Atlantic Canada Mathematics Curriculum

New Brunswick
Department of Education
Educational Programs & Services Branch

New Nouveau Brunswick

CURRICULUM

Patterns and Relations 113

(Implementation Edition)

2002

Additional copies of this document (*Patterns and Relations 113*)
may be obtained from the Instructional Resources Branch.
Title Code (840920)

Acknowledgements

The departments of education of New Brunswick, Newfoundland and Labrador, Nova Scotia, and Prince Edward Island gratefully acknowledge the contributions of the following groups and individuals toward the development of this *Patterns and Relations 113* curriculum guide.

- The Regional Mathematics Curriculum Committee; current and past representatives include the following:

New Brunswick
Greta Gilmore, Mathematics Teacher,
Belleisle Regional High School

John Hildebrand, Mathematics Consultant,
Department of Education

Pierre Plourde, Mathematics Consultant,
Department of Education

Nova Scotia
Richard MacKinnon, Mathematics Consultant,
Department of Education & Culture

Lynn Evans Phillips, Mathematics Teacher,
Park View Education Centre

Newfoundland and Labrador
Sadie May, Distance Education Coordinator for Mathematics
Department of Education

Patricia Maxwell, Program Development Specialist,
Department of Education

Prince Edward Island
Elaine Somerville, Mathematics/Science Consultant,
Department of Education

- The Provincial Curriculum Working Group, comprising teachers and other educators in Nova Scotia, which served as lead province in drafting and revising the document.
- The teachers and other educators and stakeholders across Atlantic Canada who contributed to the development of the *Patterns and Relations 113* curriculum guide.

Table of Contents

I. Background and Rationale

A. Background

Mathematics curriculum reform in Atlantic Canada is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society. Curriculum reform has been motivated by a desire to ensure that students in Atlantic Canada benefit from world-class curriculum and instruction in mathematics as a significant part of their learning experiences.

The *Foundation for the Atlantic Canada Mathematics Curriculum* (1996) firmly establishes the *Curriculum and Evaluation Standards for School Mathematics* (1989) of the National Council of Teachers of Mathematics (NCTM) as a guiding beacon for pursuing this vision, which embraces the principles of students learning to value and become active "doers" of mathematics and advocates a curriculum which focusses on the unifying ideas of mathematical problem solving, communication, reasoning, and connections. These principles and unifying ideas are reaffirmed with the publication of NCTM's *Principles and Standards for School Mathematics* (2000). The *Foundation for the Atlantic Canada Mathematics Curriculum* establishes a framework for the development of detailed grade-level documents describing mathematics curriculum and guiding instruction.

Mathematics curriculum development has taken place under the auspices of the Atlantic Provinces Education Foundation (APEF), an organization sponsored and managed by the governments of the four Atlantic Provinces. APEF has brought together teachers with department of education officials to co-operatively plan and execute the development of curricula in mathematics, science, language arts, and other curricular areas. Each of these curriculum efforts has been aimed at producing a program that would ultimately support the Essential Graduation Learnings (EGLs), also developed regionally. These EGLs and the contribution of the mathematics curriculum to their achievement are presented in the "Outcomes" section of the mathematics foundation document.

B. Rationale

The *Foundation for the Atlantic Canada Mathematics Curriculum* provides an overview of the philosophy and goals of the mathematics curriculum, presenting broad curriculum outcomes and addressing a variety of issues with respect to the learning and teaching of mathematics. This curriculum guide is one of several which provide greater specificity and clarity for the classroom teacher. The *Foundation for the Atlantic Canada Mathematics Curriculum* describes the mathematics curriculum in terms of a series of outcomes—General Curriculum Outcomes (GCOs), which relate to subject strands, and Key-Stage Curriculum Outcomes (KSCOs), which articulate the GCOs further for the end of grades 3, 6, 9, and 12. This guide builds on the structure introduced in the foundation document, by relating Specific Curriculum Outcomes (SCOs) to KSCOs for *Patterns and Relations 113*. Figure 1 further clarifies the outcome structure.

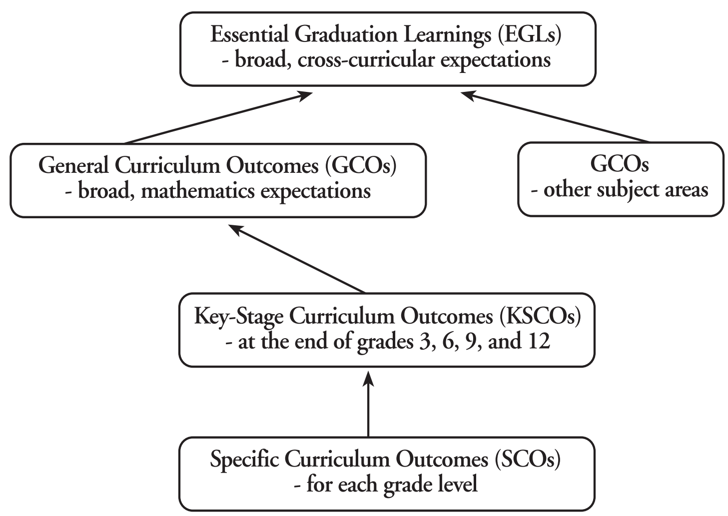


Figure 1: Outcome Framework

This mathematics guide is based upon several key assumptions or beliefs about mathematics learning which have grown out of research and practice, including the following: (i) mathematics learning is an active and constructive process; (ii) learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles and at different rates; (iii) learning is most likely when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and nurtures positive attitudes and sustained effort; (iv) learning is most effective when standards of expectation are made clear and assessment and feedback are ongoing; and (v) learners benefit, both socially and intellectually, from a variety of learning experiences, both independent and in collaboration with others.

II. Program Design and Components

A. Program Organization

As indicated previously, the mathematics curriculum is designed to support the Atlantic Canada Essential Graduation Learnings (EGLs). The curriculum is designed to significantly contribute to students meeting each of the six EGLs, with the communication and problem-solving EGLs relating particularly well with the curriculum's unifying ideas. (See the "Outcomes" section of the *Foundation for the Atlantic Canada Mathematics Curriculum*.) The foundation document then goes on to present student outcomes at key stages of the student's school experience.

This curriculum guide presents specific curriculum outcomes for *Patterns and Relations 113*. As illustrated in Figure 2, these outcomes represent the step-by-step means by which students work toward accomplishing the key-stage curriculum outcomes, the general curriculum outcomes, and, ultimately, the essential graduation learnings.

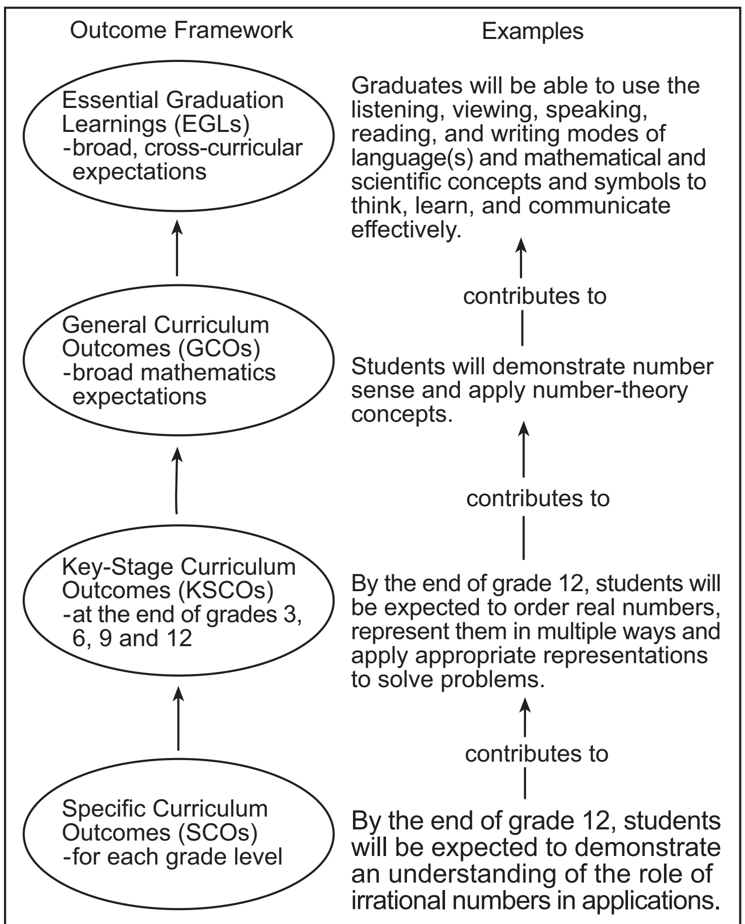


Figure 2: Examples of Outcomes

It is important to emphasize that the initial presentation of the specific curriculum outcomes for this course (pp. 17-28) follows the outcome structure established in the *Foundation for the Atlantic Canada Mathematics Curriculum* and does not represent a natural teaching sequence. In *Patterns and Relations 113*, however, a suggested teaching order for specific curriculum outcomes has been given within a sequence of four topics or units (i.e., Applications in Trigonometry; Patterns; Quadratics; and Exponential Growth). While the units are presented with a specific teaching sequence in mind, some flexibility exists as to the ordering of units within the course. It is expected that teachers will make individual decisions as to what sequence of topics will best suit their classes. In most instances, this will occur in consultation with fellow staff members, department heads, and/or district level personnel.

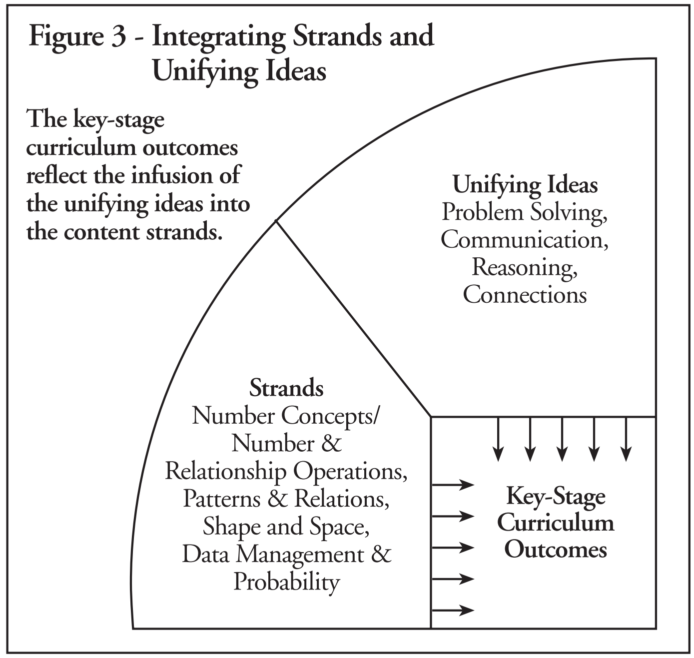
Decisions on sequencing will depend on a number of factors, including the nature and interests of the students themselves. For instance, what might serve well as a "kickoff" topic for one group of students might be less effective in that role with a second group. Another consideration with respect to sequencing will be co-ordinating the mathematics program with other aspects of the students' school experience. An example of such co-ordination would be studying aspects of measurement in connection with appropriate topics in science. As well, sequencing could be influenced by events outside of the school, such as elections, special community celebrations, or natural occurrences.

B. Unifying Ideas

The NCTM *Curriculum and Evaluation Standards* (1989) and *Principles and Standards* (2000) establishes mathematical problem solving, communication, reasoning, and connections as central elements of the mathematics curriculum. The *Foundation for the Atlantic Canada Mathematics Curriculum* (pp. 7-11) further emphasizes these unifying ideas and presents them as being integral to all aspects of the curriculum. Indeed, while the general curriculum outcomes are organized around content strands, every opportunity has been taken to infuse the key-stage curriculum outcomes with one or more of the unifying ideas. This is illustrated in Figure 3.

These unifying concepts serve to link the content to methodology. They make it clear that mathematics is to be taught in a problem-solving mode; classroom activities and student assignments must be structured so as to provide opportunities for students to communicate mathematically; via teacher encouragement and questioning, students must explain and clarify their mathematical reasoning; and mathematics with which students are involved on a day-to-day basis must be connected to other mathematics, other disciplines, and/or the world around them.

Students will be expected to address routine and/or non-routine



mathematical problems on a daily basis. Over time, numerous problem-solving strategies should be modelled for students, and students should be encouraged to employ various strategies in many problem-solving situations. While choices with respect to the timing of the introduction of any given strategy will vary, strategies such as try-and-adjust, look for a pattern, draw a picture, act it out, use models, make a table or chart, and make an organized list should all become familiar to students during their early years of schooling, whereas working backward, logical reasoning, trying a simpler problem, changing point of view, and writing an open sentence or equation would be part of a student's repertoire in the later elementary years. During middle school and the 9/10 years, this repertoire will be extended to include such strategies as interpreting formulas, checking for hidden assumptions, examining systematic or critical cases, and solving algebraically. *Patterns and Relations 113* will continue to develop students' problem-solving reportoires.

Opportunities should be created frequently to link mathematics and career opportunities. Students need to be aware of the importance of mathematics and the need for mathematics in so many career paths. This realization will help maximize the number of students who strive to develop and maintain the mathematical abilities required for success in higher-level mathematics programming in senior high mathematics and beyond.

C. Learning and Teaching Mathematics

The unifying ideas of the mathematics curriculum suggest quite clearly that the mathematics classroom needs to be one in which students are actively engaged each day in the doing of mathematics. No longer is it sufficient or proper to view mathematics as a set of concepts and algorithms for the teacher to transmit to students. Instead, students must come to see mathematics as a vibrant and useful tool for helping them understand their world, and as a discipline which lends itself to multiple strategies, student innovation, and, quite often, multiple solutions. (See the "Contexts for Learning and Teaching" section of the foundation document.)

The learning environment will be one in which students and teachers make regular use of manipulative materials and technology, and actively participate in discourse and conjecture, verify reasoning, and share solutions. This environment will be one in which respect is given to all ideas in which reasoning and sense making are valued above "getting the right answer." Students will have access to a variety of learning resources, will balance the acquisition of procedural skills with attaining conceptual understanding, will estimate routinely to verify the reasonableness of their work, will compute in a variety of ways while continuing to place emphasis on mental computation skills, and will engage in homework as a useful extension of their classroom experiences.

D. Meeting the Needs of All Learners

The *Foundation for the Atlantic Canada Mathematics Curriculum* stresses the need to deal successfully with a wide variety of equity and diversity issues. Not only must teachers be aware of, and adapt instruction to account for, differences in student readiness, but they must also remain aware of avoiding gender and cultural biases in their teaching. Ideally, every student should find his/her learning opportunities maximized in the mathematics classroom.

NCTM's *Principles and Standards* (2000) cites equity as a core element of its vision for mathematics education. "All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students"(p. 12).

At grade 11 in New Brunswick, variations in student readiness, aptitude, and post-secondary intentions are addressed in significant part by the provision of courses at levels 1, 2 and 3. Students at all levels will work toward achievement of the same key-stage and general curriculum outcomes, and many of the course-specific curriculum outcomes will also be the same or similar. As well, the instructional environment and philosophy should be the same at all levels, with high expectations maintained for all students. The significant difference between levels will be the depth, breadth, and degree of sophistication and formalism expected with respect to each general outcome. Similarities between courses should allow some students to move from one course level to another.

By and large, Level 3 courses will be characterized by a greater focus on concrete activities, models, and applications, with less emphasis given to formalism, symbolism, computational or symbol-manipulating facility, and mathematical structure. Level 1 and 2 courses will involve greater attention to abstraction and more sophisticated generalizations, while Level 3 courses would see less time spent on complex exercises and connections with advanced mathematical ideas. Level 1 courses, which are designed for particularly talented students of mathematics, will be characterized by both more sophisticated engagement with mathematical concepts and techniques, and the extension of some topics beyond the scope provided at Level 2. These extensions will be included in Level 2 curriculum guides and identified with a symbol.

By way of a brief illustration, students at all levels should develop an understanding of exponential relationships. Students taking Level 3 courses have as much need as others to understand the nature of exponential relationships, given the central place of these relationships in universal, everyday issues such as investment, personal and government debt, and world population dynamics. The nature of exponential relationships can be developed through concrete, hands-on experiments and data analysis that does not require a lot of formalism or symbol manipulation. The more formal and symbolic operations on exponential relationships will be much more prevalent in Level 1 and 2 courses.

Finally, within any given course at any level, teachers must understand, and design instruction to accommodate, differences in student learning styles. Different instructional modes are clearly appropriate, for example, for those students who are primarily visual learners versus those who learn best by doing. Further, the practice of designing classroom activities to support a variety of learning styles must be extended to the assessment realm; such an extension implies the use of a wide variety of assessment techniques, including journal writing, portfolios, projects, presentations, and structured interviews.

E. Support Resources

This curriculum guide represents the central resource for the teacher of *Patterns and Relations 113*. Other resources are ancillary to it. This guide should serve as the focal point for all daily, unit, and course-long planning, as well as a reference point to determine the extent to which the instructional outcomes should be met.

Nevertheless, other resources will be significant in the mathematics classroom. Textual and other print resources will be significant to the extent that they support the curriculum goals. Teachers will need professional resources as they seek to broaden their instructional and mathematical skills. Key among these are the NCTM publications, including the *Principles and Standards for School Mathematics, Assessment Standards for School Mathematics, Curriculum and Evaluation Standards for School Mathematics*, the *Addenda Series*, *Professional Standards for Teaching Mathematics,* and the various NCTM journals and yearbooks. As well, manipulative materials and appropriate access to technological resources (e.g., software, videos) should be available. Calculators will be an integral part of many learning activities.

F. Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different than were those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss with parents changes in mathematical pedagogy and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their children in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their children's lives, assisting children with mathematical activities at home, and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

G. Connections Across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating learning experiences—through teacher-directed activities, group or independent exploration, and other opportune learning situations. However, it should be remembered that certain aspects of mathematics are sequential, and need to be developed in the context of structured learning experiences.

The concepts and skills developed in mathematics are applied in many other disciplines. These include science, social studies, music, technology education, art, physical education, and home economics. Efforts should be made to make connections and use examples which apply across a variety of discipline areas.

In science, for example, the concepts and skills of measurement are applied in the context of scientific investigations. Statistical concepts and skills are applied as students collect, present, and analyse data. Examples and applications of many mathematical relations and functions abound.

In social studies, knowledge of confidence intervals is valuable in intrepreting polling data, and an understanding of exponential growth is necessary to appreciate the significance of government debt and population growth. As well, students read, interpret, and construct tables, charts, and graphs in a variety of contexts such as demography.

Opportunities for mathematical connections are also plentiful in physical educaiton, many technological courses and the fine arts.

III. Assessment and Evaluation

A. Assessing Student Learning

Assessment and evaluation are integral to the process of teaching and learning. Ongoing assessment and evaluation are critical, not only with respect to clarifying student achievement and thereby motivating student performance, but also for providing a basis upon which teachers make meaningful instructional decisions. (See "Assessing and Evaluating Student Learning" in the *Foundation for the Atlantic Canada Mathematics Curriculum*.)

Characteristics of good student assessment should include the following: i) using a wide variety of assessment strategies and tools; ii) aligning assessment strategies and tools with the curriculum and instructional techniques; and iii) ensuring fairness both in application and scoring. The *Principles for Fair Student Assessment Practices for Education in Canada* elaborate good assessment practice and serve as a guide with respect to student assessment for the mathematics foundation document. (See also, Appendix A, "Assessing and Evaluating Student Learning.")

B. Program Assessment

Program assessment will serve to provide information to educators as to the relative success of the mathematics curriculum and its implementation. It will address such questions as the following: Are students meeting the curriculum outcomes? Is the curriculum being equitably applied across the region? Does the curriculum reflect a proper balance between procedural knowledge and conceptual understanding? Is technology fulfilling a proper role?

