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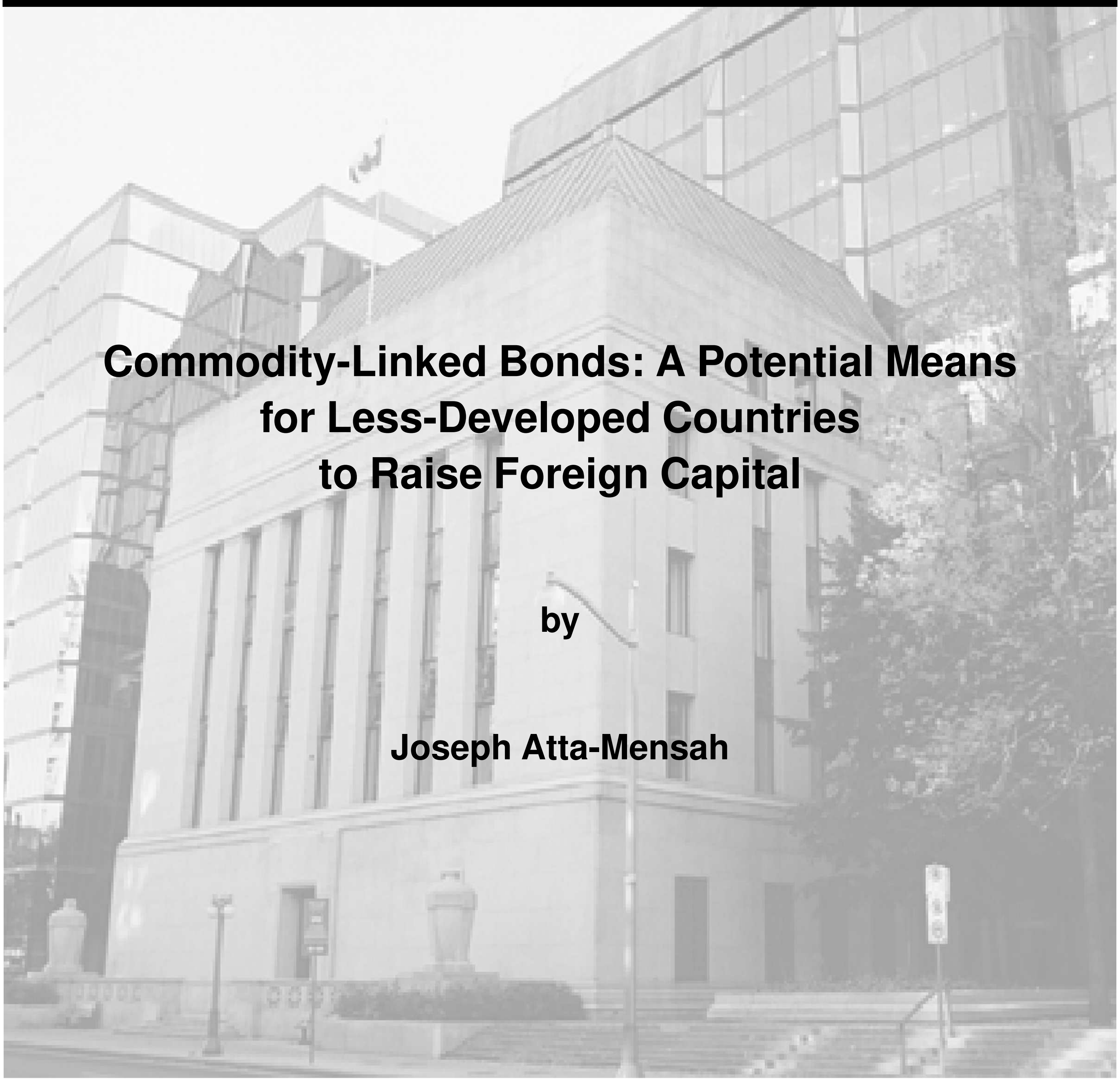

# Commodity-Linked Bonds: A Potential Means for Less-Developed Countries to Raise Foreign Capital

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## Abstract

The author suggests that commodity-linked bonds could provide a potential means for less-developed countries (LDCs) to raise money on the international capital markets, rather than through standard forms of financing. The issue of this type of bond could provide an opportunity for commodity-producing LDCs to hedge against fluctuations in their export earnings. The author's results show that the value of a commodity-linked bond increases as the price of the commodity indexed to the bond rises; this suggests that, if LDCs had issued debt contracts that were tied to their main export commodities, then their debt load would decline along with plummeting export prices (or export revenues). A simple portfolio rule derived by the author suggests that LDCs should issue more commodity-linked bonds than conventional debt if the variance of the portfolio is greater than twice the spread between the expected total return of the conventional debt and the commodity-linked bond. This rule supports the view that, if more of the LDCs' debt were issued in the form of commodity-linked bonds, then the debt-service payment of the LDCs would decline along with export prices (or export revenues), thus lightening their debt load.



*JEL classification: F30, F34, F49, G13, G11, O16*
*Bank classification: Development economics; Financial markets; International topics*



## Résumé

L'auteur voit dans les obligations indexées sur les prix des produits de base un levier susceptible d'aider les pays en développement à se procurer des capitaux sur les marchés financiers internationaux, de préférence aux méthodes classiques de financement. L'émission de titres de ce genre pourrait offrir à ceux de ces pays qui sont riches en matières premières un moyen de se prémunir contre les fluctuations de leurs recettes d'exportation. Les résultats de l'étude montrent que la valeur de ces obligations augmente avec le cours du produit de base sur lequel elles sont indexées. Cela donne à penser que, si les pays en développement émettaient des contrats d'emprunt référencés sur leurs principaux produits d'exportation, le fardeau de leur dette s'allégerait quand les cours de ces produits (ou leurs recettes d'exportation) diminuent. Selon la règle simple que propose l'auteur, les pays en développement devraient recourir davantage à l'émission d'obligations indexées sur les prix des matières premières qu'à celle d'obligations ordinaires si la variance de leur dette est deux fois plus élevée que l'écart entre les rendements totaux espérés des deux types d'obligations. Cette règle tend à confirmer les bienfaits qu'un recours accru aux émissions d'obligations indexées aurait sur le fardeau de la dette des pays en développement, du fait que l'évolution du service de la dette suivrait alors celle des prix des produits exportés (et des recettes correspondantes).



*Classification JEL: F30, F34, F49, G13, G11, O16*
*Classification de la Banque: Économie du développement; Marchés financiers; Questions internationales*

# 1. Introduction

Less-developed countries (LDCs) have for years been faced with colossal foreign debt. This debt, which is denominated in U.S. dollars at floating interest rates, became impaired in the 1970s and 1980s when interest rates were very high. Moreover, unfavourable terms of trade, due to volatile prices of export commodities and falling export revenue, have hampered the ability of LDCs to retire and/or service their debts. Consequently, the debt "overhang" has limited their access to new foreign capital, forcing them to adjust their domestic investment and consumption. Unfortunately, the LDCs are still mired in a debt crisis, which is seriously stifling their economic growth.

The purpose of this paper is to examine whether commodity-linked bonds could provide a potential means for LDCs to raise money on the international capital markets, rather than through standard forms of financing. Commodity-linked bonds differ from conventional bonds in terms of their payoffs to the holder. The bearer of the conventional bond receives fixed coupon (interest) payments during the life of the bond, and face value (principal) at maturity. The principal of a commodity-linked bond, however, is paid in either the physical units of a reference commodity or its equivalent monetary value. Similarly, the coupon payments may or may not be in units of the commodity to which the bond is indexed. Therefore, the structural difference between the two bonds is that the nominal return of the conventional bond held to maturity is known with certainty, although the real return is unknown due to inflation uncertainty, whereas both the nominal and real returns of the commodity-linked bond are not known.

In both the conventional and the commodity-linked bonds, the payments referred to are promised (or contractual). If the issuer is unable or unwilling to make the contractual payments, default occurs, and the bearer receives a smaller or zero payment. In the event of default, substantial bankruptcy, legal, and renegotiating costs may be incurred, and new uncertainties may be introduced (especially in international borrowing). These are dead-weight losses (as opposed to simple wealth transfer) to the parties involved in the contract. Derivative securities may serve to minimize these dead-weight losses, in that the state-contingent payments may be tailored to the risk preferences of either borrower or lender. This tailoring would avoid the transaction costs of using other markets for the same purpose, and would also minimize the probability of default.

