Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

Mathematics

Grade 5

CURRICULUM

2001

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- The regional mathematics curriculum committee, current and past members, include the following:

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- The Elementary Mathematics Curriculum Development Advisory Committee, comprising teachers and other educators in New Brunswick, lead province in drafting and revising the guides
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the finalization of the entry-grade 6 mathematics curriculum guides

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Introduction

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers," and they advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers and Department of Education officials to plan and develop cooperatively the curricula in mathematics, science, and language arts in both official languages.

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)

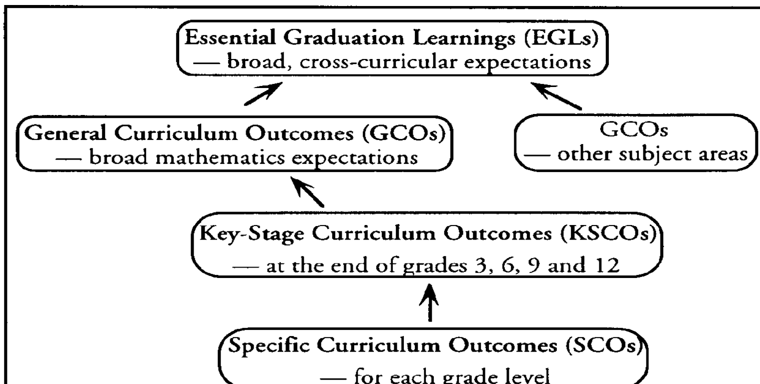


Figure 1: Outcome Framework

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. This curriculum guide is supplemented by others that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO.

The Atlantic Canada Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; and iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

II. Program Design and Components

A. Program Organization

As already indicated, the mathematics curriculum is designed to support the six Essential Graduation Learnings (EGLs). While the curriculum contributes to students' achievement of each of these, the communication and problem solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then presents outcomes at four key stages of the student's school experience.

This particular curriculum guide presents specific curriculum outcomes for each grade level. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.

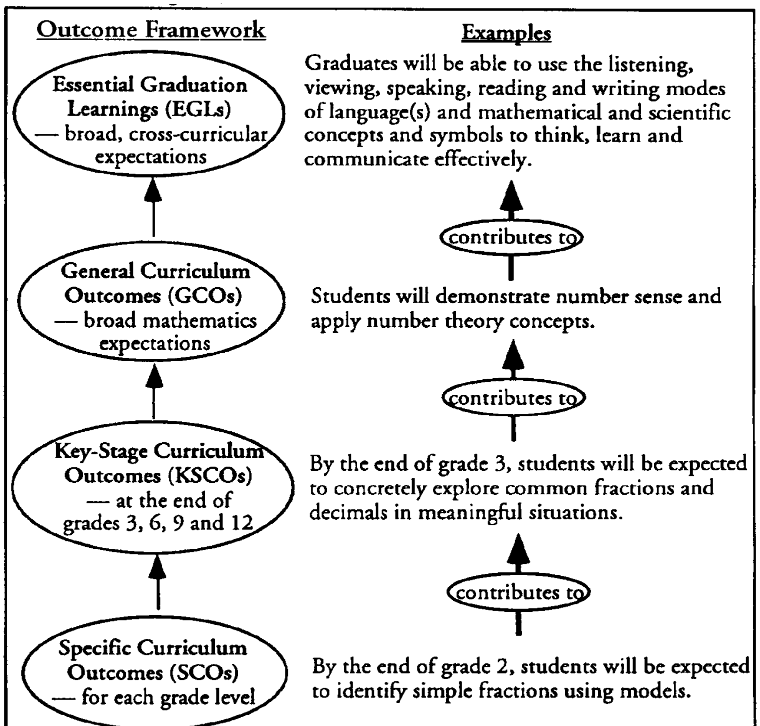


Figure 2: Examples of Outcomes

It is important to emphasize that, while the grade level outcomes (SCOs) provide a framework on which educators will base decisions regarding instruction and assessment, they are not intended to limit the scope of learning experiences. Although it is expected that most students will be able to attain the outcomes, some students' needs and performance will range across grade levels. Teachers will need to take this variation into consideration as they plan learning experiences and assess students' achievement.

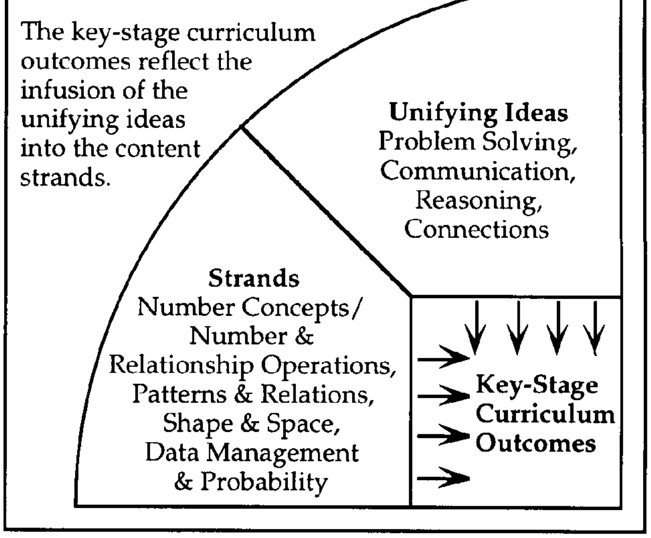
The presentation of the specific curriculum outcomes follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes like those pertaining to patterns and data management may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province like an election, an exhibition, or a fair.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)

Figure 3 — Integrating Strands and Unifying Ideas



These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Adapting to the Needs of All Learners

The F*oundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student development as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

E. Support Resources

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. These guides should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in as much as they support the specific curriculum outcomes. Many manipulative materials need to be readily at hand, and technological resources, e.g., software and videos, should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM) and the *Addenda Series* and *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* or *Elementary and Middle School Mathematics: Teaching Developmentally* (John van de Walle), *Developing Number Concepts Using Unifix Cubes* (Kathy Richardson), and *About Teaching Mathematics; A K-8 Resource* (Marilyn Burns).

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessment and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment would include i) the use of a wide variety of assessment strategies and tools, ii) aligning assessment strategies and tools with the curriculum and instructional techniques, and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborates good assessment practices and it served as a guide for student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

IV. Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for each grade. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled “Elaboration–Instructional Strategies/ Suggestions,” provides a clarification of the specific curriculum outcome(s), as well as suggestions for possible strategies/activities which could be used to help students achieve the outcome(s). While the strategies/activities presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s). They will also illustrate ways to work toward the achievement of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activites are introduced in this column of the layout by the symbol ❒ .

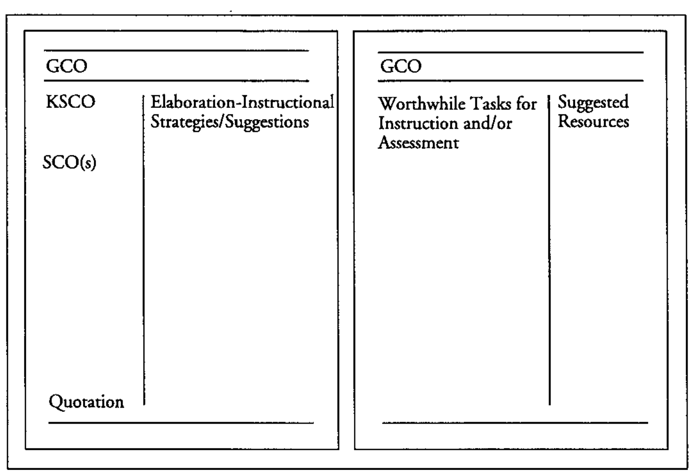


Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled "Worthwhile Tasks for Instruction and/or Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Suggested Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).

FIVE-NUMBER

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number theory concepts.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 5, students will be expected to

A1 represent whole numbers to the millions

Elaboration - Instructional Strategies/Suggestions

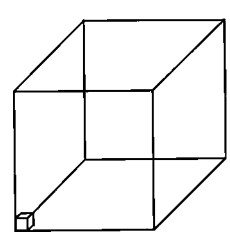
A1 Students will continue to use whole numbers as they perform computations or measurements and as they read and interpret data. To have a better understanding of large numbers, such as a million, students need opportunities to investigate problems involving these numbers.

❒ For example, students will better appreciate how much a million is by thinking about problems such as the following:
- How many $100 bills would take it to make $1 million?
- How long would a line of 1 million centicubes be?
- How many garbage bags would be needed to hold 1 million 2-litre pop bottles?
- How much grid paper would be needed to show 1 million square centimetres?

How Much Is A Million? by David Schwartz is a useful children's book to explore number meanings.

A visual model based on the cubic centimetre is a valuable way to help students conceptualize one million. Given that the large cube of the base-ten blocks represents 1000 cubic centimetres, 1000 of the large cubes represent a million.

Work with the students to construct a cubic metre using metre sticks or metre-long sticks. On the bottom layer place as many large base-ten cubes as can be gathered. Help students to determine that one would need 100 large base-ten cubes to complete one layer (10 x 10 = 100 large cubes). Then discuss how many cubic centimetres would be in the layer (100 large cubes with 1000 cubic centimetres in each, total 100 000 cubic centimetres). Next work with students to determine how many layers of the large cubes would be needed to fill the cubic metre. By stacking ten of the large cubes, students should realize that it would take ten layers or 1 000 000 cubic centimeters (10 layers with 100 000 cubic centimetres in each layer, total 1 000 000).



GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A1.1 Ask students to predict whether the gymnasium could hold a million cereal boxes. Have students do enough measurements to check their predictions.

Interview

A1.2 Ask the student to explain how he/she knows that 1 345 121 is greater than 1000 thousands and to suggest what this number might be used to represent. Ask, also, how a number such as this might be written in a newspaper.

A1.3 Ask the student to decide if he/she has lived 1 000 000 hours yet and to explain her/his thinking.

A1.4 Ask: How does a million compare to a thousand? to ten thousand?

Portfolio

A1.5 Ask the students to use newspapers and/or catalogues to find items to buy that would total $1 million. Limit to five the number one can purchase of any one item. Students might follow this up by interviewing a senior citizen to find out what could have been purchased with $1 million fifty years ago. Ask the students to write a report on their findings.

A1.6 Students could work in pairs to create 2-page spreads for a class book about a million. Each spread could begin, "If you had a million __________, it would be ___________."

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

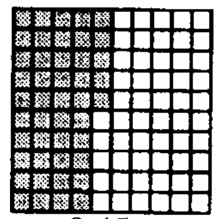
SCO: By the end of grade 5, students will be expected to

A2 interpret and model decimal tenths, hundredths and thousandths

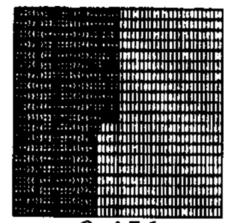
Elaboration - Instructional Strategies/Suggestions

A2 Students should continue to use physical materials to represent or model decimals. In this way, they can better see the relationship between hundredths and thousandths.

For example, students might use thousandths grids (of the same size as hundredths grids) to model decimals to the thousandths.

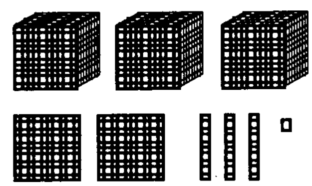


0.45



0.456

Alternatively, base-ten blocks might be used to illustrate the relationship. Within a given context, the large block could represent 1, then the flat would represent 0.1, the rod 0.01 and the small cube 0.001. The model for 3.231 would be as shown. It is helpful to vary which block represents 1 so students develop flexibility in thinking about decimal fractions.



Since 1 mm = 0.001 m, students can also represent thousandths using length measurements. For example, 0.423 m can be represented as 423 mm, 42.3 cm (a little more than 42 cm), and 4.23 dm (about 4 and one-quarter dm).

Students should be encouraged to view decimals in a variety of ways. For example: 0.452 is $\frac{452}{1000}$, $\frac{45}{100} + \frac{2}{1000}$, $\frac{4}{10} + \frac{52}{1000}$.

To help students develop a sense of number, encourage them to use reference points, e.g., 0.452 m is a little less than half a metre. Some students may recognize that it is only 0.048 m less than half a metre.

❐ Provide opportunities for students to find and share how large numbers are represented in newspapers and magazines.

Students should recognize that decimals to thousandths can represent something quite small or something very large. For example, 0.025 m is only 2.5 cm, which is a small measurement; however, 0.025 of the population of Canada refers to 25 out of every thousand people, 250 of every 10 000 people, 2500 of every 100 000 people and 25 000 of every million people, or a very large number of people. If one were to round Canada's population to 30 million, 0.025 would represent 30 x 25 000, which is 750 000 people or about the number of people in all of New Brunswick. Discussions such as these help students to develop number sense.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

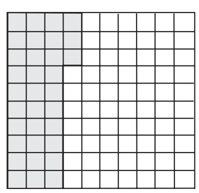
Performance

A2.1 Have the student model 0.025 using decimal squares. Then ask: How does this model differ from the model for 25 hundredths? Ask the student to model the same amounts using base-ten blocks. Ask: What block did you use to represent the ones place? Why?

A2.2 Tell the student that a new bakery slices its loaves of bread into 10 equal pieces, makes bread sticks by cutting each slice of bread into 10 equal pieces, and makes croutons by cutting each bread stick into 10 equal pieces. Ask him/her to model this using base-ten blocks. Pose questions such as: What part of the loaf is 1 slice? 3 slices? 1 stick? 5 sticks? 1 crouton? 9 croutons? 3 slices and 2 sticks? What part of 1 slice is 4 sticks? 6 croutons? 2 sticks and 3 croutons? Then have the student use the blocks to show quantities such as 0.2 loaf, 0.14 loaf, 1.5 loaves, 0.5 slice, 0.25 slice, 0.7 stick and 0.3 stick.

Paper and Pencil

A2.3 Have the students identify the decimal represented by the shaded portion of the diagram if the hundredths grid represents 1 whole. Ask how much more is required to make a whole.



Interview

A2.4 Show the student cards on which decimals have been written (e.g., 0.75 m and 0.265 m). Ask the student to place the cards appropriately on a metre stick.

A2.5 Ask the student to express 0.135 in at least three different ways.

A2.6 Ask the student to identify a situation in which 0.25 represents a small amount and one in which it represents a very large amount.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 5, students will be expected to

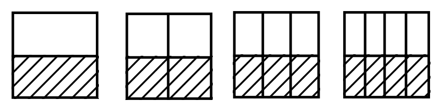
A3 interpret, model and rename fractions

Elaboration - Instructional Strategies/Suggestions

A3 Developing number sense with fractions takes time and is best supported with a conceptual approach and the use of materials. Using a variety of manipulatives helps students understand properties of fractions and realize that the relationship between the two numbers in a fraction is the focus. The manipulatives might include, among other materials, pattern blocks, fraction circles, geoboards, coloured tiles, counters and egg cartons.

Provide the students with a variety of activities that include the three interpretations of fractions: 1) part of a whole ($\frac{1}{3}$ of a chocolate bar); 2) part of a set ($\frac{2}{5}$ of 30 marbles); and 3) part of a linear measurement ($\frac{3}{4}$ of a 4 m baseboard).

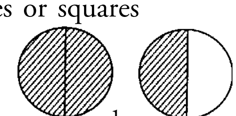
It is important that students are able to visualize equivalent fractions as the naming of the same region partitioned in different ways as shown here.



$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

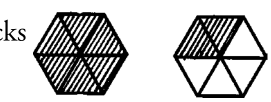
Some manipulatives which are valuable to illustrate equivalent fractions include:

-fraction circles or squares



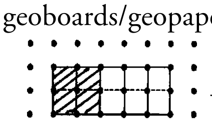
$1\frac{1}{2} = \frac{3}{2}$

-pattern blocks



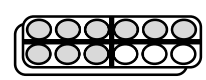
$1\frac{1}{3} = \frac{8}{6} = \frac{4}{3}$

- geoboards/geopaper

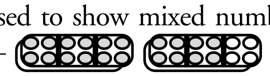


$\frac{2}{5} = \frac{4}{10}$

- egg carton

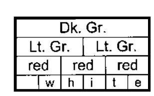


$\frac{9}{12} = \frac{3}{4}$

More than one egg carton can be used to show mixed numbers and fraction equivalents, e.g., $1\frac{2}{3} = \frac{5}{3}$ 

At this stage, a rule about multiplying numerators and denominators to form equivalents should not be offered. Such a rule could be confirmed if students observe it; however, the explanation should be connected to manipulatives.

❒ Ask the students to use colour strips or Cuisenaire rods to find equivalent fractions (e.g., the dark green rod represents the whole).



❒ Show the green triangle in the pattern blocks and tell students it is $\frac{1}{3}$. Ask them to show 1 using pattern blocks.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A3.1 Ask the student to use his/her fingers and hands to show that $\frac{1}{2}$ and $\frac{5}{10}$ are equivalent fractions. Alternatively, the student might be asked to choose a manipulative of choice to show this or some other equivalence.

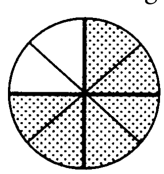
A3.2 Ask the student to use Cuisenaire rods or colour strips to show $1\frac{2}{3} = \frac{5}{3}$ and $1\frac{3}{4} = \frac{7}{4}$.

A3.3 Tell the student that you have a pan of squares represented by a geoboard. Ask him/her to use the geoboard to explain the equivalence of $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ and to make the connection to the pan of squares.

A3.4 Have pairs of students use colour tiles to show equivalences as written on selected cards. For example, $\frac{3}{4}$ and $\frac{6}{8}$, $\frac{1}{3}$ and $\frac{3}{9}$, $\frac{2}{3}$ and $\frac{8}{12}$.

Paper and Pencil

A3.5 Point out to the student that to rename $\frac{6}{8}$ as $\frac{3}{4}$, you can "clump" the 8 sections of the whole into 2s. There are then four groups of 2 sections; three of the four groups are shaded.



Have the student make a diagram and identify the "clump size" that should be used to show that $\frac{10}{15} = \frac{2}{3}$. Ask how one might predict the "clump size" without drawing the diagram.

Portfolio

A3.6 Have students prepare a poster showing all the equivalent fractions they can find using a set of no more than 30 pattern blocks.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

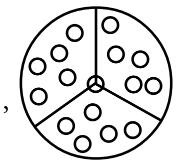
i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 5, students will be expected to

A4 demonstrate an understanding of the relationship between fractions and division

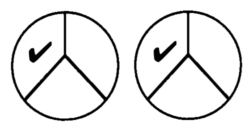
Elaboration - Instructional Strategies/Suggestions

A4 It is valuable for students to understand the relationship between fractions and division. This knowledge helps them interpret fractions of sets and later to convert fractions to decimals.



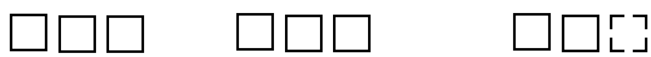
• When looking at a division situation, such as 16 ÷ 3, students can visualize it as $\frac{1}{3}$ of 16, or the share on $\frac{1}{3}$ of a mat ($\frac{16}{3}$ or $5\frac{1}{3}$), if the 16 is shared equally among the 3 parts of the mat.

• When looking at a fraction, students can think of it as an alternative way of expressing a division. For example, $\frac{2}{3}$ is the amount each person would get when 3 people share 2 items.



Each person gets $\frac{1}{3}$ of each item. $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Alternatively, $\frac{8}{3}$ tells how many groups of 3 are in 8.



1 group of 3 2nd group of 3 $\frac{2}{3}$ of a group of 3

❐ Remind students that the blue rhombus pattern block represents $\frac{1}{3}$ of a hexagon. Ask students to put together 14 of these pieces, 3 at a time, to make hexagon shapes. When the task is complete, ask students to discuss why another name for $\frac{14}{3}$ is $4\frac{2}{3}$, and why you can think of it as 14 ÷ 3.

A problem that would be represented in this way is as follows: If each person at a party of 14 could eat $\frac{1}{3}$ of a large pizza, how many pizzas would I have to buy?

As the students work through a number of these problems, they will see that dividing the numerator by the denominator is a procedure for changing an improper fraction to a mixed number. It would be inappropriate just to tell students to divide the denominator into the numerator to change an improper fraction to a mixed number before developing the conceptual understanding for such a procedure.

GCO A: Students will demonstrate number sense and apply number theory concepts.

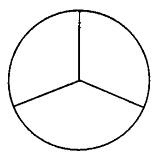
Worthwhile Tasks for Instruction and/or Assessment

Performance

A4.1 Tell the students that there were 3 pizzas left from the class party. Everyone agreed that the organizing committee would share what was left. Ask: What part of the pizzas would each of the 4 members receive if the pizzas were divided equally?

A4.2 Ask the students to use pattern blocks to explain how they know $\frac{17}{3}$ is the same as $5\frac{2}{3}$.

A4.3 Ask the student to use the circle below to find $\frac{2}{3}$ of 18.



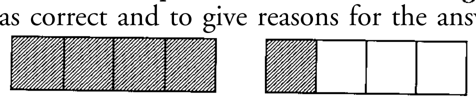
Paper and Pencil

A4.4 Tell the student that you divided one number by another and the result was $2\frac{1}{2}$. Ask him/her what the two numbers might have been.

A4.5 Have students draw two different pictures of squares to show $\frac{4}{5}$, one picture to represent part of a whole, and one to show sharing. Ask them to write a possible story that each picture could represent.

Interview

A4.6 Show the student the following and tell him/her that one person said it represented $\frac{5}{4}$ and another said it was $\frac{5}{8}$. Ask him/her which one was correct and to give reasons for the answer.



A4.7 Ask the student how many buckets of water one would need to water 9 plants if each plant needs $\frac{1}{2}$ bucket.

A4.8 Ask the student to explain why one divides 16 by 3 to find the mixed number name for $\frac{16}{3}$. Invite him/her to use drawings or models in the explanation.

