Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

Mathematics

10

CURRICULUM

2001

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- The Provincial Curriculum Working Group, comprising teachers and other educators in Nova Scotia, which served as lead province in drafting and revising the document.
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the development of the grade 10 mathematics curriculum guide.

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I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their learning experiences.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision, which embraces the principles of students learning to value and become active "doers" of mathematics and advocates a curriculum which focusses on the unifying ideas of mathematical problem solving, communication, reasoning, and connections. The *Foundation for the Atlantic Canada Mathematics Curriculum* establishes a framework for the development of detailed grade-level documents describing mathematics curriculum and guiding instruction.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers with department of education officials to co-operatively plan and execute the development of curricula in mathematics, science, language arts, and other curricular areas. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the Essential Graduation Learnings (EGLs), also developed regionally. These EGLs and the contribution of the mathematics curriculum to their achievement are presented in the "Outcomes" section of the mathematics foundation document.

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. This curriculum guide is one of several which provide greater specificity and clarity for the classroom teacher. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—General Curriculum Outcomes (GCOs), which relate to subject strands, and Key-Stage Curriculum Outcomes (KSCOs), which articulate the GCOs further for the end of grades 3, 6, 9, and 12. This guide builds on the structure introduced in the foundation document, by relating Specific Curriculum Outcomes (SCOs) to each KSCO at Year 10. Figure 1 further clarifies the outcome structure.

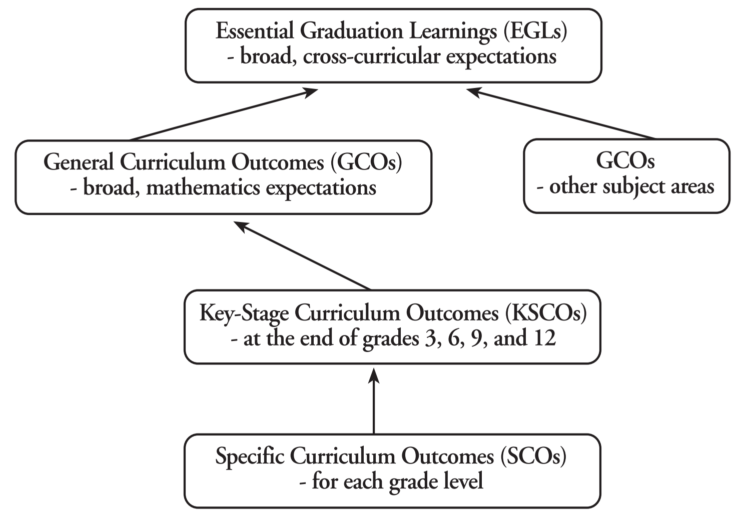


Figure 1: Outcome Framework

This mathematics guide is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice, including the following: (i) mathematics learning is an active and constructive process; (ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; (iii) learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort; (iv) learning is most effective when standards of expectation are made clear and assessment and feedback are ongoing; and (v) learners benefit, both socially and intellectually, from a variety of learning experiences, both independent and in collaboration with others.

II. Program Design and Components

A. Program Organization

As indicated previously, the mathematics curriculum is designed to support the Atlantic Canada Essential Graduation Learnings (EGLs). The curriculum is designed to significantly contribute to students meeting each of the six EGLs, with the communication and problem-solving EGLs relating particularly well with the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then goes on to present student outcomes at key stages of the student's school experience.

This curriculum guide presents specific curriculum outcomes at Year 10. As illustrated in Figure 2, these outcomes represent the step-by-step means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and, ultimately, the essential graduation learnings.

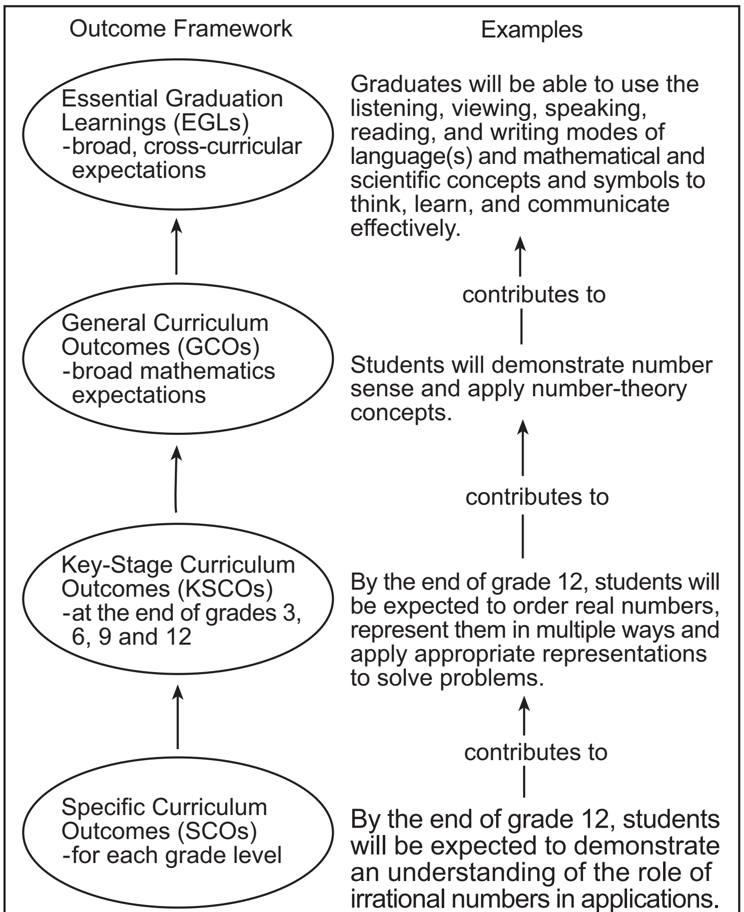


Figure 2: Examples of Outcomes

It is important to emphasize that the initial presentation of the specific curriculum outcomes at this grade level (pp. 15-41) follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a natural teaching sequence. In Mathematics 10, however, a suggested teaching order for specific curriculum outcomes has been given in terms of a sequence of seven topics or units (i.e., Data Management; Networks and Matrices; Patterns, Relations and Equations; Modeling and Functions; Geometry/Trigonomety; Geometry/Packaging; and Linear Programming). While some topics will of necessity need to be addressed before others due to prerequisite skill requirements, some flexibility exists as to the structuring of the program. It is expected that teachers will make individual decisions as to what sequence of topics will best suit their classes. In most instances, this will occur in consultation with fellow staff members, department heads, and/or district level personnel.

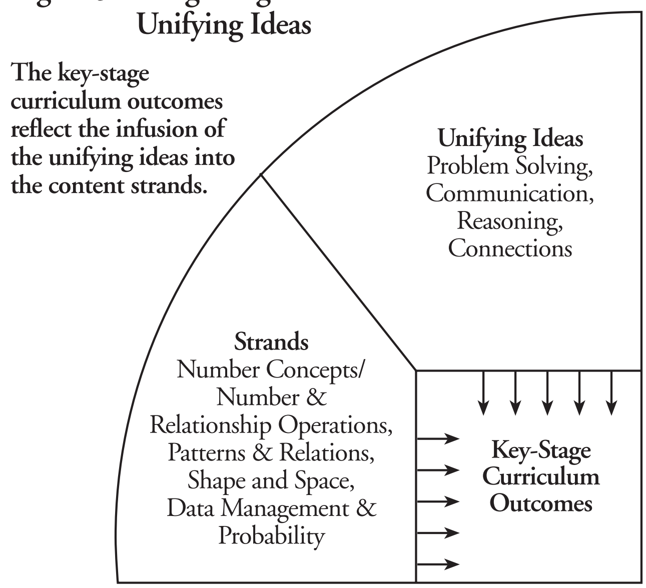
Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" topic for one group of students might be less effective in that role with a second group. Another consideration with respect to sequencing will be co-ordinating the mathematics program with other aspects of the students' school experience. An example of such co-ordination would be studying aspects of measurement in connection with appropriate topics in science. As well, sequencing could be influenced by events outside of the school, such as elections, special community celebrations, or natural occurrences.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. This is illustrated in Figure 3.

These unifying concepts serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode; classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically; via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning; and mathematics with which students are involved on a day-to-day basis must be connected to other mathematics, other disciplines, and/or the world around them.

Figure 3 - Integrating Strands and Unifying Ideas



Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling, whereas working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire in the later elementary years. During middle school and the 9/10 years, this repertoire will be extended to include such strategies as interpreting formulas, checking for hidden assumptions, examining systematic or critical cases, and solving algebraically.

Opportunities should be created frequently to link mathematics and career opportunities. During these important transitional years, students need to become aware of the importance of mathematics and the need for mathematics in so many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in higher-level mathematics programming in senior high mathematics and beyond.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline which lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, and actively participate in discourse and conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas in which reasoning and sense making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Meeting the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness, but they must also remain aware of avoiding gender and cultural biasses in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes which comprise the core curriculum, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

Since students differ in terms of ability to represent mathematical ideas with symbolic abstractions, ability to manipulate and interpret these symbols, and the pace at which they can process mathematicl ideas and concepts, this curriculum guide presents both the core curriculum and suggestions for differentiation for some students who may be encountering an unacceptably high level of challenge. These suggestions for differentiation are identified as they occur (look for the symbol) and will be found within the detailed elaborations of the curriculum which comprise the bulk of this guide. It should be noted, however, that, while suggestions for differentiation are extensive in some portions of the guide, major sections of the curriculum are appropriate for the large majority of students with little or no adaptation.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Further, the practice of designing classroom activities to support a variety of learning styles must be extended to the assessment realm; such an extension implies the use of a wide variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This curriculum guide represents the central resource for the teacher of mathematics for these grade levels. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes should be met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent that they support the curriculum goals. Teachers will need professional resources as they seek to broaden their instructional and mathematical skills. Key among these are the NCTM publications, including the *Principles and Standards for School Mathematics, Assessment Standards for School Mathematics, Curriculum and Evaluation Standards for School Mathematics*, the *Addenda Series*, *Professional Standards for Teaching Mathematics,* and the various NCTM journals and yearbooks. As well, manipulative materials and appropriate access to technological resources (e.g., software, videos) should be available. Calculators will be an integral part of many learning activities.

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different than were those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their children's lives, assisting children with mathematical activities at home, and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through learning centres, teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

The concepts and skills developed in mathematics are applied in many other disciplines. These include science, social studies, music, technology education, art, physical education, and home economics. Efforts should be made to make connections and use examples which apply across a variety of discipline areas.

In science, the concepts and skills of measurement are applied in the context of scientific investigations. Likewise, statistical concepts and skills are applied as students collect, present, and analyse data.

In social studies, measurement is used to read scale on a map, to measure land areas, and in various measures related to climatic conditions. As well, students read, interpret, and construct tables, charts, and graphs in a variety of contexts such as demography.

In addition, there are many opportunities to reinforce fraction concepts and operations in music, as well as opportunities to connect concepts such as symmetry and perspective drawings of art to aspects of 2-D and 3-D geometry.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are critical, not only with respect to clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers make meaningful instructional decisions. (See "Assessing and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment should include the following: i) using a wide variety of assessment strategies and tools; ii) aligning assessment strategies and tools with the curriculum and instructional techniques; and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborate good assessment practice and serve as a guide with respect to student assessment for the mathematics foundation document. (See also, Appendix A, "Assessing and Evaluating Student Learning.")

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address such questions as the following: Are students meeting the curriculum outcomes? Is the curriculum being equitably applied across the region? Does the curriculum reflect a proper balance between procedural knowledge and conceptual understanding? Is technology fulfilling a proper role?

IV. Designing an Instructional Plan

It is important to develop an instructional plan for the school year. Without such a plan, it is easy to run out of time in a school year before all aspects of the mathematics curriculum have been addressed. A plan for instruction that is comprehensive enough to cover all outcomes and topics will help to highlight the need for time management.

It is often advisable to use pre-testing to determine what students have retained from previous grades relative to a given topic or set of outcomes. In some cases, pre-testing may also identify students who have already acquired skills relevant to the current grade level. Pre-testing is often most useful when it occurs one to two weeks prior to the start of a a topic or set of outcomes. When the pre-test is done early enough and exposes deficiencies in prerequisite knowledge/skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of adequate development or coverage in the previous grades. This may imply that an adjustment is required to the starting point for instruction, as well as a meeting with other grade level teachers to address these concerns as necessary.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in an overall savings in time for both disciplines. The most obvious of these connections relate to the use of measurement in science and the use of a variety of data displays in social studies.

V. Curriculum Outcomes

The pages that follow provide details regarding both specific curriculum outcomes and the seven topics/units that comprise Mathematics 10. The specific curriculum outcomes are presented initially, then the details of the units follow in a series of two-page spreads. (See Figure 4 on next page.)

This guide presents the mathematics curriculum for Year 10 so that a teacher may readily view the scope of the outcomes which students are expected to meet during the year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development. (See Appendix B for a complete listing of the SCOs for grades 8 and 9.) Given that the specific curriculum outcomes at each grade level are related to the key-stage curriculum outcome framework, it is relatively easy to access a given KSCO at the previous grade and/or the next one to see how the development of particular mathematical ideas are taking place.

Within each unit, the specific curriculum outcomes are presented on two-page spreads. At the top of each page, the overarching topic is presented, with the appropriate SCO(s) displayed in the left-hand column. The second column of the layout is entitled "Elaboration-Instructional Strategies/Suggestions" and provides a clarification of the specific curriculum outcome(s), as well as some suggestions of possible strategies and/or activities which might be used to achieve the outcome(s). While the strategies and/or suggestions presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning, and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol ❐. As well, potential differentiations of the core curriculum for some students are indicated with the ◤ ◢ symbol. This symbol not only brackets text discussing differentiation, but also appears at the top of each page on which such text is located.

The third column of the two-page spread, "Worthwhile Tasks for Instruction and/or Assessment," might be used for assessment purposes or serve to further clarify the specific curriculum outcome(s). As well, those tasks regularly incorporate one or more of the four unifying ideas of the curriculum. These sample tasks are intended as examples only, and teachers will want to tailor them to meet the needs and interests of the students in their classrooms. The final column of each display is entitled “Suggested Resources” and will, over time, become a collection of useful references to resources which are particularly valuable with respect to achieving the outcome(s).

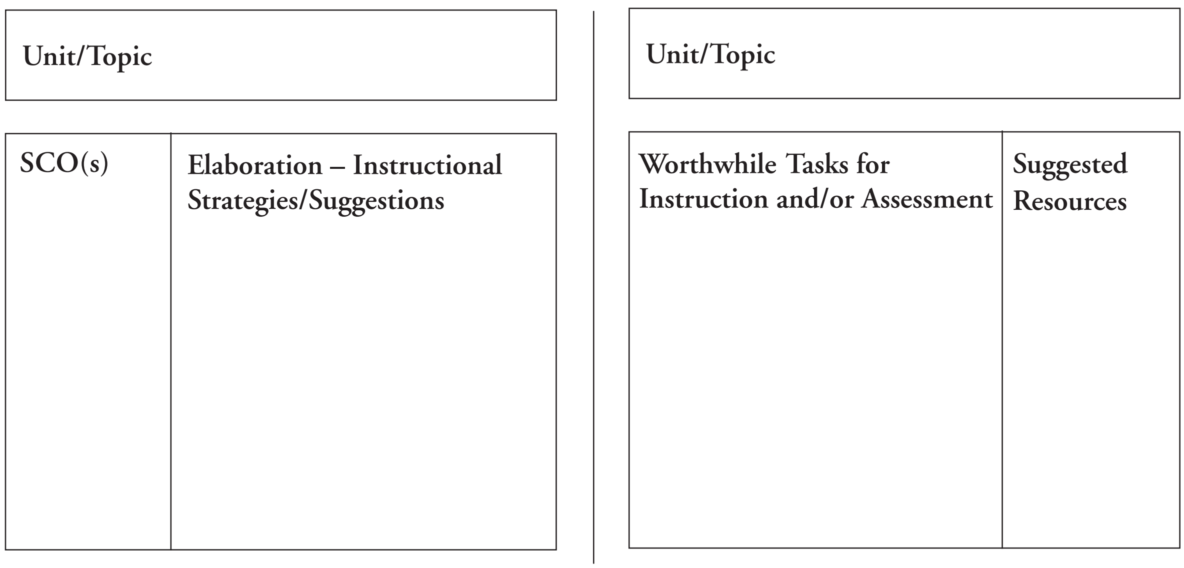


Figure 4: Layout of a 2-Page Spread

Specific Curriculum Outcomes

(by GCO)

GCO A: Students will be expected to demonstrate number sense and apply number theory concepts

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) demonstrate an understanding of number meanings with respect to the real numbers*	
A1 relate sets of numbers to solutions of inequalities	**A1** This outcome should be treated in context when students deal with problems that require a restricted solution set. Restrictions would include those related to context (e.g., negative values not allowed), as well as those relating to choice of number system (e.g., integers). Unit 7, p. 202
A2 analyze graphs or harts of situations to derive specific information	**A2** Students will analyze and interpret graphs of relationships, and of data, and will make inferences and predictions using real numbers. This would include analyzing stem-and-leaf plots, box-and-whisker plots and histograms, as well as looking at the nature of the number system, domain and range, intercepts and slopes, in linear and non-linear two-variable contexts. Unit 1, pp. 44, 52, 54, 56, 58, 60; Unit 3, pp. 104, 110; Unit 4, p. 126; Unit 7, pp. 198, 200, 202, 204
ii) order real numbers, represent them in multiple ways, and apply appropriate representations to solve problems	
A3 demonstrate an understanding of the role of irrational numbers in applications	**A3** Students will see that many calculations generate irrational numbers (radicals) and will need to be aware that using decimal approximations introduces round-off errors; consequently, they will have to make appropriate decisions as to when to approximate and when to continue to work with the radical expression. Students will be briefly introduced to conversions between entire and mixed radicals, within the context of the application of the Pythagorean Theorem. (See also outcome B2 regarding operations on irrational numbers.) Unit 5, pp. 168, 170; Unit 6, p. 190
A4 approximate square roots	**A4** Students should see that square roots are often irrational and that it will be useful to make appropriate approximations in some situations. (This outcome would be addressed in connection with A3.) Unit 5, pp.168, 170

GCO A: Students will be expected to demonstrate number sense and apply number theory concepts

	Elaboration
A5 **demonstrate an understanding of the zero product property and its relationship to solving equations by factoring**	A5 The zero product rule states that if a • b = 0, then either a = 0 or b = 0. Students will need to apply this property when solving quadratic equations by factoring. Factoring and applying the zero product property converts a quadratic equation into a set of two linear equations which students may solve more readily. Unit 3, pp. 116, 120
iii) demonstrate an understanding of the real number system and its subsystems by applying a variety of number theory concepts in relevant situations	
A6 **apply properties of numbers when operating upon expressions and equations**	**A6** This outcome involves applying properties (such as associative, distributive, identity and inverse) in the context of solving equations. This is a continuation of similar work from previous years. Unit 3, pp. 94, 100, 102, 120
A7 **demonstrate and apply an understanding of discrete and continuous number systems**	A7 Students will need to distinguish between discreet (e.g., N) and continuous (e.g., R) number systems and apply this understanding when approximating, interpolating and extrapolating. Unit 3, pp. 82, 84, 88; Unit 4, pp. 128, 152; Unit 7, p. 202

GCO B: Students will be expected to demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations

A8 demonstrate an understanding of and apply properties to operations involving square roots

Elaboration

A8 Students will develop and apply properties in relation to operations involving square roots. This will be limited to conversions between entire and mixed radicals, within the context of the application of the Pythagorean Theorem. (See outcome A3.) Unit 5, p. 168

GCO B: Students will be expected to demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

i) *explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism*

B1 model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations

Elaboration

B1 Students will model expressions and equations using concrete materials (manipulatives) and pictorial representations so that algebraic symbols and operations may be connected to these models. Students should come to see these algebraic operations as related to similar arithmetic ones. (See also outcome B3.)
Unit 3, pp. 94, 100, 102, 118, 120

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
ii) derive, analyze, and apply computational procedures in situations involving all representations of real numbers	
B2 develop algorithms and perform operations on irrational numbers	**B2** When applying the Pythagorean Theorem, students will occasionally need to add and subtract radicals; consequently, they will develop and apply the necessary procedures. (This will be done in connection with outcomes A3 and A8.) For purposes of possible differentiation, some students may briefly explore properties of radicals, but would be expected to work strictly with approximations in application situations. Unit 5, p. 170
iii) derive, analyze, and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations	
B3 use concrete materials, pictorial representations, and algebraic symbolism to perform operations on polynomials	**B3** Students will use concrete materials, pictorial representations and algebraic symbolism to perform operations on polynomials (e.g., addition, subtraction, and multiplication of monomials, monomial times binomial, monomial times trinomial, binomial times binomial). Students will be expected to associate symbolic representations with the concrete and pictorial models, and ultimately operate symbolically, with understanding. (See also outcome B1.) Unit 3, pp. 94, 116, 118, 120

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

Elaboration

B4 identify and calculate the maximum and/ or minimum values in a linear programming model

B4 Students will use linear inequalities to graph the boundaries of feasible regions and identify maximum and/or minimum values as key points on the boundaries. (This outcome will be addressed in connection with outcomes C6 and C19.) Unit 7, p. 204

B5 develop, analyze, and apply procedures for matrix multiplication

B5 Students will develop and apply the algorithm for matrix multiplication, and explore its applications. Students will be expected to be able to multiply small matrices using paper and pencil, but to use technology for larger ones. (This outcome will be addressed in connection with outcomes B6, C7 and C37.) Unit 2, pp. 70, 72, 74

B6 solve network problems using matrices

B6 Students will represent and solve network problems using matrices. (See also outcomes C7 and C37.) Unit 2, p. 74

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) model real-world problems using functions, equations, inequalities, and discrete structures*	
C1 express problems in terms of equations and vice versa	**C1** Students will analyze and interpret a variety of situations (e.g., written problems, pictorial patterns and tables of data), expressing them as equations. These equations may be linear, quadratic or exponential. Also students will create scatter plots and determine equations of best fit, with and without technology. Unit 3, pp. 80, 84, 90, 114, 122; Unit 4, pp. 144, 146, 148, 152
C2 model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities	**C2** Students will use linear, quadratic, exponential and power equations, and linear inequalities, as appropriate, to represent real-world situations. The primary focus at this time is on linear and quadratic equations. Exponential and power equations are considered only briefly by way of contrast. Linear inequalities are addressed within the context of linear programming. For purposes of differentiating as needed for some students, modeling with exponential equations may be omitted. Unit 3, pp. 80, 84, 86, 88, 90, 110, 114, 122; Unit 4, p. 144; Unit 7, pp. 198, 200
C3 gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, and domain and range	**C3** Students will gather and represent data graphically. They will need to make decisions regarding independent and dependent variables, and consider the domain and range when determining scales. Unit 1, pp. 44, 60; Unit 3, pp. 82, 88, 96, 110; Unit 4, pp. 128, 144

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
C4 create and analyze plots using appropriate technology	**C4** Students will use graphing technology to create scatter plots. They will then consider issues such as the strength of the relationships, whether they are linear or curved, and whether they are increasing or decreasing. Unit 1, p. 60; Unit 4, pp. 144, 146, 150, 154
C5 sketch graphs from words, tables, and collected data	**C5** Students will be expected to translate among tabular, written, symbolic and graphical representations of functions. In particular here, students should be able to create graphs given information in various other formats. Unit 1, p. 60; Unit 3, pp. 82, 84, 90, 122; Unit 4, pp. 126, 128, 144
C6 apply linear programming to find optimal solutions to real- world problems	**C6** Linear programming is a technique used in some optimization problems. It involves using linear inequalities to graph the boundaries of feasible regions and identifying maximum and/or minimum values. Students will learn to apply this technique in real-world situations. Unit 7, pp. 198, 202, 204
ii) represent functional relationships in multiple ways and describe connections among these representations.	
C7 model real-world situations with networks and matrices	**C7** Students will model some real-world situations using networks (i.e., connected graphs consisting of vertices and edges). They will also represent networks using matrices, to facilitate answering questions about these situations. (See also E6 and C37.) Unit 2, pp. 66, 68
C8 identify, generalize, and apply patterns	**C8** Students will examine situations presented as written descriptions, diagrams, graphs, and/or tables of data in order to identify patterns. They will then generalize (algebraically) and/or apply these patterns. Unit 3, pp. 80, 84, 86, 88, 90, 114, 122; Unit 4, pp. 128, 144, 154
C9 construct and analyze graphs and tables relating two variables	**C9** Students will analyze graphs and tables to determine mathematical characteristics, such as slope/rate of change and intercepts. As well, these characteristics will be interpreted in relation to given contexts. Analysis of rate of change (both constant and otherwise) will be of particular significance. Unit 1, p. 60; Unit 3, pp. 82, 84, 88, 90, 108, 114, 122; Unit 4, pp. 128, 132, 144, 146; Unit 7, p. 198

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions	**C10** Students need to be able to describe real-world situations in multiple ways. Consequently, they should be able to translate with facility among tables, equations, graphs and written descriptions. Unit 1, p. 44; Unit 3, pp. 80, 84, 86, 90, 114; Unit 4, p. 126
C11 write an inequality to describe its graph	**C11** Students will express inequalities algebraically by examining their graphs. Unit 7, p. 200
C12 express and interpret constraints using inequalities	**C12** Constraints are limits that are sometimes given in mathematical situations. Students will express constraints both in words and algebraically. Unit 7, pp. 198, 200
C13 determine the slope and y-intercept of a line from a table of values or a graph	**C13** Given a table of values or a graph that represents a linear situation, students should be able to determine the slope (rate of change, ratio of the rise to the run) and y-intercept (offer the initial value) through the analysis of the table or graph. Unit 3, pp. 104, 110, 112
C14 determine the equation of a line using the slope and y-intercept	**C14** The student should be able to determine the equation of a line ($y= mx+b$) if the slope (m) and y-intercept (b) are known or can be determined (see C13). Unit 3, pp. 104, 110, 112; Unit 4, p. 146

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

Outcomes	Elaboration
iv) *solve problems involving relationships, using graphing technology as well as paper-and-pencil techniques*	
C15 develop and apply strategies for solving problems	**C15** Students will solve routine and non-routine problems and discuss the strategies used. Open-ended problems with multiple solutions (more than one answer, more than one appropriate strategy) will be examined. Strategies will include applying algebraic and geometric procedures, solving equations using technology and solving network problems using matrices. Unit 1, p. 62; Unit 2, pp. 66, 68, 70; Unit 3, pp. 84, 92, 96, 98, 108
C16 interpret solutions to equations based on context	**C16** Students will need to express solutions in terms of given context and decide whether or not they make sense, or are reasonable, in terms of context. Unit 3, pp. 82, 84, 92, 94, 96, 98, 114; Unit 7, p. 202
C17 solve problems using graphing technology	**C17** Students will solve problems using technology to analyze data (e.g., mean, median, standard deviation), construct data displays (e.g., histograms, box plots), create scatter plots and lines of best fit, analyze graphs (e.g., tracing, finding points of intersection), and perform matrix operations. Unit 1, pp. 50, 52, 54, 62; Unit 2, p. 76; Unit 3, pp. 98, 108; Unit 4, pp. 150, 152, 154; Unit 7, pp. 200, 206
C18 investigate and find the solution to a problem by graphing two linear equations with and without technology	**C18** Students will solve a variety of problems by graphing pairs of linear equations, with and without technology. These problems will range from identifying the conditions under which one option is better than another (e.g., When is plan A better than plan B?) to graphically solving complex linear equations as linear systems (i.e., graphing the left hand side of the equation against the right). Unit 3, p. 98
C19 solve systems of linear equations using substitution and graphing methods	**C19** Students will solve systems of linear equations graphically, but will realize that exact solutions are not always easily obtained by this method. Consequently, students will also solve linear systems by the algebraic method of substitution (including comparison). The method of elimination (addition/subtraction) is addressed in a subsequent course. Unit 7, pp. 204, 206

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

Elaboration

C20 evaluate and interpret non-linear equations using graphing technology

C20 Students will analyze the graphs of non-linear relations. This will include finding roots of quadratic equations from the graphs of the corresponding functions, and deciding whether non-linear scatter plots represent quadratic or exponential relationships. Unit 3, pp. 116, 122; Unit 4, p. 154

vi) perform operations on and between functions

C21 explore and apply functional relationships and notation, both formally and informally

C21 Students should understand the relationship that exists between a relation and a function. They should begin by informally exploring functional relationships and then clarify and apply the mathematical concept of function, along with notation and vocabulary. For purposes of differentiation for some students, the study of functions may be kept informal. Unit 4, pp. 130, 132, 142

C22 analyze and describe transformations of quadratic functions and apply them to absolute value functions

C22 Students need to familiarize themselves with the shape of the "model" quadratic function $y=x^2$. Students will then examine and apply the graphical transformations (i.e., reflections, stretches, and translations) resulting from changes in the parameters of the function. Students will also apply these techniques to absolute value functions. For purposes of differentiation for some students, the study of transformations may be omitted. Unit 4, p. 134, 136, 140

C23 express transformations algebraically and with mapping rules

C23 When given a graph that is the image of a known graph (such as that of $y=x^2$), students should be able to express transformations either algebraically or with a mapping rule. Also, students should be able to express mapping rules algebraically and vice versa. For purposes of differentiation for some students, the study of transformations may be omitted. Unit 4, pp. 134, 136, 138

C24 rearrange equations

C24 Students should be able to transform equations from one form to another. For example, they should be able to transform linear equations in standard form ($ax+by+c=0$) to y-intercept/slope function form ($y=mx+b$), and vice versa. Likewise, quadratic equations should be transferable between transformational (e.g., $2(y-3)=x^2$) and function (e.g., $y=\frac{x^2}{2}+3$) forms. Unit 3, pp. 106, 112; Unit 4, p. 138; Unit 7, p. 206

C25 solve equations using graphs

C25 Students should learn to use graphs to solve equations. For example, solutions to both linear and quadratic equations can be determined by identifying the x-intercepts of the graphs of the corresponding functions. Also, students will discover that the solution to an equation can be determined by graphing one side of the equation against the other and identifying the intersection point. Unit 3, pp. 84, 92, 98, 108, 116

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
C26 solve quadratic equations by factoring	**C26** Students should understand the method of solving a quadratic equation by factoring and applying the zero product property (i.e., if the product of two factors is zero, then one or both of the factors must be zero). Equations to be solved by factoring include those involving common factors, those that are made of regular (ax^2+bx+c) and perfect square trinomials, and those described as being factored by the difference of squares method. Unit 3, pp. 116, 120
C27 solve linear and simple radical, exponential, and absolute value equations and linear inequalities	**C27** This outcome builds on work from previous grades. See also SCOs C18, C19, C25, C26 and C34. For purposes of differentiating as needed for some students, solving may be limited to linear equations and inequalities. Unit 3, pp. 94, 96, 100, 102, 108, 122; Unit 4, p. 140; Unit 5, p. 172; Unit 7, pp. 202, 206
v) analyze and explain the behaviours, transformations, and general properties of types of equations and relations	
C28 explore and describe the dynamics of change depicted in tables and graphs	**C28** Students should be able to analyze tables and or graphs to determine how changes in one variable affect a second. They should come to recognize the differences in the change patterns between linear, quadratic and other relationships. Unit 3, pp. 82, 88, 104, 112, 114, 122; Unit 4, p. 126
C29 investigate, and make and test conjectures concerning, the steepness and direction of a line	**C29** Through investigation, students should conclude that the sign of the slope is related to whether a line rises or falls and that the magnitude of the slope is related to the steepness of the line. Unit 3, pp. 106, 112
C30 compare regression models of linear and non-linear functions	**C30** Students should understand that not all data suggests a trend that can be represented using a regression model, and that the nature of the data will determine the choice of the linear versus a non-linear model. Unit 4, p. 154

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

	Elaboration
C31 graph equations and inequalities and analyze graphs both with and without graphing technology	**C31** Students should be able to graph equations and inequalities by constructing tables of values, by using knowledge of transformations, and by directly identifying characteristics (e.g., slope, y-intercept) from the equations and inequalities. Unit 3, p. 108; Unit 4, pp. 134, 136, 138; Unit 7, pp. 200, 202, 204
iii) interpret algebraic equations and inequalities geometrically and geometric relationships algebraically	
C32 determine if a graph is linear by plotting points in a given situation	**C32** By plotting and examining the coordinates of points, students should be able to determine, both visually and numerically, whether or not there is a constant ratio of rise to run. Unit 1, p. 62; Unit 3, pp. 82, 88, 106, 110, 112
C33 graph by constructing a table of values, by using graphing technology, and when appropriate, by intercept-slope method	**C33** See C31. Unit 3, p. 104; Unit 4, pp. 128, 132; Unit 7, p. 200
C34 investigate and make and test conjectures about the solution to equations and inequalities using graphing technology	**C34** Students should understand that all solutions to an equation in two variables fall upon a line, and that solutions to an inequality fall within a feasible region. Unit 7, pp. 200, 202

GCO C: Students will be expected to explore, recognize, represent, and apply patterns and relationships, both formally and informally

Elaboration

C35 expand and factor polynomial expressions using perimeter and area models

C35 The student should expand and factor polynomial expressions using perimeter and area models as a conceptual basis for expanding and factoring algebraically. Unit 3, pp. 116, 118

C36 explore, determine, and apply relationships between perimeter and area, surface area, and volume

C36 These relationships would include how to maximize area for a given perimeter and volume for a given surface area, as well as how to minimize perimeter for a given area and surface area for a given volume. Unit 6, pp. 186, 188, 190

C37 represent network problems using matrices and vice versa

C37 Networks can sometimes be usefully analyzed by representing them as matrices and performing matrix operations. Consequently, students should be able to both represent a network as a matrix and interpret such a matrix in terms of the corresponding network situation. (See also C7.) Unit 2, p. 68

GCO D : Students will be expected to demonstrate an understanding of and apply concepts and skills associated with measurement

Elaboration

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

i) *measure quantities indirectly, using techniques of algebra, geometry and trigonometry*

D1 determine and apply formulas for perimeter, area, surface area, and volume

D1 Students will apply formulas in a number of contexts. They will need to remain aware of precision and accuracy issues when working with measurement data. As well, students will need to determine areas and volumes in some situations by developing non-routine formulas. Unit 1, p. 48; Unit 6, pp. 182, 184, 186, 188, 190

D2 apply the properties of similar triangles

D2 Students will apply side and angle relationships of similar triangles when developing the primary trigonometric ratios and when considering three-dimensional dilatations. Unit 5, pp. 158, 162; Unit 6, p. 192

D3 relate the trigonometric functions to the ratios in similar right triangles

D3 Students will determine that the primary trigonometric ratios are constant for the equal angles in similar right triangles. Unit 5, p. 174

D4 use calculators to find trigonometric values of angles and angles when trigonometric values are known

D4 Students will do this in connection with outcomes such as D5, D6, D8 and D12. Unit 5, p. 176

GCO D : Students will be expected to demonstrate an understanding of and apply concepts and skills associated with measurement

	Elaboration
D5 apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation	**D5** Contextual situations are one of the principal motivations for studying right-triangle trigonometry. As well, students will use right-triangle trigonometry to assist in finding areas of polygons. Unit 5, p. 176; Unit 6, pp. 186, 188
D6 solve problems involving measurement using bearings and vectors	**D6** Many problems involving bearings and vectors can be solved by means of right-triangle techniques, including the application of the Pythagorean Theorem and/or trigonometric ratios. Unit 5, pp. 162, 178
ii) determine measurements in a wide variety of problem situations, and determine specified degrees of precision, accuracy, and error of measurements	
D7 determine the accuracy and precision of a measurement	**D7** Accuracy depends upon the skill with which a measurement instrument is used. Precision depends upon how finely an instrument is graduated. Students will need to take both into consideration when collecting data for analysis purposes. As well, students will need to address precision issues when performing calculations on measurement data (i.e., consider significant digits). Unit 1, pp. 46, 48; Unit 5, pp. 160, 176, 178; Unit 6, p. 182
D8 solve problems involving similar triangles and right triangles	**D8** Students will solve contextual problems using the proportionality relationship among sides in similar triangles and/or the Pythagorean Theorem. Unit 5, pp. 158, 160, 164

GCO D : Students will be expected to demonstrate an understanding of and apply concepts and skills associated with measurement

	Elaboration
D9 determine whether differences in repeated measurements are significant or accidental	**D9** When any experiment is repeated, differences in results are to be expected. Students will need to consider whether these differences are the result of accidental variation or the result of the influence of an unidentified variable and, therefore, significant. For purposes of possible differentiation, this outcome may be omitted for some students. Unit 1, p. 58
D10 determine and apply relationships between the perimeters and areas of similar figures, and between the surface areas and volumes of similar solids	**D10** Students will determine how changes in linear dimensions affect the 2-D and 3-D characteristics of shapes. They will also apply the determined relationships in problems involving areas and volumes. Unit 5, p. 160; Unit 6, p. 192
D11 explore, discover, and apply properties of maximum areas and volumes	**D11** Students will discover how to maximize area while restricting perimeter, and volume while restricting surface area, and apply these techniques in contextual situations. Unit 6, pp. 186, 190
iii) apply measurement formulas and procedures in a wide variety of contexts	
D12 solve problems using trigonometric ratios	**D12** See SCOs D5 and D6. Unit 5, pp. 176, 178; Unit 6, p. 186
D13 demonstrate an understanding of the concepts of surface area and volume	**D13** Students will need a clear understanding of the concepts of surface area and volume to successfully achieve SCOs D1, D10 and D11. Unit 6, pp. 182, 188

GCO D : Students will be expected to demonstrate an understanding of and apply concepts and skills associated with measurement

Elaboration

D14 apply the Pythagorean Theorem

D14 Students will need to apply the Pythagorean Theorem to solve problems. (See, for example, SCOs D6 and D8.) To do this, students will need to develop appropriate techniques for dealing with irrational numbers. Unit 5, pp. 164, 170

GCO E: Students will be expected to demonstrate spatial sense and apply geometric concepts, properties, and relationships

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *v)* *make and test conjectures about, and deduce properties of and relationships between, 2- and 3-dimensional figures in multiple contexts*	
E1 explore properties of, and make and test conjectures about, 2- and 3-dimensional figures	**E1** Making and testing conjectures are fundamental aspects of mathematical reasoning. Making conjectures is inherently inductive in nature, while testing them is more deductive. Students will be expected to perform these reasoning processes while expanding their understanding of 2- and 3-D shapes. This outcome will be addressed in conjunction with SCOs D1, D10, D11, D13, E8 and E9. Unit 6, pp. 184, 188, 190, 194
ii) *interpret and classify geometric figures, translate between synthetic and coordinate representations, and apply geometric properties and relationships*	
E2 solve problems involving polygons and polyhedra	**E2** Problems involving polygons and polyhedra range from locating centers of triangles to determining the most efficient shapes for 3-D packages. This outcome will be addressed in connection with numerous others, including E3 and E8. Unit 6, pp. 184, 186, 188, 190, 194

GCO E: Students will be expected to demonstrate spatial sense and apply geometric concepts, properties, and relationships

	Elaboration
E3 construct and apply altitudes, medians, angle bisectors, and perpendicular bisectors to examine their intersection points	**E3** Students will perform constructions in polygons to determine such key points as incentres, circumcentres, and centers of graviy, and consider their significance in various situations. To differentiate as needed for some students, this outcome may be omitted. Unit 6, p. 194
iii) analyze and apply Euclidean transformations, including representing and applying translations as vectors	
E4 apply transformations when solving problems	**E4** Transformational techniques may be applied to solve problems in situations involving coordinate geometry and 3-D dilatations. For purposes of differentiation for some students, the study of transformations may be omitted. Unit 3, p. 108; Unit 6, p. 192
E5 use transformations to draw graphs	**E5** When using transformations to draw graphs, students will be connecting algebraic and geometric ideas. (See also C23.) For purposes of differentiation for some students, the study of transformations may be omitted. Unit 4, p. 138
iv) represent problem situations with geometric models and apply properties of figures	
E6 represent network problems as digraphs	**E6** Students will represent network problems (e.g., problems involving situations such as mail, snowplow and airline routes) as connected graphs, with vertices representing key points and edges representing routes between them. Graphs that also show direction of travel along routes are known as directional graphs, or digraphs. (See also C7.) Unit 2, pp. 66, 68

GCO E: Students will be expected to demonstrate spatial sense and apply geometric concepts, properties, and relationships

Elaboration

vi) demonstrate an understanding of the operation of axiomatic systems, and the connections among reasoning, justification, and proof

E7 demonstrate an understanding of and write a proof for the Pythagorean Theorem

E7 Students will be applying the Pythagorean Theorem in conjunction with SCOs D6, D8, and D14. This outcome requires students to develop their mathematical reasoning (see E8 and E9) to also understand and prove this theorem.
Unit 5, p. 166

E8 use inductive and deductive reasoning when observing patterns, developing properties and making conjectures

E8 Students will use inductive and deductive reasoning in a number of settings. These would include developing operational procedures, generalizing relationships, and proving theorems. Related SCOs include D10, D11, E1 and E9. Unit 3, pp. 104, 112; Unit 5, pp. 166, 170; Unit 6, pp. 184, 186, 188, 190, 194

E9 use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid

E9 When focusing on deductive reasoning (see also SCO E8), students will need to not only develop solid deductive arguments, but also be able to examine the validity of the arguments of others. To differentiate as needed for some students, this outcome may be omitted. Unit 5, p. 166; Unit 6, p. 194

GCO F: Students will be expected to solve problems involving the collection, display, and analysis of data

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *v) design and conduct relevant statistical experiments, and analyze and communicate the results using a range of statistical arguments*	
F1 design and conduct experiments using statistical methods and scientific inquiry	**F1** In designing and conducting experiments, students will address issues relating to cause and effect (independent vs. dependent variables), controlling variables, and data collection. (See also F2.) Unit 1, pp. 44, 50
F2 demonstrate an understanding of the concerns and issues that pertain to the collection of data	**F2** Issues around which students should display understanding include what variables affect data outcomes, accuracy and precision of data, and what happens to data when an experiment is repeated. Unit 1, pp. 44, 50, 58
F3 construct various displays of data	**F3** Students will construct stem-and-leaf plots, box-and-whisker plots, histograms, and scatterplots, with a view to considering data distributions and trends. Consequently, this outcome will be considered in conjunction with others such as F4, F5, A2, C9, F8 and F10. Unit 1, pp. 52, 54, 60
F4 calculate various statistics using appropriate technology, analyze and interpret data displays, and describe relationships	**F4** Students will determine measures of central tendency and quartiles, identify extreme values, and calculate measures of dispersion (i.e., range and standard deviation). These statistical measures will be determined in conjunction with the development of data displays (see F3) and statistical analysis and the identification of appropriate conclusions (see F5). Unit 1, pp. 50, 52, 54, 56

GCO F: Students will be expected to solve problems involving the collection, display, and analysis of data

Elaboration

F5 analyze statistical summaries, draw conclusions, and communicate results about distributions of data	**F5** Having collected and displayed data, and developed appropriate statistical summaries, students will be expected to draw conclusions about the nature of the data and its implications with respect to the situation/experiment which generated it. Unit 1, pp. 50, 52, 54, 56, 58
F6 solve problems by modeling real-world phenomena	**F6** Students will solve problems by creating mathematical models of situations. This may involve conducting experiments, collecting and organizing data, summarizing/generalizing data mathematically (e.g., with tables, graphs, symbolic relationships), and using these mathematical tools to provide solutions to problems. Unit 1, p. 44; Unit 3, pp. 84, 90; Unit 4, pp. 144, 146, 150
F7 explore non-linear data using power and exponential regression to find a curve of best fit	**F7** Students will explore curve fitting for non-linear data. This should be limited to achieving an understanding that non-linear models (such as power and exponential relations) often provide better models, and hence better serve predicting and problem solving, than linear models. Unit 1, p. 62; Unit 4, p. 154
F8 determine and apply the line of best fit using the least squares method and median-median method with and without technology, and describe the differences between the two methods	**F8** Students should be able to determine the line of best fit (for data which suits a linear model) using both the median-median and least squares techniques. Students will be expected to understand and be able to execute the median-median method manually, as well as be able to use technology to execute either method. Ultimately, students should be developing mathematical models (relations in symbolic form) which they will apply to answer various questions/solve problems. For purposes of differentiating for some students, it is sufficient to use linear regression (with technology) only. Unit 1, p. 62; Unit 4, pp. 148, 150

GCO F: Students will be expected to solve problems involving the collection, display, and analysis of data

	Elaboration
iii) use curve fitting to determine the relationship between, and make predictions from, sets of data and be aware of bias in the interpretation of results	
F9 demonstrate an intuitive understanding of correlation	**F9** Students should understand correlation as a description of "how well" data fits an identifiable pattern, and correlation coefficients as a means of quantifying the degree of "wellness" of fit. Students should develop understandings with respect to the relative strength of relationships, negative vs. positive correlations, and the fact that correlation coefficients can be misleading. Unit 4, pp. 146, 150, 152, 154
F10 use interpolation, extrapolation and equations to predict and solve problems	**F10** There are many different strategies for predicting and problem solving. Here, students are expected to interpolate and/or extrapolate from data presented in tables or graphs to make predictions or answer questions. As well, students will use mathematical models of situations (i.e., equations) to predict and solve problems. Unit 1, pp. 60, 62; Unit 3, pp. 84, 90, 92; Unit 4, pp. 128, 146, 148, 152
iv) determine, interpret and apply as appropriate a wide variety of statistical measures and distributions	
F11 describe real-world relationships depicted by graphs and tables of values	**F11** Students will develop symbolic mathematical models to describe relationships that are depicted in words, graphically, or tables of values. This outcome will be done in conjunction with outcomes such as C1, C2 and C8. Unit 4, p. 144

GCO F: Students will be expected to solve problems involving the collection, display, and analysis of data

Elaboration

F12 **explore measurement issues using the normal curve**

F12 Students will see the normal curve as an extension of histograms in certain situations, and understand its pattern in relation to the standard deviation of the data being displayed. (See also SCO F13.) For purposes of possible differentiation, this outcome may be omitted for some students. Unit 1, p. 58

F13 **calculate and apply mean and standard deviation using technology, to determine if a variation makes a difference**

F13 Students will apply mean and standard deviation to determine if the degree of variation in particular situations is reasonable and to be expected by chance, or if the variation is likely to have been caused by (an)other factor(s). For purposes of possible differentiation, the study of standard deviation may be omitted for some students. Unit 1, pp. 56, 58

Unit 1
Data Management
(15–20 Hours)

In this unit students will address data collection, display and analysis. More specifically, the core curriculum addresses experimental design, issues of accuracy and precision, the construction of various types of data displays (including box plots, histograms and scatter plots), calculation and analysis of various statistics (including standard deviation), characteristics of the normal curve, and lines of best fit. All of the concepts and techniques are developed in relation to meaningful contexts and to help make predictions and/or solve problems.

For purposes of possible differentiation for some students, work on standard deviation and normal curves may be omitted. (See p. 56 for more detail.)

Data Management (15–20 hours)

SCO: In grade 10 students will be expected to

A2 **analyse graphs or charts of situations to identify specific information**

C3 **gather data, plot the data using appropriate scales and demonstrate an understanding of independent and dependent variables and domain and range**

F1 **design and conduct experiments using statistical methods and scientific inquiry**

F6 **solve problems by modeling real-world phenomena**

F2 **demonstrate an understanding of concerns and issues that pertain to the collection of data**

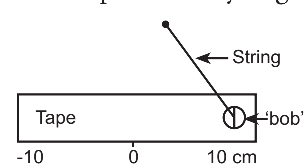
C10 **describe real-world relationships depicted by graphs, tables of values, and written descriptions**

Elaboration – Instructional Strategies/Suggestions

A2/C3 Students might begin this unit of study by considering factors that affect growth. In any relationship, factors that can change are called variables. Students should analyse charts and/or tables of situations to consider what the variables are that might affect change. Students will need to show understanding of independent and dependent variables.

F1/F6 In their efforts to understand independent and dependent variables, students might design and conduct experiments to explore which variables affect the outcomes of the experiment. In any experiment, there are multiple factors that vary, dependently and independently. All factors need to be controlled except for the one being measured. Students will gain insight into statistical methods and scientific inquiry by conducting experiments. In designing their own experiments they should be encouraged to connect the experiment to another discipline or to current issues and career applications.

By way of illustration, in the Grandfather's Clock Problem (see next page) students must determine how to build a pendulum that will swing with a period of one second. They may determine that there are certain variables that will affect the period, such as mass, length and amplitude of swing. Students should freely explore these variables by designing and conducting their own experiments. For example, students might want to determine if the amplitude affects the period. They might decide that each group in the class should take a different amplitude measure (10 cm, 20 cm, ... 80 cm) and test its effect on the period. Each group might want to conduct their experiment 10 times and then graph their results. When they attempt to graph the results, there would be a natural discussion about which variable is being manipulated (independent) and which variable is measured (dependent) as a result of the manipulation. Students should understand that the manipulated data is the amplitude (independent variable), while the period (dependent) is measured to see if different amplitudes affect it. (Amplitude and 'bob' mass will not affect the period, but length will.)



F2 When students carry out experiments, it is important that they are aware of all the variables that will affect the results and what degree of variation must occur in the data before they can conclude that the variable being studied makes a difference in the outcome of the experiment. Issues, besides variation, that must be studied include measurement techniques, error and accuracy, and types of distribution. Students must be able to answer questions like:

- If a person measures the same event repeatedly, will the results of the measurement always be the same? will they differ?
- If there is a difference in the measurement of the measured variable, is it due to measurement error or due to a change in the manipulated variable?

