# Atlantic Canada Mathematics Curriculum 

New Brunswick<br>Department of Education

Educational Programs \& Services Branch

New 后质 Nouveau Brunswick

Grade 2

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## Introduction

## I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.
The Foundation for the Atlantic Canada Mathematics Curriculum firmly establishes the Curriculum and Evaluation Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers," and they advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers and Department of Education officials to plan and develop cooperatively the curricula in mathematics, science, and language arts in both official languages.

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)


Figure 1: Outcome Framework

## B. Rationale

## II. Program Design and Components

## A. Program

Organization

The Foundation for the Atlantic Canada Mathematics Curriculum provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes-general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. This curriculum guide is supplemented by others that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes ( SCOs ) to each KSCO.

The Atlantic Canada Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risktaking, and critical thinking and that nurtures positive attitudes and sustained effort; and iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

As already indicated, the mathematics curriculum is designed to support the six Essential Graduation Learnings (EGLs). While the curriculum contributes to students' achievement of each of these, the communication and problem solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the Foundation for the Atlantic Canada Mathematics Curriculum.) The foundation document then presents outcomes at four key stages of the student's school experience.

This particular curriculum guide presents specific curriculum outcomes for each grade level. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.


It is important to emphasize that, while the grade level outcomes (SCOs) provide a framework on which educators will base decisions regarding instruction and assessment, they are not intended to limit the scope of learning experiences. Although it is expected that most students will be able to attain the outcomes, some student's needs and performance will range across grade levels. Teachers will need to take this variation into consideration as they plan learning experiences and assess students' achievement.

The presentation of the specific curriculum outcomes follows the outcome structure established in the Foundation for the Atlantic Canada Mathematics Curriculum and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes like those pertaining to patterns and data management may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what

## B. Unifying Ideas

might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province like an election, an exhibition, or a fair.

The NCTM Curriculum and Evaluation Standards establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The Foundation for the Atlantic Canada Mathematics Curriculum (pp. 711) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)


These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time numerous problemsolving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

## C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

## D. Adapting to the Needs of All Learners

## E. Support Resources

The Foundation for the Atlantic Canada Mathematics Curriculum stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student development as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. These guides should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in as much as they support the specific curriculum outcomes. Many manipulative materials need to be readily at hand, and technological resources, e.g., software and videos, should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the Curriculum and Evaluation Standards for School Mathematics (NCTM) and the Addenda Series and Yearbooks (NCTM), Elementary School Mathematics: Teaching Developmentally or Elementary and Middle School Mathematics: Teaching Developmentally (John van de Walle), Developing Number Concepts Using Unifix Cubes (Kathy Richardson), and About Teaching Mathematics; A K-8 Resource (Marilyn Burns).

## F. Role of Parents

## III. Assessment and Evaluation

## A. Assessing Student Learning

## B. Program Assessment

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessment and Evaluating Student Learning" in the Foundation for the Atlantic Canada Mathematics Curriculum.)

Characteristics of good student assessment would include i) the use of a wide variety of assessment strategies and tools, ii) aligning assessment strategies and tools with the curriculum and instructional techniques, and iii) ensuring fairness both in application and scoring. The Principles for Fair Student Assessment Practices for Education in Canada elaborates good assessment practices and it served as a guide for student assessment for the mathematics foundation document.

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

## IV. Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for each grade. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled "Elaboration-Instructional Strategies/ Suggestions," provides a clarification of the specific curriculum outcome(s), as well as suggestions for possible strategies/activities which could be used to help students achieve the outcome(s). While the strategies/activities presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s). They will also illustrate ways to work toward the achievement of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activites are introduced in this column of the layout by the symbol $\square$.


Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled "Worthwhile Tasks for Instruction and/or Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Suggested Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).

# Number Concepts/ Number and Relationship Operations <br> General Curriculum Outcome A: 

Students will demonstrate number sense and apply number-theory concepts.

# GCO A: Students will demonstrate number sense and apply number-theory concepts. 

KSCO: By the end of grade 3, students will be expected to
i) construct and communicate number meanings, and explore and apply estimation strategies, with respect to whole numbers

SCO: By the end of grade 2, students will be expected to
A1 order numbers and use ordinal language
A2 count in a variety of ways

## Elaboration - Instructional Strategies/Suggestions

A1 By the end of grade 2, students should be comfortable using ordinal language up to "thirty-first."
$\square$ To extend early experiences with ordinal language, on-going use of the calendar is a worthwhile practice. Ask questions such as the following:

- Today is the eighth. What date will it be a week from Monday?
- This is the seventeenth of April. Marc's birthday was two weeks ago today. When was his birthday?
- Ten days from yesterday will be the $\qquad$ —.
- This is the third of the month. Explain why it is easy to find two weeks from today using the calendar.
- What is the sixth month of the year?
- January is the first month. Say the months in order and stop at the eleventh month.
$\square$ Ask pairs of students to create their own set of calendar questions involving ordinal language.
A2 Students should count
- backwards and forwards
- by $2 \mathrm{~s}, 3 \mathrm{~s}, 4 \mathrm{~s}, 5 \mathrm{~s}, 10 \mathrm{~s}, 25 \mathrm{~s}$, and 100 s
- from various starting points (e.g., $30,25,20 \ldots$ or $50,75,100$ )

It is through a wide range of activities, presented regularly throughout the school year, that students begin to develop number sense and number concepts, which form the foundation required for understanding more advanced mathematical concepts.
$\square$ Students at this level enjoy rhythmic skip counting activities. These include clapping, marching, drumming with hands on the desk, and striking instruments.
$\square$ Have students use their fingers to separate counters as they count by 2 s or 3 s .

Students should recognize how to count coins (e.g., for 3 quarters, 2 dimes and a penny $-25,50,75,85,95,96$ ). Before students are able to do this, they must practise counting nickels, dimes and quarters separately. As well, they should come to recognize that to find the total for a collection of coins, sorting them and counting the larger coins first is easiest.

Many number patterns can be used to encourage skip counting; for example:
$25,50, \ldots, \ldots, 125, \ldots, \ldots$
450, _ , 350, _ , 250, _
95, 90, _ , _ , 75, _ , _ ,

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A2.1 Begin to count, " $25,50,75,100,125,150$." Ask the student to continue to count to 500 by 25 s.

A2.2 Give the student 4 quarters, 3 dimes, 2 nickels and 6 pennies. Ask $\mathrm{him} / \mathrm{her}$ to count the coins to find if an item costing $\$ 1.50$ can be bought.

A2.3 Provide coins for the student. Ask: Can you use 6 of these coins to make 43 cents? Can you make a total of $\$ 2.00$ with 6 of these coins?
A2.4 Provide a number of beans ( 60 , for example). Ask the student to separate the beans from the pile as he/she counts them by 2 s , then by 3 s .
A2.5 Provide students with play coins. Tell them that you have, for example, 5 coins in your hand that total 81 cents. Ask: What coins am I holding? (This is a problem situation and may require time.)
A2.6 Use the calculator counting constant to count by $2 \mathrm{~s}, 5 \mathrm{~s}, 10 \mathrm{~s}, 25 \mathrm{~s}$, and 100 s . Ask the student to say each number as it appears on the display.
A2.7 Have students count beans in a jar. Ask them how they grouped the beans (e.g., by $2 \mathrm{~s}, 5 \mathrm{~s}, 10 \mathrm{~s}$ ) for ease of counting.

## Interview

A1.1 Tell the student that 24 students in the class are standing in a row. Ask: If you were next to the last, in what position would you be?
A1.2 Show the student a calendar. Ask: Why is it easy to find the date which is two weeks from the sixth of the month?

A2.8 Ask: Why do you say fewer numbers when counting to 100 by 10 s than when counting by 5 s?

A2.9 Tell the student to decide which starting point, 3 or 6 , is easier when counting by 2 s to 30 . Ask the student to explain his/her choice.
A2.10 I say " $25,50,51,52,53$." What coins am I counting?
A2.11 Play "What's in the Can?" Tell the student that you are going to drop nickels (or dimes or quarters) into a can. Have the student listen as the coins drop and count to find the total. As an extension, tell the student that there is, for example, 45 cents in the can. Tell him/her that you are going to add nickels (or dimes) and ask him/her to keep track to find the total.

## GCO A：Students will demonstrate number sense and apply number－theory concepts．

KSCO：By the end of grade 3， students will be expected to
i）construct and communicate number meanings，and explore and apply estimation strategies，with respect to whole numbers

SCO：By the end of grade 3， students will be expected to A3 estimate the size of numbers to the nearest multiple of 10

## Elaboration－Instructional Strategies／Suggestions

A3 Students should explore situations which require them to estimate in terms of the nearest multiple of 10 ．Encourage the use of examples from school， home，and community environments．For example：
－About how much money will you need to ．．．？
－About how many packages will you need to buy if ．．．？
－About how much time will it take to ．．．？
－About how many Popsicles will we need to buy so everyone in the school will get at least two？

Note：Skill in rounding off is but one of the useful strategies for estimation．
It is important to use visual models such as
－a number line（e．g．，Which number is 23 closer to？）

－base－ten material For example：

|  |  |
| :---: | :---: |
| is closer to |  |
| than | 自自自自 |

－dots on a page，or pictures of objects
For example，present the strategy of dividing into sections of twenty－five．

－a hundreds chart
$\square$ Present a situation in which a student would get a dollar for every 10
flyers（or major portion thereof）delivered．Have the student use a hundreds chart to calculate how much money he／she would earn in neighbourhoods in which there are 56 houses， 34 houses， 119 houses，etc．

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A3.1 Using a number line to 100 , put "gas stations" at the multiples of ten. Place a car at various points on the line and ask the student to identify the location of the closest gas station.
A3.2 Ask the student to use base-ten materials to show why 143 is closer to 140 than to 150 .

A3.3 Tell the student that you are trying to model the number 57 with base-ten blocks but have no unit cubes. Ask him/her to use the blocks to show, and to explain to you, what number would be the closest.

A3.4 Show a picture of 40 to 70 items (e.g., dots, hearts). Ask the student to estimate the total and to explain how he/she arrived at this estimation.
A3.5 Display 40 to 70 beans, randomly placed on the table. Ask the student to estimate how many there are and to explain his/her strategy.

## Interview

A3.6 Ask the student to name five numbers that are about 100 ... are about 50 .
A3.7 Tell the student: To estimate $48+33$, Jeff said, "I'll use 50 and 30 ." Ask him/her to explain Jeff's thinking.
A3.8 Ask the student to name all the numbers that he/she might round to 100.

A3.9 Model a number line as shown below and put counters at various points on it. Ask the student which mark on the number line is nearest each counter.

A3.10 Celia collected 262 autographs from students in her school. June said Celia had about 300 autographs, but Leo said she had about 250 . Which estimate do you think is better? Why?


# GCO A: Students will demonstrate number sense and apply number-theory concepts. 

KSCO: By the end of grade 3, students will be expected to
ii) concretely explore common fractions and decimals in meaningful situations

SCO: By the end of grade 2, students will be expected to A4 identify simple fractions using models

Models must be used at all grade levels to develop fraction concepts adequately. Further, ... children should have experiences with a wide assortment of models. (Elementary School Mathematics, pp. 222-23)

## Elaboration - Instructional Strategies/Suggestions

A4 Students should investigate the more common fraction families such as halves, fourths, thirds, fifths and tenths. Provide opportunities to explore other fractions in problem situations.

It is important that the concept of "the whole," "one whole," or "one" is developed. As well, the concept of "fair share," or "equal share," should be stressed throughout. Students should explore various ways of finding fair shares of the same "whole" (for example, various ways of fairly sharing a chocolate bar among four students).
Include situations which show differing models of fractions:

- part of a whole (e.g., $\frac{1}{2}$ of a chocolate bar or $\frac{2}{3}$ of a strip of paper)
- part of a set (e.g., $\frac{3}{4}$ of the students in the class)

Students should explore these models in a variety of contexts. Both meanings can be explored using pizzas; for example:
$\frac{1}{4}$ of a pizza
$\frac{1}{10}$ of the slices have pepperoni
$\square$ You might use the book Eating Fractions, by Bruce MacMillan.
Provide many opportunities to explore and discuss fractions orally before the symbols are introduced. Continue to use, for example, the " 1 of 3 equal parts" language and help students connect the language with its symbol. Note: This is the first time that fractions are presented symbolically. Fractions such as $\frac{1}{3}$, $\frac{2}{5}$ and $\frac{4}{10}$ are relatively easy for students to read since familiar ordinal language is used for the denominator of each - third, fifth and sixth. Point out to students, however, that $\frac{1}{2}$ is read one-"half" (not one "second") and that $\frac{1}{4}$ may be read either one-"fourth" or one-"quarter". (Note: The money application of " 4 quarters make a whole dollar" can be conveniently presented in this connection.)

To assist with clarity of meaning, always write fractions with a horizontal bar.
$\square$ Provide copies of pizza outlines divided into 10 pieces. Describe this problem to the students: You invite 6 friends to your pizza party. While you are making the pizza, you find out that 2 people do not like mushrooms, the other 2 do not like pepperoni, and another person hates onions. Challenge pairs of students to draw and describe how they are going to distribute the ingredients on their pizzas. Invite them to express their ideas in terms of fractions.

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A4.1 Show the student 15 marbles. Ask: If you were given one-third of them, how many would you have? Can you find one-half of the marbles? Why or why not?

A4.2 Ask the student to show how to divide a chocolate bar so that each of four people gets a fair share. Ask: What part of the whole bar would each person get? (Encourage the student to find alternative ways to divide the bar so that each person gets $\frac{1}{4}$.)
A4.3 Provide paper shapes. Ask the student to cut or fold them to show $\frac{1}{2}$ and $\frac{1}{4}$.


