Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

CURRICULUM

Mathematics

Grade 6

2001

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Acknowledgements

The Departments of Education of New Brunswick, Newfoundland and Labrador, Nova Scotia and Prince Edward Island gratefully acknowledge the contributions of the following groups and individuals toward the development of the entry-grade 6 mathematics curriculum guides:

- The regional mathematics curriculum committee, current and past members, include the following:

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- The Elementary Mathematics Curriculum Development Advisory Committee, comprising teachers and other educators in New Brunswick, lead province in drafting and revising the guides
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the finalization of the entry-grade 6 mathematics curriculum guides

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Introduction

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their school learning experience.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision. These publications embrace the principles of students learning to value mathematics and of being active "doers," and they advocate a meaningful curriculum focussing on the unifying ideas of mathematical problem solving, communication, reasoning and connections. The foundation document subsequently establishes a framework for the development of detailed grade-level guides describing mathematics curriculum, assessment, and instructional techniques.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers and Department of Education officials to plan and develop cooperatively the curricula in mathematics, science, and language arts in both official languages.

Each of these curriculum initiatives has produced a program, using a learning-outcome framework as outlined in Figure 1, that supports the regionally-developed Essential Graduation Learnings (EGLs). (See the "Outcomes" section of the mathematics foundation document for a detailed presentation of the Essential Graduation Learnings, and the contribution of the mathematics curriculum to their achievement.)

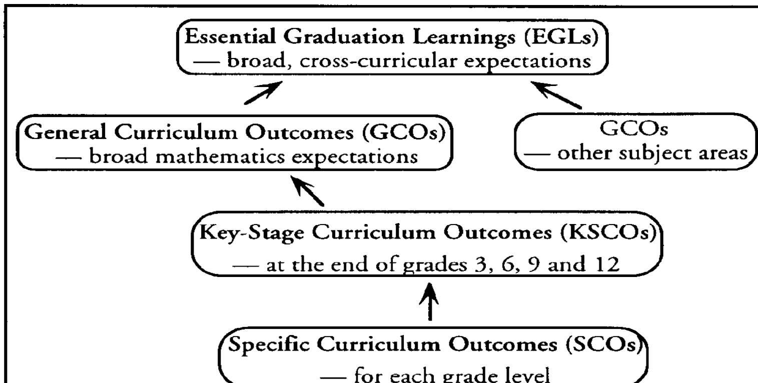


Figure 1: Outcome Framework

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. It describes the mathematics curriculum in terms of a series of outcomes—general curriculum outcomes (GCOs) which relate to subject strands and key-stage curriculum outcomes (KSCOs) which further articulate the GCOs for the end of grades 3, 6, 9 and 12. This curriculum guide is supplemented by others that provide greater specificity and clarity for the classroom teacher by relating grade-level specific curriculum outcomes (SCOs) to each KSCO.

The Atlantic Canada Mathematics Curriculum is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice. These beliefs include: i) mathematics learning is an active and constructive process; ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; iii) learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk-taking, and critical thinking and that nurtures positive attitudes and sustained effort; and iv) learning is most effective when standards of expectation are made clear with on-going assessment and feedback.

II. Program Design and Components

A. Program Organization

As already indicated, the mathematics curriculum is designed to support the six Essential Graduation Learnings (EGLs). While the curriculum contributes to students' achievement of each of these, the communication and problem solving EGLs relate particularly well to the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then presents outcomes at four key stages of the student's school experience.

This particular curriculum guide presents specific curriculum outcomes for each grade level. As illustrated in Figure 2, these outcomes represent the means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes and, ultimately, the essential graduation learnings.

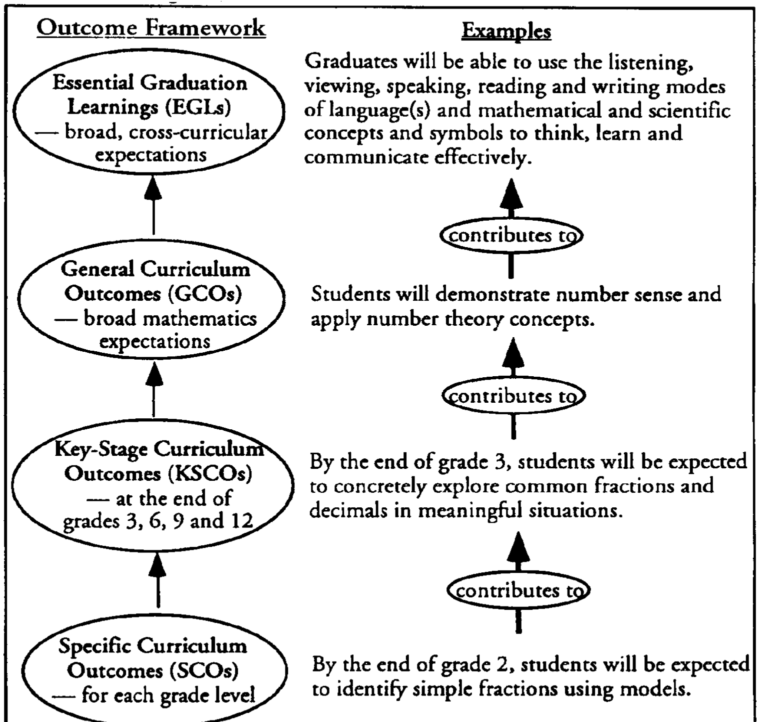


Figure 2: Examples of Outcomes

It is important to emphasize that, while the grade level outcomes (SCOs) provide a framework on which educators will base decisions regarding instruction and assessment, they are not intended to limit the scope of learning experiences. Although it is expected that most students will be able to attain the outcomes, some students' needs and performance will range across grade levels. Teachers will need to take this variation into consideration as they plan learning experiences and assess students' achievement.

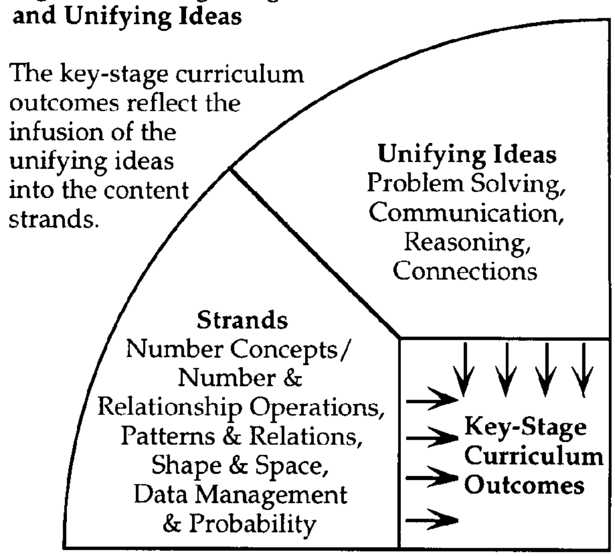
The presentation of the specific curriculum outcomes follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a suggested teaching sequence. While some outcomes will need to be addressed before others, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes like those pertaining to patterns and data management may best be addressed on an ongoing basis in connection with other strands. It is expected that teachers will make individual decisions regarding the sequencing of outcomes. Many lessons, or series of lessons, could simultaneously address many outcomes across a number of strands.

Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" strand for one group of students might be less effective in that role with a second group. Another consideration will be coordinating the mathematics program with other aspects of the students' school experience. For example, they could study facets of measurement in connection with appropriate topics in science, data management with a social studies issue and an aspect of geometry with some physical education unit. As well, sequencing could be influenced by other factors such as a major event in the community or province like an election, an exhibition, or a fair.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. (See Figure 3.)

Figure 3 — Integrating Strands and Unifying Ideas



These unifying ideas serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode, that classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically, that via teacher encouragement and questioning students must explain and clarify their mathematical reasoning, and that the mathematics with which students are involved on any given day must be connected to other mathematics, other disciplines and/or the world around them.

Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart and make an organized list should all become familiar to students during their early years of schooling, while working backward, logical reasoning, trying a simpler problem, changing point of view and writing an open sentence or equation would be part of a student's repertoire upon leaving elementary school.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the "doing of mathematics." No longer is it sufficient or appropriate to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline that lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching Mathematics" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, actively participate in discourse, conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas and in which reasoning and sense-making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Adapting to the Needs of All Learners

The F*oundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers adapt instruction to accommodate differences in student development as they enter the public school and as they progress, but they must also avoid gender and cultural biases. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes by grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Designing classroom activities to support a variety of learning styles must also be reflected in assessment strategies.

E. Support Resources

This and other curriculum guides represent the central reference for teachers of mathematics at various grade levels. These guides should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes have been met.

Texts and other resources will have significant roles in the mathematics classroom in as much as they support the specific curriculum outcomes. Many manipulative materials need to be readily at hand, and technological resources, e.g., software and videos, should be available. Calculators will be an integral part of many learning activities. Also, professional resources will need to be available to teachers as they seek to broaden their instructional and mathematical understandings. Key among these are the *Curriculum and Evaluation Standards for School Mathematics* (NCTM) and the *Addenda Series* and *Yearbooks* (NCTM), *Elementary School Mathematics: Teaching Developmentally* or *Elementary and Middle School Mathematics: Teaching Developmentally* (John van de Walle), *Developing Number Concepts Using Unifix Cubes* (Kathy Richardson), and *About Teaching Mathematics; A K-8 Resource* (Marilyn Burns).

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their students become confident, independent learners of mathematics.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to learning and teaching. Ongoing assessment and evaluation not only are critical for clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers may make meaningful instructional decisions. (See "Assessment and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment would include i) the use of a wide variety of assessment strategies and tools, ii) aligning assessment strategies and tools with the curriculum and instructional techniques, and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborates good assessment practices and it served as a guide for student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators on the relative success of the mathematics curriculum and its implementation. It will address whether or not students are meeting the curriculum outcomes, whether or not the curriculum is being equitably applied across the region, whether or not the curriculum reflects a proper balance between procedural knowledge and conceptual understanding, and whether or not technology is fulfilling its intended role.

IV. Curriculum Outcomes

This guide provides details regarding specific curriculum outcomes for each grade. As indicated earlier, the order of presentation does not prescribe a preferred order of presentation for the classroom nor does it suggest an isolated treatment of each outcome; rather, it organizes the specific curriculum outcomes in terms of the broad framework of GCOs and KSCOs developed in the mathematics foundation document.

The specific curriculum outcomes are presented on two-page spreads (see Figure 4). At the top of each page the overarching GCO is presented, with the appropriate KSCO and specific curriculum outcome(s) displayed in the left-hand column. As well, the bottom of many left-hand columns contains a relevant quotation. The second column of the layout, entitled "Elaboration–Instructional Strategies/ Suggestions," provides a clarification of the specific curriculum outcome(s), as well as suggestions for possible strategies/activities which could be used to help students achieve the outcome(s). While the strategies/activities presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s). They will also illustrate ways to work toward the achievement of the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning and connections. To readily distinguish between activities and instructional strategies, activites are introduced in this column of the layout by the symbol ❐ .

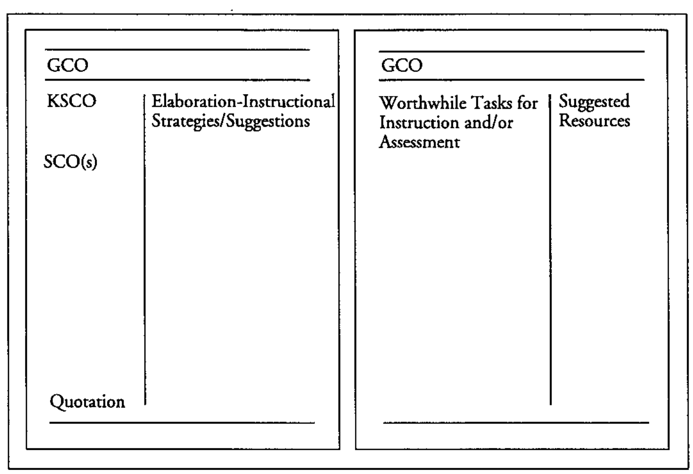


Figure 4: Layout of a 2-Page Spread

The third column of the two-page spread, entitled "Worthwhile Tasks for Instruction and/or Assessment," serves several purposes. While the sample tasks presented may be used for assessment, they will also further clarify the specific curriculum outcome(s) and will often represent useful instructional activities. As well, they regularly incorporate one or more of the four unifying ideas of the curriculum. While these tasks have headings (performance, paper and pencil, interview, observation, presentation, and portfolio), teachers should treat these headings only as suggestions. These sample tasks are intended as examples only; teachers will want to tailor items to meet the needs and interests of the students in their classrooms. The final column of each display, entitled "Suggested Resources," is available for teachers to collect useful references to resources which are particularly valuable in achieving the outcome(s).

SIX-NUMBER

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number theory concepts.

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, the students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 6, students will be expected to

A1 **represent large numbers in a variety of forms**

Elaboration - Instructional Strategies/Suggestions

A1 Students at this level may still need practice, or further experience, in reading and recording very large numbers (which may include fractions or decimals).

In grade 6, an emphasis should be placed on representing large whole numbers using rounding and decimal notation. For example, rather than writing 34 518, students should be encouraged to approximate it as 34.5 thousands; 12 340 000 might be recorded as 12.34 million. (Normally the "unit" would be millions, although the number could be written as 123.4 hundred thousand if comparisons were being made to other numbers in the hundred thousands.)

Have students practise recording numbers that are presented to them orally and rounding each one to the nearest tenth or hundredth of a million. For example, one hundred eight million ninety-three thousand, forty-six might be estimated as 108.1 million or 108.09 million.

Students should also be able to recognize and represent fractional parts of large numbers.

43 489 784 is about $43\frac{1}{2}$ million
247 986 is about $\frac{1}{4}$ million
8 762 154 375 is about $8\frac{3}{4}$ billion

- ❐ Use real data when possible. For example: The population of Atlantic Canada, as recorded in the 1991 census, is two million, three hundred seventy-eight thousand two hundred ninety-seven. (Newfoundland - 568 474, Prince Edward Island - 129 765, Nova Scotia - 920 000 and New Brunswick - 760 058.) Have students work in groups to approximate the populations of each of the four provinces.
- ❐ Ask students to find various representations for large numbers in newspapers and magazines. Encourage discussion on the need for accuracy in reporting and the appropriate use of rounded numbers.
- ❐ Have the students prepare a report on populations of Canadian cities. Ask that the report include a graph of the five most populous cities, comparing them to the five most populous cities of another country.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A1.1 Prepare and shuffle 5 sets of number cards (0-9 for each set). Have the student select nine cards and ask him/her to arrange them to make the greatest possible and least possible whole number. Have the student read each of the numbers. Consider extending the activity by asking the student determine

- how many different whole numbers could be made using the nine digits selected
- the number of $1000 bills one would get if the greatest and least numbers represented money amounts.

Interview

A1.2 Discuss the idea of counting to 100 by 10s. Ask: How many numbers are named? Then ask: How many numbers are named when counting to 1 000 000 by thousands? by hundreds? by tens? Then ask: How many numbers are named when counting to 10 000 000 000 by hundred thousands? by thousands? by hundreds?

A1.3 Tell the student that light from a star takes 7000 centuries to reach the earth. Ask: How many years is that?

A1.4 Present this library information to students

Metropolitan Toronto Library	3 068 078 books
Bibliotheque de Montreal	2 911 764 books
North York Public Library	2 431 655 books

Ask them to rewrite the numbers in a format such as □.□ million or □.□□ million books. Then ask them to make comparison statements about the number of books.

Portfolio

A1.5 Have the students collect newspaper and magazine clippings in which large numbers are used. Discuss the type of situations in which one is most likely to encounter large numbers and why that might be.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, the students will have achieved the outcomes for entry-grade 3 and will also be expected to

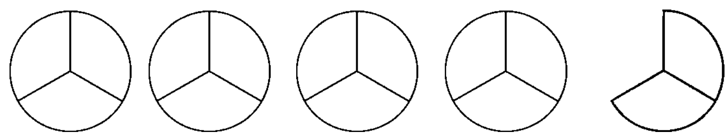
i) *demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 6, students will be expected to

A2 represent fractions and decimals

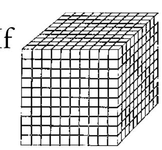
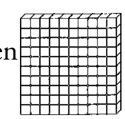
Elaboration - Instructional Strategies/Suggestions

A2 Mixed number notation and improper fractions seem to be more problematic for students than proper fractions. Students should fluently move between mixed number and improper fraction formats of a number. Rather than only applying a rule to move from one format to the other, students should be encouraged to focus on the meaning. For example, since $\frac{14}{3}$ means 14 thirds and it takes 3 thirds to make 1 whole, $\frac{14}{3}$ represents 4 wholes and another 2 thirds or $4\frac{2}{3}$.



Often it is easier for students to grasp the size of mixed numbers than improper fractions. For example, a student knows that $4\frac{1}{3}$ is a bit more than 4; he/she might have little sense of the size of $\frac{13}{3}$.

Students should be familiar with base ten block representations of decimals.

If  = 1, then  = 0.1,  = 0.01 and  = 0.001.

These representations help students visualize the relative sizes of decimals.

Measurement contexts continue to be natural ones for decimal situations. For example, a student might consider how many kilograms of ground beef one would need for four hamburgers, or how many kilometres one could walk in a minute.

❒ As a class activity, have the students use a place value chart that is divided into 6 sections representing numbers to the thousandths. Select number cards, one at a time, (or toss a 10-sided die) and have each of the students decide in which section to place the digit as it is selected (or tossed), in order to try to make the greatest (least) possible number. This activity encourages students to take risks and to think about probability. After all six digits are called, have the students compare their numbers. Extensions might include estimating how far each student's number is from a target number or determining how the number would be rounded for a newspaper article and what it might represent.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A2.1 Ask students to use coloured squares to show why $3\frac{1}{3} = \frac{10}{3}$. Observe whether or not they make wholes of 3 (or 6 or 9...) squares.

Paper and Pencil

A2.2 Contexts that lend themselves to using large numbers include astronomical data and demographic data. Contexts that lend themselves to decimal thousandths include sports data and metric measurements. An interesting activity involving decimals might require students to complete a chart such as:

In 0.1 years, I could...
In 0.01 years, I could...
In 0.0001 years, I could...

A2.3 Ask the student to determine the number of whole numbers between 2.03 million and 2.35 million.

A2.4 Provide thousandths grids. Ask students to shade the grids, one at a time, to show the following decimals:

0.004
0.203
0.023
1.799

Ask them to tell which was easiest to do and why.

Portfolio

A2.5 Ask the student to write a report on what he/she has learned about decimals and what questions he/she may now have concerning the topic.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore integers, ratios and percents in common, meaningful situations

SCO: By the end of grade 6, students will be expected to

A3 write and interpret ratios, comparing part-to-part and part-to-whole

Both part-to-whole and part-to-part ratios are comparisons of two measurements of the same type of thing. The measuring unit is the same for each value. (Elementary School Mathematics, p. 275)

Elaboration - Instructional Strategies/Suggestions

A3 Use the students themselves, counters, or other simple models to introduce the concept of ratio as a comparison between two numbers. For example, in a group of 3 boys and 2 girls,

3:2 tells the ratio of boys to girls
3:5 tells the ratio of boys to the total group
2:5 tells the ratio of girls to the total group
2:3 tells the ratio of girls to boys.

Students should read "3:2" as "3 to 2" or "3 __ for every 2 ___."

Exploring ratios can productively take place in everyday situations (e.g., the ratio of water to concentrate to make orange juice is 3:1 or "3 to 1"), or in relation to other topics in mathematics.

For example, students might examine the
- ratio of the perimeter of a square to its side length
- ratio of the length of the diagonal of a square to its side length
- ratio of the corresponding sides of similar shapes.

Ratios and fractions are both comparisons. Sometimes the fraction or ratio, compares a part to a whole. (For example, if $\frac{3}{5}$ of a rectangle is shaded, the ratio of shaded part to the whole is 3:5.) Sometimes the fraction describes a multiplicative relationship. (For example, if there are 4 red circles and 8 white circles, there are $\frac{4}{8}$ as many red circles as white circles and the ratio of red to white is 4:8.)

To further illustrate, consider that there are fourteen boys and eleven girls in a class. The ratio (fraction) of boys to all the students (part-to-whole) is $\frac{14}{25}$. This can be expressed as "fourteen twenty-fifths of the students are boys." The ratio of boys to girls is 14:11 (read "fourteen to eleven"), can be written $\frac{14}{11}$, and describes how many times greater the number of boys is than the number of girls.

Note: While the concept of ratio (in which the units are identical) may be contrasted with the concept of rate (for which each quantity has a different unit (e.g., km/h)), rate is not an outcome at grade six.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A3.1 Ask the students to model two situations which could each be described by the ratio 3:4. Specify that the situations must involve a different total number of items.