IV. Designing an Instructional Plan

It is important to develop an instructional plan for the duration of the course. Without such a plan, it is easy to run out of time before all aspects of the curriculum have been addressed. A plan for instruction that is comprehensive enough to cover all outcomes and topics will help to highlight the need for time management.

It is often advisable to use pre-testing to determine what students have retained from previous grades relative to a given topic or set of outcomes. In some cases, pre-testing may also identify students who have already acquired skills relevant to the current course. Pre-testing is often most useful when it occurs one to two weeks prior to the start of a a topic or set of outcomes. When the pre-test is done early enough and exposes deficiencies in prerequisite knowledge/ skills for individual students, sufficient time is available to address these deficiencies prior to the start of the topic/unit. When the whole group is identified as having prerequisite deficiencies, it may point to a lack of adequate development or coverage in previous grades. This may imply that an adjustment is required to the starting point for instruction, as well as a meeting with other grade level teachers to address these concerns as necessary.

Many topics in mathematics are also addressed in other disciplines, even though the nature and focus of the desired outcome is different. Whenever possible, it is valuable to connect the related outcomes of various disciplines. This can result in an overall savings in time for both disciplines. The most obvious of these connections relate to the use of measurement in science and the use of a variety of data displays in social studies.

V. Curriculum Outcomes

The pages that follow provide details regarding both specific curriculum outcomes and the four topics/units that comprise *Patterns and Relations 113*. The specific curriculum outcomes are presented initially, then the details of the units follow in a series of two-page spreads. (See Figure 4 on next page.)

This guide presents the curriculum for *Patterns and Relations 113* so that a teacher may readily view the scope of the outcomes which students are expected to meet during the year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how the students' learnings in this course are part of a bigger picture of concept and skill development. (See Appendix B for a complete listing of the SCOs for grades 9 and 10.)

Within each unit, the specific curriculum outcomes are presented on two-page spreads. At the top of each page, the overarching topic is presented, with the appropriate SCO(s) displayed in the left-hand column. The second column of the layout is entitled "Elaboration-Instructional Strategies/Suggestions" and provides a clarification of the specific curriculum outcome(s), as well as some suggestions of possible strategies and/or activities which might be used to achieve the outcome(s). While the strategies and/or suggestions presented are not intended to be rigidly applied, they will help to further clarify the specific curriculum outcome(s) and to illustrate ways to work toward the outcome(s) while maintaining an emphasis on problem solving, communications, reasoning, and connections. To readily distinguish between activities and instructional strategies, activities are introduced in this column of the layout by the symbol ❒.

The third column of the two-page spread, "Worthwhile Tasks for Instruction and/or Assessment," might be used for assessment purposes or serve to further clarify the specific curriculum outcome(s). As well, those tasks regularly incorporate one or more of the four unifying ideas of the curriculum. These sample tasks are intended as examples only, and teachers will want to tailor them to meet the needs and interests of the students in their classrooms. The final column of each display is entitled "Suggested Resources" and will, over time, become a collection of useful references to resources which are particularly valuable with respect to achieving the outcome(s).

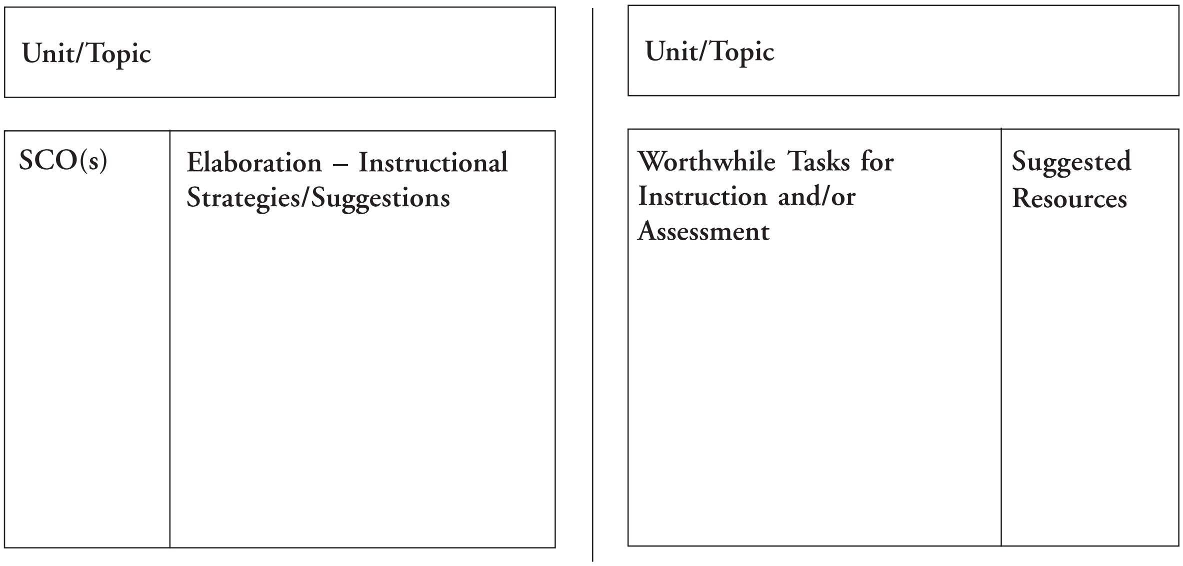


Figure 4: Layout of a 2-Page Spread

Specific Curriculum Outcomes
(by GCO)

GCO A: Students will demonstrate number sense and apply number theory concepts.

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i) demonstrate an understanding of number meanings with respect to real numbers* SCO: By the end of *Patterns and Relations 113*, students will be expected to	
A1 demonstrate an understanding of and apply zero and negative exponents	**A1** Students will understand the properties governing zero and negative exponents and apply them to rewrite and evaluate expressions. These properties will take their place among other exponent properties (see SCO A2), and should be developed using the characteristics of exponential relationships (C13) and patterns (C5). Unit 4, pp. 74, 76
A2 develop, demonstrate an understanding of, and apply properties of exponents	**A2** Students will build upon previous knowledge of exponents and identify the properties of zero and negative exponents (see SCO A1). Students will apply the properties to rewrite, simplify and evaluate expressions involving exponents. Unit 4, pp. 74, 76
A3 demonstrate an understanding of the role of irrational numbers in applications	**A3** Students will encounter irrational numbers in the context of solving problems involving quadratic equations (see SCOs B3 and C23). They will need to make appropriate decisions with respect to expressing roots exactly (e.g., $-2+\sqrt{7}$) or as rational approximations (e.g., 0.65), and be aware of the inaccuracies produced by rounding. Unit 3, p. 60
ii) order real numbers, represent them in multiple ways and apply appropriate representations to solve problems	
A8 demonstrate an understanding of the exponential growth nature of compound interest	**A8** One of the compelling reasons for studying exponential growth is so that students will have a realistic appreciation of the accelerating nature of the growth of both debt and investments, when they are subject to compound interest (see SCO B5). Studying exponential relationships, both numerically and graphically, will assist students in understanding that debt and investment (in either a personal or governmental context) must be carefully managed. Unit 4, p. 72

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

	Elaboration
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to *i)* *explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism*	
SCO: By the end of *Patterns and Relations 113*, students will be expected to	
B1 demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations	**B1** In general, students will need to understand that many arithmetic properties (e.g., distributive property) and principles (e.g., adding equals to equals produces equals) apply in algebraic situations. In particular, students will need to manipulate trigonometric expressions and equations, quadratic equations, and expressions involving exponents. This outcome will not be addressed in isolation but rather in connection with SCOs such as B4, C28, B9 and C23. Unit 1, pp. 30, 34; Unit 3, pp. 60, 62
B4 use the calculator correctly and efficiently	**B4** Students will need to use numerous calculator functions correctly, and make good choices with respect to their efficient use, in a variety of situations. These include trigonometric applications (see SCOs D4, C28, B1 and D5) and graphing relations and performing regression analysis (see SCOs such as C1, C21, F2 and F3). Unit 1, pp. 30, 32, 34, 36
ii) *derive, analyze and apply computational procedures in situations involving all representations of real numbers*	
B3 apply the quadratic formula	**B3** Students should be presented with the quadratic formula as an alternative means (other than factoring or reading x-intercepts from a graph) of solving quadratic equations. Students will apply the formula to given equations, determine the roots, and make decisions with respect to admissible vs. inadmissible roots. This outcome will be addressed in conjunction with SCO C23. Unit 3, p. 58

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration

B5 demonstrate an understanding of and apply compound interest

B5 Students will understand the connection between exponential growth and compound interest (see SCO A8) and how it differs from simple interest. They will also apply compound interest in problem contexts (C26), including those involving annuities (B6). Unit 4, pp. 72, 78, 80

B6 determine the amount and present value of annuities

B6 Students will use time diagrams, formulas and/or finance features on calculators to solve problems with respect to annuities (e.g., mortgage problems). This outcome will be connected to SCOs B5 and C26. Unit 4, pp. 78, 80

iii) derive, analyze and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations

B9 perform operations on algebraic expressions and equations

B9 Students will rewrite and/or solve expressions and equations. In the case of quadratic equations, for example, this might be done to identify the a, b and c values for the quadratic formula or to enter the equation in a graphing calculator. Addressing this outcome will be closely associated with SCOs such as B1 and C23.
Unit 3, pp. 60, 62

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

i) model real-world problems using functions, equations, inequalities and discrete structures

SCO: By the end of *Patterns and Relations 113*, students will be expected to

C1 model real-world phenomena using quadratic equations

C1 Mathematical modeling consists of providing a (simplified) mathematical description of a particular phenomenon. While mathematical models take many forms (e.g., scale diagrams, tables of values, graphs), in meeting this outcome students will describe phenomena using quadratic equations. The expectation here is that students will generate equations by applying quadratic regression to data/scatter plots. This outcome will be addressed in connection with SCOs such as C21, C8, C5 and C23. Unit 3, pp. 54, 56, 62

ii) represent functional relationships in multiple ways (e.g., written descriptions, tables, equations and graphs) and describe connections among these representations

C4 demonstrate an understanding of patterns that are arithmetic, power and geometric

C4 Among other ways, students should distinguish among arithmetic, power and geometric patterns in terms of levels of common differences between terms in sequences (e.g., arithmetic sequences have common differences at the first level while common differences are found at the second level in quadratic patterns). Also, students should understand that geometric patterns never show common differences but, instead, are characterized by common multiples. As well, students should relate arithmetic, power and geometric patterns to linear, power (especially quadratic) and exponential relations. This outcome will be addressed in connection with SCOs such as C7, C5 and C29.
Unit 2, pp. 46, 48, 50

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

	Elaboration
C5 determine and describe patterns and use them to solve problems	**C5** Mathematics is well described as the study of patterns. Identifying and taking advantage of patterns is a most effective problem-solving strategy. Students will describe numeric and other patterns, identify patterns in tables of differences (see SCO C29), identify symmetric patterns (E2), describe exponential growth patterns (C13), and use patterns to describe properties of exponents (A1). Many identified patterns will be used to answer questions (solve problems). Unit 2, p. 42; Unit 3, p. 54; Unit 4, pp. 66, 68, 76
C6 explore, describe, and apply the Fibonacci sequence	**C6** The Fibonacci sequence is one of the best known and most interesting sequences in mathematics. Students will explore it, learn how to generate it, and solve problems related to its many applications. Unit 2, p. 44
C7 relate arithmetic patterns to linear relations	**C7** Students will connect arithmetic patterns with linear relations by relating common difference to slope and identifying arithmetic patterns in tables of values of linear relations. (See also SCO C4.) Unit 2, p. 46
C8 describe and translate between graphical, tabular, and written representations of quadratic relationships	**C8** Describing and translating between various representations of quadratic relationships is a significant aspect of mathematical modeling (see SCO C1). While working towards achieving C8, students will also be sketching graphs (C21) and analyzing tables and graphs (C29). Unit 3, p. 56
C11 describe and translate between graphical, tabular, and written representations of exponential relationships	**C11** Describing and translating between various representations of exponential relationships contributes in a significant way to understanding them. While working towards achieving C11, students will also be focusing on patterns associated with exponential growth (C5), describing characteristics (C13), and analyzing tables and graphs (C29). Unit 4, pp. 66, 68, 74
iii) interpret algebraic equations and inequalities geometrically and geometric relationships algebraically	
C12 describe and apply the characteristics of quadratic relationships	**C12** Students will describe and apply such characteristics as common second-level differences, parabolic shapes, symmetry, and maximum/minimum values. This outcome will be connected with a number of others, including SCOs C29, C1, C21, C5, E2 and C8. Unit 3, pp. 54, 56

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

	Elaboration
C13 describe and apply the characteristics of exponential relationships	**C13** Students will describe and apply such characteristics as common multiples (ratios), accelerating growth/decelerating decay, asymptotes, and focal point. This outcome will be connected to others, including SCOs C29, C11, C5 and A1. Unit 4, pp. 66, 68, 74
C14 determine and interpret x-intercepts of quadratic relations	**C14** In Year 10 students began the study of quadratic relations, investigating the relationship between the factors of a quadratic equation and the x-intercepts of the graph of the corresponding relation. In this course students will determine x-intercepts as a primary means of solving quadratic equations and, hence, of solving contextual problems modeled by quadratic relations (see SCO C23). Students will need to interpret solutions in relation to contexts in order to determine which are meaningful (i.e., admissible) and which are not. Unit 3, p. 58
iv) solve problems involving relationships, using graphing technology as well as paper-and-pencil techniques	
C21 create and analyze scatter plots and determine the equations for curves of best fit, using appropriate technology	**C21** Rather than deducing the equations of relations, students in this course will use appropriate technology to determine equations for curves of best fit. Producing scatter plots of assembled data, and superimposing curves of best fit, provides representations of mathematical situations that are particularly important for visual learners and which make numerical values (like slope) more meaningful (see SCO C7). Generating equations of relations (linear, quadratic and exponential) by regression using technology (see F2, F3 and C1) makes equations of relations readily available to all students and, thus, increases the potential for interpolation and extrapolation (see F4). SCO C21 also connects to C29, C12 and C5. Unit 2, p. 46; Unit 3, pp. 54, 56, 62; Unit 4, p. 70
C23 solve problems involving quadratic equations	**C23** Students will solve a variety of problems involving quadratic equations (see SCO C1). This will often be conveniently done graphically, although using the quadratic formula (B3) will sometimes be an option. Students will identify maximum/minimum values, find roots (C14), and interpolate and/or extrapolate (F4). Students should also be prepared to identify inadmissible roots when dealing with problem contexts. Unit 3, pp. 58, 62

GCO C: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

	Elaboration
C25 solve problems involving exponential equations	**C25** Students will solve problems involving exponential equations in a variety of real-world contexts. They will solve by considering basic patterns (see SCO C5), modeling (C21 and F3), translating between representations (C11), and interpolating and extrapolating (F4). Unit 4, p. 70
C26 solve problems involving compound interest	**C26** Students will solve a variety of problems involving compound interest, both those relating to investment and those associated with debt. In particular, students will deal extensively with annuity situations (see SCO B6). C26 will be addressed in connection with A8 and B5. Unit 4, pp. 72, 78, 80
v) analyze and explain the behaviours, transformations and general properties of types of equations and relations	
C29 analyze tables and graphs to distinguish between linear, quadratic and exponential relationships	**C29** Students will differentiate between linear, quadratic and exponential relationships in a number of ways. In tables, they will identify number patterns (e.g., common multiples) and investigate first-, second- and third-level differences. With respect to graphs, they will not only recognize general shapes, but also identify key characteristics such as vertices, symmetry and asymptotes. This SCO will be addressed in connection with numerous others such as C1, C21, C5, E2, C12 and C13. Unit 3, p. 54; Unit 4, pp. 66, 68
vi) perform operations on and between functions	
C28 solve simple trigonometric equations	**C28** Students will solve simple trigonometric equations, both substituting trigonometric values for known angles to solve for unknown measurements, and using known information to determine trigonometric values for unknown angles and then solving for the angles using inverse trigonometric relationships. This outcome will be addressed in connection with SCOs B4, D4 and D5. Unit 1, p. 30, 34

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

iii) *apply measurement formulas and procedures in a wide variety of contexts*

SCO: By the end of *Patterns and Relations 113*, students will be expected to

D4 **solve problems using the sine, cosine, and tangent ratios**

D5 **apply the Law of Sines, the Law of Cosines and the formula 'area of a triangle ABC = ½ bcsinA' to solve problems**

Elaboration

D4 In Year 10 students solved problems involving right triangle trigonometry. In this course the primary trigonometric ratios will be applied with respect to the Sine Law, Cosine Law, and area of a triangle. (See SCOs B4, C28 and D5 for connections.) Unit 1, p. 30

D5 Students will apply the Law of Sines, Law of Cosines, and formula for the area of a triangle in a variety of problem situations. This outcome is closely connected with SCOs B4, D4 and C28. Unit 1, pp. 32, 34, 36, 38

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

ii) *interpret and classify geometric figures, translate between synthetic (Euclidean) and coordinate representations, and apply geometric properties and relationships*

SCO: By the end of *Patterns and Relations 113*, students will be expected to

E2 describe and apply symmetry

Elaboration

E2 Students will identify, both in tables and on graphs, the symmetric nature of quadratic relationships. They will subsequently apply this knowledge to answer questions such as those dealing with maximum/minimum values. This outcome will be addressed in connection with SCOs C5 and C12. Unit 3, p. 54

GCO F: Students will solve problems involving the collection, display and analysis of data.
KSCO: By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to

iii) use curve fitting to determine the relationship between, and make predictions from, sets of data and be aware of bias in the interpretation of results

SCO: By the end of *Patterns and Relations 113*, students will be expected to

F2 use curve-fitting to determine the equations of quadratic relationships

F3 use curve fitting to determine the equations of exponential relationships

F4 interpolate and extrapolate to predict and solve problems

Elaboration

F2 Students will use curve fitting (with technology) to determine the equations of quadratic relationships (see also SCOs C21 and C1). This curve-fitting exercise will facilitate interpolation and extrapolation (F4). Unit 3, pp. 56, 62

F3 Students will use curve fitting (with technology) to determine the equations of exponential relationships (see also SCOs C21 and C25). This curve-fitting exercise will facilitate interpolation and extrapolation (F4). Unit 4, p. 70

F4 Students will use graphs and/or equations to interpolate and extrapolate to answer questions/solve problems. This outcome will be addressed in connection with SCOs such as F2, F3, C21, C23 and C25. Unit 3, p. 56; Unit 4, p. 70

Unit 1
Applications of Trigonometry
(15-20 Hours)

Trigonometry

Outcomes

SCO: In this course students will be expected to

D4 **solve problems using the sine, cosine, and tangent ratios**

B4 **use the calculator correctly and efficiently**

C28 **solve simple trigonometric equations**

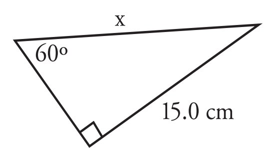
B1 **demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations**

Elaboration—Instructional Strategies/Suggestions

Trigonometry is the study of the relationships of the measures of the sides and angles of triangles; all problems that deal with geometric situations where length or angle measure is required can be modeled with diagrams or constructions of the geometric figures.

D4 In Year 10, students developed the trigonometric ratios $\sin\theta$, $\cos\theta$, and $\tan\theta$ and applied them to problems involving right triangles. Students may need to review the use of trigonometric ratios to solve problems involving right triangles.

B4/C28/B1 Students will use their calculators correctly and efficiently for various procedures while working with trigonometric relationships. For example, when finding a missing side in a situation modeled by a right triangle, some students may set up an equation like the following;



$$\sin(60°) = \frac{\text{length of side opposite}}{\text{length of hypotenuse}} = \frac{15.0}{x}$$

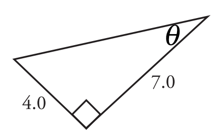
then multiply both sides by x: $x\sin(60°) = 15.0$

then divide by $\sin(60°)$, they would have the expression: $x = \frac{15.0}{\sin(60°)}$

Using their calculator in degree mode, they would divide 15.0 by $\sin 60°$ to obtain 17.3 cm, the length of the hypotenuse. This would be efficient use of the calculator.

