Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

CURRICULUM

Mathematics

Grade 9

2000

Additional copies of this document (Mathematics 9) may be obtained from the Instructional Resources Branch.
Title Code (842440)

Ce document est également disponible en français auprès des Ressources pédagogiques.
Code du Titre (842460)

Acknowledgements

The departments of education of New Brunswick, Newfoundland and Labrador, Nova Scotia, and Prince Edward Island gratefully acknowledge the contributions of the following groups and individuals toward the development of this grade 9 mathematics curriculum guide.

- The Regional Mathematics Curriculum Committee; current and past representatives include the following:

New Brunswick
Greta Gilmore, Mathematics Teacher,
Belleisle Regional High School

John Hildebrand, Mathematics Consultant,
Department of Education

Nova Scotia
Richard MacKinnon, Mathematics Consultant,
Department of Education & Culture

Sharon McCready, Mathematics Teacher,
Sherwood Park Educational Centre

Newfoundland and Labrador
Roy Hodder, Acting Vice Principal,
I. J. Samson Junior High School

Patricia Maxwell, Mathematics Consultant,
Department of Education

Prince Edward Island
Clayton Coe, Mathematics/Science Consultant,
Department of Education

Joan Kennedy, Mathematics Teacher,
Stonepark Intermediate School

- The Provincial Curriculum Working Group, comprising teachers and other educators in Newfoundland and Labrador, which served as lead province in drafting and revising the document.
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the development of the grade 9 mathematics curriculum guide.

Table of Contents

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their learning experiences.

The *Foundation for the Atlantic Canada Mathematics Curriculum* firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision, which embraces the principles of students learning to value and become active “doers” of mathematics and advocates a curriculum which focusses on the unifying ideas of mathematical problem solving, communication, reasoning, and connections. The *Foundation for the Atlantic Canada Mathematics Curriculum* establishes a framework for the development of detailed grade-level documents describing mathematics curriculum and guiding instruction.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers with department of education officials to co-operatively plan and execute the development of curricula in mathematics, science, and language arts in both official languages. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the Essential Graduation Learnings (EGLs), also developed regionally. These EGLs and the contribution of the mathematics curriculum to their achievement are presented in the “Outcomes” section of the mathematics foundation document.

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. This curriculum guide is one of several which provide greater specificity and clarity for the classroom teacher. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—General Curriculum Outcomes (GCOs), which relate to subject strands, and Key-Stage Curriculum Outcomes (KSCOs), which articulate the GCOs further for the end of grades 3, 6, 9, and 12. This guide builds on the structure introduced in the foundation document, by relating Specific Curriculum Outcomes (SCOs) to each KSCO at each grade level. Figure 1 further clarifies the outcome structure.

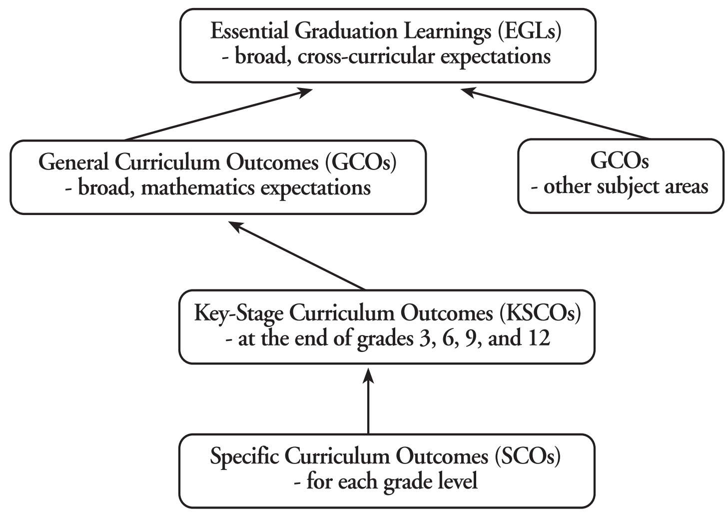


Figure 1: Outcome Framework

This mathematics guide is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice, including the following: (i) mathematics learning is an active and constructive process; (ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; (iii) learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort; (iv) learning is most effective when standards of expectation are made clear and assessment and feedback are ongoing; and (v) learners benefit, both socially and intellectually, from a variety of learning experinces, both independent and in collaboration with others.

II. Program Design and Components

A. Program Organization

As indicated previously, the mathematics curriculum is designed to support the Atlantic Canada Essential Graduation Learnings (EGLs). The curriculum is designed to significantly contribute to students meeting each of the six EGLs, with the communication and problem-solving EGLs relating particularly well with the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then goes on to present student outcomes at key stages of the student's school experience.

This curriculum guide presents specific curriculum outcomes at individual grade levels. As illustrated in Figure 2, these outcomes represent the step-by-step means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and, ultimately, the essential graduation learnings.

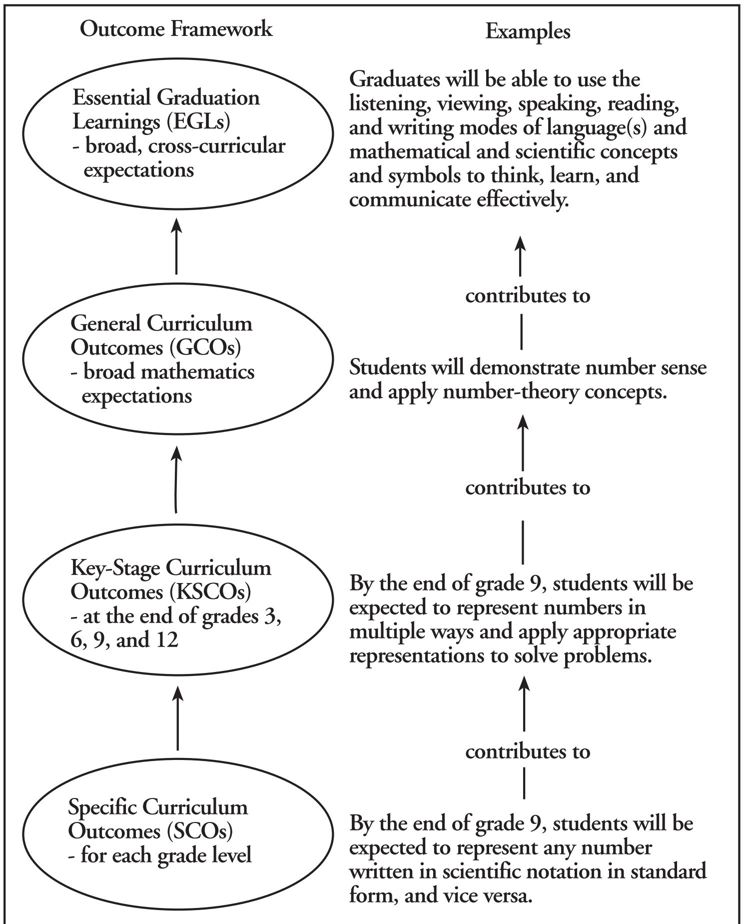


Figure 2: Examples of Outcomes

It is important to emphasize that the presentation of the specific curriculum outcomes at each grade level follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and **does not necessarily represent a natural teaching sequence**. While some outcomes will of necessity need to be addressed before others due to prerequisite skill requirements, a great deal of flexibility exists as to the structuring of the program. As well, some outcomes (e.g. Patterns and Data Management) may be best addressed on an on-going basis in connection with other topics. It is expected that teachers will make individual decisions as to what sequence of topics/outcomes will best suit their classes. In most instances, this will occur in consultation with fellow staff members, department heads, and/or district level personnel.

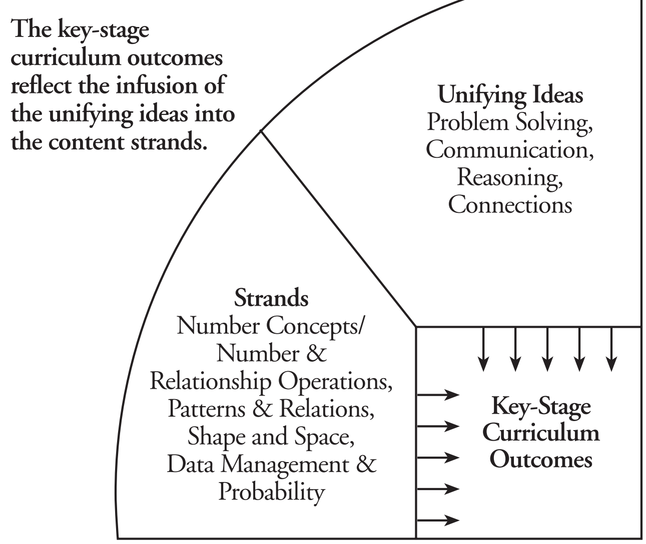
Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a “kickoff” topic for one group of students might be less effective in that role with a second group. Another consideration with respect to sequencing will be co-ordinating the mathematics program with other aspects of the students’ school experience. Examples of such co-ordination include studying aspects of measurement in connection with appropriate topics in science, data management with a social studies issue, and some aspect of geometry with some physical education unit. As well, sequencing could be influenced by other events outside of the school, such as elections, special community celebrations, or natural occurrences.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. This is illustrated in Figure 3.

These unifying concepts serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode; classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically; via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning; and mathematics with which students are involved on a day-to-day basis must be connected to other mathematics, other disciplines, and/or the world around them.

Figure 3 - Integrating Strands and Unifying Ideas



Students will be expected to address routine and/or non-routine mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling, whereas working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire in the later elementary years. In grades 7-9, this repertoire will be extended to include such strategies as interpreting formulas, checking for hidden assumptions, examining systematic or critical cases, and solving algebraically.

Opportunities should be created frequently to link mathematics and career opportunities. During these important transitional years, students need to become aware of the importance of mathematics and the need for mathematics in so many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in higher-level mathematics programming in senior high mathematics and beyond.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline which lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, and actively participate in discourse and conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas in which reasoning and sense making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on basic mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Meeting the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness as they enter the intermediate setting and as they progress, but they must also remain aware of avoiding gender and cultural biasses in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

The reality of individual student differences must not be ignored when making instructional decisions. While this curriculum guide presents specific curriculum outcomes for each grade level, it must be acknowledged that all students will not progress at the same pace and will not be equally positioned with respect to attaining any given outcome at any given time. The specific curriculum outcomes represent, at best, a reasonable framework for assisting students to ultimately achieve the key-stage and general curriculum outcomes.

As well, teachers must understand, and design instruction to accommodate differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Further, the practice of designing classroom activities to support a variety of learning styles must be extended to the assessment realm; such an extension implies the use of a wide variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This curriculum guide represents the central resource for the teacher of mathematics for these grade levels. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and yearly planning, as well as a reference point to determine the extent to which the instructional outcomes should be met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent that they support the curriculum goals. Teachers will need professional resources as they seek to broaden their instructional and mathematical skills. Key among these are the NCTM publications, including the *Principles and Standards for School Mathematics, Assessment Standards for School Mathematics, Curriculum and Evaluation Standards for School Mathematics*, the *Grades 5-8 Addenda Series*, *Professional Standards for Teaching Mathematics,* and the various NCTM yearbooks. As well, manipulative materials and appropriate access to technological resources (e.g. software, videos) should be available. Calculators will be an integral part of many learning activities.

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different than were those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their children's lives, assisting children with mathematical activities at home, and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through learning centres, teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

The concepts and skills developed in mathematics are applied in many other disciplines. These include science, social studies, music, technology education, art, physical education, and home economics. Efforts should be made to make connections and use examples which apply across a variety of discipline areas.

In science, the concepts and skills of measurement are applied in the context of scientific investigations. Likewise, statistical concepts and skills are applied as students collect, present, and analyse data.

In social studies, measurement is used to read scale on a map, to measure land areas, and in various measures related to climatic conditions. As well, students read, interpret, and construct tables, charts, and graphs in a variety of contexts such as demography.

In addition, there are many opportunities to reinforce fraction concepts and operations in music, as well as opportunities to connect concepts such as symmetry and perspective drawings of art to aspects of 2-D and 3-D geometry.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are critical, not only with respect to clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers make meaningful instructional decisions. (See “Assessing and Evaluating Student Learning” in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment should include the following: i) using a wide variety of assessment strategies and tools; ii) aligning assessment strategies and tools with the curriculum and instructional techniques; and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborate good assessment practice and serve as a guide with respect to student assessment for the mathematics foundation document.

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address such questions as the following: Are students meeting the curriculum outcomes? Is the curriculum being equitably applied across the region? Does the curriculum reflect a proper balance between procedural knowledge and conceptual understanding? Is technology fulfilling a proper role?

IV. Designing an Instructional Plan

It is important to design an instructional plan for the school year. This plan should reflect the fact that specific curriculum outcomes (SCOs) falling under any given general curriculum outcome (GCO) should not be taught in isolation. There are many opportunities for connections and integration across the various strands of the mathematics curriculum.

Consideration should be given to the relative weighting for outcomes under each GCO so that this can be reflected in the amount of time devoted to each aspect of the curriculum. Naturally, time spent must be sensitive to the background of students as well as to cross-curricular issues. Without an instructional plan, it is easy to run out of time in a school year before all aspects of the mathematics curriculum have been addressed. A plan for instruction that is comprehensive enough to cover all outcomes and strands will help to highlight the need for time management.

It is often advisable to use pre-testing to determine what students have retained from previous grades relative to a given set of outcomes. In some cases, pre-testing may also identify students who have already acquired skills relevant to the current grade level. Pre-testing is often most useful when it occurs one to two weeks prior to the start of a set of outcomes. In this case, a set of outcomes may define a topic or unit of work, such as fraction concepts and operations. When the pre-test is done early enough and exposes deficiencies in prerequisite knowledge/skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of adequate development or coverage in the previous grades. This may imply that an adjustment is required to the starting point for instruction, as well as a meeting with other grade level teachers to address these concerns is necessary.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in an overall savings in time for both disciplines. The most obvious of these connections relate to the use of measurement in science and the use of a variety of data displays in social studies.

V. Curriculum Outcomes

The pages that follow provide details regarding specific curriculum outcomes. As indicated earlier, the order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching GCOs and KSCOs of the mathematics foundation document. The specific curriculum outcomes are presented on individual two-page spreads. See Figure 4 on next page.

This guide presents the mathematics curriculum by grade level so that a teacher may readily view the scope of the outcomes which students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings at a particular grade level are part of a bigger picture of concept and skill development. Given that the specific curriculum outcomes at each grade level are related to the key-stage curriculum outcome framework, it is relatively easy to access a given KSCO at the previous grade and/or the next one to see how the development of particular mathematical ideas are taking place.

Within a grade level, the specific curriculum outcomes are presented on individual two-page spreads. At the top of each page, the overarching GCO is presented, with the appropriate KSCO(s) and SCO(s) displayed in the left-hand column. The KSCO(s) are in italics while the SCO(s) are bold-face. The second column of the layout is entitled "Elaboration-Instructional Strategies/Suggestions" and provides a clarification of the specific curriculum outcome(s), as well as some suggestions of possible strategies and/or activities which might be used to achieve the outcome(s). While the strategies and/or suggestions presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning, and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol ❑.

The third column of the two-page spread, "Worthwhile Tasks for Instruction and/or Assessment," might be used for assessment purposes or serve to further clarify the specific curriculum outcome(s). As well, those tasks regularly incorporate one or more of the four unifying ideas of the curriculum. These sample tasks are intended as examples only, and teachers will want to tailor them to meet the needs and interests of the students in their classrooms. The final column of each display is entitled "Suggested Resources" and will, over time, become a collection of useful references to resources which are particularly valuable with respect to achieving the outcome(s).

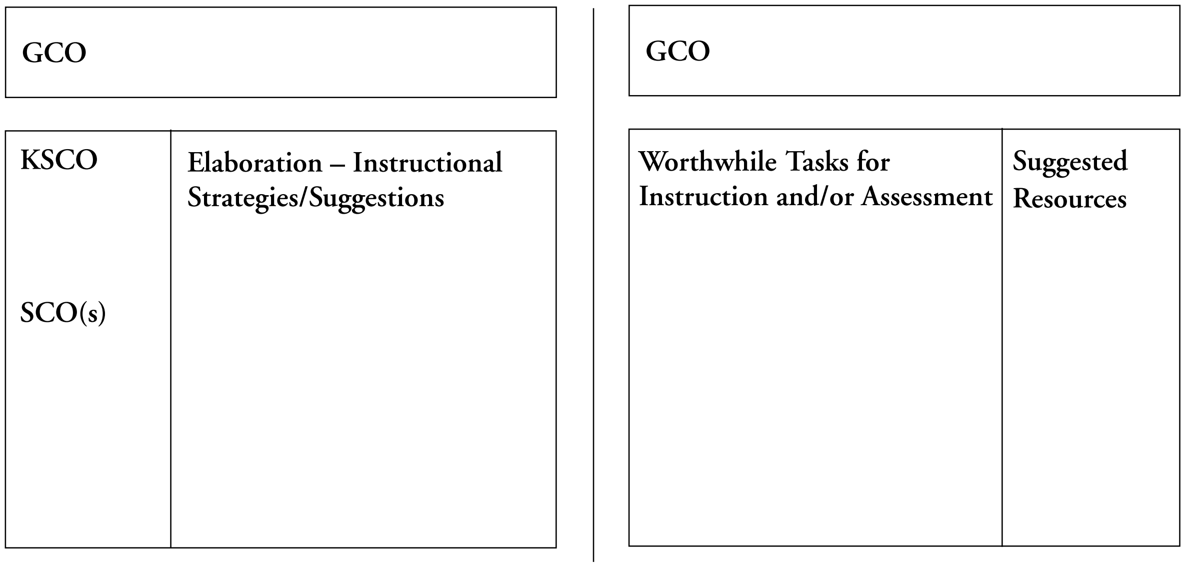


Figure 4: Layout of a 2-Page Spread

NINE - NUMBER

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome A:

Students will demonstrate number sense and apply number-theory concepts.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 9, students will be expected to

A1 solve problems involving square root and principal square root

Elaboration – Instructional Strategies/Suggestions

A1 Students will be expected to investigate situations where a decision is needed regarding whether the solution involves both values for the square root or just the principal square root. Consider a problem such as the following to introduce the topic.

- ❑ One integer is double another, and the sum of their squares is 45. What are the integers? [Students may solve this algebraically or by using guess and test. Some students will find the positive solution and assume, if there is no teacher intervention, that it is the only solution. Solutions are 3 and 6, and -3 and -6.]

Finding square roots using prime factorization, mental computation, estimation, and the calculator has been explored in grade 8. Students in grade 9 should come to realize that because $(-5)^2$ and $(+5)^2$ both equal 25, the square root of 25 is ± 5. Until this grade level, most students assumed that the square root of a number was positive. Mathematicians use $\sqrt{\ }$ to represent only positive square roots, so when the question is written as $\sqrt{25}$, the answer is 5. However, when solving the equation $x^2 = 4$, x can equal $\frac{1}{243} = \pm 2$. The issue of principal square root should be treated as a mathematical convention as opposed to a focus of attention.

Students should understand that in real-world problems the answer is almost always the positive square root of a number because it is the only value that makes sense in most contexts. It is useful, however, for students to recognize that both the positive and the negative value can solve equations such as $x^2 = 16$. This will be useful when solving equations in higher grade levels. The study of square roots is an opportunity to revisit the Pythagorean relationship. Students can work with points on a coordinate plane and find the distance between them, using the Pythagorean relationship, and come to realize that these distances are always positive, regardless of the signs of the coordinates.

- ❑ A town council set up the town map in such a way that the Town Hall was at the centre (0, 0). This was then overlaid by a four-quadrant grid so that all locations were determined using positive and negative coordinates. The hospital is located at (-5, -4), and the community swimming pool is located at (1, 4). One unit on the grid represents 1 km of actual distance.
  a) How far are the hospital and the swimming pool from city hall?
  b) How far is the hospital from the pool?
  [Solution to part b): $c^2 = 6^2 + 8^2$
  $c^2 = 100$
  $c = \pm\sqrt{100}$
  $c = \pm 10$

  Since -10 cannot represent a distance, 10 km must be the answer.]

This might be an appropriate time to introduce informally the notion of absolute value.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A1.1 When solving the equation $x^2 = 4$, Jason discovered that $(-2)^2 = 4$, and $(+2)^2 = 4$. He concluded through guess-and-check that there are two solutions to this equation. Sarah solved the same problem, using the " $\sqrt{\ }$ " button on the calculator. Her solution produced only one answer.

a) Is Jason's conclusion correct? Explain why or why not.

b) How can you explain the fact that Sarah's method produced only one solution? [The symbol $\sqrt{\ }$ represents positive square root. When Sarah used the square root button on the calculator, only the positive square root resulted. It is often necessary to interpret solutions produced by calculators.]

A1.2 A square has an area of 109 cm^2. What are the lengths of its sides?

Portfolio

A1.3 Jim lives in the downtown area of a city where the houses are very close together. He wants to paint a window on the second floor. The window sill is 3.5 m above the ground. The only ladder available is 5 m long. The space between the houses is only 2 m, and the window is on the side of the house. Ask students

a) if he places a ladder at the height of the window sill, how far away from the house the base of the ladder will need to be [Ask them, when calculating, to consider the two possible answers and justify which one is reasonable.]

b) if he places the ladder as far away from the house as the house next door will allow, how far up the side of the house the ladder will reach

c) to comment on whether the length of this ladder makes it suitable for painting the window

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 9, students will be expected to

A2 **graph, and write in symbols and in words, the solution set for equations and inequalities involving integers and other real numbers**

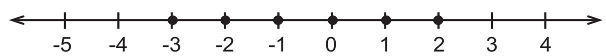
Elaboration – Instructional Strategies/Suggestions

A2 The symbols for less than or equal to, $\leq$, and greater than or equal to, $\geq$, should be included as well as <, >, and =. Also, students should graph sets that are described using upper and lower bounds.

❑ Consider the set of real numbers greater than -3 and less than or equal to 2. The expression $\{x \mid -3 < x \leq 2, x \in R\}$ describes this set. This expression is read as, the set of values x where x is greater than -3, x is less than or equal to 2, and x belongs to the set of real numbers.

The language of set notation should be developed in conjunction with the topic. This need not be taught as a discrete topic; it can easily be integrated as a way of representing solutions to equations and inequalities. This is a good opportunity to review the various number sets studied in previous grades and the symbols which are used to describe or represent them. Set notation provides a concise means of describing a set of numbers that are infinite.

When a set is shown on a number line, students should be able to accurately describe the set both in words and by using symbols. For example, students should be able to describe, using formal notation or in words, the set represented in the diagram below.



This set can be described in a number of ways such as $\{y \mid -4 < y \leq 2, y \in I\}$, or as $\{y \mid -3 \leq y \leq 2, y \in I\}$. Likewise, when students are given a description such as $\{x \mid -2.5 < x \leq \sqrt{7}, x \in R\}$, they should be able to produce the graph shown.

In words, this graph can be described as the set of values x where x is greater than -2.5, x is less than or equal to $\sqrt{7}$, and x is real. The language of inequality, such as "at least," "at most," "more than," and "less than," should be discussed so that students can relate this language to the symbols of inequality.

Through comparison of the two graphs shown, students can continue to develop an understanding of the difference between discrete and continuous data.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A2.1 In the last history test, Jane's grade was more than a passing grade (50%), and less than an A^- (80%).

a) Represent Jane's possible grade on the test, using set notation.

b) In the upcoming test she hopes to increase her grade by 10. Represent Jane's possible grade on the second test, using set notation. [Possible answers: first test $\{x \mid 50 < x < 80, x \in W\}$, second test $\{y \mid 60 < y < 90, y \in W\}$]

A2.2 On Tuesday the *Clothes are Us* store received a shipment of 90 leather jackets. If Sue sells more than 10 jackets in a week, she will get a bonus of $10 for every additional jacket sold.

a) If Sue **did** receive a bonus check, use both set notation and a number line to represent the possible number of jackets Sue sold in the week the jackets arrived.

b) Use both set notation and a number line to represent the possible values of the bonus that Sue could receive.

c) If Sue **did not** receive a bonus check, use both set notation and a number line to represent the possible number of jackets Sue sold in the week the jackets arrived.

