The Drivers and Pricing of Liquidity in Interest Rate Option Markets

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ABSTRACT

The objectives of this paper are to examine the effect of liquidity on interest rate option prices, and to determine whether it is driven by a common systematic factor. Using daily bid and ask prices of euro (€) interest rate caps/floors, we document a negative effect of liquidity on option prices – illiquid options trade at *higher* prices relative to liquid options, after controlling for the volatility smile and term structure variables. This is opposite to the evidence for other assets such as equities, bonds and currency options. We also identify a systematic common factor that drives liquidity, across option maturities and strike rates. This liquidity factor is driven by the changes in uncertainty in the equity and fixed income markets. Our results have important implications for the pricing and hedging of liquidity risk in derivatives markets.

JEL Classification: G10, G12, G13, G15

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LIQUIDITY HAS LONG BEEN RECOGNIZED as an important factor driving prices in any market. Financial economists have been concerned with quantifying the impact of liquidity on the prices of financial assets. In an early paper, Amihud and Mendelson (1986) conclude that stocks with higher transaction costs command higher expected returns. Longstaff (1995a) derives an analytical upper bound for the discount in security prices for lack of marketability. More recently, financial economists have focused on the commonality in liquidity across various assets and its implications for asset pricing. For example, Chordia et al. (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2001) find that there is significant commonality in liquidity across different stocks. Amihud (2002) shows that this common component in liquidity has a role in explaining the return on the market portfolio. Pastor and Stambaugh (2003) and Acharya and Pedersen (2005) find evidence that liquidity risk, captured by the variation in the common liquidity component, is important for explaining the cross-section of stock returns.

Liquidity effects have been explored in the context of other markets as well. Amihud and Mendelsen (1991) show that illiquidity affects bond prices adversely. Chordia et al. (2003) provide evidence that common factors drive liquidity in the stock and bond markets. Elton et al. (2001) and Longstaff et al. (2005) investigate the impact of commonality in liquidity in the corporate bond market. They show that a significant part of the corporate bond spreads (over benchmark treasury and swap rates) can be explained by a common liquidity factor. In the Treasury bond market, Krishnamurthy (2002) shows that investors prefer liquid assets for which they are willing to pay a premium, while Longstaff (2004) shows that Refcorp bonds trade at higher yields compared to Treasuries due to the flight-to-liquidity premium in Treasury bonds.

Thus far, the literature has identified several stylized facts about liquidity in the stock and bond markets and its impact on the prices and returns of the respective assets. However, very little is known about the commonalities in liquidity or their implications for pricing in derivatives markets, such as those for equity or interest rate options. An exception in this relatively sparse literature is the study by Brenner, Eldor and Hauser (2001), who report that non-tradable currency options in Israel are discounted by 21 percent on average, as compared to exchange-traded options.[1]

We investigate the impact of liquidity in the market for interest rate derivatives. We raise and answer three questions: Does liquidity have an effect, positive or negative, on option prices? Is there a common factor that explains liquidity across strike rates and maturities? How is this common factor related to macro-economic variables such as the parameters of the term structure and volatilities in other markets? These are important questions in the very large market for over-

[1] In other related studies, Vijh (1990) documents the trade-off between depth and bid-ask spreads in equity options, Mayhew (2002) examines the effects of competition and market structure on equity option bid-ask spreads, while Bollen and Whaley (2004) present evidence on the impact of supply and demand effects on equity option prices.

the-counter interest rate options such as caps/floors and swaptions, which are among the most liquid options that trade in the global financial markets, with about $27 trillion in notional principal outstanding, as of December 2004.[2]

According to the existing literature, the impact of illiquidity on asset prices is *overwhelmingly* presumed to be negative, since potential holders of an asset demand to be compensated for the lack of immediacy they face, if they wish to sell the asset. Thus, the liquidity premium on the asset is expected to be positive – other things remaining the same, the more illiquid an asset, the higher is its liquidity premium and its required rate of return, and hence, the lower is its price. For example, in the case of a bond or a stock, which are assets in positive net supply, the buyer of the asset demands compensation for illiquidity, while the seller is no longer concerned about the liquidity of the asset after the transaction. In fact, within a two-asset version of the standard Lucas economy, Longstaff (2005) shows that a liquid asset can be worth up to 25 percent more than an illiquid asset, even if both have identical cash flow dynamics.

