Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

CURRICULUM

Mathematics

Grade 7

1999

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Acknowledgments

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- The Provincial Curriculum Working Group, comprising teachers and other educators in Newfoundland and Labrador which served as lead province in drafting and revising the document.
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the development of the grade 7 mathematics curriculum guide.

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I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their learning experiences.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision, which embraces the principles of students learning to value and become active "doers" of mathematics and advocates a curriculum which focusses on the unifying ideas of mathematical problem solving, communication, reasoning, and connections. The *Foundation for the Atlantic Canada Mathematics Curriculum* establishes a framework for the development of detailed grade-level documents describing mathematics curriculum and guiding instruction.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers with department of education officials to co-operatively plan and execute the development of curricula in mathematics, science, and language arts in both official languages. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the Essential Graduation Learnings (EGLs), also developed regionally. These EGLs and the contribution of the mathematics curriculum to their achievement are presented in the "Outcomes" section of the mathematics foundation document.

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. This curriculum guide is one of several which provide greater specificity and clarity for the classroom teacher. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—General Curriculum Outcomes (GCOs), which relate to subject strands, and Key-Stage Curriculum Outcomes (KSCOs), which articulate the GCOs further for the end of grades 3, 6, 9, and 12. This guide builds on the structure introduced in the foundation document, by relating Specific Curriculum Outcomes (SCOs) to each KSCO at each grade level. Figure 1 further clarifies the outcome structure.

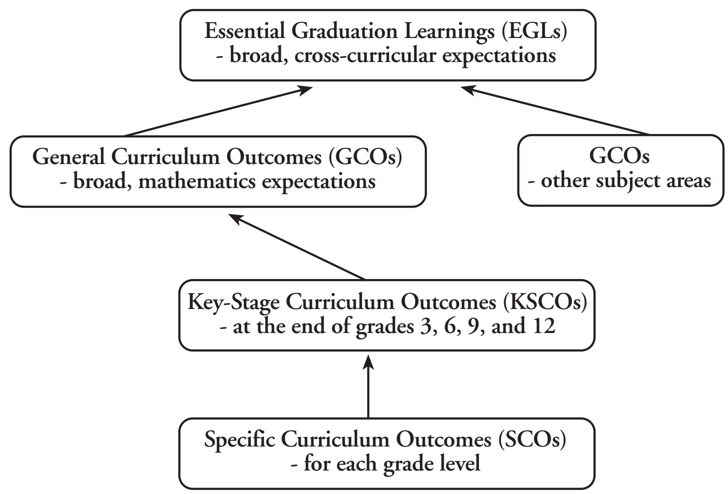


Figure 1: Outcome Framework

This mathematics guide is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice, including the following: (i) mathematics learning is an active and constructive process; (ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; (iii) learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort; (iv) learning is most effective when standards of expectation are made clear and assessment and feedback are ongoing; and (v) learners benefit, both socially and intellectually, from a variety of learning experinces, both independent and in collaboration with others.

II. Program Design and Components

A. Program Organization

As indicated previously, the mathematics curriculum is designed to support the Atlantic Canada Essential Graduation Learnings (EGLs). The curriculum is designed to significantly contribute to students meeting each of the six EGLs, with the communication and problem-solving EGLs relating particularly well with the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then goes on to present student outcomes at key stages of the student's school experience.

This curriculum guide presents specific curriculum outcomes at individual grade levels. As illustrated in Figure 2, these outcomes represent the step-by-step means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and, ultimately, the essential graduation learnings.

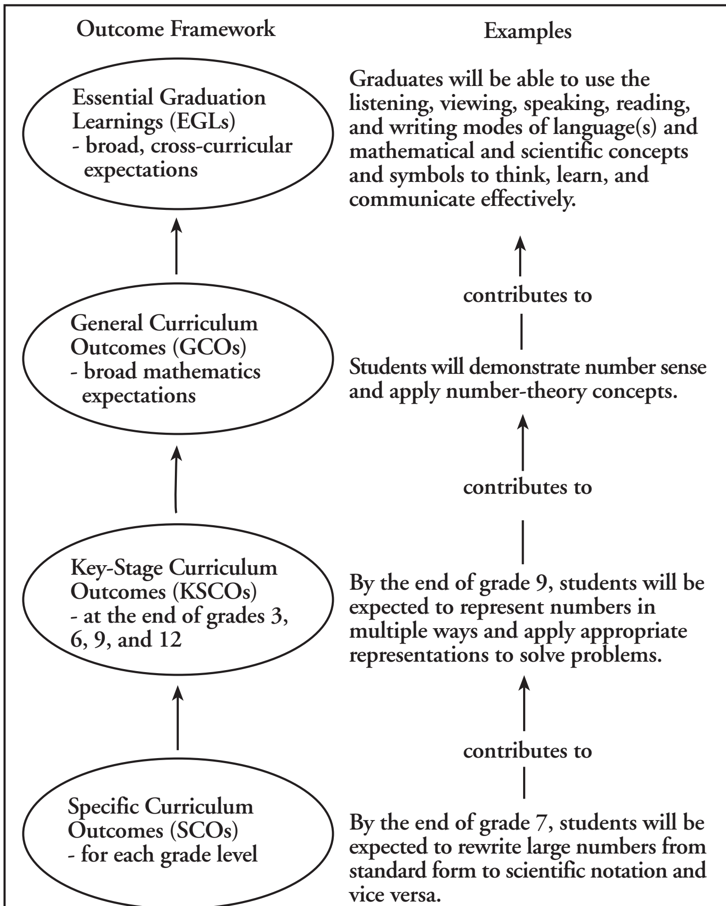


Figure 2: Examples of Outcomes

It is important to emphasize that the presentation of the specific curriculum outcomes at each grade level follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and **does not necessarily represent a natural teaching sequence**. While some outcomes will of necessity need to be addressed before others due to prerequisite skill requirements, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes (e.g. Patterns and Data Management) may be best addressed on an on-going basis in connection with other topics. It is expected that teachers will make individual decisions as to what sequence of topics/outcomes will best suit their classes. In most instances, this will occur in consultation with fellow staff members, department heads, and/or district level personnel.

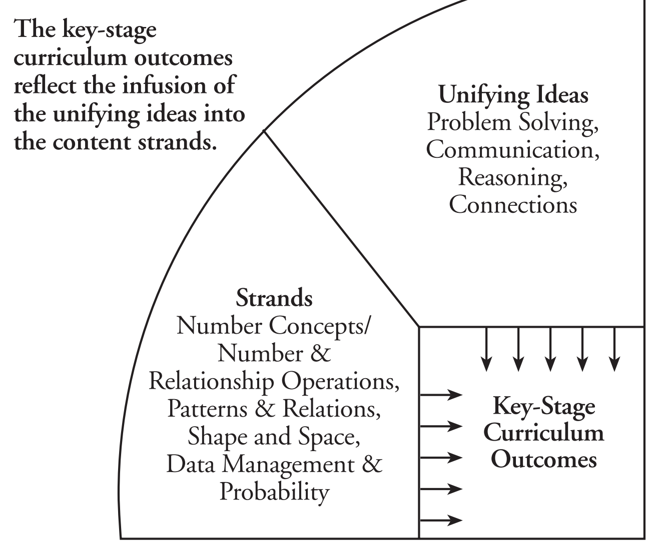
Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a “kickoff” topic for one group of students might be less effective in that role with a second group. Another consideration with respect to sequencing will be co-ordinating the mathematics program with other aspects of the students’ school experience. Examples of such co-ordination include studying aspects of measurement in connection with appropriate topics in science, data management with a social studies issue, and some aspect of geometry with some physical education unit. As well, sequencing could be influenced by other events outside of the school, such as elections, special community celebrations, or natural occurrences.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. This is illustrated in Figure 3.

These unifying concepts serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode; classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically; via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning; and mathematics with which students are involved on a day-to-day basis must be connected to other mathematics, other disciplines, and/or the world around them.

Figure 3 - Integrating Strands and Unifying Ideas



Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling, whereas working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire in the later elementary years. In grades 7-9, this repertoire will be extended to include such strategies as interpreting formulas, checking for hidden assumptions, examining systematic or critical cases, and solving algebraically.

Opportunities should be created frequently to link mathematics and career opportunities. During these important transitional years, students need to become aware of the importance of mathematics and the need for mathematics in so many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in higher-level mathematics programming in senior high mathematics and beyond.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline which lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, and actively participate in discourse and conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas in which reasoning and sense making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Adapting to the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness as they enter the intermediate setting and as they progress, but they must also remain aware of avoiding gender and cultural biasses in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes for each grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Further, the practice of designing classroom activities to support a variety of learning styles must be extended to the assessment realm; such an extension implies the use of a wide variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This curriculum guide represents the central resource for the teacher of mathematics for these grade levels. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes should be met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent that they support the curriculum goals. Teachers will need professional resources as they seek to broaden their instructional and mathematical skills. Key among these are the NCTM publications, including the *Assessment Standards for School Mathematics, Curriculum and Evaluation Standards for School Mathematics*, the *Grades 5-8 Addenda Series*, *Professional Standards for Teaching Mathematics,* and the various NCTM yearbooks. As well, manipulative materials and appropriate access to technological resources (e.g. software, videos) should be available. Calculators will be an integral part of many learning activities.

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different than were those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their children's lives, assisting children with mathematical activities at home, and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through learning centres, teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

The concepts and skills developed in mathematics are applied in many other disciplines. These include science, social studies, music, technology education, art, physical education, and home economics. Efforts should be made to make connections and use examples which apply across a variety of discipline areas.

In science, the concepts and skills of measurement are applied in the context of scientific investigations. Likewise, statistical concepts and skills are applied as students collect, present, and analyse data.

In social studies, measurement is used to read scale on a map, to measure land areas, and in various measures related to climatic conditions. As well, students read, interpret, and construct tables, charts, and graphs in a variety of contexts such as demography.

In addition, there are many opportunities to reinforce fraction concepts and operations in music, as well as opportunities to connect concepts such as symmetry and perspective drawings of art to aspects of 2-D and 3-D geometry.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are critical, not only with respect to clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers make meaningful instructional decisions. (See "Assessing and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment should include the following: i) using a wide variety of assessment strategies and tools; ii) aligning assessment strategies and tools with the curriculum and instructional techniques; and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborate good assessment practice and serve as a guide with respect to student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address such questions as the following: Are students meeting the curriculum outcomes? Is the curriculum being equitably applied across the region? Does the curriculum reflect a proper balance between procedural knowledge and conceptual understanding? Is technology fulfilling a proper role?

IV. Designing an Instructional Plan

It is important to design an instructional plan for the school year. This plan should reflect the fact that specific curriculum outcomes (SCOs) falling under any given general curriculum outcome (GCO) should not be taught in isolation. There are many opportunities for connections and integration across the various strands of the mathematics curriculum.

Consideration should be given to the relative weighting for outcomes under each GCO so that this can be reflected in the amount of time devoted to each aspect of the curriculum. Naturally, time spent must be sensitive to the background of students as well as to cross-curricular issues. Without an instructional plan, it is easy to run out of time in a school year before all aspects of the mathematics curriculum have been addressed. A plan for instruction that is comprehensive enough to cover all outcomes and strands will help to highlight the need for time management.

It is often advisable to use pre-testing to determine what students have retained from previous grades relative to a given set of outcomes. In some cases, pre-testing may also identify students who have already acquired skills relevant to the current grade level. Pre-testing is often most useful when it occurs one to two weeks prior to the start of a set of outcomes. In this case, a set of outcomes may define a topic or unit of work, such as fraction concepts and operations. When the pre-test is done early enough and exposes deficiencies in prerequisite knowledge/skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of adequate development or coverage in the previous grades. This may imply that an adjustment is required to the starting point for instruction, as well as a meeting with other grade level teachers to address these concerns is necessary.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in an overall savings in time for both disciplines. The most obvious of these connections relate to the use of measurement in science and the use of a variety of data displays in social studies.

V. Curriculum Outcomes

The pages that follow provide details regarding specific curriculum outcomes. As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching GCOs and KSCOs of the mathematics foundation document. The specific curriculum outcomes are presented on individual two-page spreads. See Figure 4 on next page.

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development. Given that the specific curriculum outcomes at each grade level are related to the key-stage curriculum outcome framework, it is relatively easy to access a given KSCO at the previous grade and/or the next one to see how the development of particular mathematical ideas are taking place.

Within a grade level, the specific curriculum outcomes are presented on individual two-page spreads. At the top of each page, the overarching GCO is presented, with the appropriate KSCO(s) and SCO(s) displayed in the left-hand column. The KSCO(s) are in italics while the SCO(s) are bold-face. The second column of the layout is entitled "Elaboration-Instructional Strategies/Suggestions" and provides a clarification of the specific curriculum outcome(s), as well as some suggestions of possible strategies and/or activities which might be used to achieve the outcome(s). While the strategies and/or suggestions presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning, and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol ❐.

The third column of the two-page spread, "Worthwhile Tasks for Instruction and/or Assessment," might be used for assessment purposes or serve to further clarify the specific curriculum outcome(s). As well, those tasks regularly incorporate one or more of the four unifying ideas of the curriculum. These sample tasks are intended as examples only, and teachers will want to tailor them to meet the needs and interests of the students in their classrooms. The final column of each display is entitled "Suggested Resources" and will, over time, become a collection of useful references to resources which are particularly valuable with respect to achieving the outcome(s).

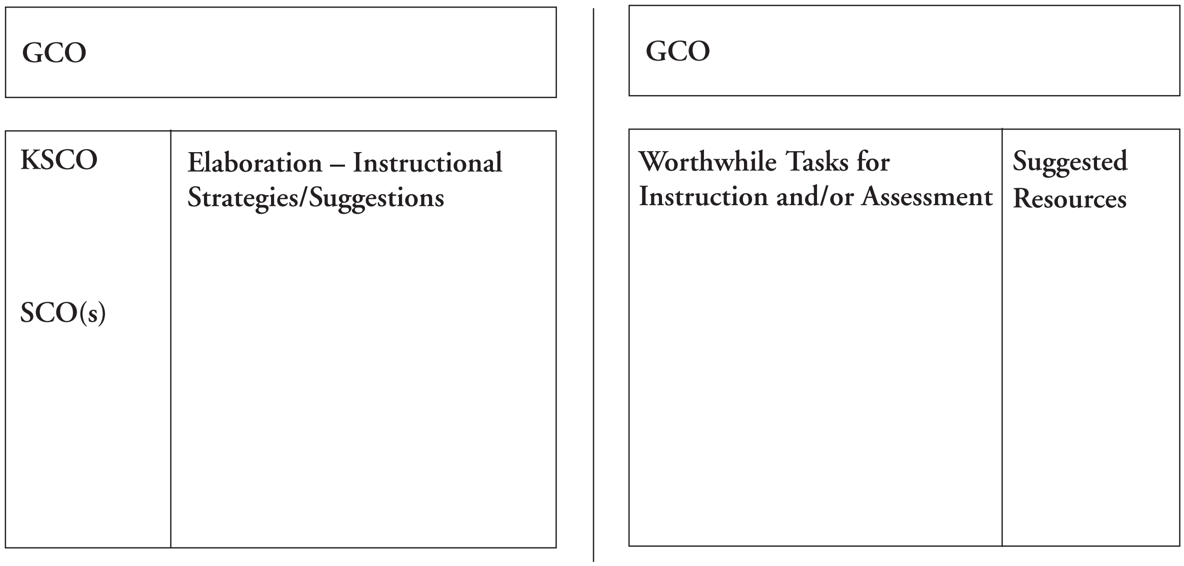


Figure 4: Layout of a 2-Page Spread

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number-theory concepts.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A1 model and use power, base, and exponent to represent repeated multiplication

A2 rename numbers among exponential, standard, and expanded forms

Elaboration – Instructional Strategies/Suggestions

A1/2 In grade 7, students are introduced to exponents as a means of expressing factors in a compact form. This knowledge will be essential in working with numbers represented in scientific notation. In fact, students have probably already had experience with exponents informally. The terms exponent, base, and power will require explanation. Students should be exposed to the terms "squared" and "cubed" to describe powers of two and powers of three. They should be able to link the term "squared" with a 2-D area model and "cubed" with the 3-D volume model. This will provide support for aspects of measurement and geometry that are also studied at this grade level and will add understanding to the ways of writing area and volume units (e.g., square centimetres as cm^2, cubic metres as m^3). It should be emphasized that the same number can be expressed in multiple ways using exponents (e.g., $64 = 8^2$ or 4^3 or 2^6).

Since students have already worked with base-ten blocks to model place value, they can relate this understanding to powers of 10. 10^1 can be represented by a base-ten rod, 10^2 by a base-ten flat, and 10^3 by a large cube. Students can then extend this model to visualize a long, made up of ten large cubes to represent 10^4, and a flat, made from a hundred large cubes to represent 10^5. Students can work backwards from the use of this model to develop the understanding that 10^0 is represented by the small cube (unit cube) and therefore $10^0 = 1$.

This model can be extended to numbers less than one by choosing the large cube to represent one. In this case, 0.1 or $\frac{1}{10}$ would be represented by a flat, 0.01 or $\frac{1}{100}$ or $\frac{1}{10^2}$ would be represented by a long, and so on.

Students can use expanded forms of numbers to demonstrate understanding of place value as well as exponents. Students can show their understanding of place value when they start with a number written in standard form, such as 40 502 or 400.03, and expand the numbers as follows:

40 502 $4\times10^4+5\times10^2+2\times10^0$

400.03 $4\times10^2+3\times\frac{1}{10^2}$

Note: Students will study negative exponents in grade 8.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A1.1 Ask students to

a) demonstrate the first three powers of ten, using base-ten blocks
b) explain how to model the second three powers of ten
c) sketch a model to demonstrate the first four powers of 2

A1.2 Ask students to model the difference between 3^2 and 2^3.

Pencil and Paper

A2.1 Find three ways to express the number 48 that involve the use of exponents.

A2.2 The following numbers are written in expanded form. Write the standard number for each.

a) $4\times10^6+4\times10^5+2\times10^3$
b) $6\times10^2+4\times10^0+2\times\frac{1}{10^3}$
c) $4\times10^3+4\times10^1+37\times10^0$

A2.3 Write in expanded form the distance from Earth to the moon, which is given as 384 000 km.

A1.3 John wants to use his calculator to find 9^4, but the 4 key is missing.

a) Explain how he can use the calculator to find the answer to this question even though the 4 is missing.
b) Suppose the 9 key is missing instead. Explain how he might now use the calculator to find the answer.

Interview

A2.4 Explain to students that Susan was asked to find the volume of a cube that was 4 cm on a side. She wrote: 4^3 as $3\times3\times3\times3=81$.
Ask students if the solution is correct and also to explain why or why not.

A1.4 Tell students that 10^3 has 4 digits. Ask them how many digits 20^3 and 40^3 have. Ask why.

Portfolio

A1.5 Ask students to find a value for □ and a value for * which would make the following sentence true: $3^{\square} = 9^{*}$. Ask if there are other values for □ and * that would work.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A3 rewrite large numbers from standard form to scientific notation and vice versa

Elaboration – Instructional Strategies/Suggestions

A3 This is the first time scientific notation has been studied. Discussion should focus on when large numbers, typically written in scientific notation, occur. Examples include world population, land areas for large countries or continents, national budgets, national debts, distances between planets, and distances of various planets from the sun.

❒ Give students two numbers such as 55 000 000 and 8 000 000 and ask them to multiply the numbers, using the calculator. They will probably see an answer written as 4.4 e14.

This answer provides a good basis for discussion about the reason scientific notation is so important. Through multiplying very large numbers with a calculator, students can see how the calculator changes the form of a number. They should eventually conclude that, when most calculators display numbers with more digits than the calculator can hold, the format for the result is consistent. It will be necessary to explain to students what this calculator format means. That is, when they see 4.4e14, it means 4.4×10^{14}. When a number is expressed in scientific notation, it is written as a number between 1 and 10 multiplied by a power of 10.

In earlier grades, students learned about the various ways large numbers can be expressed; for example, 6 200 000 can be written as 6.2 million. Writing the number as 6.2 million provides a good starting point for an introduction to scientific notation, since 6.2 million = 6.2×10^6.

Study will be limited to large numbers and thus will only require positive exponents. While students do significant work with negative numbers at this grade level, the concepts associated with negative exponents will be introduced in grade 8.

Significant attention should be given to how various calculators display scientific notation. Students should be aware of the different ways that calculators display error messages and exponents in scientific notation so that they do not confuse them.

Students should also have experience changing numbers such as 34.5×10^6 to scientific notation. They should be able to immediately relate 34.5×10^6 and 3.45×10^7 as being the same number and understand why only one of them is written in scientific notation. Students should understand that recording numbers in a consistent format makes comparison easier.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A3.1 Write a sentence, using each of the following numbers, to provide a reasonable context for these numbers. Give each of the numbers a unit of measure and make sure that the context and the unit used are a reasonable match for each other.

a) 3.25×10^3
b) 3.1×10^5
c) 3.06×10^8

A3.2 Rewrite each of the following, using scientific notation:

a) Light travels at 300 000 000 m/s.
b) The sun is 14 600 000 000 m from Earth.
c) The human body has about 100 billion cells.

A3.3 Rewrite each of the following, using scientific notation and also in standard form:

a) The population of the earth is about 57×10^8 .
b) The size of the universe is estimated to be 40×10^9 light years.

Presentation

A3.4 Ask students to explain to the class the usefulness of writing numbers in scientific notation, and to research situations where numbers are so large that they are more conveniently handled in this form.

Portfolio

A3.5 Ask students to include a sample of numbers written in scientific notation which they find in magazines or other subject areas such as science.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *apply number-theory concepts in relevant situations and explain the interrelated structure of whole numbers, integers, and rational numbers*

SCO: By the end of grade 7, students will be expected to

A4 **solve and create problems involving common factors and greatest common factors (GCF)**

Elaboration – Instructional Strategies/Suggestions

A4 An understanding of common factors and GCF will be useful to students in renaming fractions in lowest terms and in problem-solving situations.

It is often useful to pose a problem as a starting point for instruction; for example,

❒ Sue's uncle donated 96 juice packs and 64 chocolate treats for her party. What is the largest number of people that can be at the party and share the food equally (without breaking any packs or treats)?

Students worked with factors, common factors, and prime numbers in previous grades. It may be necessary to review the terms factor, common factor, and prime number before extending to prime factorization. The prime factorization method and the listing of factors method should be considered in developing GCF as well as methods developed by students. To find the GCF of 12 and 18 using prime factorization, first write the prime factors for each number: $12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$

Then choose factors common to both numbers and multiply them to get the GCF. The GCF of 12 and 18 is $2 \times 3 = 6$.

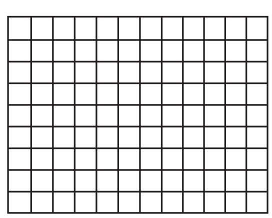
To find the GCF of 24 and 36 using listing of factors, list the factors for each number: 24 = 1, 2, 3, 4, 6, 8, 12, 24
36 = 1, 2, 3, 4, 6, 9, 12, 18, 36

Circle the factors in common and select the greatest common factor, which in this case is 12.

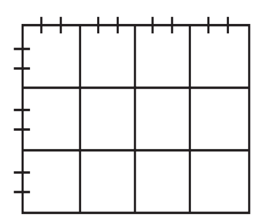
Students should be exposed to a variety of methods. Some students may relate better to a visual explanation of GCF; for example,

❒ Ask students to create a rectangle of dimensions $a \times b$ where a and b are the numbers for which we want to find the GCF. The GCF is the dimension of the largest square with whole number side lengths that can be used to exactly tile this rectangle.

I



II



The 9×12 rectangle shown above (diagram I) can be covered exactly using 1-unit squares. However, the largest square which can be used to tile the area is shown as diagram II. Each unit tile is a 3 by 3 square. The dimensions of this square equal the GCF of 9 and 12.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A4.1 Chris is filling loot bags for his sister's birthday party. She has 24 pogs, 36 caramels, and 60 chocolate treats. What is the largest number of bags which can be filled if all the treats are to be used, no treats are left over or subdivided, and all children receive the same things in their loot bags?

A4.2 John is creating a miniature, quilted wall hanging which is made up of square blocks. He wants the wall hanging to be exactly 15 cm by 20 cm.

a) Find the size of the largest finished block with whole number side lengths which can be used to exactly cover the area.
b) Find the GCF of 15 and 20.
c) Compare your answers in b(i) and b(ii) and describe what you notice.

A4.3 Sarah decides to make a quilt for the bed based on the design John used in A4.2. However, she feels that she should enlarge the block size so that it will not require as many blocks. The bed quilt must be 200 cm by 250 cm.

a) Make a list of possible sizes for the square blocks.
b) What is the largest square block that can be used?
c) Sarah decided that the blocks should be larger than 15 cm but smaller than 30 cm. What size would the finished blocks need to be to make this work?

A4.4 The GCF of 8 and an unknown number is 4. Find possible values for the missing number. Describe what all the values have in common.

Note: Assessment items used in the next two pages sometimes incorporate GCF and LCM in the same item.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *apply number-theory concepts in relevant situations and explain the interrelated structure of whole numbers, integers, and rational numbers*

SCO: By the end of grade 7, students will be expected to

A5 **solve and create problems involving common multiples and least common multiples (LCM)**

Elaboration – Instructional Strategies/Suggestions

A5 Although the notion of multiple is not formally addressed in previous grades, students have worked with multiples when developing multiplication facts. To find LCMs, the prime factorization and listing of multiples methods can be developed and discussed for appropriate usage; for example, to find the LCM of 12 and 16

- **prime factorization method:**

$12 = 2 \times 2 \times 3$
$16 = 2 \times 2 \quad \times 2 \times 2$
$2 \times 2 \times 3 \times 2 \times 2 = 4$

Select common factors, and then all other factors for each number and find the product.

- **listing of multiples method:**

12 = 12, 24, 36, 48, 60, 72, 84, 96
16 = 16, 32, 48, 64, 80, 96, 112, 128

Both 48 and 96 are common multiples, but 48 is the least common multiple.

An interesting variation on this method is to find the LCM with a calculator.

❒ Have students work in pairs with each using a calculator. Ask them to find the LCM of 9 and 12. They would use the following keystrokes:

Student A: 9 + = = ...

Student B: 12 + = = ...

Ask them to record the values for each press of the equal sign in a table until a common value appears. (It should be noted that not all calculators have this feature.)

An understanding of common multiples and LCM will be useful in grade 8 when adding and subtracting fractions and in many problem-solving situations. It might be interesting to lead students to discover that the LCM of two numbers can be found by dividing the product of the numbers by the GCF (e.g., LCM of 6 and 8 is $(6 \times 8) \div 2$). Students may discover why this relationship works by studying closely the prime factorization method. They can observe that the repeated portion of the factors is only included once in the LCM. The repeated part is the GCF. When we multiply the two numbers 6×8 and divide by the GCF 2, it produces the LCM.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A4/5.1

a) The LCM of two numbers is 24. Find possible values for the numbers.

b) The GCF of 8 and an unknown number is 4, and the LCM of the two numbers is 24. Find a possible value for the unknown number.

A4/5.2 If the GCF of two numbers is 8, and the LCM is 80, what are the possibilities for the pair of numbers.

A5.1 Joe and Pat work part time at a music rental store. Joe works every four days, and Pat works every six days.

a) If they both started work on September 27^{th} and the store is open every day, what will be the next date they work together?

b) Find two more dates on which they will work together.

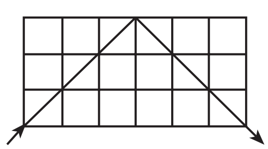
A5.2 Sarah has three aunts who live in other provinces of Canada. Her aunt living in Vancouver comes home every fourth summer, her aunt living in Calgary comes home every third summer, and her aunt living in Toronto comes home every second summer. The family had a reunion when Sarah was 6 years old. Sarah's dad is planning another reunion when all his sisters are home again. How old will Sarah be at the next reunion?