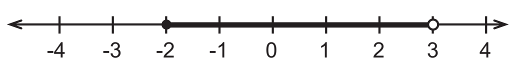
A2.3 Represent each of the following, using a number line.

a) all integers that are greater than 5

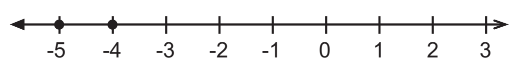
b) all real numbers that are less than or equal to $-\pi$

A2.4 Match each set with the line number that represents it.

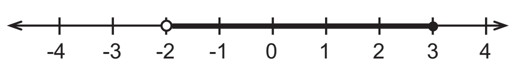
a) $\{x \mid x \geq -3, x \in I\}$



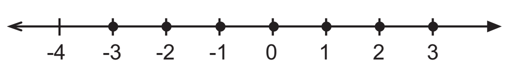
b) $\{x \mid x < -3, x \in I\}$



c) $\{x \mid -2 \leq x < 3, x \in R\}$



d) $\{x \mid -2 < x \leq 3, x \in R\}$



Portfolio

A2.5 Chris wants to make a tablecloth for a circular plant table. The cloth must, at the very least, cover the top of the table, and, preferably, hang down over the edge. The diameter of the table is 40 cm. She has two metres of lace edging which will go around the edge of the cloth. Ask students, if Chris uses all of the lace, to use set notation and a number line in representing the approximate

a) diameter of the tablecloth

b) number of centimetres the cloth hangs down over the edge

c) radius of the cloth

[Consider physical or computer modelling for this question.]

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *demonstrate an understanding of number meanings with respect to integers and rational and irrational numbers, and explore their use in meaningful situations*

SCO: By the end of grade 9, students will be expected to

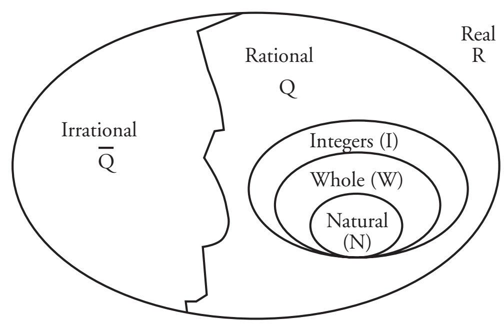
A3 demonstrate an understanding of the meaning and uses of irrational numbers

A4 demonstrate an understanding of the interrelationships of subsets of real numbers

Elaboration – Instructional Strategies/Suggestions

A3 Students have already worked with square roots in grade 8, so many of them may have been exposed to the term irrational number. Students can look at the square roots of numbers, such as 2 or 443, using a computer or calculator. They can discuss whether there is any pattern apparent in the decimal representation. Work with this outcome should be done in conjunction with outcome A2. Students can also look at the decimal representation for π. Long lists of decimal places for π can be found in many older textbooks. One means of demonstrating understanding of irrational numbers involves placing them on a number line relative to known rational numbers. The lengths of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc. can be found by using the spiral which is illustrated on the next two-page spread. Investigation of the Pythagorean relationship will be the primary use of the irrational number at this level.

A4 Given any real number, students need to be able to determine whether it is rational or irrational, and justify why. Through the use of Venn diagrams, the interrelationship between natural, whole, integer, rational, irrational, and real numbers should be established. Students should be asked to produce a diagram which shows the relationship among the various subsets of the real number set. For example,



Students should be able to give examples and explain why numbers satisfy the conditions of natural, whole, integer, rational, and irrational numbers. For example, students can be asked to identify a number that is real but not rational, or rational but not an integer, and justify the selection.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A3.1 To find how far away the horizon is, use the formula $d = \sqrt{2rh}$, where h represents the height, in metres, above the ground and r is the radius of the Earth. The mean radius of the Earth is 6400 km.

a) Find the distance to the horizon for a height of 45 m and then for a height of 400 m.
b) Are the answers exact values? Explain.
c) Describe a set of values for h which will make the distances, d, exact values.

A4.1 Place a ✓ in the spaces to indicate that the number belongs to the number set, and justify.

	N	W	I	Q	$\bar{Q}$	R
5						
-2						
$\frac{3}{4}$						
-1.3						
$\sqrt{7}$						
$\sqrt{9.5}$						

Interview

A3.2 Ask students if there are more irrational or rational numbers, and to explain their answers.

A3.3 Ask students if π is irrational, if $\frac{\pi}{2}$ is irrational, if 2π is irrational. Ask them how they know.

Presentation

A4.2 Ask students to sort the list of numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, … $\sqrt{20}$ into two sets, those that are rational and those that are irrational, and to explain how they know their sorting is correct.

Journal/Portfolio

A4.3 Ask students to use a diagram to show how the following sets of numbers are related: natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

A4.4 Ask students to identify each of the following as sometimes true, always true, or never true, and to justify their choices.

a) All whole numbers are integers.
b) All integers are whole numbers.
c) If a number is a rational number then it is also an integer.
d) If a number is an integer then it is also a rational number.
e) There is a number which is both rational and irrational.
f) The number $\sqrt{0.16}$ is irrational.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *read, write, and order integers, rational numbers, and common irrational numbers*

iv) *apply number-theory concepts in relevant situations, and explain the interrelated structure of whole numbers, integers, rational, and irrational numbers*

SCO: By the end of grade 9, students will be expected to

A5 **compare and order real numbers**

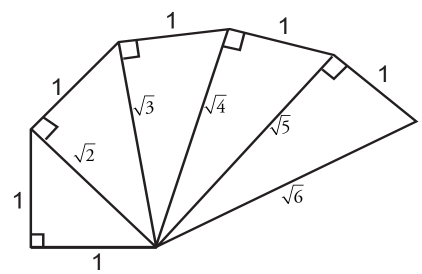
Elaboration – Instructional Strategies/Suggestions

A5 In many respects outcome A3 is very closely linked with comparing and ordering, since this is another way of demonstrating understanding of the meaning of an irrational number. Students can be asked to construct segments of various lengths, for example, $\sqrt{2}$, $\sqrt{3}$, and $\sqrt{5}$. This can be done using the Wheel of Theodorus (also known as the Spiral of Archimedes). The Wheel of Theodorus is named for its creator, Theodorus of Cyrene, who was a student of Pythagoras. This wheel starts with a right triangle with legs 1 unit in length. The hypotenuse of this triangle is

$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

A second right triangle is constructed so that the hypotenuse of the first triangle is a leg for the second triangle, along with a second leg of 1 unit in length. The third triangle uses the hypotenuse of the second triangle, which is $\sqrt{3}$, as one leg and 1 unit as the other leg. This is illustrated below:

❑ Ask students to use a compass to construct a spiral such as the one started at the right. Ask them to predict how many triangles would be constructed before the triangles overlap.



This construction should help students establish some relative size for irrational numbers and can be of help when comparing and placing irrational numbers along with rational numbers on a number line.

While students are expected to work with all real numbers, the focus should be on incorporating irrational numbers into ordering activities, most particularly square roots. Cube roots, fourth roots, and beyond will be addressed in subsequent years.

When finding square roots, use estimation where possible and, otherwise, use the calculator. Pencil and paper calculation of square root is not expected, though it can be explored as an optional activity for interested students.

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Performance

A5.1

a) Ask students to arrange the following in ascending order, and to justify their ordering:
π, $\frac{22}{7}$, $\frac{355}{113}$, 3.141, 592, $\sqrt{10}$, $\sqrt{9.5}$, $-\pi$, $\frac{-22}{7}$, $\frac{-355}{113}$

b) Ask them to group the numbers as rational and irrational, and to justify their groupings.

A5.2 Ask students to use less than, greater than, or equal to make each of the sentences true and justify their choices.

a) $1\frac{732}{1000}$ __ $\sqrt{3}$

b) $-\sqrt{6}$ ____ $-\sqrt{5}$

c) 1.4142135 ____ $\sqrt{2}$

Portfolio

A5.3 For an unknown reason Bill decided that the area of his new deck would be equal to his oldest daughter's age and would be completed for her birthday party. His daughter will be 15 on her birthday so he wants to build the deck 15 square metres. He also wants the shape of the deck to be square.

a) Ask students what the length of each side would be for this square deck.

b) Since measurements can only be as accurate as the measuring instruments, Bill decided that he wanted his deck to be measured as accurately as possible to the nearest hundredth of a metre, but he would tolerate a measurement error of ± 0.01 . Ask students to write a sentence to describe the greatest and the least possible dimensions of the deck.

A5.4 Kay made a triangular flower garden which had sides of 1.00 m and 2.00 m that were supposed to meet to form a right triangle. When Kay measured the length of the third side she found it to be 2.50 m.

a) Kay immediately said, "This is not a right triangle after all." Ask students what thinking may have led Kay to this conclusion.

b) Ask students to compare the new value of the third side when the triangle is made into a right triangle with the original value that Kay had determined and use it to decide whether the right angle in the first case was less than 90° or more than 90° degree. Ask them to justify their decisions.

Suggested Resources

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent numbers in multiple ways and apply appropriate representations to solve problems*

SCO: By the end of grade 9, students will be expected to

A6 represent problem situations using matrices

Elaboration – Instructional Strategies/Suggestions

A6 A matrix is a rectangular arrangement of numbers into rows and columns. For example, a Girl Guide group held two fund-raisers – a car wash and a swim-a-thon. Each event involved some expenses, and each event resulted in some income. The following presents this data:

	Income	Expenses
Car Wash	354	78
Swim-a-thon	460	122

The number array shown above is a matrix that has two rows and two columns. This matrix can be described as a 2 by 2 matrix.

This is a new topic for students, one that is continued through the high school program. In grade 9, matrices will be used as a means of storing data, and simple problems will be presented which require the addition and subtraction of matrices to solve them. Operations with matrices are expanded upon in B7.

The Melville Mantis Soccer team had 4 wins and 6 losses in June, 5 wins and 5 losses in July, and 7 wins and 3 losses in August. This information can be displayed very clearly in a matrix as follows:

	Wins	Losses
June	4	6
July	5	5
August	7	3

This soccer data is represented by a 3 by 2 matrix. Some time should be spent discussing how the rows and columns of the matrix are identified, as well as how specific entries are identified. In the previous matrix, the 6 is in the first row and the second column. This position can be described as 1, 2. The 7 is in the third row and the first column. This position can be described as 3, 1.

- ❑ Ask students to give the dimensions of the following matrix.

$$\begin{bmatrix} 15 & 8 & 22 & 12 \\ 12 & 4 & 5 & 14 \end{bmatrix}$$

[Answer: 2 by 4]

GCO (A): Students will demonstrate number sense and apply number-theory concepts.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

A6.1 In a particular library there are 10 000 books and 425 magazines identified for general circulation and 3000 books and 2500 magazines identified as reference materials. Represent this information in a matrix.

A6.2 A video rental store rents comedies, dramas, horrors and cartoons. They carry the following new releases: 25 comedies, 45 dramas, 38 horrors, and 30 cartoons. In regular selections they have 125 comedies, 300 dramas, 178 horrors, and 146 cartoons.

a) Represent this information using a 2 by 4 matrix.
b) Represent this information using a 4 by 2 matrix.

A6.3 There are four teams in Jan's soccer league. The Gremlins had 8 wins, 6 losses, and 4 ties. The Monarchs had 5 wins, 8 losses, and 5 ties; the Aces had 12 wins, 3 losses, and 2 ties; and the Aquas had 3 wins, 11 losses, and 3 ties.

a) Represent this information in two different ways using matrices.
b) Identify the dimensions for each matrix.

Portfolio

A6.4 To raise money for their soccer club Sarah's team sold food at the fair. The matrix shown represents the foods sold and the expenses and income related to each type.

	Income	Expenses
Hotdogs	67	32
Burgers	78	36
Cookies	78	25
Pop	56	28

Ask students

a) to represent this data using a matrix
b) what type of food appears to be most profitable
c) what the total profit for the fund raising project was

Suggested Resources

NINE - OPERATIONS

Number Concepts/ Number and Relationship Operations

General Curriculum Outcome B:

Students will demonstrate operation sense
and apply operation principles
and procedures in both
numeric and algebraic situations.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers*

SCO: By the end of grade 9, students will be expected to

B1 **model, solve, and create problems involving real numbers**

Elaboration – Instructional Strategies/Suggestions

B1 Students may have some difficulty identifying real-life uses for operations with rational numbers in fractional form. This may provide an opportunity to discuss the changes in the use of fractional numbers in our society. Students should come to realize the relative importance of decimal fractions in a society that uses SI measurement and calculators.

It is often an interesting exercise to have students model a problem that requires division of rational numbers in fractional form, and then ask students to re-state that problem so that it can be solved by multiplication.

The following are contexts which apply rational numbers and are worthy of consideration: stock market problems which apply decimals or fractions, depending on whether it is a Canadian or an American stock exchange; temperature problems; altitude problems; time problems, such as in sports races, that measure to a fraction of a second; mixture problems; exchange-rate problems, particularly from Canadian to American and vice versa.

Students have worked with the four operations involving rational numbers in decimal form in grade 8. They have also worked with the four operations with positive rational numbers in fractional form. When students pose and model problem situations which involve negative fractions, some students may be able to integrate previously-learned skills to solve them, while others will require more direct instruction in this area. These specific skills are addressed in B2.

❑ Claude acquired a recipe from his grandmother which makes 5 dozen chocolate chip cookies. The recipe requires $2\frac{1}{2}$ cups of oatmeal, 2 cups of chocolate chips, 2 cups of flour, 2 eggs, 1 cup of butter, 5 ml of baking powder, and 5 ml of salt.

a) Adjust the recipe to make 2 dozen cookies.
b) Adjust the recipe to make 12 dozen cookies.
c) Discuss with students what units these measurements would be represented by if all units were properly recorded in SI.

Students are not responsible for operations on irrational numbers in radical form but will be expected to solve problems using irrational numbers by converting them to decimal approximations.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Note: B1 is also incorporated into the problems on the next 2-page spread.

The following is an excerpt from a report of an American stock exchange. The first column identifies the company, the second identifies the number of shares (stocks) sold, the third and fourth identify the highest and lowest price per share paid during the day, the fifth identifies the day's final price, and the sixth represents the change in price (in dollars) from the closing price of the day before. [It may be necessary to explain to students what the stock exchange is. It would be sufficient to explain that many companies offer stocks to the public, who then buy the stocks and thus own a share in the company. These stocks are sold on the stock exchange, and the information on these sales are reported in the financial section of most newspapers.]

Stock	Sales	High ($)	Low ($)	Close ($)	Net Change ($)
Eastwick	580	$10\frac{1}{8}$	$9\frac{3}{8}$	$9\frac{5}{8}$	$-\frac{1}{4}$
Northfield	1200	$17\frac{1}{2}$	$16\frac{1}{4}$	$16\frac{7}{8}$	$\frac{1}{2}$

Pencil and Paper

B1.1 If Sarah bought 200 shares of Eastwick when it was at its lowest point of the day,

a) how much were the shares worth at closing? [$1925]

b) how much would she have made had she sold the stocks at the highest price for the day? [$2025 - $1875 = $150]

B1.2 Write a problem that can be solved by using the following expressions that are based on the information in the stock market report.

a) $300 \times 16\frac{7}{8}$ [e.g., What is the value of Aaron's 300 stocks of Northfield at closing time?]

b) $200\,(9\frac{3}{8} - 10\frac{1}{8})$ [e.g., Simon bought 200 shares of Eastwick at its high point of the day, and sold them at its low point. How much did he lose?]

c) $-\frac{1}{4} \times 1000$ [e.g., If Annon bought 1000 shares of Eastwick yesterday at closing, and sold them today at closing, how much money would he have lost?]

B1.3 Clare is making a flower garden that is shaped like a right triangle. Two of the sides of the triangle are 2 m in length.

a) What is the perimeter of the flower garden?

b) Can one of the given sides be the side opposite the right angle? Explain why or why not.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers*

SCO: By the end of grade 9, students will be expected to

B2 **add, subtract, multiply, and divide rational numbers in fractional and decimal forms, using the most appropriate method**

Elaboration – Instructional Strategies/Suggestions

B2 For this topic it is particularly important to consider the development that has occurred in grades 7 and 8. Significant work has taken place using concrete models and pictorial representations for fractions. Most students should be able to work reasonably well with all four operations with fractions at the symbolic level by grade 9. However, some students may require a brief review of operations with fractions using concrete and pictorial models.

Students worked with the four operations with integers, starting in grade 7, and with operations related to rational numbers in decimal and fractional form in grade 8. Operations with negative rational numbers in fractional form is new at grade 9.

Mental computation should reinforce the student's use of the algorithms. It should be noted that the outcome refers to using the most appropriate method. As was the case for other number sets, students should always consider mental computation first. Regular opportunities should be provided for students to practise mental computation. Problems which students should be able to tackle, using the mental computation strategies developed in earlier grades, include the examples below:

$-\frac{2}{3} \times 12$ $\quad$ $\frac{2}{3} \times -\frac{3}{4}$ $\quad$ $-0.5 \div -2.5$

$6 \div -\frac{1}{2}$ $\quad$ $-\frac{2}{3} \div -\frac{1}{3}$ $\quad$ $2 \div -0.5$

$-\frac{1}{3} \div -2$

In solving $6 \div -\frac{1}{2}$, students should see that this problem is the same as $-6 \div \frac{1}{2}$. They should ask the question, How many halves are in 6 and likewise how many halves are in -6?

In solving $-\frac{1}{3} \div -2$, students should realize that this has the same answer as $\frac{1}{3} \div 2$. They should then ask the question, If $\frac{1}{3}$ is divided into two equal parts, what is the size of each part?

Students need to make decisions on whether an exact or approximate answer is required. When an exact answer is required, they need to decide whether to use mental math, pencil and paper computation, or the calculator.

It is important that students become proficient with standard pencil and paper algorithms. These algorithms provide fundamental building blocks for algebraic manipulations and add to general computational efficiency.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B2.1 Some of Nikhil's stocks are reported on the New York Stock Exchange using fractions, while the rest are reported on the Toronto Stock Exchange using decimals. How much did he lose if one holding of 150 shares reported a net change of $-4\frac{1}{2}$ and a second holding of 6000 shares reported a net change of -0.25? [$150 \times -4\frac{1}{2} + 6000 \times -0.25$]

B2.2 A submarine recorded temperature readings during the $3\frac{1}{4}$ hours of its descent to the bottom of the ocean. The total change in temperature was $-18\frac{3}{4}$°C.

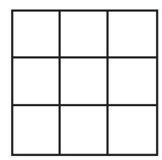
a) Find the mean hourly change in temperature.
b) Find the change that would be expected for each quarter hour.
c) What assumptions did you make?

B2.3 Brigette works $37\frac{1}{2}$ hours a week and earns $19.85 per hour. How much does she make in a year? What assumptions did you make?

Portfolio

B2.4 In a magic square each row, column, and diagonal adds to give the same value. Have students

a) create a magic square which uses a mixture of positive and negative rational numbers written in fractional form
b) create a magic square which uses a mixture of positive and negative rational numbers written in decimal form



Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers*

vi) *select and use appropriate computational techniques in given situations, and justify the choice*

SCO: By the end of grade 9, students will be expected to

B3 apply the order of operations in rational number computations

Elaboration – Instructional Strategies/Suggestions

B3 Problems involving the use of multiple operations with rational numbers provide teachers with a good opportunity to observe whether students understand the four basic operations on rational numbers instead of applying them in rote fashion. It is extremely important that the students have a solid foundation in operations with rational numbers, since it is a requirement fundamental to the study of algebra. Teachers should check with the grade 7 and 8 curriculum guides when students experience difficulty with the operations, to help plan and properly sequence remedial efforts.

In grades 7 and 8 students learned the following rules for the order of operations:

- perform the operations in brackets first,
- find the value of expressions involving exponents,
- multiply or divide in the order they appear from left to right,
- add or subtract in the order they appear from left to right.

Use of the order of operations usually involves only two or three operations combined within the same problem. However, when several operations are combined in the same problem situation, students can engage in mini competitions to solve them accurately without the aid of a calculator. Shown below are sample problems students can try.

$$\frac{\frac{3}{5} - \frac{-1}{10}}{-\frac{1}{4} - 1\frac{2}{3}} \times (-3)^2 \div 2$$

$$\frac{-2\frac{1}{2} \div \frac{2}{3}}{-1\frac{1}{4} - (-\frac{7}{4})} \times [-0.25 \div 0.05]$$

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B3.1 A company drilling near an established mine site found core samples for copper at -122.5 m, -87.6 m, -84.3 m, -105.4 m, and -0.5 m.

a) What was the mean depth of the copper finds?

b) What was the median depth of the copper finds?

B3.2 Leif loved following clues and playing games like Treasure Hunt. Helga used the answer to the question below as one of the clues in the treasure-hunt game that she made up.

$$-\frac{3}{4}\,[-2(\frac{-3}{4} - \frac{2}{3}) - 5 \div -\frac{1}{4}]$$

Leif solved it correctly. What was Leif's answer?

B3.3 Arrange the following from least to greatest.

a) $\frac{-3}{4} - (-\frac{3}{4} + \frac{4}{-5})$

b) $\frac{-3}{5} - \frac{-3}{4} - \frac{9}{-10}$

c) $6 \div \frac{-1}{5} - \frac{1}{-2}$

d) $\frac{3}{5} - (\frac{-3}{5} - \frac{-2}{3})$

Portfolio Entry

B3.4 The two formulas for converting from Celsius or Fahrenheit temperature are as follows: $C = \frac{5}{9}(F - 32^{o})$, $F = \frac{9}{5}C + 32^{o}$

a) Have students find the temperature in Fahrenheit which is equivalent to -8°C.

b) Bill was watching a Canadian television channel and heard that a record low temperature for Jan. 6th in Alaska was -40°C. The same report was shown on an American channel, but the record low for that date was reported as -40°F. Bill was confused because he knew that these two scales were quite different. Ask students to verify whether the temperatures reported on both channels were accurate.

c) Have students make a table to show the corresponding Fahrenheit temperature for each Celsius temperature from -50°C to 50°C in multiples of 10. Ask them to
   i) graph the Celsius versus Fahrenheit temperature
   ii) describe the pattern they observe

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

SCO: By the end of grade 9, students will be expected to

B4 **demonstrate an understanding of, and apply the exponent laws for, integral exponents**

Elaboration – Instructional Strategies/Suggestions

B4 Students have worked with exponents in grades 7 and 8 and should be able to recognize readily the meaning of 2^3 and 2^2. In grade 8, they were exposed to negative exponents for base 10 so that there would be a basis for working with numbers less than one written in scientific notation. That is, students should recognize 2.04×10^{-4} as the same as 0.000204.

The primary focus at grade 9 should be placed on the development of an understanding of the laws of exponents. Emphasis on attaching names to the laws should **not** be the focus of instruction.

- ❑ Ask students to evaluate $2^3 \times 2^2$ by evaluating the two powers separately and then multiplying the results. That is, students should write 2^3 as $2 \times 2 \times 2$ and 2^2 as 2×2 and multiply all the 2's together. Ask students to write $2 \times 2 \times 2 \times 2 \times 2$ with a single base and compare with $2^3 \times 2^2$. Have the students repeat the process for other powers so that they may predict the rule.
- ❑ Have students calculate $(2^2)^4$ as follows: $(2^2)^4 = 2^2 \times 2^2 \times 2^2 \times 2^2 = 2^8$. Students may apply the rule $a^m \times a^n = a^{m+n}$, or simply write $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$. After repeating this with other examples, students should be able to predict a rule that applies when an exponent is raised to an exponent.

Students used patterning with base 10 to establish meaning for negative exponents in grade 8. This should be revisited to ensure understanding.

- ❑ Ask students to look for a pattern when the following are calculated: 10^3, 10^2, 10^1, 10^0, 10^{-1}, and 10^{-2}. Ask them to also compare values of 10^{-1} and $\frac{1}{10^1}$, 10^{-2} and $\frac{1}{10^2}$, and 10^{-3} and $\frac{1}{10^3}$, and discuss possible conclusions.

Similar investigations can be designed to establish the other rules/laws. The following should be developed at grade 9:

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(ab)^n = a^n b^n \qquad (a^m)^n = a^{mn} \qquad a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

Whenever possible, instruction should be designed so that students discover rules/relationships and verify their discoveries. Otherwise, students may get the impression that the rules of mathematics are no more than "magic boxes."