However, derivative instruments differ from their underlying assets, such as stocks and bonds in three important respects. First, as pointed out earlier by Brenner, Eldor and Hauser (2001), derivative assets are generally in zero net supply, i.e., the net amount outstanding across agents in the market is zero. Second, in the case of derivatives, the risk exposures of the short side and the long side may not be the same. Third, since derivatives can be hedged by taking offsetting positions in the underlying asset as well, the liquidity effects of the latter may also play a role in determining the impact of liquidity in the derivatives market. For example, in the case of an option, both the buyer and the seller continue to have exposure to the asset after the transaction, until it is unwound. The buyer demands a *reduction* in price to compensate her for the illiquidity, while the seller demands an *increase.* In addition, due to the asymmetry of the payoffs, the seller has higher risk exposures than the buyer. The net effect of the illiquidity is determined in equilibrium, and one cannot presume *ex ante,* that it will be either positive or negative.

In the Black-Scholes world, both the buyer and the seller hedge costlessly in the underlying market; consequently, illiquidity should not have an affect on the price of a derivative asset. However, one cannot derive any general conclusions, if there is some asymmetry between the two parties in terms of their motivations for engaging in the derivatives transaction and consequently their motivation for hedging. For instance, the buyer may be a corporation attempting to reduce its exposure to interest rate fluctuations, while the seller may be a bank, which hedges its position in the derivative by taking offsetting positions in the underlying interest rate instruments – cash instruments or other derivatives such as futures contracts or swaps. Further, the seller may be concerned about the costs of maintaining a long-term hedge, given the transactions costs in the market for the underlying asset. In this case, the buyer is more concerned about buying protection against interest rate fluctuations, while it is the seller who is

[2] BIS Quarterly Review, June 2005, Bank for International Settlements, Basel, Switzerland.

more concerned about liquidity. Consequently, it is quite possible that the impact of illiquidity in the derivatives market as well as the market for the hedge instrument may cause the seller to *raise* his price. [3] Hence, illiquidity in this case may have a *positive* impact on the price, rather than the conventional *negative* impact identified in most of the liquidity literature.

These general observations can be interpreted in the context of the specific institutional structure of interest rate option markets. These markets are almost entirely institutional, with hardly any retail presence. Most interest rate options, particularly the long-dated ones such as caps, floors and swaptions, are sold over-the-counter (OTC) by large market makers, typically international banks.[4] The customers are usually on one side of the market (the ask-side), and the size of individual trades is relatively large. Many popular interest rate option products, such as caps, floors and collars are relatively long-dated portfolios of options (up to ten years maturity or more), which creates enormous transactions costs if the seller just dynamically hedges using the underlying interest rate markets. These features lead to significant issues relating to supply/demand and asymmetric information about the order flow. Since interest rate options are traded in an OTC market, there are also important credit risk issues that may influence the pricing of these options, especially during periods of crisis. Thus, the determination of the direction of the impact of illiquidity on the prices of these interest rate options is a complex issue, which is best resolved through empirical means.[5]

We make three important contributions to the literature. First, contrary to the findings in the existing literature about other asset markets, we find that higher bid-ask spreads (i.e. greater illiquidity) *increase* the prices of interest rate options. This effect goes in the opposite direction to what is observed for stocks, bonds, and even for some exchange traded currency options, as pointed out in the discussion above. To the best of our knowledge, our paper is the first paper to document such a liquidity effect in any market. This result has important implications for incorporating liquidity risk in derivative pricing models, since we show that the conventional intuition, which holds in other asset markets, may not hold in some derivative markets.

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[3] The results in Brenner, Eldor, and Hauser (2001), that illiquid currency options were priced *lower* than traded options, can also be explained by the same argument. In their case, the liquidity effect goes in the opposite direction - since these options were auctioned by the Central Bank of Israel, the buyers of these options are the ones who are concerned about illiquidity, and not the seller.

[4] Unlike exchange traded option markets, the only metric of liquidity available in the OTC interest rate option markets is the bid-ask spread – there are no volume, depth, or open interest data available. Therefore, in spite of its potential shortcomings, we are constrained to rely on the bid-ask spread alone for all our liquidity analyses, which still lead to important findings.

[5] In this context, Constantinides (1997) argues that, with transaction costs, the concept of the no-arbitrage price of a derivative is replaced by a range of prices. From a theoretical standpoint, he argues that transaction costs are more likely to play an important role in the pricing of the customized, over-the-counter derivatives (which include most interest rate options), as opposed to plain-vanilla exchange-traded contracts, since the seller has to incur higher hedging costs to cover short positions, if they are customized contracts. In a similar vein, Longstaff (1995b) shows that in the presence of frictions, option pricing models may not satisfy the martingale restriction.

