4 estimate, measure, and calculate dimensions, volumes, and surface areas of pyramids, cones, and spheres in problem situations

D5 demonstrate an understanding of and apply proportions within similar triangles

By the end of grade 10, students will be expected to

D1 determine and apply formulas for perimeter, area, surface area, and volume

D2 apply the properties of similar triangles

D3 relate the trigonometric functions to the ratios in similar right triangles

D4 use calculators to find trigonometric values of angles and angles when trigonometric values are known

D5 apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation

D6 solve problems involving measurement using bearings and vectors

D7 determine the accuracy and precision of a measurement

D8 solve problems involving similar triangles and right triangles

D9 determine whether differences in repeated measurements are significant or accidental

D10 determine and apply relationships between the perimeters and areas of similar figures, and between the surface areas and volumes of similar solids

D11 explore, discover, and apply properties of maximum areas and volumes

D12 solve problems using the trigonometric ratios

D13 demonstrate an understanding of the concepts of surface area and volume

D14 apply the Pythagorean Theorem

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as (i) the direction, orientation, and perspectives of objects in space; (ii) the relative shapes and sizes of figures and objects; and (iii) how a change in shape relates to a change in size. Geometric concepts, properties, and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

By the end of grade 9, students will be expected to

E1 investigate, and demonstrate an understanding of, the minimum sufficient conditions to produce unique triangles

E2 investigate, and demonstrate an understanding of, the properties of, and the minimum sufficient conditions to guarantee, congruent triangles

E3 make informal deductions, using congruent triangle and angle properties

E4 demonstrate an understanding of and apply the properties of similar triangles

E5 relate congruence and similarity of triangles

E6 use mapping notation to represent transformations of geometric figures, and interpret such notations

E7 analyse and represent combinations of transformations, using mapping notation

E8 investigate, determine, and apply the effects of transformations of geometric figures, on congruence, similarity, and orientation

By the end of grade 10, students will be expected to

E1 explore properties of, and make and test conjectures about 2- and 3-dimensional figures

E2 solve problems involving polygons and polyhedra

E3 construct and apply altitudes, medians, angle bisectors, and perpendicular bisectors to examine their intersection points

E4 apply transformations when solving problems

E5 use transformations to draw graphs

E6 represent network problems as digraphs

E7 demonstrate an understanding of, and write a proof for, the Pythagorean Theorem

E8 use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures

E9 use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid

GCO F: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display and analysis of data involves (i) attention to sampling procedures and issues, (ii) recording and organizing collected data, (iii) choosing and creating appropriate data displays, (iv) analysing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean), and (v) formulating and evaluating statistical arguments.

By the end of grade 9, students will be expected to

F1 determine characteristics of possible relationships shown in scatter plots

F2 sketch lines of best fit and determine their equations

F3 sketch curves of best fit for relationships that appear to be non-linear

F4 select, defend, and use the most appropriate methods for displaying data

F5 draw inferences and make predictions based on data analysis and data displays

F6 demonstrate an understanding of the role of data management in society

F7 evaluate arguments and interpretations that are based on data analysis

By the end of grade 10, students will be expected to

F1 design and conduct experiments using statistical methods and scientific inquiry

F2 demonstrate an understanding of the concerns and issues that pertain to the collection of data

F3 construct various displays of data

F4 calculate various statistics using appropriate technology, analyse and interpret data displays, and describe relationships

F5 analyse statistical summaries, draw conclusions, and communicate results about distributions of data

F6 solve problems by modeling real-world phenomena

F7 explore non-linear data using power and exponential regression to find a curve of best fit

F8 determine and apply the line of best fit using the least squares method and median-median method with and without technology, and describe the differences between the two methods

F9 demonstrate an intuitive understanding of correlation

F10 use interpolation, extrapolation and equations to predict and solve problems

F11 describe real-world relationships depicted by graphs and tables of values

F12 explore measurement issues using the normal curve

F13 calculate and apply mean and standard deviation using technology, to determine if a variation makes a difference

GCO G: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails (i) determining probabilities by conducting experiments and/or making theoretical calculations, (ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment, and (iii) analysing problem situations to decide how best to determine probabilities.

In grade 9, students will be expected to

By the end of grade 9, students will be expected to

G1 make predictions of probabilities involving dependent and independent events by designing and conducting experiments and simulations

G2 determine theoretical probabilities of independent and dependent events

G3 demonstrate an understanding of how experimental and theoretical probabilities are related

G4 recognize and explain why decisions based on probabilities may be combinations of theoretical calculations, experimental results, and subjective judgements