Note: The elaboration for B4 is continued on the next 2-page spread.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B4.1 The dog chewed John's calculator and completely destroyed the 9 key.

a) Explain how he could find the value of 9^4 without using this key.

b) Suppose instead that the 4 key were destroyed instead of the 9 key. Explain how he could then find the value of 9^4.

B4.2 Use patterning to help you find the last digit in

a) 4^{100}

b) $(-2)^{101}$

c) 5^{50}

B4.3

a) Make graphs of $\frac{1}{2^n}$ and 2^n when n = 1, 2, 3, ...

b) What do you notice about the shapes?

Interview

B4.4 Ask students to explain two ways that $(2 \times 5)^4$ can be calculated.

B4.5

a) Ask students to explain why the following is easy to solve mentally: $2^4 \times 2^{-4} \times 5^3 \times 5^{-3} \times 10^5 \times 10^{-4}$.

b) Ask students to solve the problem mentally.

c) Ask them to write a similar problem involving six bases and exponents which is also easy to solve mentally.

B4.6 Ask students to solve each of the following mentally.

a) $4^6 \times 4^{-4} \times 4^0$

b) $7^9 \div (7^7 \times 7^1)$

c) $145^3 \times 145^2 \times 145^{-4}$

d) $(32^2)^4 \div 32^9$

e) $(5^7 \times 5^5 \times 5^4 \times (5^{-7}))^0 \div 14^1$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) model problem situations involving rational numbers and integers

iii) apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents

SCO: By the end of grade 9, students will be expected to

B4 demonstrate an understanding of, and apply the exponent laws for, integral exponents

Elaboration – Instructional Strategies/Suggestions

B4 (Cont'd) $a^o = 1$ was developed in grade 8 in conjunction with $a^{-n} = \frac{1}{a^n}$, and these laws should be revisited in grade 9 as well. Practice should involve numerical bases with some extension to literal bases. Discussion should consider what happens to $\frac{1}{a^n}$ when n becomes very large or when n approaches zero for $a \in N$. It should be noted that work with fractional exponents is **not** done formally at this grade level. When investigating $\frac{1}{a^n}$, values should be generated using the calculator, and applications should be kept straightforward.

Students can explore different solutions to problems, such as $3^4 \times 3^{-5} \times 3^3$, to develop an appreciation for the efficiency that the exponents laws provide. For example,

$$\begin{aligned} & 3^4 \times 3^{-5} \times 3^3 \\ = \; & 3^{4+(-5)+3} \\ = \; & 3^2 \\ = \; & 9 \end{aligned}$$

$$\begin{aligned} & 3^4 \times 3^{-5} \times 3^3 \\ = \; & 3 \times 3 \times 3 \times 3 \times \frac{1}{3^5} \times 3 \times 3 \times 3 \\ = \; & 81 \times \frac{1}{3 \times 3 \times 3 \times 3 \times 3} \times 27 \\ = \; & 81 \times \frac{1}{243} \times 27 \\ = \; & 2187 \div 243 \\ = \; & 9 \end{aligned}$$

$$\begin{aligned} & 3^4 \times 3^{-5} \times 3^3 \\ = \; & 3^{4+3} \times 3^{-5} \\ = \; & 3^7 \times \frac{1}{3^5} \\ = \; & \frac{3^7}{3^5} \\ = \; & 3^{7-5} \\ = \; & 3^2 \\ = \; & 9 \end{aligned}$$

Each of these solutions uses the exponent laws in different ways. Students can consider the three solutions and discuss which solution is most efficient and why it is most efficient. Students should be encouraged to use the laws as efficiently as possible; however, students should not be penalized for using other methods.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B4.7 Simplify:

a) $4^2 \div 4^{-2}$

b) $\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^{-2}$

c) $\left(\frac{-24m^2n^2}{4m^3n^{-3}}\right)^0$

B4.8 Consider each of the following:

a) If $a^m = a^n$, then what must be true of m and n?

b) If $a^m = b^m$, then what must be true of a and b?

c) If $2^{3n} = 2^{n-4}$, then what must be true of 3n and n - 4?

d) Use what you found in c) above to find the value of n.

e) Solve for y: $6^{4y-3} = 6^5$

B4.9 The following shows the values for various powers of 2.

$2^0 = 1$	$2^6 = 64$	$2^{12} = 4096$	$2^{18} = 262\ 144$
$2^1 = 2$	$2^7 = 128$	$2^{13} = 8192$	$2^{19} = 524\ 288$
$2^2 = 4$	$2^8 = 256$	$2^{14} = 16\ 384$	$2^{20} = 1\ 048\ 576$
$2^3 = 8$	$2^9 = 512$	$2^{15} = 32\ 768$	$2^{21} = 2\ 097\ 152$
$2^4 = 16$	$2^{10} = 1024$	$2^{16} = 65\ 536$	$2^{22} = 4\ 194\ 304$
$2^5 = 32$	$2^{11} = 2048$	$2^{17} = 131\ 072$	$2^{23} = 8\ 388\ 608$

Use the above values to help solve the following without the aid of a calculator.

a) $\frac{32\ 768 \times 262\ 144}{2048 \times 65\ 536}$

b) $\frac{(8192)^2\ 4096}{4\ 194\ 304}$

c) $\frac{512^3 \times 2048^2}{4096^3}$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers*

vi) *select and use appropriate computational techniques in given situations, and justify the choice*

SCO: By the end of grade 9, students will be expected to

B5 **model, solve, and create problems involving numbers expressed in scientific notation**

Elaboration – Instructional Strategies/Suggestions

B5 There are many applications of exponents which involve very large or very small numbers. These include distances from the Earth to other planets or to the moon or sun, representation of large dollar amounts, such as the estimated worth of Bill Gates, the size of the national debt, the mass of an electron, and various atomic masses. Specifically, for example, the mass of an electron is 9.2×10^{-28} g, and the national debt of Canada is about 2.3×10^{11} dollars. Students should be exposed to measurement of large quantities, using units such as the kilolitre 10^3, megalitre 10^6, gigalitre 10^9, teralitre 10^{12}, and small quantities, using units such as the millilitre 10^{-3}, microlitre 10^{-6}, and nanolitre 10^{-9}. Recalling such units from memory should not be the focus, although students generally like to be able to attach a specific name to very large and very small quantities.

Students should be exposed to problems involving addition, subtraction, multiplication, and division with numbers in scientific notation. Through solving problems involving the four basic operations, students will see how the use of scientific notation is helpful in solving multiplication and division problems, but not always as helpful in addition and subtraction situations. All of the laws/rules for exponents should be explored in the context of numbers written in scientific notation. They should explore and develop a process/convention to apply to addition or subtraction of numbers in scientific notation. This convention might be as follows:

Two numbers written in scientific notation can be added or subtracted only when both numbers are written to the same power of ten. Therefore,
$2.3 \times 10^{-3} + 4.7 \times 10^{-3}$ can be added as follows: $(2.3 + 4.7) \times 10^{-3}$; whereas
$2.3 \times 10^{-3} + 2.30 \times 10^{-2}$ cannot be added conveniently as written. However, they can be added if the exponents of ten are made the same. For example,
$2.3 \times 10^{-3} + 2.30 \times 10^{-2}$ can be written as $2.3 \times 10^{-3} + 23.0 \times 10^{-3}$ and now they can then be added as follows:

$(2.3 + 23.0) \times 10^{-3.}$

$= 25.3 \times 10^{-3}$

$= 2.53 \times 10^{-2}$

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B5.1

a) Determine which of the following can be readily calculated when the numbers are written in their current form.

i) $2.3 \times 10^{-3} \times 4.0 \times 10^{-2}$ ii) $2.3 \times 10^{-3} \times 4.0 \times 10^{5}$

iii) $6.3 \times 10^{6} \div 3.0 \times 10^{5}$ iv) $2.53 \times 10^{6} + 3.00 \times 10^{6}$

v) $6.5 \times 10^{-2} - 3.0 \times 10^{-3}$ vi) $6.50 \times 10^{-2} + 4.02 \times 10^{2}$

b) Calculate each of the questions in part a). If the questions cannot be readily solved in their current form, modify the format of one of the numbers to make it more convenient to solve.

c) Explain why modification was necessary to the format of some of the numbers.

B5.2 The national debt for Canada is approaching $\$10^{12}$. If the estimated number of people in Canada is 3.12×10^{7}, what is each person's share of this debt?

B5.3 A computer is advertised as offering 25 gb of storage (gb refers to gigabytes). How many bytes of storage is this? When personal computers first came on the market, a memory of 180 kb was considered quite a large storage capacity. If you tried to store the same data in this early computer as compared with the advertised computer, how many such computers would be required?

B5.4 The diameter of an electron is approximately 0.000 000 000 000 56 cm.

a) How big would it look using a magnification of 120 000 000?

b) Do you think we would be able to see it? Explain.

B5.5 A leaky faucet drips at a rate of one drop every 4 seconds. One drop of water is about 0.07 ml. Suppose that in the city of Halifax there are 3000 homes that have leaky faucets.

a) Calculate the amount of water wasted in a year.

b) Write your answer in scientific notation.

Portfolio

B5.6 Ask students to determine their current ages in seconds and write their answers in scientific notation.

B5.7 Light travels at a speed of 300 000 000 m/s. Ask students

a) how far it will travel in a year

b) to write their answers in scientific notation

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers*

vi) *select and use appropriate computational techniques in given situations, and justify the choice*

SCO: By the end of grade 9, students will be expected to

B6 **determine the reasonableness of results in problem situations involving square roots, rational numbers, and numbers written in scientific notation**

Elaboration – Instructional Strategies/Suggestions

B6 Students have developed estimation skills in relation to irrational and rational numbers in grade 8. They should continue to use these skills and apply them to large numbers.

- ❑ Suppose the richest person in the world had a net worth of approximately 63 billion dollars. The national debt of the United States is approaching 10 trillion dollars. Approximately how many people of comparable wealth would have to turn over their entire wealth to completely wipe out the national debt of the United States? [Students might do the following: compare 6.3×10^{10} with 1.0×10^{13}, divide to eliminate the 10^{10} that is in common, and then compare 6.3 with 1.0×10^{3}. Students can employ a number of strategies to reach an answer, such as rounding 6.3 to 6, and conclude that the result is something greater than 100 and less than 200 and the answer is closer to 200 than to 100. This might lead to an estimate of 160.]

A review of the laws of exponents in the context of numbers written in scientific notation will be valuable. Students should explore questions such as $(2.84 \times 10^{-4}) \times (3.20 \times 10^{6}) \div (3.02 \times 10^{3})$. They should be able to compute accurately, as well as perform quick computational estimation to acquire an estimate of the answer. For the problem above, students might round 2.84, 3.2, and 3.02 all to 3 and go through the following process mentally to estimate the answer:

$$(3 \times 3 \div 3) \times (10^{-4} \times 10^{6} \div 10^{3})$$
$$= 3 \times 10^{-4+6-3}$$
$$= 3 \times 10^{-1}$$
$$= 0.3$$

- ❑ The diameter of the red blood cell is 0.000 079 mm. How many would it take to cover a distance of 1 centimetre if they were lined up end to end?

Students have done significant work with estimation of square root in grade 8. This will be continued in grade 9, particularly as it relates to right triangles.

- ❑ A rectangular deck is 8.2 m by 4.8 m. The diagonal of the deck was found to be 8.1 m. Is this a reasonable finding?

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Interview

B6.1 Ask students to solve the following mentally:

$$(4 \times 10^3) \times (4 \times 10^3) \div (4 \times 10^8)$$

B6.2 Ask students to estimate the answer to the following:

$$(3.985 \times 10^4) \times (4.087 \times 10^{-3}) \div (4 \times 10^2)$$

B6.3 In 1990, Pluto was at its closest point to Earth, a distance of 4 290 000 000 km. A space probe can travel at a speed of 25 000 km/hr. Ask students to write each number in scientific notation and then approximate to complete each of the following sentences:

a) I know the space probe will take more than ? hours to reach Pluto because . . .
b) I know the space probe will take less than ? hours to reach Pluto because . . .

B6.4 A ladder leans against a building and exactly reaches the roof. The base of the ladder is 1.5 m away from the base of the building, and the length of the ladder is 5 m. Using the measurements given, Brigette found the height of the building to be 5.2 m. Ask students if this answer is reasonable. Ask them to justify their decisions without formal calculation.

B6.5 When an umbrella is completely closed it is 0.84 m in length. Ask students if it will fit into a rectangular box with dimensions 0.8 m and 0.4 m, and to explain their answers.

Portfolio

B6.6 The world population is approximately 6 000 000 000. The total land area of the Earth is approximately 148 940 000 km^2.

a) Ask students approximately how many people are on this planet per square kilometre of land.
b) Have students use their calculators to compute the answer. Ask them if the size of the numbers presented any problems and to explain.
c) Have students convert the numbers to scientific notation and again use their calculators to compute the answer. Ask if there were there any advantages or disadvantages of using scientific notation in solving the problem and to explain.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

SCO: By the end of grade 9, students will be expected to

B7 **model, solve, and create problems involving the matrix operations of addition, subtraction, and scalar multiplication**

Elaboration – Instructional Strategies/Suggestions

B7 Often data sets will exist for very similar situations. Sometimes it is necessary to add or subtract data sets to solve problems. For example, one set of data might represent expenses for month one, and a second set of data might represent expenses for month two. In order to find the total expenses, it would be necessary to add the two sets of data. These sets of data can be displayed in matrices.

To add two matrices it is important to note that they can only be added when they have the same number of rows and columns. A matrix with 2 rows and 3 columns is described as a 2 by 3 matrix (2 × 3). This is called the **order** of the matrix. Matrices can be added when they have the same order. For example,

$$\begin{bmatrix} -2 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} -2+4 & 5+6 \\ 6+3 & 7+(-3) \end{bmatrix}$$

Simply add the corresponding entries. The answer is $\begin{bmatrix} 2 & 11 \\ 9 & 4 \end{bmatrix}$

In contrast, give students two matrices where the order of the two matrices is different and ask them to add. They should see readily why this would pose a problem. For example,

$$\begin{bmatrix} 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 5 \\ 1 & 0 & 4 \end{bmatrix}$$

- ❑ In 1997 the Red team had 4 wins, 6 losses, and 3 ties, while the Blue team had 6 wins, 4 losses, and 3 ties. In 1998 the Red team had 8 wins, 4 losses, and 1 tie, while the Blue team had 6 wins, 7 losses, and 3 ties. In 1999 the Red team had 7 wins, 4 losses, and 5 ties, and the Blue team had 6 wins, 7 losses, and 2 ties. Create three matrices to represent the data sets. Create a single matrix that combines the three data sets to show the overall performance of the Red team and the Blue team.
- ❑ Ask students to add the following mentally:

$$\begin{bmatrix} 0.5 & \frac{1}{2} \\ \frac{3}{4} & 0.3 \end{bmatrix} + \begin{bmatrix} 1.45 & \frac{3}{8} \\ 0.25 & \frac{1}{5} \end{bmatrix}$$

Note: The elaboration for B7 is continued on the next 2-page spread.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B7.1 The following data represents the first quarter financial information for two clothing stores:

	January		February		March	
	Income	Expenses	Income	Expenses	Income	Expenses
Store 1	120 00	90 000	130 000	100 000	145 000	110 000
Store 2	90 000	100 000	112 00	107 000	138 000	125 000

a) Write one matrix to represent the first quarter income and expenses for both stores.

b) Use the information in the new matrix to find the first quarter profit for each store.

c) As owner, you feel it necessary to close one store. Which one will you close and why?

d) Compare the income and expenses for January and March. How much more profit was realized in March than in January?

e) Why do you think the lowest income month was January?

B7.2 In one province of Canada the following data was collected to find the yearly cost for tuition and room and board for a post-secondary education.

Tuition	1984	1988	1992	1996
4-year university degree	1230	1650	1957	2408
3-year college public	1023	1230	1590	1950
2-year college public	768	1178	1489	1705
2-year college private	4756	5123	6835	8012

Room and Board	1984	1988	1992	1996
4-year university degree	2344	2456	2678	2789
3-year college public	2023	2130	2290	2450
2-year college public	2768	2 978	3089	3105
2-year college private	2756	3123	3123	3012

a) Write a matrix for each data set.

b) Find the total cost matrix.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *model problem situations involving rational numbers and integers*

iii) *apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers, and exponents*

SCO: By the end of grade 9, students will be expected to

B7 **model, solve, and create problems involving the matrix operations of addition, subtraction, and scalar multiplication**

Elaboration – Instructional Strategies/Suggestions

B7 (Cont'd) When working with matrices, a real number multiplier is often called a scalar. To multiply a matrix by a scalar, you simply multiply each entry in the matrix by the scalar. For example, to multiply the matrix below by the scalar 2, you simply multiply each entry in the matrix by 2 as shown:

$$2\begin{bmatrix} 2 & 11 \\ 9 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 22 \\ 18 & 8 \end{bmatrix}$$

- ❑ Mr. Turner owns two electronics companies. The data below shows the number of units of each product that can be produced in each company in one week.

	Company 1	Company 2
Amplifiers	34	25
Video cameras	14	23
Speakers	32	27

Mr. Turner decides to increase productivity by 25%. Write the matrix to represent the approximate productivity level.

It is through modelling problem situations which involve addition, subtraction, and scalar multiplication that these operations and the reason for using matrices become relevant.

Once students have solved a few problems involving matrix addition, subtraction, and scalar multiplication, they should be challenged to create problems of their own that are suited to a solution involving matrices.

- ❑ Ask students to solve the following:

$$4\begin{bmatrix} 0.4 & \frac{1}{2} \\ 0.3 & \frac{1}{3} \\ 0.5 & \frac{2}{3} \end{bmatrix} - \begin{bmatrix} 0.8 & \frac{1}{4} \\ 1.2 & \frac{2}{3} \\ 1.5 & \frac{3}{4} \end{bmatrix}$$

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B7.3 You own three factories which produce hockey equipment. Last year's production is represented in the table.

	Factory 1	Factory 2	Factory 3
Sticks	5500	4600	8900
Shin Pads	1400	2300	1800
Helmets	2300	2000	1200

a) Suppose that next year you want to double your productivity. Create a matrix to represent the current production rate and use scalar multiplication to represent the situation when productivity is doubled.

b) As an interim measure you set a target to increase your productivity by 25% in the first quarter of next year. Create a matrix to represent the production when a 25% increase is achieved.

c) If the productivity was increased by 25% in each quarter for the full year, would this be the same as doubling the productivity? Justify.

B7.4 Simplify:

$$5\begin{bmatrix} 0.9 & \frac{5}{6} \\ 1.6 & 2.4 \end{bmatrix} + \begin{bmatrix} 4.7 & \frac{5}{6} \\ 2.7 & \frac{1}{2} \end{bmatrix}$$

Portfolio

B7.5 Belinda usually does a mid-season and an end of season report of the statistics on a softball league. The season is 20 games long.

	First Ten Games		Last Ten Games	
Team	Wins	Losses	Wins	Losses
Bluebells	7	3	6	4
Daisies	8	2	7	3
Tulips	5	5	4	6
Sweet Grass	3	7	4	6
Wild Flowers	2	8	4	6

a) Have students record the data in a matrix to reflect the combined data.

b) Ask students what team would qualify for first place standing in the league.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 9, students will be expected to

B8 add and subtract polynomial expressions symbolically to solve problems

Elaboration – Instructional Strategies/Suggestions

B8 Students have worked with addition and subtraction of terms concretely, pictorially, and symbolically and have explored the difference between addition and subtraction of polynomials, using concrete materials and diagrams in grade 8. In grade 9, work with concrete and pictorial representation is continued, but students are expected to reach the symbolic level with respect to the addition and subtraction of polynomial expressions. This topic can be developed concretely using base 10 blocks or algebra tiles.

It is important for students to view subtraction in a variety of ways.

- *Comparison* simply refers to comparing the quantities, and the difference between them is the answer.
- *Taking away* simply refers to starting with a quantity and removing or taking away a specified amount to arrive at an answer. For example, $(x^2 + 2x - 2) - (x^2 + x + 1)$

  if we start with  and add zero represented by ■ □, then

  take away  we are left with 

- *Adding the opposite* refers to approaching subtraction by first changing the question to an addition question and adding the opposite of a quantity. For example, instead of subtracting x, one might add -x.
- *Missing addends*, ask the question, What could be added to the number being subtracted to get to the first value? In 3p - (-p) we ask the question, What would you add to -p to get 3p?

All four of these meanings for subtraction should have been developed in previous grades. Perimeter is a very useful application of addition and subtraction of polynomials.

Magic-number problems such as the one below and calendar problems (B8.5) are both fun ways to practise the addition and subtraction of polynomials.

❑ Start with any number, n, add 6, subtract 2n, add 10, subtract -n + 10. What will be the final answer? Students can practise this problem by choosing specific values for n and realizing that the answer is always 6. They can then prove algebraically that it is always 6, as follows

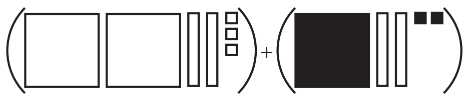
$$\begin{aligned} &= n + 6 - 2n + 10 - (-n + 10) \\ &= -n + 16 + n - 10 \\ &= 6 \end{aligned}$$

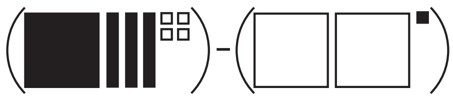
GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

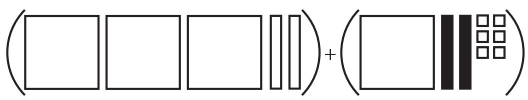
Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B8.1 Write an algebraic expression for each of the following and simplify (shaded areas represent positive):

a) 

b) 

c) 

B8.2 Simplify:

a) $(2x^2 - 5x) - (-3x^2 + 2x)$

b) $(3y^2 - 2xy) + (y^2 + 4xy)$

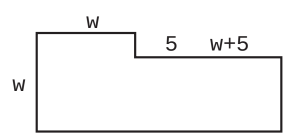
B8.3 List 3 different pairs of polynomials that might be

a) added to give $3w^2 - 5w + 4$

b) subtracted to give $3w^2 - 5w + 4$

Portfolio

B8.4 Have students use the diagram to answer the questions below:



a) Find a polynomial expression that represents the perimeter.

b) If w = 8, find the perimeter using two forms of the polynomial expression. Which calculation was easier? Why?

B8.5 The following array represents a calendar for September:

	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

Have students note that when any two-by-two array is selected from this calendar the sum of the diagonal numbers is always the same. For example,

12 13 12 + 20 = 19 + 13

19 20

a) Ask students, if we let x equal the first number in the two-by-two array, how the other numbers would be represented.

b) Have them express the sum of the diagonal numbers algebraically.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 9, students will be expected to

B9 **factor algebraic expressions with common monomial factors, concretely, pictorially, and symbolically**

Elaboration – Instructional Strategies/Suggestions

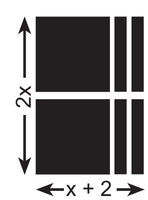
B9 Factoring should start by presenting students with a collection of algebra tiles such as the tiles associated with the expression $4x + 8$ or $2x^2 + 4x$. Students should spend some time reflecting on whether it is possible to create a rectangle from the given materials. They should create rectangles from the tiles and record the dimensions, always considering if more than one rectangle is possible. They should repeat this process several times with several trinomials until they are used to how to place the tiles. Discussion should lead to the fact that the dimensions are the factors. This can be related back to the modelling of multiplication in the elementary grades. If more than one rectangle is possible, there is more than one way to write the pair of factors; however, only one of these represents removing the greatest common factor (GCF).

A review of GCF may be necessary. Emphasis should be placed on factoring and multiplying as reverse operations. Outcomes B9, B11, and B12 should be addressed together, since multiplication and factoring are closely related, as are factoring and division.

Since the elementary grades students have also used the sharing model for division. This can also have some application for factoring when the common factor is numerical. For example, consider the expression $3x + 12$, which can be modelled as

These can be organized into a rectangle as shown. This can be easily seen as three rows with the same amount in each row. The number of rows represents one factor, and the amount in each row represents the other factor. There are 3 rows with →$x + 4$ in each row. The factors are 3 and $x + 4$, and the product of $3(x + 4)$ can be expressed as $3x + 12$.