Interview

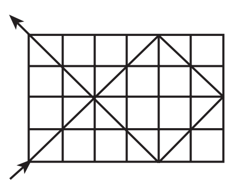
A4/5.3 Ask students why the GCF has to be a factor of the LCM.

Extension

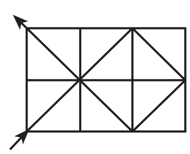
A visual means of finding LCM involves the use of the gridded pool-table model. Count the number of squares the ball passes through before it reaches a corner pocket (using 45 degree angles and assuming that angle of incidence equals angle of reflection); for example,



The ball passes through 6 squares, so LCM of 3 and 6 is 6.



The ball passes through 12 squares, so LCM of 4 and 6 is 12.



The ball passes through 6 squares, so LCM of 2 and 3 is 6.

A5.3 Ask students to use the gridded pool-table model to find the LCM of:

a) 8 and 12

b) 16 and 20

c) 5 and 10

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *apply number-theory concepts in relevant situations and explain the interrelated structure of whole numbers, integers, and rational numbers*

SCO: By the end of grade 7, students will be expected to

A6 develop and apply divisibility rules for 3, 4, 6, and 9

Elaboration – Instructional Strategies/Suggestions

A6 Exploration of the divisibility rules serves as an excellent opportunity to extend number sense. Instruction should be organized so that the students can arrive at the divisibility rules themselves. Students should be reminded of the divisibility rules for 2, 5, and 10, which most should recall readily. Knowledge of divisibility rules will provide a valuable tool for mental arithmetic and general development of operation sense.

❒ Have students explore divisibility rules for 3, 6, and 9. Ask them to write the first 10 multiples of 3. Ask what they notice about the numbers. If no student mentions the sum of the digits, ask them to find the sum of the digits and describe what they notice. Ask them which numbers in the same list are divisible by 6. Ask what they notice about these numbers. Test the conclusions, using numbers such as 393, 504, and 5832.

The divisibility rules are as follows:

A number is divisible by

- 2 if it is even
- 5 if it ends in a 5 or a 0
- 10 if it ends in a 0
- 3 if the sum of the digits is divisible by 3
- 9 if the sum of the digits is divisible by 9
- 4 if the number formed by the last two digits is divisible by 4
- 6 if the number is divisible by 3 and even

It may also be interesting to explore why the divisibility rules work; for example,

Divisibility test for 4: Consider 2346. 2346 = $2300 + 46$. Since 100 is divisible by 4, all multiples of 100 are divisible by 4, therefore, 2300 is divisible by 4. All that remains is to determine if 46 is divisible by 4.

Divisibility tests were much more useful before calculators were readily available. Today they are mainly studied as an opportunity to provide additional number sense, and because they provide a tool that is useful in mental computation activities.

It is also important to learn how to test for divisibility on a calculator. That is, students should realize that the test for divisibility on a calculator involves dividing to see if the quotient is a whole number. For example, to find if 276 is divisible by 8, have students use a calculator to calculate $276 \div 8$. Since the calculator shows 34.5, it tells them that 276 is not divisible by 8. The understanding of divisibility rules is closely tied to the development of an intuitive understanding of mathematics. Once students understand divisibility for 2 and 3, they can use this knowledge to develop a means of testing for divisibility by 6. This should be seen as a problem-solving opportunity for students.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A6.1

a) Once you know that a particular number is divisible by 3 and 5, how does this help you in determining whether it is divisible by 15?

b) Why does knowing that a number is divisible by 3 and 6 not ensure that the number is divisible by 18?

Presentation/Portfolio

A6.2 A newspaper article reported that two new prime numbers had just been discovered. The numbers presented in the article were 235 345 678 125 237 215 and 456 347 235 567 327. Ask students to write and/or orally discuss whether the numbers reported were indeed prime, as the article stated, and explain how they know.

A6.3 After working with all the divisibility rules, ask students to group them according to characteristics and present their conclusions to the class.

Portfolio

A6.4 Ask students to complete the number by filling in each blank with a digit. Ask them to explain how they know their answer is correct.

a) 26_ is divisible by 10
b) 154_ is divisible by 2
c) _6_ is divisible by 6
d) 26_ is divisible by 3
e) 1_ 2 is divisible by 9
f) 15_ is divisible by 4

A6.5 Tell students that Jack's teacher often assigns group work, and that sometimes she uses groups of 2, and other times groups of 3, 4, or 8. Ask students to find the least number of students in the class in order that groups of the identified sizes can be formed evenly. Ask them to justify their answers.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A7 apply patterning in renaming numbers from fractions and mixed numbers to decimal numbers

Elaboration – Instructional Strategies/Suggestions

A7 Much work has already been done in previous grades in relation to this outcome, and it should not require extensive treatment. While the concepts should be developed by using concrete and pictorial models, other forms of representation (symbolic, verbal, and contextual) should also be employed. Base-ten blocks and decimal squares are useful tools when working with decimal fractions. This study should include terminating and repeating decimal fractions.

In working with equivalence of mixed numbers and improper fractions, a number of models should be explored. Use materials such as pattern blocks, fraction bars, and various fraction kits based on circle, rectangular, and polygonal models. The number line is also useful in illustrating and connecting a variety of subsets of the real number system.

Many fractional numbers produce decimals that will not terminate, such as $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{9}$. Students should be introduced to the terminology "repeating" and "period" as well as bar notation to show repeating periods. The patterns produced by fractions with denominators such as 7, 14, and 17 should be explored since they have particularly interesting periods. Students should use calculators to find the decimal form for $\frac{1}{7}$, $\frac{2}{7}$, etc. and, observing these patterns, predict the decimal for other fractions having a denominator of 7. How well the pattern for $\frac{1}{7}$ is shown by a calculator display is determined by the number of digits on the calculator display. Sometimes students will need to use pencil and paper in conjunction with the calculator to help uncover patterns for decimals with long periods. Students should also be aware of the effect of calculator rounding (i.e., automatic rounding caused by the limit on the number of digits which the calculator can display).

A fractional part of a whole is sometimes easier to visualize than the equivalent decimal representation. A review of some of the work done in converting from fractions to decimals should help students relate decimals, especially repeating decimals, to their fractional form.

Students should recognize that only fractions which can be re-expressed with denominators that are powers of 10, such as 10, 100, and 1000, will be terminating when written in decimal form. For example, $3\frac{2}{5} = 3\frac{4}{10} = 3.4$, $2\frac{3}{8} = 2\frac{375}{1000} = 2.37$, $\frac{7}{80} = \frac{875}{10\ 000} = 0.087$. The terminating decimal, 0.312, is read as three hundred twelve thousandths. When a student reads a terminating decimal, it should be clear how to write it in fractional form.

Note: Outcomes A7 and A8 are closely related and should be addressed together for instruction.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance/Interview

A7.1 Ask students to explain how to use a calculator to find the decimal equivalent for $\frac{2}{3}$.

A7.2 Ask students to represent $1\frac{3}{4}$, using base-ten blocks or decimal squares.

A7.3 Ask students to represent $\frac{1}{13}$ and $\frac{2}{7}$ as decimals, using repeating notation with the aid of a calculator.

A7.4 Ask students to compare the decimals for the following pairs and have them discuss the similarities and differences they observe.

a) $\frac{1}{12}$ and $\frac{1}{120}$

b) $\frac{3}{8}$ and $\frac{3}{80}$

Since the decimal for $\frac{3}{16}$ is 0.1875, ask students to predict the fraction that would produce a decimal of 0.01875.

Pencil and Paper

A7.5 A certain candy bar can easily be broken into 8 equal pieces. There are 27 students in Sue's class, and she has $3\frac{1}{2}$ candy bars. Sue found how many eighths there were in $3\frac{1}{2}$, using equivalent fractions. Is there enough for each student to have a block of the candy bar? Explain, using pictures, how the answer is found. Represent as a decimal the fractional part that is left over.

A7.6 John created a game for his birthday party, with the winner getting a prize. The winner picked from the following list the decimal which simplifies to produce the smallest denominator:
0.135, 0.375, 0.225, 0.250, 0.950, 0.500, 0.125, 0.040, 0.150
Pat won the prize. What fraction did she pick?

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A8 **rename single-digit and double-digit repeating decimals to fractions through the use of patterns, and use these patterns to make predictions**

Elaboration – Instructional Strategies/Suggestions

A8 Given that division will show students that $\frac{1}{9} = 0.\bar{1}$, $\frac{2}{9} = 0.\bar{2}$, and $\frac{3}{9} = 0.\bar{3}$, they should see that decimals of the form $0.\bar{a}$ are fractions of the form $\frac{a}{9}$ (e.g., $\frac{4}{9} = 0.\bar{4}$), and, ultimately that $\frac{9}{9} = 0.\bar{9} = 1$.

Similarly, by learning that $\frac{1}{99} = 0.\overline{01}$, $\frac{2}{99} = 0.\overline{02}$, and $\frac{3}{99} = 0.\overline{03}$, students should see that decimals of the form $0.\overline{ab}$ are fractions of the form $\frac{ab}{99}$ (e.g., $\frac{54}{99} = 0.\overline{54}$). This is an application of patterns and is closely linked with GCO (C). The calculator should be used to develop these patterns. (Note: Since it impacts on decimals with longer periods, students will need to be aware of automatic calculator rounding and the impact which it can have on patterns.)

❐ Give students a set of fractions such as $\frac{1}{13}, \frac{2}{13}, \frac{3}{13}$. Ask them to find a pattern and then use the pattern to predict the decimal for other fractions such as $\frac{4}{13}, \frac{5}{13}$, and $\frac{10}{13}$.

Fractions with denominators of 15 may be interesting to consider because the pattern changes from repeating to terminating as the numerators increase—that is, $\frac{1}{15} = 0.0\bar{6}$, $\frac{2}{15} = 0.1\bar{3}$, and $\frac{3}{15} = 0.2$. Students should be able to predict and confirm what other numerators will produce terminating decimals and also predict the repeating decimal for fractions like $\frac{4}{15}$ and $\frac{7}{15}$.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Interview

A8.1 Ask students if the fraction $\frac{1}{27}$ produces a repeating pattern and, if so, to describe it and use it to predict decimal values when doubling and tripling the numerator of the fraction. Have them explore the decimal pattern for fractions with a denominator of 27, using multiples of 3 for the numerator. Ask them what they notice. Ask them to describe the overall decimal pattern for fractions with denominator 27.

A8.2 Chris had a calculator which displayed 2.3737374. Chris concluded that it was not a repeating decimal. Ask students to explain why Chris drew this conclusion and whether or not they believe it to be a correct conclusion.

Performance

A8.3 Given the following decimal representations of fractions as produced by a calculator: $\frac{1}{11}$ = 0.090909..., $\frac{2}{11}$ = 0.181818..., $\frac{3}{11}$ = 0.272727..., ask students to

a) predict the decimals for $\frac{5}{11}$ and $\frac{9}{11}$
b) predict the fraction which will have 0.636363... as a decimal
c) predict what the decimal for $\frac{8}{11}$ would look like on a calculator display if the calculator is set to display 8 places after the decimal. Ask them to write the decimal in compact form. Have them describe the pattern for decimals with denominator 11.
d) predict the fraction which will have 0.909090… as a decimal

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *read, write, and order integers, rational numbers, and common irrational numbers*

SCO: By the end of grade 7, students will be expected to

A9 compare and order proper and improper fractions, mixed numbers, and decimal numbers

Elaboration – Instructional Strategies/Suggestions

A9 Students have compared and ordered decimals as well as fractions in previous grades. At this grade level, they should be able to deal with relative size when numbers are presented in a variety of forms within the same question. Number lines are particularly useful in comparing and ordering fractions. Several different approaches can be developed or considered to help students to compare fractions. Each one may be particularly useful for certain situations. The following strategies should be considered. (Note: the first three strategies were developed in previous grades and may only require review).

- Compare in relation to a particular benchmark point such as $\frac{1}{2}$ or 1 (e.g., $\frac{2}{5} < \frac{3}{4}$ because $\frac{2}{5} < \frac{1}{2}$ and $\frac{3}{4} > \frac{1}{2}$).
- Compare based on common denominators (e.g., if both fractions have the same denominator, the larger numerator represents the larger fraction, $\frac{5}{8} > \frac{3}{8}$). If denominators differ, it may be useful to re-express the fractions with equivalent denominators (e.g., when comparing $\frac{2}{5}$ and $\frac{3}{7}$, change $\frac{2}{5}$ to $\frac{14}{35}$ and $\frac{3}{7}$ to $\frac{15}{35}$ to obtain equivalent denominators, and then compare the numerators).
- Compare based on common numerators (e.g., if both fractions have the same numerator, the one with the smallest denominator is larger, $\frac{3}{4} > \frac{3}{5}$).
- Change all numbers to decimal values and compare by use of place-value comparisons (e.g., $\frac{1}{8} < 0.13$ because $0.125 < 0.130$). It is sometimes useful for students to list all numbers to the same number of decimal places when comparing decimals, so that students are comparing tenths to tenths or hundredths to hundredths.
- Since students have learned to represent decimals by using base-ten blocks in earlier grades, they can draw upon this knowledge in comparing decimals. By visualizing the concrete and/or pictorial representation of the decimal, students should be able to make decisions pertaining to relative size of decimal numbers.
- Model the fraction situation using various models such as Fraction Factory Pieces, pattern blocks, base-ten materials, and decimal squares.

❒ Have students play FRIO (Fractions in Order) to reinforce understanding. Each student gets five fraction (or decimal) cards out of a pack of cards and is not allowed to change the order. Each student takes turns trading one of the cards for a new one from the pack. They place the new card in whichever location best helps in getting the cards in order. The object is to be the first to get one's cards in order. The pack should have sufficient cards to allow the game to run smoothly—for groups of 3, there should be at least 30 cards per group, and for groups of 4, at least 40 cards per group.

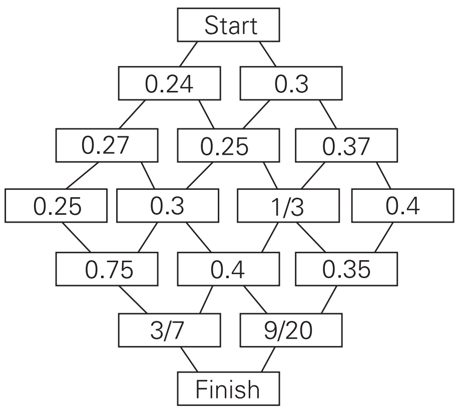
GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A9.1 Ask students to write a variety of numbers on file cards, expressing some as decimals and others as fractions. Place the cards in random order on the chalkboard ledge, facing the students. Tell students the object of the activity is to arrange the numbers from least to greatest. Ask each student to move one card and justify the move. Continue until students are satisfied the numbers are arranged as desired.

A9.2 Ask students to find as many paths as possible to go from the start to the finish, always moving to a larger number with each move.



This diagram can be used as a template, and students can create their own maze using different numbers. They can exchange them to find possible paths from start to finish.

Interview/Presentation

A9.3 Ask students to estimate a whole number value for □ to make each of the following true:

a) $0.4 < \frac{\square}{8} < 0.7$

b) $0.\square < \frac{1}{4}$

c) $\frac{3}{11} < 0.\square < \frac{4}{11}$

A9.4 Ask students to explain how to order the following numbers from least to greatest:

$\frac{3}{7},\ 1\frac{1}{3},\ \frac{5}{9},\ \frac{13}{12},\ 1\frac{4}{9},\ 0.45,\ 0.93$

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A10 illustrate, explain, and express ratios, fractions, decimals, and percents in alternative forms

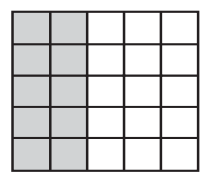
Elaboration – Instructional Strategies/Suggestions

A10 Students should understand that percent on its own does not represent a specific quantity; instead, it represents a special ratio. For example, 90% might represent 9 out of 10, 18 out of 20, 45 out of 50, and 90 out of 100.

The most obvious link between fractions and percent relates to the students' experiences with grades on a test. A second connection to percent involves sales tax and discount. These are probably the earliest experiences students have with the use of percent.

Shading portions of a 10×10 grid, or using commercially produced decimal squares or base-ten blocks, helps in developing a concrete or pictorial model for percent. Students should be given many opportunities to visualize percent, using both circles and 10×10 grids. Students should realize that in most cases percent is used as a special ratio to describe a part out of a whole rather than one part to another part.

Example:



Some students may read this diagram as $\frac{10}{15}$ or $66\frac{2}{3}$ % instead of $\frac{10}{25}$ or 40%. It is important to emphasize the part-to-whole ratio in connection with percent.

Emphasis should be placed on recognizing the fractional equivalent to certain percents and on developing mental facility with regard to changes from one form to another. The following are common fractions which students should be able to link mentally with their decimal and percent equivalents: $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{10}, \frac{2}{10}, \ldots \frac{9}{10}, \frac{1}{20}, \frac{3}{20}, \ldots \frac{1}{25}, \frac{2}{25}, \frac{3}{25}, \ldots$. Such mental links will be essential in order to estimate the percent of a given number. Further, it is also important for students to recognize that certain fractions (e.g., $\frac{4}{9}$) represent slightly less than 50%, whereas others (e.g., $\frac{5}{9}$) represent slightly more than 50%.

Note: It may be valuable to consider addressing D5 in conjunction with A10.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A10.1 Give students 10 two-coloured counters and ask them to turn 4 to the red side and 6 to the yellow side. Ask students to write as many expressions as possible, utilizing all formats. Ask them what would happen to the expressions if one counter was turned over to show a red side instead of a yellow side.

Pencil and Paper

A10.2 John made a strip of blocks, using red and white blocks as shown:

R	R	W	W	W	W	R	R	W	W

a) Describe these ratios from the strip:
- 6:4
- 6:10
- 4:10

b) In making a quilt, 5 more strips of blocks were made exactly as shown in the diagram for part a) above. When these are added to the strip shown, what will be the new ratio of red to white? of red to the total number of strips? Did the ratio change? Why or why not?

Interview

A10.3 Ask students to choose the item which does not belong to the list and explain their choice.

$\frac{3}{4}$ 0.75 0.34 75%

A10.4 Ask students to represent each of the following in as many different forms as they can think of: 60%, 0.75, 2:4, $\frac{2}{5}$

A10.5 While there are many forms which can be used to express most numbers, certain forms are associated with various contexts or situations. Ask students which form is typically associated with each of the following:
a) a special end-of-season sale at a clothing store
b) the batting average of a baseball player
c) the part of a cup which is used in a typical recipe
d) the teeth on the wheels and the gears in a bicycle
e) the sales tax

A10.6 Ask students to estimate a percent which is a close approximation for each of the following, and to indicate why their estimate is larger or smaller than the exact value. (They do not need to find the exact value to do this.)
a) $\frac{7}{11}$
b) 4:9
c) $\frac{6}{13}$
d) 7:16

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A11 demonstrate number sense for percent

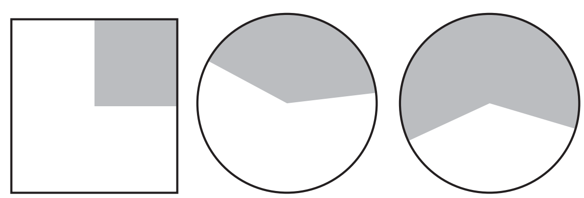
Elaboration – Instructional Strategies/Suggestions

A11 The focus should be on an intuitive understanding about percent. Number sense for percent should be developed through the use of benchmarks:

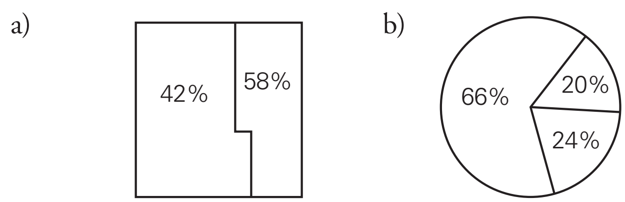
- 99% is almost all
- 49% is almost half
- 10% is not very much
- 1% is very small in relation to total

Discussion should also focus on the contexts in which 1% would be considered high and the contexts in which 90% would be considered low. For example, consider the mercury content of fish that might be hazardous to humans versus the error rate of an air traffic controller. Students might wish to research the lethal dose of such familiar substances as caffeine, nicotine, or aspirin. For such common substances, a quantity of 0.1% of body weight might be beyond the lethal dosage. A 90% rate for opening of parachutes would be considered seriously low.

As well, students should relate to percent visually. They should recognize that, for any usage of percent, parts should always add to give 100%. They should also recognize the relative percentage of a figure and be able to match this with what is shown visually. For example, they should be asked to estimate the percent that the shaded portion of the following diagrams represent.



❒ Ask students what is incorrect about each of the following:



c) Sarah wrote a test and had 8 questions right and 10 questions wrong. Sarah announced, 8 out of 10 is 80%—that's not a bad grade!

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

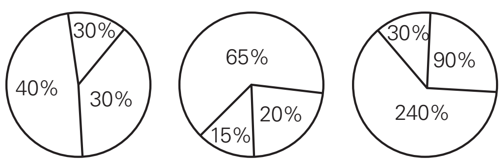
A11.1 Ask students to draw three circles of radius 3 cm. Ask them to shade a portion of the first circle and write on a separate sheet their estimate of the percentage which they shaded. Have them exchange circles with a partner and estimate what percentage of their partner's circle is shaded, compare answers, and discuss any differences. They should try to come to consensus on who is closer. Have them get a third party opinion if consensus cannot be reached. Ask students to repeat with the second and third circles. Discuss whether students feel their ability to estimate improved as they went from the first to the third estimate.

Pencil and Paper

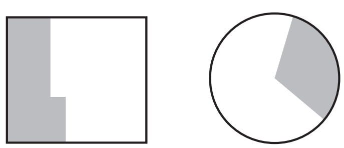
A11.2 A newspaper article reads: 45% of the people in P.E.I. are under 25 years of age, 35% are between 25 and 45, and 40% are over 45. Do you think the article is correct? Why or why not? If you think the article is not correct, suggest how it might be corrected so that the information at least sounds more reasonable.

Interview

A11.3 Ask students which of the following seem(s) reasonable. Ask them to explain their answer.



A11.4 Ask students to estimate the percentage that is shaded of each of the following diagrams.



Portfolio

A11.5 Ask students to critique this situation and to explain why the reasoning is flawed.

> Jim found out that on his test the ratio of questions answered correctly to questions answered incorrectly was 12:13. He concluded that he should get a very good grade.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *read, write, and order integers, rational numbers, and common irrational numbers*

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 7, students will be expected to

A12 represent integers (including zero) concretely, pictorially, and symbolically, using a variety of models

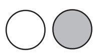
A13 compare and order integers

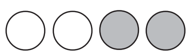
Elaboration – Instructional Strategies/Suggestions

A12 Exposure to negative numbers began in previous grades. Negative numbers have also been part of the day-to-day life of many students through their experiences with temperatures below zero and distances below sea level, and through playing certain card games. A brief review of the meaning of negative numbers is all that should be necessary before beginning work on this outcome.

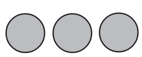
Students should be able to represent the same integers in multiple ways, including the use of coloured chips, where one colour represents negative and another represents positive. Students should recognize that zero can have many interpretations. For example, zero may be a point on a scale (e.g., temperature, sea level), the absence of any quantity, or the balance of positive and negative values. This balance of positive and negative values is known as the zero principle. In temperature, zero can have two different meanings. Zero degrees Celsius is not the absence of all temperature, but rather a balance point between ice and water. On the other hand, zero Kelvin does represent an actual absence—of molecular motion.

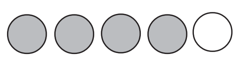
For the diagrams which follow, the solid dot is used to represent positive, and the open dot is used to represent negative; for example,

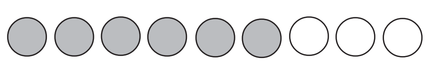
 represents zero, and

 represents zero.

The principle that zero can be represented in multiple ways forms the basis for representation of other integers. When students grasp that any one integer can be represented in multiple ways, this understanding will provide a basis for subtraction; for example, +3 can be represented as

 ,

 ,

 , etc.

These diagrams can also be interpreted by using the concept of 'net worth,' that is, one's wealth if all assets were used to pay all debts. Students should understand that they might have $3 for lunch money at the same time that they owe $3. A variety of combinations can be used to show any net worth.

A13 Comparing and ordering integers can be done using coloured counters; however, students should also work with other models such as the number line and other real-life models which are closely linked to the number line; for example, students should compare and order heights above or below sea level, as well as arrange temperatures from coldest to warmest, and vice versa.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance (Materials)/Pencil and Paper (Pictures)

A12.1 Ask students to represent

a) -4, using 6 coloured counters
b) zero, using 6 coloured counters
c) +2, using 6 coloured counters

Pencil and Paper

A12.2 Plot points +5 and -5 on a number line. What do you notice about them? Why do you think number pairs such as -5 and +5 are called opposites?

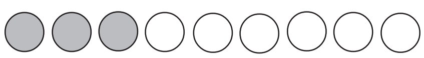
A12.3 Write an integer for each of the following situations:

a) A person walks up 8 flights of stairs.
b) An elevator goes down 7 floors.
c) The temperature falls by 7 degrees.
d) Sue deposits $110 dollars in the bank.
e) The peak of the mountain is 1123 m above sea level.

Interview

A12.4 Ask students if -3 can be represented using 6 counters and to explain why or why not.

A12.5 Ask students what number is represented by the counters shown.



A12.6 Sandy owes $4 dollars to his friend, has $12 in savings, and just received his $5 weekly allowance. Ask students to find Sandy's net worth and explain how it was found.

Interview/Presentation

A13.1 Ask students to tell which

a) golf score is better, -7 or +3. Explain why.
b) temperature is higher, -7 or +3. Explain why.
c) altitude is higher, -342 or +23. Explain why.

A13.2 Ask students to explain why it is true that

a) a negative number further from zero is less than a negative number that is close to zero
b) a negative number is always less than a positive number
c) a positive number is always greater than a negative number

Portfolio

A13.3 Ask students to use a newspaper to find +/- records of hockey players and rank them according to their +/- records.

Suggested Resources

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving rational numbers and integers

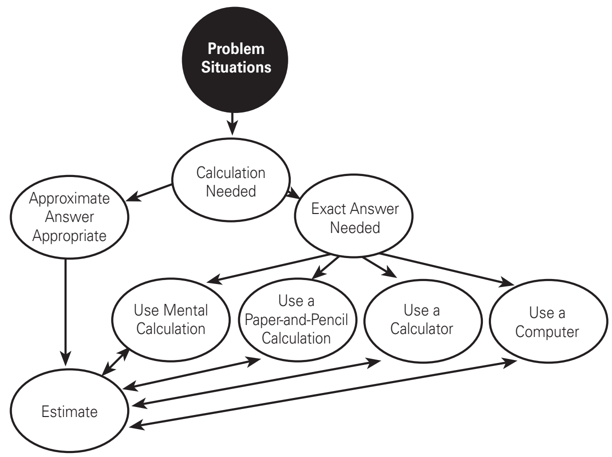
SCO: By the end of grade 7, students will be expected to

B1 use estimation strategies to assess and justify the reasonableness of calculation results for integers and decimal numbers

B2 use mental math strategies for calculations involving integers and decimal numbers

Elaboration - Instructional Strategies/Suggestions

B1/2



Whenever a problem situation which requires computation is encountered in mathematics, the first decision to be made is whether it requires an exact or an approximate answer. If an approximate result is sufficient, an estimate can usually be arrived at mentally (see B2). If an exact answer is needed, working it out mentally should be attempted—that is, do a mental calculation. However, a mental calculation is not always possible, owing to the nature of the numbers involved or the lack of mental calculation skills, thus, other alternatives such as pencil and paper algorithms, the calculator, or the computer are considered. Pencil and paper is used only when the numbers are not too tedious, there are few numbers to work with, or a calculator is not readily available. The calculator is usually the preferred method, for it generally reduces the chance of an error and speeds up the process. The computer is an asset when there are numerous repetitive calculations. The decision-making process for computation is illustrated in the diagram above. It is important to remember that pencil and paper algorithms should be developed carefully; otherwise, they will not exist as a computational option for students.

