Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

CURRICULUM

Mathematics

Grade 3

2001

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Table Of Contents

Introduction

Background and Rationale 1

Background 1
Rationale 2

Program Design and Components 2

Program Organization 2
Unifying Ideas 4
Learning and Teaching Mathematics 5
Adapting to the Needs of All Learners 6
Support Resources 6
Role of Parents 7

Assessment and Evaluation 7

Assessing Student Learning 7
Programs Assessment 7

Curriculum Outcomes 8

Grade 3

Number Concepts/Number and Relationship Operations 3-1

Number Sense and Number Concepts 3-2
Operation Sense and Number Operations 3-20

Patterns and Relations 3-37

Shape and Space 3-45

Measurement 3-46
Geometry 3-56

Data Management and Probability 3-73

Data Management 3-74
Probability 3-82

Index 3-85

Correlations 3-89

Introduction

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers," and they advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers and Department of Education officials to plan and develop cooperatively the curricula in mathematics, science, and language arts in both official languages.

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)

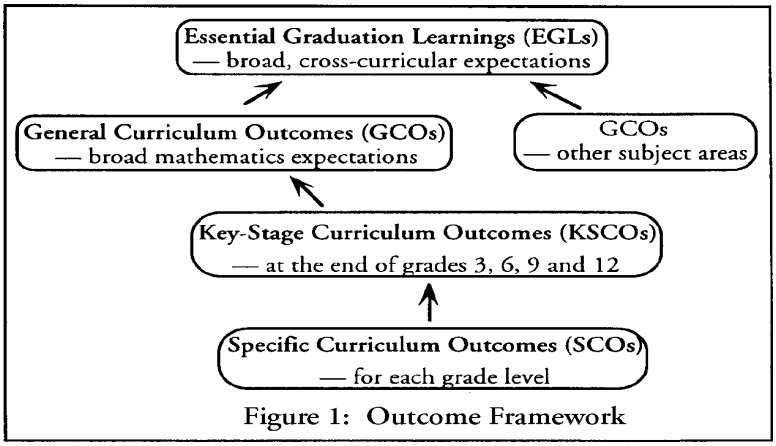


Figure 1: Outcome Framework

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. This curriculum guide is supplemented by others that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO.

The Atlantic Canada Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; and iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

II. Program Design and Components

A. Program Organization

As already indicated, the mathematics curriculum is designed to support the six Essential Graduation Learnings (EGLs). While the curriculum contributes to students' achievement of each of these, the communication and problem solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then presents outcomes at four key stages of the student's school experience.

This particular curriculum guide presents specific curriculum outcomes for each grade level. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.

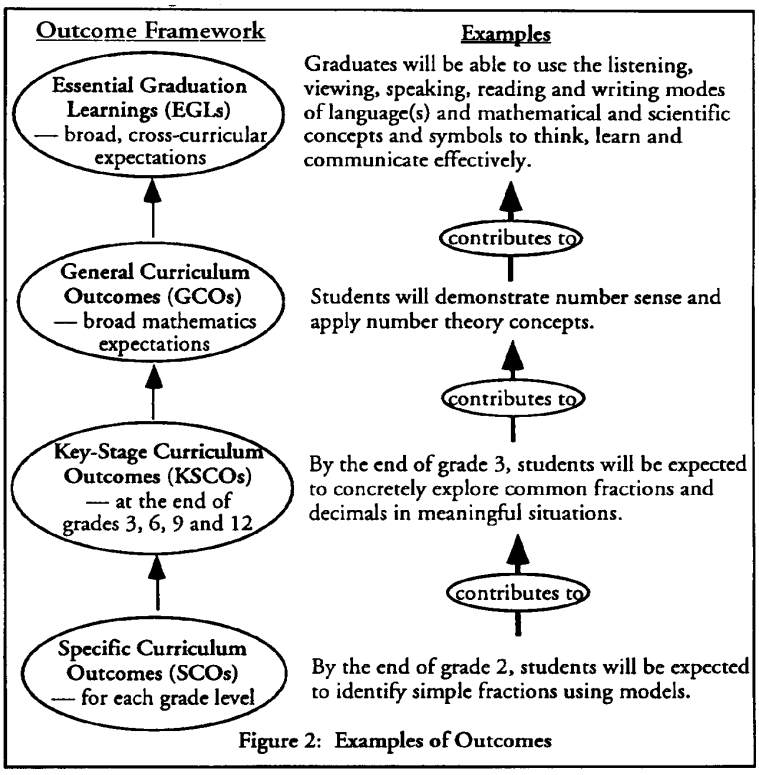


Figure 2: Examples of Outcomes

It is important to emphasize that, while the grade level outcomes (SCOs) provide a framework on which educators will base decisions regarding instruction and assessment, they are not intended to limit the scope of learning experiences. Although it is expected that most students will be able to attain the outcomes, some students' needs and performance will range across grade levels. Teachers will need to take this variation into consideration as they plan learning experiences and assess students' achievement.

The presentation of the specific curriculum outcomes follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes like those pertaining to patterns and data management may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

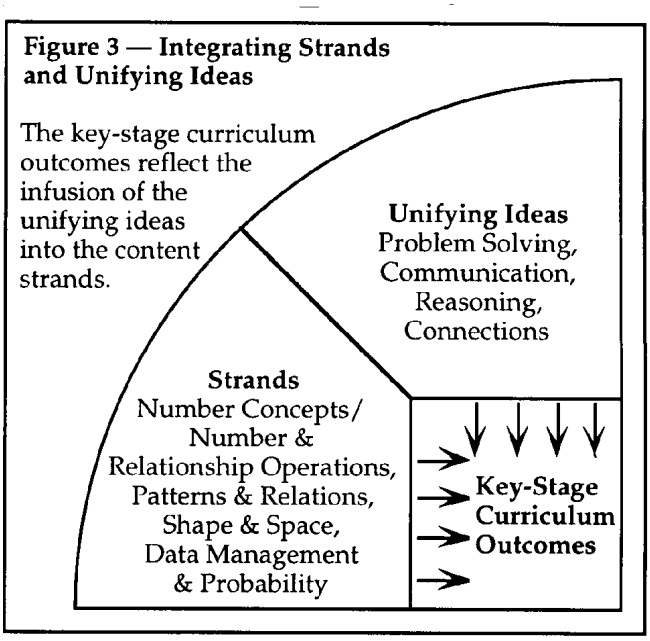
Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what

might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province like an election, an exhibition, or a fair.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)

Figure 3 — Integrating Strands and Unifying Ideas



These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Adapting to the Needs of All Learners

The F*oundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student development as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

E. Support Resources

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. These guides should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in as much as they support the specific curriculum outcomes. Many manipulative materials need to be readily at hand, and technological resources, e.g., software and videos, should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM) and the *Addenda Series* and *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* or *Elementary and Middle School Mathematics: Teaching Developmentally* (John van de Walle), *Developing Number Concepts Using Unifix Cubes* (Kathy Richardson), and *About Teaching Mathematics; A K-8 Resource* (Marilyn Burns).

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

III. Assessment and Evaluation

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessment and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

A. Assessing Student Learning

Characteristics of good student assessment would include i) the use of a wide variety of assessment strategies and tools, ii) aligning assessment strategies and tools with the curriculum and instructional techniques, and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborates good assessment practices and it served as a guide for student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

IV. Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for each grade. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled "Elaboration–Instructional Strategies/ Suggestions," provides a clarification of the specific curriculum outcome(s), as well as suggestions for possible strategies/activities which could be used to help students achieve the outcome(s). While the strategies/activities presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s). They will also illustrate ways to work toward the achievement of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activites are introduced in this column of the layout by the symbol ❒ .

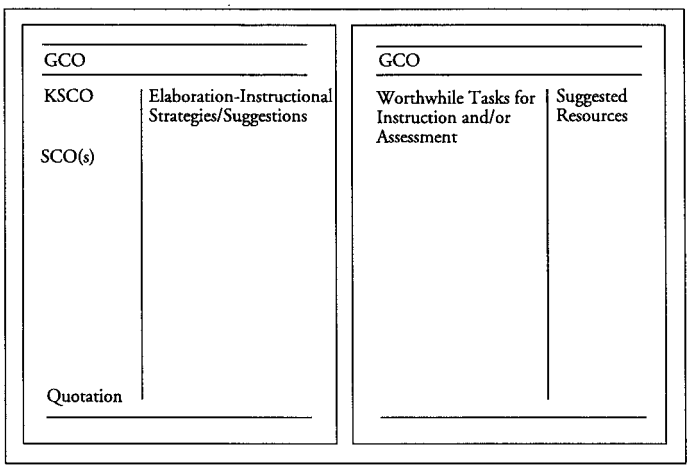


Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled "Worthwhile Tasks for Instruction and/or Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Suggested Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number-theory concepts.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

i) construct and communicate number meanings, and explore and apply estimation strategies, with respect to whole numbers

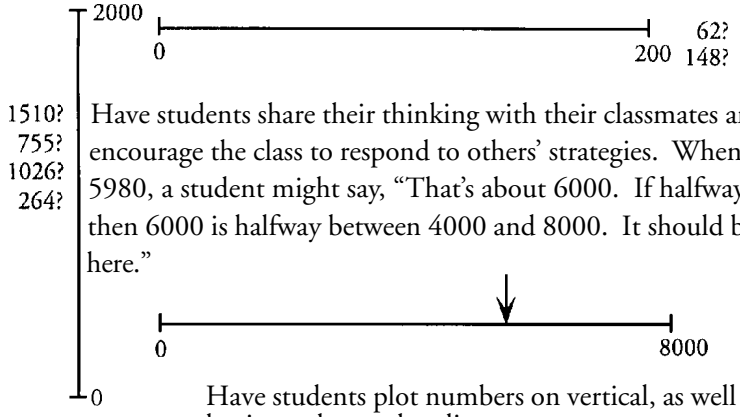
SCO: By the end of grade 3, students will be expected to

A1 compare and order whole numbers to thousands

Elaboration - Instructional Strategies/Suggestions

A1 Students should be able to identify the greater of two whole numbers and order a set of numbers from greatest to least (or least to greatest). When modelling comparisons (e.g., 500 and 489), stress the importance of the digit in the hundreds place and that "89" is not enough to make 1 more in the hundreds place.

- ❒ Prepare a deck of number cards which contain both 2- and 3-digit numbers. Have the students deal all the cards face down to the players. Have each player turn the top card over; the one who has the greater (greatest) number "wins" both or all the cards in play. The winner is the one who has collected the most cards when all the cards are turned over.
- ❒ Ask: If ☐ 39 > 422, what can you say about ☐?
  If ☐ 39 > ☐87, what do you know about the missing digits?
- ❒ Provide a set of cards (10 - 15) with each card having a 2- or 3-digit number (4-digit when students are ready) on it. Ask the student to order the number cards from least to greatest and to explain how he/she determined the relative number size.
- ❒ Shuffle a prepared deck of 40 number cards (4 sets of 0-9). Have the student select three or four of the cards and arrange them to make the greatest possible number and the least possible number. Ask the student to model these numbers with base-ten materials.
- ❒ As a class activity, repeatedly roll a die and have the students fill in the digits, one at a time, on a place-value chart. Alternate by having them try to make the greatest number or the least number. Model the task by placing your digits on an overhead chart. Regularly ask questions such as, What do you need? What don't you want me to roll?
- ❒ A good estimation activity is plotting numbers on number lines.



Have students share their thinking with their classmates and encourage the class to respond to others' strategies. When plotting 5980, a student might say, "That's about 6000. If halfway is 4000, then 6000 is halfway between 4000 and 8000. It should be about here."

Have students plot numbers on vertical, as well as horizontal, number lines.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

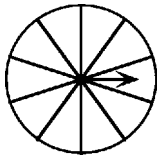
Performance

A1.1 Give the student some prepared cards with 3- and 4-digit numbers on them. Ask him/her to order the number cards from least to greatest.

A1.2 Show the numbers 501 and 398 and ask which is greater. Have the students explain their answers. Encourage them to use base-ten models in their explanations.

A1.3 Ask the students to enter a certain number on a calculator (e.g., 2235). Ask: How can you, without clearing the calculator, make the number 2435? (2446? 1234?)

A1.4 Give each of two students a spinner (as shown) with 10 numbers that are in the hundreds and/or the thousands. Have them spin at the same time. The one who spins the higher number gets a token. The students play until someone has gathered 10 tokens. Select numbers according to the students' level of understanding. (Numbers such as 345, 354, 381, 309, 1008, 1800, 1080, 1335, 1353, and 2000 would work well.)



Paper and Pencil

A1.5 Ask the student to write a number that is

- ten more than 165
- one hundred greater than 655
- between 463 and 474
- a little less than 300
- two hundred less than 206
- greater than 348 but less than 360, etc.

Interview

A1.6 Ask the student to explain why a 3-digit number is always greater than a 2-digit number.

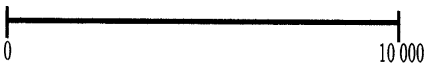
A1.7 Ask the student to select 5 numbers between 600 and 630, and to write them in increasing order.

A1.8 Tell the student that *The Guinness Book of Records* reports each of the following to be the largest ever of its kind.

hamburger	2509 kg	doughnut	1700 kg
salami	678 kg	popcorn ball	1080 kg
ice cream sundae	377 kg	lollipop	1369 kg
Easter egg	4765 kg		

Ask the student to order the items from least to greatest.

A1.9 Provide number lines for students, ask them to estimate where some numbers might lie, and to give their reasoning; for example, 2465.



Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

i) construct and communicate number meanings, and explore and apply estimation strategies, with respect to whole numbers

SCO: By the end of grade 3, students will be expected to

A2 estimate the size of numbers to the nearest ten or hundred

Elaboration - Instructional Strategies/Suggestions

A2 Rather than rounding numbers in isolation, emphasis should be on rounding to estimate results of calculations in problem-solving contexts. Instead of simply applying the "5" rule (i.e., 5 or greater rounds up, less than 5 rounds down), students need to learn to do what makes sense in each situation. For example, it makes sense to

underestimate (rounding down)

- the number of items you can buy with a fixed amount of money
- the distance you can travel on one quarter of a tank of gas

overestimate (rounding up)

- how much food to prepare for a party, to make sure there is enough
- the amount of string required to wrap a parcel

❒ Have students map out a street on which the houses are numbered in intervals of 10, with a bus stop at every tenth house. Ask students to choose an address and decide which bus stop they would use if they were tired.

Encourage students to use appropriate compensation techniques when estimating in calculation situations. For example,

- when adding, rounding one number up and the other down is often a good technique (e.g., 32 + 45 becomes 30 + 50, while 27 + 45 becomes 30 + 40)
- when subtracting, rounding both numbers in the same direction tends to preserve the difference best (e.g., 45 - 32 becomes 40 - 30, while 45 - 27 becomes 50 - 30)

Note: It is important that these types of rounding situations be presented in real contexts.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

A2.1 Ask the student to explain how he/she would round two or three 2-digit numbers for an estimated sum. To assess the student's understanding of the compensation strategy, the following problem might be used: 38 + 48 + 35

A2.2 Ask the student to name some numbers that might be rounded to 120.

A2.3 Point out that when you go grocery shopping, you sometimes round each price to the nearest 50¢. Ask: Using this system, how much should you have estimated as the total for products priced at 79¢, $1.38 and $2.59?

Interview

A2.4 Tell the student that a number is rounded to 40. Ask: What might it have been?

A2.5 Ask the student to describe a situation in which he/she would use the number 500 as an estimate for 475.

A2.6 Tell the student that Jane estimated 82 - 47 by using 80 - 50. Without actually finding the answer, ask how he/she knows that Jane's estimate is low.

A2.7 Tell the student that Marla said, "To estimate 46 + 25, I would add 50 + 20." Mark said, "It should be 50 plus 30." Ask: Whose estimate was closer? Why?

A2.8 Tell the student that Patrick went to the store with a loonie. He estimated the cost as he picked up a 12¢ pencil, a 25¢ eraser, a 29¢ notepad, and a 19¢ pen. Ask: How might Patrick have estimated? Without finding the actual sum, do you think he had enough money?

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

ii) concretely explore common fractions and decimals in meaningful situations

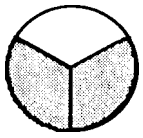
SCO: By the end of grade 3, students will be expected to

A3 use simple fractions to describe situations

Elaboration - Instructional Strategies/Suggestions

A3 Students should continue to use simple fractions such as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{10}$, and any other fractions that come up in context. These fractions should describe

- parts of a whole



(e.g., thirds)

- parts of a set

(e.g., fourths)

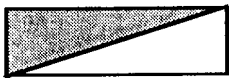
Presenting fractions in context will make them much more meaningful to students. It will be natural also, when examining a situation involving a fraction such as $\frac{1}{4}$, to show the related fraction $\frac{3}{4}$. Always use a horizontal line when writing fractions.

❒ Explore fraction relationships among pattern blocks.