Second, we find that there is a single common factor determining the liquidity of interest rate options at different strike rates and maturities that explains about one-third of the variation in liquidity across these options. Third, this systematic liquidity factor appears to be related to the changes in uncertainty in equity and fixed income markets. This finding has important implications for the measurement and hedging of liquidity risk in interest rate option portfolios. Perceptions of greater uncertainty in these markets result in greater illiquidity. Further, these uncertainty shocks shift liquidity from in-the-money and at-the-money options to out-of-the-money options, on average. We are not aware of any other study that has documented a common liquidity factor in any derivatives market. This investigation of the commonality of liquidity complements similar studies for other markets such as those for stocks, Treasury and corporate bonds.

The structure of our paper is as follows. In Section I we describe the data set and present summary statistics. Section II documents the time-variation in euro interest rate option prices as summarized by the scaled implied volatility. This section also provides evidence on the pattern of implied volatility across strike rates as described by the volatility smile. After controlling for the term structure and volatility factors, a simultaneous equation system is estimated to examine the relationship between the price (scaled implied volatility) and the liquidity (scaled bid-ask spread) of interest rate options. Section III explores the commonality in the liquidity of interest rate options, across strike rates and maturities, and links this systematic factor to changes in macro-economic variables. Section IV concludes with a summary of the main results and directions for future research**.**

I. Data

The data for this study consist of an extensive collection of euro (€) cap and floor prices over the 29-month period, January 1999 to May 2001, obtained from WestLB (Westdeutsche Landesbank Girozentrale) Global Derivatives and Fixed Income Group. These are daily bid and offer quotes over 586 trading days for nine maturities (2 years to 10 years, in annual increments) across twelve different strike rates ranging from 2% to 8%. (Prices are not available for all of the maturity-strike combinations each day.)[6] Therefore, this dataset allows us to control for strike price biases in the liquidity analysis of caps and floors. These caps and floors are portfolios of European interest rate

[6]The Euro OTC interest rate derivatives market is *extremely* competitive, especially for plain-vanilla contracts like caps and floors. The BIS estimates the Herfindahl index (sum of squares of market shares of all participants) for Euro interest rate options (which includes exotic options) at about 500-600 during 1999-2004, which is even lower than that for USD interest rate options (around 1,000). Since a lower value of this index (away from the maximum possible value of 10,000) indicates a more competitive market, it is safe to rely on option quotes from a top European derivatives dealer (reflecting the best information available with them) like WestLb during our sample period. Thus, any dealer specific effects on price quotes are likely to be small and unsystematic across the over 30,000 bid and ask price quotes each that are used in this paper.

options on the 6-month Euribor with a 6 monthly reset frequency.[7] Along with the options data, we also collected data on € swap rates and the daily term structure of euro interest rates curve from the same source. These are key inputs necessary for checking cap-floor parity, as well as for conducting the subsequent empirical tests.

Table I provides descriptive statistics on the midpoint of the bid and ask prices for caps and floors over our sample period. The prices of these options can be almost three orders of magnitude apart, depending on the strike rate and maturity of the option. For example, a deep out-of-the-money, two-year, cap may have a market price of just a few basis points, while a deep in-the-money, ten-year, cap may be priced above 1500 basis points. Since interest rates have varied substantially during our sample period, the data have to be reclassified in terms of moneyness ("depth in-the-money") to be meaningfully compared over time. In Table I, the prices of options are grouped together into "moneyness buckets," by estimating the Log Moneyness Ratio (LMR) for each cap/floor. The LMR is defined as the logarithm of the ratio of the par swap rate to the strike rate of the option. Since the relevant swap rate changes every day, the moneyness of the same strike rate, same maturity, option, as measured by the LMR, also changes each day. The average price, as well as the standard deviation of these prices, in basis points, is reported in the Table. It is clear from the Table that cap/floor prices display a fair amount of variability over time. Since these prices are grouped together by moneyness, a large part of this variability in prices over time can be attributed to changes in volatilities over time, since term structure effects are largely taken into account by our adjustment.