Mental computation strategies, both for exact and approximate answers, are frequently the most useful of all strategies and develop over years of practice; therefore they should be frequently revisited throughout the year. Attention to mental computational estimation strategies helps students to develop number and operation sense, check reasonableness of solutions, recognize errors on calculator displays, and develop confidence when dealing with number. Students should always estimate before calculating in order to judge the reasonableness of results obtained using other methods.

Note: The elaborations for B1 and B2 are continued on the next two 2-page spreads.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B1.1 Ask students to use rounding to find an estimate and explain the rounding used.

a) sum of -2392 and 4899
b) difference of $134.63 and $19.15
c) product of 125 and 7.9
d) quotient of -$435 and 73

B1.2 Ask students to use the front-end strategy to find an estimate for each of the following and explain their process.

a) -692 - 460
b) $14.32 + $27.25 + $ 11.13
c) 1.8×387

B1.3 Ask students to use the compatible number strategy to find an estimate for

a) 13.4 + 11.9 + 26.5 + 7.3
b) $19 \times 37.2 \times 4.8$

Pencil and Paper

B1.4 Mary said to Sharon, "I'm thinking of a number that when multiplied by 8.7 gives a product of about 7.2." Give five numbers that Sharon could have used to answer Mary's question. Show how Sharon's estimates are reasonable.

B1.5 The attendance at a hockey game for five successive nights was as follows: 34 235, 28 678, 31 345, 27 398, 30 281. Each person paid $9.95. John used his calculator and found $151 177 315.00 as the exact amount of money taken in for the five nights.

a) Use estimation to check the reasonableness of John's solution and explain your method. Share your method with the rest of the class.

b) What error do you think John made which caused him to get his answer?

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving rational numbers and integers*

SCO: By the end of grade 7, students will be expected to

B1 **use estimation strategies to assess and justify the reasonableness of calculation results for integers and decimal numbers**

Elaboration - Instructional Strategies/Suggestions

B1 Assessment for estimation done through performance tasks or student oral presentation or interview gives great insight into students' understanding. However, estimation can also be assessed in written form by asking students to explain the process used to find an estimate. Students should be given specific practice related to individual strategies before they are asked to choose a strategy appropriate to a given situation. When students are exposed to a variety of strategies, they will become better at mental math and estimation. Facility with basic facts and mental computation skills are required for estimation. Estimation should not be considered an exercise one does only when called upon to do so, but an integral part of doing any computation, whether it is with a calculator or with pencil and paper. Strategies that may be used for estimation include

- **rounding:** Change the problem to one that is easier to work with mentally by substituting 'nicer' numbers (e.g., 5, multiples of 10) of similar magnitude; for example, $213 \times 48 \approx 210 \times 50 = 10\ 50$ or $213 \times 48 \approx 200 \times 50 = 10\ 00$.
- **front-end strategy:** Perform operations from left to right; for example,
  $2.39 + $4.56 + $2.97 + $2.28 + $5.78 = ?
  front - $2 + $4 + $2 + $2 + $5 = $15
  end - group cents to form dollars,
  $39 + 56 \approx \$1 \quad 97 \approx \$ \quad 28 + 78 \approx \$$
  $1 + $1 + $1 = $3
  total the front and end, $15 + 3 = $18.
  For a second example, -4×702 = ?
  front - 4×700 = -28 000 and **end** - 4×29 is about -4×30 = -120;
  the answer is about -28 120.
- **special number strategy:** Turn one of the numbers into 1, 10, 100, etc., for ease of multiplication or division; for example, $324.4 \div 0.9$ = ?
  Since 0.97 is almost 1 then the estimate would be $324 \div 1 = 32$.
  Another special number strategy is to double both the dividend and divisor if dividing by 5; for example, $342.5 \div 5$,
  think: $342.5 \div 5 \approx 680 \div 10 = 68$.
- **clustering strategy:** Round a quantity of numbers to the same number and multiply the quantity by the rounded number; for example,
  $\$389.22 + \$420.27 + \$396.45 + \$403.67 + \$395.50$,
  think: all these numbers are about $400 so $5 \times \$400 = \$200$.
- **compatible numbers:** Look for number combinations that result in 10, 100, 1000, etc.; for example,
  467 + 281 + 241 + 325,
  think: 467 + 241 is close to 700 and 281 + 325 is close to 600,
  700 + 600 = 1300. (Note: These pairs of numbers are deemed compatible because the two ten's digits together make another hundred in each case.)

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B1.6 Give students a column of numbers and ask them to estimate the sum. Have students exchange their question with a partner. Ask students to check the estimate of their partner and explain whether or not they feel the estimate of their partner is a reasonable one. Do the same estimation activity, using a long list of numbers and a calculator. Again, ask students to use estimation to check the reasonableness of results.

B1.7 During a field trip, have students do an estimate of the number of seats in a hockey rink, theatre, or stadium. Ask them to explain how they arrived at their estimate and then compare the estimate to the actual number as identified by the box office.

B1.8 Have students work with a partner and play the range game. Game explanation: This is an estimation game for any of the four operations. First, pick a start number and an operation. The start number and operation are stored in the calculator. Students then take turns entering a number and pressing the equal button to try and achieve a result in the target range. The winner is the student who gets the first number that will produce the answer in the target range. See example below.

Start with 17; range is 800 to 830; use multiplication.
Press: 17 $\times$ (Estimate) = ???? (looking for $800 \leftrightarrow 830$)
Student 1: $17 \times 50 = 850$
Student 2: $17 \times 40 = 680$
Student 1: $17 \times 45 = 785$
Student 2: $17 \times 48 = 816$ (Winner)
Variations of this game can be found in *Elementary and Middle School Mathematics*, 3rd Edition, Van de Walle.

Interview

B1.9 Ask students to estimate the answer for each of the following. Ask them to compare the estimate with the answer provided to determine reasonableness. Students should be able to explain their strategy and why they feel their answer is reasonable.

a) $83.658 \div 8.9 = 0.919$
b) $103.56 \times 4.9 = 507.512$
c) $3456 + 3567 + 3450 + 3300 + 3712 + 3645 = 21\ 524$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will be expected to

v) apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving rational numbers and integers

SCO: By the end of grade 7, students will be expected to

B2 use mental math strategies for calculations involving integers and decimal numbers

Elaboration - Instructional Strategies/Suggestions

B2 Many of the strategies for estimation can also be used in mentally calculating exact answers. These include

- **front-end strategy**: Perform calculations from left to right. Three ways of using the front-end strategy are demonstrated using the example, -46 + (-38):
  1) Add the tens and ones separately and combine.
     -40 + (-30) = -70 -6 + (-8) = -14 -70 + (-14) = -84
  2) Add tens of one number to the other complete number, and then add the ones.
     -46 + (-30) = -76 -76 + (-8) = -84
  3) Round one number, add the other, and adjust the rounding.
     -46 + (-40) (-38 rounded to -40) = -86
     -86 + 2 (since an extra -2 was added) = -84
- **compatible numbers:** Analyse the numbers to see if compatible sums resulting in 10, 100, 1000, etc. are present. For example,
  -28 + 63 + 37 + 33 + (-72)
  = (-28 + (-72)) + (63 + 37) + 33
  = -100 + 100 + 33
  = 33
- **compatible factors:** Analyse the numbers to see if compatible products resulting in 10, 100, 1000, etc. are present. For example,
  $-8 \times 137 \times 125 = (-8 \times 125) \times 137 = -1000 \times 137$ = -137 000.

  Another variation of the same strategy is to break one or more of the numbers into pairs of factors that may be compatible.
  For example, 75×28,
  think: $75 = 25 \times 3$ and $28 = 4 \times 7$, $25 \times 4 = 100$ and $3 \times 7 = 21$, $21 \times 100 = 2100$.
- **working by parts:** Break a number into two parts and find the missing factor; one (or both) of the parts would be a multiple of 10, 100, 1000, etc. For example, $9\overline{)5472}$,
  think: 5472 is 5400 + 72, 5400 = 9×600 and $72 = 9 \times 8$;
  so, 600 + 8 = 608.

  The numbers themselves dictate what strategy or strategies to use; students should be expected to use these types of strategies whenever they do a calculation. (Note: Estimation and mental calculations can also utilize the properties identified in B3.)
- **double and halve:** Double one factor and halve the other. (Half as many groups which are twice as large results in the same product.) For example, 486×500 is the same as $243 \times 1000 = 243\ 000$.

Note: Estimation and mental calculations should also utilize the properties identified in B3.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Interview/Presentation

B2.1 Ask students to explain why an estimate or an exact answer is likely to be required in these situations. John's bill for dinner at a Pizza Delight is \$36.58. Is an estimate enough when

a) the waiter finds the tax? Explain why.
b) the waiter finds the total of the bill? Explain why.
c) John figures out the tip? Explain why.
d) John checks the bill for accuracy? Explain why.

B2.2 Ask students to explain a relatively quick way to find the sum of \$43.52, \$24.31 and \$57.48.

B2.3 Ask students to use the double-and-halve strategy to find the products of the following factors:

a) 24.6 and 20
b) 5 and 144

Ask them to explain why they think it is easier to double 5 and halve 144 than to halve 5 and double 144.

B2.4 Jeremy multiplied 25×72 by thinking 100×18. Ask students to explain the strategy that Jeremy used.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

SCO: By the end of grade 7, students will be expected to

B3 demonstrate an understanding of the properties of operations with decimal numbers and integers

Elaboration - Instructional Strategies/Suggestions

B3 The mathematical properties—*commutative* (order), *associative* (grouping), and *distributive*—should be revisited. As well, students should reconfirm the properties of zero and one as applied to the basic operations. Instruction should focus on the usefulness of these properties rather than recognition of names or matching exercises. Refer to the properties by their formal names and encourage students to use terminology such as commutativity. Discussion should include why certain properties do not apply to subtraction and division. The distributive property is used when students first learn the multiplication algorithm in previous grades. Since these properties have been previously addressed, they should only be revisited in context.

These properties should be revisited so that students can confirm their relevance to the number systems that are new to this grade level. That is, they should understand that in the same way that $6+0=6$, the property holds for $-8+0=-8$, and if $5\times 4=4\times 5$, then the same property holds for $-15\times 4=4\times(-15)$; likewise, since $1\times 53=53$, then $1\times(-57)=-57$.

When working with properties such as commutativity and associativity, students should be exposed informally to the concept of closure (i.e., that the number which results from the application of any particular operation always belongs to the same set of numbers as the original numbers). It is through the discussion of whether sets are closed to particular operations that students realize the need for extension to other number systems; for example, they should understand that 2 - 5 has no meaning in the set of whole numbers but has meaning in the set of integers.

Closure can be explored as it relates to situations such as

- addition within the set of even numbers
- addition within the set of odd numbers
- division within the set of inegers

The distributive property can be very useful in mental computation; for example, in multiplying $6\times(-84)$, the student might think -84 = -80+(-4) so

6(-80+(-4)) = $6\times(-80)+6\times(-4)$
= -480 + (-24)
= -504

or $-7\times 68+(-7)\times 32$ might be thought of as follows:
68 + 32 is 100 and $(-7)\times 100$ is -700.

Likewise, the associative and commutative principles can be useful in mental computation as well. For example, $25\times(-37)\times 4$,
think $25\times 4=100$
then $100\times(-37)=-370$.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance/Interview

B3.1 Ask students to find the answer to this problem mentally and to explain their strategy.

$25\times(-102)\times4$

B3.2 The following problem was written on the chalkboard, and, before the teacher could turn around, Jill had the solution. Ask students to explain how Jill was able to solve the problem so quickly.

$-34\times17\times624\times0$

B3.3 The following problem was presented to the class:

$-4\times27+(-4)\times73$

Jan was asked to explain how she had solved this problem mentally. Here is how she explained her solution to the class:

I saw that 27 and 73 added to give 100, so I just multiplied -4 by 100.

Ask students if this solution is correct, and, if so, to explain why Jan's solution works and what property is applied here.

B3.4 Ask students to explain how to mentally compute each of the following:

a) $58\times(-7)$
b) 8×73
c) $-6\times73+(-6)\times27$

Portfolio

B3.5 Have students use examples to help show whether the following sets are closed for addition, subtraction, multiplication, and division:

a) the even whole numbers
b) the odd whole numbers
c) the set of whole numbers
d) the set of integers

B3.6 Ask students to complete the following pattern, and to identify and explain the pattern they observe. Ask them how this pattern relates to the properties.

$\frac{900}{300}=$

$\frac{90}{30}=$

$\frac{9}{3}=$

$\frac{0.9}{0.3}=$

$\frac{0.09}{0.03}=$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

B4 determine and use the most appropriate computational method in problem situations involving whole numbers and/or decimals

B5 apply the order of operations for problems involving whole and decimal numbers

Elaboration - Instructional Strategies/Suggestions

B4 Students should continue to consciously make decisions about the most appropriate method of computation for a given situation. These methods include mental computation, computational estimation, pencil and paper algorithms, and the calculator. For divisors or multiples of more than two digits, the use of technology is expected. Addition and multiplication facts are very important and should be revisited as needed. This might happen as part of 3- to 5-minute mental computation activities at the beginning of class. *Mental computation and computational estimation are not very effective if students do not have a strong facility with basic facts.*

In addition, the problem situations should draw upon and incorporate previously learned problem-solving strategies. These include creating problems similar to given problems, looking for the possibility of more than one solution, supplying missing information to solve problems, and solving problems which include information not relevant to the solution. These and other problem-solving strategies should be supported on an ongoing basis.

B5 Students should realize that rules for order of operations are necessary in order to maintain consistency of results. It is important to provide students with situations in which they can recognize the need for the order of operations.

❒ Ask students to write a number sentence for the following: the total cost for a family with two parents and three children for theatre tickets if children's tickets cost \$8.50 and adult tickets cost \$14.80. When students write a number sentence such as, $3 \times \$8.50 + 2 \times \$14.80$, ask if this solution makes sense:
$3 \times \$8.50 = \$25.50 + 2 = \$27.50 \times \$14.80 = \$407.00$
This should help them relate to why the operations are not performed in the order they appear.

The order of operations is as follows: **B**rackets, **E**xponents, **D**ivision/ **M**ultiplication (order of appearance from left to right), and **A**ddition/ **S**ubtraction (order of appearance from left to right). The acronym, **BEDMAS,** is a good memory device.

Specific instruction should be given on calculator use with regard to the order of operations, especially as it pertains to the use of the memory function of the calculator. Students should be alerted to the different forms of division questions such as $3.7\overline{)33.3}$, $33.3 \div 3.7$, $\frac{33.3}{3.7}$, and recognize the necessity of preparing problems for calculator entry. Students should also be aware that different calculators process the order of operations in different ways. Some calculators are programmed to address the order of operations automatically, and others are not.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper/Journal

B4/5.1 Ask students to write a number sentence for the following and solve it, using the order of operations.

a) Ms. Janes bought the following for her project: 5 sheets of pressboard at \$8.95 a sheet, 20 planks at \$2.95 each, and 2 litres of paint at \$9.95. What was the total cost?

b) Three times the sum of \$34.95 and \$48.95 represents the total amount of Jim's sales on April 29. When his expenses, which total \$75.00, were subtracted, what was his profit?

c) Consider solving the number sentences for a) and b) by ignoring the order of operations. Would the solution make sense in terms of the problem? Discuss.

Pencil and Paper

B5.1 Owing to some faulty keys, the operation signs in these problems did not print. Use the information which is supplied to help determine which operations were used.

a) $(7.4 \square 2) \square 12.6 = 2.2$

b) $2^{\square} \square\ 7.2 = 8.8$

B5.2 Because the shift key of the keyboard did not work, none of the brackets appeared in these problems. If the student has the right answers to both problems, identify where the brackets must have been.

a) $4 + 6 \times 8 - 3 = 77$

b) $26 - 4 \times 4 - 2 = 18$

B5.3 Billy had to answer the following skill-testing questions to win the contest prize. What are the winning answers?

a) $234 \times 3 - 512 \div (2 \times 4)^2$

b) $18 + 8 \times 7 - 118 \div 4$

Billy was told that the correct answer for b) is 16, but Billy disagreed. What did the contest organizers do in solving the question which caused them to get 16 for the answer? Explain why you think they made that error.

Presentation

B5.4 Ask students to explain why it is necessary to know the order of operations to compute $4 \times 7 - 3 \times 6$. Ask them to compare the solution of the previous problem with the solution of $4 \times (7 - 3) \times 6$. Ask whether the solutions are the same or different and why.

Portfolio

B5.5 Have students use their calculator to answer the following question: Chris found the attendance reports for hockey games at the stadium to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539, and 4602. If tickets were sold for \$12.75 each, and expenses amounted to \$258,712.00, what was the profit for the stadium?

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

B6 **estimate the sum or difference of fractions when appropriate**

Elaboration - Instructional Strategies/Suggestions

B6 An estimate is sometimes all that is required to satisfy a given situation. At other times, an estimate would be used to determine if an answer, found using an algorithm or a calculator, is reasonable. *Formal algorithms for the addition and subtraction of fractions will be a significant focus in grade 8.* At this grade level, students will work only with estimates of the sum and difference of fractions. This work, done in advance of the algorithm, will allow students time to become efficient in determining a "ballpark answer." This skill will permit students to make quick and efficient judgments about the reasonableness of answers acquired through algorithms when they are studied in grade 8.

For example, students should be able to readily decide that $\frac{2}{3} + \frac{1}{2}$ is greater than 1 because $\frac{2}{3}$ is greater than $\frac{1}{2}$, and, since another $\frac{1}{2}$ is being added, the answer must be greater than 1. Estimation of this nature will primarily utilize benchmarks such as 0, $\frac{1}{2}$, and 1.

Similarly, students should be able to reach and justify the approximate value in situations such as those shown below.

$$4\frac{5}{6} - \frac{4}{5} \approx 4$$

$$4\frac{3}{7} + 5\frac{5}{8} \approx 10$$

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B6.1 $10\frac{5}{6} - 4\frac{7}{8}$ and $2\frac{1}{2} + 2\frac{3}{4}$ are examples of calculations involving fractions which have answers of about 5. Write two addition and two subtraction questions involving fractions which have the same answer.

Interview/Presentation

B6.2 Ask students to use estimation to answer the following:

a) Susan has 1 cup of brown sugar. One layer of the squares she is making requires $\frac{2}{3}$ of a cup of brown sugar, while the second layer requires $\frac{1}{2}$ of a cup. Does she need to go to the supermarket to get more brown sugar or does she have enough? Justify.

b) Jane added $3\frac{3}{4}$ and $4\frac{7}{8}$. Jane's sister in grade 8 said the answer was $7\frac{5}{6}$. Jane wanted to determine if her sister's answer was reasonable. Explain how she might go about this.

c) John added $3\frac{5}{6}$ and $\frac{3}{4}$. He obtained an answer which was between 6 and 7. Is this reasonable? Explain why or why not.

B6.3 Ask students to decide whether the answer to each of the following is less than or greater than one and to give a reason for each decision.

a) $\frac{3}{4} + \frac{1}{8}$

b) $\frac{7}{8} + \frac{1}{5}$

c) $1\frac{4}{5} - \frac{7}{8}$

B6.4 A recipe uses $2\frac{1}{4}$ cups of flour, $1\frac{1}{3}$ cups of sugar, and $\frac{1}{4}$ cup of nuts. Al wanted to put all the dry ingredients into one measuring cup. Ask students what size measuring cup they would recommend.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

B7 multiply mentally a fraction by a whole number and vice versa

Elaboration - Instructional Strategies/Suggestions

B7 The ability to multiply a fraction by a whole number is necessary for working mentally with calculation of percents. It is therefore necessary to develop multiplication of fractions to the point where students can apply mental strategies when working with simple cases of multiplying a whole number by a fraction. Full development of multiplication of fractions will occur in grade 8.

Mental computation with fractions is certainly aided when students can visualize the fractions which they are multiplying. It is therefore important to utilize concrete models and pictorial representations to aid in this visualization process. Such materials include Fraction Factory Pieces, circle models, fraction bars, fraction strips, fraction tiles, pattern blocks, tangrams, geoboards, and money.

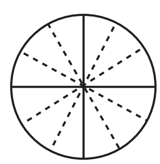
When students work with multiplication of a fraction by a whole number, they should start with situations such as $\frac{1}{4}$ of 12.

This is viewed as dividing 12 objects into 4 sets; how many are in each set?

When multiplying $8 \times \frac{1}{4}$, this can be viewed as eight sets of one-quarter, as follows:

Students should be able to form the 8 quarters to make 2 whole circles.

Another means of visualizing multiplication that uses the circle model follows.



$3 \times \frac{1}{12} = \frac{1}{4}$

The circle is divided into 12 sections. Each section is $\frac{1}{12}$ of the circle. $3 \times \frac{1}{12} = \frac{3}{12}$ or $\frac{1}{4}$ of the circle.

These are models that were typically used when students first developed a concept of division in the early years of schooling. Students should be able to make a quick link between, for example, finding $\frac{1}{4}$ and dividing by 4. They can then move from this understanding to the realization that $\frac{3}{4}$ of 12 represents 3 of the 4 groups. Students should move from the materials to visualization of this process for simple fractions. The formal pencil and paper algorithms are not the goal at this grade level, but will be developed in grade 8.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper/Portfolio

B7.1 Ask students to sketch diagrams to illustrate how to find

a) $\frac{3}{4}$ of a set of 20 items

b) $\frac{2}{5}$ of a set of 15 items

B7.2 At the school concert there were 600 people in attendance. $\frac{1}{4}$ of those attending were men, $\frac{1}{3}$ of those attending were women, and the rest were children. Ask students to determine

a) how many children attended

b) how much money was taken in at the door, if adults paid $4.00 per ticket and children paid $2.00 per ticket

Interview

B7.3 Ask students how they would go about estimating $\frac{3}{4}$ of a set of 21 items.

B7.4 Ask students how they would rewrite $\frac{1}{6}$ of 48 as a division problem.

B7.5 Ask students to mentally find the product of $\frac{4}{5} \times 20$ and explain their thinking.

B7.6 Ask students to use mental computation to answer each of the following:

a) $\frac{1}{2}$ of Joan's markers are used up. How many in her pack of 12 still work?

b) George had $\frac{2}{3}$ of the questions on the test correct. Since there were 30 questions on the test, how many did he get correct?

c) Only $\frac{1}{8}$ of the rooms at the hotel are vacant. There are 64 rooms at the hotel. How many rooms are full?

d) $\frac{2}{5}$ of the students in Sarah's class have to take the bus to school. There are 30 students in Sarah's class. How many do not take the bus?

B7.7 Tell students that you need to make 26 cheerleader shirts, and each shirt requires $\frac{1}{2}$ m of fabric. Ask them to determine how many metres of fabric are required.

Portfolio

B7.8 As a homework assignment have students find examples of price discounts that are expressed as fractions, using supermarket and/or department-store flyers as well as newspaper ads.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving rational numbers and integers*

SCO: By the end of grade 7, students will be expected to

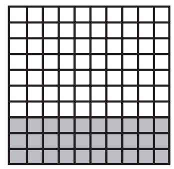
B8 **estimate and determine percent when given the part and the whole**

B9 **estimate and determine the percent of a number**

Elaboration - Instructional Strategies/Suggestions

B8/9 Students should be able to readily identify percent from a picture in situations for which both accurate and approximate values are provided. That is, students should be able to give an accurate value for the percentage shaded in diagram A below; and should be able to estimate the percentage shaded in diagram B.

A.



B.

Students should recognize, for instance, that 21 out of 40 represents slightly more than 50%. In some special cases such as finding 25% of a number, students might recognize 25% as $\frac{1}{4}$ and then divide by 4 as a means of finding $\frac{1}{4}$ of the number. Students should make immediate connections between certain percentages and their fraction equivalents, such as 50%, 25%, 75%, and 20%, 30%, 40%, etc. As well, for other percents students should be encouraged to recognize that percents such as 51% or 49% are close to $\frac{1}{2}$ and therefore use $\frac{1}{2}$ for estimation purposes.

When exact answers are required, students should be able to employ a variety of strategies in calculating percent of a number, including

- changing percent to a decimal and multiplying
  12% of 80 = 0.12×80 = ?
- computing 1% and then multiplying
  A 10×10 grid can provide students with a visual image of the 1% method. To find 6% of 400, tell students you have $400 and you want to share it equally among the 100 cells. Ask them how much would be in each cell? In 2 cells? In 6 cells? After working with other multiples of 100, try the same problem but with quantities such as $450 or $750. Students can also use this method to estimate; for example, they can estimate 8% of 619 by first mentally finding 8% of 600, using this method.
- changing to a fraction and dividing
  25% of 60
  = $\frac{1}{4} \times 60$
  = $60 \div 4$
- using the percent key on a calculator

For outcome B8, students would typically answer problems such as: Jane scored 32 out of 40 on a test. What percent does this represent?

For outcome B9, students would answer questions such as: Joe bought a pair of pants for $49.98. What did the pants cost after a 15% tax was added?

Note: The elaborations for B8 and B9 are continued on the next 2-page spread.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B8.1 $\frac{7}{8}$ of the tickets were sold for a concert held in a concert hall that has a 925-seat capacity. What percentage of the total capacity was utilized?

B8.2 The manager of a concert hall indicated that, in order to make a profit, the hall must be filled to at least 70% capacity or else the price of each ticket increased. The seating capacity is 1200, and advance ticket sales are at 912. Will a profit be made based on the number of tickets sold in advance sales?

B9.1 Describe more than one method that could be used to mentally estimate 22% of 310. How could you find the exact answer by calculating mentally?

Pencil and Paper/Project

B9.2 In a particular situation Sarah found that she had to repeatedly find various percentages of $440. To help, Sarah decided to make a table as follows:

50% of $440 =	25% of $440 =
10% of $440 =	5% of $440 =
1% of $440 =	

a) Ask students to help Sarah complete the table. Ask them to explain how Sarah might use the table to help find the following:
4% of $440 21% of $440
49% of $440 95% of $440
b) Ask students what other percentages can be found easily, using the information from Sarah's table, and discuss.
c) Ask students to make up their own table for finding various percents of $1600. Have them use the table to find 6%, 15%, 30%, 70%, and 99% of $1600.

B9.3 Tell students that Mrs. Nugent gave a test and marked it out of 40. To help convert grades to percents, she made a table to show the percentage for $\frac{1}{40}, \frac{5}{40}, \frac{10}{40}, \frac{15}{40} \ldots$, and used it to calculate the other percent grades. Ask students to make the table and use it to find $\frac{29}{40}$.

Interview

B9.4 Tell students that a number is between 30 and 50. Ask them to explain how to find what two numbers 20% of that number would fall between.

B9.5 Ask students to explain how to estimate 48% of something.

B9.6 Jane found 52% by finding 50% + 1% + 1%. Ask students to explain why this works. Have them use this strategy to find 52% of 160.

Interview/Presentation

B8.3 Ask students to
a) explain why 70% is not a good estimate for 35 out of 80
b) explain how to estimate the percentage when a test score is 26 correct out of 55
c) change each of the following to a percent mentally and explain their thinking: $\frac{2}{5}, \frac{4}{25}, \frac{6}{50}, \frac{7}{20}$
d) estimate the percent for each of the following and explain their thinking: $\frac{7}{48}, \frac{5}{19}, \frac{7}{24}$
e) indicate what percent of a book is left to read if they have read 60 out of 150 pages and explain their thinking

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict and justify the reasonableness of results in relevant problem situations involving rational numbers and integers*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

B8 **estimate and determine percent when given the part and the whole**

B9 **estimate and determine the percent of a number**

B10 **create and solve problems that involve the use of percent**

Elaboration - Instructional Strategies/Suggestions

B8/9 (Cont'd) Most important of all are those methods which students arrive at themselves. Before teachers provide specific methods, students should be given an opportunity to explore and come up with their own unique methods. Student-developed methods/approaches should be valued and, when appropriate, added to those which would normally be provided through instruction. Students should be able to compute 10%, 50%, and 1% mentally. This skill can be used to help with other mental computations.