❒ Use coloured tiles to make a rectangle that is $\frac{1}{3}$ yellow and $\frac{2}{3}$ green. Make another rectangle that is $\frac{2}{4}$ red and $\frac{1}{2}$ blue.

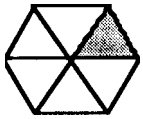
❒ Students might examine fractions of particular geometric shapes. For example:

$\frac{1}{2}$ of a rectangle



$\frac{1}{4}$ of a square

$\frac{1}{6}$ of a hexagon



Models must be used at all grade levels to develop fraction concepts adequately. Further, . . . children should have experiences with a wide assortment of models. (Elementary School Mathematics, pp. 222-23)

GCO A: Students will demonstrate number sense and apply number-theory concepts.

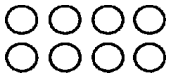
Worthwhile Tasks for Instruction and/or Assessment

Performance

A3.1 Ask the student to use multilink cubes to show that $\frac{1}{2}$ is less than $\frac{3}{4}$.

Paper and Pencil

A3.2 Ask the student to colour $\frac{1}{4}$ of the circles.



A3.3 Tell the student that Jack has a total of ten 2- and 4-hole buttons. Ask: What is the greatest fraction that can have 2 holes? What is the least fraction that can have 2 holes?

A3.4 Tell the student that Lee and Teddy bought their mother a gift for Christmas which cost $20. Lee paid $\frac{3}{4}$ of the cost, and Teddy paid the balance. Ask: How much did each pay? Provide coloured counters to help him/her solve the problem.

A3.5 Pair each student with a partner to solve this problem: Eight-year-old Samantha, whose birthday is January 25th, said, "I can't wait until I'm 8 and $\frac{11}{12}$." Ask: Why was she excited?

Interview

A3.6 Ask the student to tell why, whenever you see a representation of $\frac{1}{3}$, there is always a $\frac{2}{3}$ associated with it.

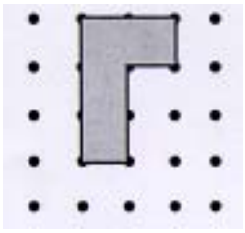
A3.7 Ask the student to describe how to find $\frac{1}{3}$ of a strip of 9 squares.

Portfolio

A3.8 Tell the student that you have 8 coins. Half of them are pennies. More than $\frac{1}{8}$ of them are quarters. The others are nickels. Have the student use coins to represent the situation. Ask: How much money might you have?

Presentation

A3.9 Ask pairs of students to explore the following problem and to present their findings to the class: The shape below is $\frac{1}{2}$ of a larger one. What could the larger one look like? How many different possibilities can you find?



Invite students to create similar problems to challenge other groups.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

iii) read and write whole numbers and demonstrate an understanding of place value (to four places)

SCO: By the end of grade 3, students will be expected to

A4 demonstrate an understanding of base-ten groupings (units, tens, hundreds, thousands)

Elaboration - Instructional Strategies/Suggestions

A4 Although some students will have a clear understanding of the base-ten pattern of our place-value system, many will still be in the early stages of its development. It is important that students be provided with regular opportunities to strengthen their knowledge. It takes time for the students to thoroughly understand our base-ten system and to recognize and use the place-value notation.

Students should recognize that one thousand is just another expression for ten hundreds. (This may be extended to "two thousand is another way to say twenty hundreds," etc.) Continue to expect students to model numbers and to engage in trading activities.

- ❒ Say: Show me four hundred eighty-nine with your materials. If you add seven, how many do you have? What if you were to add another four? Explain your trading.
- ❒ Provide a shuffled set of 50 cards (5 sets numbered 0 to 9). Have a pair of students draw two cards and decide the greatest number they can make. Ask them to add this number in base-ten materials to the value currently on their place-value mat, trading when necessary. (For example, if pair one were to draw a 5 and a 7, they would add 75 to the chart.) Three cards may be drawn when students are capable of trading at this level.
- ❒ Have students play "Race For A Loonie." Ask each student to repeatedly toss a die and count out pennies on a mat. Ten pennies are exchanged for a dime, and ten dimes for a loonie.
- ❒ Ask the students to enter a number (e.g., 4567) on the calculator. Have them change the 5 to an 8 in one step (the 4 to a 6, the 6 to a 9, etc.).

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A4.1 Ask the student to show twenty-eight hundred seventy-six with base-ten materials. Have him/her roll a die and add to his/her model until reaching at least three thousand.

A4.2 Have the students use a mat with sections marked off for $1, 10¢ and 1¢. Ask them to toss two dice, find the sum, and place the total on the mat. Have them exchange 10 pennies for one dime and continue until they have reached a dollar.

A4.3 Tell the student that Mary knows that she needs to collect 300 bottle caps for the contest and she already has 287. Show with the base-ten materials how many more she must collect to reach her goal.

Interview

A4.4 Have the students pretend they won three thousand dollars. Ask them to determine how many hundred dollar bills that would be and have them explain their method.

A4.5 Tell the student that Martin said the car cost thirty-four hundred dollars, while Sam said he thought it cost over three thousand dollars. Ask: Are they disagreeing? Explain.

A4.6 Show the student a number (8317, for example). Ask: What does the 3 represent in this number? What does the 1 represent? How many thousands are there?

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

iii) read and write whole numbers and demonstrate an understanding of place value (to four places)

SCO: By the end of grade 3, students will be expected to

A5 record, model, and interpret numbers up to and including the thousands

Number sense develops over many years, and there may be a wide disparity between the concepts and skills of your third-grade students. The understanding they possess will be fostered if they model numbers in many ways and use numbers to describe real-world situations. (Curriculum and Evaluation Standards, Addenda Series, Third-Grade Book, p. 9)

The key instructional tool for developing the conceptual knowledge of place value and also for connecting these concepts to symbolism is the use of base-ten models. (Elementary School Mathematics, p. 157)

Elaboration - Instructional Strategies/Suggestions

A5 "Building" larger numbers with concrete materials helps students develop a better sense of those numbers. For example, when introducing 1000, it is beneficial to present it first as ten stacked flats. This helps students visualize 1000 more easily. Students who have had extended experiences with base ten materials will be able to visualize the models of numbers with minimal difficulty. In some cases, it is sufficient for them to draw the models. It is important, however, that proportional models continue to be used, rather than nonproportional ones such as an abacus.

It is also important to spend time developing the concept of zero in numbers. For some students, the number "3002" looks like "three hundred two." Students need many experiences using base-ten material to model numbers with zeros as digits.

After extensive work with the base ten materials, ask students to record numbers (e.g., three hundred forty; nine hundred eight; seven thousand sixteen; sixty-four hundred thirty-two; a number that has twenty-two tens and three ones). Experiences which involve interpreting numbers in different ways will help students to write numbers of the latter type. Invite students to take turns giving numbers for the class to write.

Introduce the "read, model, and record" triad. Students should be able to move easily from one form to another.

- Read a number and have them record and model it.
- Model a number and have them record and read it.
- Show the written number and have them read and model it.

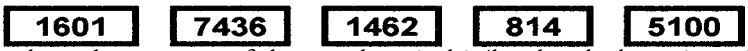
Provide opportunities for students to interpret numbers in different ways. For example, 2936 may be interpreted as

2 thousands, 9 hundreds, 3 tens, 6 ones
OR
29 hundreds, 3 tens, 6 ones
etc.

Note: These variations can and should be modeled using base ten materials.

Exercises such as 432 = __h, __t, __o should be avoided because students do not need to understand the value of each digit to complete them correctly. Learning experiences such as the following are more useful.

- ☐ Create a number greater than 500 with 3 in the tens place.
- ☐ Provide a set of number cards, such as



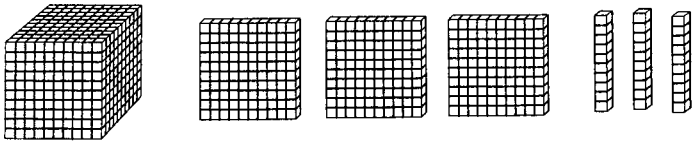
One student chooses one of the numbers in his/her head, then gives clues to another student, based on different ways to read the number. The student should try to use clues which could apply to more than one number. For example, the clue, "there are more than 15 hundreds," eliminates 814 and 1462 from the group of five numbers.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A5.1 Provide the student with a base-ten block representation of a number. Ask him/her to write and read the number that is modelled.



A5.2 Ask the student to model 2230 in more than one way, using base-ten blocks. For example, 1 thousand, 12 hundreds and 3 tens or 22 hundreds and 30 ones.

A5.3 Show the student 2006 written symbolically and ask him/her to model it. Then have the student read the number and express it in other ways.

A5.4 Ask the student to write a number which is a) 10 greater, b) 100 greater than a given number.

A5.5 Invite students to make up number riddles. Other students show their solutions to the riddles by building models with base-ten blocks.

For example: I am between 1100 and 1200.
My last digit is an odd number.
What is the least number I can be? The greatest?

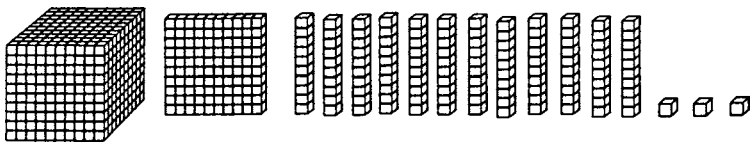
Paper and Pencil

A5.6 Read numbers or show models of numbers and have the student record them.

A5.7 Ask the student to describe different items that might cost about $1250.

Interview

A5.8 Ask the student to tell why the model below shows 1223.



A5.9 Ask the student to describe 4000 in more than one way.

A5.10 Ask: How many tens make 1000?

A5.11 Tell the student that Mary won $5000 in a contest. If she wants all her prize money in $10 bills, how many would she receive?

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

iv) order whole numbers and represent them in multiple ways

SCO: By the end of grade 3, students will be expected to

A6 read numbers in several ways

Elaboration - Instructional Strategies/Suggestions

A6 Students should understand that numbers can be read in more than one way. Experiences which involve interpreting numbers differently help students develop this idea. For example, 1236 may be read "one thousand two hundred thirty-six" or "twelve hundred thirty-six."

It is important to point out that both responses are correct, but sometimes one may be more appropriate or used more frequently. For example:

- She was born in 1900. (nineteen hundred)
- He will graduate in 2004. (two thousand four)
- The paint job on the car cost $2400. (Twenty-four hundred is used more often, but two thousand four hundred is also correct.)

Although students at this level are unlikely to be dealing with numbers to ten thousands, they should be aware that numbers greater than 9999 are written with a space and no comma (e.g., 10 453 which is read ten thousand four hundred fifty-three). Also, they need to understand that, when reading a number, the word "and" is reserved for the decimal and avoided in the middle of the number.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A6.1 Ask the student to use base-ten materials to represent 2047 in different ways.

A6.2 Ask the student to show with base-ten blocks that 132 is 13 tens and 2 ones.

Paper and Pencil

A6.3 Ask the student to record the number made up of 15 tens and 15 ones.

A6.4 Explain to the student that a number is made up of 42 tens and fewer than 5 ones. Ask him/her to write what it could be.

Interview

A6.5 Ask the student to read 3241 without using the word "thousand."

A6.6 Provide the student with the following table:

Dog Breeds In Canada

Breed	No. Registered 1995
Labrador Retriever	9471
Golden Retriever	8699
German Shepherd	8453
Shetland Sheepdog	4775
Rottweiler	4242
Poodle	4050
Shih Tzu	2895
Yorkshire Terrier	2458
Miniature Schnauzer	2381
Siberian Husky	2220

Ask the student to read the number of registered dogs both ways (i.e., 2441 is two thousand four hundred forty-one or twenty-four hundred forty-one).

A6.7 Ask the student to explain why 320 is the same as 32 tens.

A6.8 Tell the student that, to subtract 132 from 500, Anne said, "500 is 50 tens, so I'll regroup one ten and that leaves 49 tens." Ask why this was a good strategy.

Presentation

A6.9 Ask pairs of students to discuss the following problem: Jane said that 421 has more tens than 139. Peter said that it has fewer. How can they both be correct? Share ideas in a class discussion.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

v) apply number-theory concepts (e.g., place-value pattern) in meaningful contexts, with respect to whole numbers and commonly used fractions and decimals

SCO: By the end of grade 3, students will be expected to

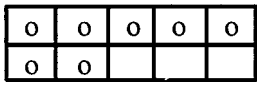
A7 extend the place-value system to model and record numbers involving tenths

Elaboration - Instructional Strategies/Suggestions

A7 To help students extend the place-value system to decimals, focus on the basic pattern of ten. Remind students that 10 ones make 1 ten, 10 tens make 1 hundred, etc. Then, extend this pattern to help students understand that it takes 10 equal parts (tenths) to make 1. Explain that the place to the right of the ones is tenths.

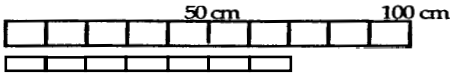
It is helpful for students to see tenths in a variety of models.
For example, 0.7 (7 tenths) may be modelled using

-ten frames

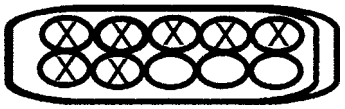


-circle patterns

- base-ten materials. For example, 0.7 (7 tenths) of a metre would be 7 rods (7 decimetres)



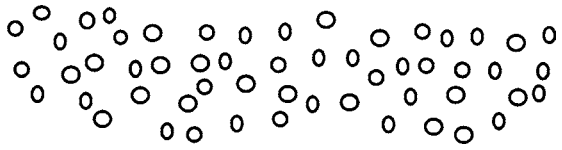
-egg cartons with 2 sections removed



Note: The shift from a ten-frame base-ten, or other similar model representing 10 to it representing 1 is challenging for many students and will need to be reinforced through various activities.

To foster understanding of decimals, it is important that they be read correctly. For example, 3.4 should be read as 3 and 4 tenths, not 3 point 4, or 3 decimal 4. It is also important that students understand the relationship between fractions and decimals.

- ❒ Invite students to predict the number of times a coin will land tails up when flipped 10 times. Have them check their predictions and record their data as a decimal and a fraction. Invite them to share their results with others.
- ❒ Ask students to circle about 0.4 of the dots without counting.



- ❒ Provide items and ask the students to measure them in tenths of centimetres.

Encourage students to explain the estimating strategies they used.

GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A7.1 Ask the student to model 0.2 and 1.2, using ten frames.

A7.2 Show the student a handful of beans. Ask him/her to estimate how many would make up about 0.1 of the amount. Have the student explain the thinking involved.

A7.3 Ask the student to estimate and colour 0.1 of a rectangle.

Paper and Pencil

A7.4 Ask the student to continue the pattern.

0.7, 0.8, 0.9, ___, ___,

A7.5 Show a 2 x 5 grid and ask the student to colour 0.3 of it.

A7.6 Ask the student to record the number which is 0.2 less than 1.

A7.7 Ask the student to record numbers, such as four tenths, sixteen and seven tenths, four thousand ninety-nine and nine tenths, twenty-four hundred six and five tenths.

Interview

A7.8 Ask: Why does the model below not show 0.4?

A7.9 Ask: What fraction (in decimal form) of the letters in the word HAMBURGERS is vowels?

A7.10 Explain to the student that someone forgot to put the decimal in the number below. Ask where it could be if the number is less than 100.

1427

A7.11 Ask: How do you know 1.1 is greater than 0.4?

Portfolio

A7.12 Have the student explain why 3.2 is greater than 3, but less than 4.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 3, students will be expected to

v) apply number theory concepts (e.g., place-value pattern) in meaningful contexts, with respect to whole numbers and commonly used fractions and decimals

SCO: By the end of grade 3, students will be expected to

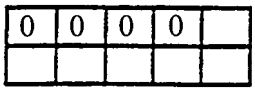
A8 order and compare decimals to tenths

Elaboration - Instructional Strategies/Suggestions

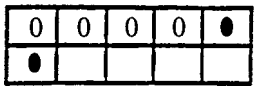
A8 When comparing or ordering decimal numbers, as with performing these tasks with whole numbers, students must realize that one must keep in mind several points.

- The whole number part of the number is a critical part for comparison. (e.g., 2.39 < 4.2 because 2.39 is not even 3 whole units.)
- When numbers have the same whole part, the decimal part becomes the critical part for comparison. (e.g., 4.3 is 4 and 3 tenths. This is less than 4.7 which is 4 and 7 tenths.)
- It is important to examine the placement of digits, not just the number of digits. (e.g., 6.2 and 40 both have 2 digits, but 6.2 is not even 7 whole units so it is much less than 40.)

Initially, when comparing two decimal numbers, it is helpful to use manipulative materials. By matching corresponding parts, students are able to see which number has more parts. The ten frame represents one unit, each box represents a tenth:



0.4



0.6 is 2 tenths more

It is appropriate at this level to use the symbols < and > to represent comparisons (e.g., 3.4 < 5.6).

Students should have opportunities to order a list of numbers.