We also document the magnitude and behavior of the liquidity costs in these markets over time, for caps and floors across strike rates. We use the bid-ask spreads for the caps and floors as a proxy for the liquidity of the market. As mentioned earlier, in an OTC market, this is the *only* measure of liquidity available for these options; other measures of liquidity common in exchange-traded markets such as volume, depth, market impact etc., are just not available. The data on even the bid-ask spreads are not widely available for the market as a whole. In our sample, we do observe the bid-ask spread for a particular dealer for each option every day. Therefore, we settle for using this metric as a meaningful, although potentially imperfect, proxy for liquidity.

It is important to note that these are measures of the liquidity costs in the interest rate options market and not in the underlying market for swaps. Although the liquidity costs in the two markets may be related, the bid-ask spreads for caps and floors directly capture the effect of various frictions in the interest rate options market, in addition to the transaction costs in the underlying market, as well as the imperfections in hedging between the option market and the underlying swap market. In Table II, we present the bid-ask spreads scaled by the average of the bid and ask price of the option, defined as the ScaledBAS, grouped together into moneyness

[7] In the appendix, we provide the details of the contract structure for these options.

buckets by the LMR. Close-to-the-money caps and floors have proportional bid-ask spreads of about 8% - 9%, except for some of the shorter-term caps and floors that have higher bid-ask spreads. Since deep in-the-money options (low strike rate caps and high strike rate floors) have higher prices, they have lower proportional bid-ask spreads (3% - 4%). Some of the deep out-of-the-money options have large proportional bid-ask spreads - for example, the two year deep out-of-the-money caps, with an average price of just a couple of basis points, have bid-ask spreads almost as large as the price itself, on average about 80.9% of the price. Part of the reason for this behavior of bid-ask spreads is that some of the costs of the market makers (transactions costs on hedges, administrative costs of trading, etc.) are absolute costs that must be incurred whatever may be the value of the option sold. However, some of the other costs of the market maker (inventory holding costs, hedging costs, etc.) would be dependent on the value of the option bought or sold. It is also important to note that, in general, these bid-ask spreads, are much larger than those for most exchange-traded options.

For our empirical tests, we pool the data on caps and floors, since it allows us to obtain a wider range of strike rates, covering rates that are both in-the-money and out-of-the-money, for both caps and floors. Before doing so, we check for put-call parity between caps and floors, and find that, on average, put-call parity holds in our dataset, although there may be deviations from parity for some individual observations. However, these apparent deviations are unsystematic, and they may not be actual violations due to the high cost of carrying out the arbitrage using "off-market" swaps. These parity computations are a consistency check as well, which assures us about the integrity of our dataset.

II. How do Liquidity and Option Prices vary?

We use implied volatilities from the Black-BGM model to characterize option prices throughout the analysis from here on.[8] The raw implied volatility obtained from the Black model removes underlying term structure effects from option prices.[9] Therefore, a change in the implied volatility of an option from one day to the next can be attributed to changes in interest rate uncertainty, or other effects not captured by the model, and not simply due to changes in the underlying term structure. Further, we scale the implied volatility of an option at a particular

[8] The use of implied volatilities from a variant of the Black-Scholes model, even though it is a model dependent measure, is in line with all prior studies in the literature, including Bollen and Whaley (2004). The details of the calculation of implied volatility are provided in the appendix.

[9] Our implied volatility estimation is likely to have much smaller errors than those generally encountered in equity options (see, for example, Canina and Figlewski (1993)). We pool the data for caps and floors, which reduces errors due to mis-estimation of the underlying yield curve. The options we consider are much longer term (the shortest cap/floor is 2 year maturity), which reduces this potential error further. In addition, for most of our empirical tests, we do not include deep ITM or deep OTM options, where estimation errors are likely to be larger. Furthermore, since we consider the implied flat volatilities of caps and floors, the errors are further reduced due to the implicit "averaging" in this computation.

strike rate by the implied volatility of an at-the-money option with the same maturity on the same day, to obtain the scaled implied volatility (ScaledIV) of that option on that particular day. The ScaledIV is a cleaner measure of option prices, since even the at-the-money option volatility has been factored out of the implied volatility of each option contract. In addition, in the empirical tests where we use ScaledIV, we control for the shape of the volatility smile (using functions of LMR), and use several term structure variables as approximate controls for the skewness and excess kurtosis in the underlying interest rate distribution. In the presence of these controls, the changes in the ScaledIV for a particular option cannot be attributed to changes in the underlying term structure or to changes in the level of volatilities at that maturity. Therefore, the ScaledIV can be effectively used to examine factors such as liquidity, *other than* the underlying term structure or interest rate uncertainty that may affect option prices in this market.[10] In the rest of the paper, we use the ScaledIV as the "adjusted" price of the option, for every strike and maturity.