Presentation

A4.9 Give a group of students a fraction. Ask them to act it out in a division "skit." The rest of the class has to guess the fraction being acted out. For example, for $\frac{13}{4}$, students might pretend to make families of 4 using thirteen objects or classmates and see how many families there are and how many are left over.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *explore integers, ratios and percents in common, meaningful situations*

SCO: By the end of grade 5, students will be expected to

A5 explore the concepts of ratio and rate informally

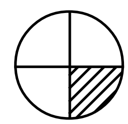
Elaboration - Instructional Strategies/Suggestions

A5 Ratio is a multiplicative comparison of two numbers or quantities of the same type (e.g., 10:1 is the ratio of the value of a dime as compared to that of penny; 3:2 is the ratio of the number of boys to the number of girls in a group of 3 boys and 2 girls). One is using ratio when stating, "She is running twice as fast now as last year." (2:1) or "She has done 3 times as much today as she did yesterday." (3:1)

In previous grades, students have often compared two quantities in a subtractive/additive way. For ratio, emphasis should be given to the multiplicative comparison. For example, if asked to compare a dime to a penny, students may say that the dime is 9 cents more than the penny; this is not a ratio. Help them view the comparison as 10 cents to 1 cent or the dime as 10 times the value of the penny.

Rate is also a multiplicative comparison of two quantities, but the quantities are described in different units (e.g., 2 cans for $0.98 is a price rate for a product or 20 km/hour a rate of speed).

It is sometimes useful to think of ratios in terms of fractions. For example: The ratio of shaded parts to the total number of parts in the circle is 1:4 or $\frac{1}{4}$. OR The ratio of shaded parts to unshaded parts is 1:3 or $\frac{1}{3}$. (There are $\frac{1}{3}$ as many shaded parts as unshaded parts.)



Geometric, numerical, and measurement situations can all be utilized to demonstrate common ratio and rate situations.

- Geometric situations
  - the ratio of the number of sides in a hexagon to the number of sides in a square (6:4)
  - the ratio of the number of vertices to the number of edges in a rectangular prism (8:12)
  - the ratio of the number of vertices in a hexagon to the number of sides (6:6)
- Numerical situations
  - the ratio comparing the value of a quarter to that of a dime (25:10)
  - the rate of pay for a job (e.g., $5/hour)
  - the ratio comparing the number of multiples of 2 to the multiples of 4 for numbers from 1 to 100 (2:1 or 50:25)
- Measurement situations
  - the rate describing the "crowdedness" of a classroom (e.g., 25 people/60m^2)
  - the ratio of perimeter to side length of a square (4:1)
  - the ratio of describing the enlargement factor on a Xerox copy (3:2)
  - the rate comparing area to side length of a square (e.g., 1:1 or 4:2 or 9:3...)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A5.1 Ask the students to model a situation in which there are six of one thing for every two of another.

Paper and Pencil

A5.2 Ask the students to fill in the blanks in as many ways as possible to create a true statement about mathematical situations. For example,

For every ______ ______ , there are _______ _______ .
(Sample response: For every one dozen, there are 12 items.)

A5.3 Ask the students to make one drawing that shows both of the following:

For every one pencil, there are three pieces of paper.
For every three pencils, there are nine pieces of paper.

Ask the students what they would say for every five pencils.

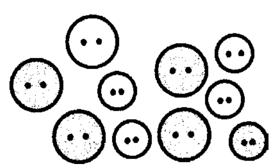
Interview

A5.4 Ask the student to give a number of ratios that relate to sports. For example, for every 5 players on the starting lineup in basketball, there are 9 players in baseball. Answers could be shared.

A5.5 Ask the student to tell what ratio would always be four to one; generally, but not always, four to one.

A5.6 Ask: How can you predict the length of an object in centimetres if you know the length in millimetres? Would it be possible to predict the length in centimetres if you knew the length in metres? Which do you find easier and why?

A5.7 Ask the student to give as many ratios as he/she can using a set of buttons or a picture of buttons.



A5.8 Tell the student that the ratio of cans to cases is 96 to 4. Ask what the number of cans would be when there is one case.

Portfolio

A5.9 Ask the students to write out a number of ratios that they notice in their homes using this pattern:

For every ______ ______, there are _______ _______.

A5.10 Ask the students to describe one or more situations that depict a rate of 3:1.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) read and write whole numbers and decimals, and demonstrate an understanding of place value (to millions and to thousandths)

SCO: By the end of grade 5, students will be expected to

A6 read and represent numbers to millions

Two important ideas developed for three-digit numbers should be carefully extended to larger numbers. First, the grouping idea should be generalized. That is, ten in any position makes a single thing in the next position, and vice-versa. Second, the oral and written patterns for numbers in three digits is duplicated in a clever way for every three digits to the left. (Elementary School Mathematics, p. 173)

Elaboration - Instructional Strategies/Suggestions

A6 Students should be made aware of the pattern in the place-value system whereby each set of three digits is read as a number (up to 999) with the appropriate unit. For example, 42 135 456 is read as 42 million, 135 thousand, 456.

A place-value chart blocked into sets of 3 digits is useful to demonstrate this idea.

Millions			Thousands			Units		
H	T	O	H	T	O	H	T	O

Students should be exposed to reading whole numbers in a variety of ways. For example:

- 6 200 000 as six million, two hundred thousand or 6.2 million (6 and 2 tenths million), since $\frac{1}{10}$ of a million is 100 000.
- 2 153 456 as 2 million, one hundred fifty-three thousand, four hundred fifty-six; or two thousand one hundred fifty-three thousand, four hundred fifty-six

❐ Students require practice placing counters or digits on a place value chart to represent a number stated orally. The digital form can be written once the chart is filled in for each number. Ask the student to read orally the number he/she forms.

❐ When students have had sufficient practice with the place value chart, have them write only the digital form of numbers read aloud to them. Vary the difficulty by including numbers that have several zeroes. Ask students to read the numbers back to the class. Whenever an activity such as this is designed for a class, it is recommended that more be done with the numbers than simply writing them properly. An obvious extension is to look at the numbers individually and have a discussion of what each might represent. This will help students develop a sense of large numbers. It is important to use data where possible.

❐ As an extension to reading and writing numbers, ask the students to practise telling how many more must be added to make a particular number. For example, have students write "nine hundred eighty thousand, four" (980 004) and ask how many more would be needed to make a million. The idea is for the students to find a way that makes sense to them to find the difference. Some may recognize that twenty thousand less four would make a million, or nineteen thousand nine hundred ninety-six. Such learning experiences provide practice with mental math strategies.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A6.1 Have the student use the digits 2, 0, 4, 0, 5 and 3 to record three different whole numbers. Ask him/her to read each one. As an extension, ask the student to determine how many numbers can be made using these digits.

A6.2 Ask the students to make the largest possible number on their calculators. Have them read the number. (They will probably point out the fact that numbers without spaces are more difficult to read.) Ask: If you were to subtract 98 765 432 from the number displayed, predict what the calculator will show when you press the equal sign. Check it.

A6.3 Have students rearrange the cards below in various orders. Ask them to write the digital form of the number for each arrangement.

five	hundred	two	million	three	thousand	four

Paper and Pencil

A6.4 Ask the student to write in numerals the population of British Columbia, which is "three million, two hundred eighty thousand," and the population of Quebec, which is "6.9 million."

Interview

A6.5 Tell the student that a number has 8 digits and ask what he/she knows about it.

Portfolio

A6.6 Have the students write a report on the different ways numbers are written in newspapers and magazines. Ask them to include a section on "Estimation in the Media."

A6.7 Ask the students to make a list of whole numbers that take, for example, three words to say. (Some examples include 9 000 080, 600 000, 403).

A6.8 Tell the students that a company has 1.45 million paperbacks. Ask: How many boxes, and what size, would be needed to hold these books? Could the students in the school read this many books and, if so, how long would it take them? Have them determine how many library shelves this many paperbacks would fill.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) read and write whole numbers and decimals, and demonstrate an understanding of place value (to millions and to thousandths)

SCO: By the end of grade 5, students will be expected to

A7 read and represent decimals to thousandths

Elaboration - Instructional Strategies/Suggestions

A7 It is important that students recognize numbers with three places after the decimal represent thousandths, and numbers with two places or one after the decimal represent hundredths or tenths. Students should be able to read decimal numbers in print and record decimal numbers upon hearing them orally.

Students should also be able to place decimal numbers on a number line. For example, given a segment with end points labelled 2 and 4, students should mark where they think the following numbers would be and defend their positions: 2.3, 2.51, 2.999, 3.01, 3.75, 3.409 and 3.490.

It is recommended that students state the quantitative value of the digits when reading decimal numbers. The decimal is read as "and."

For example:

16.8 is read "sixteen and eight-tenths."
0.57 is read "fifty-seven hundredths."
2.091 is read "two and ninety-one thousandths."

Reading decimal numbers in context, such as kilograms of hamburger or litres of juice, is useful when making the connection between fractions and decimals. 6.25 L is six and twenty-five hundredths litres or can be thought of as $6\frac{25}{100}$, which some might recognize as $6\frac{1}{4}$. Students should also be able to interpret whole numbers written in decimal format (e.g., 5.1 million as 5 100 000).

18.5 can be read "18 and 5 tenths" but is often read "18 and a half." Have students practise reading numbers in this way. For example, 6.497 may be read as about 6 and a half, 48.73 as about 48 and 3 quarters, and 12.254 as about 12 and a quarter.

❒ Have the students discuss when a fraction rather than a decimal number is likely to be preferred and when the decimal representation would seem more appropriate.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A7.1 Give a pair of students three dice and ask them to take turns tossing all three. Ask them to make the largest number they can using the digits to represent tenths, hundredths, and thousandths. Ask them how much would need to be added to their number to make one whole. For example, if they were to toss 3, 6, and 2, the largest possible number would be 0.632; 0.368 would need to be added to make one whole.

A7.2 At a centre, place five different displays of combinations of base-ten blocks. Ask the students to visit the centre and record the five decimals displayed. Explain that the large cube represents one.

Paper and Pencil

A7.3 Tell the student that gasoline is priced at 56.9¢ per litre. Ask: What part of a dollar is this?

A7.4 Ask the student to write the numerals for "two hundred fifty-six thousandths" and "two hundred and fifty-six thousandths." Ask him/her to explain why watching and listening for "and" is important when interpreting numbers.

Interview

A7.5 Ask the student to explain why newspapers might record a number as 2.5 million instead of 2 500 000. Ask him/her to discuss whether or not this is a good idea.

A7.6 Show the student a written request that Samuel's teacher gave him: "Please cut 3.25 m of ribbon for me." Ask the student to read the note and tell how many centimetres of ribbon this would be.

A7.7 Tell the student that you drank 0.485 L of juice. Ask: About how much more would I have to drink to equal 0.5 L?

Portfolio

A7.8 Ask the students to write a report on the use of 0.5 and $\frac{1}{2}$. Have them survey adults and check newspapers and magazines to find when each is used.

A7.9 Ask the students to write 10 different decimal numbers that have tenths, hundredths, and/or thousandths. Have them make base-ten block pictures that would represent their numbers.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 5, students will be expected to

A8 compare and order large numbers

Elaboration - Instructional Strategies/Suggestions

A8 Students should be able to compare two whole number quantities in a variety of formats. For example:

- both numbers are written out in full (e.g., 34 256 876 > 34 255 996)
- both numbers are written in decimal form (e.g., 34.25 < 34.3)
- one number is written in full and the other using decimal notation (e.g., 34 256 876 < 35.2 million)
- numbers are written using different units (e.g., 3423 thousand 453 > 3 325 146)

The latter two formats tend to be more challenging for students. It is important, therefore, that they have practise renaming numbers in different ways. Once students recognize that there are 1000 thousands in a million, they realize that 6 million and 6000 thousands are the same. Students need to have a sense of the size of large numbers. Simply being able to point out the place value of digits is not sufficient.

Provide questions in context, such as:

❒ Mrs. McKinnon won $2 435 752 in the lottery. She already had $2.5 million saved. Has she doubled her money? Explain.

❒ Order the populations of the following metropolitan cities from least to greatest:
New York - 17.95 million   Paris - 8 720 000
Tokyo - 28.4 million   London, England - 7 000 000
Ask the students to make comparative statements about the populations. For example: The population of Tokyo is about four times that of London. The population of Paris is about half the population of New York City. The populations of New York and Paris together are less than the population of Tokyo.

❒ One school district ordered 1.2 million sheets of paper to be used for copying in the schools and another school district ordered 11 hundred thousand sheets. Which district ordered the most paper? Approximately how many boxes would be needed to hold this quantity of paper? (Ensure students indicate the size of the boxes.) About how long would one of the quantities of this paper last in your school?

Some attention may need to be given to rounding to the nearest million or hundred thousand for the purpose of comparing numbers easily.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A8.1 Ask the student to order the following number cards from least to greatest.

7 406 397	0.9 million	950 606
2.13 million	1000 thousands	38 hundred thousand

A8.2 Tell the students that Jason spent $980 every day for 5 years, Sue spent $854 every day for 6 years, and Keri spent $1156 every day for 4 years. Ask them to predict who spent the most and the least money. Students may use calculators to check their predictions.

A8.3 Have students predict which 3 cities in the world they think have the most people. Then provide students with data (or have them find the data on the Internet) on the population of ten of the most populous cities around the world. Have them order the data to verify their predictions.

Interview

A8.4 Tell the student that Maggie compared 3 425 630 and 3 524 013 by explaining that 34 hundred thousand is less than 35 hundred thousand. Ask the student to explain Maggie's reasoning and to identify other approaches for making the comparison.

Portfolio

A8.5 Have students design a game for their classmates that requires them to compare and order large numbers.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 5, students will be expected to

A9 compare and order decimals

Elaboration - Instructional Strategies/Suggestions

A9 Students should be able to determine which of two decimal numbers is greater by comparing the whole number parts first and then the amounts to the right of the decimals. It is important that students understand decimal numbers do not need the same number of places after the decimal to be compared. For example, one can quickly conclude that 0.8 > 0.423, without converting 0.8 to 0.800, because the former is much more than half and the latter is less than half. Students should also understand that a number having more places after the decimal than another does not mean it is smaller nor does it mean it is larger—these are common misconceptions. That is, some students think 0.101 is larger than 0.11 because 101 is larger than 11; others think it is smaller just because it has thousandths while the other number has only hundredths. (These same students would also say 0.101 is smaller than 0.1 because it has thousandths while 0.1 has only tenths.) Such misconceptions are best dealt with by having students make base-ten block representations of numbers that are being compared.

Students should recognize that thousandths are generally small in comparison to other numbers (e.g., 0.003 is much smaller than 3). One-thousandth is one-tenth of one-hundredth, and one-hundredth of one-tenth. Thousandths make little difference when two numbers are compared, unless the numbers are very small (e.g., 0.014 m and 0.009 m). There are times, however, when thousandths are not particularly small. For example, if 3.124 million was a population figure, 4000 would not be considered small. The size is relative to the context.

Encourage students to round decimals to the nearest tenth or hundredth to get a relative sense of their size.

Measurement contexts provide valuable learning experiences for decimal numbers because any measurement can be written in an equivalent unit that requires decimals (e.g., 345 mL is 0.345 L).

❒ Give students eight blank cards and ask them to write a decimal on each. Have them challenge a partner to order the number cards.

❒ Give students the number cards 0.99, 0.987, 0.9 and 1.001, and ask them which decimal number they think is closest to 1. Have them explain how they made their decisions.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A9.1 Provide a set of "digit cards." Ask students to place cards in the missing spaces, in as many ways as possible, to make the sentence below true.

$$9.\square 8 < \square.2\,\square 0$$

A9.2 Have students copy a template of the type shown.

$$\square.\square\;\square > \square.\square\;\square$$
$$\square\;\square.\square > \square\;\square.\square$$
$$\square\;\square.\square > \square.\square\;\square$$

Roll a die. As the number is called, each student fills in a blank on his or her paper. Roll the die 18 times. The students who end up with three true sentences win a point. Repeat the process.

A9.3 Give partners of students six cards with different base-ten block pictures on them. Ask them to order the numbers represented and read the decimals to one another. For this activity, explain that the flat equals one.

Interview

A9.4 Ask the student to explain why you cannot compare two decimal numbers by simply counting the number of digits in each.

A9.5 Give students the number cards 9.023, 10.9, 9.05, 10.11 and 9.8, and ask them which decimal they think is closest to 10. Have them explain how they made their decisions.

Portfolio

A9.6 Provide examples of some of the best javelin throw distances that have occured in past Olympics. For example:

1972: 90.48 m
1980: 91.20 m
1988: 84.28 m
1992: 89.66 m

Ask students to arrange the distances in order and determine whether records always improve. Students can follow up by choosing records from a different Olympic event to order and include in their portfolios.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 5, students will be expected to

A10 compare and order fractions using conceptual methods

Elaboration - Instructional Strategies/Suggestions

A10 Students should continue to use conceptual methods to compare fractions. These methods include i) comparing each to a reference point, ii) comparing the two numerators when the fractions have the same denominator, and iii) comparing the two denominators when the fractions have the same numerator.

A common error made by students at this level is to think, for example, that $\frac{10}{7}$ is greater than $\frac{10}{6}$ because of their experience comparing whole numbers. Considerable time needs to be spent on activities and discussion to develop number sense of fractions. The "pizza model" works well. Ask students: Which would you rather have, a piece of pizza divided into 6 equal parts or a piece of the same pizza divided into 7 equal parts? This is basic, but not always recognized by all. Pose questions such as the following:

-Which is greater, $\frac{3}{10}$ or $\frac{3}{8}$? A possible answer: "I know $\frac{3}{8} > \frac{3}{10}$ because eighths are larger than tenths."

-Which is greater, $\frac{3}{8}$ or $\frac{7}{10}$? A possible answer: "I know $\frac{7}{10}$ is greater than $\frac{3}{8}$ because $\frac{3}{8}$ is less than half and $\frac{7}{10}$ is greater than half."

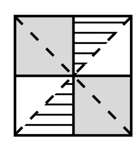
-Which is greater, $\frac{4}{5}$ or $\frac{3}{4}$? A possible answer: "I know $\frac{4}{5}$ is greater than $\frac{3}{4}$ because $\frac{4}{5}$ is only $\frac{1}{5}$ away from a whole and $\frac{3}{4}$ is $\frac{1}{4}$ away from a whole."

Students should be encouraged to compare fractions greater than one by considering them as mixed numbers.

For example, which is greater, $\frac{10}{8}$ or $\frac{7}{5}$? A possible answer: "I know $\frac{7}{5}$ is greater because $\frac{10}{8}$ is $1\frac{2}{8}$, $\frac{7}{5}$ is $1\frac{2}{5}$, and $\frac{2}{5}$ is greater than $\frac{2}{8}$."

❒ Invite students to create patches (made of paper) for a class patchwork quilt in which the colours on their patches show a particular comparison.

For example, this patch could be used to illustrate that $\frac{1}{4} < \frac{3}{8}$.



Some students may be ready to compare pairs of fractions by finding equivalents that share the same numerator or same denominator. For example: i) When comparing $\frac{3}{5}$ and $\frac{7}{10}$, $\frac{3}{5}$ can be renamed as $\frac{6}{10}$.

ii) To compare $\frac{5}{3}$ and $\frac{10}{7}$, rename $\frac{5}{3}$ as $\frac{10}{6}$. A focus on a procedure for finding equivalent fractions is not important at this level.

An understanding of fraction concepts and order and equivalence relations is a prerequisite for success in computation with fractions. (<u>NCTM 1989 Yearbook</u>, pp. 160-61)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A10.1 Provide students with pattern blocks. Ask them to arrange the blocks to model two different fractions, one being less than the other. Have them record the number sentence that describes the model.

Paper and Pencil

A10.2 Ask the student to use the digits 1 to 9 to fill in the boxes in 3 different ways to make true statements.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

Interview

A10.3 Ask: How do you know that $\frac{1}{3} < \frac{3}{4}$?

A10.4 Ask: If you know that $\frac{2}{\square} > \frac{2}{7}$, what do you know about the value of $\square$? Explain.

A10.5 Give the student cards on which the following fractions are written: $\frac{2}{5}$ $\frac{1}{4}$ $\frac{6}{5}$ $\frac{7}{8}$ $\frac{5}{10}$

Ask him/her to order the fractions from least to greatest and to give reasons for the order. This task could be modified to include some decimals, particularly tenths, and common fractions. For example $\frac{5}{10}$ could be written as 0.5.

Presentation

A10.6 Have students conduct an experiment rolling a pair of coloured dice. The number on the red die is used as the numerator of a fraction and the number on the blue die is used as the denominator. Have them predict whether or not the fraction will usually be less than half. Allow students to conduct the experiment to verify their predictions and present their findings to the class.

Paired Discussion

A10.7 Ask pairs of students to find a way of showing which of $\frac{7}{8}$ and $\frac{5}{6}$ is greater. Ask them to provide an explanation that is easily understood without the use of materials. Have them list pairs of fractions that they find more difficult to compare and to explain why.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals

SCO: By the end of grade 5, students will be expected to

A11 recognize and find factors of numbers

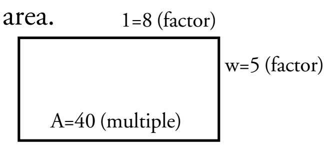
Elaboration - Instructional Strategies/Suggestions

A11 Because students have been working formally with multiplication since grade 3, they should be familiar with the term "factor." At this point, students should also recognize that:

- the factors of a number are never greater than the number
- the number is always a multiple of any of its factors
- division can be used to find factors
- the greatest factor is always the number itself
- the least factor is always 1
- the second greatest factor is always $\frac{1}{2}$ the number or less

To find factors, students might use various strategies.

• They can create rectangles of a particular area. (The use of square tiles or grid paper facilitates this process.) The length and width of the rectangle are factors of the number representing the area.