There are two types of commodity-indexed bonds: forward and option. With the forward type, the coupon and/or principal payment to the bearer of the bond are linearly related to the price of a stated amount of the reference commodity.[1] With the option type, the coupon payments are

1. Technically, the forward type is known as the commodity-indexed bond, and the option type is known as the commodity-linked bond. Unless otherwise stated, however, the terms commodity-indexed bond and commodity-linked bond are used interchangeably.

similar to that of a conventional bond, but at maturity the bearer receives the face value plus an option to buy or sell a predetermined quantity of the commodity at a specified price. Alternatively, to minimize the default risk, the borrower may be given the option to pay the minimum of the face value and the value of the reference amount of the commodity at the maturity date.

In this paper, two approaches are taken to examine the potential benefits of LDCs issuing commodity-linked bonds. First, the theory of option pricing is applied to determine the market price of a commodity-linked bond. An assessment is then made as to whether the value of the commodity-linked bond decreases with the decrease in the underlying commodity price. Second, the model of Myers and Thompson (1989) is extended to determine the optimal proportion of an LDC's total external debt that must be issued by the country in the form of commodity-linked bonds. The relationship between the commodity price and the demand for the bond is also determined.

The results reported in this paper show that the value of the commodity-linked bond increases as the price of the commodity indexed to the bond rises, which suggests that if LDCs had issued debt contracts that were tied to their main export commodities, then their debt load would have declined along with plummeting export prices (or export revenues).

It is also demonstrated in this paper that the coupon rate for a conventional debt with a face value identical to that of a commodity-linked bond is generally less than the coupon rate for a commodity-linked bond that pays holders, on maturity, the minimum of the face value and the monetary value of a pre-specified unit of a commodity. This implies that LDCs or corporations in need of investment funds could share the appreciation of the market value of the underlying commodity with the bondholders, in return for a lower coupon rate.

The results reported in this paper also show that the coupon rate for the conventional bond is greater than its counterpart for a commodity-linked bond whose terminal payoff is the greater of the face value and the monetary value of a pre-specified unit of a commodity. Through the issue of such a bond, an LDC could share the depreciation of the market value of its commodity with bondholders in exchange for higher coupon rates. This result corroborates Caballero (2003), who argues that bonds of this nature act as a hedge for LDCs in times when the commodity prices collapse.

A simple portfolio rule a country could follow in its allocation of debt instruments and the level of imports is also derived. The rule suggests that LDCs should issue more commodity-linked bonds than conventional debt. It supports the view that, if more of LDCs' debts were issued in the form of commodity-linked bonds, the debt-service payment of the LDCs would decline along with export prices (or export revenues), thus lightening the debt load of the LDCs.

This paper is organized as follows. Section 2 provides a brief background on past experiences with the issue of commodity-linked bonds. Section 3 discusses avenues available to LDCs to protect their export commodity prices. Section 4 constructs a model of external debt allocation by an LDC. Section 5 offers some conclusions.

## 2. Experiences with Commodity-Linked Bonds

In this section, previous experiences with commodity-linked bonds are summarized.[2]

### 2.1 Gold-linked bonds

The most popular form of commodity-indexed bond is referenced to specified units of gold. A well-known example of gold bonds was issued in 1973 by the French government and accepted in the financial markets as the "Giscard." The "Giscard" carried a 7 per cent nominal coupon rate and a redemption value indexed to the price of a 1 kilogram bar of gold. The bearers of the "Giscard" were protected by a safeguard clause, which stated that interest and the face-value payments would be indexed to a 1 kilogram bar of gold should the French franc lose its parity with gold and other currencies. In 1977, to the disappointment of the French government, the French franc was forced by other European currencies to float. Furthermore, in 1978, the International Monetary Fund (IMF) abolished the linkage of currencies to gold. As a consequence of these two economic events, the safeguard clause became operative and therefore, in 1980, the government of France paid 393 francs in interest payments for every single bond issued, instead of the 70 francs originally planned for. Moreover, each of the issued bonds, which was traded at par in 1977, matured in January 1988 with a redemption value of 8,910 francs. Thus, the bonds increased in value by about 700 per cent over 10 years.

After the "Giscard," other types of gold-linked securities were issued. Unlike the "Giscard," which had only its redemption value indexed to a specified amount of gold, they had their principal and/or interest payments indexed to gold. One type was issued in 1981 by the Refinement International Company: the gold bonds were 3.29 per cent gold-linked, with an aggregate principal of 100,000 ounces of gold. The maturity date for the bonds was February 1996. Interest payments were made annually. Bearers of these bonds had the option to receive both interest and principal in either the monetary value of the specified amount of gold indexed to

2. This section has been influenced by Fall (1986) and Privolos and Duncan (1991).

the bond, or the physical quantity of gold referenced. Claims for the units of gold could be made in London or Zurich.

The gold warrants issued by Echo Bay Mines Ltd. of Canada in 1981 were another type of gold-indexed securities: they issued 1,550,000 preferred voting shares. Holders of these shares were entitled to an annual dividend of US$3 and four gold warrants per share. Each warrant, when exercised, guaranteed the holder 0.0706 ounces of gold from Echo Bay Mines at a price of US$595 per ounce. The four warrants had to be exercised on different dates: 31 January 1986, 31 January 1987, 31 January 1988, and 31 January 1989, respectively. Holders of the warrants were allowed to trade them to a third party before 30 December 1983. The exercise of the warrants was dependent on the completion of the Lupin Gold project.

## 2.2 Silver-linked bonds

In 1980, the Sunshine Mining Company, a large silver mine in the United States, issued US$25 million worth of silver-indexed bonds to hedge against the fluctuations in the price of silver. Each US$1,000 bond was indexed to 50 ounces of silver, payed a coupon rate of 8.5 per cent, and had a maturity of 15 years. At each bond's maturity, its bearer received the maximum of the face value of US$1,000 or the market value of 50 ounces of silver. The bonds were redeemable on or after 15 April 1995 only if the average silver price for 30 consecutive days was above US$40 per ounce.

Silver-indexed bonds were also issued by the Sunshine Mining company in April 1985. Each US$1,000 bond was referenced to 58 ounces of silver and the coupon rate was increased to 9.75 per cent. On the maturity date of April 2004, the holders of the bonds had the option of choosing the face value of US$1,000 or the market value of 58 ounces of silver.

Unlike the gold bonds, there are not many silver-linked securities, for the economic reason that the market price of silver has not fluctuated very much. Hence, silver producers do not have an incentive to issue silver bonds for the sole purpose of hedging against changes in silver prices.

## 2.3 Oil-linked bonds

Oil-backed bonds appeared in the financial market during the late 1970s. The government of Mexico is believed to have been the first to issue such bonds. These bonds, known in the financial markets as Petrobonds, were issued on behalf of the government by the National Financiere S.A. (NAFINSA), a development bank owned by the Mexican government. Each 1,000 peso bond was linked to 1.95354 barrels of oil.

The coupon rate was 12.65823 per cent per annum and matured at the end of three years. On the maturity date, the Petrobonds were redeemed at a value equal to the maximum of the face value or the market value of the referenced units of oil plus all coupons received during the life of the bond. With this issue, the government was not only raising new money at low nominal cost, but was also hedging part of its oil production against fluctuations in oil prices. On the other hand, bearers of the Petrobonds were hoping to benefit from an upswing in the price of crude oil.

In 1981, Petro-Lewis Corporation of Denver issued US$20 million worth of oil-indexed notes. Each note had a lifetime of five years and paid an annual coupon rate of 9 per cent. As Fall (1986) explains, each note was expected to pay the face value (principal), the accrued interest, and a contingent interest on the maturity date. The contingent interest, which had a feature of a cap, was defined as the increase over US$668.96 of (i) the average crude oil price of 18.5 barrels of crude oil for the three months ending 28 February 1986 or, (ii) if greater, the highest average price of 18.5 per cent barrels of crude oil, up to a maximum of US$1,258 or US$68 per barrel for any calendar quarter through the quarter ending 31 December 1985. This feature enabled an investor to make at most an additional US$589 per bond. The oil notes of Petro-Lewis differed from the Petrobonds in that the repayment of the face value included a call option on oil prices, and therefore offered protection to the bearers from a fall in oil prices. In the case of Petrobonds, the payment of the principal was fully indexed to specified units of oil.