A. Time-variation in Liquidity and Option Prices

Figure 1 presents the surface plots for the ScaledIV over time, by moneyness represented by LMR.[11] The plots are presented for three representative maturities - 2-year, 5-year, and 10-year - for the pooled cap and floor data. These plots clearly show that there is a significant smile curve in interest rate options in this market, across strike rates. The smile curve is steeper for short-term options, while for long-term options, it is flatter and not symmetric around the at-the-money strike rate. Both the curvature and the slope of the volatility smile show significant time-variation, sometimes even on a daily basis. The changes in the curvature and slope over time are more pronounced for the 2-year maturity options, although they are also perceptible for the longer maturity options. Figure 1 also presents the surface plot of the Euro spot rates from two to ten years maturity over our sample period. Similar to the volatility surfaces, the Euro term structure surface also shows significant time variation. It is clear that there is an increase in spot rates in the early part of our sample, followed by a flattening of the term structure due to an increase, primarily in the rates at the shorter end of the term structure, during the latter part of our sample period. Therefore, both the level of interest rates and the slope of the term structure exhibit significant time variation over our sample period.

Figure 2 presents the time series plots of the scaled bid-ask spreads for each maturity bucket, by moneyness. The out-of-the-money bucket contains caps with LMR less than -0.1 and floors with LMR greater than 0.1. Similarly, the in-the-money bucket contains caps with LMR greater than

[10] Changes in the ScaledIV, in the presence of these controls, are somewhat analogous to the excess returns used in asset pricing studies.

[11] These plots are presented for representative maturities of 2, 5, and 10 years, since the plots for the other maturities are similar. In addition, since 3-D plots require the data to be complete over the entire grid, we present the volatility smiles over the LMR range from -0.3 to +0.3, which is the subset of strikes over which complete data are available over a substantial number of days in our dataset.

0.1 and floors with LMR less than -0.1. The at-the-money bucket contains caps and floors with LMR between -0.1 and 0.1. For each day and each maturity, the scaled bid-ask spreads within each bucket are averaged, and then plotted over time. Each plot present the time-series of the scaled bid-ask spreads for the nine option maturities, separately for the three moneyness groups. These plots clearly indicate that there is significant time-variation in scaled bid-ask spreads, across maturity and moneyness. In addition, within each moneyness group, there appear to be both systematic and unsystematic components (across maturities) to the time variation in the scaled bid-ask spreads. Indeed, the extent of commonality in the time-variation in bid-ask spreads in this market is one of the primary questions we investigate in this paper.

B. Smile Across the Strike Rates

As argued in the introduction, the relationship between the liquidity of an asset and its price is of fundamental importance in any asset market. This relationship for common underlying assets like stocks and bonds usually predicts that more liquid assets have lower returns and higher prices. However, for derivative assets, especially options, this relationship may go the other way. In addition, there is no reason to expect that either liquidity or price is exogenously determined. Both liquidity and price may be *endogenously* determined by some set of exogenous variables. Therefore, in this section, we estimate a simultaneous equation model of liquidity (scaled bid-ask spreads) and price (ScaledIV), using an array of macro-financial variables as the exogenous determinants of these two endogenous variables.

However, unlike underlying asset markets, option markets have another dimension (the strike price/rate), along which both liquidity and price change, as shown in the figures before. Therefore, we must control for these strike rate effects, in order to correctly interpret the effect of the exogenous variables on price and liquidity. In order to correctly control for strike rate effects, we must estimate the relationship between the option price and the strike rate, i.e., the overall form of volatility smiles in this market. Therefore, we estimate various functional forms for volatility smiles, using pooled time-series cross-sectional regressions of ScaledIV on various functions of LMR. The most common functional form for the volatility smile is a quadratic function of LMR, which is also supported by the plots in Figure 1. In order to account for the asymmetry, if any, in the smile curve, we allow the slope of the smile curve to differ for in-the-money and out-of-the-money options, as follows:

$$ScaledIV = c1 + c2 * LMR + c3 * LMR^2 + c4 * \left(1_{LMR<0} * LMR\right) \qquad (1)$$

In Table III, we report the results for this quadratic functional form with the asymmetric slope term, since it fits the observed volatility smiles the best.[12] The regression coefficients in all the

