Monetary Policy in a Small Open Economy

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Introduction

The past decade has seen widespread application of New Keynesian models to the study of monetary policy. These models are derived from first principles and provide a rigorous setting in which to re-examine conventional wisdom and well-known puzzles in economics. A number of authors have examined the special case of small open economies, but their results are typically formulated in general theoretical terms.[1] This paper applies the New Keynesian modelling approach to the context of the Canadian economy.

Canada is a small open economy with distinctive features; in particular,

(i) it is highly dependent on a single large foreign economy;

(ii) it comprises a significant primary goods sector geared largely to exports;

(iii) its other traded goods sector (e.g., manufacturing) is highly integrated with the United States.

Following the global trend, the Canadian economy has become more open and integrated, both domestically and with the rest of the world. The volume of foreign trade in non-primary goods has grown substantially, while primary goods prices, and perhaps the size of that sector, have generally fallen. Labour mobility between sectors and across countries has increased, and the exchange rate pass-through may well have declined.

1. See surveys in Lane (2001) and Bowman and Doyle (2003).

Canada has pursued a flexible exchange rate regime since the early 1970s, and an inflation-targeting regime since 1991. While there is strong evidence that a flexible exchange rate has helped Canada to weather commodity price shocks, the persistent decline of the Canadian dollar and the increasing level of economic exchange between Canada and the United States, have led some economists to question the benefits of a floating exchange rate in the current environment.[2] Several authors have even argued that the current regime stifles productivity.

Accordingly, in this paper, we examine how some of the features of the Canadian economy might affect monetary policy and particularly the exchange rate. We ask whether past trends call for a less variable exchange rate other than through the direct effect of transactions costs.[3]

We develop a simple one-period model of a small open economy with nominal-wage rigidities, price-taking firms, and decreasing returns-to-scale technology. Similar models have been used in the literature, although the technology is usually assumed to have constant returns to scale, and firms are monopolistic-competitive and set their prices.[4] We treat the equilibrium outcome that obtains if wages were perfectly flexible as a benchmark for an efficient monetary policy. For simplicity, we examine alternative cases separately,[5] and focus on the monetary response to relative price shocks.

We first consider a one-sector case, whereby there is only one domestic and one foreign good, and all prices are set in the world market. We then show that the efficient monetary policy calls for a flexible exchange rate and conforms to conventional wisdom. Specifically, following a negative shock to the relative price of domestic goods, the local currency must depreciate and domestic prices rise, to counteract the rigidity of wages and emulate the drop in real wages that would have obtained if wages were flexible. In this case, monetary policy can reproduce the flexible-wage outcome.

We go on to examine a two-sector model. The goods in one sector are thought of as primary goods, the prices of which are set in the world market and exogenously given. The goods in the other sector are considered other traded goods (e.g., manufacturing) and have a relatively low elasticity of substitution with foreign (non-primary) goods. We consider the two extreme

2. See Courchene (1998), Harris (1993), Laidler (1999), and Murray (1999) for arguments on both sides of the debate.

3. For the effects of transactions costs, see Macklem, Osakwe, Pioro, and Schembri (2001).

4. See Devereux (2002) and Obstfeld and Rogoff (1999).

5. It is possible to describe one model that nests many cases. However, this unduly complicates the presentation and algebra. Moreover, the functional specifications in the extreme cases are more acceptable than a single artificial specification.

cases of perfect labour mobility and no labour mobility between the two sectors, as well as two cases where prices of non-primary goods are set in the world market and where they adjust endogenously to make demand and supply equal.

We show that, when labour is mobile, monetary policy can reproduce the flexible-wage outcome: as in the one-sector model, the domestic currency needs to depreciate following a negative shock to the relative price of primary goods. Output and employment in the domestic non-primary goods sector increase to take up the slack, and hence, if assumed endogenous, prices in that sector must fall relative to foreign goods. In this last case, higher economic integration, represented by a higher volume of international trade or higher elasticity of substitution between domestic and foreign (non-primary) goods, induces smaller currency depreciations. When labour is not mobile, it is shown that monetary policy cannot reproduce the flexible-wage outcome at the sectoral level, but it can reproduce the outcome at the aggregate level.

Finally, we again consider the one-sector model, but where firms are assumed to incur fixed costs in production along the same lines as described in Blanchard and Kiyotaki (1987). Aside from the literal interpretation, such fixed costs can also be thought of as a measure of a firm’s degree of output efficiency. The presence of fixed costs means that, following negative shocks, some firms may not find it profitable to produce, and hence a portion of the economy’s capacity may be left idle. Under these conditions, as long as some capacity remains idle, an improvement of the terms of trade would raise capacity utilization and output, but not necessarily the markup of domestic prices over wages. We then show that in this environment, monetary policy cannot always reproduce the flexible-wage outcome. In fact, in some circumstances, monetary policy may have to sustain production in inefficient firms, that is, in firms that would make negative profits in a perfectly flexible environment. In this sense, therefore, monetary policy can stifle productivity.

The literature on New Keynesian open-economy macroeconomics has usually focused on one-sector (albeit monopolistic-competitive) models. Two papers by Tille (1999, 2002) are noteworthy in relation to our work. As in our study, Tille (1999) examines the effect of the elasticity of substitution on exchange rate policy. Our work extends Tille’s results in that we allow for two domestic sectors, and we examine the effect of a change in the elasticity of substitution between the goods in one sector and its foreign counterpart, keeping the elasticity of substitution between the two domestic sectors unchanged. Tille, on the other hand, considers one domestic sector and examines the effect of a country-wide change in the elasticity of substitution between domestic and foreign goods. In contrast, Tille (2002)[6] examines monetary policy in a two-sector model, but his focus is on the different implications for monetary policy between country-wide and sectoral productivity shocks. His model assumes a representative household, so that issues of labour mobility do not arise, and the volume of trade as well as the elasticity of substitution between goods are fixed.

The paper is organized as follows. The baseline model is described in section 1. The one-sector economy is examined in section 2, and the two-sector economy in section 3. The case of fixed costs in production is considered in section 4. The final section concludes and offers suggestions for future research.

1 The Baseline Model

We consider a one-period small open economy that trades with the rest of the world. The domestic economy consists of a primary-goods sector, which takes world prices as given, and a non-primary-goods sector, the characteristics of which will be described below. Both types of goods can be traded.[7] Home-produced and foreign-produced primary goods are assumed to be perfect substitutes for each other, whereas non-primary goods are differentiated.

1.1 Households

The total number of households in the economy is assumed constant and normalized to equal 1. Households have identical preferences over the consumption of goods, leisure, and real money balances. The periodic utility of a household is

$$\frac{1}{1-\sigma}C^{1-\sigma} - \frac{1}{1+\phi}N^{1+\phi} + \chi \ln\left(\frac{M}{P}\right),$$

where N is the number of hours worked, $\frac{M}{P}$ are real money balances, and C is consumption of a composite of goods:

$$C = \left(\frac{C_X}{\gamma_X}\right)^{\gamma_X}\left(\frac{C_T}{\gamma_T}\right)^{1-\gamma_X},$$

6. We became aware of this study while working on our model.
7. The model can easily be extended to the case that includes non-traded goods.

$$C_T = \left[\gamma_H^{\frac{1}{\eta}} C_H^{\frac{\eta-1}{\eta}} + (1-\gamma_H)^{\frac{1}{\eta}} C_F^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}.$$

C_X, C_H, and C_F denote, respectively, consumption of primary goods, home-produced non-primary goods (*H*-goods), and imported non-primary goods (*F*-goods). The elasticity of substitution, η, between *H*-goods and *F*-goods is assumed to be greater than or equal to 1.[8]

Households choose consumption allocations and money balances after shocks are realized, subject to the budget constraint

$$P_X C_X + P_T C_T + M = WN + \Pi + T,$$

where Π denotes dividends, and W is the wage.

Standard optimization then implies that a household's holding of money balances and expenditures on the various types of good are:

$$P_X C_X = \gamma_X PC,$$

$$P_T C_T = (1-\gamma_X)PC,$$

$$P_H C_H = \gamma_H \left(\frac{P_H}{P_T}\right)^{1-\eta} P_T C_T,$$

$$P_F C_F = (1-\gamma_H)\left(\frac{P_F}{P_T}\right)^{1-\eta} P_T C_T,$$

and

$$\frac{M}{P} = \chi C^{\sigma},$$

where P_i is the (domestic-currency) price of goods in sector *i*, P is the aggregate price index, and P_T is the price index of non-primary goods,[9]

$$P = P_X^{\gamma_X} P_T^{1-\gamma_X},$$

8. The limit case $\eta = 1$ is identified with the Cobb-Douglas functional form.

9. The price index of a composite good can be defined as the minimum expenditure needed to buy one unit of the composite good.

$$P_T = [\gamma_H P_H^{1-\eta} + (1-\gamma_H) P_F^{1-\eta}]^{\frac{1}{1-\eta}}.$$

1.2 Production

Production in sector i $(i = X, H)$ requires a continuum of differentiated types of labour, ij, $j \in [0, 1]$. Each household is associated with one type of labour, but several households may be associated with the same type. The number of households of each type and the number of firms in any sector are assumed constant. To examine the effect of labour mobility between sectors, we will consider the case where both sectors employ the same types of labour, i.e., type *Xj* and type *Hj* are identical. Otherwise, type *Xj* and type *Hl* are assumed to be different for all *j* and *l*.

All firms in a given sector are identical and have decreasing returns-to-scale production technologies. Specifically, output by a firm in sector i is

$$Y_i = \frac{1}{1-\alpha} A_i L_i^{1-\alpha},$$

$$L_i = \left(\int_0^1 l(ij)^{\frac{\lambda-1}{\lambda}} dj \right)^{\frac{\lambda}{\lambda-1}},$$

where $l(ij)$ is labour input of type ij, $0 \le \alpha < 1$, $\lambda \ge 1$, and A_i is a sector-wide technological shock.