A3.2 Ask the student to find the following body ratios, comparing results with others:

wrist size: ankle size
wrist size: neck size
head height: full height

A3.3 Ask the student to select 20 tiles of four different colours so that pairs of colours show the following ratios: 4 to 3, 2:1, $\frac{1}{3}$.

Paper and Pencil

A3.4 Give students the following information and ask them to write/read ratio comparisons and to identify those that can be expressed as fractions.

4 cats 3 goldfish 2 hamsters

Interview

A3.5 Ask the student whether or not he/she believes that the ratio of the population of any city in Canada to the total population of Canada could be 1:2. The student should explain his/her response.

A3.6 Ask: Why might you describe the ratio below as 4:1? as 1:4? Are there other ratios to describe the boys and girls?

B B B B G
B= boy G=girl

Presentation

A3.7 Students might investigate the number of deaths in Canada caused by various diseases and examine the ratios involved. If the students are interested, they might further investigate the funding for research in the study of these various diseases to see if the ratio of money spent is similar to the ratio of death caused by the diseases.

Portfolio

A3.8 Students might create a report on ratios found in the classroom. They could include such ratios as boys:girls; teacher:pupils; desks:students; tables:students; pencils:students; m^2 of classroom:student; etc.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

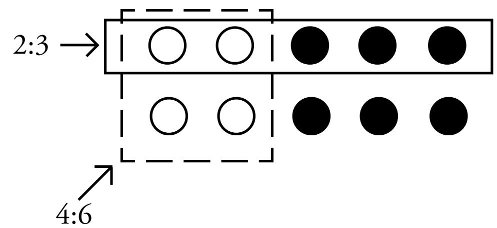
ii) *explore integers, ratios and percents in common, meaningful situations*

SCO: By the end of grade 6, students will be expected to

A4 **demonstrate an understanding of equivalent ratios**

Elaboration - Instructional Strategies/Suggestions

A4 Students should understand why, for example, the ratios 2:3 and 4:6 represent the same relationship, i.e., if for every 2 of one item, there are 3 of another, then, automatically, for every 4 of the first item, there are 6 of the second.



Many students will recognize the similarity between the concept of equivalent ratios and the concept of equivalent fractions. For example, in the diagram above, $\frac{2}{5}$ of the counters in the top row are white, but $\frac{4}{10}$ of the counters in total are white, so $\frac{2}{5} = \frac{4}{10}$. Also, the ratios 2:5 and 4:10 are equivalent, since if 2 of every 5 are white, then 4 of every 10 would also be white.

Students should use the concept of equivalent ratios primarily to make interpretation of situations easier. For example, if in a large bag of marbles, the ratio of blue marbles to the total number of marbles is 4:10 (i.e., 4 out of every 10 marbles are blue), then to answer, “how many blue marbles would you expect in 100 selections?”, it would be useful to use the equivalent ratio 40:100.

- ❒ Have students work in pairs or small groups to discuss equivalent ratios if Sue received 36 votes and Sam received 9 votes.
  - 36:9 or 4:1 (Sue received 4 votes for every 1 vote Sam received.)
  - 9:36 or 1:4 (Sam received 1 vote for every 4 Sue received.)
  - 36:45 or 4:5 (Sue received 4 votes for every 5 votes cast.)
  - 9:45 or 1:5 (Sam received 1 vote for every 5 cast.)
- ❒ Ask students to work in pairs writing situations for which classmates would practise dealing with equivalent ratios.
- ❒ Have students use Cuisenaire rods to find rods in the ratio of 1:2 (e.g., white to red). By writing the values of the rods, equivalent ratios are generated (e.g., 1:2 = 2:4).

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A4.1 Ask the students to set up a display of counters which shows the ratio 5:6. Set up another display to show an equivalent ratio so that the reason for the equivalence is clear.

Paper and Pencil

A4.2 Present the diagram below.

x x x o

x x x o

x x x o

Ask: What equivalent ratios does this diagram show?

A4.3 For each of the following, ask the student to find an equivalent ratio in which one of the terms is 20.

4:6 10:30 3:5 4:5 3:6

A4.4 Ask the student to list all the ratios that are equivalent to 1:2 in which the second term is less than 50.

Interview

A4.5 Tell the student that in a class of 30 students, there are 20 girls. Ask him/her to explain why the ratio of boys to girls is 1:2.

A4.6 Ask the student to explain how a place value chart can be used to generate equivalent ratios.

A4.7 Ask: Why do you get an equivalent ratio by multiplying both terms of a ratio by 3?

A4.8 Ask: Can the ratio 4:5 be equivalent to any other ratio in which the two terms are only one apart? Why or why not?

Presentation

A4.9 758 people were surveyed and 248 responded that they used Brighto detergent. Ask the students (working in pairs) to estimate to find what ratio would best describe the number of people who use Brighto. Have the students make up similar situations for their classmates to solve.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *explore integers, ratios and percents in common, meaningful situations*

SCO: By the end of grade 6, students will be expected to

A5 demonstrate an understanding of the concept of percent as a ratio

Elaboration - Instructional Strategies/Suggestions

A5 Percent should be viewed as a special ratio, a ratio for which the second term is 100.

Students should not be computing with percentages at this time and need not work with percentages greater than 100, but should recognize

• situations in which percent is commonly used

• diagrams that represent various percentages (e.g., 2%, 35%)

• the relationship between the percent and decimal names of ratios (e.g., 48% and 0.48)

• the percent equivalents for common ratios like $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$

• the fact that finding a percentage is the same as finding an equivalent ratio.

❒ Students might explore a variety of geographic or social studies data expressed in terms of percentages. For example:
- about 70% of the earth is water
- about 68% of Canadian households own microwaves
- over 80% of car passengers wear seat belts

❒ Students might cut sheets of paper and/or lengths of string to show 50%, 10%, 25%, etc.

❒ Have the students predict percentages, give their prediction strategies, and then check their predictions. For example, ask them to estimate the percentage of
- red counters when fifty 2-coloured counters are shaken and spilled
- each colour of Bingo chips, if a total of 100 blue, red, and green chips are shown on an overhead for 10 seconds
- a hundredths grid that is shaded in to make a picture.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A5.1 Have the student draw a design in a hundredths grid (or partially cover a flat) and describe the percentage of the grid covered. Ask further questions such as: How many more squares would you have to shade in (or cover) to cover $\frac{1}{2}$ ($\frac{1}{4}$, 0.68, 80%) of the squares?

A5.2 Ask the student to shade in hundredths grids to show particular percentages.

Paper and Pencil

A5.3 Describe a family of 5 which includes 3 children. Have the student indicate the percentage of the family the children represent and the percentage each child represents. Ask the student to describe a family of a different size with the same percentage of children.

Interview

A5.4 Ask: Which is least? most? Explain your answer.

$\frac{1}{20}$ 20% 0.020

A5.5 Ask: What percent of a metre stick is 37 cm?

A5.6 Ask the students to name percents that indicate
- almost all of something
- very little of something
- a little less than half of something

A5.7 Ask: Why might teachers use percentages to indicate marks on tests rather than just indicating the number right? Why is it not necessary to have 100 marks on a test to use percent?

Portfolio

A5.8 Have students create a pencil crayon quilt made of patches of various colours. They can describe the approximate or exact percentages of each colour within the patch and then estimate the percent of the total quilt that is each colour.

A5.9 Ask each student to write a letter to a friend/relative/teacher telling what he/she learned about ratio.

A5.10 Ask students to collect (from newspapers, flyers, magazines) examples of situations in which percent is used and have them make a collage for a class display.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

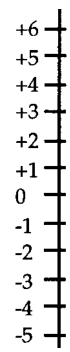
ii) *explore integers, ratios and percents in common, meaningful situations*

SCO: By the end of grade 6, students will be expected to

A6 **demonstrate an understanding of the meaning of a negative integer**

Elaboration - Instructional Strategies/Suggestions

A6 All students will have previously encountered negative integers informally, as in dealing with winter temperatures. To build on this informal understanding, it might be useful to start with a vertical number line (which resembles a thermometer).



The main ideas for students to understand are that

- each negative integer is the mirror image of a positive integer with respect to the 0 mark
- 0 is neither positive nor negative
- negative integers are all less than any positive integer.

Students should be encouraged to read -5 as "negative 5" rather than "minus 5," to minimize confusion with respect to the operation of subtraction.

Other useful contexts for considering negative integers are

- elevators which go both above and below ground, so floors can be given both positive and negative labels
- golf scores above and below par
- money situations involving debits and credits
- height above and below sea level.

Note: Addition and subtraction situations involving integers should only be dealt with informally.

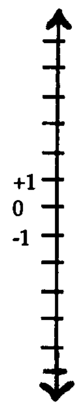
Negative numbers are an important set of numbers. They can and should be explored before they are encountered in algebra. In fact, students almost every day either have some interaction with negative numbers or experience a phenomenon that negative numbers can model. (Elementary School Mathematics, p. 411)

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A6.1 Have the student place the numbers -4, -3, and +2 at the appropriate places on the number line.



Paper and Pencil

A6.2 Ask: How many negative integers are greater than -7?

A6.3 Tell the student that a number is 12 jumps away from its opposite on a number line. Ask: What is the number?

Interview

A6.4 Explain why -4 and +4 are closer to each other than -5 and +5.

A6.5 Ask: In what situations might you encounter negative numbers?

A6.6 Have the student explain ways in which -4 is like +4.

A6.7 Ask: Why is an integer never 11 away from its opposite on a number line?

Portfolio

A6.8 Ask the students to design a simple game for which positive and negative points might be awarded. Have the students play and keep track of their total scores.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) read and write whole numbers and decimals and demonstrate an understanding of place value (to millions and to thousandths)

SCO: By the end of grade 6, students will be expected to

A7 read and write whole numbers in a variety of forms

A8 demonstrate an understanding of the place value system

Elaboration - Instructional Strategies/Suggestions

A7 Students have already had opportunities to work with numbers in the millions. This work may need to be reviewed. Whole numbers should be written out

- fully (e.g., 345 321 400, read as 345 million, 321 thousand, 400)
- rounded using decimal notation (e.g., 345.3 million).

Some students may be interested in exploring exponential notation. For example, one writes 10^2 to mean 10 x 10, and $10^2 = 100$. Also, 10^3 is written to mean 10 x 10 x 10, and $10^3 = 1000$. Therefore, 3422 may be written as $3 \times 10^3 + 4 \times 10^2 + 2 \times 10 + 2$.

Opportunities should be provided in which students record the numerical form of a number that is either spoken or written out in word form. Conversely, students should have the opportunity to both write out and say the word form of a number expressed in its symbolic form. Special attention should be given to numbers for which the numerical expression includes a number of internal zeroes, since these tend to cause the most difficulty for students (e.g., nine hundred two million thirty thousand three).

A8 Students should understand that the place value system follows a pattern such that

- each position represents 10 times as much as the position to its right
- each position represents $\frac{1}{10}$ as much as the position to its left
- positions are grouped in 3s for purposes of reading them, both before and after the decimal point.

All students should be aware that numbers extend to the left at least into the billions group and to the right into the ten thousandths, hundred thousandths and millionths places. If students inquire about these extensions, a discussion is in order.

Although "billions" refers to numbers rarely found in students' experiences, they may be interested in investigating numbers of this magnitude as they relate to national debt, personal fortunes, populations, pieces of trivia (e.g., "How long is a billion millimetres?"), etc.

Base-ten blocks can be used to model larger numbers and these patterns. For example, a long rod of ten large cubes represents 10 000 (to parallel the rod, which represents 10) and a flat of 100 large cubes arranged in a 10 x 10 rectangle represents 100 000 (to parallel the flat, which represents 100).

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A7.1 Ask the student to read these numbers:

105 020 003
64 203 006
920 000 029

A7.2 Ask the student to arrange the cards shown below in at least 3 ways and record the numeral for each.

4	2	million	5	3	thousand	billion

A7.3 Provide students with "pretend" cheques for which the dollar amounts have been listed. Ask them to write out the word form of each amount.

Paper and Pencil

A8.1 Ask the student to use only the digits 2, 3, and 4 to create three numbers with values between 42 million and 43 million. (Each digit can be used more than once.)

A7.4 Ask the student to write the number 3 thousand as millions.

Interview

A8.2 Ask the student to explain the difference(s) in how these numbers are written:

two thousandths
two thousand
twenty thousand
twenty thousandths

Portfolio

A7/8.1 Have each student prepare a "lesson plan" to teach a grade 5 student what a billion means. They may wish to actually do the teaching and report on the experience.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) order whole numbers, fractions and decimals and represent them in multiple ways

SCO: By the end of grade 6, students will be expected to

A9 relate fractional and decimal forms of numbers

To connect the two numeration systems, fractions and decimals, students should make concept-oriented translations from one system to another . . . The calculator can also play a significant role in decimal concept development. (Elementary School Mathematics, p. 262)

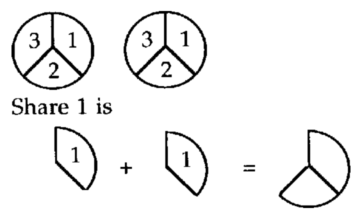
Elaboration - Instructional Strategies/Suggestions

A9 A few students will already know the decimal equivalents of some simple fractions (e.g., $\frac{1}{2}$ = 0.5, $\frac{1}{4}$ = 0.25, $\frac{1}{5}$ = 0.2) as well as any fraction with a denominator of 10, 100, or 1000. For example, to locate 8.75 on a number line, many students think of 0.75 as being three quarters of the way from 8 to 9.

Many grade 6 students, however, believe that the only fractions which can be described by decimals are those with denominators which are a power of 10 or a factor of a power of 10.

By building on the connection between fractions and division, students should be able to represent any fraction in decimal form, using the calculator as an aid.

For example, $\frac{2}{3}$ means 2 wholes shared among 3, so $\frac{2}{3}$ = 2 ÷ 3. The calculator display would show 0.6666666. Physically, the demonstration could be 2 pizzas shared by 3 people.



or $\frac{2}{3}$ of one whole

Students should recognize when a decimal repeats, but need not deal with the symbolism for handling repeating decimals at this time.

Base ten blocks can be used to explain the decimal equivalents to fractions, even when these decimals repeat. For example, 1 ÷ 3 could be modelled as follows: The large cube represents one whole. It must be shared by 3 people. Trade the cube for 10 flats (10 tenths). Each of the 3 people gets 3 tenths, so the decimal begins with 0.3. Trade the leftover thenth for 10 rods (10 hundredths). Each of the 3 people gets 3 hundredths, so the next digit in the decimal is 3(i.e., the decimal begins with 0.33). Continue the process.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A9.1 Have the student use a calculator to find the decimal forms for $\frac{4}{6}$ and $\frac{1}{6}$ and then subtract. Ask: How might you have predicted the difference?

Paper and Pencil

A9.2 Ask students to explain how they know that the decimal form of $\frac{4}{15}$ cannot begin with 0.6.

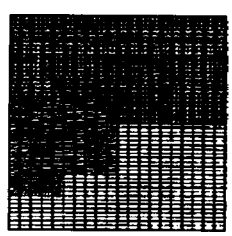
A9.3 Have the student describe a fraction which is a bit less than 0.4 and to justify the selection. Ask: Can you name another that is between these two?

Interview

A9.4 Ask the student to identify how the decimal forms of $\frac{1}{8}$ and $\frac{1}{4}$ are related and explain what this tells about fractions. Have the student provide another pair of fractions with the same relationship.

A9.5 Ask how the diagram below shows that

$0.625 = \frac{5}{8} = \frac{1}{2}\left(\text{i.e., }\frac{4}{8}\right) + \frac{1}{2}$ **of** $\frac{1}{4}\left(\text{i.e., }\frac{1}{8}\right)$



Ask: What fraction/decimal equivalence would be shown if twice as much were shaded?

A9.6 Ask: How does knowing that $\frac{1}{4} = 0.25$ help you find the decimal form of $\frac{3}{4}$? of $\frac{5}{4}$?

Portfolio

A9.7 Ask the students to use calculators to find the decimal forms for a group of fractions and make as many observations as possible about the decimals obtained. A sample group is

$\frac{1}{8} \quad \frac{2}{8} \quad \frac{3}{8} \quad \frac{4}{8} \quad \frac{5}{8} \quad \frac{6}{8} \quad \frac{7}{8} \quad \frac{8}{8}$

A9.8 Ask students to respond in writing to the following question: How are fractions and decimals alike and how do they differ?

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) *apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals*

SCO: By the end of grade 6, students will be expected to

A10 determine factors and common factors

Number theory is the study of relationships found among the natural numbers. At the elementary level, number theory includes the concepts of prime number, odd and even numbers, and the related notions of factor, multiple, and divisibility. (Elementary School Mathematics, p. 404)

Elaboration - Instructional Strategies/Suggestions

A10 Most students should have little difficulty with the concept of one number being a factor of another. These can be found by dividing by smaller numbers and looking for remainders of zero. This concept extends directly from previous work in multiplication and division. The concept of common factors, however, will be new to most students. It may be useful to ensure that students understand that "common" is used in the sense of "joint", rather than "ordinary"; this is a typical misunderstanding on the part of students.

To introduce the concept of common factors, have students compare the factors of 2 numbers (e.g., 16 and 18) and note any factors which are factors of both numbers, i.e., common factors.

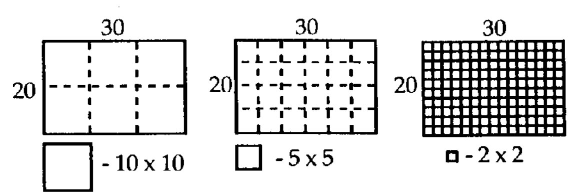
16- 1,2,4,8,16

18- 1,2,3,6,9,18

Upon examining these lists, the students will see that 1 and 2 are the only factors common to both 16 and 18.

Students should soon conclude that 1 is always a common factor of any two numbers.

Another way to find common factors of a pair of numbers is to begin by creating a rectangle, using the two numbers as the length and width. A common factor is the side length of any square which can be used to tile (or cover) the rectangle exactly. For example, for a 20 x 30 rectangle, 10, 2 and 5 are common factors since the following tilings are possible:



Another interesting approach to factors and common factors involves putting different coloured cubes on a hundredths chart as one skip counts by different amounts. For example, skip count by 2s, putting a red cube on each number; skip count by 3s, putting a blue cube on each number; skip count by 5s, putting a yellow cube on each number. Ask: What numbers have both a red and blue cube? All three cubes? What does this mean about their factors?

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A10.1 Ask the student to draw one or more rectangles to show that 8 is a common factor of 16 and 24.

Paper and Pencil

A10.2 Have students create a number which has 4, 7, 28 and 12 as factors. Ask them if there is a smaller number which will meet the conditions and to explain why or why not.

A10.3 Ask the student to find a number with 6 factors.

A10.4 Ask: If 3 and 4 are common factors of a pair of numbers, what might the numbers be? List 3 possibilities.

Interview

A10.5 Tell the student that the common factors of a particular pair of numbers include 10. Ask him/her to explain how this guarantees that 2 and 5 are also common factors.

A10.6 Ask: Why is it not possible for a common factor of 38 and 90 to be greater than 20?

Portfolio

A10.7 Have the students design a test they think could be used to determine students' understanding of common factors.

Suggested Resources

GCO A: Students will demonstrate number sense and apply number theory concepts.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals

SCO: By the end of grade 6, students will be expected to

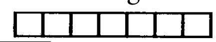
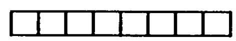
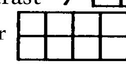
A11 distinguish between prime and composite numbers

Prime numbers can be viewed as fundamental building blocks of the other natural numbers. (Elementary School Mathematics, p. 405)

Elaboration - Instructional Strategies/Suggestions

A11 A prime number can be defined in two ways:

• a number which has only 2 factors, 1 and itself, (e.g., 29 (with factors of 1 and 29) is prime and 28 (with factors of 1, 2, 4, 7, 14 and 28) is not)

• a number for which that same number of squares can be arranged in a rectangle in only one way. For example, contrast 7 , which is prime, with 8  or , which is not.

❐ Have the class use up to 36 coloured tiles to explore the different rectangles that can be made for each number through 36. Pairs of students may be assigned 2 or 3 numbers each (e.g., one pair might explore the numbers 21, 22, and 23). Have the numbers (1-36) written horizontally across the front of the room or on the board. Ask each pair to cut out of squared paper all the rectangles that can be made for each of their numbers and to display them under the numbers. When the display is complete, only the prime numbers (except for 1) will have but one rectangle.

Students should recognize that the concept of prime numbers applies only to whole numbers. Although students should have strategies for determining whether or not a number is prime, it is not essential for them to be able to quickly recognize whether or not a number is prime. Exceptions would be that students should be able to identify i) even numbers (other than 2) and ii) numbers ending in 5 or 0 (other than 5) as non-primes right away.

Many students do not realize that 1 is not a prime number. There are many explanations for this, but it is sufficient for students to realize that 1 has only 1 factor, whereas prime numbers have 2 factors.

