Extraction of Expected Inflation from Canadian Forward Rates

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Introduction

Svensson (1993), Söderlind (1995), and Söderlind and Svensson (1997) suggest that forward interest rates (nominal interest rates agreed upon today for an investment period starting in the future) could serve as one indicator of the stance of monetary policy. They argue for the use of a forward rate as an information variable because it can indicate market expectations of future short-term interest rates, the future path of inflation, and risk premiums in general.

Although forward rates contain the same information as the yield curve, the forward-rate curve presents the information in a manner more easily interpreted for monetary policy purposes. Whereas the yield curve can be interpreted as expected future averages of the variables of concern, Svensson argues that the forward rate curve can be interpreted as indicating the expected time path of the variables in question. Forward rates therefore

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allow a separation of expectations for the short, medium, and long term more easily than the yield curve.

Theoretically, forward rates consist of four unobservable components: expected future inflation, expected real interest rates, the inflation-risk premium, and the forward-term risk premium. Extracting any of these components is therefore a difficult exercise.[1] Information on expected inflation could be obtained more readily if both nominal and inflation-indexed bonds were available. Unfortunately, the market for indexed bonds in Canada is not liquid enough, and the bonds have maturity dates only at the long end of the term structure. A more general approach some researchers have taken to extract expected inflation is to assume that real interest rates are constant, and that the inflation-risk and forward-term risk premiums are small and negligible.

The purpose of this paper is to extract expected future inflation from Canadian forward rates based on a rule (or technique) proposed by Söderlind (1995). The rule allows us to consider the question, If the nominal forward interest rate increases by 100 basis points, by how much have expectations of future inflation risen?

Under the forward-rate rule, inflation expectations and expected real interest rates are assumed to have a joint normal distribution and the risk premiums (inflation-risk and forward-term risk) are time-invariant or small.[2] Under this framework, extracting expected inflation from forward rates becomes a standard signal-extraction problem (Sargent 1987). The optimal estimate of the change in inflation expectations, based on implied forward rates, is a linear rule where the change of forward rate is multiplied by a coefficient.

One may ask why a central bank would be interested in inflation expectations extracted from forward interest rates when other methods of extracting inflation expectations are readily available. The answer lies with the frequency of the data. Most of the forecasting models for inflation rely on variables whose data are available monthly or, in some cases, quarterly. Policy-makers, using these models, can therefore have monthly information on inflation expectations. Forward rates are, however, available daily. Hence, under the framework we propose in this paper, policy-makers could use forward rates to obtain a daily read on the markets' expectations of future inflation.

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1. Söderlind and Svensson (1997) survey alternative techniques for extracting the components of forward rates.

2. It must be noted that studies by Evans (1998), Canova and Marrinan (1996), Bekaert, Hodrick, and Marshall (1997), Hardouvelis (1988), and Jorion and Mishkin (1991) suggest that risk premiums vary over time.

In the first part of the paper, we use a standard consumption-based capital-asset-pricing (C-CAPM) model to examine the size and time-series properties of the risk premiums embedded in Canadian forward rates. In the rest of the paper, we examine the relationship between expected inflation and forward rates.

Based on the estimated coefficients for the nominal forward rates at different horizons, one can use observed daily forward rates to construct a term structure of expected inflation on a particular day. From the term structure of inflation expectations, one can also assess some effects of monetary policy. For example, to assess the effects of inflation target announcements one can examine the behaviour of the term structure of inflation expectations before and after the announcement of a central bank's intention to embark on a policy of targeting inflation.

The main results of the paper are as follows.

- By assuming a joint process of inflation and consumption growth, the inflation-risk and forward-term premiums are estimated to be very small (less than 1 basis point for inflation and 12 basis points for the term premiums) and have negligible effects on the projection coefficient in the forward-rate rule for Canada.
- Over a 4-year horizon, for every 100-basis-point increase in forward rates, inflation expectations rise between 40 and 50 basis points.
- Canadian real interest rates are far from constant and are generally more volatile than inflation expectations. The real interest rates are more volatile at the shorter end than at the longer end.
- At the shorter end (1 to 5 quarters), real interest rates and expected inflation tend to move in the same direction, consistent with the interpretation that the policy authorities are reacting to the increase in expected inflation. However, at the longer end (6 to 16 quarters) the two variables move in opposite directions, consistent with the interpretation that higher real short-term interest rates, which contribute to higher real long-term interest rates, are also expected to depress economic activity and cause inflation to fall.
- Immediately following the Eric J. Hanson Memorial Lecture, delivered by Governor John Crow at the University of Alberta on 18 January 1988, and the joint announcements, in February 1991 and December 1993, by the Bank of Canada and the Government of Canada to set targets for inflation with a general commitment to price stability, the term structure of inflation expectations shifted down.

In the next section, we present a brief background of selected studies on the subject. Section 2 presents a C-CAPM model of the forward-rate term structure. We also estimate future inflation-risk premiums and forward-term premiums. Section 3 presents the rule for the extraction of expected inflation from forward rates. In Section 4 we examine the impact of major Bank announcements on the term structure of inflationary expectations. Conclusions are drawn in the last section.

1 Background

Svensson (1993) suggests that forward interest rates contain information about future inflation. In Söderlind and Svensson (1997), the forward rate is theoretically shown to consist of: expected future inflation, expected real interest rates, and inflation-risk and forward-term risk premiums. From U.S. and U.K. data, Söderlind (1995) finds that most, but not all, movements in nominal forward interest rates reflect changes in inflation expectations. Söderlind also finds that real interest rates are far from constant, but less volatile than inflation expectations. Furthermore, Söderlind observes a negative correlation between inflation expectations and real interest rates. Based on a general-equilibrium model, Söderlind calculates the implied nominal term premium and inflation-risk premium and finds them to be small—25 and 30 basis points, respectively, for a large risk-aversion parameter value that is set to 5.

Before the work of Svensson (1993), other researchers found that the slope of the yield curve is a good predictor of expected inflation. Studies in this area include Fama (1990), Mishkin (1988, 1990, and 1991), Lowe (1992), Frankel and Lown (1994), and Day and Lange (1997). In this body of work, the authors run simple regressions of inflation changes on interest rate spreads and examine the statistical significance of the estimated coefficient of the spread variable; a significant coefficient implies that the spread is a good predictor of future changes in the rate of inflation. The difference between these papers and ours is that we focus on the market expectation of the path of future inflation rather than on average expected inflation.

Bekaert, Hodrick, and Marshall (1997), Hardouvelis (1988), and Jorion and Mishkin (1991) observed in U.S. data that risk premiums are significant and time-variant. Evans (1998) studied the term structure of real rates, expected inflation, and inflation-risk premiums in the United Kingdom. Evans' analysis is based on new estimates of the real term structure, which is derived from U.K. index-linked bonds. Evans finds strong evidence to reject the Fisher hypothesis, the expectations hypothesis for real rates, and to support the presence of time-varying inflation-risk premiums throughout the term structure.

2 A General-Equilibrium Term Structure Model of Forward Rates

In this section, we employ a standard C-CAPM model to derive a term-structure expression for a forward rate and its components, inflation and consumption growth. To extract information on expected future inflation from forward rates, we examine the size and time-series behaviour of the risk premiums embedded in the Canadian forward rate. For this, we specify a joint process, a VAR(1)-ARCH(1)[3] model for inflation and consumption growth, which allows time variations in variance-covariances. The C-CAPM model parameters are estimated by fitting the theoretical forward rates to estimated forward rates from observed treasury bill and bond data (which will be discussed in Section 3). Then the model parameter estimates are used in the calculation of risk premiums.