[12] We also tested a specification with an asymmetric term for the curvature of the smile, but it did not add any significant explanatory power over the specification with the asymmetric term for just the slope of the

alternative specifications, for each maturity, are highly significant. In addition, the specification we report explains over half of the variability in the scaled implied volatilities. In most specifications, the asymmetry term for the slope of the smile is statistically significant, indicating that the shape of the volatility function is different for in-the-money options, as compared to that for out-of-the-money options. We also considered including other option Greeks in the above specifications. We did not do so for two reasons. First, the squared term for the LMR included above is an (approximate) proxy for the convexity term. Second, introducing other option Greeks explicitly may introduce potential collinearity, since, to a first order approximation, these risk parameters can be modeled as linear functions of volatility and the square root of the time to expiration.[13] Therefore, in all the tests that follow, we control for the strike price effects using the asymmetric quadratic function of LMR.

C. *How are Liquidity and Price Related in the Interest Rate Options Market?*

We first estimate the correlation between ScaledBAS and ScaledIV, within each of the three moneyness buckets defined earlier (OTM/ATM/ITM), separately for each maturity. For example, the correlation between the ScaledBAS and the ScaledIV for the 5-year maturity caps/floors is 0.78 for OTM options, 0.46 for ATM options, and 0.43 for ITM options. Figure 3 presents the sample scatter plots for the 5-year maturity options, for all the three moneyness buckets. The plots for the other maturities are similar. Across all the nine maturities, we find that the average of the correlation coefficients (between the ScaledBAS and ScaledIV) is 0.68 for OTM options, 0.50 for ATM options, and 0.46 for ITM options. Although these are just raw correlations between illiquidity and price, they do indicate that, on average, more illiquid options appear to have higher prices, across all moneyness buckets and maturities. However, there is no reason to expect that either price or liquidity is exogenously determined – most likely, both price and liquidity affect each other endogenously, while being jointly determined by a set of exogenous macro drivers.

Next, we estimate a simultaneous model that endogenizes both price and liquidity, using several macro-financial variables as fundamental exogenous drivers of these two endogenous variables, controlling for strike price effects, for each of the 9 option maturities individually:

$$\begin{aligned} ScaledIV &= c1 + c2 * ScaledBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + \\ &\quad c6 * 6Mrate + c7 * Slope + c8SwpnVol \\ ScaledBAS &= d1 + d2 * ScaledIV + d3 * DefSprd + d4 * DAXVol \end{aligned} \tag{2}$$

smile. We got similar results when we tested a polynomial specification with higher order terms, which turned out to be statistically unimportant.

[13] See, for example, Brenner and Subrahmanyam (1994), who provide, in the context of the Black-Scholes model, approximate values for the risk parameters of options that are close to being at-the-money on a forward basis.

The intuition for examining these macro-financial variables is as follows. In the first equation of the simultaneous equation model, we include variables that are more likely to affect option prices *directly* rather than through the liquidity effect. These include term structure variables, as well as LMR controls for strike price effects. In the second equation, we include variables that are more likely to directly affect the liquidity of these options. These include variables reflecting uncertainty in the equity markets and the aggregate credit risk in the banking sector. Of course, due to the simultaneous nature of the model, all the exogenous variables, including the LMR controls for strike price effects, affect both price and liquidity.[14] These five macro-financial variables, taken together, incorporate most of the relevant information about fundamental economic indicators, like expected inflation, GDP growth rate, and risk premia. Since these fundamental economic variables are available at most monthly, we must rely on daily proxies for the expectations of these economic factors in the financial markets.

The spot 6-month Euribor (*6Mrate*) and the slope of the yield curve (*Slope*, defined as the difference between the 5-year and 6-month spot rates) are included as proxies for general economic conditions and the stage of the business cycle, as well as the direction of interest rate changes in the future. For example, if interest rates are mean-reverting, very low interest rates are likely to be followed by rate increases. Similarly, a steeply upward-sloping yield curve also signals rate increases. This would manifest itself in a higher demand for out-of-the-money caps in the market, thus affecting the prices and liquidity of these options. These variables also proxy for the expectations in the financial markets about future inflation and money supply. The swaption volatility (*SwpnVol*) is added to examine whether the patterns of the smile vary significantly with the level of uncertainty in the interest rate options market. During uncertain times, information asymmetry issues are likely to be more important than during periods of lower volatility. If there is significantly greater information asymmetry, market makers may charge higher than normal prices for away-from-the-money options, since they may be more averse to taking short position at these strike rates. This will lead to a steeper volatility smile, implying higher scaled implied volatilities of options. Also, during times of greater uncertainty, a risk-averse market maker may demand higher compensation for providing liquidity to the market, which would affect the shape of the smile. Since we have divided the volatility of each option by the volatility of the corresponding ATM cap to obtain the scaled IV, we use the ATM swaption volatility as an explanatory variable here, in order to avoid having the same variable on both sides of the regression equation. The ATM swaption volatility can be interpreted as a general measure of the future interest rate volatility.[15]

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[14] We considered other macro-financial variables as well, such as yields on speculative grade long-term debt, the short term repo rate as a proxy for money supply, and stock returns in European equity markets. These variables were eliminated due to collinearity with the five macro variables included in the model.