A4.4 Provide a number of different pictures and ask students to sort them into groups illustrating halves, thirds, fourths and fifths. (Be certain to provide numerous examples of both "part of a whole" and "part of a set." For thirds, for example, you might have pictures such as the following:


You may also wish to include some pictures which incorrectly represent the same fractions.

## Paper and Pencil

A4.5 Provide the child with a strip of circles as shown.

$$
0000
$$

Ask the student to draw happy faces on $\frac{1}{4}$ of the circles.

## Interview

A4.6 Ask the student what fraction would suggest a "fair" share if 4 children were sharing.
A4.7 Ask the student when one might talk about $\frac{1}{2}$.
A4.8 Ask: Is $\frac{1}{2}$ a lot or a little? Explain.
A4.9 Ask: If you are really hungry and want a large piece of cake, would you cut the cake into thirds, fourths or tenths? Explain.

## Portfolio/Presentation

A4.10 Have the students ask family members when they use fractions. Make lists and share them with the class.

A4.11 Ask the students to find and copy a recipe that they could easily make. Have them choose one that contains some fractions.

# GCO A: Students will demonstrate number sense and apply number-theory concepts. 

KSCO: By the end of grade 3, students will be expected to
iii) read and write whole numbers and demonstrate an understanding of place value (to four places)

SCO: By the end of grade 2, students will be expected to A5 describe numbers in a variety of ways

## Elaboration - Instructional Strategies/Suggestions

A5 The development of number sense must be a focal point of mathematics instruction. Number sense cannot be properly developed, however, without a sound understanding of place-value concepts. These concepts develop slowly, and it is important that students experience a wide variety of worthwhile activities in order to build a firm foundation for place value.

Many students who can state that " 36 is 3 tens and 6 ones" are simply following a pattern that has been presented to them and do not have a clear understanding of the concept of place value, that is, that it is convenient to group numbers consistently and that, because we use only ten symbols, the placement of a digit in a numeral determines its meaning.

It is important that students have many and varied experiences with materials that they can count and group in a variety of ways. These activities should be purposeful and presented throughout the year as students build their sense of number.
$\square$ Ask students, working in pairs with a given number of counters, to group them in several different ways and to record each grouping, using charts. For example, 24 is

| Number <br> of groups <br> of five | Number <br> left |
| :---: | :---: |
|  |  |
| 4 | 4 |


| Number <br> of groups <br> of eight | Number <br> left |
| :---: | :---: |
|  |  |
| 3 | 0 |


| Number <br> of groups <br> of ten | Number <br> left |
| :---: | :---: |
|  |  |
| 2 | 4 |

$\square$ A daily activity that teachers have found to be valuable for the development of this concept is the "Number of the Day." Beginning with the first day of school in September, students express the number in as many ways as they can. For example, day 26 , probably sometime in October, may be expressed as $5+5+5+5+5+1 ; 20+6 ; 10+$ $10+6 ; 10+16 ; 26$ ones; 2 tens and 6 ones; etc. By spring students will have had their "Day 100" celebration and be describing numbers such as 139 as $25+25+25+25+25+10+4$; five 25 s and 14 more; 5 quarters, 1 dime and 4 pennies; $100+39$; 139 ones; 13 tens and 9 ones; 1 hundred, 3 tens and 9 ones; etc. It is not unreasonable to expect many students to express a number in 20 or 30 different ways. Activities such as these help students become comfortable with number and provide necessary practice towards the development of both number sense and the concept of place value.

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance <br> A5.1 Show <br> 

Ask: How many does this represent? How else might this many be modelled? Encourage the student to talk about the groupings. Does he/she recognize that each grouping represents the same number? Ask the student to identify the grouping that is easiest to read and to explain why.

A5.2 Show


Ask questions similar to those in A5.1.
A5.3 Show the student

Provide additional dimes and pennies for the student. Ask: How might this amount be represented in other ways?
A5.4 Provide 26 counters and ask the student to count them and record the amount. Point at the " 2 " and ask him/her to show you that part using the counters. Do the same with the " 6 ."

## Paper and Pencil

A5.5 Ask the student to write the number 43 in at least 6 different ways.

## Portfolio

A5.6 Have students design a page for their portfolios on which they have expressed their favourite 2-digit number in a number of creative ways.


# GCO A: Students will demonstrate number sense and apply number-theory concepts. 

KSCO: By the end of grade 3, students will be expected to
iii) read and write whole numbers and demonstrate an understanding of place value (to four places)

SCO: By the end of grade 2, students will be expected to
A6 demonstrate an understanding of baseten groupings

## Elaboration - Instructional Strategies/Suggestions

The early place-value focus must be on the development of an understanding of base-ten groupings and a knowledge of how these groupings are recorded. Students should notice how much easier it is to count when the numbers are grouped. As with many topics presented at this level, it is important that this development not be rushed. Later problems related to number can often be traced back to an improper introduction to this concept.

The foundation for the development of the concept of place value centres around grouping activities. Students should have experiences with different-sized groupings. Place value focusses on groupings of ten. Activities must begin with proportional models so students can "see" that the piece representing the ten is actually ten times the size of the unit piece. Examples of pregrouped proportional models are bean sticks (ten beans glued to a popsicle stick or tongue depressor), teacher made "rods," and manufactured base-ten materials.
$\square$ Have students use a die to play "Race For A Ten." They roll the die and count out the number of unit cubes. When they get exactly ten, they trade them in for a rod. Play "Race For A Fifty" or "Race for a Hundred," using the same rules.
$\square$ Ask students to arrange counters (46, for example) into groups of ten, and to represent the number both symbolically and with base-ten rods and unit cubes.
$\square$ Direct the students to count the children in the class, represent the number symbolically, and model the number, using the base-ten models.
$\square$ Have students model 2-digit numbers in various ways; for example, 46 as 46 ones, or 4 tens and 6 ones, or 3 tens and 16 ones. (Note: This last example is useful with respect to the regrouping in the subtraction algorithm.)

Once students understand the grouping concept for 2-digit numbers, grouping for 3-digit numbers should not be difficult.
$\square$ Provide unit cubes (132, for example) to pairs of students. Ask the students to trade their units for rods and to write the numeral. It is important that there be a lot of teacher-directed dialogue regarding the groupings. Ten rods can then be traded for a flat for easy counting. As an extension, have each pair combine their flats, rods, and units with those of another pair of students and discuss the new groupings.
Ultimately, students should realize that both 10 tens ( 10 rods) and 100 ones (100 unit cubes) make one hundred ( 1 flat ).
Note: It is suggested that standard column headings (i.e., Tens and Ones) not be used at this point as these headings can be misleading to the students.

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A6.1 Observe students as they play "Race For A Hundred." Question them to assess their understanding of the "trading" concept.

A6.2 Give the student a number of counters. Ask him/her to make groupings of ten counters and to represent the number both symbolically and with base-ten materials.

Ask him/her to show a list of numbers that might be rolled on a die to reach a total of fifty. Have the student show the trading and explain what he/she is doing.

A6.4 Show the student


Ask him/her to show at least one other way of representing the number with base-ten materials.

A6.5 Provide a place-value mat and base-ten materials. Have students work in pairs rolling a die, counting out the ones and trading for tens.

Observe the students.

- Do they use counting on or counting from the

| Number of <br> groups of ten | Number <br> leff |
| :---: | :---: |
|  |  |
|  |  | start (e.g., count the " 5 " in the units place and add the " 2 " that has been rolled)?

- Do they arrange the unit cubes for easy recognition (e.g., arrange 8 in a 2 by 4 array)?
- Have some progressed to using mental strategies (e.g., " 6 " on the mat, roll a " 4 ," clear the ones place and put a rod on the tens side of the mat)?
- With continued practice, more sophisticated strategies surface. For example, " 5 " on the ones side, roll a " 6 ," clear all but one of the units and add the rod. Not all students are able to do this; however, those who can should not be discouraged from proceeding in this way, as they have clearly grasped the trading concept.


## Interview

A6.6 Show the student some numbers modelled with base-ten materials. For example:


Ask: What do the models represent? How do you know the second one has a greater value than the first one?

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to
iii) read and write whole numbers and demonstrate an understanding of place value (to four places)

SCO: By the end of grade 2, students will be expected to
A7 model numbers to three places

The key instructional tool for developing the conceptual knowledge of place value and also for connecting these concepts to symbolism is the use of base-ten models. (Elementary School Mathematics, p. 157)

## Elaboration - Instructional Strategies/Suggestions

A7 Students should be able to create and interpret a model representing either a 2-digit or a 3-digit number, using proportional materials. Examples:


It is important to provide practice with numbers containing zeros. For example:


Students should express numbers symbolically, verbally, and with models. They should be expected to move from any one to both of the other representations. For example, " 43 " is the symbol for "forty-three" and is modelled

When shown

?
, the student is expected to write " 302 " and read the number as "three hundred two."

When hearing "two hundred thirty-four," the student should be able to write" 234 " and model it as


It is important that students realize a number can be expressed in different ways. For example, 234 can mean 2 hundreds, 3 tens and 4 ones; 23 tens and 4 ones; or 234 ones.
$\square$ Have the students explore all the numbers that can be modelled, given a fixed number of base-ten blocks. For example, 6 base-ten blocks can represent 600, 501, 510, etc.

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A7.1 Ask the student to use base-ten materials to show 314 in three different ways.
A7.2 Ask the student to model a number which contains more hundreds than tens but more ones than hundreds. Once the model is complete, have the student write the number which the model represents.

A7.3 Invite students to make a robot with base-ten blocks and symbolize its value. For example:

## 143



A7.4 Ask the student to model "five hundred one" and "five hundred ten," write the numbers symbolically, and discuss their similarities and differences.

A7.5 Show the student a number written symbolically. Ask him/her to read the number and model it with base-ten materials.

## Paper and Pencil

A7.6 Ask students to write numbers symbolically as you read them. Be certain to include examples that have zeros in their symbolic representation.

## Interview

A7.7 Ask the student to describe how much 100 is.
A7.8 Show 4 base ten flats, 15 rods and 6 units. Ask: What number is this showing? Ask the student to model the number in another way.
A7.9 Tell the student that Ann said there are 3 tens in 135, but Jeff said that there are 13 tens. Ask: Do you agree with Ann or Jeff? Explain.
A7.10 Ask: How is 500 like 50 and how is it different?
A7.11 Ask: Why is 234 not the same as 324 ?

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to
iv) order whole numbers and represent them in multiple ways and
v) apply number theory concepts (e.g., place-value patterns) in meaningful contexts, with respect to whole numbers and commonly used fractions and decimals

SCO: By the end of grade 2, students will be expected to A8 compare and order numbers by size
A9 recognize, extend, and create simple place-value patterns

If children have number sense, they understand the relationship of numbers to each other, are able to tell when an answer or unit of measurement is reasonable, and can use numbers effectively in many situations.
Number sense takes a long time to develop; even adults continue to grow in this ability. To establish an environment that nurtures the development of number sense in children, you will need to provide interesting questions to explore, suitable materials with which to investigate those questions, and a classroom climate that encourages the discussions and the display of the results of mathematical investigations. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p.10)

## Elaboration - Instructional Strategies/Suggestions

A8 Students should be able to compare two or more numbers, each less than 1000 , to determine relative sizes. Include situations in which numbers are located on a hundreds chart and/or number line.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

Students should be shown how to use the symbols "<" and ">" to describe the relative sizes of numbers. They should be able to name numbers greater than, less than, or between given numbers.
$\square$ Give groups of students small pumpkins. Ask the groups to first estimate the number of seeds in their pumpkins, and then to remove and count the seeds. Have the groups arrange their data on a single graph for easy comparison.
$\square$ Have students compare the number of students in several schools to see which schools have populations greater or less than their school. The data can be arranged on a graph for easy comparison.


A9 Students should extend and create simple place-value patterns. For example:
$28,38,48,58, \ldots$
47, 147, 247, ...
258, 248, 238, ...
312, 412, 512, ..
512, 502, 492, ...

## GCO A: Students will demonstrate number sense and apply number-theory concepts.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

A8.1 Ask the student to use materials to show why 243 is less than 324.
A8.2 Ask the student to find a number between 312 and 387 that can be represented using 8 base-ten blocks.

## Paper and Pencil

A8.3 Ask the student to use the symbols $5,2,4,3,<$, and $>$ to create two true number sentences.

A8.4 Ask the student to fill in the box with different symbols or, in different ways, to make this statement true.

$$
3 \square 4<352
$$

A8.5 Ask: How many different ways can the following blanks be filled to make a true statement?

A9.1 Have the student continue each of the patterns below for 3 or 4 numbers.

32, 42, 52,...
378, 388, 398,...

## Interview

A8.6 Ask: What do you do to compare the size of two numbers?
A8.7 Ask: Why are there more numbers greater than 123 than less than 123 ?
A9.2 Ask: What number would probably come tenth in each pattern below? twentieth?

141, 142, 143,...
$141,151,161, \ldots$

# Number Concepts/ Number and Relationship Operations <br> General Curriculum Outcome B: 

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to
i) demonstrate an
understanding of the
connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations

SCO: By the end of grade 2, students will be expected to
B1 recognize that multiplication can be used to determine the total amount in groups of equal size
B2 recognize that division can mean determining how many groups of a fixed size are in a larger group or representing fair sharing

For all four of the operations, models (usually sets of counters and number lines) and word stories or word problems are the two basic tools the teacher has to help students develop operation concepts. (Elementary School Mathematics, $p$. 109)

## Elaboration - Instructional Strategies/Suggestions

B1/B2 Multiplication at this level is very informal and should deal strictly with the concept of putting together groups of equal size - not the memorization of facts. Encourage students to use language that makes sense to them (e.g., 3 groups of 2 , sets of 2 , piles of 2 , as opposed to "three times two"). Teachers should model language such as "I have four groups of five; that's twenty."

These early experiences with the multiplication concept should include skip counting (e.g., $4,8,12, \ldots$ ) and repeated addition (e.g., four plus four plus four"). Continue regular practice with skip counting, including using the hundreds chart and the calculator constant.