Firms take prices and wages as given and choose their volume of output after shocks are realized. Profit maximization then implies that demand by a firm in sector *i* for composite labour and individual labour of type *ij* is

$$L_i = \left[\frac{A_i P_i}{W_i} \right]^{\frac{1}{\alpha}} \text{ and } l(ij) = \left[\frac{W_i}{W(ij)} \right]^{\lambda} L_i,$$

where $W(ij)$ is the wage for labour of type ij, and W_i is a wage index for labour employed in sector *i*,[10]

10. W_i can be interpreted as the minimum cost of a unit of composite labour.

$$W_i = \left(\int_0^1 W(ij)^{1-\lambda} dj \right)^{\frac{1}{1-\lambda}}.$$

Thus, all else being equal, the higher the price markup over wages, the larger the demand is for labour and the larger output is by each firm.

1.3 Price determination

We assume throughout that the law of one price holds for all goods, i.e., $P_i = SP_i^f, i = X, H, F$, where S is the nominal exchange rate, expressed as the domestic price of a unit of foreign currency, and *f* superscript indicates foreign variables. Furthermore, on the basis that the domestic markets are small, prices of primary goods and foreign goods will be assumed to be determined in the world market and exogenously given. We will consider, however, the case where the price of home-produced non-primary goods is endogenously determined.

1.4 Wage-setting

We assume that households of the same type pool their resources and jointly set their wages. We will also consider the case where wages are pre-determined and the case where they are flexible. In the former, households are assumed to set wages at the beginning of the period, before shocks are realized, to maximize expected utility. The first-order condition then implies

$$W(ij) = \frac{\lambda}{\lambda - 1} \frac{E[l^h(ij)^{1+\phi}]}{E\left[\frac{l^h(ij)}{PC(ij)^{\sigma}}\right]},$$

where E is the expectations operator, and $l^h(ij)$ denotes the demand for labour per household of type *ij*.

In the case of flexible wages, households set wages after shocks are realized to maximize their utility. The first-order condition then implies

$$W(ij) = \frac{\lambda}{\lambda - 1} PC(ij)^{\sigma} l^h(ij)^{\phi}.$$

2 One-sector economy

In this section, we assume that there is only one sector in the economy, identified with the primary goods sector. Accordingly, households consume only primary goods and imported non-primary goods or, in other words, $\gamma_H = 0$. The domestic price level is then completely determined by the exchange rate and the exogenously given world prices, and relative prices and the terms of trade are exogenously given:

$$P = S(P_X^f)^{\gamma}(P_F^f)^{1-\gamma} \qquad (\gamma \equiv \gamma_X)$$

$$\frac{P_X}{P_F} = \frac{P_X^f}{P_F^f} \qquad \frac{P_X}{P} = \left(\frac{P_X^f}{P_F^f}\right)^{1-\gamma}.$$

Domestic monetary policy therefore has no effect on the terms of trade in this scenario, and the ratio of primary goods consumption to imported goods consumption depends only on world prices:

$$\frac{C_X}{C_F} = \frac{\gamma}{1-\gamma}\frac{P_F}{P_X} = \frac{\gamma}{1-\gamma}\frac{P_F^f}{P_X^f}.$$

2.1 Equilibrium

We focus on symmetric equilibria, whereby all households supply the same amount of labour and earn equal wages, i.e., $L = l(ij)$ and $W = W(ij)$ for all *j*. (The subscript *X*, which refers to the domestic sector, will be omitted from now on when there is no ambiguity.)

The home country sells primary goods in the world market and buys foreign goods in return. In equilibrium, prices adjust to equate total expenditures with income,

$$PC = nP_X Y,$$

where *n* is the number of firms in the sector. Hence, equilibrium aggregate consumption is

$$C = \frac{n}{1-\alpha}A\frac{P_X}{P}L^{1-\alpha} = \frac{n}{1-\alpha}\left(A\frac{P_X}{P}\right)^{\frac{1}{\alpha}}\left[\frac{P}{W}\right]^{\frac{1-\alpha}{\alpha}}.$$

2.1.1 Flexible wages

Since there is only one sector in the domestic economy (and the total number of households equals 1), the demand for labour per household is $l^h = nL$, and therefore real wages are

$$\frac{W}{P} = \frac{\lambda}{\lambda - 1} C^{\sigma} (nL)^{\phi}.$$

Substituting the expressions for equilibrium consumption and labour demand, it follows that

$$\left(\frac{W}{P}\right)^{\Theta} = \left[\frac{\lambda}{\lambda - 1}\right]^{\alpha} \left(\frac{1}{1-\alpha}\right)^{\alpha\sigma} \left(\frac{n^{\alpha} A P_X}{P}\right)^{\phi + \sigma},$$

$$\left(\frac{W}{P_X}\right)^{\Theta} = \left[\frac{\lambda}{\lambda - 1}\right]^{\alpha} \left(\frac{1}{1-\alpha}\right)^{\alpha\sigma} (n^{\alpha} A)^{\phi + \sigma} \left(\frac{P_X}{P}\right)^{(-\alpha)(1-\sigma)},$$

$$C^{\Theta} = \left[\frac{\lambda - 1}{\lambda}\right]^{1-\alpha} \left(\frac{1}{1-\alpha}\right)^{\alpha + \varphi} \left(\frac{n^{\alpha} A P_X}{P}\right)^{(1+\varphi)},$$

$$L^{\Theta} = \left[\frac{\lambda - 1}{\lambda}\right] (1-\alpha)^{\sigma} \frac{1}{n^{\phi + \sigma}} \left(\frac{A P_X}{P}\right)^{1-\sigma},$$

where

$$\Theta \equiv 1 + \phi - (1-\sigma)(1-\alpha) = \phi + \alpha + \sigma(1-\alpha).$$

All else being equal, a lower relative price of primary goods leads to lower income from exports, hence lower consumption, output, and demand for labour. Under flexible wages, the lower demand for labour is partly offset by lower real wages. Nevertheless, wages relative to primary goods prices increase because of the higher marginal product of labour.

2.1.2 Predetermined wages

Substituting equilibrium aggregate consumption in $M = \chi P C^{\sigma}$, it follows that

$$P^{\alpha + \sigma(1-\alpha)} = \chi^{-\alpha} \left(\frac{1-\alpha}{n}\right)^{\sigma\alpha} \left(A_X \frac{P_X}{P}\right)^{-\sigma} W^{\sigma(1-\alpha)} M^{\alpha},$$

$$P_X^{\alpha+\sigma(1-\alpha)} = \chi^{-\alpha}\left(\frac{1-\alpha}{n}\right)^{\sigma\alpha} A_X^{-\sigma}\left(\frac{P_X}{P}\right)^{\alpha(1-\sigma)} W^{\sigma(1-\alpha)} M^{\alpha},$$

$$S^{\alpha+\sigma(1-\alpha)} = \chi^{-\alpha}\left(\frac{1-\alpha}{n}\right)^{\sigma\alpha} A_X^{-\sigma}\left(\frac{P_X}{P}\right)^{\alpha(1-\sigma)} W^{\sigma(1-\alpha)} M^{\alpha} (P_X^f)^{-(\alpha+\sigma(1-\alpha))}.$$

From the first-order equation for predetermined wages,

$$W = \frac{1}{\chi}\frac{\lambda}{\lambda-1}\frac{E[(nL)^{1+\phi}]}{E\left[\frac{nL}{M}\right]},$$

it follows that

$$W^{\frac{\alpha+\phi}{\alpha}} = \frac{1}{\chi}\frac{\lambda}{\lambda-1}\frac{E\left[n^{1+\phi}(AP_X)^{\frac{1+\phi}{\alpha}}\right]}{E\left[\frac{n(AP_X)^{\frac{1}{\alpha}}}{M}\right]},$$

$$(\chi W)^{\frac{\Theta}{\alpha+\sigma(1-\alpha)}} = K\frac{E\left[\left(A\frac{P_X}{P}\right)^{\frac{(1+\phi)(1-\sigma)}{\alpha+\sigma(1-\alpha)}} M^{\frac{1+\phi}{\alpha+\sigma(1-\alpha)}}\right]}{E\left[\left(A\frac{P_X}{P}\right)^{\frac{(1-\sigma)}{\alpha+\sigma(1-\alpha)}} M^{\frac{(1-\alpha)(1-\sigma)}{\alpha+\sigma(1-\alpha)}}\right]},$$

where

$$K = \frac{\lambda}{\lambda-1}(1-\alpha)^{\frac{\sigma\phi}{\alpha+\sigma(1-\alpha)}} n^{\frac{\phi\alpha(1-\sigma)}{\alpha+\sigma(1-\alpha)}}.$$

The above equations for P, P_X, S, and W permit one to derive various monetary policies. In particular, a fixed exchange rate policy, say $S = 1$, requires

$$M = \chi\left(\frac{1-\alpha}{n}\right)^{-\sigma} A^{\sigma}\left(\frac{P_X}{P}\right)^{-(1-\sigma)} W^{\frac{-\sigma(1-\alpha)}{\alpha}} (P_X^f)^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} .$$

A policy that keeps the price of primary goods constant at $\bar{P}_X$ requires that the exchange rate fully offset any change in the world price of primary goods, but otherwise keeps the exchange rate constant. Such a policy obtains if

$$M = \chi\left(\frac{1-\alpha}{n}\right)^{-\sigma} A^{\sigma}\left(\frac{P_X}{P}\right)^{-(1-\sigma)} W^{\frac{-\sigma(1-\alpha)}{\alpha}} \bar{P}_X^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} .$$

Likewise, a policy that keeps the price level constant at $\bar{P}$ requires that the exchange rate fully offset any change in the foreign price index, $P^f \equiv (P_X^f)^{\gamma}(P_F^f)^{1-\gamma}$, but otherwise keep the exchange rate constant. Such a policy obtains if

$$M = \chi\left(\frac{1-\alpha}{n}\right)^{-\sigma}\left(A_X\frac{P_X}{P}\right)^{\sigma} W^{\frac{-\sigma(1-\alpha)}{\alpha}} \bar{P}^{\frac{\alpha+\sigma(1-\alpha)}{\alpha}} .$$

None of the three policies just described is efficient in general, although they are all efficient regarding foreign shocks that do not affect the terms of trade. Indeed, in this particular case, it is best to let the exchange rate fully offset the foreign shock and insulate domestic prices and the domestic economy (see below). A similar claim, however, cannot be made with regard to shocks to the terms of trade, say a drop in the world price of primary goods. If such a shock is reflected fully in the exchange rate, and domestic prices of primary goods are kept constant, then the marginal cost of labour would be too high, since employment is kept constant. Nor should the exchange rate be kept constant and allow the shock translate in a one-for-one drop in the home-currency price of primary goods, because this would imply too low a markup over wages, and therefore lead to an inefficient loss of employment and consumption. Likewise, a policy that targets a constant price level would keep the real wage inefficiently constant.[11]