It should be noted that this method only works well when the greatest common factor is numerical. When the common factor is literal, the focus should be on creating a rectangle with tiles, and recording the dimensions from reading the tile measures along the two dimensions of the rectangle. For example,



While significant work is expected at the concrete and pictorial level, it is expected that students will be able to work symbolically at the end of the process. Students will begin to wean themselves as they begin to see the patterns in the symbols and are able to visualize the tiles without actually building rectangles with them. It is important for teachers to provide a logical sequence of questions for students so that the patterns become obvious. As students begin to see the patterns they should be encouraged to share their findings with others. In the end, teachers should be hearing from students the procedures that teachers have in the past told the students. The weaning process will be slow, and some may continue to use the tiles.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B9.1 Use an area model to factor each of the following:
a) $3x + 3$ b) $4x + 8$ c) $5r - 10$ d) $2p - 2$

B9.2 Use the sharing model for division to factor B9.1.

B9.3 Use the area model to factor each of the following:
a) $a^2 + 3a$ b) $2b^2 + 4b$

B9.4 Explain why sharing is not a good model to use to help factor in B9.3.

B9.5 Factor each of the following symbolically:
a) $3a^2 + 12a$ b) $6a^2 - 12a$ c) $4xy^2 - 8x^2y$

Performance

B9.6 The area of a rectangle is $4p^2 - 12p$.

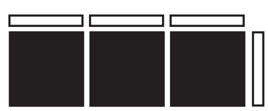
a) Have students create all possible rectangles which have the given area and write the dimensions of each rectangle.
b) Ask students which rectangle represents the expression when the GCF has been removed.
c) Ask students what the GCF of $4p^2$ and $12p$ is.

B9.7 Have students create rectangles, using the materials shown, record the dimensions and the area of each rectangle, and write a mathematical sentence involving the dimensions and the area for each rectangle.

a)


b)


c)


d)


Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 9, students will be expected to

B10 **recognize that the dimensions of a rectangular area model of a polynomial are its factors**

B11 **find products of two monomials, a monomial and a polynomial, and two binomials, concretely, pictorially, and symbolically**

Elaboration – Instructional Strategies/Suggestions

B10/11 Students should first study multiplication of a monomial by a monomial, extend to multiplication of a scalar by a polynomial, then move to multiplication of any monomial by a polynomial, and finally to multiplication of a binomial by a binomial. Only multiplication of binomials is new at this grade level. One method of illustrating multiplication of polynomials is through the use of area models. Base 10 blocks or algebra tiles are very helpful. It would be useful to review the distributive property as an introduction to this topic. This can be done particularly well using algebra tiles. Applications should be emphasized, especially in relation to area problems.

Start by giving students a collection of algebra tiles representing an expression such as $x^2 + 8x + 12$ and ask them to create a rectangle. When they create the rectangle, they should spend some time reflecting on whether it is the only possible rectangle for the given materials. Students should record the dimensions of the rectangle. They should repeat this process several times with several trinomials until they are familiar with how to place the tiles.

❑ Give students a collection such as the one shown.

a) Ask them to create a rectangle.

b) Ask students to write an expression to represent the collection of tiles and then an expression for each dimension.

c) Ask them to record symbolically an equation which links the product of dimensions to area.

d) Ask them to explain how each component of the product relates back to the factors and to try to tie this to what they know about the distributive property.

Answer to a):
Rectangle:

Note: The elaboration for B10/11 is continued on the next 2-page spread.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance/Pencil and Paper

B10.1 The area of a rectangle is $x^2 + 6x + 8$

a) Find the possible dimensions of the rectangle.
b) Are there other rectangles which can be formed that have the same area?
c) For the rectangle(s) that you have formed, find the perimeter.
d) Can you find other rectangles which have the same perimeter? Explain.
e) Repeat a) - d) for a rectangle of area $x^2 - 6x + 8$.

B10.2 Explain why a rectangle cannot be formed with an area of

$$x^2 + 6x + 6 \qquad x^2 + 3x + 1$$

a) Use tiles to help you create two other trinomials so that no rectangle is possible.
b) What can you conclude about these trinomials?

B10.3 The product of two factors of a trinomial is formed using 10 algebra tiles. Model and record symbolically as many possibilities as you can find which fit this criterion.

Portfolio

B10.4 Have students examine the pattern of the factors for each of the following:

$$x^2 + 2x + 1,\ x^2 + 4x + 4,\ x^2 + 6x + 9,\ x^2 - 8x + 16$$

Have them

a) explain the pattern that they observe
b) create at least two other polynomials whose factors will be consistent with this pattern [e.g., $x^2 + 8x + 16$, $x^2 + 10x + 25$, $x^2 - 10x + 25$]
c) think of a concise way of writing the factors

B10.5 Ask students to examine the pattern of the factors for each of the following:

$$x^2 + 3x + 2,\ x^2 + 4x + 3,\ x^2 + 5x + 4$$

Have them

a) explain the pattern that they observe
b) create at least two other polynomials whose factors will be consistent with this pattern

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

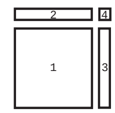
SCO: By the end of grade 9, students will be expected to

B10 recognize that the dimensions of a rectangular area model of a polynomial are its factors

B11 find products of two monomials, a monomial and a polynomial, and two binomials, concretely, pictorially, and symbolically

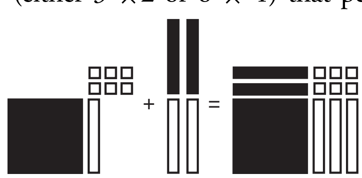
Elaboration – Instructional Strategies/Suggestions

B10/11 (Cont'd) It is important to model rectangles using a consistent format. The format recommended is to build from the lower left-hand corner and to use the larger pieces working outward from that corner. Teachers should be careful to always model the proper use and placement of tiles when building rectangles so that students become comfortable with the four general regions. This will prevent students from trying to use a series of unit tiles to fill a space that could be occupied by an x-tile, or a series of x-tiles to fill a space that could have been filled by an x^2-tile. Region 1: x^2-tiles, Region 2: horizontal x-tiles, Region 3: vertical x-tiles, Region 4: unit tiles. For example, a rectangle for $x^2 + 3x + 2$ is built as



B11 It will be necessary to give students the dimensions for a rectangle and ask them to fill in the area and record the value of that area. Since students should already know that l x w = A, they can record their findings as a product and begin establishing the product and factors relationship. By comparing the symbols representing dimensions with those representing area, using several examples of both cases, and recording their observations, students should establish a pattern. It is through clarifying and applying this pattern that students are empowered to move confidently to the symbolic level.

All initial work should be done using positive signs. Gradually ease students into working with negative signs, first with middle term negative, then with third term negative, and finally with both terms negative. It is important to remember that with trinomials, such as $x^2 - x - 6$, the zero principle applies, since more than one negative rod will be required. That is, given the collection shown, in order to create a rectangle, additional pieces are necessary. Arrange the pieces as shown and then add the additional pieces, applying the zero principle, to form the complete rectangle. Students will experiment with the unit tiles trying to form a rectangle (either 3×2 or 6×1) that permits the zero principle to be used.



For $x^2 - x - 6$, 2x and - 2x must be added to complete the rectangle. The dimensions of the rectangle are x + 2 and x - 3.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

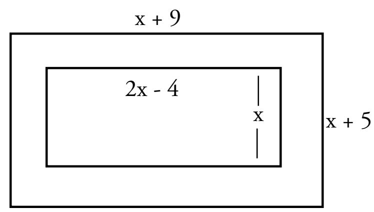
Pencil and Paper

B11.1 Draw a rectangle to show the area represented by each product:

a) length 2x, width 3x
b) length 2x + 1, width 4
c) length 4x + 2, width 2x +1
d) length x + 3, width 2x - 1
e) Record each product for parts a) through d), using symbols.

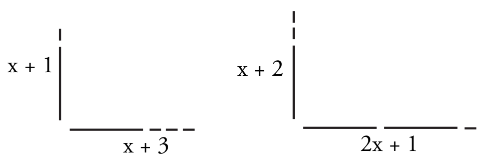
B11.2 Write an expression that can be used to find the area of the matting around the picture.

a) Simplify the expression.
b) Evaluate the expression when x = 5.



Performance

B11.3 Have students create the product rectangle with these dimensions, using algebra tiles, and record the factors and the product symbolically.



B11.4 In the first diagram, have students create a rectangle for the dimensions shown, and in the second, have them record the dimensions for the given area. Ask them to compare and discuss the results.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 9, students will be expected to

B12 **find quotients of polynomials with monomial divisors**

B13 **evaluate polynomial expressions**

Elaboration – Instructional Strategies/Suggestions

B12 The study of division should begin with division of a monomial by a monomial, progress to a polynomial by a scalar, and then to division of a polynomial by a monomial. Problems with remainders are to be avoided. Students should be given situations where they have a specific collection of tiles and asked to create a rectangle with one dimension given. A parallel should be drawn between division and factoring. Essentially, in factoring students are expected to find both of the factors, whereas in division one of the factors (dimensions) is given, and students are expected to find the other factor (dimension).

- ❑ Create a rectangle, using four x^2 tiles and eight x tiles where 4x is one of the dimensions, and ask students to find the other dimension.

The most commonly used symbolic method of dividing a polynomial by a monomial at this level is to break the polynomial apart and solve individual monomial division problems, for example, $\frac{3x + 12}{3} = \frac{3x}{3} + \frac{12}{3}$. This method can easily be modelled using tiles where students use the sharing model for division. They start with a collection of three x-tiles and 12 unit tiles and divide them into three groups. The quotient is the number of tiles in each group. For this problem x + 4 tiles will be a part of each group so the quotient is x + 4.

B13 Students should be exposed to the evaluation of polynomials before and after they have been simplified so that the advantage of simplifying prior to evaluating can be emphasized. It is through such comparisons that students can see a need for simplifying. This outcome occurs at grade 7 and grade 8 as well. Each time students develop new algebraic expressions, equations, or computational techniques involving algebra, it is necessary to evaluate them for given or selected values of the variables.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Performance

B12.1 Have students

a) form $x^2 + 5x$ using algebra tiles
b) create a rectangle where x is one of the dimensions
c) identify the other dimension
c) write a division sentence for the situation

Pencil and Paper

B12/13.1

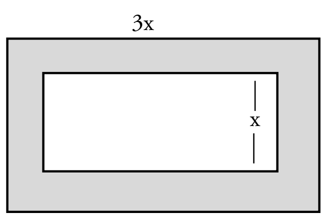
a) Evaluate the expression $(4y^3 - 12y^2 + 8y) \div 4y$ for y = -3.
b) Perform the division and evaluate the simplified expression for y = -3.
c) Compare the two results. What do you notice?
d) Was it easier to evaluate the expression before or after the division? Explain.

B12/13.2 Simplify the expression: $\frac{2x^2 + 6x}{2x}$

a) Find the value of the expression for x = 6 by replacing x in the original expression.
b) Find the value of the expression for x = 6 by replacing x in the simplified expression.
c) Compare the two results. What do you notice?

Portfolio

B12.2 The inside rectangle in the diagram below is a flower garden. The shaded area is a concrete walkway around it. The area of the flower garden is given by the expression $2x^2 + 4x$, and the area of the large rectangle, including the walkway and the flower garden, is $3x^2 + 6x$.



a) Have students use the information provided to find an expression for each of the missing dimensions of each rectangle.
b) Ask them to find the dimensions and area of the flower garden, if x = 2.3m.
c) Ask students to find the area of the walkway.

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *explore and explain, using physical models, the connections between arithmetic and algebraic operations*

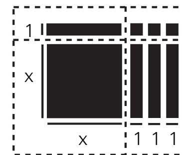
iv) *apply operations to algebraic expressions to represent and solve relevant problems*

SCO: By the end of grade 9, students will be expected to

B14 demonstrate an understanding of the applicability of commutative, associative, distributive, identity, and inverse properties to operations involving algebraic expressions

Elaboration – Instructional Strategies/Suggestions

B14 The properties should not be addressed in isolation. They should be addressed in the context of simplifying expressions and solving equations. The distributive property is of particular importance when working with multiplication and division of polynomials. Students should connect the multiplication of a monomial or binomial by a polynomial to the distributive property and to the area model. For example, when multiplying (x + 1) (x + 3), students should see from the concrete model that the bottom row of the 2 × 2 grid shown represents x (x + 3), and the top row represents 1(x + 3).



The parallels between the use of these properties with numbers and with algebra should be pointed out. For example,

$6(5 + 4) = 6 \times 5 + 6 \times 4$ and $2x(x + 3) = 2x \times x + 2x \times 3$

$5 \times 0 = 0$ and $-3y \times 0 = 0$

$\frac{1}{5} \times 5 = 1$ and $\frac{1}{-4x} \times -4x = 1$

The use of the inverse and identity properties as part of solving equations should be highlighted. For example,

$3x + 4 = -5$	
$3x + 4 + (-4) = -5 + (-4)$	Ask, Why do we add -4 to both sides?
$3x + 0 = -9$	Ask, Why is 4 + (-4) = 0?
$3x = -9$	Ask, Why is 3x + 0 = 3x?
$\frac{1}{3} \times 3x = \frac{1}{3} \times -9$	
$1x = -3$	Ask, Why is $\frac{1}{3} \times 3 = 1$?
$x = -3$	Ask, If 1x = -3, why can we say that therefore x = -3?

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

B14.1 Explain how the distributive property is used in multiplying (x + 5)(x + 6).

Interview

B14.2 Have students

a) find the product: 2(y + 3)(y - 6) × 0

b) explain why this problem can be answered quickly

B14.2 Ask students to analyse the solution to the given equation and identify the properties that have been used. [Steps are shown in more detail than is normally the case in a solution in order to highlight the properties which are utilized.]

$$\begin{aligned} -2x + 7 &= 3x + 22 \\ -2x + 7 + (-7) &= 3x + 22 + (-7) \\ -2x + 0 &= 3x + 15 \\ -2x &= 3x + 15 \\ -2x + -3x &= 3x + (-3x) + 15 \\ -5x &= 0 + 15 \\ -5x &= 15 \\ \frac{1}{-5}(-5x) &= \frac{1}{-5}(15) \\ x &= -3 \end{aligned}$$

Suggested Resources

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

vi) *select and use appropriate computational techniques in given situations, and justify the choice*

SCO: By the end of grade 9, students will be expected to

B15 select and use appropriate strategies in problem situations

Elaboration – Instructional Strategies/Suggestions

B15 As with numerical situations, algebraic situations can also be addressed using a variety of strategies. For example, consider the following problems.

- ❒ Find the area of a rectangle with a width of x + 2 and a length of 2x + 3. [A number of strategies can be employed. Some students will solve this problem, using algebra tiles. Others will recognize and apply the distributive property, while others will be able to multiply the two expressions mentally to achieve an accurate answer.]
- ❒ A baseball team won 25 more games than it lost. It played 109 games for the season. How many games did the team win?

Students must make a number of decisions concerning this problem. First of all, is an estimate sufficient? The wording of the question indicates that an exact answer is expected. Otherwise, the question would probably say, About or approximately how many games were won? Once the decision is made that an exact answer is needed, the student will choose an appropriate method. Some students may solve this problem mentally, while others may be able to solve it using patterns; some will see it as a numerical exercise; while others would apply algebra to the situation. Even when the decision is made to use an algebraic equation, students will select to show varying degrees of detail in their algebraic solutions. Often it is the intent of the question that a particular strategy be employed. For example, in grade 9, because there is a relatively heavy focus on developing algebraic solutions, it is sometimes necessary to specify a particular procedure in order to ensure it is practised. When this is the case, the instructions should be clear as to the expectations. Otherwise, it is inappropriate to penalize students for a solution that has an acceptable strategy and that leads to a correct answer.

Procedures and algorithms that are linked to problem types should be avoided. Traditionally, instruction has categorized problems into types such as the coin problem, the money problem, and the age number problem. Too much focus in this direction turns a problem-solving opportunity into a routine procedure.

GCO (B): Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

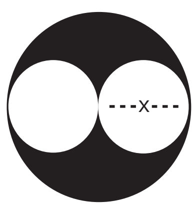
Worthwhile Tasks for Instruction and/or Assessment

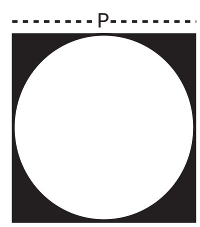
Pencil and Paper

B15.1 John is building a fence to enclose a portion of his backyard. He wishes to fence only three sides because he intends to use the back of the house as the fourth side. He is using 24 m of fencing because he feels that is all he can afford. The portion of the yard enclosed will be rectangular.

a) If w represents the width, write an expression to represent the length.
b) Identify all the possible rectangles that can be formed and use this information to help him find the one with the maximum area.
c) Graph the width and the area of all the possible rectangles and discuss the pattern that is observed.
d) Discuss the nature of the graph and comment on the maximum area in relation to the graph.

B15.2 Write an expression for the perimeter and area of the shaded region for each.





a) Find the area of the shaded region if the value of x = 4 cm.
b) Find the area of the shaded region if the value of P = 10 cm.

B15.3 A piece of lumber is to be cut to make a model so that the second piece is twice the first and the third piece is 2 cm less then twice the second. The piece of lumber is 2 m long. What is the length of each piece to the nearest centimetre?

Suggested Resources

NINE - PATTERNS

Patterns and Relations

General Curriculum Outcome C:

Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry- grade 6 and will also be expected to

i) *analyse, generalize, and create patterns and relationships to model and solve real-world and mathematical problem situations*

SCO: By the end of grade 9, students will be expected to

C1 represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

C2 interpret graphs that represent linear and non-linear data

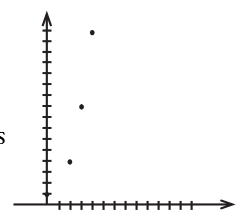
Elaboration – Instructional Strategies/Suggestions

C1 In grade 8, students described linear patterns algebraically, using mathematical expressions and/or equations. At grade 9, students will do the same, but situations will extend beyond linear to include exponential and parabolic curves. To clarify the difference between relationships that produce exponential and parabolic curves, see the elaboration on p. 50 and p. 52. Graphing calculators or graphing software should be utilized, where possible, for the purpose of exploring and extending patterns graphically.

Students should be able to move interchangeably among the various representations that describe relationships. They should describe in words, and use expressions and equations to represent patterns given in tables, graphs, charts, pictures, and/or by problem situations. Information presented in a variety of formats should be used to derive mathematical expressions and to predict unknown values. For example, students might be given information in a table such as

a	0	2	3	4	5
b	1	4	9	16	25

or a graph such as



and be asked to describe in words, or by using an expression or equation, the pattern they see. The table shown is represented by the equation $b = a^2$, and the graph shown is represented by the equation $y = 2^x$. A description for a pattern may include the use of tiles, cubes, or pictures to model what is observed. Once an algebraic description of a pattern or a graph is established, this can be used to predict unknown values. When students observe the pattern, they can use it or its equation to determine the value of y for given *x* values. For example, they can find the value of y when x is 10 or when x is 102 in the equation $y = 2^x$.

C2 In grade 8, students did a significant amount of work describing and graphing non-linear situations, using broken-line graphs. In grade 9, the focus will be on patterns that can be described algebraically and represented by smooth curves. However, at times, students may address relationships that they are able to represent graphically, but they may have to wait until later grades to describe the relationship algebraically. The terms exponential and parabolic should be used at grade 9. Such shapes were included in grade 7 as well, but no attempt was made to attach formal names to these graphs.

- ❑ After folding a piece of paper, graph the number of folds against the number of regions. Determine the area of each region after each fold, and determine what fraction of the original area the new areas represent. Graph the number of folds against the area of each region, and also against the fraction of the original area each region represents. Is there any pattern shown in the data?

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

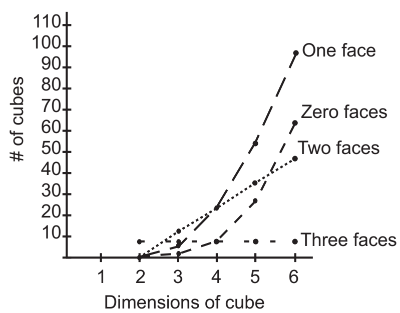
C1/2.1 Build various cubes from unit cubes, such as a 2 by 2 by 2, a 3 by 3 by 3, ... Imagine that you are to paint the exterior of the large cube and then take it apart.

a) Fill in the table with the required information.

Dimensions of the large cube	# of cubes with…			
	3 faces painted	2 faces painted	1 face painted	0 faces painted

b) Graph the dimension of the large cube against each number of faces painted, putting all 4 graphs on the same coordinate plane. Discuss the shape of the graphs.

Answer:



c) Write a general expression, in words or symbolically, for the pattern shown in the table to illustrate the relationship between the dimensions of the cubes and the number of faces with paint on them. Note: It may be necessary to make a larger table to see the pattern more clearly.

Portfolio

C1/2.2 Ask students to make a table of all the possible whole number dimensions for a rectangular garden utilizing exactly 48 metres of fencing. Ask them to create a graph to show how change in width is related to the area of the garden. Ask students to write about the minimum and maximum area, and how they are read from the graph. Have students repeat, using 40 metres of fencing, and compare the graphs.

Extension

C1/2.3 Clayton is riding a ferris wheel at the fair. Ask students to make a graph of Clayton's height above the ground over time for a wheel of radius 7m, and where one rotation takes place every 15 seconds. Discuss with students how the graph would be different if the speed of rotation or the radius were increased. Ask what assumption(s) they made. [Accuracy is not important for this graph. Only the general shape of the graph would be expected.]

Suggested Resources

Addenda Series - Grades 5-8, "Patterns and Functions"

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *analyse, generalize, and create patterns and relationships to model and solve real-world and mathematical problem situations*

ii) *analyse functional relationships to explain how the change in one quantity results in a change in another*

SCO: By the end of grade 9, students will be expected to

C3 **construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity**

Elaboration – Instructional Strategies/Suggestions

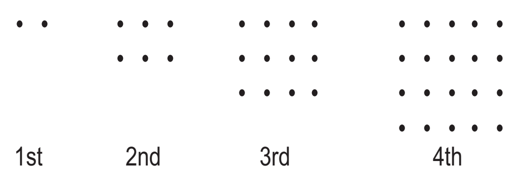
C3 Students have already worked with rate of change in grade 8. They will revisit this notion in grade 9 and extend it to look at issues such as patterns which lead to a constant rate of change (a linear graph), and patterns which lead to other shapes.

❐ Debbie was riding her bicycle when a dog ran out in front of her, causing her to brake sharply. The speed v at different times t after she began to brake is shown in the table.

Time, t(s)	0	0.1	0.2	0.3	0.4	0.5
Speed, v($\frac{m}{s}$)	10	8	6	4	2	0

a) Describe the relation between speed and time.
b) Draw a graph of the relationship between v and t.
c) Write an equation for the relation. [This part requires knowledge of C4.]
d) As time is measured at a constant interval, what do you notice about the differences in speed? [Discussion with students should bring out the relationship between constant difference and linear relationships, and how this relates to slope.]

❐ The dot patterns below are arranged as rectangles.



a) What are the first four rectangular numbers and what are the dimensions of these rectangles? [2, 6, 12, 20], [1 × 2, 2 × 3, 3 × 4, 4 × 5]
b) Find the next two rectangular numbers and the dimensions.
c) Graph the relationship.
d) Comment on the change (difference) from one number to the next in the number pattern.
e) Look at the pattern of the differences in the numbers and discuss their observations. [They should notice a pattern in the differences.]
f) Write an expression for the nth term. [Students may need to make a table for this such as the one shown.]

diagram #	1	2	3	4	5	n
# of dots	1 × 2	2 × 3	3 × 4	4 × 5	5 × 6	n × (n + 1)

It may be useful to introduce the terms dependent and independent variables.