Students will also use trigonometric equations, like the one to the right, to find missing angles in right triangles.

$$\tan\theta = \frac{4.0}{7.0}$$



Most students would first change the ratio to a decimal, $\tan\theta = 0.57$.

Then students would find the angle measure θ by using "tan $^{-1}$."

$$\theta = \tan^{-1}(.57)$$
$$\doteq 29.68$$
$$\doteq 30°$$

Some students may simply enter $\tan^{-1}\left(\frac{4.0}{7.0}\right)$ into their calculator and solve for theta with this efficient use of the calculator.

Since this chapter deals with measurement, students should be careful to properly use precision, accuracy, and significant digits. Teachers may wish to discuss these at this time.

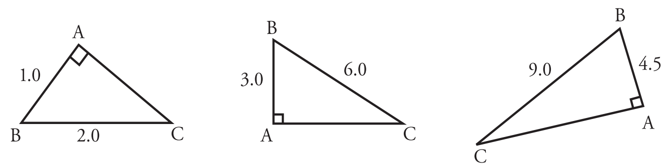
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

D4/B4/C28/B1

Performance

1) Find the measure of angle C for each of the following:



a) Complete this table:

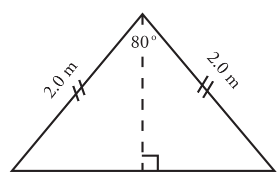
Length of hypotenuse	Length of shortest side	$m\angle C$

b) Look for a relationship that seems to be true from examining the pattern in the table.

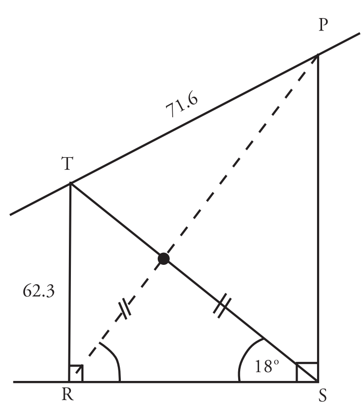
c) Make a conjecture.

d) Test your conjecture in a situation that you make up.

2) Oliver makes a shelter in the shape of an isosceles triangle. Using the given measures, find how high the shelter is.



3) Richard leans the top of his 7.3 m ladder against the sill of a window that is 6.5 m above the ground. At what angle to the ground will his ladder be?



Suggested Resources

Brueningsen, Chris, et al. *Real-World Math with the CBLTM System.* Texas Instruments, 1994.

Meiring, Steven P. A Core Curriculum: Making Mathematics Count for Everyone. Addenda Series *9–12*, Reston, VA: NCTM, 1992.

The Geometer's Sketchpad Software. Emeryville, CA: Key Curriculum Press.

Trigonometry

Outcomes

SCO: In this course students will be expected to

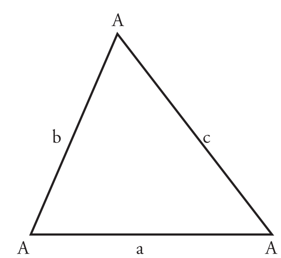
D5 **apply the Law of Sines, the Law of Cosines, and the formula "area of a triangle $ABC = \frac{1}{2}bc \sin A$" to solve problems**

B4 **use the calculator correctly and efficiently**

Elaboration—Instructional Strategies/Suggestions

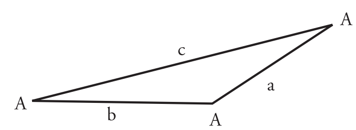
D5/B4 Students will interact with the teacher to develop a procedure for obtaining measurements of triangles that are not right angled (oblique triangles). They should develop through teacher-led discussion, or a directed activity, the Law of Cosines and the Law of Sines, as well as a new formula for finding the area of any triangle.

To develop the Law of Sines students might begin like this (or they might use a geometry software package to explore in the same way):



Questions 1–3 refer to the acute triangle ΔABC on the right.

1) Measure each side to the nearest 10th of a centimetre.
   a) c b) a c) b
2) Measure each angle to the nearest degree.
   a) $\angle A$ b) $\angle B$ c) $\angle C$



3) Calculate each of the following to 1 decimal place.
   a) $\frac{a}{\sin A}$ b) $\frac{b}{\sin B}$ c) $\frac{c}{\sin C}$
4) Repeat questions 1–3 for the obtuse ΔABC.
5) Draw an acute triangle, ΔABC, of your own and repeat questions 1–3.
6) Draw an obtuse triangle, ΔABC, of your own and repeat questions 1–3.
7) Based on the results of question 3, what can you conclude about the relationship between $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$

Students should conclude that these ratios are equal. Any one proportional statement is the formula called the Law of Sines.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \text{ or } \frac{b}{\sin B} = \frac{c}{\sin C}$$

Students should realize that some students may use the formula in this form:

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

Have students think about when they would use the Law of Sines. They should conclude that it will be used either when they know one angle and are looking for another, or they are given two angles and need to find a side.

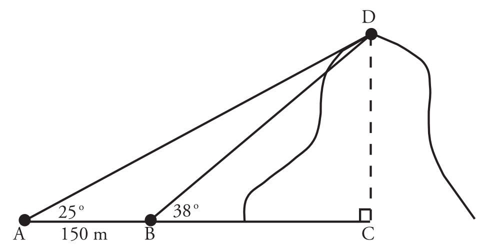
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

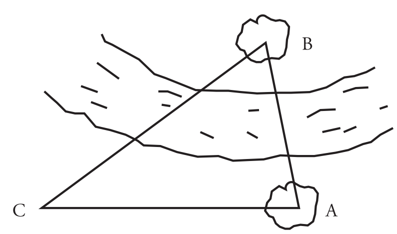
D5/B4

Performance

1) Surveyors cannot get to the inside centre of a mountain easily. Therefore, a mountain's height must be measured in a more indirect way. Find the height of the mountain.



2) Surveyor Sally had to determine the distance between two large trees situated on opposite sides of a river. She placed a stake at C, 100.0 m from point A. Sally then determines the angle measures at points C and A to be 45° and 80° respectively.
   a) Ask students to help her find the distance between the trees.
   b) Ask students to create a different problem using the above diagram.



Journal

3) In class, Marlene said that she could find AB in question 2) (above) using the Law of Sines. Is she correct or not? Explain.

4) Billy is using $\frac{a}{\sin A} = \frac{b}{\sin B}$ to solve a problem. Billy's dad said that in his day he would have used $\frac{\sin A}{a} = \frac{\sin B}{b}$. Is Billy or his dad correct?

   Explain.

Suggested Resources

Meiring, Steven P.. A Core Curriculum: Making Mathematics Count for Everyone. Addenda Series 9–12, Reston, VA: NCTM, 1992.

Geometers' Sketchpad Software. Emeryville, CA: Key Curriculum Press.

Trigonometry

Outcomes

SCO: In this course students will be expected to

D5 **apply the Law of Sines, the Law of Cosines, and the formula "area of a triangle $ABC = \frac{1}{2}bc\ \sin A$" to solve problems**

B4 **use the calculator correctly and efficiently**

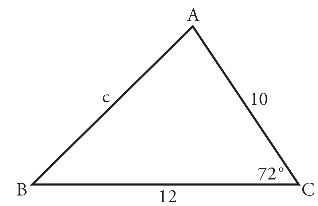
C28 **solve simple trigonometric equations**

B1 **demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations**

Elaboration—Instructional Strategies/Suggestions

D5/B4/C28/B1 Teachers should help students develop the Law of Cosines which, stated symbolically in one form, is
$c^2 = a^2 + b^2 - 2ab \cos C$

Applied to the given diagram, the Law of Cosines would state:



$AB^2 = BC^2 + AC^2 - 2(BC)(AC) \cos C$

$AB^2 = 12^2 + 10.^2 - 2(12)(10) \cos 72^o$

$AB^2 = 144 + 100. - 240(0.3090)$

$AB \doteq 13.0$

Usually this formula is used to find a missing side, given the other two sides and the angle opposite the given side. The formula can be restated as $b^2 = a^2 + c^2 - 2ac \cos B$, or $a^2 = b^2 + c^2 - 2\ bc \cos A$, depending on which side the students wants to find.

Sometimes students will use the formula to find an angle measure when all three side lengths are given. They don't need to arrange the formula first. For example, given a = 12.0 cm, b = 9.5 cm, and c = 7.2 cm, students could find any of the angle measures. If they wanted $\angle A$:

$a^2 = b^2 + c^2 - 2\ bc \cos A.$

$12.0^2 = 9.5^2 + 7.2^2 - 2(9.5)(7.2) \cos A$

$144 = 90.25 + 51.84 - 136.8 \cos A$

Next, they need to isolate the variable term:

$144 - 90.25 - 51.84 = -136.8 \cos A$

then isolate the variable:

$$\frac{1.91}{-136.8} = \frac{-136.8 \cos A}{-136.8}$$

$$-0.01396 = \cos A$$

$$\angle A = \cos^{-1}(-0.01396)$$

$$\angle A = 91°$$

Students should notice the similarity of the last two steps in the above calculation to the steps used when solving right-triangle trigonometric equations. They would conclude that the angle A measures about 91°.

Students need to think about using the Law of Cosines when they have a situation where there is no right angle, and they need a third side length, given the other two, or an angle measure given all the side lengths.

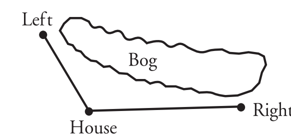
Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

D5/B4/C28/B1

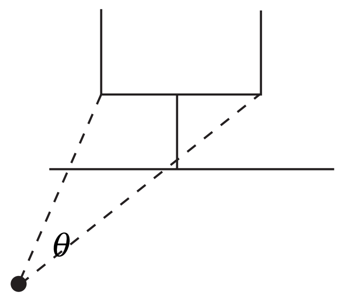
Performance

1) A farmer wants to find the length of the back of his property which is all bog land. He knows that his left and right boundary lines connect near his house at an angle of 147°. The left boundary length is 90 m and the right is 110 m. Suggest a means of finding this length.



2) Terry is building an A-frame cabin in the woods. The length of each of two rafters is 8.50 m. If the angle of the apex of the frame is to be 46°, calculate the proposed width of the cabin at the base.

3) a) A football player is attempting a field goal. His position on the field is such that the ball is 7.5 m to the left upright of the goal post and 10.0 m to the right up-right of the goal post. The goal posts are 4.3 m apart. Find the angle marked θ.



b) If the ball is moved to the middle of the field, position P, then the ball is equidistant to both uprights, approximately 8.5 m each. Find the angle corresponding to θ from this position.

D5

Journal

4) Explain why the Law of Cosines might be a useful relationship to try to remember.

Suggested Resources

Brueningsen, Chris, et al. *Real-World Math with the CBLTM System.* Texas Instruments, 1994.

Meiring, Steven P. A Core Curriculum: Making Mathematics Count for Everyone. Addenda Series *9–12*, Reston, VA: NCTM, 1992.

Trigonometry

Outcomes

SCO: In this course students will be expected to

D5 **apply the Law of Sines, the Law of Cosines, and the formula "area of a triangle $ABC = \frac{1}{2}bc \ \sin A$" to solve problems**

B4 **use the calculator correctly and efficiently**

Elaboration—Instructional Strategies/Suggestions

Teachers may want to discuss with students that the Law of Cosines 'looks' like the Pythagorean Theorem with an adjustment factor to make up for the lack of a right angle.

$c^2 = a^2 + b^2 -$ adjustment factor

They may want to have students examine several situations where the measure of $\angle C$ has various values, say 40°, 50°, 70°, 85°, 90°, 100°, 110°, 120°, ... Students would evaluate 2ab cos C and note that as $m\angle C \rightarrow 90°, 2ab\cos C \rightarrow 0$. As m $\angle C$ gets larger and larger beyond 90°, 2ab cos C gets smaller and smaller.

Talk to students now about

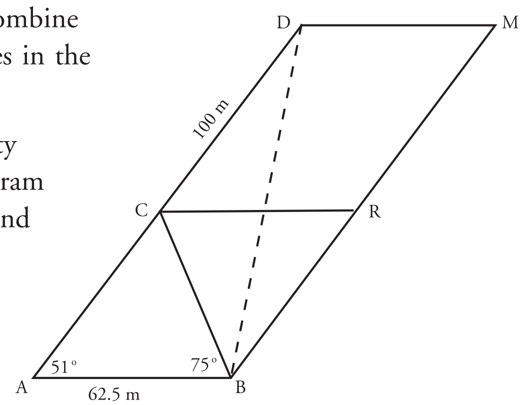
if m$\angle C$ =90°, then c=$\sqrt{a^2 + b^2}$ (Pythagorean Theorem)

if m$\angle C$ =90°,, then c should get smaller: $c^2=a^2+b^2$ - adjustment factor

if m$\angle C$ =90°,, then c should get larger: $c^2=a^2+b^2$ - a larger and larger negative value.

Students should be prepared to combine both the Laws of Sines and Cosines in the same question when required.

For example, farmer Jones' property (ABRC) is shaped like a parallelogram (see diagram). He is given more land (DMRC). He needs to know the distance from D to B. He would first have to use the Law of Sines to get BC, then the Law of Cosines to get BD.



Trigonometry

Worthwhile Tasks for Instruction and/or Assessment

D5/B4

Activity

1) Given $\triangle ABC$ with b = 4.0 cm and c = 3.0 cm:
   a) Ask students to determine "a" if $m\angle A = 90°$.
   b) Ask students to evaluate 2bc cos A, if $m\angle A = 90°$.
   c) Ask students to determine "a" again using $a^2 = b^2 + c^2 - 2bc \cos A$, given $m\angle A = 90°$. Make a conjecture about this new formula.
   d) Ask students to construct an accurate diagram of $\triangle ABC$ with b = 4.0 cm, c = 3.0 cm, and $m\angle A = 80°$°.
   e) Ask students to
      i) predict if a < 5.0 cm, a = 5.0 cm, or $a \geq 5.0\text{cm}$ and explain their choice.
      ii) measure with a ruler the length "a" and record it.
      iii) calculate the length "a" using $a^2 = b^2 + c^2 - 2bc \cos 80°$.
   f) Ask students to repeat step (e) given $m\angle A$ = 60°, 65°, 85°, 89°.
   g) Ask students to describe the pattern and how it fits with their conjecture in (c).
   h) Ask students to repeat step (e) given $m\angle A$ = 91°, 95°, 100°, 120°.
   i) Ask students to describe the pattern and how it fits with their conjecture in (c).
   j) Ask students to make a statement about how the formula $a^2 = b^2 + c^2 - 2bc \cos A$ can be used.
   k) Glen conjectured that if he knew the side measures, he could determine the $m\angle A$ using this new formula. Do you agree or disagree? Explain.
   l) Colin conjectured that if he knew the length of "b" and "a" and the $m\angle C$ he could determine the length c. Explain what Colin must be thinking.

2) In the town where Simon lives some roads were constructed as in the diagram. Simon needs to know how much longer the road from S to P is than the road from R to T.

D5

Journal

3) How can you help yourself remember the formula for the Law of Cosines? (**Hint**: it is the Pythagorean Theorem, plus or minus some adjustment factor. How can I remember the adjustment factor?)

Suggested Resources

Brueningsen, Chris, et al. *Real-World Math with the CBLTM System.* Texas Instruments, 1994.

Meiring, Steven P. A Core Curriculum: Making Mathematics Count for Everyone. Addenda Series *9–12*, Reston, VA: NCTM, 1992.

Trigonometry

Outcomes

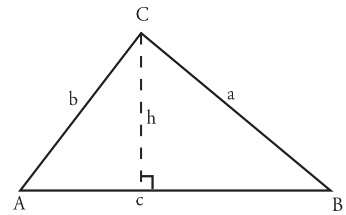
SCO: In this course students will be expected to

D5 apply the Law of Sines, the Law of Cosines, and the formula "area of a triangle $ABC = \frac{1}{2}bc\ \sin A$" to solve problems

Elaboration—Instructional Strategies/Suggestions

D5 To determine the area formula, area of a triangle $= \frac{1}{2}bc\ \sin A$, students should be asked to explain how they would find the area of triangle ABC.

Students would write $A = \frac{1}{2}\ base \times height$.



In this triangle, the base is c, so

$A = \frac{1}{2}\ base \times height$

The teacher would then ask students to replace the "h" with an expression using sin A.

Students would write $\sin A = \frac{h}{b}$

or, $h = b \sin A$

so, $A = \frac{1}{2}c(b \sin A)$

or without brackets $A = \frac{1}{2}cb \sin A$

Students should apply this formula in various problem-solving situations involving area. To use this formula to find area, students should realize that they need any two sides and the included angle measure of any triangular shape. When the area of a triangular shape is given, students can use this formula to find any of the missing three measures, if the other two are given. For example, if the area of a triangular region on a stage was to be carpeted with 37 square metres of carpet, and two adjacent sides measures of the carpet were given as 12.0 m and 6.7 m, then the angle between these sides could be found:

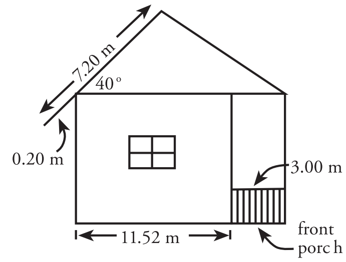
$$\begin{aligned} Area &= \tfrac{1}{2}bc \sin A \\ 37 &= \tfrac{1}{2}(12.0)(6.7) \sin A \\ \tfrac{37}{40.2} &= \sin A \\ A &\doteq 67^\circ \end{aligned}$$

Trigonometry

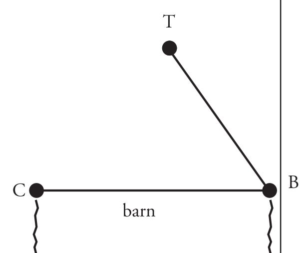
Worthwhile Tasks for Instruction and/or Assessment

D5

Performance



1) Hilary wants to paint the triangular gable ends of her log cabin. She knows that a can of paint will cover 39 m^2. She expects to have to paint two coats. If a can costs \$29.95. how much money will she have to spend?
2) Cousin Barney is building a new corral on the side of his barn for his new lamb, Huey. He measures the barn length to be 15.25 m. There is already a fence from one end of the barn to a tree (T) with a length 21.62 m. Barney has just spread seed that covers 120.50 m^2 inside the triangular region C - B - T. How long will the fence be that goes from C to T?



Journal

3) Ask students to explain why "c sin A" from the formula area of a triangle $= \frac{1}{2} bc \sin A$ is the same as "h" in the formula area of a triangle $= \frac{bh}{2}$.

Suggested Resources

Meiring, Steven P. A Core Curriculum: Making Mathematics Count for Everyone. Addenda Series *9–12*, Reston, VA: NCTM, 1992.

Unit 2
Patterns

(10 Hours)

Patterns

Outcomes

SCO: In this course, students will be expected to

C5 determine and describe patterns and use them to solve problems

Elaboration—Instructional Strategies/Suggestions

C5 Mathematics curriculum should be organized around problem solving. Classroom environments should be created in which problem solving can flourish and problem-solving strategies are developed and discussed. One such strategy is to search for and describe patterns. In the following example students should make an organized list to help them look for the pattern. Once the pattern is discovered, students should use it in solving the problem. For example,

❐ Scientists have invented a time machine (1985). By setting the dial, you can move forward in time. Set it forward 6 minutes and you will be in the year 1988. Set it forward 16 minutes, and you will be in the year 1993; Set it forward 26 minutes, and you will be in the year 1998; 36 minutes, year 2003. If the machine continues in this manner, in what year will you be if the timer is set ahead 65 minutes?