Nevertheless, monetary policy under predetermined wages can achieve the outcome that obtains under flexible wages. To that effect, it can ensure that real wages under both regimes are equal, i.e.,

11. One can, in fact, show directly that household utility in each of these cases is lower than the utility that obtains under flexible wages.

$$\left(\frac{W}{P}\right)^{\Theta} = \left[\frac{\lambda}{\lambda-1}\right]^{\alpha}\left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\left(\frac{n^{\alpha}A_X P_X}{P}\right)^{\phi+\sigma}.$$

Hence,

$$P^{\Theta} = W^{\Theta}\left[\frac{\lambda}{\lambda-1}\right]^{-\alpha}\left(\frac{1}{1-\alpha}\right)^{-\alpha\sigma}\left(\frac{n^{\alpha}A_X P_X}{P}\right)^{-(\phi+\sigma)}$$

and

$$M^{\Theta} = [\chi W]^{\Theta}\left[\frac{\lambda-1}{\lambda}\right]^{\alpha+\sigma(1-\alpha)}\left(\frac{1}{1-\alpha}\right)^{\sigma\varphi}\left[\frac{n^{\alpha}A_X P_X}{P}\right]^{-\varphi(1-\sigma)}$$

$$S^{\Theta} = \left(\frac{W}{P_X^f}\right)^{\Theta}\left[\frac{\lambda}{\lambda-1}\right]^{-\alpha}\left(\frac{1}{1-\alpha}\right)^{-\alpha\sigma}(n^{\alpha}A_X)^{-(\phi+\sigma)}\left(\frac{P_X}{P}\right)^{\alpha(1-\sigma)}.$$

Written differently,

$$\hat{m} = \frac{-\phi(1-\sigma)}{\Theta}[\hat{a}_X + (1-\gamma)(\hat{p}_X - \hat{p}_F)]$$

$$\hat{s} = -\hat{p}_X^f + \frac{\alpha(1-\sigma)(1-\gamma)}{\Theta}(\hat{p}_X^f - \hat{p}_F^f) - \frac{\phi+\sigma}{\Theta}\hat{a}_X,$$

where hat superscripts denote percentage changes from the equilibrium outcome that obtains when no shocks occur. Note that

$$\frac{\alpha(1-\sigma)(1-\gamma)}{\Theta} \le 1-\gamma \le 1.$$

As expected, in response to a shock that does not affect the terms of trade, the money supply is kept constant, and the exchange rate adjusts to fully offset the effect on domestic prices. However, in response to decreases (respectively increases) in the relative price of primary goods, the money supply must expand (respectively contract), and the domestic currency depreciate (respectively appreciate), to offset the shock on the domestic sector. The larger the relative share, $1-\gamma$, of foreign goods in total domestic expenditures, or equivalently, the larger the share of exports in total output, the greater the effect of a change in primary goods prices on domestic income and consumption, and therefore the greater the required change in the exchange rate and money supply.

Section 2 examined monetary policy in a one-sector economy. Relative prices referred to the price of all domestic goods relative to foreign goods, and accordingly, a relative price shock is a shock across all domestic sectors. One would therefore expect monetary policy to respond fairly substantially to such shocks. In the following section, we examine how monetary responses are altered when there are two domestic sectors in the economy and the shocks affect prices in one sector relative to the other.

3 Two-Sector Economy

We now suppose that the domestic economy consists of a (non-primary) traded goods sector in addition to the primary goods sector (i.e., $\gamma_H \neq 0$). We continue to assume that prices of primary goods and prices of imported goods are determined in the world market and exogenously given. The mechanism by which prices of (non-primary) home-produced goods are determined, however, is more open to question, since the domestic market for such goods is likely to be relatively important. We therefore examine the case where it is exogenously given and the case where it is endogenously determined. We also consider alternative scenarios regarding labour mobility across sectors.[12]

All prices and wages are assumed equal in the long run,[13] hence all firms produce the same output, and

$$\begin{aligned}\frac{\bar{C}_X}{\gamma_X} &= \frac{\bar{C}_T}{1-\gamma_X} = \frac{\bar{C}_H}{(1-\gamma_X)\gamma_H} = \frac{\bar{C}_F}{(1-\gamma_X)(1-\gamma_H)} = \bar{C} \\ &= n_X\bar{Y}_X + n_H\bar{Y}_H = (n_X+n_H)\bar{Y},\end{aligned}$$

where bar superscripts denote long-run values. The (long-run) share of exports of primary goods in total output is then

$$\alpha_X \equiv \frac{n_X\bar{Y}_X - \bar{C}_X}{(n_X+n_H)\bar{Y}} = \frac{n_X - \gamma_X(n_X+n_H)}{(n_X+n_H)};$$

the share of exports of non-primary goods in total output is

12. Not treated in this paper, but not difficult to incorporate, are alternative assumptions regarding labour mobility across national borders.

13. This is reasonable, since all firms are identical and should therefore make equal profit in the long run; and labour is mobile in the long run, hence wages cannot be systematically higher in one sector.

$$\alpha_H = \frac{n_H\bar{Y}_H - \bar{C}_H}{(n_X + n_H)\bar{Y}} = \frac{n_H - (1-\gamma_X)\gamma_H(n_X + n_H)}{n_X + n_H};$$

the share of imports of non-primary goods must balance the share of total exports

$$\alpha_F = \alpha_X + \alpha_H = \frac{\bar{C}_F}{(n_X + n_H)\bar{Y}} = (1-\gamma_X)(1-\gamma_H);$$

whereas the share of the volume of trade in non-primary goods is

$$\alpha_H + \alpha_F = \alpha_X + 2\alpha_H = \frac{n_H}{n_X + n_H} + (1-\gamma_X)(1-\gamma_H) - (1-\gamma_X)\gamma_H.$$

Given the changes over time in the composition of trade in Canada and the evidence of greater integration with the United States, we will be particularly interested in the implications for monetary policy of changes in the:

- relative size of the primary goods sector driven by changes in the relative number of firms;
- volume of trade in non-primary goods driven by changes in the share of consumption of home-produced non-primary goods relative to foreign-produced goods; and
- elasticity of substitution between home-produced and imported non-primary goods.

We first examine the case where prices of all goods are set in the world market and labour is perfectly mobile across domestic sectors.

3.1 Prices of all goods set in the world market, and labour perfectly mobile

All relative prices are then exogenously given, and the ratios of consumption of the various products are independent of the exchange rate and monetary policy:

$$\frac{C_X}{C_T} = \frac{\gamma_X}{\gamma_T}\frac{P_T}{P_X},$$

$$\frac{C_H}{C_F} = \frac{\gamma_H}{1-\gamma_H}\left(\frac{P_H}{P_F}\right)^{-\eta}.$$

The domestic sectors are assumed to employ the same types of labour. Wages are therefore equal across the domestic sectors, and relative employment and output across sectors depend only on relative prices and technological shocks and not on the exchange rate policy:

$$\frac{n_X L_X}{n_H L_H} = \frac{n_X}{n_H}\left[\frac{A_X P_X}{A_H P_H}\right]^{\frac{1}{\alpha}},$$

$$\frac{n_X Y_X}{n_H Y_H} = \frac{n_X}{n_H}\left[\frac{A_X}{A_H}\right]^{\frac{1}{\alpha}}\left[\frac{P_X}{P_H}\right]^{\frac{1-\alpha}{\alpha}}.$$

Total demand for labour per household is

$$L = n_X L_X + n_H L_H = \Omega\left[\frac{P}{W}\right]^{\frac{1}{\alpha}},$$

where

$$\Omega = n_X\left[\frac{A_X P_X}{P}\right]^{\frac{1}{\alpha}} + n_H\left[\frac{A_H P_H}{P}\right]^{\frac{1}{\alpha}}.$$

In equilibrium, total expenditures must equal total income:

$$PC = n_X P_X Y_X + n_H P_H Y_H,$$

hence

$$C = \frac{1}{1-\alpha}\Omega\left[\frac{P}{W}\right]^{\frac{1-\alpha}{\alpha}}.$$

3.1.1 Flexible wages

$$W = \frac{\lambda}{\lambda - 1} P C^{\sigma} L^{\phi},$$

hence

$$\left(\frac{W}{P}\right)^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\left[\frac{\lambda}{\lambda-1}\right]^{\alpha}\Omega^{\alpha(\phi+\sigma)}$$

$$C^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha+\varphi}\left[\frac{\lambda-1}{\lambda}\right]^{1-\alpha}\Omega^{\alpha(1+\varphi)}$$

$$L^{\Theta} = (1-\alpha)^{\sigma}\frac{\lambda-1}{\lambda}\Omega^{\alpha(1-\sigma)},$$

(recall $\Theta = \alpha + \phi + \sigma(1-\alpha)$).