2.1 A model of theoretical forward rates

We consider a standard capital asset-pricing (CAPM) model with a time-separable utility function, with consumption (c_t) as the argument, of the form

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{1}$$

where β is the discount factor, γ is the relative risk-aversion parameter, and the unit of t is quarters. Assume that the economic agent can invest in, among other assets, nominal and real bonds with different maturities. Let $r_{t,k}$ and $R_{t,k}$ be the annualized yields to maturity on k-period real and nominal bonds. It can be shown that:

$$r_{t,k} = -\frac{4}{k}\ln E_t\left[\beta^k\left(\frac{c_{t+k}}{c_t}\right)^{-\gamma}\right], \tag{2}$$

and

$$R_{t,k} = -\frac{4}{k}\ln E_t\left[\beta^k\left(\frac{c_{t+k}}{c_t}\right)^{-\gamma}\left(\frac{P_{t+k}}{P_t}\right)^{-1}\right]. \tag{3}$$

3. Vector autoregression of order 1—autoregressive conditional heteroscedasticity of order 1.

We now define the forward rate $f_{t,k,q}$ as the yield on a forward contract with maturity of q quarters and settlement $k+q$ quarters ahead. The forward rate can be written as

$$f_{t,k,q} = \frac{k+q}{q} R_{t,k+q} - \frac{k}{q} R_{t,k}. \tag{4}$$

By assuming conditional normality of consumption growth and inflation, we can derive the relationship between forward rates and expected future inflation as

$$f_{t,k,q} = E_t r_{t+k,q} + E_t \pi_{t+k,q} + \varphi^f_{t,k,q} + \varphi^\pi_{t,k,q}, \tag{5}$$

where

$$E_t r_{t+k,q},\ E_t \pi_{t+k,q},\ \varphi^f_{t,k,q},\ \text{and}\ \varphi^\pi_{t,k,q}$$

are the expected future real interest rate, expected future inflation, future forward-term premium, and future inflation-risk premium. The last three components are defined as

$$\begin{aligned} E_t \pi_{t+k,q} &= E_t \ln(P_{t+k+q}/P_{t+k}), \\ \varphi^f_{t,k,q} &= f_{t,k,q} - E_t R_{t+k,q}, \\ \varphi^\pi_{t,k,q} &= E_t R_{t+k,q} - E_t r_{t+k,q} - E_t \pi_{t+k,q}. \end{aligned} \tag{6}$$

Equation (5) shows that the predictability of expected inflation based on forward rates depends on three other terms: the expected future real interest rates; the future forward-term premium; and the future inflation-risk premium. If one makes extreme assumptions, such as constant expected real interest rates, future forward-term premiums, and future inflation risk-premiums, then any fluctuation in the forward rates can be assigned to the changes in expected inflation. However, studies show that real interest rates are not constant over time and are correlated with expected inflation. The two premium terms may also affect the relationship between forward rates and expected inflation.

Equation (5) forms the basis of the forward-rate rule used by others, including Söderlind (1995), to extract expected future inflation from forward rates. The rule, however, assumes that the risk premiums in equation (5) are either constant or negligible. In the next section, we will attempt to quantify the size of the risk premiums in Canada and examine whether it affects the projection coefficients in the forward-rate rule.

2.2 Examining the inflation-risk and forward-term premiums

In order to estimate inflation-risk and forward-term premiums, we first specify a joint process of inflation and consumption growth by a VAR(1)-ARCH(1) model. Then we can get closed-form solutions for the forward rates and term premiums, which are functions of the discount factor and risk-aversion parameter in the model as well as the conditional mean and variance-covariances of inflation and consumption growth. The discount factor and risk-aversion parameter are estimated by fitting the theoretical forward rates to the estimated forward rates based on Svensson's model (see Section 3). The parameter estimates are then used in the calculation of inflation-risk and forward-term premiums. We present the details next.

2.2.1 A VAR(1)-ARCH(1) process of inflation and consumption growth

We denote $z_{1t} = \Delta \log P_t$, $z_{2t} = \Delta \log c_t$ and $z_t = [z_{1t} \ z_{2t}]'$. Assume z_t follows the VAR(1)-ARCH(1) process, and is expressed as:

$$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix} + \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$\text{and } \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} h_{1t} & h_{12t} \\ h_{12t} & h_{2t} \end{bmatrix}\right), \tag{7}$$

where the conditional variance-covariances of inflation and consumption growth follow the ARCH(1) process

$$h_{1t} = a_{01} + a_{11} u_{1t-1}^2,$$

$$h_{2t} = a_{02} + a_{12} u_{2t-1}^2,$$

$$h_{12t} = \rho (h_{1t} h_{2t})^{1/2}. \tag{8}$$

Equation (7) states that inflation and consumption growth are governed by a vector-autoregressive process of order 1. The innovations in the process have the time-varying variance-covariance shown in equation (8). Also we assume the correlation coefficient between inflation and consumption growth (ρ) is constant over time.

To simplify the notation, denote

$$B_0 = \begin{bmatrix} A_{01} \\ A_{02} \end{bmatrix}, \quad B_1 = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix}, \quad u_t = \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}, \quad h_t = \begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix},$$

$$a_0 = \begin{bmatrix} a_{01} \\ a_{02} \end{bmatrix}, \quad a_1 = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{12} \end{bmatrix}, \text{ and } H_t = \begin{bmatrix} h_{1t} & h_{12t} \\ h_{12t} & h_{2t} \end{bmatrix}.$$

Then, the simplified form of the VAR(1)-ARCH(1) model shown in equations (7) and (8) becomes

$$z_t = B_0 + B_1 z_{t-1} + u_t, \quad u_t \sim N(0, H_t),$$

$$h_t = a_0 + u'_{t-1} a_1 u'_{t-1},$$

$$h_{12t} = \rho(h_{1t} h_{2t})^{1/2}. \tag{9}$$

2.2.2 Closed-form solution for forward rates and risk premiums

Given the above VAR(1)-ARCH(1) process of inflation and consumption growth, Appendix 1 shows that the real and nominal yield to maturity and expected future inflation are functions of conditional mean and variance-covariances of inflation and consumption growth in the following forms:

$$E_t r_{t+(k,q)} = -4\ln\beta + \frac{4}{q}\begin{bmatrix} 0 & \gamma \end{bmatrix} E_t \sum_{i=k+1}^{k+q} z_{t+i}$$

$$-\frac{4}{2q}\begin{bmatrix} 0 & \gamma \end{bmatrix} E_t \mathrm{Var}_{t+k}\left(\sum_{i=k+1}^{k+q} z_{t+i}\right)\begin{bmatrix} 0 & \gamma \end{bmatrix}',$$

$$R_{t,k} = -4\ln\beta + \frac{4}{k}\begin{bmatrix} 1 & \gamma \end{bmatrix} E_t \sum_{i=1}^{k} z_{t+i}$$

$$-\frac{4}{2k}\begin{bmatrix} 1 & \gamma \end{bmatrix} \mathrm{Var}_t\left(\sum_{i=1}^{k} z_{t+i}\right)\begin{bmatrix} 1 & \gamma \end{bmatrix}',$$

$$E_t R_{t+k,\,q} \;=\; -4\ln\beta + \frac{4}{q}\begin{bmatrix}1 & \gamma\end{bmatrix} E_t \sum_{i=k+1}^{k+q} z_{t+i}$$

$$-\frac{4}{2q}\begin{bmatrix}1 & \gamma\end{bmatrix} E_t \mathrm{Var}_{t+k}\left(\sum_{i=k+1}^{k+q} z_{t+i}\right)\begin{bmatrix}1 & \gamma\end{bmatrix}',$$

$$E_t \pi_{t+k,\,q} \;=\; \frac{4}{q}\begin{bmatrix}1 & 0\end{bmatrix} E_t \sum_{i=k+1}^{k+q} z_{t+i}, \qquad (10)$$

where expressions for the conditional mean and variance-covariances of inflation and consumption growth

$$E_t \sum_{i=k+1}^{k+q} z_{t+i}$$

and

$$E_t \mathrm{Var}_{t+k}\left(\sum_{i=k+1}^{k+q} z_{t+i}\right)$$

are also derived in Appendix 1.

These formulas are then substituted into equations (4) and (6) to get the forward rate, inflation-risk premium, and forward-term premium.[4] The expressions for the inflation-risk premium and forward-term premium include only conditional variances and covariances of inflation and consumption growth with risk-aversion parameter γ.

2.2.3 Estimation results

First, we estimate the VAR(1)-ARCH(1) process of inflation and consumption growth using Canadian data. We use the CPI as the price index and real personal consumption expenditure as consumption. The estimation period is from 1961Q4 to 1997Q4. Table 1 reports the estimation results. The table shows that most of the parameter estimates are significant at the 5 per cent level. We can infer from Table 1 that the unconditional standard

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4. It could be shown that the expression for nominal interest rates (equation [10]) is similar to the two-factor model of the term structure of interest rates developed by Longstaff and Schwartz (1992), where the two factors are the short-term interest rate and volatility.