Similarly, it is important to focus on the meaning of division rather than the symbolization. Division is used either to determine the number of smaller groups which fit into a larger group or to represent fair sharing. Situations might or might not involve leftovers; for example: How many packs of 2 cookies can you make out of a group of 24? 23?

It may be appropriate at this time to deal with situations without symbols or, alternatively, to allow students to develop their own notation to describe a multiplication or division situation.
Multiplication and division concepts should often be presented simultaneously. For example: Share the given cookies fairly among 4 students. How many will each of the four get? How many are there altogether? What would happen if you were to share among six students? eight? Introduce the concepts by having students explore meaningful problem situations.
$\square$ Ask the children to arrange themselves into groups of four. Have them skip count the number of children in the class. Try groups of three, five, etc.
$\square$ Tell the students that each member of their group requires four pieces of paper. Have them distribute the paper and tell how many pieces they needed in all. Observe how they "count."
$\square$ Provide students with stickers and have them give a certain number to each member of their group. How many stickers does the group have altogether?

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B1.1 Ask the students to count out 8 raisins for each member in their group. How many raisins are there altogether? Encourage the students to use a hundreds chart if they have difficulty adding 8 s or skip counting by 8 .

B1.2 Tell the student that you have bought a number of packages, each containing 3 juice boxes. Ask the student to model the problem and tell you how many boxes you might have.
B1/2.1 Simulate a bake sale. Provide trays and cut-out squares and cookies. Have the students group the sweets on the trays for the sale - maybe 4 squares/tray and half a dozen cookies/tray. Encourage the students to talk about what they are doing and to use informal multiplication and division language such as "group of . . .," "shared among . . .," etc.

B2.1 Provide 12 counters. Tell students to think of them as a dozen eggs. Ask them to model different ways to divide up the eggs so that all the groups formed have the same size.

## Paper and Pencil

B1.3 Ask: Which of the following situations could be described using multiplication? Why?


B1.4 Ask students to draw a picture to show why the number of wheels on 4 bicycles is the same as the number of wheels on 2 cars.

## Interview

B1.5 Point out that you might say " $2,4,6,8, \ldots$ " when counting boots. Ask: How high would you have to count if there were 8 pairs of boots? Explain your thinking.

B2.2 Present a situation in which there are 15 pencils. The pencils are to be put into piles with the same number in each. Ask: How many piles might there be and how many pencils would be in each pile? Show another way that this might be done.

## Portfolio

B1/2.2 Ask the student to create a series of problems involving bicycle and tricycle wheels, and identify which involve the idea of either multiplication or division.

Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to
ii) recognize and explain the relationships among the four basic operations and
iii) create and model problem situations involving whole numbers, using one or more of the four basic operations

SCO: By the end of grade 2, students will be expected to
B3 demonstrate an understanding that addition can be used to solve subtraction problems and vice versa
B4 create word problems involving addition and subtraction
[A] very important way to help children construct operation meanings is from word problems or word stories. The word problem provides an opportunity for examining a much more diverse set of meanings for each operation. Models remain a critical part of the development. (Elementary School Mathematics. p. 110)

## Elaboration - Instructional Strategies/Suggestions

B3/B4 Creating word problems based on contexts of interest requires students to think about the meaning of the operations more thoroughly than does simply solving problems. Although creating word problems may seem difficult to some students at first, facility will increase with experience. Students might be encouraged to dramatize the situations, as this is very helpful in clarifying their thinking.
When dealing with subtraction, encourage students to create problems involving all three meanings - take away, comparison, and missing addend. (Note: While students have had previous experiences with these types of situations, many will still need to be convinced that subtraction is the means to solve comparison and/or missing-addend problems.) It is particularly by using the missing-addend meaning that students see the relationship between addition and subtraction, and how subtraction problems can be solved by adding. Teachers can model this connection by re-uniting parts in an addition after solving a subtraction problem.
$\square$ An effective activity is to invite groups of students to choose books which have addition or subtraction contexts that they especially like. Give them time to prepare dramatizations of the books (changing whatever variables they like) to present to others. Their presentations should include a number of problems for others to solve.
Provide activities involving the use of money to purchase items. For example:
$\square$ Fran had 25 cents. She spent 16 cents. How much change does she get back? Encourage the students to explain how they go about solving the problem; for instance: " 16 and 4 more are 20 , plus 5 is 25 . She gets 9 cents change." Or, " 16 cents and 10 cents are 26 cents, so she gets only 9 cents back."
$\square$ Set up a "store" within the classroom and have the students take turns being the cashier. Model for them how to "count on" when making change.
$\square$ Paul has 78 cents, and his brother has 92 cents. How much more money does Paul's brother have? (Providing a visual aid for the students may be helpful.)


## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B3.1 Tell the student that Eric had two quarters. He bought a package of baseball cards for 39 cents. Ask him/her to explain (using coins provided) how to determine how much change Eric got back.
B3.2 Tell the student that Janet read 18 books and Frederica read 42. Ask him/her to use the number chart shown below to explain how to find the difference.


## Paper and Pencil

B3.3 Ask students to write the addition fact that would help them solve

$$
\begin{aligned}
& 18-9=\square \\
& 50-\square=20
\end{aligned}
$$

B3.4 Tell the student that $15+25=40$. Ask: What subtraction questions might this information help to answer?

## Interview

B3.5 Tell a student that someone told you that you do not have to learn to subtract if you know how to add. Ask: Do you agree? Why or why not?
B4.1 Ask the student to create a subtraction problem involving the numbers 23 and 12.

B4.2 Ask the student to create a word problem about the heights of two students.

## Presentation

B4.3 Have a group of students create and present a skit involving a number of addition and subtraction situations.

## Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to iv) demonstrate proficiency with addition and subtraction facts

SCO: By the end of grade 2, students will be expected to B5 develop and apply strategies to learn addition and subtraction facts B6 recall addition facts involving two addends, each less than 10, and the related subtraction facts

## Although we may have

 experienced mathematics lessons in a passive way, research has shown that children learn best when they are active learners, trying new ways to find answers to problems and talking with their classmates about what they are learning. Many different materials can be used in these mathematical investigations: beans glued onto craft or Popsicle sticks, dominos, paper clips, small cubes, or just about any other easily handled item. Computers and calculators, pencils and paper, prepared worksheets, and student textbooks all have a place in the modern second-grade mathematics classroom. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 10)
## Elaboration - Instructional Strategies/Suggestions

B5 Developing and applying strategies significantly reduce the number of facts students have to memorize independently. Students should understand

- how to work from doubles - e.g., $4+5$ is $4+4$ and one more; 9-4 is 1 more than 8-4
- that by adding a certain amount to the first addend and subtracting the same amount from the second addend, the sum does not change - e.g., $9+8=10$ ( 1 more) +7 (1 less)
- that subtraction or addition can be done in stages by bridging to 10 e.g., $8+7=(8+2)+5=15 ; 13-8=13-3-5$
- how to work with addends that differ by two - e.g., $6+8$ (take one from the 8 and add it to the 6 ) equals $7+7 ; 7+9$ is the same as $8+8$, etc.
- the "think addition" strategy for subtraction - e.g., for 16-9 the student may start with 9 , add 1 to make 10 , then 6 more, for a total of 7
- how to use the "subtract a ten" strategy - e.g., to find 17-9, think "1710 is 7 , but I only need to subtract 9 , so the answer is one more, or 8 ."
- how to "add to each" in subtraction situations - e.g., to find 17-9, use 18-10, since adding one to each preserves the difference (distance) between them.
It is very important that students have the opportunity to discuss their strategies with others.
B6 Facts with a sum to 10 should be easily recalled by mid year. By year end, it is expected that most students will be able to recall most addition facts to 18 , although some may rely (quite effectively) on strategies for some sums between 10 and 18 ; for example, $8+7$ is the same as $10+5$, or $15 ; 7+9$ is the same as $8+8$, or 16 .

Although many students will be able to recall facts with a fair amount of ease, others will require more practice. It is important to provide opportunities for this practice, using games and meaningful contexts as much as possible, rather than only requiring the memorization of facts in isolation. Board games in which students are required to find the sum of 2 dice to determine how far to move are good examples.

Note: Fact learning is a mental exercise with an oral and/or visual prompt; therefore, the focus is oral, rather than on paper and pencil practice. Also, drills should be short with immediate feedback (e.g., 3-5 minutes per day) over an extended period of time. Ultimately, each child may be "doing their facts" in a unique way; this could be a combination of visualization, quick strategy application and memory. It really does not matter how children operate as long as recall is immediate (i.e., within about three seconds).

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B5/6.1 Invite students to play a game of concentration. Provide 18 cards with 9 different sums ( 2 different facts for each sum) which have to be matched (e.g., $9+5$ and $7+7 ; 6+1$ and $0+7$ ). Students turn over cards to find matches (i.e., cards which have the same sum).

B5/6.2 Have students work in pairs to sort addition facts into groups of facts which are related. Give them time to share with others and to explain their sortings.

## Paper and Pencil

B5.1 Have the student list the doubles facts which might help him/her solve $8+9$.

B5.2 Students should list all the subtraction questions they can for which both numbers are less than ten and their difference is 3 .

B6.1 Provide a price list with items costing 18 cents or less. Ask: Which items could you buy with two dimes? How much would be left over after each purchase?

## Interview

B5.3 Ask: How does knowing $4+4=8$ help you to know other facts?
B6.2 Students roll 2 dice. They either add or subtract these values. For example: if a 5 and a 2 are rolled, they will work with either 5, 2, 7 or $5,2,3$. Ask the students to make up a subtraction story based on these numbers, and write the corresponding number sentence. (Note: Ten-sided dice work well, as do prepared number cards.)
B6.3 Ask the student to tell why $\square+5$ has to be 2 greater than $\square+3$.

## Portfolio

B5.4 Have the student describe in as many ways as he/she can why $15-8=7$.

Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to v) apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers

SCO: By the end of grade 2, students will be expected to
B7 demonstrate an understanding of basic principles of addition
B8 add 3 single-digit numbers

## Elaboration - Instructional Strategies/Suggestions

B7 Understanding the basic principles behind addition allows children greater flexibility in their thinking. Recognizing these principles also reduces the number of facts that children need to memorize.

When dealing with the commutative property, or order principle (e.g., $3+4=4+3$ ), it is helpful for children to use visual models (e.g., hands, elevator beads) to physically reverse the order of the two groups.
Students need to understand that adding 1 changes the units digit of a number, whereas adding 10 changes only the tens digit. As well, students should recognize that adding 0 has no effect on the value of a number.
$\square$ To practise using some of these principles, invite students to participate in a game of mental math that involves adding 1 or 10 each time; for example, begin with 3 , add $10,10,1,10,1,1$, etc. Although some students may be able to do the computation entirely in their heads, others may require a hundred chart to follow the pattern of numbers. It is through this practice that students will begin to visualize the position of the numbers, and further develop their sense of number.
B8 When finding totals for 3 addends, regrouping into 5 s or 10 s is often useful for easier handling. For example:
could become



The associative principle, which allows a group of addends to be arranged in different orders, might also be used by students with strength in particular facts. For example, a student who knows that $4+4=8$ might regroup $4+5+4$ as $4+4+5$.

Encourage students to make decisions about which sequence to use to add a series of digits. For example, to add $3+8+1+7$, many students might find that first adding 3 and 7 together, and then adding on $8+1$, is easiest.

Students should investigate real-life situations in which they are required to add 3 numbers (each less than 10) to get a total. Include contexts such as - totalling the number of marbles among three students

- totalling the number of points scored in a game
- totalling the number of members in 3 students' families
- finding the number of people in each of 3 different families if the total is known to be 13 (or some other number) people. In this case, students should be encouraged to find as many different combinations as possible.


## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B7.1 Ask the student to use materials to show why the units digit does not change when 10 is added to a number.

B7.2 Ask the student to explain, using a model, why he/she knows that $3+4$ has to equal $4+3$ even before finding the total.

B8.1 Using a model, ask the student to find as many different ways as he/she can to make a total of 10 , using 3 numbers.
B8.2 Have students work in pairs. Present a series of questions involving the sum of single-digit numbers. One student determines a sum using a calculator, while the other calculates mentally. The object is to determine who gets the answer first. (Roles should be reversed at some point during the activity.) Sums should be included that encourage students to look for combinations that lend themselves to efficient calculation.
Examples might be: $8+6+2+4+7=$ $\qquad$ ;
$5+5+4+4=$ $\qquad$ ; $4+5+5+1+6+9+3=$ $\qquad$

## Pencil and Paper

B8.3 Present a situation similar to the one illustrated. A student throws 3 darts. Each lands on the board. What might the total score be?


B8.4 Ask students to find, in as many ways as possible, 3 numbers that total 6 .

## Interview

B7.3 Ask: Why is it easy to add 0 and 1 to numbers?
B8.5 Ask: Why is it easy to add the numbers $5+5+6+4+8+2$ ?

## Presentation

B8.6 Ask the student to explain to a classmate the way he/she would add $5+9+1+5$.

Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to v) apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers.

SCO: By the end of grade 2, students will be expected to
B9 model and perform the addition of two 2-digit numbers, with and without regrouping

## Elaboration - Instructional Strategies/Suggestions

B9 Using models is helpful for students to relate the physical action of joining two groups to the symbols. For example:


Students should always estimate first. Encouraging students to add the tens first will help with estimation. Adding in this way also strengthens place-value concepts (e.g., when adding 43 and 23 , the student says " 40 plus 20 (not $4+2$ ) is 60 ").

By writing a number sentence horizontally, you can encourage more divergent thinking. For example, a student might think of $31+52$ as

$$
\left.\begin{array}{lrl}
31,41,51,61,71,81,82,83 \text { OR } & \begin{array}{l}
31,32,33,43,53,63,73,83 \\
(31+5 \text { tens }+2)
\end{array} & (31+2+5 \text { tens })
\end{array}\right\}
$$

You might encourage students to use a hundreds chart to perform 2-digit additions. For example, for $32+45$, start at 32 and move 4 squares down and 5 right. For $34+18$, students might start at 34 , go down 2 squares and back 2 squares.