Note that the behaviour of Ω governs the behaviour of the macro variables, and

$$\begin{aligned}\alpha\hat{\Omega} &= \alpha_X(\hat{p}_X - \hat{p}_T) + \frac{n_H}{n_X+n_H}(\hat{p}_H - \hat{p}_T) + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H} \\ &= \alpha_X\hat{p}_X^f + \alpha_H\hat{p}_H^f - \alpha_F\hat{p}_F^f + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}.\end{aligned}$$

3.1.2 Predetermined wages

$$W = \frac{1}{\chi}\frac{\lambda}{\lambda-1}\frac{E[L^{1+\phi}]}{E\left[\frac{L}{M}\right]}.$$

As in the one-sector model, monetary policy can reproduce the outcome that obtains under flexible wages: for this purpose, it suffices to ensure that real wages under both regimes are equal. This would require

$$P^{\Theta} = W^{\Theta}\left[\frac{\lambda}{\lambda-1}\right]^{-\alpha}\left(\frac{1}{1-\alpha}\right)^{-\alpha\sigma}\Omega^{-\alpha(\phi+\sigma)},$$

hence

$$M^{\Theta} = (\chi W)^{\Theta}\left[\frac{\lambda}{\lambda-1}\right]^{-\alpha-\sigma(1-\alpha)}\left(\frac{1}{1-\alpha}\right)^{\sigma\varphi}\Omega^{-\alpha\varphi(1-\sigma)}.$$

Equivalently, in terms of the exchange rate,

$$\begin{aligned}
\hat{s} &= \hat{p}_X - \hat{p}_X^f = \hat{p}_X - \hat{p} + \hat{p} - \hat{p}_X^f = (1-\gamma_X)(\hat{p}_X^f - \hat{p}_T^f) - \frac{\alpha(\phi+\sigma)}{\Theta}\hat{\Omega} - \hat{p}_X^f \\
&= -\gamma_X \hat{p}_X^f - (1-\gamma_X)\hat{p}_T^f - \frac{(\phi+\sigma)}{\Theta}\left(\alpha_X \hat{p}_X^f + \alpha_H \hat{p}_H^f - \alpha_F \hat{p}_F^f + \frac{n_X \hat{a}_X + n_H \hat{a}_H}{n_X + n_H}\right) \\
&= -\left[\gamma_X + \frac{(\phi+\sigma)}{\Theta}\alpha_X\right]\hat{p}_X^f - \left[(1-\gamma_X)\gamma_H + \frac{(\phi+\sigma)}{\Theta}\alpha_H\right]\hat{p}_H^f - \\
&\quad -\left[\frac{\alpha(1-\sigma)}{\Theta}\alpha_F\right]\hat{p}_F^f - \frac{(\phi+\sigma)}{\Theta}\frac{n_X \hat{a}_X + n_H \hat{a}_H}{n_X + n_H}.
\end{aligned}$$

Not surprisingly, under flexible wages, the effect of changes in relative prices on the domestic economy depends on the share of imports/exports of the goods directly affected. For instance, assuming the home country is a net exporter of primary goods, a lower relative price of primary goods entails a lower income from exports of that good, hence lower employment in the primary goods sector as labour shifts to the other sector. Decreasing returns to scale then entail lower real wages and lower aggregate employment.

Accordingly, the monetary response to a relative price shock depends on the share of imports/exports of the good affected directly by the shock. In particular, all else being equal, the volume of trade of non-primary goods has no bearing on the monetary response to changes in the relative price of primary goods.

3.2 Prices of all goods set in the world market, and no labour mobility

The domestic sectors are now assumed to employ different types of labour, so wages can be different across sectors. We nevertheless assume that consumption is the same across households, because of, for instance, non-distortionary taxes or transfers. A balanced account in equilibrium then implies

$$PC = n_X P_X Y_X + n_H P_H Y_H,$$

hence

$$C = \frac{n_X}{1-\alpha}\left(A_X \frac{P_X}{P}\right)^{\frac{1}{\alpha}}\left[\frac{P}{W_X}\right]^{\frac{1-\alpha}{\alpha}} + \frac{n_H}{1-\alpha}\left(A_H \frac{P_H}{P}\right)^{\frac{1}{\alpha}}\left[\frac{P}{W_H}\right]^{\frac{1-\alpha}{\alpha}}.$$

3.2.1 Flexible wages

$$\left(\frac{W_i}{P}\right)^{\frac{\alpha+\phi}{\alpha}} = \frac{\lambda}{\lambda-1}n^{\phi}C^{\sigma}\left[A_i\frac{P_i}{P}\right]^{\frac{\phi}{\alpha}}.$$

Substituting the expressions for sectoral wages into the expression for equilibrium consumption,

$$C^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha+\phi}\left(n^{\phi}\frac{\lambda}{\lambda-1}\right)^{-(1-\alpha)}\Omega^{\alpha(1+\varphi)},$$

$$\left(\frac{W_i}{P}\right)^{\Theta} = \left[\frac{\lambda}{\lambda-1}\right]^{\alpha}n^{\alpha\phi}\left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\Omega^{\alpha(1+\varphi)\frac{\alpha\sigma}{\alpha+\phi}}\left[A_i\frac{P_i}{P}\right]^{\frac{\phi\Theta}{\alpha+\phi}},$$

$$L_i^{\Theta} = \frac{\lambda-1}{\lambda}n^{-\phi}(1-\alpha)^{\sigma}\Omega^{-(1+\varphi)\frac{\alpha\sigma}{\alpha+\phi}}\left[\frac{A_iP_i}{P}\right]^{\frac{\Theta}{(\alpha+\phi)}},$$

where

$$\Omega^{\alpha(1+\varphi)} \equiv \left[n_X\left(A_X\frac{P_X}{P}\right)^{\frac{1+\phi}{\alpha+\phi}} + n_H\left(A_H\frac{P_H}{P}\right)^{\frac{1+\phi}{\alpha+\phi}}\right]^{\alpha+\phi}.$$

Note that the variable Ω is different from that in the previous section, but to a first-order approximation, it has the same percentage deviations from initial conditions,

$$\alpha\hat{\Omega} = \alpha_X\hat{p}_X^f + \alpha_H\hat{p}_H^f - \alpha_F\hat{p}_F^f + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X + n_H}.$$

Consequently, the outcome at the sectoral level is different from the result that obtains when labour is mobile. Following a negative shock to primary goods prices, real wages for households in that sector fall more, and employment falls less than it would if labour were mobile, assuming the home country is a net exporter of primary goods. By the same token, real wages rise more and employment rises less in the other sector, because unemployment in the primary goods sector does not translate directly into an increased supply of labour in the other sector. Rather, the lower income from exports of primary goods, and increased transfers due to higher unemployment, translate into lower wages, hence higher employment and output in the non-primary goods sector. Nonetheless, at the aggregate level, and to a first-order approximation, deviations in real wages,[14] employment, consumption, and output are the same in both scenarios (see details in Appendix). Also, the volume of trade in the non-primary goods sector still has no bearing on the effects that such shocks have on the economy.

3.2.2 Predetermined wages

Note that under flexible wages, the ratio of wages across the two sectors depends on relative prices,

$$\frac{W_X}{W_H} = \left[\frac{A_X}{A_H}\frac{P_X}{P_H}\right]^{\frac{\phi}{\alpha + \phi}},$$

whereas, under predetermined wages, the ratio is, of course, constant. Thus, in general, monetary policy cannot reproduce the outcome that obtains under flexible wages and no labour mobility at the sectoral level. It can, however, reproduce the outcome at the aggregate level. To that effect it suffices to ensure that aggregate real wages equal those that prevail under flexible wages.[15] Under these conditions, output, in the aggregate as well as in each sector, and hence consumption and aggregate employment, would be the same as in the flexible case with labour mobility. But naturally, employment per household of each type will be different, since households in one sector would be called on to work more or less to offset the fluctuations in employment caused by inflexible wages in the other sector.

3.3 Prices of home-produced non-primary goods endogenous, labour fully mobile

In this scenario, prices of primary goods and foreign-produced non-primary goods are still determined in the world market; however, prices of home-produced non-primary goods adjust to make supply and demand equal.[16] Specifically, the demand for exports of non-primary goods is assumed to satisfy

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14. The aggregate wage index is

$$\frac{W}{P} = \left(\frac{W_X}{P}\right)^{\frac{n_X}{n_X + n_H}}\left(\frac{W_H}{P}\right)^{\frac{n_H}{n_X + n_H}}.$$

15. With or without labour mobility.

16. An alternative scenario with similar consequences is that prices of imported goods depend on domestic demand, i.e., there is pricing to market by foreign firms.

$$EX_H = \theta\left(\frac{P_H}{P_T}\right)^{-\eta},$$

where θ is the volume of exports of *H*-goods that obtain in the long-run, i.e.,

$$\theta = \alpha_H\left(\frac{1}{1-\alpha}\right)^{\frac{\alpha+\phi}{\Theta}}\left[\frac{\lambda-1}{\lambda}\right]^{\frac{1-\alpha}{\Theta}}(n_X+n_H)^{\frac{\alpha(1+\varphi)}{\Theta}}.$$

The same derivations as in section 3.1 imply

$$L = \Omega\left[\frac{P}{W}\right]^{\frac{1}{\alpha}},$$

$$C = \frac{1}{1-\alpha}\Omega\left[\frac{P}{W}\right]^{\frac{1-\alpha}{\alpha}},$$

with Ω defined as in section 3.1.

Now, however, relative prices, and hence Ω, are not exogenously given. Rather, the price of *H*-goods adjusts so that total demand for home-produced traded goods equals supply, i.e.,

$$C_H + EX_H = n_H Y_H,$$

hence

$$(1-\gamma_X)\gamma_H\frac{P}{P_T}C + \theta = \frac{1}{1-\alpha}n_H A_H^{\frac{1}{\alpha}}\left(\frac{P_H}{P_T}\right)^{\eta}\left[\frac{P_H}{W}\right]^{\frac{1-\alpha}{\alpha}}.$$

All else being equal, an increase in P_H raises output of home-produced non-primary goods, which can be liquidated only if their price relative to foreign non-primary goods falls so that domestic consumption and exports of *H*-goods increase.

3.3.1 Flexible wages

The same derivations as in section 3.1 give

$$\left(\frac{W}{P}\right)^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\left[\frac{\lambda}{\lambda-1}\right]^{\alpha}\Omega^{\alpha(\phi+\sigma)}$$

$$C^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha+\varphi}\left[\frac{\lambda-1}{\lambda}\right]^{1-\alpha}\Omega^{\alpha(1+\varphi)}$$

$$L^{\Theta} = (1-\alpha)^{\sigma}\frac{\lambda-1}{\lambda}\Omega^{\alpha(1-\sigma)}.$$

Substituting into the market equilibrium condition for *H*-goods, it follows (see Appendix for details) that

$$\Psi(\hat{p}_X - \hat{p}_F) = \left(\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right)\left(\frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right) + \frac{1}{\alpha}\hat{a}_H + \Gamma(\hat{p}_H - \hat{p}_F),$$

and hence

$$\alpha\hat{\Omega} = \left(\alpha_X + \alpha_H\frac{\Psi}{\Gamma}\right)(\hat{p}_X - \hat{p}_F) + \left[1 - \frac{\alpha_H}{\Gamma}\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right]$$
$$\left(\frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right) - \frac{\alpha_H}{\Gamma}\frac{1}{\alpha}\hat{a}_H,$$

$$\hat{w} - \hat{p}_X = \left[\frac{\phi+\sigma}{\Theta}\alpha_X - (1-\gamma_X) + \left[\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H} + \left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right]\frac{\Psi}{\Gamma}\right](\hat{p}_X - \hat{p}_F) +$$
$$+\left[\frac{\phi+\sigma}{\Theta} - \left(\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H} + \left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right)\frac{(1+\varphi)}{\Gamma\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right]\left(\frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right)$$
$$-\left(\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H} + \left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right)\frac{1}{\Gamma\alpha}\hat{a}_H,$$

where the coefficients Ψ and Γ are positive if the home country is a net exporter of primary goods.