[15] Although swaption implied volatilities are not exactly the same as the cap/floor implied volatility, they both tend to move together. Hence, swaption implied volatilities are a valid proxy for the perceived uncertainty in the future interest rates, as reflected in the option market. The data on the ATM swaption volatility in the Euro market was obtained from DataStream.

The ATM volatility and term structure variables act as approximate controls for a model of interest rates displaying skewness and excess kurtosis. Typically, in such models, the future distribution of interest rates depends on today's volatility and the level of interest rates. Thus, by including the contemporaneous volatility and interest rate variables in the regression, we try to ensure that the relationship of the scaled implied volatilities to liquidity is separate from the effect arising out of a more detailed structural model for the interest rates.

In the second equation of the simultaneous equation model, the 6-month Treasury-Euribor spread (*DefSprd*) is included as a measure of the default risk of the constituent banks in the Euribor fixing. The volatility of the DAX index (*DAXVol*) proxies for the level of uncertainty in the European equity markets, which could also have an impact on market expectations of the future liquidity. Since stock prices reflect expectations about future cash flows and discount rates, the average volatility in equity market also proxies for the level of uncertainty in the expectations about future cash flows and discount rates.

This simultaneous equation model is estimated using three-stage least squares, since the residuals in each equation are correlated with the endogenous variables, and the residuals are correlated across the two equations. We use instrumental variables to produce consistent estimates, and generalized least squares (GLS) to account for the correlation structure of the residuals across the two equations. In the first stage, we develop instrumented values for both the endogenous variables, using all exogenous variables in the system as instruments. In the second stage, based on a two-stage least squares estimation of each equation, we obtain a consistent estimate of the covariance matrix of the equation disturbances. Using this covariance matrix of residuals from the second stage, and the instrumented values of the endogenous variables from the first stage, we then do a GLS estimation as the third stage of the three stage least squares estimation.

The results for this model are presented in Table IV. Our primary inference is regarding the sign of the coefficients c2 and d2. Both these coefficients are positive and statistically significant for all option maturities. This shows that, within an endogenous framework specified above, controlling for strike price effects and potential exogenous drivers of price and liquidity in this market, higher values of ScaledBAS are associated with higher values of ScaledIV, and vice-versa. In other words, more liquid options are priced *lower*, while less liquid options are priced *higher*, after taking into account the effects of other macro variables. This is an important result, and is quite different from the joint behavior of price and liquidity observed in underlying asset markets, like those for stocks and bonds. For example, in the equity markets, it has been shown that more liquid stocks have lower returns (higher prices) – what we observe here is the opposite, i.e., more liquid options have *lower* prices, and higher liquidity is actually associated with a discount, not a premium. The primary explanation for this result is the fundamental difference between derivative assets and underlying assets alluded to in the introduction – the fact that derivatives are in zero net supply, have asymmetric risk exposures, and are tied to their underlying assets in terms of price and liquidity. As argued earlier by Brenner, Eldor and Hauser (2001), for an asset in zero net supply, both the buyer and the seller are concerned about illiquidity pushing the prices in opposite direction. Depending on the risk exposure and the hedging needs of each side, either the "buyer-effect" (lower prices for illiquid assets) or the "seller-effect" (higher prices for illiquid assets) could dominate. In this market, we find that the "seller-effect" dominates and the more illiquid options have higher prices. In addition, we see that the coefficients c2 and d2 are generally increasing in the maturity of these options. This indicates that the longer maturity options exhibit a stronger liquidity effect, perhaps to compensate the seller for the illiquidity over a longer time frame. This sheds some light on the term structure dimension of liquidity effects in this market.