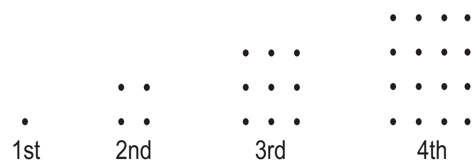
Note: The elaboration for C3 is continued on the next 2-page spread.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C3.1 The number pattern below represents the first four square numbers.



a) What are the next four square numbers?

b) Make a table and a graph to observe the pattern. How can the pattern be used to determine whether the relationship is linear?
[Students might say that it is not linear because there is no constant change apparent in the difference. If they have studied exponential relations, they should also look for a common ratio.]

c) Write an expression for the nth term, using the patterns observed in the table.

C3.2 Determine whether the tables below represent graphs that are linear, parabolic, or exponential, and justify the choice.

a)

x	0	1	2	3	4	5	6	7	8
y	2	3	6	11	18	27	38	51	66

b)

x	0	1	2	3	4	5	6	7	8
y	4	8	12	16	20	24	28	32	36

c)

x	0	1	2	3	4	5	6
y	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	$\frac{27}{2}$	$\frac{81}{2}$	$\frac{243}{2}$	$\frac{729}{2}$

C3.3 Using the following equations, make a table of values. Analyse the table of values to determine whether the equation represents graphs that are linear, parabolic, or exponential. [Note: If students have difficulty making decisions based on the pattern in the table, they should be encouraged to construct the graph to help. When graphing calculators are available, they can be useful here.]

a) $y = 4x - 3$

b) $y = 3x^2$

c) $y = 7x - 4$

d) $y = 3^x$

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent patterns and relationships in multiple ways (including the use of algebraic expressions, equations, inequalities, and exponents)*

SCO: By the end of grade 9, students will be expected to

C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

Elaboration – Instructional Strategies/Suggestions

C3 (Cont'd) In observing these two patterns, students should realize that when a relationship is linear, the y-values (dependent variables) increase by a constant amount for a constant increase in x. When the relationship is not linear, students may observe that there is still a pattern to the differences. For example,

2 6 12 20 30 42 56 number pattern

4 6 8 10 12 14 differences

[Students may observe that even though the difference is not a constant, there is a constant increase in the differences. This indicates a parabolic curve, which will be described in later grades as a quadratic relationship.]

❑ Fido did not have fleas when he went to the dog pound. However, the number of fleas he had in subsequent days grew as shown in the table, where n represents the number of days and f the number of fleas.

n	1	2	3	4	5
f	1	3	9	27	81

a) How many fleas did Fido have on the seventh day?
b) Is there a common difference in the values of the dependent variable?
c) Is there a constant increase in the difference values?
d) Would it be meaningful to look at the number of fleas on the tenth day? [Discuss the danger of overextrapolation.] [Students should observe that there is no common difference and there is no constant increase in the differences.]

1 3 9 27 81

2 6 18 54

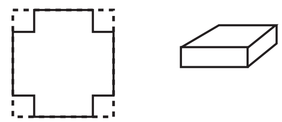
At this point, students should explore to find other relationships between the numbers. If no suggestion is forthcoming, direct the students thinking and ask them to look at the ratios of the values of the dependent variable in the table. If the pattern is exponential, the ratio will be the same for all pairs of consecutive values of the dependent variable. It is important to note that these patterns are only easily observable if consecutive values of the independent variable are selected. For the table above, $81 \div 27 = 3$ $27 \div 9 = 3$ $9 \div 3 = 3$. This indicates an exponential relationship.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Portfolio

C3. 4 Give students a square piece of grid paper (it should be a fairly large grid, about $\frac{1}{2}$ or 1 cm blocks). Ask them to cut a square of 1 unit per side from each corner and fold up the edges to make an open box.



Another group should cut a square of 2 units per side from each corner, and likewise for squares of 3 units per side, and 4 units per side, until the limit of the size of the square cut from each corner has been reached.

a) Ask students to look at all the boxes they have produced and predict which has the greatest volume and which the least volume.
b) Ask students to fill in the information in the table below:

Length of side of cut-out square	Dimensions of the box	Volume (in cubic units)

c) Ask students to graph the length of the side of the cut-out square on the x axis and the volume of the box in cubic units on the y axis, and discuss and describe the shape of the graph.
d) Suppose the length of the side of the cut-out square is x units in length. Ask students to fill in a row of the table with the rest of the appropriate information. Ask them to write a general expression for the volume of the box. Have them use graphing technology to graph the relationship and compare it to the one produced earlier.

Extension/Enrichment

e) Once students have worked on the multiplication of polynomials, which is also part of this course, they should be able to multiply length by width by height to produce an expression which they can be told is a cubic. Ask students why they think it is called a cubic. Some students may want to create other cubics and graph them, using graphing technology. From this they may be able to write a conclusion about the graphs of cubics. [Students might say that graphs of cubics appear to change direction twice, or that they have two bumps.]

N.C.T.M. Addenda Series, "Patterns and Functions," Experiment #2, page 64.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *represent patterns and relationships in multiple ways (including the use of algebraic expressions, equations, inequalities, and exponents)*

SCO: By the end of grade 9, students will be expected to

C4 **determine the equations of lines by obtaining their slopes and y-intercepts from graphs, and sketch graphs of equations using y-intercepts and slopes**

Elaboration – Instructional Strategies/Suggestions

C4 Students worked with slope in grade 8; however, it was addressed less formally. In grade 8, the outcome related to slope is as follows: *Explore change by comparing vertical change and horizontal change, and link visual characteristics of slope with its numerical value.* At grade 9, slope is referred to as slope = $\frac{\text{rise}}{\text{run}}$. However, it is still important to make the link between the terminology rise and vertical change, and run and horizontal change. The rise and run should still be determined from the graph. It should be noted that students do not work with the notation $\frac{\Delta y}{\Delta x}$. Students should be able to determine the equation of any linear relationship by extracting the slope and the y-intercept from a graph.

❑ Graph each of the equations $y = \frac{2}{3}x - 1$ and $y = -2x + 3$ by using a table of values.
  a) Find the slope of each of the graphs.
  b) Find the y-intercept for each graph.
  c) Compare the slopes and y-intercepts with the original equations and explain what you notice.

The discussion of y-intercept will be new at this grade level. Students should know that the y-intercept refers to the place where the graph intersects the y-axis. They should make the connection that it is a place where the x-coordinate of a point is equal to zero. Students should be able to find the y-intercept readily from the equation by realizing that in an equation such as $y = -2x + 6$, when $x = 0$, then $y = -2(0) + 6$, $\therefore$ $y = 6$. Thus the ordered pair (0, 6) represents the coordinates of the y-intercept. They should also recognize the slope-intercept form of an equation and be able to sketch the graph from the equation. Students can also explore the x-intercept and how it can be determined from the equation. Once students realize that the x-intercept is the place where $y = 0$, they can put $y = 0$ into the equation to determine the x-intercept as well.

For $y = -2x + 6$

$0 = -2x + 6$

$2x = 6$

$x = 3$, $\therefore$ the x-intercept is (3, 0)

Students can also explore the equation of horizontal and vertical lines and the equations which describe them.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C3/4.1 Let s represent the number of sides of a figure.

a) Draw a series of diagrams, beginning with a triangle and increasing the number of sides by 1 for each figure up to 10 sides.
b) Draw all possible diagonals from one vertex in each figure.
c) Draw all possible diagonals which can be drawn in the figure.
d) Make tables with the following headings and record the data:

# of sides	diagonals from one vertex	total # of diagonals

e) Graph the data from columns 1 and 2, and from columns 1 and 3.
f) Write a general expression for the number of diagonals from one vertex and the number of diagonals in total based on n sides [diagonals from one vertex, n - 3, diagonals in total, $n(n - 3) \div 2$].
g) Determine the slope and y-intercept for the linear relationship.
h) Consider the common difference of the linear relationship and compare it with the slope of the line. What do you notice?

Interview

C4.1 Ask students if it is possible to construct more than one line with a slope of -4. Ask them to explain why or why not, using diagrams to aid in their explanation.

C4.2 Ask students what it means when the slope of a line is equal to zero. Ask them what the line would look like. Ask what they think the slope would be for a line that is vertical. [Sample answer: Students might say, since there is no horizontal change, the horizontal change equals 0, and you cannot divide by 0.]

Portfolio

C4.3 Ask students to graph each relation from a table of values, and to find the slope and y-intercept for each graph. Ask them to compare the slope and y-intercept for each graph with the original equation. Ask what they notice.

$y = \frac{1}{2}x - 5$	$y = -3$	$y = -2x + 1$
$y - 3 = -4x$	$x = 4$	$2y = -3x - 4$

[Equations in non-standard form may need to come after some equation work.]

C4.4 The base of a mountain is at (-2, 1). The summit is at (-6, 13). Ask students,

a) if a mountain climber is at (-4, 7), what the distance to the summit is
b) what distance the climber had already covered when she reached (-4, 7)
c) to find the slope of the climb from the base of the mountain to (-4, 7) and use it to find the equation of the line
d) to find the slope of the climb from (-4, 7) to the summit and use it to find the equation of the line

[It is expected that students will graph the points to help them write the equation instead of using purely algebraic methods.]

Suggested Resources

NCTM Addenda Series

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iv) explain the connections among algebraic and non- algebraic representations of patterns and relationships

SCO: By the end of grade 9, students will be expected to

C5 explain the connections among different representations of patterns and relationships

Elaboration – Instructional Strategies/Suggestions

C5 One of the goals of algebra is to recognize patterns and relationships in real life. Students need to use models such as tables, graphs, and symbolic statements to help investigate patterns and relationships that cannot easily be solved by arithmetic alone. Data points often will not fall along a straight line. Students will need to expect that not all relationships are linear. At this point they should be able to recognize that some relationships do not appear to be linear, and also to predict what type of shape is apparent from the graph.

Students drew a line of best fit in grade 8; however, this was done somewhat informally. In grade 9, they can use a combination of formal and informal methods to determine the line of best fit. The slope and y-intercept can be used to help write an equation for a best-fit line. Teachers should be ready to accept different equations from different students, since this is not an area where there is only one acceptable answer.

At this stage, students will not have the algebraic background to determine the equation of a great variety of non-linear situations. However, they should be able to predict shape to the point where they can decide the shape is parabolic (especially if both sides of the parabola are shown). It is more difficult for students to recognize parabolic curve when only one side of the parabola is apparent, since these may be difficult to distinguish from exponential graphs. Students may also explore other relationships without attaching formal names to the graphs.

Students should be able to explain why tabular data represent a linear, parabolic, or exponential relationship. They should be able to say that since the first difference is a constant, the pattern is linear, or that, since there is a common ratio, the pattern is exponential. They should be able to explain why certain equations, such as $y = 2x$, $y = -3x - 4$, and $y = \frac{1}{2}x + 3$, all represent linear relationships, while $y = x^2$ and $y = 2x^2$ represent parabolic curves, and $y = 2^x$ and $y = 3^x$ represent exponential curves.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Portfolio/Project

C5.1 A person's blood sugar is raised by drinking regular pop. Most people can cope with certain raised levels of blood sugar, but some become quite hyperactive. It was determined that the level of blood sugar that triggers hyperactivity is reached when specific quantities of pop are consumed, and this critical number depends on body mass. This chart is based on the number of ml of pop consumed in 2 hours that triggers hyperactivity for certain body masses.

body mass (kg)	ml of pop
46	750 ml
55	800 ml
64	1000 ml
73	1200 ml
82	1400 ml
91	1500 ml
100	1650 ml
109	1700 ml

Ask students to

a) make a graphical record of the data and draw the line of best fit
b) find the slope of the line and explain its meaning in this context
c) determine the y-intercept and discuss whether it is meaningful
d) write an algebraic model that represents the line
e) compare/contrast their model with that of a classmate

C5.2 Ask students to draw the graph for $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$, using a table of values, making sure the table contains both positive and negative values of x. Ask them to describe each graph and explain what appears to be the relationship between them. If graphing technology is available, ask them to graph each equation again, using technology to confirm that the graphs produced on paper are accurate.

Extension

C5.3 Assign one group to each of several research topics, such as the following:

i) tidal information over a 24-hour period
ii) tidal information for a 6-month period
iii) average rainfall per month for a three year period
iv) average monthly temperature for a three year period

Ask each group to make a graph of the data collected and to compare graphs. Ask them what the graphs have in common. [Most of i) - iv) should produce results which are periodic in nature and thus go beyond the intent of the outcomes.]

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *apply algebraic methods to solve linear equations and inequalities and investigate non-linear equations*

SCO: By the end of grade 9, students will be expected to

C6 **solve single-variable equations algebraically, and verify the solutions**

Elaboration – Instructional Strategies/Suggestions

C6 The following represents the types of single variable equations which students should be able to solve by the end of grade 9. In some cases, the examples indicated are actually subsets of other examples. Also, elements of these examples may appear together in specific equations.

$-2x + 12 = 4x$ $\quad$ $-3(x + 7) = -30$ $\quad$ $\frac{x}{4} = \frac{7}{10}$

$3x + 5 = -x - 4$ $\quad$ $\frac{1}{3}x + 8 = -1.4$

$2(3x - 6) = \frac{1}{2}(4x + 2)$ $\quad$ $x^2 + 25 = 169$

Students should work with questions involving decimal fractions as well as common fractions, but most questions in the initial development should apply the integer subset of rational numbers.

A brief review of the various informal methods to solve equations developed in grades 7 and 8 may be warranted. These include the use of algebra tiles, and methods such as the cover-up method and the use of a balance.

❑ Solve the following, using tiles, and record each step algebraically.

Emphasis in practice should be on application problems. It is particularly important to incorporate opportunities to make use of percentages, as well as measurement concepts and measurement units.

Formula rearrangement could be considered as a possible extension of this topic. Since students have already worked with the equation of a line, this might be a good place to start. Students often have difficulty with linear equations when not written in a familiar form. Students can start with forms such as $3y = -6x + 9$ and $y - 3 = 4x$. These involve a single step to change the equation to slope-intercept form. This can lead to situations where additional rearrangement is necessary. Experiences can also be provided in rearranging familiar formulas, such as

$A = lw$ $\quad$ $V = lwh$ $\quad$ $A = bh$

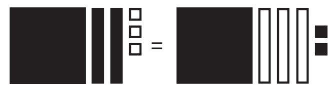
$P = 2l + 2w$ $\quad$ $C = 2\pi r$

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Performance

C6.1 Ask students to solve the following, using tiles, and to record each step in the solution algebraically.



Pencil and Paper

C6/7.1 Your school is selling almonds as a fund-raiser. The almonds sell for \$1.00 per box, and the school receives 40% of the proceeds.

a) How many boxes must be sold to make \$12 000.00?
b) How many boxes must be sold to make at least \$12 000.00?
c) How many boxes must be sold to make more than \$10 000.00 but less than or equal to the maximum permitted by the district regulations for fund-raising? District regulations set a maximum for \$15 000.00 on any one fund-raising project.

C6.2 Each side of the quadrilateral is 2 cm longer than the preceding side. If the perimeter is 44 cm, what is the length of the longest side of the quadrilateral?

C6.3 Each of the following equations represents a situation which is involved with two amounts of money being invested at different interest rates, and the amount of interest earned.

i) $0.08(1000 - x) = 0.06x + 10$
ii) $0.085(3000 - x) + 0.09x = 230$

a) Write two problems, each of which can be solved using one of the given equations.
b) Solve each equation and relate the solution back to the problem which you created.

Interview

C6.4 Ask students to explain how the four given equations are related:

$2p + 4 = 4p + 8$, $-2p + 4 = 8$, $4p + 8 = 8p + 16$, $-2p = 4$

Portfolio Entry (Extension)

C6.5 The volume of a cone is found by using the formula $V = \frac{1}{3}\pi r^2h$. Sarah is experimenting to find how high she needs to build a conical storage tower. She knows the volume of material to be stored is 112 m^3. She decides to rewrite the formula so that h is the subject and then experiment with different values of r.

a) Ask students how the formula can be written with h as the subject.
b) Ask students to choose varied values of r and find corresponding values of h.

Suggested Resources

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *apply algebraic methods to solve linear equations and inequalities and investigate non-linear equations*

SCO: By the end of grade 9, students will be expected to

C7 **solve first-degree single-variable inequalities algebraically, verify the solutions, and display them on number lines**

C8 **solve and create problems involving linear equations and inequalities**

Elaboration – Instructional Strategies/Suggestions

C7 Solving inequalities is new at the grade 9 level. Students will only work with inequalities where the variable has an exponent of 1. Solving inequalities of the form $x^2 \geq 25$ will not be core at this grade level. An understanding of how various operations affect the truth of an inequality can be developed before introducing variables.

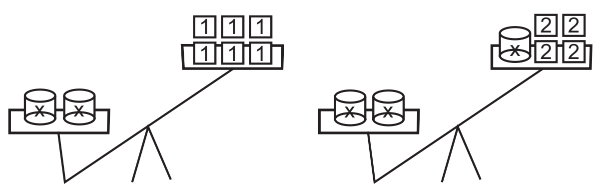
❑ Students can start with true sentences such as $-2 < 4$ and $5 > 1$. They can make a chart which shows each inequality and investigate how the truth of each is affected when the following operations are performed on both sides of the inequality:

- add a positive number	- add a negative number
- subtract a positive	- subtract a negative
- multiply by a positive	- multiply by a negative
- divide by a positive	- divide by a negative

Emphasis should be placed on having students graph their solutions to algebraic inequalities on a number line to understand clearly what the answer represents; that is, a set of values rather than a single number.

A concrete method of introducing inequalities would be to show students a balance scale such as the following.

❑ Find several true values for the unknown in each case. Graph the set of answers on a number line and describe what they notice about the set of answers.



A link should be made between C7/C8 and outcome A2.

C8 Students should not only be provided with opportunities to solve problems, they should also be asked to create problems of their own. They can be given problems that represent equality situations and asked to rewrite them as inequality problems. These problems can be exchanged and solved.

Students should also solve problems which involve an upper and a lower limit, such as the following.

❑ Suppose a certain product will bend at temperatures over 50° C and crack at temperatures below 0° C. Write a problem based on this information, and solve it.

GCO (C): Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

C7.1 Taylor received 77%, 70%, 81%, and 78% on her first four mathematics tests. What mark will she need to get on the fifth test in order to achieve at least an 80% average?

C7.2 Verify whether {-2, +3, +5, -1, +9, -9, -14} are solutions to the inequality $-2x - 5 > 7$. Solve the inequality and graph the solution on a number line. Check to determine how many of the numbers from the set above would be part of the graphical solution.

C8.1 For each of the following equations or inequalities

- i) $d + d + 2 + d + 4 = 39$
- ii) $x + 5(x + 2) + 10(2x) = 192$
- iii) $3x + 2 < 45$

a) create a problem which can be solved using it
b) solve the equation or inequality and relate the solution to the problem

C8.2 Two drivers left the city of Summerside 1 hour apart. The first driver was driving at a speed of 80 km/h. The second driver overtook the first driver after a distance of 320 kilometres. Create and solve questions that are based on the information provided.

Journal

C7.3 Ask students to explain why $3n - 2 > 8$ and $3n + 4 < 14$ do not have any solutions in common. Ask them to modify one of the inequalities so that they have exactly one solution in common.

C8.3 Ask students to

a) create three equations that are equivalent, and explain why they are equivalent
b) create three inequalities that are equivalent, and explain why they are equivalent

Suggested Resources

Shape and Space
(*Measurement*)

General Curriculum Outcome D:

Students will demonstrate an understanding of
and apply concepts and skills
associated with measurement.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *demonstrate an understanding of the concept of rates; use direct and indirect measurements to describe and make comparisons and read and interpret scales; and describe how a change in one measurement affects other, indirect measurements*

SCO: By the end of grade 9, students will be expected to

D1 **solve indirect measurement problems by connecting rates and slopes**

Elaboration – Instructional Strategies/Suggestions

D1 There are many opportunities in grade 9 to explore how change in one quantity will affect a related quantity. One of the most obvious topics of grade 9 that deals with change is the study of slope. Students deal with slope in grade 9 as $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in y}}{\text{change in x}}$. (Note: There is **no** intent to use the delta (Δ) notation or to formalize calculation of slope through the use of an expression such as $\frac{y_2 - y_1}{x_2 - x_1}$).

There are many opportunities for students to collect data, use data from patterning situations, or use data from prepared tables and charts to create graphs and determine the slope (see outcome F2). One simple experiment involves testing 'the bounciness of balls.'

❑ Collect several balls such as ping-pong balls, tennis balls, and basketballs. Select a ball and drop it from a range of heights. Record the drop height and the bounce height (a measuring tape attached to a wall will help measure these heights more accurately). Graph the data and fit a line to it. Find the slope of the line and use the slope to write an equation in the form $y = mx$ where m is the slope. Note that the slope represents the $\frac{\text{change in bounce height}}{\text{change in drop height}}$. Have different groups perform the same experiment with different balls. Since we control the drop height in all cases, students should be able to infer that the change in bounce height is the factor affecting the slope of the line. They should consider which ball has the most bounce and compare this with the slope of the line. Students should be asked to write about how bounciness and slope are related. [This activity can also be used in meeting outcomes F1 and F2.]

Indirect measurement situations will also arise in the study of similarity (see outcomes D5 and E4). Students will apply ratio and proportion in practically all application problems pertaining to this area of study.

There are many opportunities to link this outcome with SCO's C1 - C4.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

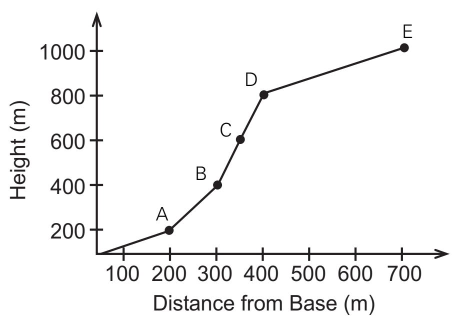
Performance

D1.1 Have students perform the following experiment to find the amount of water lost through a leaking faucet. This can be simulated using paper cups, a timer, and a glass measuring cylinder. Punch a small hole in the bottom of the cup and cover it with a finger until ready to start. Remove the finger and record the amount of water at regular intervals. (If the water drips into a thin graduated cylinder, these measurements can be more easily made.) Graph the data and draw the line of best fit. Find the slope of the line. Try this again with two small holes in the cup. Have students respond to the following questions.

a) Do you expect the same slope? Explain.
b) Will the data continue to be linear? Explain.
c) How does the change in flow rate affect the slope of the line?

Pencil and Paper

D1.2 The graph represents the side view of a ski hill. Find the slope of each segment of the hill and compare the number for the slope with the segment. What do you notice? Write a statement that relates the magnitude of the slope to the steepness of the corresponding segment.



Portfolio

D1.3 Sarah made a rectangular blanket for her sister's doll. The dimensions of the blanket were 40 cm by 60 cm. After washing, the dimensions of the blanket shrank by 4% uniformly. Have students complete the following assignment.

a) What are the new dimensions?
b) Write the ratio of change in dimensions to original dimensions.
c) Make a scale drawing of the original blanket and the shrunken blanket on grid paper. Find the area of the blanket before and after washing and write the ratio of the change in area to original area as a percentage. How does this percentage relate to the original percentage of shrinkage?
d) Try this again using a shrinkage of 10%. Compare the shrinkage percentage of the dimensions with the affect on area. What do you notice? Develop a plan that might be used to observe if there is any consistent relationship here.

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *communicate using a full range of SI units and select appropriate units in given situations*

SCO: By the end of grade 9, students will be expected to

D2 **solve measurement problems involving conversion among SI units**

Elaboration – Instructional Strategies/Suggestions

D2 In grade 9, this outcome is not meant to be taught in isolation, but rather students should be exposed to problem situations that will require them to make conversions from one SI unit to another. This outcome should be integrated most particularly into outcomes D3 to D5.

- ❑ Ask students to find the capacity of a water line that is 20 metres long and two centimetres in diameter. [Students will have to convert to either metres or centimetres, but the focus of the question is on finding the volume.]

There should be opportunities to revisit mass and capacity units, as well as linear, area, and volume units, throughout the full SI scale (from milli to kilo). Because students are involved with scientific notation in grade 9, this should also be extended to include the use of prefixes such as micro, nano, and gigo. This is referred to in the instructional implications associated with scientific notation in GCO B, specifically B5 and B6.