## 2.4 Other forms of commodity-indexed securities

Other bonds have been indexed to other types of precious metals. As Privolos and Duncan (1991) report, Inco, which is one of the world's largest producers of nickel, copper, silver, cobalt, and platinum, in 1984 raised Can$90 million on the financial market through the issuance of bonds linked to the price of nickel or copper. The bonds, which matured in 1991, paid a coupon rate of 10 per cent per annum. Holders of the bonds had the option of receiving at the maturity date the face value or the monetary value of a specified amount of nickel or copper. This issue enabled Inco to get out of its financial difficulties in 1984.

Cominco Ltd. of Canada also raised US$54 million in 1981 by issuing preferred-share and commodity-indexed warrants (CIS). Holders of the CIS had the right to exchange each warrant on or before August 1992 for a number of common shares of the corporation, based on the average market price of zinc or copper and the market value of common stocks on the exercise date.

The largest producer of copper in the United States, Magna, issued copper-indexed notes in 1988. The notes matured in 1998 and linked the interest payments to the price of copper. The interest rates ranged between 21 per cent per annum at average copper prices of US$2 per pound and above, and 12 per cent per annum at average copper prices of 80 cents (US) per pound and below. The indexation of the interest payments to copper prices enabled Magna to reorganize its liabilities and therefore to control the risk to, and the net worth of, the company.

Commodity-indexed bonds have also been used in the LDCs to finance development projects. The government of Malaysia accepted a loan from Citibank that was indexed to prices of palm oil. Similarly, Metallgesellschaft used copper-indexed financing to invest in the copper belt of Papua New Guinea.

## 3. Ways to Protect Export Commodities from Price Volatility

For years, LDCs have been faced with colossal foreign debt. The retirement and/or servicing of this debt has been a major problem for LDCs and their creditors due to the volatility of the prices of export commodities and hence their export revenues. The crisis created by these debt “overhangs” has drawn academics and practitioners to research ways and means for creditors to receive, if not the principal, at least the interest payments on the debt. The crisis has also made it difficult for LDCs to obtain new loans.

The difficulty that LDCs face in meeting their debt obligations would be reduced if they could embark on measures that would protect their export commodities from price volatilities. One measure suggested in the literature is that LDCs adopt hedging strategies. Whereas some researchers suggest the use of futures markets by these countries, other researchers call for LDCs to shift the risk that their commodity prices face to the financial markets. Fall (1986) describes three methods LDCs use to hedge against the risk their export commodity prices face: international commodity agreements (ICAs), the futures markets, and countertrade.

### 3.1 International commodity agreements

ICAs, which cover commodities such as cocoa, coffee, natural rubber, olive oil, sugar, and tin, have been in effect for a number of years. Through these agreements, the LDCs and consumer countries sign a pact that seeks to stabilize world prices of commodities. This stabilization scheme is carried out to attract importers and satisfy the interest of producing countries. Fall (1986) states that producing countries prefer price-supporting systems that are achieved through export quotas or buffer stocks. ICAs allow prices of commodities to fluctuate freely within an agreed range. Whenever prices fall through the floor, export quotas are applied or the buffer-stock manager enters the market and purchases sufficient amounts of the commodity. Either action raises the price of the commodity to fall within the predetermined range. On the other hand, should prices go through the ceiling, the export quotas are relaxed or the buffer-stock manager sells the commodity in the spot market to drive the price down to within the range.

ICAs have been fraught with three problems. First, there is asymmetry in the incentives of the importers and the producing LDCs in entering into the agreement. Whereas the consumers (importers) are mainly concerned that higher prices will reduce their purchasing power of imports, the producers are concerned with low prices. Second, the buffer-stock manager is faced with limited funds to purchase the commodity whenever the price falls through the floor. Third, it is extremely difficult to get all the signatories to ICAs to abide by the quotas whenever the price falls through the floor.

## 3.2 Futures market

By entering into the futures market, LDCs can lock in the price at which the commodity will be sold in the future. However, futures contracts have their limitations. First, their term to maturity is about two years. Second, regulations at the futures exchanges restrict investors (and therefore LDCs) from taking huge positions in the markets, to prevent them from cornering the market or manipulating prices. These limitations suggest that LDCs may not be in a position to hedge all their exports through the futures market.

## 3.3 Countertrade

Countertrade, defined as a financing scheme in which settlements are made in the form of physical goods instead of money, can take three forms. The first is the barter system: LDCs can have bilateral or multilateral arrangements with developed economies in which they exchange their export commodities for other goods produced by the developed countries. The transactions can take place instantaneously or within a year. The weakness of the barter system is its difficulty in matching the interests of participating parties. This problem is known as the "double coincidence of wants."

The second form of countertrade strategy is "buyback arrangements." In this strategy, LDCs import production facilities and agree to deliver at some future date a specified amount of output. These arrangements most often involve the financing of processing plants in LDCs. Under this scheme, LDCs are able to lock in the present the future earnings of output. Although the scheme does not insulate producer countries from the risk of volatile commodity prices, it is project-specific.

The third form of countertrade strategy is known as a "counterpurchase agreement." In this strategy, an LDC imports certain commodities from a developed country and simultaneously commits itself to export to that country a specific amount of commodities at an agreed date. Under this arrangement, LDCs are protected against export-commodity price risk. Furthermore, the transactions made under this arrangement are similar to the importing LDC entering into a mixture of spot and forward contracts with the developed economies. Hence, LDCs enjoy similar advantages offered by forward contracts.

## 3.4 The Baker plan

Despite the availability of the above-noted hedging schemes, an enormous debt continues to "overhang" LDCs, which has prompted a call for debt reorganization. The United States, a major creditor of LDCs, has tried to use two different plans to help relieve and solve the debt crisis. The first plan, known as the "Baker plan," was proposed by Mr. James Baker, the U.S. Secretary of the Treasury, at the October 1985 annual meeting of the IMF and World Bank in Seoul, South Korea. The Baker plan consisted of three parts and aimed to solve the debt problem through a program of sustained growth for the economies of LDCs. In the first part of the plan, international financial institutions encouraged debtor countries to develop comprehensive macroeconomic and structural policies that would enhance their growth, adjust their balance of payments, and reduce their inflation rates. The second part of the plan called on the international financial institutions to continue lending to LDCs that embarked on structural adjustment policies. In the third part of the plan, the private banks increased their lending in support of comprehensive economic adjustment programs.

It was Secretary Baker's aim that, by implementing his plan, LDCs would be encouraged to use austere economic measures to curb inflation, and encouraged to produce trade surpluses needed to service foreign debt. The structural adjustment and new foreign lending would spur economic growth for the LDCs and consequently reduce their debt load.

The Baker plan, however, failed to achieve its purpose, because the private and the multilateral banks did not increase their lending, and the LDCs, for political reasons, were not able to implement the structural adjustment policies. As a result of this failure, the United States, in March 1985, implemented a scheme known as the "Brady plan."

The plan, which was announced by Mr. Nicholas Brady, the U.S. Secretary of the Treasury, called for the forgiveness of part of the LDC's debt. It also proposed that the IMF and the World Bank extend credit to debt-burdened nations so that they could collateralize debt-for-bond exchanges at discounts and cash buybacks of debt, and ameliorate the interest payments on new or modified debt contracts. Kenen (1990) notes that, in accordance with the Brady plan, the IMF and the World Bank extended new credit to Mexico, Costa Rica, and the Philippines.

## 3.5 Research on the policy of debt relief for LDCs

The Baker and the Brady plans led to academic research on the policy of debt relief for LDCs. Advocates of debt relief, such as Krugman (1989), suggest that reducing the debt of an LDC that has a debt overhang could increase that country's economic efficiency and consequently its real income, which would in turn lead to a reduction in its default risk. Kenen (1990) supports the position of Krugman (1989) and Sachs (1988) by arguing that a country with a large debt overhang suffers in two ways from a fall in economic efficiency. First, the high debt-service payments made by debt-laden countries require high tax rates that discourage capital formation and the repatriation of capital. Second, since governments of heavily indebted LDCs are responsible for making the debt-service payments and, therefore, those payments appear in their budgets, they may not institute a devaluation policy that could be required to improve their foreign reserve positions and consequently the debt crises.

Dornbusch (1988) notes that governments of LDCs use inefficient economic methods to produce the trade surpluses needed to service their foreign debt. One reason for the inefficient methods may be the fact that devaluation increases the domestic-currency cost of servicing foreign debt. The higher cost raises the budget deficit and the growth rate of the money supply, and consequently the inflation rate rises.