C10 Students should practise determining if variables are dependent or independent by using charts (mind maps, for example), given data in a table, or graphs. For example, before conducting the pendulum experiment, students might have assumed that period depends on amplitude, but from this table of results students should conclude that the period of a pendulum is not affected by the size of the amplitude, and thus amplitude is not a variable that will affect the outcome in the problem.

amplitude (cm)	10	20	30	40	50	60
period (sec)	0.9	0.8	0.9	0.9	0.8	0.8

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F1/F6/F2/C3/A2/C10

Performance

1) Grandfather's Clock Problem
   I have always remembered my grandfather's clock. My hope is that some day I will build a grandfather's clock so that I, too, bask in its glory. Unfortunately, all I know about this type of clock is that the pendulum is going to control a gear that drives the second hand. The gear that I have has 60 teeth. I assume that each swing of the pendulum must move one tooth on the gear and that each swing must take one second. How should I construct my pendulum? Ask students to do the following:
   a) List several variables that might affect the period (one complete swing) of the pendulum.
   b) Design an experiment to determine if the mass of the pendulum has an affect on the period. What variables need to be controlled?
   c) Conduct the experiment, gather and interpret the data, explain which variable is dependent and which is independent, and communicate the outcome.

Performance/Presentation/Assignment

2) Ask students to design an experiment, or a series of experiments, to determine how best to construct a ski-jump ramp to maximize the length of the jump. Conduct at least one experiment and communicate the results.

F6/C3/A2/C10

Pencil/Paper

3) In an experiment to determine if there is a relationship between the diameter of a circular balloon and the amount of time it shoots through the air when released before it hits the ground, the following data was collected.

puffs to fill the balloon	7	9	3	10	5	8
diameter (cm)	18	20	9	22	12	19.2
time in air (sec)	3.5	4.2	2	4	3.1	3.7

Ask students to do the following:
   a) Organize the data.
   b) Identify the independent and dependent variables.
   c) Describe any values that could not be used in the domain or range.
   d) Describe a different experiment to determine a relationship, and identify the dependent and independent variables.

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10 students will be expected to

D7 determine the accuracy and precision of a measurement

Elaboration – Instructional Strategies/Suggestions

D7 While performing the experiments described on the previous pages, students are using measurement devices that include centimetre/millimetre rulers, metre sticks, and stop watches. Students have to record their measurements during each experiment.

When a tool is used to measure a quantity, the precision of the measurement depends on how fine the scale divisions are on the measuring tool. Students' answers can only be as precise as the least precise measured value. Consequently, if they measure an amplitude to be exactly 10 cm, using a ruler marked in millimetres, they should record the measurement as 10.00 cm or 100.0 mm to indicate as much precision as possible.

The accuracy of a measurement depends on the skill of the student when using the instrument (for example, placement of tool and angle of sight when reading the tool). Other factors such as temperature and humidity may also affect accuracy.

Because the precision of all measuring devices is limited, the number of digits that are valid for any measurement is also limited. The valid digits are called the significant digits. Examples:

56.0 m—three significant digits
0.0026 kg—two significant digits
0.002060 kg—four significant digits

When a larger number, such as 186 000, is written without a decimal point, the number of significant digits is uncertain; it could be three, four, five, or six. This is one situation in which scientific notation is important. Writing large numbers using scientific notation allows for a clear indication of the number of significant digits that are appropriate in a given situation. Example:

186 000 m $\rightarrow 1.860 \times 10^5$—four significant digits
$\rightarrow 1.86 \times 10^5$—three significant digits

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

D7

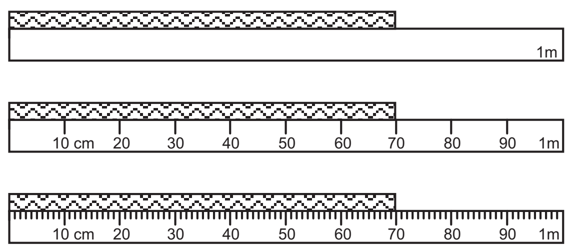
Pencil/Paper

1) Rick recorded a measurement as 76 000 mm. Ask students to respond to the following:
   a) Why is it difficult to tell how many significant digits there are?
   b) How can the number of significant digits in such a number be made clear?

2) Two students use a metre stick to measure the same width of a lab table. One records 84 cm being as precise as possible, the other student records 83.78 cm being as precise as possible. Ask students to describe how this is possible.

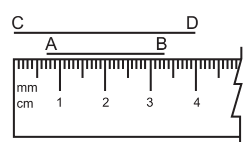
Pencil/Paper, Interview

3) Three different metre sticks could be used in a measurement. Ask students: What measurement do you get in each case? Is there a difference in the number of significant digits? Why or why not?



Performance

4) Ask students to
   a) find the length of $\overline{AB}$.
   b) draw the segment $\overline{PQ}$ that has the measure 1.63 cm.
   c) find the length of $\overline{CD}$.



Interview

5) Ask students what determines the precision of a measurement.

6) Ask students to give an example of a measurement that is
   a) accurate but not precise
   b) precise but not accurate

7) Ask students how the last digit differs from the other digits in a measurement.

Suggested Resources

The Geometer's Sketchpad, Key Curriculum Press, software, 1995.

Bennett, Dan, "Exploring Geometry with the Geometer's Sketchpad", Key Curriculum Press, 1993.

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

D7 determine the accuracy and precision of a measurement

D1 determine and apply formulas for perimeter, area, surface area, and volume

Elaboration – Instructional Strategies/Suggestions

D7

When operations are performed on numbers representing measurement data, the results cannot be more precise than the least precise values involved in the calculations. As illustrated in the first example below, when adding or subtracting, the answer is expressed with the same precision as the least precise number involved in the calculation. The second example illustrates that, when multiplying or dividing, the answer has no more significant digits than the least number found in the values involved in the operation. (Note: These are examples of "conventions", and have been agreed upon by the mathematics and science partners in Atlantic Canada.)

Adding and Subtracting:

$$\left.\begin{array}{r} 24.686\text{ m} \\ 2.343\text{ m} \\ \underline{3.21\ \ \text{m}} \end{array}\right\} \text{least precise is } 3.21\text{ m}$$

30.239 m $\longrightarrow$ 30.24 m (same precision as the least precise)

Multiplying and Dividing:

3.22 cm
2.1 cm $\longrightarrow$ two significant digits $\longrightarrow$ smallest number of significant digits
6.762 cm $\longrightarrow$ the answer should have two significant digits $\longrightarrow$ hence, 6.8 cm.

D1 Students should apply precision and accuracy when performing calculations using area and perimeter formulas. For example, when using $A = \frac{bh}{2}$, the 'b' and the 'h' have a degree of precision, but the '2' has no effect on the answer (the '2' is not measured data). Also, students should be encouraged to use the 'pi' (π) key on their calculators, since they should be using at least as many digits as their least precise measure. This outcome is addressed again in the 'Geometry/ Packaging' unit of this course.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

D7

Pencil/Paper

1) Have students express each of the answers with the appropriate significant digits.
   a) 8.7 g +15.43 g +19 g = 43.13 g (ans = 43 g)
   b) 4.32 cm × 1.7 cm = 7.344 cm^2 (ans = 7.3 cm^2)
   c) 853.2 L – 627.443 L = 225.757 L (ans = 225.8 L)
   d) 38.742 kg ÷ 0.421 = 92.023 75 kg (ans = 92.0 kg)
   e) 5.40 m × 3.21 m × 1.871 m = 32.431 914 m^3 (ans = 32.4 m^3)
   f) 5.47 m^3 + 11 m^3 + 87.300 m^3 = 103.770 m^3 (ans = 104 m^3)

2) Have students find the answers to these exercises:
   a) Find the area of a rectangle 2 mm by 30 cm.
   b) Find the perimeter of a rectangle 25 cm by 2.00 m.

Journal

3) Have students comment on this statement: "When two measurements are added together, the answer can have no more significant digits than the measurement with the least number of significant digits."

D7/D1

Portfolio

4) Have students solve the following problem: The radius of Earth at the equator is 6378 km. Imagine a stiff wire wrapped around the equator of a perfectly smooth Earth. Suppose we now increased the length of the wire by 15 m, and shaped the wire into a circle centred at the centre of Earth. Predict, then determine, how far above Earth's surface the wire would be. Explain your reasoning. If any information that you did not need was given in the problem, indicate it.

Suggested Resources

The Geometer's Sketchpad. Key Curriculum Press, 1995.[Software.]

Bennett, Dan. "Exploring Geometry with the Geometer's Sketchpad." Key Curriculum Press, 1993.

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

F1 **design and conduct experiments, using statistical methods and scientific inquiry**

F2 **demonstrate an understanding of concerns and issues that pertain to the collection of data**

F4 **calculate various statistics, using appropriate technology, analyse and interpret displays, and describe the relationships**

F5 **analyse statistical summaries, draw conclusions, and communicate results about distributions of data**

C17 **solve problems, using graphing technology**

Elaboration – Instructional Strategies/Suggestions

F1/F2/F4/F5 As students conduct experiments like those described on previous pages, they will be performing frequent measurements. When experimenting to determine whether a variable is having an effect on the dependent variable, the same experiment is often repeated many times. Students could become curious as to why they are getting 'different' results each time they repeat the same experiment. For example, in the pendulum experiment, when the amplitude is set at 10 cm, the pendulum swung, and the experiment repeated five times, students are likely to record five different measurements for the period. (When there is a difference in the measurement of the measured (dependent) variable, students must determine if it is due to measurement error or due to a change in the manipulated (independent) variable.)

At first, students will be expected to express the measured phenomenon with a particular value. To do this, students are expected to average the measurements obtained over several repetitions, and discuss their confidence in using the resulting 'average' to represent the measured phenomenon. The discussion might include discussing how their confidence might change if the number of measurements of the phenomenon increased.

F4/F5/C17 Students should take time to discuss central tendency. They have dealt with this topic before, so the treatment should not be exhaustive. They need to understand the difference between mean, median, and mode, and when each is appropriately used. For example, it is generally not appropriate to discuss mean and median when working with discrete data (such as the number of each type of pet owned by students in a class). Mode, on the other hand, is very appropriate for discrete data. There are times, however, when discrete data is collected (e.g., readings every 3 seconds), and then treated as if it is continuous data for curve-fitting.

Students should be aware that mean and median can be determined using technology, and should become very comfortable using technology for this purpose. For example, if students enter data into a list on a graphing calculator, they could easily find values for the mean and the median.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F1/F2/F4/F5/C17

Performance

1) Ask students to work in groups of three. One student will take another student's pulse rate, while the third student records the results. The students should exchange roles as the experiment is repeated. Ask students to design and conduct an experiment from which they might be able to determine the pulse rate for any individual.
2) Ask students to work in pairs to collect data on reaction time. One student will collect data, the other will record. Students should reverse roles and repeat the experiment. Ask students to use a stopwatch to determine if it is possible to time how long it takes another clock to show 5 seconds have passed.

Pencil/Paper

3) When an experiment is repeated many times and different measurements for a phenomenon are recorded, ask students to explain how they can determine one measurement that reflects the phenomenon.
4) A marble is rolled down a ramp 5.0 cm in length and out onto the floor. Students measured how far along the floor the marble rolled. The data below were collected by repeating this over and over again. Given these measurements (made using a centimetre ruler), ask students to use the data to answer this question: "How far will a marble roll along a horizontal surface after it leaves that ramp."

   15.4, 12.8, 16.1, 15.3, 14.7, 13.2, 15.1, 16.4, 13.2, 17.1, 15.6, 12.8, 13.3, 14.7, 12.8, 14.6, 15.5

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

F4 **calculate various statistics, using appropriate technology; analyse and interpret displays; and describe the relationships**

C17 **solve problems, using graphing technology**

A2 **analyse graphs or charts of situations to identify specific information**

F3 **construct various displays of data**

F5 **analyse statistical summaries, draw conclusions, and communicate results about distributions of data**

Elaboration – Instructional Strategies/Suggestions

F4/C17 Students will be calculating various statistics including mean, median, and mode; upper and lower quartiles and extreme values when interpreting box-and-whisker plots (or box plots). Calculators should be used when appropriate. Students should be aware of how to use technology properly to do these calculations, whether using a scientific calculator, graphing calculator, or a statistical software program.

A2 Sometimes students will observe points that follow the general pattern of the data but are far removed from other points. Other times they will find data values that are inconsistent with the general trend. Such points may indicate errors in measurement that need to be corrected, or they may indicate the presence of some factor that deserves special attention. Whatever the cause, students should look for, and attempt to explain, unusual points, called "outliers."

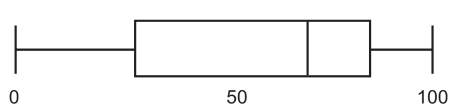
F3 Students should construct stem-and-leaf plots, box-and-whisker plots, and histograms to display data. All of these displays have been presented at earlier grade levels in the Atlantic Canada curriculum, so teachers should check with students to determine how much instruction or help is required.

F3/C17/A2/F5 When constructing stem-and-leaf plots, they may have to be reminded to order their plots and align them vertically so as to easily count and interpret data. Students should be encouraged to interpret the plots for clusters of data, gaps in the data, and the amount of spread in the data.

1	0
2	
3	08
4	
5	017
6	1255689
7	34799
8	0185

Another good way to compare distributions of data is by constructing box-and-whisker plots (box plots). Students should be expected to construct these not only by hand, but also with the use of technology. Students should also be comfortable using technology to interpret the box plots. For example, tracing allows students to move the cursor from the lower extreme, through the quartiles, to the upper extreme, indicating their values at the bottom of the screen. Students should learn to interpret the distributions according to the width of the box (which contains at least 50 percent of the data), the position of the median, and the length of the whiskers.

For example, if this box plot includes all the exam marks from the first term math exam, what could be concluded about how well the class did?



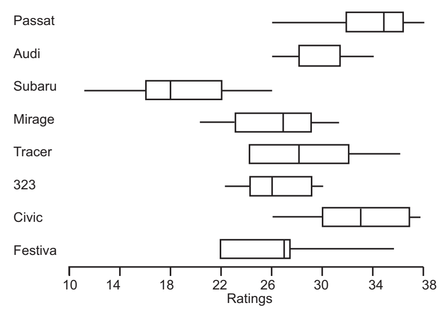
Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F5/A2

Pencil/Paper

1) Give students the chart showing the ratings for comfort and style of various car types, and ask them to answer the following questions:
   a) Which car seems to have the most consistent ratings?
   b) Explain the meaning of the placement of the median bar in the box for the Festiva, the Subaru, and the Audi.



F5/A2/F3

Performance

2) The following data represents the bowling scores of Lisa and Ruth in their last 10 games.
   Lisa: 65, 105, 90, 95, 72, 85, 110, 88, 92, 95
   Ruth: 125, 110, 81, 62, 98, 115, 68, 72, 118, 69

   Ask students to use a display of data to help argue for putting
   a) Lisa on their team next year
   b) Ruth on their team for the play-off.

F4

3) The Golf Tournament: The very first annual golf tournament is under way, and the prize up for grabs is a graphing calculator for each math student in the winning school, to be paid for by the student councils of the losing teams' schools. Each school selects five golfers and two alternates for their team. Two 18-hole games of golf will be played twice a month, beginning in May and ending in September, with each team entering five players to play. The course to be played each half-month will be selected at random by the principal of the winning school in the previous game. At the end of September, the school team with the lowest accumulated score over the year will be declared the winner, and the other schools will supply them with their prize.

   In early August the accumulated scores for each of the teams are totalled, and the results show that any of the teams could go on be the winner. But wait, the school's nurse has announced that your school's best golfer has developed a serious back problem and will not be able to play for the rest of this year. An alternate must replace her for the remainder of the tournament. The following tables represent the previous 20 golf scores recorded for each of your alternates. (Luckily they were members of the same golf course.)

Ronnie:	83	75	77	82	95	93	91	101	103	92
	82	72	90	88	85	81	95	97	105	91
Bobbie:	68	89	101	67	107	110	98	89	72	100
	91	69	105	101	65	87	86	92	91	104

   Ask students to select the alternate they would choose to replace your star for the rest of the games this year. They must present arguments and diagrams to support their decision.

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

A2 **analyse graphs or charts of situations to identify specific information**

F3 **construct various displays of data**

F4 **calculate various statistics, using appropriate technology; analyse and interpret displays; and describe relationships**

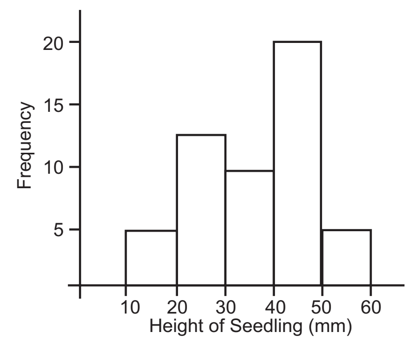
F5 **analyse statistical summaries, draw conclusions, and communicate results about distributions of data**

C17 **solve problems, using graphing technology**

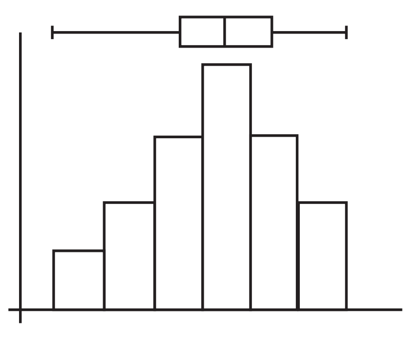
Elaboration – Instructional Strategies/Suggestions

A2/F3/F4 Students should be able to explain that a histogram is similar to a stem-and-leaf plot turned on its side. The stems of a stem-and-leaf plot determine the intervals that are marked along the horizontal axis of a histogram, the leaves become the vertical bars, and the number of leaves determines the height of each bar. In a stem-and-leaf plot the number of data points in each bar and the actual values are clearly stated.

C17/A2/F3/F4/F5 Students should be expected to use technology to construct histograms. They should understand that each bar contains a continuous set of data with a lower extreme and a higher extreme. The difference between these extremes defines the 'bin width.' For example, the histogram to the right is marked in tens of millimetres along the horizontal axis. The data in the first bar includes all measurements greater than or equal to 10 mm and less than 20 mm. This would be called a bin width of 10. The vertical axis represents the frequency.



C17/F3/F5 Students might want to superimpose box plots over histograms, using technology to help interpret distribution. For example, by extending quartile lines, students could easily see from the box plot where 50 percent of the data is distributed in the histogram. By extending the median line from the box plot, students can tell where the median falls in the histogram.



Given a histogram of the results of an experiment, students could be asked to interpret the histogram and then present the results in a box plot. Students would need to determine from the histogram where 50 percent of the data points lie, where the median is, and extend whiskers to the extremes.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F3/A2/F4/F5/C17

Performance

1) In a study of the longevity of a particular species of cat, biologists recorded the life spans of 30 cats. Their results are presented in the following table.

Life Spans of Cats (in years)									
12.9	13.2	14.1	13.9	12.8	13.1	13.1	13.2	13.6	13.0
13.4	13.6	12.9	13.3	11.8	12.8	14.6	12.8	10.4	14.8
11.5	13.5	13.6	12.9	9.6	14.5	13.5	13.8	14.4	13.3

a) Have students create both a stem-and-leaf plot and a histogram for these data. Have them explain which display is more helpful when asked to determine the median life span for a cat. How are the displays the same? How are they different?

b) Have students create a box plot, using the same horizontal scale as their histogram, and use both to answer this question: How likely is it that a cat will live 14 years? 13.8 years? Explain.

2) The lifetimes (in years) of 30 Brand A and 30 Brand B batteries are given in the tables below. Have students answer the following questions.

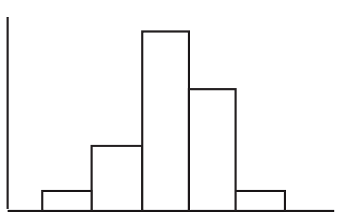
a) Which of these brands of battery is more reliable?

Measured Lifetimes of 30 Brand A Batteries				
5.1	7.3	6.9	4.7	4.6
6.2	6.4	5.5	4.9	6.9
6.0	4.8	4.1	5.3	8.1
6.3	7.5	5.0	5.7	9.3
3.3	3.1	4.3	5.9	6.6
5.8	5.0	6.1	4.6	5.7

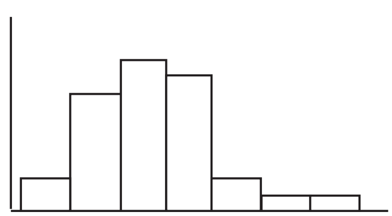
Measured Lifetimes of 30 Brand B Batteries				
5.4	6.3	5.0	5.9	5.6
4.7	6.0	3.3	6.6	6.0
5.0	6.5	5.8	5.4	4.9
5.7	6.8	5.6	4.9	6.0
4.9	5.7	6.2	7.5	5.8
6.8	5.9	5.3	5.6	5.9

b) One of the histograms below displays the lifetimes of the 30 Brand A batteries, and the other displays the lifetimes of the 30 Brand B batteries. (The window settings are $2 \leq x \leq 11$, $0 \leq y \leq 16$.) Plot the histogram of the lifetimes of either brand, using these settings, and

i)



ii)



identify the matching histogram. Use a bin interval (Xscl) of 1.

c) What is the mean in histogram i)? Explain what the mean value tells you about the batteries.

d) I just bought a battery that lasted 3.4 hours. How likely is it that is Brand A? Brand B? neither? Explain.

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

F4 **calculate various statistics, using appropriate technology; analyse and interpret displays; and describe relationships**

F5 **analyse statistical summaries, draw conclusions, and communicate results about distribution of data**

F13 **calculate and apply mean and standard deviation, using technology to determine if a variation makes a difference**

A2 **analyse graphs or charts of situations to identify specific information**

Elaboration – Instructional Strategies/Suggestions

F4/F5 Students must be able to answer questions like:

- If a person measures the same event over and over again, would the results for the measurement always be the same?
- If there is a difference in the measurement of an event, is it due to measurement uncertainty or due to some change in a variable?

To help answer these questions, students should perform experiments, pool their results, construct histograms, box plots and stem-and-leaf plots, and study the distributions of the data.

F4/F5/A2

❒ Perform the following experiment to determine how accurately people can time events. A student will watch the second hand of a clock on the wall and stop his/her stopwatch after five seconds, without looking at the stop watch (looking only at the wall clock). Repeat many times (with numerous students) and pool the class data. The class should then make a histogram of the data, and a stem-and-leaf plot. The two displays could be compared and contrasted with respect to the distribution information they provide.

In studying the distribution of data, students should understand that experimental results about a variable often differ from the results expected.

❒ For the above experiment, ask the students if they measured the five-second interval the same each time. If not, why did the measurements vary? Were any differences due to a problem with the wall clock or due to measurement fluctuations? How much deviation from the expected is reasonable before it can be concluded that a variable is affecting the outcomes?

F4/F5/F13/A2 Students have learned that an average alone is not adequate to describe a set of data effectively. The description of the data should include information about how the data are spread out. Understanding this dispersion of data includes knowing the range of the data and something about its variation. Students should learn that standard deviation is a way to measure variation. It helps explain how far each data value is from the mean. If most of the data clusters around the mean, then there is little variation and the standard deviation will be low. If, however, the data are more widely dispersed, then there is a lot of variation, resulting in a higher standard deviation.

Students should first learn to calculate standard deviation by hand to help them understand how the value is determined. However, students should quickly learn to use technology to determine the standard deviation of any set of data. Once the standard deviation is obtained, explaining and applying it become the focus.

F4 Note: Some calculators offer a choice when calculating standard deviation. When arranging the variance, s^2, some divide by n - 1, others by n, where n is the number of observations. The difference in these two approaches is small, unless there are very few observations. Encourage students to use lots of data so that 'n' is useful. Statistical summaries on calculators often include median, mean, standard deviation, extreme and quartile values.

The core curriculum calls for students to study standard deviation and normal curves, and their role in determining the significance of data variations (SCOs F13, F12 and D9). For purposes of differentiation, however, these outcomes may be omitted for some students, with the study of the distribution of data confined to the analysis of box plots, stem-and-leaf plots and histograms.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F13/A2/F4/F5

Performance

1) In 1798 the English scientist Henry Cavendish repeatedly measured the density of the earth in a careful experiment with a torsion balance. Here are his 23 measurements of the same quantity (the density of the earth relative to that of water), made with the same instrument. (From S. M. Stigler, Do robust estimators work with real data? Annals of Statistics 5 [1977]: 1055-1078.)

5.36	5.62	5.27	5.46	5.53	5.57
5.29	5.29	5.39	5.30	5.10	5.79
5.58	5.44	5.42	5.75	5.34	5.63
5.65	5.34	5.47	5.68	5.85	

   a) Make a histogram of these data.
   b) Describe the distribution.

2) A fisheries researcher compiled the following data on lengths (in millimetres) of 6-year-old pond goldfish:

217	230	220	221	225	223
219	217	225	228	234	222
231	222	220	222	222	223
225	214	221	233	227	234
223	225	253	220	213	224
235	283	210	218	235	231

   a) Make a histogram of the data, using technology.
   b) Describe the distribution.
   c) Change data values 253 and 283 to 203 and 207, respectively. Describe how these changes affect the distribution.
   d) Assuming that all of these data were obtained from goldfish in the same pond, how likely would it seem to you that a goldfish of each of the following lengths would be found in this pond? Explain.

   i) 205 mm ii) 215 mm iii) 225 mm iv) 235 mm

Journal

3) Construct box plots using the data in questions 1 and 2 (with corrections) above. From the box plots describe between what values the middle 50% of the data can be found. Write a short article about your findings for each distribution.

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10, students will be expected to

F5 **analyse statistical summaries, draw conclusions, and communicate results about distribution of data**

A2 **analyse graphs or charts of situations to identify specific information**

F12 **explore measurement issues using the normal curve**

D9 **determine whether differences in repeated measurements are significant or accidental**

F13 **calculate and apply mean and standard deviation, using technology to determine if a variation makes a difference**

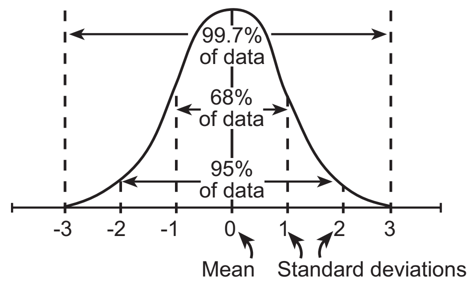
F2 **demonstrate an understanding of concerns and issues that pertain to the collection of data**

Elaboration – Instructional Strategies/Suggestions

F5 As students continue to study the distribution of data they should come to understand that the distribution of any random phenomena tends to be normal if it is described over a large number of independent repetitions.

F5/A2 If students repeat a controlled experiment many times, pool data and create progressive histograms, they should notice that the shape of the histograms becomes more bell-like as the number of data represented in the distribution increases. They should come to understand that repeated measurements which are subject to accidental or random effects only, will produce bell-shaped distributions which exhibit the characteristics of a normal curve.

F12 Students should note that the central part of the normal curve concaves down, and that the ends are concave-up. Students will discover that the central region includes about 68% of the data, falls under the concave-down part of the curve, and lies within one standard deviation of the mean. They should also understand that, in a "normal" situation, 95% of all the data should fall within two standard deviations of the mean.



F12/D9/F13/A2/F5/F2 In studying the distribution of data, students should understand that experimental results with respect to a variable often differ from those expected. Since two standard deviations from the mean represents 95% of the data, any data that falls outside two standard deviations is often considered suspect. That is, data values outside two standard deviations only occur about 5% of the time ($2\frac{1}{2}$% above the mean, and $2\frac{1}{2}$% below the mean), thus any value this extreme may not be occurring by chance, and some other variable may be affecting the outcome of the experiment. Students should also understand that area underneath the normal curve and probabilities are associated, and that probabilities can be measured with standard deviation.

When communicating the results of experiments, students would benefit from making should make classroom presentations and/or producing meaningful written reports. They would

- identify the type of experiment
- declare the variables and identify any concerns, errors, or issues associated with the collection of the data
- describe how they collected the data and identify possible sources of error
- calculate appropriate statistics, including total amount of data, mean, range, and standard deviation
- construct tables, plots, and graphs that will assist in the interpretation and presentation of the data.

See the note at the bottom of page 56 for suggestions on possible differentiation.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

F5/A2/F12/D9/F13/F2

Activity

1) To determine whether or not amplitude affects the period of a pendulum, students measured the period of a 1m pendulum, using various amplitudes. Each group repeated their own experiment using a particular amplitude and a 1m pendulum. They recorded the results and were surprised that they obtained different measurements even though they were conducting the same experiment over and over again. Students then decided to redo the experiment, controlling the amplitude, length of pendulum and mass of 'bob'. They wanted to study the variation of the period measurements.
   a) Each pair of students collects 10 period measurements.
   b) Each pair works with another pair, pooling their data and constructing a histogram.
   c) Frequency polygons are drawn by joining consecutive midpoints at the top of each bar.
   d) Students estimate the area between the frequency polygon and the horizontal axis.
   e) Mean and standard deviation values are added to the histogram.
   f) Students estimate the area bounded by the two first standard deviations, the frequency polygon and the horizontal axis, and express this as a percentage of the total area.
   g) Students repeat steps b) through f) for larger sets of data, by combining the data from two groups, four groups and, ultimately, the whole class.
   h) Students then take the normal curve, place it on a grid and estimate the area within one standard deviation of the mean.
   i) Students write about what they have found.

Performance

2) Tree Farm Problem
   To determine a selling price, Dale, the tree farmer, wants to know the typical height of 5 year-old fir trees. Dale took a sample of readings (in cm) from a field of 5 year-old trees.

39	45	14	36	23	36	12	32	25	35
46	10	49	31	34	12	61	92	51	26
24	45	57	41	42	56	50	33	77	32
32	22	21	31	45	8	38	15	57	20
43	60	48	28	33	55	55	56	42	65

   a) Find a single number to represent the typical height of a 5-year- old fir tree from Dale's field.
   b) Discuss how confident you are that the 92 cm tree was measured properly. What about the 8 cm tree? What about the 77 cm tree?
   c) Change the 92 cm tree to 50 cm, and redo parts a) and b).

Suggested Resources

For all Practical Purposes, 4th edition, pg 192, Solomon Garfunkel, consortium for Mathematics and its applications (COMAP), WH Freeman and Company.

Data Management (15–20 hours)

SCO: In grade 10 students will be expected to

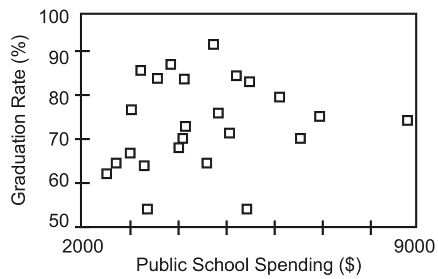
- **C4** **create and analyse scatter plots using appropriate technology**
- **C3** **gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, and domain and range**
- **C5** **sketch graphs from words, tables, and collected data**
- **F3** **construct various displays of data**
- **C9** **construct and analyse graphs and tables relating two variables**
- **A2** **analyse graphs or charts of situations to identify specific information**
- **F10** **use interpolation, extrapolation, and equations to predict and solve problems**

Elaboration – Instructional Strategies/Suggestions

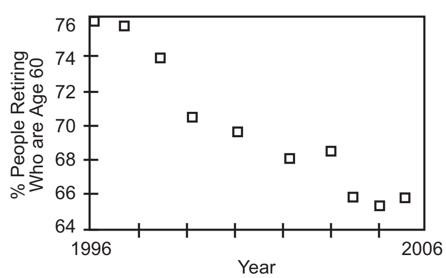
C4/C3/C5/F3/C9/A2/F10 Sketching graphs from written descriptions is detailed in Unit 4 of this guide. In this unit, students might conduct experiments to gather data or be given data already collected. After organizing the data in a table and determining which set of data is the independent data and which is the dependent, students will plot the data, selecting appropriate scales carefully. Help students realize that the independent data is the data being manipulated or changed, and that the dependent data respond to those changes.

In general students will examine the plotted data to identify any pattern or trend, so that they can use the graph to predict answers and solve problems.

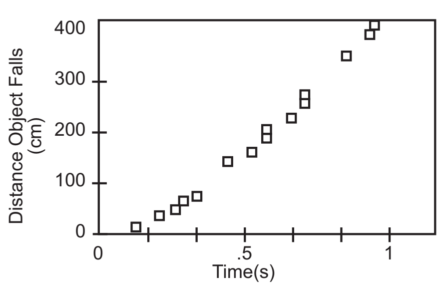
Sometimes the plotted data shows no trend, as in the graph below which shows the graduation rate of a province versus the amount of money (from all sources) spent per student on schooling.



Sometimes the data form a fairly linear pattern, and students might be able to discuss the trend that the linearity suggests. In the example below, students might be able to predict that about 64 percent of people age 60 will retire in 2006.



F10 Sometimes it may not be obvious to students if the data are linear or non-linear. However, they should still be able to interpolate to predict answers. For example, after 0.45 seconds an object has fallen about 100 cm.



Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

C5/C3/F3/C9/A2

Pencil/Paper, Performance

1) a) Sketch a reasonable graph comparing
   i) the amount of water in the bathtub and the time in seconds since the plug was pulled
   ii) the temperature of the water coming out of the hot water faucet and the time the water has been flowing
   iii) the number of drink containers collected and the number of dollars refunded
   iv) the volume of a balloon (starting fully blown up) and the time since it has been released (unfastened)

   b) Explain in each case how the dependent variable and independent variable were identified.

C4/C3/C5/F3/C9/A2/F10

2) **Tree Growth**

Ask students how they can predict the diameter of a tree, knowing its age. The data below were collected from chestnut trees growing in a relatively poor site.

Age (years)	Diameter (cm)	Age (years)	Diameter (cm)
4	2.03	23	11.94
5	2.03	25	16.51
8	2.54	28	15.24
8	5.08	29	11.43
8	7.62	30	15.24
10	5.08	30	17.78
10	8.89	33	20.32
12	12.45	34	16.51
13	8.89	35	17.78
14	6.35	38	12.70
16	11.43	38	17.78
18	11.68	40	19.05
20	13.97	42	19.05
22	14.73		

Ask students the following questions:

a) What diameter do you think a 32-year-old tree on this site would have?
b) How confident are you in your prediction? Explain.
c) Discuss outliers in these data.
d) Sketch a curve or line that best fits this data.
e) Use the sketch to answer (a), then (b).

Suggested Resources

Data Management (15–20 hours)

SCO: In grade 10 students will be expected to

C15 **develop and apply strategies for solving problems**

C32 **plot points, given a situation or a table of values, to help determine if a graph is linear**

C17 **solve problems using graphing technology**

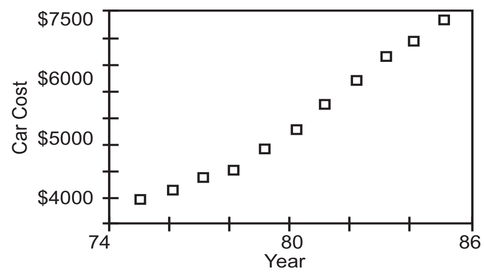
F7 **explore non-linear data using power and exponential regression to find a curve of best fit**

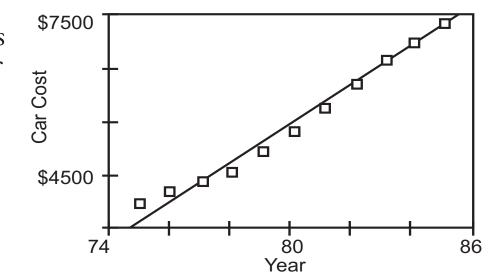
F8 **determine and apply a line of best fit using linear regression with technology**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

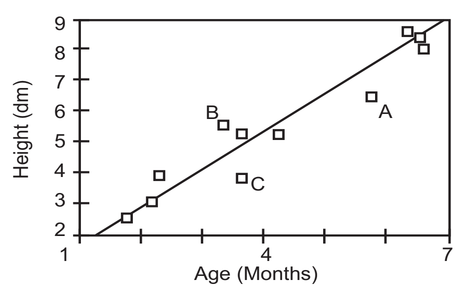
Elaboration – Instructional Strategies/Suggestions

C15/C32/C17/F7/F8/F10 Once students think that the data show a trend, they might want to draw the line of best fit that is a mathematical model for the given data. This line should make it easier for students to interpolate or extrapolate. For example, given the scatterplot that represents the rising cost of a particular car model between the years 1974 and 1986, a student might decide that this pattern looks linear and try to draw a line of best fit. Before drawing the line, he/she could use a piece of uncooked spaghetti (or other straight edge) and place it so that it forms a line of best fit. Some students might try various positions until they are satisfied that the position of their spaghetti represents the line of best fit. Others may ultimately place it differently. This variation will motivate the need for a better method. Students should discuss why their placements would be inconsistent with others in the class. By using the edge of the spaghetti, students can pencil in their lines of best fit. If they extend their lines, they can use them to predict that in 1986 the cost of the car would be approximately $7 800.





Students should use technology to construct scatter plots and lines of best fit. Students can trace the line of best fit, as well as the scatter plot, to predict answers. Curves such as power regression, exponential regression, and quadratic regression can also be considered. Note: The use of regression is introductory at this time. The modeling unit will focus on this topic and help students develop understanding of the regression procedure.



Sometimes scatter plots do not show a strong relationship, and the line of best fit is difficult to draw. Students should discuss points like A, B and C (in the example above) and decide whether these data are outliers or not.

Data Management (15–20 hours)

Worthwhile Tasks for Instruction and/or Assessment

C15/C32/F10

Pencil/Paper

1) On a summer day in Halifax, the following temperatures were recorded at various altitudes:

Altitude (m)	0	300	1500	3000	4500	6000	9000	10826
Temperature (°C)	15	13	5	–5	–15	–26	–44	–56

Have students do the following:

a) Draw a scatterplot of the data.
b) Find the equation of the line of best fit.
c) Predict the temperature for a balloonist flying at 3800 m. (What assumption(s) are you making?)
d) Interpret the rate of change of the temperature as the altitude increases.

C15/C32/F10/F7/F8/C17

Pencil/Paper, Portfolio/Project

2) The number of chirps crickets make per second is related to air temperature.

Temperature (°C)	15	17	16	18	15	16	16	15	14	16	16
Chirps per second	20	27	22	30	19	21	20	24	22	24	25

Have students do the following:

a) Draw a scatter plot and find the line of best fit.
b) At what temperature do crickets stop chirping? (What assumptions(s) are you making?)

3) Have students contact a local real estate company to gather information to determine if the selling price of a house is related to the living area (floor space). (Students might also try number of rooms vs. selling price, floor space metrage vs. selling price, etc.)

4) Data collected near Hanford, Washington, relate the rate of cancer deaths and an index of exposure to leaking radioactive contamination at their nuclear power plant over several years.

Exposure index	2.5	2.6	3.4	1.3	1.6	3.8	11.6	6.4	8.3
Death rate (per 100 000)	147	130	130	114	138	162	208	178	210

a) Have students use linear regression on a graphing calculator to find the equation for the line of best fit.
b) Ask students if this is a good model and to write several sentences to support their answer. Have them discuss and interpret the meaning of the death rate with respect to the index of exposure.

Suggested Resources

Unit 2
Networks and Matrices
(10-15 Hours)

In this unit students will solve problems by modeling network situations with connected graphs, digraphs and matrices. Developing and applying procedures for matrix multiplication (both with and without technology) will be an integral part of the process. (Note: There are no differentiations suggested for this unit.)

Networks and Matrices (10-15 hours)

SCO: In grade 10, students will be expected to

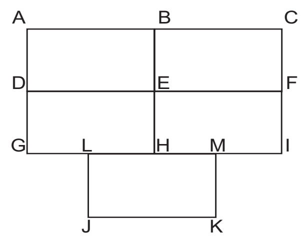
C7 **model real-world situations with networks and matrices**

C15 **develop and apply strategies for solving problems**

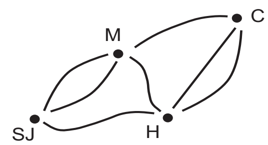
E6 **represent network problems as digraphs**

Elaboration – Instructional Strategies/Suggestions

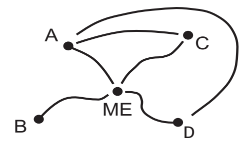
C7 A network is a group of people, places, objects, or ideas that are connected in some way. For example, in our bodies, blood flows through a network of veins and arteries. Trains and planes take people and supplies to various locations, and their routes form a maze comprising a network. Sometimes representations of these networks are called graphs, or digraphs. (Digraphs have an extra feature that gives information with respect to direction.) Following are some examples of networks:



Downtown Streets



Airline Routes

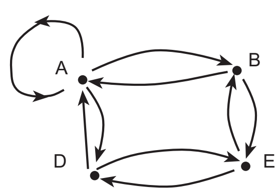


Direct Friendships

Vertices and edges are the two important features of a network. The vertices are the points at which the paths or edges of the network intersect. Even vertices are those at which an even number of edges come together, whereas odd vertices are those where an odd number of edges meet.

Students should explore networks for their efficiency. Can each edge of the network be travelled without repetition? (Note: A network with this characteristic is called "Eulerian" or "traceable".) Can each vertex be visited without repetition? What does this have to do with even and odd vertices?

C7/C15/E6 Students should be able to convert from a given situation to a graphical model and vice versa. For example, from the downtown network above students might be asked to sketch a graph for a snowplow to follow. They would have to take into account that each street would have to be travelled twice for plowing, as the plow throws snow to each edge of the road. Their graphs may look like the one given below for the block labelled ABED. In this example, the plow starts at A, travels to B-E-D-A, then turns around at A, and proceeds to D-E-B-A.



Students might also be asked to describe in words the situations depicted in the "Airline Routes" and "Direct Friendships" graphs.

Networks and Matrices (10-15 hours)

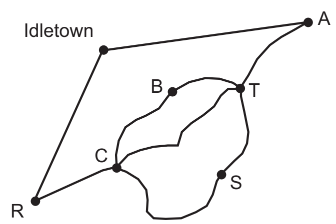
Worthwhile Tasks for Instruction and/or Assessment

C7/C15

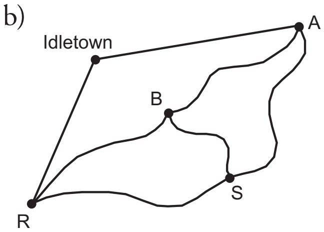
Pencil/Paper

1) The mailman lives in Idletown and delivers mail along each road illustrated below. Can he start in Idletown, deliver the mail, and return to his home without going along any road twice? Explain.

a)



b)



C7/C15

Performance

2) Four towns are located in such a way that the roads that join them form a quadrilateral. A fifth town is located at the intersection of the two diagonals of the quadrilateral. Have students do the following:
   a) Draw a network graph.
   b) Determine if a snowplough operator can plough both sides of all roads without driving along a section that he/she has already ploughed.
   c) Create a new network (involving at least four towns) such that the driver can plough each road, one side at a time and without retracing a ploughed section, and end in the same town as he/she starts.

Suggested Resources

Mathematical Investigation. Book 2. “The Mail Carrier Problem.” Dale Seymour Publication, 1990.

Mathematics Teacher. “From Graphs to Matrices”, NCTM February, 1990

Networks and Matrices (10-15 hours)

SCO: In grade 10, students will be expected to

C7 **model real-world situations with networks and matrices**

C37 **represent network problems using matrices and vice versa**

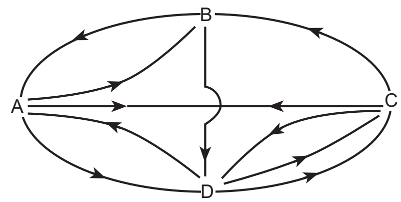
E6 **represent network problems as digraphs**

C15 **develop and apply strategies for solving problems**

Elaboration – Instructional Strategies/Suggestions

C7/C37/E6/C15 Transportation networks are a rich source of situations that lend themselves to graphical analysis. Matrices can also be used to represent these networks. Performing operations on matrices produces other matrices. These resulting matrices can then be interpreted in relation to the given situation(s).

For example, consider a transportation network between four towns A, B, C, and D. The arrows represent direct routes.



This information can be represented in a matrix, R, in the following way. (Note: The matrix represents routes from towns shown beside the rows to towns shown above the columns.)

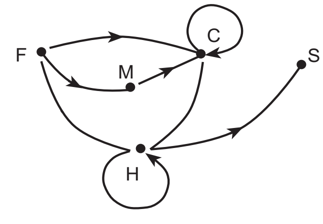
$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix} \end{array}$$

A “0” represents no direct path, a “1” represents one direct path, and the “2” represents two direct paths (as the diagram shows from D to C). The matrix has four rows (horizontal) and four columns (vertical). Each element may be identified by its (row, column) location. So, the element (4, 3) is 2.

Students should use matrices to represent simple networks and/or digraphs. (Note: a digraph is a graph with arrows added to indicate direction.)

For example, if, for a schedule of local airline flights, “0” indicates no flight connection, and “1” indicates a one-way connection, the network could be represented with a digraph, and the digraph by a matrix. (Note: No directional arrow on the digraph indicates both directions are possible. Also, a circular edge that leads back to its starting position might indicate a flight made by a small tourist business or a flying club that returns to the same airport.)

$$\begin{array}{c} \\ C \\ H \\ F \\ M \\ S \end{array} \begin{array}{c} \begin{array}{ccccc} C & H & F & M & S \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$



Students should have practice translating between written situations, graphical representations of networks, and matrices.

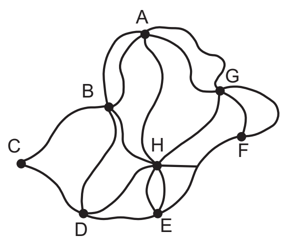
Networks and Matrices (10-15 hours)

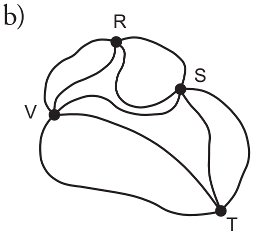
Worthwhile Tasks for Instruction and/or Assessment

C37/C7

Pencil/Paper

1) The following networks represent ski trails at a ski resort just outside Majortown.

a) 

b) 

Ask students to represent each as a matrix, using a “0” for no connection between vertices, “1” for one connection between vertices, “2” for two connections between vertices, and so on. Identify and interpret element (4, 3) in each matrix.

E6/C37/C7

Performance

2) Some airports in Atlantic Canada are connected by direct flights. Represent the following information using
   a) a digraph
   b) a matrix

   There are four direct flights from Sydney to Halifax and six direct flights returning to Sydney. There are two direct flights from Deer Lake to Halifax and return. There is one direct flight from Halifax to Moncton. From Halifax to Fredericton there is one direct flight, and one which returns from Fredericton to Halifax. There are two direct flights from Charlottetown to Moncton, but only one from Moncton to Charlottetown.

Suggested Resources

Networks and Matrices (10-15 hours)

SCO: In grade 10, students will be expected to

B5 **develop, analyse, and apply procedures for matrix multiplication**

C15 **develop and apply strategies for solving problems**

Elaboration – Instructional Strategies/Suggestions

B5/C15 Matrix multiplication has many uses. Here, students will represent network problems using matrices, and perform matrix multiplication to solve the problems. How matrices are multiplied, however, is not obvious. Students should develop an understanding of this calculation through a context. For example, consider a situation in which a contractor builds four model houses (A, B, C, and D) in three different locations: the North End, the South End, and the East Side. Teachers might ask students to find the total number of doors required for new houses in the North End. Give students tables like these, which detail number of houses and number of doors.

	House models			
	A	B	C	D
North End	10	5	1	2

	Doors
A	2
B	2
C	3
D	3

From interpreting the given information, most students would quickly decide to perform this operation: (10 • 2 + 5 • 2 + 1 • 3 + 2 • 3) = 39. When students are given the South End table, they could also find the total number of doors to be sent to the South End. A series of similar questions leads to a need to organize all the information. The use of labels for rows and columns is critical for students to understand how to place numbers in an answer matrix.

Matrix X gives the quantity of each house model built last year, and matrix Y gives the number of exterior doors and windows in each of the four models.

$$\text{Matrix X}\quad \begin{array}{ll} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{l} \text{North End} \\ \text{South End} \\ \text{East Side} \end{array} & \begin{pmatrix} 10 & 5 & 1 & 2 \\ 5 & 10 & 2 & 5 \\ 6 & 4 & 5 & 3 \end{pmatrix} \end{array} \qquad \text{Matrix Y}\quad \begin{array}{ll} & \begin{array}{cc} \text{Doors} & \text{Windows} \end{array} \\ \begin{array}{l} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 2 & 12 \\ 2 & 20 \\ 3 & 15 \\ 3 & 20 \end{pmatrix} \end{array}$$

If the contractor wants to know how many doors to ship to the North End, he/she should multiply the first row of matrix X by the first column of matrix Y. To find windows to the East Side, he/she would multiply the third row of matrix x by the second column of Y.

$$\underset{X}{\begin{pmatrix} 10 & 5 & 1 & 2 \\ 5 & 10 & 2 & 5 \\ 6 & 4 & 5 & 3 \end{pmatrix}} \bullet \underset{Y}{\begin{pmatrix} 2 & 12 \\ 2 & 20 \\ 3 & 15 \\ 3 & 20 \end{pmatrix}} = \underset{XY}{\begin{pmatrix} 39 & 275 \\ 51 & 390 \\ 44 & 287 \end{pmatrix}}$$

Because each element in row 1 of X represents the number of houses of each model in the North End, and each element in column 1 of Y represents the number of doors in each model, the element in row 1, column 1 of matrix XY represents the total number of doors in the houses in the North End. By extension, the entire matrix XY gives the number of doors and windows for each housing development. Each element in matrix XY is found by multiplying the appropriate row of matrix X by the appropriate column of matrix Y.

Teachers should have students focus on the labels used for rows and columns to bring meaning to the numbers in the answer matrix. For example, element (2, 2) in the answer matrix, which is 390, represents each element of row 2, matrix X, multiplied by each element in column 2, matrix Y, and the products added, giving the total number of windows in the South End.