Students who find all possible rectangles of that area will have found all the factors.

• They can divide by each number that is less than or equal to the given number and in that way may find all the possible factors.

• They might use knowledge of special properties of multiples of specific factors. For example, there are only even digits in the ones place of multiples of 2, or the ones digit of multiples of 5 is either 5 or 0.

Some students confuse the terms "factor" and "multiple." Model this language consistently for students. Statements like "2 is a factor of 4; 4 is a multiple of 2" are helpful. Also, provide a variety of experiences that will require students to use the words factor, multiple and product, both orally and in writing. For example, show students a rectangle with its dimensions and area marked. Ask them as partners to take turns making as many different statements as possible about the factors, multiples and product shown by this rectangle.

❐ Ask students to tell what they know about multiplication sentences such as, 22 x 12 = 264. It is expected that they will know 22 and 12 are factors of 264; 264 is the product of 22 and 12; 264 is a multiple of 22 and 12. Some students may recognize that 22 x 12 is the same thing as 11 x 24, consequently deducing that 11 and 24 are factors of 264.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A11.1 Ask students to use 24 colour tiles to find all of the factors of 24. They might then use more or fewer tiles to discover another number that has the same number of factors. Students should record their results.

A11.2 Ask students to use colour tiles to determine which of 24, 36, and 45 has the greatest number of factors and to keep a record of how they arrived at their answer.

Paper and Pencil

A11.3 Tell the student that a certain number has 2, 3 and 4 as factors. Ask: What might the number be?

A11.4 Have the student compare the factors of a number and its double (e.g., 12 and 24) and describe what he/she notices.

A11.5 Ask the student to express 36 as the product of two factors in as many ways as possible.

Interview

A11.6 Ask the student to explain how he/she knows, without dividing, that 2 cannot be a factor of 47.

A11.7 Ask the student to describe how he/she would go about finding the factors of a number.

A11.8 Ask the student why he/she would know without finding all factors of 42 that the two greatest ones are 21 and 42.

Portfolio

A11.9 Ask the students to explain, in a few sentences, why every whole number greater than one has at least two factors.

A11.10 Ask the students to use 24 colour tiles to make rectangles of various shapes and then to write about their observations using the words “factor,” “product,” and “multiple” in their work.

A11.11 Tell the students that a marching band has 120 members. Ask them to explore the many different ways one might arrange the band into equal rows for marching.

Suggested Resources

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

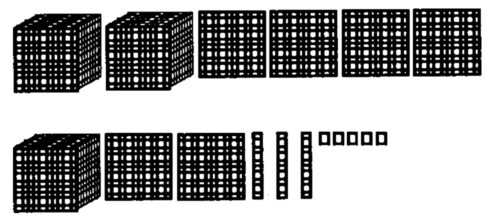
SCO: By the end of grade 5, students will be expected to

B1 find sums and differences involving decimals to thousandths

Elaboration - Instructional Strategies/Suggestions

B1 Students should be encouraged to use materials (e.g., thousandths and hundredths grids and base-ten blocks), if needed, to help them solve addition and subtraction questions involving tenths, hundredths and thousandths.

When using base-ten blocks, if the large cube is used to represent the whole, 2.4 + 1.235 is modelled as:



Addition and subtraction questions should be presented both horizontally and vertically to encourage alternative computational strategies. For example, for 1.234 + 1.990, students might calculate: 1.234 + 2 = 3.234 followed by 3.234 - 0.01 = 3.224.

Students have to make choices when doing computations. First, they have to determine whether the answer must be exact or if an approximation is sufficient. If an exact answer is required, other decisions must be made after they estimate, such as, "Can I compute this mentally?" If not, "Will I use a paper-and-pencil method or a calculator?" Students do not automatically make these decisions; they must be encouraged to look at the possibilities for all computation questions. Only when this selection of procedures is encouraged will students begin to make appropriate decisions. It is important to remember, however, that because students must become proficient with paper-and-pencil methods, the use of a calculator should not always be an option. Pencil-and-paper calculations are generally used if the computation is not overly tedious. If it proves to be, a calculator would probably be selected.

Students should be able to use algorithms of choice when they calculate with pencil-and-paper methods. While it is important that the algorithms developed by students are respected, if they are cumbersome or inefficient, students should be guided toward more appropriate ones.

Before performing a pencil-and-paper calculation, students need to estimate the result. Estimation is also important when using a calculator to determine if the solution in the display is reasonable. Students who have developed good estimation strategies generally have a good sense of number. It is important to model this estimation behaviour for the students.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B1.1 Provide base-ten blocks or thousandths grids. Have the student choose addition or subtraction questions involving decimal numbers to represent with the models.

B1.2 Model 4.23 and 1.359 with base-ten blocks or thousandths grids. Ask the student to use the materials to explain how to find the difference between the two numbers.

B1.3 Give students a variety of labels from canned goods or pictures of such items from flyers, which show either the contents in millilitres or the mass in grams. Ask students to find three items with a total capacity or total mass greater than a specified number of litres or grams.

Paper and Pencil

B1.4 Provide students with the batting averages of some baseball players. Have them calculate the spread between the player with the highest average and the one with the lowest. Have students create problems using the averages on the list.

B1.5 Ask the students to fill in the boxes so that the answer for each question is 0.4. The only stipulation is that the digit 0 cannot be used to the right of the decimal points.

$$\begin{array}{r} \square.\square\square \\ +\,\square.\square\square \\ \hline \end{array} \qquad \begin{array}{r} \square\square.\square \\ -\,\square\square.\square \\ \hline \end{array}$$

B1.6 Request that the students provide examples of questions in which two decimal numbers are added and the answers are whole numbers.

Interview

B1.7 Tell students that you have added 3 numbers, each less than 1, and the result is 2.4. Ask if all the decimal numbers could be less than one-half and to explain why or why not. Once students realize the numbers cannot all be less than one-half, ask them how many could be.

B1.8 Present the following situation in which Jane made an error when she subtracted. Ask the students what one might say to help Jane understand why the answer is incorrect.

$$\begin{array}{r} 5.23 \\ -\ \underline{1.453} \\ 3.783 \end{array}$$

Presentation

B1.9 Have students conduct research to find situations in which decimals are added and subtracted in everyday life and present their findings in a video, as an oral presentation or as a written report.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 5, students will be expected to

B2 multiply 2-, 3- and 4-digit numbers by 1-digit numbers

B3 find the product of two 2-digit numbers

Elaboration - Instructional Strategies/Suggestions

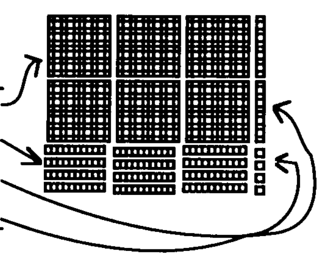
B2 Students should be encouraged to continue to use the language of models to explain the multiplication algorithm. For example: “4 sets of 2 units are 8 units. 4 sets of 5 rods are 20 rods, which when regrouped are 2 flats, leaving no rods. 4 sets of 4 flats are 16 flats. The 2 additional flats make 18 flats, which when regrouped are a thousand cube and 8 flats. 4 sets of 3 thousand cubes are 12 thousand cubes; the additional 1 thousand makes 13 thousand.”

$$\begin{array}{r} {\scriptstyle 12} \\ 3452 \\ \times \quad 4 \\ \hline 13\,808 \end{array}$$

Encourage mental math strategies such as: 5 x 66 is the same as 10 x 33 (double/half strategy); 44 x 25 = 11 x 100 (÷4, x 4); 3 x 213 is 639 using the front-end strategy (3 x 200 + 3 x 10 + 3 x 3).

B3 Similarly, models should be used for the multiplication of 2-digit by 2-digit numbers. In this case, it is efficient to model the product as the area of a rectangle with the dimensions of the two numbers. This can be done using base-ten blocks or grid paper. Students should relate the model to the algorithm. The symbolic steps should be recorded and related to each physical manipulation.

$$\begin{array}{r} 31 \\ \times\,24 \\ \hline 600 \\ 120 \\ 20 \\ 4 \\ \hline 744 \end{array}$$



When the students understand the area model, they may choose to use a grid-paper drawing as an explanation. The standard algorithm might be presented, but it is important that an explanation with models be provided, not just procedural rules. As always, students should be given the choice of using the standard algorithm or an alternative one. If, however, students are using inefficient algorithms, they should be guided to select more appropriate ones. And, as for all computational questions, students should estimate before calculating.

Immediate recall of basic multiplication facts is a necessary prerequisite not only for paper-and-pencil algorithmic procedures, but also for estimation and mental computation.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B2.1 Ask the student to use a model to explain how three theatres, each having 243 seats, could hold 729 people.

B3.1 Have the student use a model to show how to find the amount of money collected for photos if 43 students each bring in $23.

Paper and Pencil

B2.2 Have the students compare the results of 36 x 4 and 46 x 3 and the results of 74 x 5 and 54 x 7. Ask them to make general statements about the switching of the tens digit in the 2-digit factor with the single-digit factor.

B3.2 Ask the students to explain why the product of two different 2-digit numbers is always greater than 100.

Interview

B2.3 Tell the student that a number is multiplied by 8 and the product is 11 384. Ask how he/she knows that the number was greater than 1000 and had a 3 or an 8 in the ones place.

B2.4 Tell the student that to find 7 x 513, Anne began with 3500. Ask what she would do next to find the product.

B2.5 Tell the student that 24 x 5 = 120. Ask him/her how this could be used to find the product of 34 and 5.

Portfolio

B3.3 Ask students to explore the pattern in these products: 15 x 15, 25 x 25, 35 x 35, etc. Have them describe the pattern and tell how the pattern could be used to predict 85 x 85 or 135 x 135. They might then test their predictions using a calculator where appropriate. Alternatively, students might explore the pattern in these products: 19 x 21, 29 x 31, 39 x 41, and use it to make a prediction for 79 x 81 and 109 x 111.

B3.4 Ask students to explore the following:

24 + 35 is the same as 25 + 34. Is 24 x 35 the same as 25 x 34? Provide an explanation.

19 + 32 is the same as 20 + 31. Is 19 x 32 the same as 20 x 31? Explain.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 5, students will be expected to

B4 divide 2-, 3- and 4-digit numbers by single-digit divisors and investigate division by 2-digit divisors

Elaboration - Instructional Strategies/Suggestions

B4 Division problems can involve either sharing or finding how many groups.

$$\begin{array}{r} 15 \\ 3\overline{)4532} \\ -\underline{3} \\ 15 \\ -\underline{15} \\ 3 \end{array}$$

"4 thousands shared among 3, each gets 1 thousand with 1 thousand left. Trade 1 thousand for 10 hundreds; now 15 hundreds to share, each gets 5 hundreds, etc."

OR

$$\begin{array}{r} 3\overline{)4532} \\ -\underline{3000} \\ 1532 \\ -\underline{1500} \end{array}$$

"Make 1000 sets of 3, using 3000; 1532 left. Make 500 sets of 3 using 1500; 32 left..." The number of sets at each stage, tends to be a multiple of 10, 100 or 1000 to facilitate computation.

The traditional long-division algorithm, whether modelled with base-ten blocks or not, is best described using "sharing words." For example, in the algorithm to the right, it is important that students realize the "4" represents 4 thousands and if three share, each will get 1 (thousand). One thousand is left and when put with the 5 hundreds gives 15 hundreds to share among 3, etc.

$$\begin{array}{r} 15 \\ 3\overline{)4532} \\ -\underline{3} \\ 15 \\ -\underline{15} \\ 03 \end{array} \quad \text{etc.}$$

Students should understand why the number of units leftover after the sharing must be less than the divisor. Models help to clarify this idea. A common mistake of students is to write a remainder as a decimal when the divisor is other than 10 (e.g., a remainder of 7 is written as .7). This should be addressed through a discussion of remainders and the meaning of tenths.

Students at this level should also have opportunities to investigate division by 2-digit divisors. At this introductory stage, it is important to focus on providing a good estimate for the quotient. Division by 10, 20, 30, etc. is a good place to begin. Given the question, 869 ÷ 20, one might think, "20(2 tens) x 40 (4 tens) = 800 (8 hundreds); 69 remains. 20 (2 tens) x 3 = 60 (6 tens); giving a solid estimate of 43." From examples of this nature, it is logical to explore questions such as 2713 ÷ 31. 2713 is close to 2700 and 31 is close to 30. 30 (3 tens) x 90 (9 tens) = 2700 (27 hundreds). A good estimate is 90. A knowledge of multiplication facts is key to estimating quotients.

Strategies, such as using compatible numbers and compensating, are also helpful to estimate quotients. In the example, 9118 ÷ 16, 9118 is close to 9000 and 16 is close to 15; the 90 and 15 form a compatible number pair and the estimate would be about 600. Students should have sufficient experience with 2-digit divisors that they understand the process.

Dividing by 2-digit divisors can be tedious and many times a calculator would be used. Students should be taught to interpret the decimal remainder. For example: When can the remainder be ignored? When must one round up? When does it form an important part of the solution?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

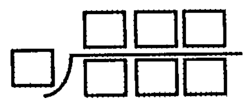
Worthwhile Tasks for Instruction and/or Assessment

Performance

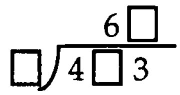
B4.1 Ask the student to use materials to model how to divide 489 by 7.

Paper and Pencil

B4.2 Ask the students to fill in the boxes so that no digit is used more than once, there is no remainder, and the resulting division is correct.



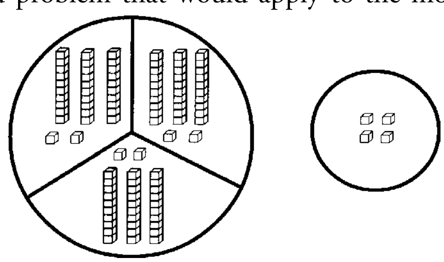
B4.3 Ask the students to fill in the boxes to make the division sentence true.



Interview

B4.4 Ask the student to tell what division is being modelled below and to provide a word problem that would apply to the model.

100 ÷ 32 = 3 R4



B4.5 Tell the student that 612 students in a school are being bussed to a museum. The law states that a maximum of 45 students is allowed on each bus. Ask them to estimate and then calculate how many busses will be needed.

Portfolio

B4.6 Ask students to write a word problem involving division by a 2-digit number for each of the following:

a) a situation in which the remainder would be ignored
b) a situation in which the remainder would be rounded up
c) a situation in which the remainder would be part of the answer

Sample situations:

a) A person has 78¢. Pens cost 19¢ each. How many pens can the person buy?
b) 126 people wanting to go on a trip. Twelve passengers can travel in each van. How many vans are needed?
c) 75 metres of ribbon are to be shared among 10 students. How much ribbon will each student get for crafts?

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

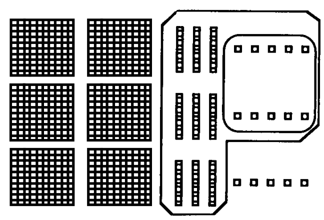
SCO: By the end of grade 5, students will be expected to

B5 find simple products of whole numbers and decimals

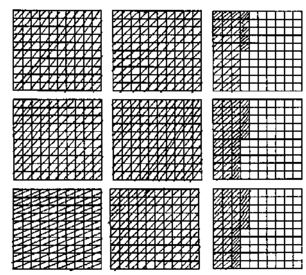
Elaboration - Instructional Strategies/Suggestions

B5 To help students develop a conceptual understanding of the multiplication of whole numbers and decimals, it is important initially to model such questions. For example, the following situation may be represented in several ways. "Each costume for the class play requires 2.35 metres of fabric. How much fabric should be purchased to make 3 costumes?" 3 x 2.35 = 7.05 may be modelled using:

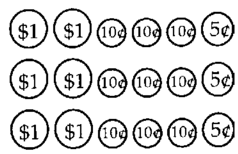
- base ten blocks
  (the flat represents 1, the rod 0.1, etc.)



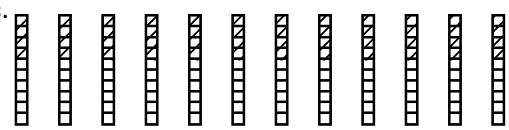
- decimal grids



- money



Students also need to work with problem-solving situations in which decimals are multiplied by whole numbers. Such questions require taking part of a set and are modelled differently. Given the question 0.4 x 12, one needs to take 0.4 of the set of 12. This may be modelled using base-ten blocks.



Let the rod represent 1, and it is easy for students to take 0.4 of each rod. 48 tenths equals 4.8.

Note: Before modelling procedures or calculating using paper-and-pencil, students should estimate the product.

It is important to point out that for some multiplication questions a mental strategy should be employed and that the front-end method is most often selected. (See page 5-40, for more information on the front-end method.) For example, 4 x 20.12.

❒ Provide students with examples of calculations and ask them to identify those numbers that lend themselves to the front-end strategy.

Examples:	23.31 x 3	67.9 x 7	2 x 435.24 8 x 35.48 4 x 25.21

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B5.1 Request that the students use materials to model 8 x 2.03.

B5.2 Ask the students to draw a model to show 0.3 x 15.

Paper and Pencil

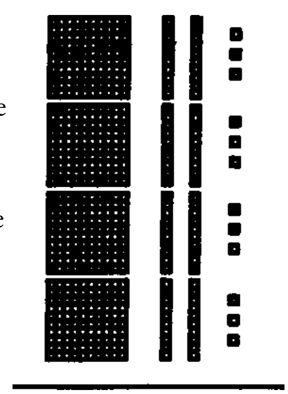
B5.3 Ask the students to fill in the boxes with digits that make the computation true.

$$\begin{array}{r} 4.\square 8 \\ \times \quad \square \\ \hline \square 6.28 \end{array}$$

Interview

B5.4 Ask the student to explain how and why the answers to 435 x 7 and 43.5 x 7 are alike and how they are different.

B5.5 Tell the student that a flat represents 1 whole. Ask what multiplication question is represented by the base-ten blocks. Have the student do the pencil-and-paper computation that is represented, relating each step in the process to the blocks in the display.



B5.6 Tell the student that to find 2.25 x 8, Jane said, "16 + 2 = 18." Ask the student to explain Jane's thinking.

B5.7 Ask the student to find the answer to 3 x $2.13 using the front-end method. 5 x $4.25?

Portfolio

B5.8 Ask students to make a list of ten computations that require multiplication of decimal numbers by whole numbers. Have them include some examples that their classmates could solve using the front-end strategy. These lists could be exchanged in class.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 5, students will be expected to

B6 **divide decimal numbers by single-digit whole numbers**

Elaboration - Instructional Strategies/Suggestions

B6 With the support of models, such as base-ten blocks, students will come to see that the process of dividing decimal numbers by whole numbers is identical to that involving the division of whole numbers. For example:

```
    11.35
4 ) 45.4
  - 40
     5.4
  -  4
     1.4
  -  1.2
     0.20
  -  0.20
     0
```

"4 tens shared among 4, each gets 1 ten using 40; 5.4 left to share; each gets 1 whole using 4; 1.4 is left to share; 14 tenths shared among 4, each gets 3 tenths; 2 tenths left or 20 hundredths; each gets 5 hundredths."

Students need to be reminded of the importance of estimating before modelling a question or using paper-and-pencil procedures.

It is also important to discuss the nature of the remainder when dividing decimal numbers. For example, in the division question shown at the right, the remainder is 0.1, not 1. If desired, this leftover could be traded for 10 hundredths making the quotient more accurate, i.e., 115.13+.

```
    115.1 +
3 ) 345.4
```

- ❐ Using store flyers, prepare a list of specific items such as: 5 items for \$4.65, 8 items for \$16.88, etc. Provide students with the list and the flyers and ask them to determine the specific items.
- ❐ Students could research winning relay times in Olympic events. They could then compute the average time taken by each of the 4 competitors in these events. The *Canadian Almanac* and other references provide detailed Olympic data.
- ❐ Explain to the students how one rounds numbers when determining the price of single items. For example, if you wanted to buy one can of peas that is priced at 2/99¢, your price per can would be 50¢.
  3/\$1 - individual price is 34¢
  2/\$1.49 - individual price is 75¢

  Ask the students to calculate the cost of:
  2 cans of peas, priced 6/\$4.50
  4 pkg. tissue, priced 10/\$7.95
  1 kg ground beef @ 3 kg/\$10
  1 can ginger ale @ 12/\$4.99

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B6.1 Tell the student that four people are sharing a pizza which costs \$14.40. Ask what each person's share of the cost should be. Have the student prove that his/her answer is correct.

Paper and Pencil

B6.2 Ask the student to fill in the box so that the amount of money is one that may be shared equally with none remaining.

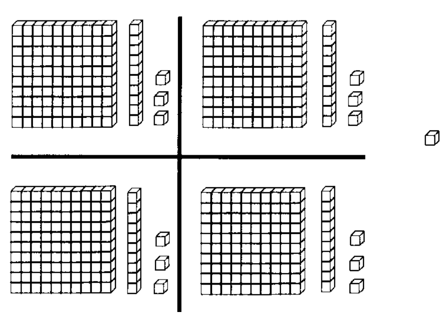
$$4\overline{)\$3.4\square}$$

B6.3 Ask the student for some possible values for the dividend if the result is 5.2 when you divide by a single-digit divisor.

B6.4 Have students fill in the boxes below, in more than one way, making certain not to use the same digit twice within one division question.

$$\begin{array}{r} \square.\square \\ \square\overline{)\square.\square} \end{array}$$

B6.5 Tell student that the flat represents one whole in the division question represented below. Ask him/her to express the division symbolically and to create an accompanying problem.