Other economists, such as Krugman (1988 and 1999), have used the debt Laffer curve to argue when debt forgiveness would be beneficial to LDCs. Krugman asserts that, if the LDC is on the correct (inclining) side of the debt Laffer curve, then debt forgiveness will lead to a reduction in the market value of outstanding debt, and therefore will be detrimental to creditors. The reverse holds true when the debtor country is on the wrong (declining) side of the Laffer curve. This calls for the debtor country's position on the Laffer curve to be determined before a decision on forgiveness is made.

Froot, Scharfstein, and Stein (1989) point out the moral-hazard effect of forgiveness. They argue that the amount of relief required to induce investment in the LDCs may depend on a variety of factors, some of which may be known only by the borrowing country. A borrowing country would know the level of austere economic measures it can impose on its citizens without causing serious disruptions. Hence, in negotiating for debt relief, the country might conceal part of the private information it has on its citizens in order to receive more relief. Froot, Scharfstein, and Stein believe that these problems could be resolved if the forgiven countries would index their future debt-service payment to commodity prices.

The failure of the Baker and Brady plans has led to recent calls for better management of LDCs' debt. Krueger (2003) catalogues how some developing countries are attempting to restructure their debts and the potential challenges they face. Caballero (2003) calls on the IMF to set up a Contingent-Markets Department and a Crisis Department. Caballero's proposal, which is close to that of Williamson (2000), calls for the Contingent-Markets Department to be responsible for identifying a country's sources of capital-inflow volatility that are potentially contractible. The Crises Department would be responsible for handling non-contractible shocks, such as unexpected events and blunders. Through the creation of these departments, Caballero indicates that the debt of the emerging countries would be better managed. Other experts have also called on the IMF to be focused, transparent, predictable, and quick to intervene in countries facing debt crisis (Meltzer 2000, Williamson 2000, and Fischer 2002).

Avenues available to LDCs for hedging against fluctuations in the prices of their export commodities are fraught with great difficulties for them. It is therefore important that the LDCs find alternative means to deal with their growing external debt crisis. This paper proposes that LDCs consider raising capital on the financial markets through the issue of commodity-linked bonds.

The use of commodity-indexed bonds, as O'Hara (1984) notes, dates as far back as 1863, when the Confederate States of America (CSA) issued bonds payable in bales of cotton. In recent years, several commodity-indexed bonds have been issued on the financial markets. There are a number of economic reasons why LDCs should be encouraged to issue commodity-linked bonds. First, by issuing commodity-indexed bonds, governments and corporations that need investment funds could share the appreciating market value of underlying commodities with bondholders in return for a lower coupon rate. Furthermore, as Budd (1983) argues, the issuing of commodity-linked bonds offers an opportunity for commodity-producing issuers and international commodity organizations to borrow at below-market interest rates. Through this process, LDCs could place themselves in an advantageous position by being linked to the international markets, such as the U.S. commodity markets and Eurobond markets.

Second, countries with a higher chance of defaulting on the final payment of a bond, because of serious balance-of-payment problems, could minimize the probability of default by asking for higher coupon payments during the life of the bond, in exchange for paying the minimum of the bond's face value and the monetary value of a pre-specified unit of the commodity indexed to the bond. The default probability is reduced because the contractual debt payments are reduced in precisely those circumstances when balance-of-payments problems occur. Also, under this arrangement the maximum the issuer would pay on the maturity date is the face value.

Third, the LDCs could, through the issue of bonds linked to their main exports, hedge against fluctuations in their export earnings. Myers and Thompson (1989) note that the debt crisis faced by the LDCs are due to a fall in export revenues and a simultaneous rise in world interest rates and debt-service payments. Myers and Thompson argue that, if the LDCs' debt had been issued in the form of commodity-linked bonds, then the debt-service payment of the LDCs would have declined along with export prices (or export revenues), thus lightening their debt load. Those who oppose LDCs issuing commodity-linked bonds suggest that LDCs should use the futures market to control for commodity price risk. Regulators of the futures markets, however, impose limits on the movements of the futures price in a single day. Thus, futures prices cannot move quickly to accommodate new information. Such limits are not in place for commodity options; therefore, commodity-linked bonds, which are a combination of straight bonds and commodity options, would react to new information to form the equilibrium price. Another advantage of commodity-linked bonds over futures contracts is that futures contracts have a maturity of less than one year and exist for a limited number of commodities. By issuing commodity-linked bonds, LDCs can have longer-term maturity and also index the bonds to any commodity of their choice.

Fourth, the issuance of commodity-linked bonds minimizes the default risk faced by financiers of LDC loans. A way still must be found, however, to reach the necessary collateral arrangements between LDCs and the developed nations that are major holders of the bond. One way is a legal contract between the LDCs and investing nations such that holders of a commodity-linked bond are empowered to seize any proceeds from the LDCs' exports in any of the signatory countries in the case of default. The drawback is that such a contract is not enforceable, and enormous transactions costs would have to be incurred to settle a dispute between an LDC and a bearer of the bond. Kletzer and Wright (2000), however, demonstrate that, in the presence of credible punishment threats, sovereign borrowers would always choose to renegotiate an existing loan contract rather than default.

Fifth, the use of commodity-linked bonds for external financing would minimize the enormous transactions costs that would be incurred if LDCs were to dynamically hedge their export revenues with futures contracts. In this paper, the model of Myers and Thompson (1989) is extended to determine the optimal proportion of total external debt that must be issued by an LDC in the form of commodity-linked bonds.

Sixth, in a world of inflation, and given the general uncertainties in the markets, the availability of the commodity, indexed to the bonds, greatly reduces the default risk of the bonds. Hence, issuers of the bonds must maintain a threshold level of inventory similar to what banks hold as reserve requirements. Moreover, issuers of the bonds who do not have the commodity must back the bonds with a long position in the forward or futures contracts, whose maturity is timed with the redemption date of the bonds.

# 4. A Model of Optimal External Debt Allocation

In this section, the framework of portfolio theory is applied to derive simple rules LDCs could follow in allocating debt instruments and their level of imports.[3] Besides the usual assumptions of no taxes, continuous trading, and zero transactions costs made in the financial literature, the following assumptions are made: the LDC has a small open economy; all prices of assets are denominated in U.S. dollars; all external debt is issued by the government; there are no short sales, because a country cannot sell short its own debt; two sources of foreign finances are available to the government (the issue of conventional bonds and the issue of commodity-linked bonds); there is only one perishable and divisible imported good; and the rates of change in the price of the export commodity and the Libor rate follow a stochastic Brownian motion.

## 4.1 Conventional debt

The process followed by the price of the export commodity is postulated as:

$$\frac{dP}{P} = \alpha_p dt + \sigma_p dz_p, \tag{1}$$

where $\alpha_p$ is the instantaneous average return of holding one unit of the export commodity, $\sigma_p$ is the instantaneous standard deviation of the rate of change of the commodity price, and $dz_p$ has a standard normal distribution with a mean of zero and a variance of $dt$. Note that $\alpha_p$ and $\sigma_p$ may be functions of $P$ and $t$. For the purpose of this exercise, however, they are assumed to be constants.

The Libor rate is assumed to follow a mean-reversion stochastic process of the form:

$$dr = \kappa(\theta - r)dt + \sigma_r dz_r . \tag{2}$$

The parameters are all constants. The Libor rate tends to be pulled towards the average target, $\theta$. $\sigma_r$ is the instantaneous standard deviation for the rate of change of the Libor rate, and $dz_r$ is

3. See Merton (1971, 1973) on the methodology followed herein.

normally distributed with a mean of zero and a variance of $dt$. Also, $dz_p$ and $dz_r$ have an instantaneous correlation of $\rho_{pr}dt$.