Table 1

VAR(1)-ARCH(1) Estimation Results, 1961Q4 to 1997Q4

Parameter	Estimates	Standard error	*t*-statistic	Significance
A_{01}	0.0009	0.00046	1.92158	0.05466
A_{02}	0.0114	0.00077	14.8912	0.00000
A_{11}	0.8380	0.0288	29.11450	0.00000
A_{21}	0.1114	0.0336	3.3193	0.00090
A_{12}	−0.1409	0.0498	–2.8286	0.00468
A_{22}	−0.0557	0.0700	−0.79539	0.42639
a_{01}	0.00003	0.00000	7.13577	0.00000
a_{02}	0.00008	0.00001	7.4662	0.00000
a_{11}	0.1679	0.1161	1.4455	0.14831
a_{12}	0.1596	0.0947	1.68572	0.09185
ρ	−0.1740	0.0661	–2.6340	0.00844

deviation of inflation over the estimation period is 2.40 percentage points.[5] The correlation between inflation and consumption growth is estimated to be −0.2, which implies negative and weak correlation between these two series.

Next, with the Svensson forward-rate series (which will be discussed in Section 3) over different horizons (k), we substitute the estimated parameters in Table 1 and a risk-aversion parameter (γ) of 5 into equations (4), (6), and (10) to estimate the inflation-risk and forward-term premiums.[6] Since the Svensson forward rates are only available for the period 1985Q3 to 1997Q2, we estimate the premiums over this period with q set to 4. Note that the estimation of the premiums does not depend on β.

Figure 1 plots the inflation-risk premium and forward-term premium for two cases: $k = 4$ quarters and $k = 8$ quarters. For the 4 and 8 quarters, we find that the inflation-risk premium is estimated to be 0.74 basis points and the forward-term premium to be 11.6 basis points. The 0.74 basis point inflation-risk premium can be interpreted as corresponding to 1.22 percentage points' standard deviation of inflation, which is lower than the unconditional standard deviation (2.40 percentage points) calculated above.

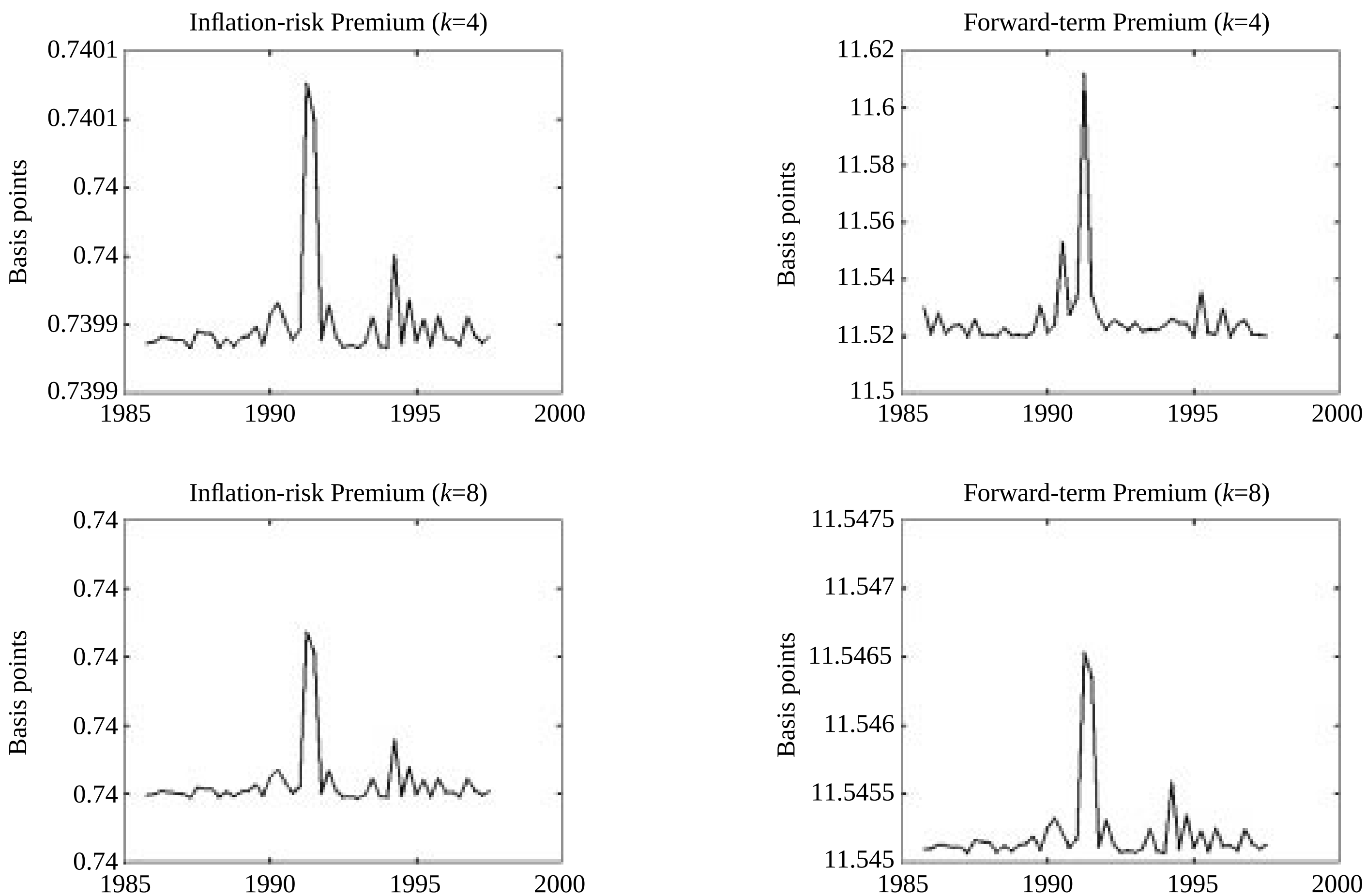
It must be noted that our estimate of the future inflation-risk premium is low compared with other estimates in the literature. Assuming a risk-aversion parameter (γ) of 5, Söderlind estimates the inflation-risk premium

5. Unconditional standard deviation of inflation at an annual rate is calculated by $sqrt(a_{01}/(1-a_{11}))$*4*10000 (basis points). The factor is 4 because the estimated model is quarterly.

6. Note that our choice of $\gamma = 5$ is based on the work of Söderlind (1995).

Figure 1

Inflation-Risk and Forward-Term Premiums at 12-Month Maturity



for the United States is around 30 basis points. Using U.S. consumption data and assuming γ to be 2, Hahm (1998) also estimates the inflation-risk premium to be about 12 basis points. Our results must be carefully compared with others', because the definitions of risk premium may be different in different models. Our definition relates to the conditional expectation of future inflation variance. For example, for $k = 4$, the inflation-risk premium is linked to the conditional variance of inflation from one year ahead to two years ahead.

The inference we draw from this exercise is that inflation-risk and forward-term premiums in Canada are economically insignificant proportions of forward rates, and should not affect the results of our next empirical exercise. In what follows, we therefore set them to 0.

3 Extracting Inflation Expectations: The Forward-Rate Rule

As shown in equation (5), the forward rate is composed of expected future inflation, expected real interest rates, and the inflation-risk and forward-term risk premiums. Because we have shown that the premiums in Canada are insignificant, we can say that Canadian forward rates are made up of two components: expected future inflation and expected real interest rates. Separating these two components is a classic signal-extraction problem.[7] We shall follow Söderlind (1995) and the forward-rate rule to extract inflation expectations from Canadian forward rates.

3.1 The forward-rate rule

To derive the rule, set the inflation-risk and forward-term risk premiums in equation (5) to 0. The rule is obtained from equation (5) as:

$$\hat{\pi}^{e}_{t,k,q} = a_k + b_k f_{t,k,q}, \tag{11}$$

where $\hat{\pi}^{e}_{t,k,q}$

is the estimate of $E_t \pi_{t+k,q}$, the expected inflation, and a_k and b_k are coefficients.