[Ans. - Students should begin to organize the information in an organized list:

Time set forward	Year
0	1985
6	1988
16	1993
26	1998
36	2003
•	•
•	•
65	?

Students might notice that the numbers (after the second) in the first column increase by 10, while the years (after the second) in the second column increase by 5. They could use this pattern to predict the answer for 66, then adjust for 65.]

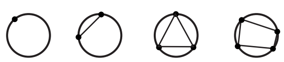
In order to help themselves, students should solve problems using the strategy of pattern description by exploring and describing many different types of patterns, including numbers, words, pictures, geometric shapes. For example, see the activities in on the page opposite.

Patterns

Worthwhile Tasks for Instruction and/or Assessment

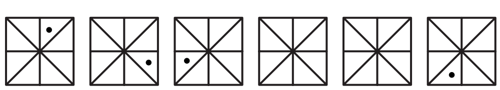
C5

Performance

1) For each of the following sets, give the next element of the set. State in your own words what you think the patterning rule is.
   a) 80, 40, 20, 10, ______
      Patterning rule: ________________
   b) James, Jill, Joan, John, _________
      Patterning rule: __________
   c) 1, 8, 27, 64, 125, _________
      Patterning rule: ____________
   e) 1, 1, 2, 3, 5, 8, 13, ____________
      Patterning rule: ________________
   f) Alvin, Barbara, Carla, Dennis, ___________
      Patterning rule: __________________
   g)  ___
      Patterning rule: ____________
   h) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
      Patterning rule: ___________
   i) 6, 30, 150, 750: ____________
      Patterning rule: ____________
   j) 

      ?
      Patterning rule: ____________

2) Given a few beginning elements and the last element, fill in the missing elements in each pattern.
   a) 

   b) 

3) An empty street car picks up five passengers at the first stop, drops off two passengers at the second stop, picks up five passengers at the third stop, drops off two passengers at the fourth stop, and so on. If it continues in this manner, how many passengers will be on the streetcar after the 16^{th} stop?

Suggested Resources

Hirsch, Christian R., and Robert A. Laing, eds. *Activities for Active Learning and Teaching: Selections from the* Mathematics Teacher. Reston, VA: NCTM, 1993.

Dolan, Daniel T., and James Williamson. *Teaching Problem Solving Strategies*. Reading, MA: Addison-Wesley Publishing Company, 1983.

Patterns

Outcomes

SCO: In this course, students will be expected to

C6 explore, describe, and apply the Fibonacci Sequence

Elaboration—Instructional Strategies/Suggestions

C6 One of the greatest mathematicians of the Middle Ages was Leonardo of Pisa, called Fibonacci. He wrote a book on arithmetic and algebra titled *Liber Abacci*. This book was influential in introducing into Europe the Hindu-Arabic numerals with which we now write numbers. One of the many interesting problems in this book was about rabbits. Students might be asked to solve this problem and look for a pattern.

❒ A pair of rabbits one month old are too young to produce more rabbits, but suppose that in their second month and every month thereafter they produced a new pair. If each new pair of rabbits does the same and none of the rabbits die, how many pairs of rabbits will there be at the beginning of each month?

The number of pairs of rabbits at the beginning of each month form a sequence: 1, 1, 2, 3, 5, 8, 13 ... known as the Fibonacci Sequence. Its terms follow a simple pattern. Ask the students to describe this pattern.

The Fibonacci Sequence has showed up in an amazingly wide variety of creations. For example:

1) the petals of many flower species—their petals commonly occur only in Fibonacci number configurations.
2) The seeds in the flower head of a sunflower spiral in two different directions—the numbers of spirals are Fibonacci numbers.
3) The same is true in the spiral of pineapples, and pine cones.
4) In many trees, the leaves spiral around the stem. The number of turns required to find a leaf in a position directly above another leaf is a Fibonacci number.
5) Musical scales
6) Reproduction of bees

Students should explore some of the above situations to find and apply patterns. See the page opposite for examples.

Students might also explore the ratio between terms in the sequence and connect it to the Golden Ratio $\left(\frac{1\pm\sqrt{5}}{2}\right)$. If the largest square is removed from a rectangle whose dimensions are the golden ratio, another rectangle will remain whose dimensions are, again, the golden ratio. To explore the ratios between successive terms, ask students to divide the larger term by the one that precedes it. For example, 5 divided by 3, or 13 divided by 8, or 34 divided by 21. Ask them to record the answers for several of these ratios as decimals and notice the pattern that seems to be developing as the terms get larger. Compare the ratios to the decimal approximation of the golden ratio. (The successive ratios seem to be approaching the value of the golden ratio.)

... continued

Patterns

Worthwhile Tasks for Instruction and/or Assessment

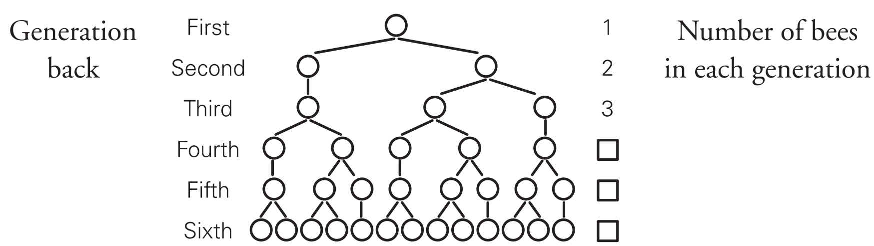
C6

Performance

1) In Fibonacci's rabbit problem,
   a) How many pairs of rabbits will there be at the beginning of the seventh month?
   b) How many pairs will there be at the beginning of the 12th month?
   c) How many pairs of adult rabbits (at least one month old) will there be at the beginning of the seventh month?
   d) How many baby rabbits (less than one month old) will there be?
2) Fibonacci numbers have some remarkable properties.
   a) Find the missing numbers in this sequence of sums.
   b) Describe the pattern.

$$
\begin{aligned}
1^2 &= 1\times 1\\
1^2+1^2 &= 1\times 2\\
1^2+1^2+2^2 &= 2\times 3\\
1^2+1^2+2^2+3^2 &= 3\times \_\\
1^2+1^2+2^2+3^2+5^2 &= \_\times \_\\
1^2+1^2+2^2+3^2+5^2+8^2 &= \_\times \_\\
1^2+1^2+2^2+3^2+5^2+8^2+13^2 &= \_\times \_
\end{aligned}
$$

3) A male bee has only one parent, his mother, whereas a female bee has both a father and a mother. In the tree below, each male is represented by the symbol [ ] and each female by the symbol [ ].
   a) Use the figure to find the numbers of bees in the fourth, fifth, and sixth generations back.
   b) What do you notice about the sequence of numbers of bees in successive generations of ancestors?
   c) How many bees do you think would be in the seventh, eighth, and ninth generations back?
4) Compare the family tree of a male bee with that of the pairs of rabbits.
   a) What does each pair of baby rabbits correspond to on the bee's family tree?
   b) What does each pair of adult rabbits correspond to on the bee's family tree?



5) Use the patterns to answer the following questions:
   a) Guess the sum of the first 10 terms of the Fibonacci Sequence without adding them.
   b) Write the next line in Pattern B.
   c) Guess the sum of the squares of the first 10 terms of the Fibonacci Sequence without adding them.
   d) Which pattern, A, B, or C, do these figures illustrate:

Pattern A

$$
\begin{aligned}
1+1 &= 2\\
1+1+2 &= 4\\
1+1+2+3 &= \_\\
1+1+2+3+5 &= \_\\
1+1+2+3+5+8 &= \_\\
1+1+2+3+5+8+13 &= \_\\
1+1+2+3+5+8+13+21 &= \_
\end{aligned}
$$

Pattern B

$$
\begin{aligned}
1^3+1^3 &= 2\\
1^3+2^3 &= 5\\
2^3+3^3 &= \_\\
3^3+5^3 &= \_\\
5^3+8^3 &= \_\\
8^3+13^3 &= \_
\end{aligned}
$$

Pattern C

$$
\begin{aligned}
1^3+1^3 &= 2-1\cdot 2\\
1^3+1^3+2^3 &= 6-2\cdot 3\\
1^3+1^3+2^3+3^3 &= 15-3\cdot 5\\
1^3+1^3+2^3+3^3+5^3 &= 40-5\cdot 8
\end{aligned}
$$

Pattern D

$$
\begin{aligned}
1^3+2^3-1^3 &= 8\\
2^3+3^3-1^3 &= 34\\
3^3+5^3-2^3 &= \_\\
5^3+8^3-3^3 &= \_
\end{aligned}
$$

Suggested Resources

Patterns

Outcomes

SCO: In this course, students will be expected to

C4 **demonstrate an understanding of patterns that are arithemetic, power, and geometric**

C7 **relate arithmetic patterns to linear relations**

C21 **create and analyse scatter plots and determine equations for the curves of best fit, using appropriate technology**

Elaboration—Instructional Strategies/Suggestions

C4 Students extend previous knowledge to describe and reason about a variety of contexts using the mathematical relationships. Students should have had experience in creating and using symbolic and graphical representation of patterns, especially those tied to linear and quadratic growth. In this course these experiences will be extended to arithmetic, power, and geometric sequences, with particular focus on quadratic and exponential relations.

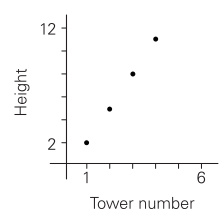
To begin this unit, students should extend their work with patterns to include investigation of sequences of numbers that fall into two categories:

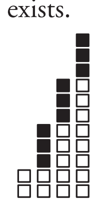
1) arithmetic sequences (a sequence where consecutive terms present a common difference)
2) power sequences (a sequence made up of consecutive terms found by raising consecutive counting numbers to the same power)

C7 Students should clearly see that arithmetic sequence leads to a relationship that is linear and can be described as a rule. They might develop this in the following way. Students could be given the sequence 2 5 8 11 14 and be asked for the 100th term. They can see that there is a common difference of 3, and by completing an organized list they might be able to predict the 100th term, then the nth term. Teachers should help students to note that numbers in an arithmetic sequence have a common difference, that the common difference is a constant, and that it is connected to the slope of the line. When the pattern is expressed with symbols, students need to make sense of the constant term in the equation and its connection to the pattern. (Note: the general formula for arithmetic sequence is not part of the course.)

1st term	(t_1)	is	$2+3\cdot 0=2$
2nd term	(t_2)	is	$2+3\cdot \_=5$
3rd term	(t_3)	is	$2+3\cdot \_=8$
5th term	(t_4)	is	$\_\_\_\_=14$
100th term	(t_{100})	is	$2+3\cdot \_=\_$
nth term	(t_n)	is	$2+3\cdot \_=\_=3n-1$

Students should always be given the opportunity to work with the five representations of a concept as the concept is being developed. They are context, concrete, pictorial, verbal, and symbolic. Students might be given the diagram of towers and be asked to construct it with cubes and to record the heights. Then, they should describe the pattern in words and perhaps attempt to create a context for which this pattern exists. They could graph this relationship. Students could talk about the height of the towers increasing by 3 as the number increases by 1. They should see that this is the slope of the graph, and it is the coefficient of the independent variable in the equation. They could examine the y-intercept and discuss why it has no meaning in this context. They might conclude that the y-intercept would be –1 and that it would represent the next height of towers (if that were possible) in the other direction. Finally, they might be asked to describe the relationship between the tower number in the sequence and the height of the tower and predict the height of the 10th tower. They might do this by obtaining an equation that would represent the relationship between the number of the towers and their heights.





C21 As a way of getting their equation, students could use graphing technology. To do this, for example, they could enter the tower number in list 1 and the height of each tower in list 2 on a TI-83 and use LinReg to obtain the equation $y = 3x - 1$. It is important to discuss with students that while LinReg produces an equation, it is meaningful only in the context for integers greater than or equal to 1.

Patterns

Worthwhile Tasks for Instruction and/or Assessment

C4/C21

Performance

1) Complete each sequence and find the nth term.
   a) 2, 4, 6, 8, 10, _, _, _, ... n^th^.
   b) 3, 6, 9, 12, _, _, _, ... n^th^.
   c) 2, 7, 12, 17, _, _, _, ... n^th^.

C4/C7

Performance

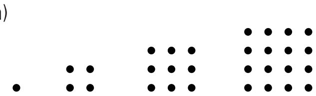
2) Explain why each of the above is called an arithmetic sequence.

C4/C7/C21

Performance

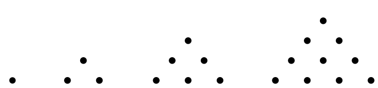
3) Explore the following dot patterns and determine if they form an arithmetic sequence or not. Explain.

a)



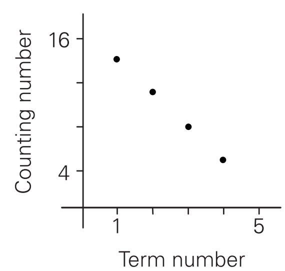
number of dots on one side: n	1 2 3 4 ... 10... 20
number of dots in array: s	

b)



number of dots on one side: n	1 2 3 4 ... 10... 20
number of dots in array: T	

4) If this graph represents a sequence of numbers,
   a) Is the sequence arithmetic?
   b) What would be the value of the 8th term?
   c) Describe the sequence in words.
   d) Describe the n^{th} term.



C7

Performance

5) a) Create an arithmetic sequence with seven terms, and explain why it could be described using a linear relation.
   b) Using $y = -3x + 10$, develop an arithmetic sequence with six terms.

Suggested Resources

Patterns

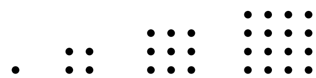
Outcomes

SCO: In this course, students will be expected to

C4 demonstrate an understanding of patterns that are arithmetic, power, and geometric

Elaboration—Instructional Strategies/Suggestions

C4 Ask students to examine the dot pattern given below and complete the table of values.



n	1	2	3	4 ...	10 ...	20 ...	n
s	1	4	9	16			

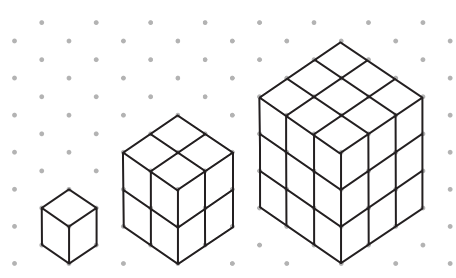
They might notice that each n-value (the number of dots on each side of the square) is squared to get each s-value (the number of dots in the whole array). They could use this to predict the answers for n = 10 and 20. Students could then describe the pattern in words and arrive at $s = n^2$.

They should be able to describe the difference between this pattern and the arithmetic patterns looked at previously. Here, there is no common difference between successive terms. The differences are 3, 5, and 7. However, if the common differences are subtracted students would see a common difference occur at the second level. When a common difference occurs at the second level students should understand that an equation with an n^2 (quadratic) will result. Coming up with the equation is not the goal, however. They should understand that there is a way to describe the relationship with an equation. The sequence where there is a common difference but it is not at the first level is called a power sequence.

$t_n \rightarrow$ 1 4 9 16 25 36
$D_1 \rightarrow$ 3 5 7 9 11 13
$D_2 \rightarrow$ 2 2 2 2 2

Students might use cubes to build towers (see the discussion on the previous two-page spread) to compare the growth rate visually between a quadratic relationship and that of an arithmetic sequence (linear relationship).

As suggested above, power sequences have common differences only after the first level of differences. Have students examine the following diagram, extend the diagram, and complete a table.



terms $\rightarrow$ 1 8 27 64
$D_1 \rightarrow$ 7 19 37 61 91
$D_2 \rightarrow$ 12 18 24 30
$D_3 \rightarrow$ 6 6 6

Ask students to determine if this pattern is quadratic (common difference at the second level). Ask the students to find a third set of differences in the same way that they found the second set of differences. This common difference at the third level denotes a cubic relationship. Ask them to describe the pattern in words. Ask students to draw a graph of this situation and the rectangular dot pattern at the top of the page and discuss what is the same about the graphs and what is different. They should now be able to contrast the power sequence that represents a cubic relationship with the power sequence that represents a quadratic sequence.

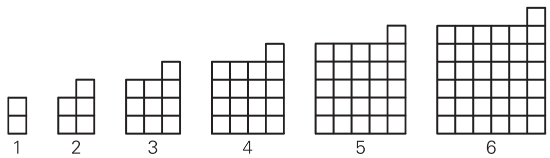
Patterns

Worthwhile Tasks for Instruction and/or Assessment

C4

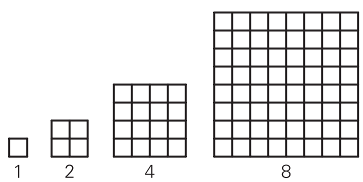
Performance

1) The area of the following regions is the number of units inside.



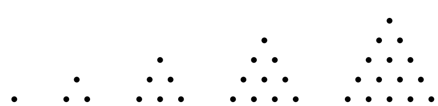
   a) Copy and complete the following number sequence of the areas of these squares.
      2 5 __ __ __ __
   b) What kind of sequence do the areas form?
   c) Copy and complete the number sequence of the perimeters 7, 11, _, _, _
   d) What kind of sequence do the perimeters form? Explain.

2) These figures illustrate a sequence of squares in which the length of the side is successively doubled.



   a) What are the perimeters of these four squares?
   b) What happens to the perimeter of a square if the length of its side is doubled?
   c) What are the areas of these four squares?
   d) What happens to the area of a square if the length of its side is doubled?

3) The first terms in the sequence of triangular numbers are illustrated by the figures below.



   a) Write the five numbers illustrated and continue the sequence to show the next five terms.
   b) Find the differences between the terms. Is there a common difference?
   c) Find the difference between the terms in the differences.
   d) Are triangular numbers terms in an arithmetic sequence? Explain.
   e) Are they terms in a power sequence?

4) Ask students to explain how finding common differences in patterns helps determine what kind of sequence, and how that might help them identify what kind of equation might represent the sequence.

Suggested Resources

Patterns

Outcomes

SCO: In this course, students will be expected to

C4 demonstrate an understanding of patterns that are arithmetic, power, and geometric

Elaboration—Instructional Strategies/Suggestions

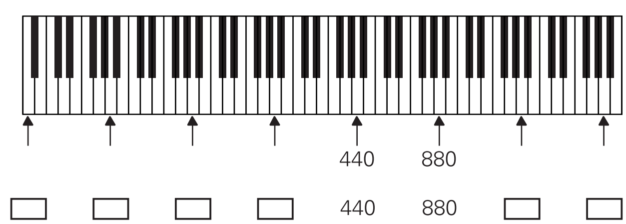
C4 Students should continue the study of patterns to note that not all sequences of number have common differences. A geometric sequence is a number sequence in which each successive term may be found by multiplying by the same number. Students should be able to contrast geometric sequences with power sequences.

The figure at the right shows three stacks of "matho" chips. The stacks illustrate the first three terms of a geometric sequence. To discover this, students might answer the following questions.

1) What are the first three terms?
2) Is there a common difference?
3) Is there a common ratio?
4) What are the next three terms of the sequence?

If the number by which each term is multiplied (common ratio) is greater than 1, the sequence grows at an increasing rate. Students should be asked to describe what happens to the terms in the sequence if the common ratio is less than 1. Students should look at various contexts in which geometric sequences occur.

When a piano is tuned, the first note to be tuned is the A above middle C. It has a frequency of 440 cycles per second. Then the other seven As on the keyboard are tuned so that their frequencies form a geometric sequence. For example, given the piano keyboard and two of the frequencies:



Students could be asked:

1) What is the common ratio of this sequence?
2) Find the frequencies of the other As.
3) Write all eight terms of the sequence and then cross out every second term. Do the remaining terms form a geometric sequence? Explain.
4) Sketch a graph of all eight terms. How does it compare to the graphs of arithmetic and power sequences?