Students should be introduced to the term "composite" (for non-prime numbers other than one) and encouraged to accurately use language such as multiple, common multiple, factor, common factor, prime and composite. As well, encourage students to explore numbers and become familiar with their composition.

❐ Ask students to write about the number 36, describing it using "factor language" in as many ways as they can. Answers might include: 36 is a composite number with 9 factors (1, 2, 3, 4, 6, 9, 12, 18, 36). Two of its factors are prime numbers. Five rectangles can be made with 36 tiles, one of which is a square.

GCO A: Students will demonstrate number sense and apply number theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A11.1 Ask the student to draw a diagram to show why 10 is not a prime number.

A11.2 Ask students to express even numbers greater than 2 in terms of sums of prime numbers. (Sample answers include 4 = 2 + 2, 6 = 3 + 3, 8 = 3 + 5, ..., 48 = 43 + 5, 50 = 47 + 3, ...)

A11.3 Have students identify the prime numbers to 100 by exploring the sieve of Eratosthenes, using a hundreds chart. Before they start, have the students review the concept that "1" is neither prime nor composite, but in a separate category by itself. Have the students begin by circling the first prime number (2), and then cross out every second number (the multiples of 2). These must be composites. Have them circle the next prime number (3) and cross out every third number thereafter (the multiples of 3, some of which have already been crossed off). The students then proceed to 5, 7, 11, etc. At the end of the process the circled numerals will be the prime numbers up to 100.

Paper and Pencil

A11.4 Ask: Are there more prime numbers between 50 and 60 or between 60 and 70?

A11.5 Ask the student to find 3 pairs of prime numbers that differ by two (e.g., 5 and 7).

Interview

A11.6 Ask: Why is it easy to know that certain large numbers (e.g., 4 283 495) are not prime, even without factoring them?

A11.7 Tell the student that the numbers 2 and 3 are consecutive numbers, both of which are prime numbers. Ask: Why can there be no other examples of consecutive prime numbers?

Portfolio

A11.8 Have students use a computer or calculator to help them determine the prime numbers up to 100. Ask them to prepare a report describing as many features of their list as they can.

Suggested Resources

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 6, students will be expected to

B1 **compute products of whole numbers and decimals**

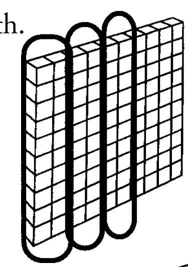
Elaboration - Instructional Strategies/Suggestions

B1 Students should be able to compute products of whole numbers using an algorithm. They should also know when it is appropriate to use a paper/pencil algorithm, a mental procedure, or a calculator for the operation. Students require practice estimating products and a knowledge of the multiplication facts is essential.

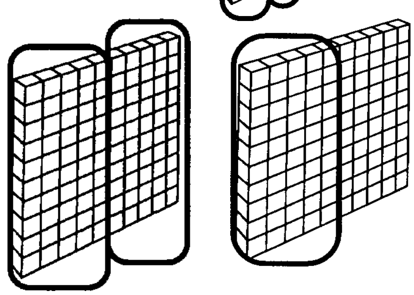
They should continue to use base ten blocks and money models to make sense of the multiplication algorithm involving decimals.

It is not enough to tell students to multiply, estimate and decide where to put the decimal point; they need to see why the procedure works.

Base ten materials are effective models for calculations involving whole numbers and decimals. If a flat represents one unit, each column is a tenth.



3 x 0.2 = 6 tenths (0.6)

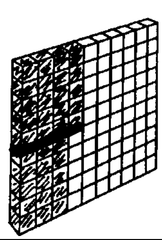


3 x 0.5 = 15 tenths or 1.5

Students might also rewrite the multiplication. For example,

$$\begin{array}{r} 5.4 \\ \times\ 2 \\ \hline \end{array} \quad \text{as} \quad \begin{array}{r} 54 \text{ tenths} \\ \times\ 2 \qquad \\ \hline 108 \text{ tenths or } 10.8. \end{array}$$

Similarly, base ten materials can be used to represent the multiplication of hundredths by a whole number. If the flat represents one, each of the hundred squares would represent 0.01.



0.05 x 7 = 7 times 5 hundredths

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B1.1 Ask the student to draw or build a model to illustrate 4 x 3.453.

B1.2 Have the student describe how to calculate 3 x 4.23 using a money model.

Paper and Pencil

B1.3 Ask the student to determine how much more five cans of juice cost at $1.29 each than six cans at $0.99 each.

B1.4 Have the students identify a decimal which when multiplied by 500 will produce a result of 200. (This could be checked using a calculator.)

B1.5 Ask the students to find the missing digits:

$$\begin{array}{r} 5.\square\,3 \\ \times\ \square \\ \hline 3\square.58 \end{array}$$

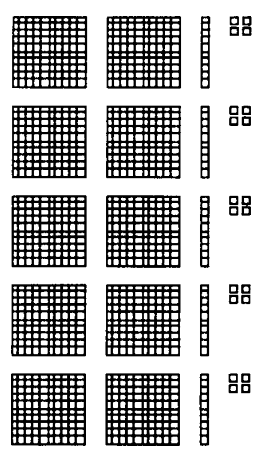
Ask them to "think about their thinking" and be prepared to explain what steps they took, and why, in finding the missing digits.

Interview

B1.6 Ask the student to respond to the following: Jane said 3.45 x 4 must be 1.380. There is only one digit before the decimal place in 3.45, so there must be one digit before the decimal place in the product.

B1.7 Ask the student if the result of multiplying a decimal by a whole number can be a whole number.

B1.8 Present the arrangement of blocks shown at right. Ask: What multiplication is shown if the flat is assumed to represent one whole?



Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 6, students will be expected to

B2 model and calculate the products of two decimal numbers

Elaboration - Instructional Strategies/Suggestions

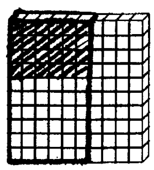
B2 Patterns can be used to help students understand the placement of the decimal in the product of two decimal amounts. For example, consider:

$$\begin{array}{r} 42 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 4.2 \\ \underline{\times\ 4} \end{array} \qquad \begin{array}{r} 4.2 \\ \underline{\times\ 0.4} \end{array}$$

Since each product may be rewritten as follows, and each is one tenth of the previous:

$$\begin{array}{r} 42 \\ \underline{\times\ 4} \\ 168 \end{array} \qquad \begin{array}{r} 42 \text{ tenths} \\ \underline{\times\ 4} \\ 168 \text{ tenths} \end{array} \qquad \begin{array}{r} 4.2 \\ \underline{\times\ \ 4 \text{ tenths}} \\ 16.8 \text{ tenths} \end{array}$$

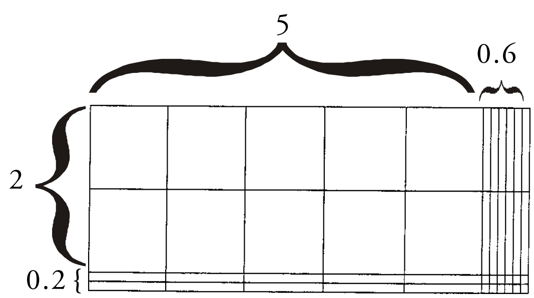
An area approach can also be used to explain the placement of the decimal. For example, 0.4 x 0.6 is $\frac{4}{10}$ of $\frac{6}{10}$.



Each rod is 0.1 of a flat. Six rods are placed on top of the flat. Four cubes are placed on top of each of the six rods. This shows that $\frac{4}{10}$ of $\frac{6}{10}$ is $\frac{24}{100}$ or 0.24.

An example of another variation of an area model follows: To find 2.2 x 5.6, consider 2.2 cm and 5.6 cm to be the dimensions of a rectangle.

Area = 10 cm^2+ 1.2 cm^2 + 1.0 cm^2 + 0. 12 cm^2 = 12. 32 cm^2



Students should interpret the symbolism in meaningful ways. For example, 0.6 x 34.5 is $\frac{6}{10}$ of 34.5, so the product is more than half of 34.5, but not much more. Rather than simply providing a rule about "counting decimal places," a rule which students often mix up, it is better if students understand why and how the whole number calculation can be used and adjusted for different decimal multipliers.

Students must be encouraged to estimate products before calculating. For example, one might round each of the decimal numbers 2.86 x 8.153 for an estimate of 24 (3 x 8). When estimation is an automatic response students will, when faced with a calculation, not depend on the "counting back decimal places," rule.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B2.1 Tell the students that two decimals are multiplied. The product is 0.48. Ask: What might they have been? Give two other pairs.

B2.2 Ask students to work in pairs sharing strategies for estimating and calculating in situations such as

- 6.15 m of material at $4.95 a metre
- area of a rectangular plot of land 24.78 m x 9.2 m
- 0.5 of a length of rope 20.6 m long.

Have them write similar questions to share with their classmates.

Interview

B2.3 Ask: Why is the answer to 0.6 x 0.4 a whole number of hundredths?

B2.4 Ask: Is it possible to multiply 2 decimals and get the same result as if you had multiplied 2 whole numbers?

B2.5 Ask: When you multiply two decimals, how does the result compare in size to the numbers you multiplied?

Presentation

B2.6 Ask the students to find their own heights in metres. Have them research some animal sizes and prepare a report:

An animal that is about 0.1 of my height is ______.
An animal that is about 0.2 of my height is ______.
Etc.

Portfolio

B2.7 Present the student with the following problem: A multiplication involving two decimal amounts can be modelled using exactly 13 base ten pieces. Determine what numbers might have been multiplied.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

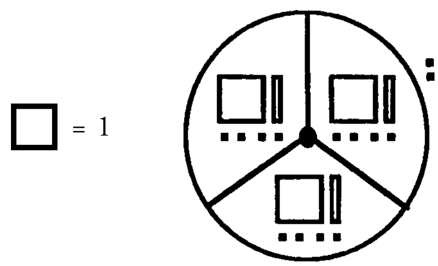
i) *model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures*

SCO: By the end of grade 6, students will be expected to

B3 compute quotients of whole numbers and decimals

Elaboration - Instructional Strategies/Suggestions

B3 Students should continue to use manipulative materials to model division of a decimal by a whole number.
For example: 3.44 ÷ 3



Common contexts in which this type of calculation would emerge, and to which the students would relate, are the sharing of money and unit pricing. Other possible contexts are sharing metres of ribbons, litres of juice or kilograms of meat. Students should be expected to estimate quotients. For example, 4.28 ÷ 3 will be a bit more than 1, but 4.28 ÷ 5 will be close to $\frac{4}{5}$ or 0.8. Some may think of 4.28 ÷ 5 as 428 hundredths divided by 5 (about 85 hundredths), or as 42.8 tenths ÷ 5 (about 8 tenths).

Students should understand that the "remainder" when they perform the division of a decimal number is different than with whole numbers. For example, when dividing 3.4 by 3, the remainder "1" at the end of the algorithm is really 0.01, not 1. They should recognize that for more accuracy, they could continue the process.

$$\begin{array}{r} 1.13 \\ 3\overline{)3.40} \\ \underline{-33} \\ 10 \\ \underline{-\ 9} \\ 1 \end{array}$$

Many students will be ready to use short division when dividing by a single digit number. For example,

$$\begin{array}{r} 0.62 \\ 4\overline{)2.48} \end{array} \quad \text{or} \quad \begin{array}{r} 1.42 \\ 3\overline{)4.^{1}26} \end{array}$$

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

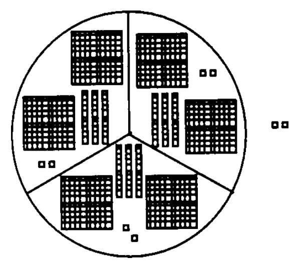
B3.1 Have the student draw a model to show how to find 5.28 ÷ 4.

Paper and Pencil

B3.2 Tell the students that a can of pop holds 0.355 L. Ask them to determine the number of cans of pop required to total 5 L.

Interview

B3.3 Present the display shown at right and ask what division is being modelled, assuming that a flat represents 1.



B3.4 Ask: How are the results of 423 ÷ 3 and 42.3 ÷ 3 related?

B3.5 Ask: Why is the remainder not really 1 when you divide 2.1 by 4?

B3.6 Ask the student to complete the calculation at right. Observe how the remainder is handled.

$$\begin{array}{r} 8. \\ 4\,\overline{)\,34.6} \\ \underline{-\,32} \\ 2 \end{array}$$

Ask him/her to create a problem for the computation and to explain what to do with the remainder.

Presentation

B3.7 Ask the student to use a store flyer to find items that are sold in twos, threes or other groupings, and provide unit prices for the various items. Ideally, these unit prices will be compared to prices of the same or similar products from another store's flyer.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures

SCO: By the end of grade 6, students will be expected to

B4 model and calculate the quotients of two decimals

Elaboration - Instructional Strategies/Suggestions

B4 As with multiplication, students should relate the division of a decimal by a decimal to the corresponding division of a decimal by a whole number. Approaches include

• considering the unit - For example, 43.2 ÷ 0.5 = 432 tenths ÷ 5 tenths. How many 5 tenths are in 432 tenths? (432 ÷ 5) Similarly, 43.25 ÷ 0.5 = 432.5 tenths ÷ 5 tenths. How many 5 tenths are in 432.5 tenths? (432.5 ÷ 5)

Students may notice that since 43.2 ÷ 0.5 = 432 ÷ 5, they could physically change the problem before solving it, i.e.,

$$0.5\overline{)43.2} \longrightarrow 5\overline{)432}$$

• money model - Students might find it productive to use a money model. For example, 43.2 ÷ 0.4 might be interpreted as determining the number of sets of 4 dimes in $43.20. Since 10 sets of 4 dimes each is $4, 100 sets of 4 dimes is $40. Another eight sets (8 x 40¢) would be needed to make the extra $3.20. Therefore, 43.2 ÷ 0.4 is 108.

❐ To practise division, have students compete in paper airplane races. Each student flies his/her plane 3 times, measuring each distance travelled in metres. The score is determined for each by finding the average distance.

For example:	2.43 m (or 2 m, 43 cm)
	1.89 m
	2.25 m
Average distance	= 6.57 m ÷ 3
	= 2.19 m

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B4.1 Ask students to use a number line or base ten blocks to show why 4.2 ÷ 0.2 is the same as 42 ÷ 2.

B4.2 Ask students to use a metre stick to explain why 3.4 ÷ 0.2 is the same as 34 ÷ 2 (3.4 dm ÷ 0.2 dm = 34 cm ÷ 2 cm).

Paper and Pencil

B4.3 Ask students to fill in the missing digits. The boxes may represent different amounts.

$4.\square \div 0.\square = 14.\square$

B4.4 Have students describe the situation by referring to coins.

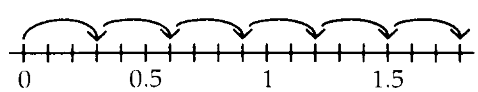
2.40 ÷ 0.1 = 24

Interview

B4.5 Ask: Which question has an answer different from the others? How could you know in advance of complete calculations?

42.5 ÷ 0.5 425 ÷5 85 ÷ 1 0.425 ÷ 0.05

B4.6 Ask the student to explain how the diagram shows that 1.8 ÷ 0.3 = 6



B4.7 Why might someone find it easier to divide 8.8 by 0.2 than 1.1 by 0.3?

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) model problem situations involving the addition and subtraction of simple fractions

SCO: By the end of grade 6, students will be expected to

B5 add and subtract simple fractions using models

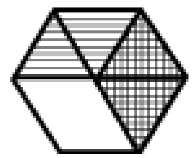
When area models are used for addition and subtraction, common denominators are frequently not involved at all. (<u>Elementary School Mathematics</u>, p. 244)

Elaboration - Instructional Strategies/Suggestions

B5 It is important to continue to provide concrete experiences to help students build an understanding of simple fraction operations. Models to use include

- pattern blocks

$\frac{4}{6} - \frac{1}{3}$

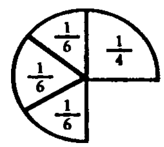


$\frac{2}{6}$ or $\frac{1}{3}$ remaining

(Let the yellow hexagon represent the whole; each green triangle will be $\frac{1}{6}$ and the blue rhombus will be $\frac{1}{3}$.)

- fraction circles

$\frac{1}{4} + \frac{3}{6}$



$= \frac{3}{4}$

Students have no difficulty adding or subtracting fractions with like denominators. For example,

$$\frac{3}{5} + \frac{2}{5} = 3 \text{ fifths} + 2 \text{ fifths}$$
$$= 5 \text{ fifths}$$
$$= \frac{5}{5}$$

Students should realize, however, (as the first examples clearly show) that fractions with unlike denominators can be added and subtracted.

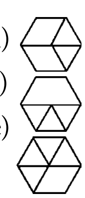
❐ Students might enjoy finding many sums for 1 by covering the hexagon pattern block in as many different ways as possible.

For example:

$1 = \frac{2}{3} + \frac{2}{6}$ (2 blue + 2 green)

$= \frac{1}{2} + \frac{3}{6}$ (1 red + 3 yellow)

$= \frac{4}{6} + \frac{1}{3}$ (4 green + 1 blue)



Note: The intent at this level is to perform operations using models, not algorithms.

Some students may be ready to explore the repeated addition model of multiplication via an activity such as: Ask the student to show $\frac{1}{4}$. Show 4 one-quarters. How might that be written? How much is 4 one-quarters? Show six one-quarters. What can you say about this? How might this be written?

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

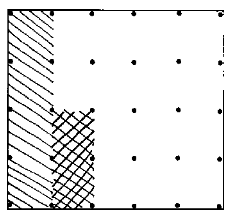
Worthwhile Tasks for Instruction and/or Assessment

Performance

B5.1 Ask the student to create a pattern block model for $\frac{2}{3} + \frac{1}{2}$.

Paper and Pencil

B5.2 Ask the student to state the fraction addition modelled on dot paper, assuming that the portion shown is the whole.



B5.3 Have the student explain how the diagram at right shows that $\frac{4}{12} + \frac{5}{12} + \frac{3}{12} = 1$.

R	R	R	R
B	B	B	B
B	Y	Y	Y

R=red B=blue Y=yellow

B5.4 Have the student list a group of 3 fractions that add to $\frac{1}{2}$.

Interview

B5.5 Ask the student to explain why $\frac{2}{3} + \frac{4}{5}$ has to be greater than 1.

B5.6 Tell the student that Jane said: $\frac{4}{8} + \frac{2}{8} = \frac{6}{16}$. Ask: Is she right or wrong? Why?

B5.7 Tell the student that you have subtracted two fractions and the result is less than $\frac{1}{2}$. Ask: Can both fractions be greater than $\frac{1}{2}$? less than $\frac{1}{2}$?

Portfolio

B5.8 Using triangular grid paper, have students copy the models they create to show the different ways that a hexagon can be covered. For each model, the student is to write the appropriate fraction addition that goes with it.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

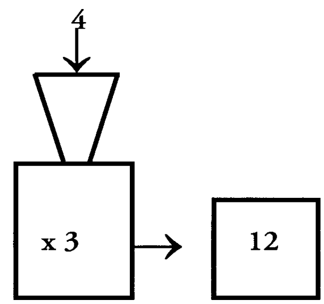
iii) explore algebraic situations informally

SCO: By the end of grade 6, students will be expected to

B6 demonstrate an understanding of the function nature of input-output situations

Elaboration - Instructional Strategies/Suggestions

B6 In introducing the concept of a function, the "function machine" works well. Show students both how inputs are acted upon by functions and the resulting outputs.



Grade six students can try to decide what the function machine does if they are given a series of inputs and outputs. For example, if 4 → 12, 6 → 16, and 10 → 24, what did the machine do? (Answer: Doubled and added 4)

Number tricks provide an enjoyable context for students to practise the concept of function, where a given input results in a specified output.

For example, consider this "trick:"	Explanation
Choose a number.	□
Add 8.	□ + 8
Multiply by 2.	2 □ + 16
Subtract 14.	2 □ + 2
Divide by 2.	□ + 1
Subtract 1.	□

Students should recognize that the result depends on the original number chosen. After discussing the "trick" and seeing why it works, students will enjoy making up their own, thereby further exploring the concept of function.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B6.1 Tell the students that you intend to create a number "trick" in which you perform a sequence of calculations. In each case, ask what one calculation could be substituted for the sequence of calculations.

(i) add 5, add 20, subtract 10
(ii) multiply by 5, multiply by 20, divide by 10
(iii) add 4, double, add 6, divide by 2

Interview

B6.2 Tell the student that you put 25 into the function machine and 77 came out. Ask for four possible functions that would do this.

B6.3 Ask pairs of students to each provide an explanation as to why the following number "trick" works:

Pick a number between 0 and 10
Add 7 to the number.
Double the new number.
Add 11.
Subtract 25
Divide by 2.