Networks and Matrices (10-15 hours)

Worthwhile Tasks for Instruction and/or Assessment

B5/C15
Activity

1) a) In many situations involving chance, matrices can be used to represent probabilities, and matrix operations can be used to predict future events. Suppose, for example, that data gathered about the weather in a particular location show that 68% of rainy days are followed by another rainy day and that 35% of days without rain are followed by rainy days. This allows the development of the following matrix relating today's weather with tomorrow's weather. Fill in the percents as decimals.

$$\begin{array}{ll} & \text{Tomorrow} \\ & \begin{array}{cc}\text{Rain} & \text{No rain}\end{array} \\ \text{Today} \begin{array}{r}\text{Rain} \\ \text{No rain}\end{array} & \begin{pmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{pmatrix} \end{array}$$

b) The matrix $\begin{pmatrix}1 & 0\end{pmatrix}$ (yes no) is used to indicate rain today, and the matrix $\begin{pmatrix}0 & 1\end{pmatrix}$ (no yes) is used to indicate no rain today. Now suppose that today is Monday, there is no rain, and the probability of rain for Wednesday is needed. The matrix $\begin{pmatrix}0 & 1\end{pmatrix}$ represents today's weather. Tuesday's forecast is achieved by multiplying $\begin{pmatrix}0 & 1\end{pmatrix}$ by the matrix of percentages in (a) above.

Determine and interpret the result. $\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{pmatrix}$

c) Wednesday's forecast is achieved by a similar calculation using the matrix representing Tuesday's forecast. Determine the probability of rain for Wednesday.

d) What percent chance is there of no rain Wednesday?

e) Explain why in (c) you're multiplying the matrix for Tuesday's forecast by the matrix from (a).

f) If this process continues, what eventually happens to the portion of days on which it will rain?

Suggested Resources

Connecting Mathematics, Addenda Series, NCTM, 1992

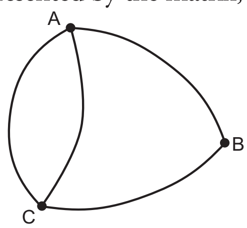
Networks and Matrices (10-15 hours)

SCO: In grade 10, students will be expected to

B5 develop, analyse, and apply procedures for matrix multiplication

Elaboration – Instructional Strategies/Suggestions

B5 Matrix algebra has applications in many areas of mathematics. Here a connected graph may be represented by a matrix with elements representing the number of edges that connect one vertex to another. For example, the graph at left is represented by the matrix, N.



$$N = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \end{array} \qquad N^2 = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{pmatrix} 5 & 2 & 1 \\ 2 & 2 & 2 \\ 1 & 2 & 5 \end{pmatrix} \end{array}$$

If the matrix is squared, the new entries give the number of paths composed of two edges that link one vertex with another. For example, the second entry of the first row of N^2 indicates that there are two paths composed of two edges that connect A to B. (One is A-C-B inside, the other is A-C-B outside.) Once students have shown that they understand N^2, they could try to interpret N^3.

Part of developing an understanding of multiplication of matrices is understanding that the dimensions of the matrices are very important. Students should be involved in activities which encourage them to make conjectures such as:

"Two matrices can be multiplied only if the number of rows in the first matrix is the same as the number of columns in the second matrix, and the number of elements in each row matches with the number of elements in each column."

(Note: The first part of this conjecture is not critical for matrix multiplication, while the second part is.)

They might also conjecture that the answer matrix takes on the dimensions (number of rows in first matrix $\times$ number of columns in second matrix).

For example:

$$\underset{\substack{\uparrow \\ 2\times 3}}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}} \bullet \underset{\substack{\uparrow \\ 3\times 2}}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}} = \underset{\substack{\uparrow \\ 2\times 2}}{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

Networks and Matrices (10-15 hours)

Worthwhile Tasks for Instruction and/or Assessment

B5

Performance

1) Have students explain in their own words when it is possible and when it is not possible to multiply two matrices together.
2) Ask students to describe what type of matrices can be squared (i.e., multiplied by themselves).

B5

Pencil/Paper

3) a) For the following matrices, find the answer matrix and record the dimensions below each matrix.

i) $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix} \bullet \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{pmatrix} =$

iii) $\begin{pmatrix} 1 & 2 & 3 & -1 \\ 0 & 5 & 2 & 6 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} =$

ii) $\begin{pmatrix} 0 & 2 \\ -1 & -2 \end{pmatrix} \bullet \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix} =$

iv) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \bullet \begin{pmatrix} 2 & 3 \\ 3 & 1 \end{pmatrix} =$

b) From the dimensions recorded, make a conjecture about how to predict the size of the answer matrix.

c) Repeat (a) and (b) for these matrices, if possible. If not, explain why not.

i) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} =$

ii) $\begin{pmatrix} 0 & 1 & 3 \\ 1 & 1 & 4 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} =$

iii) $\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} =$

d) What conjecture can you make about the dimensions and matrix multiplication?

B5

Journal/Pencil/Paper

4) Create two matrices, A and B, to show that $AB \neq BA$, and another pair of matrices, C and D, that show that $CD = DC$. What conclusion can you make from this?

Journal/Performance

5) Have students create a problem that deals with transportation between five towns or cities; represent it with a graph and a matrix; cube the matrix and interpret the resulting matrix; then explain how their graphs justify their interpretation.

Suggested Resources

Networks and Matrices (10-15 hours)

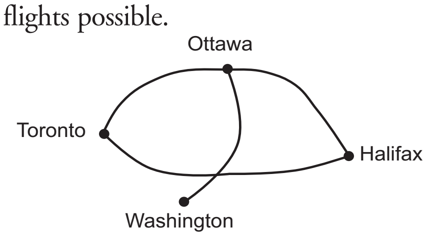
SCO: In grade 10, students will be expected to

B6 **solve network problems, using matrices**

B5 **develop, analyse, and apply procedures for matrix multiplication**

Elaboration – Instructional Strategies/Suggestions

B6/B5 Students will represent network problems, using matrices, and perform multiplication on the matrices to solve problems. For example, given the network below showing local airline flights, if "0" represents no flight connection and "1" represents a connection, students should be able to write matrix N representing all flights possible.



$$\begin{array}{c} \\ T \\ O \\ H \\ W \end{array}\begin{array}{c} \begin{array}{cccc} T & O & H & W \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

N^2 is the result of multiplying: [N] [N] $[N]^2$

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \bullet \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 3 & 1 & 0 \\ 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

Students should be expected to interpret the meaning of matrix N^2.

To help students do this, have them do a simpler activity. For example, build a 3×3 matrix, A, and label it with Hamilton, Toronto, and Ottawa. Have students make a corresponding digraph.

$$\begin{array}{c} \\ T \\ O \\ H \end{array}\begin{array}{c} \begin{array}{ccc} T & O & H \end{array} \\ \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \end{array} = A$$

Then, have students find A^2: $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

In A^2 the (1,1) entry is 2. This represents the number of ways to get from Toronto (row 1) to Toronto (column 1). Looking at the digraph, students should see a flight from Toronto to Ottawa and one from Ottawa to Toronto; these combine for one of the two Toronto-to-Toronto one-stopover (or two-leg) flights. The other is Toronto to Hamilton and Hamilton to Toronto – a second, one-stopover flight. Thus, A^2 represents one-stopover (or two-leg) flights.

Returning to the original problem, have students compare N^2 to the digraph for N and find the one-stopover flights. For example, there is a 3 in the N^2 matrix. What does it tell students? (Ans: Since it is in the second row and second column, it must mean there are three, one-stopover flights from Ottawa to Ottawa. The digraph shows that they are Ottawa to Toronto and back, Ottawa to Washington and back, and Ottawa to Halifax and back.)

❑ The digraph shows Washington to Ottawa, then on to Halifax as the only one-stopover flight from Washington to Halifax. Where is this value represented in the matrix N^2?

Networks and Matrices (10-15 hours)

Worthwhile Tasks for Instruction and/or Assessment

B6/B5

Performance

1) Refer to the transportation network between four towns, A, B, C, and D, which is presented at the top of page 68. Ask students to use their calculators to find R^2 and interpret elements (3, 1), (2, 3), and (1, 2).

$$R = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{pmatrix}$$

2) $$M = \begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 1 \\ 1 & 5 & 0 \end{pmatrix}$$ Matrix M represents the number of direct flight paths between three cities.

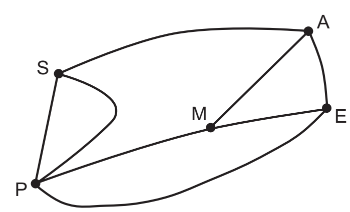
a) Interpret the meaning of the element in i) row 1, column 1
ii) row 3, column 2

b) Draw a graph to represent the matrix.

c) Explain the meaning of the element in row 2, column 3 of M^2.

Pencil/Paper

3) The roads between some towns in rural New Brunswick are displayed in this network.



Ask students to do the following:

a) Represent the network with a matrix, using a “0” for no connection, a “1” for one connection, etc.

b) Show on the graph the three ways to go from S to M, each passing through one other town. Show, using matrices, how this can be verified.

c) Doris, who is in town E, wants to get to town M, but wants to pass through one other town before getting to M. Produce a matrix that will show the answer to how many possible routes she can take.

d) Create a problem for which it would be important to know the number of routes through one town before getting to another. Solve the problem, then pass the problem to your partner to see if he/she can solve it.

Suggested Resources

Networks and Matrices (10-15 hours)

SCO: In grade 10, students will be expected to

C17 solve problems, using graphing technology

Elaboration – Instructional Strategies/Suggestions

C17 Students should develop an understanding of matrix multiplication by working through problems and activities by hand, using matrices with small dimensions. Once they understand matrix multiplication, they need to explore this procedure, using technology. This is very important when the dimensions of matrices get quite large, because multiplication by hand becomes very tedious, and it is easy to make errors.

Students should enter data into matrices, using technology, and then use the technology to multiply and square matrices. To square a matrix, students can use the x^2 key, or the caret (^) and the 2.

Students should recognize error messages and learn how to interpret them.

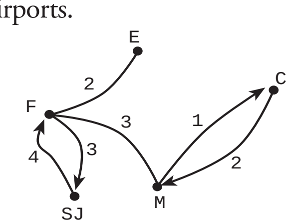
Networks and Matrices (10-15 hours)

Worthwhile Tasks for Instruction and/or Assessment

B5/C17

Performance

1) Ask students to do the following with respect to the network sketched below:
   a) Represent this network of direct flights between airports with a matrix. (Note: The numbers indicate the number of direct flights between connected airports.)
   b) Explain how Henry can tell from the graph that there are 12 one-stopover flights from Saint John to Fredericton to Saint John. Show, using matrices, how this can be verified.
   c) Elaine is in Charlottetown and wants to travel to Edmundston.
      i) Can she get there with a direct flight? Explain.
      ii) Can she get there with a one-stopover flight? Explain.
      iii) Can she get there with a two-stopover flight? Explain.
      iv) Show a matrix that represents all the two-stopover flights available for all airports.



Suggested Resources

Unit 3
Patterns, Relations, and Equations
(25-30 Hours)

In this unit students will examine situations to identify patterns and relationships, and represent these relationships in multiple ways (i.e., with words, concrete models, pictures, tables, graphs and equations). The core curriculum places particular emphasis on the construction and analysis of tables, graphs and equations to determine some of their key characteristics and to use them to solve problems. Key concepts and skills include the nature of linear and quadratic relations, rate of change and slope, and solving linear and quadratic equations graphically and algebraically.

Extensive suggestions for possible differentiation are provided in this unit. These focus primarily on more emphasis on concrete examples and techniques, less abstraction, and reduced technical analysis and symbol manipulation. Notes regarding differentiation are found on pp. 80, 104, 118 and 122. As well, detailed alternatives for portions of the unit are found on pp. 86-93 and 110-113.

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10, students will be expected to

C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

C8 identify, generalize, and apply patterns

C1 express problems in terms of equations and vice versa

C2 model real-world phenomena with linear, quadratic, exponential, and power equations, and linear inequalities

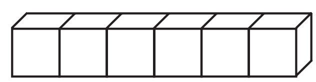
Elaboration – Instructional Strategies/Suggestions

The initial portion of this unit involves representing situations in various ways (e.g., words, tables, graphs and equations), analyzing and interpreting these representations, and using them to answer questions, make predictions and/or solve problems. Pages 80-85 present a development of these ideas and techniques for the core curriculum. Some students, however, may find it beneficial to spend more time with concrete models and numerical situations before generalizing with symbols (equations), and be best served by placing less emphasis on technical analysis. Consequently, for purposes of possible differentiation, the opening portion of the unit is presented in a second form on pages 86-93.

C10 Research in learning shows that observations of patterns and relationships (which may be represented in context, concretely, pictorially, symbolically or verbally) lie at the heart of acquiring deep understanding in many areas of mathematics – algebra and function in particular. Mathematical tasks such as *Activity* 1) on the page opposite help develop students' algebraic thinking and reasoning, and can lead to student insights and invention that exceed our expectations.

C8 Many situations provide opportunities to generalize and represent mathematical ideas and processes. For example, the cube investigation (*Activity* 1) opposite) offers a geometric context from which mathematical ideas can be developed. Algebra is a way to represent and generalize these ideas.

Six cubes have been placed end to end on a flat surface to form a "train" with six "cars". Students are to count the visible faces (the bottom faces are not visible) and record in a table:



number of cubes	1	2	3	4	5	6
# visible faces	5	8	11	1	17	20

C8/C1/C2 When asked to describe patterns, many students will identify the horizontal pattern, i.e., that the numbers increase by 3 each time. They should be encouraged to explain how this is visible in the train. This should result in having them describe the pattern in this way, "...as each cube is added to the train, three more visible faces result..."

Students should be particularly encouraged to describe the vertical pattern, or the pattern that relates the two sets of numbers. Some may describe this as "...three times the number of cars, add two." Again students should relate this to the train – "each additional car adds three more visible faces, and two visible faces (one at each end of the train) are always visible."

The writing prompt in *Activity* 1f) gives all students an opportunity to describe the patterns in their own words. Exchanging written statements (in g)) forces students to interpret another's writing, thus encouraging the writer to be as clear as possible. Changing the words into symbols (in part i)) should occur naturally ($f = 3c + 2$).

Continued...

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C10/C8/C1/C2/C5/C9/C16/C28

Activity/Performance

1) a) Begin with a cube shape to represent a car of a train.
   b) Ask the students to count the visible faces. (Do not include the bottom, which should be sitting on a flat surface and, hence, not be visible.)
   c) Ask students to build a train using six cubes and complete this table:

number of cubes	1	2	3	4	5	6
# visible faces						

   d) Ask students to describe any patterns they see in the table and describe how the patterns are visible in the cubes.
   e) Ask students to add five cars to their trains and predict the new value to add to the table.
   f) Ask students to complete the following writing prompt: "If you tell me how many cars are in the train, I can tell you the number of faces by ..."
   g) Have students exchange their written statements and then use the statement written to predict the number of faces of a train with 100 cars.
   h) Ask students to describe and explain their patterns in words, if there are 'c' cars in the train.
   i) Ask students to change the words into mathematical symbols, describing the pattern in an algebraic sentence ($f = 3c + 2$).
   j) Ask students to explain the meaning of the '3' in the equation.
   k) Ask students to explain the meaning of the '2' in the equation.
   l) Ask students to graph the values in the table.
   m) Have students describe where the '3' is in the graph, and its meaning from the graph with reference to the context.
   n) Have students describe where the '2' is in the graph, and its meaning.
   o) Have students explain whether or not they should sketch a straight line through all the data points.

Performance

2) Have students look at their train of cubes from a 'bird's-eye' view. If each cube has a perimeter of 4 units, ask the students what the perimeter of the whole train (6 cubes) would be. Using what they have learned, express the perimeter as an expression with respect to 'c', the number of cars. Graph the relationship between the perimeter of the train and the number of cars. Compare the steepness of the two graphs (in 1) and 2)) and discuss any differences.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C5 **sketch graphs from words, tables, and collected data**

C9 **construct and analyse graphs and tables relating two variables**

C28 **explore and describe the dynamics of change depicted in tables and graphs**

C32 **plot points, given a situation or a table of values, to help determine if a graph is linear**

C16 **interpret solutions to equations based on context**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

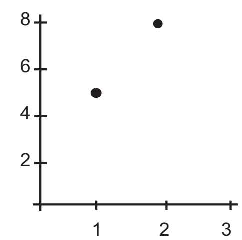
C3 **gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range**

Elaboration – Instructional Strategies/Suggestions

...continued

C5/C9/C28/C32 Students should describe the '3' in the equation f=3c+2 as the number of visible faces for each car. On the graph they should describe the '3' as placing their finger on the first dot, sliding horizontally to below the next dot ("when a car is added to the train") then sliding up to the dot ("the number of visible faces increase by 3"). Students should be encouraged to speak about constant growth, steepness and slope, a ratio of 3:1, and $\frac{\text{rise}}{\text{run}} = \frac{3}{1} = 3$.

C5/C9/C16 The '2' in the equation represents the two "special" visible faces, one at each end of the train. The '2' on the graph requires students to think about the graph extending to the left ("down three, left one"), which would result in a possible dot on the y-axis at 2. Students should discuss the meaning of this on the graph ("for no cars, 2 visible faces") and that this does not make sense, which is why the point was not plotted (restriction on the domain).



A7/C3 There should be discussion about joining the dots and why this would not be appropriate in this context. For example, have students try to interpret what would be implied if there was a point located between the first two points, say (1.5, 6.5). The 1.5 indicates that 'half' a car is added to the train which, of course, is not possible. The conclusion is that this graph is continuous within a restricted domain of natural numbers only, and that it appears as a set of discrete points that have particular meaning within the context of the problem or situation.

Patterns, Relations, and Equations (25-30 hours)

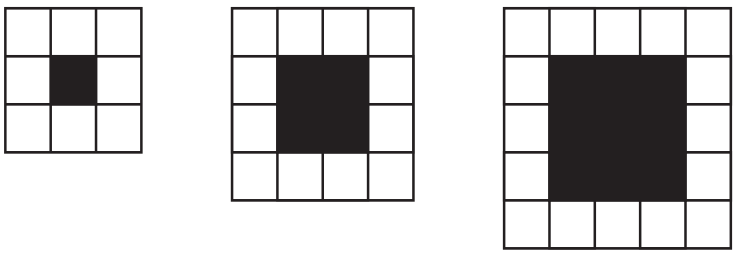
Worthwhile Tasks for Instruction and/or Assessment

...Continued

C5/C9/C28/C32/C16/A7/C3

Performance

3) In the design of a particular set of swimming pools (which we will call 'square pools') and surrounding patios, the water surface of each pool is in the shape of a square. Around the pool there is a border of square-shaped patio tiles. The pools come in many sizes. Here are pictures of the three smallest pools available.



a) Have students (using two-coloured square tiles) build models of the above pools (using red for the water surface and white for the patio tiles), then have them

i) organize data into a table with the following headings:

Pool # (length of one side)	# of red tiles	# of white tiles	total # of tiles

ii) plot graphs of
   a. # of red tiles vs pool #
   b. # of white tiles vs pool #

iii) describe the shapes of the two graphs. How are they the same? How are they different? (Include discussion about the domains.)

b) Have students (using cube-a-links) build towers to model the growth in columns 2 and 3 of the table.

c) Ask students to answer the following question, and explain the answer: When will the number of red tiles and the number of white tiles be the same? Interpret this in the context of the problem.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C10 **describe real-world relationships depicted by graphs, tables of values, and written descriptions**

F6 **solve problems by modelling real-world phenomena**

C9 **construct and analyse graphs and tables relating two variables**

C8 **identify, generalize, and apply patterns**

C1 **express problems in terms of equations and vice versa**

C5 **sketch graphs from words, tables, and collected data**

C2 **model real-world phenomena with linear and quadratic equations**

C16 **interpret solutions to equations based on context**

C25 **solve equations using graphs**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

C15 **develop and apply strategies for solving problems**

Elaboration – Instructional Strategies/Suggestions

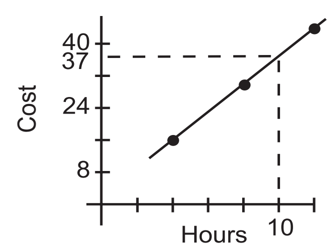
C10/F6/C9/C8/C1/C5/C2 Sometimes the context of a situation states the relationship in words. For example, Ralph rents snowboards for $3.50 per hour, but requires a $2 non-refundable deposit. How much money will it cost you to rent Ralph's snowboard and use it for 2 hours? 3 hours? 6 hours? 10 hours? Students could create a table or make a graph to respond to these questions. From looking at the patterns that develop, or from the wording of the problem, students should be able to state that the total cost = $2 deposit + 3.50 per hour or, in symbols, $c = 2 + 3.50h$. In creating a graph or table it should be clear that, as each hour passes, the rental cost increases by $3.50. Students should be able to see how an increase of $3.50 per hour determines a certain steepness for the graph, and how this connects to the slope of the line.

Students can use the equation to predict answers. For example, the cost of renting for 10 hours would be

$$\begin{aligned} c &= 2 + 3.50\,(10) \\ &= 2 + 35.0 \\ &= 37 \rightarrow \$37.00 \end{aligned}$$

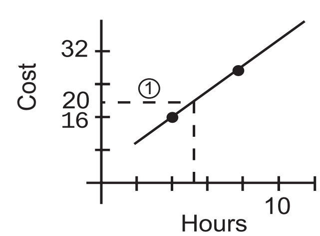
C16 Students may not need the equation to get the solution. Renting for 10 hours means $3.50 times 10, and adding a $2.00 deposit gives $37.00.

C25/F10/C16 Students should also be encouraged to use their graphs to solve. For example, begin at h=10, rise vertically to the model line, move horizontally to the vertical axis to read the value $37.00, then interpret what this means.



A7 Have students explain why it makes sense to join the dots on this graph.

C15/C25/C16 In the process of describing patterns with equations, and using graphs, tables, and equations to interpolate and extrapolate answers, students are developing 'solving' strategies. For example, when predicting how many hours of rental time are available for a cost of $20, students could interpolate from the graph as shown in the diagram. They could also use the equation $c = 2 + 3.5h$, where $c = 20$, and solve $20 = 2 + 3.5h$. Students should examine the graph and try to connect it to the equation $20 = 2 + 3.5h$. The broken horizontal line on the graph (marked 1) represents the equation $c = 20$; have students add this to their graph. The line through the points is the linear pattern $c = 2 + 3.5h$. The point where they intersect has the approximate co-ordinates (5.1, 20). Have students interpret the meaning of this point. (Note: Students might use graphing technology and the intersect feature to find the point of intersection, or use the table feature.)



Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

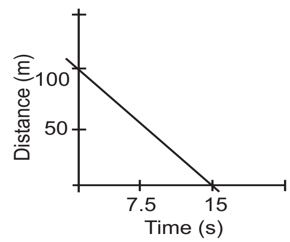
C2/F6/C5/C8/C9/C25/F10/C15

Performance

1) Mary is getting in shape. The first day she does 9 situps, the second day she does 13, the third day 17, the fourth day 21. If she continues in this way, how many situps will she do on the 5th day? 6th day? 10th day? 20th day? 50th day? 60th day?
What restrictions come into play as this pattern continues?

C16/F10/C25

2) The graph at right represents the relationship between time and distance from the finish line in a race. Respond to the following prompts, as Belinda comes into view, approaching the finish line.
   a) How far is she from the finish line when she comes into view? Explain how you know.
   b) Does she increase her speed near the end (sprint to the finish)? Explain how you know.
   c) How long did it take Belinda to complete the last 100 m? Explain.
   d) With five seconds left, how much further did she have to run?
   e) How long did it take her to do the last 60 m? Explain how you know.
   f) Assuming she did not change her speed, how long did it take her to complete the last 150 m? Explain how you know.



C10/F6/C1/C2

Pencil/paper

3) The Save-More gasoline station attracts customers by offering coupons worth \$0.04 on every \$1.00-worth of gasoline purchased.

Value of gasoline purchase, v (\$)	1	2	■	10	■
Value of coupons, c (\$)	■	■	0.24	■	0.60

   a) Copy and complete the table.
   b) Express in words the relation between the value of coupons and the value of gasoline purchased.
   c) State the equation.

C1/C2

4) Given the equation $y = \frac{1}{2}x + 5$,
   a) ask students to describe this relation in words.
   b) ask students to make up a problem which this equation could be used to solve.

C1/C2/F6/C5/C8/C9/F10/C16/A7/C25

Portfolio

5) A taxi cab charges the rates shown in the accompanying table.

length of trip (km)	5	10	15
total cost (\$)	9.25	15.50	21.75

   a) Plot these points on a co-ordinate system.
   b) Discuss if these points should be joined.
   c) Determine the equation.
   d) Explain why the graph does not start at the origin.
   e) Based on the equation, predict how much 7 km and 30 km rides will cost.
   f) Explain the significance of the slope of the line.
   g) From the graph find the length of a trip which costs \$25.

C25

Journal/Pencil/Paper

6) Manny missed class on Friday. Ask students to write to Manny, explaining how to solve $-5n + 3 = 13$ using graphs.

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C10 **describe real-world relationships depicted by graphs, tables of values, and written descriptions**

C2 **model real-world phenomena with linear and quadratic equations**

C8 **identify, generalize, and apply patterns**

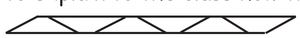
Elaboration – Instructional Strategies/Suggestions

The next eight pages present a possible differentiation of the preceding material for some students. See the note on page 80 for details.

C10/C2/C8 Research in learning shows that observations of patterns and relationships (which may be represented in context, concretely, pictorially, symbolically or verbally) lie at the heart of acquiring deep understanding in many areas of mathematics – algebra and function in particular. Mathematical tasks such as *Activity* 1) on the page opposite help develop students' algebraic thinking and reasoning, and can lead to student insights and invention that exceed our expectations.

Many situations provide opportunities to generalize and represent mathematical ideas and processes. For example, the pattern block investigation (*Activity* 1) opposite) offers a geometric context from which mathematical ideas can be developed. Algebra is a way to represent and generalize these ideas.

Using trapezoid pattern blocks to represent cars in a train, students could construct a train of six cars. A number sentence can be used to describe the perimeter of the train, given that one trapezoid has a perimeter of five units. Students are likely to offer several different number sentences. As they present these number sentences, ask them to explain to the class how the number sentence works. For example,

 6 (5) – 5 (2) = P is a number sentence that gives perimeter. There are six cars, each with five units of perimeter, but there are five places where two units are no longer included in the perimeter. Other sentences may include:

4 (3) + 2 (4) = P, where there are four cars in the train, each with three exposed units of perimeter, and two cars each with four units of perimeter; and 6 (3) + 2 = P, where there are six cars in the train, each with three exposed units of perimeter on the sides, and there are two more units of perimeter, one on each end.

Students should then be asked to add two cars to their train and make changes in the number sentences.

$6(5) - 5(2) = P \Rightarrow 8(5) - 7(2) = P$

$4(3) + 2(4) = P \Rightarrow 6(3) + 2(4) = P$

$6(3) + 2 = P \Rightarrow 8(3) + 2 = P$

Students should each explain how the change in the length of the train results in a change in the number sentence. Then tell students to change their number sentences to reflect the perimeter if the train has 20 cars. To answer this students will use the patterns they observe in the recorded data. Have them complete the following writing prompt: "If you tell me how many cars are in the train, I can tell you the perimeter by ..." Have the completed prompts interchanged. Tell the students the train has 100 cars, and have them use the writing prompt to tell the perimeter. If the writing is not clear enough for them, have them return the prompt to the original writer for clarification. Everyone should get the same answer. Next, have students change the number sentences to represent a train with 'n' cars. They can use the patterns observed previously, or the writing prompt, to record algebraic patterns. The three patterns above would eventually become

1) $5n - 2(n - 1) = P$
2) $3(n - 2) + 2(4) = P$
3) $3n + 2 = P$

Have students discuss equivalent equations and the use of variables. Have them each explain their algebraic sentences to make sure they are talking about their variable and the number of cars in the train.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C10/C2/C8

Activity

1) a) Begin with a trapezoid from a pattern block set (or an alternative source) in which the longer side of the isosceles trapezoid is two times the length of each of the other sides.
   b) Ask students to explore the relationships that deal with the side lengths of all the sides.
   c) Ask students to discuss what relationships they discovered.
   d) Ask students what the perimeter of the trapezoid is.
   e) Ask students to build trains of six cars. 
   f) Ask students to write number sentences that represent the perimeter, record their sentences on chart paper at the front of the class, and explain to the class how each sentence makes sense.
   g) Ask students to add two cars to their trains.
   h) Ask students to change their number sentences so that they work for the new trains, and explain them.
   i) Ask students to complete the following writing prompt: "If you tell me how many cars are in the train, I can tell you the perimeter by ..."
   j) Have them use the prompt of another student to tell the perimeter of a 100 car train.
   k) Ask students to change their number sentences to work for trains in which there are 'n' cars, and explain their new sentences.
   l) Ask students to explain why all the sentences are the same (equivalent), even though they look different.

Portfolio

2) Ask students to repeat the above activity using the square shape, and then again using the hexagon shape. Have them describe what is different and what is the same.

Suggested Resources

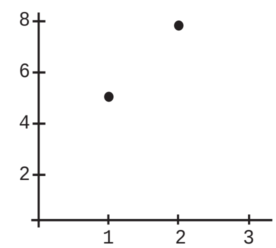
Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

- **C2** model real-world phenomena with linear and quadratic equations
- **C9** construct and analyse graphs and tables relating two variables
- **C3** gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range
- **C28** explore and describe the dynamics of change depicted in tables and graphs
- **C32** plot points, given a situation or a table of values, to help determine if a graph is linear
- **A7** demonstrate and apply an understanding of discrete and continuous number systems
- **C8** identify, generalize, and apply patterns

Elaboration – Instructional Strategies/Suggestions

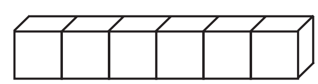
C2/C9/C3/C28/C32/A7/C8 In the trapezoid activity described on the previous pages, students may have obtained several different-looking equations. Each of these equations can be simplified to 3n + 2 = P (see pg 86). Students should graph P = 3n + 2. They should talk about why the number of cars should go on the horizontal axis (independent variable) and the perimeter should go on the vertical axis (dependent variable). Students might plot the points by beginning with "for one car the perimeter is five units" and put a point at (1, 5). "For two cars the perimeter is ...", and so on. The teacher should discuss with the students what is happening between each pair of points on the graph. Pointing at (1, 5), the teacher might ask a student to explain what is happening in the context as the teacher moves to (2, 8). The student should talk about adding another car to the train, the perimeter changing from five to eight, and why that happens.



Students and teachers should discuss the linearity of the points and that the change from one to another is constant (perimeter increases by three units with each additional car). The teacher might discuss how the constant change of the three perimeter units shows up in the equation. Have students understand that the 3 is connected to the 3 'exposed' perimeter units on each car of the train. If students are tempted to join the points, ask them what these 'new' points would represent in the context. Teachers might discuss with students the perimeter of the train if no cars are in the train, but also why this point is not on the graph.

If the trapezoid shape is changed to a square shape, the equation changes to 2n + 2 = P; a regular hexagon gives 4n + 2 = P. Students can discuss steepness of the related graphs with respect to these three situations. They can talk about the '3', the '2', and the '4' in the equations, and their meaning on the graph and in the train (exposed perimeter units per car).

Consider having students construct cube trains by placing cubes end to end on a flat surface. Students are to count the visible faces (the bottom faces are not visible) and record in a table:



number of cubes	1	2	3	4	5	6
# visible faces	5	8	11	14	17	20

When asked to describe patterns, many students will identify the horizontal pattern, i.e., that the numbers increase by 3 each time. They should be encouraged to explain how this is visible in the train. This should result in having them describe the pattern in this way: "...as each cube is added to the train, three more visible faces result..."

Patterns, Relations, and Equations (25-30 hours)

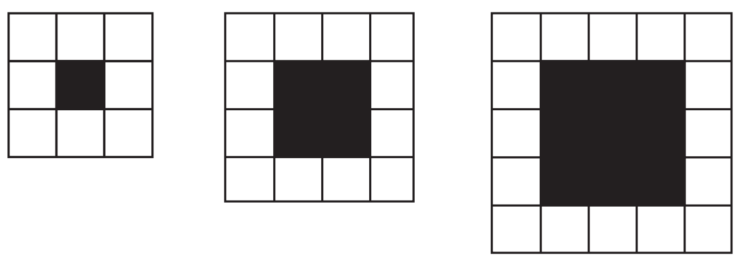
Worthwhile Tasks for Instruction and/or Assessment

C2/C9/C3/C28/C32/A7/C8

Performance

Have students do the following:

1) a) Continue the trapezoid activity from page 87.
   b) Construct a graph showing a relationship between the number of cars and the perimeter of the train.
   c) Each time you move your pencil from one point to the next, describe what is happening in the situation. Is it the same occurrence no matter what point you start with? Explain.
   d) $P = 3n + 2$ describes the perimeter of a train with 'n' cars. How does the 3 show up on the graph? How do you see the 3 in the train of cars?
   e) Explain why you cannot join the dots on the graph.
   f) Use your graph to predict the perimeter when there are 12 cars.
   g) Use your graph to predict how many cars give a perimeter of 32 units.
2) In the design of a particular set of swimming pools (which we will call 'square pools') and surrounding patios, the water surface of each pool is in the shape of a square. Around the pool there is a border of square-shaped patio tiles. The pools come in many sizes. Here are pictures of the three smallest pools available.



   a) Have students (using the two-coloured square tiles) build models of the above pools (using red for the water surface and white for the patio tiles), then have them
      i) organize data into a table with the following headings:

Pool # (length of one side)	# of red tiles	# of white tiles	total # of tiles

      ii) plot graphs of
         a. # of red tiles vs pool #
         b. # of white tiles vs pool #
      iii) describe the shapes of the two graphs. How are they the same? How are they different? (Include discussion about the domains.)
   b) Have students (using cube-a-links) build towers to model the growth in columns 2 and 3 of the table.
   c) Ask students to answer the following question, and explain the answer: When will the number of red tiles and the number of white tiles be the same? Interpret this in the context of the problem.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

- **C1** express problems in terms of equations and vice versa
- **C2** model real-world phenomena with linear and quadratic equations
- **F6** solve problems by modelling real-world phenomena
- **C9** construct and analyse graphs and tables relating two variables
- **C5** sketch graphs from words, tables, and collected data
- **C8** identify, generalize, and apply patterns
- **C10** describe real-world relationships depicted by graphs, tables of values, and written descriptions
- **F10** use interpolation, extrapolation, and equations to predict and solve problems

Elaboration – Instructional Strategies/Suggestions

C1/C2/F6 Previously, students examined patterns and developed equations to represent the relationships found in the patterns. For example, from the number sentences

$$\left.\begin{array}{l} 6\,(5) - 5\,(2) = P \\ 4\,(3) + 2\,(4) = P \\ 6\,(3) + 2 = P \end{array}\right) \quad \text{generalizations can be made} \quad \left(\begin{array}{l} 5n - 2\,(n - 1) = P \\ 3\,(n - 2) + 2\,(4) = P \\ 3n + 2 = P \end{array}\right.$$

C9/C5/C8 Students are expected to create or complete tables of values and to analyse tables for patterns. For example, in the "square swimming pool" situation (see page 89), students create a table (like the one to the right) and fill in the numbers. When asked to describe the patterns, students often notice the vertical pattern first; for example, in column 3 the number of white tiles is increasing by steps of four. So, if asked to predict the number of white tiles for pool number 7, students would likely start with the '20' (20 white tiles for pool number 4) and add 4 tiles to it three times, resulting in 32 white tiles for pool number 7.

Pool # (p)	# red tiles (r)	# white tiles (w)
1	1	8
2	4	12
3	9	16
4	16	20
.	.	.
.	.	.
7	49	32

Some students, on the other hand, will describe the pattern that relates the number of red tiles to the pool number. The horizontal pattern would be described as "the number of red tiles is the square of the pool number." It is this pattern that relates the two variables and can be expressed as an equation or a relation. As students look for a pattern relating the pool number with the number of white tiles, they may note (especially if they have the diagram to look at, or the tiles to manipulate) that the pool number and the number of units of perimeter on one side of the square pool are the same. So if they find the perimeter of the pool (pool number times four) and add the four corner tiles, they get the total number of white tiles, i.e., four times pool number + four = number of white tiles. This leads to the equation $w = 4p + 4$.

C10 Sometimes the context of a situation states the relationship in words. For example, Ralph rents snowboards for $3.50 per hour, but requires a $2 non-refundable deposit. How much money will it costs you rent Ralph's snowboard and use it for 2 hours? 3 hours? 6 hours? 10 hours? Students could create a table or make a graph to respond to these questions. From looking at the patterns that develop, or from the wording of the problem, students should be able to state that total cost = $2 deposit + 3.50 per hour, or $c = 2 + 3.50h$.

F10 Students can use the equation to predict answers. For example, the cost of renting for 10 hours would be

$$\begin{aligned} c &= 2 + 3.50\,(10) \\ &= 2 + 35.0 \\ &= 37 \Rightarrow \$37.00 \end{aligned}$$

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C1/C2/F6/C9/C5/C8/C10/F10

Performance

1) Draw some rectangles with perimeter of 18 cm. Create a table with the following headings: length, width, area.
   a) Sketch a graph that shows how the width varies with the length of the base.
   b) Write the equation that represents this relationship.
   c) Complete a table of values to show how the area varies with the length of the base.
   d) Sketch a graph that shows this relationship.
   e) Describe the pattern.
   f) Discuss discrete vs. continuous data in the above situation.

2) The Car Rental Problem
   The Griswald family arrived in Europe for their holiday. At the car rental agency, they are offered two options:
   Option 1: $25/day plus $0.05 per kilometre OR
   Option 2: $60/day with unlimited number of kilometres
   Ask students to use a spreadsheet to examine the costs of the two options for various driving distances. At what distance per day would Option 2 become the better option? If the rate per kilometre was $0.12 and they planned to travel an average of 450 km every other day for 5 days, which option should the family choose? Explain.

3) Mary is getting in shape. The first day she does 9 situps, the second day she does 13, the third day 17, the fourth day 21. If she continues this way, how many situps will she do on the
   a) 5th day? b) 6th day? c) 10th day?
   d) 20th day? e) 50th day? f) 60th day?
   What restrictions come into play as this pattern continues?

Pencil/Paper

4) The Save-More gasoline station attracts customers by offering coupons worth $0.04 on every $1.00-worth of gasoline purchased.
   a) Copy and complete the table.

Value of gasoline purchase, v ($)	1	2	■	10	■
Value of coupons, c ($)	■	■	0.24	■	0.60

   b) Express in words the relation between the value of coupons and the value of gasoline purchased.
   c) State the equation.

5) Given the equation $y = \frac{1}{2}x + 5$,
   a) ask students to describe this relation in words.
   b) ask students to make up a problem which this equation could be used to solve.

Portfolio

6) A taxi cab charges the rates shown in the accompanying table.

length of trip (km)	5	10	15
total cost ($)	9.25	15.50	21.75

   a) Plot these points on a co-ordinate system.
   b) Discuss if these points should be joined.
   c) Determine the equation.
   d) Explain why the graph does not start at the origin.
   e) Based on the equation, predict how much 7 km and 30 km rides will cost.
   f) Explain the significance of the slope of the line.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C16 **interpret solutions to equations based on context**

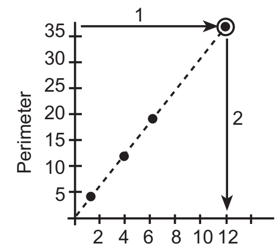
C15 **develop and apply strategies for solving problems**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

C25 **solve equations using graphs**

Elaboration – Instructional Strategies/Suggestions

C16/C15/F10/C25 In the process of describing patterns with equations, and using graphs, tables, and equations to interpolate and extrapolate answers, students are developing 'solving' strategies. For example, when predicting how many cars of the trapezoid train give a perimeter of 38 units, students could extrapolate from the graph as shown in the diagram. They could also use the equation p = 3n + 2, let p = 38, and solve 38 = 3n + 2. However, some students at this level have difficulty manipulating the symbols to solve this equation. They might better begin by examining the graph and trying to connect it to the equation 38 = 3n + 2. The horizontal line on the graph (marked 1) represents the equation p = 38; have students add this to their graphs. The dotted line through the points is the linear pattern 3n + 2 = p. Have students put this on their own graphs. The point where they intersect has the co-ordinates (12, 38). Have students interpret the meaning of this point. (Note: Students might use graphing technology and the intersect feature to find the point of intersection.)

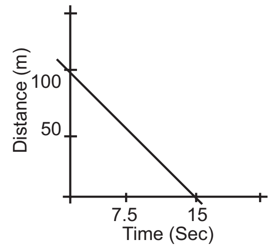


Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C16/F10/C25/C15

Performance



1) The graph at right represents the relationship between time and distance from the finish line in a race. Respond to the following prompts, as Belinda comes into view, approaching the finish line.
   a) How far is she from the finish line when she comes into view? Explain how you know.
   b) Does she increase her speed near the end (sprint to the finish)? Explain how you know.
   c) How long did it take Belinda to complete the last100 m? Explain.
   d) With five seconds left, how much further did she have to run?
   e) How long did it take her to do the last 60 m? Explain how you know.
   f) Assuming she did not change her speed, how long did it take her to complete the last 150 m? Explain how you know.

2) "Avid Rent-a-Car" charges fees for car rental based on the formula $c = 0.12d + 39.95$, where c is the cost of rental in dollars, and d is distance travelled in km.

   "Intrapride" charges are based on the formula $c = 0.14d + 29.95$.
   a) If you had $150, from which agency should you rent? Explain.
   b) If your trip was 1 day and 400 km, which company should get your business? Explain.

Journal/Pencil/Paper

3) Manny missed class on Friday. Ask students how they would explain to Manny how to solve $-5n + 3 = 13$ using graphs.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

B1 **model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations**

B3 **use concrete materials, pictorial representations, and algebraic symbolism to perform operations on polynomials**

A6 **apply properties of numbers when operating upon expressing and equations**

C16 **interpret solutions to equations based on context**

C27 **solve linear and simple radical, exponential and absolute value equations and linear inequalities**

Elaboration – Instructional Strategies/Suggestions

B1/B3/A6/C27 To help students develop an understanding for symbol manipulation, have them model an equation like 38 = 3n+ 2 with tiles (on a balance place mat). Students need to

understand that they should end up with  on one side of the balance, because it represents the variable ‘n’. Whatever is left on the other side represents the value of ‘n’.

First, students must remove the  on the right side. This is done using the zero principle (adding the opposite to both sides).

The symbols to represent removing 2 $\rightarrow$ 38 + (–2) = 3n + 2 + (–2)

Perform the simplification.

Symbols $\rightarrow$ 36 = 3n

Separate into three equal rows, so that division can take place

Symbols $\rightarrow$ $\frac{36}{3} = \frac{3n}{3}$

Read one of the three equal rows

Symbols $\rightarrow$ 12 = n

C16 If the equation arose from a given context, students should relate the result to the context. Students should practise solving equations using concrete materials and recording the correct symbols.

C27 In their previous experiences students have solved equations (like the one above) using tiles, and may be able to solve without the use of tiles. Ultimately, students should be able to solve equations symbolically, without the use of tiles. Let them leave the tiles behind when they show readiness to do so.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

B1/B3/A6/C27

Pencil/Paper

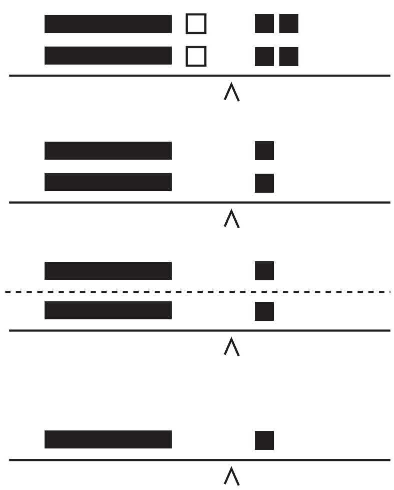
1) Manny said he understands how to use graphs to solve equations, but that he needs help doing it algebraically. Have students use pictures and/or manipulative materials to help Manny solve
   a) $-5n + 3 = 13$
   b) $2(x + 3) = -2$

2) Tricia said that she didn't understand how to model "divide by 3" in the equation $3x = 6$. She said that she understood at $3x = 6$ can be modelled

   Ask students to help her continue so that it is clear how $\frac{3x}{3} = \frac{6}{3}$ is connected to the manipulation of the tiles.

B1/B3/A6/C27

3) Ask students to find the error in the solution below, and show how to correct it. Question: Solve $2(x-1) = 4$
   Solution:



$2(x - 1) = 4$

$2x - 2 - 2 = 4 - 2$

$2x = 2$

$\frac{2x}{2} = \frac{2}{2}$

$x = 1$

C16

4) a) Create a real-life question that can be modelled with the equation given in 3) above.
   b) Explain what the solution means with respect to the context.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C15 **develop and apply strategies for solving problems**

C27 **solve linear and simple radical, exponential and absolute value equations and linear inequalities**

C3 **gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range**

C16 **interpret solutions to equations based on context**

Elaboration – Instructional Strategies/Suggestions

C15/C27 Students should add to their repertoire of problem solving strategies by solving non-routine problems and discussing the various strategies used. It is, therefore, important that open-ended problems with multiple solutions (more than one answer, more than one appropriate strategy) be tackled. As well, it is important that students have opportunities to present their approaches and solutions to the class so that all students can share and experience different ways to solve problems.

One of the strategies that should be continued and reinforced as the year progresses is the algebraic approach. While some students may continue longer than others using tiles, often the tiles will not be a good model to use, due to size of numbers, etc. Encourage students not only to use an algebraic approach, but also to develop proficiency with algebraic manipulation of all kinds. Students should be encouraged to look for patterns and to generalize them in the situations being explored, in order to have equations to solve.

When using an algebraic approach, students should carefully identify the variables before attempting to set up their equation(s). Doing this helps clarify the problem in the students' minds. After solving an equation, students should check the reasonableness of their answer and report it with respect to the problem context.

(Note: Outcome **C27** addresses a variety of equations and inequalities. At this stage students will only be solving linear equations. Other equations and inequalities are solved later in this unit and/or in subsequent units.)

C3/C16 Students will be expected to write equations that represent relevant situations and determine the equations of graphical models of linear relations. Once linear equations are obtained, they will be used to make predications. In modeling situations, students must understand which variable is independent and which is dependent, what domain is suitable, and what the slope and intercepts represent in relation to the context.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C15/C27

Performance

1) Draw some rectangles with a perimeter of 18 cm. Create a table with the headings length and height.
   a) Sketch a graph that shows how the height depends upon the length of the base.
   b) Write the equation that represents this relationship.
   c) Use the equation to find the height, if the length is 5 cm.
   d) Use the equation to find the length, if the height is 6 cm.
   e) Use the equation to find the length, if the height is
      i) two times the length
      ii) three more than twice the length
      iii) two thirds of half the length, less two fifths cm.

2) The Car Rental Problem
   The Griswald family arrived in Europe for their holiday. At the car rental agency, they are offered two options:
   Option 1: $45/day plus $0.12 per kilometre OR
   Option 2: $39.95/day plus $0.15 per kilometre
   a) State both options as equations.
   b) Which option costs less for a trip of
      (i) 50 km? (ii) 150 km? (iii) 300 km?
   c) Which option allows the Griswalds to travel further if they have each of these amounts to spend?
      i) $80 ii) $150 iii) $50

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C18 investigate and find the solution to a problem by graphing two linear equations with and without technology

C25 solve equations using graphs

C15 develop and apply strategies for solving problems

C16 interpret solutions based on context

C17 solve problems using graphing technology

Elaboration – Instructional Strategies/Suggestions

C18/C25/C15/C16 Students will briefly investigate the solution to a problem by graphing two linear equations and determining their point of intersection. (This is dealt with in greater detail in a later unit in this course.)

C18/C25/C17 Students could solve an equation of the form $3(x + 1) = 5(x - 1) + 3$ by graphing each side of the equation and exploring the point of intersection. The intersection x-value satisfies the equation. This procedure would be simplified if explored using the intersect feature of the graphing calculator.

C18 In some contexts students might be given two situations and have to choose the one that better suits their purpose. To do this they might have to graph each and interpret the intersection point. For example, from Ralph's snowboard rental problem (p. 84), students would graph $c = 2 + 3.50\ h$. If Rachel opened a similar business with the cost relationship described as $c = 8 + 1.50\ h$, students could also graph this equation and interpret the intersection point to help them decide from which business to rent the snowboard.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C18/C25/C15/C16

Performance

1) Two taxi companies charge their fares in different ways:
   Taxi A: $2.00 at pick-up, then $1.50 per kilometre travelled.
   Taxi B: $2.50 at pick-up, then $1.40 per kilometre travelled.
   Each of these methods of charging can be expressed as an equation. Have students
   a) state the equation for each
   b) sketch a graph for each
   c) answer the following prompts:
      i) Which taxi would you take to a meeting 6.5 km away? Explain.
      ii) Which taxi would you take to a meeting 4 km away? Explain.
      iii) Which taxi would you take to a meeting 5 km away? Explain.

2) Ralph rents snowboards for $2.75 per hour, plus a $20 flat fee. Rachel rents her snowboards for only $1.50 per hour, but asks for a $50 flat fee. Zeno rents his snowboards for $10 per hour, plus a $5 flat fee.
   a) Interpret the intersection value of Ralph's and Rachel's graphs.
   b) If you were going snowboarding for the weekend, from whom would you rent? Explain.
   c) Under what conditions are you likely to rent from Zeno? Explain.
   d) Make up a rental scheme for your own snowboard business, and ask questions to which your fellow students could respond.