Patterns

Worthwhile Tasks for Instruction and/or Assessment

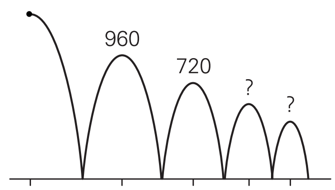
C4

Performance

1) Copy the following geometric sequences, writing in the missing terms.

4 12 36 __ __
11 121 __ __
5 20 __ 320 __
__ __ 72 144 288
16 40 100 __ __
81 54 __ __ __

2) When a rubber ball is dropped from a height of 1280 cm, the heights in cm of the first few bounces form a geometric sequence:



960 720 __ __ __

a) What is the common ratio of this sequence?
b) Find the lengths of the third and fourth bounces.

3) Tell whether each of the following number sequences is arithmetic, geometric, or power. If the sequence is arithmetic, give the common difference. If it is geometric, give the common ratio. If it is a power sequence, explain why.

5	10	15	20	25	…
2	8	32	128	512	…
1	3	6	10	15	…
80	40	20	10	5	…
3	20	37	54	71	…
2	6	24	120	720	…
60	51	42	33	24	…
32	48	72	108	162	…

4) Use graphs to show how there is a difference in any three patterns above. Explain how the graph helps you decide if the sequence is arithmetic, power, or geometric.
5) Stella explained that the sequence {7, 43, 125, 271, 499, 827, 1273, 1855 ... } is linear because it has a common difference. Discuss whether you agree with Stella's reasoning.

Suggested Resources

UNIT 3:
QUADRATICS

Unit 3
Quadratics

(20-25 Hours)

Quadratics

Outcomes

SCO: In this course, students will be expected to

C29 **analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships**

C12 **describe and apply the characteristics of quadratic relationships**

C1 **model real-world phenomena using quadratic equations**

C21 **create and analyse scatter plots and determine equations for the curves of best fit using appropriate technology**

C5 **determine and describe patterns and use them to solve problems**

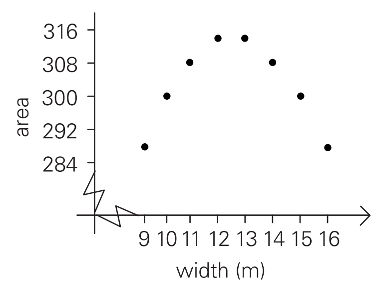
E2 **describe and apply symmetry**

Elaboration—Instructional Strategies/Suggestions

C29/C12/C1 In Year 10 and in the preceding Patterns unit of this course, students have analysed and applied arithmetic sequences and have connected them to linear relations, reaffirming their understanding that a linear relation represents a constant growth rate. In this course, as students begin to study quadratic relationships, they should note connections to these power sequences (common difference at the second level), examined in the previous unit.

C29/C12/C21 In this unit students should examine situations presented in graphs and tables. They should determine if the situation can be described as a linear relationship. If it is not linear, they should be able to explain why, and state and explain whether the relation is quadratic. Students should be able to determine equations for patterns using regression. They should then be able to tell if the relationship has a maximum or minimum value by examining the numerical coefficient of the x^2 term. If the numerical coefficient of x^2 is negative, then the graph is reflected in the x-axis causing the vertex to be at the highest point of the graph, giving a maximum value. They should also be able to get this information from a table. Finally, students should be able to solve problems by interpolating or extrapolating using the graph or equation.

C29/C12/C1/C5 The campsite problem (on the opposite page) asks campers to stake out their campsite with 50 metres of string with which they are to create the rectangular boundary. One side does not require string. being a river bank. Students must find the length and width measurements to maximize the area of their campsite. As students plot a graph of width versus area they will note that as they increase the width a unit at a time, the area of the campsite does not increase at a constant rate. They will see as they continue to plot ordered pairs that the graph will take on a parabolic shape. Since the graph has its vertex at the highest point, students should expect the coefficient of x^2 in the equation to be negative.



width	9	10	11	12	13	14	15	16
area	288	300	308	312	312	308	300	288

12 8 4 0 -4 -8 -12

C5/E2 From the table of width versus area, students should notice the symmetry in the area values. They should also notice that as the width increases metre by metre the area increases at a different rate each time.

They should be able to say that the relationship is quadratic and that the graphical representation would be parabolic.

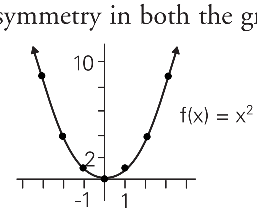
x	-5	-4	-3	-2	-1	0	1
y	30	18	8	0	-6	-10	-12

-12 -10 -8 -6 -4 -2

A similar pattern can be used in table 2. Students should be able to predict the next y-value using common differences between successive y-terms, the next two y-values will be –12 and -10.

Students should focus on the visual patterns and the symmetry in both the graph and the table.

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



Quadratics

Worthwhile Tasks for Instruction and/or Assessment

C29/C12

Pencil and Paper

a)

x	0	1	2	3	4	5
y	2	4	6	8	10	12

b)

x	–5	–3	–1	1	3	5
y	16	12	8	4	0	–4

c)

x	–3	–2	–1	0	1	2
y	18	8	2	0	2	8

1) Which of the following tables do you think will produce a i) linear relationship, ii) quadratic relationship, iii) neither. Explain.
2) Compare the data in these tables. Which table(s) do you think represents a quadratic relationship and explain your thinking.

a)

x	1	2	3	4	5	6	7
y	90	75	60	45	30	15	0

b)

x	5	10	20	30	40	50
y	8	11	16	20	23	25

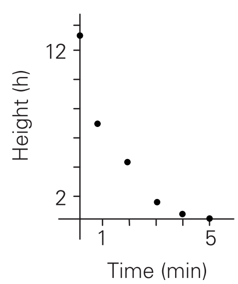
c)

x	0	1	2	3	4	5	6	7	8	9	10
y	0	10	18	24	28	30	30	28	24	18	10

C1/C29/C12

Pencil and Paper

3) The following graph represents the height of the water in a tank as the tank is being drained. Do you think the pattern in the graph represents a quadratic relationship? Explain.



4) Create a graph and/or a table of values that represents a relationship that is linear, quadratic, or neither. Give it to your partner and ask him/her to determine which it is and explain why.

C21

Pencil and Paper

5) For questions 1, 2, and 3 above, determine the equation for the curve of best fit using technology.

C1/C5/E2

Performance

6) As campers arrive at By the River campsite, they are given string (50 m) and four stakes with which they are to mark out a rectangular region for their tent. Antoine suggests that they use the river for one boundary, which would give more string for the other sides.
   a) Andrée wants to make the width (the sides perpendicular to the river) 10 m. What will be the length of the other side?
   b) Describe fully in words how the length of the boundary changes as the width increases through all possible values.
   c) Find the area enclosed by the boundary for each different width.
   d) Sketch a graph to show how the area enclosed changes as the width increases.
   e) Crystal wants to find the dimensions that produce the greatest area.
      i) Describe in words a method by which you could find this length and width.
      ii) Use your method to help Crystal.

Suggested Resources

The "Language of Function and Graphs", Shell Materials. Nottingham England, 1985, Camping

Quadratics

Outcomes

SCO: In this course, students will be expected to

C21 **create and analyse scatter plots and determine equations for the curves of best fit using appropriate technology**

F2 **use curve fitting to determine the equations of quadratic relationships**

F4 **interpolate and extrapolate to predict and solve problems**

C12 **describe and apply the characteristics of quadratic relationships**

C1 **model real-world phenomena using quadratic equations**

C8 **describe and translate between graphical, tabular, and written representations of quadratic relationships**

Elaboration—Instructional Strategies/Suggestions

C1/C21/F2 To get the equation for a parabola, students should create a scatter plot from data. The focus should be on using the given information to generate enough data points so that students can determine the curve of best fit for the scatter plot using appropriate technology. They will use "quadratic regression" to determine the equation, unless the pattern is obvious.

F2/F4 In the situation described in the above paragraph, students would choose quadratic regression when they are quite sure of its parabolic shape.

The emphasis should be on exploring the visual display of the relationship. Students should be aware that this relationship can be represented by an equation that they could generate using technology. They should use the graph and the equation to make predictions and answer questions. Students should have a variety of experiences exploring the use of quadratic regression. Students should be given contexts where it is of interest to them to interpolate and extrapolate. For example, students could investigate how the price of pizza relates to its diameter.

❒ A company is planning to make a 15-cm personal pizza and a party pizza (50 cm). They want to determine what price to charge for each new size pizza. Their current price list is given by the data below:

Diameter (cm)	25	30	38	46
Price in dollars	5.25	6.33	9.00	12.93

Ask students to create a graph of the relationship and estimate the price of the 15-cm and 50-cm using the graph. Have them predict the price of a 40-cm pizza. Students should note that it makes sense that this relationship is quadratic, since the area of a circle is obtained by squaring the radius, and squaring a variable leads to a quadratic relationship. An extension of this activity might be to provide various brochures from local pizza places and have different groups of students find the mathematical model for each place. They should present their findings as an advertisement for a pizza place that is introducing their new personal and party-size pizzas.

C12 Students should, through avariety of experiences with relations, come to recognize the elements in a real-world problem that suggest a particular model, e.g., area suggests a quadratic function, since it changes at a different rate as the width of the rectangle increases. Trajectory suggests a quadratic function just in its natural going-up-and-down pattern, with a maximum value, denoted in the equation by a negative x^2.

C1/C8 Students must be able to model situations with and solve problems using the quadratic relation. Situations may be presented in words (or words and equations) or by graphs and/or tables of values. When solving a problem, students might be expected to use an equation to predict or to get a table to see the maximum value. Being presented with a situation, students might begin by collecting data or reading the given data, creating graphs using appropriate scales, domains, and ranges. For example, the problem might be to calculate the "hangtime" of a punt in a football game. Students can picture the ball after it is kicked. The path it follows "looks" parabolic. A proper domain can easily be selected since the ball is not likely to hang longer than 6 seconds. The range is limited by the height the ball will reach.

Quadratics

Worthwhile Tasks for Instruction and/or Assessment

C1/C21/F2/F4/C8

Performance

1) Have students collect data, sketch a graph, and model the following situation in order to predict answers and solve the problem:

Extend a 3 m wire from the back of a desk to the top of the chalkboard and mark 10 cm intervals. For a starting position, let a carrier roll along the wire, and time how long it takes for the carrier to reach each of the marks along the wire up to 2 m. From this data, predict how long it will take to reach 2.5 m and 3 m. Verify your prediction.

2) Chantal pulled the plug in her bathtub and watched closely as the water drained. As the water drained she made marks on her tub and used them later to determine the quantity of water remaining in the tub at various time intervals. The table contains the data she determined.

Time in seconds	15	25	48	60	71	100	120	130	150	180	190
Water (litres) remaining	55	51.1	42.6	38.6	35	26.5	21.4	18.8	14.6	9.5	7.9

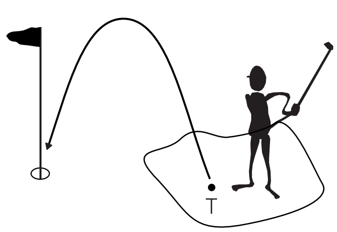
a) Ask the students to model the data with an equation using technology expressing litres of water remaining in terms of time.

b) Ask the students how much water was in the tub when the plug was pulled how long will it take to empty, and to explain why they chose to model the data with the function they chose.

C1/C12/C8

Performance

3) This picture represents the path of a ball as it flies through the air.



a) Ask students to describe how the height of the golf ball changes.

b) Ask them to sketch a graph to illustrate their description and explain why they drew it like they did.

c) When a golf ball travels through the air (goes way up into the sky, then comes back to land on the ground) do you think it maintains the same speed at all times? Explain.

d) Where in its flight is the speed of the ball the slowest? Explain.

4) After retrieving a ball from the roof of my house I threw it up into the air towards the street. Sketch a picture of the flight of the ball. How is it the same as the flight of the golf ball in question? How is it different?

5) From the table below describe in words the relationship between the embryo's length and age.

Age (months)	2	3	4	5	6
Length (cm)	1	9	14	18	20

Suggested Resources

The Language of Function and Graphs. Nottingham, UK: Shell Material, 1985, p. 20

Quadratics

Outcomes

SCO: In this course, students will be expected to

C14 **determine and interpret x-intercepts of quadratic functions**

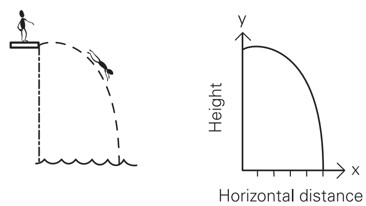
B3 **apply the quadratic formula**

C23 **solve problems involving quadratic equations**

Elaboration—Instructional Strategies/Suggestions

C14 In Year 10, students explored the factors of a quadratic equation and their connection to the horizontal axis intercepts on graphs. In this course students will be expected to solve all quadratic equations by using graphs (e.g., reading horizontal axis intercepts) or by using the quadratic formula.

Students should explore horizontal axis intercepts in a meaningful context. For example, a problem involving diving off a cliff gives students an opportunity to explore the vertex coordinates and horizontal axis intercepts in a meaningful way. Because it is a graph of height versus horizontal distance, this graph is in fact a picture of the event.



The vertex coordinates represent the maximum height by the diver, and the horizontal distance travelled to obtain to the maximum height. The horizontal axis intercepts represent the horizontal distance travelled to the water-entry point.

The equation $h = 88 + d - 0.85\ d^2$ expresses the distance above the water h versus d, the horizontal distance travelled. Students should graph the relationship and explore the values at the h-intercept, the top of the curve, and the d-intercept. Interpret each of these values with respect to the problem.

B3/C23 In the above example, most students would use technology to enter the equation, draw its graph, and trace to get values for the vertex and the *d*- and *h*-intercepts. In Year 10 students connected horizontal axis intercepts to solve equations where $h = 0$. To motivate the need for the quadratic formula to be introduced, students should be given a quadratic equation to solve for which technology can produce only an approximate answer. To get the exact answer requires an algebraic process. So, give the students the quadratic formula (see below) and ask them to use it to obtain values for the independent variable, then compare these unknowns to the values approximated from the graph. Previously, students also learned to find the horizontal axis intercepts (x-intercept) using factoring. Students should understand that the quadratic formula can be used instead of factoring and especially, when factoring is not possible. When $ax^2 + bx + c = 0$ and $a \neq 0$ is solved for x, the result is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (It is not necessary to develop the quadratic formula.)

The quadratic formula, then can be used to solve any quadratic equation since $ax^2 + bx + c = 0$ represents any quadratic equation. Students might then be asked to use the quadratic formula to determine where the diver enters the water. Since the quadratic equation formula has $\pm$ symbol, two possible roots will result. Only one would represent where the diver enters the water. The other is an inadmissable root. Take this opportunity to talk about inadmissable roots, and how in real context, often the use of only one of the roots of the equation is appropriate. Students need to use the formula several times to get used to substituting the values for *a*, *b*, and *c*. As well, they should check the answers with the graphing calculator to ensure that the use of the formula is correct and that the formula really does work.

... continued

Quadratics

Worthwhile Tasks for Instruction and/or Assessment

Suggested Resources

C14/C23

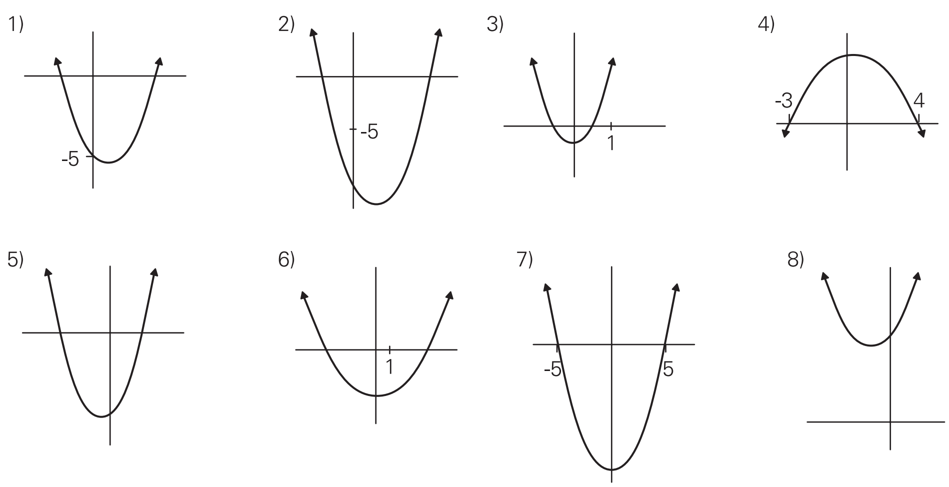
Performance

1) Richard and Elaine shot two missiles at a target whose coordinates are (30, 0). Missile R follow a path defined by the equation $h = -t^2 + 60t - 828$, while missile E follow a path defined by the equation $h = -t^2 + 60t - 892$, where h is height in metres and t is time in seconds. Both missiles overshot the target. Which missile overshot the target the most? Richard said that his missile was three times further from the target than Elaine's. Is he correct? Justify.

C14/B3

Performance

2) a) Solve the following quadratic equations, then use the solution(s) to match each equation with its corresponding graph, if possible:
   - i) $x^2 - 3x - 10 = 0$
   - ii) $x^2 = 2x + 15$
   - iii) $x^2 - 25 = 0$
   - iv) $x^2 + x = 12$
   - v) $x^2 + x + 12 = 0$
   - vi) $x^2 = 3x + 5$



C14/C23

Performance

3) In 1919, Babe Ruth hit a very long home run in a baseball game between the Boston Red Sox and the New York Giants. The trajectory of the ball is given by the equation $y = x - .0017x^2$, where x represents the horizontal distance (in feet) and y the vertical distance (in feet) of the ball from home plate.
   a) What was the greatest height reached by the ball?
   b) How far from home plate did the ball land?
   c) At what height was the ball when it crossed the plate?

... continued

Quadratics

Outcomes

SCO: In this course, students will be expected to

A3 **demonstrate an understanding of the role of irrational numbers in applications**

B1 **demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations**

B9 **perform operations on algebraic expressions and equations**

Elaboration—Instructional Strategies/Suggestions

... continued

A3 Since irrational numbers arise when solving quadratic equations, discussion should centre around whether an exact or an approximate solution is appropriate. Students will always be expected to give their answers using significant digits correctly. Students should be fully cognizant of the inaccuracies caused by round-off error.

B1/B9 Consider this example,

$$\begin{aligned} &\text{if } x^2 = 25 \\ &\quad x = \pm 5, \text{ and discuss the reasoning that should occur:} \\ &\text{if } x = 5, \text{ then } x^2 = 25 \\ &\text{if } x = -5, \text{ then } x^2 = 25 \end{aligned}$$

The above brief activity provides opportunity for worthwhile discussion that solidifies students' understanding of operation sense and the underlying importance of definitions and "order of operations." For example, the difference between -2^2 and $(-2)^2$ can be understood by recognizing what the base is and then performing the squaring operation first.

Students should also understand that if $(x + 2)(x + 3) = 0$, then one or both of the factors $(x + 2)$ and $(x + 3)$ must be zero, and how this leads to the solution thathas two possible answers. For example,

$$\begin{aligned} &\text{if } x + 2 = 0 \\ &\quad x = -2 \\ &\text{if } x + 3 = 0 \\ &\quad x = -3 \\ &\quad \therefore \{-2, -3\} \end{aligned}$$

Quadratics

Worthwhile Tasks for Instruction and/or Assessment

... continued

B3/A3

Pencil/Paper

4) Hector has been told that the width (x) of a rectangular field can be found using the equation $3x = 5 - 2x^2$. On the right, he is using the quadratic formula to find the width.

$3x = 5 - 2x$

step 1: $2x^2 + 3x - 5 = 0$

step 2: $x = \dfrac{3 \pm \sqrt{6 - 40}}{4}$

   a) State the equation
   b) Explain what Hector did to get the equation in step 1
   c) Explain what Hector did to get the equation in step 2. Is he correct? Explain.
   d) Ask students to solve the original equation. Ask them what number system is represented in the solution?