Presentation

B6.4 Ask the student to prepare a set of five "number tricks," the trickier the better. Each trick should be based on doing various computations with an initial number that the other person chooses. Students should prepare a "manual" of the tricks with explanations for the user.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) *apply computational facts and procedures (algorithms) in a wide variety of problem situations involve whole numbers and decimals*

SCO: By the end of grade 6, students will be expected to

B7 solve and create relevant addition, subtraction, multiplication and division problems involving whole numbers

Elaboration - Instructional Strategies/Suggestions

B7 Students should continue to use the four operations to solve mathematical and real-world problems. They should also have the opportunity to create problems for others to solve.

Students should also be encouraged to estimate answers to test for reasonableness and whenever the calculation can be done mentally, children should do so.

There are many interesting sources of data, both on the Internet and in reference books. Some of the most useful print resources include the Canadian Global Almanac, the Guiness Book of World Records and the Top Ten of Everything.

Internet searches can be done for data relating to any topic of student interest, such as sports, populations or food.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B7.1 Provide the student with appropriate data and ask him/her to determine how much farther away Jupiter is from Earth than from Mars. This might be reported as a ratio or in terms of absolute distance.

B7.2 Ask the student to determine all possible sums using any combination of the numbers 389, 243, 301, 332 and 91.

Interview

B7.3 Ask: When might one multiply to find the perimeter of a shape?

Presentation

B7/8.1 Have the students pretend to work in a post office. Institute a "rule" that states that the total of the length, height, and width of a package has to be less than 100 cm to mail it. The students determine whether or not various packages will work. In addition, ask them to create a list of package sizes that would "just make it."

B7.4 Have students plan a trip with various stops. Using distance charts or maps, they find the distances between the stops. They then determine the length of the entire round trip. Alternatively, students could plan a trip of a given length (between 1200 and 1500 km, for example). An activity such as this could extend to calculations which could include total cost of trip (gas, lodging, meals, etc.).

Portfolio

B7/8.2 Ask the students to create a variety of "outlandish" problems involving lengths. For example:

- How many toothbrushes are required to make a line that is 2 km long?
- How many pennies must be lined up to make a kilometre?

B7.5 Have students create problems based on the information provided.

Urban/Rural Population 1991

	RURAL	URBAN
Canada	6 389 724	20 909 135
NF	264 023	304 451
NS	418 434	481 508
PE	77 952	51 813
NB	378 686	345 214

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) apply computational facts and procedures (algorithms) in a wide variety of problem situations involve whole numbers and decimals

SCO: By the end of grade 6, students will be expected to

B8 solve and create relevant addition, subtraction, multiplication and division problems involving decimals

Elaboration - Instructional Strategies/Suggestions

B8 Provide problems involving decimals. Contexts might include money and measurement problems.

Demographic data can be a good source of material for problem solving and creation. Provide students with information about the populations of various places in Canada. Have them create authentic problems based on data provided in decimal form. For example:

- How many people in B.C. are living in urban areas?
- How many more people live in Edmonton than Saskatoon?
- How many more people live in Halifax than Saint John?
- How many people/km^2 are found in Canada's largest cities?
- How would you graph the populations of Canada's capital cities?

Some problems can be strictly mathematical challenges. For example, students might be asked to find as many combinations of digits as they can to fill in the templates below so that the sum or difference is 10.0.

$$\begin{array}{r} \square.\square \\ +\ \square.\square \\ \hline 1\ 0.0 \end{array} \qquad \begin{array}{r} \square\square.\square \\ -\ \square.\square \\ \hline 1\ 0.0 \end{array}$$

Pieces of children's literature often provide interesting contexts for problems. A good example is <u>Code Red at the Supermall</u> by Eric Wilson. The picture book, <u>Counting on Frank</u> by Rod Clement, is a favourite of students, as well.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B8.1 Have the student draw a quadrilateral with a perimeter of 16.3 cm.

Paper and Pencil

B8.2 Ask the student to create a measurement problem involving the addition or subtraction of hundredths of metres.

B8.3 Ask the student to create addition, subtraction, multiplication and division word problems, each with an answer of 4.2.

B8.4 Ask the student to describe a situation involving both addition and multiplication of decimals.

Interview

B8.5 Tell the student that you have bought 1.362 kg of cheese and 0.485 kg of ham. Ask him/her to compute the total amount of food mass and the difference between the amounts. Also ask the student to estimate how many ham and cheese sandwiches could be prepared with these amounts.

B8.6 Ask the student to create and solve a multiplication problem involving a particular item (e.g., bananas) using decimals (not in a money context).

Portfolio

B7/8.3 Have the students each create a "Problem Sheet of the Week" based on a predetermined topic or holiday. Select one or more to assign to the class for homework. Be sure to credit the author. They may wish to publish a class problem booklet.

B8.7 Have students create problems involving this data:

Population by Official Language 1991 (%)

	English	French	Bilingual
NF	96.5	0.04	3.3
NS	91.1	0.02	8.6
PE	89.6	0.2	10.1
NB	57.9	12.5	29.5

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers and decimals*

SCO: By the end of grade 6, students will be expected to

B9 **estimate products and quotients involving whole numbers only, whole numbers and decimals, and decimals only**

Number sense is a critical component of our students' education. Encouraging students to estimate and check answers as an integral part of any numerical exercise, discussing common measurement situations with them, and asking them to justify their mathematical choices will help students develop this crucial ability. (Curriculum and Evaluation Standards, Addenda Series, Sixth-Grade Book, p.10)

Elaboration - Instructional Strategies/Suggestions

B9 Estimation should precede all calculations so results can be tested for reasonableness.

When considering multiplication by a decimal, students should recognize that, for example, 0.8 of something will be almost that amount, but not quite, and 2.4 multiplied by an amount will be double the amount with almost another half of it added on.

It is important for students to realize estimation is a useful skill in their lives; therefore, regular emphasis on real-life contexts should be provided. On-going practice in computational estimation is a key to developing understanding of number and number operations and increasing mental process skills. Although rounding has often been the only estimation strategy taught, there are others (many of which provide a more accurate answer) that should be part of a student's repertoire.

- Rounding:

  - Multiplication: 688 x 79 is easily rounded to 700 x 80 to give 56 000, which is a good estimate. Consider, however, 653 x 45. If one were to round according to the "rounding rule," the estimate (35 000) would not be close to the actual answer (29 385). Multiplying 700 by 40 would give a much more accurate estimate (28 000) and 600 x 50 provides an even closer one (30 000). It is important that students explore possible rounding combinations with their calculators and discuss the reasons for the variances.

  - Division: 789.6 ÷ 89 Think: "90 multiplied by what number would give an answer close to 800?"

- Front-end:

  - Multiplication: 6.1 x 23.4 might be considered to be 6 x 20 (120) plus 6 x 3 (18) plus a little more for an estimate of 140, or 6 x 25 = 150.

  - Division: Pencil and paper division involves front-end estimation. The first step is to determine in which column the first digit of the quotient belongs. For $8\overline{)424.53}$, the first digit is a 5 and is placed over the 2 of the 42 tens. The front-end estimate is, therefore, 5 tens or 50.

❒ Have the students estimate each of the following and tell which of their estimates is closer and how they know:
9.7 kg of beef at $4.59/kg,
4.38 kg of fish at $12.59/kg.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B9.1 Have the student compute the approximate number of hours in 10 000 seconds or 100 000 seconds.

B9.2 Tell the student that you have multiplied a decimal by a whole number and the estimate is 5.5. Ask: What might the numbers be?

B9.3 Provide the student with a supermarket checkout slip on which the total has been removed. Have the student estimate the total amount.

B9.4 Tell the student that it takes about 0.08 kg of beef to make a hamburger patty. Sue checks the label on the package and finds she has 2.456 kg of beef. Ask: About how many patties can she make?

Interview

B9.5 Tell the student that two numbers have been multiplied together. The result is about 40 000. Ask: What might the numbers be?

B9.6 Ask: Which is the best estimate for 37 x 94 and why?
30 x 90 40 x 100 35 x 95 40 x 95 40 x 90

B9.7 Ask the student why someone might estimate 516 x 0.48 by taking half of 500.

B9.8 Tell the student that the cashier told Samantha that her total for 3 kg of grapes at $3.39/kg was $11.97. Ask: How did Samantha know right away that the cashier had made a mistake?

B9.9 Tell the student that Sandy's marks for English, Math, Science and French were all about the same. When she added them up, she had a total of 319. Ask the student to estimate Sandy's average mark.

B9.10 Ask the student for an estimate of the total cost of 25 pens at $0.79 each. Ask what estimating strategy he/she used and if there is another way to easily estimate the answer.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

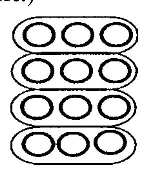
SCO: By the end of grade 6, students will be expected to

B10 divide numbers by 0.1, 0.01 and 0.001 mentally

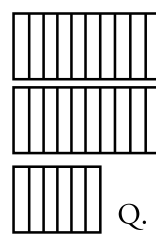
Elaboration - Instructional Strategies/Suggestions

B10 By the end of grade 5 students have mentally multiplied and divided numbers by 10, 100 and 1000, and multiplied numbers mentally by 0.1, 0.01 and 0.001. Students will now add division by 0.1, 0.01, and 0.001 to their mental repertoires.

Since students generally expect the division process to result in a quotient which is smaller than the dividend, it is important that students understand why that is not the case here. One way to illustrate this is by analogy. For example, students will understand that one way to illustrate 12 ÷ 3 is to consider how many 3s (i.e., groups of 3) there are in 12. Obviously, there are 4. (See diagram at left below.) Similarly, to illustrate 2.6 ÷ 0.1, consider how many 0.1s (i.e., one-tenths) there are in 2.6. Clearly, there are 10 in each unit and another 6 in 0.6, for a total of 26 one-tenths. (See diagram at right.)



Q. How many 3s in 12?
A. 4



Q. How many one-tenths in two and six-tenths?
A. 26

Ultimately, students would see that dividing by 0.1 (one-tenth) increases the number of parts (and, hence, the answer) by a factor of 10 (ten times). Similarly, they should understand that dividing by 0.01 (one-hundredth) results in increasing the answer by a factor of 100, and dividing by 0.001 (one-thousandth) increases the answer by a factor of 1000.

Students should be able to describe these changes in terms of place value. For example, they should be able to explain that, when dividing by 0.01, each hundredth becomes a unit, each tenth becomes a ten, each unit becomes a hundred, each ten becomes a thousand, etc.

❐ Include dividing by 0.1, 0.01 and 0.001 as part of regular mental math activities. As students become comfortable with questions of this type, include a mix of questions involving multiplication and division by 1000, 100, 10, 0.1, 0.01 and 0.001.

In fact, there are relatively few workable divisions that can be done mentally compared with the other three operations . . . That does not mean that division is less important as a mental computation skill. However, mental division is more of a tool for estimation. (Elementary School Mathematics, p. 209)

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B10.1 Ask the student to divide 0.0034 by 0.1. Tell him/her to divide the result by 0.1, that result by 0.1, and the final result by 0.1. Ask: Is the result greater than or less than 1? Will repeatedly dividing by 0.1 always lead to a number greater than 1? Why or why not?

Paper and Pencil

B10.2 Present the following:

4□6.□ ÷ 0.1 = □5.□3 ÷ 0.01

Ask: What digits belong in the boxes?

B10.3 Ask: Which answer will have a 3 in the tenths place?

42 345 ÷ 0.1   42.345 ÷ 0.01   42.345 ÷ 0.001

Interview

B10.4 Ask: What digit would be in the tens place after dividing 453.2 by 0.01. Why?

B10.5 Tell the student that you have divided a decimal number by 0.001 and the answer is a decimal number also. Ask: What do you know about the original decimal number?

B10.6 Ask the student to explain why dividing by 0.01 produces the same result as multiplying by 100.

B10.7 Ask: Why does multiplying a number by 0.1 usually give a lesser answer than dividing the same number by 0.1?

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO vi) By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 6, students will be expected to

B11 calculate sums and differences in relevant contexts by using the most appropriate method

Elaboration - Instructional Strategies/Suggestions

B121 Students should recognize the need for different approaches to computation depending on the problem situation. Estimation must be used with all computations, but when an exact answer is required, students need to decide whether it is more appropriate to use a mental strategy, a pencil-and-paper calculation, or some form of technology, most often the calculator.

Students should continue to practise mental math strategies. One objective is for them to use it in their daily lives, not just in math class. It is also important to point out that the benefits students gain from exploring number and number patterns while developing mental math strategies are immeasurable. It is recommended that regular, maybe daily, practice be provided. This could take the form of out-of-context exploration of number at times (drill/practice or strategies), but would be most effective when addressed in a problem-solving context. For example, the following question might be presented to see if students can not only solve problems, but recognize that a mental strategy could be employed:

> Mason bought 32.5 m of fencing that was on sale for \$3 a metre. How much change did he receive from a \$100 bill?

Students should perform mental computations with facility using strategies as outlined in the grades 4 and 5 guides.

- Front-end (left-to-right):

$$\begin{array}{r} 24\ 345 \\ 3\ 214 \\ \underline{10\ 116} \end{array}$$

Beginning at the front, or ten thousands place in this case, this sum might be read thirty-seven thousand (2 + 1 and 4 + 3 or 24 + 10 + 3 for 37), six hundred sixty, no, that's seventy (6 + 4)-five.

a) $\begin{array}{r} 28\ 164 \\ \underline{-12\ 052} \end{array}$ b) $\begin{array}{r} 15\ 347 \\ \underline{-9\ 579} \end{array}$

a) can easily be solved with the front-end strategy. A quick glance at the digits in b) would lead one to consider something other than a mental strategy.

- Compensation:

\$25.95 + 3.99 + 11.98 is \$26 + \$4 + \$12, or \$42 subtract \$0.08 for \$41.92.

For the following subtraction, one might use a combination of front-end and compensation:

$$\begin{array}{r} 7683 \\ \underline{-5249} \end{array}$$

24 hundred forty, less 6(9 - 3) is 24 hundred thirty-four or 2434.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Interview

B11.1 Ask the student to describe a way to compute 3000 - 2898 mentally. (The counting-on strategy would work well.)

B11.2 Ask the student to explain how to solve each of the following using a mental strategy:

$75 + $12.25 + $5.75 =   470 + 1068 + 30 =

$$\begin{array}{r} 2435.7 \\ 304.1 \\ \underline{1050.2} \end{array} \qquad \begin{array}{r} 4579.25 \\ \underline{-2134.14} \end{array}$$

B11.3 The numbers "75, 25," "45, 55," "340, 660" are examples of what are termed "compatible" or "nice" or "partner" numbers. Ask the student to give the "partner" number for each of the following:

40(60)   49(51)   730(70 or 270)   21(79)

B11.4 Provide examples of computations and ask the students which method they would use. Some examples are:

a) 2 x 22.3	b) 7.64 x $2.38	c) 100 - 12
d) $4.63 x 11	e) 24.8 x 0.5	f) $126.48 - 14.20
g) 1097.6 ÷ 39.2	h) $1.99 + $2.99 + $5.98 + $0.99	

Many students at this grade should be able to mentally compute "a" (maybe using the front-end method), "c" (subtracting 10 and 2 more), "e" (finding half of the number), "f" (using front-end, although this is easier to see when written vertically) and "h." Examples "b," "d," and "g" would be solved with a paper/pencil algorithm or with a calculator. Some may wish to calculate "d" ($4.63 x 11) mentally using the "eleven" strategy.

Portfolio

B11.5 Have the students make up an exercise sheet of computation examples that would lend themselves to mental strategies. Ask that they include a range of the strategies used in class. These could serve as practice sheets for the class.

Suggested Resources

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

vi) select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

SCO: By the end of grade 6, students will be expected to

B12 calculate products and quotients in relevant contexts by using the most appropriate method

Elaboration - Instructional Strategies/Suggestions

B12 Students will have much experience with multiplication and division situations by grade 6. They will have mastered the mental strategies of multiplying and dividing whole and decimal numbers by multiples of ten and it is expected that they will know their multiplication facts.

- Front-end

Multiplication:

3 x 325.15 Using the front-end strategy, one would say, 9 hundred (3 x 300) seventy-five (3 x 25) and 45 (3 x 15) hundredths (975.45).

Students very often proceed to find solutions using a paper/pencil algorithm without first checking to see if a mental strategy could be employed. It is important that there is a mix (mental and paper/pencil) when problem situations are presented.

Division:

Calculation has traditionally proceeded from left to right in division algorithms. Students should be able to divide using the "short" method. To provide practice, give students a mix of division exercises. Ask students to decide which examples can be calculated quickly using the mental front-end strategy, and which would require the use of the "short" division algorithm (which can also be a mental strategy).

$3\overline{)120.96}$ $5\overline{)176.28}$ $12\overline{)2400}$ $4\overline{)248.04}$

- Compensation

Multiplication: Students should be able to recognize that 9 x \$4.95 is 9 x \$5.00, or \$45, less \$0.45 (9 x 5) for a total of \$44.55.

Division: Students should recognize that dividing by 5 can often be easier if the dividend is doubled and one divides by 10. For example 1632 ÷ 5 is the same as 3264 ÷ 10, or 326.4.

One other useful multiplication strategy is "double/halve," or "halve/double." An explanation must be provided as to how, for example, 22 groups of 15 gives the same product as 11 groups of 30.

It is important that students be taught to use some of the calculator features, other than the basic operations. Because students need to practise paper/pencil calculations, the use of the calculator should be monitored.

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

B12.1 Have the student give a written explanation for how the double/halve strategy works.

Interview

B12.2 Ask the student how to calculate 14 000 ÷ 50 mentally.

B12.3 Have the student describe two numbers greater than 100 that would be easy to multiply mentally and to explain how.

B12.4 Give the students a variety of computation questions and ask which could be easily computed mentally and which they would choose to calculate using a paper-and-pencil algorithm. Have the student give an estimate for each one.

2 x 315.2   35 x 18   99 x 85   47 x 58

B12.5 Ask the student how to use a calculator to help find 999 999 x 343 343.

B12.6 Have the student describe two numbers greater than 100 that would be easy to divide mentally.

B12.7 Give the student a variety of calculations and ask him/her to tell which could be done easily mentally.

B12.8 Ask: How would you calculate 90 316 248 x 10.1? (An 8-digit caluator display makes this question problematic.)

Presentation

B12.9 Ask pairs of student to look for a pattern when squaring 8-digit numbers ending in 5. (For example, 55^2 is 3025, 35^2 is 1225.) Have them give an explanation as to why the pattern works this way.

Suggested Resources

Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *describe, extend and create a wide variety of patterns and relationships to model and solve problems involving real-world situations and mathematical concepts*

SCO: By the end of grade 6, students will be expected to

C1 solve problems involving patterns

C2 use patterns to explore division by 0.1, 0.01 and 0.001

. . . Students can explore patterns that involve a progression from step to step. In technical terms these are called "sequences". We will simply call them "growing patterns". With these patterns, students not only extend patterns but look for a generalization or algebraic relationship that will tell them what the pattern will be at any point along the way. (Elementary School Mathematics, p. 376)

Elaboration - Instructional Strategies/Suggestions

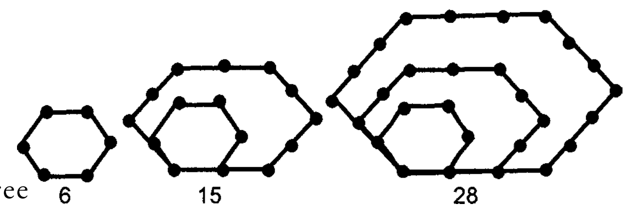
C1 Students should continue to use patterns to help them solve mathematical problems. For example:

- Ask the student to use a calculator to compute answers to the following pairs of questions:

$$\frac{1}{2}+\frac{1}{4} \quad (\text{and } 1-\frac{1}{4})$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8} \quad (\text{and } 1-\frac{1}{8})$$
$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} \quad (\text{and } 1-\frac{1}{16})$$

What do you think $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}$ **will be?**

❒ Have students continue the pattern to find the next three hexagonal numbers.



❒ Find the number of factors of 10, 20, 40, and 80. Predict the number of factors for 640.

❒ Two grade 6 students started an environmental club. They agreed to each get a new member every month. Each new member would recruit a new member by the end of their first month, and every month thereafter. How many members will there be at the end of one year?

C2 By the end of grade 5 students have mentally multiplied and divided numbers by 10, 100 and 1000; multiplied numbers mentally by 0.1, 0.01 and 0.001; and examined the patterns connected with these operations. In grade 6 (SCO B10) students are expected to mentally divide numbers by 0.1, 0.01 and 0.001. This outcome (C2) encourages students to recognize that the pattern of changes produced by dividing by 0.1, 0.01 and 0.001 is the same as that produced by multiplying by 10, 100 and 1000. For example:

4.71 x 10 = 47.1	4.71 ÷ 0.1 = 47.1
4.71 x 100 = 471	4.71 ÷ 0.01 = 471
4.71 x 1000 = 4710	4.71 ÷ 0.001 = 4710

It is important that students be able to describe these patterns with respect to place value changes, not just in terms of a rule involving moving the decimal point n places.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C1.1 Have students explore the potential difficulty in using a calculator to multiply very large numbers, for which the product is greater than the display capability. For example, in lieu of using a calculator to find 999 999 999 x 999 999 999, students might study a pattern and generalize:

99 x 99 =
999 x 999 =
9999 x 9999 =
....