Pencil/Paper

3) Solve the following using graphing technology and explain how you use the technology.
   a) $2(2x - 1) + 5 = 3(x - 2) - 1$
   b) $3 - 2(3x - 2) = 5 + 3(4x - 1)$
   c) $\frac{2}{3}(x - 6) + \frac{1}{5}(2x - 3) = \frac{4}{5}(2x + 3)$

   Where appropriate, express your answers as fractions.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

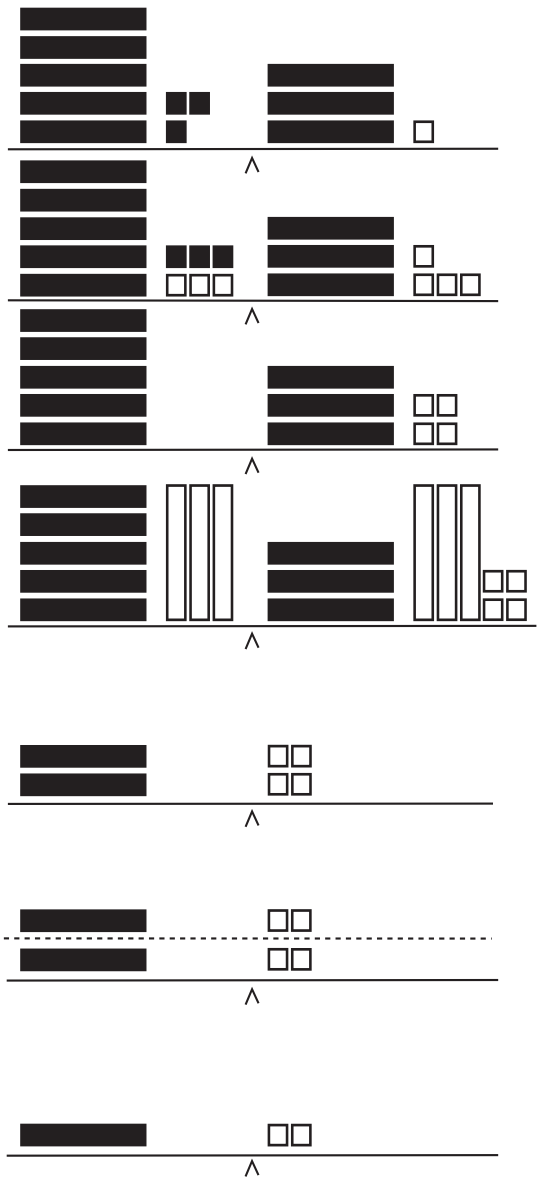
A6 **apply properties of numbers when operating upon expressions and equations**

B1 **model (with concrete materials and pictorial representations), and express the relationships between arithmetic operations and the operations on algebraic expressions and equations**

C27 **solve linear and simple radical, exponential and absolute value equations and linear inequalities**

Elaboration – Instructional Strategies/Suggestions

A6/B1/C27 Students continue to use concrete materials to model more complex equations and manipulate the materials to solve the equations. They should connect the manipulation of the materials to the symbols. For example, the diagram below shows students' work and recordings as they solve the equation $5x + 3 = 3x - 1$.



$5x + 3 = 3x - 1$

$5x + 3 + (-3) = 3x - 1 + (-3)$

$5x = 3x - 4$

$5x + (-3x) = 3x + (-3x) - 4$

$2x = -4$

$\frac{2x}{2} = \frac{-4}{2}$

$x = {}^{-}2$

(See notes on page 94 and 96 about the importance of encouraging students to be proficient with algebraic manipulative skills and to solve using symbols.)

. . . continued

Patterns, Relations, and Equations (25-30 hours)

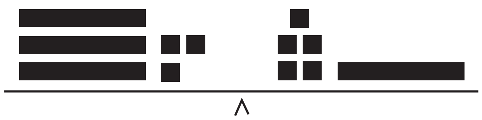
Worthwhile Tasks for Instruction and/or Assessment

B1/A6/C27

Pencil/Paper

1) In the example below, Rita subtracts numbers from both sides differently than Sheila does. Are both methods acceptable? Explain.

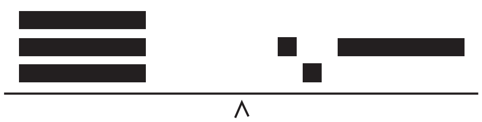
Rita:



3x + 3 = 5 + x

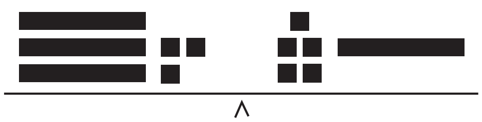
–3 –3

Rita:



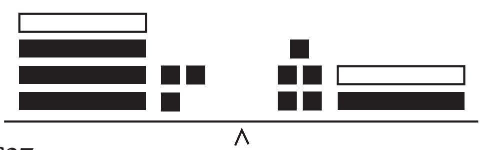
3x = 2 + x

Sheila:



3x + 3 = 5 + x

Sheila:



3x + (–x) + 3 = 5 + x + (–x)

B1/A6/C27

Performance

2) In the question above, Rita and Sheila are solving the equation 3 (x + 1) = 5 + x differently. Ask students to use materials or pictures, and symbols, to complete Rita's solution (using Sheila's step) and to complete Sheila's response (using Rita's step).

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

A6 **apply properties of numbers when operating upon expressions and equations**

B1 **model (with concrete materials and pictorial representations), and express the relationships between arithmetic operations and the operations on algebraic expressions and equations**

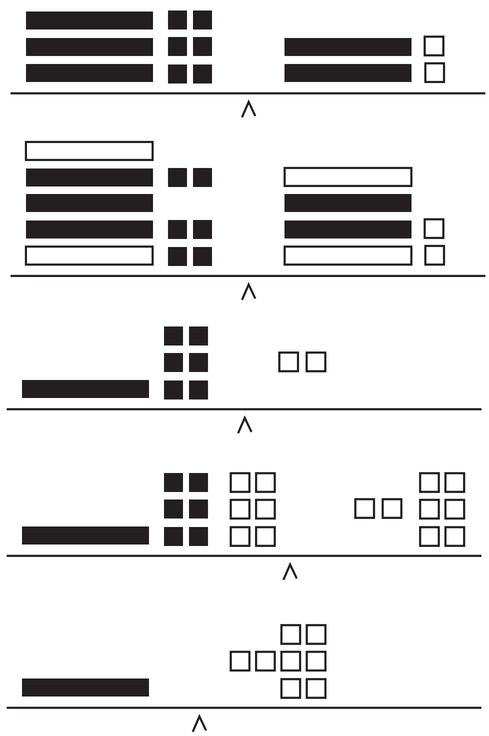
C27 **solve linear and simple radical, exponential and absolute value equations and linear inequalities**

Elaboration – Instructional Strategies/Suggestions

A6/B1/C27. . . continued

This diagram shows a similar equation-solving situation, involving an equation containing bracketed expressions.

3 (x + 2) = 2 (x – 1)



3 (x + 2) = 2 (x – 1)
or
3x + 6 = 2x – 2

3x + (–2x) + 6 = 2x + (–2x) – 2

x + 6 = –2

x + 6 + (–6) = –2 + (–6)

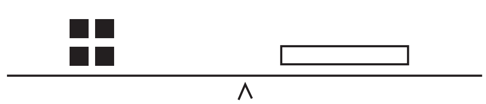
x = –8

Students may sometimes be left with a black x-rod on the right-hand side or a white x-rod either on the right or left-hand side. Have students address this by using tiles and symbols to solve the two situations below:

a) 

Symbols:

[–3x + 3 = 2x – 2]

b) 

[4 = –x]

Since there are several correct ways to complete and record these, students should be encouraged to present different solutions to the class.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

B1/A6/C27

Performance

1) Have students solve the following equations using materials and symbols or pictures and symbols, showing all of their steps.
   a) $3(x + 1) = 6(x - 1)$
   b) $2(x - 2) = 4(x + 1)$
   c) $2(2x - 5) = 3(2 - x)$
   d) $3(5 - 2x) = 2(x - 2) + 1$
2) Explain, using pictures, why $3(2x - 1)$ is the same as $6x - 3$.

A6

Pencil/Paper

3) Sadie wants to take a taxi but is unsure as to which taxi company to call. A brochure describes the fare for Yellow Taxi as $2.50 at pickup, then 5 cents for every 15 seconds the taxi is used. Green Taxi charges $3.00 at pickup, then 3 cents for every 10 seconds the taxi is used.
   a) Which company has the best deal if
      i) Sadie has to travel for about 15 minutes?
      ii) Sadie has to travel for about 30 minutes?
   b) Is there a number of minutes for which both taxis charge the same amount? Explain.

B1/C27/A6

Journal

4) Explain how you finish solving an equation when the step you just completed looks like:
   a) $7 = 1 - 5x$
   b) $3 = \frac{2}{3}x$
   c) $-3x + \frac{1}{2} = 4$
   d) $-5 = \frac{2}{5}(-3) + x$

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

A2 **analyse graphs or charts of situations to identify specific information**

C33 **graph by constructing a table of values, by using graphing technology, and when appropriate by the y-intercept-slope method**

C28 **explore and describe the dynamics of change depicted in tables and graphs**

E8 **use inductive and deductive reasoning when observing patterns, developing properties and making conjectures**

C13 **determine the slope and y-intercept of a line from a table of values or a graph**

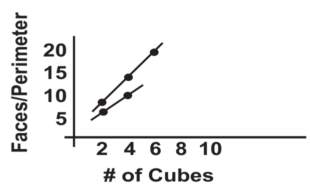
C14 **determine the equation of a line using the slope and y-intercept**

Elaboration – Instructional Strategies/Suggestions

The next portion of this unit addresses the analysis of linear relations and their graphs, as well as solving linear equations by working with corresponding relations. For the core curriculum, the development of these concepts and techniques takes place on pages 104-109. Some students, however, may find it beneficial to focus on the less abstract representations of these mathematical ideas, with reduced emphasis on technical analysis and symbol manipulation; therefore, pages 110-113 are included as an alternative for purposes of possible differentiation.

A2/C33/C28/E8/C13/C14 In the 'cube train' activity (pg. 81), students determined two equations by examining patterns in tables. The number of visible faces depends on the number of cars in the train ($3c + 2 = f$), and the perimeter of the top of the train depends upon the number of cars in the train ($2c + 2 = p$).

When these equations are graphed, students should discuss the meaning, in each case, of the coefficient of c (c representing the number of cars). For example, the number of visible faces can be represented with the equation $3c + 2 = f$ because the 3 represents the three 'exposed' visible faces of each cube in the train. On the graph, the 3 represents the increase in the number of visible faces per unit change in the number of cubes. Students should find that the graph representing the perimeter of the top of the cube train is less steep. The perimeter is increasing by only two units for every unit change in the number of cubes. They should conjecture that the coefficient of the independent variable represents the growth rate of the situation, or the slope of the line.



Similarly, students might conjecture that the '+2' in each of the above equations is the initial value (or y-intercept) if the lines were extended to the left. They should discuss the validity of that possibility.

Sometimes equations can be easily constructed from the words describing a situation. For example, in the snowboarding problem (pg. 84), students determined that since a deposit of $2 is required before even getting the board, the co-ordinate (0, 2) had to be on the graph. Then, each point that followed showed a $3.50 increase in cost (rise $3.50, run 1 hour). So, the steepness or slope is determined by this ratio and the equation can be written as $c = 2 + 3.50h$.

C14/C33 Students should be able to describe the slope of a graph as the ratio of the rise to the run and recognise that it is the coefficient (m) of the independent variable when the equation is in the form $y = mx + b$. The 'b' value, or y-intercept, is the value for y when $x = 0$ and sometimes is the initial value in a situation. From making these connections, students should be able to write equations from graphs, and also use the 'm' and 'b' values to help them graph equations.

$$\text{slope} \Rightarrow \frac{\text{rise}}{\text{run}} \Rightarrow \frac{\text{vertical change}}{\text{horizontal change}}$$

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

A2/C33/C28/E8

Performance

1) A) Use the slope and y-intercept to graph the following, then check using technology.
   - i) $y = 3x + 2$
   - ii) $y = 5x + 2$
   - iii) $y = -1x + 2$

   Respond to the following prompts:

   a) Look at the three graphs. Describe what is the same and what is different.
   b) Look at the three equations. Describe what is the same and what is different.
   c) What is the connection between the equations and the graphs? What affects the steepness of a graph?
   d) What is the y-intercept? Explain.
   e) Make up an equation to test your ideas.

   B) Use the slope and y-intercept to graph the following, then check using technology.
   - i) $y = 2x - 5$
   - ii) $y = 2x - 1$
   - iii) $y = 2x + 3$

   Respond to the following prompts:

   a) Look at the three graphs. Describe what is the same and what is different.
   b) Look at the three equations. Describe what is the same and what is different.
   c) What conjecture might you make?
   d) Make up an equation to test your conjecture.
   e) Since the m–values are all the same, what conjecture might you make about graphs that have all the same slopes?

C13/C14/C33

Pencil/Paper

2) Each table of values for x and y describes a relation. Each equation defines a relation. Ask students to match each table to its corresponding equation, if possible.

a)

x	–2	–1	0	1	2
y	–6	–3	0	3	6

b)

x	–2	–1	0	1	2
y	–7	–4	–1	2	5

c)

x	–1	3	7	11	15
y	5	2	–1	–4	–7

d)

x	–2	–1	0	1	2
y	–7	–2	3	8	13

i) $y = 3x - 1$

ii) $y = 5x + 3$

iii) $4y = -3x + 17$

iv) $y = 3x$

v) $2y = 5x + 3$

3) After Michael collects these data, he must use the table to determine the equation for the line that represents them. Ask students to explain how Michael would do this.

x	12	14	16	18	20
y	35	41	47	53	59

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C32 determine if a graph is linear by plotting points in a given situation

C29 investigate, and make and test conjectures concerning the steepness and direction of a line

C24 rearrange equations

Elaboration – Instructional Strategies/Suggestions

C32 By this time, students should be able to tell if a set of points is linear or not by examining the graph. They should be able to note a pattern in points that are linear, i.e., the ratio of rise to run remains constant. They have discovered the m-value in $y = mx + b$ to be the value that identifies the slope of a line. If many lines have the same m-value, then those lines have the same slope and must be parallel. When an equation can be expressed in $y = mx + b$ form, it is linear. When the ratio of rise to run between successive points is constant, then those points determine a line.

C29 When examining several line graphs, students have noticed that as the m-values change, the steepness of the graphs changes. Horizontal lines have slopes equal to zero, and as the value of m increases from zero, the steepness (from lower left to upper right) continues to increase. When the graph becomes vertical it has an undefined slope (since the run value between successive points is zero, and division by zero is undefined).

Negative slopes behave in a similar way. As the magnitude of the m-values increases, the slope (high in left, low in right) continues to get steeper.

Students should realize that graphs with slopes that are additive inverses (e.g., the graphs of $y = \frac{3}{4}x$ and $y = -\frac{3}{4}x$) are reflections of one another across the y-axis.

C24 Students should be able to transform linear equations into the form $y = mx + b$ for graphing purposes, and for obtaining slopes and y-intercepts. Sometimes contexts require equations to be expressed with integral coefficients, so students should also be able to transform equations of the form $y = mx + b$ to the general form $ax + by + c = 0$.

For example,

1)
$$\begin{aligned} 3x - 2y - 5 &= 0 \\ 3x - 2y - 5 - 3x &= -3x \\ -2y - 5 &= -3x \\ -2y - 5 + 5 &= -3x + 5 \\ -2y &= -3x + 5 \\ y &= \frac{3}{2}x - \frac{5}{2} \end{aligned}$$
[divide by –2]

2)
$$\begin{aligned} y &= \frac{2}{3}x - \frac{1}{2} \\ 6y &= 6\left(\frac{2}{3}\right)x - 6\left(\frac{1}{2}\right) \end{aligned}$$
[multiply each term by 6]
$$6y = 4x - 3 \quad \text{[simplify]}$$
$$6y - 4x + 3 = 0 \quad \text{[rearrange]}$$

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C32/C24

Performance

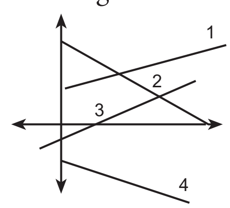
1) Ask students to determine three ordered pairs for each equation (3 pairs will make it more obvious whether it is linear or not), graph the three ordered pairs, conjecture whether they are linear or not, and check their conjectures using graphing technology.

a) $y = x^2 + 2$	f) $y = 4x - 2$
b) $2x = y$	g) $\frac{2}{x} = y$
c) $3x^2 - 2x + 1 = y$	h) $y + 3 = 0$
d) $y = \frac{1}{2}x + \frac{5}{2}$	i) $y = \frac{1}{2} - x$
e) $y = 2 - x^3$	j) $y = \sqrt{2} - x$

C29/C32

Interview

2) Have students match each line segment in the graph to one of the given expressions.
   a) positive growth rate, positive initial value
   b) negative growth rate, smallest initial value
   c) positive growth rate, negative initial value
   d) negative growth rate, positive initial value



C32

Pencil/Paper

3) Erica saves \$2 in one week, \$4 the following week, \$6 the next week, and so on for a number of successive weeks. Ask students to
   a) complete the table

Week, *w*	1	2	3	4	5	6	7
Total Savings, *t* (\$)	2	6	■	■	■	■	■

   b) draw the graph
   c) determine if the graph is linear and explain why/why not.

Portfolio

4) a) Ask students to describe what happens to the graph of the equation $y = mx + 1$ as the value of m changes.
   b) Ask students what they think would be the slope of i) a vertical line and ii) a horizontal line. Explain in each case.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C25 solve equations using graphs

C31 graph equations and inequalities and analyse graphs both with and without graphing technology

C27 solve linear and simple radical, exponential and absolute value equations and linear inequalities

C9 construct and analyse graphs and tables relating two variables

E4 apply transformations when solving problems

C17 solve problems using graphing technology

C15 develop and apply strategies for solving problems

Elaboration – Instructional Strategies/Suggestions

C25/C31/C27/C9 Students have learned that, when a linear equation is expressed in the form $y = mx + b$, 'b' represents the y-intercept. Consequently, when stating the y-intercept of a line, students need only read the 'b' value from the equation in $y = mx + b$ form. They have learned that this value is also sometimes called the initial value. They should understand that it is the value of y when $x = 0$ (thus, it could be found by substituting $x=0$ into the equation).

Students should also understand that the x-intercept can be found algebraically, by substituting $y=0$ into the equation. The x-intercept is not "visible" in some linear equations.

Students should understand that they can solve equations by using the x-intercept on graphs. For example, when asked to solve $3(2x – 5) + 2 = 2(2x – 1) + 4$, students could simplify and rearrange this equation to equal zero:

$6x – 15 + 2 = 4x – 2 + 4$

$6x – 4x – 13 = 2$

$2x – 15 = 0$

They could then graph $y = 2x – 15$ and locate the x-intercept (we let $y = 0$) to solve the equation. Students could also solve this equation by graphing the left side as one relation, the right side as another, and finding the x-value of the point of intersection.

E4 Students should also understand that the same equation could be expressed as $2x – 13 = 2$ and this could be solved by graphing $y = 2x – 13$ and translating the x-axis up to $y = 2$. Students would then read the intercept of the translated x-axis to get the solution.

C15/C17 Students should learn to use technology to solve equations by evaluating x-intercepts. For example, on a TI-83 students could trace to find the x-intercept, or they could use 1:value or 2:zero in the CALC menu.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C25/C31/C27/C9/E4/C17/C15

Activity

1) The cost of printing the school newspaper can be represented by the equation $c = 23 + .25p$, where p is the number of pages and c is the cost in dollars.
   a) Which variable is the independent variable? Why?
   b) Graph the equation.
   c) From the graph, read the p-intercept and interpret its meaning.
   d) From the graph, find the number of pages used if the cost was $28.
   e) Solve the equation $.25p + 23 = 32.5$ using the graph. Interpret the solution.
   f) Make a statement about the values in the domain of this equation.
   g) Make up an equation or a situation, and pose a question or two, such that your partner will have to use his/her knowledge of p-intercepts to solve.

Pencil/Paper

2) a) Use a graph to solve each of the following. Do each in more than one way. Explain how you did each one, each way.
      i) $1.3 (2m - 1) - 2.7 (3m - 5) = (5.1 (-2m + 1.3))$
      ii) $\frac{2}{5}\left(10x + \frac{2}{3}\right) - \frac{3}{4}\left(6x - \frac{1}{3}\right) = \frac{2}{3}\left(\frac{3}{4}\ x + 2\right)$
   b) How many ways are there of solving these equations?
   c) Use a symbolic approach to check the answers.

3) Ask student to draw a graph of each of
   a) $y = \frac{-3}{4}x + 6$
   b) $y = \frac{2}{3}\ x - 2$

   and use the graphs to solve each when
      i) $y = 0$
      ii) $y = -2$
      iii) $y = 4$

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

A2 **analyse graphs or charts of situations to identify specific information**

C3 **gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range**

C2 **model real-world phenomena with linear and quadratic equations**

C13 **determine and interpret the slope and y-intercept of a line from a table of values or a graph**

C14 **determine the equation of a line using the slope and y-intercept**

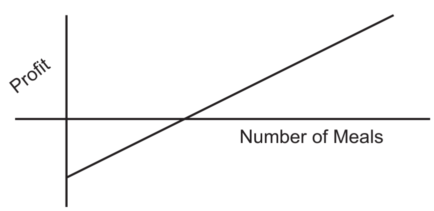
C32 **plot points, given a situation or a table of values, to help determine if a graph is linear**

Elaboration – Instructional Strategies/Suggestions

The next four pages present a possible differentiation of the material on pages 104-109. See the note on page 104 for details.

A2/C3/C2 As described earlier (p. 86) students should have opportunities to graph relationships and discuss where the points appear on the graph and whether or not that placement can help determine a description of the relationship. Studying graphs of discrete data presents opportunities to talk about why the domains and ranges in some contexts only include whole numbers or integers. Why is it that sometimes points are joined on a graph? How does joining the points relate to the kinds of numbers in the domain and range? Appropriate domains and ranges for situations should be discussed while working with linear and non-linear models, and while using the graphing calculator's window feature. For example: The profit from a restaurant can be shown using a graph based on the number of meals served.

a) Why can negative numbers be included on the y-axis (in the range)?



b) What is the significance of the point where the graph crosses the horizontal axis?

c) The graph implies that the situation uses continuous data. Is that the case? Explain.

C13/C14/C3/C32 Students could also be given the table of values for a situation like the above and use it to determine the slope and y-intercept. They should first determine whether or not the points are linear. This can be done by plotting the points onto a graph to see if it looks linear. They should notice that as the number of meals increases by 5 each time, the profit increases by $50 (i.e., that the rate of growth is constant).

Number of meals	Profit
5	–150
10	–100
15	–50
25	50
30	100

By extending the pattern, students could add to the table 0 number of meals and the -$200 profit which it gives. This would determine the y-intercept. Then, taking two convenient points, determine the slope:

slope = $\frac{\text{rise}}{\text{run}} = \frac{100 - 50}{30 - 25} = \frac{50}{5} = 10$

From this they should form the equation p = 10m – 200. They should discuss why it is appropriate to include negative real numbers in the range, but not in the domain.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

A2

Performance

1) Using graphing technology, graph the following:
   i) $y = 3x + 2$
   ii) $y = 5x + 2$
   iii) $y = -1x + 2$

   Then ask students to respond to the following prompts:

   a) Look at the three graphs. Describe what is the same and what is different.
   b) Look at the three equations. Describe what is the same and what is different.
   c) What is the connection between the equations and the graphs? What affects the steepness of a graph.
   d) What is the y-intercept? Explain.
   e) Make up an equation to test your ideas.

Portfolio

2) a) Ask students to describe what happens to the graph of the equation $y = mx + 1$ as the value of m changes.
   b) Ask students what they think would be the slope of i) a vertical line and ii) a horizontal line. Explain in each case.

C13/C14

Pencil/Paper

3) Each table of values for x and y describes a relation. Each equation defines a relation. Ask students to match each table to its corresponding equation, if possible.

a)

x	−2	−1	0	1	2
y	−6	−3	0	3	6

b)

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

c)

x	−1	3	7	11	15
y	5	2	−1	−4	−7

d)

x	−2	−1	0	1	2
y	−7	−2	3	8	13

i) $y = 3x - 1$
ii) $y = 5x + 3$
iii) $4y = -3x + 17$
iv) $y = 3x$
v) $2y = 5x + 3$

C2/C3/C32

Pencil/Paper

4) Erica saves $2 in one week, $4 the following week, $6 the next week, and so on for a number of successive weeks. Ask students to
   a) complete the table

Week, *w*	1	2	3	4	5	6	7
Total Savings, *t* ($)	2	6	■	■	■	■	■

   b) draw the graph
   c) determine if the graph is linear and explain why/why not.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students wll be expected to

C13 **determine and interpret the slope and y-intercept of a line from a table of values or a graph**

C32 **plot points, given a situation or a table of values, to help determine if a graph is linear**

C14 **determine the equation of a line using the slope and y-intercept**

C28 **explore and describe the dynamics of change depicted in tables and graphs**

C29 **investigate, make and test conjectures concerning the steepness and direction of a line**

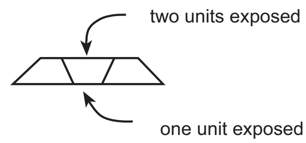
E8 **use inductive reasoning when observing patterns, developing properties, and making conjectures**

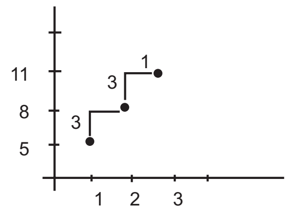
C24 **rearrange equations**

Elaboration – Instructional Strategies/Suggestions

C13/C32 In the 'train' activity (p. 86), students determined that the relationship between perimeter and the number of cars in a train depends on the shape of the car. When the cars are trapezoids, then the relationship can be represented by 3n + 2 = P; when squares, by 2n + 2 = P; and when hexagons, by 4n + 2 = P.

C13/C32/C28/C29/E8 When the above equations were graphed, students discussed the meaning of the coefficient of *n* with respect to the shape of the car. For example, when the car was represented by a trapezoid, the perimeter could be represented with the equation 3n + 2 = p because the 3 represented the three 'exposed' perimeter units of each trapezoid in the train. On the graph, the 3 represented the increase in perimeter per unit change in the number of cars. Students found the graph representing the train made of hexagon shapes to be steeper, because the perimeter was increasing by four units for every unit change in the number of cars. They could have conjectured that the coefficient for the independent variable provides the growth rate of the situation, or the slope of the line.





Similarly, students might conjecture that the '+2' in the above equation is the initial value (or y-intercept) if the line was extended to the left. They should discuss the validity of that possibility.

Sometimes equations can be easily constructed from the words describing a situation. For example, in the snowboarding problem (p. 90) students determined that, since a deposit of $2 is required before even getting the board, the co-ordinate (0, 2) had to be on the graph. Then, each point that followed showed a $3.50 increase in cost (rise $3.50, run 1 hour). So, the steepness or slope is determined by this ratio and the equation can be written as c = 2 + 3.50h.

C14/C32/C24 Students should be able to describe the slope of a graph as the ratio of the rise to the run

$$\text{slope} \Rightarrow \frac{\text{rise}}{\text{run}} \Rightarrow \frac{\text{vertical change}}{\text{horizontal change}}$$

and recognize that it is the coefficient (m) of the independent variable when the equation is in the form y = mx + b. The 'b' value, or y-intercept, is the initial value in a situation. From making these connections, students should be able to write equations from graphs, and also use the 'm' and 'b' values to help them graph equations. Students should also be able to transform linear equations into the form y = mx + b for graphing purposes, and for obtaining slopes and y-intercepts.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C32/C24/E8

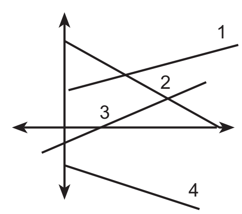
Performance

1) Ask students to determine three ordered pairs for each equation, graph the three ordered pairs, conjecture whether they are linear or not, and check their conjectures using graphing technology.
   a) $y = x^2 + 2$
   b) $2x = y$
   c) $3x^2 - 2x + 1 = y$
   d) $y = \frac{1}{2}x + \frac{5}{2}$
   e) $y = 2 - x^3$
   f) $y = 4x - 2$
   g) $\frac{1}{2} = y$
   h) $y + 3 = 0$
   i) $y = \frac{1}{2} - x$
   j) $y = \sqrt{2} - x$

C29

Interview

2) Have students match each line segment in the graph to one of the given expressions.
   a) positive growth rate, positive initial value
   b) negative growth rate, smallest initial value
   c) positive growth rate, negative initial value
   d) negative growth rate, positive initial value



Paper/Pencil/Journal

C13/C14

3) After Michael collects this data, he must use it to determine the equation for the line. Ask students to explain how Michael would do this. Begin by explaining how Michael knows this data is linear.

x	y
12	35
14	41
16	47
18	53
20	59

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C1 **express problems in terms of equations and vice versa**

C2 **model real-world phenomena with linear, quadratic, exponential, and power equations, and linear inequalities**

C8 **identify, generalize, and apply patterns**

C9 **construct and analyse graphs and tables relating two variables**

C10 **describe real-world relationships depicted by graphs, tables of values, and written descriptions**

C16 **interpret solutions to questions based on context**

C28 **explore and describe the dynamics of change depicted in tables and graphs**

Elaboration – Instructional Strategies/Suggestions

C1/C2/C5/C8/C9/C10/C16 In previous activities students recorded data in tables and looked for patterns in data to try to describe relationships between two variables. One of those situations involved square swimming pools (p. 83) and resulted in students' graphing data (# of red tiles vs. pool #) that they described as non-linear.

Students could go back to that graph (# of red tiles vs pool #) and describe the patterns in more detail. Students will note that the growth rate is not constant, and that the value for the dependent variable gets larger more quickly as the independent variable increases (i.e., the growth rate is not constant). Students might notice that the number of red tiles is the square of the pool number, and so can be recorded in an equation as $r = n^2$. Using this they would be able to predict the area of a pool (number of red tiles), given the pool number (the length of one side of the square pool).

Pool # (n) length of one side	# red tiles (r)	# white tiles (w)
1	1	8
2	4	12
3	9	16
4	16	20
.	.	.
.	.	.
7	49	32

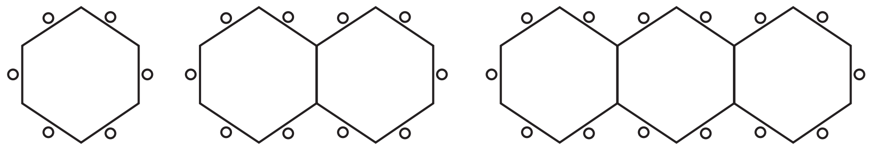
C28 Students might be asked to explore other equations in which the independent variable is squared, or tables that contain quadratic relationships. The purpose of this type of investigation would be to explore the parabolic shape of the graphs of quadratic relationships, and the patterns in corresponding tables of values. Students should realize how the patterns are different from those that are linear, and begin to use them to predict values.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C1/C2/C5/C8/C9/C10/C16

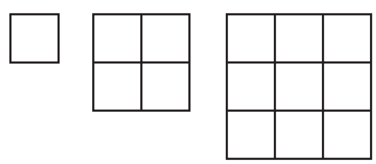
Performance



1) A restaurant has hexagonal tables that each seat 6 people when standing alone. When two tables are joined, they seat 10 people. Ask students to refer to the diagrams above and
   a) create a table relating the number of tables to the number of seats
   b) using the table, predict how many seats there are for 5 tables joined; 10 tables joined
   c) describe the pattern using an algebraic equation
   d) create a graph that shows the relationship between the number of tables joined, and the number of seats at the tables
   e) predict how many joined tables must be used to seat 30 and explain how they got their answer
   f) make up a problem relating to this situation to examine their partner's understanding.

Tables	Seats
1	6
2	10
.	.
.	.

2) A restaurant using square tables arranges them in squares as shown.



   a) Determine an equation that relates the number of square tables and the number of seats along one side.
   b) Predict how many tables are needed to seat 36 people. Explain.

Seats on one side	# of tables
1	1
2	4
3	9
.	.
.	.

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C25 solve equations using graphs

C26 solve quadratic equations by factoring

A5 demonstrate an understanding of the zero product property and its relationship to solving equations by factoring

C35 expand and factor polynomial expressions using perimeter and area models

B3 use concrete materials, pictorial representations, and algebraic symbolism perform operations on polynomials

C20 evaluate and interpret non-linear equations using graphing technology

Elaboration – Instructional Strategies/Suggestions

C25/C26/A5/C35 Students should understand that if they want to solve a quadratic equation they should try to break it down to be represented with linear equations. They do this by factoring. When the product of the factors is zero, then at least one of the factors must be zero. This leads them to both possible roots. So, the factors are connected to the roots (which are 3 and 1 in the example at right). Students must now connect the roots to the x-intercepts (or zeros) on the graph of the corresponding quadratic relation.

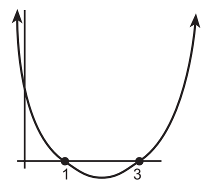
$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$\therefore$ either $x - 3 = 0$ or $x - 1 = 0$

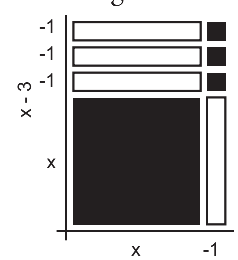
$\therefore$ $x = 3$ or $x = 1$

So, the roots are 3 and 1.



B3/C25/C20/C35 Many students at this level will still benefit from using concrete materials to model the equations (see below and the next two-page spread) and to find the factors. The purpose of the use of tiles is for students to form mental images of the operations, which helps them make connections to the use and manipulation of the symbols. Students strive for mastery of symbolic manipulation, and many of them should, relatively quickly, be ready to move away from dependence on the tiles. Students should also use their graphing technology to help them draw graphs and visualize roots.

diagram 1



- ❐ Ask students to use algebra tiles to model expressions as follows. Ask half the students to build a model using dimensions $(x - 3)$ by $(x - 1)$, as shown in diagram 1. (Note: Some may use dimensions $(x - 1)$ by $(x - 3)$, resulting in a different position of the model.) Ask the other half of the students to model the expression $x^2 - 4x + 3$, as shown in diagram 2, without using any factors. (Ultimately, all students will get the same model.)
- ❐ Ask some students to solve $(x - 1)(x - 3) = 0$, and others to solve $x^2 - 4x + 3 = 0$. All students should graph the corresponding relations for each of these equations. (They should get the same graphs and see the x-intercepts of 1 and 3 on the graphs, as shown at the top of the page.)

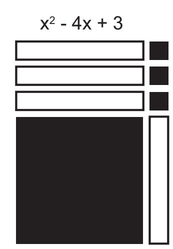


diagram 2

Initially, in the process of solving quadratic equations by factoring and exploring the x-intercepts of their graphs, students should model the expressions in the equations with concrete materials. They should use the materials and the area model to find the factors of the quadratic expressions, and connect these factors to the roots of the equations and zeros of the graphs.

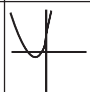
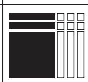
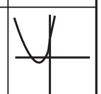
Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

Performance

C25/C26/C35/B3

1) a) Ask students to complete the table below:

expression	pictorial model	factors	function to graph	x-intercept	y-intercept	equation = 0	roots	graph/ picture
$x^2 + 3x + 2$		(x+2) (x+1)	$x^2 + 3x + 2 = y$	–2, –1	2	$x^2 + 3x + 2 = 0$	–2, –1	
$x^2 + 2x + 1$								
		(x+4) (x+1)						
$x^2 + 4x + 4$								
$x^2 – 2x + 1$								
								
				2, 1	2			
			$x^2 – 5x + 4 = y$					
							–1, –3	

Then ask students to respond to the following prompts:

b) Describe the pattern between the factors of the given expression and the x-intercepts of the corresponding graph. Do you think it will always be true? Make up an expression and graph it when it equals zero to test your conjecture.

c) Make a conjecture concerning identifying the y-intercept by looking at a quadratic equation. Test your conjecture.

d) Using the results of (b) and (c) above, state the x- and y-intercepts for each of the following (if possible). If not possible, explain why not.

i) $x^2 – 1 = 0$

ii) $x^2 – 4 = 0$

iii) $x^2 + 1 = 0$

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

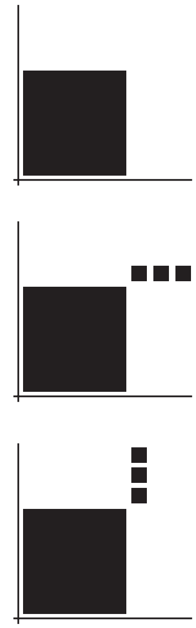
B1 **model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and the operation on algebraic expressions and equations**

B3 **use concrete materials, pictorial representations and symbolism to perform operations on polynomials**

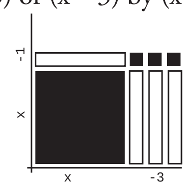
C35 **expand and factor polynomial expressions using perimeter and area models**

Elaboration – Instructional Strategies/Suggestions

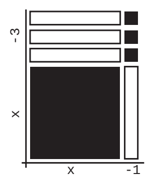
B1/B3/C35…continued



Students should begin to factor using expressions with positive signs only, then move to expressions with negative signs. Early in this process students might factor an expression like $x^2 - 4x + 3$. To model it they should begin with one dark x^2 tile, four white x-tiles and three dark unit tiles and try to construct a rectangle. Once students have some experience, they may use the following systematic procedure. They place the x^2 at the origin in the first quadrant, then place the three unit tiles in the upper right corner. Some may make a 3×1 rectangle with them, some a 1×3 rectangle—both are correct. Then, it is clear where the white x-tiles go to complete the rectangle. The dimensions are read along the axes: $(x - 1)$ by $(x - 3)$ or $(x - 3)$ by $(x - 1)$.

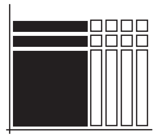


or



As students practise this factoring process, the teacher should lead them to more challenging quadratic expressions. For example, $x^2 + 8x + 12$ would cause more challenge to students, since the placement of the unit tiles could be in several different rectangular arrangements (4×3, 6×2, 12×1), only one of which leads to having eight x-tiles.

Eventually, students should attempt some with negative signs (e.g., $x^2 - 2x - 8$). The challenge now is to place the white unit tiles 8×1 or 4×2, both of which lead to having more than two white (negative) x-tiles. The zero principle must be used to allow the rectangle to be constructed.



2 shaded + 2 white x-tiles = zero

Students should understand that dividing and factoring are closely related processes, the difference being that when dividing, one factor is given (that is, one dimension of the rectangle is provided).

Students should be factoring trinomials of the form $ax^2 + bx + c$, with $a \geq 0$, including special forms that involve perfect square trinomials, common factors (e.g., $(x^2 + 4x) = x(x + 4)$) and difference of two squares (e.g., $(x^2 - 4) = (x + 2)(x - 2)$). Keep the value of 'c' small for easy use of tiles and remember the use of tiles is to help students understand the concept. Once students develop the concept and understand the manipulation of symbols, they may choose to skip the use of tiles. Encourage them to move away from dependence on tiles as you observe their understanding.

For purposes of differentiation, some students may work only with trinomials with a=1.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C35

Performance

1) Have students match the expressions on the left with the equivalent expressions on the right and, for each, explain how they determined the match (tables, graphs, tiles, etc.).

a)	$a^2 - 4a - 5$	i)	$4a\,(2a - 3)$
b)	$a^2 - 3a - 10$	ii)	$(a - 6)\,(a + 1)$
c)	$a^2 + 7a + 6$	iii)	$(a + 1)\,(a - 5)$
d)	$a^2 - 4a + 4$	iv)	$3(a - 4)\,(a + 2)$
e)	$a^2 - 5a - 6$	v)	$4\,(a + 3)\,(a - 3)$
f)	$8a^2 - 12a$	vi)	$(a - 2)^2$
g)	$4a^2 - 36$	vii)	$(a - 2)\,(a + 5)$
h)	$3a^2 - 6a - 24$	viii)	$(a + 6)\,(a + 1)$
		ix)	$(a - 5)\,(a + 2)$

B3

2) Amber is playing a game. For each of the following she must draw an area representation (or build with tiles) to find the matching pairs. Ask students how many matching pairs there are and to explain with pictures or tiles.

a)	$3x - 12$	i)	$x^2 + 2x - 8$
b)	$2x + 1 - (5x + 3) + 2x$	ii)	$x^2 - 9$
c)	$4\,(3x - 1)$	iii)	$3\,(x - 4)$
d)	$-3x\,(2x + 1)$	iv)	$-3x - 6x^2$
e)	$(x + 4)\,(x - 2)$	v)	$(x + 3)\,(x + 3)$
f)	$x^2 + 6x + 9$	vi)	$12x - 4$
g)	$(x + 3)\,(x - 3)$	vii)	$-6x^2 + 1$
		viii)	$9x + 4$
		ix)	$-x - 2$

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

SCO: In grade 10 students will be expected to

C26 **solve quadratic equations by factoring**

B3 **use concrete materials, pictorial representations and symbolism to perform operations on polynomials**

B1 **model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and the operation on algebraic expressions and equations**

A6 **apply properties of numbers when operating upon expressions and equations**

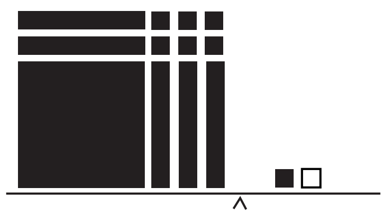
A5 **demonstrate an understanding of the zero product property and its relationship to solving equations by factoring**

Elaboration – Instructional Strategies/Suggestions

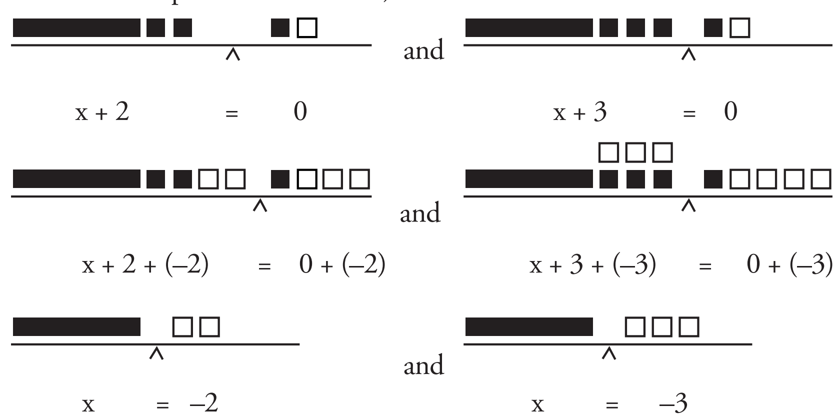
C26/B3/B1/A6/A5 Students should model the solving of quadratic equations, as they did linear, using concrete materials. For example, given the equation $x^2 + 5x + 6 = 0$, they could use the tiles to build the equation on a balance (any representation for zero could be used),

$x^2 + 5x + 6 \quad = \quad 0$

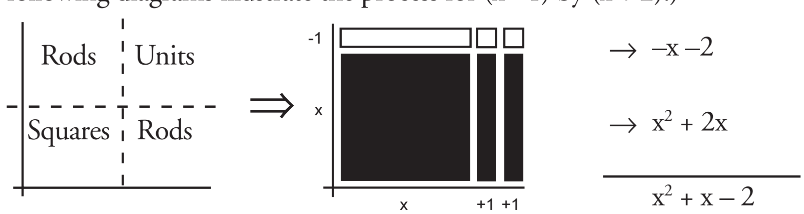
then make a rectangle,



then apply the zero-product property (using the dimensions of the rectangle, or factors of the expression $x^2 + 5x + 6$)



(Note: When students reverse the process, (i.e., expand factors to find the area), have them use the length of the x-tile to trace the length 'x' along the axes, then the width of the x-tile to mark the units. Once the dimensions are indicated, they just have to complete the rectangle. They should be familiar with the idea of what shapes go into what regions based on their experiences with factoring. The following diagrams illustrate the process for (x - 1) by (x + 2).)



Encourage students to move away from dependence on tiles as you observe their understanding of the equation-solving process.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C26/B3/B1/A6/A5

Performance

1) Ask students to solve the following equations using tiles. Have them check their solutions using graphs and/or algebraic methods.
   a) $x^2 - 9x + 8 = 0$
   b) $p^2 - 10p + 16 = 0$
   c) $y^2 = 5y + 14$
   d) $2x^2 = 18$
   e) $x^2 = -8x$
   f) $2x^2 + 2x - 5 = x^2 - x + 5$

2) A baseball is hit into fair territory. The height, h (in metres), of the ball is described by $h = 30x - 6x^2$, where x is the time, in seconds.
   a) How long does it take before the ball is caught?
   b) What player do you think catches the ball? Explain.
   c) Approximately how high did the ball go? Explain.

3) Draw a graph of each of the following:
   a) $y = x^2 - 5x$
   b) $y = 3x^2 - 2x - 1$

   Use the graphs to solve each when
   i) $y=0$
   ii) $y=2$
   iii) $y=4$

   Discuss your approach to this problem with a classmate.

4) Given the following equations to solve, predict which method would be best to use for each. Explain in each case.
   a) $3x^2 - x - 2 = 0$
   b) $6x^2 + 7x = 5$
   c) $x^2 - 2x + 10 = 0$
   d) $2x^2 + 8x + 8 = 0$
   e) $5x^2 - 25 = 0$
   f) $3x^2 = 6$
   g) $x^2 - \sqrt{2}\,x + 1 = 0$
   h) $x^2 = 3x + 10$

Suggested Resources

Patterns, Relations, and Equations (25-30 hours)

C1 **express problems in terms of equations and vice versa**

C28 **explore and describe the dynamics of change depicted in tables and graphs**

C5 **sketch graphs from words, tables, and collected data**

C8 **identify, generalize, and apply patterns**

C9 **construct and analyse graphs and tables relating two variables**

C2 **model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities**

C20 **evaluate and interpret non-linear equations using graphing technology**

C27 **solve linear and simple radical, exponential and absolute value equations and linear inequalities**

Elaboration – Instructional Strategies/Suggestions

C1 Now that students have an understanding that not all patterns can be expressed with a linear equation, it is important for students to be aware that not all non-linear patterns are quadratic. Consequently, students will look briefly at non-linear patterns that are exponential.

C28/C5/C8/C9/C2/C20

❒ Determine the thickness of one sheet of paper. (Let students discuss how they might do this. One method they could use is to measure a stack of 500 sheets, then divide by 500, giving the thickness of one sheet. They might approximate this measurement as 0.01 cm.)
Fold a piece of paper in half and record its thickness. Fold the paper in half again and record its thickness. Continue this process to complete the table below. (Note: The entries in the third row are a re-statement of the entries in the second row, to enable students to better recognize the pattern.) Look for a pattern to help predict the thickness after 10, 20, and 30 folds.

fold #	0	1	2	3	4	5…	30
thickness (cm)	.01	.02	.04	.08			
thickness (cm)	1(.01)	2(.01)	4(.01)	8(.01)			

Students should notice the pattern $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$ from examining the third row. Have students plot these values to produce the graph $y = 2^x$. Have them predict the thickness after 20 folds (if 20 folds were possible). Students could also produce graphs for $y = 3^x$ and $y = 10^x$ and discuss how the graphs are the same and how they are different.
Students could produce the graph of another relationship for the same activity. The graph would show the relationship between the area of the sheet of paper and the number of folds as the paper is folded in half each time. This graph could be expressed using $y = \frac{1}{2}x$ or $y = 2^{-x}$. Teachers and students might take some time here to discuss how taking half of something is a limiting process and will never reach zero. Consequently, the graph approaches $y = 0$ but will never reach it.
This particular family of equations and graphs will be studied in greater detail in a subsequent course.
C27 Students should learn to solve simple equations for y-values that can be expressed with the same base as the base of the independent variable. Examples:

a) Solve $y = 2^x$ for $y = 8$
Soln:
$$8 = 2^x$$
$$2^3 = 2^x$$
$$\therefore x = 3$$

b) Solve $y + 1 = 3^x$ for $y = 8$
Soln:
$$8 + 1 = 3^x$$
$$9 = 3^x$$
$$3^2 = 3^x$$
$$x = 2$$

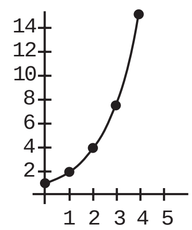
c) Solve $y = 10^x - 2$ for $y = 8$
Soln:
$$8 = 10^x - 2$$
$$10 = 10^x$$
$$\therefore x = 1$$

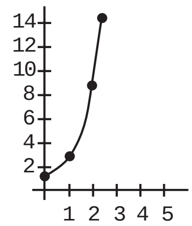
Modeling with exponential relations and solving simple exponential equations (SCOs C2 and C27) are clearly part of the core curriculum. For purposes of differentiating as needed for some students, however, this work with exponential relations and equations may be omitted.

Patterns, Relations, and Equations (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

1) Given the graphs of $y = 2^x$ and $y = 3^x$,





a) find y for the following x-values:
i) 4 ii) 2.5 iii) $\frac{1}{2}$ iv) $3\frac{1}{3}$

b) describe how you find the x-values, given the following y-values:
i) 9 ii) 25 iii) 36 iv) 0.5
v) 0.3

c) check each of your answers in (a) and (b) by solving algebraically.

2) a) If a penny was put in your bank account on day 1, then twice as much as the previous day was added to your account each day afterward, how much money would you have after a month of 31 days?

b) If instead the amount added was tripled each day, but only lasted for 10 days, which amount would you rather have?

c) After how many days would the arrangement in (b) be better than the arrangement in (a)? Show this graphically.

Suggested Resources

Unit 4
Modeling and Functions
(25-30 Hours)

In this unit students will develop an understanding of functions and their value in modeling real-world relationships. More specifically, the core curriculum addresses exploring, representing and applying functional relations (in particular, linear and quadratic), the analysis and description of transformations of quadratic functions (algebraically and with mapping rules), the use of transformations to draw graphs, and the application of transformations to absolute value functions. As well, collecting data, constructing scatter plots and determining (with and without technology) lines of best fit are addressed, along with related issues of correlation and variance.