❒ A possible activity, is to have students examine skating scores from a competition and decide who was in first, second, and third place. As a variation, invite three students to perform some "stunt" and have other students give a decimal score between 1 and 10 for each performance. Then ask students to decide first, second, and third place.

❒ Prepare a deck of cards with numbers such as 0.1, 0.2, ... 0.9, 1.0, 1.1 ... 1.9, 2.0, 2.1 ... 2.9 for a pair of students. Each student gets half the deck. They both turn over one card at a time. The student with the card showing the greater number keeps both cards. Play continues until someone has all the cards.

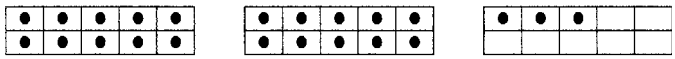
GCO A: Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

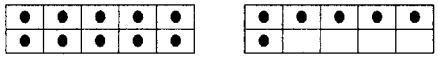
Performance

A8.1 Provide the student with models of two decimals. Ask him/her to model an amount between the two and tell you the decimal it represents.

A8.2 Provide ten-frame models of several decimal amounts. Ask the student to identify the decimals modelled and put them in order.



2.3



1.6

A8.3 Show the student a decimal amount such as 0.9. Ask him/her to record and model a greater amount.

Paper and Pencil

A8.4 Have the student complete a given place-value pattern.

(e.g., 2.9, ___, 3.1, 3.2, ___, 3.4)

Alternatively, ask the student to complete a list of numbers by considering the two numbers surrounding each blank.

(e.g., 3.4, ___, 4.7, 5.9, ___, 8.1)

Any number between 3.4 and 4.7 is acceptable for the first blank, and any number between 5.9 and 8.1 is appropriate for the second.

A8.5 Ask the student to create 4 different number sentences involving decimals in which the < sign is used. Ask him/her to rewrite each sentence so that the > sign can be used.

A8.6 Ask the student to draw two line segments, 4.3 cm and 4.7 cm, and to compare the two lengths.

Interview

A8.7 Tell the student that you are thinking of a number which is greater than 1.5. Ask him/her to ask relevant questions which would help to determine the number.

Portfolio

A8.8 Ask the student to describe or draw pictures of as many items as he/she can find which have a length between 0.8 m and 1.8 m.

Suggested Resources

THREE-OPERATIONS

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

i) demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of four basic operations

SCO: By the end of grade 3, students will be expected to

B1 recognize several meanings for multiplication

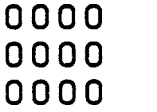
B2 recognize several meanings for division

When children have number sense, they understand not only the relationships between numbers but also the effects of arithmetical operations on numbers. They exhibit confidence in their answers and willingness to investigate new situations. (Curriculum and Evaluation Standards, Addenda Series, Third-Grade Book, p. 9)

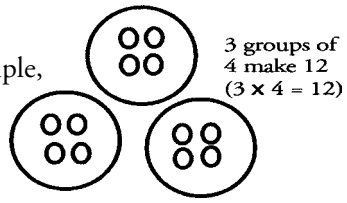
Elaboration - Instructional Strategies/Suggestions

B1/B2 At this level, it is important that students understand that there are different ways of looking at the concept of multiplication. They need to be aware that the product may be determined by

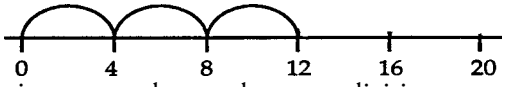
- the total number in an array - for example,



- making sets of equal groups - for example, students can create equal-sized groups with actual items



- repeated addition - for example, 4 + 4 + 4 can be written as 3 x 4



It is equally important that students see division can mean

- sharing - A number of objects is shared fairly among the members of a group. How many objects will each member of the group receive?
For example: Twelve cookies are shared fairly among four people. How many will each person get?

- how many groups - We know how many objects will be in each group. The question is, how many groups will there be?
For example: There are 12 cookies. Each package must contain 4 cookies. How many packages of cookies will there be?

In both cases, students need to understand that a ÷ b is equivalent to repeatedly subtracting b from a and counting the number of subtractions to reach zero.

To help students understand the relationship between the two meanings of division, it is beneficial to use concrete materials. Demonstrate that, in sharing 12 items among 3 people, for example, the actual giving of 1 item to each person is the same as creating a group of 3. In other words, sharing among 3 people is equivalent to finding how many groups of 3 can be formed. Give students situations in which there are remainders and ask them to determine how to deal with them.

Remember to delay the formal writing of multiplication and division sentences until students understand the meaning of the operation.

When creating multiplication and division problems, use contexts in which things actually come in groups, for example, packaged foods - eggs (6 x 2), juice packs (1 x 3), hamburger buns (2 x 4).

❒ Invite students to brainstorm activities suitable for various numbers of people, such as 1 person (reading, drawing, etc.); 2 people (tennis, chess, etc.); 3 people (skipping, marbles, etc.); and so on. Ask students to choose an activity and decide how many equal groups are possible for the whole class. Discuss which group sizes work well and why.

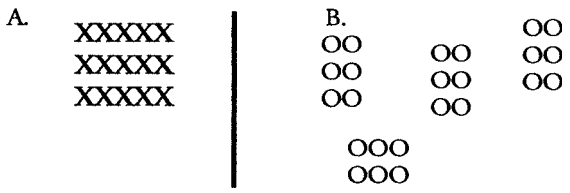
GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B2.1 Have the student enter 35 on a calculator and repeatedly subtract 7. Ask him/her to record these subtractions on paper until they reach 0. Have him/her express what happened in terms of a division.

B1/2.1 Ask the student to record multiplication and division statements based on the drawings shown.



B1.1 Provide the student with some toothpicks. Ask him/her to use them to make 5 squares and then state a multiplication sentence which describes how many toothpicks have been used.

B1.2 Ask the student to use counters to show why the result is even if you multiply two even numbers.

Pencil and Paper

B2.2 Ask the student to write a division story about 30 ÷ 5.

B1.3 Have the student draw a picture to show what 4 x 6 means.

Interview

B1.4 Ask the student what 6 x 3 tells about tricycle wheels.

B2.3 Ask the student to describe a situation for which you might have to find the answer to 16 ÷ 2.

Portfolio

B1/2.2 Ask the student to draw pictures showing various situations in which either multiplication or division might be used.

Presentation

B1/2.3 Invite a group of students to act out a skit modelling either a multiplication or division situation. Ask other students to guess the number sentence being dramatized.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

ii) recognize and explain the relationships among the four basic operations

SCO: By the end of grade 3, students will be expected to

B3 recognize the relationship between multiplication and division

Elaboration - Instructional Strategies/Suggestions

B3 It is important students understand that for every multiplication situation there is a related division situation and vice versa.

For example, the array below can be thought of as

x x x x
x x x x • 2 groups of 4 (2 x 4)

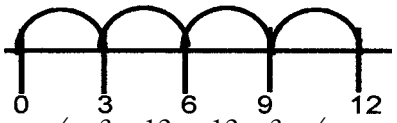
- 8 shared among 2, which is written 8 ÷ 2, or how many groups of 4 can be made from 8, which is written 8 ÷ 4.

Similarly, this diagram could be interpreted as

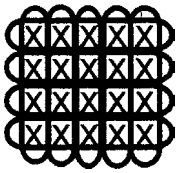
0 0 0 0 0 0
0 0 0 0 0 0

- 3 x 4 = 12 (There are 3 groups of 4.)
- 12 ÷ 3 = 4 (Using the concept of sharing, 12 shared among 3 gives each person 4.)
- 12 ÷ 4 = 3 (Using the "how many groups" concept, one might ask, "How many sets of 4 can you make with 12 ?")

Using a number line also helps students see the relationship between multiplication and division.



4 x 3 = 12 12 ÷ 3 = 4



Use arrays to illustrate the commutative (order) property.

This array shows 4 rows of 5 or 4 x 5. If we were to rotate it, or look at it from the side, it would show 5 rows of 4 columns or 5 x 4; therefore, 4 x 5 = 5 x 4.

Point out that 2 x 9, for example, is the same as 9 x 2, but that it tends to be easier to visualize and figure out the answer for 2 groups of 9.

The models that we can use to illustrate division concepts are exactly the same as those for multiplication. In fact, when a division is modelled, the result always looks like a multiplication model. (Elementary School Mathematics, p. 124)

❒ Give students lots of practice with materials to help them visualize the operations. Ask questions as they form sets. For example: "Show me 4 groups of 5. How many do you have altogether? How did you divide up the counters?" Encourage the students to respond using the language for describing multiplication and division. For example: "I have 4 groups of 5, which is 20. I started with 20 and divided them into 4 groups: twenty divided by 4 is 5." Extend this learning experience by asking the students to use the same number of counters and divide them in another way.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

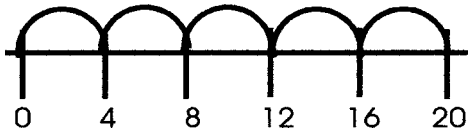
Worthwhile Tasks for Instruction and/or Assessment

Performance

B3.1 Provide the student with some toothpicks and ask him/her to use 12 to make 4 identical shapes. Ask the student what division and multiplication sentences could describe the creation of the shapes.

Pencil and Paper

B3.2 Show the following number line. Ask the student to record what multiplication and division sentences it might be showing.



B3.3 Show the student the multiplication sentence 5 x 8 = 40. Ask the student to write related division sentences.

Interview

B3.4 Set up a 3 x 4 array and ask the student to give two multiplication and two division sentences which describe it.

B3.5 Explain to the student that 26 ÷ 4 tells you something about the way the students are grouped in the classroom. Ask him/her to talk about the size of the group and the number of groups.

Portfolio

B3.6 Ask the student to explain how the relationship between multiplying and dividing is like the relationship between adding and subtracting.

B3.7 Ask the student to write problems in which one has to multiply or divide to find the answer. Have him/her illustrate the solutions and describe the multiplication/division relationship.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

iii) create and model problem situations involving whole numbers, using one or more of the four basic operations

SCO: By the end of grade 3, students will be expected to

B4 solve and create problems involving addition and/or subtraction

B5 solve and create problems involving multiplication and division with small numbers

Elaboration - Instructional Strategies/Suggestions

B4/B5 Presenting the four operations through meaningful contexts helps students develop a better sense of which operation is required. Encourage the use of calculators to solve real problems which involve calculations with very large or decimal numbers. It is important that students begin to understand when it is appropriate to use a calculator and the importance of estimating when using the technology.

Asking students to create their own word problems requires them to focus on the meaning of the four operations. Encourage them to create problems which include different meanings for all four operations.

Addition/subtraction:	215 and 28 more. How many altogether? 215. One group of 28. Size of other group? 215. 28 gone. How many left? 215 in one group. 28 in another. How many more?
Muliplication/division:	24 items. Groups of 4. How many groups? 24 items. 4 must share. Size of share? 4 groups of 6. How many altogether?

Have students work together in groups to invent problems which are meaningful and of interest to them. Some may like to present their problems for others to solve. Student-created problems are also a valuable tool for teachers to pinpoint areas of difficulty.

Present multi-step problems for the students and encourage them to create some of their own for their classmates.

- ❒ Have students solve and create strategy problems involving the operations. For example, arrange 3, 4, and 5 in the indicated spaces to make the sentence true. ☐ x ☐ + ☐ = 19
- ❒ To encourage problem solving and problem creation, set up a second-hand shop of student-donated items. The store could be open for certain times each day, and students could role-play buying items, adding totals, making change, etc. This would provide an opportunity for students to practise the "adding on" concept for finding the difference.

For example, several items total $1.48. A student pays with a toonie. The clerk says, "$1.48," gives 2 pennies and says, "$1.49, $1.50;" gives a quarter and says, "$1.75;" gives another quarter and says, "$2.00." The difference (change) is 52 cents. The use of counting on as a strategy to solve subtraction problems should be encouraged.

- ❒ Have students solve problems involving time.
  Martha arrived at 7:45 p.m. and left 20 minutes later. What time did she leave?
  Sean finished his test at 10:50 a.m. and Jacob finished at 12:08 p.m. How much longer did it take Jacob to write the test?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance
B4.1 Ask the student to count back the change for $5.00, if the bill totalled $3.59.

Pencil and Paper
B4.2 Have the student use only 1s, 2s, and 3s, along with + signs, to get a total of 45.

B4/5.1 Ask the student to create a story problem which is represented by a particular computation, for example, 212 + 35 or 8 x 9 or 145 - 19.

B4/5.2 Provide the student with a list (e.g., food prices, lengths, scores). Ask him/her to use the list to create and solve a problem, using a particular operation.

B4.3 Ask the student to write a subtraction problem that includes $1.40 and 16¢. Solve the problem.

B4.4 Ask the student to write an addition problem that has an answer of 38 and includes the number 50.

Interview
B4.5 Observe how the student finds the time that has passed between 9:42 a.m. and 10:15 a.m.

Portfolio
B4/5.3 Have each student choose an even number between 1 and 100. Ask him/her to create and solve an addition, a subtraction, a multiplication and a division problem in which this number is either in the answer or involved somehow in the problem.

Presentation
B5.1 Have pairs of students choose a mystery object and make up a riddle about it, using a multiplication or division sentence as the clue. For example, I'm thinking of something in this room that shows 4 x 4 (legs on a group of 4 desks).

B4/5.4 Provide students with data of enrolments in a number of local schools. Ask them to create and solve problems, using the data.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

v) apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers

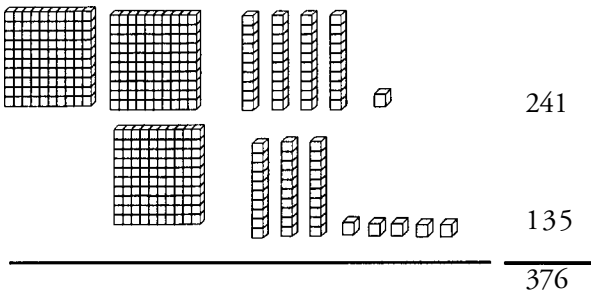
SCO: By the end of grade 3, students will be expected to

B6 add and subtract with and without regrouping (up to and including 3-digit numbers)

Elaboration - Instructional Strategies/Suggestions

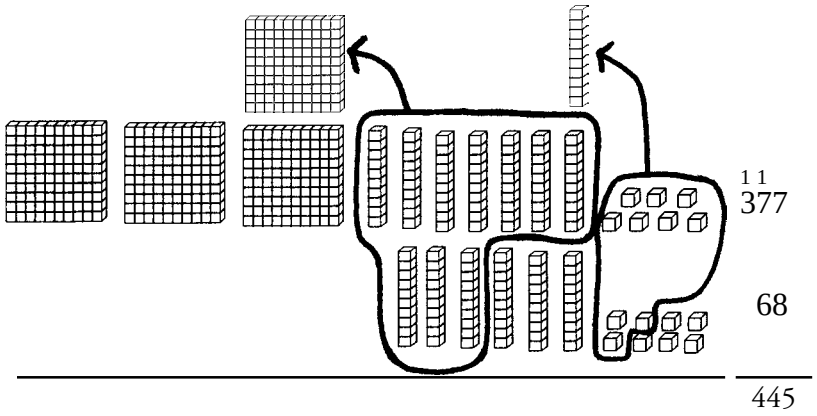
B6 Students need to develop and use alternative paper-and-pencil and mental algorithms to solve problems. They need to see these alternatives as being equally as valid as a traditional algorithm. Mixing addition or subtraction problems involving regrouping with those which do not require regrouping forces students to examine each question before selecting a strategy.

When introducing addition and subtraction with 3-digit numbers, use base-ten materials to model the operations. For 241 + 135, for example:



Students may prefer to use the front-end algorithm for the above example.

A regrouping strategy, however, might be selected for 377 + 68.



81 - 37 is the same as 84 - 40 which equals 44; or 37 and 3 more make 40, and 41 makes 81, so the difference is 44.

When adding numbers that are close to a multiple of 100, it makes sense to round the numbers and adjust later, rather than adding a number of the higher digits. 298 + 399 + 198 can be solved by adding 300 + 400 + 200, which is 900, and subtracting 5 for the answer of 895.

For subtraction, it may be easier to do the opposite. For example, 500 - 143 can be changed to 499 - 142 (or 499 - 143 and add 1 on).

Note: The word "regrouping" is preferred to terms like "borrowing" or "carrying" which conveys less meaning about the process. Many students should continue to use models to understand regrouping; for others, this will not be necessary.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B6.1 Observe the student as he/she adds 125 and 134 or subtracts 134 from 217 using base-ten materials.

B6.2 Provide the following addition or subtraction calculations for the student to complete. Ask the student to explain his/her strategy.

38 + 97 98 - 44 400 - 255

B6.3 Show the student a number of addition and subtraction questions, some of which require regouping and some which do not. Ask him/her to point out, or circle, the questions that require regrouping.