Paper and Pencil

C1.2 Have pairs of students calculate: 38 x 32
36 x 34
37 x 33

Ask: What do you notice? Predict what 58 x 52 will be. Explain your prediction and verify.

C1.3 Ask students to find the sum of the first 30 even numbers (2 + 4, 2 + 4 + 6, etc.). Check to see if they are able to detect the multiplying pattern.

C1.4 Ask students to begin with 1, then add the next odd number for a sum, followed by the sum of the first three odd numbers, etc. Ask them to predict what the sum of the first twelve odd numbers would be.

Interview

C2.1 Tell the student that Frank divided 42.8 by 0.1 and got an answer between three and four hundred. Ask the student to explain how he/she knows that Frank's answer must be incorrect.

C2.2 Ask the student to explain why dividing a number by 0.01 results in a greater number than he/she originally started with.

C2.3 Ask the student to predict (in terms of change in place value) what the effect of dividing by 0.00001 would be.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) explore how a change in one quantity in a relationship affects another

SCO: By the end of grade 6, students will be expected to

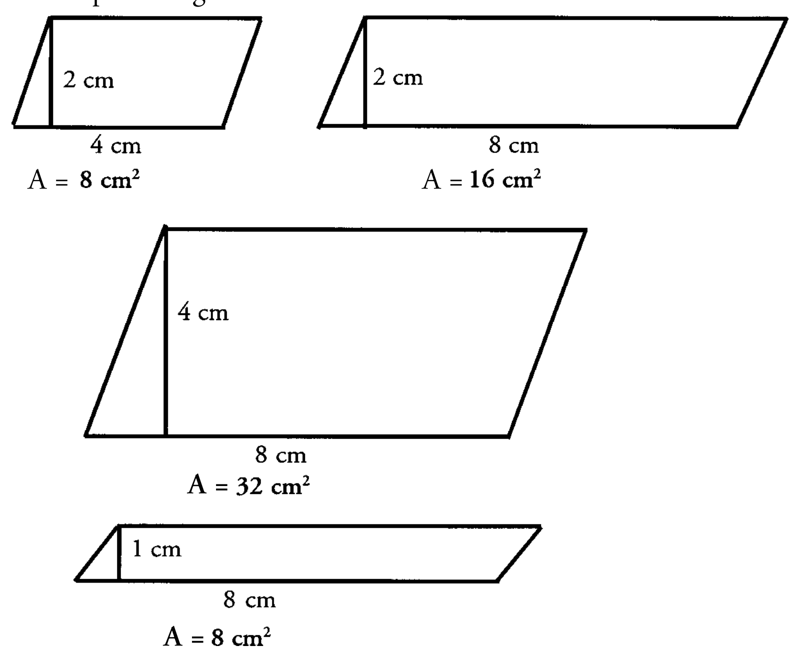
C3 recognize and explain how changes in base or height affect areas of rectangles, parallelograms or triangles

C4 recognize and explain how changes in height, depth or length affect volumes of rectangular prisms

C5 recognize and explain how a change in one term of a ratio affects the other term

Elaboration - Instructional Strategies/Suggestions

C3 As students explore the formulas for areas of rectangles, parallelograms and triangles, they need to interpret what the formulas actually mean, not just what to do with the numbers. For example, the formula for area of a parallelogram is A = bh; that should mean, to the student, that if b is doubled, then so is A; if b and h are both doubled, then the area is quadrupled; if b is doubled but h is halved, the area remains the same. Explore these relationships through models such as:



C4 Similarly, with volumes of rectangular prisms, students should be aware of the impact of changes, both singly and in combination, to each of the variables in the formula.

C5 Just as with equivalent fractions, students should be aware that as one term of a ratio is multiplied or divided by a particular quantity, so is the other. Students should also be aware that, usually, when a particular quantity is added to or subtracted from each term, the resulting ratios are not equivalent.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

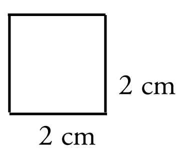
Performance

C4.1 Provide the students with multilink cubes. Have them construct rectangular prisms with the dimensions 3 x 5 x 2 and 6 x 5 x 2. Have the students find the volume of each. Ask: How could you have anticipated that the second volume would be twice the first? How do you think a 6 x 5 x 4 prism would compare to one 3 x 5 x 2?

C3.1 Ask students to draw three different triangles on grid paper, all of which have a base of 4 units and an area of 8 square units. Have them choose one of their triangles and change its base so that its area will be 16 square units.

Paper and Pencil

C3.2 Ask the student to draw a rectangle with an area exactly four times the area of the one shown and indicate the dimensions. Then ask the student to compare the factors and tell how one could have predicted the area relationship.



C5.1 Tell the student that two ratios are equivalent. The first term of the first ratio is 10 and the first term of the second ratio is 25. Ask: How are the second terms of the ratios related?

Interview

C3.3 Tell the student that a particular parallelogram has the same height as another, but the base is three times as long. Ask: How are the areas related and why?

C5.2 Ask the student to explain why halving one term of a ratio necessitates halving the other term as well in order to preserve the ratio.

Portfolio

C4.2 Ask the students to design a number of different rectangular boxes for fudge. Each design must have a volume of 1200 cm^3. Have the students write a report on their favourite designs, providing reasons for their choices.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) represent mathematical patterns and relationships in a variety of ways (including rules, tables and one- and two-dimensional graphs)

SCO: By the end of grade 6, students will be expected to

C6 represent equivalent ratios using tables and graphs

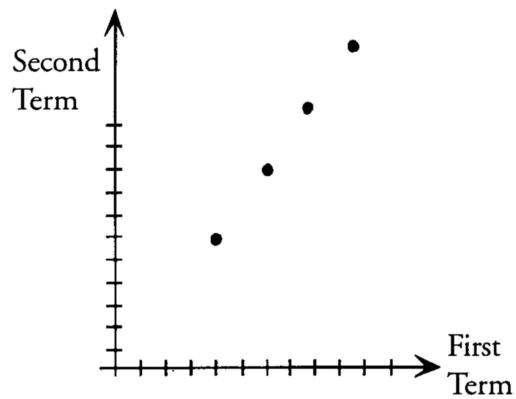
Elaboration - Instructional Strategies/Suggestions

C6 By representing equivalent ratios in tables and graphs, students will more clearly see both the relationship between the two elements of the ratio and the relationship between the two equivalent pairs.

For example, the ratio 2:3 is equivalent to:

First Term	4	6	8	10
Second Term	6	9	12	15

This can be graphed as shown:



It is easy for the student to see from the table that, in each case, the first term is $\frac{2}{3}$ of the second and that in each case, the same factor used to multiply 2 to create the new first term is used to multiply 3 to create the new second term. It is also possible to approximate from the graph that a ratio such as 6.66:10 is also equivalent to 2:3.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C6.1 Have the student draw a graph to show the equivalents of the ratio 4:5.

Paper and Pencil

C6.2 There are about 11 seniors in Canada for every 7 children under age 4. In the following table, list the approximate number of seniors for the given numbers of children:

Children under 4	7000	14 000	28 000	31 500
Seniors	11 000			

Interview

C6.3 Tell the student that a certain ratio is listed as 11:32. Ask: What simpler ratio would be a good estimate for this? How do you know?

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) *represent mathematical patterns and relationships in a variety of ways (including rules, tables and one- and two-dimensional graphs)*

SCO: By the end of grade 6, students will be expected to

C7 **represent square and triangular numbers concretely, pictorially and symbolically**

Elaboration - Instructional Strategies/Suggestions

C7 Students may have had informal exposure previously to square and triangular numbers. These special numbers have both geometric and numerical significance. It would be worthwhile to ensure that they are familiar with these patterns of numbers.

Square numbers:

			x x x x
		x x x	x x x x
	x x	x x x	x x x x
x	x x	x x x	x x x x
1	4	9	16

Square numbers may be represented in square arrays and are the products of numbers multiplied by themselves.

Triangular numbers:

x	x	x	x
	x x	x x	x x
		x x x	x x x
			x x x x
1	3	6	10

Each triangular number is half the number in an array with dimensions that are one unit apart. For example, 6, the third triangular number, is half the number in a 3 x 4 array.

```
x - - -
x x - -
x x x -
```

Likewise 10, the fourth triangular number, is half the number in a 4 x 5 array.

```
x - - - -
x x - - -
x x x - -
x x x x -
```

Triangular numbers may be displayed in triangle-like shapes and are calculated by adding the consecutive numbers beginning at 1.

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C7.1 Ask the student to draw a picture of the 8th triangular number and tell what it is.

C7.2 Ask students to draw a number pattern in which the numbers might be called doublesquares. Ask: How would you test to determine whether or not a number is doublesquare?

Paper and Pencil

C7.3 Ask the student to find out if two triangular numbers added together could ever make a square number and if two square numbers could be added together to make a triangular number.

C7.4 Ask the student to find out if the sum of two square numbers is ever a square number and whether the sum of two triangular numbers is ever triangular.

Interview

C7.5 Sara said that 100 is not a square number since, if you draw a 25 x 4 array, it's not square. Ask: How would you respond to Sara?

C7.6 Ask: Why is 8 not a square number?

C7.7 Ask the student if 144 is a square number and to give reasons for his/her answer.

Suggested Resources

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) solve linear equations using informal, non-algebraic methods

SCO: By the end of grade 6, students will be expected to

C8 solve simple linear equations using open frames

C9 demonstrate an understanding of the use of letters to replace open frames

Elaboration - Instructional Strategies/Suggestions

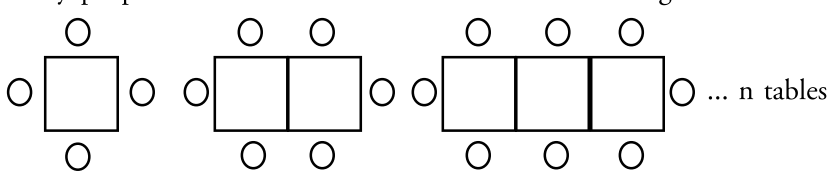
C8 Students should be able to solve simple linear open frame equations. Example: Suppose that in a class of 23, 8 students are working independently and the others are in groups of 3. How many groups are there? The information can be represented as $3 \times \square + 8 = 23$. The number of groups, 5, is the missing number.

Model the language used to express the meaning of equations. The above example might be read, "Three times a number and 8 more is 23." Encourage students to regularly read equations in meaningful ways.

C9 Having had experience for several years using open sentences, students might be introduced to the use of letters to represent variable (unknown) quantities. Students can be shown the parallel between sentences such as $5 + \square = 8$ and $5 + n = 8$. They also need to realize that the particular letter used is irrelevant.

Students should understand that this use of a letter is simply a convention and no more or less meaningful than the use of the open frame. They must also be taught that the multiplication sign is often not written when a letter is used (e.g., instead of writing $3 \times \square$, write 3n). Many students misinterpret 3n to mean a number in the thirties, so it is very important that this convention be made clear.

There are many patterns students could investigate that may be conveniently expressed using variables. For example, consider how many people can be seated at "n" tables in the following situation:



4 people a 1 table	6 people at 2 tables	8 people at 3 tables	2n + 2 people at n tables

Note: Students may well benefit from explicity describing their counts as follows:

(1 table)	2 at the ends	+	2 on the sides	=	4 people
(2 tables)	2 at the ends	+	2x2 on the sides	=	6 people
(3 tables)	2 at the ends	+	2x3 on the sides	=	8 people
(40 tables)	2 at the ends	+	2x40 on the sides	=	82 people
(n tables)	2 at the ends	+	2xn on the sides	=	2+2n people

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

C8.1 Provide a few simple linear equations using open frames for students to solve.

C8.2 Have the students create three different equations containing open frames for which the solution is 5.

Interview

C8.3 Tell the student that $\square$ represents a certain number. Ask: Why must the solutions to $2\square + 8 = 35$ and $2\square + 4 = 31$ be the same?

C9.1 Ask the student to tell what each phrase means:

$5 + n$
$3 - n$
$2n$
$n \div 2$

C9.2 Ask the student which would have the larger value, "n" or "y," and to explain.

$2n + 16 = 32$ $\qquad$ $2y + 16 = 36$

Suggested Resources

Shape and Space

General Curriculum Outcome D:

Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations

SCO: By the end of grade 6, students will be expected to

D1 use the relationship among particular SI units to compare objects

D2 describe mass measurements in tonnes

The purpose of an interview is to uncover how students think about mathematics, so provide opportunities for contradictions in students' beliefs about mathematical concepts to emerge. (Mathematics Assessment, Stenmark, ed. NCTM, 1991, p.29)

Elaboration - Instructional Strategies/Suggestions

D1 Students should review the meaning of the SI prefixes, i.e.,

milli ($\frac{1}{1000}$ of) centi ($\frac{1}{100}$ of)

deci ($\frac{1}{10}$ of) kilo (1000 of)

Using these meanings students can compare amounts. For example:

- Which is greater, 3.45 L or 345 mL?
- How many milligrams make a kilogram?
- How many metres is 45.2 cm?

Students should realize that the relationship between linear SI units is not the same as the relationship between corresponding SI area and volume units. For example, 100 cm = 1m, but 100 $cm^2 \neq 1\ m^2$ and 100 $cm^3 \neq 1\ m^3$.

Students should explore the relationships between SI area and volume units by comparing values in a series of concrete situations. For example, for the figure below, the linear dimensions, in cm, are $\frac{1}{10}$ the magnitude of the same dimensions in mm. The area, in cm^2, however, is $\frac{1}{100}$ the magnitude of the area in mm^2.

$\boxed{A = 60\ cm^2 = 6000\ mm^2}$ 5 cm = 50 mm

12 cm = 120 mm

D2 Students should be introduced to the "tonne". The tonne is equivalent to 1000 kg. (This unit should be distinguished from the "short ton" which is used in the U.S. to represent 2000 pounds.)

Students should

- be aware of the types of items which might have masses measured in tonnes
- relate the tonne to other mass units (e.g., 456 kg = 0.456 tonnes)
- solve problems involving tonnes

❒ Students might investigate to determine the number of children it would take to balance an elephant, a rhino, a blue whale, a brontosaurus, etc.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

D1.1 Ask: How many cubic decimetres are there in a cubic metre?

D1.2 Describe an object as being 0.003 dm long. Ask the student whether or not it would be a whole number of centimetres (millimetres) long.

D1.3 Tell the student that the area of a rug is 9000 cm^2 and ask how many square metres that is.

Interview

D1.4 Ask: What do you think "kilosecond" should mean (i.e., how long would one last)?

D1.5 Tell the student that Sue said "10 dm = 1 m, so then 10 dm^2 = 1m^2." Ask: Do you agree? Why or why not?

D1.6 Tell the student that the area of a rectangular rug is 10 000 cm^2. Ask: What might the dimensions be?

Presentation

D2.1 Ask students to work in pairs and decide whether or not items with the following masses would be easy to lift:

0.001 tonnes	0.001 kg
10 000 tonnes	10 000 g

Have them share their conclusions and reasoning with the class.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations

SCO: By the end of grade 6, students will be expected to

D3 demonstrate an understanding of the relationship between capacity and volume

Elaboration - Instructional Strategies/Suggestions

D3 It is important to have students explore the relationship between the cubic units of volume and capacity. Students need to solve problems involving both capacity and volume and to understand the following relationships between the two:

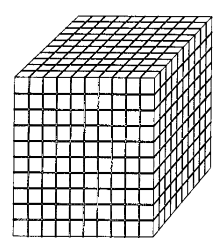
$1\ cm^3 = 1\ mL$ $1\ dm^3 = 1\ L$ $1\ m^3 = 1\ kL$

❐ Have students investigate the capacity, or volume, of moving trucks of various sizes. Let them determine how much and what furniture could be moved using trucks of various sizes.

Base ten blocks serve as good models. The small (units) block has a volume of 1 cubic centimetre and would displace 1 mL of liquid in a container; the 10 x 10 x 10 block (dm^3) has a volume of 1000 cubic centimetres and would displace 1 L of liquid. (Some may be interested in pursuing the relationship among volume, capacity, and mass. A cubic centimetre of water, equivalent to 1 mL, has a mass of about 1 g; 1000 cm^3 of water, equivalent to 1 L, has a mass of about 1 kg.)

Capacity and volume are both measures of the size of a 3-D region of space. Capacity is usually associated with measuring fluids and/or the containers which hold fluids. Volume is more commonly associated with solid objects, and is an expression built in three dimensions upon length units.

$1\ cm^3 = 1\ mL$



$1000\ cm^3 = 1\ L$

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D3.1 Have the students use base ten materials to make a structure with a volume of 1256 cm^3. Ask: What would the capacity be for your structure?

D3.2 Have the student place a 20 cm^3 structure into a full container of water. Ask: How many millilitres of water spill out?

Interview

D3.3 Ask the student to decide whether he/she would use volume or capacity units to describe

- amount of water in a pool
- amount of wheat in a barrel
- living space in a house

Portfolio

D3.4 Ask students to design a lesson plan for a grade four class in which they address the following:

- What does volume mean?
- What does capacity mean?
- How are they similar? different?

Invite them to teach their lesson to small groups of children and write a report on this experience.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units

SCO: By the end of grade 6, students will be expected to

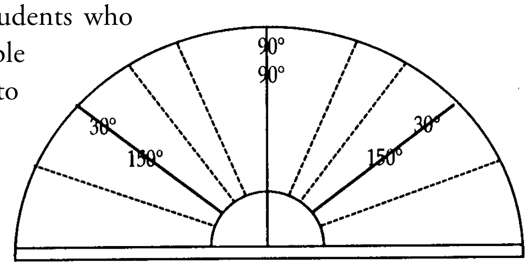
D4 estimate and measure angles using a protractor

D5 draw angles of a given size

The protractor is one of the most poorly understood measuring instruments found in schools . . . By making a protractor with a large unit angle, all of [the] mysterious features can be understood. Then, a careful comparison with a standard protractor will permit that instrument to be used with understanding. (Elementary School Mathematics, p.305)

Elaboration - Instructional Strategies/Suggestions

D4 Students should learn how to use a protractor to measure angles reasonably accurately. Students who use protractors with double scales on them will need to learn how to determine which set of numbers to use in a given situation. This is best accomplished by first having the student estimate the size of the angle and then decide which reading makes the most sense.

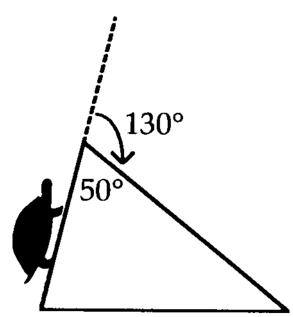


❒ Ask students to find the measures of each of the angles in various quadrilaterals. They should observe that the sum is always 360°. Similarly, they might find the sums of the angles in other types of polygons.

D5 Students need to learn how to use a protractor to draw an angle. Most work should focus on angles between 0° and 90°. There should be some discussion, however, of how to draw larger angles (e.g., 120° or 180°).

Students should be aware of the importance of positioning the 0° line on the protractor so that it coincides with the first arm of the angle in order to produce an accurate drawing.

❒ Using the computer programme, Logo, have students examine the effect of various angle turns. In situations like the one illustrated below, students should note that the size of the required turn is 130°, even though the angle of the triangle is 50°.

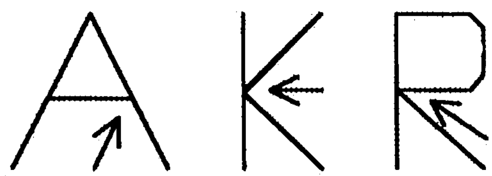


GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D4.1 Ask the student to measure the angles found in various letters of the alphabet.



D4.2 Ask the student to estimate, and show with their hands, the size of a 120° angle.

D5.1 Ask the student to make a 135° angle without using a protractor.

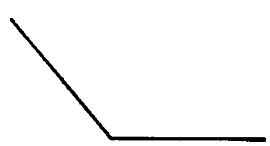
Pencil and Paper

D5.2 Draw an angle (e.g., 60°). Ask the students to draw an angle 90° greater without using a protractor.

D5.3 Ask the students to draw an angle they think might measure 150°. Ask: How close were you?

Interview

D4.3 Tell the student that Jeff measured this angle and said it measured 50°. Ask: Do you agree? Why or why not?



D4.4 Tell the student that Marc said he could make an angle bigger by extending both angle arms. Ask: What do you think of this plan?

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units

SCO: By the end of grade 6, students will be expected to

D6 solve measurement problems involving length, capacity, area, volume, mass and time

Elaboration - Instructional Strategies/Suggestions

D6 Students should regularly solve problems involving a variety of types of measurement. Many problems can and should be coordinated with the teaching of other curricular areas (e.g., using map scales and making scientific observations).