In Appendix 2, b_k is shown to have the following form:

$$\hat{b}_k = \frac{1 + \rho_k \sigma_k}{1 + \sigma_k^2 + 2\rho_k \sigma_k}, \tag{12}$$

7. See Sargent (1987, 229).

where

$$\sigma_k^2 = \frac{\mathrm{Var}(r^e_{t,k,q})}{\mathrm{Var}(\hat{\pi}^e_{t,k,q})},$$

$$\rho_k = \mathrm{Corr}(r^e_{t,k,q}, \hat{\pi}^e_{t,k,q}), \tag{13}$$

and

$$r^e_{t,k,q} = f_{t,k,q} - \hat{\pi}^e_{t,k,q}.$$

The value of b_k tells us whether the forward rate is a good indicator of expected inflation. If b_k equals 1, then the forward rate is a perfect measure of the expected inflation; any change in the forward rates reflects the change in the expected inflation. For example, if the expected real interest rate is constant, i.e., $\mathrm{Var}(r^e_{t,k,q}) = 0$, and $b_k = 1$, then any movement in the forward rate is ascribed to the change in expected inflation. The ratio σ_k^2 reflects the relative volatility of the expected forward inflation and the expected forward real interest rate, and ρ_k is the correlation coefficient of expected forward inflation and the expected forward real interest rate.[8]

To get estimates of b_k, we need data on forward rates and expected inflation. In the following subsections, we describe how we construct these series.

3.2 Estimates of Canadian forward rates

There are several ways of estimating implicit forward rates (see Chapter 2 in Anderson and et al. 1996). Most estimates are based on models that fit a discount function pioneered by McCulloch (1971). Nelson and Siegel (1987) explicitly model the implied forward-rate curve instead of the term structure of interest rates. They propose a functional form that could generate forward-rate curves of a variety of shapes, including a monotonic component and a humped component. Svensson (1994b) increases the flexibility of the original Nelson and Siegel model by adding a second hump in the forward-rate curve.

8. Söderlind (1995) summarizes several possible cases for the different variabilities of expected real interest rates and expected inflation.

3.2.1 Svensson's 1994 model

In the framework of Svensson (1994a, 1994b), it is assumed that the instantaneous forward rate is the solution to a second-order differential equation with three roots. Given a trading date *t* and a settlement date *m*, let $f(m)$ denote the instantaneous forward rate $f(t, t+m)$. Svensson's function is of the form:

$$f(m;b) = s_0 + s_1 \exp\left(-\frac{m}{\tau_1}\right) + s_2 \frac{m}{\tau_1} \exp\left(-\frac{m}{\tau_1}\right) + s_3 \frac{m}{\tau_2} \exp\left(-\frac{m}{\tau_2}\right),$$

where $b = (s_0, s_1, s_2, s_3, \tau_1, \tau_2)$ is a vector of parameters—s_0, τ_1, and τ_2 must be positive.

The forward rate in equation (13) consists of three components. The first is a constant, s_0, and the second is an exponential term, $s_1 \exp(-m/\tau_1)$, which decreases (or increases, if s_1 is negative) monotonically towards 0 as the settlement time approaches infinity. The third term generates a hump shape (or a *U* shape, if s_2 is negative) as a function of the time to settlement. When the time to settlement approaches infinity, the forward rate approaches the constant s_0 and, when the time to settlement approaches 0, the forward rate approaches the constant $(s_0 + s_1)$. Therefore, s_0 and $s_0 + s_1$ must be non-negative to ensure that the instantaneous spot and long rates do not take negative values.

Note that Svensson's addition to the Nelson and Siegel (1987) is a second hump shape, which is captured by $s_3(m/\tau_2)\exp(-m/\tau_2)$. Svensson suggests that this term is needed to add more flexibility and improve the fit of the forward-rate curve.

3.2.2 Estimation method

From observable bond prices (or yields), coupon rates, and maturity dates, we can obtain the parameter estimates of an implied forward rate curve. This is done by choosing the parameters to minimize either (the sum of squared) price errors or (the sum of squared) yield errors.

However, minimizing price errors sometimes results in fairly large yield errors for bonds and bills with short maturities (Svensson 1994a, 1994b). This is because the elasticity of the bond price with respect to one plus the bond yield is equal to the duration of a bond. This is also known as the weighted average time to cash flows including coupon payments and the redemption payment. For short maturities, prices are not very sensitive to yields. We therefore recommend that the parameters be chosen to minimize yield errors. Svensson (1994a, 1994b) also points out that in monetary policy analysis the focus is on interest rates rather than prices, and so it makes sense to minimize errors in the yield dimension rather than in the price dimension.

Our technique minimizes yield errors by assuming that the observed yield to maturity differs from the estimated yield to maturity by an error term. The parameters are estimated such that the estimated yields to maturity fit the observed yields to maturity. Also, the estimation has been done with the restriction that the forward-rate curve starts at the left end from the overnight rate. The details on the estimation procedures can be found in the appendix in Svensson (1994a, 1994b).

The implicit forward rates are estimated from raw data of Canadian treasury bills and bonds. Using the Gauss programs made available to us by Svensson, we input the yields, coupon rates, and maturity dates with maturities of overnight, 3, 6, and 12 months, and 2 to 10, 15, 20, and 30 years. The monthly estimation period is July 1985 to June 1997. For each month, we use the data on the trade date, the first Wednesday in the month (or the first Thursday if the first Wednesday was a holiday).

3.3 Estimates of expected inflation from a vector error-correction model based on M1 disequilibria

To evaluate the forward-rate rule, we estimate expected inflation from a vector error-correction model (VECM) based on Armour, Atta-Mensah, Engert, and Hendry (1996), Engert and Hendry (1998), and Engert, Hendry, and Yuan (1998). The VECM, which is based on the long-run demand for M1 estimated by Hendry (1995), is made up of a four-equation VECM for M1, CPI, real GDP, and the 90-day commercial paper rate. This model focuses on the effects on the inflation rate of deviations of M1 from its long-run demand. In addition to lagged endogenous variables, the model also includes a short-term U.S. interest rate, the exchange rate, and a simple measure of the output gap. The VECM's inflation forecasting ability has been considered in the above papers, and the authors conclude that the model provides considerable information about future inflation.

We obtained the estimates of expected inflation by running recursive regressions over the sample period. This method involves estimating the VECM from 1950Q1 to 1979Q4 and forecasting the price level $k+4$ quarters ahead, where $k = 1$, 2, 3, 4, 5, 6, 8, and 16. The estimate of expected inflation from $t+k$ to $t+k+4$ is obtained as $\ln P_{t+k+4} - \ln P_{t+k}$. The model is re-estimated by extending the sample size by one quarter at a time; $k+4$ quarter-ahead forecasts of the price level are made at each time and the corresponding expected inflation calculated. The process continues until the data points are exhausted.

To correspond with the availability of the forward-rate data, estimates of inflation were obtained from 1985Q3 to 1997Q2. Since the VECM is based on quarterly data, the estimated forward rates were converted into quarterly frequency for the evaluation of the forward-rate rule.

Figure 2 plots the expected inflation rate and the nominal forward rates at various horizons. The graph shows that expectations of future inflation follow more or less the same trends as forward rates.

3.4 Expected inflation and the forward-rate rule

Next, we examine the relationship between the expected inflation generated by the VECM and the data for forward rates that are estimated using the Svensson model. In other words, we investigate the forward-rate rule described in Section 3.1. But before doing so, we calculated the expected real interest rate as the difference between nominal forward rates and expected inflation (using forecasts from the VECM). Table 2 reports the evaluation results.

The b_k, shown in equation (8), are the focus of our evaluation of forward-rate rules. The results in Table 2 show that at the shorter end of the maturity spectrum (the 1- to 4-quarter horizon), expectations of inflation change between 38 and 41 basis points for every 100 basis point change in forward rates. For the medium term (the 5- to 8-quarter horizon), the change in expected inflation for 100 basis points' change in the forward rate is between 50 and 55 basis points. At the longer-term horizon, 12 to 16 quarters, expected inflation changes between 36 and 40 basis points for 100 basis points' change in forward rates.