Opportunities should also be created to apply measurement conversion in outcomes related to applications that involve solving equations, as well as operations on polynomials. Since one of the most common application problems for operations with polynomials relates perimeter, area, and volume, opportunities to reinforce conversion should be plentiful.

Many opportunities to apply and reinforce conversion skills will also arise in the context of similarity.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D2.1 An ice-cream cone is filled with soft ice cream, and then a scoop of hard ice cream is placed on top. The ice-cream cone has a height of 10 cm and an inside diameter of 7 cm.

a) Assuming that the scoop of hard ice cream is a perfect semi-sphere which exactly fits the cone, how many ml of ice cream is in this serving?
b) Jan has 1 litre of soft ice cream and 2 litres of hard ice cream. How many servings can Jan make?

D2.2 The Johnson family has a shed which is 180 cm high at the walls and 2.4 m high in the centre, with rectangular floor dimensions of 3 m by 4.2 m.

a) If they fill the shed with hay to the height of the walls, how much will it hold?
b) If a loft is created in the peak of the roof, how much would the loft hold? What assumption did you make?
c) If the floor dimensions were increased by x, write an expression to represent the new capacity in each of a) and b). Use these expressions to find the capacity if x = 85 cm, and if x = 1.65 m.

D2.3 KOOL drinks wants to design a tetrapak for their 600 ml serving of Fruit Punch. Find several possible sets of dimensions of the box. From the dimensions found, determine which box requires the least amount of materials to make.

Portfolio

D2.4 Mr. MacDonald decided to put a playroom in his restaurant. He decided to fill a room to a depth of 30 cm with hollow balls. Each ball has a mass of 5 g and a diameter of 7 cm. The room is rectangular with dimensions 4.2 m by 5.5 m.

a) Ask students to find the approximate number of balls necessary to fill the room to the depth desired.
b) Ask students what assumptions they made in finding the number of balls.
c) The balls are shipped in boxes which are 0.5 cubic metres in volume. Have students find the approximate number of boxes which will be needed to fill the room.
d) Ask students what it would cost to get the balls shipped, if Mr. MacDonald uses Econo Shipping to ship the boxes of balls at $0.50 per kg.

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *estimate and apply measurement concepts in relevant problem situations, and use tools and units which reflect an appropriate degree of accuracy*

iv) *develop and apply a wide range of measurement formulas and procedures*

SCO: By the end of grade 9, students will be expected to

D3 relate the volumes of pyramids and cones to the volumes of corresponding prisms and cylinders

D4 estimate, measure, and calculate dimensions, volumes, and surface areas of pyramids, cones, and spheres in problem situations

Elaboration – Instructional Strategies/Suggestions

D3 This outcome is intended to provide an informal opportunity to relate volume of a cylinder to the volume of a cone of equal base and height, as well as the volume of a pyramid to the volume of a prism of equal base and height. There are many commercial materials available that can be used to assist in making these connections, but the outcome can also be achieved using teacher- or student-made materials.

- ❑ Ask students to make a cone and a cylinder of equal height, using heavy construction paper. Ask them to put a plastic liner inside and pour water, sand, or rice into the cone until it is filled to the top. Ask them to pour this into the cylinder and observe the height of the material in the cylinder and record their findings. When this is done by several groups using different cylinder and cone combinations, all the results can be compiled and a conclusion reached. Repeat the activity, using a pyramid and a prism.

From this activity students should be able to draw the conclusion that the formula for the volume of a cone and a pyramid is $V = \frac{1}{3}Bh$, where B stands for area of the base.

D4 Students should be given a variety of hollow shapes, including the pyramid, cone, and sphere, and be asked to estimate the volume and surface area. A variety of estimation strategies should be considered. For example, in some instances students may simply round all dimensions to the nearest 10 and mentally calculate the volume or surface area. Depending on how rough an estimate is required in the case of the sphere, students can think of it as inside a cube where the diameter of the sphere is also the dimensions of the cube. Therefore, a quick estimate for volume would be d x d x d. Students should realize why this would result in an estimate that is greater than the actual volume. They could then fill the objects with water, sand, or rice and measure the actual volume.

For many objects the net of a shape makes the surface area formula fairly obvious to students. However, for some of the new shapes at grade 9, the formula is not as obvious from the net. Surface area of pyramids, cones, and spheres are new at grade 9. Surface area of the pyramid tends to be fairly straightforward because it is easily determined from the net. However, the surface area of a cone and the volume and surface area of a sphere will require more deliberate development.

Note: The elaboration for D4 is continued on the next 2-page spread.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Performance

D3.1 Ask students to make one of the following, using materials of their own choosing, a cylinder and a cone which have the same height and base, or a right prism and a pyramid which have the same height and base.

a) Have students explain and demonstrate to the class what happens when they fill the cone or the pyramid and pour the contents into the cylinder or the prism.

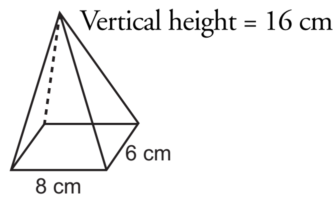
b) Ask students to write in words and using symbols the relationship between the volume of a cone and a cylinder or of a pyramid and a prism which have equal base and height.

Pencil and Paper

D4.1 Estimate the volume of the following shapes and explain your reasoning.

a)

b)



D4.2 A sphere fits exactly inside a cube which is 12 cm in diameter. Find the surface area and volume of the cube.

D4.3 A witch's hat was made for Halloween from a piece of heavy cardboard. Sandy decided that, in order to have enough room to fit the hat over her witch's wig, she would need the opening to be 56 cm in circumference. She wanted the hat to measure 30 cm from the brim to the point at the top of the cone shape.

a) What was the area of the cardboard that she needed to cut to form the hat?

b) The brim of the hat is circular and is 8 cm wide. What is the radius of the inner and outer circle which will need to be cut to make the brim?

D4.4 A sphere fits exactly inside a cylinder. The sphere is 12 cm in diameter. Find the surface area and volume of the sphere.

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *estimate and apply measurement concepts in relevant problem situations, and use tools and units which reflect an appropriate degree of accuracy*

iv) *develop and apply a wide range of measurement formulas and procedures*

SCO: By the end of grade 9, students will be expected to

D4 **measure, estimate, and calculate dimensions, volumes, and surface areas of pyramids, cones, and spheres in problem situations**

Elaboration – Instructional Strategies/Suggestions

D4 (Cont'd) The formula for the volume of a sphere can be found by using a cylinder and a sphere of equal diameter and height. This can be done using commercially-produced or teacher-made materials. Such materials can show that the volume of the sphere is $\frac{2}{3}$ of the volume of the corresponding cylinder.

Volume of sphere = $\frac{2}{3}$ of the volume of cylinder

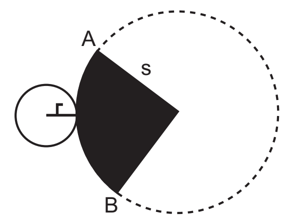
Volume of sphere = $\frac{2}{3} \times \pi r^2 h$

Volume of sphere = $\frac{2}{3} \times \pi r^2(2r)$ (since the height h = 2r)

Volume of sphere = $\frac{4}{3} \times \pi r^3$ or $\frac{4}{3} \pi r^3$

Students may not be able to produce this explanation on their own but should be exposed to the development of the formula.

❑ To find the surface area of a cone, consider the net shown. Explain why this is the net for a cone. Ask students to make this net.



The small circle is the base of the cone, and its area is πr^2. The shaded section of the large circle is the lateral surface (wrap-around part) for the cone. The ratio of the area of the sector to the area of the whole circle should be the same as the ratio of the length of arc AB to the circumference of the whole circle. That is,

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc AB}}{\text{circumference of circle}}$$

[arc AB = the circumference of the small circle]

$$\frac{\text{area of sector}}{\pi s^2} = \frac{2\pi r}{2\pi s}$$

$$\frac{\text{area of sector}}{\pi s^2} = \frac{r}{s}$$

$$\text{area of sector} = \frac{r}{s} \times \pi s^2$$

$$\text{area of sector} = \pi rs$$

(area of lateral surface)

Total surface area of cone = area of base circle + area of lateral surface $A = \pi r^2 + \pi rs$

The surface area of a sphere is found by using the formula $A = 4\pi r^2$. This formula should also be developed by students.

It is important that problem situations be included where students are asked to find the dimensions when surface area or volume is given.

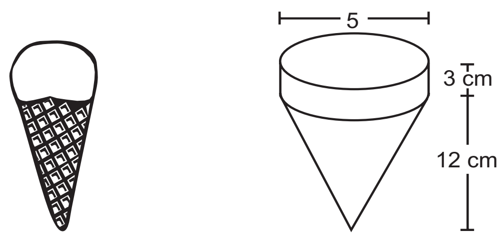
Memorization of formulas such as these is not intended.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D4.5 A commercially produced ice-cream treat has the cone filled with ice cream and has ice cream on the top.



a) If the ice cream is 2.5 cm above the cone in the shape of a perfect hemisphere, and the cone is 5 cm in diameter and 12 cm high, how much ice cream would be required to make one such ice-cream treat?

b) If the ice cream is 3 cm above the cone in the shape of a cylinder, and the cone is 5 cm in diameter and 12 cm high, how much ice cream would be required to make one such ice-cream treat?

D4.6 The Heritage Committee is restoring the old church in town. The steeple (a square pyramid) will be covered with metal sheeting. What is the minimum amount of sheeting that will be required to cover the steeple if the base is 4 m^2 and the slant height is 8m?

D4.7 The radius of a ball is 14 cm. By how much does the surface area increase if the ball is inflated so that its radius increases by 1 cm?

Suggested Resources

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *estimate and apply measurement concepts in relevant problem situations, and use tools and units which reflect an appropriate degree of accuracy*

iv) *develop and apply a wide range of measurement formulas and procedures*

SCO: By the end of grade 9, students will be expected to

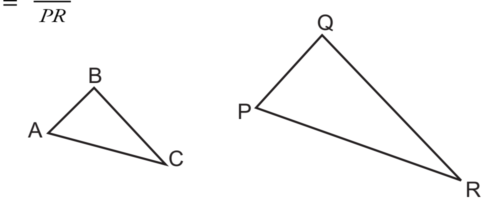
D5 demonstrate an understanding of and apply proportions within similar triangles

Elaboration – Instructional Strategies/Suggestions

D5 In E4 the ratios which relate sides of similar triangles are developed. For instructional purposes, D5 should be developed after E4.

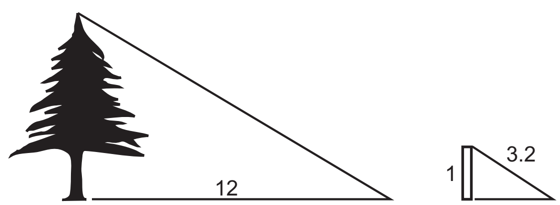
First, students should understand the relationships between corresponding sides of similar triangles. That is, for ΔABC and ΔPQR, if $\Delta ABC \sim \Delta PQR$ then

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$



Since the two triangles shown are similar, the ratios of side lengths within one triangle are equal to the ratios of the corresponding side lengths within the other triangle. That is, students should be able to conclude that $\frac{AB}{AC} = \frac{PQ}{PR}$. When similarity can be confirmed for two triangles, ratios such as these may be useful in solving problems.

- ❑ A tree casts a shadow of 12 m at the same time that a metre stick casts a shadow of 3.2 m. Ask students to find the height of the tree.



This problem can be solved using

$$\frac{\text{height of the tree}}{\text{length of tree's shadow}} = \frac{\text{height of the metre-stick}}{\text{length of metre-stick's shadow}}$$

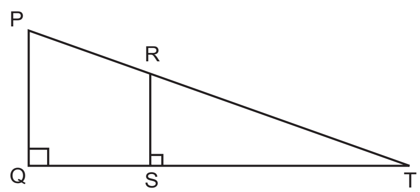
It is through using ratios of sides within triangles that the trigonometric ratios are established in grade 10.

GCO (D): Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

D5.1



a) Which triangles are similar? Why?

b) Measure the sides and determine the ratios of

i) $\frac{PQ}{QT}$, $\frac{RS}{ST}$

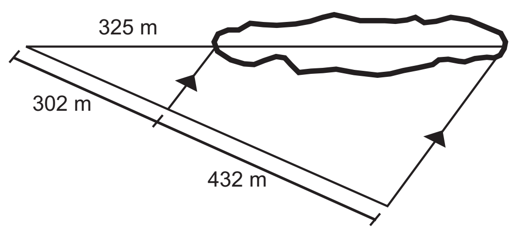
ii) $\frac{PQ}{PT}$, $\frac{RS}{RT}$

iii) $\frac{QT}{PT}$, $\frac{ST}{RT}$

c) What do you notice about the values?

d) If PQ = 8.2 cm, QS = 5.3 cm, and ST = 7.3 cm, use one of the ratios pairs in part b) to find RS.

D5.2 Use the measurements provided in the diagram to find the length of the lake.



Suggested Resources

Shape and Space (Geometry)

General Curriculum Outcome E:

Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *draw inferences, deduce properties, and make logical deductions in synthetic [Euclidean] and transformational geometric situations*

SCO: By the end of grade 9, students will be expected to

E1 **investigate, and demonstrate an understanding of, the minimum sufficient conditions to produce unique triangles**

Elaboration – Instructional Strategies/Suggestions

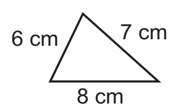
E1 A triangle is made up of three sides and three angles. Students will investigate whether all six pieces of information are needed to guarantee a unique triangle.

This topic could be introduced by using pencil and paper constructions, software, or by using manipulatives.

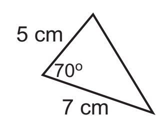
- ❑ Provide students with three pieces of drinking straw of pre-determined length and/or pipe cleaners and ask them to form a triangle. [By comparing and discussing with others, students should see that, although each of their triangles may be oriented differently, they are all the same. In other words, when given the lengths of the three sides of a triangle, there is only one unique triangle that can be produced. Students should try three different lengths of material to see if the same conclusion holds true.]

Similarly, by using angle strips (or straws and pipe cleaners), it is possible to investigate other combinations to see if any of those produce unique triangles. Through investigation, students should see that the following information is necessary in order to produce unique triangles:

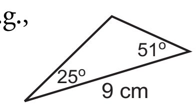
Case 1 - 3 sides; e.g.,



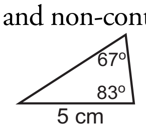
Case 2 - 2 sides and contained angle; e.g.,



Case 3 - 2 angles and contained side; e.g.,



Case 4 - 2 angles and non-contained side; e.g.,



In Cases 1, 2, and 3, students can observe the uniqueness of the triangle produced through construction. In Case 4, the uniqueness is much more difficult to observe through construction. Students should realize that, since they know two angles of a triangle, the third is also known. This allows them to apply Case 3 to the information provided in Case 4 and again produce a unique triangle. Students should also be asked to explore other possibilities as well, such as AAA and SSA, to confirm that a unique triangle is not possible in each of these cases. They should also consider whether there are any situations where only two pieces of information are sufficient to guarantee uniqueness.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Performance

E1/2.1 Students should work in groups for this activity.

a) Provide each group with 3 pieces of drinking straw measuring 5 cm, 7 cm, and 8 cm (or three geostrips of different lengths). Ask each group to form a triangle with the pieces of straw and compare it with the triangles of other groups. Ask students what they notice and to record their findings.

b) Each group should then remove the longest piece of straw and form a triangle, using the 5 cm and 7 cm pieces along with a 50º angle made from angle strips. The third side can be any length necessary to complete the triangle. Experiment by placing the 50º angle in different locations so that it is the contained and then non-contained angle.
   i) Ask students what they notice and to record their findings.
   ii) Ask them to compare their findings with those of other students for each case explored, and to state conclusions.
   iii) Ask students where the 50º must be placed relative to the other known sides in order for all groups to produce triangles that are the same.

c) Using only the 7 cm straw and 2 angles of 50º and 70º, along with 2 other pieces of straw, ask students to explore possible ways of combining them to make a triangle. [The two other pieces of straw will need to be cut appropriately to make the triangle, depending on where the angles are placed.]
   i) Ask students to compare their triangles. Did any groups produce the congruent triangles?
   ii) Ask students what arrangement of two angles and one side always produces two triangles that are alike. Ask them to summarize their findings.

E1/2.2 Tell students that you are given four parts of one triangle equal to the corresponding four parts of another triangle, and that you are not told what parts they are. Have students explore whether there is any combination of the four pieces of information for which the triangles are not congruent, and ask what they concluded.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) draw inferences, deduce properties, and make logical deductions in synthetic [Euclidean] and transformational geometric situations

SCO: By the end of grade 9, students will be expected to

E2 investigate, and demonstrate an understanding of, the properties of, and the minimum sufficient conditions to, guarantee congruent triangles

E3 make informal deductions, using congruent triangle and angle properties

Elaboration – Instructional Strategies/Suggestions

E2 Through investigation of the conditions which produce unique triangles, students should become aware that if the SSS (side-side-side), SAS (side-angle-side), ASA (angle-side-angle), or AAS (angle-angle-side) relationships exist between two triangles then the triangles are congruent and, therefore, the other corresponding parts are also congruent. For example, if the three sides of one triangle are congruent to the corresponding three sides of another triangle, then the triangles are congruent. Since the triangles are congruent, it is therefore true that the three sets of corresponding angles are also congruent. Again, confirmation of these findings may be further explored using either pencil and paper constructions, manipulatives, and/or technology.

- ❑ Ask students if knowing that two parts of one triangle are congruent to two corresponding parts of another triangle would be sufficient information to conclude that two triangles are congruent.

Students have not had experience to date with the symbolism associated with congruency. They should be exposed to the symbol "$\cong$" which is read as "is congruent to." Students should also work with the formal language of congruence. The statement AB = CD is read, "the length of AB is equal to the length of CD." If AB = CD then $\overline{AB} \cong \overline{CD}$. $\overline{AB} \cong \overline{CD}$ is read, "the segment AB is congruent to the segment CD."

E3 It is in relation to this outcome that students are first exposed to formal deductive reasoning. Most of the reasoning that has been applied to geometry up to this point has been inductive. It is useful to distinguish between inductive and deductive reasoning using both mathematical and non-mathematical examples. Inductive reasoning was used in establishing the uniqueness of triangles for E1. Students make several triangles from the same specifications and carry out measurements to conclude that they are all congruent. It is also used in non- mathematical situations such as

> Abel ate strawberries on two occasions and each time broke out in hives.
> He concluded that strawberries give him hives.

Deductive reasoning is used in situations such as the following:

- ❑ All tomato juice will stain cotton.
  Rockie spilled tomato juice on his cotton shirt.
  What can we conclude?

This type of thinking is also applied in the following:

- ❑ $\Delta ABC \cong \Delta PQR$
  What can we conclude about $\angle A$ and $\angle P$?

Note: The elaboration for E3 is continued on the next 2-page spread.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

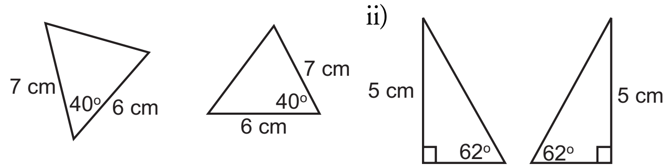
Pencil and Paper

E1/2.3 Is it possible to construct a unique triangle ABC with $\angle B = 60^\circ$, AB = 5 cm, and AC = 4 cm? Explain. Support your explanation with a diagram or model.

Interview

E1/2.4 Ask students to explain why the two triangles are congruent.

i) ii)



Extension

E1/2.5 For right triangles, it is possible to produce a unique triangle when the hypotenuse and one other side is known.

a) Ask students to explore this possibility using straws or geostrips.
b) This property is often identified as the HS (hypotenuse-side) property or postulate. When the right angle is included, this becomes two sides and the non-included angle which was rejected, as a set of minimum sufficient conditions. Ask students why it provides a set of minimum sufficient conditions in the case of right triangles.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) draw inferences, deduce properties, and make logical deductions in synthetic [Euclidean] and transformational geometric situations

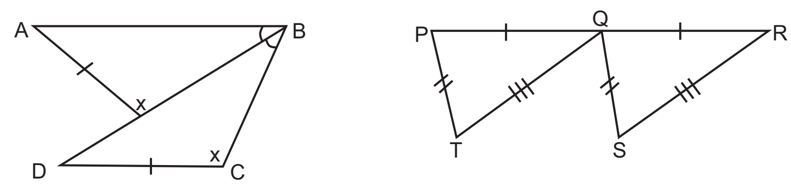
SCO: By the end of grade 9, students will be expected to

E3 make informal deductions, using congruent triangle and angle properties

Elaboration – Instructional Strategies/Suggestions

E3 (Cont'd) Students should be able to use the congruent triangles relationships developed in E1 and E2, as well as angle properties studied in previous grades, such as properties related to parallel lines, vertically opposite angles, complementary angles, and supplementary angles, to determine if a congruency relationship exists between pairs of triangles. They can then use the fact that two triangles are congruent to find missing side and angle measures.

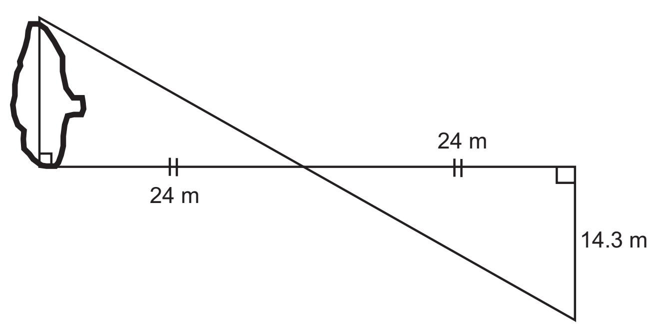
- ❏ Given the information indicated in the diagrams,
  a) why is $\overline{AB} \cong \overline{DB}$?
  b) why is $\angle TPQ \cong \angle SQR$?



[Sample Solution 1: Students might say, since two angles and one side of one triangle are congruent to the corresponding parts of the second triangle, the triangles are congruent. Because the triangles are congruent, we know that the other corresponding pairs of angles and sides within these triangles are also congruent. Therefore $\overline{AB} \cong \overline{DB}$ and $\angle TPQ \cong \angle SQR$. Students might also apply transformational geometry in building an argument to justify this conclusion.]

[Sample Solution 2: Translate P onto Q, then Q will translate to R, since P - Q - R is straight, and PQ = PR. T will map to S, since PT = QS and QT = RS. The triangles are congruent since the image and object are congruent under a translation.]

- ❏ Using the information provided, find the length of the lake, and justify your findings (base the justification on one of the four conditions of congruency).



Students should be aware that it is very important when naming the vertices of congruent triangles that the vertices of the triangles are written in an order so as to correspond with the congruent parts.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

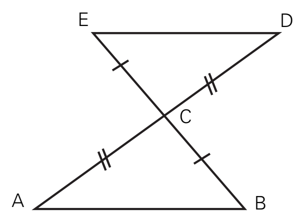
Pencil and Paper

E2/3.1 A rectangle ABCD has its diagonals drawn. The diagonals meet at point E.

a) Name four pairs of triangles which are congruent.

b) Explain why you know they are congruent (base your explanation on one of the four conditions for congruency).

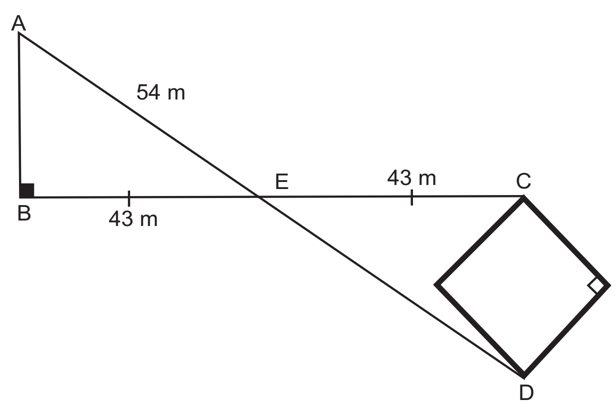
E2/3.2 Study the diagram below and the information that it provides. Determine whether the information about the two triangles is sufficient to conclude that they are congruent. If yes, explain why, using one of the conditions for congruence. If no, explain why not.