Time problems should include the use of a variety of time units, exploration of the recording of time using the 24-hour clock, and exploration of the idea of time zones. This provides opportunities for students to plan "worldwide" trips using travel schedules and taking time zones into account.

There are many opportunities to link measurement with relevant problem solving. For example:

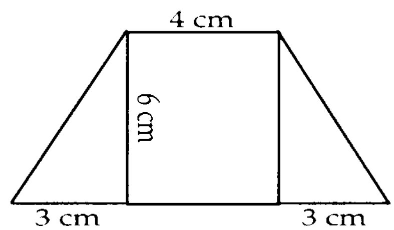
- time and length measures to determine speed
- area and number measures to calculate population densities
- area and length measures to find ratios in similar figures
- mass and capacity measures to conclude that the mass of 1L of water is about 1kg.

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D6.1 Ask the student to find the area of the trapezoid and explain his/her method.



D6.2 Provide students with airline schedules and ask that they work in pairs to discuss the times of flight arrivals and departures. They may wish to plan a cross-Canada trip. Have them visit at least 6 major cities. What is the quickest way to go to Victoria, B.C.?

Interview

D6.3 Ask the student how he/she might estimate the volume of a beach ball.

Portfolio

D6.4 Have the students plan a trip (car and ferries only) starting on a Monday morning at Saint John, N.B. They can assume that they will average 90 km/h on land. Ask: When would you expect to arrive in St. John's, Newfoundland? Include all times and schedules.

D6.5 Ask pairs of students to design pens for 6 gorillas in a zoo. The animals need an exercise area and a watering hole. Encourage them to be creative in their designs. Ask the students to calculate the cost of tiling the floor of their pens.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

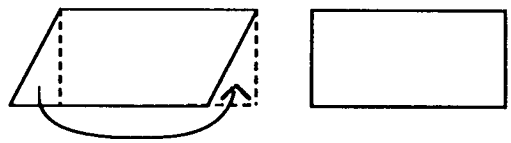
iv) develop and apply rules and procedures for determining measures (using concrete and graphic models)

SCO: By the end of grade 6, students will be expected to

D7 demonstrate an understanding of the relationships among the bases, heights and areas of parallelograms

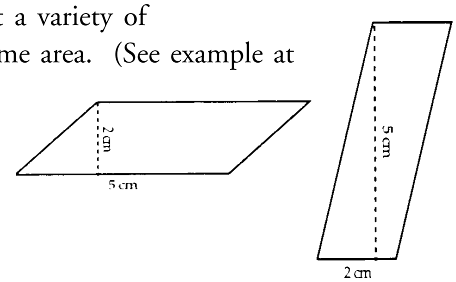
Elaboration - Instructional Strategies/Suggestions

D7 Students should recognize that the area of a parallelogram is the same as the area of a related rectangle, i.e., one with the same base and height.

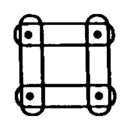


Students should be able to determine the base or height, given the area and the other dimension.

Students should recognize that a variety of parallelograms can have the same area. (See example at right.)



❒ Try the following "trick." Make a flexible rectangle using geostrips or cardboard strips and brads. Begin to deform (tilt) the rectangle. Ask the students whether or not the area has changed. Keep tilting until students see that the area has decreased. Discuss how with each additional deformation, a new parallelogram was created with the same base, but less height; therefore, the area decreased.



GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

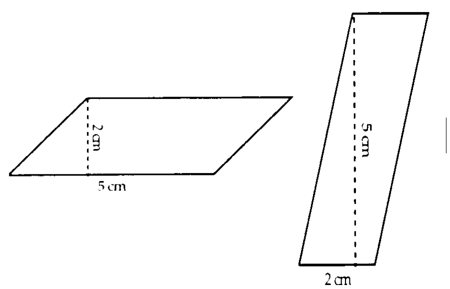
D7.1 Ask the student to draw on grid paper a parallelogram with an area of 24 cm^2. Then ask him/her to create three other parallelograms with the same base length and area.

Paper and Pencil

D7.2 Ask: If two parallelograms have the same area, do they have to be similar?

Interview

D7.3 Ask: Do these parallelograms have the same area? How do you know?



D7.4 Ask the student to determine which of the two shapes below has the greater area and to provide an explanation.

Suggested Resources

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iv) develop and apply rules and procedures for determining measures (using concrete and graphic models)

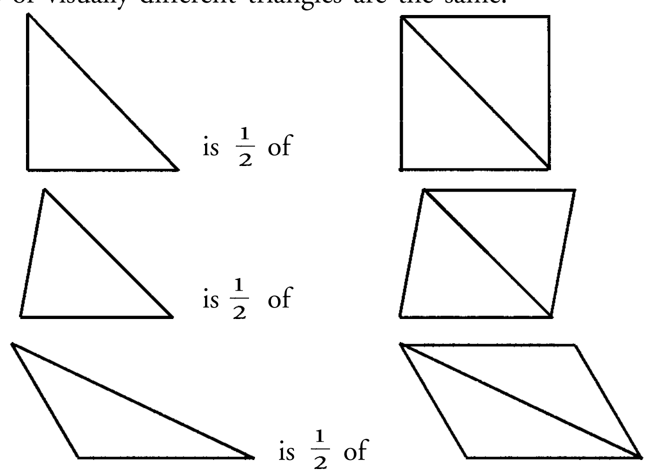
SCO: By the end of grade 6, students will be expected to

D8 demonstrate an understanding of the relationship between the area of a triangle and the area of a related parallelogram

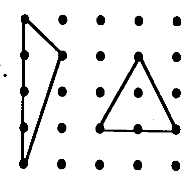
D9 demonstrate an understanding of the relationships between the three dimensions of rectangular prisms and volume and surface area

Elaboration - Instructional Strategies/Suggestions

D8 Students should recognize that any triangle is one half of a parallelogram. Thus, students should see that the area of the triangle is just one-half of the area of the related parallelogram. Students can use this relationship to find areas of simple triangles. Students should understand that, as long as the base and height are the same, the areas of visually-different triangles are the same.



❒ Have students locate on a geoboard as many triangles as possible which have an area of 2 cm^2. Students should understand that any triangle with base 4 and height 1, or base 2 and height 2, will qualify.



D9 To determine volumes and/or surface areas students should build structures or fill containers with centimetre cubes, always estimating before calculating.

Although students need not commit formulas to memory, their experiences should indicate to them that each of the three dimensions of the prism, i.e., the height, depth, and width, affects the volume and surface area. For example, in a 3 cm x 6 cm x 2 cm box, 3 x 6 or 18 cubes fill a layer. Since there are 2 layers, the volume must be 2 x 18 = 36 cm^3.

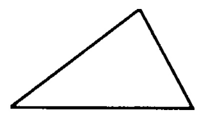
❒ Milk cartons can be cut and fashioned into cubes with side lengths of 10 cm and used as building blocks. Each block has a volume of 1 dm^3 (or 1000 cm^3, or 0.001 m^3) and each face a surface area of 100 cm^2 (or 0.01m^2).

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D8.1 Ask the student to draw a parallelogram with twice the area of the triangle at right.



Paper and Pencil

D9.1 Ask the student to write an explanation for why a prism with a base that is 5 cm x 3 cm and a height of 4 cm has to have a volume of 60 cm^3.

D9.2 Ask: Which has the greater effect on the volume of a prism, doubling the area of the base or doubling the height?

Portfolio

D8.2 Present the following scenario: The area of a certain triangle is found. The area of another triangle is 2 units less and its height is 1 unit less than the height of the first triangle. Ask: What do you know about the bases? Explain your thinking in a report for your portfolio.

Suggested Resources

Shape and Space

General Curriculum Outcome E:

Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) identify, draw, and build physical models of geometric figures

iv) solve problems using geometric relationships and spatial reasoning

SCO: By the end of grade 6, students will be expected to

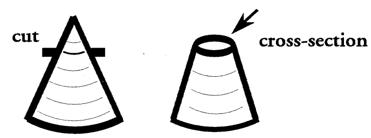
E1 describe and represent the various cross-sections of cones, cylinders, pyramids and prisms

Students should be challenged to analyze their thought processes and explanations. They should be allowed sufficient time to discuss the quality of their answers and to ponder such questions as, Could it be another way? What would happen if ... ? (Curriculum and Evaluation Standards for School Mathematics, p. 113)

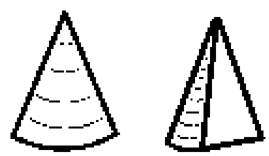
Elaboration–Instructional Strategies/Suggestions

E1 A cross-section is the 2-D face produced when a plane cut is made through a 3-D shape. For example, consider a right-circular cone.

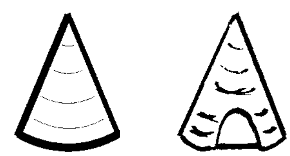
a) If it is cut in any plane parallel to its base, the face produced is a circle.



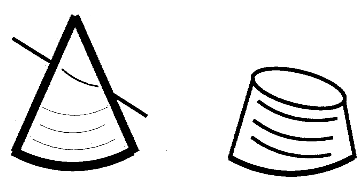
b) If it is cut down through its vertex, the exposed face is a triangle.



c) If it is cut in a plane parallel to a plane of symmetry, the shape below is produced.



d) If the cone is cut obliquely—not parallel to its base—the face produced is an oval (ellipse).



Cross-sections should be investigated by actually cutting shapes or by observing water surfaces in models of shapes. Students will come with prior experiences with cross-sections of cubes, square prisms, and rectangular prisms.

It is not intended at this stage for students to remember the various cross-sections for each 3-D shape without the presence of the shape; however, they should try to draw their predictions of the cross-sections before they cut the shape. Using elastic bands on the shapes to represent cuts is a way to help students visualize.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E1.1 Have students make some triangular prisms with Plasticine. Ask them to predict and draw the polygonal cross-section(s) that would result with each of the following cuts and to check their predictions by making the appropriate cuts:

(a) cut parallel to its bases
(b) cut parallel to one of its rectangular faces
(c) cut obliquely (slanting) towards its bases
(d) cut obliquely to a rectangular face

E1.2 Ask students to explain and demonstrate how a square pyramid could be cut to produce each of the following cross-sections:
(a) a circle (b) a rectangle (c) a trapezoid

Presentation

E1.3 Have students stack four hexagonal pattern blocks to make a hexagonal prism. Ask them to discuss with a partner some ways this prism could be cut and what shapes would be produced. Have them present their ideas to the class, including their pictures of the different cross-sections.

Interview

E1.4 Provide a variety of 3-D shapes for students to examine. Explain to them that you have a mystery 3-D shape that has been cut to make a triangular cross-section. Ask them to think of four possibilities for this mystery shape and to describe the cuts that would have been made to produce this cross-section.

E1.5 Ask students to describe how a cylinder could be cut to produce each of the following cross-sections:
(a) a circle (b) a rectangle (c) an oval

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *identify, draw, and build physical models of geometric figures*

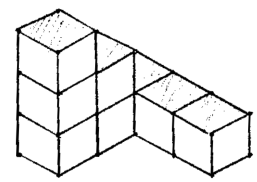
iv) *solve problems using geometric relationships and spatial reasoning*

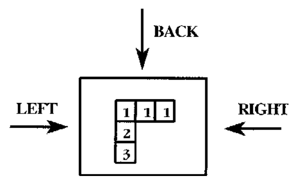
SCO: By the end of grade six, students will be expected to

E2 make and interpret orthographic drawings of 3-D shapes made with cubes

Elaboration–Instructional Strategies/Suggestions

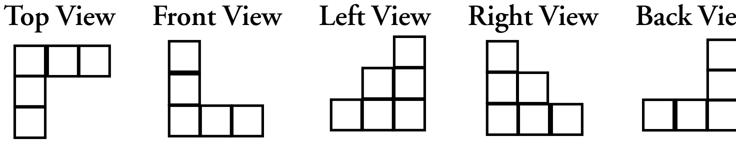
E2 Orthographic drawings are a series of 2-D views of a 3-D shape drawn by looking at the shape straight down (to get a top view) and straight on (to get front, back, right, and left views). The figure at right could be interpreted in a mat plan that shows its base with numbers indicating the number of cubes high.



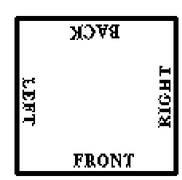


Students could build this shape with cubes, place them on mat plans, and draw the various orthographic views on square dot paper. The following are the views for this shape:

Top View **Front View** **Left View** **Right View** **Back View**



The use of mats often helps students with these drawings. A square of plain paper appropriately marked with directions would be a simple mat for this purpose. The students could then place a 3-D shape on the mat and move the mat to make the drawing of each view. Note: Left and right are always relative to the front.



Some students might find it helpful to close one eye and place themselves so that they are at eye level with the shape; they should then see only one face of the 3-D shape.

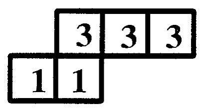
- ❒ Provide students with directional mats and 3-D shapes made of eight cubes. Have them draw mat plans and make top, front, and right orthographic views of these shapes.
- ❒ Provide students with top, front, and left orthographic views of various 3-D shapes. Have them use cubes to build the shapes with these views and to draw mat plans.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

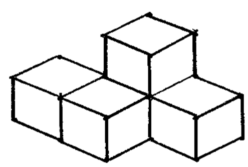
Performance

E2.1 Have students use cubes to construct the shape that has this mat plan.

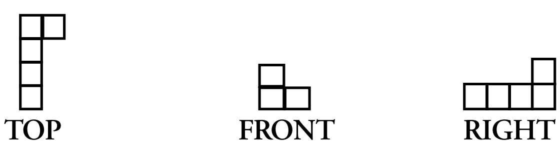


Ask them to place it on a mat and to draw the various orthographic views, labelling each.

E2.2 Ask students to use cubes to construct the shape shown at right. Ask them to draw the mat plan for this shape and the top, front, and left orthographic views, labelling each. Ask: Would the back and right orthographic views of this shape be needed for someone to be able to build this shape or would the three views you have be sufficient?



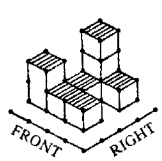
E2.3 Ask students to construct a building with cubes that would have these orthographic views.

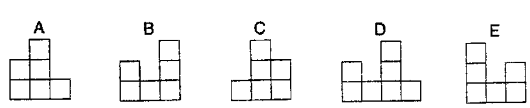


Ask them to draw its mat plan.

Paper and Pencil

E2.4 Provide students with this picture of a building drawn from its front-right corner. Ask them which one of A–E is the right orthographic view.





Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) describe, model, and compare 2- and 3-D figures and shapes, explore their properties and classify them in alternative ways

SCO: By the end of grade 6, students will be expected to

E3 make and apply generalizations about the sum of the angles in triangles and quadrilaterals

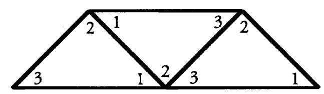
E4 make and apply generalizations about the diagonal properties of trapezoids, kites, parallelograms and rhombi

E5 sort the members of the quadrilateral "family" under property headings

Modeling, mapping, and engaging in activities and spatial experiences organized around physical models can help students discover, visualize, and represent concepts and properties of geometric figures in the physical world. (Geometry in the Middle Grades, p. 1)

Elaboration–Instructional Strategies/Suggestions

E3 Students should investigate these generalizations in a variety of ways. For example, if each student cuts out a triangle, tears off its three angles, and places them together, the three angles will form a straight line (180°); if each student cuts out three copies of a triangle, labels the three angles, and starts a tessellation, they can see that the sum of the angles is 180°;



if students measure the three angles in a variety of triangles and add them, the 180° relationship might be revealed, although measurement often produces only approximate results.

Similarly, students could be lead to generalize the 360° sum of the angles of any quadrilateral.

E4 Generalizations about diagonal properties should result from guided investigations.

a) For a rhombus, the diagonals are perpendicular bisectors of each other, form four congruent right triangles, bisect the angles of the rhombus, and are its two lines of reflective symmetry.

b) For a parallelogram, the diagonals bisect each other and form two pairs of congruent triangles.

c) For a kite, the diagonals are perpendicular and form two pairs of congruent right triangles; one of the diagonals is bisected, and the other diagonal is a line of reflective symmetry and bisects two opposite angles of the kite.

d) For a trapezoid, there are no special properties of its diagonals.

These properties should be developed for each figure, applied in a variety of ways, compared to the others, and combined with the side and angle properties of the figures.

E5 Students should be able to sort pictures or cutouts of the various quadrilaterals into sets according to one or more properties. These properties include: diagonals bisect each other; opposite sides congruent; four right angles; diagonals perpendicular to each other; opposite angles congruent; has reflective symmetry; and diagonals form two pair of congruent triangles.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E3.1 Tell students that you heard someone say, "Since any quadrilateral can be divided into two triangles and a triangle has a sum of angles of 180°, it is obvious that the sum of the angles of a quadrilateral is 360°. Have students draw pictures of various quadrilaterals to verify the truth of this statement. Have them extend this thinking to find the sum of the angles of a pentagon.

E4.1 Have students draw a scalene right triangle and use a mira to draw the reflected images of this triangle in both arms of the right angle to make a pentagon. Have them join two vertices to produce a quadrilateral with its two diagonals showing. Ask them to name this quadrilateral. Ask if everyone in the class got the same type of quadrilateral. Have them list the properties of this quadrilateral that they can confirm from the way they drew it.

E5.1 Make a set of cards with a variety of pictures of different members of the quadrilateral family. Distribute them to the students. Choose an attribute card (e.g., opposite sides parallel). Have students put their shape cards under this attribute, if appropriate, and to discuss why or why not. Choose another attribute card (e.g., diagonals bisect each other) and place it with the first card. Have students discuss which shapes should now be removed and why.

Paper and Pencil

E3.2 Ask students to find the size of the missing angle(s) for each triangle and to draw the triangle if (a) two of its angles are 70° and 45°, (b) two of its angles are each 75°, (c) it is a right triangle with a 60° angle, and (d) it is an isosceles triangle with an angle of 102°.

E3.3 Ask students to draw parallelograms on square dot paper. Have them measure one of the angles of the parallelograms and determine the other three angles by using known relationships.

E4.2 Ask students to list the properties of a rhombus that are the same as those of a kite and the ones that are different.

E4.3 Ask each student to draw a segment that is 6–10cm long. Have them use only a Mira to draw a rhombus that has this segment as one of its diagonals, explaining the process that they used.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) describe, model, and compare 2-D and 3-D figures and shapes, explore their properties and classify them in alternative ways

SCO: By the end of grade 6, students will be expected to

E6 recognize, name, describe and represent similar figures

Elaboration–Instructional Strategies/Suggestions

E6 Students have an intuitive sense of similar shapes—shapes that are enlargements or reductions of each other. Students' experiences with negatives of photographs that can be developed in different sizes, with maps or pictures that are drawn to scale, and with images produced by magnifying glasses provide natural connections for this concept. Overhead projectors, photocopiers, and film projectors are other sources of real-world contexts to relate to similar figures.

Students should discuss what the word *similar* might mean to them in everyday contexts. Compare these meanings to the specific meaning of the word in mathematics (i.e., corresponding angles equal and pairs of corresponding sides equal multiples of each other).

- ❒ Prepare pairs of shapes some of which are similar and some which are not. Tape the larger onto the board and place the smaller on the overhead projector. Have a student move the projector until the image coincides, or does not, with the one taped on the board. They are similar if a match can be made.
- ❒ Place a red pattern block on the overhead projector. Have students compare the projected image to the actual block, asking them what is the same and what is different. Have a student place the block in the corresponding angles of the projected image. (This should emphasize the role angles play in making shapes similar.) Informally compare the lengths of corresponding sides of the block and of the projected image, seeing approximately how many times longer the image sides are than the sides of the block. (This will be easier if you move the projector in advance so the sides will be a whole number times larger rather than a fractional times larger.)

All dilatation images of a shape (see SCO E9) are similar; however, not all similar figures are merely dilatation images of one another—similar figures can be on different planes and/or be the result of a dilatation in combination with other transformations.

Students are likely to recognize the similarity of different sizes of regular polygons (e.g., equilateral triangles and squares). Also, because all triangles with equal angles are similar, they are more likely to easily recognize similar triangles than most similar quadrilaterals.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E6.1 Have students examine the 3 different-sized triangles in a tangram set for similarity. Ask them if they think these triangles are similar. Have them compare the angles and the lengths of the corresponding sides of these triangles to confirm or refute their answer.

E6.2 Provide students with a sheet of rectangles, most of which are similar. Ask them to cut out the rectangles they think are similar and to draw in the diagonals. Ask them to lay the similar rectangles on top of one another, starting with the largest and in such a way that they share one common vertex. Ask them what they notice about the diagonals of the similar rectangles. Have them check one of the rectangles that is not similar to see the difference in the diagonals.