In sum, the results indicate that for every percentage point increase in the forward rate, expected inflation has risen by one-half a percentage point or less. If real interest rates were constant, forward rates and expected inflation should move 1 for 1. The fact that b_k is less than 1 implies that real interest rates are far from constant. Söderlind (1995), who uses U.S. data, estimates that b_k is equal to 0.56 for $k = 4$ and 0.27 for $k = 8$. With a smaller sample size, Söderlind improves his estimates to 0.98 for $k = 4$ and 0.56 for $k = 8$. Mishkin (1990) studies the relationship between interest rate spreads and future inflation rates and concludes that interest rates contain information about inflation rates for periods between 6 and 12 months ahead. Mishkin's estimate of b_k is between 0.7 and 1.5. Our results suggest that forward rates in Canada contain information about future inflation.

Table 2 shows results concerning information in forward rates of different maturities. The measures of relative volatility between expected real interest rates and expected inflation show that expected real interest

Figure 2

Forward Rates and Expected Inflation at the 1-, 8-, and 16-Quarter Horizons

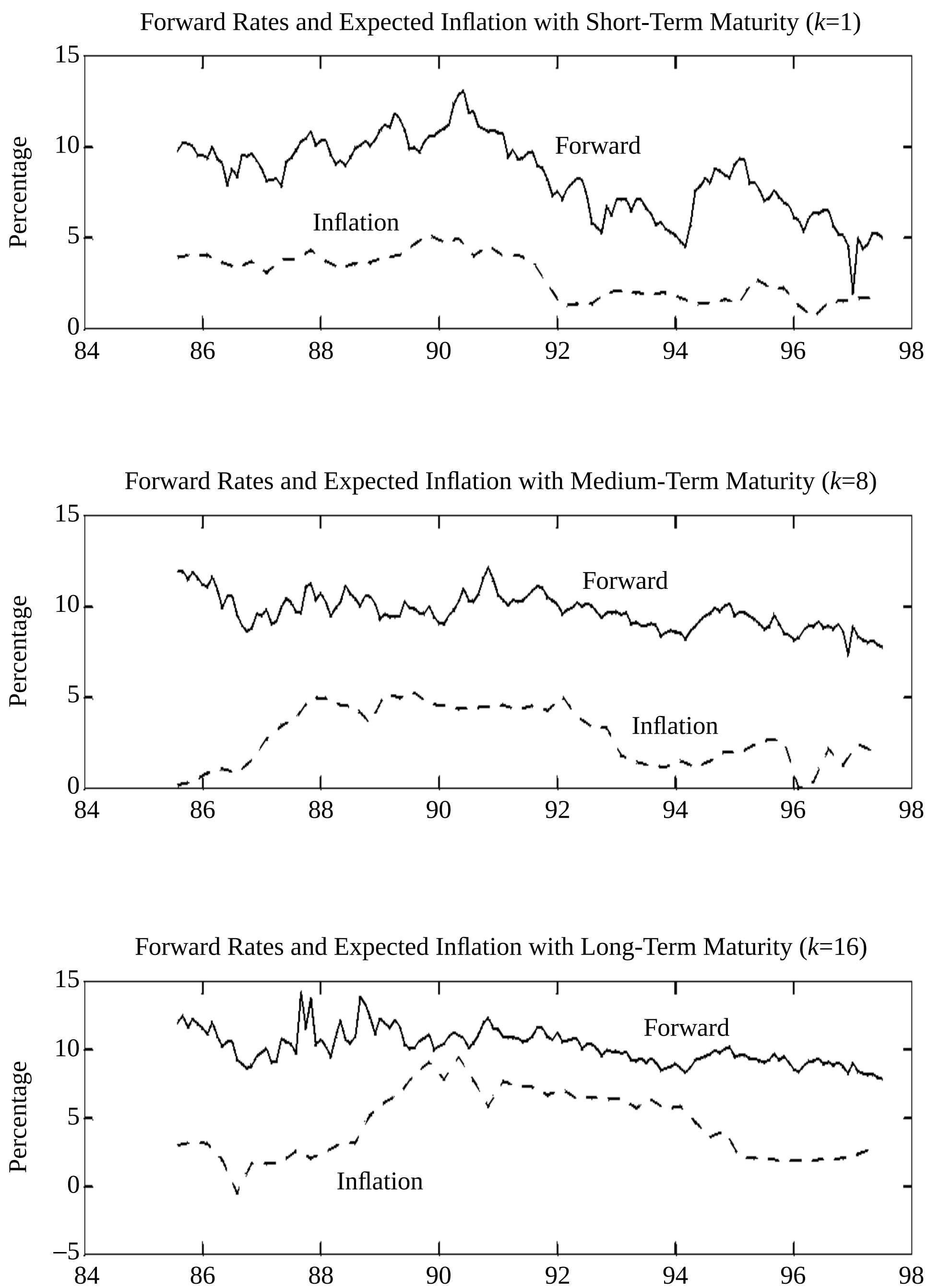


Table 2

Evaluation of Forward-Rate Rules, 1985Q3 to 1997Q2

k (Quarters)	b_k	σ_k	ρ_k	MSE(b_k)/MSE(1)
1	0.3806	1.5099	0.6885	0.0982
2	0.3810	1.4692	0.5813	0.1361
3	0.4001	1.3228	0.3793	0.2281
4	0.4136	1.2285	0.1778	0.3287
5	0.4972	1.0057	0.0035	0.4955
6	0.5543	0.9149	−0.1849	0.6444
8	0.4903	1.0089	−0.5447	0.7655
12	0.3957	1.0356	−0.8331	0.8819
16	0.3650	1.0237	−0.9134	0.9316

rates fluctuate more than expected inflation at the shorter end of the maturity spectrum (the 1- to 4-quarter horizon). From the medium to longer term (the 5- to 16-quarter horizon), we observe that the real interest rate and expected inflation fluctuate about the same amount. The σ_k values are slightly different from the findings in Söderlind (1995)—0.59 for $k = 4$, and 1.32 for $k = 8$ when the sample is from 1958Q2 to 1990Q4, excluding 1982 to 1985. The σ_k values are 0.93 and 1.32 for $k = 4$ and 8 when the full sample is used. The results imply that Canadian real interest rates not only vary over time but are very volatile, in particular at the shorter end of the maturity spectrum.

The results in Table 2 also show that the correlations between expected real forward interest rates and expected inflation are positive at the shorter end of the maturity spectrum (1 to 5 quarters) and negative at the longer end. Our result is consistent with the view that the monetary authorities react, in the short run, to an increase in expected inflation by raising nominal rates by more than the increase in expected inflation, which in turn increases the real interest rate. Thus, in the short run, real rates and expected inflation move in the same direction (a positive correlation coefficient). In the longer run, the higher future real interest rates depress future economic activity. The negative correlation between real rates and real economic activity, which comes from the real sector, combines with a negative relation between expected inflation and real activity, which is traced to the monetary sector, thus inducing the negative relation between expected inflation and expected real rates. Hence, in the long run, real rates and expected inflation move in opposite directions (a negative correlation coefficient).

We also provide the ratio of mean squared errors with the estimated b_k and $b_k = 1$, MSE(b_k)/MSE(1). This is a “measure of efficiency” using forward rates as indicators of expected inflation. It can be shown that $\text{MSE}(b_k)/\text{MSE}(1) = (1-\rho_k^2)/(1+\sigma_k^2+2\rho_k\sigma_k)$. The results indicate that our estimates of the relative efficiency of the forward rule are generally very strong for the 6- to 16-quarter horizon. The estimates are between 0.64 and 0.93 for these maturity periods.

In this section we have shown empirically, through the forward-rate rule, that there is a link between forward rates and expected inflation. However, we find that the relationship between forward rates and expected inflation is not 1 for 1. We argue that the failure of the relationship to obey the Fisher rule is because real interest rates are not constant.

4 The Policy of Price Stability and the Behaviour of Expected Inflation

In this section, we examine the behaviour of the term structure of expected inflation based on the estimates obtained by the rule. In particular, we assess three events concerning the evolution of the Bank of Canada's policy towards price stability. These dates are: January 1988 (Governor Crow's Hanson lecture); February 1991 (the joint announcement by the Bank and the Government of Canada to adopt inflation targets); and December 1993 (the second joint announcement by the Bank and the Government of Canada to extend the inflation targets).