Provide examples of situations in which students will have to devise some method for regrouping. For example, tell them that one student found that there were $55 \mathrm{M} \& \mathrm{M}$ 's in one bag and 58 in another. Ask them to determine how many there were in the two bags. Ask them to model the question and to explain how they solved it.

Some students will combine the 2 fifties to make 10 tens, or a hundred flat, and then add the 5 and 8 together to get 13 , for a total of 113 . Others may add the units digits first and recognize that they can make a ten to add to their tens digits and have 3 extras.

By looking at addition conceptually, rather than procedurally, you will find that many students will have no more difficulty with problems involving regrouping than with those that do not. Once students choose to use a procedure, it is important for teachers to accept the different choices some will add from the left, others from the right.

Note: When providing problems or practice exercises, be certain that there is a mix of questions - some that require regrouping and some that do not.

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B9.1 Ask the student to use bundles of ten Popsicle sticks and single sticks to show why $42+26=68$.

B9.2 Ask the student to select 6 base-ten blocks (any combination of units and rods). Have him/her add 4 blocks. Ask: What answers can you get and what addition goes with each one?

## Paper and Pencil

B9.3 Ask the students to arrange the digits $2,3,4$, and 5 in the boxes below to have the greatest possible answer.


B9.4 Have the student fill in the boxes below in at least four ways so that the addition will be true.


## Interview

B9.5 Present an addition model with ten blocks as shown. Ask:
What addition question is being modelled?

B9.6 Tell the student that to add 36 and 29, Jake said, " 30 and 20 are 50 , and 10 more are 60 , plus 5 is 65 ." Ask him/her to describe Jake's thinking.

Tell the student that to add 36 and 29, Sara said, " $36,46,56,66,65$ - the answer is 65 ." Ask him/her to explain what Sara was doing.
B9.7 Show the student a variety of addition exercises and ask him/her to indicate/circle the ones that require regrouping.

## Portfolio

B9.8 The students might prepare a display showing as many ways as they can think of to perform a particular addition of their choice.


## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to v) apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers

SCO: By the end of grade 2, students will be expected to B10 model and perform the subtraction of two 2-digit numbers, with and without regrouping

## Elaboration - Instructional Strategies/Suggestions

B10 Continue to use models to relate the physical action of separating or comparing the two groups to the symbols. For example, 32-21 may be modelled as

- take away
- a comparison


Again, it is important for students to estimate first. As with addition, when using the front-end method to estimate, it is often as easy to get the actual answer as it is to estimate.

Also, in the case of addition, it is important to provide a mix of problems, some that require regrouping and some that do not.

When students are recording their work, you may see one or more of the following methods, or others. All of these are acceptable general procedures because they reflect understanding of the subtraction concept. It is important to honour all expressions of correct reasoning.
A. 45
$\frac{-27}{x}$
18
B. $\begin{array}{r}45 \\ -27 \\ \hline\end{array} \begin{array}{r}25 \\ \hline 18\end{array}$
C. $\quad 3_{1}^{1}$
$\frac{-27}{18}$
D. 45
E. 45
$-\underline{27}$ 18

Student A might think, "Well, 40-20 is 20. I'll take one of the leftover tens and add it to the 5 ones, so there's one ten left and $15-7=8$."

Student B thinks, " $45-20=25$. I still have to subtract 7. $25-7=25-5$ -2 , which is 18 ."

Student C reasons, "I can't take 7 from 5. So I'll take one of the four tens, leaving three, and subtract 7 from that 10 . That's 3 to add to the 5 already there, to make 8 ones. Then 2 tens from 3 tens is 1 ten."
Student D thinks, "What do I add to 7 to end in a 5? It's 8. That gives 15 ; write down the 1 ; regroup the ten ones to 1 ten. What do I add to 2 tens and 1 ten to get 4 tens? 1 ten."

Student E works it out mentally and reasons, " 27 plus 20 is 47 , subtract 2 is 45 . The difference is 18 ."

This particular problem might also be approached by subtracting 30 from 45 and then adding on the extra 3 , or taking 30 from 48 , which gives the same answer. Also, a student might think, " 3 more to get 30 and 15 more to 45 , for a total of 18 added on."

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B10.1 Organize a class yard sale for which students bring in items priced under a dollar. Have each student start with a dollar and record each purchase they make in a subtraction sentence. (There should be a banker who will change money for dimes and pennies.)
B10.2 Ask the student to use bundles of ten Popsicle sticks and single sticks to show why 46-22 $=24$.
B10.3 Tell the student he/she must begin with 6 base-ten blocks (any combination of units and rods). He/she must then remove 4 blocks. Ask: What answers can be achieved and what subtraction goes with each choice?

## Paper and Pencil

B10.4 Ask the students to arrange the digits 2, 3, 4, and 5 in the boxes below, to have the greatest possible difference.


B10.5 Have the student fill in the boxes below in at least four different ways so that the subtraction will be true.


## Interview

B10.6 Ask: How are the answers to the following questions related?
43-21
53-21
63-21
63-31

B10.7 Tell the student that Caroline was subtracting two numbers and said, " $48,38,35$." Ask: What might the numbers have been? Why?
B10.8 Have the student explain two different ways to solve 54-23.

## Portfolio

B10.9 The students might prepare a display showing as many ways as they can think of to perform a particular subtraction of their choice.

Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to vi) apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers

SCO: By the end of grade 2, students will be expected to B11 estimate the sum or difference of two 2-digit numbers

## Elaboration - Instructional Strategies/Suggestions

B11 It is important that estimation be considered by the student as a way to get a sense of the size of a quantity, rather than as just another set of rules. The goal for educators is to stress this importance and provide the needed practice so that students intuitively and automatically estimate, not just doing so when required to by the teacher or textbook instruction. Regular questioning, such as "Is your answer reasonable? How do you know?" will help students to focus on this skill.

Rounding each number to the nearest multiple of ten is only one way to estimate. For example, for $46+35$, a student using just a rounding rule might estimate the sum as $50+40=90$, whereas a student who increases one number and decreases the other $(50+30=80)$ has a better estimate.

When estimating $44+36$, students might choose to use the front-end method and think, " $40+30$ is 70 , the units make about another 10 , for a sum of 80 ."

Sometimes a student may select a combination of strategies. For example, for $48+27$, he/she might combine rounding and front-end strategies and think, " $50+27$ for a total of 77 ." Eventually, with sufficient practice, these skills with estimation will form the foundation for mental math computation.

Using base-ten blocks or a hundreds chart is helpful for students as they begin estimating. For example, if the student thinks of the estimate of a two-digit number as a set of rods only, he/she might see that 37 ( 3 rods and 7 ones) is closer to 4 rods than to 3 rods.
 Eventually, students should realize that estimating can easily be performed without the blocks.

Using a number line, whether vertical or horizontal, is also helpful for estimation. For example, if students imagine gas stations located at the multiples of 10 , they can put an imaginary car on any given number and determine the closest gas station. Forty-
 two is closer to the gas station at 40 than 50 , whereas 48 is closer to the ${ }^{42} 40$ gas station at 50 . Clearly, 45 can be estimated by either 40 or 50 .
Using ten frames for relatively small numbers may also prove worthwhile for some students. Displaying 23 on ten frames, for example, clearly shows that it is closer to 20 than to 30 .


Note: Estimation is usually a mental process. Consequently, students need to be able to mentally calculate sums involving numbers such as multiples of 10. (For example, a student should be able to determine $30+50$ by thinking, " 3 tens plus 5 tens makes 8 tens, or 80 .")

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B11.1 Ask students to use toy cars on a number line to estimate the sum of $48+37$.

B11.2 Provide counters and ten frames, number lines, and base-ten materials. Ask the student to choose one model, or a combination of models, to explain how he/she might show a grade one student that 27 is closer to 30 than to 20 .

## Paper and Pencil

B11.3 Tell the student that two numbers have been added and the sum is about, but not exactly, 40. Ask the student to list 4 possible pairs of numbers that might have been added.
B11.4 Ask the student to estimate each of the following. Have the student explain, without actually finding the answer, if the estimate is probably too high or too low.

$$
48+57 \quad 39-18 \quad 60-38
$$

## Interview

B11.5 Tell the students that a number between 30 and 40 (in the thirties) is added to a number between 40 and 50 (in the forties). Ask: What might be a good estimate for the answer? Why?

B11.6 Ask: Do you believe that 48 might ever be used as an estimate for a sum or difference? Explain your answer.

B11.7 Ask the student to explain why a good estimate for a subtraction might be greater than the actual answer sometimes, but less other times. Encourage the student to use examples to help her/him explain.

B11.8 Tell the student that José thinks she will need about 33 hot dogs for her party. Ask: If the hot dogs come in packages of ten, how many packages should José buy? Explain your answer.
B11.9 Tell the student that Jason knew there were 35 members in his Campers' Club and about 28 in the club in the neighbouring town. When asked to estimate the number of name tags to make for members of both clubs, Jason said, "I think I should make 65." Ask: How do you think Jason estimated? Was it a good estimate?

Suggested Resources

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to
vii) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 2, students will be expected to
B12 use technology to solve problems involving sums or differences of larger numbers

## Elaboration - Instructional Strategies/Suggestions

B12 Students solving authentic problems (e.g., using a catalogue to find the total price for a set of furniture for their dream bedroom) may encounter numbers beyond their capabilities to add or subtract. In these situations, rather than abandoning the problems, students should use calculators or computers to assist them in finding the required sums or differences. For some students, this may involve teaching the fundamentals of what the various buttons on their calculators mean and when to use them.

As necessary, provide opportunities for students to become familiar with calculators. Setting up a play store, and using play money, opens up a range of opportunities. The "shopper" can estimate when selecting items, while the "cashier" practises both using the calculator and using estimation to check the reasonableness of the calculator result.

Calculator practice may also be connected to the development of other outcomes (e.g., SCOs A2, B5 and B7). Examples include the following:

- Show 69 on your display. What one step must you take to change this to 49?
- Mary wanted to add 24 and 37 on her calculator, but the " 7 " button was not working. What might she do to calculate the answer accurately?
- $\quad$ Show 228. How can you make your display show 238?
- Show 345. What must you subtract to have your display show 300?
- Continue use of the calculator constant (e.g., $8+8===$ etc.).

Provide students with problem situations for which it makes sense to select a calculator to compute. Discuss the technology and stress the importance of estimating to check reasonableness. As well, provide situations for which it is appropriate to estimate only, and an exact answer is not required.

Note: Although students at this grade will routinely use calculators in computational situations involving numbers with more than two digits, they may easily work with multiples of one hundred mentally. If students are led, for example, to see 3 hundred +2 hundred as no different than joining 3 and 2 of any other unit, it will be easy to mentally calculate 300 +200 as 500. Similarly, students should be encouraged to mentally calculate sums such as $50+30$ or differences like $80-20$ rather than feeling that they must record the numbers and work in a procedural way. In fact, if students have been estimating, they will realize that they have been thinking like this all along.

## GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

B12.1 Ask students to show how they would use their calculators to find $487+209$.

B12.2 Provide calculators for the students and say: Do not use your "clear" button for this exercise. Show 443. How can you make it 543? How can 543 be changed to 523 ? What must you do to 523 to make it 530? Etc.
B12.3 Present the following problem: The school sold chocolate bars to raise money for field trips. A teacher reported that grade 1 sold 86 bars, grade 2-118, grade 3-74, and grade 4-98. How many chocolate bars were sold by the four grades? Ask the students for suggestions as to how to go about solving the problem.
B12.4 Simulate a store by providing a wide selection of items, each priced under a dollar. Ask each student "shopper" to use estimation to select as many items as possible without exceeding a given spending limit (e.g., 200 cents). Have other students take turns being the "cashier," using a calculator to total and calculate change for each selection of purchases.

## Interview

B12.5 Tell the student that you were using your calculator to find $48+37$. You accidentally pressed $48+27$. Ask: What should I do now?

B12.6 Tell the student that you need to know the answer to 70-40. Ask: Would you use a calculator, do it mentally or do it on paper? Why?
B12.7 Ask the student why it might be easy to do a question like $52+20$ mentally.
B12.8 Ask the student to tell you a problem that he/she would use a calculator to solve.

B12.9 Tell the student that Jacob used a calculator to add $18+36$. He got an answer of 64 and knew immediately that he had made a mistake. Ask: How do you think that Jacob knew this?

## Portfolio

B12.10 Have the students make a chart that would help another student decide whether to do a computation mentally, work it out on paper, or use a calculator.

Suggested Resources

## Patterns and Relations

## General Curriculum Outcome C:

Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to i) recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement)

SCO: By the end of grade 2, students will be expected to C1 compare and contrast patterns C2 demonstrate an understanding that there are often many ways to continue a pattern, unless a pattern rule is provided

Mathematics empowers students. Because mathematics is in part the study of patterns, recognizing patterns and predicting what comes next are crucial problemsolving skills that children must develop early if they are to succeed. Children at this grade vary greatly in their intuitive recognition of patterns. Some children immediately see patterns, whereas others recognize them only after their attention is focussed on a pattern by the teacher or another student. Because seeing relationships and making generalizations are abilities to be nurtured, pattern activities should continue throughout the year. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 1)

## Elaboration - Instructional Strategies/Suggestions

C1 Many patterns share certain characteristics. Students should take note of the ways in which a pair of patterns are similar and how they differ. Some of the characteristics a pair might share are

- repeating patterns (e.g., $1,2,1,2,1,2, \ldots$

$$
3,5,7,3,5,7,3,5,7, \ldots
$$



- "growing" patterns (e.g., 2, 4, 6, 8, 10, ...

$$
1,4,7,10,13, \ldots
$$



- some of the same numbers or shapes (e.g., $1,1,2,3,5,8,13, \ldots$

$$
2,3,5,8,13, \ldots)
$$

C2 Students should be encouraged to see that patterns which begin with a given set of numbers, shapes, colours, etc. may continue in many different ways. In fact, one can never be sure how the pattern continues unless a pattern rule is spelled out. For example, a pattern that begins 1, 2, 3, might continue
$1,2,3,1,2,3,1,2,3, \ldots$ (repeating $1,2,3$ )
$1,2,3,4,5,6,7, \ldots$ (listing the natural numbers)
$1,2,3,5,8,13, \ldots$ (each number after 2 being the sum of the two preceding ones)
$1,2,3,2,1,2,3,2,1, \ldots$ (going up and down between 3 and 1 ) etc.
Encourage students to find many ways to continue patterns beginning with two or three specific numbers or shapes.