B10 When students are creating problems that utilize percent, they can be given flyers from local supermarkets and/or department stores. These can be used to create problems which involve

- calculating the total savings when certain items are purchased at the sale price
- calculating the percent saved using the sale price and the original price

❒ Have students work in pairs. Each student first works individually to create three problems, using a newspaper flyer. This student then solves these problems on a separate sheet of paper. Partners swap problems and solve the problems created by their partner. Solutions are checked by the person who originally created the problems. When differences in solutions occur, both students work together to try to determine the source of error. (Sometimes the source may be a vaguely worded question. This can provide some food for further discussion with the small group or the whole class.)

❒ Give students data such as the following and ask them to create and solve problems involving percent which relate to the data.

The sales results for the school canteen for the first four months of the school year are as follows:
September - $1200,
October - $1460,
November - $1745, and
December - $1235.

❒ Joe, Jane, and Jill received the following grades on a science test: $\frac{32}{50}$, $\frac{36}{50}$, and $\frac{42}{50}$. In reviewing the test, Mr. Hipditch realized that one of the questions lacked sufficient information to allow for a correct answer to be found. Since nobody in the class answered it correctly, he decided to calculate grades out of a possible 48 points instead of 50 points.

Ask students to write questions which can be answered from the above situation and exchange them with a partner to solve.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B9.7 In reading a circle graph, Sarah realized that the sections of the graph did not contain any numbers or percents. She decided to use the angle measures to help read it. The section which represented the number of red cars looked as if it represented an angle of about 90^0.

a) Explain how to find what percentage of the cars are red.

b) It was more difficult to estimate the number of degrees the blue cars represented so Sarah used a protractor and found the angle to be exactly 145^0. What percentage of the cars are blue?

c) Suppose the circle graph represents the colours of the cars which pass an intersection during a one-hour period. Based on the information provided, if 400 cars passed this intersection, how many would you expect to be blue and how many would you expect to be red? How many were neither blue nor red?

Interview

B9.8 Ask students to use the fraction equivalent of 25% to calculate each of the following mentally: 25% of 800, 25% of 32.

Presentation

B10.1 Ask students to create percentage problems that might be solved, using the number sentences given.

a) $\frac{1}{5} \times 40 = ?$

b) $0.2 \times \$80 = ?$

c) $120 - 20% of $120 = ?

Portfolio

B10.2 In a flyer the following information is shown:

Jeans regularly $65 now 25% off, leather jackets regularly $239 now 30% off, watches regularly $29.99 now 20% off, study lights regularly $22 now 60% off.

Give students data such as that shown above and ask them to create at least three problems based on the data. Have them exchange problems with a partner and solve.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

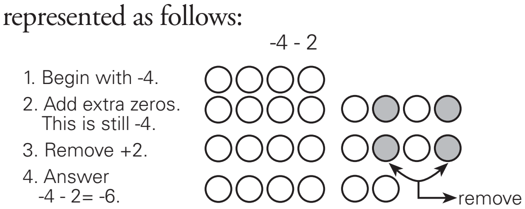
B11 add and subtract integers concretely, pictorially, and symbolically to solve problems

Elaboration - Instructional Strategies/Suggestions

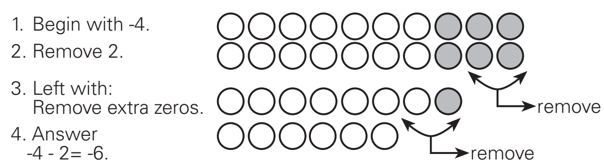
B11 Students should be given an opportunity to develop understanding of addition and subtraction of integers, using integer counters, the number line, and real-life situations that involve integers such as height above and below sea level, temperature, and net worth. Integer counters work very well for addition and subtraction problem situations. When adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros (e.g., -2 + 4).

Therefore, -2 + 4 = 2

For subtraction, Elaine owes $4 and she borrows $2 more from a friend. This may be represented as follows:



As was illustrated in A12, -4 can be modelled in a variety of ways; therefore, the same problem can be modelled as follows:

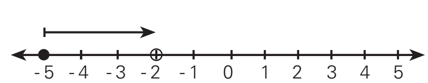


These addition and subtraction situations can also be developed using the concept of net worth. Using this model for addition, students should determine net worth in situations such as owing $3 to a friend but getting a $5 allowance. For subtraction, they should think of the removal of debt for subtraction of negative quantities.

Emphasis should be placed on problems involving time zones; temperature; heights above and below sea level; bookkeeping; profit/loss as it relates to stocks or shares; games that involve "going in the hole"; and sports records as in hockey, football, and golf.

When using the number line to describe subtraction of integers, the use of the comparison model is more meaningful than the take-away model for subtraction. For example, -3 - (-4) means how far is it from -4 to -3. The distance is 1. In going from -4 to -3, we move in a positive direction so the answer is +1. Note: When the second number of an addition or subtraction is negative, we use parenthesis, such as $-4 + (-3)$. Subtraction may be easier for some students using a missing addend approach. For example, to find -8 - (-4), ask: What would you add to -4 to get -8?

❐ Show -2-(-5) on a number line.



Start with -5.
Ask how far it is to -2,
and in what direction. Answer is 3.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B11.1 Have students use a number line, coloured counters, or the concept of net worth to explain why

a) -3 + 8 = 5
b) -5 - 3 = -8
c) -4 - (-6) = 2
d) 9 + (-2) = 7
e) 6 - 4 = 2
f) 8 - (-3) = 11

B11.2 Have students work in pairs. Ask them to roll two dice of different colours. Assign negative to one colour and positive to the other, and write a number sentence for the sum. Have them roll the two dice again, find the sum mentally, and add the result to their previous score. Have them exchange turns until one person reaches +20 or -20. Ask why it would be fair to accept +20 or -20 as the winning score.

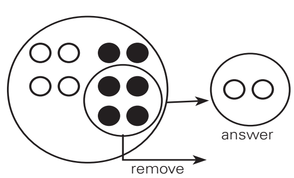
This activity can be modified and used with other operations. It can also be modified by assigning the negative or positive to specific colours after each roll, instead of maintaining the same designation throughout the game. Ask students if this change would allow them to get to 20 more quickly. Have them consider other possible rule changes, such as "You can't go over 20."

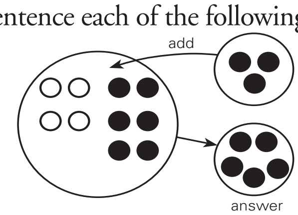
A similar game can be played using a deck of cards, where red represents one sign and black the other sign.

Pencil and Paper

B11.3 Make a budget that shows income and expenditures for one month. Create some integer problems based on your budget. (This could be done using a spreadsheet.)

B11.4 Explain in words and as a number sentence each of the following:

a) 

b) 

B11.5 John saved $50 during the fall. He owes $15 to his friend. Because he had a good term report his father gave him $20. What is John's net worth?

Interview

B11.6 Ask students if the sum of a negative number and a positive number is always negative and to explain why or why not.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

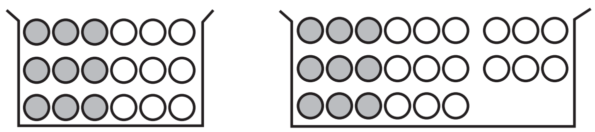
SCO: By the end of grade 7, students will be expected to

B12 multiply integers concretely, pictorially, and symbolically to solve problems

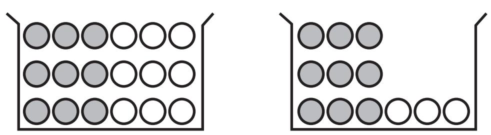
Elaboration - Instructional Strategies/Suggestions

B12 Addition of integers helps to establish some of the initial groundwork for multiplication of integers. Multiplication of integers should start with examining multiplication as repeated addition, as in
4 sets of (-3) = (-3) + (-3) + (-3) + (-3).

The following is one way of using counters to model multiplication. Start with a container having an equal number of positives and negatives.



+2 x (-3) implies adding 2 sets of -3. What is the total in the container?



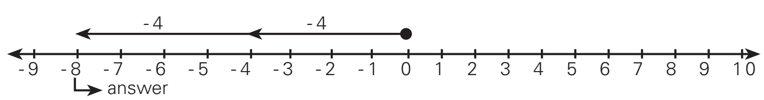
-2 x (-3) implies removing 2 sets of -3. When 2 sets of -3 are removed, what remains in the container?

When the first factor of the multiplication is positive, the operation is conceptualized as repeated addition. When the first factor of the multiplication is negative, the operation is conceptualized as repeated subtraction.

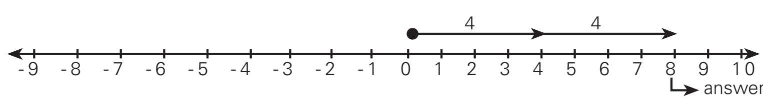
Net worth can also be used as a context for multiplication. Consider, for example, the impact on net worth if a person owes $6 to each of 3 friends, or if a debit of $6 to each of 3 friends is forgiven.

The number line can also be used to model problems such as:

$2\times(-4) \rightarrow$ 2 sets -4



$2\times(+4) \rightarrow$ 2 sets



Patterning can then be used to justify the result for a negative by a negative.

$3\times(-2)=-6$

$2\times(-2)=-4$

$1\times(-2)=-2$

$0\times(-2)=0$

$-1\times(-2)=2$

$-2\times(-2)=?$

$-3\times(-2)=?$

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B12.1 Start with a neutrally charged container (equal number of positives and negatives). [could use a glass container with two colours of marbles]

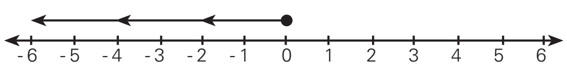
a) Have students take out 4 groups of -2 from the container. Ask what charge is left in the container. Have them sketch diagrams to illustrate this situation and write a number sentence to model the situation described.
b) Have students add 3 groups of -3 to the container. Ask what charge is in the container. Have them sketch diagrams to illustrate this situation and write a number sentence to model the situation described.

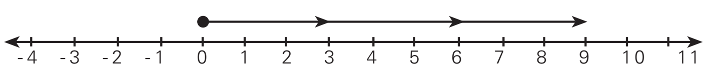
Pencil and Paper

B12.2 Write a number sentence for each of the following problems and use a diagram to model the situations.

a) Fran lost 3 points in each round (hand) of cards that was played. If she played 4 rounds (hands), what was her score at the end of the game?
b) Bill owed $5 to each of 3 friends. What was his net worth, based on this situation?

B12.3 Write number sentences for each of the following:

a) 

b) 

Interview

B12.4 Ask students to name as many pairs of integers as possible that have a product of -16 and then a product of +16. Ask what they notice about the number of possible pairs for the positive product versus the negative product.

Portfolio

B12.5 Sarah borrows $8 from each of her two friends, Chris and Jo. Because it was Sarah's birthday, her two friends each forgave Sarah's debt. Ask students to explain how this affected Sarah's net worth.

Journal Entry

B12.6 Tell students that a friend missed class the day that multiplication of integers was first introduced. Ask them to write a detailed explanation for the friend to help him/her understand how to solve

$-2\times(+5)$ and $-3\times(-4$

B12.7 Tell students that a list of temperatures for seven days was prepared, but the ink became wet and one temperature was unreadable. The mean temperature was -3°C , and the six known temperatures were 2°C , -4°C , -6°C , -2°C , 4°C , and -5°C . Ask students to find the missing temperature.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

B13 divide integers concretely, pictorially, and symbolically to solve problems

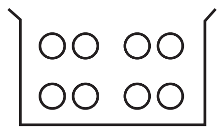
Elaboration - Instructional Strategies/Suggestions

B13 The following provides a starting point for modelling division.

❒ Start with an empty container

$-8 \div (-2)$

Ask students, how many groups of -2 are in -8.



Add successive groups of -2 to the container until there are -8 in it. Record the number of groups. Four groups of -2 were added to the container; therefore, $-8 \div (-2) = 4$.

This model can be used with some modification for other division situations.

Patterning is also useful in division; for example,

$6 \div 2 = 3$	$-6 \div (-2) = 3$
$4 \div 2 = 2$	$-4 \div (-2) = 2$
$2 \div 2 = 1$	$-2 \div (-2) = 1$
$0 \div 2 = 0$	$0 \div (-2) = 0$
$-2 \div 2 = ?$	$2 \div (-2) = ?$
$-4 \div 2 = ?$	$4 \div (-2) = ?$

Comparison of multiplication and division situations can also be very useful in helping students understand division of integers. After multiplication has been fully developed, the fact that multiplication and division are inverse operations can be utilized. For example, since $-4 \times 3 = -12$, it must be true that the product divided by either factor should equal the other factor; therefore, $-12 \div (-4) = 3$ and $-12 \div 3 = -4$. Likewise, if $-4 \times (-3) = 12$, then $12 \div (-4) = -3$ and $12 \div (-3) = -4$.

Using a missing factor can also be useful. For example, in the case of $-16 \div (-4)$, ask: what multiplied by -4 gives -16?

The concept of net worth can be linked with division as well. Students can think of owing $12 when an equal amount is owed to each of three friends. Students can determine how much is owed to each friend.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B13.1 Complete the following patterns:

$9 \div 3 = 3$	$-9 \div (-3) = 3$
$6 \div 3 = 2$	$-6 \div (-3) = 2$
$3 \div 3 = 1$	$-3 \div (-3) = 1$
$0 \div 3 = 0$	$0 \div (-3) = 0$
___ $\div 3 = -1$	___ $\div (-3) = -1$
___ $\div 3 =$ ___	___ $\div$ ___ $=$ ___
___ $\div$ ___ $=$ ___	___ $\div$ ___ $=$ ___
	___ $\div$ ___ $=$ ___

Based on the patterns, write a conclusion that can be drawn about division of a positive integer by a positive integer, a negative by a negative, a positive by a negative, and a negative by a positive. Confirm your conclusion by creating similar patterns beginning with

a) $20 \div 4$	b) $-20 \div (-4)$
$16 \div 4$	$-16 \div (-4)$

B13.2 Write a number sentence to model the following situation. Chris and his three friends together owe $32. They agreed to share the debt equally. What is each person's share of the debt?

B13.3 Write a division or multiplication sentence to solve each problem.

a) $? \div (-3) = -9$
b) $57 \div ? = -3$

Interview

B13.4 Ask students to name as many division problems as possible, based on each of the following multiplication sentences, using the fact that multiplication and division are inverse operations.

a) $-5 \times (-4) = 20$
b) $6 \times (-3) = -18$

Journal Entry

B13.5 Tell students that a friend missed class the day that division of integers was first introduced. Ask them to write a detailed explanation for the friend to help him/her understand how to solve

a) $-10 \div 5$
b) $-24 \div (-6)$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

vi) *select and use appropriate computational techniques in given situations and justify the choice*

SCO: By the end of grade 7, students will be expected to

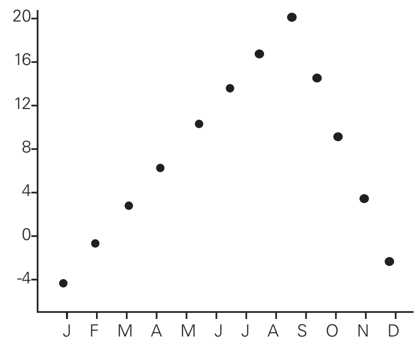
B14 solve and pose problems which utilize addition, subtraction, multiplication, and division of integers

B15 apply the order of operations to integers

Elaboration - Instructional Strategies/Suggestions

B14 This outcome should be addressed concurrently with outcomes B11, B12, and B13. The contexts mentioned in relation to the addition and subtraction of integers will apply equally well for multiplication and division situations. Students could be given data about temperature, height above or below sea level, net worth, as well as various games that involve score keeping that is above and below zero (often referred to as "going in the hole"), and be asked to create and solve problems.

❐ Give students a temperature graph such as the one shown and ask them to create questions which can be answered from the graph. Have them exchange questions and solve them.



Questions created might include: What was the change in temperature from February to May? from September to December? What was the average temperature?

B15 The order of operations was already developed and utilized with whole and decimal numbers. Students should understand how to use the +/- key on the calculator and how negatives are dealt with differently on certain calculators. However, the focus should be on working with models and pictorial representations. Numbers should be kept small enough that most computations can be handled mentally, once students understand the issues pertaining to the sign of the numbers. In grade 8, students will work with rational numbers in decimal form, and the need for calculator use will become more evident.

The proper use of brackets in writing expressions involving signed numbers should be discussed. Students should understand, for example, the need for the brackets around (-4) in the expression -5 - (-4), and why that same need is not so great for the -5 in the same expression.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B14.1 Find a temperature graph from social studies and create problems which require the use of addition and subtraction of integers. Exchange the problems with a partner and solve.

B14.2 Write a problem that can be solved, using each of the following number sentences.

a) $4 \times (-5) = -20$

b) $-12 \div 3 = -4$

c) $-5 \times (-6) = 30$

B15.1 The following is a record of Jerry's weight change per month:

Jan.	Lost 4 kg	Feb.	Lost 2 kg
March	Lost 3 kg	April	Gained 2 kg
May	Gained 1 kg	June	Lost 2 kg

a) If Jerry's weight was 90 kg before the diet, what was his weight after the 6-month diet?

b) What was his average change of weight per month?

B15.2 Pam recorded the daily high temperatures for one week and found the average high temperature for the week to be $-4°C$. The temperatures from Sunday to Friday were $11°C$, $-17°C$, $0°C$, $-23°C$, $-13°C$, and $9°C$. Estimate the temperature on Saturday. What was the actual temperature on Saturday? Compare the answer and your estimate.

Portfolio

B15.3 Tell students that to win a free trip, they must answer the following skill-testing question correctly:

$-3 \times (-4) + (-18) \div 6 - (-5)$

Tell students that the contest organizers say that the answer is +4. Ask them to write a note to the organizers explaining why there is a problem with their solution.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

SCO: By the end of grade 7, students will be expected to

B16 create and evaluate simple variable expressions by recognizing that the four operations apply in the same way as they do for numerical expressions

Elaboration - Instructional Strategies/Suggestions

B16 In mathematics, quantities that change are called variables. It is important to develop with students a sense of why we need variables. To this end, they can start thinking about variability by thinking about things that change or vary. This concept of variability is further developed as part of GCO (C). In fact, it is recommended that the outcomes B16, B17, and B18 be developed in conjunction with the outcomes of GCO (C).

This concept of variability can be developed by looking at simple patterning. Students are familiar with the concept of perimeter, so a simple example to consider would be the perimeter of a square for various side lengths, where P = 4S.

Side Length	1	2	3	4	5
Perimeter	4	8	12	16	20

This outcome focusses on evaluating simple variable expressions by substituting a value for the variable in the expression. Students should understand that everything that was true in evaluating numerical expressions applies to variable expressions, once the variable has been given a numerical value. It should be emphasized that there is one clear distinction in the way variable expressions are written, in comparison to numerical ones. Students should understand that $4 \times n$, $4 \cdot n$, and 4n are all acceptable ways of writing products, whereas 4×5 and 45 have very different meanings. Some students may also confuse expressions such as 3m with three metres, since their common experiences with the use of a single letter have often been in association with measurement units.

Evaluating open expressions provides practice that will help students become more efficient when solving equations, using such methods as guess and check or systematic trial. Be careful that the problem situations are restricted to those for which the students have developed the operations on the number sets involved. For example, since fraction operations have not been developed symbolically, it will be necessary to use primarily decimal numbers and integers as replacement values or coefficients. Fractional number replacement values should be limited to those which can be calculated mentally.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B16.1 Evaluate the following:

a) $\square \times 2 + 5$ when $\square = \frac{1}{2}$

b) $5 + 4m$ when $m = 4.2$

c) $\frac{y}{4} + 22$ when $y = 60$

d) $-3 + 5p$ when $p = -2$

B16.2 Make a table to find the length of a rectangle for each value of width (m). The length is $4m + 5$ and m=1, 2, 3, ..., 7.

B16.3 Using the following algebraic expressions

$\frac{p}{3}$ 3p + 1 -2p + 3

a) make a chart and generate at least six values for each expression
b) make a graph to display each set of values, where p is used on the horizontal axis and the values generated from each of the expressions are graphed vertically

Portfolio

B16.4 Ask students to evaluate, using a spreadsheet or by creating a table, 5x + (-4) where x = 3, 4, 5, 6, ... 10. Ask students to create a question which this set of values might answer.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 7, students will be expected to

B17 distinguish between like and unlike terms

B18 add and subtract like terms by recognizing the parallel with numerical situations, using concrete and pictorial models

Elaboration - Instructional Strategies/Suggestions

B17/18 Work with addition and subtraction should focus on the use of concrete and pictorial representations and should be kept fairly straightforward. A number of concrete materials, such as algebra tiles or pattern blocks, can be used to model variable use.

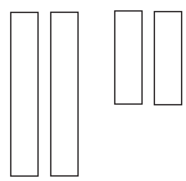
Students should also be able to apply modelling to familiar relationships. For example, ask students to consider the perimeter of a rectangle. They should be able to relate easily to the fact that the perimeter can be found using $P = L + L + W + W$.

However, to simplify the situation, the perimeter can also be written as

$P = 2 \times L + 2 \times W$

or $P = 2L + 2W$.

Using a set of algebra tiles that includes both x tiles and y tiles, model this for the students as follows:



Activities should provide an opportunity for students to demonstrate an understanding about why we use a different tile to represent L and W, why we can combine the L's and the W's, and why the L's and W's cannot be combined to make "4LW." Students should see a parallel here with numerical situations. For example, the comparison should be made between $2 \times 7 + 2 \times 5$ and $2L + 2W$, and it should be made clear why neither can be re-written as $4 \times 7 \times 5$ or $4 \times L \times W$. Activities should also include situations for which such combinations are possible, such as $3p + 2p = 5p$ and $3 \times 7 + 2 \times 7 = 5 \times 7$.

Once materials are used to help students combine and distinguish variables from one another, this concept can be extended to the introduction of other tiles such as x^2, y^2, and xy tiles. The important link should be made between the area of a tile and its name; for example, the xy tile is called an xy tile because its dimensions are x and y and its area is xy.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

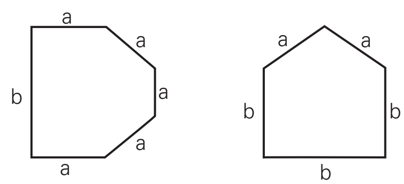
Performance

B17/18.1 Ask students to write an expression, in compact form, for each of the following:

a) ☆+☆+☆+❀+❀+❀+❀+❀+❀

b) ☆+☆+☆+❀+❀+☆+☆+☆+❀

B17/18.2 Ask students to write an expression that is as compact as possible for the perimeter of each of the following figures:



B17/18.3 Ask students to show 4p + 2q using algebra tiles or pictures.

B17/18.4 Ask students to use materials to simplify

a) p + p + p + p + p
b) 2p + 3q + p + 4q
c) 4p + 5p + (- 3p)

B17/18.5 The cost of renting a Seadoo is $12 per hour. Ask students to find the cost of a rental for 3 hours, 4 hours, 6 hours, and h hours.

B17/18.6 Ask students to find the perimeter for each polygon when the side length is 6. when it is 7.2.

a) pentagon
b) hexagon
c) heptagon
d) octagon

Note: B17/18.5 and B17/18.6 contain elements connecting to C2.

Suggested Resources

Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *analyse, generalize, and create patterns and relationships to model and solve real-world and mathematical problem situations*

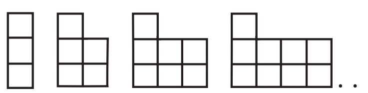
SCO: By the end of grade 7, students will be expected to

C1 describe a pattern, using written and spoken language and tables and graphs

Elaboration - Instructional Strategies/Suggestions

Mathematics is in many ways the study of patterns. Patterns are utilized repeatedly as a means of developing concepts and as a tool in solving problems. Students recognize, represent, and apply patterns in other strands of this curriculum. (When there is a connection to patterning in the development of other strands, it is often highlighted in the elaboration.) Essentially, outcomes C1 and C2 can be treated together for instruction.

C1 Given a number, geometric, pictorial, or situational pattern, students should be able to describe the pattern in spoken and written language. Very often, students will need to extend the pattern to fully understand it. (Note: This may be possible in more than one way.) Students should then be able to represent the pattern in a table of values and continue on to create its graph, which could be done using technology. It is important to make connections among contextual, pictorial, concrete, symbolic, graphical, and verbal representations. Students should also be asked to create their own patterns and represent them in multiple ways. When using geometric patterns, students might be asked to describe how to build the next model or other models in the series. This helps to connect the pattern and the model, and provides a link with the geometry strand.



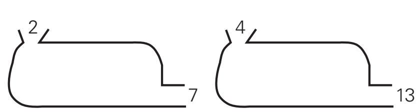
For the above pattern, students could be asked to relate the number of units along the bottom to the number of units in the area. Using this relationship, they can generate a table of values and construct a graph. Students can be assisted in arriving at a relationship by making a table of values and examining the horizontal relationship within it. Many students at this level will need to look at many cases before they are able to generalize. For the pattern above, students may make a table as follows:

b	area
1	3
2	3+2
3	3+2(2)
4	3+2(3)
5	3+2(4)
n	3+2(n-1)

Students do not have the algebraic skills to confirm that $3+2(n-1)$ simplifies to $2n+1$. They can, therefore, accept that there is more than one format for the correct answer. Extending this to an expression or equation connects with outcome C2.

It is important that students have a sense of how the value of an expression changes with the value of the variable. Number-in/number-out games can be used to develop this sense. These types of activities are often referred to as "function machines."

❐



What function do you think this machine has? (Valid answers may vary.)

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C1/2.1 The diagram below shows a series of triangular supports for a bridge.

a) Continue the pattern above for up to seven triangles.

b) Complete the chart to show pattern growth.

# of triangles	# of line segments
1	
2	
3	
4	
5	
.	
.	

c) Describe in writing how the pattern grows.

d) Predict the number of line segments for 10 triangles and 20 triangles.

e) Write an algebraic expression to show the number of line segments for n triangles.

f) Draw a graph to show the pattern. Does it make sense to join the points? Discuss the shape of the graph.

C1/2.2 Given 2, 5, 10, 17, 26, 37, ... ,

a) continue the pattern for the next three numbers

b) describe, in words, how the pattern grows

Interview

C1/2.3 For the table in C1/2.1, a student was asked to explain the relationship between number of triangles and number of line segments. The student described the pattern as follows: “it goes up by 2.” Ask students if they agree or disagree, and have them explain their reasons.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *analyse, generalize, and create patterns and relationships to model and solve real-world and mathematical problem situations*

SCO: By the end of grade 7, students will be expected to

C2 summarize simple patterns, using constants, variables, algebraic expressions, and equations, and use them in making predictions

Elaboration - Instructional Strategies/Suggestions

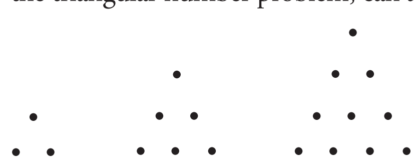
C2 While students have had some exposure to the use of variables to summarize patterns, grade 7 may be the students' first formal exposure to their use. Students will need some time to grasp the meaning of variable. In mathematics, variables are, typically, quantities that change. Students should also understand, however, that while in many situations variables can take on many values (e.g., P=4S for any value of S), in other situations they represent a single value (e.g., x+3=9). Students might relate variables to things which change over time that are part of their own experiences, such as their height or hair length.

In the early stages of variable usage it may be wise to avoid the use of "x" as a variable, since students often get "x" mixed up with the multiplication symbol. It is important, when reading aloud to students, to read expressions such as 3m as "a number m multiplied by 3," or "3 times m." Care must also be taken since students with limited understanding of, or exposure to, symbolism will often view an expression such as 3m as being 36 when m = 6, or as 3 metres. It is also easy for students to confuse the placement of variables when writing expressions or equations; for example, if there are 6 notebooks for each student, they might write s = 6n instead of n = 6s. (Also, see B16, B17, and B18 for related outcomes.)