Pencil and Paper

B6.4 Have the student explain in writing why someone might first subtract 30 from 67 in order to calculate 67 - 26. Ask what would be done next.

Interview

B6.5 Display the numbers 124 and 75 with base-ten blocks. Ask the student to describe the addition process as he/she manipulates the models.

B6.6 Tell the student that Sue was to add 36 + 59 and said, "36, 96, 95." Have the student explain Sue's thinking.

B6.7 Ask why someone might find it easier to subtract 123 - 99 than 123 - 87.

B6.8 Ask the student to explain an easy way to find the sum of $1.99, $2.98 and $4.99 without using a calculator.

Portfolio

B6.9 Ask the student to prepare a display showing a variety of ways to calculate 57 - 18, indicating his/her preference and the reason for it.

B6.10 Ask the students to use a sales flyer to create some problems for his/her classmates. Have them include both problems and solutions in their portfolios.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

v) apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers

SCO: By the end of grade 3, students will be expected to

B7 recognize principles of multiplication and division

B8 relate multiplication and division facts

Children are likely to use the corresponding multiplication fact to think of a division fact... It is an interesting question to ask, "When children are working on a page of division facts, are they practising division or multiplication?" (Elementary School Mathematics, p. 150)

Elaboration - Instructional Strategies/Suggestions

B7/8 Students should explore multiplication and division situations which will help them to recognize that

- sets can be broken down into subsets

For example, 5 sets of 3  can be

4 sets of 3 + 1 set of 3  , or

3 sets of 3 + 2 sets of 3  , or

5 sets of 2 + 5 sets of 1 

To determine how many sets of 3 there are in 15, one might find

- the number of sets of 3 in 12, plus the number of sets of 3 in 3

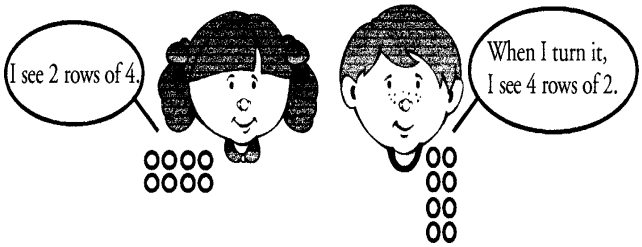
or

- the number of sets of 3 in 6, plus the number of sets of 3 in another 6, plus the number of sets of 3 in 3

- 1 x ☐ simply means one group of ☐

- order doesn't matter when you multiply because the same array can be viewed differently; for example:

2 x 4 = 4 x 2



- ☐ x 0 = 0 since many zeros still equal zero

- ☐ ÷ 1 simply means "how many 1's in ☐"

Students learn division facts by thinking about corresponding multiplication facts. They can reduce the number of separate multiplication facts to be learned by drawing on a relationship previously explored. For example, any multiple of 4 is twice the same multiple of 2. To help students learn to find one fact based on what they know about another, include, on a regular basis, questions such as How does knowing 5 x 4 = 20 help you to know 6 x 4? What other division fact could help you solve 30 ÷ 6?

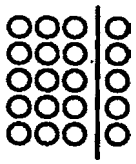
After students have had extensive experiences modelling multiplication and division operations and can use strategies to determine muliplication or division facts, they are ready to begin to commit the facts to memory. Games involving number cubes/dice help reinforce facts. It is expected that most students will have mastered the multiplication facts to at least 6 x 6 by the end of grade 3.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B7.1 Display a 5 x 4 array of objects and place a ruler on the line shown.



Ask how this shows that 5 x 4 = 5 x 3 + 5 x 1. Then ask the student to move the ruler to show another way to find 5 x 4 and explain his/her thinking, and/or provide the student with a piece of paper upon which an array (6 x 5) has been drawn. Ask him/her to fold the paper to show different ways the multiplication can be expressed.

B8.1 Have the student colour in all the places where 12 appears on a multiplication table. Ask him/her to describe what he/she observes.

Pencil and Paper

B7.2 Ask how the diagram below, which shows 3 x 4, also shows 6 x 2.

x x x x x x
x x x x x x

B7.3 Ask the student to explain how knowing 4 x 3 = 12 helps him/her solve the following questions:

4 x 6 = ☐
8 x 3 = ☐
4 x 4 = ☐

Interview

B7.4 Have the student explain why it is easy to multiply by 1 or 0.

B7.5 Ask the student what facts he/she might use to help solve 7 x 4.

B8.2 Ask the student how knowing that 5 x 6 = 30 could help in knowing other multiplication or division facts.

B7.6 Tell the student that to solve 42 ÷ 7, Alan said, "21 ÷ 7 = 3." Ask what he would do next?

B7.7 Provide the student with a copy of a multiplication grid. Point out the row beginning with 6, 12... . Ask the student to explain why the numbers increase by 6 each time.

Presentation

B8.3 Ask the student to consider the relationships between multiplication and division facts in order to decide the minimum number of facts he/she thinks it might be necessary to memorize in order to know all of them. The presentation should include an explanation of how he/she arrived at the minimum.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

vi) apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers

SCO: By the end of grade 3, students will be expected to

B9 continue to estimate in addition and subtraction situations

Elaboration - Instructional Strategies/Suggestions

B9 Estimation in addition and subtraction situations in grade 3 will extend to working with 3-digit numbers. One method for estimating is rounding. Students might sometimes round to the nearest hundred, but other times it is more appropriate to round to the nearest ten. It is important they understand that the situation should dictate how closely to estimate.

For example, if you have $255 and have to estimate the total of 2 items costing $79 and $172, one might round to the nearest ten dollars. However, if the items cost $96 and $88, estimating to the nearest hundred is sufficient.

Base-ten blocks and number lines are both suitable models to use to assist students in their initial consideration of estimation.

• Number line model: Gas stations are imagined at each multiple of 100 (or 10 if rounding to the nearest 10). To estimate 475 + 392, you would place your "vehicle" on 392 and see that you are almost at 400. Therefore, 475 + 392 is about 875. It is not always necessary to round both numbers.

• Left to right or front-end method:
The following are examples in which this method makes sense.

$$\begin{array}{r} 138 \\ +\ 245 \\ \hline \end{array}$$

370 (200 + 100 is 300, 30 + 40 is 70, for an estimate of 370.) Some students may consider the ones in their estimate. For example, the ones are a little more than 10, making the estimate 380.

$$\begin{array}{r} 476 \\ -\ 348 \\ \hline \end{array}$$

130 (400 - 300 is 100, 70 - 40 is 30, 6 and 8 are about the same, so I'll ignore them; my estimate is 130.)

Sometimes a combination of front-end and clustering works well. In the following example, add the hundreds digits first (300 + 300 + 200, or 800) and then cluster the 29, 35 and 42 together to make an additional 100, for an estimate of 900.

$$\begin{array}{r} 329 \\ 335 \\ +\ 242 \\ \hline \end{array}$$

This method is often closer to the actual answer than in rounding situations. It is important for the students to examine the question first in order to select a strategy that will not only give them an estimate near the actual answer, but one that is efficient and makes sense to them.

The goal is for students to estimate on their own, not only when the teacher or textbook requires it. They have to value the skill and they must know the teacher values it. Estimation must precede all pencil/paper and calculator calculations; therefore, it is important that students have efficient estimation strategies. Estimating is a mental activity, one that improves with regular practice. Encouraging students to share their thinking will help provide a range of strategies from which students can choose.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B9.1 Have the student toss three dice and list the six possible numbers that can be formed. Ask the student to give an estimate of how much should be added to each number to get a sum of about 1000.

Pencil and Paper

B9.2 Tell the student that 3☐4 + 5☐3 is about 900. Ask what might go in the blanks.

Interview

B9.3 Tell the students that a teeter-totter will hold up to 300 kg. Ask them to estimate the number of children their age who could safely play on it.

B9.4 Ask the student which of the following solutions is close to 500.

329 + 189 329 + 217 329 + 287

Ask the student which of the following solutions is closest to 50.

125-30 168 - 115 103 - 82

In both cases, ask the student to explain his/her thinking.

B9.5 Tell the student that you were given a subtraction question for which you estimated the answer to be 100. Ask him/her to list some numbers you might have been subtracting.

B9.6 Tell the student that you subtracted a number in the three hundreds from a number in the five hundreds. Ask what the numbers might have been if you have correctly estimated the answer to be 100.

B9.7 Ask him/her to identify situations in which an exact answer would be required and some in which an estimate is sufficient.

B9.8 Show the student the number of sports cards in James' collection.

Baseball	247
Football	124
Hockey	138

Ask the student to estimate the total number of cards in the collection and to describe the strategy used.

Portfolio

B9.9 Ask the students to interview parents/relatives/neighbours to find out when they use estimation. Have them prepare a report and share their findings with the class.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

vi) apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers

SCO: By the end of grade 3, students will be expected to

B10 begin to estimate in multiplication and division situations

Elaboration - Instructional Strategies/Suggestions

B10 Generally, in multiplication and division situations, students in grade 3 are still working with products less than 100. Therefore, estimates might more likely focus on whether the product is closer to 20, 40, 60, or 80, or the quotient closer to 1, 5, or 10.

Examples such as the following will help students develop strategies for estimating in multiplication and division contexts.

- 7 x 9 will be less than 70 because 7 tens equal 70.
- 56 ÷ 6 is less than 10 because one would need 60 to make 10 groups of 6.
- An estimate for 48 ÷ 8 would not be close to 10 because 10 groups of 8 equal 80. It will be closer to 5 because 5 groups of 8 (one half as much) are 40.
- 3 x 17 is closer to 3 x 20, or 60, than 3 x 10, or 30. A good estimate would be 50.

Many multiplication and division estimation situations deal with money; for example:
The Popsicles in the cafeteria cost 15¢ each. I have a loonie. Can I buy one for myself and each of my five friends?

Student thinking: At 10¢ each, I could buy 10 Popsicles. At 20¢ each, I could buy only 5 Popsicles. So, at 15¢ each, I can buy somewhere in the middle, between 5 and 10 Popsicles.

If erasers are on sale for 19¢, how many would you estimate one could buy with a loonie?

Estimation is a mental activity, one that becomes more precise with practice. Regular attention to estimation activities and the sharing of strategies is necessary and promotes the use of mental math.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B10.1 Have the student estimate whether the products of the following questions are closer to 20 or 60 and record how he/she knows, without actually calculating the exact answer.

3 x 8.9 7 x 8 5 x 9

Interview

B10.2 Ask the student to tell how he/she knows whether the answers to the following questions are closer to 1, 5, or 10 without actually performing the calculations.

75 ÷ 8 25 ÷ 6 23 ÷ 4

B10.3 Show the student a picture of a hexagon with a side measurement of 19 cm. Ask him/her to estimate the number of centimetres it would be around the outside of the hexagon.

B10.4 Tell the students that you bought 2 dozen hotdogs for a party for seven. Ask them to estimate to decide if this will be enough.

B10.5 Tell the students that Marta has a loonie and wants to buy three more party favours that cost 29¢ each. Ask them if Marta has enough money and to explain their thinking.

B10.6 Tell the student that you have a loonie. Ask: How do you know that I am unable to buy 5 packages of stickers that cost 21¢ each?

B10.7 Show the student the number of library books signed out by each grade level.

Grade 1 - 21
Grade 2 - 20
Grade 3 - 19
Grade 4 - 22
Grade 5 - 18

Explain that the librarian said about 100 books were signed out by the five classes. Ask the student to explain the librarian's thinking.

Presentation

B10.8 Have the students ask a grown-up to describe some situations in which they might use estimates involving multiplication and division at a food store. Ask the students to share their findings with their classmates.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 3, students will be expected to

vii) select and use appropriate computational techniques (including mental, paper-and-pencil, and technological) in given situations

SCO: By the end of grade 3, students will be expected to

B11 mentally add and subtract two-digit and one-digit numbers

B12 mentally add and subtract rounded numbers

B13 use technology to solve problems involving larger numbers

Each mental algorithm can be presented to your class, discussed, and practised briefly on different days. As new methods are introduced, some children will select different approaches for the same task. These should certainly be discussed and accepted. (Elementary School Mathematics, p. 203)
Mental algorithms develop and improve in both quality and quantity over years of practice. (Elementary School Mathematics, p. 203)

Elaboration - Instructional Strategies/Suggestions

B11 When an exact answer is required, students may choose to use a mental math strategy, a pencil and paper algorithm, or a calculator.
Continue oral drill of addition and subtraction facts required for mental calculations. To become more efficient in performing mental calculations, students need to develop a variety of strategies, including the following:

- Making a ten - For example, for 26 + 7, one might think 26 and 4 make 30, and 3 more is 33.
  This can be extended to 2-digit numbers. For example, for 37 + 26, 37 plus 20 is 57, and 3 more is 60, plus 3 is 63.
- Front-end - For example, for 47 + 8, think 40 plus 15 (7 + 8), or 55.

  Consider the following for adding a number of 2-digit numbers using the front-end approach:

  24 Students might say "20, 10, and 30 makes
  12 60. 64, 66, 67 - the answer is 67."
  31
- Counting on - When mentally calculating 50 - 19, think, "One more is 20, and 30 more make 50, so the difference is 31."
- Subtract 10 and compensate - For 31 - 8, many students will subtract 10 and add 2 back on.
- Balancing - When subtracting 29 from 54, the difference between the two numbers is the same as subtracting 30 from 55.
- Using the nearest multiple of ten and compensating - For 31 - 8, think, "30 minus 8 is 22 and 1 more is 23."
- Partner numbers - Students should know the number combinations which go together to give a sum of 10 (2 and 8, 3 and 7, etc.) and begin to recognize those that sum to 100 (25 and 75, 60 and 40, 45 and 55, 49 and 51, etc.).

It is useful to use a hundreds chart to help students visualize mental calculations. For example, when adding 11, go down one and over one, which is really 10 more plus one.

B12 Students should be expected to continue to mentally add and subtract rounded numbers; for example, 400 + 500, 100 - 80, 300 - 30, etc.

B13 Students should be encouraged to use calculators as tools in problems requiring tedious calculations or those beyond their capabilities.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B11.1 Ask the student to add mentally as you draw numbers from a bag, and to stop you when the sum has passed 40.

B13.1 Ask the student to show how he/she would use a calculator to find 4234 + 187.

Pencil and Paper

B11.2 Have the student make a list of calculations involving 2- and 3-digit numbers which would be quicker to do mentally than on paper or with a calculator.

B12.1 Present calculations, such as the following, orally (or on an overhead), and ask the student to write only the answer. Allow only a few seconds for each question. (e.g., 300 + 600, 200 - 40, 200 + 80 + 30, 220 - 40)

Interview

B11.3 Ask the student to describe a strategy for calculating 48 - 9 (or 76 + 11) mentally.

B11.4 Show pictures of several items with prices less than a dollar. Have the student start with $2.00 and buy as many items as possible. Ask him/her to tell you how much is left after each purchase.

B11.5 Tell the student that to find 37 - 8, Rita said, "37, 27, 29." Have the student explain Rita's thinking.

B11.6 Tell the student that you had 3 quarters and spent 48¢. Ask him/her to explain how one might mentally figure out the change.

B11.7 Ask the student to give the partner number for

82 (18)	49 ( )	65 ( )
75 ( )	60 ( )	91 ( )

B11.8 Tell the student that to subtract 7 from 51 John said that he would rather subtract 6 from 50. Ask him/her if this works and why.

Portfolio

B11.9 Have the student explain why, when adding a single-digit number to a number in the fifties, the answer has to be either in the fifties or sixties.

Suggested Resources

THREE-PATTERNS

Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to

i) recognize, describe, extend, and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric, and measurement)

SCO: By the end of grade 3, students will be expected to

C1 recognize the pattern implicit in our place-value system

C2 recognize and create geometric patterns

Almost all patterning activities should involve some form of physical materials to make up the pattern. This is especially true of repeating patterns in grades K - 4 . . . (Elementary School Mathematics, p. 373)

Elaboration - Instructional Strategies/Suggestions

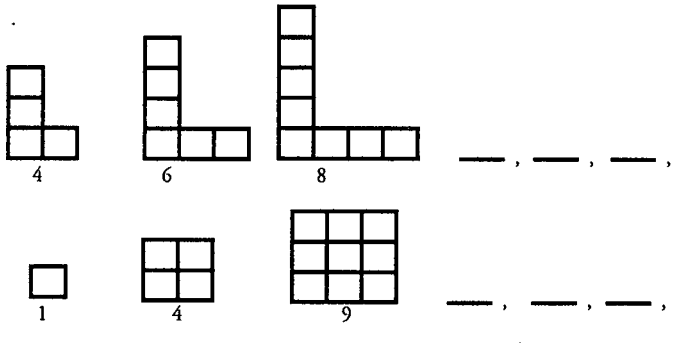
C1/C2 Students at this level have learned that the digit representing hundreds is to the left of the one representing tens in our place-value system, and it takes ten of one unit to be equivalent to one of the units to its left. Students can use this pattern to understand how we extend the place value system to decimals; that is, if 10 ones make 1 ten, 10 tens make 1 hundred, etc., and since 10 tenths make 1, the place to the right of the ones should be tenths.