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

C1.1 Ask students to create two different patterns using pattern blocks, both of which focus on the number of sides in the shapes.

C1.2 Request that the student create two different patterns, using counters. Ask him/her to find two ways in which the patterns are the same and two ways in which they differ.
C1.3 Provide green pattern blocks $(\boldsymbol{\Delta})$ to students working in pairs. Ask them to work together to produce a growing pattern of triangles.
C2.1 Provide the start of a pattern, using base-ten blocks. Ask the student to continue the pattern in more than one way, and to describe the pattern rule in each case.

## Interview

C1.4 Ask the students to create two patterns which are similar, but not exactly the same. Ask them to comment on the similarities and differences.
C2.2 Tell a student that the first two numbers in a pattern are 5, 10. Ask for several different ways in which the pattern might be continued.

C2.3 Tell the student that another child continued the pattern 1, 2, 3, 4, by saying 2,3 . Ask: Do you think that the other child is incorrect or is there a rule which might explain continuing the pattern in this way?

## Portfolio

C1.5 Students collect photographs or drawings of patterns they observe in the environment. They might comment on these patterns and what makes them patterns.

Suggested Resources

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to
ii) use patterns to solve problems

SCO: By the end of grade 2, students will be expected to
C3 identify and use patterns in an addition table
C4 identify and extend place-value patterns

## Elaboration - Instructional Strategies/Suggestions

C3 Students should be encouraged to identify and explain patterns that can be found in an addition table. These patterns can then be used to help students determine an unknown sum or difference.

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 6 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 7 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| 9 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |

For instance, students might note that

- only even numbers are located on the main diagonal (upper left to lower right), so the sum of a number with itself is always even
- the numbers increase by ones across a row, since one more is added for each step right
- all of the 8 s are on one diagonal line, since each time an addend is one greater, the other must be one less
- there are three 2 s , four 3 s , five 4 s , etc.
- the diagonals of any four numbers that form a square will have the same sum


C4 Students should be able to explain how to transform the model of a 3-digit number to that of another number in which only one digit is different; for example:

432


442


Students can then extend given place-value patterns and/or identify their own; for example: 512, 502, 492, ...

$$
214,314, \ldots, \ldots, \ldots
$$

$\square$ Have students convert several hundreds charts so they can see from 1 to 100, 101 to 200, up to 999 . On these charts, use coloured counters to cover numbers forming a pattern and encourage the students to explore the place value representation of the covered numbers; for example, the pattern $13,23,33,43, \ldots$, depicted as a vertical column of counters, represents increasing the number by 10 each time.

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

C3.1 Request that the student cover a pattern on the addition table and then describe how the pattern could help him/her remember a forgotten fact.

C4.1 Ask the student to use base-ten materials to help him/her construct a place-value pattern in which the second term is 302 .
C4.2 Have the student count by 20s to 200, recording each number said. Ask: How often do you say a number that has a 0 in the tens place?

## Paper and Pencil

C3.2 Cover all the 9 s within an addition table. Ask students to describe the pattern in the numbers that make up the 9 s .
C4.3 Show the first and second terms of a pattern, using base-ten blocks.


Ask: What do you think the 10th term probably is? Explain why.
C4.4 Have the student continue the patterns below for 3 or 4 terms:
132, 142, 152, ...
378, 388, 398, ...
213, 313, 413, ...

## Interview

C4.5 Present the following place-value blocks:


Ask the child what you should show next. Explain why.

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to iii)
represent mathematical patterns and
relationships in informal ways, including via open sentences (i.e., statements with missing addends)

SCO: By the end of grade 2, students will be expected to
C5 represent patterns, using their own notation or symbolism
C6 solve simple open sentences involving addition and subtraction facts

## Elaboration - Instructional Strategies/Suggestions

C5 To explain a pattern such as $\mathrm{oxoxxoxxx} \ldots$, students might use their own notational systems; for example:

| 1 | 1 |
| :--- | :--- |
| 1 | 2 |
| 1 | 3 |
| etc. |  |

In order to continue a numerical pattern, a student might want to make notations between numbers (e.g., 2, ${ }^{+1} 3,{ }^{+2} 5,^{+3} 8$, $\qquad$ , $\qquad$ , __ ).

It is helpful for students to verbalize what they see. For example, when using attribute blocks, a student might say,

"Big blue circle, small red circle, big blue triangle, small red triangle. Well, I guess that next I could show a big blue square and then a small red square . . ."
C6 Students should solve open sentences of the following 6 forms:

```
\(\mathrm{a}+\mathrm{b}=\square\) (e.g., \(6+3=\square\) )
\(a+\square=c\) (e.g., \(2+\square=8\) )
\(\square+b=c \quad(\) e.g., \(\square+4=5)\)
\(\mathrm{c}-\mathrm{a}=\square\) (e.g., \(7-2=\square\) )
\(\mathrm{c}-\square=\mathrm{b}\) (e.g., \(4-\square=2\) )
\(\square-\mathrm{a}=\mathrm{b}\) (e.g., \(\square-8=1\) )
```

Generally, students find the first type of sentence in each group easiest to solve. Many students find the last type of sentence in each group, one without an obvious starting place, the most difficult. Note: Students should have manipulatives available to them when solving these open sentences.
The solving of open sentences should regularly be connected to problem situations. Students should not come to see it as a process distinct from useful applications.

A re-emphasis on the link between addition sentences and subtraction sentences will help students solve open sentences; for example,
$\square-3=7$ and $7+3=\square$

## GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

## Worthwhile Tasks for Instruction and/or Assessment

## Paper and Pencil

C6.1 Ask the student to find the number that makes each sentence true.
$5+\square=13$
$16-\square=7$
$6+4=\square$
Ask: Can there be more than one answer for each? Why or why not?

## Interview

C5.1 Clap a pattern, such as $*_{* * * * * * * * * * * \text {. Ask: How might }}^{\text {* }}$ you describe the pattern to another student if he/she couldn't clap it?

C5.2 Provide the beginning of patterns; for example:

- small yellow $\Delta$, big red $\bigcirc$, small blue $\Delta$, big red $\bigcirc, \ldots$
- big thick yellow $\triangle$, small thin yellow $\boldsymbol{\Delta}, \ldots$
- (10) (16) (56) (10) (10) (10)

Have the student continue the pattern and tell how he/she knew what to do.
C6.2 Ask: How might you use counters to find the number to make this sentence true?
$\square+8=15$
Have the student write a story problem for this number sentence.

## Shape and Space

## General Curriculum Outcome D:

## Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to
i) measure and understand basic concepts and attributes of length, capacity, mass, area, and time

SCO: By the end of grade 2, students will be expected to D1 identify procedures (not involving units) to be used to compare areas

One of the purposes of early comparison activities with areas is to help students distinguish between size (or area) and shape, lengths, and other dimensions. (Elementary School Mathematics, p. 298)

## Elaboration - Instructional Strategies/Suggestions

D1 Children should recognize that area tells about the space an object takes up on a flat surface. Children's first comparison activities should be by perception, that is, by deciding that "this one looks larger or feels larger." Subsequently, children should investigate strategies to compare the areas of two or more objects. Situations should involve both direct and indirect measurement.

- direct measurement, i.e., comparing areas by covering one item with another
- indirect measurement, i.e., comparing areas by reshaping one item so that it fits on top of the other, or by comparing both items to a third referent

Students should be led to see that the definition of area is such that shape is irrelevant; the area does not change just because the object is cut up and rearranged on the surface. Students might think, for example, of how a space might be covered by a textbook, a scribbler, and a small notebook. Even though the objects may switch their positions, the same space is being covered.


Provide opportunities for students to order the areas of a number of shapes or flat objects, as well as opportunities for students to select the item most appropriate to cover a particular space.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

D1.1 Provide students with 5 identical squares. Have them create a single shape with them, trace around it and then cut it out. Once this task is complete, have the students create a different single shape using the squares, trace around it and cut it out. Ask how they know that different-looking shapes can have the same area.

D1.2 Have students estimate where to cut a piece of adding-machine tape so that it has the same area as an envelope (which is available and which is wider). Suggest to the students that they cut up and rearrange the tape on top of the envelope to test their estimate.

D1.3 Provide students with two sheets of paper, as below. Ask them to decide which has the greater area and provide proof.


## Interview

D1.4 Ask students whether or not an item which is long and thin can have the same amount of area as a short, fat item.

D1.5 Ask the student to name or draw an item which is long but does not have much area.

D1.6 Ask: Why is a tablecloth's area usually greater than the area of the table it is supposed to cover?

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to
ii) identify and use nonstandard and standard units of measurement and appreciate their role in communication

SCO: By the end of grade 2, students will be expected to D2 demonstrate a sense of how long 1 cm and 1 m are
D3 estimate and measure length in non-standard and standard units
D4 recognize and explain why standard units are used

A developmentally appropriate curriculum encourages the exploration of a wide variety of mathematical ideas in such a way that children retain their enjoyment of, and curiosity about, mathematics. It incorporates real-world contexts, children's experiences, and children's language in developing ideas. It recognizes that children need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. (Curriculum and Evaluation
Standards, p. 16)

## Elaboration - Instructional Strategies/Suggestions

D2 Students should develop a sense of the length of a metre and a centimetre.
$\square$ One way to do this is to have students relate lengths to their own bodies. For example: "My legs are about half a metre long, my nose is 4 cm long, and 8 of my footprints would make a metre."
$\square$ Frequently ask students to "show me one centimetre ( $6 \mathrm{~cm}, 20 \mathrm{~cm}$, etc.) with your fingers or hands" and "show me one metre with your arms." Focussing on estimation as an essential part of the measurement process helps students develop a better sense of the size of the units.

D3 Students should estimate and measure lengths in metres and centimetres, as well as using non-standard units.
$\square$ Include situations which are of interest to the students and that provide useful information, such as buying carpet for the reading corner, measuring book heights for a new bookcase, or determining if a large piece of furniture can fit through the door.

D4 Students should recognize that standard units are used in order to ensure that we understand one another when describing measurements.
$\square$ Students should be provided with experiences in which they measure an item's length with, for example, a pencil of their own choosing and tell others how many pencils long the item is. Inevitably, different numbers will arise, as different pencils have different lengths. Students should discuss how this might cause a problem if they had to call someone on the phone and tell them the length of the item. Following this introduce the centimetre cube and have everyone measure the item again with a line of centimetre cubes or rods. Students will see that this time everyone arrives at the same number; thus, communication of that number to others would be meaningful.
The book, How Big is a Foot?, would be appropriate to read to students to dramatize the situation described above.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

D2/3.1 Ask the students to cut a length of about 1 m from a ball of string. Have them verify their estimates.

D3.1 Ask the students to estimate the number of times a paper-clip chain that just fits around a wrist would fit along the length of an arm. Have them check their estimates.

## Interview

D4.1 Ask: Why does it make more sense to measure the length of a running track for a meet in metres than in foot paces?

D4.2 Ask: Why is it not a good idea to tell someone how long a table is by using pieces of paper as a measurement unit?

## Presentation

D3.2 Ask groups of students to each develop an appropriate plan to compare the height of the wastepaper basket to the distance around its top, and to explain each plan to the other groups.

## Portfolio

D2/3.2 Have students develop a book on measurement that they can add to over time.

D2/3.3 Give each student a metre-long piece of twine and ask them to use it to measure objects in their homes. Have them make lists of items that are almost a metre, one metre, or a little more than a metre.

Have the students enter their findings in a table such as the one shown below.

| almost a metre | one metre | more than a metre |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

Ask: How could one use the twine to identify objects that are about half a metre?

Suggested Resources



## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to
ii) identify and use nonstandard and standard units of measurement and appreciate their role in communication

SCO: By the end of grade 2, students will be expected to
D5 demonstrate a sense of how much 1 L is
D6 estimate and measure capacity in non-standard and standard units
D7 demonstrate a sense of how much 1 kg is
D8 estimate and measure mass, using non-standard and standard units

## Elaboration - Instructional Strategies/Suggestions

D5 Students should develop a sense of how much a litre container will hold.
$\square$ Include situations in which students will come to realize that containers can have different shapes but still hold a litre. For example, fill an empty 1 litre milk container with sand. Have students estimate which other container would hold the same amount, and check by pouring the contents into it.

To provide a common personal referent for a litre, use the large base-ten cube ( 1000 block) which holds a litre of liquid.
D6 Students should estimate and measure capacity in litres, as well as using non-standard units.
$\square$ Provide students with several containers of different sizes and shapes. Ask them to order the containers by capacity, starting with the one that will hold the least. Have students verify their estimations by using a non-standard unit of measurement.
$\square$ How many litres of trash will the wastebasket hold?
$\square$ Provide 5 or 6 large containers (fish tank, pail, jug, baby bath, roasting pan). Tell the students that one of the containers holds (_) litres. Have them estimate and vote for the container they think has the correct capacity. Encourage students to talk about their reasoning and confirm their estimates.