In the real world, problems are seldom presented algebraically. They are usually described as a situation or observed as a pattern, and students have to translate the situation or pattern into an expression or equation. There is usually a need to use variables because the result of the situation can vary. This is especially true of patterns.

Students should use tables to organize the information that a pattern provides. In tabular form, students can better observe what the pattern is and extend it to find missing values. This can be handled well using a spreadsheet program. While some students may only be able to make use of the spreadsheet as a table organizer, others may be able to produce the formulas necessary to extend patterns. When using tables to discover relationships, it is important for students to realize that they are looking for the relationship between the two variables.

Patterns used should include counting patterns, patterns involving exponential growth, and patterns involving geometric figures such as polygonal trains. Using polygonal trains, students can be asked to develop variable expressions and equations and predict number of sides, perimeter, and area as the pattern is extended. Many students will need to use a series of number sentences organized in a table of values to lead them to a generalization. Various growth patterns represented by various dot arrays, such as the triangular number problem, can also be considered.


, etc.

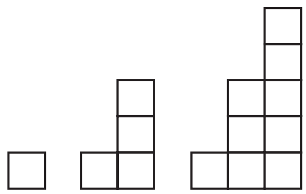
Dot patterns such as the one above can motivate the study of Gauss, who is credited for discovering a formula for the series $1+2+3+4+\ldots$ at a very early age.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C1/2.4 Have students use cubes to copy and extend these shapes to the fifth shape in the pattern.



a) Have students construct a chart to record and reveal the pattern.
b) Ask them to predict the total number of cubes needed to make the 10th and 25th shapes in the pattern and explain their prediction.
c) Ask them to explain in words how the pattern grows.
d) Ask: If n is the number of cubes along the bottom of one shape in the pattern, what would be the total number of cubes in the shape?
e) Have students make a graph of the pattern and ask what shape the graph has. Discuss the shape of the graph.

C1/2.5 The table shows the relationship between the number of riders on a tour bus and the cost of providing boxed lunches.

Customers	1	2	3	4	5
Cost	4.25	8.50	12.75	17.00	21.25…

a) Ask students to explain how the lunch cost is related to the number of riders.
b) Have them write an equation for finding the lunch cost (l) for the number of customers (n).
c) Ask them to use the equation to find the cost of lunch if there were 25 people on the tour.
d) Ask how many people were on the bus if the tour-bus leader spent $89.25 on lunch.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) apply algebraic methods to solve linear equations and inequalities, and investigate non-linear equations

SCO: By the end of grade 7, students will be expected to

C3 explain the difference between algebraic expressions and algebraic equations

C4 solve one- and two-step single-variable linear equations, using systematic trial

Elaboration - Instructional Strategies/Suggestions

C3 Explaining the difference between algebraic expressions and algebraic equations should be a natural flow from the use of these in working with patterns, as opposed to providing a definition at the beginning of instruction. A number sentence is called an equation. A number sentence with a variable is an algebraic equation. The major difference between an equation and an expression is that an equation is a complete sentence and therefore contains a verb. For example, p = 3 reads 'p is equal to 3,' whereas p + 3 reads 'p plus three.' p + 3 contains no verb and is therefore considered an expression. Equations containing more than one variable such as b = 2n - 1 can have a variety of values which make them true. For each value of n , a corresponding value of b can be found. The expression 2n - 1 behaves in a similar way as the previous equation, whereas 2a - 1 = 7 is only true for a single value of a.

C4 At first, students might begin by using the guess-and-check (try-and-adjust) strategy. By observing patterns in their results, students become more systematic in the guesses they make.

❐ Provide students with a problem such as the following:
Anne is making a geometric pattern with some toothpicks. The pattern can be described using t = 3s - 2, where t stands for the total number of toothpicks and s stands for the number of toothpicks along the bottom. If you knew that the total number of toothpicks used was 82, what would be the number of toothpicks along the bottom?
Start with a guess such as $30 \therefore 3\times 30-2=88$.
Since this guess is too large, try $20 \therefore 3\times 20-2=58$.

Students should realize that the desired result is between the two guesses, and the first guess was closer to the desired answer. For a third guess the student might select $26 \therefore 26\times 3-2=76$. Students should then realize that the desired answer is between 26 and 30 and select new guesses until they arrive at the correct solution. Since each guess is made in a systematic way by using the information acquired from previous guesses, the process used is called systematic trial.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C3.1 Below there are three expressions and/or equations in algebraic form:

4p - 5 = B
4p - 5 = 55
4p - 5

The three expressions and/or equations have some similarities and some differences. List ways in which they are similar and ways in which they differ. Find some possible values for the variables in the three situations. Which are equations and which are expressions? Explain why.

Interview

C3.2 Ask students to give three examples of algebraic equations and three examples of algebraic expressions. Ask them to explain what it is about the expressions that makes them algebraic. Ask students to explain what it is that makes the examples they created equations or expressions. Ask whether an algebraic expression and an algebraic equation can be demonstrated using a balance. Ask students to explain or demonstrate.

Presentation

C3.3 Ask students if it can be true that 3b - 1 is equal to 5 under some conditions and equal to 29 in other conditions. Ask students to explain their reasoning to the class.

C4.1 Ask students to explain how to find the value of f in the given equation by using systematic trial:

154 + 2f = 340

Portfolio

C3/4.1 Ask students to find the value of p in the following equations by using systematic trial:

a) 5p + 8 = 63
b) 6p - 9 = 81

Ask them to explain if each equation has just one value for p or if they think there are others in addition to the one found. Have them justify their answer.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *apply algebraic methods to solve linear equations and inequalities, and investigate non-linear equations*

SCO: By the end of grade 7 students will be expected to

C5 illustrate the solution for one- and two-step single-variable linear equations, using concrete materials and diagrams

Elaboration - Instructional Strategies/Suggestions

C5 One way to show a solution to a simple equation concretely is through the use of envelopes and counters.

❒ Using a small envelope, place a number of counters inside. On the outside of the envelope write a variable such as W. Make an equation such as

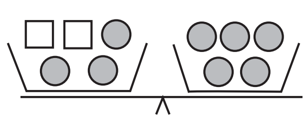
$$\boxed{W} + 3 = 7$$

W represents the number of counters inside the envelope. Ask students to guess the number of counters in the envelope to make the equation true and then verify by checking the envelope.

This model can be used when the variable is repeated or when positive and negative integer counters are involved; for example,

$$\boxed{M} + \boxed{M} + (-3) = 9$$

Ask students what other ways the equation can be written. It is through rewriting an equation in alternative forms that we arrive at the notion of solving equations using formal methods. Students should be made familiar with such notions as adding or subtracting the same value from both sides of an equation and with why equality is maintained. This can be illustrated by using a balance, since a balance illustrates equality extremely well. Students can be asked to find what each ☐ represents in situations such as the following:



Students can be led through a process of neutralizing the positives or negatives on one side of the balance so that the balance is left with only the unknown value(s) on one side of the balance. If only one unknown exists, its value should be apparent. In the situation above, students would then go through a process of sharing to determine the value of the unknown.

As an extension, the **cover-up method** can also be explored. The cover-up method is named for the way it is typically applied. For example, using the formula 4m + 5 = 25, cover up the 4m and ask the question, "What added to 5 makes 25?" Next, cover the m and ask, "What multiplied by 4 makes 20?"

❒ Give students the equation 3p + 5 = 17. Ask, "What number plus 5 = 17?" When students respond that 12 plus 5 = 17, ask, "What multiplied by 3 must equal 12?" Since 3 x 4 = 12, then p must be equal to 4.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

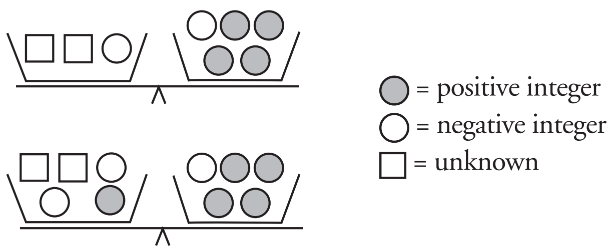
Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C5.1 Find the integer represented by □ for the following balances and explain each step in finding the solution.

a)

b)



Interview

C4/5.1 Ask students what integer is hidden in each envelope to make this equation true.

$\boxed{W} + \boxed{W} + 2 = 12$

Ask them how to rewrite the equation so that the two no longer exists on the left-hand side of the equation and yet equality is still maintained.

Ask them to use sharing to determine the value of W.

Extension

C5.2 Sandy started with the equation 4p + 14 = 46. She covered up the 4p and asked herself the question: What added to 14 gives 46?

a) Ask students what her answer should have been.
b) Using her answer in part a), Sandy wrote a new equation, which was 4p = ____. She then asked herself: What multiplied by 4 equals ___? Ask students what the value of p is.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent patterns and relationships in multiple ways (including the use of algebraic expressions, equations, inequalities, and exponents)*

iv) *explain the connections among algebraic and non-algebraic representations of patterns and relationships*

SCO: By the end of grade 7, students will be expected to

C6 **graph linear equations, using a table of values**

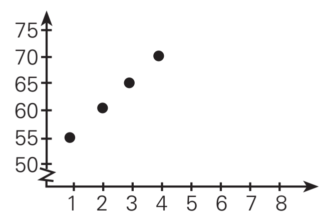
C7 **interpolate and extrapolate number values from a given graph**

Elaboration - Instructional Strategies/Suggestions

C6 In previous grades, students plotted ordered pairs, using all four quadrants of the coordinate plane. The terms x-axis and y-axis should be used for the vertical and horizontal axes. Most patterns studied thus far lead to graphing in the first quadrant. Plotting points on the coordinate plane is often included as part of the study of integers. Students should be encouraged to view the axes as two number lines that are perpendicular to each other, intersecting at their origins. Graphing of linear equations should be done using a table of values, but this can be facilitated through the use of technology. Spreadsheet software and/or the graphing calculator can and should be utilized where it is available, in addition to work with pencil and paper. Much of the graphing will be incorporated into the work with C1 and C2.

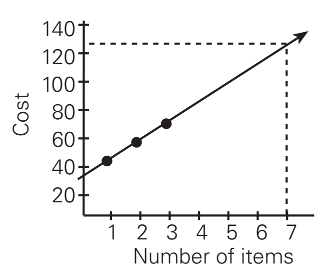
❒ Ms. Stead's class decided to order t-shirts that had the signature of all students from the class on the back. It cost \$50 to set up the printing and then \$5 per shirt printed. Have students create a table of values and graph for this situation, as follows:

n	5n+50
1	55
2	60
3	65
.	.
.	.
.	.
10	?
n	?



Ask students if negative values make sense in this situation.

C7 Work should be done on interpolating (finding a point between two known points); for example, if (2,5) and (3,7) are two points on a straight line graph, students might judge the y-value for x = 2.5 to be half way between 5 and 7. This can be done using the actual numbers, or estimating from the graph. Work should also be done on extrapolating (finding a point that lies beyond the known data). This would be done mainly by "eye-balling" or estimating from the graph. If several points are known, they can be used to draw the graph, which can then be extended to extrapolate to an unknown value. This process will be particularly important in data management (GCO F). In fact, whenever students use a graph to extend or predict beyond given values, they are using extrapolation. Such experiences occur naturally in any patterning activities. For example, in the graph below, to find a cost for 7 items, Jim sketched a line through the three known points, then drew a vertical line from 7 up to the line he just sketched, and then drew a horizontal line from the intersection point across to the y-axis. The point of intersection with the y-axis allowed Jim to extrapolate a cost when x = 7.



GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

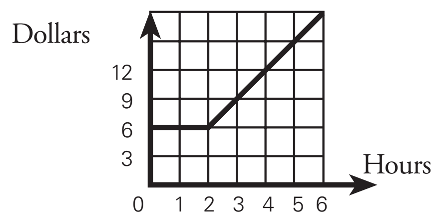
Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C6/7.1 The cost, C, of a pizza is $3, plus $1 for each topping, t.

a) Construct a table to show the cost of the pizza for 1, 2, 3, 4, 5, 6, 7, and 8 toppings.
b) Write a relationship to represent the cost of the pizza for t toppings.
c) Explain whether or not t can be equal to -2.
d) Graph this situation and use the graph to find how many toppings are on a pizza costing $14.00.

C6/7.2 The following graph shows how much Pat charges to babysit.



a) How much would Pat earn when babysitting for $1\frac{1}{2}$ hours? for 3 hours? for $3\frac{1}{2}$ hours? for 5 hours?
b) Estimate how much she would earn for 7 hours. Extend the graph to check your estimate.
c) Describe, in words, how Pat gets paid.

Portfolio

C6/7.3 Bill noted that the temperature was increasing at a very regular rate. From his measurements, he created the following table.

time	temp(°C)
0	-10
1	-7
2	-4
3	-1

Assuming the pattern continues, ask students to graph the values and use the graph to predict the temperature when time equals 18.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *analyse functional relationships to explain how the change in one quantity results in a change in another*

SCO: By the end of grade 7, students will be expected to

C8 **determine if an ordered pair is a solution to a linear equation**

C9 **construct and analyse graphs to show how change in one quantity affects a related quantity**

Elaboration - Instructional Strategies/Suggestions

C8 Given various ordered pairs, students should determine if the ordered pairs satisfy (i.e., are solutions to) a given equation. They should be able to determine this both by plotting the points to see if they are is in keeping with the rest of the points in the pattern and by substituting them into the equation to see if they make the equation true or false. They should also equate an ordered pair that makes an equation true with the fact that it is a solution to the equation.

C9 Since rates naturally involve the comparison of two different quantities, this provides an ideal context for the study of linear graphs. Also, in many situations which can be described graphically, the rate varies, and a non-linear graph is produced. Students should be given situations which are not linear, as well as situations for which the rate is consistent and a linear graph is produced.

It is important when making tables of values to emphasize that data only produces a line when equal increases in x result in equal increases in y. For example, in the table below, as x increases by 1, y increases by 3.

x	1	2	3	4
y	2	5	8	11

The curriculum is rich with opportunity to study change. In terms of number theory, students can investigate such things as what happens to the size of 2^x as x increases. Many situations in the strand of measurement provide an excellent context to study the effect change in one quantity will have on a related quantity. For example, students can relate change in the length of a side of a square to change in perimeter, or to change in area. Given a fixed perimeter, it is interesting for students to study change in width of a rectangle and its impact on the area of the figure. Students can also investigate different perimeters when the area is kept constant.

While perimeter, area, and volume are not formally studied in grade 7, students have studied these in previous grades. Using perimeter, area, and volume as a context for the study of functional relationships will provide a means of keeping them current for students until they revisit and extend them in grade 8.

Rates such as cost per kilogram and pay per hour can also be used to illustrate and support the development of this outcome.

❐ A certain brand of oranges regularly sells for $0.59 each. Graph the cost of 3, 6, 9, and 12 oranges. This week these oranges are on sale for 3 for $1.00. Graph using the same axes the cost of 3, 6, 9, and 12 oranges. Describe the differences and similarities in the two graphs. Explain why these differences and similarities exist.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Portfolio

C8.1 The equation y = 3n - 1 describes a pattern.

a) Ask students to identify three points on a line that belong to this pattern.
b) Ask if (8, 23) belongs to this pattern. Ask students to explain why or why not.
c) Have students graph the equation, using a table of values.
d) Ask them if (15, 40) lies on the line. Ask if (15, 40) is a solution to the equation.

C9.1 Tell students that the perimeter of a rectangle is fixed at 22 units. Ask them to complete the following chart, showing some possible dimensions for the rectangle.

width	length	perimeter	area
1		22	
2		22	
3		22	
.		.	
.		.	

a) Ask students to construct a graph, using width on the horizontal axis and length on the vertical axis. Ask them to describe the graph and explain why they think it has its shape. Ask how change in width affects length.
b) Ask students to construct a graph, using width on the horizontal axis and area on the vertical axis. Ask them to describe the graph and explain why they think it has its particular shape. Ask what conditions of the rectangle would produce the greatest area.
c) Ask students to consider the graph produced in part b) and predict what the graph would look like if width were replaced by length on the horizontal axis. Ask students to graph to verify their prediction. Have them compare the graphs in b) and c) and write a conclusion.
d) Using the table and graphs, ask students what conclusions they can draw with respect to the following:
   - the sum of the length and width in relationship to the perimeter
   - the shape of the rectangle when the area is at its maximum

   Ask if the maximum area is shown in the table or do they think it is possible to construct a rectangle with the same fixed perimeter that has an even greater area than any of the rectangles in the table. Ask why or why not.

   Ask what is the smallest area they observed. Ask if it is possible to produce a rectangle with an even smaller area for a fixed perimeter of 22 units.

Suggested Resources

Measurement

General Curriculum Outcome (D):

Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *communicate, using a full range of SI units (e.g., mm, cm, dm, m, hm, Dm, km), and select appropriate units in given situations*

SCO: By the end of grade 7, students will be expected to

D1 identify, use, and convert among the SI units to measure, estimate, and solve problems that relate to length, area, volume, mass, and capacity

Elaboration - Instructional Strategies/Suggestions

D1 Students need to understand the approximate nature of measurement. The idea should be developed that all measurement includes some error, but that measuring with a device having smaller units or subdivisions can achieve a greater degree of precision.

Teachers should avail themselves of every opportunity to make measurement real. Such experiences should have been common in earlier grades, so teachers should confirm students' knowledge of SI before spending time on such activities. Consider the following:

❒ Have students estimate the distance to the local post office or bank, to the next town, or from their own house to the school. Have them also estimate and measure mass and capacity in the context of common objects or containers. After students estimate, ask them to measure whenever possible to confirm their estimate, and discuss with them the approximate nature of the measurement obtained.

It would be useful to clarify the difference between weight and mass since they are often used interchangeably, although they have very different meanings and are measured in different ways. Students should be aware that weight is a measure of gravitational pull, whereas mass is a measure of the amount of matter. Students may be familiar with the newton, a unit of measure of weight in SI.

The prefixes deca- and hecto- are introduced at grade 7. In previous grades, students worked primarily with the commonly used units. The prefixes milli-, centi-, deci-, deca-, hecto-, and kilo- should be examined as part of the study of length, mass, and capacity. Examples of benchmarks for the newly-introduced units are: a classroom is approximately a decametre in length, whereas a soccer field is approximately a hectometre in length. Once students have been introduced to the hectometre, they can become familiar with the area unit hectare. One hectare is 1 hm^2. This unit is commonly used to measure land area. Students should be aware that the full metric scale also applies to capacity and mass. Knowledge of the complete metric scale will help students relate the units to each other as multiples of 10.

Practice related to conversion should take place within the common units, using real problem situations. There will be more formal opportunities to deal with area and volume units in the context of working with area, surface area, and volume in grade 8. Area and volume units should be introduced in simple estimation and measurement situations which also draw upon more intuitive methods of measuring; for example, overlaying a centimetre grid on an irregular shape to measure area, and filling a container with centimetre cubes to measure or estimate its volume. Again, these activities should not be new to students and should, therefore, not require much instructional time. Study of measurement units in isolation from realistic problem situations is not a meaningful learning experience for students.

Note: The elaboration for D1 is continued on the next two 2-page spreads.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D1.1 Start with a deck of cards which has a different segment or shape printed on each card. Organize students in groups of four. Ask students to write down their estimate of length or perimeter for the figure or segment shown on the first card. One student then measures. The student whose estimate is closest to the measured value scores one point. The winner is the student with the most points at the end of the game.

D1.2 Have students use a tape measure or trundle wheel to measure an area of 100 m^2 and an area of 10 000 m^2. Have them use these areas to help estimate the answers to the following questions:

a) What is the total area of the school? The school grounds?
b) What is the area of the block or community where you live?

Interview

D1.3 Ask students to identify the most reasonable unit for each situation:

a) The water bottle John took on the trip held 600 __.
b) The length of Sarah's house is 22__.
c) The playground at school has a length of 0.8__.
d) The mass of a mouse (the rolling type, not the squeaking type) is 70 __.

D1.4 Bobbi-Jo said she was exactly one hundred and forty-two centimetres tall. Ask students to evaluate her statement. (The issue to focus on is the meaning of the word exact.)

D1.5 Ask students to estimate, and explain how they arrived at their estimate, for the following situations:

a) How long would it take you to walk 100 km?
b) What is the mass of one million paper clips?
c) What is the capacity of a container which would hold 10 000 pennies?
d) What is the area of your favourite book, a tabletop, the seat of your chair, the classroom floor, or a classroom wall?
(Some of these estimation activities may seem a little far-fetched but are intended as a means of getting students to think about how they can use a smaller quantity to estimate with respect to larger quantities.)

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *communicate, using a full range of SI units (e.g., mm, cm, dm, m, hm, Dm, km), and select appropriate units in given situations*

SCO: By the end of grade 7, students will be expected to

D1 identify, use, and convert among the SI units to measure, estimate, and solve problems that relate to length, area, volume, mass, and capacity

Elaboration - Instructional Strategies/Suggestions

D1 (Cont'd) Volume and capacity units are often used without distinguishing between them. Volume is the space an object occupies, whereas capacity is the amount of material that would fill a given volume. A container with a volume of 1 cm^3 has a capacity of one millilitre of a liquid. Of course, real containers must be designed to allow for expansion owing to temperature, and therefore the container's nominal capacity is actually less than its absolute capacity. This can be observed by filling a container with liquid and pouring the contents into a graduated container. Capacity indicated on the label is usually less than the amount the container is capable of holding.

The study of measurement, while treating all prefixes (milli, centi, deci, deca, hecto, kilo), should focus on the most commonly used units; for example, the common units for

- length and perimeter include the mm, cm, m, and km
- area include mm^2, cm^2, m^2, km^2, and hectare
- volume include cm^3 and m^3
- mass include mg, g, kg, tonne
- capacity include mL, L, and kL

It is important to make the link between the cubic units and capacity, specifically 1 mL = 1 cm^3, 1 L = 1000 cm^3, 1 kL = 1 m^3. A link can be made between capacity and mass by using the mass of pure water, since 1 mL of water has a mass of 1 g and 1L of water has a mass of 1 kg when mass is measured at a temperature of 4°C.

As a part of studying any aspect of measurement, estimation should be considered as a vital component. In many real-life situations a reasonable estimate of a measure is sufficient to satisfy the need. Many problems related to measurement will be solved by using formulas, and the actual calculations done by using a calculator. The ability to judge the reasonableness of results will serve as a check to determine if the calculator and the formula are used correctly. However, estimation based on visual and hands-on judgment should also be emphasized in relation to measurement.

Students often reduce conversion to a process of shifting decimals to the left or right. An effort should be made to relate the size of a number to the size of the unit. For example, start with 12.5 cm and change the unit to metres. It should be understood that, since the metre is a larger unit, the associated number must be reduced. For example:

12.5 cm $\rightarrow$? m. The unit is larger so the number of them must be reduced; therefore, divide to get the answer.

13 m$\rightarrow$? mm. The unit is smaller so the number of them must be increased; therefore, multiply to get the answer.

It should be noted that many aspects of measurement are addressed through science and social studies curricula. When overlap occurs there is no need to address the topic twice. Care should be taken, however, to ensure that all aspects of the outcome are addressed.

Note: The elaboration for D1 is continued on the next 2-page spread.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D1.6 Ask students to trace their hands and feet on square centimetre grid paper. Ask them to estimate the area of the feet and hands. Ask: If the bottom of one foot is approximately 1% of the surface area of your body, what is an estimate of the body's surface area?

D1.7 Have students collect leaves in October and press them in a book. During the winter, the leaves can be used as irregular shapes. Have students place them on a centimetre grid and use counting to approximate the area.

Pencil and Paper

D1.8 A rectangular aquarium is 30 cm high with a base of 24 cm by 50 cm.

a) How many litres of water would it take to fill the tank to 6 cm from the top?
b) How high would the water be if 10 L of water were poured into the tank? if 40 L of water were poured into the tank?

D1.9 Find the missing value for each of the following:

a) 22 cm is the same as __ mm.
b) 56 mm is the same as __ m.
c) 45 mm is the same as 4.5__.

D1.10 John wants to make Kool-Aid for his party. He invites 6 friends. The jug used to mix the Kool-Aid holds two litres. If each glass holds 200 mL, will there be enough for John and his friends? If there is any Kool-Aid left, how much can each person have when it is shared equally? Explain your strategy.

Interview/Presentation

D1.11 Ask students to explain how volume units and capacity units are related. Ask them to use examples to help in their explanation.

D1.12 Ask students to make a list of all the different ways they can think of to estimate the volume of a container.

D1.13 Ask students to estimate the amount of air in the classroom per person. Ask them to explain how they arrived at their estimate.

Portfolio

D1.14 Ask students to find the volumes of three cubes with side lengths of 1cm, 3cm, and 5cm. Ask them to double the dimensions of each cube and find the volumes once again. Ask them to use the information obtained to predict how the volume changes when the dimensions are doubled. Ask students to explain how they arrived at their conclusion.

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *communicate, using a full range of SI units (e.g., mm, cm, dm, m, hm, Dm, km), and select appropriate units in given situations*

iii) *estimate and apply measurement concepts in relevant problem situations, and use tools and units which reflect an appropriate degree of accuracy*

SCO: By the end of grade 7, students will be expected to

D1 identify, use, and convert among the SI units to measure, estimate, and solve problems that relate to length, area, volume, mass, and capacity

D2 apply concepts and skills related to time in problem situations

Elaboration - Instructional Strategies/Suggestions

D1 (Cont'd) It may be interesting to explore how measurement and measurement instruments are used in the local community and the wider world. Some examples include police radar, ship radar, uses in the home such as kitchen scales and bathroom scales, weather stations, coast guards, recording studios, hospitals, and pharmacies.

It is also interesting to look at the degree of precision required when using measurement in manufacturing. Students can explore such issues as tolerance, and what kinds of measuring instruments are necessary to ensure the accuracy required for machine parts such as ball bearings.

❒ Bring in a bathroom scales, a two-pan balance, and a small object such as a carrot, an apple, or a wrist watch. Ask students which scales they would use to find the mass of the wrist watch or other small object and to justify their answer. (Two issues might emerge here—the precision of the measure and the fact that one instrument measures weight and the other mass.)

D2 Students' knowledge of the facts related to the calendar (e.g., number of months in a year, order of months, number of days in each month, number of weeks in a year, number of days in a year and leap year, and time zones) should be confirmed. Most students have acquired this knowledge during previous grades. This should be discussed informally in class or built into problem-solving activities. This is a topic which should not require any formal lessons or specific class periods. Gaps in the students' knowledge of calendar facts can be filled by many indirect means such as the preparation of a bulletin board display that highlights certain important facts. Many aspects of calendar and clock time can be integrated with and supported by the social studies and science curricula to avoid unnecessary repetition.

Problems related to calculation with time can be integrated into problem-solving sessions. Such problems should require students to add or subtract using time, to determine elapsed time, and to convert a given time from the 24-hour clock to the 12-hour clock and vice versa. Many students initially have difficulty computing with the base-60 structure of time units. This aspect may need deliberate development.

❒ Enrichment: Research how the date for Easter is determined and why its date varies from one year to another (first Sunday after the first full moon following the spring equinox).

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D2.1 Sarah finished her homework at 8:45 p.m., and her bedtime is at 10:15 p.m. Would it be wise for her to rent a video to watch after her homework if the video has a playing time of 105 minutes? Why or why not?

D2.2 If you left Gander at 10:15 a.m. (Newfoundland Time) and arrived in Toronto at 1:15 p.m. (Eastern Time), how long was your flight?

D2.3 Sarah Lee makes breaded fish sticks at the local fish processing plant. She makes four 1-kg boxes in 10 minutes. How many boxes can she make in a typical workday of 6 hours and 45 minutes?

D2.4 How many seconds are in a year? an average life span? Write your answer in scientific notation.