E6.3 Have each student make a triangle on a geoboard using the bottom left peg and the pegs directly above and to the right of it. Ask them to make four different triangles, all similar to this first one.

E6.4 Using a flashlight and a shape in a dimmed classroom, move the flashlight to cast shadows on the wall asking students to identify if the shadow is similar, or not, to the shape. Repeat using other shapes. Ask: Where does the flashlight need to be held to produce a similar shape?

E6.5 Have students use only the triangles in the pattern blocks to make other larger triangles. Ask: Are these larger triangles similar to the green pattern block? Have them hold a green block close to one of their eyes, stand over one of the larger triangles staring down at it using the eye with the block in front of it, and move the block until it coincides with the larger triangle. (This is another way that students can test for similarity.) Have them compare the sizes of the angles in the triangles and the lengths of the corresponding sides. Repeat these tasks using the other pattern blocks.

Paper and Pencil

E6.6 Have students draw scalene triangles, cut them out, and use these triangles to help draw smaller and larger similar triangles.

E6.7 Provide students with shapes or pictures drawn on one-sized grid paper. Have them use a different-sized grid paper to draw similar shapes or pictures.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

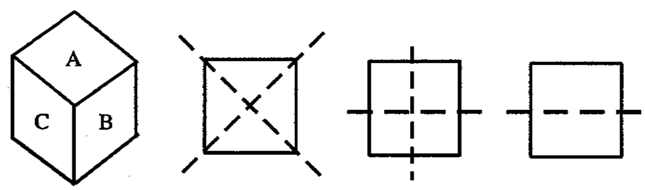
iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

SCO: By the end of grade 6, students will be expected to

E7 **make generalizations about the planes of symmetry of 3-D shapes**

Elaboration–Instructional Strategies/Suggestions

E7 In the same way that some 2-D shapes have lines of reflective symmetry, some 3-D shapes have planes of reflective symmetry, i.e., planes that bisect 3-D shapes such that all points in one-half are mirror images of the corresponding points in the other half. These planes of symmetry should be connected to cross-sections as examples of special cuts. A cube, for example, has nine different planes of symmetry as shown in the figure below. (Note: Although faces B and C could also be cut by perpendicular planes as shown on face A below, only one cut on face B would produce a different result.)



6 planes of symmetry (2 cuts on each of faces A, B, and C)

2 planes of symmetry (2 cuts on face A)

1 plane of symmetry (1 cut on face B)

Students should investigate planes of symmetry of triangular, square, rectangular, pentagonal, and hexagonal pyramids. Students should discover the pattern that the number of planes of reflective symmetry for these pyramids is equal to the number of lines of reflective symmetry of their bases (e.g., a square pyramid has 4 planes of symmetry and its square base has 4 lines of symmetry).

❒ Provide students with triangular, rectangular, square, pentagonal, and hexagonal prisms. Have them investigate the number of lines of reflective symmetry of the bases of these prisms. Ask: Will planes through these lines of symmetry be planes of symmetry of these prisms? Do these prisms have other planes of symmetry? Have students explain how to find the number of planes of symmetry of a prism.

When students examine cones and cylinders for planes of symmetry, the concept of infinite number will likely arise as students notice that there are “a whole bunch” of planes that would cut through the centre of the circular bases to be planes of symmetry. Also, spheres will be found to have infinitely-many planes of symmetry.

Note: The cones, cylinders, prisms, and pyramids used should be right, such as the ones typically found in basic sets of solids.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E7.1 By stacking pattern blocks construct a 3-D configuration that has two planes of symmetry. Ask students to show where its two planes of symmetry are.

E7.2 Ask students to use a set of pyramids that have regular polygonal bases to help complete this table.

Pyramid	No. of Planes of Symmetry
triangular (one equilateral face)	
square	
pentagonal	
hexagonal	

Ask: Do you see a pattern that you could use to predict how many planes of symmetry an octagonal pyramid has?

E7.3 Have students examine real-world objects of a variety of shapes (e.g., boxes, containers, toys, candies, and candles) for planes of reflective symmetry.

E7.4 Ask students to use 12 multi-link cubes to build a shape that has two planes of reflective symmetry.

Interview

E7.5 Provide pictures of houses, garden sheds, gazebos and other structures. Ask students to choose which structures have plane(s) of symmetry and describe the location(s) of these planes.

E7.6 Ask students what is meant by this statement: "The number of planes of symmetry that a cone has is related to the number of diagonals a circle has."

E7.7 Ask students to compare the number of planes of symmetry of a square prism (non-cubic) and a rectangular prism. Have them explain why the square prism has more planes of symmetry than the rectangular prism. Have them explain why they both have fewer planes of symmetry than a cube.

E7.8 Show students a water glass that has an interestingly shaped bottom. Ask them to describe any plane(s) of symmetry that this glass might have.

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

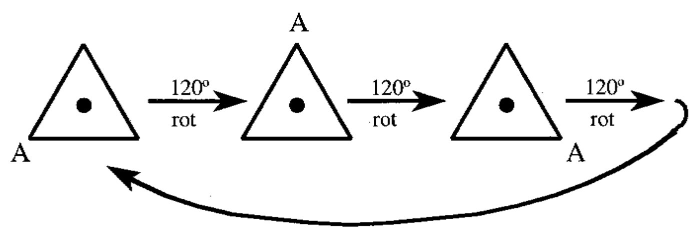
SCO: By the end of grade 6, students will be expected to

E8 make generalizations about the rotational symmetry property of all members of the quadrilateral "family" and of regular polygons

Symmetry in two and three dimensions provides rich opportunities for students to see geometry in the world of art, nature, construction, and so on. (Curriculum and Evaluation Standards for School Mathematics, p.115)

Elaboration–Instructional Strategies/Suggestions

E8 If a shape can be turned about a point so that it exactly coincides with its original position at least once in less than a complete rotation, it is said to have rotational symmetry. The number of times it appears in the identical position during one complete rotation is the order of rotational symmetry. For example, if an equilateral triangle is turned clockwise 120° about its centre point, the image is identical; if it is turned another 120°, again the image is identical. It is said to have rotational symmetry of order 3.



If a shape has to be rotated 360 degrees before it fits its traced image, it does not have rotational symmetry.

❒ Have students use a blue block from the pattern blocks, tracing it on paper. Have them place the block on its traced image and lightly put a pencil mark in the upper left corner of the block. Ask them to turn the block within its traced image to the right until it fits its traced image again. Have them notice where the mark on the block is now. Have them continue turning the block to the right until it fits its traced image again, noticing where the mark is on the block. (Through this activity, students should conclude that a rhombus has rotational symmetry of order 2.)

Through activities such as the one above with other members of the quadrilateral family of shapes, students should generalize that a square has rotational symmetry of order 4; a rhombus, parallelogram, and rectangle each have rotational symmetry of order 2; a kite and a trapezoid do not have rotational symmetry.

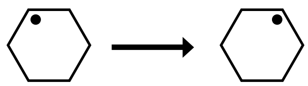
Note: There are many handy contexts for exploring rotational symmetry. For example, consider the common toddler toy which involves fitting blocks through openings. In how many ways can the hexagonal block be fitted through the hexagonal opening?

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

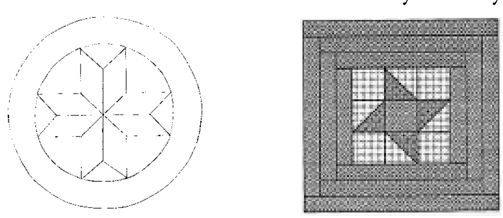
E8.1 Have students trace around a hexagonal pattern block and mark a dot at one vertex on the block. Have them rotate the hexagon clockwise until it fits the traced hexagon exactly.



Have them continue until the marked vertex returns to its original position. Ask them how many times the block appeared in the identical position. Have them describe its rotational symmetry.

E8.2 Repeat E8.1 for square, rhombus, rectangle, parallelogram, kite, and trapezoid.

E8.3 Provide students with pictures of designs and quilt patterns such as the ones below. Ask them to predict whether they have rotational symmetry. Have them use tracing paper to confirm their predictions by tracing the patterns and rotating the tracing paper on top of the pictures. Have them check as well for reflective symmetry.



E8.4 Ask students to use geoboards to make shapes that have rotational symmetry of order 2.

Interview

E8.5 Ask the student to explain how someone would know the order of rotational symmetry for any regular polygon.

Portfolio

E8.6 Have students examine newspapers and magazines for pictures and logos that have rotational symmetry. Ask them to select four of their favourite ones, paste them on paper, and write short descriptions of their symmetry, including comments on their reflective symmetry if they have it. (Many companies (e.g., Chrysler and Mercedes Benz) have logos that are symmetric.)

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

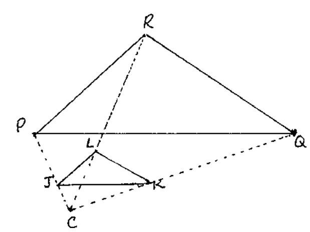
SCO: By the end of grade 6, students will be expected to

E9 recognize and represent dilatation images of 2-D figures and connect to similar figures

Elaboration–Instructional Strategies/Suggestions

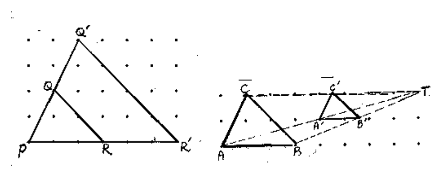
E9 Introduce dilatations by having students participate in activities such as the one below.

- ❒ Have each student draw a scalene triangle, labelled JKL, on a sheet of paper and select a point C outside this triangle. Ask them to measure the distance from C to J, and triple this distance for CP; measure the distance from C to K, and triple this distance for CQ; and measure the distance from C to L, and triple this distance for CR. Have them draw a triangle by joining P, Q, and R and ask them to compare the two triangles.



Explain to students that if lines through all corresponding vertices of two shapes on a plane converge at a single point (as they do in the activity above), these shapes are dilatation images of one another, the point of convergence is the centre of dilatation, and the two shapes are similar.

$\Delta A'B'C'$ is the dilatation image of with T as the centre of dilatation; is the dilatation image of with P as the centre of dilatation. Comparing the distances from P to the corresponding points shows that they are all twice as far out and from T to the corresponding points shows that they are all half as far.



The centre of dilatation could be connected to the concept of vanishing point if students have done perspective drawing in art.

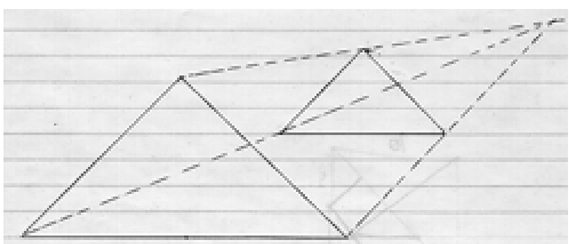
Students should be encouraged to compare dilatation images and to see if they can see any relationships between them. Some students might notice that corresponding sides are parallel and that each pair of corresponding sides have the same ratio.

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

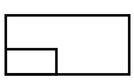
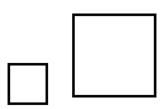
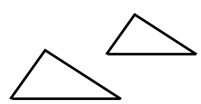
Worthwhile Tasks for Instruction and/or Assessment

Performance

E9.1 Have students each trace the large triangle from tangram sets on lined paper so that its longest side lies along a line (see below). Have them also trace one of the smaller triangles from the sets so that its longest side lies along a different line. Ask them to draw lines between corresponding vertices of the two triangles and extend them. Ask: What is the point where the three lines intersect called? Have them investigate whether lined paper could be used in the same way to set up any two similar figures to be dilatation images.



E9.2 Have students try to visualize the existence of a centre of dilatation for each of the following pairs to predict if they are dilatation images of one another. Have them check by actually locating the centre using a straight edge.

a)  b)  c) 

E9.3 Have students each trace a red pattern block on a sheet of plain paper. Ask them to draw for this pattern block a dilatation image of their choice. Ask them to compare the angles and sides of their two trapezoids.

E9.4 Have students draw a 9cm x 12cm rectangle on lined paper, with a 12cm length lying along one of the lines on the paper. Have them draw a 3cm x 4cm rectangle somewhere else on the paper with a 4cm length lying along a line. Ask: Are the two rectangles dilatation images of one another? How do you know?

Suggested Resources

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts

SCO: By the end of grade 6, students will be expected to

E10 predict and represent the result of combining transformations

Computer software allows students to construct two- and three-dimensional shapes on a screen and then flip, turn, or slide them to view them from a new perspective. (Curriculum and Evaluation Standards for School Mathematics, p.114)

Elaboration–Instructional Strategies/Suggestions

E10 Students should understand that two congruent shapes on the same plane are images of one another under a translation, reflection, rotation, or any combination of these three transformations. If two similar shapes are on the same plane, they are dilatation images or dilatations in combination with translations, reflections, or rotations. Students should investigate a variety of combinations, each time trying to visualize the result to make a prediction before actually carrying out the transformations. These combinations should include a reflection followed by a translation, two translations, two reflections, a translation followed by a rotation, two rotations, and a dilatation followed by a translation.

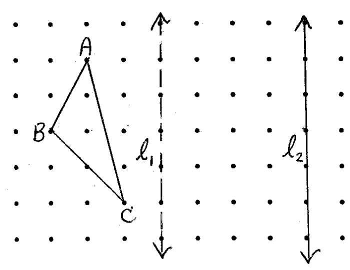
- ❐ Place three geoboards side by side. Have one student make a scalene triangle on the first geoboard. Ask another student to construct on the second geoboard the image of this triangle if the right side of the first geoboard is used as a mirror line. Ask another student to construct on the third geoboard the image of the triangle on the second geoboard under a 90 degree counterclockwise rotation. (Repeat this activity using other shapes.)
- ❐ Provide each student with grid paper marked with a co-ordinate system and three pattern blocks of the same type. Ask students to place one block on the system so that one of its vertices is at (-5,3). Ask them to place a second block so that it would be the image of the first block under a horizontal translation of 10 units. Then ask them to place the third block so that it is the image of the second block under a reflection in the x-axis. Have them compare the first and third blocks. Repeat this activity using two other transformations. (Extension: Have each student carry out two transformations of their choice on the co-ordinate system and leave only the first and third blocks in place. Have them exchange co-ordinate systems with a partner and have them try to predict the two transformations that took place. Share their predictions and actual transformations.)
- ❐ Have students investigate such questions as:
  - If a shape undergoes 2 translations, does it matter in which order they take place?
  - Does a reflection followed by a translation produce the same result as the translation followed by the reflection?

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

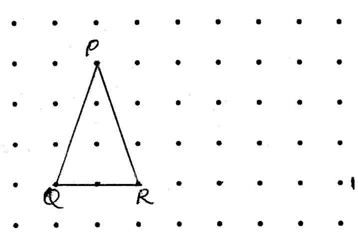
Performance

E10.1 Have students locate the image of after a reflection in line 1 followed by a reflection in line 2. Ask them what single transformation of would have the same result.



E10.2 Ask students to draw an isosceles triangle on dot paper and to translate it 4 units horizontally. Ask them to describe the reflection that would produce the same result.

ΔABC



E10.3 Present students with the pictures on grid paper of two congruent shapes—the first and the third (after two transformations were performed). Ask students to predict what two transformations were performed. Ask: Could this have been done in more than one way? Could this have been done by a single transformation?

Paper and Pencil

E10.4 Have students each draw a 4 x 4 square on grid paper and shade some of the small squares inside to create a design. Have them draw another 4 x 4 square attached under the first square. Ask them draw in this square the image of their design under a reflection in the bottom side of the first square. In another 4 x 4 square attached under the second square, have them draw the half-turn rotation image of the design in the second square. Ask them to compare the designs in the first and third squares.

Suggested Resources

Data Management and Probability

General Curriculum Outcome F:

Students will solve problems involving the collection, display and analysis of data.

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *collect, organize and describe relevant data in multiple ways*

SCO: By the end of grade 6, students will be expected to

F1 choose and evaluate appropriate samples for data collection

F2 identify various types of data sources

Elaboration - Instructional Strategies/Suggestions

F1 One often wishes to gather information about a large population, but does not have the ability to check every person involved. In situations such as these, samples are used. One then generalizes to the entire population, recognizing that conclusions drawn from the sample may not be perfectly true for the entire group, but trying to choose the sample to minimize the degree of error.

Students should consider both how to choose samples and how safe it is to generalize to the full populations. For example, suppose one wanted to determine people's favourite take-out food. It would not be wise to choose a sample of patrons of Pizza Palace. Clearly, that sample could be biased in favour of pizza.

In choosing a sample, students should carefully consider the information being sought and how a person answering (a) question(s) could be biased. For example, if students want to find what radio station is most popular, they should probably consider

- the mix of ages within the sample
- the sex distribution within the sample
- the availability of a variety of stations to those sampled
- the time of day (Some stations are likely more attractive to listeners at a particular time of day).

A sample should be constructed to deal with such potential biases.

F2 Students will realize that although some data is collected first- hand by interviewing or observing, much of the data to which they are exposed is second-hand data. Students should explore, through discussion, how such data might be collected and how reliable they feel it is. For example, if students read that 30% of children in Canada are not physically fit, what might they wonder about the data source? Was a sample used? Were children tested directly or was data collected by asking doctors or teachers? Students should realize that they must be careful about drawing conclusions from reported data. Becoming familiar with sources for different types of data would be valuable to students.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Interview

F1.1 Ask students to describe a situation in which they feel that a sample might be biased.

F1/2.1 Ask students what sample/data source they would use to determine the amount of water an average Canadian uses in a day.

F1.2 Tell students that to judge the popularity of the prime minister, news reporters talked to a number of visitors to Parliament Hill. The reporters felt that their sample was unbiased since people of all age groups were there. Ask the students if they agree? Why or why not?

F1.3 Ask: How would you construct a sample to interview in order to predict a provincial election result?

F2.1 Ask students where they might go to find data about the number of school-aged children in their province.

F1.4 Ask students why a sample of 5-year-olds might not be the best one to find out what playground equipment a school should have.

Presentation

F1/2.2 Have students locate a media story which presents numerical information of some sort about Canadians. The students are to tell how they believe the data was collected and whether or not they believe the public can be reasonably certain that the data is reliable.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) construct a variety of data displays (including tables, charts and graphs) and consider their relative appropriateness

SCO: By the end of grade 6, students will be expected to

F3 plot coordinates in four quadrants

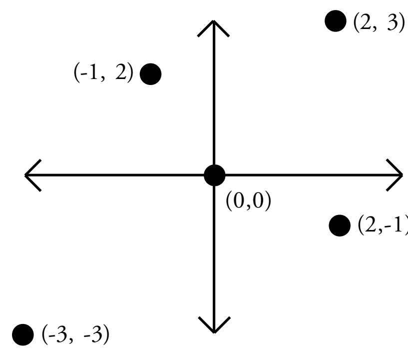
Elaboration - Instructional Strategies/Suggestions

F3 Students should plot data points in all four quadrants.

This could be motivated by using linking geoboards. If one board has all the pegs labelled with coordinates and another board is attached to the left, the new pegs must be labelled.

Students should recognize that

- a negative number for the first coordinate indicates that the point is to the left of the vertical axis
- a negative number for the second coordinate indicates that the point is below the horizontal axis
- the point at which the axes intesect has coordinates (0,0) and is known as the origin.

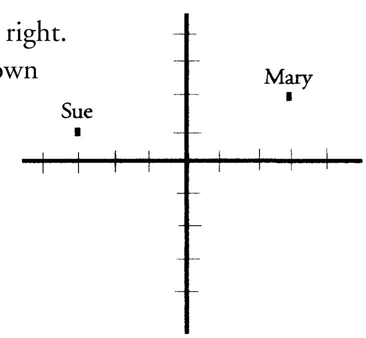


❒ Students might create drawings using all four quadrants of the coordinate grid. They could then provide other students with a list of the vertices, in order, for each drawing created. The other students would subsequently re-create the drawings.

Situations which might be modelled using 4-quadrant graphs include

- high vs low temperatures for different days
- mathematical relationships (e.g., a number vs its double)
- locations, as blocks north, south, east, and west from the town centre.

❒ Show a map (graph) like the one at right. Ask: How many blocks north of town centre does Mary live? How many blocks east? Write Sue's location as an ordered pair.



GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F3.1 Ask students to determine what happens to a plotted shape if all the first coordinates are switched with the corresponding second coordinates (e.g., (3,-2) becomes (-2,3)).

Paper and Pencil

F3.2 Ask the student to describe the relationship among the points (-4, 2), (-2, 1), (0,0) and (2,1).

F3.3 Ask the student to describe where each of these points would be located following a half-turn about the origin: (-3, -5), (3, 6), (-2, 4).

F3.4 Ask students to plot 10 points in quadrant 1 for which the difference between the first and second coordinate is 3. Ask them to identify other points along the continuation of this line which have coordinates with negative values and to list 3 pairs of such coordinates.

F3.5 Ask students to name 5 coordinate pairs in the top left quadrant of a graph.