5) Revisit question 1) on the last two-page spread. Ask students to use the quadratic formula to determine the time it would take for each missile to hit the ground. Compare the answers obtained this time to the answers obtained the first time. Compare your answers. How are they the same? Different?

6) a) Explain how the graphing calculator displays the x-intercepts, and how this is different than the answer you get using the quadratic formula.
   b) Redo question 2) on the last two-page spread using the quadratic formula. Compare your answers.

Suggested Resources

Quadratics

Outcomes

SCO: In this course, students will be expected to

C1 **model real-world phenomena using quadratic equations**

C23 **solve problems involving quadratic equations**

B9 **perform operations on algebraic expressions and equations**

B1 **demonstrate an understanding of the relationships that exist between arithmetic operations and the operations used when solving equations**

C21 **create amd analyze scatter plots and determine equations for the curves of best fit, using appropirate technology**

F2 **use curve fitting to determine the equations of quadratic relationships**

Elaboration—Instructional Strategies/Suggestions

C1/C23 Students should be involved in solving a variety of problems using various techniques that involve quadratic equations. The expectation is that students would often use technology as a tool in solving the problems. If an equation is required, students should determine the equation using quadratic regression. To solve the equation, they can let $y = 0$ and then use the x-intercepts of the graph, or the quadratic formula to obtain the roots. Spreadsheets might be a tool that could help students solve problems.

The “campsite” problem discussed on p. 50, and “pasture problem” problem discussed on the opposite page provide the opportunity for students to use a trial and error approach, and perhaps a spreadsheet to help with the calculations. In the campsite problem, they might begin by trying a “width” dimension of 5 m. This would lead them to reason that the length would have to be 50 m less twice the 5 m or 40 m. Students might organize the trials into a table with headings Width, Length, and Area. Some students may wish to go further and set up a spreadsheet, or use the “table” feature on the graphing calculator. This approach would require them to establish formulas for the three categories “x” for width, “$50 - 2x$” for length, and “$x\ (50 - 2x)$”, for area. Then as they enter different width values, the length and area would automatically appear on the spreadsheet.

Others may graph the width versus the area and find the maximum by tracing the path to the curve until they find the highest point. They would then be expected to interpret it as the greatest area.

C23/B9/B1 When using a quadratic equation to solve a problem students might first have to rearrange the equation into general form ($ax^2 + bx + c = 0$), so that they can determine the values for a, b, and c. In so doing, they should be aware of the equation-solving process that allows them to manipulate the equation into the appropriate form.

For example, if given the equation $-\frac{1}{2}(x+5) = 3x^2$, students might first multiply all terms by –2, thus enabling them now to rearrange the equation so that the sum of the terms will be zero.

$$-2\left[-\frac{1}{2}(x+5)\right] = \left[3x^2\right](-2)$$

$$x+5 = -6x^2$$

$$6x^2 + x + 5 = 0$$

C1/C21/C23/F2 A trajectory problem usually includes an equation that represents the path of the object. Students might evaluate this equation for various heights. The graph of each equations can be traced to find solutions (x-intercepts) or maximum heights. Other trajectory problems may give certain information and expect students to determine the equation that describes the path. This can be done by entering the data points in lists, creating a scatter plot, and finding the curve of best fit using quadratic regression. Sometimes students are asked to find the maximum height. Knowing that the time it takes to reach a maximum height is halfway between the x-intercepts (symmetry of the parabola) might provide another way for students to calculate the maximum height (assuming the x-intercepts exist).

Quadratics

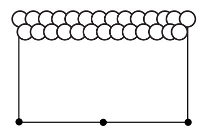
Worthwhile Tasks for Instruction and/or Assessment

C1/C23/B9/B1

Performance

1) A football is kicked into the air and follows the path $h = -2x^2 + 16x$, where x is the time in seconds and h is the height in metres.
   a) What is the maximum height of the football?
   b) How long does the ball stay in the air?
   c) How high is the ball after 6 seconds?
   d) How long does it take the ball to reach a height of 15 m?

2) Farmer Brown has many hectares of pasture that have not been fenced. His sister Ethel asks her brother if she can use some of his land to keep her cattle. He decides that he can spare some pasture area down by the old stone wall. He has 440 m of fencing. Ethel wants to use the 440 m of fence to create three walls (the stone wall will be the fourth) of a rectangular area.



   a) Create a table with the following headings: width, length, area.
   b) Create a graph of area versus width.
   c) Find the measurements of this rectangle so that she will have a maximum amount of grass.
   d) What will be the area? How do you know it's a maximum? Explain.

3) Tracie and Nathaniel are doubling the floor area of their camp, which now measures 48 square metres. The equation $x^2 - 8x - 16 = 96$ represents the new floor (enlarged area) whose width is x metres. Use the equation to find the dimensions of the new floor.

C1/C23/B9/B1/C21/F3

Enrichment

1) An ice cream specialty shop currently sells 240 ice cream cones per day at a price of \$3.50 each. Based on results from a survey, for each \$0.25 decrease in price sales will increase 60 cones per day. If the shop pays \$2.00 for each ice cream cone, what price will maximize the revenue?
2) A ball was hit by a bat 1 dm above home plate and reached a maximum height of eight metres, a horizontal distance of two metres from home plate.
   a) State three coordinates from the above information.
   b) Find the best equation of the path of the ball. How far from home plate was the ball caught?

Extension

3) The trajectory of a cannon ball is defined by the equation $y = ax - 0.1(1 + a^2)x^2$, where $x, y > 0$ 'a' is the slope of the barrel of the cannon, and (x, y) represents the coordinates (in kilometres) of the cannon ball at any point in its trajectory.
   a) Graph the above equation using the following a-values: 0.6, 0.8, 1, and 1.2.
   b) Which of these values yields the greatest horizontal distance for the ball?

Suggested Resources

UNIT 4:
EXPONENTIAL GROWTH

Unit 4
Exponential Growth

(25-30 Hours)

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C29 **analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships**

C13 **describe and apply the characteristics of exponential relationships**

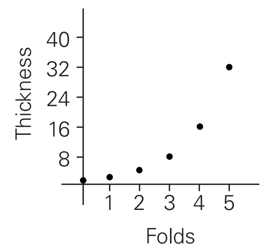
C11 **describe and translate between graphical, tabular, and written"representations of exponential relationships**

C5 **determine and describe patterns and use them to solve problems**

Elaboration—Instructional Strategies/Suggestions

C29/C13/C11/C5 Having already studied linear and quadratic relations in this and previous courses, students should extend their study of relationships to those that are exponential. Students have connected linear functions with arithmetic sequences and quadratic functions with power sequences, and now they should connect exponential functions with geometric sequences. For example, they might want to revisit the paper folding activity that produced data for the thickness of a simple sheet of paper. When they graph this relationship, they may not be able to distinguish it from a quadratic relationship at first.

Number of folds	0	1	2	3	4	5
Thickness	1	2	4	8	16	32
Powers of 2	2^0	2^1	2^2	2^3	2^4	2^5



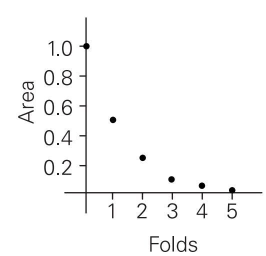
The students fold the paper in half, in half again, etc. They can easily see from the table that after folding the paper in half the fifth time, they have a thickness 32 times what they started with. Ask them to extend the table using their understanding of the pattern. Have students create and graph a similar table for the quadratic relationship $y = x^2$. Ask them to describe what happens to both graphs and tables after the seventh value. When asked to find a pattern in the data, students will try constant growth and see that the data is not linear. They might try common difference and find that the data is not quadratic. Students should note the common ratio, e.g., and hence connect this kind of relationship to geometric sequence.

C13 Teachers should talk to students about the growth characteristic of exponential relationships. As one variable increases at a constant rate, the other increases or decreases as a multiple of the previous one, e.g., there is a common ratio between terms as described above as 2.

C29/C13/C11/C5

Return to the paper-folding activity and focus on the area of the paper. Students should notice that as the paper is folded in half, the area gets smaller by a factor of $\frac{1}{2}$ each time (e.g., $\frac{1}{2}$ the original, $\frac{1}{4}$ the original, $\frac{1}{8}$ the original, ...). They can compare this situation, its table, and graph with the thickness situation described above and note that both are exponential—the first, a growth; the second, a decay. Students could be asked to consider if the area would ever become zero.

Number of folds	0	1	2	3
Area	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$
Powers of 2	2^0	2^{-1}	2^{-2}	2^{-3}



This might help them understand that an exponential relationship should always approach an asymptote (a line to which the graph approaches at an extreme value but never touches).

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

C29/C13

Paper and Pencil

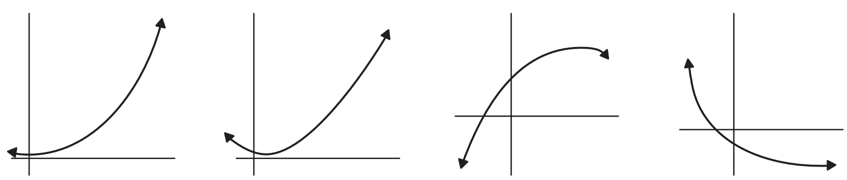
1) Given these three tables of data taken from three different experiments,
   a) which, if any, do you think represent an exponential relationship? Explain your reasoning.
   b) if any are exponential, explain whether they are growth or decay.

x	1	3	5	7	13	–11
y	–4	–32	–84	–160	–532	–340

x	1	3	5	7	–11	–13
y	13	157	1453	13177	–5	–5

x	1	3	5	7	13	–11
y	–17.5	.825	1.825	2.825	–5.825	–6.175

2) Which of these graphs might represent an exponential relationship? Explain your reasoning.



C11/C5

Paper and Pencil

3) Take a sheet of paper and fold it in half. You now have two sections. Fold in half again. You now have four sections, and so on. Complete the table

Number of folds: n	0	1	3	4	5	6
Number of sections: S	1	2	4	8		

   a) Explain how to express all the S-values with a base of 2.
   b) Use what you found in (a) to predict the number of sections after 8 folds; 10 folds.
   c) Sketch a graph using the values you have obtained so far. Should you join the dots? Explain.
   d) What kind of relationship does your graph represent—linear, quadratic, or exponential? Explain how you know. Is it growth or decay? Explain.

C29/C13/C11/C5

Paper and Pencil

4) Ima Clever is the hottest math teacher around. One major school is so anxious to hire her that they offered her a choice of three salary options.

   Option 1: $30 for the first day of work, but overall earnings double for each additional day of work
   Option 2: Three cents for the first day of work, but overall earnings triple for each additional day of work
   Option 3: A flat rate of $300 000 a day for the 195-day school year.

Ask students to determine which contract Ima should sign. Have them explain fully using graphs, tables, and written reports.

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C29 **analyse tables and graphs to distinguish between linear, quadratic, and exponential relationships**

C13 **describe and apply the characteristics of exponential relationships**

C5 **determine and describe patterns and use them to solve problems**

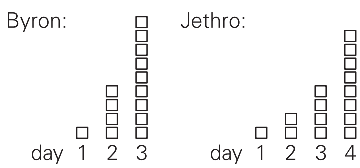
C11 **describe and translate between graphical, tabular, and written representations of exponential relationships**

Elaboration—Instructional Strategies/Suggestions

C29/C13/C5 Having just studied quadratic relationships in the previous unit, students should compare the two functions (quadratic and exponential) with particular focus on their growth characteristics. For example, consider the allowance problem:

> Byron and Jethro were comparing their allowances. Byron receives 1 cent on the first day, 4 cents on the second, 9 cents on the third, 16 cents on the fourth day, and so on—each day receiving an amount equal to the the square of the day of the month in cents. Jethro convinced his parents to pay him 1 cent on the first day of the month, 2 cents on the second day, 4 cents on the third day, and so on—each next day receiving double the amount than on the previous day. Who has more money after one week, two weeks, three weeks, a month?

C11 Students might want to model the growth of the allowances using towers of cubes and create tables to compare the accumulated amounts of money, for both Byron and Jethro as the month progresses. They should notice that the amounts are increasing at different rates. They will note that the growth rate for both

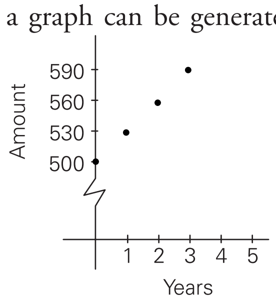


Accumulated Amounts

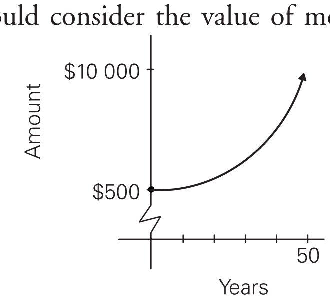
day	1	2	3	4	5
Byron	1	5	14	30	55
Jethro	1	3	7	15	31

allowances are not constant. Students should try to describe the rates of growth in words and draw graphs.

C29/C13/C5/C11 Simple and compound interest provide a good context for examining and comparing rates of growth that are linear and exponential in nature. From a practical situation of $500 growing at 6% compounded annually, a graph can be generated. Students could consider the value of money after three



Dollars	Years
500	0
530	1
560	2
590	3
.	.
	50



Dollars	Years
500	0
530	1
561.80	2
595.51	3
.	.
9210.08	50

years and compare it to the value in a simple interest situation. A graphical display should be examined.

Students should feel comfortable moving from one representation to the other (graph, table, context, verbal, symbol) regardless of what representation of the exponential relationship they are given. Given the graph, they should be able to recognize its shape and represent it with a table or in words. Given the table of values, they should be able to see the exponential pattern and describe it in words.

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

C29/C5
Performance

Time (hours)	0	1	2	3	4	5	6	7
Number	100	126	160	200	250	316	400	500

1) A culture of bacteria was grown in a laboratory. The table below shows the number of bacteria present at different times.
   a) Begin by studying the table. Can you see any pattern in the data?
   b) Compare the numbers of bacteria in the culture at zero hours and at three hours. In the same way, compare the numbers at one and four hours, two and five hours, three and six hours, four and seven hours. What do you notice? Use the pattern you have found to predict the number of bacteria after 8, 9, 10, and 11 hours.
   c) What is the **growth ratio** (population increase per hour). What do you notice? Use this to predict the number of bacteria after 8, 9, 10, and 11 hours. Do your answers agree with your earlier predictions?

C13/C5/C11
Performance

2) Why do you think that the Canadian government declared chain letters illegal? Use words, graphs, tables, and/or equations to help explain your answer.

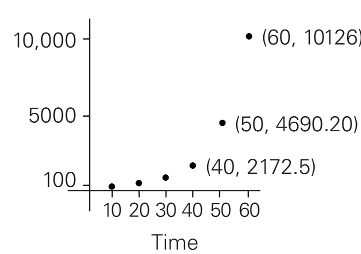
   A chain letter usually contains a message that encourages the reader to send money or a gift to the top name on a list of up to 10 names, then to erase the top name and add his or her own name to the bottom of the list. Then the reader is to send this letter to 20 friends, asking each of them to do the same.

C29/C13/C5/C11
Performance

3) Match the situation given with the relationship (linear, quadratic, exponential) that would best describe the value of the investment rate.

   Situation 1: Billy invests in his friend's new cyclo-motor machine. Billy gives $500 to his friend. His friend says that each month he would set aside 1% of Billy's investment and return the $500 and the amount set aside at the end of two years.
   Situation 2: Maria invests in Sally's ITS business. Sally says she will repay Maria the amount of money invested plus the amount earned at 12% yearly interest. The first month she will calculate 1% of the amount invested and add it to the invested amount so that the next month the 1% will be calculated on the 'new' total, thus earning interest on the amount plus interest.
   Situation 3: Sally offers Harold a different deal. She says she will repay him with the money that accumulates after she puts $1 into an account and each month triples the amount that was put in the last time.

   a) Show using a table of values, and graphs, in which of the above ways you would prefer to invest your money. Explain your reasoning.



4) Given the graph
   a) Prepare a table of values.
   b) Explain how you know that the relationship in the graph is exponential.
   c) Describe the relationship using the context of compound interest.

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C21 **create and analyse scatter plots and determine equations for the curves of best fit using appropriate technology**

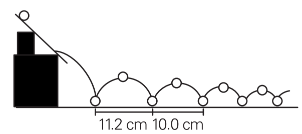
F3 **use curve fitting to determine the equations of exponential relationships**

F4 **interpolate and extrapolate to predict and solve problems**

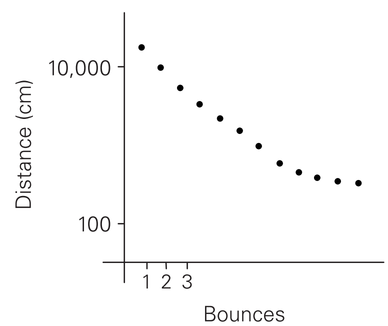
C25 **solve problems involving exponential equations**

Elaboration—Instructional Strategies/Suggestions

C21 Exponential relationships, compound interest, population growth, and the allowance problem provide interesting contexts. If appropriate technology is available, students can conduct an experiment and collect data using technology. Such experiments should result in data that looks exponential (bacteria growth or radioactive decay; heat build-up in a car on a sunny day; or the cooling of a cup of coffee as it sits untouched). Again the focus should be on identifying independent and dependent variables and recognizing that the relation has a pattern. Furthermore, the pattern can be described by a curve on a graph and by an equation. If the context suggests that the pattern approaches an asymptote, then students should choose exponential regression. All students should be able to generalize the exponential pattern to a function using graphing technology.



F3/F4C25 From the graphs, students should be able to interpolate and extrapolate answers to help them see values that indicate different growth rates. They should sketch and use technology to obtain the curve of best fit. For example, students might conduct an experiment to test how the horizontal distance that a ball travels between bounces is related to the number of bounces completed. They should recognize that the data is exponential (that is, there is a common ratio). Students should be encouraged to fit their data to the equation $y = AB^x$ where x is the number of bounces and y is horizontal distance. Ask the students to determine that A roughly represents the initial horizontal distance the ball travelled before the first bounce. They could verify this by letting $x = 0$, $B^0 = 1$ and therefore $y = A$ when the number of bounces = 0.



C21/F3/F4/C25 Ask students to enter $A * B \wedge x$ into y_1 = on the function screen. If the initial horizontal distance is 12.5 cm, have students set A equal to that value (on the calculator type $12.5 \rightarrow \text{STO} \rightarrow A$).Then have students try various values for B beginning with B = 1 and press graph to compare the equation to the data for best fit. Eventually a value for B will result in the best-fit curve. Have students interpret the B-value and come to realize that it is the same value as the common ratio.

When students need to find the equation they would be expected to use exponential regression. Ask students to check their work by finding the exponential equation using exponential regression.