Interview

D2.5 Ask students to use airline flight schedules to

a) plan a flight from St. John's to Vancouver. Have them determine the date and time of their departure and their arrival. Have them determine the apparent elapsed time when they arrive and the actual elapsed time.
b) repeat part a) for a flight to Australia

Presentation

D1.15 Make arrangements to visit businesses and services in your community. Have students prepare questions about how the people use measurement. Be sure to consider all types of measurement: linear, area, volume, capacity, mass, time, angles, and money. Ask students to look for common and unique measures and processes of measuring. Ask them to report on their findings.

D1.16 Ask students to investigate the measurement instruments in their family car and in a friend's boat.

Portfolio

D2.6 Ask students to consider how time would be different if it were totally organized using base 10. Ask them to create a clock and/or calendar for which time is organized using base 10. As part of this project, students should consider units such as milli-, centi-, deci-, and kilo-days, compare them to our common units, and decide which might become most popular and why. [It may be interesting to research how the current calendar became a 12-month calendar as opposed to a 10-month calendar.]

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *demonstrate an understanding of the concept of rates, use direct and indirect measurements to describe and make comparisons and read and interpret scales, and describe how a change in one measurement affects other indirect measurements*

SCO(s): By the end of grade 7, students will be expected to

D3 develop and use rate as a tool for solving indirect measurement problems in a variety of contexts

Elaboration - Instructional Strategies/Suggestions

D3 Rate is a very commonly used measurement. Rate is applied in daily life in numerous forms, such as breathing rate, heart rate, rate of pay, unit pricing, and speed. All these situations represent proportional thinking; that is, they show a relationship between two quantities such that the relationship is one of multiplication or division.

This topic lends itself quite nicely to a project-oriented approach.

❒ The class might be divided into groups, with each group given the task of determining the most cost-effective manner for a family of four to take a trip to a particular city. The group may wish to compare the cost of driving, going by bus, or flying. Each group may choose a location. Using maps, they can determine highway distances, and consider ferry and toll costs, as well as any other expenses. Application of rates would include kilometres per hour, litres per kilometre, and cost of fuel per litre. When considering busing and flying as options, students can determine such rates as the cost per person when given the total cost for a family, or vice versa.

Rates are usually written as in the following examples: 5 m/s, 72 beats/minute, and 80 km/h. These are read '5 metres per second', '72 beats per minute', and '80 kilometres per hour'. While initial measurements are often taken so that the second term of the rate is something other than one, usually the rate is then re-written to make the second term a one. For example, it may be determined that a swimmer has a heart rate of eight beats in ten seconds. This would usually be expressed as:

8 beats in 10 seconds $\rightarrow$ 48 beats in 60 seconds $\rightarrow$ 48 beats/min.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D3.1 Joan's family drove at 80 km per hour for 8 hours and 12 minutes. How far did they drive? What assumptions did you make?

Interview

D3.2 Ask students to identify situations which typically use each of the following:

a) amount per second
b) amount per minute
c) amount per hour

Presentation

D3.3 Ask students to explain how they would use speed of travel and a watch to determine distance travelled.

Portfolio

D3.4 Ask students to visit a supermarket and find two or three package sizes of the same product. Ask them to find items measured in L or ml and items measured in g or kg. Ask them to explain how they could estimate and calculate to determine which is the better buy, and to explain why they think particular items are measured in mass units while other items are measured in capacity units.

D3.5 Ask students approximately how long it takes to drive from Sydney, Nova Scotia, to Fredericton, New Brunswick. Ask them to justify their answers.

D3.6 Ask students to investigate how various devices make indirect measurements—for instance, a thermometer directly measures volume of the liquid inside it as this liquid expands and contracts, and this device is used to indirectly measure temperature. Ask students to explain how the notion of rate is applied in these instruments.

Mini Projects

D3.7 Ask students to collect flyers from various supermarkets in the local area and compare pricing. (If there are no major supermarkets, consider using the flyers from supermarkets in urban areas to compare cost of food in that community with cost of food in the local area.) Ask students what costs might be higher in urban areas than in rural areas.

D3.8 Suggest to students the use of sports statistics as a basis for a project. Rates, such as points per game, runs batted in, and goals against, are often applied in sports. Such statistics are usually available in the sports section of major newspapers.

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *demonstrate an understanding of the concept of rate, use direct and indirect measurements to describe and make comparisons and read and interpret scales, and describe how a change in one measurement affects other indirect measurements*

SCO(s): By the end of grade 7, students will be expected to

D4 **construct and analyse graphs of rates to show how change in one quantity affects a related quantity**

Elaboration - Instructional Strategies/Suggestions

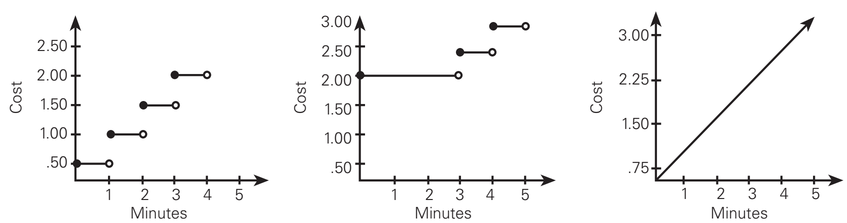
D4 Since rates involve the comparison of two quantities, they provide a natural opportunity to tie this topic to linear graphs. Also, in many situations which can be described graphically, the rate varies, and a non-linear graph can be produced. Students should be exposed to both linear and non-linear situations.

The results of each group's investigation from the project described in relation to D3 could provide meaningful data for the students to graph. A discussion might centre around linear vs. non-linear relationships, when students are considering rates such as cost per kilometre for each method of travel. Students should be encouraged to look for patterns; for example, they might explore whether cost per kilometre decreases for longer trips, and whether this holds true for all methods of transportation.

The analysis of graphs should include creating stories that describe a sketch and constructing graphs based on a story which involves changes in related quantities. Students might study graphs which represent a variety of growth patterns in nature. It is also useful to study the graphs of childhood growth. These are typically kept by public health nurses or pediatricians.

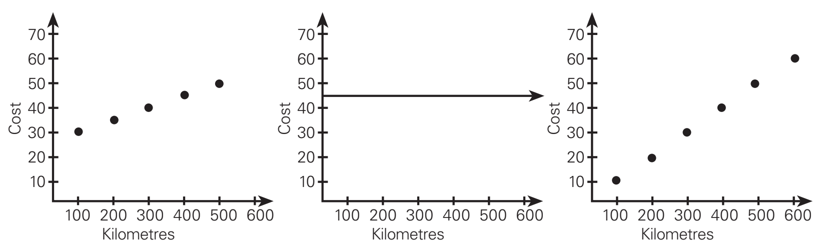
One very interesting application of rate to consider is the method used to charge for long distance.

❐ Give students graphs which relate cost of a telephone call to duration of the call, such as the following:



Ask students to explain how the long-distance companies charge, based on information in the graphs. Leading questions may be required to help them in the interpretation of the graphs.

❐ The following graphs show the cost of car rentals per day. There is, in some cases, an additional charge, depending on the distance the car is driven.



Ask students to explain the rates in each case. Ask which rental deal would be best if you were renting the car and expecting to drive about 70 km per day, and about 600 km per day.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D4.1 Bobbie-Sue ran a bath. She first turned on the water full force until the tub was half full. She then turned off the water and soaked for a while. Halfway through the bath the water became chilly, and she turned on the hot water but ran it slowly. Sketch and label a graph which might illustrate this story. Label the axes and give the graph a title.

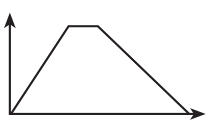
D4.2 A tour bus left the station and drove slowly through downtown; it stopped at a historic site for a short time and then travelled quickly along the highway to its next stop, where it stayed for lunch. The tour bus then took a byroad home, slowing down at several scenic locations. Sketch and label a graph illustrating this tour.

D4.3 Sketch a graph that would show the height of grass over time, for a spring and summer when the lawn is mowed whenever the grass reaches a height of 12cm; for a spring and summer when the grass is mowed every 14 days.

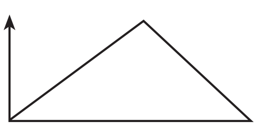
Portfolio

D4.4 Mr. Jones takes a walk each day. The three graphs below show the distance he travelled over a period of time. Ask students to use the graphs to create a story which describes the graphs, and label the axes.

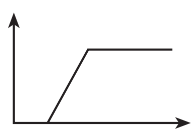
a)



b)



c)



D4.5 If available, use Calculator or Computer-Based Laboratories (CBLs) to collect data from the environment or through experiments based on topics in science. Most data of this type involves change and can be interpreted in terms of rate. Have students create graphs for the data. [The Calculator or Computer-Based Laboratories (CBLs) are data collection tools that some schools have available in the science or mathematics department.]

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *develop and apply a wide range of measurement formulas and procedures (including indirect measures)*

SCO: By the end of grade 7, students will be expected to

D5 demonstrate an understanding of the relationships among diameter, radii, and circumference of circles, and use the relationships to solve problems

Elaboration - Instructional Strategies/Suggestions

D5 The concept of π can be investigated through measurement and charting the value of $\frac{C}{d}$ for a number of circular objects. Students can bring round containers from home for this purpose. This activity can be done in groups, and results reported to the whole class. A piece of string can be used as the measuring tool. After the value of π has been established, the formulas $C = \pi d$ and $C = 2\pi r$ should be developed. Students should use these formulas to solve application problems.

It is useful to perform a similar activity using much larger circles.

- ❐ Have students measure the distance around circles or semi-circles on the gym floor and also measure the diameter of the circles. Ask them to find the ratio of $C \div d$ and compare with the ratios found for the smaller circular objects measured earlier.

Ultimately, students should understand that the ratio of circumference to diameter is constant for all circles, and that the name π is given to the value of this ratio.

Calculator use is encouraged, since multiplication by π can be tedious using pencil and paper. However, for approximations or estimates, students may use 3 as an approximate value for π. While students should be exposed to 3.14 as an approximation of π, they should study the difference in the answer provided when using 3.14 for π and when using the π on the calculator. This difference may lead to some discussion as to whether an exact value for π exists. Some students may want to investigate this further. The study of irrational numbers is addressed in more detail in grade 8.

While $\frac{22}{7}$ has been used in the past as an approximation for π, this approximation should be avoided as it may lead students to the belief that π is rational. That is, students may incorrectly draw the conclusion that π can be expressed as a quotient of two integers.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

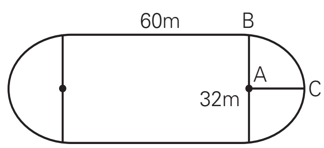
Worthwhile Tasks for Instruction and/or Assessment

Performance

D5.1 Collect a series of circular containers and ask students to sort them into those for which the circumference is about equal to the height, the circumference is less than the height, and the circumference is more than the height. Ask them to explain their choices and then measure to confirm their accuracy.

Pencil and Paper

D5.2 Ali's school has a running track which is semi-circular at each end, as shown. How many times does she have to go around the track to run 2 km?



Interview

D5.3 Ask the student to decide which of the following is the best estimate of the circumference of a circle with a radius 3.5 cm: 10.5 cm, 21cm, or 42 cm. Ask the student to justify his/her choice.

Portfolio

D5.4

a) George's parents have asked TJ's Fine Carpentry Co. to construct a circular dining-room table. They want the table large enough to seat 12 people so that each person has 60 cm of table space along the circumference. Ask students what the diameter of the table should be, and by how much the diameter will change if George's parents decide to reduce seating space to 45 cm.

b) Ask students, if each person requires only 45 cm of space around the table, what the smallest possible dimensions of the dining room are, if each chair requires at least 80 cm of space between the table and the closest wall to allow people easy access to their seating place. Ask students what assumptions they made.

c) Mary Lee's parents approached the same company with the dimensions of their dining room, which measured 4.5 m by 4.1 m. They wanted to purchase a new circular table and also wanted to know the maximum number of people that could be accommodated around their table, if each person required a minimum of 50 cm table space and 75 cm of chair space between the table and wall. Ask students, if there were five regular family members, how many guests they could invite for dinner and be able to seat everyone around the table.

Suggested Resources

Geometry

General Curriculum Outcomes E:

Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *construct and analyse 2- and 3-D models, using a variety of materials and tools*

ii) *compare and classify geometric figures, understand and apply geometric properties and relationships, and represent geometric figures via coordinates*

iv) *represent and solve abstract and real-world problems in terms of 2- and 3-D geometric models*

v) *draw inferences, deduce properties, and make logical deductions in synthetic (Euclidean) and transformational geometric situations*

SCO: By the end of grade 7, students will be expected to

E1 decide and justify which combinations of triangle classifications are possible, through construction using materials and/or technology

E2 determine and use relationships between angle measures and side lengths in triangles

Elaboration - Instructional Strategies/Suggestions

E1 In previous grades, students have worked with triangle classifications according to both angle measure and side length. They should be familiar with the classifications scalene, isosceles, equilateral, acute-angled, obtuse-angled, right-angled, and equiangular. Construction should utilize a variety of materials; for example, with straws and pipe cleaners, or geostrips and paper fasteners, students can

- use three straws/strips, all of different lengths, to construct a right triangle, an acute triangle, and an obtuse triangle
- produce all three triangles once again but ensure that two sides are the same length
- determine what type(s) of triangles can be produced if all sides are the same length

Informally, students should consider whether certain combinations of classifications can exist at the same time. For example, questions such as the following can be posed, and students can show by example or counterexample whether certain combinations can exist together, and justify their conclusions.

- Can a triangle be constructed that is isosceles and obtuse-angled at the same time?
- Can a triangle be constructed that is right equilateral?
- Can a triangle be constructed that is right isosceles?
- Can a triangle be constructed that is obtuse scalene?

Students should explore relationships among triangles.

❒ Ask students to use tangram pieces to create as many unique triangles as possible, using only two pieces, three pieces, four pieces, five pieces, and so on. Ask students to classify each triangle produced by using two classifications such as right isosceles or obtuse scalene. Ask them what type of triangle appears most often and why. (In such activities, a sketch of each triangle should be recorded to avoid repetition.)

E2 Exploration of triangles should include drawing conclusions about angle measures within an isosceles triangle and establishing relationships between the longest side and the largest angle, as well as the shortest side and the smallest angle. It is important that students make quick associations between side length and angle measure. For example, suppose a triangle has an angle of 130° and sides of 4 cm, 5 cm, and 7.8 cm. Students should know that, since 130° has to be the largest angle, it must be opposite to the 7.8 cm side. Students should also understand why the length of the longest side must be less than the sum of the other two sides. This will lead to conclusions about whether certain triangles can or cannot exist.

❒ A triangle has integral sides, with the longest side being 9 cm. Ask students to describe all possible combinations of triangles for which this is true.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E1.1 Ask students to use geostrips and paper fasteners to make three different triangles, and to compare the triangles with those of others. Ask them to identify how they are alike and how they differ, and discuss the similarities and differences.

E1.2 Start with a large supply of straws cut in only three different lengths. Ask students how many unique triangles can be made. Ask students to sort the triangles by side length and then by angle measure, and then try to give each triangle a name that is based on both side length and angle measure.

Performance/Pencil and Paper

E1.3 Ask students to use sketches, geostrips, straws, or software to investigate the following combinations to determine which are possible. Ask them to justify why they feel certain triangles from the list cannot be constructed.

right scalene	acute scalene	obtuse scalene
right isosceles	acute isosceles	obtuse isosceles
right equilateral	acute equilateral	obtuse equilateral
right obtuse	equiangular scalene	right equiangular

Pencil and Paper/Software

E2.1 Ask students to draw or make several isosceles, equilateral, and scalene triangles. Have them measure the angles in several isosceles triangles, in several equilateral triangles, and in several scalene triangles. Ask them what they conclude and to explain whether or not this conclusion would hold for all triangles of the various types constructed.

E2.2 A triangle has two sides of length 7 cm and 3 cm. If 7 cm is the longest side, ask what students know about the missing side. If 7 cm is not the longest side, ask them what they know about the missing side. Ask them to justify their answers.

E2.3 $\triangle$ABC has sides AB = 8.2 cm, BC = 5.3 cm, and the missing side is the longest side. Ask students what vertex has the largest angle, what vertex the smallest angle, and to give reasons for their answers. Ask them what they know about the length of the longest side.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *construct and analyse 2- and 3-D models, using a variety of materials and tools*

ii) *compare and classify geometric figures, understand and apply geometric properties and relationships, and represent geometric figures via coordinates*

iv) *represent and solve abstract and real-world problems in terms of 2- and 3-D geometric models*

v) *draw inferences, deduce properties, and make logical deductions in synthetic (Euclidean) and transformational geometric situations*

SCO: By the end of grade 7, students will be expected to

E3 construct angle bisectors and perpendicular bisectors, using a variety of methods

E4 apply angle pair relationships to find missing angle measures

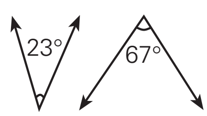
Elaboration - Instructional Strategies/Suggestions

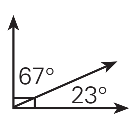
E3 Introduction to angle and segment bisection, as well as review of the concept of congruence, can be supported by the following materials: paper for folding, transparent mirror, tracing paper, geoboards and dot/grid paper, compass and straightedge, or appropriate computer software. Students will probably require basic instruction in the use of compass and straightedge. Since this is their first exposure to compass and straightedge construction, a brief look at the history of this construction method should be considered. These instruments were first used by Euclid and Plato in the 3rd century B.C. Symmetry should be reviewed in preparation for work with bisectors. Evaluation of this outcome should reflect the various methods taught.

The focus for students should be on accomplishing constructions in a variety of ways, as well as communicating, informally, how the construction was completed. Geometric designs should be used as a means of applying the constructions. Students can create interesting and colourful designs or logos for classroom display. This is usually an enjoyable activity which allows some freedom and creativity. Such activities contribute to the development of aesthetic expression, one of the *Essential Graduation Learnings*. (Enrichment: Investigate the Japanese art of paper folding, Origami.)

E4 The angle pairs to be considered include complementary angles, supplementary angles, and vertically opposite angles.

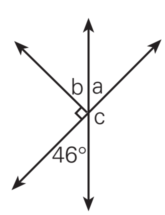
It should be noted for students that complementary and supplementary angles do not have to share a common vertex, as many students draw this incorrect conclusion. For example, the pairs of angles in each diagram below are complementary because they add to give 90°.





The notion of adjacent and non-adjacent angles is not used directly to determine angle relationships. However, an understanding of adjacency should still be developed to help students apply other angle relationships correctly. Adjacent angles share a common vertex, a common arm, and the interior of one does not overlap the interior of the other.

❒ Ask students to find the measures of the missing angles and explain how they arrived at their answers for diagrams such as the following:



GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

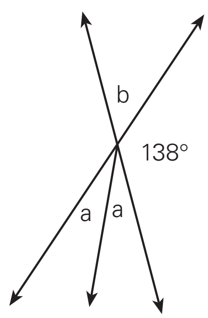
Worthwhile Tasks for Instruction and/or Assessment

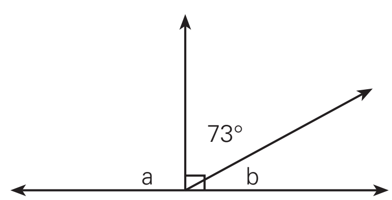
Pencil and Paper

E3.1 Use angle bisection to create a 16-point compass.

E4.1 Write a note to a friend who missed class to explain what is meant by supplementary, complementary, vertically opposite, and adjacent angles. Use diagrams to aid in your explanation.

E4.2 Find the the measure of the missing angles for each of the following:





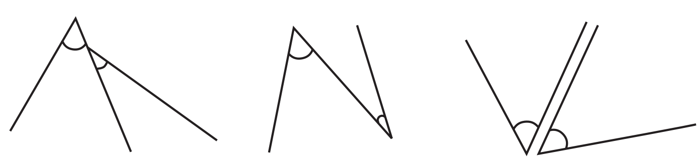
Presentation

E3.2 Ask students to demonstrate, using more than one method, how to bisect a given angle and/or line segment, and explain their method.

E3.3 Ask students to interview a carpenter and find out all the tools and methods he/she would apply to bisect a segment or angle. Ask them to report findings to the class.

Interview

E4.3 Use geostrips to form angles as shown below. Ask students to explain why these angles are not adjacent.



Journal Entry

E4.4

a) Ask students to look up the verb "complement" in the dictionary and describe how a 40° angle complements a 50° angle.

b) Ask students to look up the verb "supplement" in the dictionary and describe how a 40° angle supplements a 140° angle.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *construct and analyse 2- and 3-D models, using a variety of materials and tools*

ii) *compare and classify geometric figures, understand and apply geometric properties and relationships, and represent geometric figures via coordinates*

v) *draw inferences, deduce properties, and make logical deductions in synthetic (Euclidean) and transformational geometric situations*

SCO: By the end of grade 7, students will be expected to

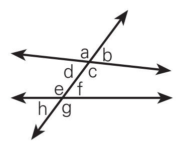
E5 identify, construct, classify, and use angle pair relationships pertaining to parallel lines and non-parallel lines and their transversals

E6 apply angle relationships to find angle measures

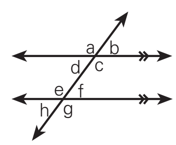
Elaboration - Instructional Strategies/Suggestions

E5 Students should be exposed to situations where a transversal crosses non-parallel lines, as well as those where the lines are parallel. They should understand that corresponding angles and alternate interior angles can exist even when the lines are not parallel, but they are equal only when a transversal intersects two parallel lines. This fact needs emphasis because, otherwise, students will later draw conclusions about two angles being equal on the basis of the appearance of parallelism, instead of verifying that two lines are parallel before drawing such a conclusion.

❒ Ask students to draw two non-parallel lines with a transversal intersecting them and to place the lower-case letters *a* through *h* inside each of the angles formed. Ask students if any angle pairs in the diagram are equal in measure. Ask them to confirm their decisions through measurement.



❒ Ask students to draw two parallel lines with a transversal and go through the process described above. Ask what they notice.

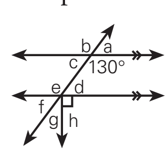


As with the constructions of bisectors, a variety of tools should be employed in the construction of parallel lines, including paper folding, transparent mirror, tracing paper, geoboards and dot/grid paper, compass and straightedge, or appropriate computer software.

This is a good opportunity to apply transformational geometry to show why the various angle pairs are equal. For example, place the mira mid-way between the parallel lines to reflect angles formed at one intersection onto angles formed at the other intersection. As well, by sliding an angle at one intersection point along the transversal, it can be shown as equal to its corresponding angle.

E6 At this stage, students should bring together various angle relationships previously developed to find and justify missing angle measures in problem situations. Such relationships include those studied in relation to parallel lines, as well as those found in E4 and E7.

❒ If the following diagram represents streets in a community, ask students at what angle each of the pairs of streets intersect.

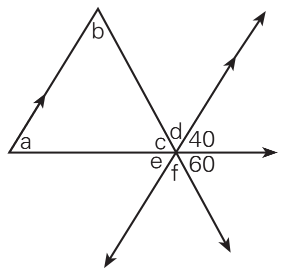


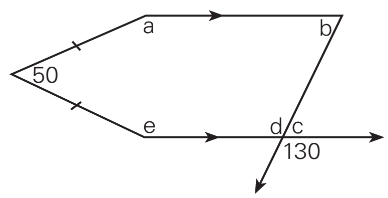
GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

E6.1 Use mathematical reasoning to find the measures of the missing angles in the given diagrams.





Interview

E5.1 Give students the descriptions of two diagrams:

- the first, where two lines p and q are cut by a transversal, but p and q are not parallel—label the angles formed as a, b, c, d, e, f, g, and h. Ask students what can be concluded about the other angles if it is given that the measure of $\angle a = 50°$.
- the second, where two lines p and q are cut by a transversal, but p and q are parallel—label the angles formed as a, b, c, d, e, f, g, and h. Ask students what can be concluded about the other angles if it is given that the measure of $\angle a = 70°$.

Ask students to explain why the two situations differ.

Portfolio

E5.2 Sue needed to draw a line parallel to the floor so that the wallpaper border was a consistent height. Ask students to explain how she might do this if the only tool available was

a) a right triangle and a straightedge
b) a straightedge ruler
c) a straightedge and a protractor
d) a Mira
e) a compass and a straightedge

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *compare and classify geometric figures, understand and apply geometric properties and relationships, and represent geometric figures via coordinates*

v) *draw inferences, deduce properties, and make logical deductions in synthetic (Euclidean) and transformational geometric situations*

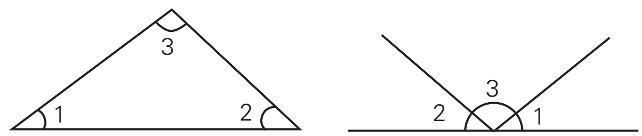
SCO(s): By the end of grade 7, students will be expected to

E7 explain, using a model, why the sum of the measures of the angles of a triangle is 180°

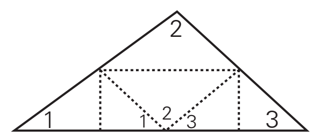
Elaboration - Instructional Strategies/Suggestions

E7 Students can explore this topic through the use of software, or through a variety of hands-on activities. Consider the following:

- ❒ Students can take a triangle and tear off the three vertices, as shown. They can then rearrange these vertices to show that they form a straight line. Since the three angles together form a straight angle, they can conclude that the sum of the angle measures is 180°.

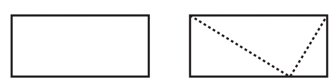


- ❒ Another approach is to have students cut out a triangle and fold it, using the dotted lines shown. Note: The horizontal dotted line is a segment that is parallel to the base, and its endpoints are midpoints of the other two sides.



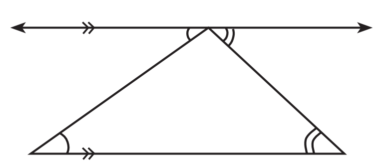
  Students can show that the three angles form a straight line at the base of the triangle.

- ❒ Similarly, start with two rectangles, as shown:



  Using one rectangle, select a point on one side and join it to the two opposite vertices. Place one rectangle on top of the other, and tape the two rectangles together on the top and sides. Cut on the dotted line of one rectangle. Open the rectangle to form a triangle.

The understanding of angle relationships and parallel lines provides another method of modelling this relationship. Students can draw a parallel line at one vertex and use the fact that the two additional angles formed at that vertex are alternate interior with the two base angles. Since the three at the top vertex form a straight line, students can deduce that the three interior angles of the triangle also add to give 180°. Transformational geometry can be applied here to show why the angles are equal.



GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

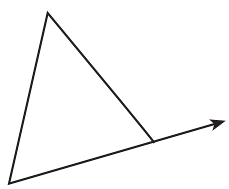
Pencil and Paper

E7.1 Work in pairs. Draw an obtuse triangle, an acute triangle, and a right triangle. Add the interior angles of the triangles and record the results in the table below:

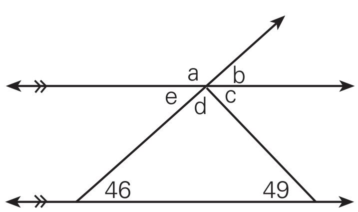
∠1	∠2	∠3	sum of measures

a) What appears common about the totals?
b) Write a conclusion. Discuss the conclusion with the whole class.

E7.2 Sketch several triangles and extend one side of each in one direction as shown. Measure all the interior angles and record each measure in a chart. Measure the angles formed by the extended sides. What conclusion can be drawn about the sum of two of the interior angles and the angle formed by the extended side?



E7.3 Use the angle relationships previously studied, as well as the sum of the measures of the angles of a triangle, to find the missing angle measures. Justify your answers.