- Students need opportunities to connect patterns to number ideas. Work with patterns found in the hundreds charts should be continued and expanded.

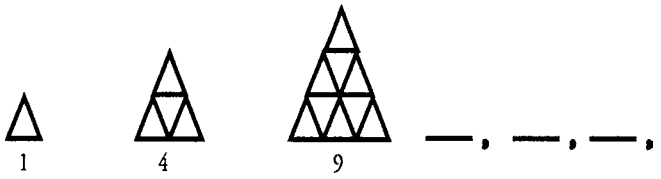
❒ Ask the students to respond orally in giving a number that is 100 more (100 less, 10 more, 10 less) than a 2- or 3-digit number that you provide.

❒ Invite students to use pattern blocks to begin a pattern for someone else to continue. The pattern should be related to the number of sides and vertices of the shapes.

❒ Have students create "L" numbers and square numbers, using tiles.



❒ Ask students to create triangular numbers with △ pattern blocks.



- When studying slides and flips, it is natural for students to observe and create patterns with shapes based on these transformations.

For example: 

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C2.1 Give the student pattern-block pieces and ask him/her to create and describe a pattern showing flips.

C2.2 Ask the student to find, in the classroom, examples of patterns created by slides, or flips, or a combination of the two.

C2.3 Ask the student to use toothpicks to continue the pattern shown below:

4 7 10

C2.4 Show a picture of the following pattern:

Ask the student to use the Δ pattern blocks to recreate the pattern and use "slide flip" language to describe the pattern.

Paper and Pencil

C1.1 Give problems such as those below and ask the student to complete to make a pattern.

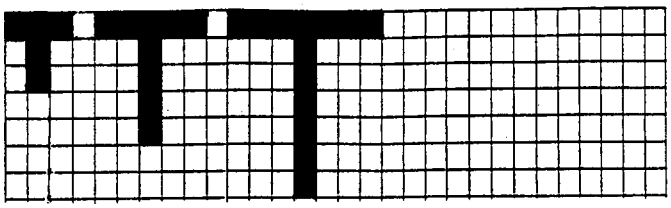
475, 575, ____, ____
233, ____, 253, ____
420, 440, ____, ____
556, ____, ____, 889

Interview

C1.2 Ask the student to explain why hundreds come directly to the left of tens in our place-value system.

Portfolio

C2.5 Provide cm^2 paper for the students. Have them use coloured pencils to continue the pattern. Ask the students to create their own growing patterns.



Suggested Resources

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to

ii) use patterns to solve problems

SCO: By the end of grade 3, students will be expected to

C3 use and recognize the patterns in a multiplication table

Elaboration - Instructional Strategies/Suggestions

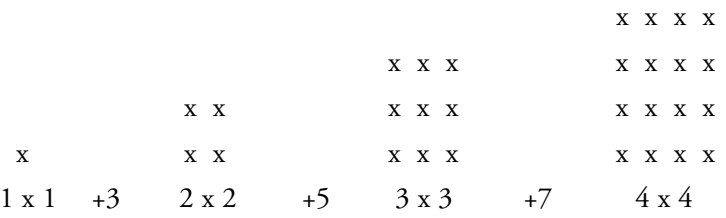
C3 Students should be encouraged to find and explain patterns that occur in the multiplication grid. It is important that students understand they can use these patterns to determine unknown products or quotients.

Multiplication Grid

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

For example, students might note that

- the numbers in each row and column increase by the same amount
- the numbers in each row increase by an amount one greater than the numbers in the previous row
- the square numbers are found on the left-right diagonal
- the numbers on the left-right diagonal increase by 1, 3, 5, 7,...



- row 4 is double row 2, row 6 is double row 3
- when you add the corresponding products of rows 2 and 3, you get the products in row 5; for example, 2 x 4 (8) plus 3 x 4 (12) is the same as 5 x 4 (20)
- when you "cross multiply" any 4 numbers that form a square on the grid, the product is always the same; for example, 2 x 6 = 3 x 4

- also, when you "cross add" these numbers and subtract the sums, you get 1
- the grid is symmetrical (i.e., numbers under the left-right diagonal are the same as the numbers above it)

❒ Ask students to look for the even and the odd numbers and see if they can find the pattern.

❒ Encourage students to examine a hundreds chart to find patterns.

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

C3.1 Ask the student to fill in the missing numbers, explaining the reason for each choice.

4, 8, ___ , 16, 20
5, ___ , 15, ___ , 25
3, ___ , ___ , 12, 15

Interview

C3.2 Provide the student with a multiplication grid. Ask him/her to describe some of the patterns he/she observes.

C3.3 Ask the student to examine the 9 times table and tell what pattern(s) he/she notices.

C3.4 Ask the student to show how one could use the multiplication grid to practise skip counting.

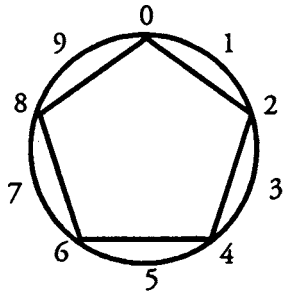
C3.5 Ask the student to explain why some columns/rows have both even and odd numbers.

C3.6 Ask the student to use the muliplication grid to explain why 4 x 5 plus 2 x 5 is the same as 6 x 5.

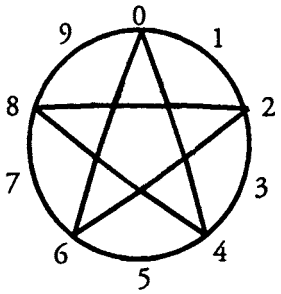
Portfolio

C3.7 Have the student create a visual pattern of the final digits of the numbers in each of the multiplication tables. In each case, the final digit of one multiple is connected by a line to the final digit of the next.

For example:



Multiples of 2



Multiples of 4

The student should record any observations that he/she makes. Ask the student to create a visual pattern for another times table.

Suggested Resources

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 3, students will be expected to

iii) represent mathematical patterns and relationships in informal ways, including via open sentences (i.e., statements with missing addends)

SCO: By the end of grade 3, students will be expected to

C4 record a repeated addition pattern using multiplicative notation

C5 recognize the meaning of open sentences of the forms:

a x b = ☐

a x ☐ = c

☐ x b = c

Elaboration - Instructional Strategies/Suggestions

C4/C5 When faced with a situation involving repeated addition, students should recognize that the addition can be written as a multiplication in which the first factor normally tells how many times the addend is repeated and the second factor represents the addend.

For example, 5 + 5 + 5 + 5 + 5 + 5 + 5 can be represented as 7 x 5.

❒ Give students questions such as the following: There were 3 muffins in each package and I bought 6 packages. How many muffins did I buy? Ask the students to model the question, to skip count to determine a response, and to write the repeated addition pattern as well as the corresponding multiplication notation: 3 + 3 + 3 + 3 + 3 + 3 or 6 x 3. (Even though 6 x 3 = 3 x 6, it is important that 6 groups of 3 be written 6 x 3.)

❒ Provide opportunities for the students to look for patterns such as

4 packages	3 + 3 + 3 + 3	4 x 3	12
5 packages	3 + 3 + 3 + 3 + 3	____	____
6 packages	_____	____	____
7 packages	_____	____	____

Although open multiplication sentences with missing factors, particularly with a missing first factor, are more challenging for students to solve than open sentences with the product missing, students need experience with both types. The following type of activity helps students to understand open multiplication sentences.

❒ Provide practice in stating the meaning for equations.

4 x ☐ = 24	Four sets of how many make 24?
☐ x 5 = 15	How many sets of 5 equal 15?
3 x 6 = ☐	3 sets of 6 is how many?

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

C5.1 Ask the student to solve the following:

4 x 3 = ☐
7 x ☐ = 21
☐ x 4 = 20

Have him/her show the meaning in a sketch or diagram.

C4.1 Have the student write problems which show the difference between 8 x 4 and 4 x 8. Ask him/her to explain why the answers (products) are the same.

C4.2 Ask the student to fill in the missing sections of the chart on baseball.

There are 9 members on a baseball team.

	9 + 9 + 9 + 9	4 x 9	
3 teams	9 + 9 + 9		
6 teams		6 x 9	
			45

Interview

C4.3 Ask the student to explain how to calculate 6 + 6 + 6 + 6 + 6 + 6 + 6 without doing the addition.

C5.2 Ask the student to create three different open multiplication sentences for which the solution is 12.

C5.3 Show the student the following equations and ask him/her to give the meaning for each.

6 x ☐ = 18
☐ x 5 = 20
4 x 4 = ☐

Suggested Resources

Shape and Space

General Curriculum Outcome D:

Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to

ii) identify and use non-standard and standard units of measurement and appreciate their role in communication

SCO: By the end of grade 3, students will be expected to

D1 estimate and measure length in metres, decimetres, and centimetres

D2 estimate and measure capacity in millilitres and litres

. . . Estimation in measurement activities helps students focus on the attribute being measured, adds intrinsic motivation, and helps develop familiarity with standard units. (Elementary School Mathematics, p. 310)

Elaboration - Instructional Strategies/Suggestions

D1 Students should continue to estimate and measure lengths in metres and centimetres. They should also be introduced to the decimetre as the unit equivalent to 10 cm or 0.1 m. Although the decimetre is not commonly used in everyday life, it is a convenient size unit for students to use in the classroom. It is represented easily with a base-ten rod. By laying out 10 centimetre cubes, students can see that the line has the same length as 1 rod. They should also line up 10 rods along a metre stick to see that the two are equal.
Note: Measurement is an ideal context for the development of decimal concepts (6 dm = 0.6 m).

The following are some worthwhile activities to reinforce measuring lengths with different units:

- Set up a mini-Olympics in which students compete in events such as a kleenex kick, a penny thumb toss, and a cotton ball puffing. Have students measure results to the nearest decimetre, and then record and compare them.
- Have the students measure the width of the front of the classroom. Ask them to estimate how many students would be needed, lying head to toe, to cover this distance. How many would be needed if they all stretched their arms over their heads, fingers to toes? Ask students to explain what things they must consider when estimating.
- Have students estimate and then determine how much money they would have if they had a metre of pennies. Nickels? Dimes? Quarters? Loonies? Ask how many "metres of dimes" they would need to buy a bicycle.

D2 Students should continue to estimate and measure capacities, using the litre. They should also be introduced to the millilitre. Graduated measuring cups can be used to provide students with a sense of 25 mL, 100 mL, 250 mL, etc. Students should realize that millilitres are extremely small units and recognize that cans and containers usually contain hundreds of millilitres. Graduated medicine cups which typically accompany children's medication or medicine droppers also provide good examples of things measured using a small number of millilitres.

Note: A cube the size of a base-ten unit cube will hold a mL of liquid; one the size of the thousand block will hold 1 litre, or 1000 mL.

- Have students discuss which unit (mL or L) is more likely to be used in measuring
  - a dose of medicine
  - pop in a can
  - punch in a punch bowl
  - water in a bathtub

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D1.1 Ask the student to estimate the heights and widths of some objects in the classroom using a metre reference (e.g., doorknob height). Have him/her check the estimates by doing the actual measurement.

D2.1 Ask the student to find the average capacity of a cereal bowl.

Interview

D2.2 Ask the student to tell how he/she might estimate the capacity of a bathtub.

D1.2 Ask the student to find objects in the classroom that are about a decimetre in length, 2 decimetres in length, etc. Ask the student to compare and describe a decimetre in relation to a ruler and to a metre stick.

D1.3 Tell the student that a bald eagle was measured to be 109 cm from beak to tail. Ask him/her to estimate and show how long that would be and then to check the estimate by measuring.

D1.4 Ask the student to show you, with his/her fingers/hands, how long 6 cm might be. 24 cm? 32 cm? 58 cm?

D1.5 Tell the student that a child is 6 dm tall. Ask him/her for an approximate age of the child.

Presentation

D1.6 Ask small groups of students to estimate the length of each person's stride. Ask them to predict who has the longest stride and to give reasons for their selection. Have students develop a table to compare long jump lengths to stride lengths and present it to the class.

D2.3 Ask the students to devise a plan to determine how much water is wasted when the tap is not turned off while brushing one's teeth. Have them use this information to find how much water is wasted over a longer period of time. Ask them to find how much water would be wasted if all the students in the class neglected to turn off the tap for one brushing.

Portfolio

D2.4 Have the student check containers at home/at the stores/in flyers and make a chart of the various sizes for different products.

D2.5 Tell the students that it has been suggested that we need to drink at least six glasses of water a day. Ask: About how many litres would that be? What would a container look like that would hold enough water for you for a week? Have the students write a report explaining how they solved the problem.

D2.6 Have students keep a personal record (in millilitres) of the beverages they consume in one week. Encourage the students to translate the total consumption for the week into litres and choose a container (or combination of containers) which would hold the amount.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to

ii) identify and use non-standard and standard units of measurement and appreciate their role in communication

SCO: By the end of grade 3, students will be expected to

D3 estimate and measure mass in grams and kilograms

D4 estimate and measure area in non-standard units and square centimetres

Let students measure to check estimates. However, it is only necessary that one or two students or one team do the measurement if the focus of the activity is on estimation. If all students are required to follow their estimates with a measure, they may correctly wonder why they bothered estimating. (Elementary School Mathematics, p. 311)

Elaboration - Instructional Strategies/Suggestions

D3 Students should continue to estimate and measure masses, using the kilogram. They should also be introduced to the gram. Students need to understand that grams are used to measure very light objects. As with all measurement units, it is important that the students have a personal benchmark/referent for gram and kilogram.

Note: A cube the size of a base-ten unit cube will hold 1 ml of water which has a mass of 1 gram; the 1000 block (dm^3) will hold a litre of water which would have a mass of 1 kilogram.

Suggested mass activities:

- ❒ Could you eat a 1 kg cantaloupe? 1 kg of popcorn?
- ❒ If sliced meat sells for $3.50/g, is it expensive?
- ❒ Measure 20 g of unpopped popcorn. Will the mass be greater, the same, or less after it has been popped? Have students compare the volume of the popped and unpopped popcorn.
- ❒ Have students discuss which unit (g or kg) is more likely to be used in measuring:
  - a bag of potatoes, a box of paper clips, a baby
- ❒ Estimate and then measure the mass of different objects in the classroom.

D4 Students should use non-standard units and square centimetres to measure the area of common items. For example, they might lay a transparent grid or index cards over an object to see about how many units it takes to cover it. They might also try filling the bottom of a box with marbles or tennis balls.

As with all measurement activities, encourage students to estimate prior to doing the actual measurement.

Eventually, students should see that, if they use different grids or objects, they will get different numbers for the areas. At this point, introduce a centimetre grid and explain that a square centimetre is one standard unit for finding area. Students should have numerous opportunities to find the area of different surfaces, using square centimetres.

They should also understand that objects of different shapes or perimeters can have the same area.

The following are some interesting activities involving area:

- ❒ Have students devise a way to measure the surface area of an apple. (They could place the peels on a transparent grid and cover with tape.) Several apples could be measured this way and ordered by surface area.
- ❒ Have students find the area of an animal drawn on grid paper.
- ❒ Have students compare the areas of different-sized pieces of paper, pattern blocks, or envelopes.
- ❒ Have students find the area of typical school photographs.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D4.1 Have the student arrange a loop of string into three shapes. Ask him/her to estimate and then determine which of the shapes has the greatest area.

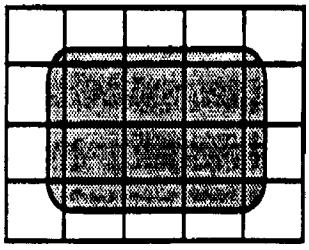
D4.2 Have the student find the area of an envelope, using postage stamps as units. Request an estimate before he/she measures.

D3.1 Ask the student to estimate and then measure, using a pan balance, the mass of a selected object.

Paper and Pencil

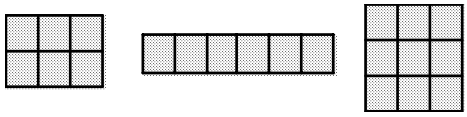
D3.2 Ask: Which unit, g or kg, would be the most appropriate to use to measure the mass of
- a box of paper clips?
- a carton of books?
- a dog?

D4.3 Ask the student to explain how he/she knows that the area of the shape shown has to be between 6 and 20 units. Ask also for an estimate.



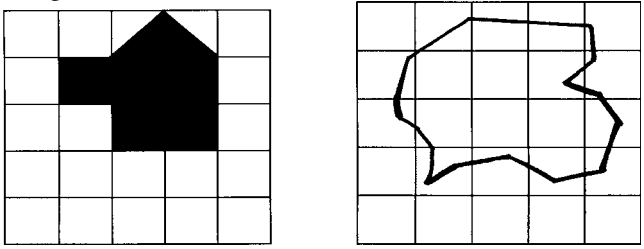
Interview

D4.4 Show 3 rectangles, such as



Ask the student which has the greatest area and to explain why.