For purposes of possible differentiation for some students, work with functions may be kept informal and transformations omitted. (See pp. 130, 142-43 for details.) Also, determining lines of best fit may be done exclusively with technology (p. 146), and discussion of variance may be omitted (p. 152).

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

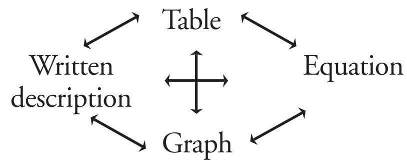
A2 analyse graphs or charts of situations to identify specific information

C28 explore and describe the dynamics of change depicted in tables and graphs

C5 sketch graphs from words, tables, and collected data

Elaboration – Instructional Strategies/Suggestions

C10



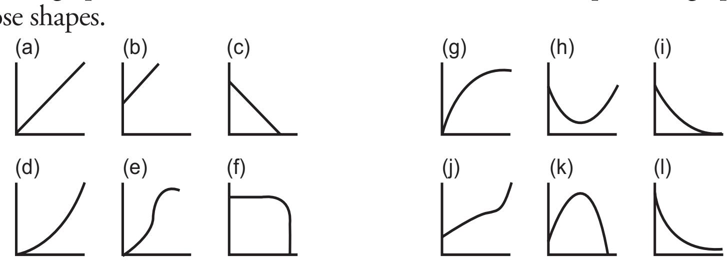
During the intermediate grades students have been encouraged to translate between different representations of a relationship. For example, from tables they should be able to describe relationships and make equations and graphs; from graphs of situations they should be able to describe the situations in words and with equations; from a written description they should be able to sketch a graph or make a table. Understanding the connections among these representations will facilitate students' learning (e.g., connecting slope to rate of change or initial values to intercepts). The situation below provides an opportunity to connect real-world relationships with graphical depictions.

C10/A2/C5/C28

❒ From the 12 graphs that follow, choose the graph that best describes each of the following situations and explain your choices.

i) John's performance maintaining his pace running up hill
ii) the amount of daylight, depending upon the time of the year
iii) the cost of a taxi cab trip is $2.00 plus $1.00 per minute
iv) the path of a golf ball
v) the amount of dough needed to make pizza crusts is calculated from knowing the diameter
vi) a runner's strategy of starting quickly, slowing to an even pace and then sprinting toward the finish
vii) the number of cigarettes smoked affecting your breathing in a negative way

For the graphs that remain, describe situations that could produce graphs of those shapes.



The situation below provides an opportunity to develop and analyze other representations of a real-world relationship depicted in a table.

❒ A taxi cab charges the following rates:

Length of trip (km)	5	10	15
Total cost ($)	9.25	15.50	21.75

a) Plot this information as points on a coordinate system.
b) Discuss if these points should be joined.
c) Determine the equation of the line.
d) Why doesn't the graph start at the origin?
e) Discuss, based on your equation, how much 7 km and 30 km rides will cost.
f) What is the significance of the slope of the line?

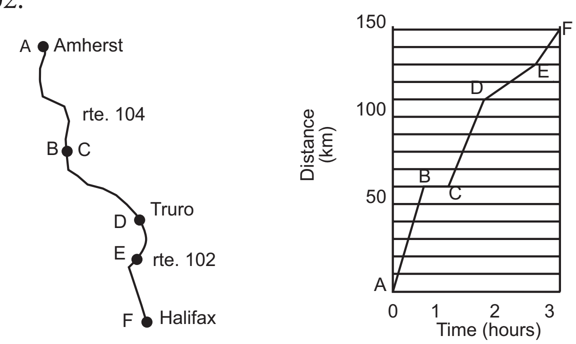
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C10/A2/C28

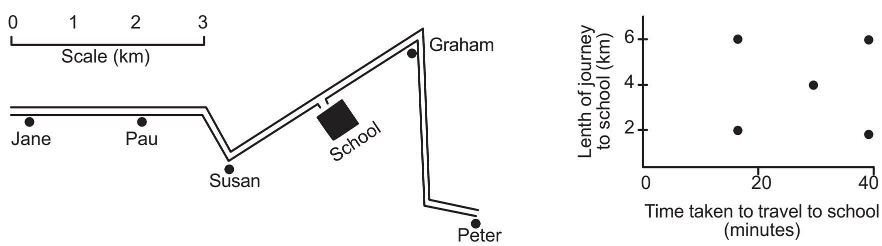
Pencil/Paper

1) The map and the graph describe a car journey from Amherst to Halifax, using Routes 104 and 102.



Describe each stage of the journey, making use of the graph and the map (i.e., describe and explain what may be happening from A to B; B to C; C to D; D to E and E to F).

2) Jane, Graham, Susan, Paul, and Peter all travel to school along the same country road every morning. Peter goes in his dad's car, Jane cycles, and Susan walks. The other two

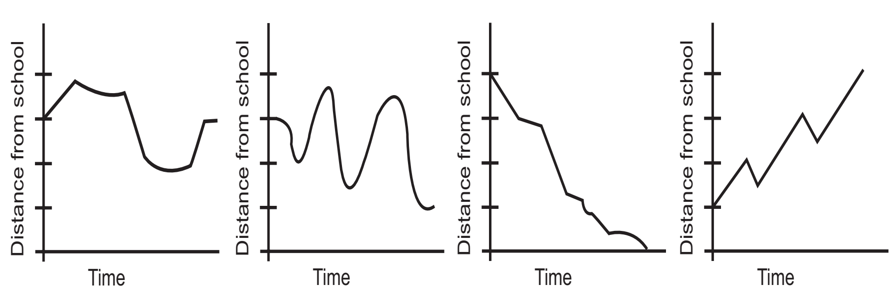


children vary how they travel from day to day. The map shows where each person lives. The graph above describes each pupil's journey to school last Monday.

a) Label each point on the graph with the name of the person it represents.

b) How did Paul and Graham travel to school on Monday? Describe how you arrived at your answer.

c) Peter's father is able to drive at 30 km/h on the straight sections of the road, but he has to slow down for the corners. Which of the following graphs is the one that most likely represents Peter's father's drive?



Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C3 **gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables and domain and range**

C5 **sketch graphs from words, tables, and collected data**

C8 **identify, generalize, and apply patterns**

C9 **construct and analyse graphs and tables relating two variables**

C33 **graph by constructing a table of values, by using graphing technology, and, when appropriate, by the slope y-intercept method**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

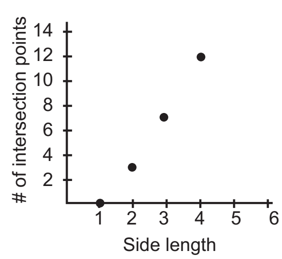
Elaboration – Instructional Strategies/Suggestions

C3/C5/C8/C9/C33/F10/A7 When given a verbal situation or data, students should be able to present it in a table and/or graph, and describe any patterns that exist.

While in the process of graphing, students should discuss which set of data is dependent and should be expressed along the vertical axis, and which is independent and expressed on the horizontal axis. Students should discuss appropriate scales for the horizontal and vertical axes. (These will often reflect domain and range considerations.) Have students interpret the graph by discussing steepness, discrete data versus continuous data, intercepts, maximum and minimum values, and linearity. As well, students should be able to predict values from the graph (using interpolation or extrapolation) and look for trends. (Note: Sometimes it is easier to predict trends and/or find results if students fit a line or curve to the data.)

By way of illustration, consider item 3) on the page opposite. Students might begin by developing a table as shown. They should be able to say that the number of intersection points depends on the side length of the equilateral triangle. It is clear that if the side length is known, they can state the number of intersection points. This relationship is very useful (functional) since there is only one correct answer for each different side length. Students could extend the table by identifying the pattern in the second column (i.e., that the difference between the second column numbers increases by one each time (e.g., the next number would be 12 + 6 = 18)). When the data are plotted, they should talk about the non-linearity of the data, and the fact that the data are discrete (since you can't talk about partial side lengths). Students could extend the table and/or the graph (perhaps change the scale) to predict the number of intersection points for greater side lengths.

Side length	# intersection points
1	0
2	3
3	7
4	12



Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C3/C5/C8/C9/C33/F10/A7

Pencil/Paper

1) On a spring day in Fredericton, the following temperatures were recorded for various altitudes:

Altitude (km)	0	0.3	1.5	3.0	4.5	6.0	9.0	10.8
Temperature (°C)	15	13	5	–5	–15	–26	–44	–56

   a) Explain which variable is dependent and which is independent and why you think so.
   b) Draw a graph.
   c) Explain whether or not you should join the points.
   d) Predict the temperature of a balloonist flying at 3.80 km. What assumptions are you making?
   e) Explain why this relationship is 'functional'.

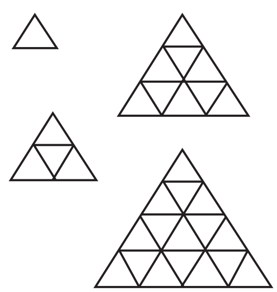
2) The number of chirps crickets made per second is related to air temperature. The following data were collected by 11 students who live in different locations in the country.

Temperature (°C)	15	17	16	18	15	16	16	15	14	16	16
Chirps per second	20	27	22	30	19	21	20	24	22	24	25

   a) At what temperature do you think crickets stop chirping? Explain.
   b) If the temperature was 7°C, how many chirps per second would you expect? How confident are you in this result? Explain.
   c) Explain how 'functional' this relationship is.

Performance

3) Use the equilateral triangle shape to form various sizes of larger triangles as shown. Then
   a) set up a table of values to record the number of intersection points within each triangle (i.e., points at which vertices of adjacent triangles meet) and triangle side length (Note: Treat the side length of the smallest triangle as 1.)
   b) predict the number of intersection points when the side length of the large triangle is 20
   c) decide whether the data display has a linear trend, a non-linear trend of some kind, or no apparent trend, and explain your choice.



4) The equation $d = -\frac{1}{3}t + 7$ describes a particular pattern found in some data.
   a) Sketch the graph for this equation.
   b) Write a short description of a real-life situation that this relationship might depict.
   c) Create a question that asks others to predict from the graph.
   d) Given the dependent variable, create a question to use the graph to predict a value for the independent variable.
   e) Is the graph continuous or discrete? Explain how you know.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

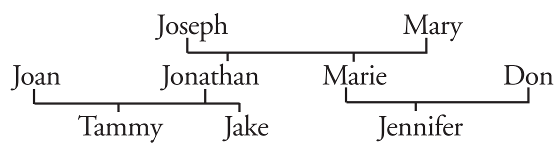
C21 explore and apply functional relationships, both formally and informally

Elaboration – Instructional Strategies/Suggestions

The following portion of the core curriculum (pp. 130-141) addresses i) the nature of mathematical functions, both informally and formally (SCO C21); ii) algebraic and graphical representations of transformations, especially with respect to quadratic functions (SCOs C22, C23 and E5); iii) the application of transformations to absolute value functions (SCO C22); and iv) the solution of equations involving absolute value (SCO C27). For purposes of possible differentiation for some students, however, the study of functions may be kept at an informal level (see pp. 142-143). Also, transformations of functions and the study of absolute value functions and equations may be omitted.

C21 Students should begin to understand the relationship that exists between a relation and a function. They should begin by exploring relations that are functions, to identify the functional aspect of these relations, i.e., that for every value in the domain of the relations there is one and only one value in the range.

Examining a family tree (perhaps a student's) is one way to illustrate this central aspect of a function. For example, consider



A relationship such as "is the parent of" is not function since a particular case (e.g., "is the parent of Jonathan") can have more than one possible answer (Joseph or Mary). On the other hand, a relationship like "is the wife of" is functional because any specific case (e.g., "is the wife of Jonathan") can have, at most, one answer associated with it (in this case, Joan).

Notationally, m(x) could be used to mean "the mother (or mother-in-law) of x", so m(Jake) would be Joan. Similary, f(x) could stand for "the father (or father-in-law) of x", so f(Marie) would be another way to represent Joseph. In this vein, students could be asked for other names for s(Jake) and b(Marie), i.e., the sister of Jake and the brother of Marie.

❒ With respect to the family tree given above,

a) determine each of the following, if possible:

i) m(Tammy)	iv) s(Joan)
ii) f(Jonathan)	v) f(Jennifer)
iii) b(Tammy)	vi) m(m(Jennifer))

b) does m(f(Jennifer)) = f(m(Jennifer))?

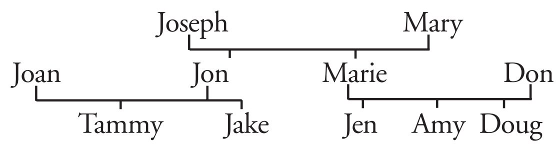
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C21

Activity

1) Consider the following family tree:



Using d(x) for "daughter of x" and n(x) for "son of x", complete the following:

a) Fill in the blanks, if possible. Are any of these relations not functional? Explain.
   i) d(   ) = Tammy   ii) m(   ) = Mary
   iii) b(   ) = Jake   iv) d(   ) = Jen

b) Fill in the blanks, if possible. Which are functional and which are not? Explain.
   i) s(Doug) =______________   ii) d(Joseph) =______________
   iii) d(Don) =______________   iv) s(Jake) = ______________

c) Create two examples(like those in (a) and (b)) that are functional and two that are not.

d) Using this notation, list all the different ways to describe Marie that are functional.

e) Does s(f(Jake)) belong to your list in d)? Explain.

Performance

2) In a game called Buzz (in which letters are randomly drawn) a relationship is depicted between letters of the alphabet and the integers 0 through 9. If a letter is in the first half of the alphabet, it is assigned a value of 2; in the latter half, a value of 5. If the letter is a vowel, a value of 3; if not, a value of 4. If letters are the independent variable, explain whether this relationship is a function or not.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C21 explore and apply functional relationships, both formally and informally

C33 graph by constructing a table of values, by using graphing technology, and, when appropriate, by the slope y-intercept method

C9 construct and analyse graphs and tables relating two variables

Elaboration – Instructional Strategies/Suggestions

C21

In mathematics, relationships are studied and discussed in terms of "one thing being a function of another." For example,

- distance is a function of time
- winning a mountain bike race is a function of how good the bike is
- talking to Robert on the phone is a function of calling the correct number.

To bring focus to the mathematical concept of function, students should be reminded that a function is a relation such that there is but one value in the range (i.e., of the dependent variable) associated with any given value of the domain (i.e., of the independent variable).

For example, for the relation $y = 2x + 1$, when $x = 5$, y will equal 11. This is the only possible y-value that can be associated with $x=5$. For each different x-value, students will find only one corresponding y-value, and, hence, this makes $y = 2x + 1$ a function. By contrast, consider the relation $y = \pm\sqrt{x}$ (graphically, a parabola lying on its side). When $x = 4$ is evaluated, $y = \pm 2$. This relation, then, is not a function since, for $x = 4$ (a value in the domain), there are two associated y-values (values in the range).

C33/C9 Looking at tables of values of relationships should help students determine if relations are functions or not.

x	y
0	0
1	±1
4	±2
9	±3

Table 1

x	y
0	1
1	2
2	3
3	8
4	8

Table 2

For example, students can see in Table 1 that there are two y-values for several of the given x-values (not a function), but in Table 2, each x-value is associated with only one y-value (a function).
(Note: Having a y-value associated with more than one x-value does not mean that a relation is not a function.)

As students graph relations, they should be encouraged to develop a visual representation of what is and isn't a function. They should also be able to provide a specific explanation as to why a particular relation is or is not a function. (Simply stating, for example, that a given graph represents a relation but not a function is not sufficient. Teachers need to probe students for their understanding about why they think this is the case. Some students might add that it is not a function because "when a vertical line is drawn through the graph it crosses the graph at more than one place, hence, I can pick an input value that has more than one output value.")

Students may use the vertical line test, but teachers must insist that students fully explain why the vertical line test demonstrates that a relation is a function or not.

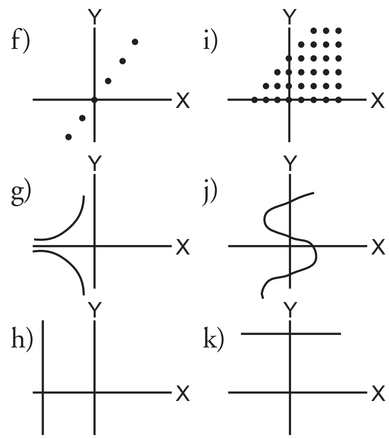
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C21/C33/C9

Performance

1) Which relations are functions? Justify.
   a) (1, 4), (2, 5), (3, 6), (4, 7), (5, 8)
   b) (–3, –3), (–2, –2), (–1, –1), (0, 0), (1, 1), (2, 2), (3, 3)
   c) (3, –3), (2, –2), (1, –1), (0, 0), (1, 1), (2, 2), (3, 3)
   d) (1, 1), (1, 2), (1, 3), (1, 4), (1, 5)
   e) (1, 1), (2, 1), (3, 1), (4, 1), (5, 1)



2) For each relation below, make a table of values (choosing integers from -3 to +3 as the domain for the variable). Which relations are functions? Justify.
   a) $y \rightarrow 3y + 2$ b) $z \rightarrow z^2 + 1$
   c) $b \rightarrow b + b^2$ d) $s^2 \rightarrow s + 1$

Pencil/Paper

3) f(x) means the function 'f' at the x-value 'x', and is often pronounced 'f of x' or 'f at x'. If $f(x) = \frac{3}{5}x - 7$ and $g(x) = -\frac{2}{3}x + 5$, find
   a) f(5)
   b) f(–2)
   c) g(–3)
   d) g(9)
   e) x, if f(x) = –7
   f) x, if $g(x) = \frac{1}{2}$
   g) f(x) + g(x)

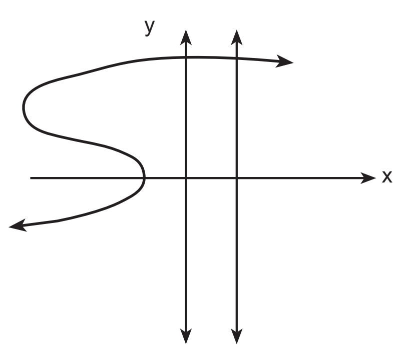
   Optional enrichment:
   viii) f(g(x))
   ix) f(g(4))
   x) $g(f(\frac{1}{2}))$

4) Does f(a + b) = f(a) + f(b), if $f(x) = -\frac{3}{4}x - 1$?

C33/C9

Interview

5) Ann was asked to determine if the graph of the relation shown at right was a function. Her response: "Yes, it is a function because it passes the vertical line test." Barb disagreed. Ask students to explain what Barb must be thinking.



6) This rule is a function: y = –3x + 5. Ask students to create a rule which is not a function.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C22 analyse and describe transformations of quadratic functions and apply them to absolute value functions

C23 express transformations algebraically and with mapping rules

C31 graph equations and analyse graphs both with and without graphing technology

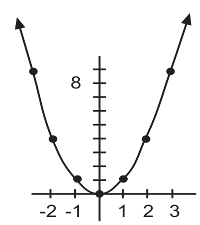
Elaboration – Instructional Strategies/Suggestions

C22/C23 The graphs of quadratic functions have parabolic shapes. Students should be able to visualize the location and shape of the graph of a quadratic function from the structure of the corresponding equation.

To do this, students should first study the table of values and graph of the ‘model’ quadratic function, $f(x) = x^2$. They should note the symmetry that is evident in both the table and graph. They should also notice the pattern in the location of the points with respect to the vertex, (0, 0).

x	y
–3	9
–2	4
–1	1
0	0
1	1
2	4
3	9

Pattern in the points (as described in relation to the vertex):
over 1 up 1, over 2 up 4, over 3 up 9, etc.



When finished this examination of the transformations of $f(x) = x^2$, students will be able to see (visual estimation) the location and shape of the graph for any quadratic equation in the form $-k(y - v) = (x - h)^2$. One at a time they will explore the affect that changes in k, v, and h have on the shape and location of the image of $f(x) = x^2$. (Note: The affect these have on $y = x^2$ is the same affect they have on other functions and relations to be studied in subsequent courses.)

C22/C23/C31 For example, students might be asked to create tables for $y = x^2$ and $-y = x^2$, using $\{-3 \le x \le 3 \mid x \in I\}$, then asked to graph both tables. They should be asked to identify similarities and differences in the graphs and tables. Their response should be that all the y-values of $-y = x^2$ are the negatives of those for $y = x^2$, and that the graph of $-y = x^2$ is a reflection in the x-axis of $y = x^2$. Students should be able to generalize that when they see a negative coefficient of ‘y’ in the equation, there will be a reflection of $y = x^2$ in the x-axis. This is clear in the corresponding mapping rule: $(x, y) \rightarrow (x, -y)$.

Similarly, students might study the tables for $y = x^2$ and $y + 3 = x^2$ and note that all the y-values in $y + 3 = x^2$ are 3 less than the corresponding y-values in the table of $y = x^2$. Students should be able to state that when they see y + 3 in the equation, there will be a vertical translation of –3 of $y = x^2$, and that the translation is the additive inverse of the number being added to y. This is visible in the mapping rule: $(x, y) \rightarrow (x, y - 3)$.

$y = x^2$		$y + 3 - x^2$	
x	y	x	y
-3	9	-3	6
-2	4	-2	1
-1	1	-1	-2
0	0	0	-3
1	1	1	-2
2	4	2	1
3	9	3	6

In a similar manner, students will determine that a non-zero value for h (in $y=(x-h)^2$) produces a horizontal translation. (See the activity on the page opposite.)

Continued...

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C22/C23/C31

Performance

1) a) Produce tables for $y = x^2$, $y = (x + 1)^2$ and $y = (x - 3)^2$, for y-values of -3,-2, -1, 0, 1, 2, 3.
   b) Sketch graphs of the relationships in the three tables.
   c) Compare the x-values in the three tables and describe what you notice.
   d) Compare the three graphs and the three equations.
   e) Describe what aspect of the equations helps you 'see' the graphs' locations.
   f) Complete: "The horizontal translation is visible in the equation. It is the ______________ *inverse* of the number being added to _________."

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C22 analyse and describe transformations of quadratic functions and apply them to absolute value functions

C23 express transformations algebraically and with mapping rules

C31 graph equations and analyse graphs both with and without graphing technology

Elaboration – Instructional Strategies/Suggestions

C22/C23/C31...continued.

Eventually, students should describe the graph of a quadratic function such as $-(y + 2) = (x - 1)^2$ as opening downward (reflection), with its vertex at (1, –2). This can also be expressed by the mapping rule: $(x, y) \rightarrow (x + 1, -y - 2)$. The pattern in the points (as described on page 120) is only affected here by the reflection (i.e., the 'up' becomes 'down') and, hence, would be over 1 down 1, over 2 down 4, over 3 down 9, etc. from the vertex, (1, -2).

Finally, students will find that the vertical stretch value is the multiplicative inverse of the coefficient of 'y'. They will also find the the vertical stretch value must be multiplied by the 'up' or 'down' values of the basic pattern in the points. So, for example, the graph of $\frac{1}{2}y = (x + 1)^2$ opens upward, with the vertex at (–1, 0) and a pattern that goes over 1 up 2; over 2 up 8; over 3 up 18. These transformations can be expressed as a mapping rule: $(x, y) \rightarrow (x - 1, 2y)$. The mapping rule can be used to obtain the coordinates of points on the image: Start with Table 1, subtract one from all x-values, multiply all y-values by 2, and produce Table 2.

$y = x^2$

x	y
–3	9
–2	4
–1	1
0	0

Table 1

$\frac{1}{2}y = (x + 1)^2$

x	y
–4	18
–3	8
–2	2
1	0

Table 2

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C22/C23

Pencil/Paper

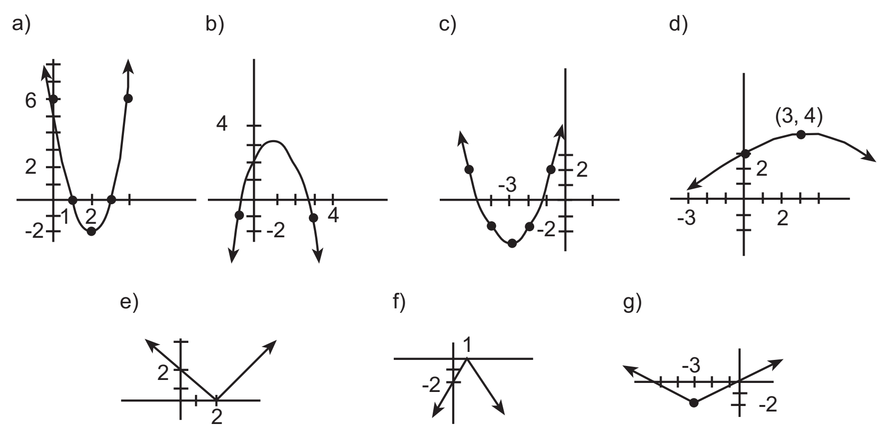
1) Describe, in words and with mapping rules, the transformations visible in these equations:

   a) i) $y + 10 = (x - 5)^2$
      ii) $-(y + 1) = x^2$
      iii) $-\frac{1}{2}(y + 2) = (x - 1)^2$
      iv) $3y = (x + 2)^2$

   b) i) $y + 5 = |x - 2|$
      ii) $-(y - 2) = |x + 5|$

2) For the following mapping rules, write the equations for the images of i) $y=x^2$ and ii) $y = |x|$.
   a) $(x, y) \rightarrow (x - 1, 3y)$
   b) $(x, y) \rightarrow (x, -\frac{1}{2}y)$
   c) $(x, y) \rightarrow (x + 2, y - 1)$
   d) $(x, y) \rightarrow (x, -\frac{3}{4}y + 1)$

3) From the graphs, describe the transformations, then write the equations.



Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

E5 **use transformations to draw graphs**

C31 **graph equations and analyse graphs both with and without graphing technology**

C24 **rearrange equations**

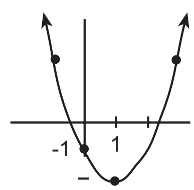
C23 **express transformations algebraically and with mapping rules**

Elaboration – Instructional Strategies/Suggestions

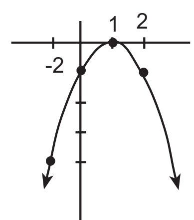
E5/C31 As students explore the transformations of $y = x^2$, they should practice sketching the images of $y = x^2$ given certain transformations, equations, or mapping rules.

They will learn to sketch graphs following a certain procedure. For example, when asked to graph $y + 2 = (x - 1)^2$, they should say to themselves, "The vertex is at (1, –2), the graph opens upward, and there are no stretches. So, I will plot the vertex at (1, –2) and (from the vertex each time), go over 1 up 1; over 2 up 4; and over 3 up 9 to create the graph." (See graph below at left).

When asked to graph $-\frac{1}{2}(y - 1) = x^2$, they would say, "There is a reflection in the x-axis (opens downward), a vertical stretch of 2, and a vertical translation of 1. Since the "–" and the $\frac{1}{2}$ are factors of y, they will affect the pattern, but the vertex is unaffected by multiplication. So, I will plot the vertex image at (0, 1), but from there go over 1 down 2 (changing "up" to "down" and multiplying by 2); then over 2, down 8 (i.e., 4 x 2); and so on." (See graph below at right.)



$y + 2 = (x - 1)^2$



$-\frac{1}{2}(y - 1) = x^2$

C24 Equations like i) $-\frac{1}{2}y + 3 = (x - 1)^2$ and (ii) $2y = x^2 + 6$ are not in transformational form and must first be re-arranged to facilitate these sketching techniques. Students should be able to state that equation i) is not in transformational form because the 3 is being added to $-\frac{1}{2}y$, not just y, so they must "separate" the $-\frac{1}{2}$ from the y. In (ii), the 6 is being added to the x^2 not the x, so they will have to move it to the other side (the y-side) of the equation.

i) $-\frac{1}{2}y + 3 = (x-1)^2$

$-\frac{1}{2}(y - 6) = (x-1)^2$

ii) $2y = x^2 + 6$

$2y - 6 = x^2$

$2(y-3) = x^2$

C23 When students are given a graph that is an image of the graph of $y = x^2$, they should be able to describe the transformations, and state the mapping rule and the equation from the graph, using their knowledge of transformations. For example, from the graph on the left above, students would describe the image in words as "a vertical translation of –2 and an horizontal translation of 1", and with a mapping rule: $(x, y) \rightarrow (x + 1, y - 2)$.

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

E5/C23/C31

Pencil/Paper

1) Describe the transformations visible in each of the following equations, then sketch the image graphs of $y = x^2$:
   a) $-3(y - 2) = x^2$
   b) $y = (x + 1)^2$
   c) $-(y + 1) = (x - 2)^2$
   d) $y + 1 = (x - 3.5)^2$

2) In each case sketch the image graph of $y = x^2$ and state the mapping rule:
   a) reflection in the x-axis, a vertical translation downward of 2
   b) a vertical stretch of $\frac{2}{3}$, a vertical translation of 3, and a horizontal translation of 1.

3) Explain the relationships between
   a) the mapping rule and the verbal description of transformations
   b) the verbal description of transformations and the equation
   c) the equation and the graph, with respect to the transformations.

4) Without sketching the graphs of the functions given below,
   a) which graph has a vertex furthest to the right and up?
   b) which graph is wider than the rest?
   c) which graph opens downward?

   Given functions:
   i) $-3y = (x + 5)^2$
   ii) $\frac{1}{5}y = (x - 2)^2$
   iii) $\frac{1}{2}(y - 3) = (x - 5)^2$
   iv) $0.75(y + 1) = (x - 2)^2$

C31/C23/C24

Journal

5) Graph $y = x^2 - 2x - 15$ using graphing technology, and describe it in as many different representations as possible, using transformations of $y = x^2$.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C22 analyse and describe transformations of quadratic functions and apply them to absolute value functions

C27 solve linear and simple radical, exponential and absolute value equations

Elaboration – Instructional Strategies/Suggestions

C22 Once they are familiar with the absolute value function, $y = |x|$, students should be able to apply their understanding of transformations with respect to $y = x^2$ to the absolute value function. (See some of the tasks on p. 137, as well as those on the page opposite.)

C27 When introducing the absolute value function, teachers might have students graph $y = x$ for $x \geq 0$, then $y = -x$ for $x \leq 0$. Have students examine their graphs and describe the function in as many ways as possible. One way to describe it symbolically is piecewise, i.e., $y = \begin{cases} x, & x \geq 0 \ / \ x \in R \\ -x, & x \leq 0 \ / \ x \in R \end{cases}$

Teachers can then introduce the absolute value notation as another way to describe the function.

In addition to graphing and transforming the absolute value function, students should learn to solve simple absolute value equations by re-expressing as piece-wise equations.

For example, when asked to solve $3|x| - 5 = 0$, they would say:

If $x \geq 0$, then $|x| = x$,
so, I solve $3x - 5 = 0$, giving $x = \frac{5}{3}$.
If $x \leq 0$, $|x| = -x$, so
$-3x - 5 = 0$, giving $x = -\frac{5}{3}$.

From these two pieces they would conclude that $x = \frac{5}{3}$ or $-\frac{5}{3}$.

Example #2: When asked to solve $2|x - 2| = 3$, students would say:
If $x - 2 \geq 0$, then $|x - 2| = x - 2$

$$\begin{aligned} \text{so, } 2(x-2) &= 3 \\ 2x - 4 &= 3 \\ 2x &= 7 \\ x &= \tfrac{7}{2} \end{aligned}$$

If $x - 2 \leq 0$ then $|x - 2| = -(x - 2)$

$$\begin{aligned} \text{so, } 2(-x+2) &= 3 \\ -2x + 4 &= 3 \\ -2x &= -1 \\ x &= \tfrac{1}{2} \end{aligned}$$

$\therefore$ solution set is $\left\{\frac{1}{2}, \frac{7}{2}\right\}$

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

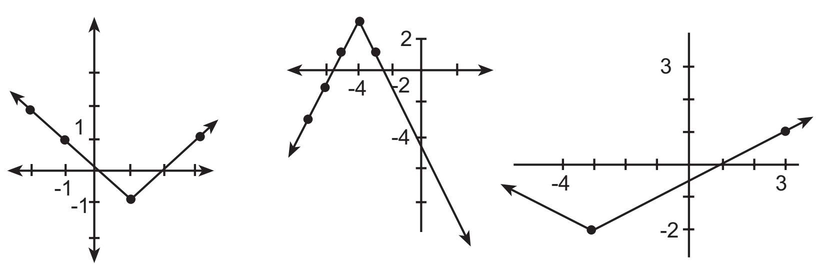
C22

Pencil/Paper

1) Describe the transformations visible in the equations, then sketch the image graphs of $y = |x|$.
   a) $-2(y-3) = |x|$ b) $\frac{1}{2}y = |x-1|$ c) $-\frac{1}{3}(y-5) = |x+2|$
   d) $y + 1 = |2x - 1.5|$ e) $-3y + 5 = \frac{1}{3}|x + 2.5|$

2) In each case, sketch the image graph of $y = |x|$ and state the mapping rule:
   a) reflection in the x-axis, a vertical translation of 2.
   b) a vertical stretch of $\frac{3}{5}$, a vertical translation of –2, and a horizontal translation of 5.

3) Write the mapping rule for the following graphs, which are images of $y = |x|$.



C27

4) a) Solve the following:
   i) $-2|x| + 5 = 4$ ii) $-|x-5| + 2 = 0$ iii) $|2x-3| - 1 = 7$
   b) Explain how you could use a graph to check your results.

Journal

5) When solving an absolute value equation, explain why you are expected to examine the expression within the absolute value symbols (the argument) for positive and zero values, and then for negative values.

Suggested Resources

Modeling and Functions (25-30 hours)

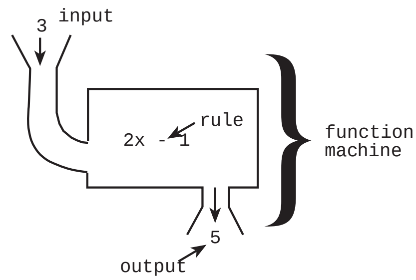
SCO: In grade 10 students will be expected to

C21 explore and apply functional relationships informally

Elaboration – Instructional Strategies/Suggestions

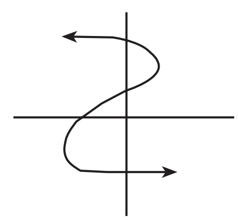
The previous section (pp. 130-141) presents the study of functions and transformations for the core curriculum. For purposes of differentiation for some students, however, the study of functions may be kept informal, and transformations omitted. (See the note on p. 130 for more detail.) This differentiation is presented here, and on the page opposite.

C21 Students should begin to understand the relationship that exists between a relation and a function. They should begin by informally exploring functional relationships. To do this they could play "What's My Rule" games. One student makes up a rule (e.g., one less than twice a number), a partner suggests a number, and the first student gives the number generated by the rule. The second student continues to give starting numbers until able to guess the rule. (Note: Students should be encouraged to make a table to organize the guesses as an aid to finding the rule.)



By engaging in this type of activity (which can be modeled with a "function machine"), students develop an understanding of input and output. When there is only one output for each input, the machine rule is functional. Students can create function machines of their own and have other students guess their functions.

As students graph these functions they should be encouraged to develop a visual representation of what is and isn't a function. They should also be able to provide a specific explanation as to why a particular relation is or is not a function. (Simply stating, for example, that this graph represents a relation but not a function is not sufficient. Teachers need to probe students for their understanding about why they think this is not a function. Some students might add that it is not a function "because when a vertical line is drawn through the graph it crosses the graph at more than one place, hence, I can pick an input value that has more than one output value.")



Students may use the vertical line test, but teachers must insist that students fully explain why the vertical line test demonstrates that a relation is a function or not.

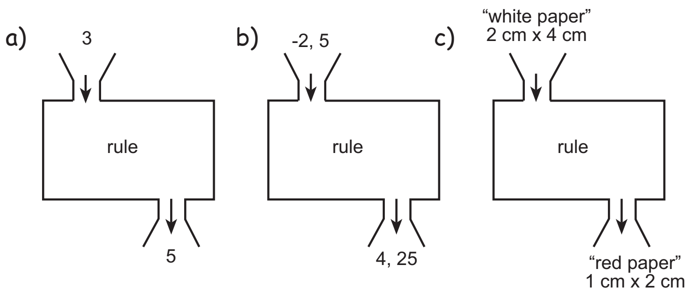
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

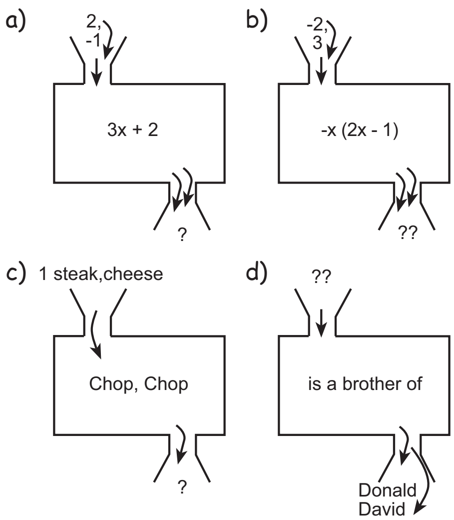
C21

Pencil/Paper

1) For each of the following, ask students to determine a 'rule' or function that is being performed on the input data to generate the output data:

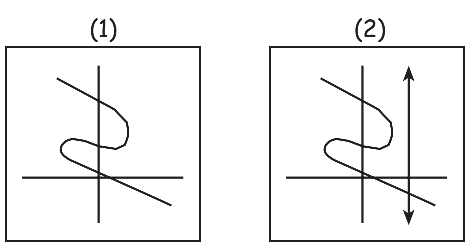


2) For each of the following, ask students to determine the unknown input or output data:



Interview

3) Ann was asked to determine if graph (1) is a relation or a function. Ann did a vertical line test. Her response: "Yes, it is a function because it passes the vertical line test (2)." Barb disagreed. Ask students to explain what Barb must be thinking.



Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C1 **express problems in terms of equations and vice versa**

C2 **model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities**

F11 **describe real-world relationships depicted by graphs and tables of values and written descriptions**

C8 **identify, generalize, and apply patterns**

F6 **solve problems by modeling real-world phenomena**

C3 **gather data, plot the data using appropriate scales and demonstrate an understanding of independent and dependent variables, domain and range**

C4 **create and analyse scatter plots using appropriate technology**

C5 **sketch graphs from words, tables, and collected data**

C9 **construct and analyse graphs and tables relating two variables**

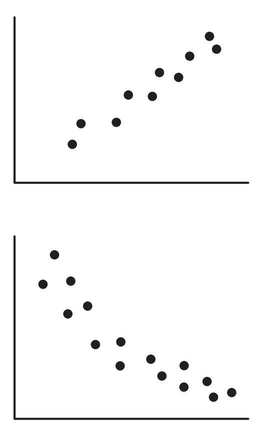
Elaboration – Instructional Strategies/Suggestions

C1/C2/F11/C8/F6 Mathematical modeling is the process of representing a relationship symbolically and using it to answer questions about the problem situation. Models help clarify the relationship between variables and make it possible to solve problems and predict results. The major value of a model is its usefulness for prediction, thus modeling is a vital real-world form of problem solving.

F6/C3/C4/C5/C9 The process of modeling often includes conducting an experiment to gather data about a situation, organizing the data into tables and graphs, and determining a mathematical rule or equation from which to predict unknown values. Students will study real-world phenomena from given data, or conduct experiments and collect data. They will graph the data and, from the graph, determine an equation that will allow them to predict results and solve problems.

C2/C4/C5/C9 As students study functions and their graphs, it is important that they develop an understanding of the general shapes of the graphs that the functions create. For example, when students study linear relations, they learn that each relation has a constant growth rate and that the growth rate could be positive, negative, zero, or infinite—each of these resulting in a linear graph, but of different steepness and direction. Some linear graphs pass through the origin, while others do not. When modeling, students should be able to tell from the context or data whether or not the graph will pass through the origin. They should also be able to tell from the growth rate or from the context, whether the graph is linear or curved. If curved, students should discuss whether it takes on a parabolic shape or some other shape. Students should begin to 'build' a function tool kit of familiar graph shapes and equation models that fit these shapes. These should include linear, power, quadratic, and exponential (See, for example, *Activity* 1) on the page opposite.)

When students see scatter plots like those shown at right, the arrangement of points may well suggest (a) particular mathematical model(s). (Here, the upper scatter plot appears to have a linear trend, while the lower appears non-linear and might indicate a quadratic, exponential or some other relationship.) Selecting a mathematical model should not depend solely on visual trends in scatter plots, however; often the context/experimental situation will also provide insight into the nature of the mathematical model.



Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C1/C2/F11/F6/C3/C4/C5/C9

Activity

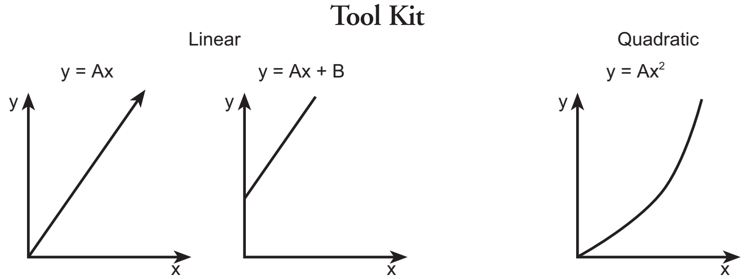
1) Dropping a Stone

Time (seconds)	0	1	2	3	4	5
Distance fallen (metres)	0	5	20	45	80	125

Ask students to do the following exercises:

a) Sketch a rough graph to illus trate these data.
b) Can you see any patterns in this table? Describe them in words and, if possible, with an equation.
c) A stone is dropped from an aircraft. How far will it fall in 10 seconds?
d) Which graph looks most like your sketch for the 'dropping stone' problem?

Tool Kit



2) A class of students measured their wrists and necks (in centimetres). The data are given in the following table:

Wrist	Neck	Wrist	Neck
15	30	17.5	33.5
15.5	31	15	33
14	34.5	16	35
15	34	14	32
15.5	31	17	40
18	35	16.5	39.5
15	33	17.5	37.5
15.5	31	15	33
14	34.5	18	42
19	40	21	42.5
14.5	35.5	20	39
16	36.5	14.5	36.5
18	38.5		

a) Plot the data (wrist size, neck size), and make an estimate of the strength of the relationship.
b) Discuss the relationship between wrist size and neck size. Does it seem to be a linear or non-linear relationship?
c) If Rodger has a 21cm wrist measurement, predict his neck size and justify your response.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

F6 **solve problems by modeling real-world phenomena**

C9 **construct and analyse graphs and tables relating two variables**

C4 **create and analyse scatter plots using appropriate technology**

F9 **demonstrate an intuitive understanding of correlation**

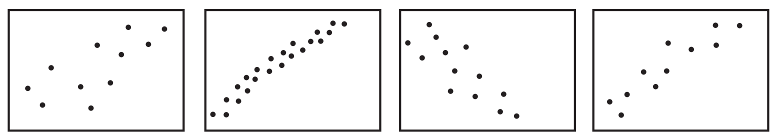
C1 **express problems in terms of equations and vice versa**

C14 **determine the equation of a line using the slope and y-intercept**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

Elaboration – Instructional Strategies/Suggestions

F6/C9/C4/F9 In the modeling process collected data is graphed to help solve a problem. The graphed data is called a scatter plot. The shape of the plot should help students describe a possible relationship between the two variables and to determine which function could be the best model. Students should be able to describe the strength of the relationship (e.g., weak, strong) by noticing how close the data points are to a line or a curve that might pass through most of the points. They might also describe the relationship as increasing or decreasing, depending on whether the data points suggest an increasing or decreasing trend.



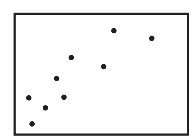
weak   strong   decreasing   increasing

By way of example, consider the data in the table below, relating the number of mosquito deaths and an index of exposure to a new insecticide.

Exposure index	2.5	2.6	3.4	1.3	1.6	3.8	11.6	6.4	8.3
Deaths	147	130	130	114	138	162	208	178	210

A scatter plot of the data is shown below. Students would likely describe it as increasing and weak. Predicting from a weak relationship is difficult. In general, however, students might describe this relationship as "the higher the exposure index, the higher the number of deaths". Predicting from it would be easier if they first approximated a line or curve of best fit.

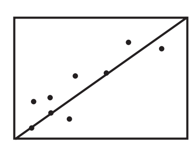
C1/C14/F10 Previously, students have approximated a line of best fit using the "eye ball" method, or a referant such as a piece of uncooked spaghetti. Placing the line so that it passes through 2 identifiable points, and using the points to find the slope and y-intercept, students can determine the equation of the line.



This line becomes the model from which students can predict values (interpolate and extrapolate).

Since many of the data points are not situated very close to the model line (weak relationship), students should be cautious when predicting. The line itself may not give an accurate prediction of the number of deaths.

When students compare their equations they will find inconsistencies. The following pages describe techniques for generating mathematical models with greater consistency and confidence.



The core curriculum requires that students determine lines of best fit using both the median-median and least squares (linear regression) methods (SCO F8). For purposes of differentiating for some students, however, it is sufficient to use linear regression (with technology) only. The median-median method (as presented on pp. 148-149) may be omitted.

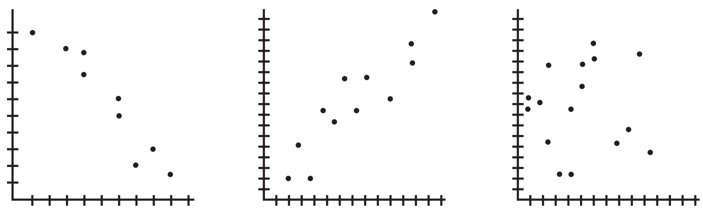
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

F9/C4/C9

Pencil/Paper, Interview

1) Ask students to describe the following scatter plots with respect to a) weak, strong, and b) increasing, decreasing:



2) Sketch the line of best fit on each of the graphs above.

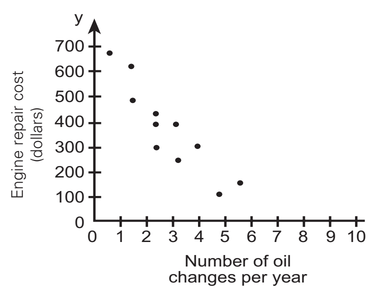
F6/C4/C9/C1/F10/C14

3) a) From the graph, if Tom changes the oil in his car 5 times a year, estimate the cost for Tom if engine trouble develops.

   b) If you wanted to keep your engine repair costs below $200 per year, how many oil changes should you make?

   c) Sketch a line of best fit and use it to answer (a) above. Compare your answers. Explain any differences.

   d) Find the equation for your line of best fit. Use the equation to answer (b) above. Explain any differences with respect to your previous answer to (b).



Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

F8 **determine and apply the line of best fit using the least squares method and median-median method with and without technology, and describe the differences between the two methods**

C1 **express problems in terms of equations and vice versa**

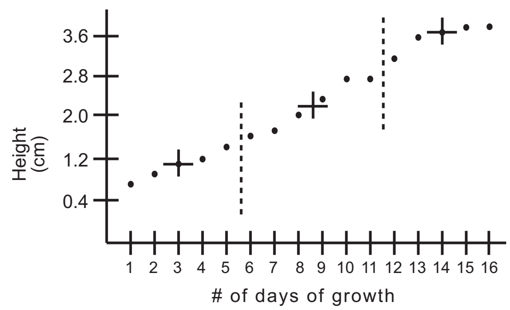
F10 **use interpolation, extrapolation, and equations to predict and solve problems**

Elaboration – Instructional Strategies/Suggestions

F8/C1/F10

To improve consistency, students should determine a best fit line for data using a standard, repeatable procedure. One such procedure is called the median-median method. This procedure produces a line of best fit based on medians from the data (organized into three groups). The median-median method is detailed below for the given data set and associated scatter plot.

Days	Heights of pea plants (cm)
1	0.7
2	0.9
3	1.1
4	1.2
5	1.4
6	1.6
7	1.7
8	2.0
9	2.3
10	2.7
11	2.7
12	3.1
13	3.5
14	3.6
15	3.7
16	3.7



To get the median-median line:

1) Organize the data into three equal groups (see dotted lines). Since there are 16 data points, put the extra one in the middle group. (If there were 17 data points, the two extra ones would be placed in the outside groups.)
2) Find the median x-value in each group (see solid vertical line in each grouping).
3) Find the median y-value in each group (see horizontal line in each grouping).
4) Three summary points have now been established. Use the two outside ones to determine the slope of the median-median line.
5) The median-median line is a transplantation of the summary line joining the two outside summary points. Translate it $\frac{1}{3}$ of the way to the 3rd summary point. Its slope will be the same as the calculated slope since the two lines are parallel.
6) The summary line can be used to determine a height for the middle summary point (at $8\frac{1}{2}$ days).
7) One third the distance between the height for $8\frac{1}{2}$ days from step 6 and the height for the middle summary point will give a value that can be added to or subtracted from the height-intercept of the summary line to get the height-intercept for the median-median line.

Students should be able to explain how medians are less affected by the values of outliers, and thus lines drawn based on median values are more likely to reflect the relationship. Even though the median-median line produces the same model for all students, they should be reminded that it still only approximates the relationship.

Note: Students should be asked to examine the scatter plot and median-median line to explain how a change in one or two data points would (and would not) affect the equation of the line.