The Bank of Canada's intention to embark on a goal of price stability was signalled with Governor Crow's Hanson lecture.[9] In his speech, Crow (1988) emphasized that monetary policy aimed to achieve price stability.[10]

To assess the impact of the Hanson lecture on economic agents' perception of future inflation, we plotted the term structure of expected inflation, derived from the forward-rate rule, for the months of January and February of 1988 in Figure 3. As the first panel of Figures 3 and 4 show, the speech appeared to have had an impact on economic agents' outlook for future inflation. Following the speech, agents immediately lowered their expectations of inflation by 30 to 40 basis points; the greatest revision was at the 6-quarter maturity.

In February 1991, the Bank of Canada and the Government of Canada jointly announced the setting of inflation targets in Canada. These targets aimed for a year-over-year rate of increase in the CPI of 3 per cent by the end of 1992, 2.5 per cent by the middle of 1994 and 2 per cent by the end of 1995. Panel 2 of both Figures 3 and 4 show that the term structure of

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9. See Laidler and Robson (1993).

10. The objective had been enunciated by Governor Gerald Bouey as early as 1984, but did not generate the same attention. See Bank of Canada (1984, 6).

Figure 3

Announcement Effects: The Term Structure of Expected Inflation around Inflation Target Announcement Dates

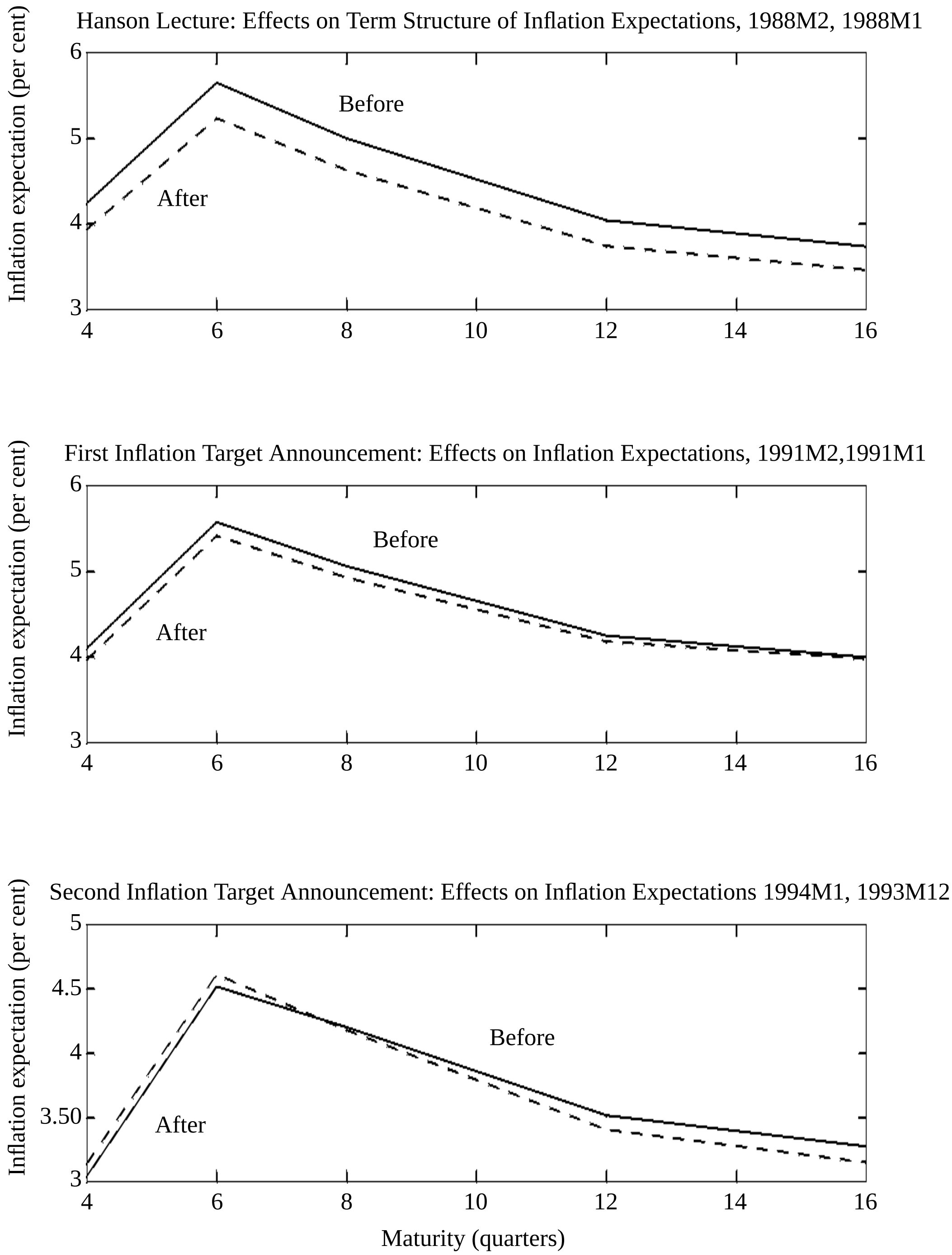
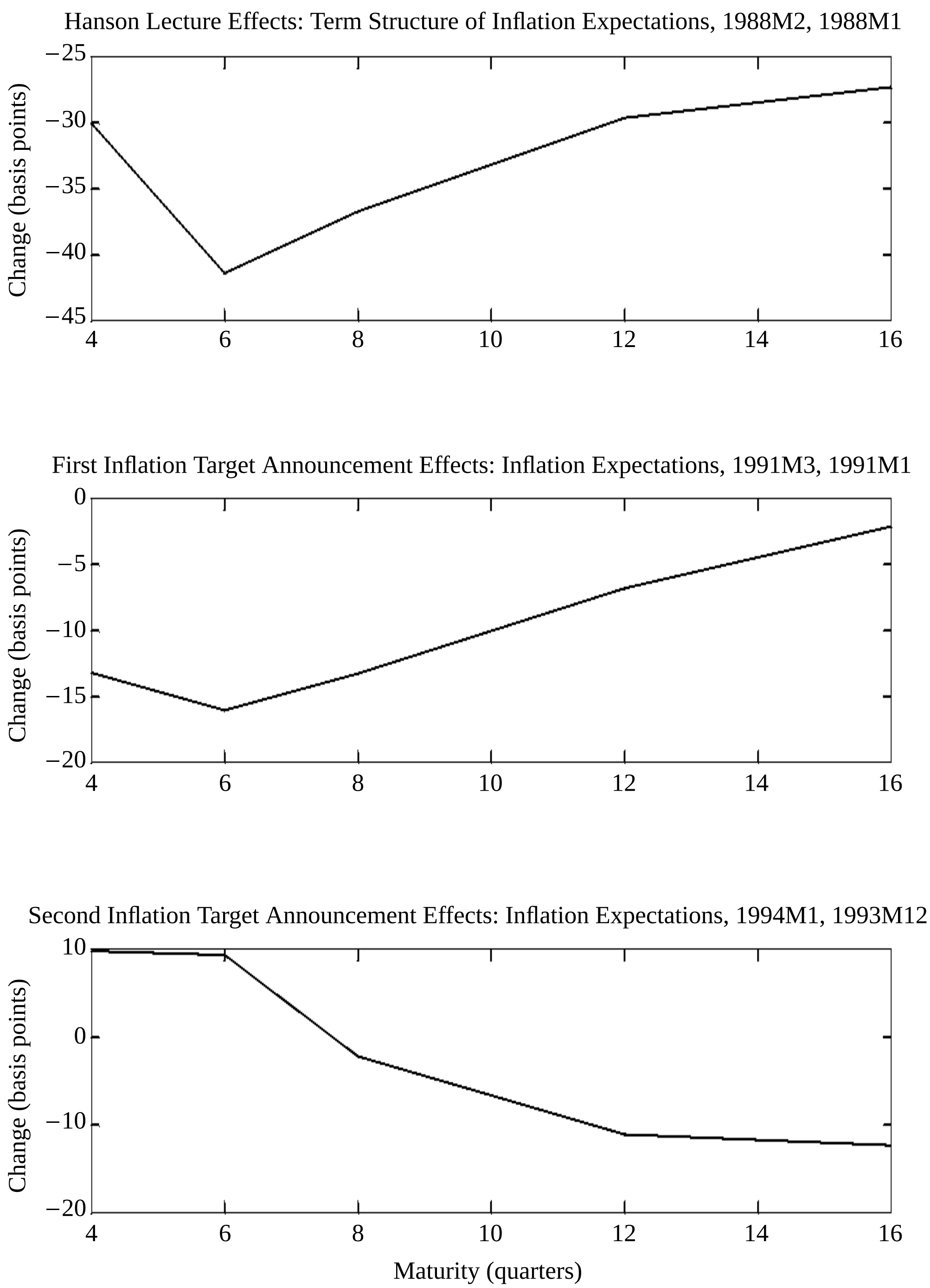


Figure 4

The Change of the Term Structure of Expected Inflation around Inflation Target Announcement Dates



inflation expectations moved down following the announcement. However, the drop in expectations was only 3 to 15 basis points; revisions were found more at the shorter end of the maturity spectrum than at the longer end. We suspect that the relatively small drop in inflation expectations may be due to the fact that the disinflationary policy of the Bank prior to this announcement had already affected expectations, and so agents were not surprised by the announcement. Evidently, the announcement alone did not strengthen confidence in price stability.