F3.6 Give the coordinates of a triangle (e.g., (1,2), (3,5) and (4,0)). Ask students to reflect the triangle in the horizontal axis and label the coordinates. Repeat in the vertical axis.

Interview

F3.7 Ask students to plot 10 points for which the first coordinate is the opposite of the second (e.g., (5,-5)). Have them describe the pattern they see and explain why they might have expected that pattern.

F3.8 Ask students to plot the scores on the various holes in this mock golf game and then explain how the graph depicts the performance of the player.

Hole	1	2	3	4	5	6	7	8	9
Score	0	-1	-2	-1	0	2	1	1	0

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *construct a variety of data displays (including tables, charts and graphs) and consider their relative appropriateness*

SCO: By the end of grade 6, students will be expected to

F4 **use bar graphs, double bar graphs and stem-and-leaf plots to display data**

Elaboration - Instructional Strategies/Suggestions

F4 Students should regularly use bar graphs, double bar graphs, and stem-and-leaf plots to display and organize data. Data can be collected in surveys, through experiments or through research. Topics may include areas of mathematics, other curricular areas and real-life situations.

For example, students might gather information about the ages of their grandparents and display it in either a double bar graph or stem-and-leaf plot.

Student's Grandparents

Student's Grandparents

4	5 6 6
5	0 0 1 1 1 2 3 5 5 5 6 6 7 7 7
6	1 1 1 2 3 5 5 8 9
7	0 0 1 2 5

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F4.1 Ask the student to create a graph which illustrates both the first and second choices of favourite sports of class members.

F4.2 Ask the student to draw a bar graph to compare the number of calories used by an adult female in one hour for each activity listed below:

Sleeping	55
Walking	180
Walking uphill	360
Running	420

Interview

F4.3 Ask: What scale would you use to graph the following data?

Category A- 25
Category B- 1000
Category C -5000.

F4.4 Ask the students to survey their classmates to find out the arm spans lengths and leg lengths of members of the class. Have a group of students create a stem-and-leaf plot for each of female arm spans, female leg lengths, male arm spans and male leg lengths. The various graphs could then be compared. Other students could create double bar graphs to contrast male and female data.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO ii) By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) construct a variety of data displays (including tables, charts and graphs) and consider their relative appropriateness

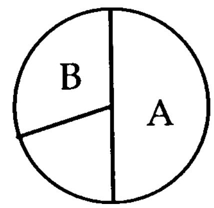
SCO: By the end of grade 6, students will be expected to

F5 use circle graphs to represent proportions

A "circle" or "pie" graph is used when a total amount has been partitioned into parts and interest is in the ratio of each part to the whole and not so much in the particular quantities. (Elementary School Mathematics, p. 396)

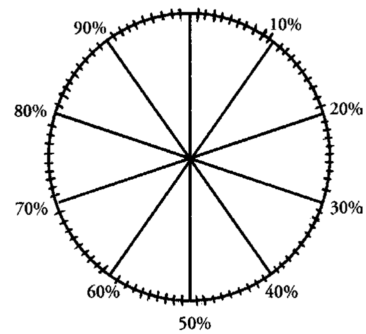
Elaboration - Instructional Strategies/Suggestions

F5 Students should realize that circle graphs are used to describe how a whole is distributed into its component parts. Students should be able to estimate percents when shown a circle graph. For example: A is about 50%. B is about 30%.



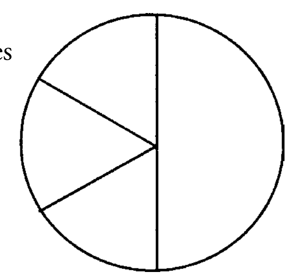
There are many easy ways to construct a circle graph:

- circle mats divided up into tenths and hundredths



- a strip of equal-sized squares shaded by categories, taped together to make a circle, with lines drawn from the centre of the circle to the positions where the categories change on the strip

- fraction circle pieces



It is important that students understand that a circle graph describes relative size, but not actual size. If, for example, circle graphs were created to show the age distributions of people in New Brunswick and Prince Edward Island, it would not be apparent from the graphs that there are more people in NB than in PEI.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

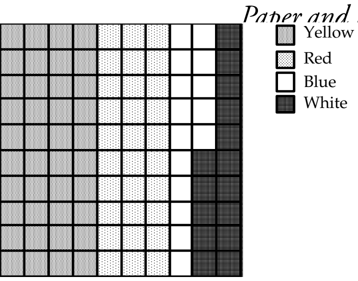
F5.1 Ask students to research the percentage of people speaking various languages in a given province in Canada. Using estimates of the percentages, ask students to create a circle graph showing the distribution.

F5.2 Ask students to draw a circle graph that they predict will show the distribution of favourite type of music of classmates, if the classmates choose among country, rock, and classical. Then let them gather data, draw the actual circle graph and see how well they predicted.

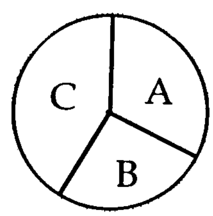
F5.3 Ask students to draw a circle graph to show the country of origin of baseball or hockey players from a favourite team.

F5.4 Have the student convert the information on the hundredths charts to a circle graph.

Paper and Pencil



:he student to estimate the percent for each segment of the
:: What might be the topic of the graph?



Why do you think circle graphs are often
ow how a government is spending its
budget?

F5.7 Ask the student to describe a situation for which he/she might use a circle graph instead of a bar graph.

F5.8 Ask whether or not the various sections of a particular circle graph can be 35%, 25%, 30%, and 15%? Explain.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

iii) read, interpret, and make and modify predictions from displays of relevant data

SCO: By the end of grade 6, students will be expected to

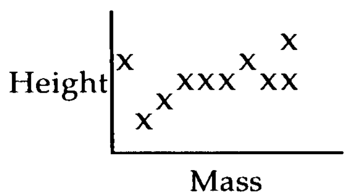
F6 interpret data represented in scatterplots

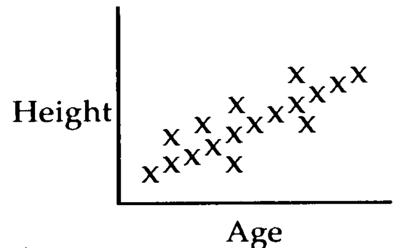
F7 make inferences from data displays

Characterizing the shape of the data, as if you were going to sculpt this shape with clay, is a prerequisite to summarizing and theory building. Paying attention to the shape of the data may be the most important idea we can communicate to students about data analysis. When you look at a table or graph, what strikes you about the data? Where are the data clumped? Is there more than one clump? Are there bumps of data in surprising places? Are there holes that contain no data? (<u>NCTM 1989 Yearbook,</u> pp. 138-39)

Elaboration - Instructional Strategies/Suggestions

F6 Scatterplots are used to show the relationship between two quantities. The plot is made up of "scattered" points, which are ordered pairs. Each ordered pair tells the simultaneous value of the two quantities. For example, the plot might show the height for people of different masses. Each ordered pair would be (mass, height of a person with that mass). Or the plot might show height for people of different ages. Each ordered pair would be (age, height for a person with that age).



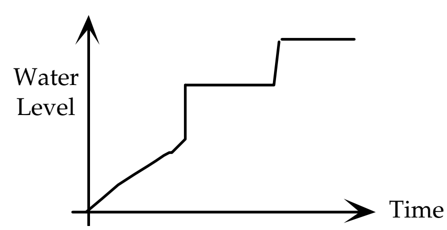


Other topics for scatterplots include

- temperature at different times of day (plotting hour against temperature) and
- tree height for different aged trees (plotting height against age).

Students should observe that fairly clear relationships are often shown (even though a few pieces of data may not fit the relationship well), and be able to describe these relationships.

F7 Students are usually intrigued by unusual graphs. For example, the graph bellow displays the level of water as someone begins to take a bath.



In this type of situation, students should each tell a "story" about the graph, describe what each thinks happened to lead to the shape of the graph.

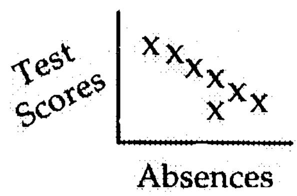
Students should also be drawing inferences from more conventional graphs and tables.

GCO F: Students will solve problems involving the collection, display and analysis of data.

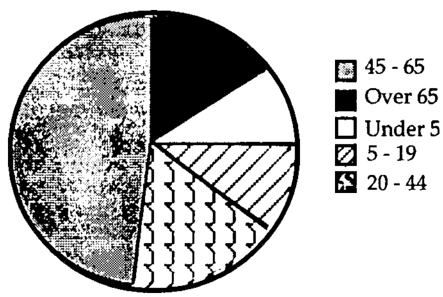
Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

F6.1 Ask: What conclusion might be drawn from this scatterplot?



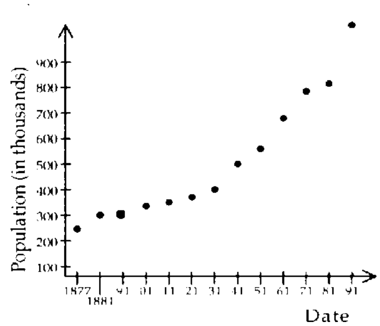
F7.1 Show this circle graph depicting ages of Canadians. Ask: What information can be gathered from the graph?



Interview

F6.2 Ask: How is a scatterplot like other graphs and how does it differ?

F6.3 Show a scatterplot of provincial populations in various years. Ask: What conclusions can be drawn?



Portfolio

F7.2 Ask students to collect a variety of graphs from the media and organize them by topic. Have them create a brief report on the information provided in the visuals.

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

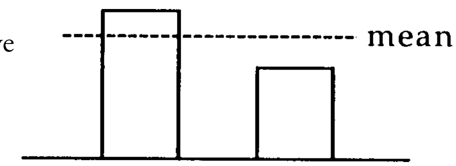
iv) develop and apply measures of central tendency (mean, median and mode)

SCO: By the end of grade 6, students will be expected to

F8 demonstrate an understanding of the difference between mean, median and mode

Elaboration - Instructional Strategies/Suggestions

F8 Students have previously encountered the concept of mean, i.e., the average calculated by taking the total amount and sharing it equally. They have also already seen that the total amount above the mean balances the total amount below.



The median is another type of average. It tells the middle number in a set of data. Students should recognize that the mean and median

- may be the same

3, 4, 5   4
↑   ↑
median   mean

- may be different

3, 4, 8   5
↑   ↑
median   mean

The mode is yet another type of average, in some ways the easiest to determine. It is the piece of data that appears most often. For example, consider the following data:

5 5 5 5 10 3 3

The mode is 5 (which appears 4 times). In this case, the median is also 5, but the mean is not.

Students might explore the "stability" of the mean and median. For example, ask them to compute both statistics for 3 , 10, 15, 22, 45, and also for 3, 10, 15, 22, 100. They will see that an "odd piece of data" has much more impact on the mean than on the median or mode.

Students might find situations for which averages are described and try to decide whether it was a mean, median, or mode that was being reported in each case. For example, a baseball player's batting average is a mean. If the average price of a house in a particular subdivision is reported, however, it might more likely be a median or a mode. Often, mode might be used to describe an average shoe size.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Paper and Pencil

F8.1 Ask the student to create a set of 3 numbers for which the mean is a lot less than the median.

F8.2 Ask students to change just one piece of the following data to increase the median:

2, 3, 4, 5, 6.

F8.3 Ask the students to create 2 different sets of data, each with a mode of 3, but one for which the mean is the same as the median and one for which it is not.

F8.4 Ask: Are the mean and median of this data the same?

30, 35, 37, 39, 49

Interview

F8.5 Ask the students to identify situations for which the mean is usually greater than the median.

F8.6 Ask the student for an example of a situation in which it might be difficult to determine a mode.

F8.7 Ask: Do you believe that the mean or the median is the most appropriate average to use to describe scores on a test? Why?

F8.8 Tell the student that an average amount of TV viewing time was reported as 20 hours a week. Ask: Which average do you think is being used? Why?

Suggested Resources

GCO F: Students will solve problems involving the collection, display and analysis of data.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

v) *formulate and solve simple problems (both real-world and from other academic disciplines) that involve the collection, display and analysis of data and explain conclusions which may be drawn*

SCO: By the end of grade 6, students will be expected to

F9 explore relevant issues for which data collection assists in reaching conclusions

There are many opportunities to include statistics in the sixth-grade curriculum. In so doing, students review many mathematical ideas, relate mathematics to the real world, and extend their ideas about statistics. (Curriculum and Evaluation Standards, Addenda Series, Sixth-Grade Book, p. 16)

Elaboration - Instructional Strategies/Suggestions

F9 At this level, students should continue to think about how to collect and display data, but might be concentrating on the analysis of data. In particular, students might consider how to and whether to interpolate or extrapolate from provided data.

Interpolating involves describing data between existing pieces of information. Extrapolating involves extending beyond the presented information.

For example, suppose students have the following information taken from a survey in a certain school district:

Grade	1	2	3	4	5	6	7
Average amount of homework (in minutes)	10	20	30	50	60	60	90

Students might describe what the data tells us (that older students generally do more homework, but there is not necessarily an increase each grade).

They might also consider whether they could use the data to determine what is an average amount of homework at grade 8 and what the dangers might be in extrapolating.

❒ Provide the following data:

Minutes spent on math homework per day	10	30	60	90	120
Score on math test	50	80	90	70	70

Ask students to draw a scatterplot and ask whether they could predict a math test score of students who do 20 minutes of homework per day. Have the students explain their reasoning.

GCO F: Students will solve problems involving the collection, display and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Performance

F9.1 The following data describes the percentage of the population of Canada that was rural in different years:

Year	1961	1966	1971	1976	1981	1986	1991
Percent rural	30.4	26.4	23.9	24.5	24.3	23.5	23.4

Ask students to draw a scatterplot and analyse the data. Ask them to predict what the value was in 1996 and in 1982, and to comment on the degree of confidence they have in their predictions.

Presentation

F9.2 Ask students to find an Internet site where data is displayed about the attendance at sports events for a particular team over a period of years. Ask the students to display the data. Ask: Could you use the information to predict the attendance in future years?

F9.3 Have students collect and display information about the change in the cost of postage stamps over the last 50 years. Then ask them to predict the cost to mail a letter in the year 2020, based on the data. Students should justify their predictions.

F9.4 Groups of students should each select a question to which they would like an answer. They should then collect data and display their findings. Examples include:

- What is the level of physical exercise of eleven-year-olds in our province?
- In what ways does one school differ from another?
- What are the proportions of various problems that a local doctor or hospital treats?

Suggested Resources

Data Management and Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

i) *explore, interpret and make conjectures about everyday probability situations by estimating probabilities, conducting experiments, beginning to construct and conduct simulations, and analysing claims which they see and hear*

SCO: By the end of grade 6, students will be expected to

G1 conduct simple simulations to determine probabilities

G2 evaluate the reliability of sampling results

G3 analyse simple probabilistic claims

A simulation is a technique used for answering questions or making decisions in complex situations where an element of chance is involved. A simulation is very much like solving a probability problem by an experimental approach. The only difference is that one must design a model that has the same probabilities as the real situation. (Elementary School Mathematics, p. 390)

Elaboration - Instructional Strategies/Suggestions

G1 Having had experience directly determining experimental probabilities, students should be introduced to simulations, experiments which indirectly model a situation. For example, if the student knows that a basketball player makes her free throws 8 times in 10, the student can determine certain probabilities related to this statistic even without the player. For example, one might create a spinner for which 0.8 of the face is labelled WIN and 0.2 is labelled MISS. By spinning a number of times, the student can find out, for example,

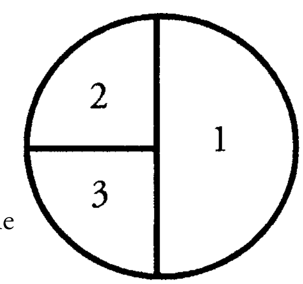
- the probability of making exactly 3 shots in the next 5 tries
- the probability of missing the first shot, but making the next 3 in a row
- the probability of missing 5 shots in a row

These probabilities may be stated as fractions, decimals or percents.

Another well-known simulation is designed to determine approximately how many boxes of cereal will need to be purchased before a consumer collects each of six possible prizes contained therein. This simulation can be performed by

- rolling a die
- recording the prize number won (based on the roll of the die)
- continuing until at least one of each number is rolled
- repeating the experiment several times and
- determining, on average, the number of rolls (purchases) required.

G2 It is important that students recognize that more data from larger samples generally produces more reliable probabilities. For example, if a student spins this spinner 10 times, it is less likely that the empirical probability of getting a 1 will be $\frac{1}{2}$ than if the student spins 100 times.



G3 Students at this age are generally sophisticated enough to interpret some of the probabilistic claims they hear. For example, if students hear that the probability of rain is 100%, yet see the sun is shining, they might wonder about the basis of the claim. Or, if students hear that the chance of rain on Saturday is 50% and on Sunday is 50%, they will realize that an interpretation of the probability for rain on the weekend of 100% does not make sense.

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G1.1 Tell the student that a particular baseball player has an average of .250, i.e., he gets 1 hit in 4 times at bat, on average. Ask the student to conduct a simulation to determine the probability that the player will get a hit each time at bat in a particular game.

G2.1 Ask the student to roll a die 12 times and record the probability of getting a 1. Then ask the student to group his/her data with other students and recalculate the probability of getting a 1. Ask: What do you notice?

Paper and Pencil

G1.2 Ask the student to develop a simulation to determine the probability that, in a family planning three children, all the children will be boys.

Interview

G3.1 Ask the student to comment on this hypothetical advertisement: "If you buy a lottery ticket, you're bound to win."

G1.3 Ask: Why would you use a die to conduct a simulation to determine the number of cereal boxes you would need to purchase to collect each of six possible prizes, but a different device if there were 10 possible prizes?

G2.2 Ask: How much data about people's hair color might it be necessary to collect before you predict the probablility that a randomly choosen individual will be blonde? Explain.

Presentation

G3.2 Ask the student to collect probability claims from news media and comment on how believable they are and why.

Suggested Resources

GCO G: Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to

ii) *determine theoretical probabilities using simple counting techniques* **and**

iii) *demonstrate an understanding of the relationship between the numerical expression describing a probability and the events which give rise to the numbers*

SCO: By the end of grade 6, students will be expected to

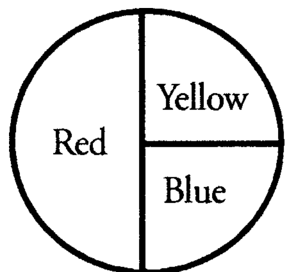
G4 determine theoretical probabilities

G5 identify events that might be associated with a particular theoretical probability

Elaboration - Instructional Strategies/Suggestions

G4 Students will often be presented with situations for which outcomes are equally likely. In these cases, they should list the outcomes and count the number of items on the list to determine probabilities. Students must also recognize, however, when outcomes are not equally likely and take this into account.

For example, using the spinner shown,



the student might list the outcomes as "red," "yellow" and "blue" and assume that since there are 3 outcomes, each has a probability of $\frac{1}{3}$. This, however, is not the case. Students might benefit from reconfiguring the situation to show equally likely outcomes by dividing the red section into two equal pieces. Now the outcomes might be "red 1," "red 2," "yellow" and "blue" and each outcome does have a probability of $\frac{1}{4}$. Because there are two red sections, the probability of red is, therefore, $\frac{2}{4}$.

Students might be encouraged to use percentages or decimals instead of, or as well as, fractions to describe probabilities. Therefore, a probability might be reported as 30% rather than as $\frac{3}{10}$.

G5 Conversely, by listing outcomes, students should consider what events might be associated with a certain probability. For example, when rolling a die, a probability of $\frac{1}{2}$ might be associated with rolling

- an even number
- a number greater than 3
- a number that is not a factor of 4

❒ Tell students that the probability of an event involving choosing a number between 1 and 100, inclusive, is $\frac{3}{4}$. Ask them to describe what the event might be. (Note: Possible answers include choosing a number greater than 25, choosing a number less than 75, and choosing a number that is not prime.)

GCO G: Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G4.1 Ask the student to create a spinner for which there are 6 equally likely outcomes and another spinner for which the 6 outcomes are not equally likely.

Paper and Pencil

G5.1 Provide a hundreds chart. Ask: What characteristic of numbers might lead you to say that the probability of choosing that type of number is $\frac{1}{5}$?

G4.2 Ask the student to list the equally likely outcomes that result when two dice are rolled and the numbers are subtracted.

G4.3 Ask the student to list the equally likely outcomes that result when two cubes are pulled from a bag with 10 red cubes and 5 blue ones.

Interview

G5.2 Ask the student to think about rolling a die. Ask: What might you expect to happen about $\frac{1}{3}$ of the time?

Presentation

G5.3 Ask students to work in groups to develop 5 different scenarios, one each to model a situation in which the probability is one of

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6}$$

Different equipment should be used in each different situation.

Suggested Resources

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