B6.6 Tell the students that five boxes have to be wrapped using 4.36 m of ribbon. Have them calculate how much ribbon should be cut off for each box. Ask them to explain what the remainder is and what could be done with it.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) explore algebraic situations informally

SCO: By the end of grade 5, students will be expected to

B7 determine whether an open sentence is always, sometimes, or never true

Elaboration - Instructional Strategies/Suggestions

B7 Students have been using open sentences since grade one. Generally, they interpret an open sentence as something that is true, if you find the one right number to make it so. In fact, if the frame is on the "other side of the equals sign," they interpret it to mean "and the answer is.... ."

However, a number of open sentences are always true and some are never true. It is important to expose students to these types of sentences and to help them realize that one cannot immediately determine the type of sentence. For example:
The sentence, "523 + $\square$ is even," is one that is sometimes true. There are many possible numbers to make it so.
The sentence, "523 + $\square$ is greater than 500," is always true, if $\square$ is a positive number.
The sentence, "523 + $\square$ is a fraction," is never true if $\square$ is a whole number.

Grade five students should be encouraged to write examples of open sentences that are always, sometimes, and never true. Encourage them to use all four operations.

Give examples of problem situations to the class and have the students write open sentences for each. It is useful to include some examples of problems that require more than one step. Students may wish to use a box, a triangle, or a letter to represent the unknown number(s). The intent is for students to practise writing open sentences, not necessarily solve them. Some students will write an all inclusive sentence, while others may need more than one open sentence. For example, in the problem situation below, more than one kind of open sentence might be presented: Jake bought a poster and 6 books for a total of \$27.12. The books were priced at \$3.69 each. How much was the poster?
$\square$ + 6 x \$3.69 = \$27.12 might be one sentence.
6 x \$3.69 = $\triangle$ and $\triangle$ + $\square$ = \$27.12 would be another way to represent the problem.

❒ Give the students open sentences and ask that they create a problem situation to match each one.
4 x $\square$ is greater than 100. $\square$ x 2 is even. 620 + $\square$ is greater than 800

Once the problems are written, discuss whether the open sentence is always or sometimes true.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B7.1 Ask: If the missing number in each of the open sentences below is a whole number, can you tell whether the open sentence is sometimes true, always true, or never true? Explain your thinking.

3 x ☐ is even.
3 x ☐ is a multiple of 3.
3 x ☐ is greater than 500.
3 x ☐ is 0.

B7.2 Have the students create an open sentence that is always true, another that is never true, and still another that is only true for one particular value replacing the open frame.

B7.3 Tell the students that some children share 43 candies equally and there is one candy left over. Have them write an open sentence that would show a way to find how many children shared the candy. Discuss the possibility of answers, one of which could be 43 ÷ ☐ = △ r1.

Interview

B7.4 Ask the student whether he/she sees a difference between the value of the ☐ in these two sentences:

5+5= ☐
5+ ☐ is greater than or equal to 15.

B7.5 Show the student the following open-sentences and ask which ones can be solved if the box represents a whole number. Have the student create a problem to match those that he/she selected:

$7.45 + ☐ = $9.22
45 ÷ ☐ = 18
76 + ☐ + 27 = 100
216 - ☐ = 44
☐ x 0 = 49

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) apply computational facts and procedures (algorithms) in a wide variety of problem situations involving whole numbers and decimals

SCO: By the end of grade 5, students will be expected to

B8 solve and create addition and subtraction problems involving whole numbers and/or decimals

B9 solve and create multiplication and division problems involving whole numbers and/or decimals

Elaboration - Instructional Strategies/Suggestions

B8 Students should continue to use addition and subtraction to solve relevant problems which are presented to them or created by them.

An example of a mathematical problem is the cryptarithm, in which each letter stands for a different digit. These problems require students to apply knowledge about the concepts of addition and subtraction and to perform numerous calculations. Students can create and share such problems. It is helpful to repeat some letters/numbers. Example:

Question	Response	Question	Response
ABC	(123)	S E N D	(9567)
+ CBD	(328)	+ M O R E	(1085)
EFA	(451)	M O N E Y	(10652)

Students should be solving multi-step word problems involving some combinations of the four operations as well as creating their own. Requiring students to create their own problems provides opportunities for them to explore the operations in depth. It is a more complex skill requiring conceptual understanding and must be part of the student's problem solving experiences.

B9 Students should also use multiplication and division procedures regularly to solve relevant mathematical problems which they encounter or create. Students should be encouraged to solve and create problems that focus on the various meanings of the two operations, i.e., multiplication as:

- sets of
- arrays
- areas of rectangles
- combinations (For example, with 3 types of ice-cream flavours and 2 types of cones, there are 3 x 2 cone/flavour combinations.)

and division as:

- how many sets of
- sharing
- finding a fraction of

Sports salaries might be an interesting context for multiplication and division problems. For example, if a baseball pitcher has a $500 000 a year contract, how much money does he earn per game pitched? per pitch?

It is important that among the problems presented to or created by students, some lend themselves to mental computation, others require paper-and-pencil computation, and still others call for the use of calculators.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B9.1 Ask the students to create a word problem, incorporating the numbers 64.2 and 3, for which division will be required to find the solution.

B8.1 Have the students create a realistic word problem involving subtraction for which the answer is 1359.2.

B8.2 Ask the students to make up a problem that would require one to compute 1000 - 385.

B8/9.1 Provide students with a variey of shapes that have clearly marked dimensions, some of which include decimal measurements. Have students find the perimeter of each shape.

B9.2 Ask the students to write and solve problems that involve the division of a decimal by a whole number.

Interview

B9.3 Ask students to explain how someone could use multiplication to help them estimate the number of words in a book.

Presentation

B8/9.2 Provide students with store flyers and ask them to create a series of problems based on the information. Have them present their problems to the class.

B8/9.3 Provide data about product sales in Atlantic Canada in various categories. Ask the students to produce a series of meaningful word problems that relate to the data. (*Canadian Global Almanac* or the Internet are possible sources for this information.)

Portfolio

B8.3 Present the following challenge and ask students to describe, in writing, their thinking process: Add all the odd numbers from 1 to 101 and then subtract all the evens from 2 to 100. What is the result?

B8/9.4 Ask students to create money word problems that involve more than one step and more than one kind of operation.

B8.4 Have the student create an addition problem that avoids using cue words such as “altogether” or “in all.”

B8.5 Have the student create a subtraction problem that involves decimals and ask that cue words such as "more than" or "less than" be avoided.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers and decimals

SCO: By the end of grade 5, students will be expected to

B10 estimate sums and differences involving decimals to thousandths

Children with number sense use numbers flexibly and choose the most appropriate representation of a number for a given circumstance. When solving problems, they are able to select from various strategies and tools - they know when to estimate, when to use paper and pencil, and when to use a calculator. They predict with some accuracy the result of an operation and describe the relationships between various forms of numbers. This 'friendliness with numbers' goes far beyond mere memorization of algorithms and number facts and implies an ability to recognize when operations are required and when they have been correctly performed. (Curriculum and Evaluation Standards, Addenda Series, Fifth-Grade Book, p. 8)

Elaboration - Instructional Strategies/Suggestions

B10 Estimation should not be considered a procedure one does only when called upon to do so; students should estimate automatically whenever faced with a calculation, regardless if the answer required is to be exact or an estimate. Facility with basic facts and mental math strategies is key to estimation.

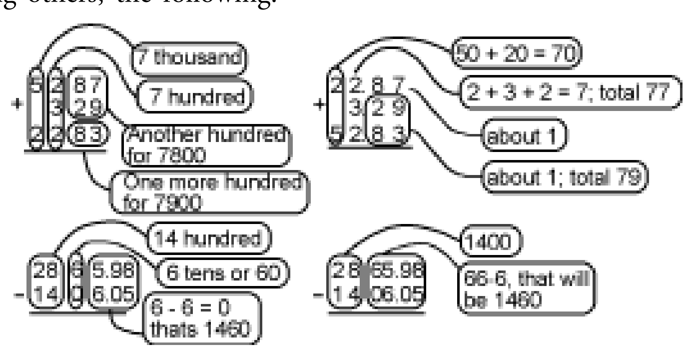
• Rounding: Rounding is a strategy commonly used for estimation in problem situations.
Addition: For the question, \$2.99 + \$7.98 + \$4.98, one can round to \$3 + \$8 and \$5 for a total of \$16. (Subtracting 5 cents gives the actual answer of \$15.95.) Subtraction: 2794 - 1616 (28 hundred - 16 hundred, or 1200). Provide students with examples where it is prudent to round up with one (or more) numbers and down with others. For example, when adding 148 and 247, the "rule" that one rounds up for "50 and more" really does not provide a close estimate; students should be providing more accurate estimations at this level. Have the students find actual answers and compare estimation strategies.

$$3.97 + \underbrace{86 + 16}_{100} + 6.04 + \underbrace{97.085}_{100} + \underbrace{25 + 72}_{100} = 310$$

10

• Compatible numbers: This involves looking for number combinations that go together to make approximately 10, 100, or 1000. For example, the question above might be estimated as shown:

• Front-end: This estimation strategy is often the one of choice. It involves an estimate of the left most digits and an adjustment for the rest, which may include "clustering" (e.g., 24 and 73 together to make 100) or further front-end strategies. In the examples below the front-end strategy could take different forms, which may include, among others, the following:



GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B10.1 Tell the students that □ 2.89 + □ 7.17 is estimated to be 48. Ask them to provide three sets of numbers that could go in the boxes to give this estimate.

B10.2 Ask the students to create a problem involving addition for which it would be appropriate to ignore the three digits after the decimal when estimating. Ask for an example for which the digits should be considered when estimating the sum.

Interview

B10.3 Have the student estimate the difference: 13.240 - 1.972. Ask him/her to describe two other subtraction questions for which one would suggest the same estimate for the difference.

B10.4 Provide the following calculations and ask the student to explain how he/she would estimate the answers:

24.3 + 39.16 + 75.03 + 62.2 998.201 - 249.6

Portfolio

B10.5 Have the students use grocery store flyers to select at least eight items that total close to, but less than, $20. Ask that they list the items, provide estimates for the total, and an explanation for their estimates.

B10.6 Tell the students that Hank always uses the "rounding rule." Ask the students to write to Hank to try and convince him that he may want to reconsider always using this strategy.

B10.7 Ask students to estimate totals of their purchases, or their parents' grocery orders. Have them report on the strategies they used and their progress with estimating.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers and decimals

SCO: By the end of grade 5, students will be expected to

B11 estimate products and quotients of two whole numbers

B12 estimate products and quotients of decimal numbers by single-digit whole numbers

Elaboration - Instructional Strategies/Suggestions

B11/B12 Students should continue to use a variety of estimation techniques for multiplication and division problems.

• Rounding: There are a number of things to consider when rounding to estimate for a multiplication problem. If one of the factors is a single digit, consider the other factor carefully. For example, when estimating 8 x 693, rounding 693 to 700 and multiplying by 8 is a much closer estimate than multiplying 10 by 700. Explore rounding one factor up and the other one down, even if it does not follow the "rule". For example, when estimating 77 by 35, compare 80 x 30 and 80 x 40 to the actual answer of 2695. Look for compatible numbers when rounding for a division estimate. For 4719 ÷ 6, think "4800 ÷ 6". For 3308 ÷ 78, think "3200 ÷ 80.

• Front-end: Begin at the left with the largest place value. For example, 8 x 823.24 would be 6400 (8 x 800). For a more accurate estimate, use additional place values (e.g., include 200 [8 x 25] for an estimated product of 6600). When estimating division questions involving 2-digit divisors, students might round sufficiently to convert the problem to a single-digit divisor calculation. For example, 7843 ÷ 30 is about 750 tens divided by 3 tens, so one can simply divide 750 by 3. Sometimes it is convenient to double both the dividend and divisor for a division estimate. 2223.89 ÷ 5 can be thought of as 4448 ÷ 10, or about 445. It is important that the students understand how this works; since twice as many people are sharing twice as much, each still gets the same share. It would be inappropriate to simply teach the rule.

❐ Have students play the Range Game for all the operations. For example, for multiplication have students enter a "start" number into their calculators, press "x," an estimated factor, and "=" to get a product within the target range. A point system could be devised by the players.

Start	Target
12	550 - 630
48	2500 - 2700
126	1000 - 1100

For division, put a "start" number into the calculator, press "÷," an estimated divisor and "=".

Start	Target
135	5 - 6
278.4	7 - 8
12.26	23 - 25

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B12.1 Ask the students to provide estimates in metres for the lengths of a ladybug and a caterpillar. Using their estimates, have them calculate how much longer 10 caterpillars would be than 10 ladybugs.

Interview

B12.2 Ask the student to give an estimate for the cost of six packages of cheese at $3.49 each.

B12.3 Tell the student that Jacques estimated the cost of three packages of gum at $0.79 each and four packages of potato chips at $0.79 each to be about $7. Hillary gave an estimate of $5.60. Ask the student how he/she thinks each estimate was determined and which estimate was closer to the actual cost.

B11.1 Tell the student that □ 834 ÷ 6 is about 300. Ask the student to decide what should go in the box.

B11.2 Tell the student that you have multiplied a 3-digit number by a 1-digit number and the answer is about 1000. Ask the student to describe three possible multiplications that you might have used.

B11.3 Ask the student for an estimate if a number between 300 and 400 is divided by a number between 60 and 70.

B11.4 Tell the student that a bus holds 72 students. Ask how he/she would estimate how many buses are required to transport 3000 students.

Portfolio

B11.5 Present Susan's approach to estimate 4598 ÷ 36, which is to replace all the digits except the first ones with 0. Since 4000 ÷ 30 is about 130, the answer is about 130. Ask students to comment on Susan's approach and to provide examples to back up their conclusions.

B11/12.1 Direct students to write to a classmate who has been absent because of an operation and has missed the classwork on estimation. Ask them to write an explanation for the absent student on what was missed.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil, and technological) in given situations

SCO: By the end of grade 5, students will be expected to

B13 perform appropriate mental multiplications with facility

Elaboration - Instructional Strategies/Suggestions

B13 A mental computation is one that produces the actual answer, not an estimate. By grade 5, students should possess a variety of strategies to compute mentally. It is important to recognize that these strategies develop and improve over the years with regular practice. This means that mental mathematics must be a consistent part of instruction in computation from primary through the elementary and middle grades. Sharing of computational strategies within the context of problem-solving situations is essential.

Students should perform and discuss the following types of mental multiplication on a regular basis:
- multiplication of two single-digit numbers, i.e., multiplication facts
- multiplication by 10, 100 and 1000
- multiplication of single-digit multiples of powers of ten (e.g., for 30 x 400, students should think “Tens times hundreds is thousands. How many thousands? 3 x 4 or 12 thousands.”
- multiplications which, upon rearrangement, make mental multiplication feasible, e.g., 25 x 40 can be arranged as 25 x 4 x 10 (or 100 x 10), 4 x 16 can be rearranged as 2 x 4 x 8 (16 ÷ 2) or 2 x 32. This is is called double/halving.
- multiplications that lend themselves to front-end strategy. For example, 3 x 2326 can be thought of as: 3 x 2000 + 3 x 300 + 3 x 20 + 3 x 6 or 6960 + 18 = 6978. This strategy is often used with some form of adjustment. For the question above, one might think: 3 x 2000 + 3 x 300 + 3 x 25 + 3 or 6975 + 3 = 6978.
- multiplication questions in which one of the factors ends in a nine (or even an eight). For such questions, one could use a compensating strategy (i.e., multiply by the next higher multiple of ten and compensate by subtracting to find the actual product. For example, when multiplying 39 by 7 mentally, one could think, “7 times 40 is 280, but there were only 39 sevens so I need to subtract 7 from 280 which gives a product of 273." For 49 x 24, one could think, "50 x 24, or 100 x 12, which is 1200. I need to subtract 24, for an answer of 1176". In order for students to be able to do a question such as this mentally, they must understand the operation, know the basic facts, and be able to recognize compatible numbers such as 24 and 76, as used in this example.

When presented with compuations, that can be done mentally, students should select the strategy that makes sense to them and is efficient.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Interview

B13.1 Tell the student that when asked to multiply 36 x 11, Kelly said, "I think 360 + 36 = 396." Ask the student to explain Kelly's thinking.

B13.2 Tell the student that to multiply 25 by 84 mentally, Stephanie thought, "100 x 21". Ask the student if this would provide the correct answer and, if so, to explain how it works. Have him/her provide other examples for which this strategy would be effective.

B13.3 Ask: How might you easily multiply 2 x 57 x 5 mentally?

B13.4 Ask: Why it is easy to calculate the questions below mentally?

48 x 20
50 x 86
242 ÷ 2

B13.5 Tell the student that Sue bought 24 cans of pop for a party. The sale price was 2 for $0.89 with a 5-cent deposit on each can. The cashier told Sue that the total was $35.90. Ask the student if this sounds reasonable and to explain his/her thinking. Ask how one might calculate the cost mentally.

B13.6 Ask the student why Lynn multiplied 11 x 30 to find 22 x 15.

B13.7 Tell the students that Mark said he would prefer to use the front-end method for finding the answers to 2 x 244 and 3 x 325 mentally, rather than using a calculator or a paper-and-pencil method. Ask the students to explain how Mark might have done these questions.

Portfolio

B13.8 Ask students to keep track of when they use their mental math strategies in the real world and to write about these experiences.

B13.9 Ask the students to keep a list of mental math strategies that they use regularly.

B13.10 Ask the students to provide explanations and examples for how one might multiply a 1-digit number by 99 mentally.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil, and technological) in given situations

SCO: By the end of grade 5, students will be expected to

B14 divide numbers mentally when appropriate

B15 multiply whole numbers by 0.1, 0.01 and 0.001 mentally

Elaboration - Instructional Strategies/Suggestions

B14 Students should regularly perform the following types of division mentally:

- division involving facts or division of 2-digit numbers by single-digit divisors (e.g., 56 ÷ 8, 75 ÷ 3)
- division by 10, 100 or 1000

Division by a power of ten results in a uniform "shrinking" of thousands, hundreds, tens, units, etc. (e.g., division by 10 involves each 10 becoming a 1, each 100 becoming a 10, each 1000 a 100, each 1 a 0.1, etc). This can be demonstrated using base-ten blocks. Dividing by 100, shrinks each 100 to 1, each 10 to 0.1, each 1 to 0.01. It would be appropriate to do short mental math activities with items such as, 453 ÷ 100, 617 ÷ 10, 213 ÷ 1000.

The "think multiplication" strategy is used regularly when dividing mentally. For example, when dividing 60 by 12, one might think, "What times 12 is 60?" For an example that requires a combination of strategies, such as 920 ÷ 40, one might think, "20 groups of 40 would be 800, leaving 120, which is 3 more groups of 40, for a total of 23 groups."

Students should be able to find the answer to division questions mentally when the divisor is a multiple of ten and the dividend is a multiple of the divisor, for example, 1400 ÷ 70. It is important for students to see the relationship between multiplication and division, and, for the above example, to ask themselves what must one multiply by 70 to give 1400.

B15 Students should relate multiplication by 0.1, 0.01, 0.001 to division by 10, 100, or 1000, respectively. To facilitate this understanding, it may be useful to remind students that multiplication indicates the number of groups of something. Therefore, just as 2 times a number is two of that number, 0.1 times a number is one-tenth of that number.

Another approach is based on the following pattern:

1000 x 4 = 4000	As the first number is divided successively
100 x 4 = 400	by 10, so is the product. It is only
10 x 4 = 40	reasonable, therefore, that 0.1 x 4 = 0.4.;
1 x 4 = 4	0.01 x 4 = 0.04; 0.001 x 4 = 0.004

Students need to use and understand the place value language described previously and avoid phrases such as, "adding two zeroes when multiplying by 100," or "move the decimal one place to the left when dividing by 10."

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B15.1 Tell the students to start with 3452 displayed on their calculators and predict how many times they would need to press "x," "0.1," and "=" to get a number less than 1. Ask them to confirm their predictions.

Paper and Pencil

B14.1 Have the student fill in the box to produce a true statement.

$$(345 \div 10 = \square \div 100)$$

Interview

B15.2 Tell the student that 178 is multiplied by 0.01. Ask: What digit will be in the tenths place? Why?

B14.2 Tell the student that when a certain number is divided by 10, the result is 45.95. Ask what the result would have been if the division had been by 100. What if it had been by 1000?

B14.3 Ask the student for a quick way to divide a number by 2 and then to divide the result by 5. Have him/her give other examples for which this strategy could be used.

B15.3 Ask the student to explain why, when you multiply a number by 0.01, the product is always less than the number.

B15.4 Explain to the student that Jacob was told one-tenth of the student body bring their lunches. Jacob said that he would divide to find how many students bring lunches. Sammy disagreed and said that he should multiply. Jane said that they were both right. Ask the student who he/she thinks chose a correct strategy and to explain why.

Presentation

B14.4 Ask the students to prepare a list of strategies one might use to calculate various division questions mentally.

Suggested Resources

Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) describe, extend, and create a wide variety of patterns and relationships to model and solve problems involving real world situations and mathematical concepts

SCO: By the end of grade 5, students will be expected to

C1 use place value patterns to extend understanding of the representation of numbers to millions

C2 recognize and explain the pattern in dividing by 10, 100 and 1000 and in multiplying by 0.1, 0.01 and 0.001

C3 solve problems using patterns

Earlier work with patterns has enriched each student's basic understanding of mathematics. Investigating additional patterns develops and refines their mathematical abilities and enables them to describe, extend, create, analyze, and predict knowledgeably. (Curriculum and Evaluation Standards, Addenda Series, Fifth-Grade Book, p. 1)

Elaboration - Instructional Strategies/Suggestions

C1 By this time, students have learned that numbers are represented in groups of 3 (i.e., there are ones, tens, and hundreds and then ones, ten and hundreds of thousands). Students need to extend this understanding to the next group of numbers (i.e., ones, tens, and hundreds of millions).