Thus, as expected, a drop in the world price of primary goods causes a drop in the price of *H*-goods relative to foreign goods. Numerical approximations show that the larger the volume of foreign trade (induced by a smaller share, γ_H, of consumption of home-produced non-primary goods in total consumption of non-primary goods), the smaller the coefficients $\frac{\Psi}{\Gamma}$ and $\alpha_X + \alpha_H\frac{\Psi}{\Gamma}$ (in $\alpha\hat{\Omega}$). And therefore, the smaller the drop in the price of home-produced non-primary goods relative to imported goods following a negative shock to the price of primary goods, the larger the drop in the price of primary goods relative to wages, and the smaller the drop in aggregate real wages, employment, and consumption. A higher elasticity of substitution, η, between home-produced and foreign-produced non-primary goods, has the same effects as a larger volume of trade, but for different reasons. A higher elasticity of substitution implies that a smaller reduction in prices is needed to clear the market for non-primary goods, whereas a larger volume of trade implies that a drop in prices is more costly in terms of income, and therefore induces greater labour supply.

3.3.2 Predetermined wages

As in section 3.1, since labour is assumed mobile between sectors, monetary policy can reproduce the equilibrium outcome under flexible wages. Again, it suffices to ensure that real wages equal those that obtain when wages are flexible. The results above showed that when wages are flexible, the larger the volume of foreign trade and the larger the elasticity of substitution between home-produced non-primary goods and imported goods, the larger the drop in the price of primary goods relative to wages following a negative shock to the price of primary goods, and therefore the exchange rate should offset the shock to a lesser degree.

In all the cases considered so far, there is no sense in which monetary policy promotes low productivity in the primary goods sector following a negative shock. When labour was assumed to be perfectly mobile, it was found that monetary policy can, in fact, reproduce the flexible-wage outcome, and when labour was not mobile, it was found that, if anything, monetary policy would sustain higher marginal productivity of labour in the primary goods sector than in the flexible-wage case. The next section describes a model whereby monetary policy can sometimes sustain inefficient productivity.

4 One-Sector Economy with Fixed Costs in Production

In this section, we make the same assumptions as in the one-sector model described in section 2, except that now firms can incur a fixed cost of production along the same lines as in Blanchard and Kiyotaki (1987).[17] Specifically, output by a firm in the domestic sector is now given by

$$Y = \frac{1}{1-\alpha} A L^{1-\alpha} - c,$$

where c is a fixed cost (expressed in the firm's own good) that is allowed to be different from one firm to another. Aside from the obvious interpretation, differences in fixed costs between firms can also be thought of as differences in efficiency of production.[18]

17. One difference in our model, with important implications, is that the total number of firms in the sector that can potentially produce output is fixed and firms have decreasing returns-to-scale technology.

18. In a partial-equilibrium setting, the fixed cost can also be thought of as an opportunity cost for doing business in a particular sector.

The presence of fixed costs means that some firms may choose not to produce following a negative shock. More formally, profits by a firm with fixed costs c are

$$\Pi = P_X Y - WL = \frac{\alpha}{1-\alpha} A_X^{\frac{1}{\alpha}} \left(\frac{P_X}{W}\right)^{\frac{1}{\alpha}} W - P_X c .$$

It follows that such a firm admits non-negative profits, and therefore produces output if

$$\frac{P_X}{W} \geq \left(\frac{1-\alpha}{\alpha} c\right)^{\frac{\alpha}{1-\alpha}} A_X^{-\frac{1}{1-\alpha}},$$

i.e., if the price markup over wages is above a certain threshold or, equivalently, if

$$c \leq \frac{\alpha}{1-\alpha} A_X^{\frac{1}{\alpha}} \left(\frac{P_X}{W}\right)^{\frac{1-\alpha}{\alpha}},$$

i.e., if the fixed cost is below a certain threshold.

The total domestic output following a shock is

$$totY = \frac{n}{1-\alpha} A_X L^{1-\alpha} - totc,$$

where n is now the number of firms that do produce output,[19] which may be smaller than the total number $\bar{n}$ of firms in the sector, and $totc$ is the total fixed cost of production incurred in the sector (e.g., $totc = nc$ if firms have identical fixed costs). Accordingly, market equilibrium is now written as

$$PC = P_X totY,$$

and the equilibrium level of aggregate consumption is

$$C = \frac{n}{1-\alpha} A_X \frac{P_X}{P} L^{1-\alpha} - \frac{P_X}{P} totc = \frac{n}{1-\alpha}\left(A_X \frac{P_X}{P}\right)^{\frac{1}{\alpha}} \left[\frac{P}{W}\right]^{\frac{1-\alpha}{\alpha}} - \frac{P_X}{P} totc .$$

19. To simplify the derivations, this number is allowed to be a fraction.

4.1 Flexible wages

Households are assumed to take the number, *n*, of firms in operation as given when they set their wages.[20] The first-order equation for wages is

$$W = \frac{\lambda}{\lambda - 1} P C^{\sigma} (nL)^{\phi}.$$

Substituting equilibrium consumption gives

$$W = \frac{\lambda}{\lambda - 1} P \left(\frac{n}{1-\alpha} A_X \frac{P_X}{P} L^{1-\alpha} - \frac{P_X}{P} totc \right)^{\sigma} (nL)^{\phi}.$$

On the other hand,

$$L = \left(\frac{A P_X}{P} \right)^{\frac{1}{\alpha}} \left[\frac{P}{W} \right]^{\frac{1}{\alpha}}.$$

It follows that the markup satisfies

$$\left[\frac{P_X}{W} \right]^{\frac{\alpha + \varphi}{\alpha\sigma}} n^{\frac{\varphi}{\sigma}} \left(\frac{n}{1-\alpha} A^{\frac{1}{\alpha}} \left[\frac{P_X}{W} \right]^{\frac{1-\alpha}{\alpha}} - totc \right) = \left(\frac{\lambda}{\lambda - 1} \right)^{-\frac{1}{\sigma}} A^{-\frac{\varphi}{\alpha\sigma}} \left(\frac{P_X}{P} \right)^{\frac{(1-\sigma)}{\sigma}}.$$

The larger the number, *n*, of firms in operation, the higher the demand for labour, the higher the real wage and the smaller the markup (see proof in Appendix). If the threshold for positive profits is binding for some value $\bar{c}$ of the fixed cost, then

$$n^{\varphi} \left(\frac{n\bar{c}}{\alpha} - totc \right)^{\sigma} = \left(\left(\frac{\lambda - 1}{\lambda} \right) \left(\frac{1-\alpha}{\alpha} \bar{c} \right) \right)^{\frac{-(\alpha + \varphi)}{1-\alpha}} A^{\frac{(1+\varphi)}{1-\alpha}} \left(\frac{P_X}{P} \right)^{(1-\sigma)}.$$

(In particular, if all firms admit the same fixed costs, then

$$n^{\varphi + \sigma} = \left(\left(\frac{\lambda - 1}{\lambda} \right) \left(\frac{1-\alpha}{\alpha} \bar{c} \right) \right)^{-\frac{(\alpha + \varphi) + \sigma(1-\alpha)}{1-\alpha}} A_X^{\frac{(1+\varphi)}{1-\alpha}} \left(\frac{P_X}{P} \right)^{(1-\sigma)}.)$$

20. It would be interesting to examine the case where households take into consideration the effect of their wage demands on the subsistence of the firm.

If the total number of firms in the sector, $\bar{n}$, is smaller than the number n that satisfies the equation above with the highest possible value of the fixed cost chosen for $\bar{c}$, then all the firms in the sector produce output. Otherwise, there is an upper limit $\bar{c}$ on c, such that only firms with fixed costs less than or equal to $\bar{c}$ can make non-negative profits, and the number of such firms cannot exceed the number n that satisfies the equation above. A favourable change in the terms of trade increases the number of firms in operation. But as long as this number is no greater than the total number of firms in the sector with fixed costs less than or equal to $\bar{c}$, the markup and employment per firm remain constant at

$$\frac{P_X}{W} = \left(\frac{1-\alpha}{\alpha}c\right)^{\frac{\alpha}{1-\alpha}} A_X^{-\frac{1}{1-\alpha}} \text{ and } L = \left(\frac{A_X P_X}{W}\right)^{\frac{1}{\alpha}}.$$

More generally, if the firms' fixed costs do not differ too much, then the markup ought to change relatively little with changes in the terms of trade. Aggregate production would have close to constant returns to scale as long as some firms remain idle, and changes in the terms of trade would be passed on to wages. However, once the economy reaches capacity, in the sense that all the firms are in operation, then higher aggregate production can obtain only by means of increases in the production of individual firms, in which case, domestic goods prices would increase faster than wages because of decreasing returns to scale.[21] The Appendix illustrates the case where firms' fixed costs are distributed uniformly on the interval [0,1].

4.1.1 Predetermined wages

One can think of two reasons why, in the flexible-wage case, a firm may not produce output following a negative shock to the terms-of-trade prices. Either its fixed costs are too high to operate at the level to which wages adjust in equilibrium, or the level of wages at which it can attract (or retain) labour is too high. If firms are sufficiently differentiated by their fixed costs so that those firms that choose not to produce do so strictly because of their higher fixed costs, then, as in the case without fixed costs, monetary policy can reproduce the flexible-wage outcome by causing the local currency to depreciate and domestic prices to rise until real wages equal those that obtain under flexible wages.