D4.5 Ask why it is easier to find the area of the shape on the left than the one on the right.



D4.6 Ask the student why the area of the shape does not show 6 units.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to

iii) estimate and determine measurements in everyday problem situations and develop a sense of the relative size of units

SCO: By the end of grade 3, students will be expected to

D5 solve problems involving kilometres

D6 use appropriate units for capacity and mass

The purpose of an interview is to uncover how students think about mathematics, so provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (Mathematics Assessment, p.29)

Elaboration - Instructional Strategies/Suggestions

D5 Students should know that 1 kilometre is 1000 metres. To fully appreciate how far 1000 metres is, it is important that they have a benchmark for the distance; for example, from the school to the town hall. Experiences with a trundle wheel are worthwhile.

Students should solve problems which are meaningful and of interest to them, involving kilometres.

It is important to include situations in which they can estimate a response, then check by doing. For example:

- Could you run 1 km without stopping?
- How many trips around the playground would you need to walk to cover 1 km?
- Are there enough students in your school to reach 1 km if they stood side by side, arms extended?
- How many of your classrooms placed end-to-end would make a kilometre?

❒ Ask the students to predict how long it will take them to walk 1 kilometre. Then take the students on a 1 km walk, and check their estimates.

❒ Students might like to design a table of other distances familiar to them (e.g. distances from homes to school) and predict how long it would take to walk, bike, etc.

D6 Students should recognize which capacity unit (mL or L) is appropriate in a given situation. Include situations in which something is measured in litres in one case and millilitres in another case. For example, a container of juice for a baby is measured in millilitres while a container of juice for a family could be measured in litres.

❒ Have students check food containers to make a list of things measured in millilitres and another of things measured in litres.

❒ Arrange a display of food containers, some with labels indicating millilitres and others with labels indicating litres.

Students should recognize which mass unit (gram or kilogram) is appropriate for measuring the mass of a specific item. It is helpful for students to investigate how everyday items are measured (e.g., food items). Include items which are small and dense as well as those which are large and porous.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D6.1 Ask the student to estimate, then verify, how much water will pass through a given funnel in 10 seconds.

D6.2 Ask the student to estimate and then find the capacity of a cereal bowl.

D6.3 Ask the student to estimate the mass of a textbook and verify the estimate.

Paper and Pencil

D6.4 Have the student list five items that would be measured in grams rather than kilograms.

D6.5 Have the student draw a picture of an object that he/she believes to have a mass of about 5 kg.

Interview

D5.1 Have the student name a place that is approximately 100 km away. (It is important that some benchmark distances have been presented in class.)

D6.6 Ask the student to name three items normally measured in millilitres and three measured in litres.

D5.2 Tell the student that the tiger shark can travel about 50 km in an hour. Ask: About how far is 50 km from the school? About how long would it take you to walk that far?

Portfolio

D5.3 Provide a list of distances (in kilometres) to various places. Ask the student to plan a trip that does not exceed 500 km.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 3, students will be expected to

iii) estimate and determine measurements in everyday problem situations and develop a sense of the relative size of units

SCO: By the end of grade 3, students will be expected to

D7 read digital and analog clocks to the nearest five minutes

D8 continue to solve a wide variety of measurement problems

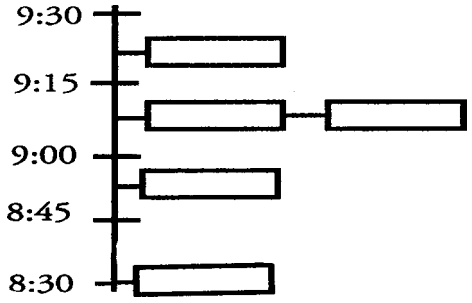
Elaboration - Instructional Strategies/Suggestions

D7 Students should read times on both digital and analog clocks to provide information about relevant situations, such as

- comparing start and finish times to determine how much time has passed
- focussing on times when special events are going to happen

To help them read time to the nearest five minutes, it is important that students are comfortable with skip counting by 5. This provides the opportunity for students to relate the numbers on a clock to the five times table. It is also helpful to use a clock that shows not only the numbers from 1 to 12, but also the minute amounts from 5 to 55 beside the numbers from 1 to 11.

❒ Have students track events throughout a specific day by means of a time line divided into 15-minute segments.
Students should record the time of the activity or event and note it at the appropriate spot on a time line; for example:



D8 Students should continue to explore interesting, real-life measurement situations. It is important for students to recognize the approximate nature of measurement, in that we are limited by the precision of the measuring instrument. To encourage this, use approximate language (about 12 cm long, a bit more than 1.5 litres, not quite a kg).

Students could, for example,

❒ plan a class party and determine the number of 1 litre cans of juice they need to buy
❒ make a cover for a chosen book from an oversized sheet of paper
❒ use a roll of dental floss to measure, approximately, the perimeter of the gym
❒ determine the size of the containers required to hold 1 kg of potatoes, macaroni, pattern blocks, etc.
❒ compare start and finish times of situations to determine how much time has elapsed (e.g., time between first bell and recess)
❒ track events throughout a specific day by means of a time line divided into 15-minute segments

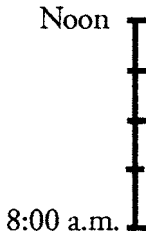
GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D7.1 Ask the student to move the hands of an analog clock to match the time shown on a digital clock.

D7.2 Ask the student to record the temperature on a chart at five different times during the morning.



Paper and Pencil

D7.3 Have the student make a list of the times when the minute hand and the hour hand are just about on numbers that are one apart.

D7.4 Have students work in pairs to set up a schedule in which every student will get 10 minutes on the computer, starting at 8:30 a.m. Ask them if all students can have time on the computer before noon and, if not, how long it will take to finish after lunch. At what time will the last one finish? (Remind them to leave time for recess.)

Interview

D7.5 Ask: What time might it be if the minute hand and hour hand are opposite one another?

Presentation

D8.1 Have students work in pairs to figure approximately how many children would have to lie down in a line to make a distance of 1 km. Ask them to record how they found their answers and to share it with the class. (Different line formations are likely to be presented, for example, head-to-toe, arms outstretched over their heads.)

Portfolio

D8.2 Tell the students that you have a map that takes only one page the size of a sheet of loose-leaf. On the map, 1 cm represents 1 km. Ask them if all of their community can fit on the map and have them provide a written rationale.

Suggested Resources

Shape and Space

General Curriculum Outcome E:

Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 3, students will be expected to

E1 continue their development of spatial sense with emphasis on perceptual constancy

Spatial understandings are necessary for interpreting, understanding, and appreciating our inherently geometric world. Insights and intuitions about two- and three-dimensional shapes and their characteristics, the interrelationships of shapes, and the effects of changes to shapes are important aspects of spatial sense. Students who develop a strong sense of spatial relationships and who master the concepts and language of geometry are better prepared to learn number and measurement ideas, as well as other advanced mathematical topics. (Curriculum and Evaluation Standards for School Mathematics, p. 48)

Elaboration - Instructional Strategies/Suggestions

E1 Perceptual constancy is the ability to recognize figures or objects in space regardless of size, position, or orientation. It involves the recognition of shape or size as stable even if it appears to be different to the observer. For example, a table top can appear to be a parallelogram or a trapezoid when viewed in certain ways, but our experience has taught us it is a rectangle. Similarly, if a shape is reduced or enlarged (as in photographs), the shape is maintained.

- ❒ Give students different shapes made from five or six centicubes. Have them make the same shapes, using larger multilink cubes. Have students make complex figures on geoboards and make copies on much smaller geopaper. Discuss what has changed and what has stayed the same.
- ❒ Show a pattern-block design that uses 10-12 blocks on an overhead projector. Have students replicate the design at their desks using pattern blocks. Discuss what aspects are constant between the overhead version and those created by students.
- ❒ Give the students three isometric drawings of shapes, two of which are different views of the same shape (see E1.4). Have them predict which are the two views of the same shape and then build the shape with cubes to check their prediction.
- ❒ Have the students draw what they see when they look at familiar objects from the top. Have the other students see if they can determine what the objects are, given only these top views. Some objects will be difficult to determine with only a top view.
- ❒ Put three different shapes made from five multilink cubes on display. In a bag, place a replica of one of these shapes made from centicubes. Have the students determine by feeling which one of the three it is.

Teachers should continue to provide experiences that focus on other spatial abilities. For example, using pattern blocks, a visual- discrimination activity could be to find the difference(s) between two designs made with 8-10 blocks; a visual-memory activity could be to have the children replicate, from memory, a design that they have been shown only briefly; a spatial-relations activity could be a pattern-block puzzle. As well, these abilities are inherent in the development of geometric concepts.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

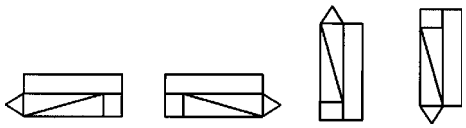
E1.1 Make a design on the overhead, using four or five tangram pieces. Have the students replicate this design, using their own pieces. Extend by showing the design for shorter periods of time before the students try to build it from memory.

E1.2 Place a variety of 3-D shapes in a bag. Have students feel the shapes in the bag, and describe them; have other students try to name them.

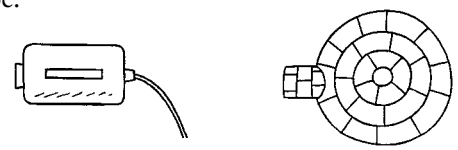
Paper and Pencil

E1.3 Ask: What 3-D shape has these faces?

E1.4 Ask: Which of the following are pictures of the same figure? Have the students predict and use tracing paper to check.



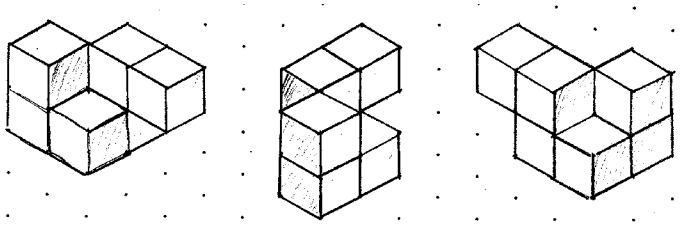
E1.5 Tell the student that these are top views of objects. Ask what objects they might be.



(Answer: While most students would see the second picture as the top view of an igloo, there could be a variety of suggestions for the first picture, including a gas pump, a toaster, or a camcorder.)

Interview

E1.6 Ask the student to predict which of the following are two views of the same shape made with six cubes? Have them build them to check.



Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

i) explore and experiment with geometric shapes and relationships

SCO: By the end of grade 3, students will be expected to

E2 recognize and represent angles that are less than/ more than right angles

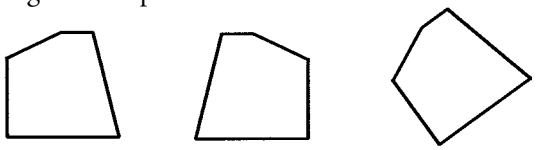
E3 recognize, name, describe, and represent congruent angles and congruent polygons

Elaboration - Instructional Strategies/Suggestions

E2 Students are not measuring angles in degrees at this stage; they are comparing angles by sight. The concept of right angle (corner) was introduced in grade two, and now other angles in shapes will be described as less or more than a right angle. Teachers should have a variety of convex polygons, including familiar triangles and quadrilaterals for students to examine.

- ❒ Provide the students with a small rectangular piece of paper or card. Discuss right angles in relation to the corners of this. Have pairs of students gather a set of six pattern blocks and, using the sheet of paper or card, find shapes that have right angles, angles less than right angles, and angles more than right angles.
- ❒ Examine the different members of the quadrilateral "family" (trapezoid, parallelogram, rectangle, rhombus, square, and kite) for these angles. Have students arrange toothpicks to make right angles and a variety of angles less/more than right angles. Find examples of angles in the classroom/school that would be right, less than right, and more than right. Students could use an index card or sheet of paper as a referent for the right angle to do comparisons.

E3 Through a variety of experiences, students should establish that congruent polygons are a perfect match (they would fit on top of one another exactly) regardless of their relative positions or orientations. Congruent angles in shapes occur when the corners match.



Polygon | Congruent but different orientation | Congruent but different position

Have students match the corners (angles) of the pattern blocks to find ones that are congruent. Compare the angles in squares and rectangles. Compare the angles in the three different-sized triangles in the tangram set.

On a sheet with pictures of a variety of polygonal picture frames in different positions and orientations, ask students to find sets of congruent frames. They should try by sight and then check, either using tracing paper or cutting them out to match.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E2.1 Have the students make shapes on a geoboard that meet different criteria. For example, make a shape that has two right angles; make a shape that has one angle that is more than a right angle; make a shape that has all angles less than a right angle.

E2.2 Give the students pictures of 12 different angles in different positions on the paper. Have them sort the angles into the three sets - right angles, more than right angles, and less than right angles.

E2.3 Have the students classify the angles on the faces of 3-D shapes.

E3.1 Ask a group of 10 students to make a rectangle anywhere on their geoboards. Have the students sort these rectangles into congruent groups.

E3.2 Prepare a set of cards with drawings of different polygons, some of them congruent but in different positions/orientations, and some similar (reductions, enlargements). Have the students match the congruent polygons. Discuss why they are/are not congruent.

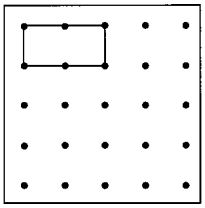
E3.3 Show the students a polygon on a geoboard. Have them make a slide, flipped, or rotated image of it on their own geoboard. Are they congruent? Compare by copying on geopaper and cutting them out.

Paper and Pencil

E3.4 Give the students a picture of a shape (for example, a parallelogram) and some tracing paper. Ask them to draw three parallelograms congruent to this one but in different positions/orientations. Have them draw one that is not congruent and describe how it is different from the others.

Presentation

E3.5 Have the students investigate how many rectangles congruent to this one can be made on a geoboard.



Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

ii) describe, model, draw, and classify 2-D figures and 3-D shapes

SCO: By the end of grade 3, students will be expected to

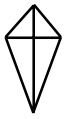
E4 recognize, name, describe, and represent kites, and some concave, convex, and regular polygons

Elaboration - Instructional Strategies/Suggestions

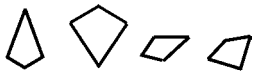
E4 Show the students a real traditional flying kite and read a story about kites to help the students make strong associations with the shape called kite.

Show them a variety of kite shapes, and some shapes that are not kites, asking the students why they think they are not kites.

Traditional kite



Other examples of kites:



These are not kites:

❒ Prepare two ropes of 1m and two ropes of 1.5m. Have four students choose ropes and, standing, hold them to make a kite. Which ropes are the same length? Have them make a rectangle using the same ropes. How were the positions of the ropes changed?

The students will have worked with some of the regular polygons (the triangle, square, and hexagon in the pattern blocks, for example); however, the name "regular" will be new. Familiarity with the term could be developed by displaying shapes that are regular and those that are not, asking the students to formulate the term's meaning. All sides must be congruent, and all angles must be congruent for the polygon to be named "regular." Make associations to things in the real world; for example, a STOP sign is a regular octagon.

❒ Have the students make all of the different polygons that can be formed with the four triangles made by cutting a square along both diagonals. These should be sorted into two groups (concave and convex polygons) that you name. Ask the students to try to describe concave and convex. At this stage, students would probably define them by their overall appearance. For example, concave polygons have an indentation or are "caved in."

Note: A kite is a quadrilateral with two pairs of congruent adjacent sides. While a rhombus is a kite, at this level treat rhombus and kite as separate classes. Students should not be given formal definitions at this stage of their development.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E4.1 Give students two long straws and two shorter straws. Have them arrange them on their desks to make a kite. Have them examine each other's to see how many different kites they can find. Ask: How are they the same and how are they different?

E4.2 Have the students use pattern blocks to make polygons that are concave and polygons that are convex.

E4.3 Give the students 15 toothpicks (all the same length). Have them arrange five of them to make a convex pentagon that is not regular, five to make a concave pentagon, and five to make a regular pentagon.

E4.4 Ask students to examine a collection of 3-D shapes to find ones that have faces that are regular polygons. Have them record the ones they find by tracing them on paper.

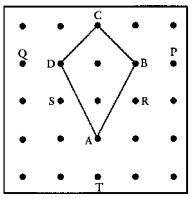
Interview

E4.5 Ask the student to compare a kite to a parallelogram and tell why they look different.

Presentation

E4.6 Have the students use a five-pin by five-pin geoboard to make as many different kites as they can. (Remind them that kites are not different if they are congruent to each other.) Record each kite found on geopaper.

E4.7 Have the student make kite *ABCD* on a geoboard as shown. If the elastic is moved from peg *A* to peg *T*, will the shape be a kite? If *B* is moved to *P*, will the shape be a kite? (How could you move another peg to make it a kite?) If the elastic is then moved from peg *P* to peg *R*, would the shape be a kite? (Move the elastic from peg *Q* to make the shape a kite.)