C2 Students should recognize and be able to explain the pattern in dividing by 10, 100 and 1000 respectively and multiplying by 0.1, 0.01 and 0.001. When initially presenting these multiplication and division operations, it is inappropriate to teach the "rule" of moving the decimal point to the right or left by counting spaces. The pattern which emerges shows that, depending on the division or multiplication, the digits move to appropriate positions. For example, dividing by 100 causes the digit in the hundreds place to move to the ones place, with all other digits moving along with it. It is important that students understand the reasons for the patterns. Have students explain these patterns and the meaning of the operations. For example, they need to understand that the answer to 2341 x 0.001 is one-thousandth of 2341, and can be thought of as a division question.

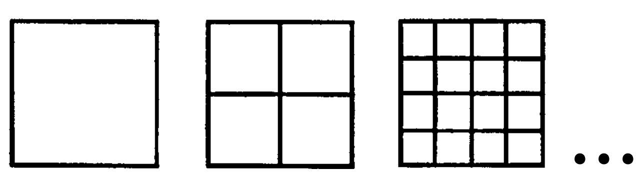
$2341 \div 10 = 234.1$	$2341 \times 0.1 = 234.1$
$2341 \div 100 = 23.41$	$2341 \times 0.01 = 23.41$
$2341 \div 1000 = 2.341$	$2341 \times 0.001 = 2.341$

C3 Many problems solved easily through the use of patterns are appropriate for grade five students. Examples include:

❒ Use the following pattern to figure out what 9 x 999 would be.

2 x 999
3 x 999
4 x 999

❒ If you keep dividing the square as shown, how many sections will there be in the tenth picture?



GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C3.1 Tell the students that when a piece of paper is folded once, the result is 2 sections. When it is folded twice, the result is 4 sections. Ask them to investigate the number of sections one would get with 3 folds and with 4 folds and have them predict the number of sections with 5 folds. Have students check their predictions and explain how one would predict the number of sections for 8 folds, if it were possible to do it.

Paper and Pencil

C3.2 Have the students predict the numbers that would occur in the next two rows:

1
1 1
1 2 1
1 3 3 1

C3.3 Ask the students to find the products below, to determine the pattern and then use the pattern to predict 1111 x 2222.

1 x 2 =
11 x 22 =
111 x 222 =

Ask them if there is a similar pattern for the following questions:

1 x 3
11 x 33
111 x 333

Interview

C1.1 Ask the student to create a place-value mat for numbers as high as hundred millions. Have him/her explain what is meant by the places being grouped in threes.

C2.1 Tell the student that a number is divided by 100 and the result is 427.4. Ask how he/she knows that the original number could not have had a 3 as one of its digits.

C2.2 Ask the student to tell what he/she knows about each the following without giving the answers: 4567 x 0.1
4567 x 0.01
4567 x 0.001

C2.3 Tell the student that Jake said, "You always get a larger answer when you multiply." Ask him/her to respond to Jake's observation.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore how a change in one quantity in a relationship affects another

SCO: By the end of grade 5, students will be expected to

C4 rearrange factors to make multiplication simpler

C5 recognize and explain how a change in one factor affects a product or quotient

C6 predict how a change in unit affects an SI measurement

C7 manipulate the dimensions of a rectangle so that the area remains the same

Elaboration - Instructional Strategies/Suggestions

C4/C5 Students should recognize that a rearrangement of factors can often make a multiplication question easier to solve. For example, 28 x 250 seems much more difficult than 7 x 1000, yet they are essentially the same. By dividing one factor by four or taking one-fourth of it, and multiplying the other factor by 4, one is taking one-fourth of the groups, but making each group 4 times as great, thereby not changing the total. In other words, if one divides one factor by the same amount as he/she multiplies the other, the answer does not change.

Alternatively, a student might think, "28 x 1000 would be 28 000, but 250 is only one-fourth of 1000, so the answer is only one-fourth of 28 000 or 7000. This strategy is particularly useful when numbers are halves or fourths of 10, 100 and 1000 (i.e., 2.5, 5, 25, 50, 250 and 500).

C6 Students should appreciate that using a smaller measurement unit will increase the number of those units or that using a larger unit will decrease the number of those units. Understanding this helps students to think that 28 cm must be 0.28 m, not 2800 m, since metres are longer than centimetres or that 352 m must be 352 000 mm, not 0.352 mm, since millimetres are shorter than metres.

C7 One of the interesting features of the square as a special rectangle to explore with students is that it is the most economical (i.e., for rectangles of a given area, it has the least perimeter). Students should be aware that many rectangles can have the same area and that if this is to be true, a longer length must accompany a shorter width. In fact, students may recognize that if one dimension is multiplied by any factor, the other dimension must be divided by that factor to retain the area. At this grade level, this relationship is best understood through a guided investigation of examples.

❒ Tell pairs of students to find as many different rectangles as they can with the area of 16 m^2. Ask them to display their findings on grid paper and to identify the one that would represent the best "room" dimensions for a bedroom and to explain why.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C7.1 Ask the student to use square tiles to show that if the length of a rectangle is halved and the width doubled, the area remains the same.

Paper and Pencil

C5.1 For each of the following, ask the student to tell how many times as great the second product is than the first:

44 x 25 compared to 44 x 100
75 x 20 compared to 75 x 100
10 x 70 compared to 90 x 70
3 x 100 compared to 12 x 250

Interview

C6.1 Ask the student whether each of the numbers below will increase or decrease as the measurement unit is changed as indicated:

0.04 m to centimetres
3.02 cm to metres
0.002 L to millilitres
2.005 km to metres

C4.1 Ask the student to explain why the result of 320 x 500 has to be half of 320 000.

C5.2 Ask the student to explain whether 5600 ÷ 5 is double or half of 5600 ÷ 10 and to explain why.

C4.2 Ask: "How does multiplying 44 by 100 help to compute 44 x 25? What must be done next in order to find the answer? Is there another way to compute 44 x 25 mentally?"

Portfolio

C4.3 Ask students to investigate the calculator exercise below to find an explanation for how it works. Encourage them to look for a pattern. Have them write about their findings.

Enter a one-digit number.
Multiply it by 7.
Multiply the answer by 143.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) represent mathematical patterns and relationships in a variety of ways (including rules, tables, and one- and two-dimensional graphs)

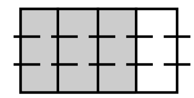
SCO: By the end of grade 5, students will be expected to

C8 demonstrate an understanding that the multiplicative relationship between numerators and denominators is constant for equivalent fractions

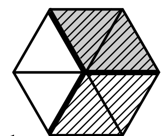
Elaboration - Instructional Strategies/Suggestions

C8 Students have already learned that equivalent fractions can be generated by either equally subdividing or equally grouping the fractional pieces that make up a whole.

For example, by subdividing each fourth into 3 sections, students can see that $\frac{3}{4} = \frac{9}{12}$.



Grouping the sixths in groups of 2, students can see that $\frac{4}{6} = \frac{2}{3}$.



If students create tables of equivalent fractions, they can observe that the multiplicative relationship between the numerators and the denominators for equivalent fractions is constant. They can also see that when numerators of equivalent fractions differ by a constant amount, the denominators also differ by a fixed amount.

For example:

Names for $\frac{1}{2}$

(a) The denominator is always twice the numerator
(b) If the numerators increase by 1, the denominators increase by 2.

Numerator	1	2	3	4	5	6	7
Denominator	2	4	6	8	10	12	14

Names for $\frac{1}{3}$

(a) The denominator is always three times the numerator
(b) If the numerators increase by 1, the denominators increase by 3.

Numerator	1	2	3	4	5	6	7
Denominator	3	6	9	12	15	18	21

Some students will be able to recognize the multiplicative relationship between the numerator and the denominator for fractions with numerators other than 1. For example, equivalents to $\frac{2}{3}$ all have denominators which are one and one-half of the numerators. Students may also notice that whenever the numerator increases by 2, the denominator increases by 3.

Students might be interested in observing what happens if the numbers which form the numerators and denominators of equivalent fractions are plotted on a coordinate grid. They will note that the points lie on a line.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C8.1 Ask the students to create a table or draw a graph to show equivalents for the fraction $\frac{3}{4}$.

C8.2 Ask the students to create tables to show the equivalents to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ with numerators of 2, 3, 4, 5 and 6. Have them predict which table would have a denominator of 60 first, if the patterns were continued, and explain why.

Paper and Pencil

C8.3 Ask the students to fill in the missing numbers to create a table of equivalent fractions.

Numerator	?	2	3	?	5
Denominator	?	8	12	?	?

C8.4 Ask the students to put the numbers 2, 4, 4, 5, 12, 20 and 40 in the correct spots in the tables of equivalent fractions shown below.

Numerator	1		3	
Denominator	4	8		16

Numerator	2		8	16
Denominator		10		

Interview

C8.5 Ask the student why $\frac{2\frac{1}{2}}{5}$ is another name for $\frac{1}{2}$ and $\frac{2}{4}$.

C8.6 Present this table of equivalent fractions.

Numerator	2	4	8	16	32
Denominator	5	10	20	40	80

Ask why the numerators increase by a lesser amount than the denominators.

Portfolio

C8.7 Provide students with sheets containing pictures of eight congruent rectangles. Ask them to shade $\frac{3}{4}$ of each of the eight rectangles in the same way. Have them leave one rectangle showing $\frac{3}{4}$ but have them subdivide the other seven rectangles in different ways to show seven different fractions that are equivalent to $\frac{3}{4}$.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) *represent mathematical patterns and relationships in a variety of ways (including rules, tables, and one- and two-dimensional graphs)*

SCO: By the end of grade 5, students will be expected to

C9 **represent measurement relationships using tables and two-dimensional graphs**

Elaboration - Instructional Strategies/Suggestions

C9 As students explore perimeters, areas and volumes, they should be encouraged to display some of the data in tables and graphs. These displays will help students to make inferences about the data.

For example, suppose students examine the perimeter of regular hexagons with various side lengths. They could record the data in a table and draw a graph as shown below.

Side Length (in cm)	1	2	3	4	5	6	7
Perimeter (in cm)	6	12	18	24	30	36	42

From either of these displays, students can easily see, for example, why the perimeter for a regular hexagon with a side length of 5.5 cm is 33 cm.

Other types of measurement graphs include:
- areas of squares for different side lengths
- volumes of cubes for different side lengths
- areas of rectangles (with length of 10 cm) for different widths

Students might compare the perimeters of rectangles (with areas of 24 square units) for different widths, and notice how perimeter shrinks and grows.

	Rectangles of Area 24 square units							
Width (in cm)	1	2	3	4	6	8	12	24
Other Side (in cm)	24	12	8	6	4	3	2	1
Perimeter (in cm)	50	28	22	20	20	22	28	50

Alternatively, students might compare the areas of rectangles (with perimeters of 24 units) for different widths, and notice how area grows and shrinks.

	Rectangles of Perimeter 24										
Width (in cm)	1	2	3	4	5	6	7	8	9	10	11
Length (in cm)	24	12	8	6	4	3	2	1	3	2	1
Area (in cm)	50	28	22	20	20	22	28	50	27	20	11

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C9.1 Provide the students with multilink cubes and have them build cubes of different sizes. In each case, ask them to record the various side lengths and volumes in a table and to draw a graph to display the information. Then ask them to predict the volume of a cube with a side length of 2.5 units.

C9.2 Provide square tiles. Ask students to build rectangles 2 tiles wide but of different lengths. Have them record the perimeter and area of each rectangle in a table and to look for any patterns shown by the data.

Paper and Pencil

C9.3 Ask the students to create graphs showing the relationship between side length in centimetres and area in square centimetres. Ask them to use the graph to determine the area of a square with side length 5.5 cm.

C9.4 Ask the students to explore the relationship between areas of rectangles of different sizes and the area of the rectangles created by doubling both dimensions. For example:

Length (in cm)	1	3	5	6		
Width (in cm)	4	4	6	6		
Area (in cm^2)	4					
Double Length (in cm)	2					
Double Width (in cm)	8					
New Area (in cm^2)						

Interview

C9.5 Ask the student to describe what happens to the area of a rectangle if its width stays the same, but its length increases by one unit.

C9.6 Ask the student to discuss the relationship between the shapes described by these measurements.

Length (in cm)	16	8	4	2
Width (in cm)	4	2	1	0.5

C9.7 Provide the following table and ask the student to discuss the shapes described by the table.

Side Length (in cm)	1	2	3	4	5
Perimeter (in cm)	4	8	12	16	20

Suggested Resources

Shape and Space

General Curriculum Outcome D:

Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter and angle*

SCO: By the end of grade 5, students will be expected to

D1 solve simple problems involving the perimeters of polygons

D2 calculate areas of irregular shapes

Elaboration - Instructional Strategies/Suggestions

D1 Students should conceptualize perimeter as the total distance around an object or figure. They might observe that, for certain figures, the perimeter is particularly easy to compute. For example:

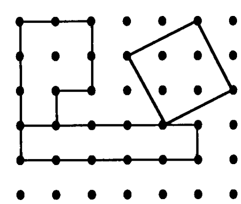
- for an equilateral triangle, the perimeter is 3 times the side length
- for a square, the perimeter is 4 times the side length
- for a rectangle, the perimeter is double the sum of the length and the width

❒ Provide students with a loop of string of fixed perimeter. Have them form the loop into various polygons on a piece of grid paper, and estimate the areas. Ask the students to determine which shape seems to have the most area for that perimeter.

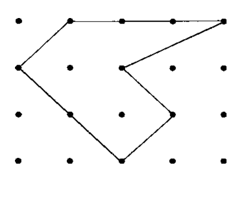
D2 Students should use transparent grids and geoboards to help them calculate the areas of a variety of shapes in their environment (e.g., shapes of hands, feet, leaves, etc.).

❒ Students might create shapes on geoboards and challenge other students to find the areas.
Areas involving square metres can be calculated by marking off an area of 1 square metre and comparing this to other areas. Alternatively, students can model a larger area involving square metres or square kilometres using a scale drawing in which, for example, 1 cm might represent 1 m or 1 km.

❒ Challenge students to find as many shapes as possible on a geoboard with a given area. For example, given an area of 5 square units:



When determining the area of the polygon on the dot paper to the right, students may think of it as the 2 whole square units and 5 obvious half squares ($2\frac{1}{2}$ square units), for a sum of $4\frac{1}{2}$ square units. Remaining is the upper right triangle, formed by the diagonal of 2 squares. It would represent half of this, or 1 more square unit, for a total of $5\frac{1}{2}$ square units. Ask the students to find another way of determining the area.



GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

D1.1 Ask the students to draw three different polygons with the same perimeter.

D2.1 Ask the students to use dot paper to compare the areas of rectangles with the following dimensions:

2 cm x 3 cm, 4 cm x 3 cm, 6 cm x 3 cm

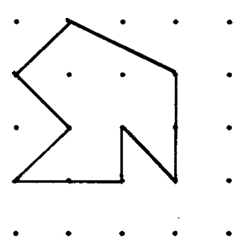
Ask what they observe and have them give another set of dimensions that follows the same pattern.

Interview

D1.2 Tell the student that the longest side of a triangle is 10 cm. Ask: Why must the perimeter be greater than 20 cm?

D1.3 Ask the student to explain why the perimeter of rectangles with whole number side lengths is always even.

D2.2 Show the student a shape (such as the one below) on a geoboard or dot paper, and ask him/her to give the area and explain how it was calculated.



Activity

D2.3 Provide activities in which students relate perimeters to areas. For example, give pairs of students 24 colour tiles and ask them to find different shapes, each with the area of 24 square units, but with different perimeters. Ask them to find a way to keep track of their shapes and perimeters. What shape has the largest perimeter? The smallest? (Have the students reach a consensus on the rules for shape formation. For example: Can there be shapes other than rectangles? Must each tile have a full side butting a side of another tile?)

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *extend understanding of measurement concepts and attributes to include volume, temperature, perimeter and angle*

SCO: By the end of grade 5, students will be expected to

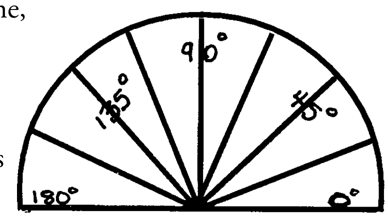
D3 determine the measure of right, acute and obtuse angles

The purpose of an interview is to uncover how students think about mathematics, so provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (Mathematics Assessment, Stenmark, ed., NCTM, 1991, p. 29)

Elaboration - Instructional Strategies/Suggestions

D3 Students often find the measurement of angles a challenging task due to the fact that degree units are so small and most protractors have double numbering running clockwise and counter-clockwise. Before students begin to use a standard protractor it is helpful if they move from the non-standard unit wedges, presented at the grade four level, to an intermediary protractor. The problems encountered with angle measurement will be reduced considerably by involving students in making their own protractors.

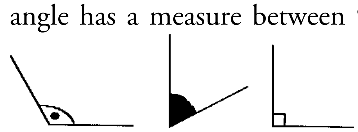
Provide the students with semicircular shapes cut from tracing paper, or construction paper. (Tracing paper allows students to see the angle vertex and follow the arms, thus enabling them to measure more easily.) Have them fold the semicircle in half, forming a right angle or square corner. Explain that angles are measured in degrees and that a right angle is made up of 90 of these degrees. Have this fold named 90°. Ask them to fold once again and have them determine, and name, the size of the new angles created by the folds. One further fold provides angles half-way between 0° and 45° and 45° and 90°. Discuss the measurement of these folds with the class and how they can assist with estimation of angle sizes.



Once students have had practice estimating (see SCO, D7) and measuring angles using homemade protractors, they can be introduced to the standard protractor. (Some students find it helpful to work with a protractor on which the numbering occurs only counter-clockwise, before using a standard protractor. See sample on next page. These protractors can be made from overhead transparencies.) Before students make any measurement with a protractor, they should be able to estimate a measurement within 5-10 degrees. This ability will make protractor reading much easier.

Students have previously classified angles as right, acute and obtuse by their overall appearances. They should now understand that

- a right angle has a measure of 90°
- a straight angle, made by two right angles, has a measure of 180°
- an acute angle has a measure less than 90°
- an obtuse angle has a measure between 90° and 180°



GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D3.1 Have the student create an acute angle which is less than half the size of a right angle.

D3.2 Ask the student to create a shape containing an odd number of right angles.

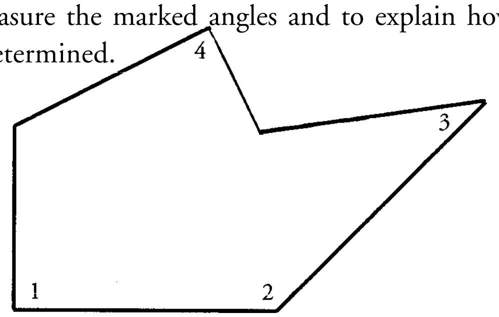
Paper and Pencil

D3.3 Ask the student to draw a shape with two obtuse angles and three acute angles. Then have them measure two of the angles.

Interview

D3.4 Ask the student where obtuse angles would be found in the classroom.

D3.5 Show the student the diagram below and ask him/her to identify and measure the marked angles and to explain how the answers were determined.



D3.6 Ask: Why do you think right angles are more common in our world than acute or obtuse angles?

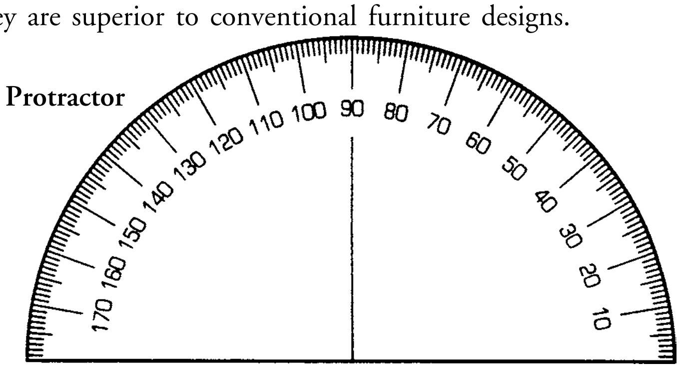
Portfolio

D3.7 Ask the students to design a room of furniture, such that none of the furniture contains right angles. Have them write a report listing advantages and disadvantages of their designs.

or

Have the students write an advertisement for their designs, detailing why they are superior to conventional furniture designs.

Sample Protractor



Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations

SCO: By the end of grade 5, students will be expected to

D4 demonstrate an understanding of the relationship among particular SI units

Elaboration - Instructional Strategies/Suggestions

D4 Students should recognize that

- 1 metre is 10 dm, 100 cm or 1000 mm;
- 1 km is 1000 m;
- 1 litre is 1000 mL;
- 1 gram is 1000 mg;
- 1 kg is 1000 g.

They will use these relationships to rename measurements when comparing them.

❒ Share a short paragraph describing the measurements of a variety of classroom items. Ask the students to insert the appropriate unit for each. For example: The table was 1524 ___ long. On it was a pencil that was 0.17 ___ long.

❒ Discuss with students the scale (metric) on a map of Canada. Invite them to investigate how the map would differ if the scale were changed (e.g., 1 cm/mm/dm represented 100 km).

❒ Ask students to investigate favourite beverages (individual portions of pop, juice, milk, etc.) and to predict and test to see what portion of a 1 litre container each would fill.

It is important to help the students develop mental images of various measurement standards. To provide estimation practice, involve students in activities such as: Show me (with hands or arms):

73 centimetres	14 millimetres
4 decimetres	0.001 kilometre

It is helpful for students to think of their ruler, as well as a metre stick or base ten blocks, when estimating length. Most rulers are 30 cm (or 300 mm) long and serve as good benchmarks. For example, 62 cm can be thought of as the length of about 2 rulers.

❒ Line up a variety of containers and ask which one would hold:

3000 millilitres	2 litres
500 millilitres	0.45 litres

❒ Have the students match objects to cards on which measurements of mass are written. In some cases, use different names for the same object (e.g., 1.5 kg and 1500 g).