Nonetheless, monetary policy ought to be different depending on whether or not the economy has reached full capacity. Since, under flexible wages, the elasticity of the markup to changes in the terms of trade is lower when the

21. To see this, differentiate the expression for wages in terms of $\frac{P_X}{P}$.

economy is below full capacity, the exchange rate must depreciate more in response to the negative shock in that case.[22] The Appendix illustrates the case where the fixed costs of firms are distributed uniformly on the interval [0,1].

If firms are not sufficiently differentiated by their fixed costs to begin with, then fixing the real wage to that which prevails under flexible wages would sustain production in inefficient firms, i.e., in firms that would not have produced if wages could adjust. Under such conditions, therefore, monetary policy cannot reproduce the flexible-wage outcome. What then, is an efficient policy?

If, in fact, all firms have equal fixed costs, then following a shock, either all firms produce output or none does. However, the latter outcome clearly cannot be efficient since it entails no income and hence no consumption by households. In this case, monetary policy always sustains production by all firms, even if some firms are inefficient.[23]

This result depends, however, on all firms having equal fixed costs. The presence of firms with different fixed costs makes it possible that for some range of the terms of trade it is optimum, as in the previous case, to offset shocks to the terms of trade and keep the price of primary goods at a level that sustains inefficient production. For another range of relative prices, however, it is optimum to allow the price of primary goods to fall to a level that thwarts production in certain firms in spite of the fact that they would be efficient if wages could adjust.[24]

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22. By the same token, this also implies that prices must be allowed to change more when the economy is at full capacity. Of course this does not take into consideration the risks of inflation. One must introduce dynamics in the model to tackle such questions.

23. Heuristically, the policy that seems closest to reproducing the flexible-wage outcome is the policy that keeps the price of primary goods (hence the markup) constant as long as the terms of trade are below the threshold that entails positive profits in the flexible-wage case, i.e., as long as the potential number,

$$n^{\varphi+\sigma} = \left(\frac{\lambda-1}{\lambda}\right)\left(\frac{1-\alpha}{\alpha}c\right)^{-\frac{(\alpha+\varphi)+\sigma(1-\alpha)}{1-\alpha}} A_X^{\frac{(1+\varphi)}{1-\alpha}}\left(\frac{P_X}{P}\right)^{(1-\sigma)},$$

of firms that can be sustained is no greater than the total number of firms in the sector. Over that range of values for the terms of trade, employment is therefore higher than it would be under flexible wages. Conversely, for the equilibrium to exist, employment likely has to be lower than it would be under flexible wages for values above the threshold. A more formal proof of this claim is outside the scope of this paper.

24. Alternatively, one can obtain similar outcomes even if all firms are identical, if one incorporates a second sector in the economy that can provide a substitute income.

Conclusions

This paper applies the New Keynesian open-economy modelling approach to examine monetary policy in the context of the Canadian economy. For that purpose, it develops a simple one-period model of a small open economy with nominal-wage rigidities, price-taking firms, and decreasing returns-to-scale technology.

The paper first looks at a one-sector case and shows that the efficient monetary policy conforms to conventional wisdom. Specifically, following a negative shock to the relative price of domestic goods, the local currency must depreciate to counteract the rigidity of wages. Next, it examines a two-sector model, with one sector standing for primary goods, the prices of which are set in the world market, and another sector standing for traded non-primary (e.g., manufactured) goods. It is subsequently shown that, as in the one-sector model, the domestic currency needs to depreciate following a negative shock to the relative price of primary goods, but that typically, higher economic integration, represented by a higher volume of international trade or higher elasticity of substitution between domestic and foreign (non-primary) goods, would induce smaller currency depreciations. Finally, the paper considers a one-sector model with fixed costs in production, and shows that monetary policy cannot always reproduce the flexible-wage outcome in such an environment. In fact, in some circumstances, monetary policy may stifle productivity, in the sense that it may have to sustain production in firms that would make negative profits in a perfectly flexible environment.

The model can be easily extended to examine a number of other questions, for instance, the effect of labour mobility across countries on the conduct of monetary policy, or the effects of adjustment costs in labour markets. While it allows one to abstract from dynamics, the restriction of the model to a single period strongly limits the breadth of the results. Extension to an infinite horizon is the natural next step in this research.

Appendix
Two-Sector Economy

All prices set in the world market, labour not mobile

Claim: Under flexible wages, real wages in the primary goods sector following a shock to prices in that sector deviate less, and employment deviates more when labour is mobile than when it is not, assuming that the home country is a net exporter of primary goods. However, to a first-order approximation, aggregate real wages and employment deviate equally in both scenarios.

Indeed, real wages (in either sector) when labour is mobile are

$$\left(\frac{W}{P}\right)^{\Theta} = \left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\left[\frac{\lambda}{\lambda-1}\right]^{\alpha}\Omega^{\alpha(\phi+\sigma)},$$

and hence, in percentage deviations from initial conditions,

$$\Theta(\hat{w}-\hat{p}) = \alpha(\phi+\sigma)\hat{\Omega} = (\phi+\sigma)\left(\alpha_X\hat{p}_X^f + \alpha_H\hat{p}_H^f - \alpha_F\hat{p}_F^f + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right);$$

whereas, when labour is not mobile,

$$\left(\frac{W_i}{P}\right)^{\Theta} = \left[\frac{\lambda}{\lambda-1}\right]^{\alpha} n^{\alpha\phi}\left(\frac{1}{1-\alpha}\right)^{\alpha\sigma}\Omega^{\alpha(1+\varphi)\frac{\alpha\sigma}{\alpha+\phi}}\left[A_i\frac{P_i}{P}\right]^{\frac{\phi\Theta}{\alpha+\phi}},$$

hence

$$\begin{aligned}\Theta(\hat{w}_i-\hat{p}) &= \alpha(1+\varphi)\frac{\alpha\sigma}{\alpha+\phi}\hat{\Omega} + \frac{\phi\Theta}{\alpha+\phi}(\hat{a}_i+\hat{p}_i-\hat{p})\\ &= (1+\varphi)\frac{\alpha\sigma}{\alpha+\phi}\left(\alpha_X\hat{p}_X^f + \alpha_H\hat{p}_H^f - \alpha_F\hat{p}_F^f + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right) + \frac{\phi\Theta}{\alpha+\phi}(\hat{a}_i+\hat{p}_i-\hat{p}).\end{aligned}$$

Recall that, in either case,

$$\begin{aligned}\alpha\hat{\Omega} &= \alpha_X(\hat{p}_X-\hat{p}_T) + \frac{n_H}{n_X+n_H}(\hat{p}_H-\hat{p}_T) + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\\ &= \alpha_X\hat{p}_X^f + \alpha_H\hat{p}_H^f - \alpha_F\hat{p}_F^f + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\end{aligned}$$

Thus, the elasticity of real wages with respect to $\hat{p}_X^f$ is

$(\phi + \sigma)\alpha_X$ when labour is mobile, and

$$(1 + \varphi)\frac{\alpha\sigma}{\alpha + \phi}\alpha_X + \frac{\phi\Theta}{\alpha + \phi}(1 - \gamma_X)$$

when labour is not mobile, and their difference is negative:

$$\begin{aligned}
&\frac{1}{\alpha + \phi}\{[(\phi + \sigma)(\alpha + \phi) - (1 + \varphi)\alpha\sigma]\alpha_X - \phi\Theta(1 - \gamma_X)\} \\
&= \frac{1}{\alpha + \phi}\{\phi(\alpha + \phi + \sigma(1 - \alpha))\alpha_X - \phi\Theta(1 - \gamma_X)\} \\
&= \frac{\phi\Theta}{\alpha + \phi}\{\alpha_X - (1 - \gamma_X)\} = -\frac{\phi\Theta}{\alpha + \phi}\frac{n_H}{n_X + n_H}.
\end{aligned}$$

The claim for sectoral employment follows immediately, since

$$L_i = \left[\frac{A_i P_i}{P}\right]^{\frac{1}{\alpha}}\left[\frac{A_i P}{W_i}\right]^{\frac{1}{\alpha}}$$

under either scenario.

The deviation in aggregate real wages when labour is not mobile is

$$\begin{aligned}
\Theta(\hat{w} - \hat{p}) &= \Theta\left[\frac{n_X}{n_X + n_H}(\hat{w}_X - \hat{p}) + \frac{n_H}{n_X + n_H}(\hat{w}_H - \hat{p})\right] \\
&= (1 + \varphi)\frac{\alpha\sigma}{\alpha + \phi}\alpha\hat{\Omega} + \frac{n_X}{n_X + n_H}\frac{\phi\Theta}{\alpha + \phi}(\hat{a}_X + \hat{p}_X - \hat{p}) + \frac{n_H}{n_X + n_H}\frac{\phi\Theta}{\alpha + \phi}(\hat{a}_H + \hat{p}_H - \hat{p}) \\
&= (1 + \varphi)\frac{\alpha\sigma}{\alpha + \phi}\alpha\hat{\Omega} + \frac{\phi\Theta}{\alpha + \phi}\alpha\hat{\Omega} = (\phi + \sigma)\alpha\hat{\Omega},
\end{aligned}$$

which is identical to that when labour is mobile; similarly for aggregate labour.