Portfolio

E4.8 Ask the students to cut a square as shown:

Have them use the three pieces to make two examples of concave polygons and trace them; also make two examples of convex polygons and trace them as well.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

ii) describe, model, draw, and classify 2-D figures and 3-D shapes

SCO: By the end of grade 3, students will be expected to

E5 recognize, name, describe, and represent different prisms and pyramids

E6 cut and assemble net patterns for pentagonal and hexagonal prisms and pyramids

E7 build skeletons of various prisms and pyramids to focus on edges and vertices

Virtually every activity that is appropriate for K-8 geometry should involve some form of hands-on materials, models, or at least paper such as graph paper or dot paper that lends itself to easy spatial explorations. (Elementary School Mathematics, p. 324)

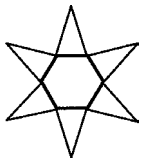
Elaboration - Instructional Strategies/Suggestions

E5 The following suggested activities will build on the students' previous experiences with prisms and pyramids. Prisms and pyramids should be presented together to have the students form impressions of these two "families" of shapes, both of which have "first names" based on the shape of their bases. These could include trapezoidal, pentagonal, hexagonal, and octagonal.

Commercial 3-D shapes usually have a variety of prisms and pyramids. While the pattern-block pieces are prisms, they have been treated as 2-D shapes; however, stacking a number of triangles, squares, or hexagons would provide examples of different prisms. Stacking the various polygons from the logic blocks would also provide examples of prisms. This stacking would help students visualize/conceptualize the uniform nature of prisms.

E6 Students should be given copies of nets of these shapes to cut out and fold up. They should be encouraged to unfold them and examine the 2-D shapes that are connected to make each net. Have them visualize the folding up and unfolding.

For example, a net for a hexagonal pyramid would be



E7 Miniature marshmallows/baking gums and toothpicks are one source of material for students to build skeletons. Each marshmallow would serve as a vertex and each toothpick, an edge.

- ❒ Have students make skeletons of each prism and the corresponding pyramid. Discuss how they started off the same but then proceeded differently. Compare the number and shapes of faces, the number of vertices, and edges. Can they detect any patterns while they are building them? For example, they always used one more marshmallow as a vertex after the base of a pyramid was made.
- ❒ Give students face, edge, and vertex clues to have them name the 3-D shape. Have students feel a shape in a bag and describe it by its faces, edges, and vertices while others try to guess what the shape is.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

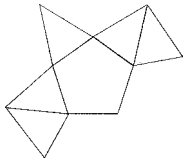
Performance

E5.1 Put a variety of prisms and pyramids in a bag. Have the students, using only their sense of touch, describe each shape and name it before bringing it out of the bag to check.

E5.2 Place a hexagonal prism and a hexagonal pyramid beside one another. Ask the students to name them. Ask them to tell you some things that are the same about them; some things that are different.

E6.1 Present the students with nets of a prism and a pyramid that have the faces joined in a different way from the ones they have cut out before. Ask them to predict what shape it would fold up to make. Have them cut it out and fold to check their prediction.

Example:



E7.1 Ask the students to use toothpicks and marshmallows to build a prism and a pyramid, using the same number of marshmallow vertices for each.

E7.2 Play "Who am I?" by giving clues based on faces, edges, and vertices. (For example: I have five faces. I have six vertices. What prism or pyramid am I?)

Paper and Pencil

E7.3 Give the student two shapes and ask them to write a comparison between them on the basis of edges, faces, and vertices.

Interview

E7.4 Show the students a toothpick and marshmallow pentagon. Ask them to describe what they would need to make a pentagonal pyramid.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

iii) investigate and predict the results of combining, subdividing, and transforming shapes

SCO: By the end of grade 3, students will be expected to

E8 predict the results of combining triangles and/or quadrilaterals

Elaboration - Instructional Strategies/Suggestions

E8 Ultimately, students should try to visualize what shape would be formed by combining triangles and quadrilaterals to make a variety of polygons and to make a variety of 3-D shapes. They might be able to do this for some combinations; however, many others will require a trial-and-error approach. Teachers should encourage the students to try to predict before they physically combine the shapes. Afterwards, the students should take the shapes apart and "in their minds" put them back together. The importance of this predicting and reconstructing in the development of spatial visualization cannot be emphasized enough.

Activities that would help develop this outcome include the following:

- Using pattern blocks, initially two at a time (for example, a trapezoid and a triangle), have students predict what polygons could be made if the blocks must share a common side. Check predictions and trace the blocks to record the new polygons. This should be extended to using three or four blocks; however, many students would have difficulty predicting when more than two blocks are used.
- Using the tangram pieces, have the students predict, and then check, the result(s) of combining
  - two small triangles
  - a small triangle and the square
  - a small triangle and the parallelogram
  - the parallelogram and the square
  - the square and two small triangles
- Make a set of cards, each card a face of a prism or pyramid, plus two each of squares, regular pentagons, and regular hexagons; four equilateral triangles; six rectangles; and six isosceles triangles. Put out a set of faces for one of the prisms or pyramids. Have the students predict the shape. Check by finding the shape and compare its faces to the cards. Cardboard 2-D shapes (like those mentioned for the cards above) could be used for the students to predict and experiment with in both 2- and 3-dimensions. There are commercial products that consist of a variety of 2-D shapes which can be manipulated to form 3-D shapes.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E8.1 Provide the students with two triangles (formed by cutting a rectangle along one diagonal). Have them make and name all the shapes that can be made by combining these two triangles with equal sides matching.

E8.2 Show the students two congruent isosceles triangles (one in each hand). Ask them to predict what polygons could be made by combining them. Sketch their predicted shape. (Remind them to visualize them coming together.) Ask for suggestions and have them explain how they would join. Ask for a volunteer to make the polygons, using your two triangles.

E8.3 Provide the students with two congruent squares, one of them cut along a diagonal. Have them investigate the different shapes that can be made by using the three pieces with equal sides matched. Record all the shapes found by tracing around them.

E8.4 Have the students use six cardboard congruent squares to build a cube, taping some, but not all, of the edges. (Each face should be attached to another along one edge). Have them unwrap the cube and lay it out flat on the desk to make a 2-D shape with six squares. (This is a net for the cube.) Ask: Is your shape the same or different from others in your class? Ask them to investigate different ways that the net for the cube could be made, and draw to record each one.

Paper and Pencil

E8.5 Ask the student to predict, check the predictions, and trace to record for each of the following. What shapes can be made by combining two congruent squares? Two congruent rectangles? Two congruent parallelograms? Two congruent rhombi?

Portfolio

E8.6 Provide the students with two of each of the six pattern blocks. Have them investigate how many new polygons can be made by using two of the same block (equal sides should be matched). Trace to record the different polygons. Which of the blocks could make only one polygon? Which could make the most different polygons? Observe that the students recognize the same shape in different positions or orientations.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

iii) investigate and predict the results of combining, subdividing, and transforming shapes

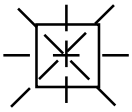
SCO: By the end of grade 3, students will be expected to

E9 find the lines of reflective symmetry of polygons

A very useful device for studying symmetry and transformations is the Mira, a piece of red Plexiglass, that stands perpendicular to the table surface. (Elementary School Mathematics, p. 350)

Elaboration - Instructional Strategies/Suggestions

E9 Students should begin to appreciate that reflective symmetry is a characteristic of some polygons and not others. These polygons can be described by stating how many lines of reflective symmetry they have. For example, after investigations, students should find that a square has 4 lines of reflective symmetry.



Experiences should provide students with the understanding that a symmetry line is where a polygon can be folded onto itself so that each half matches exactly, or where a mirror can be placed so that the reflection on one side matches the shape on the other.

- Provide students with a variety of polygons, some of which have reflective symmetry. Have the students cut them out and try to fold them onto themselves. When a perfect match is made, have the students crease on the fold line(s). Have them discuss the polygon and the number of lines of reflective symmetry.

A mira is a good instrument to have students find lines of symmetry in a variety of polygons.

Students should also examine some shapes that do not have lines of reflective symmetry; for example, the diagonal of a parallelogram is often mistaken for a line of symmetry.

Connect reflective symmetry to regular polygons (E4). Students might discover the pattern, that is, that the number of lines of reflective symmetry for regular polygons is the same as the number of sides it has. These lines either join vertices that are opposite each other, or midpoints of sides that are opposite each other, or from a vertex to the midpoint of the opposite side.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E9.1 Have the students fold a piece of rectangular paper to show all of its lines of reflective symmetry. Ask: Why does folding it along a diagonal not make a line of symmetry?

E9.2 Have the students make three polygons on a geoboard that have only two lines of reflective symmetry. Ask them to record their polygons on geopaper. Suggest that not all three be the same type of shape.

E9.3 Have the students fold a piece of paper in half. Ask them to cut out (along the fold) a shape that, when unfolded, makes a polygon with only one line of symmetry.

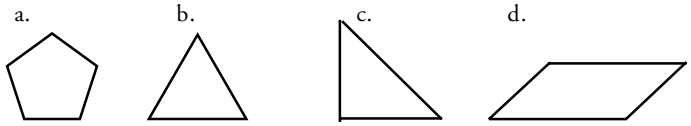
E9.4 Ask the student to examine the six shapes in the pattern blocks for lines of symmetry.

E9.5 Have the student combine various pieces in the tangram puzzle to make polygons with one line of symmetry.

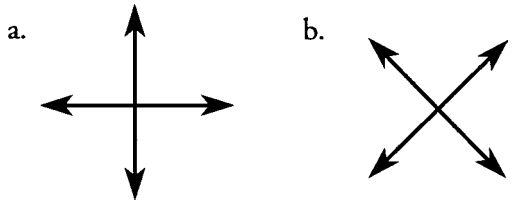
E9.6 Ask the student to sort by symmetry the shapes formed by the four triangles made by cutting a square along both diagonals (see Elaboration for E4).

Paper and Pencil

E9.7 Ask the students to use a mira to find the lines of reflective symmetry in the figures below (if there are any):



E9.8 Ask the student to draw a shape, given its two lines of reflective symmetry, as shown.



Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

iii) investigate and predict the results of combining, subdividing, and transforming shapes

SCO: By the end of grade 3, students will be expected to

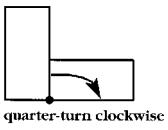
E10 recognize, name, describe, and represent half and quarter turns of 2-D figures

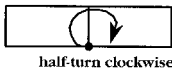
Elaboration - Instructional Strategies/Suggestions

E10 Initial experiences with these turns should be with students' own bodies, turning on the spot, one quarter at a time until a complete revolution is made. Teachers could connect quarter/half turns to the rotation of the minute hand of a clock from 12 to 3 and 12 to 6. They should connect a quarter turn to a right angle as well.

Rotations are the most perceptually challenging of the transformations. Students will need many experiences actually rotating (turning) a shape through quarter and half turns before they are able to recognize and describe such turns. It is suggested that, at this level, experiences start with cut-out 2-D shapes with right angles (for example, squares and rectangles), and turning points be one of the vertices of the shape. These turns can be clockwise or counter-clockwise. Using squared paper makes quarter and half turns easier to do.

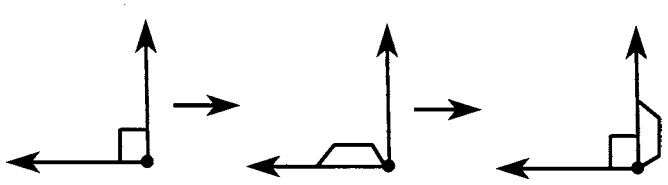
Examples:





Experiences could then involve other shapes and pictures of shapes, using tracing paper to find the rotated images, still keeping the vertices as the centres of rotation. Students should make slide and reflected images of the same shapes and be encouraged to compare them to the turned (rotated) images.

To make a quarter turn of a pattern block on plain paper, draw a right angle; place one side of the block along one arm of the angle and rotate the block until the same side of the block lies along the other arm of the angle. (The block could be traced in both positions.)



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E10.1 Have the students draw the half turn clockwise and counter-clockwise images of the trapezoid in the pattern blocks. Ask: What do you notice?

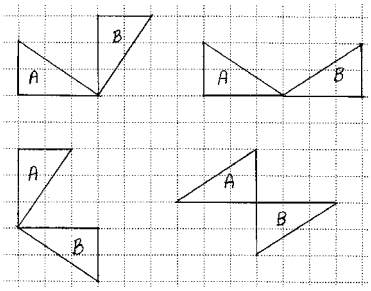
E10.2 Have the students make an acute triangle on a geoboard and draw it on geopaper. Ask them to follow these instructions: Rotate the geoboard a quarter turn clockwise and draw the image on geopaper. Rotate the geoboard another quarter turn and draw this image on geopaper. Choose one angle in the original triangle on geopaper and colour it red; find its corresponding angles in the two image triangles and colour them red as well. Compare the locations of the red angles.

Paper and Pencil

E10.3 Have students draw pictures of a square, rectangle, and a parallelogram on squared paper. Have them use tracing paper to make half-turn images of each, using one of their vertices as a turning point. (Hold a pencil point firmly on the chosen vertex as you turn the tracing paper the half turn. Using a sharp pencil, press through the tracing paper to locate the vertices of the turned shape.) Could these images have been the result of a slide? A reflection? Explain.

E10.4 Using a rectangle from the logic blocks, have the students trace it on paper. Using one vertex as a turn centre, rotate the rectangle a quarter turn counter-clockwise and trace it; using the same vertex, move the rectangle another quarter turn and trace it; continue. What happened the fourth time? What would happen if you were to continue?

E10.5 For each pair below, describe what motion (slide, reflection, rotation) of triangle A would result in triangle B.



Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 3, students will be expected to

iv) relate geometric ideas to number and measurement ideas and recognize and apply geometric principles in real-world situations

SCO: By the end of grade 3, students will be expected to

E11 recognize and identify different polygons, prisms, and pyramids in real-world contexts

E12 make the connection between arrays of squares forming rectangles and describing their dimensions

Geometry is an important component of the K-4 mathematics curriculum because geometric knowledge, relationships, and insights are useful in everyday situations and are connected to other mathematical topics and school subjects. (Curriculum and Evaluation Standards, p.48)

Elaboration - Instructional Strategies/Suggestions

E11 In the development of geometric concepts, initial experiences should include their real-world uses to help students make associations and form visual images. For example, regular polygons are used in many logos and designs. Students should be encouraged to find examples in magazines, newspapers, cars, stores, fabric, artworks, wallpaper, etc. Native handicrafts and quilt patterns are other good sources of geometric shapes in use. A class collection of boxes and containers can provide examples of various prisms and pyramids.

Teachers should collect an assortment of pictures (for example, streetscapes, formal gardens, buildings, oriental carpets). Students could find examples of 2-D and 3-D shapes in the pictures.

Students should be encouraged to use geometry in design and art projects. For example, they could design a quilt, using construction paper shapes in different colours; they could design a logo; they could make 3-D ornaments/ mobiles, using pyramids/prisms with decorated faces.

E12 One of the concrete/pictorial representations of multiplication is arrays. For example,  is one way to show 2 x 3. This essentially geometric way of representing multiplication, by combining 6 squares to form this rectangle, is also the way we often describe a rectangle, by giving its dimensions (2 by 3 rectangle in this example). Have the students use 12 square tiles to build all possible rectangles. Record them on grid paper and label with the appropriate multiplication.

The connection of arrays to area of squares and rectangles should also be made.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

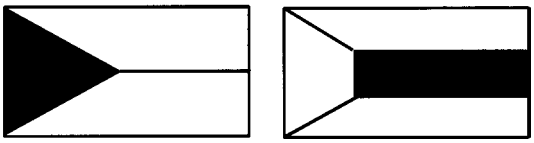
Performance

E11.1 Tell the student many unsharpened pencils are shaped like prisms. Ask him/her to find one and name the prism.

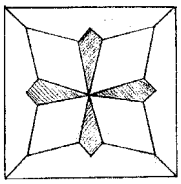
E11.2 Show the students a collection of boxes of various shapes. Have them identify and name the shape of the boxes, using geometric vocabulary.

Paper and Pencil

E11.3 Tell the students that the following flags of different countries were designed by using polygons. Ask them to find, draw, name, or describe the different polygons used in each flag.



E11.4 Tell the student that this is one square of a quilt pattern. Ask him/her to identify, draw, and name some of the different shapes that are used.



Presentation

E11.5 Have the students work in groups to explore the different rectangles that can be made, using from 1 to 36 squares. Ask them to record their results by giving the possible dimensions in a chart.

Number of Squares	Dimensions of Rectangles
1	1x1
2	1x2, 2x1
3	1x3, 3x1
4	1x4, 4x1, 2x2

Discuss which number of squares had the most possible number of rectangles; which had the least possible number of rectangles; which had rectangles with 1 as one of its dimensions.