Encourage students to estimate measurements before actually verifying them using a measuring device.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D4.1 Ask the students to show, with fingers or arms, the following lengths:

550 mm, 68 cm, 0.025 m, 4.5 dm

Ask them to state another way of expressing each of the lengths.

Paper and Pencil

D4.2 Ask the student to determine the length of time it might take to walk 1 000 000 millimetres.

D4.3 Have the student use metric units to fill in the blanks in as many ways as possible:

1000_____ = 1 ____

D4.4 Ask the student to rewrite 2.3 dm using other metric units.

Interview

D4.5 Ask the student if it would be possible to walk 0.001 km in a minute and to explain his/her thinking.

D4.6 Ask: If you change metres to centimetres, will the numerical value become greater or less? Why?

D4.7 Show the student a variety of containers and ask him/her to identify the one probably designed to hold 500 mL. Ask for reasons for the choice.

D4.8 Ask the student to describe the size of cage one might need to hold a 50 000 mg animal comfortably.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) develop and apply rules and procedures for determining measures (using concrete and graphic models)

SCO: By the end of grade 5, students will be expected to

D5 develop formulas for areas and perimeters of squares and rectangles

The development of area formulas is a fantastic opportunity to follow the spirit of the NCTM Standards. First, a problem-solving approach can meaningfully involve students and help them see that mathematics is a sense-making endeavor. Second, the connectedness of mathematics can be clearly seen. (Elementary School Mathematics, p. 313)

Elaboration - Instructional Strategies/Suggestions

D5 Students need to have many opportunities to experiment with developing their own formulas for calculating the perimeter and area of rectangles. (Remember: a square is a special rectangle.)

The concept of area (i.e., amount of surface) of a rectangle should be elicited from the students through their working with a variety of materials; it should not be taught simply in terms of a formula (i.e., multiplying the length by the width or A = l x w).

When students investigate the distance around a rectangle, they will produce their own expressions for perimeter. These might include: l + w + l + w, 2 (l + w), and 2l + 2w.

The reason for teaching this way is pragmatic; it is more difficult for students to remember ideas in the long term if they have not assimilated them into some conceptual framework.

❒ Students could work in pairs to solve the following problem: You need to determine the amount of fencing required to build a dog pen that has a length twice as long as its width. What might be the dimensions of the pen? How can the perimeter of the pen be found without adding every length? Also, you want to cover the floor of the pen with square paving stones. How many (and of what size) will you need? Find a way to calculate this without counting each stone.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D5.1 Ask the student to find the ratios of the perimeters of various regular polygons to the lengths of their diagonals.

Paper and Pencil

D5.2 Tell the student that a parallelogram has a perimeter of 42 cm. Ask the student to draw what it might look like.

D5.3 Tell the student that the perimeter of a square is 38 cm. Ask: What is its side length?

Interview

D5.4 Tell the student that the length of a rectangle is increased by 1 and the width decreased by 1. Ask: What happens to the perimeter? area?

D5.5 Ask: Can the number describing the perimeter of a polygon be less than the number describing its area? Explain your answer.

D5.6 Tell the student that you measured a particular area in square centimetres and square metres. Ask: Which number will be greater? Why?

D5.7 Ask: How many base-ten flats can fit in a square metre?

Portfolio

D5.8 Ask: Can you point out anything in the classroom that would have an area of 4 dm^2? Explain your choice(s).

D5.9 Tell the student that the area of a classroom is 600 m^2 and its perimeter is 100 m. Ask: What are the dimensions of the classroom?

D5.10 Have each student calculate how much carpet would be needed to cover the floor of a room in his/her home. Also, ask the students to include a floor plan indicating where the furniture is located. Ask: How much of the floor space is taken up by furniture?

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units

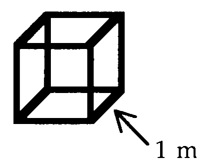
SCO: By the end of grade 5, students will be expected to

D6 solve simple problems involving volume and capacity

Elaboration - Instructional Strategies/Suggestions

D6 It is useful for students to recognize the difference between volume (the amount of space occupied by a three-dimensional object) and capacity (the amount a container is capable of holding). The volume units which they will generally encounter are cubic centimetres (cm^3), cubic decimetres (dm^3) and cubic metres (m^3), while capacity units will be millilitres (mL) and litres (L). Capacity units are usually associated with measures of liquid (e.g., litres of milk, juice and gasoline).

Students should develop personal referents for units. The use of personal referents helps students establish the relationships between the units (e.g., the small cube in the base-ten blocks is 1 cm^3 and would hold 1 mL and the large cube is 1 dm^3 and would hold 1 L). Students should realize that a cubic centimetre is the size of a cube 1 cm on a side and a cubic metre the size of a cube 1 m on a side. If they explore the size of a million using visualization of base-ten blocks (see SCO, A6) and build a cubic metre with metre sticks, they will have a good mental image of 1m^3.



Students should have a sense of which volume or capacity unit is the more appropriate to use in various circumstances.

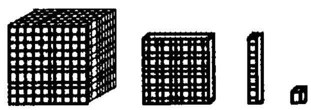
- ❐ List a variety of situations on the board (e.g., taking cough syrup, buying gasoline, finding a box in which to wrap a necklace, building a crate in which to ship a bicycle, getting a flu shot). Ask students to choose the unit of measurement that would be used for each. Have them compare their answers and defend their choices.
- ❐ Have students measure the volume of small rectangular prisms by counting the number of cubes it takes to build a duplicate of it.
- ❐ Invite groups of students to investigate the capacities of various beverage containers to determine which size container is found most often. They might record their findings in a graph or table and present it to the class.
- ❐ Have students use base-ten blocks to build several different structures, each with a volume of 60 cm^3. Students are likely to build both irregular shapes and regular prisms. This will offer an opportunity to discuss finding the volumes of irregularly-shaped, composite figures.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D6.1 Ask the student to calculate the volume of each size of base-ten block, i.e., the volumes of



Paper and Pencil

D6.2 Tell the student that a container holds 1.5L. Ask if it is large enough to make a jug of orange juice, if the concentrate is 355 ml and you have to use the concentrate can to add three full cans of water.

Interview

D6.3 Ask the student how he/she could use a 1L milk carton to estimate 750mL of water.

D6.4 Ask the student to estimate the number of cubic metres in the classroom and to give an explanation as to how the estimate was determined.

D6.5 Tell the student that you need a box with a volume of 4000 cubic centimetres to hold a gift you have purchased. Ask: What might that gift be?

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

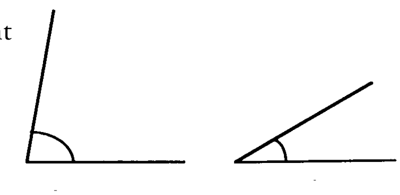
iii) *estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units*

SCO: By the end of grade 5, students will be expected to

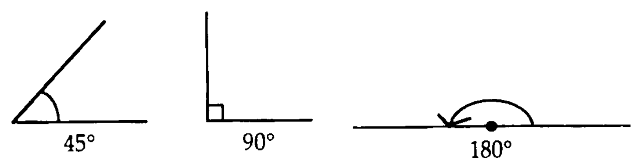
D7 **estimate angle size in degrees**

Elaboration - Instructional Strategies/Suggestions

D7 Students should be able to estimate, within a reasonable range, the measurement of an angle in degrees. For example, a student should be able to say which of the angles shown to the right is nearer to 30º and be able to provide an estimate for the other.



To estimate properly, students need to have a feel for at least the following angle sizes:



- ❒ Provide students with pipe cleaners that they can bend to form angles. Ask them to make an angle at about 30º. Have them compare their estimates with neighbours. Draw an angle with a measure of 30º on the overhead and allow students to compare their pipe-cleaner angles to the projected image. Continue by asking them to make other angles giving them the degree measures.

Geo-strips and straws are other sources of materials that can be used to make or show angles. With many different experiences over time, students will develop good estimation skills. The goal is to be able to estimate the measure of angles to within 5-10 degrees of their actual sizes.

- ❒ After students are quite capable of estimating angle size, have them write the numbers 1-10 in a column in their notebooks. Show them ten angles, one at a time, and ask them to estimate each and to record their estimates. Be sure to show the angles in a variety of postions and with arms of varying length. Afterwards go over their solutions and ask students to share the strategies they used. Repeat this activity a few days later and note any improvements students have made.
- ❒ If a computer is available with the Logo language, students can play a game in which a circle target is placed randomly. The turtle is aimed at the target by indicating an angle at which to shoot and a distance. Students could get a number of turns to try to shoot the target.

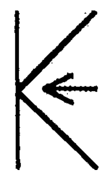
GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D7.1 Have the students estimate and then use their handmade protractors to measure the different angles found in pattern blocks.

D7.2 Ask the students to estimate the size of angles found in different printed letters of the alphabet.



D7.3 Have pairs of students work together. One student makes an angle and the other estimates the size. They check the measurement using their protractors.

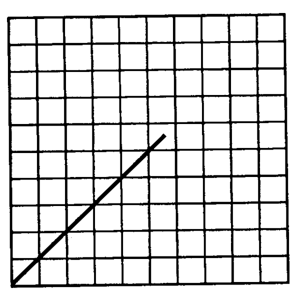
Paper and Pencil

D7.4 Ask the student to draw angles of approximately 45° and 135°.

D7.5 Show the students a 60° angle. Tell them that it measures 60° and ask them to draw one that is about $\frac{1}{2}$ the size, etc.

Interview

D7.6 Show the student the diagram below and ask why it is easy to tell that it is 45°.



D7.7 Show the student an angle of, for example 135°, and tell him/her that someone said that it was 45°. Ask the student to explain how he/she thinks such an error could be made.

D7.8 Ask: Which angle sizes do you find easy to estimate? Why?

D7.9 Show the student an 80° angle wedge and ask him/her to estimate its size.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units

SCO: By the end of grade 5, students will be expected to

D8 determine which unit is appropriate in a given situation and solve problems involving length and area

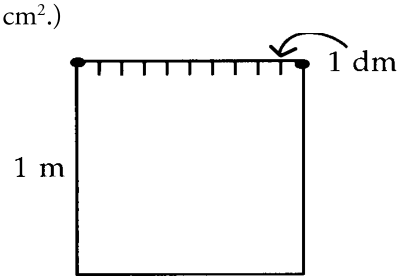
Elaboration - Instructional Strategies/Suggestions

D8 Students should be familiar with the three standard units used to measure area: the square centimetre, the square metre and the square kilometre. They also need to realize that each represents the size of the surface of a square with a side length of 1 cm, 1 m, or 1 km, respectively.

Students should have a sense of which unit would be more appropriate to measure certain areas (e.g., a postage stamp, a farm field, a classroom) and have some sense of what areas such as 100 cm^2 and 1000 cm^2 might look like.

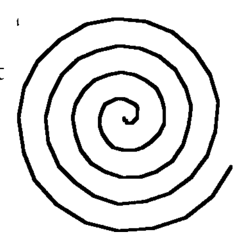
Students should be able to write measurements of area in terms of the standard units, using decimals where necessary. (For example, the area of a paperback might be 348.5 cm^2.)

Students can benefit by observing how many of one area unit it takes to create another. For example, 100 dm^2 are required to make a square metre since it takes 10 dm to make a metre.



Students can solve and create problems involving a variety of measurements in their everyday experience. Ideally, some problems will focus on particular measurements (e.g., area or length) and others will combine measurements. For example:

- ❒ The perimeter of a rectangle is 18 cm; the area is 20 cm^2. What are the dimensions of the rectangle?
- ❒ A rope 1.2 m long is wound around and around to form a spiral. In what area might it fit?



An interesting way to determine the length of a piece of wire which has been wound is to compare its mass to the mass of a known length of the wire. For example, if a ball of wire has a mass of 36 g, and a 10 cm strip of the same wire has a mass of 3.4 g, then the ball probably contains a bit more than 1 m of wire.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

D8.1 Have the students use centimetre grid paper to design a floor plan for a home. Have each square centimetre of their grid represent a square metre of the floor, and ask that the area and perimeter of each of the rooms be calculated. Ask the students to discuss how using formulas helps with these calculations.

D8.2 Tell the students that the grade five class has decided to sell fudge that has been cut into 3 cm x 3 cm pieces. Ask: How might you determine what different sizes of cardboard could be cut to hold a layer of a dozen pieces?

Interview

D8.3 Tell the student that Keri says the way to find the perimeter of a rectangle is to use the formula (l + w) x 2, but Ted says you must use (l x 2) + (w x 2). Ask: Who is right? Why? Is there any other way to find the perimeter of a rectangle?

D8.4 Tell the student that Maryann said she could figure out the length of a rectangle if she were given the width and the area. Ask: Is this possible? Explain.

D8.5 Ask the student what he/she can tell you about a rectangle that has a length of 12 cm and an area of 144 square centimetres.

D8.6 Tell the student that the area of a shape is 24 square units. The length is 6 units. Ask: How could you determine the width?

Presentation

D8.7 Ask pairs of students to work together to develop a strategy to determine the cost of carpeting the classroom floor. Invite them to present their strategies to the rest of the class.

Suggested Resources

Shape and Space

General Curriculum Outcome E:

Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *identify, draw and build physical models of geometric figures* and

iv) *solve problems using geometric relationships and spatial reasoning*

SCO: By the end of grade 5, students will be expected to

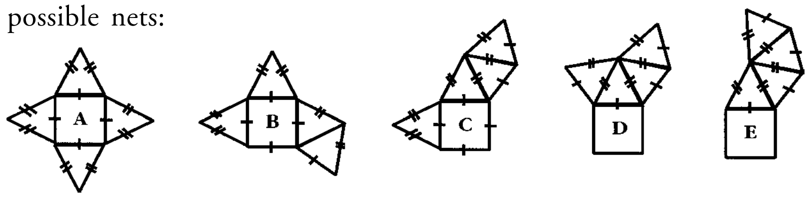
E1 draw a variety of nets for various prisms and pyramids

E2 identify, describe and represent the various cross-sections of cubes and rectangular prisms

Building solid or three-dimensional shapes presents a little more difficulty than two-dimensional shapes but is perhaps even more important. Building a model of a three-dimensional shape is an informal way to get to know and understand the shape intuitively in terms of its component parts. (<u>Elementary and Middle School Mathematics</u>, p. 355)

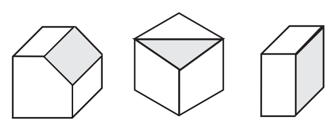
Elaboration - Instructional Strategies/Suggestions

E1 Students will have cut out and assembled prepared nets for prisms and pyramids in previous grades. They should now investigate and draw a variety of possible nets for them. These nets should be drawn by “rolling and tracing”[1] faces, cutting out, and wrapping actual 3-D solids. For example, for a square pyramid, these are some possible nets:



Note: It is not a different net if it is a reflection or rotation of one you already have.

E2 A cross-section is the 2-D shape of the face produced when a plane cut is made through a solid. For example, a cube could be cut to produce these shapes (among others):



Students should investigate cross-sections made by plane cuts parallel to faces and obliquely (not parallel); and starting at a vertex or different points along the edges of the prism. Possible sources for these prisms are plasticine, styrofoam, cheese, brownies, and rice crispie squares. Cuts could be made with piano wire or wire cheese cutters; the teacher could use a knife.

Students should try to visualize the shape made by a cut and then check their prediction. Wrapping a shape with an elastic band where the cut would be made might help some students with this visualization.

If geoblocks are available, cubes, square prisms, and rectangular prisms can be built in a variety of ways; thereby, some cross-sections of these prisms can be demonstrated without having to cut.

[1]This means placing the 3-D shape on a sheet of paper; tracing around the face of the shape with a pencil; rolling the shape over so that another face is on the paper; tracing that face, being careful that it is attached to the first face; and so on until all faces have been drawn.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

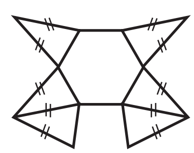
Performance

E1.1 Provide each group of four students with a triangular prism, a triangular pyramid, a square prism, and a square pyramid. Each student in the group should use one of these shapes to roll and trace a net which is then cut out. The group should discuss other possible nets for each shape and assign responsibility for creating some of them.

E1.2 Have students draw all the possible nets for a triangular pyramid with all faces equilateral triangles. Repeat for one with an equilateral base and three isosceles triangular faces. Ask: Did you get more nets for one of them? Why do you think this happened?

E1.3 Provide students with a prism or pyramid and wrapping paper. Ask them to roll and trace a net for the shape, cut it out, and actually wrap the shape to check it. Unwrap the shape, cut off one face, and ask them for the possible places this face could be reattached to produce other nets. Use tape to reattach and check. Extension: If centimetre graph paper is used for this activity, a good connection to surface area can be made.

E1.4 Show students the picture at right. Ask students to predict if it is a net, check their predictions by cutting it out, and make any changes needed to create a true net.



E2.1 Cut off the top of a 1L milk carton to make an open square prism. Fill it half full of water. Have the students tip the carton in different ways, examining the shapes of the surface of the water. Have them draw the shapes they find and discuss these as cross-sections of a square prism. (You could also use a clear plastic cube.)

E2.2 Ask the student to find three different cross-sections of a Playdoh cube. Have him/her try to visualize to predict the shape of the cross-section before the cut is made.

E2.3 Ask the student to draw the shape he/she would see if the corner of a rectangular prism were cut off.

Interview

E1.5 Show the student a net for a 3-D shape. Ask him/her to point and describe how the net would fold up to make the shape, and to name the shape.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *identify, draw and build physical models of geometric figures* and

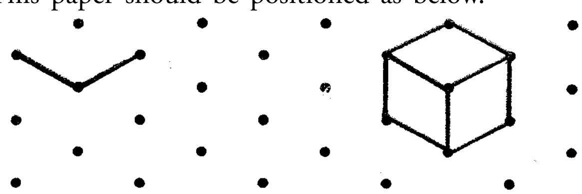
iv) *solve problems using geometric relationships and spatial reasoning*

SCO: By the end of grade 5, students will be expected to

E3 make and interpret isometric drawings of shapes made from cubes

Elaboration - Instructional Strategies/Suggestions

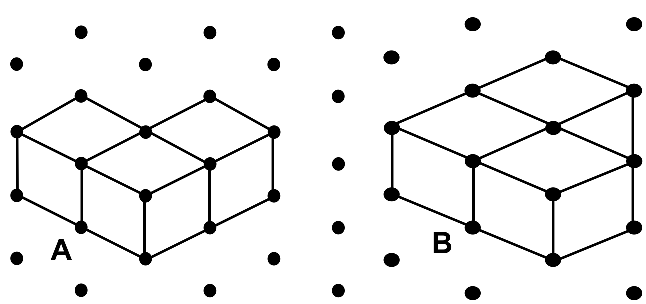
E3 For these drawings, teachers will need isometric (triangular) dot paper. This paper should be positioned as below.



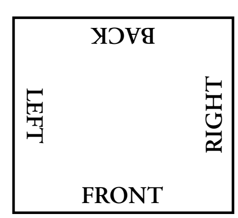
It is interesting to note that while the angle shown above is 120°, in the context of drawing cubes it will appear to be 90° (see above). This is another example of perceptual constancy in spatial visualization.

Isometric drawings are always done from a front right, front left, back right, or back left view, never looking straight on.

❒ Start with a simple shape (like A below). Have the students make the shape with three cubes, and replicate this drawing of it. They should start by drawing the foremost (front) cube and then draw the other two attached to it. This is the front left view. Have the students turn their shape so they can look at it from the front right (B) and draw it. Again, they should start by drawing the foremost (front) cube, then the one attached to it, and finally the third. (Care will need to be taken because only $1\frac{1}{2}$ faces of this one are visible!)



It might be helpful to have the students use a paper mat (at the right) so they can place their structures on it and move it to the desired position (e.g., front right) as they draw.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

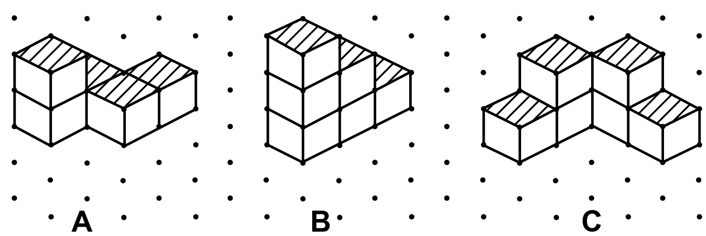
Worthwhile Tasks for Instruction and/or Assessment

Performance

E3.1 Have students make a tower using three cubes and an isometric drawing of this tower. Have them topple the tower and make isometric drawings from the front left and the front right.

E3.2 Using five cubes, have students make a T-shape and four different isometric drawings of it.

E3.3 Have students construct each of the following shapes with cubes. These are drawn from the front left. Have them make isometric drawings from the front right.



E3.4 Ask students if there could be any hidden cubes in the drawings in E3.3. If so, ask them where they could be.

E3.5 Have students make a structure using 8 cubes and draw an isometric view of it. Have them exchange drawings with classmates and each of them build the other's structure using only its drawing.

E3.6 Make a centre using 8-10 of the structures the students built in E3.5 and the corresponding drawings. Ask students to match the structures and drawings.

Portfolio

E3.7 Have students find all the different shapes that can be made from four cubes. For each one, have them make an isometric drawing to record it.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *identify, draw and build physical models of geometric figures* and

iv) *solve problems using geometric relationships and spatial reasoning*

SCO: By the end of grade 5, students will be expected to

E4 explore relationships between area and perimeter of squares and rectangles

E5 predict and construct figures made by combining two triangles

A teacher's questioning techniques and language in directing students' thinking are critical to the students' development of an understanding of geometric relationships. Students should be challenged to analyze their thought processes and explanations. (Curriculum and Evaluation Standards for School Mathematics, p. 112)

Elaboration - Instructional Strategies/Suggestions

E4 Through structured exploration activities, students should conclude that all squares with the same area have the same perimeter and all squares with the same perimeter have the same area. (For example, if it is known that a square has a perimeter of 100cm, the only square possible is 25cm x 25cm and its area is 625cm^2.) However, rectangles with the same area can have different perimeters, and rectangles with the same perimeter can have different areas. (For example, if is known that a rectangle has a perimeter of 100cm, there are many possible rectangles each with a different area. Examples include 40cm x 10cm, with an area of 400cm^2, and 30cm x 20cm, with an area of 600cm^2.) The generalization about squares is often over-generalized, causing a common misconception about the relationship between area and perimeter in other polygons.