D7 Students should develop a sense of what a kilogram "feels" like.
$\square$ Include situations in which students will

- compare objects to an established 1 kg mass (For example, have students determine which objects are more than, less than or about the same as 1 kg of sugar.)
- create masses of 1 kg (For example, ask students to fill containers with various materials until they think a mass of 1 kg is reached.)
- find common 1 kg products

D8 Students should continue to use non-standard units to measure mass. These might include marbles, ball bearings, multilink cubes, base-ten materials, erasers, etc. Continue to measure mass on a balance scale, as opposed to something like a bathroom scale, for which the source of the number appearing is less clear to the student. Students should subsequently estimate and measure mass, using the kilogram as the unit.
$\square$ Investigate the number of kilograms students could comfortably carry in their backpack or the total number of kilograms of a group of books on a shelf, etc.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

D5.1 Ask the student to mark a one-litre estimate on a plastic container. Have him/her compare it with the standard litre measurement.

D6.1 Ask the student to determine the capacity of several containers and graph and discuss the results.

D7.1 From a collection of objects, ask the student to predict which one has a mass of about 1 kilogram.

D7.2 Ask the student to choose a small item. Then have the student estimate and determine how many of the items would be required to make a mass of a kilogram.

D8.1 Ask the student to find something which has the same mass as two bags of marbles.

D8.2 Ask the student to find the number of potatoes in 2 kg . Ask: Will the number always be the same? Why or why not?

## Interview

D5.2 Ask: Would 1 litre of juice be enough for the whole class? Why or why not?

D6.2 Ask the student to describe the size of container that would be needed to hold 25 litres of water.

D8.3 Ask: What unit would you use to measure the mass of a watermelon?

## Portfolio

D5/6/7/8.1 Have students develop a book on measurement that they can add to over time.

Suggested Resources

# GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement. 

KSCO: By the end of grade 3, students will be expected to
ii) identify and use nonstandard and standard units of measurement and appreciate their role in communication

SCO: By the end of grade 2, students will be expected to
D9 estimate and measure time using non-standard units
D10 read hours and half hours on a clock
D11 explore properties of the calendar

## Elaboration - Instructional Strategies/Suggestions

D9 Students should use non-standard units to measure time.
$\square$ Students might determine how many times they can clap their hands

- while a particular song is being played
- while a race is run
- in one minute

D10 Students should be able to read time in hours and half hours on

- an analog clock
- a digital clock

They should also be aware that the minute hand and hour hand on an analog clock are different lengths, and that the minute hand is at 6 for the $\square: 30$ and at 12 for the $\square: 00$. Some students will be aware that the hour hand moves during the course of the hour, and that, at the $\square: 30$, it is halfway between two numbers.
While using an analog clock, students can be introduced to the term "half past" because half of the clock has been swept by the minute hand since the corresponding "o'clock time." Some children may be confused since there is not a half-clock spread between the two hands.
Note: An analog clock can serve as a useful model to reinforce the number concept of one half. Students can visualize one half either in terms of the hour hand moving half way from one number to the next or the minute hand moving half way around the circular face of the clock.
D11 The calendar provides rich opportunities to explore not only knowledge of time, but also number concepts. (See, for example, SCOs A1 and A2.)
By the end of grade 2, students should know the days of the week, the months of the year, and the four seasons. Students may have somewhat more difficulty with the months for which they have less experience in their own lives. Ask questions regularly, such as It is March. Which month comes next? As well, students will be developing a sense of the arrangement of our year in relation to the months and seasons; for example, January is the first month of a new year and is early in our official winter season.
Using calendars throughout the school year strengthens the students' sense of time. Each month brings a new calendar to explore.
$\square$ It is worthwhile for students to learn the jingle:
Thirty days hath September, April, June and November.
All the rest have 31, etc.
$\square$ Students might enjoy the "Knuckle Method" for remembering the number of days in each month:
Make a fist showing four knuckles; start by pointing to the first knuckle and saying, "January." The space between knuckles is February, the second knuckle is March, and so on. After saying, "July," go back to the beginning, making August land on the first knuckle and continuing until year end. The months that land on the knuckles each have 31 days.
$\square$ Consider calendar patterns: -- Are there months that need only 4 lines? -- Does every month have the same number of Mondays? -- Add six by going down one week and back one day.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

D9.1 Ask the student to estimate how many times one can count to ten, while walking heel-to-toe across the classroom. Have the student verify his/her estimate. Ask why another student might get a different result.

D10.1 Ask the student to show, on an analog clock, the time (to the nearest half hour) at which they arrive at school, have lunch, go to bed, etc.

D11.1 Ask pairs of students to predict how many weeks there are in a year. Have them use a calendar for the year to check their prediction.

## Interview

D11.2 Show the student a calendar for the year. Ask him/her to point out the day's date and to find out what date it will be in six weeks.
D11.3 Show the student a calendar for the year and ask him/her to identify ways in which months are the same and ways in which they differ.
D11.4 Referring to a calendar for the year, ask the student when he/she thinks spring starts.

D11.5 Ask: How might you use a calendar to help subtract 14 from a number?
D11.6 Tell the student that it is the 11th of the month. Ask: How might you use a calendar to add 16 , and what is the date?

## Presentation

D11.7 Have students work in pairs. Tell them that Stacy was born February 29, 1992. Ask the students to determine how many birthdays Stacy has had. When would she celebrate her birthday? Ask them to write a letter outlining how unfair it is to be born on February 29th, and what they might suggest be done to change the situation.

## Portfolio

D11.8 Provide a calendar for the year, and have the students figure out how many school days each month will have. How many Friday, the 13ths, are there in the year? On what days do the birthdays of friends and family fall?, etc. Ask the students to write about their findings for their portfolios.

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to
iii) estimate and determine measurements in everyday problem situations and develop a sense of the relative size of units

SCO: By the end of grade 2, students will be expected to D12 choose appropriate units with which to estimate and measure, and perform the measurements
D13 demonstrate an understanding that the size of unit used affects the number describing the measurement
D14 demonstrate an understanding that 100 cm make up 1 m

## A portfolio is a showcase for

 student work, a place where many types of assignments, projects, reports, and writings can be collected. Progress in, attitudes toward, and understanding of mathematics can be seen in a comprehensive way. The collection exemplifies the goals of the NCTM Evaluation Standards and shows much more than will a single test. (Mathematics Assessment, p. 35)
## Elaboration - Instructional Strategies/Suggestions

D12/D13 It is important to incorporate estimation in all measurement activities and not treat it as a separate topic. Estimating and measuring in non-standard units help students focus on principles of measurement such

- the bigger the unit used, the fewer the number of units it takes (For example, your arm might be 4 pencils -- a longer unit -- or 24 paper clips -- a shorter unit -- in length.)
- some units are more appropriate than others for measuring a certain object (For example, it makes more sense to measure a large container with a large unit.)
$\square$ It is helpful for students to use body units (arm lengths, foot lengths, strides) to estimate and measure length. This provides them with a permanent referent when estimating and measuring. A centimetre referent is the width of their finger. Invite students to estimate, explore, and record the length of various classroom objects, using their own body units. (For example, the door is almost 10 hands wide.)

Students should estimate and measure in many everyday situations. For example, they might

- find the number of kilograms they could comfortably carry in a back pack
- find the number of kilograms of food they eat in a normal week
- compare the masses of various types of balls
- find the size of bag needed to hold 1 kg of flour
- estimate the capacity of the sink in litres
- estimate the height of a young tree

Students should recognize which linear unit (metres or centimetres) is appropriate in a given situation. Include situations in which something is measured in centimetres in one case and metres in another; for example, centimetres would be appropriate when measuring the length of your foot and metres when determining the distance from your classroom to the school library.

D14 Students should recognize that a metre is 100 cm long. Although many metre sticks are marked up to 100 , it is often still not clear to students that lining up 100 centimetres really produces a metre. It is worthwhile to actually interlock 100 centimetre cubes (from a base-ten kit) and measure them against the metre stick.

As new standard units are introduced, it is important to take time to teach the format of recording measurements with that unit (e.g., 5 centimetres as 5 cm ).

## GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

D12.1 Have students locate a number of items in the room that are about as long as a base-ten rod.

D12/13.1 Have students determine how many teaspoons make up a tablespoon.
D12/13.2 Have students find out how many grapes usually balance the mass of one plum.

## Interview

D12.2 Ask the student to explain why it is more appropriate to describe the length of a pencil in centimetres than in metres.
D13.1 Explain that two students have each correctly measured an item, but one says it is 18 pencils long, while the other says it is 7 pencils long. Ask: How can this be?

D12/13.3 Tell the student that Sue measured an item and found it to be 3 pencils long. Ask: How many erasers long might it be?

D12.3 Have the student name an appropriate unit for measuring the height of a brown bear.

D12.4 Ask: Why might a long piece of rope not be a good unit for measuring the thickness of a book?

## Shape and Space

## General Curriculum Outcome E:

## Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

# GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships. 

KSCO: By the end of grade 3, students will be expected to
i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 2, students will be expected to
E1 develop aspects of spatial sense, including perceptual constancy, perception of spatial relationships, and visual discrimination

## Elaboration - Instructional Strategies/Suggestions

E1 Perceptual constancy is the ability to recognize a shape when it is seen from a different viewpoint or from a different distance (enlargement/reduction). For example, many young children have formed visual images of squares, but are not able to recognize a square that is shown in a different orientation. This skill is closely connected to the ability to visually discriminate. For students who have not developed perceptual constancy, even a triangle such as the one shown below may be perceived as a different shape altogether.


The development of perceptual constancy helps the child adjust to the environment and gives stability to his/her surroundings. Perceptual constancy is dependent, to a large degree, on experiences in geometric exploration.

Perception of spatial relationships is the ability to see the relationship between two or more objects. Those who have this ability are able to recognize congruency in slides, flips, and turns. This skill is similar to position-in-space perception (as elaborated in the kindergarten guide).
$\square$ Show the student a number of arrangements of blocks, such as


Ask the student to make these same arrangements and place them beside the ones modelled.

Visual discrimination is the ability to identify the similarities and differences between or among objects. Visual discrimination should not be dependent upon position.
$\square$ Put different triangles on each of two geoboards. Ask the student to change one of the triangles to make it the same as the other.

Activities which show two pictures and ask students to find all (or a specific number) of the differences help to develop visual discrimination skills.

Note: Students should be encouraged to work with puzzles regularly. They provide practice for all the spatial skills. It is important to provide puzzles that span a wide range of complexity; some children enter grade two with a great deal of experience, while others may have had limited experience.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E1.1 Make a shape on a geoboard (or a composite of more than one shape) and place another geoboard beside it. Ask the student to make the same shape on the second geoboard and observe how easily the task is performed.

E1.2 Set up geoboards as for E1.1. Ask the student to make the reflection of this shape on the second geoboard.
E1.3 Give the students an outline of a 3-piece tangram puzzle and the seven tangram pieces. Ask them to fill in the outline with selected pieces.
Observe how they solve the problem. For example, do they select pieces that are reasonable given the shape of the outline? Are they persistent and do they have a plan for shape selection?

## Interview

E1.4 Make a display of triangles on geoboards or on $5 \times 5$ geopaper. Ensure that some of the triangles are the same but in different orientations. Point out to the student one triangle; ask him/her to find other triangles that are the same as that triangle and to explain how he/she knows.
E1.5 Some students can be challenged to an activity similar to E1.4, but with all of the shapes in different orientations.
E1.6 Give the student a picture of a number of triangles, all of which are the same, except for one that differs slightly. Ask him/her to identify the one that is different.

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 2, students will be expected to
E2 recognize 3-D shapes from drawings and from alternative perspectives

## Elaboration - Instructional Strategies/Suggestions

E2 Students should be able to recognize common 3-D shapes when they are viewed from various perspectives. As well, they should recognize drawings of these shapes.

Students should learn that the appearance of solids changes with change of viewpoint. Questions to pose might include

- What does a triangular prism look like to a bird flying over it?
- If a worm saw a square pyramid, what would it see?
- What does a square prism look like from the side?
- How does the view of a cone from the top compare to the view from the bottom?
$\square$ Build a structure out of multilink cubes. (See example at right.)


Have students describe what it looks like from the front or from the side. Discuss whether or not it would look different from the right side than from the left side.
$\square$ Use a variety of picture books that include pictures of cubes, prisms, pyramids, and spheres to allow students to see what artists or illustrators do to make 3-D pictures look real when drawn on a sheet of paper.
$\square$ Provide isometric drawings, such as those below, and ask the students to build the structures with cubes. It must be recognized that this is developmental; some students will find this activity difficult and will therefore require much practice.


## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E2.1 Present drawings of a variety of 3-D shapes. Ask the student to match these with the actual shapes which are displayed.

E2.2 Ask the students to build a structure out of multilink cubes that might look like this from the back.


E2.3 Ask the students to build a structure that looks the same from above as from below.

## Interview

E2.4 Build a structure like the one shown. Ask: What does the structure look like from above?


E2.5 Ask the student why it is more informative to draw a cube as shown on the left than as shown on the right.


E2.6 Have the student describe two different shapes that look the same from the top.

E2.7 Ask students why a prism, standing on its non-rectangular base, looks the same from above and below, but a pyramid does not.

## Presentation

E2.8 Show the child two different drawings of a cube. Ask: How many faces can be "seen" in each drawing? Which drawing do you think is the "better" drawing? Explain your choice.


## Suggested Resources

# GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships. 

KSCO: By the end of grade 3, students will be expected to
i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 2, students will be expected to E3 sort, build, and pattern with 2-D and 3-D shapes

Young children should explore geometric concepts informally and intuitively. The use of manipulatives is crucial.
Sorting and classifying geometric figures in many ways provides children with informal analyses of the properties of these figures before the more formal work in later grades. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 25)

## Elaboration - Instructional Strategies/Suggestions

E3 During their prior schooling, students will have had many opportunities to explore shapes through sorting, patterning, and building activities. At this level, activities that will further develop these skills should be provided. Children will begin at different levels of development so it will be necessary to design a range in the complexity of the activities.
$\square$ For sorting purposes, make a collection of 2-D shapes cut from posterboard. These should include not only shapes with which students are familiar, but also some less common shapes that would fit in potential sorting categories (e.g., a semi-circle or a cashew shape could be classified under "things with curved edges"). Because it is important for students to share their thinking with others, it is recommended that they work in small groups, talking about their ideas and strategies.