Let the price of conventional debt, $Q(r, t)$, be dependent on the Libor rate. By applying Ito's Lemma, the rate of change of the price, $Q(r, t)$, is given as:

$$\frac{dQ}{Q} = \alpha_q dt + \sigma_q dz_r, \tag{3}$$

where,

$$\alpha_q = \frac{\kappa(\theta - r)Q_r + 0.5\sigma_r^2 Q_{rr} - Q_t}{Q}, \tag{4}$$

and

$$\sigma_q = \frac{\sigma_r Q_r}{Q}. \tag{5}$$

Standard arbitrage arguments can be advanced to show that the partial differential equation that governs the pricing of the conventional debt with a coupon payment of $c$ is given as:

$$[\kappa(\theta - r) - \sigma_r \lambda(r)]Q_r + 0.5\sigma_r^2 Q_{rr} - Q_t - rQ + c = 0, \tag{6}$$

where $\lambda(r)$ is the market price of risk attached to all financial assets whose underlying state variable is the Libor rate. Also, $r$ is the instantaneous riskless rate of interest. For a given boundary condition, a closed-form solution for equation (6) cannot be determined. Assuming a face value of $Q_0$, no coupon payments, and a constant market price of interest rate risk (or $\lambda(r) = \lambda$), Vasicek (1977) shows that the price of the conventional debt satisfies:

$$Q(r, \tau) = Q_0 \exp\left[\frac{1}{\kappa}(1 - e^{-\kappa\tau})(A - r) - \tau A - \frac{\sigma_r^2}{4\kappa^3}(1 - e^{-\kappa\tau})^2\right], \tag{7}$$

where $\tau$ is the time left to maturity, and

$$A = \theta + \frac{\sigma_r \lambda}{\kappa} - \frac{1}{2}\frac{\sigma_r^2}{\kappa^2}. \tag{8}$$

Alternatively, if we assume a constant interest rate, then the market value of the conventional debt will be:

$$Q(P, \tau) = \frac{c}{r}(1 - e^{-r\tau}) + Q_0 e^{-r\tau} . \tag{9}$$

Based on either equation (7) or (9), the main driver for the conventional debt is found to be the level of the interest rate.

## 4.2 Commodity-linked bond

Consider a commodity-linked bond, the value of which is solely a function of the Libor rate and the price of the export commodity. Let *H(r, P, t)* be the price of a commodity-linked bond. Applying Ito's lemma, the rate of change of the commodity-linked bond is obtained as:

$$\frac{dH}{H} = \alpha_h dt + \psi_r dz_r + \psi_p dz_p , \tag{10}$$

where

$$\begin{aligned} \alpha_h = {} & \{[\kappa(\theta - r) - \sigma_r \lambda(r)]H_r + \alpha_p P H_p + 0.5\sigma_p^2 P^2 H_{pp} + 0.5\sigma_r^2 H_{rr} \\ & + \rho_{pr}\sigma_p\sigma_r P H_{pr} - H_t\}/H, \end{aligned} \tag{11}$$

and

$$\psi_p = \frac{\sigma_p P H_p}{H} , \tag{12}$$

$$\psi_r = \frac{\sigma_r H_r}{H} . \tag{13}$$

The application of standard arbitrage arguments yields the partial differential equation that governs the valuation of the commodity-linked bond, which is of the form[4]:

4. See Schwartz (1982), Atta-Mensah (1992), or Miura and Yamauchi (1998) for expanded valuation models of commodity-linked bonds.

$$[\kappa(\theta - r) - \sigma_r\lambda(r)]H_r + 0.5\sigma_p^2 P^2 H_{pp} + 0.5\sigma_r^2 H_{rr} + \rho_{pr}\sigma_p\sigma_r P H_{pr}$$
$$+ rPH_p - H_t - rH + c^B = 0. \tag{14}$$

In equation (14), $c^B$ is the coupon rate of the commodity-linked bond. Furthermore, equation (14) is restricted by the following conditions:

$$H(r, 0) = 0 \qquad \forall\ r, \tag{15}$$

$$H(r, \infty) = Q(r, t) \qquad \forall\ r, \tag{16}$$

$$H(\infty, P) = 0 \qquad \forall\ P. \tag{17}$$

### *4.2.1 The value of the commodity-linked bond*

The price of the commodity-linked bond is shown by the solution of equation (14) subject to a boundary condition. As stated earlier, a commodity-linked bond is indexed to an underlying commodity. Assume that the promised payment on the bond at maturity is set at the maximum of the face value of the bond ($F$) and the monetary value of a pre-specified unit of the referenced commodity. Let $\gamma$ be the pre-specified unit of the commodity referenced to the bond, and $H^c(\cdot)$ the value of this particular bond; then, notationally, the final payment of the bond is of the form:

$$H^c(P, r, 0) = Max[F, \gamma P], \tag{18}$$

or,

$$H^c(P, r, 0) = F + \gamma Max[0, P - F/\gamma]. \tag{19}$$

Equation (19) implies that the promised payment of the bond is equivalent to the face value of a bond ($F$) for sure, plus $\gamma$ amounts of a call option, which gives the bearer an option to buy the reference commodity bundle at a specified exercise price, $F/\gamma$.

On the other hand, to minimize default risk, the borrower could have an option to pay the minimum of the face value and the value of the reference amount of the commodity at the maturity date. In that case, the terminal value of the bond would be:

$$H^p(P, r, 0) = Min[F, \gamma P], \tag{20}$$

or,

$$H^p(P, r, 0) = F - \gamma Max[0, F/\gamma - P]. \tag{21}$$

Equation (21) indicates that a commodity-linked bond that pays the minimum of the face value, *F*, and the monetary value of a pre-specified unit of a commodity is similar to a bond of face value, *F*, and a short position on $\gamma$ amounts of a put option, which gives the bearer an option to sell the reference commodity bundle at a specified exercise price, $F/\gamma$.

A closed-form solution of equation (14), subject to the boundary conditions, equation (19) or equation (21), is not a trivial exercise. Hence, for expositional reasons, consider a case in which the interest rate is constant. For simplicity and without loss of generality, also assume that $\gamma$ is equal to unity. With these assumptions, the differential equation for pricing the commodity-linked bond and boundary conditions simplifies to:

$$0.5\sigma_p^2 P^2 H_{pp} + rPH_p - H_t - rH + c^B = 0 \ , \tag{22}$$

and

$$H^c(P, 0) = F + Max[0, P - F], \tag{23}$$

$$H^p(P, 0) = F - Max[0, F - P]. \tag{24}$$

The solution of equation (22) subject to (23) is given as:

$$H^c(P, \tau) \ = \ \frac{c^c}{r}(1 - e^{-r\tau}) + Fe^{-r\tau} + L(P, F, \tau), \tag{25}$$

where $c^c$ is the coupon payment, $L(P, F, \tau)$ the Black-Scholes (1973) formula for valuing a call option on $P$ with exercise price $F$, and $\tau$ the time left to maturity:

$$L(Q, F, \tau) \ = \ PN(d_1) - Fe^{-r\tau}N(d_2) \ , \tag{26}$$

where

$$d_1 \ = \ \frac{\log(P/F) + \left(r + \frac{1}{2}\sigma_p^2\right)\tau}{\sigma_p\sqrt{\tau}} \quad , \tag{27}$$

$$d_2 \ = \ d_1 - \sigma_p\sqrt{\tau}, \tag{28}$$

and $N(.)$ is the cumulative normal-distribution function.

On the other hand, the value of the bond could be the solution of equation (22) subject to equation (24):

$$H^{p}(P,\tau) = \frac{c^{p}}{r}(1-e^{-r\tau}) + Fe^{-r\tau} - Q(P,F,\tau), \tag{29}$$

where $c^p$ is the coupon payment, *Q(P, F, τ)* the Black-Scholes (1973) formula for valuing a put option on *P* with exercise price *F,* and $\tau$ the time left to maturity:

$$Q(P,F,\tau) = Fe^{-r\tau}N(a_1) - PN(a_2) , \tag{30}$$

where

$$a_1 = \frac{\log(F/P) + \left(-r + \frac{1}{2}\sigma_p^2\right)\tau}{\sigma_p\sqrt{\tau}} , \tag{31}$$

$$a_2 = x_1 - \sigma_p\sqrt{\tau} , \tag{32}$$

and *N*(.) is the cumulative normal-distribution function.

#### *4.2.2 Commodity price and the value of the commodity-linked bond*

Because the primary focus of this paper is to argue that LDCs could, through the issue of bonds linked to their main exports, hedge against the fluctuations in their export earnings, one would expect the value of debt issued in the form of commodity-linked bonds to fall with the falling prices of (or revenues from) exports.