E5 To promote spatial sense and to develop visualization skills, students should work through a series of activities from simple to complex that involve constructing polygons from two triangles. Teachers could, for example, plan activities that would have students investigating the various polygons they could make using each of the following pairs of triangles:

- two congruent equilateral triangles
  (possible source: pattern blocks)
- two congruent isosceles right triangles
  (possible source: tangram pieces)
- two congruent isosceles triangles
  (possible source: a rectangle cut along both diagonals)
- two congruent right triangles
  (possible source: a rectangle cut along 1 diagonal)
- two congruent acute/obtuse triangles
  (possible source: a parallelogram cut along 1 diagonal)
- two different isosceles triangles with a base of the same length

At this grade level, the emphasis should be on visualization ("seeing in their minds"). After students have done the investigations by manipulating both triangles, teachers could show one triangle on the overhead and ask students to draw the various results of combining it with another triangle.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E4.1 Provide students with grid paper. Have them draw a square that has sides of two units. Find its perimeter and its area. Share results. Repeat with squares that have other side measurements. Ask: Do you see a relationship between side length and perimeter? between side length and area?

E4.2 Ask each student to construct on his/her geoboard a square that has a perimeter of 12 units. Have them compare their squares. Ask them if all their squares look the same. Repeat the process, asking for a rectangle with a perimeter of 12 units.

E4.3 Using square tiles, have students find all the possible rectangles that can be made from 12 tiles. Have them record their findings on grid paper and find the areas and perimeters of each rectangle. Ask them what they notice about the appearance of the rectangle with the greatest perimeter and with the least perimeter. Repeat using 24 tiles.

E5.1 Have students cut a rectangle along one diagonal and investigate the other polygons that can be made using these two triangles. After the students have found the possible polygons, the teacher could have them do the following: place the two triangles to make the rectangle, keep one triangle in place and slide the other triangle to make a parallelogram. Return to the rectangle shape and use motions to create a large triangle. Return to the rectangle shape and describe the required motions to make a kite.

Paper and Pencil

E4.4 Tell students that a farmer has 100 m of fencing to make a pen for his pigs. He decides a square or a rectangle would be the best shape. Ask them for some possible sizes of pens he could make. Ask them how the areas of the pens compare and what size they would recommend and why.

E5.2 Have students describe the possible pairs of congruent triangles that would combine to make (i) a square, (ii) a rectangle, (iii) a kite, and (iv) a parallelogram.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *describe, model and compare 2- and 3-D figures and shapes, explore their properties and classify them in alternative ways*

SCO: By the end of grade 5, students will be expected to

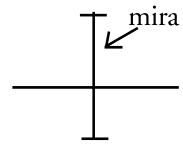
E6 **recognize, name, describe and represent perpendicular lines/ segments, bisectors of angles and segments, and perpendicular-bisectors of segments**

E7 **recognize, name, describe and construct right, obtuse and acute triangles**

Definitions should evolve from experiences in constructing, visualizing, drawing, and measuring two- and three-dimensional figures, relating properties to figures, and contrasting and classifying figures according to their properties. Students who are asked to memorize a definition and a textbook example or two are unlikely to remember the term or its application. (Curriculum and Evaluation Standards for School Mathematics, p.112)

Elaboration - Instructional Strategies/Suggestions

E6 Teach students to use a mira to draw perpendicular lines. First the student draws a line segment and places the mira across it. He/ she then moves the mira until the image of the part of the segment on one side of the mira falls on the segment on the other side. The line on which the mira sits is perpendicular to the original line segment drawn. When the image of an endpoint of the segment falls on the other endpoint, the mira is also bisecting the segment; the line on which the mira sits is the perpendicular-bisector of the segment.



❐ Have students arrange two straws or two toothpicks in various configurations (first estimating and then checking):
- parallel to one another
- intersecting
- perpendicular at an end point of one straw
- perpendicular at endpoints of each straw
- one straw perpendicular to the other straw at its midpoint
- one straw bisecting the other straw but not perpendicular
- each straw bisecting the other straw but not perpendicular
- one straw bisected by the other straw and perpendicular
- each straw bisecting the other straw and perpendicular

Have students bisect angles by folding one arm onto the other, by using a mira to find where one arm would reflect onto the other when it is placed through the vertex, and by measuring.

E7 Give students 12 cards with examples of right, acute, and obtuse triangles on them. Ask them to sort them into three groups by the nature of their angles and share how they were sorted. (This can be done even if the names are not known.) Attach the names for these classifications to the students' groups. Look for everyday examples of each type of triangle; also, examine familiar materials in the classroom, such as pattern blocks and tangrams. Have students choose straws of different lengths or geostrips to make examples of each type. These should also be connected to the side classifications (equilateral, isosceles, scalene) which were studied in grade 4.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E6.1 Have students draw a right angle, an acute angle, and an obtuse angle. Using only a mira, have them draw the bisector of each angle. Check by paper folding and/or by measuring the angles with a protractor.

E6.2 Have students make the upper case letters of the alphabet that use only segments. Have them find examples of bisectors of segments, perpendicular segments, and perpendicular-bisectors.

E7.1 Have students construct specific triangles on their geoboards and record on geopaper (e.g., an acute triangle that has one side using five pins; a right triangle that is also isosceles; an obtuse triangle that has one side using five pins).

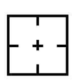
Paper and Pencil

E6.3 Have students draw, without measuring, a segment that is approximately 10cm long. Have them construct a perpendicular bisector of this segment, using only a mira.

E7.2 Have students draw three examples of each type of triangle (i.e., right, acute and obtuse).

Interview

E6.4 Have the student fold a sheet of paper in half and then in half again. Open it up to reveal the folds and draw in segments along these folds. Have him/her describe the relationship between these two segments, and how he/she could have predicted it.



Presentation

E7.3 Provide pairs of students with two 6cm straws, two 8cm straws, and two 10cm straws. Have them investigate the triangles they can make using 3 straws at a time. Complete a table of results.

Straws Used	Type of Triangle

Portfolio

E7.4 Have students investigate how many different isosceles triangles can be made on a 5 pin x 5 pin geoboard. (For this activity, "different" means the side lengths are different, not position on the geoboard.) Record the triangles on geopaper and classify them as acute, obtuse, or right.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *describe, model and compare 2- and 3-D figures and shapes, explore their properties and classify them in alternative ways*

SCO: By the end of grade 5, students will be expected to

E8 make generalizations about the diagonal properties of squares and rectangles and apply them

Elaboration - Instructional Strategies/Suggestions

E8 Teachers should plan guided investigations using paper folding, a mira, and direct measurement of lengths and angles, so that students will describe the patterns regarding diagonals in squares and rectangles.

- ❒ Provide each student in a group of 4 with a different square. Ask them how they think the lengths of the 2 diagonals of their squares compare. Have them check by measuring and share their findings. Ask them to describe the angles where the diagonals intersect. Ask them what the diagonals appear to do to each vertex angle (i.e., each corner of the square). Ask them what they think the measure of each of the smaller angles at a vertex is. Again, they check by measuring and share their findings. Have them cut out the four triangles made by the 2 diagonals. Ask them to describe and compare these triangles. Ask them to describe everything they learned about squares in this investigation. Ask them to write about all the properties of squares that they now know.

From investigations like this, students should conclude that the diagonals of a square a) are equal in length; b) bisect each other; c) intersect to form four right angles, thus are perpendicular-bisectors of each other; d) are bisectors of the vertex angles of the square, thus forming 45° angles; and e) form four congruent isosceles right triangles.

Similarly, by investigating rectangles, they should conclude that the diagonals of a rectangle a) are equal in length, b) bisect each other, c) form two pairs of equal opposite angles at the point of intersection, d) form two angles at each vertex of the rectangle that sum to 90° and have the same measures as the two angles at the other vertices, and e) form two pairs of congruent isosceles triangles.

Teachers should subsequently engage students in a variety of activities that require their knowledge of these diagonal properties.

See E11 for rotational symmetry properties which could be developed at the same time through these guided investigations.

These properties of squares and rectangles should be added to the side, angle, and reflective symmetry properties established in grade 4.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E8.1 Have students draw a square that has a diagonal of length 8 cm. Ask them what properties of a square they used to do this. Did everyone draw the same square?

E8.2 Have students cut a square along both diagonals. Have them investigate the different shapes that they can make (i) using two of the triangles formed (if equal sides must match), (ii) using three of the triangles, and (iii) using all four triangles.

E8.3 Have students draw a rectangle that has diagonals which intersect to form a 60º angle. Ask: Did everyone in the class draw the same one? How do all the rectangles compare?

E8.4 Ask: When the diagonals are drawn in a rectangle, how do you know that each triangle formed is $\frac{1}{4}$ of the rectangle?

Paper and Pencil

E8.5 Have students draw an isosceles right triangle. Have them use a mira to draw the square for which the triangle is one-quarter.

E8.6 Have them draw a segment 12 cm long. Have them, using only a mira, construct the square that has this as a diagonal.

E8.7 It has been said that "all triangles are rigid while rectangles are not. Consequently, one or both diagonals are often added in everyday constructions to make rectangular shapes rigid." Have students explain what these statements mean and give real-world examples.

Interview

E8.8 Have the student draw an isosceles triangle. Have him/her explain how a rectangle could be made that would have this isosceles triangle as one-quarter of it.

Presentation

E8.9 Explain that a family of rectangles has a perimeter of 38 cm and all of their sides are whole numbers. Have students draw this family of rectangles on graph paper. Ask: Which family member has the greatest area? the longest diagonal?

E8.10 Have students draw a rectangle showing its 2 diagonals. Have them measure one angle at a vertex and one angle at the centre. Have them find the measures of all of the other angles using only these two angle measurements and the properties of rectangles.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) *investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts*

SCO: By the end of grade 5, students will be expected to

E9 **make generalizations about the properties of translations and reflections and apply them**

Good geometry activities almost always have a spirit of inquiry or problem solving. Many of the goals of problem solving are also the goals of geometry. (Elementary and Middle School Mathematics, p. 344)

Elaboration - Instructional Strategies/Suggestions

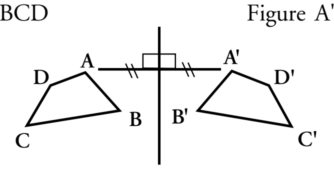
E9 After initial activities that allow students to make and recognize translations and reflections of shapes, activities such as the following should be used to identify properties of reflections.

❐ Have students draw or trace a shape, and, using a mira, draw the mirror line and the reflected image. Students then

- compare the shape and its reflected image using tracing paper or by folding over and looking through the paper at a light source. They should conclude that the shape and image are congruent.

- label the vertices of the shape (e.g., A, B, C, D) and the corresponding vertices of the reflected image (A' , B' , C' , D'). By examining the letter labels of both shapes in a clockwise direction starting at A and A' (see below), students should conclude that the shapes are of opposite orientation.

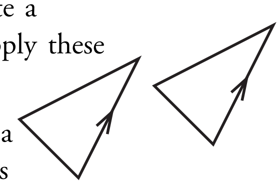
Figure ABCD Figure A'B'C'D'



- make segments by joining corresponding vertices and examine the angles made by the mirror line with these segments. Students should conclude that the mirror line is perpendicular to any segment joining corresponding image points.

- compare or measure the distance from corresponding vertices to the mirror line. Students should conclude that corresponding points are equidistant from the mirror line and, ultimately, that the mirror line is the perpendicular-bisector of all segments joining corresponding points.

These properties should be seen to hold true for a variety of shapes reflected in different mirror lines. After the students write a summary of the properties of a reflection, they should apply these properties in a variety of ways.



Similarly, through guided investigations of shapes under a translation, students should conclude that a shape and its image a) are congruent, b) have the same orientation, c) have corresponding sides parallel, and d) have segments made by joining corresponding points equal and parallel to one another.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

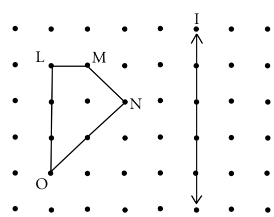
E9.1 Have groups of 3 students place 3 geoboards in a row. On the middle one, have them construct a shape. Using the edges of this geoboard as mirror lines, have them construct the reflected images on the other 2 geoboards. Have them convince one another that the figures are of opposite orientation and that corresponding points are equidistant from the mirror lines.

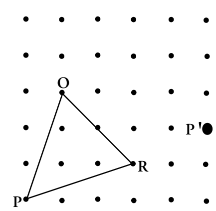
Paper and Pencil

E9.2 Have students use properties to help them draw the image under each transformation.

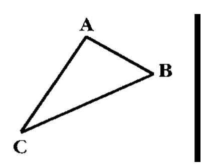
a. Figure LMNO reflected in line l

b. △ PQR under a translation where P → P'

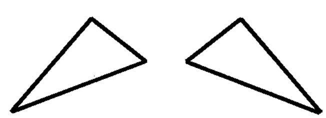




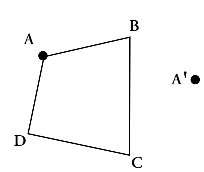
E9.3 Have students find the reflected image of ΔABC in the given mirror line using only a sheet of paper as a measuring tool. Check using a mira.



E9.4 Explain that these two triangles are reflections of one another. Have students use a ruler to find the mirror line. Check using a mira.



E9.5 Explain that Jeri started to translate Figure *ABCD*. He located the image of A and marked it. Finish the translated image for Jeri.



Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) *investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts*

SCO: By the end of grade 5, students will be expected to

E10 explore rotations of one-quarter, one-half and three-quarter turns, using a variety of centres

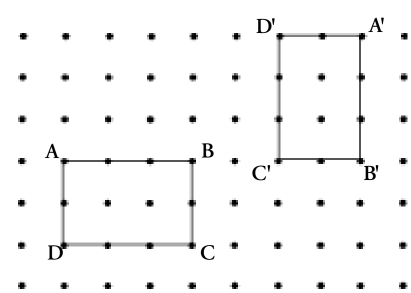
Elaboration - Instructional Strategies/Suggestions

E10 Rotations are the most perceptually challenging of the transformations. Students need many first-hand experiences making rotations and examining the results before they will be able to identify such rotations given to them. At this grade level the emphasis should be on drawing rotation images and identifying a rotation image with centres on lines containing a side of the shape and angles that are quarter, half, and 3-quarter turns.

Students' prior experiences with rotations have been quarter and half turns of triangles and quadrilaterals with their vertices as centres of rotation. Using this as a starting point, they could move on to explore quarter (90°) and half (180°) turns of other shapes with their vertices as centres. Then, shapes could be rotated about centres located along lines formed by extending the sides of the shape. Finally, some work could be done rotating 3-quarter turns.

❒ Make a large plus sign on the floor using masking tape. Have one student stand at the centre of the plus sign, holding a rope. Have a second student stand along one of the arms of the plus sign, holding the other end of the rope so that it is taut. Tell the second student to walk clockwise (keeping the rope taut) and to stop when he/she gets to another arm of the plus sign. Ask: What rotation did the second student just make? Where was the centre of rotation? Continue by giving other instructions and having students discuss the subsequent rotations.

❒ Teachers could have students use square dot paper to rotate rectangle ABCD 90° clockwise about point P. Note: P is on the line containing . Students could use tracing paper, tracing ABCD, holding a pencil firmly at point P, rotating the tracing paper 90° clockwise, and locating the rotated images of A, B, C, and D.

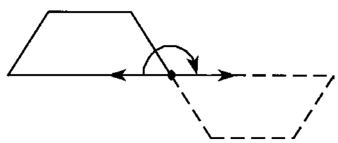


GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

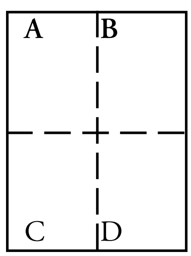
Worthwhile Tasks for Instruction and/or Assessment

Performance

E10.1 Have students trace a pattern block and choose one of its vertices as a centre of rotation. Have them extend one of the sides of the block through this vertex so that they have a straight line (an 180° angle). Have them rotate the pattern block 180° from its original position around the chosen vertex, tracing the pattern block again. Have them examine these half-turn images. Repeat using different vertices, other pattern blocks, and 90° rotations (making 90° angles at the chosen vertex.)



E10.2 Have students fold a plain sheet of paper into four quarters and label them (see below).



Have them arrange four or five pattern blocks along the horizontal segment in section A and arrange copies of the same blocks (in the same order and with the same spacing) along the left vertical segment in section B. Ask: What is the relationship between these two arrangements? Have them arrange the same blocks in section D so they it will represent a half-turn of the arrangement in section A. Have them then arrange the same blocks in section C so they represent a half-turn of the arrangement in B. Ask: What is the relationship between the arrangements in A and C?

E10.3 Have students investigate to see if there is any difference between the images made by a 180° clockwise rotation and a 180° counter-clockwise rotation.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) *investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts*

SCO: By the end of grade 5, students will be expected to

E11 **make generalizations about the rotational symmetry properties of squares and rectangles and apply them**

E12 **recognize, name and represent figures that tessellate**

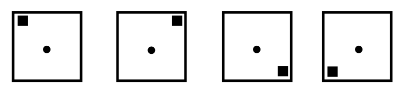
E13 **explore how figures can be dissected and transformed into other figures**

A tessellation is a tiling of the plane using one or more shapes in a repeated pattern with no holes or gaps . . . Most students will benefit from using actual tiles with which to create patterns. (<u>Elementary School Mathematics</u>, p. 339)

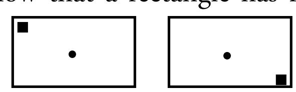
Elaboration - Instructional Strategies/Suggestions

E11

- ❒ Ask each student to use a square from the pattern blocks, mark one of its vertices, and trace around it to make a square on a sheet of paper. With the square block placed inside its picture, it is rotated clockwise, with the centre of rotation being the centre of the square (intersection point of its two diagonals), until it perfectly matches its picture again. Students should notice that the marked vertex is at the next corner. They then repeat this rotation. Because the square can appear in four identical positions during one complete 360° rotation (see below), it is said to have rotational symmetry of order 4.

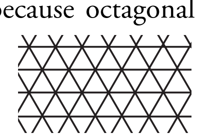


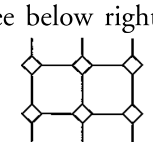
Similarly, students can show that a rectangle has rotational symmetry of order two.



These rotational symmetry properties should be combined with other properties of squares and rectangles. (See E8.)

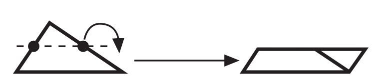
E12 A 2-D figure is said to tessellate if an arrangement of replications of it can cover a surface without gaps or overlapping. For example, if a number of triangles in the pattern blocks were used, they could be used to cover a surface; therefore, this triangle is said to tessellate (see below left). Investigations should include some shapes such as regular pentagons and regular octagons that will not tessellate. When octagons are used in flooring and tiles, squares fill the gaps because octagonal tiles will not tessellate (see below right).





E13 Students need hands-on experiences cutting polygons and transforming them to build and develop spatial visualization skills (e.g., changing a triangle to a parallelogram by cutting off the triangle formed by joining the midpoints of two sides and rotating it about either of those midpoints).

This dissection process can be helpful in developing area formulas.



GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

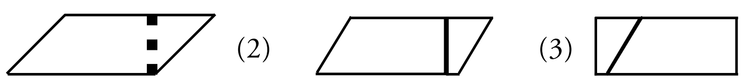
E11.1 Have students investigate if a square is the only quadrilateral with rotational symmetry of order 4.

E11.2 Have students investigate what other quadrilaterals besides a rectangle have rotational symmetry of order 2 and which ones also have 2 lines of reflective symmetry.

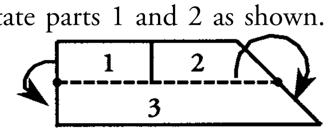
E12.1 Have students investigate all of the different pattern blocks for their ability to tessellate.

E13.1 Have students cut out a parallelogram. From any point on one side, have them draw a perpendicular segment to the opposite side. Have them cut along this segment and translate (to the left) the piece on the right until two sides match. Ask them what new shape they get.

(1) (2) (3)



E13.2 Have students cut out a trapezoid shape; fold it into two parts so that one parallel sides matches the other parallel side; unfold and draw a perpendicular anywhere in the top part (see diagram); cut it into three parts; rotate parts 1 and 2 as shown. Ask them what new shape they got.



Presentation

E12.2 Have students fold a sheet of paper in half repeatedly until they have 8 sections. With it completely folded, have them draw any triangle on the exposed surface and cut it out (cutting through all 8 sections). Using the 8 triangles, have them test to see if it tessellates. Have them share their observations. Ask: Did everyone's triangle tessellate? Did we have different triangles (acute, obtuse, right, isosceles, scalene)? What conclusion might we make about the tessellating ability of any triangle?

E12.3 Using any one of the pattern blocks and half of a sheet of plain paper, have students trace the block to completely cover this paper. Have them color one block in the centre blue. They then color the rest of the shapes so that any two shapes that share a common side are different colors (sharing a common vertex is OK). What is the smallest number of colours possible?

Suggested Resources

Data Management and Probability

General Curriculum Outcome F:

Students will solve problems involving the collection, display and analysis of data.