E3.1 Using the information provided,

a) find the distance from home to second base (that is, from C to D) in the baseball diamond, and justify your findings based on congruency

b) find the distance from first to second base – Does it matter that we don't know which base is first base?



Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) draw inferences, deduce properties, and make logical deductions in synthetic [Euclidean] and transformational geometric situations

SCO: By the end of grade 9, students will be expected to

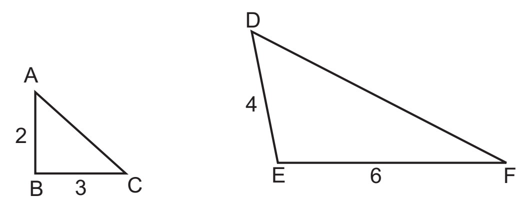
E4 demonstrate an understanding of and apply the properties of similar triangles

Elaboration – Instructional Strategies/Suggestions

E4 This topic could be explored in conjunction with enlargements and reductions (dilatations) in transformational geometry developed in grade 8. Students should recognize, through investigation, the properties of similar triangles; that is, corresponding angles that are congruent and corresponding sides that are in proportion. This should be taught concurrent or at least linked with D5.

In order to recognize that two triangles are similar, students should explore the minimum conditions necessary for two similar triangles. For example, they should come to realize that it is only necessary to know that two angles of one triangle are equal to two corresponding angles of another triangle to conclude that the triangles are similar. They should be able to justify this conclusion by using the fact that the sum of the angles in each triangle is 180°. Likewise, students should realize that knowing that two sides of one triangle are in proportion with two sides of another triangle is not sufficient information to conclude that the triangles are similar. This is illustrated below where it is given that

$\frac{AB}{DE} = \frac{BC}{EF}$



Since we do not know that any pairs of corresponding angles are congruent, it is not enough to conclude that the triangles are similar.

- ❑ Ask students if the information shown in the two diagrams above is sufficient information to conclude that the two triangles are similar. Ask students what else might be needed, and why. [Students should come to realize that information about the included angle, or the other two pairs of sides, would be necessary in order to conclude that these two triangles are similar. In fact, in the diagrams shown, it is clear that the triangles are not similar, because even though B and E are corresponding vertices, B appears to be a right angle and E appears to be obtuse.]

Two triangles are similar when two pairs of corresponding sides are in proportion and the pair of included corresponding angles is congruent. Also they are similar when two angles of one triangle are congruent to two corresponding angles of another triangle.

Note: The elaboration for E4 is continued on the next 2-page spread.

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

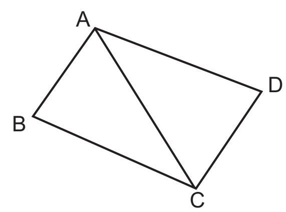
Pencil and Paper

E4.1 The shadow cast by a relay tower is 35.0 m long. At the same time, a pole which is 1.0 m tall cast a shadow of 35.0 cm. What is the height of the tower? What assumptions did you make?

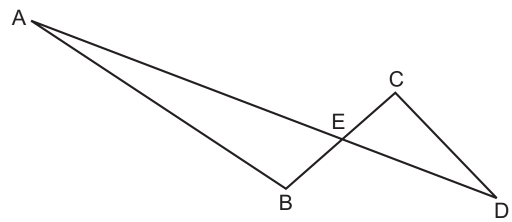
Interview

E4.2 Ask students if each set of triangles is similar. Ask them to justify their decision.

a) Given that AD = CB and AB = CD



b) Given that $\angle B \cong \angle C$



Investigation

E4.3 In groups of three, provide each student in the group with a set of plastic strips (these can be cut from stir sticks, or purchased commercially) as follows: Student A: 3 cm, 4 cm, 5 cm; Student B: 6 cm, 8 cm, 10 cm; Student C: 9 cm, 12 cm, 15 cm.

a) Ask each student to form a triangle and measure its angles. Ask them to compare angle measures.
b) Ask students to compare the lengths of each of the sides of the triangles. Ask them to predict the lengths of the sides of another triangle that will have the same angle measures, and to test their predictions.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *draw inferences, deduce properties, and make logical deductions in synthetic [Euclidean] and transformational geometric situations*

SCO: By the end of grade 9, students will be expected to

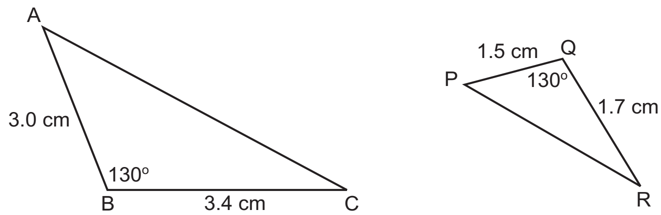
E4 **demonstrate an understanding of and apply the properties of similar triangles**

E5 **relate congruence and similarity of triangles**

Elaboration – Instructional Strategies/Suggestions

E4 (Cont'd) In the diagrams below, because two pairs of corresponding sides are in proportion, and the included angles are congruent, the triangles are similar. If PR is given, AC can be found.

❑ Ask students to find the measure of AC if PR = 2.9 cm.



Students should be exposed to a variety of situations involving similar figures, including pairs that are varied in their orientations, as well as those that are overlapping and non-overlapping. Students should be able to use the properties of similar triangles to find the measures of missing sides and angles. This topic lends itself well to real-life situations such as finding the height of buildings and trees, and finding distances which are normally difficult to measure directly; for example, the distance across a river, pond, or wetland.

E5 At this point, students are familiar with both congruence and similarity. They should now compare and contrast both of these concepts as they relate to triangles. Students should apply the minimum conditions identified in outcomes E2 and E3 for congruence and compare them to the conditions necessary to determine whether two triangles are similar. They should be able to discuss the following:

❑ If two triangles are congruent, are they also similar?
If two triangles are similar, are they also congruent?
If the ratios of the corresponding sides of two similar triangles are 1:1, what do we know about the triangles?

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

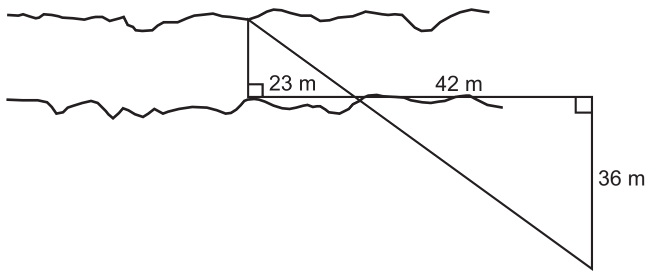
Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

E4.4

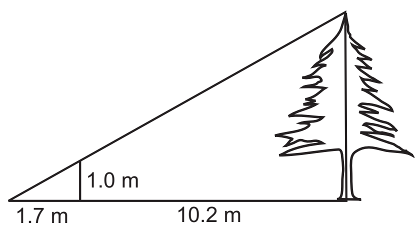
a) Are the two triangles in the diagram below similar? Justify.

b) If enough information is available, find the width of the river. If there is not enough information, what other information is needed?



c) Are the two triangles similar in the figure below? Justify.

d) If enough information is available, find the height of the tree.



Interview

E4.5

a) Ask students if pairs of congruent triangles are also similar. Ask them to explain why or why not.

b) Ask students if pairs of similar triangles are necessarily congruent. Ask them if they can be congruent, and to explain why or why not.

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) develop and analyse the properties of transformations and use them to identify relationships involving geometric figures

SCO: By the end of grade 9, students will be expected to

E6 use mapping notation to represent transformations of geometric figures, and interpret such notations

Elaboration – Instructional Strategies/Suggestions

E6 Students should already be familiar with the concepts of translations, reflections, rotations, and dilatations. The topics will now be extended so students will explore these transformations on a coordinate plane, using mapping notations. Note that the dilatation centre is restricted to the origin and rotations are restricted to 90^{o} and 180^{o}.

- ❑ Given the mapping notation $(x, y) \rightarrow (x + 5, y - 2)$ for $\triangle$ABC with vertices of A (2, 2), B (4, 2), and C (5, -3), what are the vertices of $\triangle$A´B´C´. [The vertices are (7, 0), (9, 0), and (10, -5).]
- ❑ $\triangle$ABC is reflected about the x-axis. If the vertices of $\triangle$ABC are A (3, 4), B (-2, 5), and C (-1, -4), find the vertices of $\triangle$A´B´C´. Write the mapping rule that relates the two triangles. [The mapping rule would be $(x, y) \rightarrow (x, -y)$.]
- ❑ $\triangle$ABC is enlarged by a factor of 3 using the origin as the dilatation centre. If the vertices of $\triangle$ABC are A (3, 4), B (-2, 5), and C (-1, -4), find the vertices of A´B´C´. Write the mapping rule that relates the two triangles. [The mapping rule is $(x, y) \rightarrow (3x, 3y)$.] Discuss why this mapping rule only works to determine coordinates when the dilatation centre is the origin.
- ❑ $\triangle$ABC is rotated 90^{o} clockwise about the origin. If the vertices of $\triangle$ABC are A (3, 4), B (-2, 5), and C (-1, -4), find the vertices of $\triangle$A´B´C´. Write the mapping rule that relates the two triangles. [The mapping rule would be $(x, y) \rightarrow (y, -x)$.] Discuss why this mapping rule only works to determine coordinates when the rotation centre is the origin.

Students should be given information about mapping of points, segments, or shapes and asked to interpret the mapping. That is, they should be able to describe or explain the nature of a transformation based on a given mapping.

- ❑ A transformation takes place on a quadrilateral based on the mapping $(x, y) \rightarrow (-y, x)$. Describe what happens to the figure. Suppose that one of the vertices of the quadrilateral is (4, -5). What are the coordinates of the image vertex? [This describes a rotation of 90^{o} counterclockwise about the origin. The image point would have coordinates (5, 4).]

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

E6/7.1 Construct $\triangle RST$ on a coordinate plane with vertices R (-4, 4), S(-6, 2), and T (-3, 2). Trace and cut out a copy of RST and label it $\triangle R'S'T'$.

a) Slide $\triangle R'S'T'$ four spaces to the left.
   i) What are the new coordinates of this triangle?
   ii) Compare with the vertices of the original triangle. Write the mapping rule for the translation.

b) Slide $\triangle R'S'T'$ five spaces up. Explain why $(x, y) \rightarrow (x - 4, y + 5)$ would describe the relation between the final position of the triangle and the original position.

E6/7.2 On a coordinate plane, construct $\triangle ABC$ with vertices A (2, 3), B(0, 0), and C (2, 0). Trace and cut out a copy of ABC and label it $\triangle A'B'C'$.

a) Explore these mappings and identify them as either translations, rotations (about the origin), or reflections:
   i) $(x, y) \rightarrow (x, -y)$
   ii) $(x, y) \rightarrow (-y, x)$
   iii) $(x, y) \rightarrow (x - 3, y + 2)$

b) Write the coordinates of the image triangle in each case.

c) What happens to $\triangle ABC$ when the mapping $(x, y) \rightarrow (2x, 2y)$ is applied to it?

d) Write the coordinates of the image. What assumption did you make?

e) Find the area of the image and pre-image. What do you notice?

E6/7.3 Graph $y = 2x + 1$.

a) Draw the image, using the mapping rule $(x, y) \rightarrow (x, -y)$.

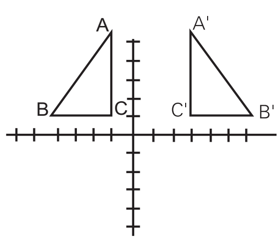
b) Find the equation of the image.

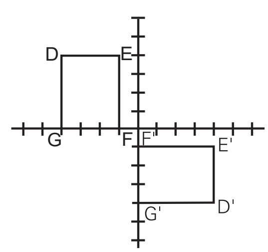
c) How does the equation of the image relate to the equation of the pre-image?

d) Starting with the equation $y = -3x - 1$, and using the same mapping rule $(x, y) \rightarrow (x, -y)$, predict the equation of the image.

Presentation

E7.1 Ask students to describe, in words and mapping notation, the transformation shown in each below.





Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) develop and analyse the properties of transformations and use them to identify relationships involving geometric figures

SCO: By the end of grade 9, students will be expected to

E7 analyse and represent combinations of transformations, using mapping notation

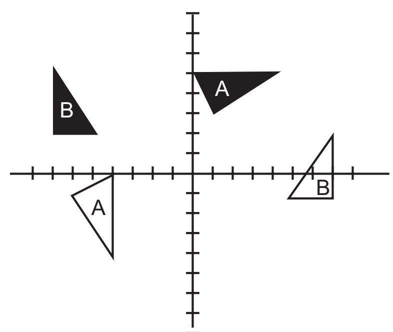
Elaboration – Instructional Strategies/Suggestions

E7 Students are expected to analyse a transformation given in mapping notation and to represent a transformation, using mapping notation.

- ❑ What mapping notation describes the movement of △PQR with vertices P (3, 1), Q (2, 4), and R (-1, 2) to △P´Q´R´ with vertices P´ (-3, 1), Q´ (-2, 4), and R´ (1, 2)?
- ❑ Using the mapping of (x, y) → (x, -y), find the coordinates of the image of △PQR. Describe the transformation in words.

Given an image and pre-image involving a combination of transformations, such as a glide reflection (slide followed by a reflection), students should be able to recognize that it is a combination of transformations and describe it, using mapping rules.

- ❑ In the figure below, the shaded diagrams are the originals (pre-images), and the non-shaded diagrams are the images. Describe a possible transformation, or set of transformations which will map the pre-image to the image in each case.



GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

E6/7.4 Draw $\triangle$BAT on a coordinate plane with B (5, 7), A (3, 3), and T(6, 2).

a) Rotate it 180° about the origin, label the image, and give the ordered pair for each new vertex.
b) Use mapping notation to describe the transformation.
c) Use the same coordinates for $\triangle$BAT above, but this time reflect it in the x-axis.
d) Compare the ordered pairs for each vertex. Describe the change, using mapping notation.

E6/7/8.1 Given the pre-image $\triangle$ABC with A (2, 3), B (-2, -1), and C (-4, 5),

a) explore if there is any relationship between the vertices when a dilatation by a factor of 2 is done, using (0, 0) as the centre of the dilatation
b) discuss the relationship between the image and pre-image relative to congruence, similarity, and orientation

Suggested Resources

GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

iii) *develop and analyse the properties of transformations and use them to identify relationships involving geometric figures*

SCO: By the end of grade 9, students will be expected to

E8 **investigate, determine, and apply the effects of transformations of geometric figures on congruence, similarity, and orientation**

Elaboration – Instructional Strategies/Suggestions

E8 In addressing outcome E8, students are given an opportunity to reflect on and consolidate several topics. For example, by using the coordinate grid to perform the various transformations, students can easily compare angle measurement and side lengths of images and pre-images to decide whether figures maintain congruence and/or similarity under the various transformations. This will provide an opportunity to revisit previously developed concepts and skills. Students can confirm, apply, and use the various properties of each transformation. In the case of reflections, students should know that

- the line segments joining points to their images are perpendicular to the reflection line and have their midpoint on the reflection line
- the reflection image of any figure is a congruent figure
- the orientation of a reflection image is the opposite of the original figure [That is, if $\triangle ABC$ is named clockwise, then $\triangle A'B'C'$ is named counter-clockwise.]

In the case of translations, students should know that

- the line segments joining points to their images are parallel and equal in length
- the translation image of any figure is a congruent figure
- the orientation of a translation image is the same as that of the original figure
- the translation images of lines or segments are parallel or collinear to their pre-images. More specifically, for 90º rotations students should know that:
  i) horizontal segments or lines become vertical, and vertical segments or lines become horizontal;
  ii) any segment or line and its image are perpendicular. For 180º rotations, students should know that segments and lines are parallel or collinear to their images.

In the case of rotations, students should know that

- a rotation of aº about a point X is such that a segment joining a point to X and a segment joining its image to X are equal in length and form an angle of aº
- the rotation image of any figure is a congruent figure
- the orientation of a rotation image is the same as that of the original figure

In the case of dilatations, students should know that

- the dilatation centre, a point, and its image form a line
- the ratio of the distance between the dilatation centre and the figure to the distance between the dilatation centre and the image is the same as the dilatation ratio
- the ratio of the length of a segment in the original figure to the length of a segment in the image is the same as the dilatation ratio
- the image is similar to the figure
- angle measures in the figure are the same as angle measures in the image

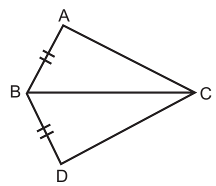
GCO (E): Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Worthwhile Tasks for Instruction and/or Assessment

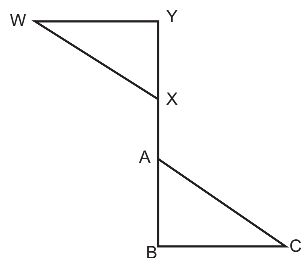
Pencil and Paper

E8.1 In each case shown, we want to show that the triangles are congruent. In order to prove that they are congruent, we decide to apply transformations. For each situation, give the transformation that seems to apply; i.e., for a reflection, give the reflection line; for a rotation, give the rotation centre and rotation angle; and for a translation, give a pair of corresponding points.

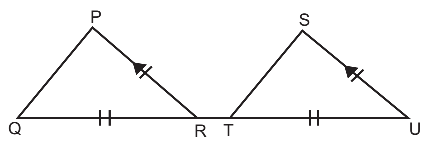
a)



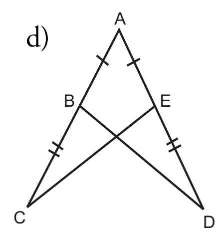
b)



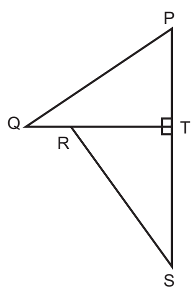
c)



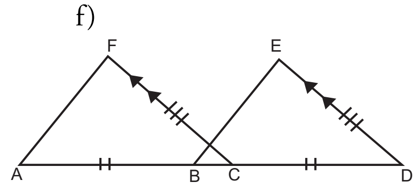
d)



e)



f)



E8.2 Refer to E8.1. If the transformation that seems to apply were done for each case,

a) what conclusions can be drawn?
b) what additional information would be necessary to conclude the triangles are congruent?

Portfolio

E8.3 Given the pre-image $\triangle ABC$ with A (2, 3), B (-2, -1), and C (-4, 5), have students find the images using each of the mapping rules (assuming that the dilatations and rotations are using the origin as the centre).

a) $(x, y) \rightarrow (x + 2, y - 3)$
b) $(x, y) \rightarrow (x, -y)$
c) $(x, y) \rightarrow (y, -x)$
d) $(x, y) \rightarrow (0.5x, 0.5y)$
e) Ask students to identify the above transformations and compare pre-images and images to determine which are congruent and which are similar.
f) Ask them to write about whether the same orientation is maintained in each of a) - d).

Suggested Resources

NINE - DATA

Data Management and Probability

General Curriculum Outcome F:

Students will solve problems involving the collection, display, and analysis of data.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *construct various data displays (both manually and via technology) and decide which is/are most appropriate*

iii) *draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)*

SCO: By the end of grade 9, students will be expected to

F1 **describe characteristics of possible relationships shown in scatterplots**

Elaboration – Instructional Strategies/Suggestions

F1 In interpreting scatterplots, students should consider whether the data being represented are continuous or discrete. Discrete data have definitive values, such as that represented by measuring the surface area of a cube train. As individual cubes are added to the train, the surface areas are represented by a set of discrete values. On the other hand, continuous data can take on all values within a given range. For example, the volume of water in a container as water flows into it over a period of time is continuous data. In establishing relationships, it is useful to draw a line to better illustrate a pattern. In the case of discrete data, students should be cautious about interpolating between the data points, since these conclusions may not be meaningful.

In the construction of scatterplots, the independent variable is always placed on the x-axis and the dependent variable is placed on the y-axis. For example, if a student wished to examine the effect of length of study time on test marks, the horizontal axis might be labelled "Number of Hours Spent Studying," and the vertical axis labelled "Mathematics Test Mark." We would expect that, as the independent variable (time) increased, then so did the dependent variable (grade). If the assumption is correct, the scatterplot should approximate a line with a positive slope. Further, if most of the points were closely grouped around the line, then we should be able to conclude that the relationship between study time and test marks is a strong one (see Figure 1). In the event the data points were dispersed but showed a generally positive trend, the relationship may be described as a weak positive relationship (see Figure 2). Figure 3 illustrates a situation where no relationship is apparent, while Figure 4 shows a strong negative relationship – that is, as the independent variable increases, the dependent variable decreases.

Consider the situations below:

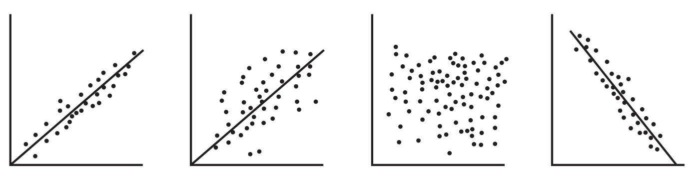


Fig. 1 Fig. 2 Fig. 3 Fig. 4

We refer to the strength of a relationship as correlation. This is elaborated on in F5.

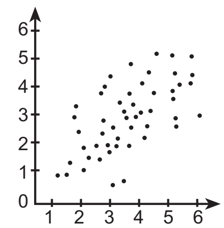
GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

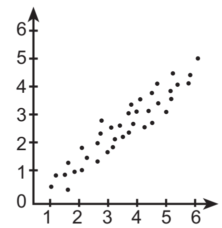
Pencil and Paper

F1.1 For the following displays,

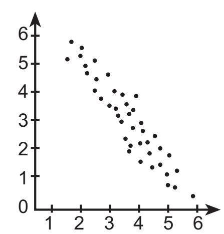
i)



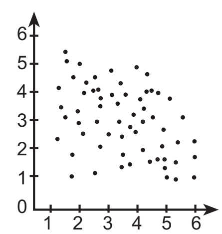
ii)



iii)



iv)



a) decide if there is a strong relationship, a weak relationship, or if there is no apparent relationship – for those that appear to have a strong or weak relationship, identify whether the slope of a line of best fit would be positive or negative.

b) complete the following statement – "As the values of the independent variable increase, the values of the dependent variable __________."

c) create a problem in which data may have been collected in order to produce each of the scatterplots

d) label the horizontal and the vertical axis on the basis of the problem that you created

e) write a conclusion based on the scatterplot and the problem situation you described in c)

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *construct various data displays (both manually and via technology) and decide which is/are most appropriate*

iii) *draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)*

SCO: By the end of grade 9, students will be expected to

F2 **sketch lines of best fit and determine their equations**

Elaboration – Instructional Strategies/Suggestions

F2 Students have worked with the slope and y-intercept method to find the equation of a line in C4. In order to get the line of best fit, they will use the eyeball method and then utilize the slope and y-intercept method to find the equation of the line. Graphing calculators can also be utilized to quickly determine a best-fit line.

By working with various examples, students should be encouraged to compare and contrast their lines of best fit and discuss reasons for possible differences that might exist for identical data. Students should see that the weaker the relationship between two variables, that is, the more dispersed the points are around the approximated line, the less reliable their conclusions will be. They should also consider the reliability of interpolations versus extrapolations when using scatterplots which show weak or strong relationships. Consider the confidence that one would have in an interpolation based on Figure 1 versus Figure 2 below.

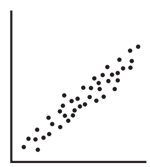


Fig. 1

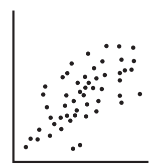


Fig. 2

There is a great opportunity to tie the study of scatterplots to science fair projects. In fact, the data collected to make a scatterplot are often more meaningful to students when they can relate them to other disciplines or to real life. Teachers should not feel the need to teach such topics twice. That is, if there is confidence that the topic has been fully developed in science, the topic may not require additional development in mathematics.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

F2.1 For each of the scatterplots shown in F1.1,

a) sketch a line of best fit
b) determine the slope of the line
c) determine the y-intercept
d) write the equation of the line
e) Compare your answers for parts a) to d) above with those of other students. Give reasons for any differences that may exist.