*Proposition 1*: *The value of the commodity-linked bond increases monotonically as the price of the commodity indexed to the bond increases.*

Proof:

Differentiating equation (25) with respect to *P*:

$$\frac{\partial H^{c}}{\partial P} = N(d_1) + \frac{1}{\sigma_p\sqrt{\tau}}N'(d_1) - \frac{Fe^{-r\tau}}{P\sigma_p\sqrt{\tau}}N'(d_2), \tag{33}$$

but,

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)x^2}, \tag{34}$$

thus,

$$\frac{\partial H^c}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[ e^{-(1/2)d_1^2} - e^{-\log(P/F) + r\tau - (1/2)d_2^2} \right] . \tag{35}$$

Substitute equation (34) in the last part of equation (35):

$$\frac{\partial H^c}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[ e^{-(1/2)d_1^2} - e^{d_1 \sigma_p \sqrt{\tau} + (1/2)\sigma_p^2 \tau - (1/2)(d_1 - \sigma_p \sqrt{\tau})^2} \right] , \tag{36}$$

which simplifies into:

$$\frac{\partial H^c}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[ e^{-(1/2)d_1^2} - e^{-(1/2)d_1^2} \right]. \tag{37}$$

Hence:

$$\frac{\partial H^c}{\partial P} = N(d_1) \geq 0 . \tag{38}$$

Alternatively, differentiating equation (29) with respect to $P$ also yields:

$$\frac{\partial H^p}{\partial P} = N(a_2) \geq 0 . \tag{39}$$

*Remarks*: The first type of commodity-linked bond is equivalent to a portfolio that consists of a discount bond with a face value of $F$ and a European call option on the commodity referenced to the bond with an exercise price of $F$. An explanation for Proposition 1 is that, as the commodity price increases, the probability that the call contained in the portfolio will end up in the money increases, which appreciates the value of the commodity-linked bond.

The second type of commodity-linked bond is equivalent to a portfolio that consists of a discount bond with a face value of $F$ and a short position on a European put option on the commodity referenced to the bond with an exercise price of $F$. The value of the commodity-linked bond rises with the increase in the price of the referenced commodity, because of the value of the put option: the chances of the put option finishing out of the money rises with the rise in the commodity price.

The two results clearly show that, if LDCs had issued debt contracts that were tied to their main export commodities, then their debt load would have declined along with plummeting export prices (or export revenues). LDCs could therefore have prevented their current debt crisis if they had issued commodity-linked bonds.

*Proposition 2*: *An LDC that has a volatile commodity price can minimize its debt burden by issuing bonds that pay holders, on maturity, the lesser of the face value of the bond and the monetary value of a pre-specified unit of a commodity, rather than the greater of these two.*

Proof:

Differentiating equation (25) with respect to $\sigma_p$ and simplifying yields:

$$\frac{\partial H^c}{\partial \sigma_p} = \sqrt{\tau} P N'(d_1) \geq 0 . \tag{40}$$

Differentiating equation (29) with respect to $\sigma_p$ yields:

$$\frac{\partial H^p}{\partial \sigma_p} = -(\sqrt{\tau} P N'(a_2)) \leq 0 . \tag{41}$$

*Remarks*: Equations (40) and (41) show that a commodity-linked bond that has an embedded put option falls in value when the volatility of the commodity price rises, whereas the opposite occurs with a bond that has an embedded call option. The increased volatility of the commodity price increases its value option attached to the bond, because a put call has no downside risk, since its value is zero irrespective of how far it finishes out of the money. Hence, an increase in the volatility of the commodity price increases the chances that the put option will expire in the money. Given that the commodity-linked bond of this type is equivalent to a regular bond and a short position on a put option, the value of the bond falls with a rise in the volatility of the commodity price. In other words, the heightened volatility of export commodity prices leads to an increase in the expected export revenue, and, with the debt burden falling with it, greatly reduces the chance of an LDC defaulting on the bond.

Alternatively, if the LDC was to issue a bond with an embedded call, then the rise in the volatility of the commodity price would increase the value of the bond. The value of the call rises with the volatility of the commodity price, because there is no downside risk to the call, since its value is zero irrespective of how far it finishes out of the money. An increase in $\sigma_p$, therefore, increases the chances that the call option will expire in the money. The implication is that an LDC increases its debt burden when it issues commodity-linked bonds that are embedded with call options on a commodity price, because the value of the bond rises with the increase in the volatility of the commodity price.

*Proposition 3*: *In an environment where interest rates are not stochastic, the coupon rate for a conventional debt with an identical face value as a commodity-linked bond is generally less than the coupon rate for a commodity-linked bond that pays holders, on maturity, the minimum of the face value and the monetary value of a pre-specified unit of a commodity. The coupon rate for the conventional bond is, however, greater than its counterpart for a commodity-linked bond whose terminal payoff is the greater of the face value and the monetary value of a pre-specified unit of a commodity.*

Proof:

Given their identical face values, an investor on the margin would be indifferent between the two types of commodity-linked bonds and a conventional bond, which implies that the current market values of the two instruments must be the same. Using equations (9) and (22), and setting $Q_0$ to $F$, we have:

$$\begin{aligned} \frac{c}{r}(1-e^{-r\tau}) + Fe^{-r\tau} &= \frac{c^c}{r}(1-e^{-r\tau}) + Fe^{-r\tau} + \\ L(P, F, \tau) &= \frac{c^p}{r}(1-e^{-r\tau}) + Fe^{-r\tau} - Q(P, F, \tau), \end{aligned} \tag{42}$$

which implies that

$$\frac{c}{r}(1-e^{-r\tau}) - \frac{c^c}{r}(1-e^{-r\tau}) = L(P, F, \tau) \ . \tag{43}$$

But $L(P, F, \tau) \geq 0$, because there is no downward risk for an option. It therefore follows that:

$$c - c^c \geq 0 \, . \tag{44}$$

Similarly,

$$\frac{c^p}{r}(1-e^{-r\tau})-\frac{c^c}{r}(1-e^{-r\tau}) = L(P, F, \tau)+Q(P, F, \tau)\geq 0\,. \tag{45}$$

Given that $L(P, F, \tau) \geq 0$ and $Q(P, F, \tau) \geq 0$,

$$c^p - c^c \geq 0\,. \tag{46}$$

Lastly,

$$\frac{c^p}{r}(1-e^{-r\tau})-\frac{c}{r}(1-e^{-r\tau}) = Q(P, F, \tau)\geq 0\,, \tag{47}$$

or

$$c^p - c \geq 0\,. \tag{48}$$

Putting equations (44), (46), and (48) together, we have:

$$c^p \leq c \leq c^c\,. \tag{49}$$

*Remarks*: Proposition 3 strengthens the economic rationale for the issue of a commodity-linked bond. It demonstrates that LDCs or corporations in need of investment funds could share the appreciation of the market value of the underlying commodity with the bondholders, in return for a lower coupon rate. In this case, LDCs would benefit by issuing commodity-linked bonds that pay, on maturity, the greater of the face value or the monetary value of a pre-specified unit of the underlying commodity. This supports Budd (1983), who argues that the issue of commodity-linked bonds offers an opportunity for commodity-producing issuers and international commodity organizations to borrow at below-market interest rates.

On the other hand, an LDC could share the depreciation of the market value of its commodity price with bondholders in exchange for higher coupon rates. The LDC would issue a commodity-linked bond whose final payoff is the lesser of the face value or the monetary value of a pre-specified unit of the underlying commodity. The issuance of such bonds would act as a hedge for an LDC during times when the commodity price experiences a collapse (Caballero 2003).

## 4.3 Net foreign debt

Without external financing, the value of imports must equal the value of exports, so that the current account is in balance each period. The assumption made in this paper, however, is that the government of the LDC has access to two sources of external financing: one is to issue conventional debt and the other is to issue a commodity-linked bond.

Let $D(t) = \int_0^t D(t-1)d\tau$ be the quantity of conventional debt outstanding to the government of the LDC.[5] The new quantity of debt issued in each period is, therefore, $\dot{D}(t) = dD/dt$. Similarly, the total quantity of commodity-linked bonds outstanding is $B(t) = \int_0^t B(t-1)d\tau$ . The quantity of new commodity-linked bonds issued is $\dot{B}(t) = dB/dt$. Furthermore, assume that both the conventional debt and the commodity-linked bond are of the console type. Also, the coupon payments to bearers of conventional debt and the commodity-linked bonds are, respectively, $c$ and $c^B$. Hence, in each period, the contributions of the conventional debt and commodity-linked bond to the net foreign debt of the government are, respectively, $Q\dot{D}\,dt - Dc$ and $H\dot{B}\,dt - Bc^B$.