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *construct a variety of data displays, including tables, charts and graphs, and consider their relative appropriateness* and

iii) *read, interpret, and make and modify predictions from displays of relevant data*

SCO: By the end of grade 5, students will be expected to

F1 **use double bar graphs to display data**

F2 **use bar graphs to display and interpret data**

Certainly graphs, pictures, and charts can be used to show data to an audience once the data have been collected, summarized, and analyzed. A pictorial representation is an effective way to make a point. However, a real user of statistics employs pictures and graphs as tools to understand the data during the process of analysis . . . Representing data in a picture, table, or graph is a way to discover the features of the data, to array the data so that their shape and relationships among aspects of the data can be seen. (NCTM 1989 Yearbook, p. 142)

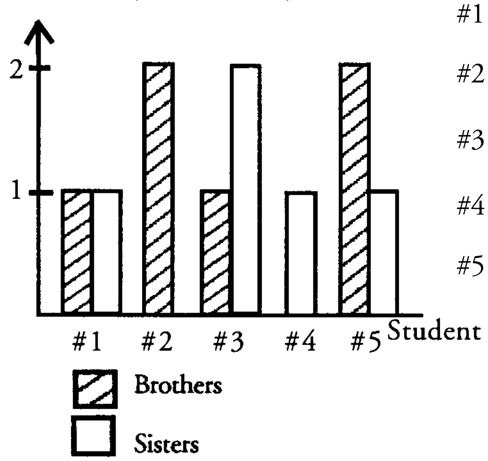
Elaboration - Instructional Strategies/Suggestions

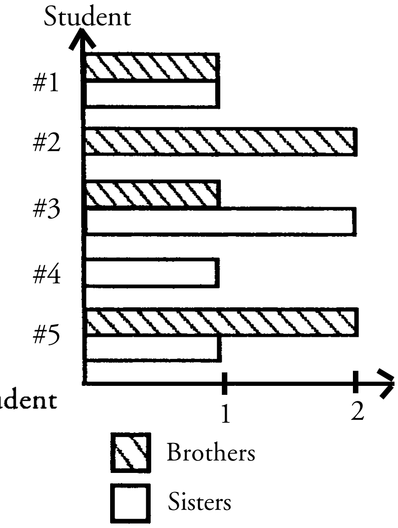
F1/F2 Students should be aware that sometimes when two pieces of data are collected about a certain population, it is desirable to display both of them simultaneously. For example, census data often shows male and female data separately for different years. Explain that this is usually done using a double bar graph.

An example is presented below. Five students in the class have been asked how many brothers and sisters they have.

	Brothers	Sisters
Student 1	1	1
Student 2	2	0
Student 3	1	2
Student 4	0	1
Student 5	2	1

The data may be displayed horizontally or vertically:





Discuss how this type of graph allows one not only to compare students in terms of how many brothers they have, or how many sisters they have, but also to compare the number of brothers versus the number of sisters.

Students might develop their own ideas about how "double pictographs" could be displayed.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F2.1 Ask the student to draw a pictograph to illustrate the results of a survey on the regular dinner times of classmates.

F2.2 Ask students to draw an appropriate graph to represent the frequency of outcomes when a die is rolled 25 times.

F1.1 Ask students to draw a double bar graph comparing how many numbers between 1 and 100 are multiples of 2, multiples of 3, and multiples of 4, with how many numbers are multiples of their doubles (i.e. multiples of 4, 6, and 8). Ask students what conclusions might be drawn.

Interview

F1.2 Ask the student to describe some data which would be appropriate to display using a double bar graph.

F2.3 Show the student an unlabelled bar graph of the populations of the Canadian provinces. Ask him/her to decide which bar goes with which province.

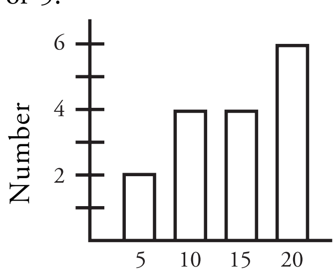
Presentation

F1.3 Have students collect information on the length and mass of various animals and display the data in a double bar graph. Ask what conclusions they might draw.

F1.4 Students might collect male/female Olympic track records, draw a double bar graph, and draw conclusions.

Portfolio

F2.4 Bill drew the graph below to display the number of factors of each multiple of 5.



Jim looked at it and said, “Hey, the number of factors is always going to be even!” “Wait,” interrupted Jan. “I don’t think that is always true.” Extend the graph to show Jan’s point. In what way does Jim need to be a bit more careful when interpreting graphs?

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *construct a variety of data displays, including tables, charts and graphs, and consider their relative appropriateness* and

iii) *read, interpret, and make and modify predictions from displays of relevant data*

SCO: By the end of grade 5, students will be expected to

F3 use coordinate graphs to display data

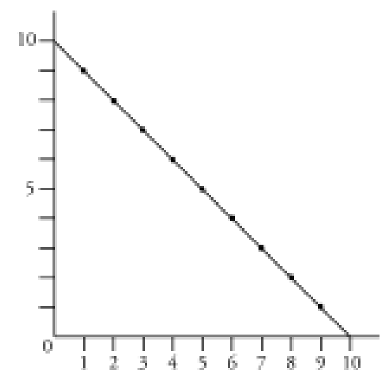
F4 create and interpret line graphs

Elaboration - Instructional Strategies/Suggestions

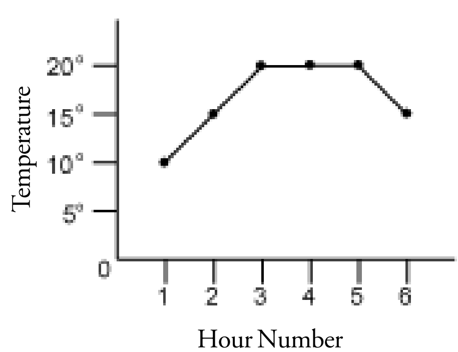
F3 Coordinate graphing will be a very useful tool for students throughout their school mathematics courses. Students should continue to use coordinates for the purposes of location. As a way to reinforce locating coordinates, students often enjoy creating "join-the-dots" pictures on a coordinate grid. After they draw their pictures, they list the coordinates in order of connection and give them to other students who figure out what the picture is.

At this level, students might also begin to consider what sort of data could be displayed using coordinate graphs.

❒ Consider the pairs of numbers that sum to ten. Use these pairs as coordinates and plot the points. (Note: Connecting the points makes sense because pairs of fractions also sum to 10.)



F4 Sometimes, coordinate points can be joined to create a line graph. The purpose of a line graph is to focus on trends implicit in the data. For example, if students measure the temperature outside every hour during a school day, they could create a graph in which the ordered pair is (hour number, temperature). By connecting the points with line segments, they see the trend in the temperature.



GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F3.1 Ask students to plot the points (2, 5) and (3, 7.5). Have them connect the points and give the coordinates of 3 other points on the line which they drew.

F3.2 Ask different groups of students to create and plot ordered pairs that satisfy these relationships:

- the two numbers in the pair add to 8
- the two numbers in the pair add to 12
- the two numbers in the pair add to 15
- the two numbers in the pair add to 17.

Have students compare their graphs and observe the similarities and differences.

F4.1 Have students collect information about the number of students in the school in grades 1, 2, 3, 4, and 5 and draw a line plot to help show whether there are "bulges" in the numbers in certain grades. It would be advisable to remind students to carefully consider the step size for the vertical scale.

F4.2 Ask students to look up the hockey scores for a favourite team over the course of 10 games and then create a line graph with the ordered pairs being (game number, number of goals scored by favourite team). Have them create a second graph with the ordered pairs being (game number, goals scored by opposing team) and then compare the two graphs.

Paper and Pencil

F3.3 Ask the student to name all the points on a coordinate graph which are as far from (1, 2) as from (3, 3), if distance is measured only along the grid lines.

Portfolio

F3.4 Ask students to compare the pictures formed by connecting these two sets of ordered pairs, in order, joining the last point to the first as the last step.

Set 1: (2, 1), (3, 1), (4, 1), (5, 1), and (4, 3)
Set 2: (1, 2), (1, 3), (1, 4), (1, 5), and (3, 4)

(Students may use geometric language, noting that there has been a flip over the diagonal of the grid.) Students could then design their own diagrams involving flip images.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *collect, organize and describe relevant data in multiple ways*

ii) *construct a variety of data displays, including tables, charts and graphs, and consider their relative appropriateness* and

iii) *read, interpret, and make and modify predictions from displays of relevant data*

SCO: By the end of grade 5, students will be expected to

F5 group data appropriately and use stem-and-leaf plots to describe the data

Elaboration - Instructional Strategies/Suggestions

F5 Stem-and-leaf plots provide a convenient organization for displaying grouped numerical data. For example, suppose the students in a class list their heights in centimetres:

140, 135, 127, 128, 131, 130, 121, 119, 124, 127, 130, 131, 139, 142, 143, 118, 129

Since the data ranges from 118 to 143, it is convenient to group it into the 110s, 120s, 130s, and 140s, producing this stem-and-leaf plot:

Heights of Students (in cm)

11	8 9
12	1 4 7 7 8 9
13	0 0 1 1 5 9
14	0 2 3

Each stem on the left is the number of tens in the data. Each leaf to the right is appended to the stem to represent the actual data value. Notice that the least values appear at the top and the greatest at the bottom of the plot. Within each grouping, the data appears in order from left to right. Students might use grid paper to help them line up their data.

Students should be encouraged to notice the shape made by the data. (For example, above there are more values in the middle than at the top or bottom.) They might also observe that the middle of the data (the median) can be found by counting the number of data pieces, identifiying the halfway point, and then counting down to find that piece of data. Above, there are 17 pieces of data. The midpoint is the 9th piece of data, which is 130.

Students will need to make decisions about what to use as the stem. For example, if data ranges from 100 to 1000, the stem might be the number of hundreds, and two-digit numbers would be the leaves. If data involves decimal amounts, they might use the whole numbers as stems and the parts of the numbers after the decimal point as the leaves.

❒ Have students gather data about any of the following, make stem-and-leaf plots, and answer questions posed by other students about the data:

- the number of marbles various students own
- the last two digits of the phone numbers of various students
- the number of pages in favourite novels.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

F5.1 Ask students to list the pieces of data from the stem-and-leaf plot and then find the mean, maximum and range of the data.

1	1 1 1 2 3
2	2 3 4
3	3 5
4	0

F5.2 Tell the student that you have collected data on the population of Canada for each year since 1867. Ask her/him to suggest how the data could be grouped for presentation.

Interview

F5.3 Ask students to describe the characteristics of a stem-and-leaf plot.

Portfolio

F5.4 Ask students to gather information about the number of phone calls that come into their homes each day over the course of a week. Groups of students could pool their data and create a stem-and-leaf plot. They should draw conclusions from the information gathered.

F5.5 Ask students to gather information on Olympic swimming records for an event of their choice. Have them create stem-and-leaf plots to show the winning times over the course of the last 30 years.

F5.6 Have students gather data on the heights of basketball players from a favourite team. The data could be displayed in a stem-and-leaf plot. Students could be asked to write a few sentences that describe the player heights as fully as possible without listing each separate piece of information.

F5.7 Have groups of students research populations for ten locations within their province, ten others in the country, and ten others in the world. Ask the students to display the data, explaining why they would organize the populations in different groupings for the three displays.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) develop and apply measures of central tendency (mean, median and mode)

SCO: By the end of grade 5, students will be expected to

F6 recognize and explain the effect of changes in data on the mean of that data

Technology provides teachers with an effective method of examining the mean with their classes. Spreadsheets are unbelievably easy to use. . . With this tool it is easy to . . . change the data in any way at all to observe the effect on the mean. (Elementary School Mathematics, p. 400)

Elaboration - Instructional Strategies/Suggestions

F6 Students should have a sense that the mean of a group of data describes its “middle.” In particular, the mean is the number such that the total of the differences from the mean for data below the mean balances the total difference for data above. For example, for 6, 9, 10, 12 and 13, the mean is 10 since the differences below 10 are 4 and 1 (for a total of 5 below), which balance the differences above 10, which are 2 and 3. (Students may find it useful to model numbers using linking cubes and redistribute them to identify the mean.)

Students should realize that the mean of a set of data

- increases if any piece of data increases
- decreases if any piece of data decreases
- increases if a piece of data below the existing mean is removed
- decreases if a piece of data above the existing mean is removed.

Working with linking cubes can help illustrate these principles.

❐ Provide a set of ten “salaries” of office workers in a certain company, for example:

5 salespersons @ $25 000
3 secretaries @ $20 000
2 clerks @ $17 500.

Ask students to determine the mean salary. Then have them predict and verify whether the mean salary goes up or down

- if a secretary resigns
- if a clerk resigns
- if 2 more salespeople are hired.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F6.1 Have students use counters (linking cubes) to show visually why the mean of 8, 10, and 15 has to be 2 more than the mean of 6, 8, and 13.

Paper and Pencil

F6.2 Ask the students to create a set of measurements that would maintain the same mean even if two pieces of the data were removed.

F6.3 Ask: Which two numbers should be removed from this set of savings amounts so that the mean remains the same?

$37, $40, $43, $20, $60, $40

Interview

F6.4 Present the data 9, 6, 8, 4, 7, 10, 5, 5, 8, 3 to a student. Ask him/her to explain the effect on the mean if the "7" is removed.

F6.5 Ask the student why the mean of this set of class sizes would not change much even if the 30 were removed from the data:

20, 20, 20, 20, 20, 20, 20, 20, 20, 20, 30

F6.6 Ask: Why is it easy to tell that the mean of the data below is 45?

43, 45, 47, 42, 48, 41, 49

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) *formulate and solve simple problems (both real-world and from other academic disciplines) that involve the collection, display and analysis of data and explain conclusions which may be drawn*

SCO: By the end of grade 5, students will be expected to

F7 **explore relevant issues for which data collection assists in reaching conclusions**

Real data are either collected by the students or obtained from real-world sources. Real data are, by their very nature, 'messy.' More data might be available than are needed to solve the problem being considered. Unusual characteristics of the data might necessitate many attempts at selection, sorting, and representation in an effort to make sense of them. (NCTM 1989 Yearbook, p. 135)

Elaboration - Instructional Strategies/Suggestions

F7 Students have had previous experience in collecting data to explore relevant issues. The focus might turn to how one chooses the best way to display that data.

For example, suppose students have collected information about the amount of fat and protein in various types of snack foods. They might then consider whether to display that information in separate bar graphs, separate pictographs, stem-and-leaf plots or double bar graphs. If bar graphs are chosen, they would consider whether the bars should be organized to go from least to greatest, greatest to least, or some other way. They should consider whether wide bars or narrow bars would be used and why the same width should be used for all bars. If pictographs are chosen, students would consider what visual should be used for the symbol and how many grams each symbol should represent. Stem-and-leaf plots would involve decisions concerning appropriate grouping and choice of stems.

Students should examine graphs from various sources (e.g., web pages, newspapers, magazines) to see what decisions have been made when showing data graphically.

Where appropriate, students should analyze the data shown to draw conclusions. If, for example, one notices that the bars for fat are always higher than the bars for protein, can one conclude that snack foods are bad for us?

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F7.1 Have students collect, display and analyze information on the nutritional value of various cereals.

Presentation

F7.2 Groups of students should determine questions to which they would like answers. They should then determine how to collect the information, and collect and display it. Sample questions include:

- What clothing and shoe sizes are most common for ten year olds?
- About how many minutes a day is a household phone generally in use?
- Do more people in Canada live within city limits, in bedroom cummunities, or in the country?

Portfolio

F7.3 Have students collect data on the prices of lettuce at different stores in a particular week. Ask them to display the data and describe whether this information would help a shopper decide the best place to shop.

F7.4 Ask groups of students to devise a way to determine how much taller grade 5 students are, on average, than grade 4 students. Have the students collect data, display it, and analyze it.

Suggested Resources

Data Management and Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *explore, interpret and make conjectures about everyday probability situations by estimating probabilities, conducting experiments, beginning to construct and conduct simulations, and analyzing claims which they see and hear*

SCO: By the end of grade 5, students will be expected to

G1 conduct simple experiments to determine probabilities

Whenever possible . . . we should try to use an experimental approach in the classroom. (Elementary School Mathematics, p. 384)

Elaboration - Instructional Strategies/Suggestions

G1 Students should continue to perform simple experiments and use fractions to indicate the experimental probabilities that result. Although dice, coins, and the like are typically used for these experiments, there are other contexts which can also be used to practise computational skills.

❒ Have pairs of students take turns rolling two dice. One number is used as the numerator of a fraction and the other as the denominator. The students can determine the probability that the fraction is in lowest terms, i.e., has no equivalent using lesser numbers.

❒ Simple experiments can be conducted on a hundred chart. Have students begin with a counter on a designated number and roll a die to determine where to move it.

# rolled on die	movement of counter
1	1 down and 1 right
2	2 down and 2 right
3	1 down and 1 left
4	2 down and 2 left
5	1 up
6	2 up

The students roll the die and move the counter a total of 5 times and record the number on which they finish. This process is repeated numerous times, always starting on the original designated number. The students then determine the probability that after 5 rolls they will land in some designated range of numbers, or on a certain type of number, such as an even number or a multiple of 3.

❒ Have the students use the random number function on a calculator or spreadsheet to generate two 2-digit numbers. They add the numbers. The event is repeated a number of times to determine the probability that the sum of the numbers will be greater than 100.

Allow students to use decimals to describe experimental results. For example, if an event occurs 9 times out of 100, the student could report the probability as $\frac{9}{100}$ or 0.09. Students should recognize that they must repeat experiments many times before reporting the probability in order to have more reliable results.

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G1.1 Have pairs of students roll a die 4 times. Ask them to create two 2-digit numbers and subtract them. They repeat the experiment 20 times. Have them calculate the probability that the difference is less than 10. Have them repeat the experiment another 20 times and compare the probability for 20 rolls compared to 40 rolls.

G1.2 A spinner is divided into 10 equal sections, labelled 0,1,2,...9 (or a 10-faced die is used). The student spins the spinner 5 times and totals the numbers spun. Ask the student to repeat this process several times, then report the probability that the sum is greater than 25. Have the students compare their findings.

G1.3 Give a student a bag containing 20 green cubes and 5 red cubes. Another student pulls out a cube, records the colour, and returns it to the bag. The experiment is repeated 20 times. Have the students indicate the probability that a green cube was chosen.

G1.4 A student places a counter at the number 50 on a hundred chart. He/she flips a coin. If the flip is heads, he/she moves down; if the flip is tails, the move is up. This is repeated 10 times to form one experiment. The experiment is repeated 20 times in total. The student is to calculate the probability that he/she moves off the board during the course of the experiment. If a student has the technological ability, he/she might use the computer to simulate coin flips and to calculate landing places.

Paper and Pencil

G1.5 Tell the student that you rolled a pair of dice 25 times and the sum of the numbers was 8 on 4 of the rolls. Ask: What is the experimental probability that the sum is 8? Does that seem reasonable?

Interview

G1.6 Ask the student how to set up an experiment to determine the probability that the difference of the numbers on a pair of dice is 1.

G1.7 Tell the student that two people performed an experiment in which a coin was tossed and the experimenter recorded the probability of tossing a head. One person got a probability of $\frac{3}{5}$. Another got a probability of $\frac{47}{100}$. Ask whether one can tell who has a more reliable result and why.

Suggested Resources

GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *determine theoretical probabilities using simple counting techniques*

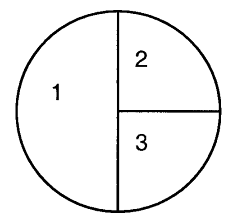
SCO: By the end of grade 5, students will be expected to

G2 determine simple theoretical probabilities and use fractions to describe them

. . . Theoretical probability is based on a logical analysis of the experiment, not on experimental results. (<u>Elementary School Mathematics</u>, p. 383)

Elaboration - Instructional Strategies/Suggestions

G2 Experimental probabilities are calculated by performing experiments. Theoretical probabilities describe what will happen, in theory, if the experiment is performed a great number of times. For example, if a die is rolled 60 times, the number 4 might come up 15 times. Thus the experimental probability is $\frac{15}{60}$ or $\frac{1}{4}$. The theoretical probability, however, is $\frac{1}{6}$. Theoretical probability is calculated by listing all of the possible equally likely outcomes and determining what fraction of them pertain to the probability being calculated. Since the only possible die rolls are 1, 2, 3, 4, 5, and 6, and since all are equally likely, then the probability of rolling a 4 is 1 out of 6, or $\frac{1}{6}$.



Consider, on the other hand, the spinner shown at right. Even though there are only 3 outcomes, they are not equally likely. The probability of spinning a 1 is $\frac{1}{2}$, not $\frac{1}{3}$.

Students should use both fractions and decimals to describe theoretical probabilities.

❒ Have students construct an array like the following to determine the theoretical probability that the product of the two numbers produced with the roll of two dice will be even.

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Counting even products will allow them to calculate a theoretical probability of $\frac{27}{36}$ or $\frac{3}{4}$ or 0.75. Students might then enjoy creating a large number of these products experimentally, determining the experimental probability, and comparing results.

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G2.1 Ask the student to put coloured cubes in a bag so that the theoretical probability of choosing a red one is $\frac{1}{2}$ and choosing a green one is $\frac{1}{4}$. Ask: Why is there more than one way to model this situation?

G2.2 Ask the student to list the first 20 multiples of 3 and determine the probability that a multiple of 3 is also a multiple of 6 (of 9).

G2.3 Ask the student to construct an array that will help him/her to determine the probability that the sum of two rolled dice will be 6, 7 or 8. Ask: What is the probability?

Paper and Pencil

G2.4 Provide a hundreds chart. Ask students to determine the probability that a number randomly chosen on the chart

- ends in a 5
- is even
- is less than 50
- is on a main diagonal

Interview

G2.5 Ask the student why the probability that the sum of the numbers on a pair of dice is 3 is not the same as the probability that the sum is 7.

Suggested Resources

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FIVE-CORRELATIONS