Modeling and Functions (25-30 hours)

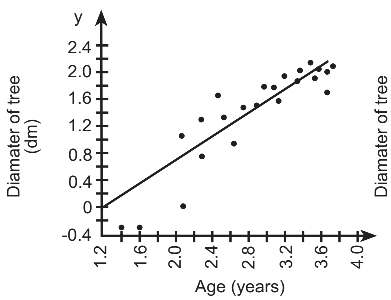
Worthwhile Tasks for Instruction and/or Assessment

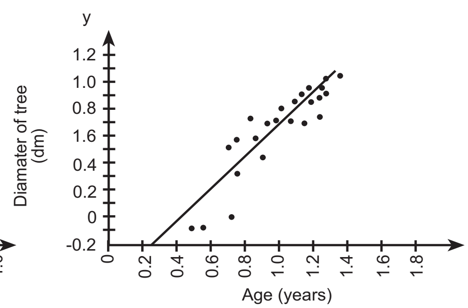
F8/C1/F10

Interview/Presentation

1) a) Have the students interpret the intercepts in these two graphs. What information is meaningful? is not meaningful? Ask students what they would say to a pulp mill owner to explain what these intercepts indicate and how meaningful they are. (Note to teacher: Students should discuss why the horizontal axis is misplaced and why the first three data points are not possible. Note also that these graphs show the dangers of extrapolating beyond the collected data.)

b) Have students interpret the slope and explain how the slope is meaningful in this situation.





c) Have students use the median-method method to determine the equation for the median-median line.

d) Have students estimate from the graph, then determine, the diameter of the two trees using the median-median line.

i) the first tree after 2.45 years

ii) the second tree after 0.55 years.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

F6 **solve problems by modeling real-world phenomena**

C4 **create and analyse scatter plots using appropriate technology**

F8 **determine and apply the line of best fit using the least squares method and median-median method with and without technology, and describe the differences between the two methods**

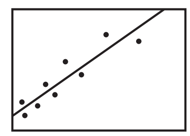
F9 **demonstrate an intuitive understanding of correlation**

C17 **solve problems using graphing technology**

Elaboration – Instructional Strategies/Suggestions

F6/C4/F8/F9/C17 Students should understand that the eyeball method of curve fitting leads to inconsistency and the median-median method can sometimes result in lines that may not best represent the data. A more consistent way to fit a line is with the use of graphing technology. The median-median line can be produced using graphing technology, but most statisticians use a least squares regression line (based on the means of the data points), that minimizes the sum of the squares of the distances from the actual to the predicted values.

Returning to the example on page 146 involving mosquito deaths, students will have already indicated that the scatter plot suggests that the more the mosquito population is exposed to the insecticide, the higher the number of deaths. When asked to predict the number of deaths for an exposure index reading of 7, students could look at the scatter plot and interpolate about 190 deaths. To get a more accurate response, they should generate a line of best fit using technology. The process is called linear regression, and the line is called the least squares regression line. Basically, the calculator positions the line such that the distances between each data point and the line are minimized. The technology will determine the equation for the line and draw it on the graph, giving an indication of how good the fit is by displaying an r-value (correlation value). The maximum value for r is +1. An r-value close to 1 indicates a strong positive association; as one variable increases, so does the other. The minimum value of r is –1. An r-value close to –1 indicates a strong negative association; as one variable decreases, the other increases. The value r = 0 indicates no association or relation between the variables; as values of x increase, some values of y increase and other values of y decrease.



Because it is possible to have a large positive correlation for data that are not really linear, the r-value alone can be misleading . This means it is important to consider the context. Students need to consider if the situation under study is one that may lead to non-linear data. For example, if data was being collected on the cooling pattern for a hot liquid sitting on the kitchen table, it will not cool beyond room temperature. This should lead to an understanding that the best model may be an exponential function (approaches one asymptote). Ultimately, deciding whether a particular model is appropriate or not will depend on consideration of the context, the visual appearance of the scatter plot, and the degree of correlation.

Continued...

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

F6/C4/F8/C1/F9/C17

Performance

1) The average aptitude test scores in mathematics in various jurisdictions are given in the following table, along with the percentage of students in each jurisdiction who took the test.

Jurisdiction	% graduates taking test	Math score	Jurisdiction	% graduates taking test	Math Score
1	5	577	12	57	438
2	10	541	13	6	559
3	10	515	14	13	524
4	10	513	15	45	462
5	59	466	16	6	536
6	60	475	17	64	472
7	73	474	18	63	472
8	13	513	19	37	494
9	17	531	20	14	496
10	4	519	21	14	534
11	14	519	22	12	527

a) Enter the data as ordered pairs (percentage, score) for jurisdictions 1–11 and determine the equation for the regression line. Have a partner enter the data for jurisdictions 12–22 and determine the equation for that regression line. How do your equations compare? Justify your equation.

b) Determine the equation and graph the regression line obtained by using all of the jurisdictions. Discuss the strength of its fit.

c) If 25 percent of the students in a jurisdiction take the test, what would you expect for the mean math score? If the mean mathematics score was 500, how many students would you expect to have taken the test?

d) What does the slope indicate about the relation between the percent of students and their average math score? Is the y-intercept meaningful?

e) From the graph of the line and the scatter plot for each column, discuss how well you think the line represents the data. What does this tell you about the difference in the scores with respect to the percentage of students who take the test?

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

F9 **demonstrate an intuitive understanding of correlation**

C1 **express problems in terms of equations and vice versa**

F10 **use interpolation, extrapolation, and equations to predict and solve problems**

C17 **solve problems using graphing technology**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

Elaboration – Instructional Strategies/Suggestions

F9/C1/F10/C17. . .continued

Sometimes statistical summaries give only an r^2-value (no r-value). The r^2-value explains the amount of variance in the data. The quantity r^2 can be used to describe the percent of variation in y that can be predicted using the least squares regression line and a given x. For example, if r = 0.8, r^2 = 0.64 or 64%; therefore the least squares regression explains 64% of the observed variation in y. A correlation of 0.6 would mean that regression explains only 36% of the variation in y.

For purposes of differentiation, discussion of variance can be omitted for some students.

In the mosquito deaths situation, linear regression produces the equation $y = 9.27x + 114.68$ (where x is the exposure index and y is the number of deaths), and an r-value of 0.9268. Students could graph the equation, trace along the line, and see that an index reading of 7 is associated with about 180 deaths. Alternative methods of calculating function values more accurately are also often available on technology. In this case, evaluating for the value 7 also gives a death rate of 180.

When data points show a strong correlation, a best-fit line will be close to all the data points and the correlation coefficient (r) will have a high reading (usually above .90). When the data points show a weak correlation, the r-value will be low and may approach zero.

When students use the equation to extrapolate they should be cautioned when interpreting the result. They should understand that when extrapolating they are assuming the data trend would continue if more data were collected. This may not be the case in many experiments. Results must be examined in terms of the given context to determine whether extrapolated values are likely to make sense. For example, in question 1), page 149, it does not make sense for a tree to have a diameter of –0.2 dm.

A7 Scatter plots often represent situations that consist of discrete data, yet lines of best fit suggest continuous data. Teachers and students should discuss this seeming conflict. For example, the mosquito population obviously suggests discrete data (one cannot talk about 85.5 mosquitos). The model, however, allows students to interpolate or extrapolate values with decimals. Students need to remain mindful of the context and discuss the number of mosquitos using whole numbers.

Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C17/F9

Performance

1) a) Using the graphs discussed in question 1) on page 151, estimate the correlation between the two data sets for each set of jurisdictions.
   b) Draw scatter plots that might be represented by correlation values of 0, – 0.5, 0.2, 0.5, –0.9.

A7

Journal

3) Have students explain why a line of best fit is considered a good model for a linear set of discrete values.

Suggested Resources

Modeling and Functions (25-30 hours)

SCO: In grade 10 students will be expected to

C8 **identify, generalize, and apply patterns**

C4 **create and analyse scatter plots using appropriate technology**

C17 **solve problems using graphing technology**

F9 **demonstrate an intuitive understanding of correlation**

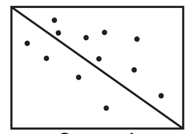
F7 **explore non-linear data using power and exponential regression to find a curve of best fit**

C20 **evaluate and interpret non- linear equations using graphing technology**

C30 **compare regression models of linear and non-linear functions**

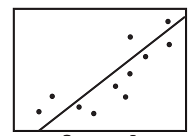
Elaboration – Instructional Strategies/Suggestions

C8/C4/C17/F9 Sometimes the data from certain situations does not suggest a clear trend. In such situations a line of best fit does not represent the data well. Students can often visualize this by examining the graphs and noticing how weak the relationships are (i.e., how scattered the data looks with respect to any line of best fit).

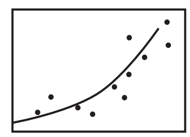


Screen 1

Mathematicians attempt to fit a line to make a trend more visual. When the fit is weak, students should expect a correlation coefficient value below 0.9. (Note: In non-mathematical contexts, correlation coefficients of 0.5 to 0.9 are often considered to be strong.) In the diagram above, Screen 1 shows a relatively weak, negative trend; however, the mathematical correlation may be between -0.9 and -1.



Screen 2



Screen 3

Sometimes a visual scan of the data suggests that a curve of best fit would be better than a straight line. For example (see Screen 2), when linear regression is used on data depicting the swing of a pendulum, the line of best fit shows a correlation value of around 0.97, but a visual scan of the graph suggests a curve might fit the data better (Screen 3).

F7/F9/C20/C30 When students look at Screen 2 they should see a pattern in the data (data to the lower left and upper right are generally above the line, while the data in the middle of the scatter plot are below the line). This pattern suggests that a curve might be a better fit. Students should try a different regression, such as power or exponential. Again, they can study both the patterns on the screen and/or the correlation coefficient to determine which might produce a better fit. When students expect a quadratic model, they should fit the data with a quadratic regression. Note: This may result in no r-value being displayed. Some technologies do not include correlation values for quadratic fits by regression. Usually an r^2 value will be displayed, indicating variance (% of variation that can be predicted). This is due to the process the calculator uses to obtain the equation.

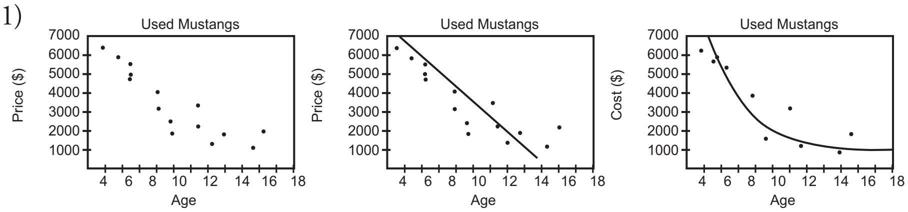
Modeling and Functions (25-30 hours)

Worthwhile Tasks for Instruction and/or Assessment

C30/F9/C8

Performance

1)



i) ii) iii)

The data in i) above represent the selling price of Ford Mustangs based on their age. Graphs ii) and iii) are the results of Chantal doing a linear regression and Andrew doing an exponential regression, respectively.

a) Which model looks like the best fit? Explain your answer.

b) Determine the price for a five-year-old and a 10-year-old Mustang from each of plots i), ii) and iii).

C20/C17/F9/C30/F7/C4

2) The following table gives the fuel economy (litres per 100 km) for selected cars in a city, and the average annual fuel cost to operate the cars.

Car	Litres per 100 km	Average annual fuel cost	Car	Litres per 100 km	Average annual fuel cost
Honda Civic CRX	4.9	$576	Honda Civic Wagon	7.8	$875
Nissan NX Coupe	8.7	901	Ford LTD Wagon	14.3	1445
Geo Metro XFL	4.6	525	Lamborghini Diablo	26.9	3045
Volkswagen	6.6	693	Porche 928 S4	18.6	2235
Chrysler Acclaim	10.1	1030	Rolls Royce Cont.	24.2	3045
Saab 9000	12.1	1314	BMW M5	22.0	2392
GM Caprice Wagon	9.3	1518	Rolls Royce Silver	24.2	3045
Volvo 240 Wagon	7.6	1204	Mercedes 300TE	15.2	1970
GM Road Master	15.2	1518	Toyota Camry Wagon	13.5	1374
Cadillac Brougham	15.2	1518			

a) Plot the fuel cost as a function of the litres per 100 km (l /100km, fuel cost), fill out the following chart for the different regression models, and draw a sketch of each line or curve of best fit.

Equation	type of regression	a	b	r	Regression equation
$y = a^x + b$	Lin				
$y = ab^x$	Exp				
$y = ax^b$	Pwr				

b) Find a model that seems to best describe the data. Be prepared to justify your claim.

c) Use your model to predict the average annual cost to operate (i) a Caprice station wagon listed at 9.3 litres per 100 km in city driving, and (ii) a Ferrari, listed at 23 litres per 100 km in the city.

Suggested Resources

Unit 5
Geometry/Trigonometry
(15 Hours)

In Unit 5 students develop and apply the primary trigonometric ratios to solve problems. In particular, this involves solving problems with respect to similar and right triangles, the Pythagorean Theorem, and bearings and vectors. The core curriculum requires that students apply deductive reasoning to understand proofs of the Pythagorean Theorem, and inductive reasoning to develop some basic operations on radicals. As well, students will solve simple radical equations.

For purposes of differentiation for some students as needed, deductive proof, operations on radicals, and solving radical equations may be omitted. (See pp. 166, 168 and 172 for more detail.)

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

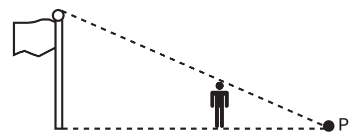
D2 apply the properties of similar triangles

D8 solve problems involving similar triangles and right triangles

Elaboration – Instructional Strategies/Suggestions

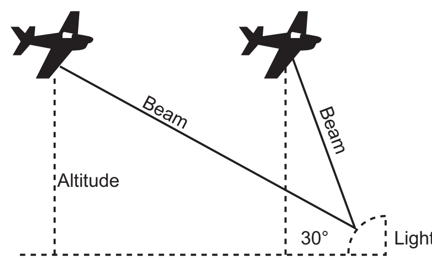
D2/D8 Students, through applications, will review techniques studied previously for indirect measurement. Many problems involving indirect measurements can be solved by using mathematical models based on similar triangles. For example, teachers might take students outside the school and determine the height of the school or a flagpole, or the width of a river or a stream, using similar triangle properties.

For example, by measuring distances from P to the base of a flagpole and to a student's feet, and knowing the height of the student, the length of the flagpole can be determined by using a scale diagram, or proportion in similar triangles.



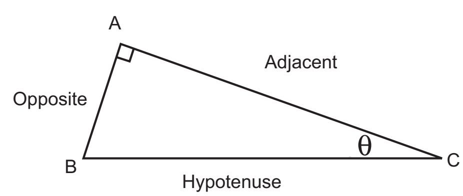
Another situtation that students might be asked to consider involves airplanes and a searchlight beam. Students will be seeking to identify the relationship that exists between the angle and the length of the light beam, and might proceed as follows:

A searchlight at an airport is set at a 30º angle of elevation. As it rotates, its beam illuminates planes flying at a fixed altitude in its path. Ask students to construct scale diagrams that involve various angles of elevation (20º, 40º, 45º, 50º, 60º, 75º) and to find a relationship between the angle of elevation and the length of the beam. For planes at a fixed altitude, students should discover that, as the angle of elevation of the beam increases, the length of the beam decreases.

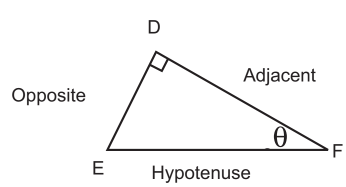


In right-angled triangles, the sides are commonly given names, as shown in the diagrams below. The naming of the opposite and adjacent sides is dependent on the reference angle, called θ in the diagram.

Students have learned that when two triangles are similar, their sides are proportional. For example, if $\triangle ABC \sim \triangle DEF$, then AB : DE = AC : DF.



Students should be encouraged to examine the ratios $\frac{\text{opposite}}{\text{adjacent}}$ (to determine if AB : AC = DE : DF), $\frac{\text{opposite}}{\text{hypotenuse}}$ (to determine if AB : BC = DE : EF) and $\frac{\text{adjacent}}{\text{hypotenuse}}$ (to determine if AC : BC = DF : EF).



(Note: Some students should also become comfortable algebraically re-expressing a similar triangle proportion such as AB : DE = AC : DF, as an equivalent comparison of ratios within triangles, i.e., AB: AC = DE : DF.)

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

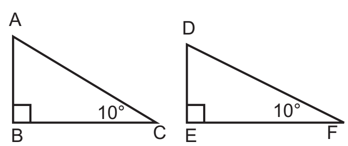
D8

Performance

1) Construct three different, but similar triangles, each with a 90º angle and a 30º angle. Use your ruler to measure the side lengths and complete the table below. (Remember to use an appropriate number of significant digits.)

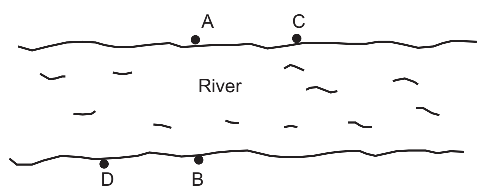
	Measure			Ratio		
Triangle #	Side opposite 30º	Hypotenuse	Side adjacent to 30º	$\frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{\text{opposite}}{\text{adjacent}}$	$\frac{\text{adjacent}}{\text{hypotenuse}}$
1 2 3						

   a) Explain what is different about triangle #1 and #2.
   b) Explain what is the same about triangle #1 and #2.
   c) Describe the pattern(s) you see in the 'Ratio' columns in the chart.
   d) If a fourth triangle has 30º and 90º angles, and the side opposite the 30º angle measures 13.2 cm, use the chart to help determine the length of the side adjacent to the 30º angle.
   e) Repeat all the above by changing 30º to 40º and answering (a) to (d).
   f) What conclusions (with respect to ratios) can you make after completing (e)?
   g) Given the two triangles below, complete this proportion statement:



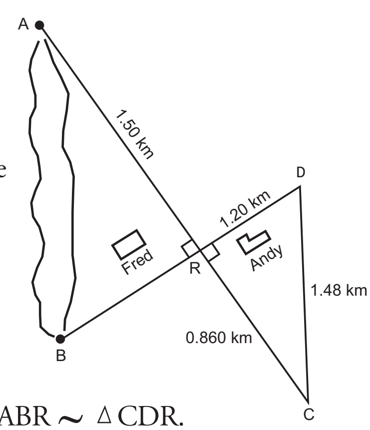
$\frac{AB}{AC} = \boxed{\frac{\quad}{\quad}}$. Explain why you think you are correct.

2) The bridge between two small towns, A and B, has been washed out. (Note: The bridge was perpendicular to each shore.) David lives 1.5 km upstream from town B. He dives into the water and swims toward the opposite shore. The current carries him as he swims so that when he gets out of the water at the point marked C, he is 0.8 km from town A.



   a) Assuming David swam in a straight line and DC intersects AB at R, with RA = 0.5 km, how far did he swim?
   b) Assuming he swam in a straight line and the distance from A to B is 1.2 km, how far did he swim?
   c) Using the same diagram, create a new situation with new measurements so that the solver must find the length of the bridge.

3) Fred's triangular property is bounded on one side by a lake with unknown length (AB). Andy has a smaller property with a triangular shape similar to Fred's, i.e., $\triangle ABR \sim \triangle CDR$.
   Fred wants to determine the length (AB) of the lake. He knows that AR = 1.50 km. His neighbour's property has these measurements: DR = 1.20 km, RC = 0.860 km, and DC = 1.48 km. Ask the students to determine the length, AB, of the lake.



Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

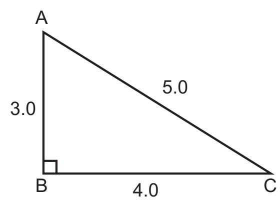
D8 **solve problems involving similar triangles and right triangles**

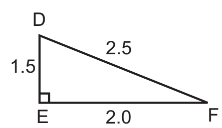
D10 **determine and apply relationships between the perimeters and areas of similar figures, and between the surface areas and volumes of similar solids**

D7 **determine the accuracy and precision of a measurement**

Elaboration – Instructional Strategies/Suggestions

D8/D10 At this point, students should extend their study of similar figures to examine the relationships between perimeters and areas of similar figures. For example, when students determine that two triangles are similar, they should know that the dimensions of one are a scale factor of the dimensions of the other. If students are given a 3, 4, 5 right triangle and a second, similar triangle with two leg measures of 1.5 and 2.0, they might predict the hypotenuse to have a length of 2.5, since the scale factor must be $\frac{1.5}{3} = \frac{1}{2}$.





The total perimeter of each triangle also reflects the scale factor (perimeter of $\triangle$ABC = 12 and perimeter of $\triangle$DEF = 6). Before calculating the areas, students might again predict a scale factor of $\frac{1}{2}$, but, when calculated,

$A_1 = \frac{bh}{2} = \frac{3(4)}{2} = 6.0$

$A_2 = \frac{bh}{2} = \frac{1.5\ (2)}{2} = 1.5$

and the ratio $\frac{A_2}{A_1} = \frac{1.5}{6} = \frac{1}{4}$

Students will find the ratio of the areas to be $\frac{1}{4}$.

D10 From an exploration such as the above, students should be able to describe the ratio between the perimeters of similar figures to be the same as the ratio between any pair of corresponding sides. The ratio of the areas, however, would be the square of the ratio of any two sides, or square of the scale factor.

D7 Throughout this section, dimensions of triangles will be given, measured or calculated. Students should be accurate when measuring and be careful to record measurements with appropriate precision (see page 46 in the Data Management section).

Geometry/Trigonometry (15 hours)

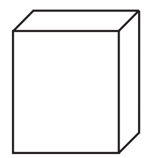
Worthwhile Tasks for Instruction and/or Assessment

D10/D8/D7

Performance

1) Using a model of a common geometric shape (e.g., pattern blocks), take any mathematical shape and trace it on a piece of paper. Create a similar shape that has four times the area. Explain how you did this. Without measuring, what can you say about the ratio of the perimeters? Explain.

2) A box containing 350 g of cereal has measurements 3.85 dm by 2.10 dm by 5.11 cm, and sells for $3.75. The business wants to create a similarly-shaped box to contain more cereal, so as to reduce the cost of manufacturing small containers. If they want the total surface area to be four times that of the original box, what will be the new dimensions of the similarly-shaped box?



Suggested Resources

Geometry/Trigonometry (15 hours)

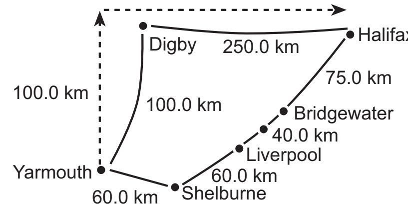
SCO: In grade 10, students will be expected to

D6 solve problems involving measurement, using bearings and vectors

D2 apply the properties of similar triangles

Elaboration – Instructional Strategies/Suggestions

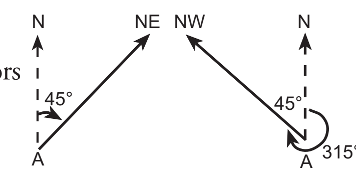
D6 Students should also determine measurements incorporating compass bearings and vectors as translation arrows or directional arrows. Consider the following example.



A pilot plans a flight in a small plane, starting from Yarmouth, Nova Scotia. (Find the airport on the map.) The pilot decides to fly north for one hour at 100 km/h (no wind). After one hour, the plane heads east for two-and-a-half hours. Ask students to indicate on the map how far the plane travels. Students would draw a vector due north (indicated with a dotted line) from Yarmouth and mark 100.0 km to represent its length. The second leg of the trip is marked with a dotted line to indicate a vector that is 250.0 km long. The plane should now be somewhere near Halifax. (Students might then be asked to create the vector direction and flight time needed to proceed to Bridgewater or Liverpool, using North, South, East, and West components only.)

Since a vector represents a directional arrow, the resultant vector in the above trip is the vector joining Yarmouth to Halifax. In other words, the result is that the plane left Yarmouth and arrived in Halifax.

D2 Students could determine the flying distance directly from Halifax back to Yarmouth by drawing a scale diagram of the situation. Some students might use the Pythagorean Theorem, since the two given vectors are perpendicular. Students could measure the angle on the scale diagram to determine the bearing for the return trip to Yarmouth. Later in the unit, students will use trigonometry for these calculations.



Directions can be read as bearings, which use angle measurements, always measured clockwise from due north. Heading northeast would be a 45° bearing; northwest would be 315°. Students will use scale drawings when solving problems.

Some students may want to investigate the use of vectors expressed as column matrices. (See, for example, *Enrichment* problem 3) on the next page.)

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

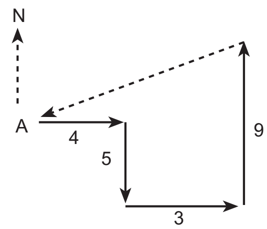
D2/D6

Pencil/Paper

1) A boat leaves port, P, travelling on a 45° bearing. It travels 6.0 km in an hour, then changes direction to a 180° bearing. After travelling 4.0 km in this direction, the boat returns to port. Ask students to use a scale diagram to find how far it has to travel to get back to port, and on what bearing.

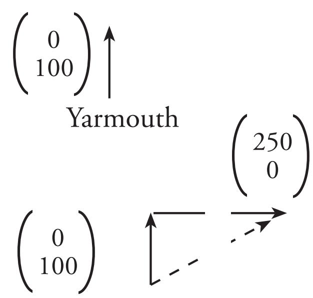
2) a) Given the vectors as marked on the diagram, ask students to describe, using bearings, the path indicated from A..

   b) Have students make up a problem from the information in this diagram for others students to solve.



Enrichment

3) While Sadie was working on the Yarmouth to Halifax flight problem on the previous page, she made the following comment, "I was reading about vectors and bearings on the Internet, and on the basis of what I read, this problem could be solved by using components." Vectors can be described by using components; one component represents the movement to the 'right,' the other, the movement 'up,' thus not only determining a direction but also a distance or 'magnitude.' For example, the vector leaving Yarmouth for Digby has a component to the 'right' of zero, since the direction was due north, and a component 'up' of 100.0 km, since the plane was travelling 100.0 km per hour.



It would be symbolized $\begin{pmatrix} \text{right} \\ \text{up} \end{pmatrix}$, or as $\begin{pmatrix} 0 \\ 100 \end{pmatrix}$.

The second vector was to the east to Halifax, so the movement 'right' was 250.0 km (2.5 hours at 100km/h) and the movement 'up' was zero.

The resultant vector (dotted line) has a 'right' movement of 250 and an 'up' of 100. Thus the resultant can be expressed as $\begin{pmatrix} 250 \\ 100 \end{pmatrix}$ and can be seen to be the sum of the two given vectors: $\begin{pmatrix} 0 \\ 100 \end{pmatrix} + \begin{pmatrix} 250 \\ 0 \end{pmatrix} = \begin{pmatrix} 250 \\ 100 \end{pmatrix}$

The magnitude (or distance in this case) of the resultant vector can be found by using the Pythagorean Theorem, since a right triangle is involved:

$$\left|\begin{pmatrix} 250 \\ 100 \end{pmatrix}\right| = \sqrt{250.0^2 + 100.0^2} = 269.3 \text{ km}$$

   a) If a resultant vector $\overrightarrow{AB}$ has components $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$, draw a diagram and explain what this means. Find the magnitude of $\overrightarrow{AB}$.

   b) If a plane flies 225.0 km north, then 125.0 km west, draw a diagram, label, using components, and show how to use components to find how far the plane is from its starting point.

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

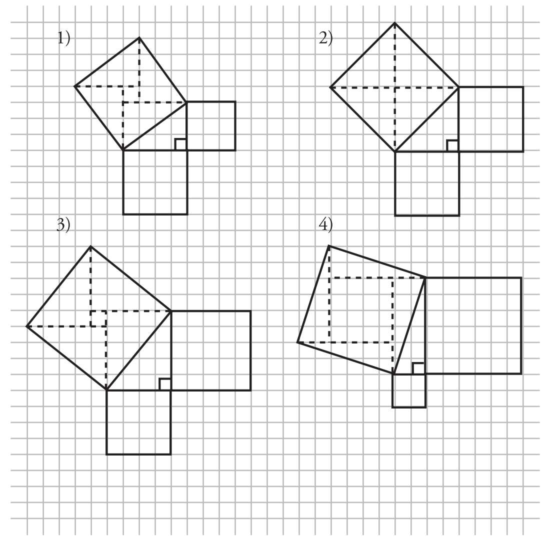
D8 solve problems involving similar triangles and right triangles

D14 apply the Pythagorean Theorem

Elaboration – Instructional Strategies/Suggestions

D8/D14 As students work with right triangles (using properties of similar triangles), they should review and apply the Pythagorean Theorem to solve problems. By way of review, students might explore the following situation:

Squares are drawn on each side of a right-angled triangle. The area of the square on the hypotenuse is compared with the areas of the squares on the other two sides.



[To do this, students should cut along the dotted lines and place the pieces onto the squares on the other two sides. Additional snipping might be required to complete the task.] What conjecture might the students make?

Students should be asked to demonstrate their understanding of the Pythagorean Theorem and how it is used to solve problems involving right triangles. For example, in the vector situation on page 162, from Yarmouth to near Digby was 100.0 km, and near Digby to near Halifax was 250.0 km. Students might determine the resultant distance from Yarmouth to Halifax to be about 269 km.

$$YH^2 = YD^2 + DH^2$$

$$= 100^2 + 250^2$$

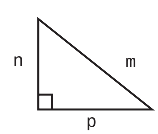
$$YH = \sqrt{72500}$$

$$\doteq 269$$

Students should be able to state and demonstrate an understanding of the theorem, "... in any right triangle, the square of the length of the hypotenuse is the same as the sum of the squares of the lengths of the two other sides (legs)." They should understand how to symbolize the theorem, regardless of the letter names given to the sides.

$$\text{hypotenuse}^2 = \text{leg}^2 + \text{leg}^2$$

$$m^2 = n^2 + p^2$$



Often, in application problems requiring the use of the Pythagorean Theorem, buildings, poles, and towers are used. It should be clear to students that, unless stated otherwise, these structures are assumed to be at right angles to the ground.

Geometry/Trigonometry (15 hours)

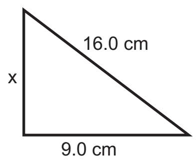
Worthwhile Tasks for Instruction and/or Assessment

D8/D14

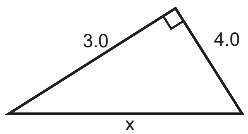
Pencil/Paper

1) Ask students to indicate in which of the following situations the Pythagorean Theorem can be applied and explain why.

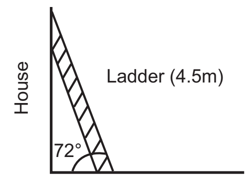
a) Find x



b) Find x

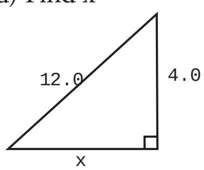


c)

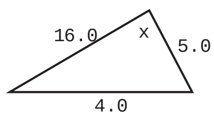


How far is the foot of the ladder from the house?

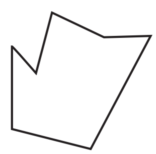
d) Find x



e) Does x = 90º?



2) a) A carpenter has a scrap piece of plywood to use as a shelf suport. It is large enough to create a triangular piece with a right angle. Ask the students to describe how the carpenter can use his knowledge of mathematics to construct a right-angle triangle from this scrap of wood.



b) Bill measures one edge of the scrap plywood to be 4 dm. Ask the students to show how he might use this measurement as a leg measurement to create a right triangle.

c) Sue uses the 4 dm as the hypotenuse measurement. Ask the students to describe what her method might be.

Journal

3) You and a friend want to determine the height of a tree in the school yard. The tree is too high to climb. Your friend has a metre stick. Explain how you and your friend can determine the height of the tree.

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

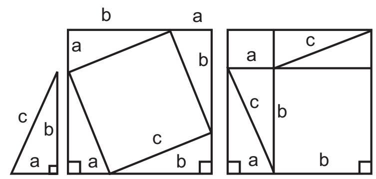
E7 **demonstrate an understanding of and write a proof for the Pythagorean Theorem**

E8 **use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures**

E9 **use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid**

Elaboration – Instructional Strategies/Suggestions

E7/E8/E9 Students have explored the Pythagorean Theorem in previous years and have seen that it works in every right-triangle situation where they have applied it. They should understand that when a relationship is proven in general, it becomes a theorem, in this case called the Pythagorean Theorem. They need to come to an agreement that their explorations and their applications of a relationship are not proofs; i.e., a proof is not the result of many cases that work. They should then try to find a way to prove beyond a doubt that the Phythagorean relationship will work for every case (turn the relationship into a theorem). For example, students might be asked to discuss how the diagram above could lead to a proof of the Pythagorean Theorem.



Teachers may have to discuss with students what constitutes a proof. Often, when teachers talk about proof, they are really talking about critical thinking. The type of proof expected in this activity might evolve from a process such as the following:

- *understand the situation* (i.e., make sense of the three diagrams and how they are connected.)
- *deal with the evidence* (Students should see that all of the triangles have the same dimensions. They should note that the large square in the middle diagram represents c^2, that the rest of the area of that middle diagram is made up of four triangles, that those same four triangles are situated differently in the right-hand diagram, ...)
- *go beyond the evidence* (Students may create other representations to try to validate their conjectures.)
- *state and support conclusions/decisions/solutions* (Students should be encouraged to record each statement that they think is correct or leads towards the validation of each of their conjectures. For example, they may begin by saying that each side of the four-sided figure in the middle diagram has a length ‘c’. This is true because each side of the square is also the ‘c’ side of the given triangle.)
- *apply the conclusions/decisions/solutions* (Students would then conclude that the large four-sided figure in the middle diagram is a square with side length ‘c’, so it represents c^2. . . .)

After students have developed what they consider to be proofs, several of them should be asked to explain and defend their proofs to the class, to test their comprehension of them. Students should then be encouraged to explore other proofs of the same theorem, and write proofs of their own. A variety of activities, such as paper/pencil construction, measuring, a Pythagorean Theorem video, paper folding, and modeling should be available as additional ways to explore the relationship and prove the theorem (see activities on the next page). Students might be interested to know that there are more proofs of the Pythagorean Theorem than of any other theorem in geometry.

The core curriculum requires that students be able to write a proof for the Pythagorean Theorem, as well as to show that they understand it. For purposes of differentiation, it would be sufficient for some students to show that they understand the theorem. As well, the core curriculum requires students to construct and evaluate the validity of logical arguments (SCO E9). To differentiate, this requirement could be dropped for some students. This might mean, for example, that while some students would evaluate “demonstrations” of the truth of the Pythagorean Theorem to determine whether or not they were valid proofs, other students would not.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

E7

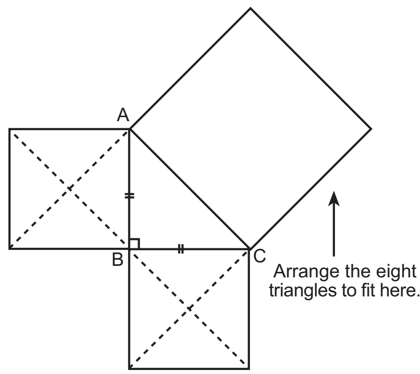
Journal

1) Pythagorean triples occur when the three side lengths of a right triangle are all whole numbers.
   a) Ask the students to explain how they would check to see if a 3 – 4 – 5 combination is a Pythagorean triple.
   b) Ask the students to find at least two more Pythagorean triples.

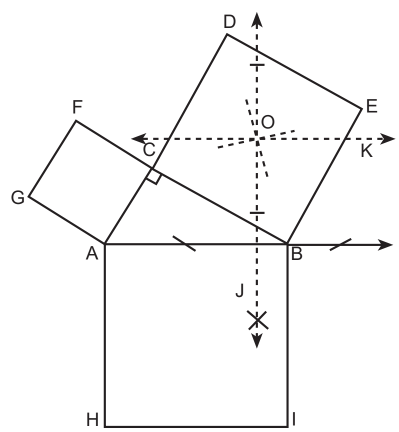
E7/E8/E9

Performance

2) Right triangle ABC is isosceles. Have the students reproduce the diagram at right, then cut out the four triangles in each of the two small squares and arrange them to exactly cover the larger square. Ask the students to describe their findings in their own words.



3) Ask the students to perform the steps below using a full sheet of paper. (Note: The arcs and segment extensions necessary to complete Steps 1 and 2 are not indicated in the diagram.)



Step 1: Construct a scalene right triangle in the middle of your paper (hypotenuse down). Label it so that the hypotenuse is $\overline{AB}$ and the longer leg is $\overline{BC}$.

Step 2: Construct a square on each side of the triangle. Label the square on the longer leg BCDE. Label the square on the shorter leg AGFC. Label the square on the hypotenuse ABIH.

Step 3: Locate the centre of BCDE (intersection of the two diagonals). Label the point O.

Step 4: Through point O, construct line J perpendicular to the hypotenuse AB.

Step 5: Through point O, construct line K perpendicular to line J. Line K is parallel to the hypotenuse. Lines J and K divide BCDE into four parts.

Step 6: Cut out the smaller square AGFC and the four parts of square BCDE. Arrange them to exactly cover the square ABIH on the hypotenuse. Describe what has occurred in your own words.

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

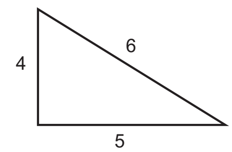
A3 **demonstrate an understanding of the role of irrational numbers in applications**

A4 **approximate square roots**

A8 **demonstrate an understanding of and apply properties to operations involving square roots**

Elaboration – Instructional Strategies/Suggestions

A3/A4 When students use the Pythagorean Theorem to solve problems, very often the resulting side length may not be a whole number. For example, when searching for Pythagorean triples, one student thought that since 3 – 4 – 5 was a Pythagorean triple, 4 – 5 – 6 might be as well. However, when attempting to verify this, the student found that $\sqrt{4^2 + 5^2} = \sqrt{16 + 25} = \sqrt{41}$. The student knew that 41 was not the square of a whole number.



Students need to be able to use, visualize, and interpret irrational numbers (e.g., $\sqrt{41}$), especially when using trigonometry and solving equations that involve radicals. Students need to know that $\sqrt{41}$ can be read as "square root of 41." They should realize that, when $\sqrt{41}$ is multiplied by itself ($\sqrt{41}^2$), the result is 41.

Students should be involved in discussion about irrational numbers. For example: "When should I use an exact value (e.g., $\sqrt{24}$), the reduced exact value ($2\sqrt{6}$), or an approximate value (4.898979...), and to how many decimal places?" This question can be answered by examining the use of the irrational number and deciding which expression is most appropriate in the given context. Being able to simplify a radical is, for instance, helpful in appreciating its magnitude.

In this course, students will operate on radicals, use mental math and use calculators. When approximating square roots, students should use the appropriate number of decimal places, determined by the measurements used in the problem. Calculations involving radicals (and other irrational numbers such as pi) can often be done by using appropriate keys on calculators.

A8 Students should understand that any whole number has two square roots, one positive and one negative (e.g., the square root of 25 is 5 or –5). They should realize that the principal square root (indicated by the radical sign) is the positive root. (Note: $\sqrt{25} = 5$ while $-\sqrt{25} = -5$)

Students should also understand that, when estimating square roots of numbers between zero and one, the square root will be larger than the number itself.
For example: $\sqrt{\frac{1}{4}} = \frac{1}{2}$ and $\frac{1}{2} > \frac{1}{4}$

When approximating square roots, students should be asked to place numbers such as $\sqrt{62}$ on the number line. They might reason thus: $\sqrt{62}$ is between $\sqrt{49}$ and $\sqrt{64}$, so $\sqrt{62}$ is between 7 and 8. Further, $\sqrt{62}$ is close to $\sqrt{64}$ or 8.

The core curriculum requires that students understand and apply properties of radicals in operations (SCOs A8 and B2). This implies that students should understand the connection and be able to move between entire and mixed radicals, and perform basic addition and subtraction procedures. (The development of these techniques is limited here by the need for them in the accompanying theoretical and contextual situations. Operations on radicals will be developed further in a later course.) For purposes of differentiation, some students may briefly explore properties of radicals, but would be expected to work strictly with approximations in application situations.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

A3/A4/A8

Pencil/Paper

1) A police cruiser is at the point (6.0, 0.0) and is returning to the harbour located at (3.0, 1.0). A sailboat sends a distress call from the point (0.0, 0.0). The police cruiser has enough fuel to go 12.0 km. Should the captain of the cruiser go to the harbour to refuel or can he go to the sailboat first and then return to the harbour? Justify your choice.
2) You know that π is approximately 3.14. Suppose you were making circular tablecloths as holiday gifts for your family. The tablecloths have a colourful fringe. You measure the diameter of the tablecloth and estimate the length of fringe to cut, using a value of 3 for π. Compared to using the π key on the calculator, do you think your estimate is an overestimate or an under-estimate? Many students use $\pi = 3.14$ instead of the π key on the calculator. How much over or under would the answer be if the diameter was measured to be 100.0 cm? Have the students explain the proper use of the π value if the diameter is measured as 152.35 cm.

Performance

3) Confederation II Bridge
   Jubilant Joey, a construction engineer, computed that the maximum safe load of a new bridge being planned from Prince Edward Island to Newfoundland could be determined by the expression
   $100\,(99–70\sqrt{2}\,)$ tons.

   A sign was made to post on the new bridge, based on Joey's safe load calculation, saying the bridge could safely hold 100 tons. On the day it opened to traffic, a section of the bridge collapsed under a load less than a 10th of the posted weight. Ask the students to write an explanation to the public to explain why the bridge collapsed.

   Discussion: If students are not sure how the error was made, ask them to evaluate the given expression, using the square root of 2 correctly on their calculators. Then ask them to perform the same calculation using an approximate value for $\sqrt{2}$, such as the very common 1.4. It is useful for students to use algebra to see why such small differences in the value of $\sqrt{2}$ are creating large differences in their answers. Have students expand the expression $100\,(99–70\sqrt{2}\,)$, using the distributive law. They should now see that any difference between $\sqrt{2}$ and their approximation for $\sqrt{2}$ gets multiplied by 7000, making a potentially huge difference in the result.

   Students might also explore changing the 100 to 1000, or the 99 to 98, or the 70 to 71, etc., to see how changes in these values might affect the calculation. They should use their findings to help them create their own problems like this one to give to another class to solve.

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

A3 **demonstrate an understanding of the role of irrational numbers in applications**

A4 **approximate square roots**

B2 **develop algorithms and perform operations on irrational numbers**

D14 **apply the Pythagorean Theorem**

E8 **use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures**

Elaboration – Instructional Strategies/Suggestions

A3 Irrational numbers may result from using the Pythagorean Theorem (as discussed on the previous page). Discussion of the use of exact and approximate solutions, and extending student understanding of irrational numbers and how they fit within the real number system, should take place within this context.

A4/B2 Students' understanding of irrational numbers and how they fit within the real number system could be strengthened by exploring and interpreting square roots visually as well as numerically. When computing with square roots, estimation will be faciliated if students learn to simplify some radicals first. Students should be competent in the use of calculators for finding square roots of numbers and understand the way approximate values for irrational numbers are displayed. Students should be careful when using approximations.

D14 Students might use geoboards or dot paper to help visualize and represent square roots. For example, when asked to find the side length of the square in Diagram 1, students might draw in the lines to form a right triangle (Diagram 2) and use the Pythagorean Theorem to determine that AB must be $\sqrt{1^2+3^2} = \sqrt{10}$. From this activity, students can 'see' that $\sqrt{10}$ has a definite length, and thus it makes sense to call it a 'real' number.

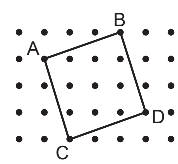


Diagram 1

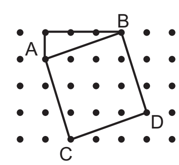


Diagram 2

B2/E8 Visually, students can also learn about simplification of radicals. For example, the square (in Diagram 3) with side AB = $\sqrt{2^2+2^2} = \sqrt{4+4} = \sqrt{8}$ and the square with side MN = $\sqrt{1^2+1^2} = \sqrt{1+1} = \sqrt{2}$ can be compared. MN is the hypotenuse of a right triangle with legs one unit long. AB is the hypotenuse of a right triangle with legs 2 units long. Thus the triangle with side MN is similar to the triangle with side AB, making AB twice as long as MN, or $2\sqrt{2}$. Students can now conjecture that $2\sqrt{2} = \sqrt{8}$, then more formally explore this idea: $\sqrt{8} = \sqrt{4 \cdot 2}$ or $\sqrt{4} \cdot \sqrt{2} = 2 \cdot \sqrt{2}$ or $2\sqrt{2}$. Thinking of a side with length $\sqrt{8}$ as a side with length $2\sqrt{2}$ might help students approximate the length as 2(1.414) or 2.828 units. Similarly, a side length of $\sqrt{75}$ could be expressed as $5\sqrt{3}$ and approximated more easily (knowing $\sqrt{3} \doteq 1.73$) as $5\,(1.73) \doteq 8.6$.

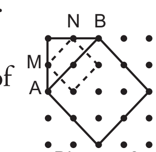


Diagram 3

Some problem situations require the addition or subtraction of lengths and, hence, may require the addition or subtraction of expressions involving radicals. Consequently, the basic principles of adding and subtracting radicals should be addressed. Students should see that, to add $\sqrt{8}$ and $3\sqrt{2}$, they must first rewrite $\sqrt{8}$ as $2\sqrt{2}$, then add $3\sqrt{2}$ to get $5\sqrt{2}$. The connection should be made to algebra (e.g., $2x + 3x = 5x$). The need to add these exact values should be made clear through the context of the problem.

For suggestions for possible differentiation, see the discussion at the bottom of page 168.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

A3/A4/B2/D14

Pencil/Paper

1) Have the students draw a diagram to show the length of each of the following and record each as the square root of the sum of two squares.

   a) $\sqrt{13}$ b) $\sqrt{29}$ c) $\sqrt{45}$

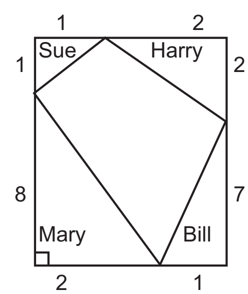
2) Ask the students to record $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$ to two decimal places, then use these to approximate the values for

   a) $\sqrt{72}$ b) $\sqrt{45}$ c) $\sqrt{27}$

Pencil/Paper/Journal

3) a) Ask students to describe $\sqrt{72}$, using a picture.
   b) Use the picture of $\sqrt{72}$ to approximate a value for $\sqrt{72}$.

4) A section of land 9 km by 3 km was sectioned off into 5 lots. Fencing was needed to define the 5 properties.
   a) If the four land owners in the corner lots have to buy the sections of fence for their own properties, how much more fencing will Bill have to buy than Sue?
   b) Find the length of the total fencing required.
   c) If fencing costs $15 per 9 metres, what will be the cost of the fencing?



Enrichment

5) Have students create a problem similar to 4), only in reverse. This is to say that the problem will give the lengths of fencing, and request the dimensions of the original rectangular plot of land.

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

C27 **solve linear and simple radical, exponential, and absolute value equations, and linear inequalities**

Elaboration – Instructional Strategies/Suggestions

C27 Students should explore how to solve simple radical equations (i.e., equations with the variable under a square root sign).

For example, when asked to find the missing side of a triangle being used to model a flower garden, they might need to solve an equation like

$$\sqrt{2x + 2} + 15.5 = 25.5$$

They should discover that to solve this they need to isolate the radical,

$$\sqrt{2x + 2} = 25.5 - 15.5$$

then square both sides to eliminate the radical,

$$(\sqrt{2x + 2})^2 = (25.5 - 15.5)^2$$

$$2x + 2 = 100$$

$$2x = 98$$

$$x = 49$$

(**Note:** Equations involving square roots on both sides of the equation will be dealt with in a subsequent course.)

Solving simple radical equations (SCO C27) is clearly part of the core curriculum. For purposes of differentiating as needed for some students, however, solving radical equations may be omitted.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

C27

Performance

1) During several years of working with a large electrical contracting firm, Nate made use of several mathematical formulas.

   a) Solve for the missing variable in $I = \sqrt{\frac{P}{R}}$

      i) when I = \$17.50 and P = \$208

      ii) when R = 6.5% and P = \$1500

   b) Use the formula $V = \sqrt{PR} + 15$ to find the value of the missing variable, given

      i) R = 8.9, V = 124.6

      ii) P = 235, V = 57.3

      iii) P = 1500, R = 9.3

2) Solve

   a) $3 = x + \sqrt{x - 3}$

   b) $\sqrt{2x + 3} = 5$

   c) $\sqrt{7 - 3x} + 3 = x$

   d) $x - 2 = \sqrt{x - 2} + 12$

   e) $\sqrt{-3x - 14} - x = 4$

   f) $3 = \sqrt{3x - 9}$

   g) $5 - \sqrt{2x} = -7$

   h) $\sqrt{\frac{2}{3}}\, x - 1 = 5$

Suggested Resources

Geometry/Trigonometry (15 hours)

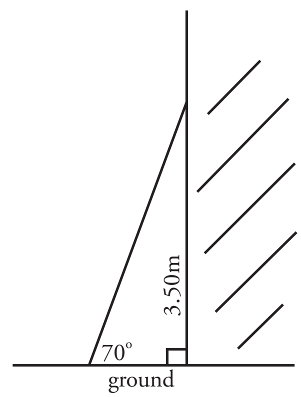
SCO: In grade 10, students will be expected to

D3 relate the trigonometric functions to the ratios in similar right triangles

Elaboration – Instructional Strategies/Suggestions

D3 Students should investigate the three ratios between the lengths of pairs of sides in right-angle triangles. They might begin this process by constructing several, similar right triangles, each with one angle of, for example, 30°. Using 30° as the reference angle, students could then measure the opposite and adjacent sides and hypotenuse for each triangle, and determine the appropriate ratios (opposite to adjacent, opposite to hypotenuse, and adjacent to hypotenuse) for each. Students should readily notice that the value of any given ratio stays constant, no matter the size of the triangle.