If $x$ is the fixed rate of commodities exported and $m(t)$ is the rate of imports consumed, then, in every instant, imports must be financed by the sum of export revenue and the value of new total debt less the total coupon payments. In other words, the government's instantaneous import bill is constrained by the following function:

$$m(t)dt = Pxdt + QdD + HdB - Dcdt - Bc^B dt . \tag{50}$$

Let $W$ be the value of the total external debt of the government of the LDC:

$$W = QD + HB . \tag{51}$$

The change in $W$ is, therefore,

$$dW = DdQ + BdH + QdD + HdB . \tag{52}$$

But the import constraint of equation (50) shows that:

$$QdD + HdB = m(t) - Pxdt + Dcdt + Bc^B dt . \tag{53}$$

Substituting equation (53) into equation (52),

$$dW = DdQ + BdH + m(t) - Pxdt + Dcdt + Bc^B dt . \tag{54}$$

Define $\omega_1$ as the fraction of the total external debt held in conventional debt and $\omega_2$ as the fraction of external debt held in commodity-linked bonds:

5. $D(t\text{-}1)$ is a conventional debt that matures in $t$-1 periods.

$$\omega_1 = QD/W \qquad \text{and} \qquad \omega_2 = HB/W.$$

Equation (54) then becomes:

$$dW = \omega_1 W \frac{dQ}{Q} + \omega_2 W \frac{dH}{H} + mdt - Pxdt + \frac{\omega_1 W}{Q} cdt + \frac{\omega_2 W}{H} c^B . \tag{55}$$

Note that $Q$ and $H$ must satisfy equations (8) or (9) and (25) or (29). Substitute equations (3) and (10) into equation (55) and note that $\omega_1 + \omega_2 = 1$. Since $\omega_2 = 1 - \omega_1$, the flow of the net external debt is:

$$\begin{aligned} dW = {} & [\omega_1 W(\alpha_q - \alpha_h + c/Q - c^B/H) + m - Px \\ & + W(\alpha_h + c^B/H)]dt + [\omega_1 W(\sigma_q - \psi_r) + W\psi_r]dz_r \\ & + (1 - \omega_1) W \psi_p dz_p . \end{aligned} \tag{56}$$

Equation (56) demonstrates that the value of the external debt of the LDC changes with the market valuations of conventional bonds and commodity-linked bonds, the import bill, and export revenue. Shocks from interest rates and commodity prices, however, make the market valuation of the debt very uncertain.

## 4.4 The government's maximization problem

The government is faced with choosing in each period the level of imports, $m$, and the fractions of total external debt, $\omega_1$ and $\omega_2$, that must be held in conventional debt and commodity-linked bonds. The government embarks on this portfolio and imports rule in a manner that maximizes the expected value of a time-additive von Neumann-Morgenstern utility function. The problem is formulated as:

$$\underset{m,\,\omega_1}{Max}\; E_0\left[\int_0^{\infty} e^{-\beta t} U(m(t), t)\right] dt , \tag{57}$$

subject to equation (56) and

$$W(0) = W_0 . \tag{58}$$

Also, the utility function $U(\cdot)$ is restricted to be concave in $m$ (i.e., $U_m > 0$ and $U_{mm} < 0$). $E_0$ is the expectations operator, conditional on $W(0) = W_0$ being known.

Using dynamic programming techniques, a $J$ function can be defined as:

$$J(W, P, r, t) \equiv \underset{m,\ \omega_1}{Max} E_0\left[\int_0^\infty e^{-\beta t} U(m(t), t)\right] dt\,. \tag{59}$$

Equation (59) is also constrained by equations (56) and (58). Equation (59) can therefore be rewritten as,

$$J(W(t_0), P, r, t_0) \equiv \underset{m,\ \omega_1}{Max} E_{t_0}\left[\int_{t_0}^{t_1} e^{-\beta t} U(m(t), t) dt + J(W((t_1), P, r, t_1))\right]\ . \tag{60}$$

As shown in the appendix, the optimization problem that faces the government is reduced to:

$$\underset{m,\ \omega_1}{Max} \Phi(\omega, m; W, P, r, t) = e^{-\beta t} U(m(t), t) + L(J)\,, \tag{61}$$

where $L$, which is known as the Dynkin operator over the variables $W$, $P$, and $r$, is defined in the appendix. The first-order condition for a maximization problem is:

$$\phi_m = e^{-\beta t} U_m + J_w = 0\ , \tag{62}$$

$$\begin{aligned}\phi_{\omega_1} &= J_w W(\alpha_q - \alpha_h + c/Q - c^B/H) \\ &+ J_{wp} WP\sigma_p(\rho_{pr}(\sigma_q - \psi_r) - \psi_p) + J_{wr} W\sigma_r((\sigma_q - \psi_r) - \rho_{rp}\psi_p) \\ &+ 0.5 J_{ww} W^2 (2\omega_1(\sigma_q - \psi_r)^2 + 2\psi_r(\sigma_q - \psi_r) \\ &+ 2\rho_{pr}\psi_p(1 - 2\omega_1)(\sigma_q - \psi_r) - 2\rho_{pr}\psi_p\psi_r - 2(1 - \omega_1)\psi_p^2) = 0.\end{aligned} \tag{63}$$

Before finding the optimum proportions of commodity-linked bonds and conventional debt that must be raised by the government externally, some comments on equation (62) should be made.

Equation (62) implies that the marginal utility of external debt to the government of an LDC is negative. The LDC, therefore, chooses an optimum level of imported goods at the point where the sum of marginal utility derived from consuming imported goods and the marginal utility of external debt is zero. In other words, LDCs will contract loans up to the point where the marginal disutility of total external debt is completely offset by the marginal utility derived from imported goods.

## 4.5 Optimal allocation of external debt

Equation (63) is used to obtain the optimum proportions of the total external debt that must be held in conventional debt and commodity-linked bonds. Thus, rearranging equation (63) and simplifying, the optimum weight of conventional debt is expressed as:

$$\begin{aligned}\omega_1^* = & -\frac{J_w}{WJ_{ww}}\left[\frac{\alpha_q - \alpha_h + c/Q - c^B/H}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] \\ & -\frac{J_{wp}P}{WJ_{ww}}\left[\frac{\rho_{pr}\sigma_p(\sigma_q - \psi_r) - \sigma_p\psi_p}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] \\ & -\frac{J_{wr}}{WJ_{ww}}\left[\frac{\sigma_r(\sigma_q - \psi_r) - \sigma_r\rho_{pr}\psi_p}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] \\ & + \left[\frac{\psi_r(\sigma_q - \psi_r) + \psi_p(\rho_{pr}\psi_r - \psi_p)}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] .\end{aligned} \tag{64}$$

Without loss of generality, the last term of equation (64) could be dropped, because it does not add much to the discussion. The optimum proportion of external debt that is in the form of commodity-linked bonds is given as:

$$\omega_2^* = 1 - \omega_1^* . \tag{65}$$

Thus,

$$\omega_2^* = 1 + \frac{J_w}{WJ_{ww}}\left[\frac{\alpha_q - \alpha_h + c/Q - c^B/H}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right]$$

$$+ \frac{J_{wp}P}{WJ_{ww}}\left[\frac{\rho_{pr}\sigma_p(\sigma_q - \psi_r) - \sigma_p\psi_p}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] \tag{66}$$

$$+ \frac{J_{wr}}{WJ_{ww}}\left[\frac{\sigma_r(\sigma_q - \psi_r) - \sigma_r\rho_{pr}\psi_p}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right] .$$

Assume that the government of the LDC has a logarithmic utility function with a constant rate of time preference $\gamma$. Also, let the ratio of the government's instantaneous import bill to the external debt be $\lambda$. Thus, $\lambda = m/W$. With these equations, we have:

$$U(m, t) = e^{-\gamma t}\log(m) \quad , \tag{67}$$

$$m^*(W, P, r, t) = \lambda W \ . \tag{68}$$

Equations (62), (66), and (67) can be used to obtain an expression for the $J(\cdot)$ value function:

$$J(W, P, r, t) = -(1/\lambda)e^{-\gamma t}\log(W) + \Gamma(P, r, t) \quad , \tag{69}$$

where $\Gamma(\cdot)$ is a function of the underlying state variables in the economy other than $W$.