Portfolio Entry

E7.4 Ask students to choose one of the methods studied for proving that the sum of the measures of the angles of a triangle add to give 180 and show that it is true for an obtuse scalene triangle. Ask them to justify their answers.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *construct and analyse 2- and 3-D models, using a variety of materials and tools*

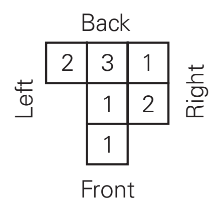
iv) *represent and solve abstract and real-world problems in terms of 2- and 3-D geometric models*

SCO(s): By the end of grade 7, students will be expected to

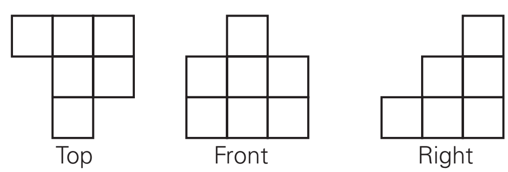
E8 sketch and build 3-D objects, using a variety of materials and information about the objects

Elaboration - Instructional Strategies/Suggestions

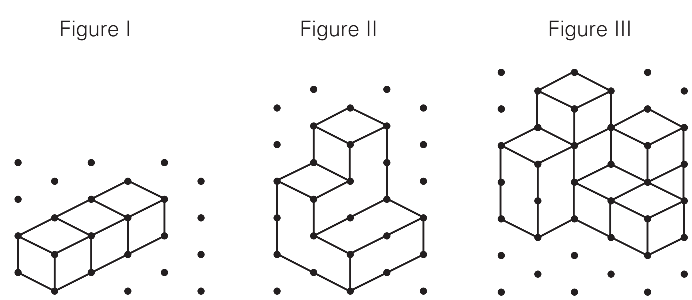
E8 Students will have had experience with nets of three-dimensional shapes in previous grades. Observing and learning to represent two- and three-dimensional figures in various positions by drawing and construction helps students to develop spatial sense. Most students' mathematical experiences with three-dimensional objects are derived from two-dimensional pictures found in textbooks. Students must be able to read and represent information from two-dimensional representations. This ability can be developed through the use of small interlocking or stackable cubes. Students can start with some information about the foundation of the object. The numbers in the blocks show the height of each part of the structure. This type of plan is often called a mat plan.



Students can also be given the top, front, and side view of an object and asked to construct the object, or they can match this information with that provided in the situation described above. The information below describes the same figure as the information above, but it is given in a different format. The three diagrams represent a set of plans for a single shape.



Depending on the students' experiences with using isometric dot paper, the above structure may be difficult for students to sketch. Figure III below shows, on isometric dot paper, one view of the object described in the mat plan given above.



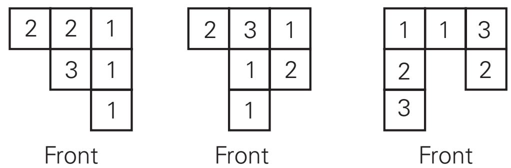
When students have had limited experience using isometric dot paper, they should start with simpler figures, such as Figures I and II, and progress to the more difficult.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

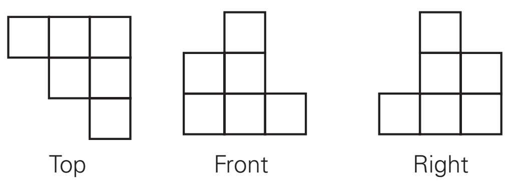
Pencil and Paper/Performance

E8.1 The following are mat plans for 3-D objects. The numbers on the blocks represent heights.



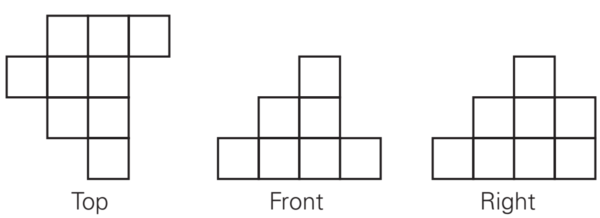
a) Ask students to use the information to construct the three objects, using cubes.

b) The following are the top, front, and side views of one of the figures shown above. Ask students to use the information about the various views of the figure to determine which of the three diagrams above matches with the three diagrams represented below:



c) Ask them to sketch the three initial figures, on isometric dot paper. (This may be challenging for some students.)

E8.2 Ask students to use the information provided to build a 3-D shape, using blocks, and create a single two-dimensional mat plan that describes the building. Ask them to discuss whether there is only one shape which can be built to meet these specifications.



Projects

E8.3 Ask students to investigate and build models of the Platonic solids. This can be done by dividing the class into groups, each of which focusses on a different Platonic solid. (Note: Some of the Platonic solids are much more challenging to build than others.)

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *develop and analyse the properties of transformations and use them to identify relationships involving geometric figures*

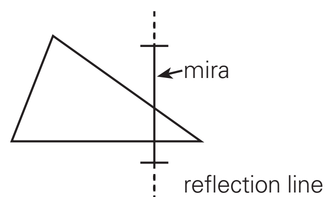
SCO(s): By the end of grade 7, students will be expected to

E9 **draw, describe, and apply translations, reflections, and rotations, and their combinations, and identify and use the properties associated with these transformations**

Elaboration - Instructional Strategies/Suggestions

E9 Students have already been introduced informally to the concept of congruence. They have also been exposed with increasing sophistication to transformational geometry in previous grades. At grade 7, we move towards a more formal language of transformations. Emphasis at this level should be on recognition of what changes and what stays the same as a result of a transformation. The more formal language, such as translation, reflection, and rotation, should begin to replace slides, flips, and turns. The terms pre-image and image should be used to describe a figure. Slide notation refers to the use of descriptions of slides, such as (4L, 3D). Grid or dot paper should be used, as well as the four-quadrant coordinate plane.

Paper folding and the Mira (transparent mirror) are encouraged in working with reflections. When using paper folding, students can fold on the reflection line and trace the image figure, or simply stab the paper at the vertices of a pre-image to leave the basic image shape. The mira can be placed on the reflection line, as shown, and students can trace the image from the reflection which appears in the mira. Line symmetry can be revisited in conjunction with the reflection.



Rotations should be done by inspection if the turn centre is inside the figure, on a vertex or edge of the figure, or on a line containing a vertex or edge. However, when the turn centre is more randomly placed, many students may still require the use of tracing paper to assist them in placing the image. Some students may still require tracing paper in both situations; however, they should be encouraged to visualize the image before the transformation is performed. Rotational symmetry should be reinforced as part of the development of this topic. When available, a number of computer programs are useful in working with transformations.

With respect to describing transformations, students should be able to recognize a given transformation as one of the following: a reflection, a translation, a rotation, or some combination of these. In addition, when given a pre-image and image, students should be able to describe

- a translation, using words, slide notation, or a translation arrow
- a reflection, by determining the location of the line of reflection
- a rotation, using degree or fraction-of-turn measures, both clockwise and counterclockwise, and identify the location of the centre of a rotation

Note: The elaboration for E9 is continued on the next 2-page spread.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E9.1 Have students work in pairs and give each student a geoboard and elastics. Have each student create a figure and its transformation (either translation, reflection, or rotation). Ask students to examine each other's geoboards to determine the transformation that has taken place. Ask students to explain the transformation, using specific transformation language. Ask them to also describe a transformation that would move the image back to the pre-image position. This process can be repeated using different figures, transformations, and combinations of transformations.

E9.2 Have students make a design by drawing the figure below on paper and then making four turn images of it.

a) Ask students to describe the turn centre used and the direction and degree of each turn so that someone else, given the same figure, can duplicate the design.
b) Ask students to choose a new turn centre and repeat the exercise. Ask them what differences they observe.

E9.3 Have students use a mira to determine the number of lines of symmetry in each of a scalene, isosceles, and equilateral triangle. Ask them to discuss whether or not the number of symmetry lines would be consistent for all triangles which are part of these classifications.

E9.4 Ask students to use a mira, or fold paper, to prove that vertically opposite angles are equal.

Pencil and Paper

E9.5

a) Sketch a quadrilateral on a four-quadrant plane.
b) Label and record the coordinates of its vertices.
c) Translate the quadrilateral [3R, 2U].
d) Label and record the coordinates of the corresponding vertices of the image.
e) Compare the coordinates of the pre-image with the coordinates of the image and record your observations.
f) Predict the coordinates when the quadrilateral is translated [3L, 3D].

E9.6 Reflect a triangle over two parallel lines and compare the image with the pre-image. Describe one transformation which would move the image back to the pre-image position.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) develop and analyse the properties of transformations and use them to identify relationships involving geometric figures

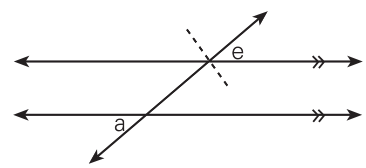
SCO(s): By the end of grade 7, students will be expected to

E9 draw, describe, and apply translations, reflections, and rotations, and their combinations, and identify and use the properties associated with these transformations

Elaboration - Instructional Strategies/Suggestions

E9 (Cont'd) There are many opportunities to apply transformations to help students better understand other elements of the geometry curriculum. For example, in working with scalene, isosceles, and equilateral triangles, students can predict the number of lines of symmetry that would exist for the various triangles, and confirm their predictions, using reflections.

Transformations can be embedded in the development and application of outcomes E1–E7. For example, in the diagram below, in order to show that $\angle a = \angle e$, a combination of transformations could be applied. Students might reflect $\angle e$ about the dotted reflection line shown, and then slide it along the transversal to overlap $\angle a$. In this way, transformations can be used to show why certain angles or segments are equal.



Tessellations provide an excellent context for applying transformations. It may also be interesting to study Islamic Art that involves repetitive patterning; such activities can often be obtained by conducting a Web search.

When investigating properties of transformations, students should consider the concepts of similarity and congruence, which were developed informally in previous grades. Various manipulatives can be used to investigate transformations, such as cardboard cutouts and geometry sets. This topic is also well suited to the use of software.

In discussing the properties of transformations, students should consider if the image

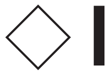
- has side lengths the same as the pre-image
- has angle measures equal to those of the pre-image
- is congruent to the pre-image
- is similar to the pre-image
- has the same orientation as the pre-image
- appears to have remained stationary with respect to the pre-image

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

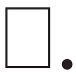
Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

E9.7 Given these figures on a geoboard, transform each as follows:



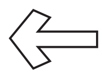
- Reflect the figure about the reflection line.



- Rotate the figure 90°, 180°, and 270° about the given rotation point.

- Translate the figure [2L, 4D].

E9.8 Use a shape, such as the one below, placed on a coordinate grid.



a) Select a turn centre and rotate the object 90 degrees. Write a paragraph describing which properties, such as congruency, area, and orientation, are maintained in the transformation. Discuss whether there would be a difference in the response if the rotation had been 180 degrees. Discuss whether there are differences in properties when different turn centres are selected.

b) Translate the object [2R, 4D] and write a paragraph describing which properties, such as congruency, area, and orientation, are maintained in the transformation. Discuss whether there would be a difference in the response if the translation had been [3L, 4U].

c) Reflect the object about the x-axis and write a paragraph describing which properties, such as congruency, area, and orientation, are maintained in the transformation. Discuss if there would be a difference if the reflection was about the y-axis.

d) Use a concluding paragraph to highlight the differences in the effects of the three transformations.

Interview

E9.9 Ask students to describe a rotation that results in a pre-image which has the same position as the image.

E9.10 Tell students that an object is reflected and the resulting image appears exactly the same in all respects as the original object. Ask them under what conditions this would be possible.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

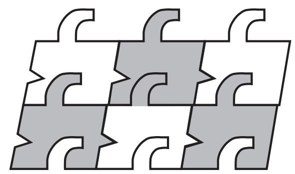
iii) *develop and analyse the properties of transformations and use them to identify relationships involving geometric figures*

SCO: By the end of grade 7, students will be expected to

E10 create and describe designs using translation, rotation, and reflection

Elaboration - Instructional Strategies/Suggestions

E10 This topic can provide an avenue for students to demonstrate their creativity. The designs produced can make interesting wall hangings for the classroom. The works of M.C. Esher would make an interesting research project using the Internet. A simple application of Esher-like tessellations might look like the following:



Islamic Art is also often geometry-based and expresses the logic and order inherent in the Islamic vision of the universe.

Wallpaper is a good source of designs which utilize transformational geometry and Esher-like transformations. If there is a wallpaper store close by, teachers can request old wallpaper books from discontinued designs. Students can look at the designs to find evidence of translations, reflections, and rotations, and record the transformations they observe. Many wallpaper designs incorporate multiple transformations, and some include interesting tessellations.

❒ A simple tessellation can be produced with a compass by following these steps:

- Make a circle, with compass placed at a point on a line.
- With the same radius, make two circles, using the intersection points of circle and line as centres.
- Make four more circles, using the new points of intersection.
- Continue outward in all directions, and then connect the centres of the circles, producing a grid of equilateral triangles.

Such a grid can be the basis for many Islamic designs. Study the transformations inherent in the design that is produced.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

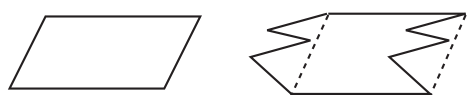
Worthwhile Tasks for Instruction and/or Assessment

Portfolio

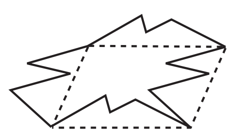
E10.1 Ask students to use wallpaper from wallpaper store discards or discontinued wallpaper books to write about the transformations evident in wallpaper designs.

Project

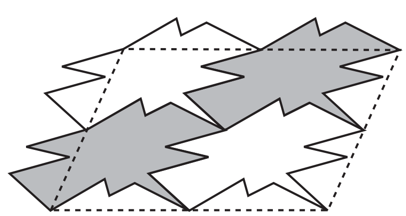
E10.2 After students have had some opportunity to review some of M.E. Esher's work, ask them to create their own design, using a similar technique. (Note: A review of many currently-existing mathematics resources will turn up examples of Esher's work.) For example, ask students to construct a parallelogram, and then construct a random shape on the left side of the parallelogram. Have them slide the random shape so that it is also on the right side, as shown:



Ask students to construct a random shape on the bottom and slide it to the top, as shown:



Translate the new polygon to create tessellations.



This can be done using pencil and paper techniques, or by using software.

Suggested Resources

Data Management

General Curriculum Outcome F:

Students will solve problems involving the collection, display, and analysis of data.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *be aware of sampling issues and understand procedures with respect to collecting data*

SCO: By the end of grade 7, students will be expected to

F1 communicate through example the distinction between biassed and unbiassed sampling, and first- and second-hand data

F2 formulate questions for investigation from relevant contexts

Elaboration - Instructional Strategies/Suggestions

F1 Students have had some informal exposure to this outcome in previous grades, but some of the terms will still require clarification. This is particularly true of biassed and unbiassed and first- and second-hand data. Class discussion should focus on when it would be appropriate to survey a sample versus the entire population. Discussion should also focus on whether a particular group is biassed or unbiassed. For example, suppose a survey is being conducted to determine Newfoundlanders' favourite way to spend a vacation, and the sample is chosen from people camping at Terra Nova National Park. Students should be able to explain why this is likely to be a biassed sample.

First-hand data is data that is collected and used for the purpose for which it was collected. Data that may have been collected for one purpose but is used for some secondary purpose is second-hand data. Students should understand that when data is needed for decision making, it is not always necessary to collect it from original sources as the data may already exist to meet the need. Good sources of data include Statistics Canada, government records and published reports, and town offices. When using secondary-source data, students should still be encouraged to consider the nature of the sample used so that they can check for bias.

F2 In formulating questions for investigation, the following should be considered:

- Will the majority of people in the survey understand the questions?
- Will the majority of people understand the questions in the same way?
- How can the questions be refined to mean the same thing to nearly everyone?
- Will the answers to the questions give the information desired? (Sometimes wording of questions is too vague to draw out the desired response.)
- What information is desired and how is it to be displayed? (Sometimes, advance consideration of how the information is to be displayed helps in reformulating the question.)

Question formulation should also include consideration of the issue of bias. Students should analyse questions to determine if there is any bias involved in the phrasing. Encourage students to test their questions before they administer them. This outcome is well suited to integration with the language arts curriculum. In fact, this outcome focusses on ensuring that the language used communicates as intended.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F2.1 Consider the following survey question:
Should elementary, intermediate, and high school students be allowed to use calculators on all their mathematics assignments? Yes ___ No ___

How could the above question be reformulated to make the information acquired more useful? Give reasons for your decision.

F2.2 The following are two sample survey questions. Which is better? Give a reason for your choice.

a) How many brothers and sisters do you have? ___
b) Are you a member of a large family? Yes ___ No ___

Interview

F1.1 Ask students to decide whether the sample is biassed or unbiassed and explain their choice.

a) Question to answer: What is your favourite sport? Sample is chosen from people attending a soccer game.
b) Question to answer: What is your favourite soft drink? Sample is chosen by picking names from a telephone book.

Journal Entry

F1.2 Tell students you want to find the average height of the students attending the school. Ask them to explain two ways a sample can be selected, one of which is biassed and the other unbiassed.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *be aware of sampling issues and understand procedures with respect to collecting data*

SCO: By the end of grade 7, students will be expected to

F3 select, defend, and use appropriate data collection methods and evaluate issues to be considered when collecting data

Elaboration - Instructional Strategies/Suggestions

F3 Class discussion should focus on the possible ways of collecting data and the advantages and disadvantages of various hypothetical situations. Such advantages/disadvantages include cost, availability of target groups, and suitability of the collection process given the nature of the desired data. The following are some possible data collection methods: questionnaire, phone interview, personal interview, probability experiment, extraction of second-hand data, and timed sampling. These techniques should be applied as part of small-group projects. Students should be able to justify their selection of a means of data collection by identifying and comparing the advantages and disadvantages of various methods.

When students select a topic for investigation and create a set of questions to be asked, they should exchange and critique each other's questions. Sometimes, it will be necessary to bring a question before the whole class for careful evaluation to ensure that the desired information will be acquired. After students have considered their classmates' questions, it is helpful to test them with someone who has had no involvement with the project, perhaps someone from outside the class. Careful consideration of questions before administration can address issues such as language appropriateness and cultural sensitivity.

Cost is as relevant to the school environment as it is in most business, public, and political applications of surveying. In the classroom, cost of surveying may often be related to whether or not photocopying is required. Cost may limit the size of the questionnaire to a half page, and it may also affect the size of the sample to be surveyed. Students should brainstorm about how cost can be minimized. For example, if they interview to get student responses, they can tally the answers directly onto the sheet of questions. This may reduce or eliminate significant costs associated with their survey.

It may be interesting to explore how much polling companies actually charge for conducting surveys and polls. In larger cities, a number of such companies usually compete for the job of conducting a poll. Students can write a letter to a polling company, asking for further information about polling, the skills required of people they employ, and the nature of career opportunities in this field.

Ask students to speculate on what kind of questions might be considered an invasion of privacy or sensitive in nature, owing to cultural or other issues. For example, specific religious beliefs about dancing, types of music and dress, restrictions on foods, and methods of food preparation can make certain questions sensitive to specific groups.

The level of privacy that can be guaranteed will influence the degree to which certain questions are answered truthfully. Indeed, certain questions can produce answers which may have legal and/or moral implications; for example, consider the implications of asking respondents if they have ever shoplifted.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Presentation

F3.1 Tell students you wish to survey adults in your town. Ask them which is more cost-effective, a telephone survey or a mail-out survey, and have them explain their reasoning.

Tell students that you want to survey a sample selected from across the province. Ask them if their choice would still be the more cost-effective, and have them justify their choice.

F3.2 When Sarah collected her data she found it necessary to rephrase some of the questions for some of the people responding. Ask students what type of data collection they think she was using and some advantages and disadvantages of this type.

F3.3 Ask students to work together to identify situations in which

a) language used in a questionnaire might be
   - inappropriate for a target group
   - leading the respondent to give a particular answer
   - upsetting to certain groups of people, owing to religious or moral sensitivity

b) the use of information collected for certain purposes might **not** be ethical

c) the cost is lower by using
   - the mail
   - the telephone

d) the question may have some cultural sensitivity

e) the respondents may be reluctant to answer the question because the issue invades their privacy

(Students may wish to create sample questions or hypothetical situations to illustrate their point.)

Portfolio

F3.4 Even that which seems the most commonplace of questions for some groups can be considered sensitive to others. Ask students why they think these may be sensitive questions for certain groups or individuals.

a) Which of the following do you consider to be the most comfortable running shoe: Nike, Adidas, or Puma? (The issue might be that these are all expensive and some children cannot afford these name brands.)

b) How often do you go on a family vacation? (The issues here include cost and nature of 'family.')

c) Which of the following is your favourite meat product: chicken, pork, or beef? (The issue here might be related to religious beliefs or the individual may be a vegetarian.)

d) What is your favourite card game? (The issue here may be cultural or religious.)

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *construct various data displays (both manually and via technology), and decide which is/are most appropriate*

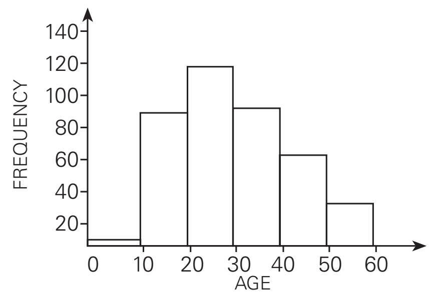
SCO: By the end of grade 7, students will be expected to

F4 construct a histogram

Elaboration - Instructional Strategies/Suggestions

F4 A histogram is similar to a bar graph. Bar graphs can be used when one dimension of the data is not numerical. Histograms can be used with ungrouped data when there are a small number classes of the data, but are most typically used to show the frequency distribution of grouped data. The histogram should be introduced by focussing on grouped whole number data and using class intervals such as 0-4, 5-9,…or 0-9, 10-19,… The following frequency table and graph illustrate a typical histogram for the ages of people at a concert.

AGES	FREQUENCY
0Ð9	4
10-Ð19	87
20Ð29	119
30Ð39	90
40Ð49	61
50Ð59	32



Notice that the numbers identifying the intervals show only the first number of each interval on the graph. The 10, 20,…mark separation points between intervals. In these situations the number at the lower bound is always included in the interval, but the number at the upper bound of the interval is excluded from that interval, and is thus included in the next interval.

When making decisions about grouping of data, it is important to remember that all intervals should be of equal size. Also, normally the number of groups is usually kept to between 4 and 10.

With a focus on whole number data, the issue of upper and lower bound should only be addressed if a context arises which requires it. Students will work with histograms in later grades where this will be more of a concern.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper/Technology

F4.1 The data in the table below shows the number of pedestrians killed in one year in a large city, by age.

AGE	FREQUENCY
0–9	84
10–19	28
20–29	9
30–39	24
40–49	19
50–59	43
60–69	63
70–79	58
80–89	38

a) Construct a histogram for the data.
b) Comment on the shape of the distribution, and give possible reasons for it.

Portfolio

F4.2 One year in December the number of hours of bright sunshine recorded at 36 selected stations is as follows:

16 25 41 20 35 20 16 8 38 23 25 38 41 34 24 39 47
45 17 42 44 47 45 51 35 37 51 39 14 14 40 44 50 40
31 22

Ask students to

a) choose an interval and create a frequency table for the data
b) use the grouped data to create a histogram
c) choose a different interval and repeat a) and b)
d) compare the two histograms and explain which they feel is more useful

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

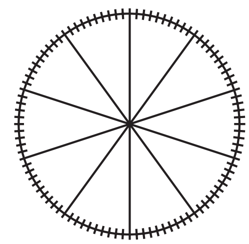
ii) construct various data displays (both manually and via technology), and decide which is/are most appropriate

SCO: By the end of grade 7, students will be expected to

F5 construct appropriate data displays, grouping data where appropriate and taking into consideration the nature of data

Elaboration - Instructional Strategies/Suggestions

F5 Students have already worked in previous grades with a variety of data displays for grouped and ungrouped data. The bar graph and pictograph have been studied extensively so should not require direct instruction. Students have also worked with line graphs, broken-line graphs, stem-and-leaf plots, scatterplots, and circle graphs. At this grade level, the circle graph is constructed only through the use of the percent circle. Students can be provided with a circle mat divided into tenths, and hundredths, as shown.



Data for circle graphs would typically be given as percents or as raw data to be converted to percents. Since construction will take place using the percent circle, students will not need to make any conversions to degrees.

Students should be able to identify advantages and disadvantages of the various forms of data display. They should develop a sense of what type of data is best displayed by particular formats and what questions can be better answered with particular displays. As well, decisions on the selection of a display method are determined by what the creator of the display wishes to communicate. For example, if it is desirable to show how large a part of the whole a particular amount represents, a circle graph might be selected. If it is desirable to make comparisons among groups or quantities, a bar graph might be selected. If it is desirable to illustrate change over time, a line or broken-line graph might be selected. Scatterplots are used to help determine whether there is (or is not) a relationship between two variables.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F5.1 Given a graph from a text, magazine, or newspaper, convert the graph to some other display form. Discuss which is the better way to display this data and why.

F5.2 What factors should be considered when determining what amount each picture in a pictograph should represent?

F5.3 Jay works part time in a shoe store. She was involved in completing the spring order. The following were ordered in relation to shoe size:

5% size 5 - $5\frac{1}{2}$ 15% size 6 - $6\frac{1}{2}$

45% size 7 - $7\frac{1}{2}$ 20% size 8 - $8\frac{1}{2}$

5% size 9 - $9\frac{1}{2}$ 5% size 10 - $10\frac{1}{2}$

a) Construct a circle graph, using a hundredths mat.
b) Write three questions which can be answered from the graph.

Interview

F5.4 Ask students which type of graph they would use to display a student council budget. Ask why they chose that graph.

Presentation

F5.5 Make a list of all the graphs students have studied. Assign each group a different type of graph. Ask students to start with raw data and construct a graph to represent them. Ask each group to prepare a presentation to explain to the class how they went about constructing their graph. Each group can also make a wall display. (Remind students that they should know how all these graphs are constructed, so they need to pay particular attention to all the presentations to learn from each other.) Ask students to critique the display method chosen by classmates in terms of its suitability for the type of data with which the group was working.

F5.6 John decided to do his project on the "Relationship Between Mathematics Grades and Other Factors." He chose to compare a person's height with their French grades and with their math grades. The following data was collected:

Pupil	Math Mark	French Mark	Height(cm)
A	28	40	160
B	38	60	152
C	49	53	170
D	62	70	180
E	71	72	166
F	72	89	150
G	80	70	147
H	84	90	182
I	87	88	190
J	93	90	171

Ask students to make two scatterplots for the data, and use them to comment on whether there appears to be any relationship between the variables considered.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)*

SCO: By the end of grade 7, students will be expected to

F6 **read and make inferences for grouped and ungrouped data displays**

Elaboration - Instructional Strategies/Suggestions

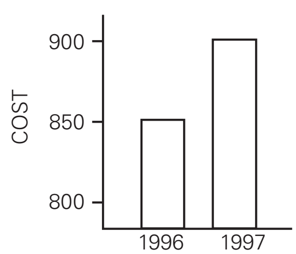
F6 Students should read and make inferences using a variety of data displays, including frequency tables, bar graphs, line graphs, broken line graphs, circle graphs, stem-and-leaf plots, and scatterplots.

Teachers should keep a file of graphs from magazines and newspapers. Students should also collect graphs, tables, and charts that link relevant and current issues to the study of statistics. Such collections allow for a more interdisciplinary and real-life approach to instruction. Ask students to write questions which can be answered using the graphs they find. They can then exchange and answer each other's questions.

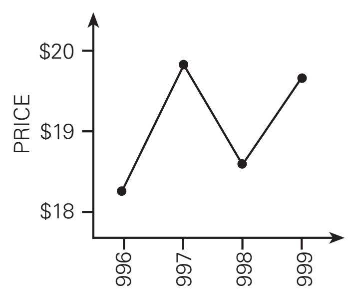
Outcomes F4, F5, and F6 should be addressed together. Construction of graphs tends to be a time-consuming activity. It is therefore important that each graph that is constructed is also analysed to get the maximum benefit from the time invested.

In reading and making inferences from graphs, students need to pay particular attention to things such as the actual range of the scale and whether or not the scale begins at zero. For example, in Graph A one might conclude cost has doubled, based on the size of the bar, and in Graph B that the price is extremely unstable, when, in fact, the nature of the vertical scale profoundly affects the conclusions drawn.