Investigation

F1/2/5.1 Ask students to attach a measuring tape to a wall and use it to measure (for numerous trials) the height of bounce of a ball (use a tennis ball, table tennis ball, or softball) and its drop height.

a) Ask students to plot the points on a grid.
b) Ask them what the two variables are.
c) Ask them if there is any apparent relationship between the two variables.
d) Ask students to use the eyeball method to fit a line to the data.
e) Ask them to estimate the y-intercept and find the slope.
f) Ask students to use the graph to find the bounce height for two pieces of data not collected.
g) Discuss with students whether the line trend that may appear in the data collected is likely to continue indefinitely.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *construct various data displays (both manually and via technology) and decide which is/are most appropriate*

iii) *draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)*

SCO: By the end of grade 9, students will be expected to

F3 **sketch curves of best fit for relationships that appear to be non-linear**

F4 **select, defend, and use the most appropriate methods for displaying data**

Elaboration – Instructional Strategies/Suggestions

F3 When students recognize that the pattern of the data does not appear to be linear, they should consider fitting a curve to the data. Construction of a curve of best fit can be done by inspection using a piece of cooked spaghetti or string. They may be able to predict that the relationship is not linear by looking at the pattern of the data in a table of values. In C3, students learned that, when the values of the independent variable are selected at regular intervals, a linear relationship produces values for the independent variable that are at or near regular intervals. When this is not the case, the data are non-linear. On the basis of the work done in C3, students may be able to predict a parabolic or exponential relationship. Teachers might wish to demonstrate to students a curve of best fit, using technology. This may extend to include finding the equation of the curve, but it should be noted that finding the equation of a curve of best fit is **not core** at this level and therefore should not be addressed as part of assessment.


Fig. 1

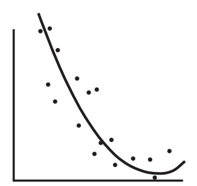


Fig. 2

In Figure 1, a line is drawn to fit the data, while in Figure 2 a curve is drawn for the same data. Students should discuss why the curve seems to represent the pattern shown in the data better than the line.

F4 This discussion should be integrated where possible into project work. Students should be asked to evaluate various situations to determine and debate why a particular display is best suited to a specific type of data, or to a given context. Students should be able to discuss this in terms of continuous versus discrete data sets. For example, given a bar graph and a line graph, ask students which would be more appropriate to display the amount of water flowing into a container. They should also be able to justify their choice.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper/Performance

F3.1 Bryce videotaped himself taking lay-up shots in basketball. He took ten lay-ups and used stop action to measure his height above the ground at various times. These data are shown in the table.

time (sec)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
height (m)	0.05	0.67	0.85	1.12	1.28	1.10	0.82	0.43	0.01

a) Graph the data and fit a line or curve to it.
b) Although the fit may not be perfect, use any patterns that are observable in the data to decide and justify whether it is parabolic or exponential.
c) Are the data in this situation discrete or continuous data? Explain.
d) Compare your answers with those of others in the class for parts a), b), and c). What do you notice?

F1/2/5.2 Sandy did a ball-bounce experiment and acquired the following data by using three trials at each height. The data are shown in the tables below.

Drop height (cm)	30.0	30.0	30.0	40.0	40.0	40.0	50.0	50.0	50.0
Bounce (cm)	20.2	20.6	19.9	23.2	23.4	23.1	29.1	29.8	31.0

Drop height (cm)	60.0	60.0	60.0	70.0	70.0	70.0	80.0	80.0	80.0
Bounce (cm)	34.4	35.0	35.3	41.0	41.4	40.6	51.0	51.5	50.8

a) Graph the data.
b) Is there any apparent relationship between the two variables?
c) Use the eyeball method to fit a line to the data.
d) Estimate the y-intercept and find the slope.
e) Use the graph to find the bounce height for 12 cm and 120 cm.
f) Is the data in this situation discrete or continuous data? Explain.
g) Find the average of the trials, plot the data again, and compare to the graph drawn of the individual trials. What do you notice?

Portfolio

F4.1 Ask students to analyse the graphs they made in response to questions on this page and decide whether the data could have been represented by a stem-and-leaf, box-and-whisker, circle graph, or histogram, instead of scatterplot. Ask them to justify their decisions.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *construct various data displays (both manually and via technololgy) and decide which is/are most appropriate*

iii) *draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)*

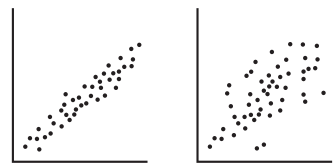
SCO: By the end of grade 9, students will be expected to

F5 **draw inferences and make predictions based on data analysis and data displays**

Elaboration – Instructional Strategies/Suggestions

F5 Students are expected to draw inferences and conclusions based on a variety of data displays, but most particularly, scatterplots, since the scatterplot is of greater focus at this grade level. The goodness of fit of data to a line is called the correlation. If the scatterplot approximates a line, students should be able to conclude that, when the slope is positive, the correlation is positive; likewise, when the slope is negative, the correlation is negative. Students should identify zero correlation as no apparent relationship, and a correlation of +1 or -1 as a perfect fit with the line, or perfect correlation. A strong relationship between the two variables increases the confidence that one can have in the predictions made based on that relationship.

❑ For the diagrams shown, in which case would you have a greater confidence in the predictions? Explain why.



It is important to discuss with students that correlation does not imply cause and effect. It is often wrongly assumed that, when there is a strong relationship between two events, one causes the other.

❑ John found that there was a strong positive relationship (correlation) between getting very high grades in the math test and having long hair. He concluded that to get a good grade in math you need to let your hair grow. Evaluate this conclusion.

(Note: **Calculation of correlation is not intended at this level**.)

The line of best fit for data should be used to make inferences about data that were not directly collected. That is, students should predict data between two known pieces of data (interpolate), and make predictions beyond the data that have been collected (extrapolate). Students should also use the equation of the line of best fit to determine data values which were not collected. That is, once the equation of the line of best fit has been found, students can select values for the independent variable and replace x in the equation to find corresponding values of y, the dependent variable.

Note: The elaboration for F5 is continued on the next 2-page spread.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper/Performance

F3/5.1 Jolene starts with a piece of paper which has an area of 128 cm^2. She folds the paper, then opens it and measures the area of each new rectangle formed. She makes a second fold which divides the paper into four rectangles and measures the area of each new rectangle again.

a) Make a table of values which relates the number of folds with the area of the rectangles which are formed, including up to 6 folds of the paper.

b) Fit a line or curve to the data and discuss whether the pattern appears to be linear, parabolic, or exponential. Justify your choice.

Portfolio

F5.1 Ask students to predict whether there is any relationship between the height of a person and the height of his or her father. Ask them to write an explanation of why this might be true or untrue in their particular case.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *demonstrate an appreciation of statistics as a decision-making tool by formulating and solving relevant problems (e.g., projects with respect to current issues and/or other academic disciplines)*

vi) *make convincing statistical arguments and evaluate those of others*

SCO: By the end of grade 9, students will be expected to

F5 **draw inferences and make predictions based on data analysis and data displays**

Elaboration – Instructional Strategies/Suggestions

F5 (Cont'd) Students should be reminded that in many situations selecting data points far beyond the collected data may not provide meaningful information. This should only be done by giving careful consideration to the context that the data represent. For example, the fact that Byron was 46 cm at birth and 1.4 m at age ten does not necessarily imply that he will be 2.34 m at age 20.

Consideration should also be given to the nature of the data, that is, whether the data is continuous or discrete. For example, it would seem most reasonable to predict the amount of water in a container after a given period of time if water is flowing into the container at a constant rate. However, predicting one student's test mark on the basis of the test marks of other students may be less reasonable.

Outliers for data were discussed in previous grades. The study of scatterplots provides a good context to revisit this notion. Students should continue to consider whether the outliers represent meaningful data in the context of any given problem. Students should be aware that an outlier may exist owing to an incorrect measurement or some other human error. Students should discuss ways of overcoming this difficulty, either by ignoring the outlier, or by retrial for the data that appear to be an outlier.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Portfolio

F1/2/3/5.1 Tell students that the fire hydrant at the end of Spruce Hill Road is leaking water. A pool of water has collected at the end of the road which is growing so that its radius is increasing at a rate of 2 cm per minute.

a) Ask students to create a table, a graph, and an equation to represent the growth in the radius of the spill for the first six minutes.
b) Ask them to create a table, a graph, and an equation to represent the growth in the circumference of the spill for the first six minutes.
c) Ask students to create a table, a graph, and an equation to represent the growth in the area radius of the spill for the first six minutes.
d) In case of a) - c), ask students if they were able to join the data points, and to explain why or why not.
e) Ask students, Which of a) - c) are linear? What is the shape of the other graphs? How did you decide?
f) In each case, ask students to use the table, graph, or equation to find the radius, the circumference, and the area when 20 minutes have passed. Ask them to consider the degree to which they have confidence in their prediction. [Students should consider two things here. Since the data are an exact fit with the line or curve, we can have a great degree of confidence in our prediction. However, we do not know anything about the land area involved. After the water reaches a certain height, it may start flowing down an incline, and the radius of the pool would cease to increase. Also, the water may be turned off by the time 20 minutes have passed. It should be noted that there is often a problem arising when extrapolation occurs which goes far beyond the collected data points. As well, there is an assumption that the depression in the ground is perfectly round, which is seldom the actual situation.]
g) Ask students if, in general, it was easier to find the radius, circumference, and area using the table, the graph, or the equation, and to explain why.

Suggested Resources

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

v) *demonstrate an appreciation of statistics as a decision-making tool by formulating and solving relevant problems (e.g., projects with respect to current issues and/or other academic disciplines)*

vi) *make convincing statistical arguments and evaluate those of others*

SCO: By the end of grade 9, students will be expected to

F6 demonstrate an understanding of the role of data management in society

F7 evaluate arguments and interpretations that are based on data analysis

Elaboration – Instructional Strategies/Suggestions

F6 It is useful to address this topic early in the school year so that students can keep a record of the information they study in other subjects or hear about in the media which is statistical in nature. In order to develop an appreciation for the role of data management in society, organize the students into small groups and have them discuss and list the decisions people make that are based on statistics. Discuss how statistics has increased our knowledge base. Ask the groups to discuss and report on how statistical information is used in areas such as smoking and/or alcohol consumption and its/their impact on health; speed limits related to type of road or number of accidents; prescription drugs and their side effects; various sports statistics which are kept and how those statistics are used; and polling as a means of predicting the outcome of an election. Discuss the overlap that exists in the study of probability and the study of statistics as it pertains to many of these areas.

F6/7 Ask students to research situations where decisions have been made that are based on some form of data collection. Students can analyse the situation from the perspective of the method of data collection, sampling procedure, method of presentation of data, and conclusions reached based on data collected. Questions can also be stimulated dealing with whether or not issues have been deliberately omitted, if there is any bias associated with the presenter, whether there are contrary arguments possible based on the same data set, whether the data show a strong enough pattern for use as a predictor, and whether extrapolation beyond the data is at all meaningful.

F7 Students can compare various methods of displaying data and evaluating their effectiveness. Comparisons of scale adjustments to indicate such things as degree of growth or loss should be explored. Discussion should take place regarding how the choice of certain graphs can lead to inaccurate judgments.

Students' understanding of statistics is enhanced by evaluating the arguments of others. This is particularly important since advertising, forecasting, and public policy are frequently based on data analysis. The media is full of representations of data to support statistical claims. These can be used to stimulate discussion.

GCO (F): Students will solve problems involving the collection, display, and analysis of data.

Worthwhile Tasks for Instruction and/or Assessment

Presentation

F7.1 Ask students to work in groups to develop a brief presentation of their arguments to each other, and then to present a critique for each other's arguments, concerning the following problem.

In a certain country the defence budget was 30 million dollars for 1980. The total budget for that country was 500 million dollars. The following year the defence budget was 35 million dollars, whereas the total budget was 605 million dollars. Inflation during the period between the two budgets was 10%.

a) You are invited to give a presentation for a pacifist society. You want to explain that the defence budget has decreased this year. Explain how you would do this.
b) You are invited to give a presentation at a military academy. You want to explain that the defence budget has increased this year. Explain how you would do this.

(Adapted from Grades 9-12 Addenda Series: Data Analysis and Statistics Across the Curriculum)

Project

F6/7.1

a) Ask students to find examples of data use from a recent newspaper or magazine. Ask them to discuss the role of the data component of the newspaper or magazine article and how the article would be changed if the reference to specific data had been removed from the article. [This can be assigned over a one- or two-week period to give students time to get to libraries.]
b) Where data displays are in an article, ask students to change the display in some way so the message conveyed by the data is modified to either enlarge upon or reduce the impact of the message conveyed by the data in the original display. [Over time, teachers can keep examples of what other students have found so that, when some students have difficulty finding anything that is suitable, teachers can refer back to their files of collected samples.]

Suggested Resources

StatsCan Website

Data Management and Probability

General Curriculum Outcome G:

Students will represent and solve problems involving uncertainty.

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations*

SCO: By the end of grade 9, students will be expected to

G1 make predictions of probabilities involving dependent and independent events by designing and conducting experiments and simulations

Elaboration – Instructional Strategies/Suggestions

G1 In grade 7, students explored independent events, using tree diagrams and area models. These may need to be revisited briefly with students.

It is important to give the students a clear understanding of the difference between events which are dependent and those which are independent. If the probability of the second event is affected by the outcome of the first event, then the two events are dependent. However, if the probability of the second event is not influenced by the outcome of the first event, then the two events are independent. Most situations that students have encountered in the curriculum that involve two events are independent. One of the most common experiments to illustrate dependence versus independence is simulated through drawing objects from a container. If you replace the first object before drawing the second, then the second event becomes independent of the first, whereas, if the first object is not replaced, then the second event is dependent on the first. Students should draw tree diagrams or area diagrams for both situations to help them understand the difference.

❑ Put two red and three whites cubes in a bag.
  a) What is the probability that two red cubes will be drawn from the bag if there is no replacement?
  b) What is the probability that two red cubes will be drawn from the bag if the first is replaced before drawing the second?

These questions can be answered experimentally by placing the cubes in a bag and conducting a series of trials. Such experiments may be done for their own sake, or to simulate some other event which cannot be modelled directly. The situation above could simulate a situation where a family has two boys and three girls and we want to find the probability that the first two children born are boys. Naturally, replacement would be required in this situation, since the events are independent of each other.

A simulation can be generated using a graphing calculator or computer software. A spreadsheet can be useful to record data. Whenever technology is available, it can be used as a support in achieving this outcome, but it should not totally replace hands-on activities. Simulation was addressed in grade 7, and more sophisticated situations involving complementary events were studied in grade 8. See the instructional implications for grade 7 for a detailed explanation of how to conduct a simulation.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G1.1 Ask students to work in groups to design a simulation for determining the probability that, in a family of three, all children will be girls. The use of a spreadsheet or graphing calculator is encouraged, but this activity can also be achieved using three coins.

a) Ask students to select an appropriate model for the simulation, and justify their choice. [The model can be two-colour counters, where one colour represents girls and the other represents boys.]
b) Ask students what the probability is that there will be at least one boy in this family of three children.
c) Ask them what the probability is that there will be all boys in this family of three children.

G1.2 Prior to class, put two counters of one colour (black) and four of another colour (red) in a bag. Ask students to work with a partner, where one person acts as recorder and the other draws objects from a bag.

a) Ask them to draw a counter from the bag, and then replace the counter and draw again, and record the results of the two draws. Ask them to repeat this activity 50 times and record the results.
    i) Have students use the results to estimate the probability of getting two blacks, two reds, and a black and a red.
    ii) Have students use the results to estimate the probability of **not** getting two that are the same colour.
b) Ask students to draw a counter from the bag, and then draw a second counter. Ask them to record the results of the two draws, and then return the two counters to the bag. Have them repeat this process for 50 trials.
    i) Ask students to use the results to estimate the probability of getting two blacks, two reds, and a red and a black.
    ii) Ask students to use the results to estimate the probability of **not** getting two that are the same colour.
c) Ask students which of part a) and b) would they describe as dependent, and which would they describe as independent. Ask them to explain their choice.
d) Ask students to pool their results and discuss any differences they notice between the pooled data and the data collected by each pair.

G1.3 Sue placed two green and two red cubes in a bag. She wanted to find the probability of drawing two green if the first one is not returned before drawing the second. Ask students to design an experiment to solve this problem and conduct it. [Have students save the data collected in each of G1.1, G1.2, and G1.3 since it will be useful for outcome G3.]

Suggested Resources

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

i) *make predictions regarding, and design and carry out, probability experiments and simulations in relation to a variety of real-world situations*

ii) *derive theoretical probabilities, using a range of formal and informal techniques*

SCO: By the end of grade 9, students will be expected to

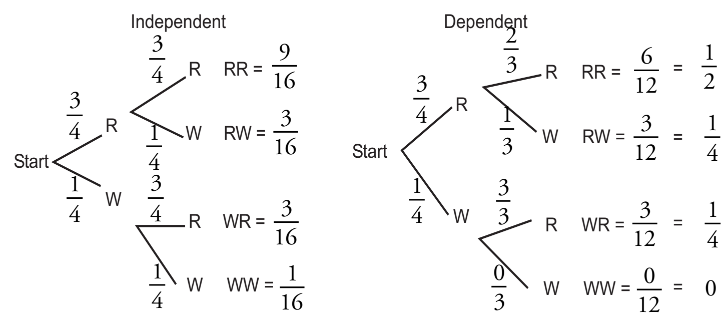
G2 determine theoretical probabilities of independent and dependent events

Elaboration – Instructional Strategies/Suggestions

G2 It is not always directly obvious when events are dependent or independent. Sometimes students will need to discuss whether two events are truly independent of each other. In general, event A is independent of event B if the probability of A is not affected by the occurrence or nonoccurrence of B. Likewise, event B is dependent on event A when the result in event B is directly affected by the occurrence or nonoccurrence of A.

- ❑ Three red and one white counter are placed in a bag. The probability of drawing two reds when the first one is replaced before drawing the second is $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$. Ask students to create a tree diagram and use it to explain why this is true.
- ❑ The probability of drawing two reds wheg the first one is not replaced before drawing the second is $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$. Once the first red is drawn, the sample size for the second event changes to two red and one white. Naturally, in this situation, the probability of getting two reds if a white is drawn on the first try is 0. Ask students to explain why this is so.

In general, for any two independent events, A and B, the probability of A and B is equal to P(A) × P(B). Students can also use tree diagrams to solve compound-event problems. This would be considered an informal method. For the examples above, probability tree diagrams are shown. The various branches represent different probabilities; therefore, the branches are labelled in order to help clarify what each branch represents.



These tree diagrams are quite different from the ones studied in grade 7. In grade 7, tree diagrams were used to identify all the possible outcomes.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Pencil and Paper

G2.1 A box contains 3 red balls and 2 blue balls.

a) You remove two balls from the box; the second one is removed without replacing the first.
   i) Draw a tree diagram to show all the possible outcomes for this situation.
   ii) What is the probability of drawing two blue balls?

b) Suppose you draw two balls from the box, and the first one is replaced before drawing the second.
   i) Draw a tree diagram to show all the possible outcomes for this situation.
   ii) What is the probability of drawing two blue balls?

G2.2 The following describes events A and B. Decide whether the events are dependent or independent and explain your thinking.

a) A. Mrs. Brown's first child was a boy.
   B. Mrs. Brown's second child will be a boy.

b) A. It snowed last night.
   B. Jon will be late for school this morning.

c) A. Leif swam 2 hours every day for the last ten months.
   B. Leif's swimming times have improved.

d) A. Allison got an A in her last math test.
   B. Allison will get an A in her next math test.

e) A. Matthew got a head in his last coin toss.
   B. Matthew will get a head in his next coin toss.

G2.3 This problem is based on the same situation as that described in G1/2.1. In this situation, students are expected to solve the problem, using theoretical probability. Prior to class, put two black counters and four red counters in a bag.

a) Ask students to find the probability of drawing two black counters, two red counters, and a red and a black counter from the bag when the first one is replaced before drawing the second.

b) Ask students to find the probability of drawing two black counters, two red counters, and a red and a black counter from the bag when the first one is **not** replaced before drawing the second.

G2.4 Sue placed two green and two red cubes in a bag. Find the probability of drawing two green cubes if the first one is not returned before drawing the second.

Suggested Resources

Dice Game, NCTM Addenda Series, 1991, pp. 12-19

GCO (G): Students will represent and solve problems involving uncertainty.

KSCO: By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to

ii) *derive theoretical probabilities, using a range of formal and informal techniques*

iii) *determine and compare experimental and theoretical results*

iv) *relate a variety of numerical expressions to the corresponding experimental or simulation situation*

SCO: By the end of grade 9, students will be expected to

G3 **demonstrate an understanding of how experimental and theoretical probabilities are related**

G4 **recognize and explain why decisions based on probabilities may be combinations of theoretical calculations, experimental results, and subjective judgments**

Elaboration – Instructional Strategies/Suggestions

G3 Once students have worked with probability experiments and derived theoretical probability, they should be able to compare the results obtained from each method. Students should be able to relate the experimental probability with results achieved, using the definition of theoretical probability. Discuss with the students when they can be comfortable that the experimental probability is a close approximation of the theoretical probability, and what can be done to increase their confidence in experimental results. Discussion here should focus on the influence of increasing sample size. For example, students can look at the results when they have collected a sample of size 50, pool their data with another student's to have data for sample size 100, and then pool the data for the whole class to see what effect a very large sample size may have on the results.

G4 Students should relate to how decision making is affected by the combination of probability and subjective judgments. For example, consider the variety of strategies people use when choosing their lottery numbers. Some use the same numbers for repeated lotteries, others use past frequencies to select their numbers, and others allow their numbers to be randomly selected.

Another example to consider is the impact that the probability of rainfall has on decisions made about whether to engage in some outdoor sports, replace a window in a house, or put the laundry out on a clothes line.

Students might engage in evaluating situations that are amenable to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Road accidents with/without seatbelts is a good example for safe prediction, while the use of airbags involves a more questionable situation. There are many situations where the unknowns are so great that probabilistic arguments only appear authoritative, such as life on other planets, dangers of transgenic animals, and the threat of global warming. Discuss with students: What are the reasons for the uncertainty? What are the important questions to ask regarding a situation in order to reduce it to probabilistic form?

- ❑ Discuss why knowing that one party is favoured to win an election by 65% of voters may or may not influence voting of individuals on election day.

GCO (G): Students will represent and solve problems involving uncertainty.

Worthwhile Tasks for Instruction and/or Assessment

Performance

G3.1 Ask students to

a) roll two dice 50 times to determine the experimental probability of rolling a total of 7
b) use a rectangular array to determine the theoretical probability of getting a seven
c) discuss the difference in theoretical and experimental probability in this case, and why there are differences

Pencil and Paper

G3.2 Compare and discuss the results in G1.2 and G2.3.

G3.3 Find the theoretical probability associated with G1.1, and compare it to the experimental results.

Interview

G4.1 Tell students that they have been told that they tested positive on a medical test that is 90% accurate. The disease for which they were tested is very rare – only one person in a million suffers from it. Ask students if they should assume they have the disease, and to explain their answer. [Students should discuss the fact that the sample size for positive must be very small because the disease is so rare. On the other hand, the sample size for negative is probably very large for exactly the same reason. We can, therefore, probably have greater confidence in negative results then positive results.]

Presentation

G4.2 Odette knows that theoretically she has a 1 in 2 chance of getting a head when she flips a coin. Claude had a particular coin that, when flipped 50 times, came up heads 40 of the 50 times. Ingrid feels that, even if there is an equal chance of getting heads, heads will appear more often because she feels it is her lucky choice. Ask students to categorize the three situations as subjective (based on opinion), experimental, or theoretical, and present to the class how each can play a part in decision making.

Portfolio

G4.3 A notorious individual was found guilty of murder, partly because of forensic testing which pointed to him. It was later discovered that the lab which did the testing knew that the sample came from a suspect, and were being asked for a judgment by the prosecutor. Ask students what biases may affect the judgment of the lab. Ask them if these biases could have changed the probability estimates of the lab. Ask students if there is any way that these biases might be reduced or eliminated. Ask if their solutions would be very difficult or costly to implement.

Suggested Resources

G4 "Montana Red Dog" card game described in "Dealing with Data and Chance," NCTM

Addenda Series, 1991, pp. 41-45

NINE - INDEX