Students should then repeat this process for right triangles containing angles of other sizes (e.g., 15°, 40°, 45°, 70°). They will see that as the reference angle changes, the values of the three ratios change. They should also be guided to conjecture, however, that in any right triangle, the ratios of the different pairs of sides remain constant for a given acute angle, regardless of the size of that angle or the size of the triangle.



Once students have a clear picture of the connection between these ratios and given angles, they can use them in a problem or two to help clarify their value. For example, given that a ladder makes a 70° angle (as shown) when touching 3.50 m up a wall, how far is the foot of the ladder from the base of the wall? To solve this, students would begin by noting that they know the length of the side opposite the 70° angle and wish to find the length of the adjacent side; hence the ratio of opposite to adjacent is 3.50 to x. From their previous work with certain angles in right triangles, they would be able to state that, for a 70° angle, this ratio should equal about 2.74. Consequently, they could say

$$2.74 = \frac{3.50}{x}$$

$$x = \frac{3.50}{2.74}$$

$$x = 1.28 \text{ m}$$

After students have a clear understanding of the concept of these ratios being fixed for any given angle, and appreciate their usefulness in problem-solving situations, the formal mathematical names (i.e., sine θ, cosine θ, tangent θ) can be introduced. As is often the case throughout mathematics, these names provide a convenient and standard means of talking about the special ratios.

This may be an appropriate time to introduce an activity in which students determine the exact values of the sine, cosine and tangent of angles in special triangles. Not only will students apply symmetry and the Phythagorean Theorem, but they will also be able to i) compare these exact values to those determined previously by measurement and calculation, ii) begin to look at the patterns of values for angles between 0° and 90°, and iii) set the stage for trigonometric studies in subsequent courses.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

D3

Performance

1) The following exploration connects the unique ratios of similar triangles to trigonometric ratios.

   Question: What is a safe angle for a ladder meeting the ground?

   a) Using a model of a ladder (such as a metre stick or board), have students determine a safe climbing position when a ladder leans against a wall. Have them estimate the safe angle as a group and measure the base and ladder length. As in the real world, the distance up the wall is an indirect measurement and should be calculated (by the Pythagorean Theorem). Have them draw a scale diagram and measure the angle with a protractor. Ask them if their estimates are reasonable.
   b) If the ladder was an extension ladder, it could be extended to different lengths. If the safe angle is to be maintained, the distance from the foot of the ladder to the wall must also change. Ask students to use similar triangles to calculate new distances from the wall for various ladder lengths.
   c) Ask students if there is a quick way a fireman can determine where he should put the foot of his ladder to safely reach the roof of a burning building. Since he knows how long his ladder is, a quick estimate of the ground distance would be easy to calculate.
   d) Have students investigate, having each group use a different safe angle and scale diagrams. (Note: This could be explored nicely with a geometry software package.) These constant ratios (in this case, adjacent to hypotenuse) can be defined as cosine θ.
   e) How far from the wall must a 10 m ladder be placed, given a safe angle of 72°. (Use of the calculator is now appropriate.)

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

D4 use calculators to find trigonometric values of angles and angles when trigonometric values are known

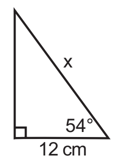
D5 apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation

D12 solve problems, using trigonometric ratios

D7 determine the accuracy and precision of a measurement

Elaboration – Instructional Strategies/Suggestions

D4/D5/D12 It is expected that students will use calculators to find trigonometric ratios ($\sin\theta$, $\cos\theta$, and $\tan\theta$) for given angles, and angle measurements for given side ratios. For example, when asked to find a side length:

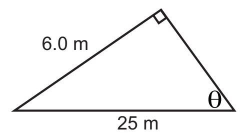


$$\cos 54^\circ = \frac{12}{x}$$
$$x = \frac{12}{\cos 54^\circ}$$

$$x \doteq 20.4156$$
$$x \doteq 20 \text{ cm}$$

When asked to find an angle measure:

$$\sin\theta = \frac{6.0}{25}$$
$$= 0.24$$
$$\therefore \theta \doteq 13.88^\circ, \text{ or } 14^\circ$$



(**Note:** Using a calulator to connect trigonometric ratios with angles obscures the patterns that can be observed in tables of trigonometric values (e.g., for acute angles, all values of $\sin\theta$, and $\cos\theta$ are between 0 and 1; the values of $\sin\theta$ increase as angle size increases from 0° to 90°). It will be valuable to undertake some brief activity to acquaint students with these general patterns.)

D7 Students should realize that an expression like $\frac{12}{\cos 54^\circ}$ is an irrational number, so that when calculated as 20.4156 cm, this is only an approximation. Rounding should occur to express the answer with an appropriate number of significant digits.

When ratios are given for trigonometric relationships, how many significant digits should students work with to achieve an answer? As always, the answer should be presented to the number of significant digits determined by applying the appropriate rule(s) for significant digits.

In general, the precision of answers (in measurement situations) depends upon the precision of the measurements. Consequently, if something measures exactly 5 metres and an answer in centimetres is wanted, represent the 5 m as 5.00 m. When angles are being measured with protractors, be sure that students get precise answers. The teacher should discuss with the students the degree of precision needed for the situation. Angle measures should be recorded using decimals rather than minutes and seconds.

Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

D12

Activity

1) The teacher plays a game with the students, in which, given two trigonometric ratios, the students must give the third as a fraction and draw a complete diagram.
   The teacher says "I have a triangle in which
   a) $\cos \theta = \frac{3}{4}$ and $\tan \theta = \frac{\sqrt{7}}{3}$
   b) $\sin \theta = \frac{4}{5}$ and $\cos \theta = \frac{3}{5}$
   Have pairs of students continue to play this game, for short time periods each day, making up their own questions and answers.

D12/D7

Pencil/Paper

2) Meteorologist Wendy Storm is using a sextant to determine the height of a weather balloon. She sights the balloon 1400.0 m from the sextant and on an angle of elevation of 44.27º. The sextant is 1.5 m above the ground. Have students answer the following:
   a) How high is the balloon?
   b) If Wendy knew that the balloon was at an altitude of 1000.0m, but 1400.0m from the sextant, what would be the sextant reading on the angle of elevation?

D4/D12

Journal

3) Ask students to describe how they would determine the height of a flagpole which they cannot climb in case A and case B.
   Case A: It is a very sunny day and you and your friend have a tape measure.
   Case B: It is an overcast day, and you and your friend have a metre stick and a clinometer (a device to measure angle of elevation).

Suggested Resources

Geometry/Trigonometry (15 hours)

SCO: In grade 10, students will be expected to

D6 **solve problems involving measurement, using bearings and vectors**

D12 **solve problems, using trigonometric ratios**

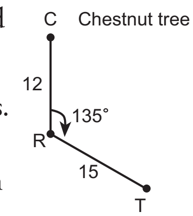
D7 **determine the accuracy and precision of a measurement**

Elaboration – Instructional Strategies/Suggestions

D6 Students should be able to use trigonometry to help solve problems involving bearings and vectors.

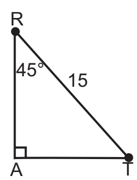
❐ Marie and Marcel found an old treasure map. It said to begin at the old chestnut tree and walk on a bearing of 180° for 12 paces. From there, count off 15 paces on a 135° bearing. At this point, if they dug down, they would find a rewarding treasure.

To find the treasure, they created the diagram at right and followed the directions:



The treasure they found was wrapped in a piece of canvas. It was a small box with another note attached. The note said if they could determine how far away they were from the chestnut tree, and on what bearing they would need to walk to go directly there, they would be given the key to open the box.

D12/D7 For their solution, Marie suggested creating a right triangle ΔRAT, because she knew the $\angle$ ART would have to be 45°.



Marcel then recorded $\sin \angle ART = \frac{AT}{RT}$

$$\sin 45° = \frac{AT}{15}$$

$$AT = 15 \sin 45° = 10.6 \text{ or about 11 paces.}$$

Marcel said that they now knew AR to be 11 paces, since ΔART is isosceles. Since CR is 12 paces, then CA would be 23 paces.

Now they could determine TC (the direct path from the treasure back to the chestnut tree) and the $\angle$ATC:

$$TC^2 = AT^2 + CA^2 \qquad \tan \angle ATC = \frac{23}{11}$$

$$= 11^2 + 23^2 \qquad \therefore \angle ATC \doteq 64°$$

$$TC \doteq 25$$

They agreed the bearing would be 270° + 64° = 334°, and the distance back to the chestnut tree would be about 25 paces.

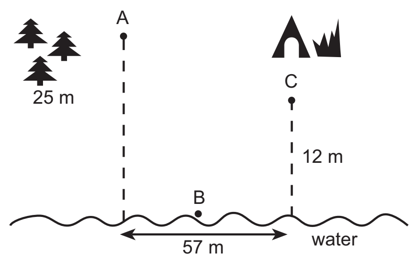
Geometry/Trigonometry (15 hours)

Worthwhile Tasks for Instruction and/or Assessment

D6/D12/D7

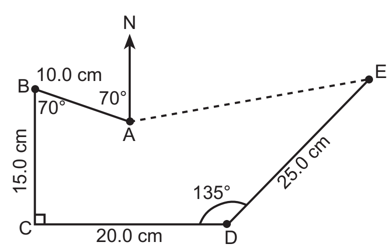
Performance

1) Having just come out of the woods at position A, Marnie noticed a fire at her campsite, C. She wanted to douse the fire with water. She quickly noticed a big rock, B, on a bearing of 160° at the water's edge, where she could fill her bucket and then run directly to the campsite.

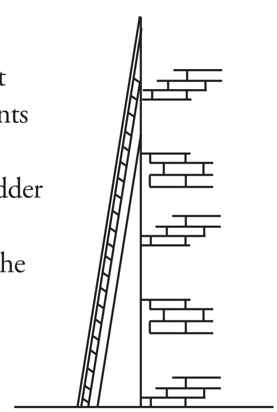


   a) How far must she run in total?
   b) What bearing must she run after filling her bucket?
   c) Is there a better place to fill her bucket that would result in the shortest distance she would have to run?
   d) What bearing is that place from her position at A?

2) If you walk on a bearing of 225° for 4 km, then walk on a 90° bearing for 1 km, how far are you from your starting point?
3) Lynn walks on a bearing of 60° while Sharilyn walks on a bearing of 300°. They each walk for an hour. Lynn walks 6.40 km, Sharilyn walks 6.56 km. How far apart are they? On what bearing should Sharilyn walk to find Lynn, who has stopped?
4) Given this map,
   a) what is the bearing from A to B?
   b) what is the bearing from B to C?
   c) explain how you can find the distance B to D.
   d) what is the bearing from D to E?
   e) is there enough information to find the length from A to E? Explain how you would do it, or why you could not.



5) Suppose a particular type of ladder is safe if the angle it makes with the ground is from 65° to 80°. Have students answer the following questions:
   a) How far up on a vertical wall can a 10.0 metre ladder of this type safely reach?
   b) How far, at minimum, should it be placed from the base of the wall?



Suggested Resources

Unit 6
Geometry/Packaging
(15-20 Hours)

In this unit of the core curriculum, students explore, and make and test conjectures concerning, the relationships between perimeter and area, and surface area and volume, in order to solve problems with respect to packaging. This involves applying concepts of similarity and trigonometric ratios while striving to determine optimal solutions (such as minimizing package surface area and cost for a fixed volume). As well, students will make and test conjectures with respect to altitudes, medians, angle bisectors and perpendicular bisectors in triangles.

For purposes of possible differentiation for some students, an exploration of nets of polyhedra may be useful (see p. 190), while similarity with respect to 3-D figures (p. 192) and properties of triangles relating to physical structures (p. 194) may be omitted.

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

D1 determine and apply formulas for perimeter, area, surface area, and volume

D13 demonstrate an understanding of the concepts of surface area and volume

D7 determine the accuracy and precision of a measurement

Elaboration – Instructional Strategies/Suggestions

D1/D13 In the study of packaging, students should realize that in many cases, the shape and dimensions of a container are determined by the shape and dimension of the product to be placed in the container. For example, a pair of shoes roughly forms a rectangular shape when fitted closely together, thus, it is not surprising to package them in a box that is a rectangular prism. However, when exploring containers for products that do not have a specific shape (e.g., cereal, milk, pop), a variety of factors needs to be examined.

One of these factors is the volume of the container. The container should have a volume that is as close as possible to the volume of the product. Students have worked with volumes of three-dimensional shapes in previous grades. They know, for example, that the volume of a prism is the product of the cross-sectional area and the height. Obviously, cross-sectional area is related to the dimensions, and hence to the perimeter of the base of the prism and, in fact, to the number of sides of the polygonal base. Volumes can also be connected to surface areas, which in turn are related to perimeters. To explore and identify relationships that exist among these factors is the primary purpose of this unit.

To begin, perimeter, area, surface area, and volume may need some review in context. Students should strengthen their intuitive understanding of the concepts of area of two-dimensional figures and of surface area and volume of three-dimensional figures. While exploring area, surface area and volume, students might discover some basic formulas for areas of various polygonal shapes. Also, they may discover a formula for the area of a regular polygon with a fixed perimeter, written in terms of the number of sides.

This should lead students to discover the fact that, for a fixed perimeter, as the number of sides of a polygon increases, the area increases. In fact, the students should conclude that the circle is the limiting figure in this sequence (i.e., the figure with the most area) and be able to explain why. As students begin to focus on the volumes of prisms with regular polygonal bases, they should explore relationships among volumes, surface areas, and perimeters and discover which prism has the largest volume for a fixed surface area.

D7 Students must be aware that in many problem situations and investigations issues of precision and accuracy with respect to direct measurement must be considered. As always, answers should be presented to an appropriate number of significant digits, as described in the opening unit of this course.

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

D1/D13/D7

Pencil/Paper

1) Groups of three students can be given shapes (cylinders, rectangular solids, spheres, cones, right prisms) to measure. Tools to use: rulers, calipers, tape measures. Procedure: Each shape is to be measured by at least three different groups. They are to calculate the volume of each shape they have been given. Results should be tabulated as a class in a table such as the one that follows:

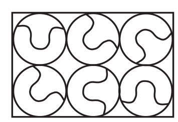
Shape	Group 1 Volume	Group 2 Volume	Group 3 Volume
A (cone) B (cylinder) C			

Discussion: Have groups explain how they measured. What were the best tools? Why are results not all the same? Who's right? Does accuracy matter? When does it matter?

2) Amherst Aerospace is facing competition and needs to upgrade. It is now manually making cylindrical pins to 1 mm, accuracy. With an automated system it could reach accuracy of 0.1 mm. What implications does this have for a pin designed to have a diameter of 3.200 cm and length of 9.500 cm? What implications are there if six of these pins are the critical component in attaching the jet engine to the aircraft?

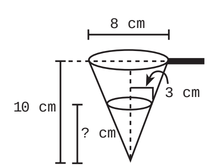
   Discussion: What percentage error is there as you construct pins of diameter from 3.200 cm - 1 mm to 3.200 + 1 mm, in 0.1 mm increments? (Students might use spreadsheets, TI-83 list or table.) How does this change if you purchase the automated system? What other industries (e.g., brain surgery, laser surgery, Hubble telescope) require this degree of accuracy?

Performance



3) Tennis balls have a radius of 4 cm. A box (which has a lid) is just big enough to contain six tennis balls. The arrangement of the balls in the box is shown at right. Have the students
   a) find the volume of the box
   b) find the volume of empty space in the box when the six balls are inside it
   c) find the percentage of the volume of the box which is empty when the six balls are in the box
   d) calculate the surface area of the box and the lid
   e) calculate the surface area and volume of a cylindrical tube (which has a lid) that is just big enough to contain three tennis balls
   f) find the percentage of the volume of the tube which is empty when the three balls are inside
   g) comment on the advantages and disadvantages of tubes and boxes for storing tennis balls.

Pencil/Paper



4) a) The diagram to the right represents a measuring scoop used for measuring soap powder for a washing machine. It has a diameter of 8 cm and a height of 10 cm. Have the students calculate the height of the powder (h cm) in the scoop when the radius of the soap powder surface is 3 cm.
   b) Suppose the same soap powder was poured into a cylindrical container with the same height (10 cm) and base diameter (8 cm). Have the students calculate the percentage of the cylinder the soap powder would fill.

Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

E2 **solve problems involving polygons and polyhedra**

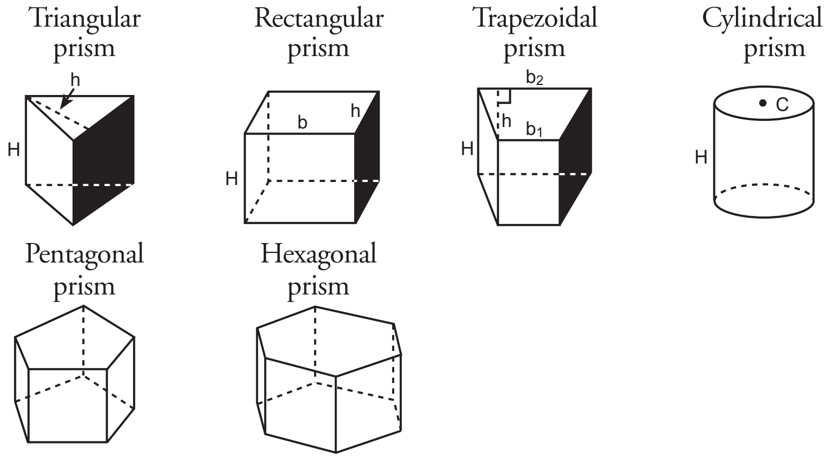
E1 **explore properties of, and make and test conjectures about two- and three-dimensional figures**

E8 **use inductive reasoning when observing patterns, developing properties, and making conjectures**

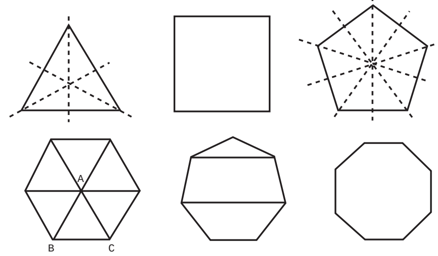
D1 **determine and apply formulas for perimeter, area, surface area, and volume**

Elaboration – Instructional Strategies/Suggestions

E2/E1/E8 The prism is a common shape in which to package items. Students should be aware that prisms are named according to the geometric shape of their bases (the parallel faces).



As students begin to study these faces they will explore the polygonal shapes in an attempt to define them using symmetry. For example, students can define an equilateral triangle as a triangle with three lines of symmetry, or rotational (point) symmetry of order 3. Students can differentiate between squares and rectangles (using lines of symmetry) – four lines of symmetry in a square, but only two in a rectangle. Also, squares have point symmetry of order 4, while rectangles are order 2. Students should note the connection between line and point symmetry in regular polygons.



Students should notice how regular polygons and lines of symmetry are related. The number of sides on the regular polygon is the same as the number of lines of symmetry. The lines of symmetry pass through the opposite vertices and the opposite midpoints when the regular polygons have an even number of sides. They pass through a vertex and the midpoint of the opposite side when a polygon has an odd number of sides. Students should also conclude that the rotational order of symmetry matches the number of sides on the regular polygon.

D1 Areas can be determined by dividing the regular polygons into smaller, more familiar shapes (e.g., a heptagon into an isosceles triangle and two trapezoids). Students could also divide any regular polygon into a number of congruent triangles by joining vertices to the centre of the polygon.

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

E2/E8

Pencil and Paper

1) With respect to symmetry, explain the difference between
   a) a parallelogram and a rectangle
   b) a rhombus and a square
   c) a rhombus and a rectangle

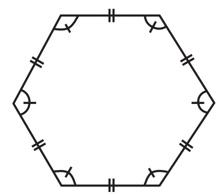
2) State and explain the connection between line and point symmetry in regular polygons.

E1/E8/D1

Journal

3) Mona has been exploring the volumes, surface areas, and symmetries of rectangular prisms. She conjectures that the volume maximizes when the rectangular prism is cube shaped. From this she conjectures that for a given volume, the rectangular prism with minimum surface area will have the greatest symmetry. Jake predicts that cylinders with minimum surface area (for a given volume) will also have the greatest symmetry. Have the students describe why he might think this.

4) When trying to determine the area of this dance floor surface, Richard drew the three lines of symmetry through the vertices. They created six triangles of equal area. Explain how Richard could determine the area of any one of the triangles.



Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

D1 determine and apply formulas for perimeter, area, surface area, and volume

E2 solve problems involving polygons and polyhedra

E8 use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures

D5 apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation

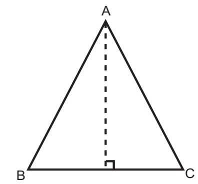
D12 solve problems using the trigonometric ratios

C36 explore and determine relationships between perimeter and area, surface area and volume

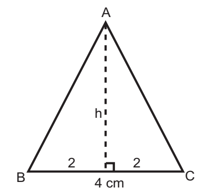
D11 explore, discover, and apply properties of maximum areas and volume

Elaboration – Instructional Strategies/Suggestions

D1/E2/E8 To determine the area of a regular hexagon, students could take one of the six triangles formed by joining vertices to the midpont of the hexagon, and determine the angle measures ($m \angle BAC = \frac{1}{6}(360^{o}) = 60^{o}$, thus $m \angle B = m \angle C = 60^{o}$ since Δ ABC is isosceles). The altitude from A to BC will bisect the angle BAC and the base BC (symmetry properties).



D5/D12 If a student is given the side length of the regular hexagon, using trigonometry, he/she can find the altitude length. Then using the area-of-the- triangle formula, the requested area is obtained. For example,



given BC = 4 cm, and

tangent $\angle$ B = tangent 60^{o} , then (by trig)

2 tangent 60^{o} = h. It follows that

Area of Δ ABC = $\frac{1}{2}(4)$ (2 tangent 60^{o})

Some students may be able to extend this to a new formula for finding the area of a regular hexagon with side length 4 by simply multiplying the above by 6.

$$\text{Area of a hexagon} = 6\left[\tfrac{1}{2}(4)(2 \text{ tangent } 60^{o})\right]$$

$$= 6(4) \tan 60^{o}$$

E8/C36/D11 In activities related to the above work on symmetry and area, students will be using inductive reasoning when observing patterns and making conjectures. For instance, some might conjecture that, based on this example, the area of any hexagon will be the same as the perimeter multiplied by tan 60^{o}. Would this be correct? (No). Ask students to restate the conjecture so that it is correct.

Students should also be examining relationships that lead to conjectures like the following:

- As the number of sides on the regular base of a prism increases, and the perimeter remains fixed, the area of the base increases; and if the height of the prism is fixed the volume also increases.
- If height and perimeter of base are held constant the cylinder is the prism with the maximum area of base and volume .

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

E8/D11

Journal

1) Bruce explores the area of triangles with a given, fixed perimeter. From his exploration, using several triangles with a perimeter of 45 units, he conjectures that the equilateral triangle will have the greatest area for a fixed perimeter. Ask the students what symmetry has to do with this.

D1/E2/E8/D5/D12/C36/D11

Performance

2) Two factors that influence the cost of building a house are the floor area and the perimeter of the house. Bruce and Anne want to build a beach house with a floor area of 64 m^2. They approach their problem by drawing a set of rectangles of different shapes, but all with area 64 $units^2$. Have students do the following activity:

   a) Complete the table.

Width	Length	Perimeter
1	64	130
2	32	68
.		
.		
.		

   b) What perimeter results in the least expensive design?

   c) As a check, Anne draws a set of rectangles, each with a perimeter of 32 units. Complete the table.

Width	Length	Area
1	15	15
2	14	28
.		
.		
.		

   d) Which rectangle has the maximum area?

   e) What kind of rectangle is described in (b) and (d)?

   f) Suggest a design for a house that has an 81 m^2 floor area with the minimum perimeter for that area.

   g) Connect symmetry to your conclusions in this activity.

3) Develop a formula to determine the area of any regular polygon, given its perimeter and number of sides.

Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

C36 **explore, determine and apply relationships between perimeter and area, and between surface area and volume**

D1 **determine and apply formulas for perimeter, area, surface area, and volume**

E8 **use inductive reasoning when observing patterns, developing properties, and making conjectures**

E2 **solve problems involving polygons and polyhedra**

E1 **explore properties of, and make and test conjectures about two- and three-dimensional figures**

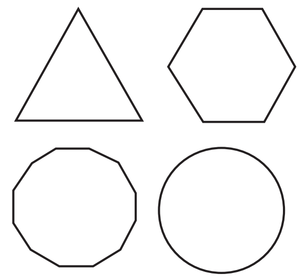
D5 **apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation**

D13 **demonstrate an understanding of the concepts of surface area volume**

Elaboration – Instructional Strategies/Suggestions

C36/D1/E8/E1/E2/D5 As students explore regular polygons of fixed perimeters and determine ways of finding their areas, they should record information to determine relationships between fixed perimeter, number of sides, and the area of the regular polygon.

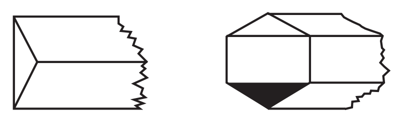
Perimeter	Number of sides	Area
24	3	28
24	5	
24	6	
24	8	
24	9	
24		



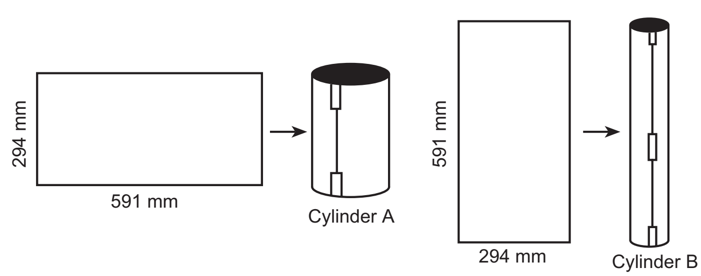
For example, given a fixed perimeter of 24 units for each of the above polygons, students would determine areas and complete a table like that above. They could use graphing technology and examine the graphs of any two sets of these data to determine if there are relationships and how strong the relationships might be.

Students should be able to predict the effect on the perimeter of a regular polygon if its area is kept constant and the number of sides is varied. Students may be able to describe the circle as the limiting shape for a sequence of regular polygons.

D13 Extending these experiences into three dimensions, students should compare capacities of prisms. For example, they might compare an equilateral trianglular prism tube (open-ended) and a regular hexagonal tube made from the same sheet of paper and having the same height. Students might speculate about the capacity of the tube as the number of sides of the regular shape of the base of the tube increases. Also students might compare the capacities of cylinders constructed from the same piece of paper, one with the width of the paper as its height, the other with the length of the paper as its height.



❐ Which of the following cylinders will have the greater volume? Check your prediction by calculation.



Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

C36/D1/E8/E1/E2/D13

Performance

1) Merle uses 27 small cubes of ice to build a large block measuring $3 \times 3 \times 3$. He wonders if this large block will melt more slowly than 27 cubes kept separate from each other. Have the students answer the following questions:
   a) How many faces will there be on 27 cubes kept separate?
   b) If made into the large block, how many of those faces are exposed?
   c) Merle concludes that a large block will melt more slowly. Do you think he is correct? Explain.

2) Merle has to deliver eight large cubes of ice to his school picnic. The freezer motor on his truck is broken, so he must arrange the eight blocks together in order to minimize the melting. Ask students to determine, by trying various arrangements, the best way for Merle to stack the ice. Comment on any symmetry they notice in their answer.

Pencil/Paper

3) Suppose a chunk of ice contains 20 L of water. Ask students to use a symmetry explanation to determine the best shape for the ice to minimize the speed of melting.

4) Have the students design two containers of different shapes that have approximately the same surface area and
   a) predict which would have the greater volume
   b) calculate the volume for each
   c) explain why it makes sense that it turns out this way

Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

C36 **explore, determine and apply relationships between perimeter and area, and between surface area and volume**

E1 **explore properties of, and make and test conjectures about two- and three-dimensional figures**

E2 **solve problems involving polygons and polyhedra**

D1 **determine and apply formulas for perimeter, area, surface area, and volume**

D11 **explore, discover, and apply properties of maximum areas and volumes**

E8 **use inductive reasoning when observing patterns, developing properties, and making conjectures**

A3 **demonstrate an understanding of the role of irrational numbers in applications**

Elaboration – Instructional Strategies/Suggestions

A large part of the cost of a container is determined by the amount of material used in its construction. The less material needed to contain a product, the more economical the container.

C36/E1/E2/D1/D11/E8 As students explore and determine surface areas and volumes of various polyhedra, they should be looking for a relationship between surface area and volume that helps them understand some decisions that might be made about packaging. Students might explore the volumes and surface areas of many different-shaped prisms, cylinders, and spheres.

Students should investigate surface areas of rectangular prisms and/or cylinders with fixed volumes to see if there is a relationship that can be described. They should find that the surface area minimizes as the prisms become more cube-like, thus the cost of material would minimize for a fixed volume.

Students will determine the most economical shape for any volume of container by comparing surface area and volume. An economy rate (ER), which is determined by a ratio or rate ($\frac{\text{volume}}{\text{surface area}}$), can be established for various containers. Students should understand and be able to explain that the higher the ER, the more economical the container will be.

Students should learn that an important factor with respect to designing containers is that containers need to be easily handled and stored. They should investigate fixing the volume and one of the dimensions (such as height) and then find ER values for various shapes. In the end they should be able to summarize what they know about the most economical rectangular prism container for a given volume and height and, in general, the most economical container for a given height and volume.

A3 Working with calculations that require students to find square roots and cube roots will occur when students are asked to find dimensions given certain surface areas or volumes. They should be reminded that the square root of a number is the length of the side of the square that represents that number. The cube root would then be the length of the side of a cube that represents that number. Sometimes students might have to use the square root and cube root features on their calculators.

By way of differentiation, some students may benefit from spending time exploring nets of polyhedra to help determine surface area of various three-dimensional shapes. Students should also try to predict from nets what certain polyhedral shapes will look like.

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

C36/E1/E2/D1/D11/E8

Pencil/Paper

1) Determine if the typical shape of a tuna can, which holds about 200 ml, is the most efficient dimensions to maximize the economy rate.

Journal

2) Explain why you think honey bees build their hives using hexagonal prisms in which to store their honey. (Answer: The hexagon is the regular shape with the most number of sides that tessellates to cover a plane.)
3) Explain what you know about the surface area and the economy rate of a polygonal prism with a fixed height, as the number of sides on the regular-shaped base increases. Use diagrams and/or tables to help in your explanation.

Pencil/Paper, Interview

4) Have students answer the following question: If a company manufactured cereal and asked you to develop a container that they should use in which to contain their cereal, how would you advise them?

Pencil/Paper, Performance

5) An engineering firm has been approached by an environmentally conscious beverage company. Ask students to design a pop can that will hold 355 ml of pop so that a minimum amount of aluminum is used and report on its dimensions by way of satisfying the company's environmental concern.
6) This building in Florence, Italy, was built in the eleventh century. It is in the shape of a regular octagonal prism with a pyramidal roof. Draw a net of the roof.
   a) Answer the following questions:
      i) How many faces does this solid have? (Don't forget the floor.)
      ii) How many corners?
      iii) How many edges?
   b) Each side of the octagon is roughly 12 m, the height of the octagonal prism is 15 m, and the height of the pyramidal roof is 4.5 m. Find the overall approximate surface area and capacity.
   c) Roughly determine the maximum number of hexagonal prism containers that could be placed on the floor of this building? The containers measure 8.5 cm on each side and are 20.0 cm high.

Alinari/Art Resource, New York

Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

D10 determine and apply relationships between the perimeters and areas of similar figures, and between the surface areas and volumes of similar solids

E4 apply transformations when solving problems

D2 apply the properties of similar triangles

Elaboration – Instructional Strategies/Suggestions

E4/D2/D10 Students should review their understanding of the properties of similar figures (sides proportional, angles equal) and apply them to 3-D shapes to clarify the characteristics of similar prisms and containers with other shapes.

- ❐ A rectangular prism is used as a cereal box and has measurements 3.0cm by 10.0 cm by 28.0 cm. To increase its capacity, the width and height of the box were doubled. Does this result in two similar prisms? Explain.

Students should then investigate how increasing the size of a container (to produce a similar-shaped container) affects its volume, surface area, and economy rate. Also, they should notice the affect of scale factors less than one on volume, surface area and economy rate.

- ❐ For the cereal box discussed in the example above, if each of the measurements was doubled, how would the capacity be affected?

To the extent which it is necessary to differentiate, this investigation of similarity with respect to 3-D shapes may be omitted for some students.

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

D10

Journal

1) State in your own words how the relationships between the perimeters and areas of similar figures are affected when extended to surface area and volumes of similar solids.

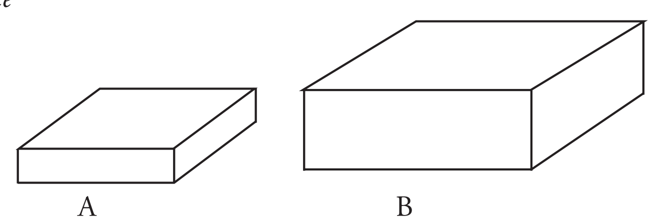
Pencil and Paper

2) Use a dilatation drawing to show how the relationship between surface areas and volumes can be determined.

D10/E4/D2

Performance

3)



a) Use a centimetre ruler to determine if container B is similar to container A.
b) Explain how you determined your answer to (a).
c) Assuming they are similar, find the ratio of the economy rates.
d) Make a container C that has the same width and depth as B, but has a height equal to the height in A.
e) Find the economy rate for container C and explain what it means with respect to containers A and B.
f) Which container (A, B or C) would you use to package breakfast cereal? Explain in detail.

Suggested Resources

Geometry/Packaging (15-20 hours)

SCO: In grade 10 students will be expected to

E1 **explore properties of, and make and test conjectures about two- and three-dimensional figures**

E2 **solve problems involving polygons and polyhedra**

E3 **construct and apply altitudes, medians, angle bisectors, and perpendicular bisectors to examine their intersection points**

E8 **use inductive and deductive reasoning when observing patterns, developing properties and making conjectures**

E9 **use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid**

Elaboration – Instructional Strategies/Suggestions

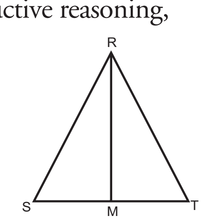
E1/E2 In the context of packaging, students need to explore reasons for the design of packages. In particular, the special properties of triangles and regular polygons will be explored.

E3 Students will consider factors like rigidity that pertain to designing containers. Mathematical concepts to be explored include the perpendicular and angle bisectors, medians and altitudes of triangles and their affect on rigidity. Also, students will determine incentres and circumcentres, and centres of gravity, and consider how these aspects might relate to design, rigidity, and strength.

Students might begin the unit by examining what factors may affect rigidity and strength. Students might use geo-strips or class-made strips that can be joined at end points, yet are flexible enough so that students can alter the shapes. For example, three strips joined to form a triangle cannot be deformed to form different triangles. On the other hand, four strips joined could produce a rectangular shape, but can be altered to produce a parallelogram that is not a rectangle. (Students could then add another strip connecting opposite vertices and see how that might affect rigidity.)

Students might also construct physical structures, as in *Activity* 1) presented on the next page.

E8/E9 Students should have the opportunity to develop logical arguments to justify their conjectures. For example, as students examine the angle bisectors and medians in various triangles, they should conjecture that these two segments become the same segment when the triangle becomes isosceles. They should then verbalize and/or write a set of logical statements, based on deductive reasoning, that would constitute a proof of the conjecture. It might look like this:



Δ RST is an isosceles triangle with RS = RT. Since $\overline{RM}$ is an angle bisector, the angles $\angle$ SRM and $\angle$ TRM are congruent, so Δ SRM and Δ TRM are congruent (SAS), making SM = MT, thus making $\overline{RM}$ a median.

In general, students could explore triangles and their medians, altitudes, and angle bisectors to discover relationships between and among these, especially as a triangle is transformed to ultimately become a regular (equilateral) triangle. They could then extend these explorations to other regular figures.

The core curriculum requires students, at several points in this unit, to make and test conjectures, explore properties, use inductive and deductive reasoning, and solve problems when dealing with 2- and 3-D figures (SCOs E1, E2, E8 and E9). At this point these are done in connection with an examination of altitudes, medians and bisectors in triangles. To differentiate as needed for some students, SCO E3 (and the associated work described in the elaboration on this page) could be omitted.

Geometry/Packaging (15-20 hours)

Worthwhile Tasks for Instruction and/or Assessment

E1/E2

Activity

1) Students will work in groups of four to construct bridges, made only of straws and straight pins. Each bridge must be constructed at a minimum cost and must support a chalk brush. The task must be finished in 20 minutes.
   Materials (given):
   5 straws (no charge)
   10 pins (no charge)

   Conditions/Costs:
   - each bridge must be at least 10 straws in width and 1.5 straws in length
   - each additional straw costs $100 000
   - each additional pin costs $10 000
   - time penalty (if not done in 20 minutes) – add $250 000 per minute

E8/E9/E3/E1

Performance

2) a) Prove that any point, P, on the bisector of an angle is equidistant from the sides of the angle.
   b) Use this theorem to help prove that a point, P, that is the intersection of two angle bisectors is the incentre of the triangle.

Journal

3) a) Explain in your own words why the intersection of the three medians of a triangle is the centre of gravity for a triangle.
   b) How would the centre of gravity be helpful when designing containers?

Suggested Resources

Geometer's Sketchpad, Key Curriculum Press

Unit 7
Linear Programming
(20–25 Hours)

In this unit students will solve certain optimization problems using linear programming techniques. More specifically, this involves constructing and analyzing tables and graphs, expressing constraints as inequalities, graphing inequalities and determining feasible regions, and solving systems of linear equations, both graphically and algebraically. (Note: More than one textual resource is available for this unit.)

While there is good potential to follow this unit of the core curriculum with all students, possible strategies for differentiation for some students include i) solving systems of equations using graphical techniques only, ii) using the alternate unit found in Appendix C to make specific adjustments at various points, and iii) strictly using the unit found in Appendix C. It should also be noted that, should time be a major factor, this unit could be omitted for some students.

Linear Programming (20–25 hours)

SCO: In grade 10 students will be expected to

C2 **model real-world phenomena with linear, quadratic, exponential, and power equations, and linear inequalities**

C6 **apply linear programming to find optimal solutions to real-world problems**

C9 **construct and analyse tables relating two variables**

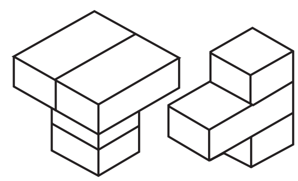
A2 **analyse graphs or charts of given situations to identify specific information**

C12 **express and interpret constraints using inequalities**

Elaboration – Instructional Strategies/Suggestions

C2/C6 Mathematical models, structures, and simulations are precisely the tools of operations research. The field of operations research is a rich source of real-world problem situations to which students can easily relate and within which mathematical concepts may be developed.

Graphing systems of linear equations and inequalities, often without a meaningful practical context, has long been the practice of beginning high school algebra courses. At the same time, however, the graphs of such systems are typically used in operations-research textbooks to develop the concepts of linear programming, which are essential to understanding the solution of many optimization problems. Applications of mathematical programming include scheduling workers to minimize labour costs, using a pattern or template to minimize waste in cutting stock, and determining the production levels of different products to maximize a company's profit. In this guide, a discussion of the following problem (from an article in "The Mathematics Teacher" by Thomas Edwards and Kenneth Chelst, February, 1999) will be used to highlight the relevant student outcomes.



❒ Suppose that a factory manufactures only tables and chairs, and that the profit on one chair is \$15 and on one table is \$20. Each chair requires one large piece of stock and two small pieces of stock (which might be modeled with Lego or other construction blocks). Each table requires two large and two small pieces of stock. If you have only six large and eight small pieces of stock, how many chairs and how many tables should you build to maximize profit?

It is important to allow students an opportunity to see what they can build using the available materials and to determine the profit for each possibility. Later, during a more abstract consideration of the problem, the abstract concepts can easily be linked to this more concrete exploration. (Students might compete in pairs or in small groups to obtain the optimal solution.)

C9/A2 To help students make sense of the given information, have them record it in a chart. For example:

	Table	Chair	Available Stock
Profit (\$)	20	15	–
Large Stock	2	1	6
Small Stock	2	2	8

As they construct various combinations of chairs and tables (using, say, Lego), have them record how many of each are possible and the corresponding profit. This data will be helpful as patterns become evident.

C12 In particular, have students pay attention to the constraints in the problem. Have them write the constraints using words. (See p. 200 for a more detailed discussion of contraints.)

Linear Programming (20–25 hours)

Worthwhile Tasks for Instruction and/or Assessment

C2/C6/C9/C12/A2

Pencil/Paper/Performance/Activity

1) A small factory produces two kinds of toys — round and square. Three kinds of materials are used to make the toys. Available are 480 units of plastic, 300 units of metal and 60 units of wood. Round toys require 4 units of plastic and 2 units of metal. Square toys require 3 units of plastic, 3 units of metal and one unit of wood. Round toys each yield \$8 profit, whereas square toys each yield \$15 profit. Suppose that all toys that are made will be sold.

   Have students
   a) organize all the information into a table.
   b) write sentences to express the constraints in the problem.
   c) determine how many constraints there are.
   d) identify five different arrangements for producing round and square toys that satisfy all the constraints, and attach a profit to each of the five arrangements.
   e) estimate what they think will be the maximum profit.

C12

Pencil/Paper

2) Write sentences to express the constraints in each of these situations:

a)

	Containers of Plastic	Profit
Available	60	—
Skateboard	5	\$1.00
Doll	2	\$0.55

b)

	Cloth (m)	Profit
Available	50	—
Shirt	3	\$5.00
Vest	2	\$3.00

A2/C12

3) For each chart in 2) above, have students find five arrangement of products that satisfy the constraints. How much profit will be made for each arrangement?

Suggested Resources

Ulep, Soledad A., *An Intuitive Approach in Teaching Linear Programming*, "The Mathematics Teacher, January, 1999, NCTM.

Linear Programming (20–25 hours)

SCO: In grade 10 students will be expected to

C2 **model real-world phenomena with linear, quadratic, exponential and power equations, and linear inequalities**

C12 **express and interpret constraints using inequalities**

C31 **graph equations and inequalities and analyse graphs both with and without graphing technology**

C33 **graph by constructing a table of values, by using graphing technology, and when appropriate, by intercept-slope method**

C34 **investigate and make and test conjectures about the solution to equations and inequalities using graphing technology**

A2 **analyse graphs or charts of given situations to identify specific information**

C17 **solve problems using graphing technology**

C11 **write an inequality to describe its graph**

Elaboration – Instructional Strategies/Suggestions

C2/C12 On the basis of their concrete exploration (using Lego), students should be able to identify two decision variables: c, the number of chairs built, and t, the number of tables built.

It is important that students define their variables clearly and precisely. The variable delarations above clearly indicate that they represent the ‘number’ of chairs and the ‘number’ of tables, not the chairs and tables themselves. The choice of variables ‘c’ and ‘t’ is made so that the variables connect clearly with the context. Students will find value in time spent interpreting the written information, and expressing it as constraints. They should also spend time becoming comfortable with all the information, clarifying the interplay between the symbols in the table, the words, the variables, and the written constraints.

Students should be able to identify two constraints on what they are able to build, because they were given only six large and eight small pieces of stock. Translating these constraints into inequalities using the decision variables may take some probing, however. Many students may present statements such as 1 table = 2 large pieces + 2 small pieces and 1 chair = 1 large piece + 2 small pieces, because that is how each item is constructed. Constraints, however, concern the consumption of limited resources, so students will need to focus separately on the limited number of large and small pieces of stock and how each of these resources is consumed in constructing tables and/or chairs. Since there are only six large pieces of stock available, and a chair requires one large piece and a table requires two, students should find that $1c + 2t \leq 6$. (This is another way of saying that the number of large pieces of stock used to make chairs and tables must be less than or equal to the total number of pieces available.) Similarly, a chair and a table each require two small pieces, so $2c + 2t \leq 8$.

(**Note:** Linear programming situations often have “unspoken” constraints as well. Here, for example, the number of tables or number of chairs cannot be less than zero, so $t \geq 0$ and $c \geq 0$ are also constraints.)

C31/C33/C34/A2/C17 After students identify the system of constraints, they should graph the system to locate a feasible region. This may be the first time that students have been asked to graph inequalities. Teachers may want to include some activities to help students decide which half-plane to include in the graph of each inequality. For example, students might graph points that satisfy the constraint using one colour and points that don’t satisfy the constraint using another colour. Students should be able to locate the edge between the two zones and identify its equation. If different groups do this for different constraints, and produce graphs on acetate, they can be overlaid on the overhead to show the feasible region. With technology, it may prove beneficial to shade the opposite regions (the regions that do not satisfy the constraints), so that the overlap of ‘unmarked’ regions becomes very clear.

A2/C11 As part of the process of connecting feasible regions (represented graphically) with inequalities or systems of inequalities (represented algebraically), it will be useful for students to describe given graphical models using inequalities, as well as graphing given inequalities.

Linear Programming (20–25 hours)

Worthwhile Tasks for Instruction and/or Assessment

C2/C12/C31/C33/A2

Activity

1) Consider the following information.

Materials	Units Needed per Round Toy	Units Needed per Square Toy	Supply Available
Plastic	4	3	480
Metal	2	3	300
Wood	0	1	60
Profit (per Toy)	$8	$15	

a) Identify the two decision variables.
b) Write sentences to express the constraints.
c) Translate the sentences expressing constraints into inequalities.
d) Assuming six groups of students, assign two groups to each of these constraints. Have them find 20 values for the variables that satisfy the constraint and indicate these with green-coloured dots on a graph sheet. Have students also find 20 values that do not satisfy the constraints and indicate these with red-coloured dots on the graph sheet.
e) Have the groups post their graph sheets on the walls and discuss the merits and aspects of each. Ask students how they would label the edge between the two coloured regions. Ask them how they would label the region that is red. the region that is green.
f) Overlap acetates of each of the constraints on the overhead projector and ask students to make statements about the feasible region.

C33/C31/C17

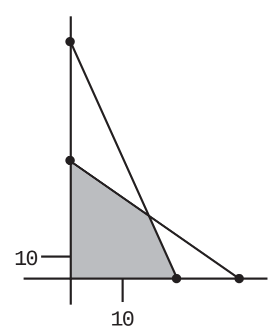
Performance

2) Have students graph each of the constraints in the above activity using graphing technology. (To make things clear, teachers may want to have students actually shade the regions that do not satisfy the constraints.)

C12/C11/A2

Journal

3) Describe the inequalities for which the graph represents the feasible region.



Suggested Resources

Linear Programming (20–25 hours)

SCO: In grade 10 students will be expected to

C6 **apply linear programming to find optimal solutions to real-world problems**

C27 **solve linear and simple radical, exponential, and absolute value equations, and linear inequalities**

C16 **interpret solutions to equations based on context**

C31 **graph equations and inequalities and analyse graphs both with and without graphing technology**

C34 **investigate and make and test conjectures about the solution to equations and inequalities using graphing technology**

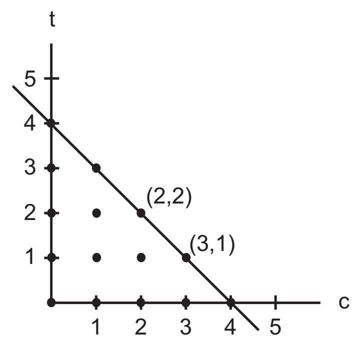
A1 **relate sets of numbers to solutions of inequalities**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

A2 **analyse graphs or charts of given situations to identify specific information**

Elaboration – Instructional Strategies/Suggestions

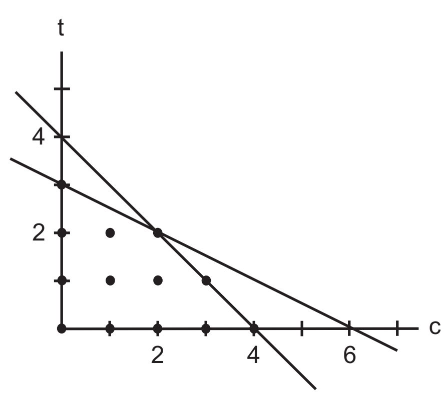
C6/C27/C16/C31/C34 A graph of a feasible region with 3 constraints determining the 3 sides of the region is shown below. Points within the region (such as (1,0), (2,2), and (3,1)) should be considered for the optimal solution. Students should be asked to interpret some of these points as they relate to the problem situation, so that they begin to link the graph with their exploration for the optimal solution. As well, students should interpret points such as (1,4) and be able to explain, in terms of the context, why they are outside the feasible region.



A feasible region

A1/A7/A2 Teachers may next want to discuss with the students the discrete nature of the feasible region for this example. Since the decision variables must take on integral values, the feasible region actually consists only of the lattice points in the "shaded" region. In real-world applications, problems of this type are usually formulated in terms of hourly or weekly production rates, and continuous variables are acceptable. The feasible region is then the entire shaded region.

C6/A2 It is also important to determine if students understand why the possibility of building two chairs and two tables renders building one chair and two tables non-optimal. The location of points on the boundary of the feasible region will play an important role in the next student exploration. (Note: The graph below describes the complete feasible region for the original problem.)



Linear Programming (20–25 hours)

Worthwhile Tasks for Instruction and/or Assessment

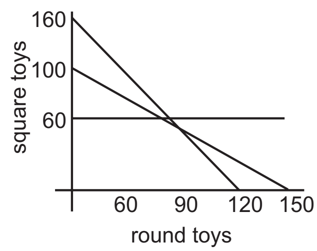
C27/C16/C31/C34/C6/A7/A2/A1

Performance

1) A small factory produces two kinds of toys, square toys and round toys. Below is a graph that includes the edges of the constraints put on the production of the toys. The equation for the constraints can be determined from the following chart:

	containers of plastic (max 480)	litres of paint (max 300)	profit
square toy	3	3	\$15
round toy	4	2	\$8

Note also that the factory's equipment limits the production of square toys to a maximum of 60.



a) Shade the feasible region and make a statement about what it represents.

b) Label some points on the edge of the feasible region and make a statement about what those points indicate.

c) Label some points in the feasible region and make a statement about what those points represent.

d) Find a point on the graph that you think represents a value for each variable that results in an optimal solution in this situation.

e) Are there points in the feasible region that do not describe values for the variables that satisfy all the constraints? Explain.