The last event we examined was the second inflation target announcement, made in December 1993, when Governor Gordon Thiessen was appointed. In the announcement, the Bank and the Government of Canada agreed to extend the inflation targets from the end of 1995 to the end of 1998 with the aim of keeping inflation inside a range of 1 to 3 per cent. The third panels of Figures 3 and 4 would suggest that expectations hardly changed at the short end but fell slightly at the long end of the maturity spectrum. These results are consistent with the interpretation that agents were confident that the Bank would maintain the status quo after the departure of Governor Crow.

Conclusions

This paper examines whether Canadian forward rates are useful indicators of expected inflation. Our results suggest that expected inflation changes less than 100 basis points for every 100 basis point change in forward rates. We also find that real interest rates vary considerably over time. Our results show that Canadian real interest rates are more volatile than expected inflation. A general-equilibrium model of the economy implies that forward-term and inflation-risk premiums are small and insignificant. Finally, an event study suggests that, immediately following the Bank's announcement to set inflation targets, economic agents seemed to lower their expectations of future inflation.

Appendix 1

The Closed-Form Solutions of the Theoretical Model

We will first derive the conditional mean and variance-covariance of a VAR(1)-ARCH(1) process, and then use them in the equilibrium conditions of the model in Section 2 to get the closed-form solutions for forward rate and its components.

1 Conditional mean and variance-covariance of a VAR(1)-ARCH(1) process

The bivariate VAR(1)-ARCH(1) process z_t is defined as:

$$z_t = B_0 + B_1 z_{t-1} + u_t, \quad u_t \sim N(0, H_t)$$

$$h_t = a_0 + u'_{t-1} a_1 u'_{t-1},$$

$$h_{12t} = \rho(h_{1t} h_{2t})^{1/2},$$

where

$$a_0 = \begin{bmatrix} a_{01} \\ a_{02} \end{bmatrix}, \; a_1 = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{12} \end{bmatrix}, \; H_t = \begin{bmatrix} h_{1t} & h_{12t} \\ h_{12t} & h_{2t} \end{bmatrix} \text{ and } h_t = \begin{bmatrix} h_{1t} \\ h_{2t} \end{bmatrix}.$$

The conditional mean and variance-covariance can be derived as:

$$E_t \sum_{s=q}^{r} z_{t+s} = B_0 \sum_{s=q}^{r} \sum_{i=1}^{s} B_1^{i-1} + \sum_{s=q}^{r} B_1^s z_t$$

$$E_t \text{Var}_{t+q-1} \sum_{s=q}^{r} z_{t+s} = \sum_{s=q}^{r} \sum_{i=q}^{s} B_1^{r-i} \text{Var}_t(u_{t+i})(B_1^{r-i})'.$$ [1]

Then,

$$\text{Var}_t(u_{t+i}) = \begin{bmatrix} h_{1t+i} & h_{12t+i} \\ h_{12t+i} & h_{2t+i} \end{bmatrix}.$$

1. Note that $E_t \text{Var}_{t+q-1} \sum_{s=q}^{r} z_{t+s}$ is different from $\text{Var}_t \sum_{s=q}^{r} z_{t+s}$, which is equal to

$$\sum_{s=q}^{r} \sum_{i=1}^{s} B_1^{r-i} \text{Var}_t(u_{t+i})(B_1^{r-i})'.$$

However, it can be easily shown that $E_t \text{Var}_{t+q-1}(u_{t+i}) = \text{Var}_t(u_{t+i})$.

It can be shown that

$$h_{1t+i} = \sigma_1^2 + a_{11}^{i-1}(h_{1t+1} - \sigma_1^2),$$

$$h_{2t+i} = \sigma_2^2 + a_{12}^{i-1}(h_{2t+1} - \sigma_2^2),$$

$$h_{12t+i} = \rho(h_{1t+1}h_{2t+1})^{1/2},$$

where

$$\sigma_j^2 = a_{0j}/(1 - a_{1j}) \text{ for } j = 1, 2.$$

2 Closed-form solutions of the model

Now we use the above formula of conditional mean and variance-covariance to derive the forward rate and its components. First, the annual yield to maturity on real bond $r_{t,k}$ is derived as

$$r_{t,k} = -4\ln\beta - \frac{4}{k}\ln E_t\left(\frac{c_{t+k}}{c_t}\right)^{-\gamma}$$

$$= -4\ln\beta + \frac{4}{k}\begin{bmatrix}0 & \gamma\end{bmatrix}E_t\sum_{i=1}^{k} z_{t+i} - \frac{4}{2k}\begin{bmatrix}0 & \gamma\end{bmatrix}E_t\mathrm{Var}_t\sum_{i=1}^{k} z_{t+i}\begin{bmatrix}0 & \gamma\end{bmatrix}'.$$

The expected future real interest rate is

$$E_t r_{t+k,q} = -4\ln\beta + \frac{4}{q}\begin{bmatrix}0 & \gamma\end{bmatrix}E_t\sum_{i=k+1}^{k+q} z_{t+i}$$

$$-\frac{4}{2k}\begin{bmatrix}0 & \gamma\end{bmatrix}E_t\mathrm{Var}_{t+k}\sum_{i=k+1}^{k+q} z_{t+i}\begin{bmatrix}0 & \gamma\end{bmatrix}'. \quad \text{(A1.1)}$$

Similarly, the annual yield to maturity on nominal bond $R_{t,k}$ is

$$R_{t,k} = -4\ln\beta - \frac{4}{k}\ln E_t\left[\left(\frac{c_{t+k}}{c_t}\right)^{-\gamma}\left(\frac{P_{t+k}}{P_t}\right)^{-1}\right]$$

$$= -4\ln\beta + \frac{4}{k}\begin{bmatrix}1 & \gamma\end{bmatrix}E_t\sum_{i=1}^{k} z_{t+i}$$

$$-\frac{4}{2k}\begin{bmatrix}1 & \gamma\end{bmatrix}E_t\mathrm{Var}_t\sum_{i=1}^{k} z_{t+i}\begin{bmatrix}1 & \gamma\end{bmatrix}'. \quad \text{(A1.2)}$$

The expected future nominal interest rate is

$$E_t R_{t+k,\,q} = -4\ln\beta + \frac{4}{q}\begin{bmatrix}1 & \gamma\end{bmatrix} E_t \sum_{i=k+1}^{k+q} z_{t+i}$$

$$-\frac{4}{2k}\begin{bmatrix}1 & \gamma\end{bmatrix} E_t \mathrm{Var}_{t+k} \sum_{i=k+1}^{k+q} z_{t+i} \begin{bmatrix}1 & \gamma\end{bmatrix}'. \qquad \text{(A1.3)}$$

The expected future inflation rate is

$$E_t \pi_{t+k,\,q} = \frac{4}{q} E_t \ln \frac{P_{t+k+q}}{P_{t+k}}$$

$$= \frac{4}{q}\begin{bmatrix}1 & 0\end{bmatrix} E_t \sum_{i=k+1}^{k+q} z_{t+i}. \qquad \text{(A1.4)}$$

Based on the nominal yield to maturity shown in equation (A1.2), it is simple to get the forward rate by

$$f_{t,\,k,\,q} = \frac{k+q}{q} R_{t,\,k+q} - \frac{k}{q} R_{t,\,k}. \qquad \text{(A1.5)}$$

The risk premium terms in forward rates are calculated by

$$\varphi^{f}_{t,\,k,\,q} = f_{t,\,k,\,q} - E_t R_{t+(k,\,q)},$$

$$\varphi^{\pi}_{t,\,k,\,q} = E_t R_{t+(k,\,q)} - E_t r_{t+k,\,q} - \pi^{e}_{t,\,k,\,q}. \qquad \text{(A1.6)}$$

Equations (A1.1), (A1.4), and (A1.6) are expressions of the four components in the forward rate (equation [A1.5]) in equation (5).