Graph A

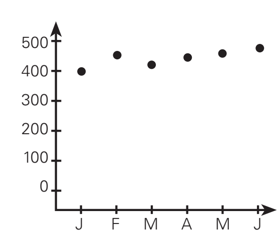


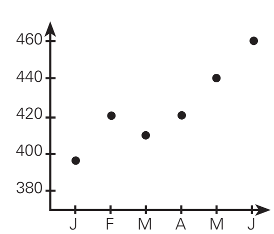
Graph B



Students can also explore how different scales used for the same set of data can create very different impressions.

❒ The two graphs below represent monthly profit for the school canteen.





What is the difference in the message they present?

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F6.1 The following data show the number of potatoes in twenty 2-kg bags, one graph representing farm A and the other farm B.

Farm A

0	8 9
1	2 4 4 6 9 9 9
2	0 1 1 3 5 6 7
3	0 1 2 3

Farm B

0	9 9
1	1 1 2 3 3 4 5 6 7 7 8 9
2	0 0 1 1 2 4
3	

a) Which farm appears to produce a greater consistency of size?
b) You are interested in larger potatoes for baking. Which farm would you choose to purchase from? Justify your decision.

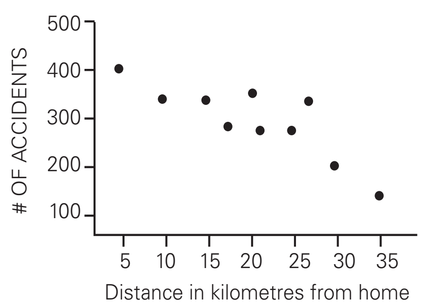
Presentation

F6.2 Have students find some graphs in the local newspaper. Working in groups, have them discuss and present to the class the following:
a) Is the graph appropriate for the data? Why or why not?
b) Do you think the message which the graph is presenting is at all distorted by the selection of scale?

F6.3 Have students find out the population of their school over the last several years, and graph the information. Ask them to use the graph to predict the population of their school in five years. Have them discuss why their prediction may not be appropriate—that is, whether there are other factors that need to be taken into consideration. Ask if the graph they selected is useful in displaying change over time. Have them discuss what other graphs were used by others in the class to display this data. Ask them to discuss which method of display is best for showing change over a period of time.

Portfolio

F6.4 The following graph shows the number of motor vehicle accidents and how far from home people were when they had the accident.



Ask students the following:
a) Based on the graph, is there a relationship between the number of accidents and the distance from home? Explain why or why not.
b) Based on the graph, is it safer to drive when you are not near home? Explain why or why not.
c) How might you justify the relationship that is apparent in the graph?

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *demonstrate an appreciation of statistics as a decision-making tool by formulating and solving relevant problems (e.g., projects with respect to current issues a/o other academic disciplines)*

SCO: By the end of grade 7, students will be expected to

F7 **formulate statistics projects to explore current issues from within mathematics, other subject areas, or the world of students**

Elaboration - Instructional Strategies/Suggestions

F7 This outcome should be addressed in conjunction with other subject areas of the curriculum, where possible, and should tie in as many of the other data management outcomes as possible. A project in a particular context can illustate and unify for students all aspects of this strand and can support many areas of the curriculum. Data collection might relate to opinions about a piece of poetry, school rules, environmental concerns, personal favourites in popular music, health issues, or to scientific data collection related to an experiment.

Problem solving should permeate the whole process, as students decide on interesting topics, formulate questions, plan the collection of data, implement plans, analyse results, and make conjectures. Research projects provide students with active experiences in dealing with information first-hand.

In some situations, it may be desirable to develop a whole-class project for which small groups take components of a larger question to work on and later put the parts together in answering the large question. For example, a large group wishing to study a common issue may split into smaller groups, with each assigned to study one of the following:

- parental or community opinions
- high school student views
- junior high school student views
- teacher or school board views

In other situations, projects may be such that each group of 3 or 4 students takes a different project, and tasks are sub-divided to individual members of this smaller group. Assessment of project work can be based on individual as well as group components.

Encourage students to formulate the project so as to capture a variety of data displays. The project will provide a vehicle for students to revisit the various display methods developed in previous grades, and keep them fresh in their memory.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Project

The following represents a list of ideas for use in the development of statistics projects. Each would require additional fleshing out by the students and can certainly be shaped by the students to better reflect their interests.

F7.1 Ask students to find out how much time is spent per week on each subject area when doing homework. Does this change when students are in grade 8? in grade 9?

F7.2 Ask students to find out what type of transportation students in their school use to get to school. Does it differ with the time of year? Does it differ by grade level?

F7.3 Ask students to find out the most popular types of after-school activities of students in their school. Does it differ by grade level? Is there a difference in preference between males and females?

F7.4 Ask students to find out how many hours of sunshine per month their community receives. Compare this with two other communities in the province and suggest reasons for the differences. (This would make a good Internet project.)

F7.5 Ask students to find out what the five favourite cereals of students are in their class or school. This question could also include adults to compare adult versus student performances in cereal consumption. Compare this with sales volume at the local supermarket to determine how 'normal' the class is relative to the rest of the community.

F7.6 Ask students to find out the favourite kind of jeans for kids in their age group. Use the result of the survey to write a recommendation to a local store regarding their ordering of types of blue jeans. A comparison could also be made for various age groups.

F7.7 Ask students to ask the student council or community council to suggest issues they would like investigated. Use this as a source for project work.

F7.8 Ask students to collect data to look for a relationship between average grade on their last report card and

a) time spent watching television
b) time spent on homework
c) hair colour
d) shoe size

F7.9 Ask students to survey or interview grade 9 students to find out about preferred part-time jobs and the amount of money typically earned. They may wish to include jobs such as babysitting, grass cutting, and paper routes.

F7.10 Ask students to conduct a survey to find out information related to

a) their favourite NHL hockey team
b) their favourite musical instrument
c) their favourite potato chip flavour or chocolate bar

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *determine, and apply as appropriate, measures of central tendency and dispersion (e.g., range)*

SCO: By the end of grade 7, students will be expected to

F8 **determine measures of central tendency and how they are affected by data presentations and fluctuations**

Elaboration - Instructional Strategies/Suggestions

F8 Mean, median, and mode have received significant development in previous grades. These are collectively referred to as measures of central tendency.

The term *mean* can be linked with real-life examples of averages in such contexts as batting average, test-score average, average temperature, and average heights of age groups. Although in everyday language the word average is sometimes used synonymously with *mean,* we should be careful to avoid this because median and mode are also averages. To find the mean for small sets of data, such as 16 30 30 37 37, students may recognize the relationships which exist between the numbers, and use mental strategies such as compensation to find the mean. That is, they can recognize that 16 is 14 smaller than 30, and the two 37's together compensate for the 14. This should allow them to conclude that the average is 30. For large data sets, add the individual values and divide by the number of values, using a calculator.

The median of a set of values is the middle value when the values are arranged in order from smallest to largest. For an odd number of values the median is easily observable. When there is an even number of values, it is necessary to take the middle two values, and find the mean of these two numbers.

The mode of a set of scores is the score that occurs most often. It is not uncommon to find a set of data that is bi-modal. Students may be familiar with the term "a la mode," which means according to the fashion or most popular choice. Mode is particularly useful for describing non-numerical data such as eye colour and type of pet, as well as for numerical data. When data is displayed in a frequency table such as the one below, identification of the mode is very straightforward. However, there are many sources of error for students in determining the median or the mean from such a table, so observation of student approaches is very important.

Score	18	20	22	24	26
Frequency	7	5	8	4	2

The measure of central tendency that is best suited to a particular situation is dependent on the situation. For example, the median is not affected as much as the mean by outliers or extreme values. When there are no extreme values, there tends to be very little difference between the median and the mean. With a basic calculator, the mean is definitely the easiest to calculate. The mode is easiest to read from a frequency table, but it might not be near the centre of the values, there might be more than one mode, it might change drastically with the addition of new values, and it is possible that in fact no mode exists.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F8.1 Susan is taking six courses this term, and her mean score is 68%. In order to play varsity sports, she is required to have a mean score of 70%. Susan scored the same results in five of the six subjects, but improved in the other one from term I to term II. However, her average did increase exactly enough to allow her to be involved in varsity sports.

a) By how much did she increase her grade in the sixth subject?
b) Show, by creating a set of possible grades for Susan, how increasing just one grade can result in an increase of 2% in the overall average.
c) Suppose she accomplished the same increase in her average, but did so by improving her marks in three subjects, all by the same amount. By how much would each mark need to increase?

F8.2 Suppose the mean of a set of test scores is 80. One of the grades is erased from the report card, and the other four are 90, 95, 85, and 100. Ask students the following questions:

a) What do you know about the grade that is missing?
b) Explain a plan that you could use to find the missing grade from the information that is available.
c) Why do you think the grade was missing?

F8.3 Ask students which of mean, median, or mode would be most helpful to know in each situation, and to justify their choice.

a) Your are ordering bowling shoes for a bowling alley.
b) You want to know if you read more or fewer books per month than most people in your class.
c) You want to know the "average" amount spent per week on junk food in your class.

Portfolio

F8.4 Present the following problem to students:

John did a survey to find out the weekly earnings of students in his class who baby-sit regularly. He found the following: \$27, \$25, \$19, \$23, \$16, \$22, \$26, \$25, \$29, \$30, \$18, \$28. John wanted to calculate the mean, but he left his calculator at home. John decided to guess the mean to be 25. To be on the safe side, John looked at the data and recorded the difference between his guess and the actual data values (using + or - to indicate whether the values were above or below his guess). The results were as follows: +2, 0, -6, -2, -9, -3, +1, 0, +4, +5, -7, +3. He then added +2, +1, +4, +5, +3 to get +15. He also added -6, -2, -9, -3 and -7 to get -27. He then added (-27) + (+ 15) to get -12. He thought: There are 12 pieces of data and the difference is -12; so he divided and decided that his guess was 1 too high and changed his guess to 24. Ask students if John's method works. Have them explain why or why not.

Ask students to test this method with a different set of data.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) determine, and apply as appropriate, measures of central tendency and dispersion (e.g., range)

SCO: By the end of grade 7, students will be expected to

F9 draw inferences and make predictions based on the variability of data sets, using range and the examination of outliers, gaps, and clusters

Elaboration - Instructional Strategies/Suggestions

F9 In order to find the range of a set of data, it is necessary to know the extreme values. The range is the difference between these two extreme values. Gaps and clusters are found through observing and analysing the data. Such observations are made clearer when the data are displayed in a frequency table, or in a graph such as a stem-and-leaf plot or bar graph.

Study of range should be closely tied to the study of mean, median, and mode. Students should consider questions such as which of the mean, median, and mode will be most influenced by an outlier.

Although the measures of central tendency of two sets of data may be quite similar, the actual data can be quite different.

❐ The following are the test scores for three math classes. Discuss the mean, median, mode, and variability of the data.

Class A: 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 55, 55, 60, 100

Class B: 50, 50, 50, 50, 50, 50, 50, 50, 50, 55, 55, 55, 55, 55, 55, 55, 55, 55, 55, 60, 60

Class C: 10, 30, 30, 30, 30, 30, 30, 30, 30, 50, 60, 70, 70, 70, 70, 70, 75, 75, 75, 80, 100

Discuss how two pieces of the data can be changed without affecting the mean, median, mode, or range of the data for each class.

❐ Consider the data set: 4, 4, 5, 5, 6, 6, 9, 9, 9, 10, 10, 10, 11, 29. Discuss whether there are any clusters, gaps, or outliers in this data. Discuss the affect on mean, median, and mode if the outlier is removed from the data.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F8/9.1 Chris received the following test scores at mid-year: 80, 96, 84, 90, 84, 60.

a) If Chris had a choice of using the mean or median for his report card, which do you think he would prefer? Why?

b) Clearly, one of the scores is quite different from the others. Do you think one low mark has a greater effect on the mean or the median? Why?

F8/9.2 Working in groups, use a page out of an old telephone book to make a frequency chart to identify the last digit of the telephone numbers on the page. Decide whether mean, median, or mode would be best in finding the "average." Identify if there are any gaps or clusters in the data.

Interview

F8/9.3 When Mr. Brown gave a science test, he found the following:

- The mean for the test was 72%.
- The mode for the test was 65%.
- The median for the test was 65%.

When he gave back the test, it was determined that his answer key was wrong, and all of the students had a certain question correct which was valued at 5%. He was then compelled to increase all the marks by 5. Ask students the following questions:

a) How did this affect the mean, median, and mode?

b) What things might be concluded about this set of test scores that would account for the mean being so much higher than the mode or median?

c) Create two possible sets of test scores for 10 students which would fit Mr. Brown's new mean, median, and mode. One set should be clustered around the mean, while the other set should have a range of 70 and no clustering.

Suggested Resources

Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations*

SCO: By the end of grade 7, students will be expected to

G1 identify situations for which the probability would be near 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1

Elaboration - Instructional Strategies/Suggestions

Many events cannot be predicted with certainty. However, over the long run, a relative frequency of occurrence can be established. The relative frequency with which an event occurs over the long run is called its *probability*. A good estimate of a probability can often be made through a data collection process. Theoretical probability can sometimes be obtained by carefully considering the possible outcomes and using the rules of probability. Often in real-life situations involving probability, it is not possible to determine theoretical probability, and therefore one must rely on observation and a good estimate.

G1 Using a scale similar to the one shown below, students should be able to relate various events to where they might fall along the scale.

Impossible ____________ Certain

0 $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ 1

Sometimes this scale is written using more general language, such as

Unlikely | Likely

Impossible | Half of the time | Certain

Students should understand from their previous study of probability that impossible events have a probability of 0 and events that are certain to occur have a probability of 1. All uncertain events have a probability between 0 and 1. Students should also be able to cite examples of situations which have a very low likelihood of occurring (probability near 0), those which have a very strong likelihood of occurring (probability near 1), and those which might have probabilities near to $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

G1.1 For each of the following weather situations, identify which probability words (impossible, unlikely, half of the time, likely, or certain) best describes the situation.

a) The probability of snow today is 60%.
b) The probability of rain tomorrow is 20%.
c) The probability of rain today is 100%.
d) The probability of a dust storm today is 0%.

Interview

G1.2 Ask students to use the scale of 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, 1 to assess the reasonable probability of the events described below and explain their choices.

a) You will have cereal for breakfast one day this month.
b) The sun will set tomorrow.
c) The sky will be red at sunset.
d) The next baby born in your town will be a boy.
e) It will snow at least once in the month of June.
f) It will snow at least once in the month of April.
g) You can live six months without water or any other liquid.
h) You will get an even number when you roll a die.
i) You will get a head when you flip a coin.
j) A fourteen-year-old girl will be shorter than a fourteen-year-old boy.

Journal

G1.3 Ask students to identify two situations which may occur in real life that they feel are

a) certain
b) highly likely to occur
c) about a 1 in 5 chance of occurring
d) about a 1 in 2 chance of occurring
e) impossible
f) highly unlikely to occur

Suggested Resources

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations*

SCO: By the end of grade 7, students will be expected to

G2 solve probability problems, using simulations and by conducting experiments

Elaboration - Instructional Strategies/Suggestions

G2 Simulation is a method of exploring and answering questions about problems by running experiments that model the actual situation. It is through such simulations that students can isolate the critical factors associated with a problem. This experimental approach to solving problems is also called the Monte Carlo method, named in honour of the casinos of Monte Carlo on the Mediterranean. The use of Monte Carlo simulation highlights the role of mathematical modelling as a problem-solving strategy. A simulation involves an experiment that is designed to model the problem situation, and simulated data are generated and analysed as though they were real data. Situations in which a simulation would be particularly useful include those that are too dangerous for direct experimentation, too long-term to experiment with in the classroom, or too costly.

It is important that a simulation accurately reflect the situation being simulated. For example, if a simulation uses green, blue, and red cubes to represent it, and if the green represents something that is twice as likely to occur as either of the others, the simulation needs to represent this accordingly—that is, twice as many green cubes should be used as either blue or red.

In conducting a simulation, the following steps are important:

- The problem and any underlying assumptions should be clearly defined.
- A model should be selected to generate the necessary outcomes.
- A large number of trials should be conducted and recorded.
- The information should be summarized to draw a conclusion.

Many examples of experiments which involve simulation can be found in science.

In everyday situations many questions involving probability can be answered using simulation.

❒ Suppose you wanted to find the probability of a family with three children having all girls. Have students simulate by tossing three coins or three two-coloured counters. One colour of a counter, or one side of a coin, can represent a girl and the other a boy. After a series of trials done by pairs of students, class data can be collected, and the information used to estimate the probability that a family of three children would be all girls.

Before conducting experiments, students should predict the probability whenever possible, and use the experiment to verify or refute the prediction. Simple experiments can be conducted through activities involving spinners, dice, coin flips, and coloured counters in a bag. These materials, which tend to be accessible and relatively inexpensive, serve to establish experimental estimates of probability.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G2.1 Munchie-Crunchie cereal has a free model airplane in each box. There are three different styles of airplanes in the boxes. Ask students to design a simulation and conduct it to determine the number of boxes they would have to buy to get a complete set of model airplanes.

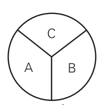
G2.2 For 12 days in a row, the forecast indicated a 50% chance of rain. Ask students to design a simulation to determine how many days were actually without rain.

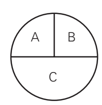
G2.3 Have students, working with a partner, roll a six-faced die 50 times, and record the number of times they get each number. Ask them to estimate the probability of rolling a four, a two, an even number, and a number greater than two. Combine all the data from the class for each event, and have students use that data to find the answer to each question again and compare with their group data. Ask what conclusion they can draw.

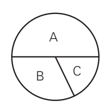
Pencil and Paper

G2.4 Design a simulation which can be used to estimate the probability that in a certain family of four children every child will have at least one sister and one brother.

G2.5 Match the spinner with the table it would most likely produce, given 50 spins, and justify your choice.







Letter	A	B	C	A	B	C	A	B	C
Tally	28	12	10	17	15	18	28	15	7

Interview

G2.6 Ask students to place the corner of a sheet of paper at the centre of a circle, and use it to trace a 90° angle, colour the inside of the 90° angle blue, and leave the rest of the circle white. Have them attach a spinner to the centre of the circle. Ask the following questions:

a) If you spin the spinner, are you just as likely to obtain blue as white? Why?
b) Are you certain to get a blue in 50 spins?
c) Is it possible to spin 50 times and never get blue? Why?
d) Estimate the probability of getting blue by spinning 50 times and recording the number of white and number of blue.

Portfolio Entry

G2.7 John has only white socks and black socks. The electricity is out, and he is getting dressed in the dark. All of his socks are in the top drawer of his dresser, but they are not paired. Ask students to design a simulation which can be used to estimate the probability that, when he pulls two socks out of the drawer, they will be the same colour.

Suggested Resources

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *derive theoretical probabilities, using a range of formal and informal techniques*

SCO: By the end of grade 7, students will be expected to

G3 identify all possible outcomes of two independent events, using tree diagrams and area models

Elaboration - Instructional Strategies/Suggestions

G3 The ability to count possibilities is useful in a wide range of occupations and hobbies. For example, a worker in the city's traffic department might want to investigate the number of routes from one point to another within the city. Many problems in probability require the use of counting techniques; that is, there is a need to determine the number of possibilities for a given situation. All techniques which involve looking for the number of possibilities fall under the branch of mathematics referred to as *combinatorics.* This branch of mathematics deals with systematic ways of counting. One concrete or pictorial means of organizing this counting is through the use of tree diagrams. Many problem situations related to counting are closely tied to graph theory. For example, a problem may involve finding the number of pathways between two points.

From the use of tree diagrams, some students will arrive at the fundamental counting principle, but this principle should not be formally introduced. Some students will undoubtedly arrive at this principle inductively. The fundamental counting principle states that, if there are m ways for one thing to occur and n ways for another thing to occur, then the two actions can be performed in this order in $m \times n$ ways.

Using the area model, students can identify the possibilities regarding one event on one dimension of a rectangle and the possibilities regarding the other event, using the other dimension. The total number of possibilities is represented by the number of divisions of the rectangle.

❒ Sarah has three sweaters and two pairs of shorts. How many outfits does she have?

	Sweater(a)	Sweater(b)	Sweater(c)
shorts (1)	1a	1b	1c
shorts(2)	2a	2b	2c

A tree diagram can also be used to organize possibilities.

❒ Suppose a menu offers a lunch special of a hot dog or a hamburger with a choice of an apple, banana, or orange for dessert. Use a tree diagram to organize this information.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

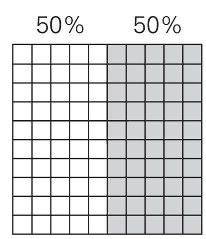
Pencil and Paper

G3.1 Each student at a girls' private school must wear a white shirt or a blue shirt with a grey or black skirt or grey or black pants. How many variations exist in the uniform? Use a tree diagram to illustrate the solution to this problem.

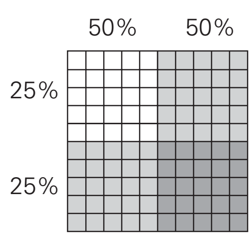
G3.2 When Sue purchased a new car, she had a choice of cloth or leather interior and a choice of black, green, blue, or red exterior.

a) Draw a tree diagram for this situation.
b) Lu's Auto always stocks at least three cars of every available colour and interior design. How many cars would Lu need to stock?

G3.3 Bill is a 50% free-throw shooter in basketball. That is, he makes his foul shot 50% of the time. An area model for his first shot would look like this:



He also has a 50% chance of making the second shot. He gets 0 points if he makes no baskets, 1 point for one basket, and 2 points for two baskets. The addition of the second shot to the model would look as follows:



a) Is he more likely to get 0 points, 1 point, or 2 points in a two-shot free-throw situation? Make the diagram on grid paper and use it to determine the probability of getting 0 points, 1 point, or 2 points.
b) Make a similar area model diagram for Jill, who is a 60% free-throw shooter in basketball, and use it to decide if she is more likely to get 0 points, 1 point, or 2 points. Make the diagram on grid paper and use it to determine the probability of getting 0 points, 1 point, or 2 points.

Suggested Resources

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *derive theoretical probabilities, using a range of formal and informal techniques*

iii) *determine and compare experimental and theoretical results*

SCO: By the end of grade 7, students will be expected to

G4 **create and solve problems, using the numerical definition of probability**

G5 **compare experimental results with theoretical results**

Elaboration - Instructional Strategies/Suggestions

G4/5 To find the theoretical probability of an event, it is necessary to determine:

$$p\text{ (event)} = \frac{\text{\# of favourable outcomes}}{\text{total \# of outcomes}}$$

This definition can be used only when dealing with equally likely events. For example, when a standard die is rolled, there should be an equal chance of getting a 1, 2, 3, 4, 5, or 6. Since all six possibilities have an equal chance of occurring, the probability of any one of them occurring is 1 in 6 or $\frac{1}{6}$.

There are many situations in which the chances are not equal. For example, one experiment often conducted is that of tossing a thumb tack to see if it lands with the point up or down. The probability of a tack landing point up versus point down is not $\frac{1}{2}$; therefore, the formula for theoretical probability would not apply. Theoretical probability can only be used to predict what will happen in the long run, when events represented are equally likely to occur. Students should realize that the probability in many situations cannot be characterized as equally likely, and therefore theoretical probability is more difficult to determine. In such cases, experiments should be limited to determining the relative frequency of a particular event.

❒ Give students a paper bag and some coloured blocks. Ask students to select two or three colours of cubes and place a certain number of each colour in the bag. Have students exchange bags and conduct experiments to determine an experimental probability, and then open the bag and use the contents to calculate the theoretical probability.

❒ Give students a styrofoam cup and ask them to find the probability it will land on its bottom if dropped. They should see that this is an example of a situation in which they are unable to find the theoretical probability.

Once students have worked with probability experiments and derived theoretical results, they should be able to compare results. In simple situations, students should be able to relate the experimental results to results achieved using the definition of theoretical probability. Discussion should take place about why differences in theoretical and experimental probability occur, and how they may in some situations relate to sample size.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

G4.1 Find the theoretical probability for each of the following situations which involve a six-faced die:

a) the probability of tossing a 4 with your die
b) the probability of tossing a 2 with your die
c) the probability of tossing an even number
d) the probability of tossing a number greater than 2

G4.2 A circle has a 90˚ angle drawn at its centre. The 90˚ angle is coloured blue, and the rest of the circle is white. A spinner is attached to the centre of the circle. Determine the theoretical probability of the spinner landing on blue.

G4.3 What is the theoretical probability

a) of randomly pointing to a prime number on a hundreds chart?
b) that the GCF of two even numbers is an odd number?
c) that a 2-digit number that ends in a 3 is also divisible by 3?

Presentation

G5.1 Note: Question G4.1 is similar to G2.3, except that in G2.3 the answer was based on experimental probability. Question G4.2 is similar to G2.6, except that in G2.6 the answer was also based on experimental probability.

a) Ask students to compare the results in G2.3 with G4.1. Ask for their observations.
b) Ask students to compare the answer they found in G2.6 with the results found in G4.2. Ask for their observations.

Suggested Resources

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) *relate a variety of numerical expressions (ratios, fractions, decimals, percents) to the corresponding experimental or simulation situation*

SCO(s): By the end of grade 7, students will be expected to

G6 **use fractions, decimals, and percents as numerical expressions to describe probability**

Elaboration - Instructional Strategies/Suggestions

G6 This outcome should be integrated with other outcomes in this strand. Students will have had direct instruction in changing numbers from one form to another when they worked with related outcomes in GCO(A) and GCO(B). It is important for students to acquire an understanding that probability can be represented in multiple forms.

One means of accomplishing this understanding is by specifying a particular form for the answer. On occasion, questions should also be given for which no form is specified, or for which different groups are given the same problem but each group is asked to present the answer in a different form. When the class discusses the results in a large group, students should observe the variation in the answers and discuss or account for the differences. Through such experiences, students should come to the realization that the various forms are alternative representations of the same value.

Probability is most often represented by using a fraction, where the numerator represents the number of favourable outcomes and the denominator represents the total possible outcomes. This representation has many advantages, since it often maintains the original numbers in simple situations. However, probability can just as easily and meaningfully be represented in decimal form. Likewise, students will often hear in news/weather reports various probability data presented as percents. For example, the likelihood of rainfall for a given day is almost always provided in percent form. In order for all situations encountered to be meaningful to the student, they should work with all three of the representations.

Although probability is also often represented as a ratio, this representation will not be a focus in grade 7. However, when this representation occurs in resource or reference materials, teachers should feel free to address it. Many authors reserve or at least favour this representation when dealing with odds in favour and odds against.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

G6.1 The average number of rainy days per year in St. John's, Nfld., is 92. For any given day chosen at random (e.g., May 22), find the probability that it will rain. Express the answer as a fraction, a decimal, and a percent.

Interview

G6.2 The weather forecast indicated that there was a 50% chance of rain. Ask students what is another way this can be expressed?

G6.3 Batting averages are always given in decimal form. Ask students why they think this is the case. Ask them how batting averages are generally stated and how they should be stated if using correct mathematical language. (The answer to the first question might be: The denominators of the fractions keep changing. Every player will have a different denominator, which is based on the number of official times at bat. When the denominators [total number of trials] vary significantly, the decimal fraction often provides a better basis for comparison. Students may also speculate as to why batting averages are not given as percents. For example, they may conclude that it sounds more impressive to be hitting "two eighty" than hitting only 28% of the time.)

G6.4 Jamie was told that a particular die appeared to produce a two more often than any other number. Jamie wanted to do an experiment to find out if the die was unfair. Before he did the experiment, he calculated the theoretical probability of getting each value on the die. Ask students what format they would use to write theoretical probability in this situation. (Answer might be as follows: Fraction - because the denominators are consistent and therefore comparisons are easy to do in fractional form, OR Decimal - because it will probably be easier to write experimental probability as a decimal and it would be best to be consistent.)

Suggested Resources