Applying equation (68), the optimum proportions of the total external debt in the form of conventional debt and commodity-linked bonds are expressed as:

$$\omega_1^* = \left[\frac{(\alpha_q + c/Q) - (\alpha_h + c^B/H)}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right], \tag{70}$$

and

$$\omega_2^* = 1 + \left[\frac{(\alpha_h + c^B/H) - (\alpha_q + c/Q)}{(\sigma_q - \psi_r)^2 - 2\rho_{pr}\psi_p(\sigma_q - \psi_r) + \psi_p^2}\right]. \tag{71}$$

From equations (69) and (70) it can be seen that the optimal proportions of the total external debt raised in commodity-linked bonds and conventional debt depend on the spread between the total returns (capital gains and coupon payments) of both bonds, adjusted by the riskiness of the portfolio.[6] The results accord with the literature on capital asset pricing. It can also be seen that the proportions respond positively to the debt's own total return and negatively to the return of the alternative debt instrument. Note that $\omega_1$ and $\omega_2$ would have to be non-negative, because a country cannot sell short its own debts. As in Merton (1971), equation (70) provides a rule of thumb that could be followed by an LDC in its investment decisions. For example, the rule suggests that an LDC should hold a larger share of commodity-linked bonds in its external debt portfolio whenever the variance of the portfolio is greater than twice the spread between the expected total return of the conventional debt and the commodity-linked bond.

## 5. Conclusion

In this paper, it has been argued that the issue of commodity-linked bonds would provide an opportunity for commodity-producing developing countries to tie their borrowing needs to an endowed resource. By issuing bonds indexed to their main export commodity, LDCs could hedge against fluctuations in their export earnings and at the same time lessen the probability of defaulting on their external debt obligation.

Results reported in this paper indicate that the value of the commodity-linked bonds increases as the price of the commodity indexed to the bonds rises. This suggests that, if LDCs had issued debt contracts that were tied to their main export commodities, then their debt loads would have declined along with plummeting export prices (or export revenues). This paper has also demonstrated that the coupon rate for a commodity-linked bond is less than its counterpart for a conventional debt instrument, if LDCs share, on maturity, the appreciation in the commodity price with the bearer. The issuance of such bonds offers an opportunity for commodity-producing issuers and international commodity organizations to borrow at below-market interest rates.

On the other hand, LDCs could issue a bond whose terminal payoff is the lesser of the face value and the monetary value of a pre-specified unit of a commodity. The coupon rate for this type of bond would have to be larger than that for a conventional bond, because investors would have to be compensated for accepting the prescribed terminal payoff. The importance of these types of bonds is that they act as a hedge for LDCs against plummeting commodity prices.

Finally, using portfolio theory, a simple rule was derived for an LDC to follow in its allocation of debt instruments and the level of imports. The rule suggests that an LDC should hold a larger

6. Riskiness is measured here as the correlation between the export price and the Libor rate, and the variances of the prices of the debt instruments.

share of commodity-linked bonds in its external debt portfolio than that of a conventional debt whenever the variance of the portfolio is greater than twice the spread between the expected total return of the conventional debt and the commodity-linked bond.

Like most economic models, there are limitations to this model. The viability of a commodity-linked bonds market cannot be guaranteed by simply letting risk-prone speculators issue these bonds to risk-averse hedgers. Hence, the commodity-linked bond market must be commercially guided and participants must be major market markers, such as corporations and governments. To reduce default risk, the issuers of the bonds must maintain a threshold level of inventory, similar to what banks hold as reserves. Furthermore, issuers that do not have the commodity must back the bonds with a long position in the forward or futures contracts, whose maturity is timed with the redemption date of the bonds.

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# Appendix

## A.1 The Dynkin Operator

Let $t = t_0 + \Delta t$ and assume that the third partial derivative of $J(\cdot)$ is bounded. By applying Taylor's series theorem, the mean value theorem for integrals, and taking the limits as $\Delta t \to 0$, equation (46) becomes:

$$J(W(t_0), P, r, t_0) \equiv \underset{m,\,\omega_1}{Max} [U(m(t), t) + E(J(W(t_0), P, r, t_0))$$

$$+ J_t dt + J_w E(dW) + J_p E(dP) + J_r E(dr) + J_{wp} E(dWdP) + J_{wr} E(dWdr) \tag{A1}$$

$$J_{rp} E(drdP) + 0.5 J_{pp} E(dP)^2 + 0.5 J_{rr} E(dr)^2 ] \ .$$

However, the net foreign debt constraint (equation (42)) and equations (1) and (2) give:

$$E(dW) = [\omega_1 W(\alpha_q - \alpha_h + c/Q - c^B/H) + m - Px$$

$$+ W(\alpha_h + c^B/H)]dt \ , \tag{A2}$$

$$E(dW)^2 = [\omega_1^2 W^2 (\sigma_q - \psi_r)^2 + 2\omega_1 W^2 \psi_r (\sigma_q - \psi_r) + W^2 \psi_r^2$$

$$+ 2\rho_{pr} W^2 (\psi_p (\omega_1 - \omega_1^2)(\sigma_q - \psi_r) + (1 - \omega_1)\psi_p \psi_r) \tag{A3}$$

$$(1 - \omega_1)^2 W^2 \psi_p^2 ]dt \ ,$$

$$E(dP) = \alpha_p P dt , \tag{A4}$$

$$E(dP)^2 = \sigma_p^2 P^2 dt , \tag{A5}$$

$$E(dr) = \kappa(\theta - r)dt , \tag{A6}$$

$$E(dr)^2 = \sigma_r^2 dt , \tag{A7}$$

$$E(dWdP) = [WP\rho_{pr}\sigma_p(\omega_1(\sigma_q - \psi_r) + \psi_r) + (1-\omega_1)WP\sigma_p\psi_p]dt , \tag{A8}$$

$$E(dWdr) = [\sigma_r(\omega_1 W(\sigma_q - \psi_r) + W\psi_r) + (1-\omega_1)W\sigma_r\rho_{rp}\psi_p]dt , \tag{A9}$$

$$E(dPdr) = \rho_{pr}\sigma_p\sigma_r P dt . \tag{A10}$$

Substituting equations (A2) to (A10) into equation (A1), and noting that $E(J(W(t_0), P, r, t)) \equiv J(W(t_0), P, r, t)$, the continuous-time version of the Bellman-Dreyfus fundamental optimality equation is obtained, which is of the form:

$$
\begin{aligned}
0 \equiv \underset{m,\,\omega_1}{Max} [ & U(m(t), t) + J_t \\
& J_w(\omega_1 W(\alpha_q - \alpha_h + c/Q - c^B/H) + m - Px \\
& + W(\alpha_h + c^B/H)) + J_p(\alpha_p P) + J_r(\kappa(\theta - r)) \\
& + J_{wp}(WP\rho_{pr}\sigma_p(\omega_1(\sigma_q - \psi_r) + \psi_r) + (1-\omega_1)WP\sigma_p\psi_p) \\
& + 0.5J_{ww}(\omega_1^2 W^2(\sigma_q - \psi_r)^2 + 2\omega_1 W^2\psi_r(\sigma_q - \psi_r) + W^2\psi_r^2 \\
& + 2\rho_{pr}W^2(\psi_p(\omega_1 - \omega_1^2)(\sigma_q - \psi_r) + (1-\omega_1)\psi_p\psi_r) + (1-\omega_1)^2 W^2\psi_p^2) \\
& + 0.5\sigma_p^2 P J_{pp} + 0.5\sigma_r^2 J_{rr}] \ .
\end{aligned}
\tag{A11}
$$

In compact form, equation (A11) can be expressed as:

$$\phi(m, D, B; W, P, r, t) \ = \ U(m(t), t) + L(J) \,,$$

where $L$ is the Dynkin operator over the variables $W$, $P$, and $r$. This operator is defined as:

$$
\begin{aligned}
L(J) = {} & J_t + J_w(\omega_1 W(\alpha_q - \alpha_h + 1/Q - (1/H)Min(1, \beta P)) + m - Px \\
& + W(\alpha_h + (1/H)Min(1, \beta P))) + J_p(\alpha_p P) + J_r(\kappa(\theta - r)) \\
& + J_{wp}(WP\rho_{pr}\sigma_p(\omega_1(\sigma_q - \psi_r) + \psi_r) + (1-\omega_1)WP\sigma_p\psi_p) \\
& + 0.5J_{ww}(\omega_1^2 W^2(\sigma_q - \psi_r)^2 + 2\omega_1 W^2\psi_r(\sigma_q - \psi_r) + W^2\psi_r^2 \\
& + 2\rho_{pr}W^2(\psi_p(\omega_1 - \omega_1^2)(\sigma_q - \psi_r) + (1-\omega_1)\psi_p\psi_r) + (1-\omega_1)^2 W^2\psi_p^2) \\
& + 0.5\sigma_p^2 PJ_{pp} + 0.5\sigma_r^2 J_r] \quad .
\end{aligned}
\tag{A12}
$$

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