Appendix 2

The Derivation of the Coefficient of the Forward-Rate Rule

The forward-rate rule suggests that the forward rate is a linear predictor of the expected inflation rate:

$$\hat{\pi}^e_{t,k,q} = a + b_k f_{t,k,q}, \tag{A2.1}$$

where

$$\hat{b}_k = \frac{\mathrm{Cov}(f_{t,k,q}, \hat{\pi}^e_{t,k,q})}{\mathrm{Var}(f_{t,k,q})}. \tag{A2.2}$$

The forward rate is known to be the sum of expected future real rates, expected future inflation, plus the inflation-risk and nominal-forward premiums. Assuming that the inflation-risk and nominal-forward premiums are constants or 0, we can express (A2.2) as:

$$\hat{b}_k = \frac{\mathrm{Cov}((r^e_{t,k,q} + \hat{\pi}^e_{t,k,q}), \hat{\pi}^e_{t,k,q})}{\mathrm{Var}(r^e_{t,k,q} + \hat{\pi}^e_{t,k,q})}. \tag{A2.3}$$

Using the rule the $\mathrm{Cov}(x+y, z) = \mathrm{Cov}(x, z) + \mathrm{Cov}(y, z)$, we can express (A2.3) as:

$$\hat{b}_k = \frac{\mathrm{Cov}(r^e_{t,k,q}, \hat{\pi}^e_{t,k,q}) + \mathrm{Var}(\hat{\pi}^e_{t,k,q})}{\mathrm{Var}(r^e_{t,k,q} + \hat{\pi}^e_{t,k,q})}.$$

Expanding:

$$\hat{b}_k = \frac{\mathrm{Cov}(r^e_{t,k,q}, \hat{\pi}^e_{t,k,q}) + \mathrm{Var}(\hat{\pi}^e_{t,k,q})}{\mathrm{Var}(r^e_{t,k,q}) + \mathrm{Var}(\hat{\pi}^e_{t,k,q}) + 2\mathrm{Cov}(r^e_{t,k,q}, \hat{\pi}^e_{t,k,q})}.$$

Alternatively:

$$\hat{b}_k = \frac{1 + \rho_k \sigma_k}{1 + \sigma_k^2 + 2\rho_k \sigma_k},$$

where

$$\sigma_k^2 = \frac{\text{Var}(r^e_{t,k,q})}{\text{Var}(\hat{\pi}^e_{t,k,q})} \qquad \rho_k = \text{Corr}(r^e_{t,k,q}, \hat{\pi}^e_{t,k,q}).$$

References

Anderson, N., F. Breedon, M. Deacon, and G. Murphy. 1996. *Estimating and Interpreting the Yield Curve*. West Sussex: John Wiley and Sons.

Armour J., J. Atta-Mensah, W. Engert, and S. Hendry. 1996. "A Distant Early-Warning Model of Inflation Based on M1 Disequilibria." Bank of Canada Working Paper 96-5.

Bank of Canada. 1984. *Annual Report of the Governor to the Minister of Finance and statement of accounts for the year 1993*. Ottawa: 1984.

Bekaert, G., R. Hodrick, and D. Marshall. 1997. "The Implications of First-Order Risk Aversion for Asset Market Risk Premiums." *Journal of Monetary Economics* 40 (September): 3–39.

Bell Canada Limited. 1969. *Computer Analysis of Yield Curves*.

Canova, F. and J. Marrinan. 1996. "Reconciling the Term Structure of Interest Rates with the Consumption-based ICAP Model." *Journal of Economic Dynamics and Control* 20 (April): 709–50.

Crow, J. 1988. "The Work of Canadian Monetary Policy." Reprinted in *Bank of Canada Review* (February): 3–17.

Day, J. and R. Lange. 1997. "The Structure of Interest Rates in Canada: Information Content about Medium-Term Inflation." Bank of Canada Working Paper 97-10.

Engert, W. and S. Hendry. 1998. "Forecasting Inflation with the M1-VECM: Part Two." Bank of Canada Working Paper 98-06.

Engert, W., S. Hendry, and M. Yuan. 1998. "An Investigation of Instability in the M1-VECM: A Time-varying Parameter Model." Bank of Canada Working Paper, forthcoming.

Evans, M. 1998. "Real Rates, Expected Inflation, and Inflation Risk Premia." *Journal of Finance* 53 (February): 187–218.

Fama, E. 1990. "Term Structure Forecasts of Interest Rates, Inflation, and Real Returns." *Journal of Monetary Economics* 25 (January): 59–76.

Frankel, J. and C. Lown. 1994. "An Indicator of Future Inflation Extracted from the Steepness of the Interest Rate Yield Curve Along Its Entire Length." *Quarterly Journal of Economics* 109 (May): 517–30.

Hahm, J.-H. 1998. "Consumption Adjustment to Real Interest Rates: Intertemporal Substitution Revisited." *Journal of Economic Dynamics and Control* 22 (February): 293–320.

Hardouvelis, G. 1988. "The Predictive Power of the Term Structure during Recent Monetary Regimes." *Journal of Finance* 43 (June): 339–56.

Hendry, S. 1995. "Long-Run Demand for M1." Bank of Canada Working Paper 95-11.

Jorion, P. and F. Mishkin. 1991. "A Multi-Country Comparison of Term Structure Forecasts at Long Horizons." *Journal of Financial Economics* 29 (March): 59–80.

Laidler, D. and W. Robson. 1993. "The Great Canadian Disinflation: The Economics and Politics of Monetary Policy in Canada, 1988–93." C.D. Howe Institute Policy Study No. 19.

Longstaff, F. and E. Schwartz. 1992. "Interest Rate Volatility and the Term Structure: A Two-Factor General Equilibrium Model." *Journal of Finance* 47 (September): 1259–82.

Lowe, P. 1992. "The Term Structure of Interest Rates, Real Activity, and Inflation." Reserve Bank of Australia Research Discussion Paper 9204.

McCulloch, J. 1971. "Measuring the Term Structure of Interest Rates." *Journal of Business* 44 (January): 19–31.

Mishkin, F. 1988. “The Information in the Term Structure: Some Further Results.” *Journal of Applied Econometrics* 3 (October to December): 307–14.

———. 1990. “What Does the Term Structure Tell Us About Future Inflation?” *Journal of Monetary Economics* 25 (January): 77–95.

———. 1991. “A Multi-Country Study of the Information in the Shorter Maturity Term Structure About Future Inflation.” *Journal of International Money and Finance* 10 (March): 2–22.

Nelson, C. and A. Siegel. 1987. “Parsimonious Modelling of Yield Curves.” *Journal of Business* 60 (October): 473–89.

Sargent, T. 1987. *Macroeconomic Theory.* Orlando: Academic Press.

Söderlind, P. 1995. “Forward Interest Rates as Indicators of Inflation Expectations.” Centre for Economic Policy Research Discussion Paper 1313.

Söderlind, P. and L. Svensson. 1997. “New Techniques to Extract Market Expectations from Financial Instruments.” *Journal of Monetary Economics* 40 (October): 383–429.

Svensson, L. 1993. “Term, Inflation, and Foreign Exchange Risk Premia: A Unified Treatment.” National Bureau of Economic Research Working Paper 4544.

———. 1994a. “Monetary Policy with Flexible Exchange Rates and Forward Interest Rates as Indicators.” National Bureau of Economic Research Working Paper 4633.

———. 1994b. “Estimating and Interpreting Forward Interest Rates: Sweden 1992–1994.” Centre for Economic Policy Research Discussion Paper 1051.

———. 1995. “Estimating Forward Interest Rates with the Extended Nelson and Siegel Method.” *Quarterly Review*, Sveriges Riksbank 3: 13–26.