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

C21/F3/F4/C25

Performance

1) Any cube larger than $2\times2\times2$ cm constructed with cubes will, when dropped in a paint can and removed, have some of its cubes with three faces painted, some with two faces painted, some with one face painted, and some with no faces painted. For any $n\times n\times n$ cube ($n > 2$), complete the table and use it to generate an equation that can be used to determine the number of faces with:
   a) no faces painted b) one face painted
   c) two faces painted d) three faces painted.

dimensions of the cube	number of faces painted 0	1	2	3
$3\times3\times3$	1	6	12	8
$4\times4\times4$				
$5\times5\times5$				
⋮				
$n\times n\times n$				

2) Conduct the following experiment to determine how long it takes a cup of boiled water to lose half its heat.
   a) boil water and pour into a cup
   b) insert temperature probe
   c) gather temperature data and create a scatter plot
   d) fit a curve to the data and find its equation

3) Tower of Hanoi Problem. Use a 1×3 grid and have available several blocks, each a different size. With one or more blocks placed in one cell (on top of one another in descending order of size—called a tower), the objective is to transfer the tower to another cell in the minimum number of moves obeying these rules: (1) move only one block at a time to constitute one move and (2) a block may only be placed on top of a larger block (or on no block) at any time.
   a) Complete the table for this game.
   b) Generate the function that would relate the number of blocks to the minimum number of moves.

number of blocks	minimum moves
1	1
2	3
3	
4	
5	

Paper and Pencil

4) A population growth equation $P = 3.8(1.017)^t$ gives an annual percentage population growth of 1.7% for Australia.
   a) Write the annual percentage growth rate of a country whose population is given by $P = 60(1.035)^t$.
   b) A country's population has an annual percentage growth of 6.4%. Its population in 1985 was 53 million. Write an equation to give the population in millions t years from 1985.
   c) If r is the annual percentage growth, and the initial population is P, write the equation that gives the population at time t.

5) Kate bought a computer for \$3000 to use in a business she is setting up. If it depreciates at a rate of 30% per year, what will be the depreciated value after one year, two years, ... five years? Find an expression for its value after n years and show it on a graph. Approximately how long does it take for the value of the computer to reduce to half the present amount?

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

A8 **demonstrate an understanding of the exponential growth of compound interest**

C26 **solve problems that require the application of compound interest**

B5 **demonstrate an understanding of and apply compound interest**

Elaboration—Instructional Strategies/Suggestions

A8 In today's financial world, most situations are not calculated using simple interest ($I = Prt$). Interest is not calculated just once during the life of a loan or investment, but quite frequently. For example, the mortgage on a house may be compounded semi-annually. Students will understand that if they invest their money in a situation that involves compound interest, they will be paid interest on their interest. As the money accumulates in this way, the growth is exponential. Have students explain how they know that the growth is exponential.

C26 Students should explore the different ways that compound interest is used in banking and through investment. This will include some work with annuities (see p. 96).

❒ Suppose $100 is invested at 10% per year compounded semi-annually and is cashed after two years, interest will be calculated four times (two times each year). At the end of six months the $100 investment will earn 5% (compounded semi-annually), so that six months later your $105 will earn 5% interest again, growing to $110.25. At the end of the fourth interest-earning period (two years) the $100 will have grown to $121.55. Ask students to create a table and plot the relationship between compound periods and total amount of money. Ask them to explain the patterns they see. What is the common ratio?

B5/A8 Students should learn that the calculation described above can be done more efficiently using the formula $A = P\,(1 + r)^{t}$. To help students understand this formula, ask them to find 5% of $100 and then add it to $100. They should get $100\times0.05\rightarrow\$5$, added to $\$100(100\times1.05\rightarrow105)$. Help them understand that the (1 + r) is the 105%. (It may help some to show that $P\,(1 + r)$ is the same as $P + Pr$ or $\$100+\$100\times0.05$.) Ask students to explain why $A = P\,(1 + r)^t$ describes an exponential relationship. How is the formula $A = P\,(1 + r)^t$ the same as $y = ab^x$? Ask students to relate the a and P, and the b and $(1 + r)$.

To help students understand the exponent t, you might have them use the constant feature on their calculators. For example, on the TI-83 have students do the first $\boxed{x}$ calculation (first compounding period) $\rightarrow\$100\times1.05$, enter, $\rightarrow1.05$. Then press 1.05, enter, $\rightarrow110.25$. Then just press enter, $\rightarrow$, 115.76. Ask students to describe what is happening and how the number of times they push "enter" relates to the value for t, the number of compounding periods.

A8/C26/B5 Students should use the compound interest formula not only to determine how much an investment will be worth some day, but also how much needs to be invested now (present value) to provide a certain sum at some future date. $\dfrac{A}{(1+r)^{1}}$

In the study of compound interest students might investigate how long it takes for an amount to double if it is invested at different rates. This should lead to the "rule of 72," which students can then use to quickly approximate doubling time given a particular situation. If an amount is invested at 7.2% it will take 10 years to double. Also, if an amount is invested at 10%, it will double in approximately 7.2 years.

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

A8

Pencil and Paper

1) Two situations are given below.

Situation 1: Jeff borrows $1000 from his brother Mike. Mike, to be fair asks Jeff to pay him back the $1000 in one year, plus the interest that accumulates at 8% over the year.
Situation 2: Sharon borrows $1000 from her banker Bill. Bill tells Sharon that he can lend her $1000 for a year at 8% per year compounded monthly. She must repay the loan in one year.

a) Explain how the debt growth in these situations is different. Explain why.
b) Which situation requires more money to satisfy? Explain why.
c) Graph these two situations and describe how the graphs differ.

A8/B5

Pencil and Paper

2) When using the formula $A = P(1 + r)^t$ describe
a) how the value used for "*r*" relates to the given yearly interest rate
b) when asked to find *A*, how to determine the value for "*t*"

A8/C26/B5

Performance

3) Islay's grandmother Sharon gave Islay's mother $1200 on the day Islay was born. Islay's mother invested the money at 11.4% per year compounded quarterly. Complete the following steps to see how much money Islay will have on her 18th birthday. ____
a) 0 months = $1200 ____
b) 3 months = $\$1200\times(114/4)$ = $1200(1.0285) = ____
c) 6 months = $\$1200\times(114/4)\times(1.114/4)$ = $1200(1.0285)^2 = ____
d) 9 months = $\$1200\times(114/4)\times$_________ = $1200(1.0285)^3 = ____
e) 1 year = $\$1200\times(114/4)\times$_____________ = $ ___________ = ____
f) 2 years = $\$1200\times(114/4)\times$_____________ = $ ___________ = ____
g) 10 years = $\$1200\times(114/4)\times$____________ = $ =
h) 18th birthday = $\$1200\times(114/4)\times$______ = $ =

4) Islay decides to cash in the money from the investment (#3) just after her 18th birthday to use it as a down payment on the purchase of a car. The car she needs will cost $14 595.27 including tax. She borrows the difference at 12.4% per year compounded semi-annually over three years. What will be her monthly payments?

5) Bobby has just won $50 000 in the lottery. He decides to invest just enough of the $50 000 so that in three years he can purchase a $30 000 car. He will invest now at 12.5% per year compounded quarterly. How much of the $50 000 will be left to buy his math teacher a present?

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C11 describe and translate between graphical, tabular, and written representations of exponential relationships

C13 describe and apply the characteristics of exponential relationships

A1 demonstrate an understanding of and apply zero and negative exponents

A2 develop, demonstrate an understanding of, and apply properties of exponents

Elaboration—Instructional Strategies/Suggestions

C11 Now that students have had some opportunity to analyse graphs and use them to explore rates of change, they should take some time to explore other patterns. In particular, they should explore the pattern that determines the shape and location of the exponential graphs and how that might change as the equation changes. By looking at graphs of $y = 2^x$ and $y = 3^x$ and $y = 10^x$, students should notice that they all pass through the point (0, 1), the "focal" point.

C13/A1/A2 Studying patterns in the graphs and tables will bring up many important concepts dealing with number sense. For example,

i) In the paragraph above students should be able to generalize that any base to the exponent 0 will result in 1.
ii) In creating the table for $y = 2^x$, when $x = -1, -2$, y will result in fractions $\frac{1}{2}, \frac{1}{4}$, ... students instead, may use their calculators and get y-values of 0.5, 0.25 ...

Upon further investigation, they should notice that $2^{-1} = \frac{1}{2}$, and $2^{-2} = \frac{1}{4}$, and $3^{-1} = \frac{1}{3}$, and $3^{-2} = \frac{1}{9}$. Pronouncing decimals as fractions will help students make the connections. For example, when students see 0.5, they should say this as "five-tenths," and 0.25 is pronounced "twenty-five hundredths." All of this will be visually reinforced as they find these corresponding values on the graphs. Students should be able to describe these equalities in words. Students should be encouraged to view numbers in their various representations $(10^{-1} \Rightarrow \frac{1}{10} \Rightarrow 0.1)$. This is a logical connection with the approach taken in earlier grades but needs to be reinforced and extended. Mental math activities should reinforce understanding.

Students should look closely at the generalized equation $y = a^x$ and explore what happens to its graph:

a) when
   i) $a = 1$ iv) $a < 0$
   ii) $0 < a < 1$ v) $a > 1$
   iii) $a = 0$

b) Students should graph $y = 2^x$ and trace for y-values when
   i) $x = 0$ iv) $x = \frac{1}{2}$
   ii) $x = -1$ v) $x = \frac{1}{3}$
   iii) $x = 1$

(A table might be helpful)

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

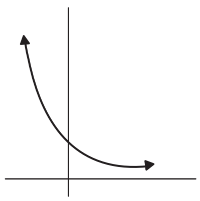
C11/C13/A1

Mental Math

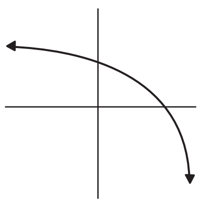
1) Match each graph in the left column to a given stimulus in the right column, if possible. Produce a graph for the stimulus that doesn't match

Graph

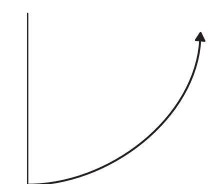
a)



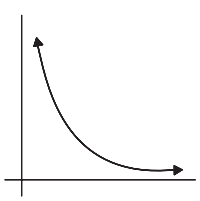
b)



c)



d)



Stimulus

i) On the first day I received one cent, the second day two cents, the third day four cents, the fifth day eight cents, ...

ii) $y = b^x$, $0 < b < 1$

iii) $y = 2^{-x}$

iv) I poured myself some coffee that was too hot to drink, so I let it cool. I fell asleep.

v) $y = \left(\frac{3}{2}\right)^x$

vi)

x	−3	0	5	6
y	30	10	5	4.99

C13/A1/A2

Pencil and Paper

2) Explain how you would know that if $2^{-1} = \frac{1}{2}$, that $\left(\frac{2}{3}\right)^{-1} = \frac{3}{2}$.

3) Use the graph $y = 4^x$ to explain why $4^0 = 1$.

4) Evaluate:

i) $\left(\frac{4}{5}\right)^{-2}$ ii) $3^0 + 2^{-2}$ iii) $\left(\frac{1}{2}\right)^{-4} - 4^2 + x^0$ if $x \neq 0$

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

A2 **develop, demonstrate an understanding of, and apply properties of exponents**

A1 **demonstrate an understanding of and apply zero and negative exponents**

C5 **determine and describe patterns and use them to solve problems**

Elaboration—Instructional Strategies/Suggestions

A2/A1/C5 Students may already have realized from their work with compound interest and money growth that, in the equation $A = 750\ (1.05)^n$, when $n = 0$, $(1.05)^0 = 1$, and since they have not yet invested their money ($n = 0$), they still have the $750 they started with.

A1 Zero and negative exponents can be explored using patterns when exponential functions are studied. For example, complete the table below for $y = 10^x$ by generalizing the pattern observed.

10^4	10000	10^{-1} ____	
10^3	1000	10^{-2} ____	
10^2	100	10^{-3} ____	
10^1	10	10^{-4} ____	rule? ____________________
10^0	___		

As students explore negative exponents, they should realize that a number with a negative exponent can always be written with a positive exponent, but one form is not "better" than another, just different. Traditional teaching of exponents has left the impression that a base raised to a negative exponent should be changed to a base raised to a positive. In many contexts (e.g., scientific notation) the negative exponent is preferred.

C5 Students have learned in previous courses that scientific notation can be used to record numbers using proper significant digits. For example, to express the number 2900 (two significant digits) with three significant digits, use this form: 2.90×10^3. An effort should be made to remind students of the patterns for scientific notation. Discussion: "If you want to express 385 as a number in scientific notation, then you could write 3.85 as $\frac{385}{100}\times100$, which simplifies to 385×100, or 3.85×10^2."

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

A2/A1

Pencil and Paper

1) Use the graph of $y = 3^x$ to explain why:

   a) $3^{-2} = \frac{1}{9}$ b) $3^0 = 1$ c) $3^{0.5} \doteq 1.732$

2) Rewrite in another form:
   a) 2^{-3} with a positive exponent
   b) 312^0
   c) 1 as a power of 10.
   d) 3^{2x} with a different base.
   e) $\left(\frac{1}{8}\right)$ with a base 2
   g) 3 250 000 with two significant digits
   f) 3 250 000 in scientific notation

3) Explain how 625 can be expressed using base 5.

A2/A1

Mental math

4) Express
   a) 32 with a base 2 d) 2^{-3} with a positive exponent
   b) 27 with a base 3 e) 4^7 with a negative exponent
   c) 16 as a power of 2. f) $\left(\frac{1}{2}\right)^{-5}$ as a whole number

5) Solve
   a) $4^x = 16$ b) $3^x = 9$ c) $2(5^x) = 50$ d) $3^{-3} = x$ e) $49x^0 = 7^x$

A2/A1/C5

Performance

6) Evaluate by subtracting exponents. Now, evaluate the same by using your calculator to expand the numerator and denominator, then dividing. Tell in words how what you have done explains why a power with exponent zero is defined the way it is.

7) Greg loses points on a test for saying that $\left(\frac{1}{2}\right)^{-8}$ equals -2^8. Tell Greg what error he made, and how to avoid it in the future.

8) Divide $\dfrac{6.42\times10^{14}}{4.91\times10^{-7}}$ using estimation. Comment on Teddy's solution:

   Teddy's solution: 6.42 rounds to 6, and 4.91 rounds to 5, so the quotient of $\frac{6}{5}$ is about 1. 10^{14} divided by 10^{-7} gives about 100, so the answer is about 100.

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C26 solve problems that require the application of compound interest

B5 demonstrate an understanding of and apply compound interest

B6 determine the amount and present value of annuities

Elaboration—Instructional Strategies/Suggestions

C26/B5 Students should solve a variety of problems using various techniques that involve exponential equations. Several of these contexts have been referred to in earlier elaboration such as compound interest, the allowance problem, population growth, and government debt load.

Compound interest should be used by students to determine:

- the amount of an investment over a period of time with different compounding periods. For example, calculate the amount of money Amanda will have at age 18 if \$5000 is invested at 10% per year, compounded monthly when she is one year old.

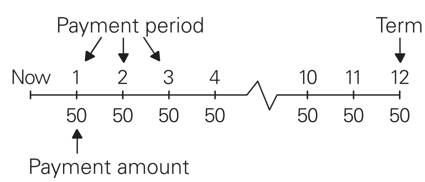
  $$\left(A = \$5000\left(1+\frac{0.1}{12}\right)^{(12)(17)}\right)$$

- the present value of a loan payment. For example, Toby decides to invest enough money now so that in 3 years, he can help his daughter buy a car. He wants to give her \$3000. How much must he invest now at 12.5%?
  $3000 = P(1 + .0625)^{2(3)}$

  $$\left(P = \frac{3000}{1.0625^6}\right)$$

B6 An annuity is a sequence of payments made at equal time intervals. The amount of the annuity is the sum of the sequence of payments, including all the interest earned. Examples of annuities include mortgage payments, pension cheques, and payments made to repay loans. Annuities can be illustrated visually with a time diagram. Consider Wibur's situation:

> Wilbur wishes to save money for a stereo system. He plans to set aside \$50 per month, beginning at the **end** of January, and to invest his money in a savings program that pays at 12% per year compounded monthly. Wilbur's last payment will be made at the **end** of December.



Note that Wilbur's first payment is at the end of the first payment period. Wilbur's January payment will earn interest at 12% per year, compounded monthly, for 11 months. Using the compound interest formula

$$A = P\left(1+\frac{i}{n}\right)^{nt}$$
$$= 50(+0.01)^{11} \text{ or } 50(1.01)^{11}$$

Similarly his second payment will be $50(1.01)^{10}$, and these payments for a sequence of values all the way to the December payment of $50(1.01)^{0}$, or just \$50.

... continued

Exponential Growth

Worthwhile Tasks for Instruction and/or Assessment

C26/B5

Performance

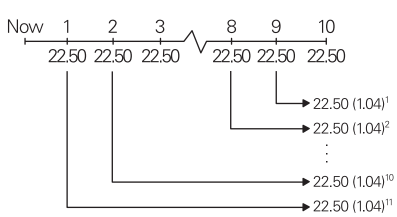
1) Dorothy buys a \$15 000 car. She is able to pay \$3000 as a down payment. She borrows the rest of the money from a bank at 10.25% per year, compounded semi-annually over five years. How much will Dorothy actually pay for the car?
2) Ronnie would like to have \$8500 available when he graduates from high school to help expand his business. How much should he invest now at 12% per year, compounded monthly, in order to have the \$8500 two years from now?
3) Mr. and Mrs. Maze wish to give their newborn son a cheque for \$20 000 on his 21st birthday. How much money must they invest at his birth if the money will earn 4.8% per year compounded semi-annually.
   a) Read Arthur's solution attempt to find any errors.
   b) Explain to Arthur what he did wrong and how to fix it.

Arthur's solution begins ... $A = \dfrac{20000(1+.048)^{21}}{2}$

C26/B5/B6

Performance

4) The following time diagram represents the amount of an annuity with a term of five years.
   a) What is the payment being made?
   b) What is the annual interest rate?
   c) What is the compounding period?



5) Cindy plans to save for her new baby that she plans to have in three years. She decides to invest \$1000 twice a year at 9.8% per year, compounded semi-annually. Draw a time diagram to represent her investment.

6) A deck can be built for \$2 300. How much should you start investing every month if the interest rate is 8% per year, compounded monthly, and you hope to have enough cash in 18 months?

Suggested Resources

Exponential Growth

Outcomes

SCO: In this course, students will be expected to

C26 solve problems that require the application of compound interest

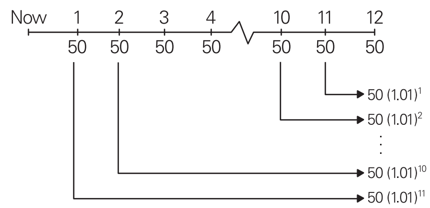
B5 demonstrate an understanding of and apply compound interest

B6 determine the amount and present value of annuities

Elaboration—Instructional Strategies/Suggestions

... continued

B6 The same pattern will continue for all Wilbur's payments. Wilbur's 11th or second-last payment will be made at the **end** of November. This $50 payment will earn interest for only one month, so it will amount to $50(1.01)^1$. His final payment will earn no interest, since he plans to take all the savings and buy the stereo.



Each of his payments earns interest except the last, and the accumulated sum, including interest, becomes the total amount he receives. To find the total amount, students should add the 12 payments

$$50(1.01)^0 + 50(1.01)^1 + 50(1.01)^2 + \ldots + 50(1.01)^{12}\ .$$

B6/C26/B5 Students should be encouraged to use the Finance feature on the TI-83 calculator to find the sum, in addition to adding the 12 payments as in the example above. Using the Finance feature, students will select 1:TVM Solver ... and enter values in appropriate places. *N* is the total number of payments (12 in the above example). I% asks for the interest rate as a percentage. PV is the present value (zero in this case since Wilbur begins with no money). PMT is the payment amount ($50 in this case). FV is the future value, which is the sum of all the payments and accumulated interest (the sum of the 12 payments in the example above). Since this is the amount students need to find, they will leave the value as zero and come back to this in a moment. *P/Y* is the number of pay periods (12), and *C/Y* is the number of compounding periods (12). The payments are at the end of the month. Now return the cursor to the FV, and press "Solve" (2nd Enter) to calculate the sum, or future value.