Prices of home-produced non-primary goods endogenous

The same derivations as above give

$$\left(\frac{W}{P}\right)^{\Theta} = \left(\frac{1}{1 - \alpha}\right)^{\alpha\sigma}\left[\frac{\lambda}{\lambda - 1}\right]^{\alpha}\Omega^{\alpha(\phi + \sigma)}$$

$$C^{\Theta} = \left(\frac{1}{1 - \alpha}\right)^{\alpha + \varphi}\left[\frac{\lambda - 1}{\lambda}\right]^{1 - \alpha}\Omega^{\alpha(1 + \varphi)}$$

$$L^{\Theta} = (1-\alpha)^{\sigma}\frac{\lambda-1}{\lambda}\Omega^{\alpha(1-\sigma)}.$$

Equilibrium in the market for H-goods requires

$$(1-\gamma_X)\gamma_H\frac{P}{P_T}C+\theta = \frac{1}{1-\alpha}n_H A_H^{\frac{1}{\alpha}}\left(\frac{P_H}{P_T}\right)^{\eta}\left[\frac{P_H}{W}\right]^{\frac{1-\alpha}{\alpha}}.$$

It follows

$$(1-\gamma_X)\gamma_H\frac{P}{P_T}\left[\left(\frac{1}{1-\alpha}\right)^{\frac{\alpha+\varphi}{\Theta}}\left[\frac{\lambda-1}{\lambda}\right]^{\frac{1-\alpha}{\Theta}}\Omega^{\frac{\alpha(1+\varphi)}{\Theta}}\right]+\theta$$

$$= n_H A_H^{\frac{1}{\alpha}}\left[\frac{P_H}{P_T}\right]^{\frac{1-\alpha+\alpha\eta}{\alpha}}\left[\frac{P_T}{P}\right]^{\frac{1-\alpha}{\alpha}}\left(\frac{1}{1-\alpha}\right)^{\frac{\alpha+\varphi}{\Theta}}\left[\frac{\lambda}{\lambda-1}\right]^{-\frac{(1-\alpha)}{\Theta}}\Omega^{\frac{\alpha(1+\varphi)}{\Theta}-1}.$$

Taking log deviations from initial conditions,

$$\left[\frac{1}{\alpha}-\frac{\alpha_H(n_X+n_H)}{n_H}\right][\gamma_X(\hat{p}_X-\hat{p}_T)] = \frac{1}{\alpha}\hat{a}_H+\frac{1-\alpha+\alpha\eta}{\alpha}(\hat{p}_H-\hat{p}_T)$$

$$+\left(-\frac{1}{\alpha}+\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right)\alpha\hat{\Omega},$$

hence

$$\left[\frac{1}{\alpha}(\gamma_X+\alpha_X)-\frac{\alpha_H(n_X+n_H)}{n_H}\left(\gamma_X+\frac{(1+\varphi)}{\Theta}\alpha_X\right)\right][(\hat{p}_X-\hat{p}_T)] =$$

$$+\left(\frac{1}{\alpha}\frac{n_X}{n_X+n_H}+\frac{(1+\varphi)}{\Theta}\alpha_H+\eta-1\right)(\hat{p}_H-\hat{p}_T),$$

$$+\left(-\frac{1}{\alpha}+\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right)\frac{n_X\hat{a}_X+n_H\hat{a}_H}{n_X+n_H}+\frac{1}{\alpha}\hat{a}_H$$

$$\Psi(\hat{p}_X-\hat{p}_T) = \left(\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right)\left(\frac{n_X\hat{a}_X+n_H\hat{a}_H}{n_X+n_H}\right)+\frac{1}{\alpha}\hat{a}_H+\Gamma_1(\hat{p}_H-\hat{p}_T),$$

or equivalently

$$\Psi(\hat{p}_X - \hat{p}_F) = \left(\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right)\left(\frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right) + \frac{1}{\alpha}\hat{a}_H + \Gamma(\hat{p}_H - \hat{p}_F),$$

where

$$\Gamma_1 = \frac{1}{\alpha}\frac{n_X}{n_X+n_H} + \frac{(1+\varphi)}{\Theta}\alpha_H + \eta - 1,$$

$$\Gamma = \Gamma_1(1-\gamma_H) + \Psi\gamma_H \text{ and}$$

$$\Psi = \left[\frac{1}{\alpha}\frac{n_X}{(n_X+n_H)} - \left(1 - \frac{(1-\gamma_X)\gamma_H(n_X+n_H)}{n_H}\right)\left(\frac{(1+\varphi)}{\Theta}\frac{n_X}{n_X+n_H} - \frac{(1-\alpha)(1-\sigma)}{\Theta}\gamma_X\right)\right].$$

In particular, if $\alpha_H = 0$, then

$$\Psi = \frac{1}{\alpha}\frac{n_X}{(n_X+n_H)} \quad \Gamma_1 = \frac{1}{\alpha}\frac{n_X}{n_X+n_H} + \eta - 1$$

$$\Gamma = \frac{1}{\alpha}\frac{n_X}{n_X+n_H} + (\eta-1)(1-\gamma_H)$$

$$\alpha_X + \alpha_H\frac{\Psi}{\Gamma} = \alpha_X,$$

while if $\gamma_H = 0$, then

$$\Psi = \frac{(\varphi+\sigma)(1-\alpha)}{\alpha\Theta}\frac{n_X}{n_X+n_H} + \frac{(1-\alpha)(1-\sigma)}{\Theta}\gamma_X,$$

$$\Gamma_1 = \frac{1}{\alpha}\frac{n_X}{n_X+n_H} + \frac{(1+\varphi)}{\Theta}\frac{n_H}{n_X+n_H} + \eta - 1 \quad \Gamma = \Gamma_1, \text{ and}$$

$$\alpha_X + \alpha_H\frac{\Psi}{\Gamma} = \alpha_X + \frac{n_H}{n_X+n_H}\left[\frac{\frac{(\varphi+\sigma)(1-\alpha)}{\alpha\Theta}\frac{n_X}{n_X+n_H} + \frac{(1-\alpha)(1-\sigma)}{\Theta}\gamma_X}{\frac{1}{\alpha}\frac{n_X}{n_X+n_H} + \frac{(1+\varphi)}{\Theta}\frac{n_H}{n_X+n_H} + \eta - 1}\right].$$

It follows

$$\begin{aligned}\alpha\hat{\Omega} &= \alpha_X(\hat{p}_X - \hat{p}_F) + \alpha_H(\hat{p}_H - \hat{p}_F) + \frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H} \\ &= \left(\alpha_X + \alpha_H\frac{\Psi}{\Gamma}\right)(\hat{p}_X - \hat{p}_F) + \left[1 - \frac{\alpha_H}{\Gamma}\frac{(1+\varphi)}{\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right]\left(\frac{n_X\hat{a}_X + n_H\hat{a}_H}{n_X+n_H}\right) - \frac{\alpha_H}{\Gamma}\frac{1}{\alpha}\hat{a}_H\end{aligned}$$

and

$$\hat{w}-\hat{p}_X = \hat{w}-\hat{p}-(1-\gamma_X)[\hat{p}_X-\hat{p}_F-\gamma_H(\hat{p}_H-\hat{p}_F)]$$

$$= \left[\frac{\phi+\sigma}{\Theta}\alpha_X+\left[\frac{\phi+\sigma}{\Theta}\alpha_H+(1-\gamma_X)\gamma_H\right]\frac{\Psi}{\Gamma}-(1-\gamma_X)\right](\hat{p}_X-\hat{p}_F)+$$

$$+\left[\frac{\phi+\sigma}{\Theta}-\left(\frac{\phi+\sigma}{\Theta}\alpha_H+(1-\gamma_X)\gamma_H\right)\frac{(1+\varphi)}{\Gamma\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right]\left(\frac{n_X\hat{a}_X+n_H\hat{a}_H}{n_X+n_H}\right)$$

$$-\left(\frac{\phi+\sigma}{\Theta}\alpha_H+(1-\gamma_X)\gamma_H\right)\frac{1}{\Gamma\alpha}\hat{a}_H$$

$$= \left[\frac{\phi+\sigma}{\Theta}\alpha_X-(1-\gamma_X)+\left[\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H}+\left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right]\frac{\Psi}{\Gamma}\right](\hat{p}_X-\hat{p}_F)$$

$$+\left[\frac{\phi+\sigma}{\Theta}-\left(\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H}+\left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right)\frac{(1+\varphi)}{\Gamma\Theta}\frac{\alpha_H(n_X+n_H)}{n_H}\right]\left(\frac{n_X\hat{a}_X+n_H\hat{a}_H}{n_X+n_H}\right)$$

$$-\left(\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H}+\left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right)\frac{1}{\Gamma\alpha}\hat{a}_H.$$

Recalling $\alpha_X = \dfrac{n_X}{n_X+n_H}-\gamma_X$, and $\alpha_H = \dfrac{n_H}{n_X+n_H}-(1-\gamma_X)\gamma_H$, one also has

$$\Psi = \left[\frac{(\varphi+\sigma)(1-\alpha)}{\alpha\Theta}+\frac{(1+\varphi)}{\Theta}\left(\frac{(1-\gamma_X)\gamma_H(n_X+n_H)}{n_H}\right)\right]\frac{n_X}{(n_X+n_H)}$$

$$+\left(1-\frac{(1-\gamma_X)\gamma_H(n_X+n_H)}{n_H}\right)\left(\frac{(1-\alpha)(1-\sigma)}{\Theta}\gamma_X\right)$$

$$= \frac{(1-\gamma_X)\gamma_H(n_X+n_H)}{n_H}\left[\gamma_X+\frac{(1+\varphi)}{\Theta}\alpha_X\right]+\frac{(1-\alpha)}{\Theta}\left[(1-\sigma)\gamma_X+\frac{(\varphi+\sigma)}{\alpha}\frac{n_X}{(n_X+n_H)}\right]$$

$$\Gamma_1 = \left(\frac{1}{\alpha}-\frac{(1+\varphi)}{\Theta}\right)\frac{n_X}{n_X+n_H}+\frac{(1+\varphi)}{\Theta}\left(\frac{n_X}{n_X+n_H}+\frac{n_H}{n_X+n_H}-(1-\gamma_X)\gamma_H\right)+\eta-1$$

$$= \frac{(1-\alpha)(\varphi+\sigma)}{\alpha\Theta}\frac{n_X}{n_X+n_H}+\frac{(1+\varphi)}{\Theta}(1-(1-\gamma_X)\gamma_H)+\eta-1.$$

Thus, Ψ, Γ_1, and Γ are positive. Furthermore, numerical approximations show that

$$\frac{\Psi}{\Gamma}$$

increases, and

$$\alpha_X+\alpha_H\frac{\Psi}{\Gamma}$$

decreases with γ_H. It follows that the elasticity,

$$\left[\frac{\phi+\sigma}{\Theta}\alpha_X-(1-\gamma_X)+\left[\frac{\phi+\sigma}{\Theta}\frac{n_H}{n_X+n_H}+\left(\frac{\alpha(1-\sigma)}{\Theta}\right)(1-\gamma_X)\gamma_H\right]\frac{\Psi}{\Gamma}\right]$$

of

$$\frac{W}{P_X}$$

with respect to $\frac{P_X}{P_F}$, which is negative, is larger in absolute value as γ_H decreases.

Note too that Ψ is independent of, while Γ_1 and Γ increase with, the elasticity of substitution η. Thus

$$\frac{\Psi}{\Gamma}\text{ and }\alpha_X+\alpha_H\frac{\Psi}{\Gamma}$$

decrease with η.