Grade 2 students benefit from using hardwood blocks for construction.
They will build increasingly more complex structures over the year.
Encourage them to be creative, experimenting with different constructions. For example, challenge them to explore constructions of increasingly greater height, and to talk about these constructions with their classmates.

Children at this level will have had numerous experiences with patterning. It is important that they continue to use concrete materials in patterning activities, and that these activities become increasingly complex.
$\square$ Have students work in pairs to examine growing patterns and to discuss the number pattern associated with them.

(Note: It is not expected that square or triangular numbers be addressed at this time; the focus here is on patterning.)

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E3.1 Give the student some toothpicks and ask him/her to build a square. Ask: How many more toothpicks do you need to make two squares? three squares? Have the student make a chart to record the number pattern and see if he/she is able to describe it.

E3.2 Provide the student with circular counters. Begin a pattern such as


Ask the student how many counters must be added to continue the pattern and have him/her record it in some way. Is he/she able to predict how many are needed to make the next size design?

## Paper and Pencil

E3.3 Give the students 10 green triangular pattern blocks. Ask them to make as many different-size triangles as they can and to record their answers by making a drawing of each.

## Interview

E3.4 Give the student a picture of two constructions, one that would stand alone and one that would not. Ask the student to talk about them, and invite him/her to use hardwood blocks to convince you as to whether or not they can or cannot be constructed.


## Presentation

E3.5 Ask students who have created a special construction to present it to the class, outlining any problems they encountered and how they were able to overcome them.

Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 2, students will be expected to E4 recognize, name, and represent parallel lines and right angles

## Elaboration - Instructional Strategies/Suggestions

E4 When two lines are drawn in the plane, they are either parallel or they intersect. If the lines intersect, they form angles. Sometimes the angle looks like the corner of a square; students should learn that we call this a right angle.


Some children perceive parallelism as meaning that the lines go in the same direction or at the same slant; others realize that it means that the lines never meet, like railroad tracks. Still others notice that the lines stay the same distance apart instead of getting closer together (as they do when an angle is formed). Any of these notions for "parallel" is appropriate. The study of parallelism should be informal at this level and should be restricted to 2-D situations.

Students have already examined angles as corners in grade 1. The new idea here is simply to name "square corners" as "right angles." Many students initially believe that the right angle must point to the right, so it is important to display right angles which are oriented in many different ways.

Students should identify lines (or line segments) that are parallel or form right angles in familiar shapes or in the real world. This might include finding the parallel sides of squares, rectangles, hexagons, trapezoids, and parallelograms, as well as pairs of their sides that meet at right angles.
$\square$ Students, using looseleaf, might be asked to observe the parallel lines and right angles on their paper.
$\square$ Have students arrange toothpicks or straws to be parallel, make an angle, make a right angle, etc.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E4.1 Ask students to make a drawing of a table top with no sides parallel (or no sides forming right angles).

E4.2 Draw a right triangle for the student. Ask: What kinds of angles do you see?

## Paper and Pencil

E4.3 Ask the student to draw a picture of parallel lines and a right angle.
E4.4 Ask the student to draw a picture of a shape with parallel sides, but no right angles.

## Interview

E4.5 Ask the student to study the corners of objects in the classroom to see how common right angles actually are. Have him/her report the findings.
E4.6 Ask students to find a pattern block that shows

- parallel sides and no right angles
- parallel sides and right angles

E4.7 Ask the student to show or draw what a shape with a lot of right angles might look like.
E4.8 Ask students to point out (or draw) three items in the room that have parallel sides.

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
ii) describe, model, draw, and classify 2- and 3-D figures and shapes

SCO: By the end of grade 2, students will be expected to E5 recognize, name, describe, and represent parallelograms

## Elaboration - Instructional Strategies/Suggestions

E5 Many students will be familiar with parallelograms (particularly the rhombus) from their experience with pattern blocks, but they may not necessarily know the name.

Students should be aware that the name "parallelogram" is applied to any 4 -sided polygon that has two pairs of parallel sides. Such shapes can be long and thin or short and fat and still be parallelograms.


Rectangles and squares are special parallelograms, but it is not necessary for grade 2 students to understand this. If a student asks, however, whether or not a rectangle is a parallelogram, it is important to be accurate.

Some students might observe that the parallel sides are the same length, but it is not essential that this point be brought out.
It is sometimes difficult for students to draw parallelograms so that both pairs of sides are parallel; they could be encouraged to use lined looseleaf to help ensure that at least one pair of lines is really parallel, making it easier to represent the other pair. Another interesting way to draw parallel lines is to use a ruler. Students can use the two sides of their rulers to form one pair of parallel sides, then turn the ruler and repeat the process to form the other two sides.
$\square$ Students might be encouraged to bring in empty paper towel rolls and asked to carefully unfold them along the seam. Most students are surprised to see that the shape formed is a long, thin parallelogram.
$\square$ It is interesting for students to form rectangles, using geostrips and paper fasteners. When the rectangles are picked up, students will note that they readily "flop" into more "conventional" parallelograms.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E5.1 Provide a drawing which incorporates many shapes. Ask the student to point out the parallelograms among the shapes.

E5.2 Use masking tape to make a parallelogram on the floor. Ask the students to walk along the shape, count the number of steps, and describe how they move to walk all the way around. Ask: How does the walk differ from one around a rectangle?

E5.3 Ask students to draw a picture of a shape that is almost, but not quite, a parallelogram. Ask why they feel it is not quite a parallelogram.

## Pencil and Paper

E5.4 Provide looseleaf. Ask each student to draw a picture of a long, thin parallelogram and a short, fat one.
Interview
E5.5 Show students two parallelograms such as the ones shown below.


Ask: How are the shapes alike and how do they differ?
E5.6 Ask a student to tell how he/she would describe a parallelogram to someone who does not know what it is.

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
ii) describe, model, draw, and classify 2- and 3-D figures and shapes

SCO: By the end of grade 2, students will be expected to E6 recognize, name,
describe, and represent triangular, square, and rectangular prisms and pyramids
E7 cut and assemble nets of cubes and triangular, square, and rectangular prisms and pyramids

## Elaboration - Instructional Strategies/Suggestions

E6 Students worked with prisms and pyramids in grade 1, but may not have fully internalized the notion that the way prisms or pyramids differ from each other is based on the particular base the prism or pyramid has. At this point, students should learn to distinguish between triangular, square, and rectangular prisms and pyramids. For example:


You might ask:

- How many different shapes of faces does the triangular pyramid have? the square pyramid?
- How many corners does the square prism have? the triangular prism?

Using a set of prisms and pyramids, have students place the correct pyramid "hat" on top of each prism.
E7 One way to become familiar with prisms and pyramids is to observe, touch, and manipulate 3-D models of these shapes. Another way is to build them from 2-D plans, called nets. Students should be provided with nets for triangular, square, and rectangular prisms and pyramids. They should then cut them out and fold and tape them into the 3-D models. Sample nets:

(It is important that students realize the difference between the lines along which they cut and the lines which indicate folds.)
Sometimes nets which are found in prepared resources include tabs to simplify gluing the shapes together. Some students may become confused by the tabs and their role in the shape. Thus, it is generally preferable to have students assemble nets from polydrons (plastic materials that interlock and hinge to form 3-D shapes) or to use cardboard nets without tabs.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E6.1 Provide a variety of prism and pyramid models. Have the students use proper terminology to name the models.

E6.2 Provide drawings of a variety of 3-D shapes. Ask the students to match these with the actual solid models.

## Observation

E7.1 Observe students as they cut out and assemble the net of a square pyramid to see if they can easily visualize which piece must be the base and which pieces come together to form the point at the top.

## Interview

E6.3 Ask students to describe how a square pyramid and square prism are alike and how they differ.
E6.4 Provide a Toblerone box. Ask: How would you name the shape?
E6.5 Ask: Why might you suggest that there are two bases on a triangular prism? Could someone call one of the rectangles the base? Explain.
E6/7.1 Ask: Why might it be more difficult to identify which part of the net of a triangular pyramid should be called the base than it would be for a square pyramid?

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
ii) describe, model, draw, and classify 2- and 3-D figures and shapes

SCO: By the end of grade 2, students will be expected to E8 recognize surfaces and faces of 3-D shapes

## Elaboration - Instructional Strategies/Suggestions

E8 Many of the 3-D shapes students see or explore have flat faces, but others, such as cylinders, spheres, and cones, include curved surfaces. Students should begin to explicitly relate the description of a 3-D shape to a description of the surfaces and faces that make it up. For example, a cylinder has 2 flat faces and a curved surface, a sphere has one curved surface, and a prism has all flat faces.
Students might observe that most faces of prisms are rectangular, while pyramids have predominantly triangular faces. (The bases may or may not be rectangular or triangular, respectively.) When dealing with shapes like cylinders and cones, which have flat as well as curved surfaces, one tends to focus on the curved surface, since it provides a better sense of the whole shape.
Geoblocks provide a good source of 3-D shapes with different surfaces. The faces of solids made with a variety of polydrons would be interesting to examine, as would prisms made by stacking identical pattern blocks. Sections can be cut out of cardboard circles to produce nets, which may be folded into curved surfaces to explore a variety of cones.


## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E8.1 Provide a variety of 3-D models. Ask students to find all the square faces or all the triangular ones.

E8.2 Have students count the number of faces for various prisms. Ask:
Can different types of prisms have the same number of faces?
E8.3 Provide a variety of 3-D models, including a triangular pyramid (with all equal faces), a cube, and other solids. Ask the student to pick out the shapes in which all the faces are exactly the same.
E8.4 Ask the student to select shapes with exactly 5 faces from among a variety of models (including some triangular prisms and square pyramids).

## Interview

E8.5 Have students describe how the curved surface of a cylinder might feel compared to the curved surface of a cone.
E8.6 Display a variety of different cylinders. Prepare a paper rectangle which could be rolled to cover the curved surface of one of the cylinders. After rolling the paper, ask the student to find the cylinder for which the paper cylinder models the curved surface.
E8.7 Have the students compare the curved surfaces of a sphere, a cone, and a cylinder and describe how they differ.
E8.8 Ask students to decide whether knowing the number of curved surfaces of an object helps tell you how well it might roll.

Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
iii) investigate and predict the results of combining, subdividing, and transforming shapes

SCO: By the end of grade 2, students will be expected to E9 sort, build, and pattern with 2-D and 3-D shapes

Constructing two-dimensional and three-dimensional figures with a variety of materials (blocks, geoboards, straws and pipe cleaners, tagboard) helps children identify specific characteristics of each figure, including the symmetries of figures and some relationships between two-dimensional and three-dimensional figures. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 25)

## Elaboration - Instructional Strategies/Suggestions

E9 Students have already had considerable experience sorting, building with, and creating patterns with 2-D and 3-D shapes. The work at grade 2 should be somewhat more sophisticated than in previous years.

For example, sorting and patterning criteria might include

- circle or not
- parallelogram or not
- parallel sides or not
- right angles or not
- curved surfaces or not.
$\square$ Ask students to continue the following patterns:

$\square$ Provide a variety of shapes, including parallelograms, trapezoids, and triangles. Ask students to describe various ways to sort the shapes.
$\square$ Ask students to combine a variety of pattern blocks to build a single prism.

Note: See also SCO E3.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E9.1 Make a pattern in which every second shape has a pair of parallel sides, and the other shapes do not. Ask students to describe the pattern.

E9.2 Provide an assortment of 2-D shapes. Ask students to find several ways to sort the shapes.

E9.3 Ask students to use different-sized square or rectangular prisms to build a pyramid-like building with steps.

## Interview

E9.4 Show a set of shapes, all but one of which contain right angles. Ask the student to identify the odd shape and tell how he/she knows it is odd.

E9.5 Provide a set of shapes, including only squares and equilateral triangles. Ask students to sort the shapes. Have them describe the sorting rule they used. Ask the students if the rule could have been "has right angles" or "has parallel sides."
E9.6 Ask students why it might be easier to build a prism than to build a sphere.

## Portfolio

E9.7 Have the student create a drawing that includes a parallelogram, a rectangle, and a triangle.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
iii) investigate and predict the results of combining, subdividing, and transforming shapes

SCO: By the end of grade 2, students will be expected to E10 subdivide and change

2-D figures

## Elaboration - Instructional Strategies/Suggestions

E10 As an extension of grade 1 activities (in which students experiment with cutting familiar shapes into two or three parts), students at this level could work with more complicated constructions. It is through these explorations that they are able to recognize the attributes of various shapes and further develop their spatial sense. Many mathematical concepts can be explored as students subdivide and change 2-D figures. For example, students can explore simple fractional ideas by dividing a shape in different ways.


Students should recognize that any polygon can be subdivided into triangles.


Students can also see the similarities and differences between figures by subdividing and reforming them. For example, students can see how parallelograms are like rectangles by cutting one end off and placing it at the other,

or that any triangle can always be divided into two triangles with right angles.


Provide each student with two identical parallelograms. Have them cut one along the shorter diagonal, the other along the longer one. Compare the parts produced. Ask students to make all the possible shapes, using the four pieces.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E10.1 Have students show different ways to cut a geoboard parallelogram in half. For example:


E10.2 Ask the student to rearrange the four quarters of a square to make other shapes.


E10.3 Ask the student to find four different ways to cut a rectangle into equal pieces.

E10.4 Ask students how to cut a trapezoid into 2 pieces to show a parallelogram and a triangle.