Suggested Resources

Data Management and Probability

General Curriculum Outcome F:

Students will solve problems involving the collection, display, and analysis of data.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 3, students will be expected to

i) collect, record, organize, and describe relevant data

SCO: By the end of grade 3, students will be expected to

F1 select appropriate strategies for collecting, recording, organizing, and describing relevant data

Third-grade children are keenly aware of what others think about certain topics. This interest in others broadens and deepens as children gather and analyse opinions from a variety of people. (Curriculum and Evaluation Standards, Addenda Series, Third-Grade Book, p. 20)

Elaboration - Instructional Strategies/Suggestions

F1 At this level, the focus should be on the selection of appropriate strategies for collecting and displaying data, rather than following a set of instructions.

- ❒ Have pairs of students decide on the procedure they will use to collect and display data that will show interesting information about class members.
- ❒ Have students conduct a survey to find out what types of things 8- and 9-year-olds like to collect. They will need to decide who to survey and how to organize and present their data.
- ❒ Other graphing activities might include
  - graphing temperature over the school week/month
  - collecting and displaying data that represent accomplishments of favourite sport figures or friends (e.g., the number of goals, hits, points)
  - collecting and displaying data that would show the number of pizzas and hot dogs ordered at the cafeteria during one week
  - graphing the distance class members can throw a ball
  - graphing prices of board games
  - collecting and displaying data to represent mass of various fruits/ vegetables
  - graphing favourite books of classmates
  - graphing the mass of subject textbooks
  - graphing the mass of different breeds of dogs

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F1.1 Ask pairs of students to design a survey that will show favourite school subjects.

F1.2 Ask small groups to brainstorm a list of interesting questions for possible surveys.

Paper and Pencil

F1.3 Ask the students to conduct a survey of the types of vehicles in the parking lot or the number of vehicles passing the school during a half-hour period.

Interview

F1.4 Tell the student that Sasha wanted to find out if students in grade 4 liked the same television programs she liked. Ask him/her to suggest a way that Sasha might get this information.

F1.5 Ask the student for different strategies one could use to find the high temperatures for each day of the school week.

F1.6 Tell the student that Derek notices the prices of chocolate bars vary from store to store. Ask him/her to suggest ways Derek could gather information in order to make a graph showing the different prices.

Presentation

F1.7 Ask students to select a topic, survey family members and/or neighbours, and present their findings to the class in an organized way.

Portfolio

F1.8 Ask the students to keep track of weather conditions over the period of one month and to design a way to present the information in an organized manner.

Suggested Resources

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 3, students will be expected to

ii) construct concrete and pictorial displays of relevant data, and

iii) read and interpret displays of relevant data

SCO: By the end of grade 3, students will be expected to

F2 interpret and create pictographs in which each symbol represents more than one item

F3 create bar graphs, using simple scales

Children sometimes have difficulty with pictographs and other graphs in which each picture or symbol stands for more than one object. But careful development here will help children make sense of units and of multiplication. (Curriculum and Evaluation Standards, Addenda Series, Third-Grade Book, p. 20)

Elaboration - Instructional Strategies/Suggestions

F2 Students should make the connection between multiplication and graphing situations in which each pictograph symbol represents more than one item. Simple fractions of symbols (e.g., halves, fourths) may be introduced.

Favourite Types of Stories

Mystery	● ●
Animal	●
Tall Tales	● ● ◖
Other	●

● represents 4 children

Students should have experiences interpreting pictographs before creating them by conducting surveys and organizing data. They might contrast data on two graphs for which different values are attached to the same symbol.
For example:

Favourite Types of Stories

Class A		Class B	
Mystery	□□	Mystery	□□□
Animal	□	Animal	□□
Tall tales	□□□	Tall tales	□□□
Other	□	Other	□□□

□ represents 4 students

□ represents 2 students

In which class do more children prefer mysteries?
Which type of story was preferred by the same number of students?

F3 Students have had some experience creating bar graphs for which each section represents a value of one. At this point, students should learn to interpret graphs for which each section represents a greater value.

Students should recognize that scales are useful in situations involving larger numbers. For example, data are collected about the number of students in each grade:

K - 48 Gr 1 - 64 Gr 2 - 56

Since it would require a minimum of 64 squares to represent this information, it is preferable to use a scale. Both horizontal and vertical graphs should be considered.

In addition to drawing bar graphs from provided data, students should continue to conduct surveys and organize data to create such graphs. For example, have students collect data on the vowels in classmates' first names (e.g., how many have "a", how many have "e").

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F2.1 Have the student redraw the pictograph so that each symbol represents 4, instead of 2. Ask the student which graph he/she prefers and to give reasons for the choice. Ask if there is another way to display the data which might be clearer.

Cafeteria Orders

Pizza	o o o o o
Hamburger	o o
Hot dogs	o o o o o o

o represents 2 people

Paper and Pencil

F3.1 Provide data for a bar graph:

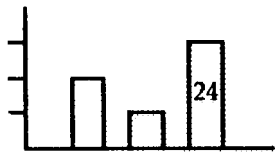
Favourite Sports

hockey	36
baseball	22
basketball	30
volleyball	16

Have the student select a scale and create a bar graph.

Interview

F3.2 Show the student this graph, which represents the favourite basketball teams of a number of grade 3 students.



Ask him/her to tell how many students each of the first two bars represents.

F2.2 Ask the student to tell how many more children prefer carrots to broccoli. Have him/her tell what other information is shown by the graph.

Carrots	o o o o
Broccoli	o o
Peas	o o o

o represents 5 children

Ask why a symbol in a pictograph usually represents more than 1.

Portfolio

F2.3 Ask pairs of students to create a pictograph describing their classmates' favourite colours, using one symbol to represent three children. Ask them to include a brief description of the strategies used to collect their data and any difficulties they encountered while creating the graph.

Suggested Resources

GCO F: Students will solve problems involving the collection, display, and analysis of data.

KSCO: *By the end of grade 3, students will be expected to*

iv) generate questions, develop and modify predictions, and implement plans, with respect to data analysis

SCO: By the end of grade 3, students will be expected to

F4 implement plans with respect to the collection of data

Children need experience in deciding what opinion questions they want answered as well as in collecting and presenting the data. Although students at this level probably will not conduct surveys scientifically, they will begin the process of making decisions on the basis of data. (Curriculum and Evaluation Standards, Addenda Series, Third-Grade Book, p. 20)

Elaboration - Instructional Strategies/Suggestions

F4 When considering how to collect data, students need to consider a variety of issues:

- Where is a good source?
- If I am surveying, how many people do I need to contact?
- Where should I go to conduct my survey?
- Does it matter when I conduct my survey?
- If I am planning to ask questions, how should I phrase them?

❒ Have a small group/class discussion. Tell the students that their assignment is to plan a survey of students in grades 3 through 5 to find out whether school uniforms should be introduced. Have the class discuss the following:

- How many students should be surveyed?
- How many from each grade/class would be surveyed?
- How many girls and how many boys should be surveyed?
- How would they decide which girls and which boys should be surveyed?

Encourage students to discuss the importance of carefully designing the survey plan to ensure a cross-section of the targeted population.

GCO F: Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Interview

F4.1 Ask the student what source(s) he/she might use to find information on the mass of different zoo animals.

F4.2 Ask the student how many people he/she should ask to determine the number of quarters adults usually carry with them.

Presentation

F4.3 Have students consider how to conduct a survey to find out the amount of TV people generally watch. They should report on all the issues to which they think they must attend.

Suggested Resources

Data Management and Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 3, students will be expected to

i) conduct informal investigations of chance and estimate probabilities, with respect to games and other simple, everyday situations

SCO: By the end of grade 3, students will be expected to

G1 predict and record results in experiments using spinners, coins, dice, coloured cubes, and other simple equipment

From a basic understanding that one event can be more or less likely than another, students can begin to predict specific ratios of outcomes of simple events. Before students have worked with part-to-whole ratios, use language such as "65 out of 100" instead of using fractional probabilities. A discussion of reasons for their predictions is always important. The experiment should then be conducted, and results compared with expected outcomes. (Elementary School Mathematics, p. 385)

Elaboration - Instructional Strategies/Suggestions

G1 Students should conduct a variety of experiments and record the outcomes. While some students might simply list frequencies of various outcomes, others may be able to use fractions to describe the probability of events.

For example, the students might roll 2 dice twenty-five times and count the number of times that the following events occur:

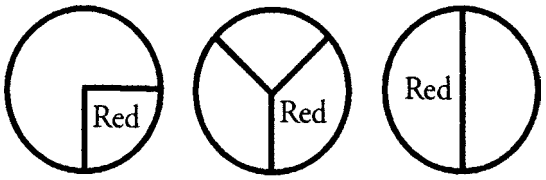
- the sum of the numbers is 7
- the difference is 2
- the product is even

Some students may be able to record the results for each outcome (sum, difference, product) on a given roll, while others may need to focus on one outcome at a time.

Once students have recorded the data, they might then make comparative statements. For example, it is more likely that the product is even than having a difference of 2.

Have them compare their findings with their classmates.

Students might use spinners and predict how often the arrow will land on particular sections in 12 spins. They can then test (and record) their predictions. Discuss how many got what they predicted. Collect the total results of the spins for a group of 10 students (120 spins) and compare the predictions to the results for the whole group. This will provide the opportunity to discuss the idea that a sample of 12 may not give the predicted results, but that larger samples, such as 120, will come closer.



Have the students devise a way to record their spins and tosses. Ask them to share their findings with their classmates.

Provide activities in which students compare their findings of a few experiments (spins/tosses) with those of many.

At this level, students should be expected to use probability language, such as “1 out of 4” or “1 chance in 3.”

GCO G: Students will represent and solve problems involving uncertainty.

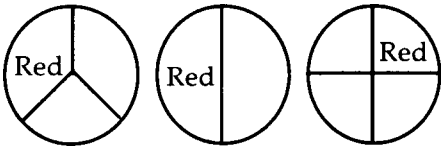
Worthwhile Tasks for Instruction and/or Assessment

Performance

G1.1 Ask the student to design a spinner on which one is more likely to spin red about eight times out of ten spins.

G1.2 Have students spill 10 two-coloured counters and record the number of each colour which occurs. Ask them to predict if it is more likely that one colour will come up more often than the other with 10 spills. Have them conduct the experiment and report on their findings.

G1.3 Show the student the spinners below.



Have the student predict the number of times red will be spun out of 10 spins for each spinner. Ask him/her to conduct the experiment to verify predictions.

Interview

G1.4 Tell the student that you tossed a coin 25 times. Have him/her predict the approximate number of heads/tails.

Presentation

G1.5 Show the students a picture of a game at the exhibition. Tell them that you can win if the ball lands on an even number. Have them discuss the chances of winning.

3	2	1	4	7
9	7	5	3	1
6	1	9	7	2
1	5	3	8	9
7	9	1	5	3

G1.6 Ask pairs of students to think of what might happen about half the time when a die is rolled. Students should experiment with the die, record outcomes, and later present their findings to classmates.

Suggested Resources

THREE-INDEX

Index

A

Addition
  Whole Numbers
    2-digit and 3-digit numbers 24-27, 34, 35
    algorithms 26, 34
    basic facts 34
    estimating 30, 31
    meanings 24
    modeling 26, 27
    regrouping 26, 27
    relation to subtraction 23
    strategies 4, 5, 26, 27, 30, 31, 34, 35

D

Data Management
  Collecting/Organizing
    surveys/experiments 47, 74-76, 78, 79
  Displaying/Interpreting
    bar graphs 76, 77
    connection to multiplication 76
    pictographs 76, 77
    scales 53, 76, 77
Division
  Whole Numbers
    basic facts 20, 28, 29
    estimating 32, 33
    meanings 20-25
    modeling 20-23, 28
    principles 28
    relation to multiplication 22-24, 28, 29
    strategies 28, 29, 32, 33

E

Estimating
  Addition 4, 5, 30, 31
  Area 48, 49
  Capacity 46, 47, 51, 52
  Division 32, 33
  Fractional amounts 14, 15
  Length 46, 47, 50-53
  Mass 48, 49, 51, 52
  Multiplication 32, 33
  Probability 82, 83
  Relative size 2
  Rounding 4, 5, 30
  Subtraction 4, 5, 30, 31
  Time elapsed 50
  When using technology 24, 30

G

Geometry
  3-D Shapes
    constructing 62-65
    edges/faces/vertices 57, 59, 61-65, 70
    nets 62, 63, 65
    prisms 62-65, 70, 71
    pyramids 57, 62-64, 70
  angles 58, 59
  combining/dividing shapes 60, 61, 64, 65, 67
  congruence 58-61, 65
  perceptual constancy 56, 57, 59, 65
  polygons
    concave/convex 58, 60, 61
    penta/hexa/octagons 6, 33, 60-64, 67
    quadrilaterals 6, 56, 58-62, 64-71
    regular 60, 61, 64, 66, 70
    triangles 58, 60, 62, 64, 65, 67, 69, 71
  spatial relations 56
  transformations
    flips/folds/reflections 38, 39, 59, 66-69
    reductions/enlargements 56, 59
    rotations 59, 68, 69
    slides 38, 39, 59, 68, 69
    symmetry 40, 66, 67
  visual discrimination 56, 59
  visual memory 56, 57
  visualizing 62-65, 70

M

Manipulatives
  base-ten materials 2, 3, 8-11, 13, 14, 26, 27, 30, 46, 48, 56
  centimeter grids 48
  clocks 52, 53
  coins 7-9, 14, 24, 25, 46, 82
  coloured tiles 6
  counters 3, 7, 21, 22, 83
  dice 2, 8, 9, 28, 31, 82, 83
  geoboards/geopaper 56, 59, 61, 67, 69
  graduated containers 46

hundreds charts 34, 38, 40
logic blocks 62, 69
Miras 66, 67
multilink cubes 7, 56
number cards 2, 3, 8, 10, 16
pan balance 49
pattern blocks 6, 38, 39, 48, 52, 56, 58, 60-62, 64, 65, 67-69
place value charts/mats 2, 8
spinners 3, 82, 83
tangram pieces 57, 58, 64, 67
trundle wheel 50

Measurement
Area
estimating 48, 49
geometric arrays 70, 71
non-standard units 7, 48, 49
square centimetre 48
Capacity
estimating 46, 47, 50, 51, 52
litre (L) 46-48, 50-52
millilitre (mL) 46-48, 50, 51
Length
centimetre (cm) 14, 17, 33, 46, 47, 52, 53
decimetre (dm) 46, 47
estimating 46, 47, 50-53
kilometre (km) 50, 51, 53
metre (m) 17, 46, 47, 50, 60
Mass
estimating 48, 49, 51, 52
gram (g) 48-51
kilogram (kg) 3, 31, 48-52
Time
elapsed 24, 25, 50
reading clocks 52, 53

Mental Computation
basic facts 28, 29, 34, 40-43
estimation 4, 5, 30-33
exact calculation 26, 27, 34, 35, 38

Modeling with
arrays 20-23, 28, 29, 70, 71
base-ten materials 2, 3, 8-11, 13, 14, 26, 27, 30
counters 7, 22
fraction circles 14
money 8, 9
number lines 2, 3, 20, 22, 23, 30
pattern blocks 6, 62
ten frames 14-17

Multiplication
Whole Numbers
basic facts 28, 29, 40-43, 52
connection to graphing 76, 77
estimating 32, 33
meanings 20-25, 42, 43
modeling 20-23, 28, 29, 70, 71
principles/properties 21, 22, 28, 29, 40, 41
relation to division 22-24, 28, 29
strategies 28, 29, 32, 33

N

Number
Decimals
comparing/ordering 15-17
modeling 14-17, 46
place value 14, 15, 17, 38
relation to fractions 14
Fractions
comparing 7
modeling 6, 7
part of a set 6, 7
part of a whole 6, 7, 76, 82, 83
relation to decimals 14
Whole Numbers
comparing/ordering 2, 3, 10
estimating relative size 2
modeling 8-11, 38, 39
place value 8-11, 38, 39
reading/writing 8-13
square numbers 38, 40
zero 10, 11

O

Operations. *See* Addition, Division, Multiplication, Subtraction

P

Patterns and Relations
geometric 38, 39, 62, 66
multiplication table 29, 40, 41, 52

number 15, 17, 29, 38-41
open sentences 42, 43
place value 17, 38, 39
repeated addition 42, 43

Probability
estimating 82, 83
experimental 14, 82, 83
sample size 82

Problem Solving 2-17, 20-35, 38-43, 46-53, 56-71, 74-79, 82, 83

S

Subtraction
Whole Numbers
2-digit and 3-digit numbers 24-27, 34, 35
algorithms 26, 34
basic facts 34
estimating 4, 5, 30, 31
meanings 24
modeling 26, 27
regrouping 13, 26, 27
relation to addition 23
strategies 4, 24-27, 30, 31, 34, 35

T

Technology, Use of
calculator 3, 8, 21, 24, 27, 34, 35
estimation in connection with 24, 30