One-sector economy with fixed costs in production

Flexible wages

Claim: The larger the number, *n*, of firms in operation, the higher the demand for labour and the higher the real wage.

Recall the expression for the markup,

$$D\left[\frac{P_X}{W}\right]^{-\frac{\alpha+\varphi}{\alpha\sigma}} = n^{\frac{\varphi}{\sigma}}\left(\frac{n}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-totc\right),$$

$$D = \left(\frac{\lambda}{\lambda-1}\right)^{-\frac{1}{\sigma}}A_X^{-\frac{\varphi}{\alpha\sigma}}\left(\frac{P_X}{P}\right)^{\frac{(1-\sigma)}{\sigma}}.$$

Differentiating both sides with respect to *n*, and letting *d* denote the partial derivative of

$$\frac{P_X}{W},$$

we obtain

$$D\left(-\frac{\alpha+\varphi}{\alpha\sigma}\right)\left[\frac{P_X}{W}\right]^{-\frac{\alpha+\varphi}{\alpha\sigma}-1} d \;=\; \frac{\varphi}{\sigma}n^{-1}D\left[\frac{P_X}{W}\right]^{-\frac{\alpha+\varphi}{\alpha\sigma}}$$

$$+\,n^{\frac{\varphi}{\sigma}}\left[\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}+\frac{1-\alpha}{\alpha}\frac{n}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}-1}d-\frac{\partial}{\partial n}totc\right],$$

hence

$$d\left[\frac{P_X}{W}\right]^{-1}\left[D\left(-\frac{\alpha+\varphi}{\alpha\sigma}\right)\left[\frac{P_X}{W}\right]^{-\frac{\alpha+\varphi}{\alpha\sigma}}-n^{\frac{\varphi}{\sigma}}\frac{n}{\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}\right] \;=\; \frac{\varphi}{\sigma}n^{-1}D\left[\frac{P_X}{W}\right]^{-\frac{\alpha+\varphi}{\alpha\sigma}}$$

$$+\,n^{\frac{\varphi}{\sigma}}\left[\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-\frac{\partial}{\partial n}totc\right],$$

hence,

$$d\frac{1}{\alpha}\left[\frac{P_X}{W}\right]^{-1}\left[(-n)A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}\left(\frac{\alpha+\varphi+(1-\alpha)\sigma}{(1-\alpha)\sigma}\right)+\frac{\alpha+\varphi}{\sigma}totc\right]$$

$$=\;\frac{\varphi}{\sigma}n^{-1}\left(\frac{n}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-totc\right)+\left[\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-\frac{\partial}{\partial n}totc\right]$$

hence,

$$-\,d\frac{1}{\alpha}\left[\frac{P_X}{W}\right]^{-1}\left[n\left(\frac{\bar{c}}{\alpha}+k\right)(1-\alpha)+\frac{\alpha+\varphi}{\sigma}\left(n\frac{\bar{c}}{\alpha}-totc+nk\right)\right]$$

$$=\;\frac{\varphi}{\sigma}n^{-1}\left[n\frac{\bar{c}}{\alpha}-totc+nk\right]+\left[\frac{\bar{c}}{\alpha}+k-\frac{\partial}{\partial n}totc\right],$$

where $\bar{c}$ is the largest fixed cost of a firm that is in operation and

$$k\equiv\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left(\frac{P_X}{W}\right)^{\frac{1-\alpha}{\alpha}}-\frac{\bar{c}}{\alpha}.$$

Note, $totc \le n\bar{c} \le \frac{n\bar{c}}{\alpha}$, $k \ge 0$ (otherwise a firm with fixed cost $\bar{c}$ would not produce), and

$$\frac{\partial}{\partial n} totc \ge \bar{c},$$

since all firms with fixed cost less than $\bar{c}$ must already be in operation. Thus, all the terms in brackets in the equation above are positive, and *d* is negative. In other words, a marginally lower number of firms in operation leads to a higher markup, which in turn implies higher, hence non-negative, profits. It follows that, as claimed, the lower the number of firms in operation, the higher the markup and the lower the real wage.

Claim: Suppose that firms' fixed costs are distributed uniformly on the interval [0,1]. Then the markup grows faster with changes in relative prices once the economy is at full capacity, i.e., once all the firms are in operation.

Indeed, if *n* is the number of firms in operation, then the highest fixed cost of a firm that does produce output is *n* and

$$totc = \frac{n^2}{2}.$$

Recall that

$$n^{\varphi}\left(\frac{n\bar{c}}{\alpha} - totc\right)^{\sigma} = \left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{1-\alpha}{\alpha}\bar{c}\right)^{\frac{-(\alpha+\varphi)}{1-\alpha}} A_X^{\frac{(1+\varphi)}{1-\alpha}} \left(\frac{P_X}{P}\right)^{(1-\sigma)}.$$

It follows

$$n^{\varphi}\left(\frac{n^2}{\alpha} - \frac{n^2}{2}\right)^{\sigma} = \left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{1-\alpha}{\alpha}n\right)^{\frac{-(\alpha+\varphi)}{1-\alpha}} A_X^{\frac{(1+\varphi)}{1-\alpha}} \left(\frac{P_X}{P}\right)^{(1-\sigma)},$$

hence

$$n^{\frac{(\alpha+\varphi)+(2\sigma+\varphi)(1-\alpha)}{1-\alpha}}\left(\frac{1}{\alpha} - \frac{1}{2}\right)^{\sigma} = \left(\frac{\lambda - 1}{\lambda}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{-(\alpha+\varphi)}{1-\alpha}} A_X^{\frac{(1+\varphi)}{1-\alpha}} \left(\frac{P_X}{P}\right)^{(1-\sigma)},$$

as long as this number is smaller than the total number of firms in the sector. It follows that a 1 per cent increase in the relative price

$$\frac{P_X}{P}$$

raises the number of firms by

$$\frac{(1-\sigma)(1-\alpha)}{(\alpha+\varphi)+(2\sigma+\varphi)(1-\alpha)}$$

and the markup by

$$\frac{\alpha(1-\sigma)}{(\alpha+\varphi)+(2\sigma+\varphi)(1-\alpha)},$$

as long as some capacity is idle. Whereas, once the economy reaches capacity, the markup grows by

$$\frac{\alpha(1-\sigma)}{(\alpha+\varphi)+\dfrac{\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}}{\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-\frac{1}{2}}\sigma(1-\alpha)},$$

which ranges over the interval

$$\left[\frac{\alpha(1-\sigma)(2-\alpha)}{(2-\alpha)(\alpha+\varphi)+2\sigma(1-\alpha)},\ \frac{\alpha(1-\sigma)}{(\alpha+\varphi)+\sigma(1-\alpha)}\right],$$

since

$$\frac{\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}}{\frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}}-\frac{1}{2}}$$

ranges over

$$\left[1,\ \frac{2}{2-\alpha}\right] \text{ as } \frac{1}{1-\alpha}A_X^{\frac{1}{\alpha}}\left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}} \geq \frac{1}{\alpha}.$$

In particular, the markup grows more rapidly once the economy reaches capacity.

Predetermined wages

Claim: Assuming firms' fixed costs are distributed uniformly on the interval [0,1], a negative shock to the terms of trade calls for a smaller expansion in the money supply when the economy is at full capacity than when it is below capacity.

From the market equilibrium,

$$C = \frac{n}{1-\alpha}\left(A_X \frac{P_X}{P}\right)^{\frac{1}{\alpha}} \left[\frac{P}{W}\right]^{\frac{1-\alpha}{\alpha}} - \frac{P_X}{P} totc .$$

Substituting this expression into $M = \chi P C^{\sigma}$, it follows that

$$M = \chi \left[\frac{P_X}{P}\right]^{\sigma - 1} \left[\frac{n}{1-\alpha} A_X^{\frac{1}{\alpha}} \left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}} - totc\right]^{\sigma} \frac{P_X}{W} W .$$

When the economy is at full capacity, the markup under flexible wages satisfies

$$\left(\frac{\bar{n}}{1-\alpha} A_X^{\frac{1}{\alpha}} \left[\frac{P_X}{W}\right]^{\frac{1-\alpha}{\alpha}} - totc\right) = \left(\frac{\lambda}{\lambda - 1}\right)^{-\frac{1}{\sigma}} A_X^{-\frac{\varphi}{\alpha\sigma}} \left(\frac{P_X}{P}\right)^{\frac{(1-\sigma)}{\sigma}} \left[\frac{P_X}{W}\right]^{-\frac{\alpha + \varphi}{\alpha\sigma}} \bar{n}^{-\frac{\varphi}{\sigma}},$$

hence

$$M = \chi \left(\frac{\lambda}{\lambda - 1}\right)^{-1} A_X^{-\frac{\varphi}{\alpha}} \left[\frac{P_X}{W}\right]^{-\frac{\varphi}{\alpha}} \bar{n}^{-\varphi} W ,$$

and therefore the money supply must expand by $\frac{\varphi}{\alpha}$ the percentage drop of the markup in response to a change in relative prices. It follows from the previous results that the money supply should expand by at most

$$\frac{\varphi(1-\sigma)}{(\alpha + \varphi) + \sigma(1-\alpha)} .$$

Whereas, when the economy is below capacity,

$$\frac{P_X}{W} = \left(\frac{1-\alpha}{\alpha}\bar{c}\right)^{\frac{\alpha}{1-\alpha}} A_X^{-\frac{1}{1-\alpha}} ,$$

hence

$$M = \chi\left[\frac{P_X}{P}\right]^{\sigma-1}\left[\frac{nc}{\alpha} - totc\right]^{\sigma}\frac{P_X}{W}W = \chi\left[\frac{P_X}{P}\right]^{\sigma-1}\left[\frac{(2-\alpha)n^2}{2\alpha}\right]^{\sigma}\frac{P_X}{W}W,$$

and therefore the money supply must expand by $\frac{\varphi}{\alpha}(2-\alpha)$ the percentage drop of the markup in response to a change in relative prices, hence by

$$\frac{\varphi(2-\alpha)(1-\sigma)}{(\alpha+\varphi)+(2\sigma+\varphi)(1-\alpha)},$$

which is greater than the upper bound in the case where the economy is at full capacity.

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