A variation on the annuity problem would happen when a future value is given and the present value is asked for. For example:

> Lucy and Pierre are saving money to celebrate their parents' 50th anniversary in three years. They would like to have $5000 for the occasion. What would they have to invest now at 10% per year, compounded monthly, to reach their objective?
> $5000 = P(1 + .083)^{36}$
> $P = 5000/(1.083)^{36}$

Exponential Growth

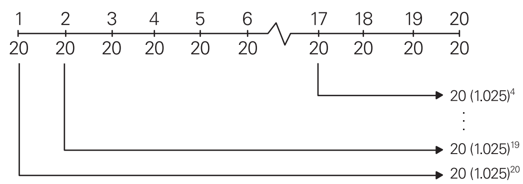
Worthwhile Tasks for Instruction and/or Assessment

... continued

B6/C26/B5

Performance

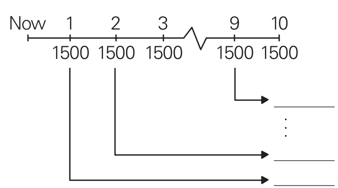
7) The following time diagram represents the amount of an annuity with a term of five years.



a) How does the diagram help you to recognize that this represents an annuity situation?
b) What is the periodic payment?
c) What is the annual interest rate?
d) How many payments are made?

8) Betty Lou begins to work part-time when she turns 16 and plans to begin the course described below at age 19. She is looking to her future costs of education. She wants to know how much money she will need to set aside each month at 10% per year, compounded monthly, in order to have $10,000 for first year of studies in an information technology course.

9) Complete the time diagram.
The interest is 7% per year, compounded annually. Payments of $1500 are made annually for 10 years.



10) Cindi plans to save for community college in three years. She decides to invest $1000 twice a year at 15% per year, compounded semi-annually. Draw a time diagram to represent her investment.

11) Joanne and Sheila set up an annuity so that in 12 years they will have enough money to purchase a new tractor for their horse farm. They deposit $4750 at the end of every six months in an account that earns 11% per year, compounded semi-annually. Set up a time diagram to illustrate their situation.

12) The Allens are interested in buying the house listed in the following ad:
Saint John, 2 storey, 3 BRs, centre hall plan, immaculate decor, hardwood floors, ground-level family room, and new kitchen. Huge mature treed lot. Asking $264 800.

They have a down payment of $150 000 and would take a mortgage with their bank at 12.5% per year, compounded monthly, and amortized over 25 years.
a) What is the value of the mortgage?
b) What is the monthly interest rate?
c) How many mortgage payments will they have to make?
d) What are their monthly payments?

Suggested Resources

Appendix A: Assessing and Evaluating Student Learning

Assessing and Evaluating Student Learning

In recent years there have been calls for change in the practices used to assess and evaluate students' progress. Many factors have set the demands for change in motion, including the following:

- ***new expectations for mathematics education as outlined in* Curriculum and Evaluation Standards for School Mathematics** (NCTM 1989)

The *Curriculum Standards* provide educators with specific information about what students should be able to do in mathematics. These expectations go far beyond learning a list of mathematical facts; instead, they emphasize such competencies as creative and critical thinking, problem solving, working collaboratively, and the ability to manage one's own learning. Students are expected to be able to communicate mathematically, to solve and create problems, to use concepts to solve real-world applications, to integrate mathematics across disciplines, and to connect strands of mathematics. For the most part, assessments used in the past have not addressed these expectations. New approaches to assessment are needed if we are to address the expectations set out in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989).

- *understanding the bonds linking teaching, learning, and assessment*

Much of our understanding of learning has been based on a theory that viewed learning as the accumulation of discrete skills. Cognitive views of learning call for an active, constructive approach in which learners gain understanding by building their own knowledge and developing connections between facts and concepts. Problem solving and reasoning become the emphases rather than the acquisition of isolated facts. Conventional testing, which includes multiple choice or having students answer questions to determine if they can recall the type of question and the procedure to be used, provides a window into only one aspect of what a student has learned. Assessments that require students to solve problems, demonstrate skills, create products, and create portfolios of work reveal more about the student's understanding and reasoning of mathematics. If students are expected to develop reasoning and problem-solving competencies, then teaching must reflect such, and in turn, assessment must reflect what is valued in teaching and learning. Feedback from assessment directly affects learning. The development of problem-solving, and higher, order thinking skills will become a realization only if assessment practices are in alignment with these expectations.

- ***limitations of the traditional methods used to determine student achievement***

Do traditional methods of assessment provide the student with information on how to improve performance? We need to develop methods of assessment that provide us with accurate information about students' academic achievement and information to guide teachers in decision making to improve both learning and teaching.

What Is Assessment?

Assessment is the systematic process of gathering information on student learning. Assessment allows teachers to communicate to students what is really valued—what is worth learning, how it should be learned, what elements of quality are considered most important, and how well students are expected to perform. In order for teachers to assess student learning in a mathematics curriculum that emphasizes applications and problem solving, they need to employ strategies that recognize the reasoning involved in the process as well as in the product. *Assessment Standards for School Mathematics* (NCTM 1995, p. 3) describes assessment practices that enable teachers to gather evidence about a student's knowledge of, ability to use, and disposition toward, mathematics and of making inferences from that evidence for a variety of purposes.

Assessment can be informal or formal. Informal assessment occurs while instruction is occurring. It is a mind-set, a daily activity that helps the teacher answer the question, Is what is taught being learned? Its primary purpose is to collect information about the instructional needs of students so that the teacher can make decisions to improve instructional strategies. For many teachers, the strategy of making annotated comments about a student's work is part of informal assessment. Assessment must do more than determine a score for the student. It should do more than portray a level of performance. It should direct teachers' communication and actions. Assessment must anticipate subsequent action.

Formal assessment requires the organization of an assessment event. In the past, mathematics teachers may have restricted these events to quizzes, tests, or exams. As the outcomes for mathematics education broaden, it becomes more obvious that these assessment methods become more limited. Some educators would argue that informal assessment provides better quality information because it is in a context that can be put to immediate use.

Why Should We Assess Student Learning?

We should assess student learning in order to

- improve instruction by identifying successful instructional strategies
- identify and address specific sources of the students' misunderstandings
- inform the students about their strengths in skills, knowledge, and learning strategies
- inform parents of their child's progress so that they can provide more effective support
- determine the level of achievement for each outcome

As an integral and ongoing part of the learning process, assessment must give each student optimal opportunity to demonstrate what he/

she knows and is able to do. It is essential, therefore, that teachers develop a repertoire of assessment strategies.

Assessment Strategies

Some assessment strategies that teachers may employ include the following.

Documenting classroom behaviours

In the past teachers have generally made observations of students' persistence, systematic working, organization, accuracy, conjecturing, modeling, creativity, and ability to communicate ideas, but often failed to document them. Certainly the ability to manage the documentation played a major part. Recording information signals to the student those behaviours that are truly valued. Teachers should focus on recording only significant events, which are those that represent a typical student's behaviour or a situation where the student demonstrates new understanding or a lack of understanding. Using a class list, teachers can expect to record comments on approximately four students per class. The use of an annotated class list allows the teacher to recognize where students are having difficulties and identify students who may be spectators in the classroom.

Using a portfolio or student journal

Having students assemble on a regular basis responses to various types of tasks is part of an effective assessment scheme. Responding to open-ended questions allows students to explore the bounds and the structure of mathematical categories. As an example, students are given a triangle in which they know two sides or an angle and a side and they are asked to find out everything they know about the triangle. This is preferable to asking students to find a particular side, because it is less prescriptive and allows students to explore the problem in many different ways and gives them the opportunity to use many different procedures and skills. Students should be monitoring their own learning by being asked to reflect and write about questions such as the following:

- What is the most interesting thing you learned in mathematics class this week?
- What do you find difficult to understand?
- How could the teacher improve mathematics instruction?
- Can you identify how the mathematics we are now studying is connected to the real world?

In the portfolio or in a journal, teachers can observe the development of the students' understanding and progress as a problem solver. Students should be doing problems that require varying lengths of time and represent both individual and group effort. What is most important is that teachers discuss with their peers what items are to be part of a meaningful portfolio, and that students also have some input into the assembling of a portfolio.

Projects and investigative reports

Students will have opportunities to do projects at various times throughout the year. For example, they may conduct a survey and do a statistical report, they may do a project by reporting on the contribution of a mathematician, or the project may involve building a complex three-dimensional shape or a set of three-dimensional shapes which relate to each other in some way. Students should also be given investigations in which they learn new mathematical concepts on their own. Excellent materials can be obtained from the National Council of Teachers of Mathematics, including the *Student Math Notes.* (These news bulletins can be downloaded from the Internet.)

Written tests, quizzes, and exams

Some critics allege that written tests are limited to assessing a student's ability to recall and replicate mathematical facts and procedures. Some educators would argue that asking students to solve contrived applications, usually within time limits, provides us with little knowledge of the students' understanding of mathematics.

How might we improve the use of written tests?

- Our challenge is to improve the nature of the questions being asked, so that we are gaining information about the students' understanding and comprehension.
- Tests must be designed so that questions being asked reflect the expectations of the outcomes being addressed.
- One way to do this is to have students construct assessment items for the test. Allowing students to contribute to the test permits them to reflect on what they were learning, and it is a most effective revision strategy.
- Teachers should reflect on the quality of the test being given to students. Are students being asked to evaluate, analyse, and synthesize information, or are they simply being asked to recall isolated facts from memory? Teachers should develop a table of specifications when planning their tests.
- In assessing students, teachers have a professional obligation to ensure that the assessment reflects those skills and behaviours that are truly valued. Good assessment goes hand-in-hand with good instruction and together they promote student achievement.

Appendix B: SCOs for Grades 9 and 10

GCO A: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers, and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, number system characteristics (e.g., density), etc.

By the end of grade 9, students will be expected to

A1 solve problems involving square root and principal square root

A2 graph, and write in symbols and in words, the solution set for equations and inequalities involving integers and other real numbers

A3 demonstrate an understanding of the meaning and uses of irrational numbers

A4 demonstrate an understanding of the inter-relationships of subsets of real numbers

A5 compare and order real numbers

A6 represent problem situations using matrices

By the end of grade 10, students will be expected to

A1 relate sets of numbers to solutions of inequalities

A2 analyse graphs or charts of situations to derive specific information

A3 demonstrate an understanding of the role of irrational numbers in applications

A4 approximate square roots

A5 demonstrate an understanding of the zero product property and its relationship to solving equations by factoring

A6 apply properties of numbers when operating upon expressions and equations

A7 demonstrate and apply an understanding of discrete and continuous number systems

A8 demonstrate an understanding of and apply properties to operations involving square roots

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 9, students will be expected to

B1 model, solve, and create problems involving real numbers

B2 add, subtract, multiply, and divide rational numbers in fractional and decimal forms, using the most appropriate method

B3 apply the order of operations in rational number computations

B4 demonstrate an understanding of, and apply the exponent laws, for integral exponents

B5 model, solve, and create problems involving numbers expressed in scientific notation

B6 judge the reasonableness of results in problem situations involving square roots, rational numbers, and numbers written in scientific notation

B7 model, solve, and create problems involving the matrix operations of addition, subtraction, and scalar multiplication

B8 add and subtract polynomial expressions symbolically to solve problems

B9 factor algebraic expressions with common monomial factors concretely, pictorially, and symbolically

B10 recognize that the dimensions of a rectangular model of a polynomial are its factors

B11 find products of two monomials, a monomial and a polynomial, and two binomials, concretely, pictorially, and symbolically

By the end of grade 10, students will be expected to

B1 model (with concrete materials and pictorial representations) and express the relationships between arithmetic operations and operations on algebraic expressions and equations

B2 develop algorithms and perform operations on irrational numbers

B3 use concrete materials, pictorial representations, and algebraic symbolism to perform operations on polynomials

B4 identify and calculate the maximum and/or minimum values in a linear programming model

B5 develop, analyse, and apply procedures for matrix multiplication

B6 solve network problems using matrices

GCO B: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 9, students will be expected to

B12 find quotients of polynomials with monomial divisors

B13 evaluate polynomial expressions

B14 demonstrate an understanding of the applicability of commutative, associative, distributive, identity, and inverse properties to operations involving algebraic expressions

B15 select and use appropriate strategies in problem situations

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 9, students will be expected to

C1 represent patterns and relationships in a variety of formats and use these representations to predict and justify unknown values

C2 interpret graphs that represent linear and non - linear data

C3 construct and analyse tables and graphs to describe how changes in one quantity affect a related quantity

C4 determine the equations of lines by obtaining their slopes and y-intercepts from graphs, and sketch graphs of equations using y-intercepts and slopes

C5 explain the connections among different representations of patterns and relationships

C6 solve single-variable equations algebraically, and verify the solutions

C7 solve first-degree single-variable inequalities algebraically, verify the solutions, and display them on number lines

C8 solve, and create problems involving linear equations and inequalities

By the end of grade 10, students will be expected to

C1 express problems in terms of equations and vice versa

C2 model real-world phenomena with linear, quadratic, exponential, and power equations, and linear inequalities

C3 gather data, plot the data using appropriate scales, and demonstrate an understanding of independent and dependent variables, and domain and range

C4 create and analyse scatter plots using appropriate technology

C5 sketch graphs from words, tables, and collected data

C6 apply linear programming to find optimal solutions to real-world problems

C7 model real-world situations with networks and matrices

C8 identify, generalize, and apply patterns

C9 construct and analyse graphs and tables relating two variables

C10 describe real-world relationships depicted by graphs, tables of values, and written descriptions

C11 write an inequality to describe its graph

C12 express and interpret constraints using inequalities

C13 determine the slope and y-intercept of a line from a table of values or a graph

C14 determine the equation of a line using the slope and y-intercept

C15 develop and apply strategies for solving problems

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally. (continued)

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 10, students will be expected to

C16 interpret solutions to equations based on context

C17 solve problems using graphing technology

C18 investigate and find the solution to a problem by graphing two linear equations with and without technology

C19 solve systems of linear equations using substitution and graphing methods

C20 evaluate and interpret non-linear equations using graphing technology

C21 explore and apply functional relationships notation, both formally and informally

C22 analyse and describe transformations of quadratic functions and apply them to absolute value functions

C23 express transformations algebraically and with mapping rules

C24 rearrange equations

C25 solve equations using graphs

C26 solve quadratic equations by factoring

C27 solve linear and simple radical, exponential, and absolute value equations and linear inequalities

C28 explore and describe the dynamics of change depicted in tables and graphs

C29 investigate, and make and test conjectures concerning, the steepness and direction of a line

C30 compare regression models of linear and non-linear functions

GCO C: Students will explore, recognize, represent, and apply patterns and relationships, both informally and formally. (continued)

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs, and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 10, students will be expected to

C31 graph equations and inequalities and analyse graphs both with and without graphing technology

C32 determine if a graph is linear by plotting points in a given situation

C33 graph by constructing a table of values, by using graphing technology, and when appropriate, by the slope y-intercept method

C34 investigate and make and test conjectures about the solution to equations and inequalities using graphing technology

C35 expand and factor polynomial expressions using perimeter and area models

C36 explore, determine, and apply relationships between perimeter and area, surface area, and volume

C37 represent network problems using matrices and vice versa

GCO D: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) and/or procedures (e.g., proportions) to determine measurements indirectly.

By the end of grade 9, students will be expected to

D1 solve indirect measurement problems by connecting rates and slopes

D2 solve measurement problems involving conversion among SI units

D3 relate the volumes of pyramids and cones to the volumes of corresponding prisms and cylinders

D4 estimate, measure, and calculate dimensions, volumes, and surface areas of pyramids, cones, and spheres in problem situations

D5 demonstrate an understanding of and apply proportions within similar triangles

By the end of grade 10, students will be expected to

D1 determine and apply formulas for perimeter, area, surface area, and volume

D2 apply the properties of similar triangles

D3 relate the trigonometric functions to the ratios in similar right triangles

D4 use calculators to find trigonometric values of angles and angles when trigonometric values are known

D5 apply trigonometric functions to solve problems involving right triangles, including the use of angles of elevation

D6 solve problems involving measurement using bearings and vectors

D7 determine the accuracy and precision of a measurement

D8 solve problems involving similar triangles and right triangles

D9 determine whether differences in repeated measurements are significant or accidental

D10 determine and apply relationships between the perimeters and areas of similar figures, and between the surface areas and volumes of similar solids

D11 explore, discover, and apply properties of maximum areas and volumes

D12 solve problems using the trigonometric ratios

D13 demonstrate an understanding of the concepts of surface area and volume

D14 apply the Pythagorean Theorem

GCO E: Students will demonstrate spatial sense and apply geometric concepts, properties, and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as (i) the direction, orientation, and perspectives of objects in space; (ii) the relative shapes and sizes of figures and objects; and (iii) how a change in shape relates to a change in size. Geometric concepts, properties, and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

By the end of grade 9, students will be expected to

E1 investigate, and demonstrate an understanding of, the minimum sufficient conditions to produce unique triangles

E2 investigate, and demonstrate an understanding of, the properties of, and the minimum sufficient conditions to guarantee, congruent triangles

E3 make informal deductions, using congruent triangle and angle properties

E4 demonstrate an understanding of and apply the properties of similar triangles

E5 relate congruence and similarity of triangles

E6 use mapping notation to represent transformations of geometric figures, and interpret such notations

E7 analyse and represent combinations of transformations, using mapping notation

E8 investigate, determine, and apply the effects of transformations of geometric figures, on congruence, similarity, and orientation

By the end of grade 10, students will be expected to

E1 explore properties of, and make and test conjectures about 2- and 3-dimensional figures

E2 solve problems involving polygons and polyhedra

E3 construct and apply altitudes, medians, angle bisectors, and perpendicular bisectors to examine their intersection points

E4 apply transformations when solving problems

E5 use transformations to draw graphs

E6 represent network problems as digraphs

E7 demonstrate an understanding of, and write a proof for, the Pythagorean Theorem

E8 use inductive and deductive reasoning when observing patterns, developing properties, and making conjectures

E9 use deductive reasoning and construct logical arguments and be able to determine, when given a logical argument, if it is valid

GCO F: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display and analysis of data involves (i) attention to sampling procedures and issues, (ii) recording and organizing collected data, (iii) choosing and creating appropriate data displays, (iv) analysing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean), and (v) formulating and evaluating statistical arguments.

By the end of grade 9, students will be expected to

F1 determine characteristics of possible relationships shown in scatter plots

F2 sketch lines of best fit and determine their equations

F3 sketch curves of best fit for relationships that appear to be non-linear

F4 select, defend, and use the most appropriate methods for displaying data

F5 draw inferences and make predictions based on data analysis and data displays

F6 demonstrate an understanding of the role of data management in society

F7 evaluate arguments and interpretations that are based on data analysis

By the end of grade 10, students will be expected to

F1 design and conduct experiments using statistical methods and scientific inquiry

F2 demonstrate an understanding of the concerns and issues that pertain to the collection of data

F3 construct various displays of data

F4 calculate various statistics using appropriate technology, analyse and interpret data displays, and describe relationships

F5 analyse statistical summaries, draw conclusions, and communicate results about distributions of data

F6 solve problems by modeling real-world phenomena

F7 explore non-linear data using power and exponential regression to find a curve of best fit

F8 determine and apply the line of best fit using the least squares method and median-median method with and without technology, and describe the differences between the two methods

F9 demonstrate an intuitive understanding of correlation

F10 use interpolation, extrapolation and equations to predict and solve problems

F11 describe real-world relationships depicted by graphs and tables of values

F12 explore measurement issues using the normal curve

F13 calculate and apply mean and standard deviation using technology, to determine if a variation makes a difference

GCO G: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails (i) determining probabilities by conducting experiments and/or making theoretical calculations, (ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment, and (iii) analysing problem situations to decide how best to determine probabilities.

In grade 9, students will be expected to

By the end of grade 9, students will be expected to

G1 make predictions of probabilities involving dependent and independent events by designing and conducting experiments and simulations

G2 determine theoretical probabilities of independent and dependent events

G3 demonstrate an understanding of how experimental and theoretical probabilities are related

G4 recognize and explain why decisions based on probabilities may be combinations of theoretical calculations, experimental results, and subjective judgements