Suggested Resources

Linear Programming (20–25 hours)

SCO: In grade 10 students will be expected to

C6 **apply linear programming to find optimal solutions to real-world problems**

B4 **identify and calculate the maximum and/or minimum values in a linear programming model**

A2 **analyse graphs or charts of given situations to identify specific information**

A7 **demonstrate and apply an understanding of discrete and continuous number systems**

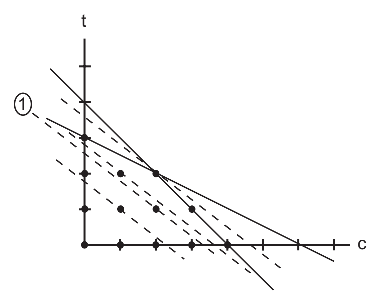
Elaboration – Instructional Strategies/Suggestions

C6 Solving a linear programming problem requires

- defining a set of *decision variables* that completely describes the decision to be made;
- identifying any *constraints* (restrictions) on the decision variables, such as the limited resources available;
- graphing the system of constraints to locate a *feasible region*;
- modeling the problem's objective by using the decision variables to define an *objective function*; and
- determining which solution within the feasible region is the *optimal solution*.

C6/B4/A2/C19 Having defined variables, identified constraints and graphed the feasible region, students should next model the objective of the original problem by defining the profit, P, as the objective function P = 15c + 20t. (Recall how c and t were defined on page 200, and that each chair has a \$15 profit, and each table a \$20 profit, as indicated on page 198.)

Students might then be asked to assume a profit of, say, \$50 and to draw a line through the feasible region that represents the objective function for a profit of \$50. (See broken line ①.) Then students should do the same for profits of, say, \$35, \$55, and \$70. (See additional broken lines on the graph.)



Students will then need only to observe that 15c + 20t = P defines a family of parallel lines, and that the further right and higher the line lies in the coordinate system, the greater the profit is. The line that is the highest and farthest right while still intersecting the feasible region passes through a corner point. Thus, this corner point corresponds to the optimal solution. The graph shows the feasible region and a family of parallel lines, including the line passing through the optimal solution (i.e., 2 chairs and 2 tables).

Note: When the family of parallel lines defined by the objective function is actually parallel to a boundary of a feasible region, the optimal solution is no longer unique. Any point lying on that portion of the boundary is optimal.

Students should also realize that it is possible, depending on the situation, for the optimal solution to lie at any point on the boundary of the feasible region. For example, if chairs garnered a \$20 profit and tables \$15, the family of parallel lines defined by P=20c + 15t would identify an optimal solution at (4,0), corresponding to 4 chairs and no tables.

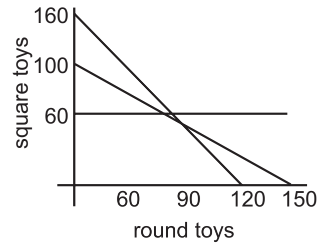
Linear Programming (20–25 hours)

Worthwhile Tasks for Instruction and/or Assessment

B4/C6/A2/A7

Activity

1) A small factory produces two kinds of toys, round and square. At right is a graph that represent the constraints put on the production of the toys. Supposing all toys that are made are sold, how many of each kind should be produced so that the total profit is as large as possible?



a) State in your own words what must be done to solve the problem.
b) Determine how much profit can be made by the toys represented by the following points, if a round toy yields \$8 profit and a square toy \$15 profit.
   i) (50, 30) ii) (80, 40)
c) Write an equation for the objective function. For each of the following points, write and sketch the objective function associated with it.
   i) (10, 20) ii) (50, 30) iii) (60,10)

   For each, determine if there are other feasible solutions that give the same objective-function value.
d) Describe how the lines sketched in c) ii) and c) iii) relate to the line drawn for c) i).
e) Find a point in the feasible region that will represent values that will result in more profit than any of the points discussed in a) to d), and graph the objective function associated with it.
f) Complete this sentence: "The further away the objective-function line is from the origin, ______________________________ ."
g) What point in the feasible region is on the objective-function line farthest from the origin?
h) Determine what the value of the objective-function will be at this point. What conclusion can be made?
i) Describe what happens to the objective-function line if the profit made for each toy reverses, i.e., the profit for each round toy is \$15 and for each square toy is \$8.
j) Sketch this new objective-function line, and determine what number of round and square toys will earn the maximum profit.

Suggested Resources

Linear Programming (20–25 hours)

SCO: In grade 10 students will be expected to

C19 solve systems of linear equations using substitution and graphing methods

C24 rearrange equations

C17 solve problems using graphing technology

C31 graph equations and inequalities and analyse graphs both with and without graphing technology

C27 solve linear and simple radical, exponential, and absolute value equations and linear inequalities

Elaboration – Instructional Strategies/Suggestions

C19/C24/C17/C31 This graphical approach to solving a linear programming problem develops the principle that a unique optimal solution to a linear programming problem generally occurs at a corner point on the boundary of a feasible region. In this case the corner point (2, 2) is the intersection of two lines, $c + 2t = 6$ and $2c + 2t = 8$. To graph these equations, students could rearrange them into function form (i.e., $t = -\frac{1}{2}c + 3$ and $t = -c + 4$). Then, using slope- y-intercept methods, they could quickly sketch the two lines. Or, they could enter these equations onto the function screen and graph using technology. The intersection point of these two lines can be estimated from the sketched graph, or can be found exactly using various methods available with graphing technology.

C19/C24/C27 Students should develop facility with the substitution method for solving systems of equations algebraically. If both equations are in function form, they can readily be combined. Previously, students concluded that when two equations are both equal to the same variable (in this case, t), then they are equal to each other, i.e., $-\frac{1}{2}c + 3 = -c + 4$. This application of the transitive property is sometimes called ‘comparison’, however students may see it as ‘substituting’ one value ($-\frac{1}{2}c + 3$) for another (t).

This equation can now be solved algebraically, giving

$\frac{1}{2}c = 1$ and

$c = 2$

With $c = 2$, then t can be determined as

$-(2) + 4 = t$

$2 = t$,

thus giving 2 chairs and 2 tables.

If students were to use the substitution method to solve the original system of equations, $c + 2t = 6$ and $2c + 2t = 8$, they might choose to solve the first equation for c ($c = -2t + 6$), substitute the expression for c in the second equation ($2(-2t + 6) + 2t = 8$) and then solve for ‘t’. This would result in determining that $t=2$. Students could then replace t with 2 in either of the original equations and determine the value of c. Again, they should find that the point of intersection corresponds to 2 tables and 2 chairs.

Linear Programming (20–25 hours)

Worthwhile Tasks for Instruction and/or Assessment

C19/C24/C27/C31

Pencil/Paper/Performance

1) The maximum profit can be determined by evaluating the intersection points of the lines given in the problem by the constraints. For each of the following sets of constraints, determine the maximum profit:

a) profit = 8x + 15y
$$\begin{cases} 2x + 3y \le 300 \\ y \le 60 \end{cases}$$

b) profit = 15x + 8y
$$\begin{cases} 4x + 3y \le 480 \\ 2x + 3y \le 300 \end{cases}$$

c) profit = 10a + 8.5g
$$\begin{cases} 3.5a + 2.5g \le 12\,000 \\ 7a + 8g \le 34\,000 \end{cases}$$

d) profit = 8.5a + 10g
$$\begin{cases} 3a + 2g \le 12\,000 \\ 7.5a + 8.2g \le 34\,000 \end{cases}$$

e) profit = \$1.00x + \$0.55y
$$\begin{cases} \frac{5}{2}x + \frac{2}{3}y \le 60 \\ 15x + 18y \le 360 \end{cases}$$

f) profit = 5.15x + 6.10y
$$\begin{cases} x \ge 8,\ y \ge 5 \\ -\frac{4}{5}x + \frac{5}{3}y \le 30 \end{cases}$$

Journal

2) a) Describe in detail the algebraic process of solving a system of equations by the substitution method.

b) If $\begin{cases} 2y - 3y = 12 \\ x + 5y = 10 \end{cases}$ explain how you would begin the algebraic process to solve the system.

Suggested Resources

Appendix A: Assessing and Evaluating Student Learning

Assessing and Evaluating Student Learning

In recent years there have been calls for change in the practices used to assess and evaluate students' progress. Many factors have set the demands for change in motion, including the following:

- ***new expectations for mathematics education as outlined in*** **Curriculum and Evaluation Standards for School Mathematics** (NCTM 1989)

The *Curriculum Standards* provide educators with specific information about what students should be able to do in mathematics. These expectations go far beyond learning a list of mathematical facts; instead, they emphasize such competencies as creative and critical thinking, problem solving, working collaboratively, and the ability to manage one's own learning. Students are expected to be able to communicate mathematically, to solve and create problems, to use concepts to solve real-world applications, to integrate mathematics across disciplines, and to connect strands of mathematics. For the most part, assessments used in the past have not addressed these expectations. New approaches to assessment are needed if we are to address the expectations set out in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989).

- *understanding the bonds linking teaching, learning, and assessment*

Much of our understanding of learning has been based on a theory that viewed learning as the accumulation of discrete skills. Cognitive views of learning call for an active, constructive approach in which learners gain understanding by building their own knowledge and developing connections between facts and concepts. Problem solving and reasoning become the emphases rather than the acquisition of isolated facts. Conventional testing, which includes multiple choice or having students answer questions to determine if they can recall the type of question and the procedure to be used, provides a window into only one aspect of what a student has learned. Assessments that require students to solve problems, demonstrate skills, create products, and create portfolios of work reveal more about the student's understanding and reasoning of mathematics. If students are expected to develop reasoning and problem-solving competencies, then teaching must reflect such, and in turn, assessment must reflect what is valued in teaching and learning. Feedback from assessment directly affects learning. The development of problem-solving, and higher, order thinking skills will become a realization only if assessment practices are in alignment with these expectations.

- ***limitations of the traditional methods used to determine student achievement***

Do traditional methods of assessment provide the student with information on how to improve performance? We need to develop methods of assessment that provide us with accurate information about students' academic achievement and information to guide teachers in decision making to improve both learning and teaching.

What Is Assessment?

Assessment is the systematic process of gathering information on student learning. Assessment allows teachers to communicate to students what is really valued—what is worth learning, how it should be learned, what elements of quality are considered most important, and how well students are expected to perform. In order for teachers to assess student learning in a mathematics curriculum that emphasizes applications and problem solving, they need to employ strategies that recognize the reasoning involved in the process as well as in the product. *Assessment Standards for School Mathematics* (NCTM 1995, p. 3) describes assessment practices that enable teachers to gather evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes.

Assessment can be informal or formal. Informal assessment occurs while instruction is occurring. It is a mind-set, a daily activity that helps the teacher answer the question, Is what is taught being learned? Its primary purpose is to collect information about the instructional needs of students so that the teacher can make decisions to improve instructional strategies. For many teachers, the strategy of making annotated comments about a student's work is part of informal assessment. Assessment must do more than determine a score for the student. It should do more than portray a level of performance. It should direct teachers' communication and actions. Assessment must anticipate subsequent action.

Formal assessment requires the organization of an assessment event. In the past, mathematics teachers may have restricted these events to quizzes, tests, or exams. As the outcomes for mathematics education broaden, it becomes more obvious that these assessment methods become more limited. Some educators would argue that informal assessment provides better quality information because it is in a context that can be put to immediate use.

Why Should We Assess Student Learning?

We should assess student learning in order to

- improve instruction by identifying successful instructional strategies
- identify and address specific sources of the students' misunderstandings
- inform the students about their strengths in skills, knowledge, and learning strategies
- inform parents of their child's progress so that they can provide more effective support
- determine the level of achievement for each outcome

As an integral and ongoing part of the learning process, assessment must give each student optimal opportunity to demonstrate what he/

she knows and is able to do. It is essential, therefore, that teachers develop a repertoire of assessment strategies.

Assessment Strategies

Some assessment strategies that teachers may employ include the following.

Documenting classroom behaviours

In the past teachers have generally made observations of students' persistence, systematic working, organization, accuracy, conjecturing, modeling, creativity, and ability to communicate ideas, but often failed to document them. Certainly the ability to manage the documentation played a major part. Recording information signals to the student those behaviours that are truly valued. Teachers should focus on recording only significant events, which are those that represent a typical student's behaviour or a situation where the student demonstrates new understanding or a lack of understanding. Using a class list, teachers can expect to record comments on approximately four students per class. The use of an annotated class list allows the teacher to recognize where students are having difficulties and identify students who may be spectators in the classroom.

Using a portfolio or student journal

Having students assemble on a regular basis responses to various types of tasks is part of an effective assessment scheme. Responding to open-ended questions allows students to explore the bounds and the structure of mathematical categories. As an example, students are given a triangle in which they know two sides or an angle and a side and they are asked to find out everything they know about the triangle. This is preferable to asking students to find a particular side, because it is less prescriptive and allows students to explore the problem in many different ways and gives them the opportunity to use many different procedures and skills. Students should be monitoring their own learning by being asked to reflect and write about questions such as the following:

- What is the most interesting thing you learned in mathematics class this week?
- What do you find difficult to understand?
- How could the teacher improve mathematics instruction?
- Can you identify how the mathematics we are now studying is connected to the real world?

In the portfolio or in a journal, teachers can observe the development of the students' understanding and progress as a problem solver. Students should be doing problems that require varying lengths of time and represent both individual and group effort. What is most important is that teachers discuss with their peers what items are to be part of a meaningful portfolio, and that students also have some input into the assembling of a portfolio.

Projects and investigative reports

Students will have opportunities to do projects at various times throughout the year. For example, they may conduct a survey and do a statistical report, they may do a project by reporting on the contribution of a mathematician, or the project may involve building a complex three-dimensional shape or a set of three-dimensional shapes which relate to each other in some way. Students should also be given investigations in which they learn new mathematical concepts on their own. Excellent materials can be obtained from the National Council of Teachers of Mathematics, including the *Student Math Notes.* (These news bulletins can be downloaded from the Internet.)

Written tests, quizzes, and exams

Some critics allege that written tests are limited to assessing a student's ability to recall and replicate mathematical facts and procedures. Some educators would argue that asking students to solve contrived applications, usually within time limits, provides us with little knowledge of the students' understanding of mathematics.

How might we improve the use of written tests?

- Our challenge is to improve the nature of the questions being asked, so that we are gaining information about the students' understanding and comprehension.
- Tests must be designed so that questions being asked reflect the expectations of the outcomes being addressed.
- One way to do this is to have students construct assessment items for the test. Allowing students to contribute to the test permits them to reflect on what they were learning, and it is a most effective revision strategy.
- Teachers should reflect on the quality of the test being given to students. Are students being asked to evaluate, analyse, and synthesize information, or are they simply being asked to recall isolated facts from memory? Teachers should develop a table of specifications when planning their tests.
- In assessing students, teachers have a professional obligation to ensure that the assessment reflects those skills and behaviours that are truly valued. Good assessment goes hand-in-hand with good instruction and together they promote student achievement.

Appendix B: SCOs for Grades 8 and 9

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, and number system characteristics (e.g., density).

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
A1 model and link various representations of square root of a number **A2** recognize perfect squares between 1 and 144 and apply patterns related to them	**A1** solve problems involving square root and principal square root **A2** graph, and write in symbols and in words, the solution set for equations and inequalities involving integers and other real numbers
A3 distinguish between an exact square root of a number and its decimal approximation **A4** find the square root of any number, using an appropriate method	**A3** demonstrate an understanding of the meaning and uses of irrational numbers **A4** demonstrate an understanding of the inter-relationships of subsets of real numbers
A5 demonstrate and explain the meaning of negative exponents for base ten	**A5** compare and order real numbers
A6 represent any number written in scientific notation in standard form, and vice versa **A7** compare and order integers and positive and negative rational numbers (in decimal and fractional forms) **A8** represent and apply fractional percents, and percents greater than 100, in fraction or decimal form, and vice versa **A9** solve proportion problems that involve equivalent ratios and rates	**A6** represent problem situations using matrices

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to

B1 demonstrate an understanding of the properties of operations with integers and positive and negative rational numbers (in decimal and fractional forms)

B2 solve problems involving proportions, using a variety of methods

B3 create and solve problems which involve finding a, b, or c in the relationship a% of b = c, using estimation and calculation

B4 apply percentage increase and decrease in problem situations

B5 add and subtract fractions concretely, pictorially, and symbolically

B6 add and subtract fractions mentally, when appropriate

B7 multiply fractions concretely, pictorially, and symbolically

B8 divide fractions concretely, pictorially, and symbolically

B9 estimate and mentally compute products and quotients involving fractions

B10 apply the order of operations to fraction computations, using both pencil and paper and the calculator

B11 model, solve, and create problems involving fractions in meaningful contexts

B12 add, subtract, multiply, and divide positive and negative decimal numbers with and without the calculator

By the end of grade 9, students will be expected to

B1 model, solve, and create problems involving real numbers

B2 add, subtract, multiply, and divide rational numbers in fractional and decimal forms using the most appropriate methods

B3 apply the order of operations in rational number computations

B4 demonstrate an understanding of, and apply the exponent laws for, integral exponents

B5 model, solve, and create problems involving numbers expressed in scientific notation

B6 determine the reasonableness of results in problem situations involving square roots, rational numbers, and numbers written in scientific notation

B7 model, solve, and create problems involving the matrix operations of addition, subtraction, and scalar multiplication

B8 add and subtract polynomial expressions symbolically to solve problems

B9 factor algebraic expressions with common monomial factors, concretely, pictorially, and symbolically

B10 recognize that the dimensions of a rectangular area model of a polynomial are its factors

B11 find products of two monomials, a monomial and a polynomial, and two binomials concretely, pictorially, and symbolically

B12 find quotients of polynomials with monomial divisors

GCO B: Students will demonstrate operaation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
B13 solve and create problems involving addition, subtraction, multiplication, and division of positive and negative decimal numbers	**B13** evaluate polynomial expressions
B14 add and subtract algebraic terms concretely, pictorially, and symbolically to solve simple algebraic problems	**B14** demonstrate an understanding of the applicability of commutative, associative, distributive, identity, and inverse properties to operations involving algebraic expressions
B15 explore addition and subtraction of polynomial expressions, concretely and pictorially	**B15** select and use appropriate strategies in problem situations
B16 demonstrate an understanding of multiplication of a polynomial by a scalar, concretely, pictorially, and symbolically	

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
C1 represent patterns and relationships in a variety of formats and use these representations to predict unknown values	**C1** represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values
C2 interpret graphs that represent linear and non - linear data	**C2** interpret graphs that represent linear and non - linear data
C3 construct and analyse tables and graphs to describe how change in one quantity affects a related quantity	**C3** construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity
C4 link visual characteristics of slope with its numerical value by comparing vertical change with horizontal change	**C4** determine the equations of lines by obtaining their slopes and y-intercepts from graphs and sketch graphs of equations using y-intercepts and slopes
C5 solve problems involving the intersection of two lines on a graph	**C5** explain the connections among different representations of patterns and relationships
C6 solve and verify simple linear equations algebraically	**C6** solve single-variable equations algebraically and verify the solutions
C7 create and solve problems, using linear equations	**C7** solve first-degree single-variable inequalities algebraically, verify the solutions, and display them on number lines
	C8 solve and create problems involving linear equations and inequalities

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) and/or procedures (e.g., proportions) to determine measurements indirectly.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
D1 solve indirect measurement problems, using proportions **D2** solve measurement problems, using appropriate SI units **D3** estimate areas of circles	**D1** solve indirect measurement problems by connecting rates and slopes **D2** solve measurement problems involving conversion among SI units **D3** relate the volumes of pyramids and cones to the volumes of corresponding prisms and cylinders
D4 develop and use the formula for the area of a circle **D5** describe patterns and generalize the relationships between areas and perimeters of quadrilaterals, and areas and circumferences of circles	**D4** estimate, measure, and calculate dimensions, volumes and surface areas of pyramids, cones, and spheres in problem situations **D5** demonstrate an understanding of and apply proportions within similar triangles
D6 calculate the areas of composite figures **D7** estimate and calculate volumes and surface areas of right prisms and cylinders **D8** measure and calculate volumes and surface areas of composite 3-D shapes **D9** demonstrate an understanding of the Pythagorean relationship, using models **D10** apply the Pythagorean relationship in problem situations	

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as (i) the direction, orientation and perspectives of objects in space, (ii) the relative shapes and sizes of figures and objects, and (iii) how a change in shape relates to a change in size. Geometric concepts, properties, and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
E1 demonstrate whether a set of orthographic views, a mat plan, and an isometric drawing can represent more than one 3-D shape **E2** examine and draw representations of 3-D shapes to determine what is necessary to produce unique shapes **E3** draw, describe, and apply transformations of 3-D shapes **E4** analyse polygons to determine their properties and interrelationships **E5** represent, analyse, describe, and apply dilatations	**E1** investigate, and demonstrate an understanding of the minimum sufficient conditions to produce unique triangles **E2** investigate, and demonstrate an understanding of the properties of, and the minimum sufficient conditions to guarantee congruent triangles **E3** make informal deductions using congruent triangle and angle properties **E4** demonstrate an understanding of and apply the properties of similar triangles **E5** relate congruence and similarity of triangles
	E6 use mapping notation to represent transformations of geometric figures, and interpret such notations **E7** analyse and represent combinations of transformations, using mapping notation **E8** investigate, determine, and apply the effects of transformations of geometric figures on congruence, similarity, and orientation

GCO F: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display and analysis of data involves (i) attention to sampling procedures and issues, (ii) recording and organizing collected data, (iii) choosing and creating appropriate data displays, (iv) analysing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean) and (v) formulating and evaluating statistical arguments.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
F1 demonstrate an understanding of the variability of repeated samples of the same population	**F1** describe characteristics of possible relationships shown in scatter plots
F2 develop and apply the concept of randomness	**F2** sketch lines of best fit and determine their equations
F3 construct and interpret circle graphs	**F3** sketch curves of best fit for relationships that appear to be non-linear
F4 construct and interpret scatter plots and determine a line of best fit by inspection	**F4** select, defend, and use the most appropriate methods for displaying data
F5 construct and interpret box-and-whisker plots	**F5** draw inferences and make predictions based on data analysis and data displays
F6 extrapolate and interpolate information from graphs	**F6** demonstrate an understanding of the role of data management in society
F7 determine the effect of variations in data on the mean, median, and mode	**F7** evaluate arguments and interpretations that are based on data analysis
F8 develop and conduct statistics projects to solve problems	
F9 evaluate data interpretations that are based on graphs and tables	

GCO G: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails (i) determining probabilities by conducting experiments and/or making theoretical calculations, (ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment, and (iii) analysing problem situations to decide how best to determine probabilities.

The following are the Specific Curriculum Outcomes (SCOs) for grades 8 and 9.

By the end of grade 8, students will be expected to	*By the end of grade 9, students will be expected to*
G1 conduct experiments and simulations to find probabilities of single and complementary events	**G1** make predictions of probabilities involving dependent and independent events by designing and conducting experiments and simulations
G2 determine theoretical probabilities of single and complementary events	**G2** determine theoretical probabilities of independent and dependent events
G3 compare experimental and theoretical probabilities	**G3** demonstrate an understanding of how experimental and theoretical probabilities are related
G4 demonstrate an understanding of how data is used to establish broad probability patterns	**G4** recognize and explain why decisions based on probabilities may be combinations of theoretical calculations, experimental results, and subjective judgements

Appendix C: Making Choices: Linear Programming

(20-25 Hours)

Making Choices: Linear Programming

SCO: In this course students will be expected to

C8 **demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions**

F5 **organize and display information in various ways with and without technology**

Elaboration – Instructional Strategies/Suggestions

C8 Students should be given situations from the real-world (in words, as graphs, and as collections of data) to explore and determine how two variables are related, and to represent those relationships in multiple ways. For example, consider the following situation: Rachel runs a snowboard rental operation at Martock Ski Resort. She asks for a non-returnable deposit of $2.00, and charges a rental fee of $3.50 per hour. Students might be asked to determine what it would cost to rent for 1, 2, 3, ...8 hours. For part hours of rental, partial amounts will be charged.

F5 As a first step in their solutions, students should identify the information given. They can see from the problem that Rachel is paid

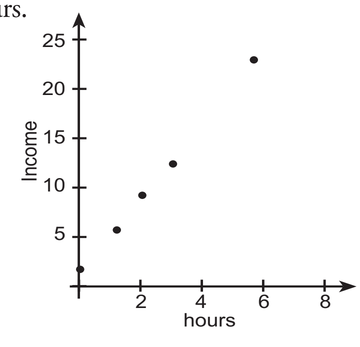
- a $2.00 non-returnable deposit
- $3.50 per hour of rental
- for partial hours of rental (e.g., $\frac{1}{4}$ hour of rental results in a partial payment of $\frac{1}{4}$ of $3.50).

They might underline the important information or make a list.

F5/C8 Students could then make tables. They should be encouraged to complete the tables using mental mathematics and explain their strategies. Students should look for patterns in their tables and use the patterns to predict other hourly rates. For example, what do they think 6 hours of rental would cost? Have them justify their answers.

number of hours	Rachel's income $
0	2.00
1	5.50
2	9.00
3	12.50

Students should plot points and sketch graphs to visualize the patterns, or to help them see the relationships. The graphs will help students predict Rachel's income for larger numbers of hours of rental, and for fractional numbers of hours.



Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

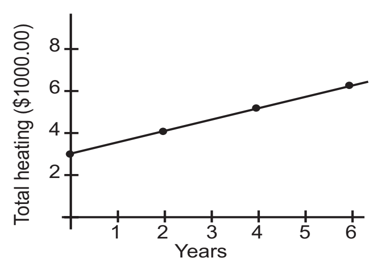
C8/F5

Performance

1) A taxi driver charges a flat fare of \$2.50, plus \$1.50/km. (Fractions of a kilometre are rounded up to the next whole number.)
   a) Create a table of values showing number of kilometres driven and amount of money paid to the driver. (If being done in the classroom, discuss with students some mental math strategies such as "add \$3.00 for every 2 km.)
   b) State the dependent and independent variables, then plot points to represent the relationship.
   c) Ask students to describe, in writing, a pattern in the table that they could use to predict the cost of travelling 9km. 21km.
   d) Ask students to graph the same information using graphing technology. Have them explain the choice of window settings.

2) A swimming pool is emptied for cleaning. The volume, v(litres), of water remaining in the pool after time, t(hours), is given by the equation $v = 50\ 000 - 2000t$.
   a) Describe, in words, the relationship between v and t that is represented in the equation.
   b) Sketch a graph of the situation.
   c) Explain how you would select your window settings when using technology to draw the graph.

3) a) Describe in words the real-life situation represented in the graph.
   b) Display the same graph using technology. This can be done by entering coordinates into lists and drawing a scatter plot or broken line graph.
   c) Predict the total cost of heating after 10 years.
   d) Create a problem which the graph can be used to solve.



Making Choices: Linear Programming

SCO: In this course students will be expected to

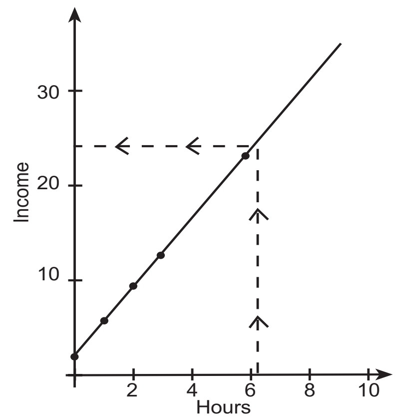
A2 **relate sets of numbers to solutions of inequalities**

C18 **interpolate and extrapolate to solve problems**

C8 **demonstrate an understanding of real-world relationships by translating between graphs, tables, and written descriptions**

Elaboration – Instructional Strategies/Suggestions

The graph to the right represents Rachel's snowboard business discussed on page 226. There is a $2.00 deposit, and a $3.50 per hour charge.



A2 As students are making their graphs they should discuss whether or not they should join the points on their graphs. The pattern of points looks linear and, since partial payments are expected for partial hours of rental, they should agree to draw a line through the points. They should also discuss proper domains and ranges. They should record their domain as all real numbers greater than or equal to zero, and their range as all real numbers greater than or equal to 2.

Ask students to find the point (4, 10) and interpret its meaning. They might respond by saying that someone rented a snowboard for 4 hours and was wrongly charged $10.00. This is less than what should have been paid and Rachel will suffer a loss of income. Students should be asked to explain how this loss of income is evident on the graph. They should respond that it is a point that lies below the line that represents what should be charged. They might examine other points that represent amounts less than that owed and decide that all points below this line represent situations involving undercharging.

C18 The line represents all possible variations of the original problem. Students can use the graph to interpolate and extrapolate. For example, if Ralph rented a snowboard for 6.3 hours, he would expect to pay $24.05. This answer could be obtained through calculation ($6.3 \times 3.50 + 2.00$) or estimated from the graph (see the dotted line). Similarly, students could estimate the payment for 10 hours rental by extrapolating from the graph.

C8 Since the graph is linear, students should take the opportunity to discuss and interpret slopes and intercepts.

❐ Ask students to
- determine the slope and what it represents in this situation
- interpret the intercept
- use these values to state the equation for the line.

Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment	Suggested Resources

C8/C18/A2

Performance

1) Revisit question 1) on page 227.
   a) Explain why you should or should not join the points plotted on the graph.
   b) Describe, in the context of the question, what is happening on the graph as you move from one point to another.
   c) What is the rate of increase in cost as you travel?
   d) From the graph predict the cost of travelling 12km. 3.5 km.
   e) Represent the relationship with an equation. Use the equation to predict the cost for a 25km taxi ride. a 5.5 km ride.

2) Revisit the swimming pool problem on page 227.
   a) Determine how much water remains after 3h. 7h. Explain how you are getting your answers. (Students may be using the graph, the equation, or mental math; encourage all three.)
   b) Interpret the slope of the line and the intercept.

3) Revisit question 3) on page 227.
   a) Represent the graph using an equation.
   b) Use the equation to determine how much the cost would be after 2.5 years.
   c) Interpret the meaning of the slope of the graph. Can you see this in the equation? Explain.
   d) By talking about the real world, explain why you think that the intercept is $3000.
   e) On the graph identify the point (5, 3000). Describe, in context, what this point represents. (Ans: After 5 years, the total heating cost is $3000.)
   f) In which region is the point (5, 3000) located? (Ans: It is in the region below the line.) Describe, with respect to the context, what is true about all the points in this region. (Ans: All the points in this region describe costs of heating less than that described by the line.)

Making Choices: Linear Programming

SCO: In this course students will be expected to

B3 **demonstrate an understanding of the relationship between arithmetic operations and operations on equations and inequalities**

B4 **use the calculator correctly and efficiently**

Elaboration – Instructional Strategies/Suggestions

B3/B4 Students will be expected to manipulate, evaluate and solve equations and inequalities as needed. They should be expected to use a variety of approaches to solving linear equations. That is, they should focus on the various ways that technology can be used to solve equations, how graphs can be used to solve equations, and at times, they should be able to use symbolic manipulation.

Symbolic manipulation may be required to change the form of an equation so that students can enter it into a calculator or computer (e.g., $2x - 3y = 9 \rightarrow y = \frac{2}{3}x - 3$). When students perform this type of manipulation, they would probably first isolate the y-term (by subtracting 2x from both sides), then isolate the y-variable (by dividing all terms by -3). The equation is now in y = mx + b form and can be graphed using slope and y-intercept or by using technology.

In the "Rachel's Snowboard" problem, students are sometimes interpolating and extrapolating as they evaluate situations to predict. Equation-solving might occur to answer many questions. For example, if they knew they could spend \$30 on rental, and wanted to know how long they could keep the snowboard, they might begin as follows:

$$\text{cost} = 2 + 3.50h$$
$$30 = 2 + 3.50h$$

Students would subtract \$2 from both sides,

$$28 = 3.50h$$

then divide both terms by 3.50.

$$8 = h$$

The snowboard could be used for eight hours.

B4 Students could get the same answer using technology by tracing the graph, or using an "evaluate and solve" feature. The graph could also be drawn with technology by entering data in lists and producing a scatter plot or broken-line graph.

Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment	Suggested Resources

B3

Pencil/Paper

1) Revisit question 1) on page 229, the taxi driver problem. Use the equation to determine how far you could travel if you only had \$9.50. \$15.00. Express each answer as an inequality.

2) Revisit question 2) on page 227, the swimming pool problem.
   a) The swimming pool owner wants to have exactly 10 000 litres of water left in his pool for the winter. After how many hours should he stop the flow of the water from the pool?
   b) How many hours will it take for the water to completely drain from the pool?

3) Revisit the equation determined for question 3) on page 227 . Determine how many years (partial years) it would take to spend \$3150 on heat.

B3/B4

4) Revisit each question above and explain or show how you could use technology to obtain all the answers.

C8/F5/A2/C18/B3/B4

Performance

5) Lonely Hearts Café provides an intimate atmosphere where people can sit, talk, and eat while enjoying each other's company. On each table a single candle provides the only light. Two waiters were heard discussing how long a candle would take to burn out completely. They decided to conduct an experiment. They began with a new 20 cm candle and checked its length every 5-10 minutes. Below are some of the data they collected.

Time (min)	0	5	10	15	30	90	200
length of candle (cm)	20.0	19.6	19.2	18.8	17.5	12.5	3.3

   a) Represent this data using a graph.
   b) State a proper domain and range.
   c) Should you join the points? Explain.
   d) Determine the slope for the line of best fit.
   e) What does the slope represent?
   f) Describe the relationship.
   g) Write an equation for the relationship.
   h) How long will the candle be after 65 minutes? 2 hours? Explain how you know.
   i) When will the candle burn out? Explain how you know.
   j) Use the equation to determine the amount of time it will take for the candle to be 8 cm long.
   k) Locate the point (75, 16). In terms of the problem context, what does this point represent? Describe this and other points in this region.

Making Choices: Linear Programming

SCO: In this course students will be expected to

C6 **apply the linear programming process to find optimal solutions**

C11 **express and interpret constraints**

F5 **organize and display information in various ways with and without technology**

Elaboration – Instructional Strategies/Suggestions

C6 Linear programming was developed during and shortly after World War II, in the 1940s. During its short history, linear programming has changed the way businesses make decisions, from guesswork and intuition to using an algorithm based on available data and guaranteed to produce an optimal decision.

Linear programming can be applied in a variety of situations. Students should understand that linear programming is a way that helps a business (no matter how large) allocate the resources it has on hand to make a particular mix of products that will maximize income and profit, or minimize expenses and costs. Linear programming has saved businesses millions of dollars.

C11/F5 Since linear programming problems can be solved using readily available computer software, or matrix methods, extracting the important data from the underlying structure may be the only problem-solving process that must be done by a human being.

Understanding the underlying structure includes being able to interpret the constraints (e.g., number or quantity of resources, recipes for creating products, unknown quantities, profit formula) and express them so that they become part of the calculation process. Students should use tables or charts to organize the given information from a linear programming problem. They should then use the chart to record trial values as they attempt to organize their thinking, and as they collect data. Then from the chart or table students should be able to derive the information that should lead to the creation of expressions for the constraints in the problem. The important skill at this point in the process is to interpret the situation and describe key elements in equations or inequalities.

❐ Suppose a toy manufacturer has 70 containers of plastic and wants to make and sell skateboards and snowboards. To make one skateboard requires 5 containers of plastic, plus paint and decals. (Paint and decals we will say are available in unlimited quantities.) To make a snowboard requires 2 containers of plastic. The manufacturer is unable to produce more than 10 skateboards, due to problems associated with shipping in wheels and bearings. The profit on one skateboard is $15 and on one snowboard is $12. How many of each should be produced to maximize the profit?

In Mathematics 9, students have solved first degree inequalities in one variable algebraically. They have tested their solutions and displayed them on number lines. In this course, students are working toward the creation of a general strategy for linear programming.

Using a table or chart to organize the information in the question above will be a useful strategy to begin. For example:

	containers of plastic for each board	maximum numbers of boards	profit per board ($)
skateboards	5	10	15
snowboards	2	–	12

Total of 70 containers of plastic

Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment	Suggested Resources

C6/C11/F5

1) A small manufacturing company produces high quality professional hockey skates. Two models of skates are produced, the Flash model (at a profit of \$30/pair) and the Streak model (at a profit of \$20/pair). The company can produce up to 50 pairs of the Flash skate per week or up to 80 pairs of the Streak skate per week. They cannot, however, produce more than 110 pairs in total per week.
   a) Describe in your own way the constraints given in the situation above.
   b) Create a table or chart with which to organize the information given to you.
   c) Find five combinations of pairs of Flashes and pairs of Streaks that satisfy all the constraints, and express the profit earned for each combination.
   d) Do you think that the company should produce more Flash or more Streak skates to maximize its profit? Explain.

Making Choices: Linear Programming

SCO: In this course students will be expected to

C11 **express and interpret constraints**

E3 **represent systems of inequalities as feasible regions**

A2 **relate sets of numbers to solutions of inequalities**

Elaboration – Instructional Strategies/Suggestions

C11/E3/A2 In developing an understanding of constraints and how to express them, students should begin by examining constraints represented by horizontal or vertical lines. For example, from the table on page 232, students should be able to state that there is a restriction on the number of skateboards that can be produced at one time. They might say that the manufacturer can produce, at most, ten skateboards. They should express this as x (the number of skateboards) is less than or equal to ten, or, in symbols, $x \le 10$.

To increase students comfort with the graphical representation of a constraint (or inequality), have them find and mark (in blue) at least ten points on a coordinate grid that satisfy this constraint. Then have them find and mark (in red) ten points that do not satisfy the constraint. They could then describe how the points show two regions. Have them describe the edge between the two regions and agree on which region represents the $x \le 10$ constraint. Have the graph re-drawn, using shading to indicate the region $x \le 10$. Students should be aware that in this situation (numbers of skateboards), even though the whole region is shaded, the context implies that only whole numbers (the points at the grid marks) in the shaded region are meaningful, i.e., $(x, y) \in W$.

Other students might follow the same procedure with the constraint $x \ge 0$. If both graphs are produced on acetate, they can be overlaid and the region that overlaps can be interpreted, then expressed as $x \le 10$ and $x \ge 0$.

An understanding of constraints represented in relation to vertical and horizontal lines is a good starting point for students. They should then explore the regions above and below oblique lines, as well as those produced by intersecting horizontal, vertical and oblique lines.

At the same time students should be gaining experience representing constraints in words and symbols and interpreting them. For example, from the column headed "containers of plastic" on page 232, students should be able to describe the situation as "One skateboard requires 5 containers of plastic. One snowboard requires 2 containers of plastic. The manufacturer only has 70 containers of plastic."

To continue, ask students to show how they would find the number of containers of plastic that are required for: 1 skateboard and 1 snowboard, 2 skateboards and 2 snowboards, 5 skateboards and 3 snowboards, 3 skateboards and 5 snowboards, x skateboards and y snowboards. Students might respond using a table, an organized list, or a chart:

number of skateboards	number of snowboards	number of containers of plastic
1	1	5(1) + 2(1) or 7
2	2	5(2) + 2(2) or 14
5	3	5(5) + 2(3) or 31
3	5	5(3) + 2(5) or 25
x	y	5(x) + 2(y)

Have students explain what $5x + 2y$ means to them. Then ask them to interpret $5x + 2y \le 70$.

Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

C11/E3/A2

Performance

1) Revisit the problem described on page 233.

Type of Skate	Max #s of pairs per week	Profit/pair ($)
Flash	50	30
Streak	80	20

max pairs per week = 110

a) Represent the number of pairs of Flash skates that can be produced in a week using one or more inequalities. (Ans: $f \leq 50$ and $f \geq 0$, or $0 \leq f \leq 50$)
b) Graph your inequality(ies).
c) Do a) and b) for pairs of Streak skates.
d) Is there a feasible region defined? If so, describe it in writing. (Ans: Discrete points in a region bounded by $0 \leq f \leq 50$ and $0 \leq S \leq 80$.)
e) Select two points in the feasible region and describe what these points represent. How much profit would result from each of these points?
f) Select two points not in the feasible region and describe how these points violate the constraints.
g) Determine an inequality to represent the third constraint (the total pairs produced in a week) and add it to the graph.
h) Describe the effect this has on the feasible region.

2) a) Given the following constraints, sketch a graph and determine a feasible region. $x \geq 1000$, $y \geq 1500$, $x + y \geq 3500$.
b) State the coordinates of the vertices of the feasible region.
c) Let x represent the number of cassettes and y the number of CDs. The inventory cost for each is \$5.00 and \$8.00, respectively. Select some points in the feasible region and determine and describe the costs with respect to the context.
d) Which combination of cassettes and CDs do you think represents the minimum cost?

Making Choices: Linear Programming

SCO: In this course students will be expected to

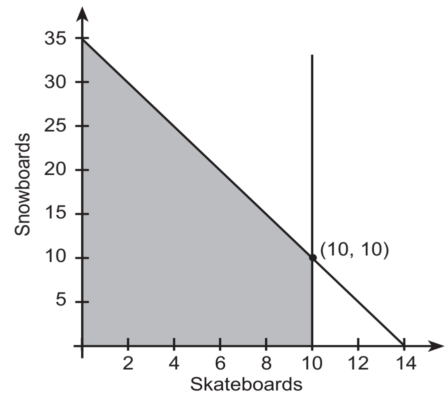
E3 **represent systems of inequalities as feasible regions**

E4 **represent linear programming problems using the Cartesian coordinate system**

C6 **apply the linear programming process to find optimal solutions**

A2 **relate sets of numbers to solutions of inequalities**

Elaboration – Instructional Strategies/Suggestions



E3/E2/C6/A2 Once students are comfortable graphing regions of inequalities they should be expected to graph all of the inequalities for a single problem and view and interpret the resulting shaded polygon region (feasible region).

Students must now find the point or points in the feasible region that produces the maximum income or profit. To begin, they should evaluate the profit for various points in the feasible region. For example, students might select the points (8, 10), (6, 18), and (9, 9).

For (8, 10):

- 8 skateboards at \$15 profit, plus 10 snowboards at \$12 profit might become 8(15) + 10(12) = \$240 profit

For (6, 18):

- 6(15) + 18(12) = \$306 profit

For (9, 9):

- 9(15) + 9(12) = \$243 profit

They should see a pattern in the profit amounts that might lead them to conclude that the more snowboards, the higher the profit. They may test this conjecture with more values, and conclude that the maximum profit occurs at the point (0, 35).

Students should continue to explore this process. Switch the profit amounts and have them now find the point that will give the maximum profit. Change the profit amounts so that the greatest profit now lies at yet another vertex of the feasible region.

C6 After several such investigations in different contexts, students should accept that the maximum profit seems to always occur at an intersection point (i.e., at a vertex of the feasible region). (Sometimes it occurs at all points on one edge of a feasible region, but this will not be the case in any problems given in this course.)

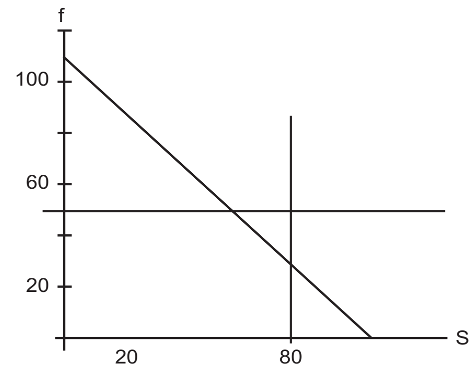
Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment	Suggested Resources

C6/A2/E3

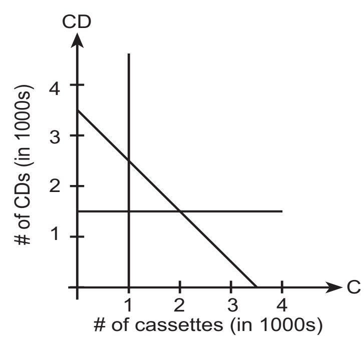
Performance

1) The graph at right represents the edges of the regions described in the skate manufacturer problem on page 233.



   a) Shade the feasible region.
   b) Describe what family of numbers the feasible region should include. (Ans: x, y ∈ W)
   c) Select 5 points that represent possible combinations of pairs of Flash skates and pairs of Streak skates. Determine the profit the company will make for each of the selections, if the Flash skates produce a profit of \$30/pair and the Streak skates produce a profit of \$20/pair.
   d) Use the pattern in the profits and the location of the points to approximate the combination that will produce the maximum profit.
   e) Describe in words how you know there is no other combination that will produce a greater profit.
   f) Suppose the profit amounts were switched so that Flash skates produced \$20 profit per pair and Streak skates produced \$30 profit per pair. Find the combination that produces the maximum profit. Describe your thinking.

2) The graph to the right represents the edges of the constraints described in the situation given in question 2), p. 235.



   a) Shade the feasible region.
   b) Describe what family of numbers the feasible region should include. (Ans: x, y ∈ W)
   c) Select 5 points that represent possible combinations of stored cassettes and CDs. Determine the cost of storage if each cassette costs \$5.00 to store and each CD costs \$8.00.
   d) Find a minimum cost. Explain how you know it is a minimum.
   e) How could you add constraints to the situation that would allow you to determine a maximum cost for the inventory?

Making Choices: Linear Programming

SCO: In this course students will be expected to

C20 **solve systems of equations and inequalities both with and without technology**

B4 **use the calculator correctly and efficiently**

B3 **demonstrate an understanding of the relationship between arithmetic operations and operations on equations and inequalities**

A2 **relate sets of numbers to solutions of inequalities**

Elaboration – Instructional Strategies/Suggestions

C20 At this point, students have a need to find the coordinates of the intersection points of the edges that form the feasible region. Students may do this by

1) estimating from a graph and checking,
2) finding the exact value from a graph, with or without technology, or
3) the algebraic method of substitution (including comparison).

C20/B4 Students should learn to use technology to show the feasible region. They might do this by shading the opposite regions than those done by hand, since the overlapped region will be clearer if left unshaded. Students would trace along edges or use intersect-calculation methods to find the coordinate of the intersection points.

C20/B3/A2 Students will often need exact answers, or want to check their estimated answers. Usually, this is cause for using an algebraic approach. Since the expressions for the constraints are sometimes in 'y =' form for graphing purposes, the two right sides can be set equal, forming a linear equation to solve. For example, consider the constraints $2x + y \leq 20$ and $y - 2x \leq 10$. For graphing, students may rearrange the equations into 'y =' form. (They should use the equality aspect since they want to graph the edge of each region.)

$$2x + y = 20 \qquad\qquad y - 2x = 10$$

$$y = -2x + 20 \qquad\qquad y = 2x + 10$$

therefore, $-2x + 20 = 2x + 10$.

To solve this equation students might begin by subtracting 10 from both sides,

$$-2x + 20 - 10 = 2x + 10 - 10$$

$$-2x + 10 = 2x$$

then adding 2x to both sides, to isolate the term with the variable.

$$-2x + 2x + 10 = 2x + 2x$$

Then, they would divide each side by 4 to isolate the variable,

$$\frac{10}{4} = \frac{4x}{4}$$

$$2.5 = x$$

producing a value for x. To find y, students should substitute this x-value into either original equation. For example, $2(2.5) + y = 20$

$$5 + y = 20$$

Subtracting 5 from both sides gives $y = 15$

Now students can conclude that (2.5, 15) is the intersection point.

When equations come from a context, students should interpret the intersection point in terms of the context. Students should be aware from the context whether the values in the solution are acceptable or not.

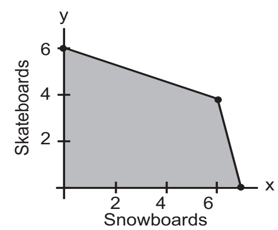
Making Choices: Linear Programming

Worthwhile Tasks for Instruction and/or Assessment	**Suggested Resources**

C20

Performance

1) a) Determine the point that represents the combination of snow and skateboards that will produce the maximum profit overall. The profit for snowboards (x-axis) is \$2.00 per board and the profit for skateboards (y-axis) is \$7.00 per board.



   b) If the profit for the skateboards changed to only \$4.00 per board, how would that affect your answer?

   c) What is the maximum profit in b)?

B3/C20/B4

2) a) Jerry wants to graph a given constraint, $3x + y \leq 28$. He decides to put it into $y = mx + b$ form so that he can see the slope and y-intercept. This is the result he gets: $y \leq 3x + 28$. Did Jerry do things correctly? Explain.

   b) Jerry knew that the point on the graph that represents the combination of 'goods' that will produce the highest profit is at or near the intersection of the two lines $y = -2x + 20$ and $2y = -10x + 64$. Find how many of each 'good' produces the maximum profit.

   c) Explain how you could use graphing technology to find the same answers for x and y as you did in b).

B3

3) Students were given two equations and asked to find the point common to both. Study the two given solutions, find any errors and explain what is wrong and how to fix it.

Elaine:

$3x - 2y = 8$

$5x - y = 12$

Solution:

$$\begin{aligned} y &= -5x + 12 \text{ ①} \\ 2y &= -3x + 8 \\ y &= -\tfrac{3}{2}x + 4 \\ -5x + 12 &= -\tfrac{3}{2}x + 4 \\ -10x + 24 &= -6x + 8 \\ -4x &= 32 \\ x &= -8 \end{aligned}$$

the point is -8

Frank:

$3x - 2y = 8$

$5x - y = 12$

Solution:

$$\begin{aligned} -y &= -5x + 12 \text{ ①} \\ y &= 5x + 12 \\ 3x - 2(5x + 12) &= 8 \\ 3x - 10x + 24 &= 8 \\ -7x + 24 &= 8 \\ -7x &= -14 \\ x &= 2 \\ y &= 5(2) + 12 \\ &= 10 + 12 \\ &= 22 \end{aligned}$$