## Interview

E10.5 Have a student draw a triangle and try to divide it into smaller triangles. Ask if it would always be possible. Then ask the same question about dividing a square into smaller squares.


## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
iii) investigate and predict the results of combining, subdividing, and transforming shapes

SCO: By the end of grade 2, students will be expected to E11 recognize, identify, describe, and represent reflective symmetry in 2-D shapes

## Elaboration - Instructional Strategies/Suggestions

E11 Figures with reflective symmetry are those in which one half of the shape is the mirror image of the other half.


To test for symmetry, students can use a mira. By placing the mira on the line that splits the two symmetric halves (the flip line), students can see if the flip image of one half falls exactly on top of the other half of the shape. Another way to test for symmetry is by folding and seeing if the two halves match perfectly.

Many students mistake symmetry for "halving." Figures with reflective symmetry can easily be split in half, but not every object with two similar halves has reflective symmetry.

has reflective symmetry

does not have reflective symmetry

Students should realize that the line of symmetry can be vertical, horizontal, or slanted.

Mira Math Activities for Elementary School provides a number of very interesting activities for students to explore symmetry. Another worthwhile book is Through the Magic Mirror by Anthony Browne.
$\square$ Have each student fold a piece of paper in half, cut out a shape and unfold to examine the result. This can become an art activity by folding more than once and trying creative cutting.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E11.1 Provide a variety of shapes and ask the students to sort them, grouping those with mirror symmetry and those without mirror symmetry.

E11.2 Have the student draw a picture to show how a shape can have symmetry, even if the right side does not match the left side.
E11.3 Provide a butterfly design. Ask students to use a mira to test the symmetry.
E11.4 Provide a design in which it would appear that one half of the design is a mirror image of the other half, but there is one small difference. Ask students to find the difference. For example:


## Interview

E11.5 Ask: Why might someone say that a circle has a lot of symmetry?

## Paper and Pencil

E11.6 Ask each student to draw a picture of a shape with symmetry.
E11.7 Ask students to draw examples of triangles with symmetry and triangles without symmetry.

E11.8 Have the student draw a picture to show what a creature that is not symmetric might look like.
E11.9 Have students create a crayon or marker design that exhibits symmetry.

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
iv) relate geometric ideas to number and measurement ideas and recognize and apply geometric principles in real-world situations

SCO: By the end of grade 2, students will be expected to E12 recognize and identify reflective symmetry in the environment

Exploring their surroundings for examples of [2-D and 3-D] figures shows the students a wide variety of geometric figures other than the ones pictured in their textbooks. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 25)

## Elaboration - Instructional Strategies/Suggestions

E12 Many of the objects (both living and inanimate) in a child's world are symmetric. Students could explore symmetry in manmade objects such as houses, furniture, food containers, tools, dinner plates, etc.
$\square$ Books with pictures of interesting buildings (such as the CN Tower) or interesting structures (such as the Confederation Bridge) would be a good source of objects with symmetry for students to explore.
$\square$ Students could also study animal books to see symmetry. They might debate, for example, whether giraffes (whose spots are not symmetric) should be called symmetric or not.
$\square$ Many fabric designs (such as those found on clothes, tablecloths, linens, etc.) exhibit symmetry. The teacher could bring in samples of fabric and discontinued wallpaper books.

Students will differ with respect to the level of precision they use in determining whether or not a shape has reflective symmetry. For example, some students will say that the number of buttons on a telephone form a symmetric design, since the placements do, whereas others will notice that the numbers are not the same on both sides of the design, so the phone pad is not totally symmetric.
$\square$ Send students on a "shape hunt" to look for 10 objects that display reflective symmetry. Encourage them to look for less obvious objects as well as more obvious ones.
$\square$ Ask students how deciding which mitten goes on which hand concerns symmetry.

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E12.1 If the school is within walking distance of a number of houses, take students on a walking tour to look for architectural examples of reflective symmetry.
E12.2 Have students check to see if their feet are exactly the same length. Ask: Would they be considered symmetric if they were not the same?

## Observation

E12.3 Observe whether or not students spontaneously point out the symmetry of various objects.

## Interview

E12.4 Ask the student to name pieces of clothing that display symmetry. Do shirts? pants? mittens?
E12.5 Ask: Does any object which is circular automatically display reflective symmetry? Why or why not?

E12.6 Ask the student to identify some number combinations (e.g., 818) that show symmetry.

## Portfolio

E12.7 Have students write a piece to describe whether or not they think animals are symmetric.

## Suggested Resources

## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to
iv) relate geometric ideas to number and measurement ideas and recognize and apply geometric principles in real-world situations

SCO: By the end of grade 2, students will be expected to E13 make the connection between reflective symmetry and using squares, rectangles, and circles
E14 make the connection between even and odd numbers and rectangles

## Elaboration - Instructional Strategies/Suggestions

E13 Students have explored simple fractions as parts of the whole. It is particularly easy for students to show halves of symmetric shapes by dividing them on the lines of symmetry.

It would be useful to ensure that symmetry lines are oriented in different ways, for example, horizontally, vertically, and slanted.


Some students will observe that rectangles can be cut in half in a variety of ways, not all of which divide them along lines of symmetry. They could fold along the lines to test. For example:


E14 Many students learn that even numbers are numbers that end in $0,2,4,6$ or 8 (and odd numbers are the others) without really understanding that "even" means "can be paired up." If numbers are displayed in rectangles in which one dimension is 2 , students can easily see the difference between even and odd numbers. Odd numbers do not partner well. This can be done using square tiles or counters arranged in a rectangular formation.


## GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

E13.1 Ask the students to trace around a small rectangular object, such as a large attribute block. Then ask them to show $\frac{1}{2}$ of the rectangle. Observe whether or not students choose to look for more than one way to do this.

E13.2 Provide pictures of circles and miras. Ask the students to find different ways to show $\frac{1}{2}$.
E13. 3 Have students fold a square on a line of symmetry to show halves.
E14.1 Ask the student to arrange 10 squares in such a way as to show that 10 is an even number.

## Interview

E14.2 Provide a copy of a rectangle which is 2 units by 8 units, subdivided into unit squares. Ask the student to explain which number(s) the picture shows is (are) even. (If the student uses just parts of the rectangle, he/she might see all of the even numbers from 2 up to 16 .)

## Suggested Resources

## Data Management and Probability

## General Curriculum Outcome F:

Students will solve problems involving the collection, display, and analysis of data.

## GCO F: Students will solve problems involving the collection, display, and analysis of

 data.KSCO: By the end of grade 3, students will be expected to i) collect, record, organize, and describe relevant data

SCO: By the end of grade 2, students will be expected to F1 conduct simple surveys and record data

## Elaboration - Instructional Strategies/Suggestions

F1 Students should develop strategies to collect and record information. This information should concern the students themselves, their school or community, and/or other topics which are meaningful to them. Attention should be paid as to how best to ask questions (if necessary) to gather information.
Students should be encouraged to use a tally system for recording data.
For example:

$\square$ Students could compare recording systems and discuss why it is easier to compare the numbers in groups $\mathrm{A} \& \mathrm{~B}$ in the second situation below:

A:
в: |||||||||||||
A: NWMXII $^{2}$
B: YK 11

One of the most important rules to follow in conducting graphing and statistics activities is to let students gather their own data. (Elementary School Mathematics, p. 391)

## GCO F: Students will solve problems involving the collection, display, and analysis of

 data.
## Worthwhile Tasks for Instruction and/or Assessment

## Interview

F1.1 Tell the students that they might be asked to conduct a survey to find the bedtimes of 7 - and 8 -year-olds. Ask: What would you have to think about when actually asking fellow students, so that the question would not be confusing (e.g., weekends or weekdays)?
F1.2 Ask students why it is easier to count the "yes" responses when they are shown like this, 状 $\mathbb{H}$, rather than like this, $||||||\mid$

## Presentation

F1.3 Have students plan and conduct an in-class survey about a favourite ___ (e.g., toy, television program, hockey player). Ask them to present the results of the survey in an organized form.

## GCO F: Students will solve problems involving the collection, display, and analysis of

 data.KSCO: By the end of grade 3, students will be expected to
ii) construct concrete and pictorial displays of relevant data, and
iii) read, and interpret displays of relevant data

SCO: By the end of grade 2, students will be expected to F2 create and interpret pictographs and symbolic bar graphs

## Elaboration - Instructional Strategies/Suggestions

F2 Students should create graphs in a variety of situations as a means of organizing collected data. While students were introduced to pictographs and symbolic bar graphs in grade 1 , it is in grade 2 , that these are particularly emphasized. It is appropriate, however, that symbols in pictographs and grid blocks on bar graphs continue to represent only one unit. (Having them represent more than one unit will be introduced in grade 3.) As well, students should be exposed to both vertical and horizontal bar graphs.
$\square$ Students could roll a die 20 times, record each number, and then create a graph showing frequencies.
$\square$ Students might collect and graph data in connection with activities in other subject areas. For example:


Students should interpret numerous graphs (both those created in class and those found elsewhere) to gain information about a variety of topics.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

F2.1 Ask the student to create a pictograph to show the favourite books of other students in the class.

## Paper and Pencil

F2.2 Provide a paragraph of text. Ask the children to draw a graph to show the number of words on each line.
F2.3 Ask the student to draw a graph showing the number of addition facts with sums of $0,1,2, \ldots$ up to 10 . Ask: What do you notice?
F2.4 Provide the student with a bar graph and ask him/her to interpret the graph by telling various things about what it tells the viewer.

Interview
F2.5 Provide a graph such as


Ask the students to write all they can about the information displayed.

## GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 3, students will be expected to iv) generate questions, develop and modify predictions, and implement plans, with respect to data analysis

SCO: By the end of grade 2, students will be expected to F3 develop and modify predictions, with respect to data collected or presented to them

## Elaboration - Instructional Strategies/Suggestions

F3 While students should continue to generate questions about data, increased attention should be paid to making predictions based on data collected or presented to them. They might state their expectations before data is collected in a certain situation, and then verify or modify their thinking as a result of seeing the actual data. Predictions might be made, and data collected about, for example,

- the number of right-handed students for each left-handed student
- whether more students get to school by riding buses, walking, or getting a drive
- whether more students swim in the winter or do not

Predictions might be based on population extensions or time extensions. For example, if students observe that more students in their class walk to school than take a bus, they might predict that the same would be true for another class in the school, but might not be true in another school elsewhere. Similarly, if students observe that fewer people take music lessons than do not, they might predict that this could be different for older students.

Predictions could be based both on information shown in tally or chart format and information shown in graphical format.

## GCO F: Students will solve problems involving the collection, display, and analysis of data.

## Worthwhile Tasks for Instruction and/or Assessment

## Performance

F3.1 Have the student predict which vowel will occur most often in a piece of text. He/she then counts and lists or graphs the number of each vowel that occurred to test his/her prediction. If the prediction is inaccurate, the student should describe why this might have occurred.
F3.2 Ask students to decide whose name they expect to hear most often when students name their favourite sports hero. Students each choose their hero and write the name. The data is represented to determine the validity of their predictions.

## Interview

F3.3 Ask: Do you think that more students in the class would choose skiing or skating for a class party? What do you think would be a good way to find out if you are correct?

F3.4 Show the student a bar graph of ice-cream sales for a store for the months of January through May. Ask him/her to predict what the graph would look like for the next three months.

## Presentation

F3.5 Show students a graph of favourite hockey teams for students in a school in Atlantic Canada. Have them work in groups and predict what a hockey graph might look like for students in British Columbia. Ask them to create the graph and to present it to the class with their reasoning.

Suggested Resources


## Data Management and Probability

## General Curriculum Outcome G:

## Students will represent and solve problems involving uncertainty.

## GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 3, students will be expected to
i) conduct informal investigations of chance and estimate probabilities, with respect to games and other simple, everyday situations

SCO: By the end of grade 2, students will be expected to
G1 demonstrate an understanding that some events are more likely than others
G2 demonstrate an understanding that probability predictions need not always come true

Probability ideas and language can be developed at this level as children make predictions about outcomes. As the children collect data about their experiments, they will need some ways to organize the data. This need connects probability to the experiences that children have had with charts and graphs. (Curriculum and Evaluation Standards, Addenda Series, Second-Grade Book, p. 18)

## Elaboration - Instructional Strategies/Suggestions

G1 Students should come to understand that some events are more likely than others. For instance, it is more likely to have snow than rain in January, more likely to get a number greater than two than less than two when rolling a die, etc.

Students should continue to use the terms "always," "never," and "probably" in appropriate situations. Devices for use could include number cards, money, and playing cards, as well as dice and spinners.
$\square$ Students might experiment with a variety of spinners to determine which sections are more or less likely to be landed on.
Following are some suggested spinner faces.


G2 Students should realize that, although one outcome may be more likely in theory, it may not happen that way in a given set of tries. For example, consider the spinner below:


While it is not very likely, it is possible to spin the B section more often than the A section during a set of 10 tries.

## GCO G: Students will represent and solve problems involving uncertainty.

## Performance

## Interview

 go first in a game? Why or why not?G1.5 Show the spinners below.

Ask: If four children are playing a game, why is the spinner on the left more likely to be used than the one on the right?

G1.6 Ask the child to think of an event that is possible, but not very likely, and another event that is very likely, but might not happen.

## Worthwhile Tasks for Instruction and/or Assessment

G1.1 Ask the child to design a spinner so that spinning red is more likely than spinning green, but spinning red is less likely than spinning yellow.

G1.2 Have the child roll a die 10 times and record each outcome. Ask: Do you think that the number which came up most often would again be the most common if you repeated the experiment? Why or why not?

G1.3 Tell the student he/she wins $\$ 1$ if the spinner lands on red and loses $\$ 1$ if it lands on blue. Ask: How would you like the spinner to be designed?

G1.4 Ask: Is flipping a coin a fair way to decide whether Anne or Sue should



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