The coefficients on the exogenous variables in the two equations provide important information about the common determinants of price and liquidity in this market. Higher spot rates are generally associated with higher scaled implied volatility, implying that when there are inflation concerns and expectation of rising interest rates, the dealers charge even higher prices (and wider bid-ask spreads) for away-from-the-money options. Note that the strike price effects are already controlled for, by including the LMR functions; so, this effect is incremental to the normal smile effects observed in this market. Once the effect of the spot rate is accounted for, the slope of the yield curve does not appear to have a significant effect on the ScaledIV. The impact of increasing interest rate uncertainty is similar – when swaption volatilities are higher, the scaled implied volatilities are also higher. This is indicative of a steepening of the volatility smile as options become more expensive. When there is more uncertainty in fixed income markets, dealers appear to charge even higher prices (and wider bid-ask spreads) for these options. Aggregate credit risk concerns, proxied by the default spread, do not appear to be significantly related to either price or liquidity in this market. However, equity market uncertainty does appear to be significantly associated with wider bid-ask spreads, and higher scaled implied volatilities. When there is greater uncertainty about future cash flows and discount rates in the economy, the scaled implied volatilities and bid-ask spreads of interest rate caps and floors is higher, adjusting for other effects. It appears that the revelation of information in the equity markets is one of the determinants of price and liquidity quotes posted by fixed income option dealers.

To analyze the relationship between price and liquidity further, we re-estimate the simultaneous equation model on first differences. In Table V, we present the results of the simultaneous equation model where daily changes in ScaledIV and ScaledBAS are regressed on each other as well as changes in LMR functions and macro-financial variables, as follows:

$$\begin{aligned} \Delta ScaledIV &= c1 + c2 * \Delta ScaledBAS + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + \\ &\quad c6 * \Delta 6Mrate + c7 * \Delta Slope + c8 * \Delta SwpnVol \\ \Delta ScaledBAS &= d1 + d2 * \Delta ScaledIV + d3 * \Delta DefSprd + d4 * \Delta DAXVol \end{aligned} \tag{3}$$

This model explicitly tests for the relationship between daily changes in the price and liquidity of options, as opposed to the relationship between levels examined earlier. As before, we estimate this model separately for each option maturity. The results in Table V are similar to the ones reported in Table IV, although these models have lower explanatory power, which is not surprising since they are estimated based on daily changes. The daily change in ScaledIV is positively associated with the daily change in ScaledBAS, controlling for changes in option specific and macro-financial variables. In addition, we find that positive shocks to the level of uncertainty in the equity as well as fixed income markets are associated with positive shocks to both price and liquidity of these interest rate options, although these effects are weaker than those observed in the simultaneous equation model in levels. One of the reasons why these effects are weaker could be the nature of the relationship between shocks to liquidity/price and the shocks to these macro-financial variables: If the relationship between them is not contemporaneous, and one affects the other with a lag, we may not observe strong significance in the contemporaneous models estimated above. We deal with the issue of lagged responses in our later sections.

The analysis above helps us understand the joint determinants of price and liquidity in this market. However, as the results indicate, there is a term structure element to the liquidity effects in this market, i.e., the variation in liquidity is not the same for all maturities. In addition, there are strike rate effects that we have controlled for. Therefore, the natural question is to what extent this liquidity is driven by common factors, across different strikes and maturities. In the next section, we explore these common drivers of liquidity in the interest rate option markets.

III. Are there Common Drivers of Liquidity in this Market?

We first examine the average correlations between scaled bid-ask spreads across different moneyness groups. As before, we categorize all the options into three moneyness groups (by LMR) – OTM, ATM, and ITM. Within each moneyness group, we have nine maturity buckets. For each maturity bucket, we have a time-series of scaled bid-ask spreads over 586 trading days. We compute the correlation between the scaled bid-ask spreads across different maturities, within each moneyness group. We average these correlations within the moneyness groups – these are reported as the diagonal elements in Table VI. For example, the OTM/OTM value of 0.68 is the average correlation between the nine maturity buckets within OTM options (so it is an average of 9x8/2, i.e., 36 correlations). This indicates that the average correlation within OTM option scaled bid-ask spreads, across all maturities, is 0.68. In addition to the average correlations within each moneyness group, we also estimate the average correlations across the moneyness groups. For example, the OTM / ITM value of 0.24 is the average correlation between each maturity bucket within the OTM options segment with the corresponding maturity within the ITM options segment (there are 9x9, i.e. 81 correlations). This indicates that the average correlation between

OTM and ITM option scaled bid-ask spreads, across all maturities, is 0.24. These correlations indicate some interesting patterns. First, the scaled bid-ask spreads seem to be fairly highly correlated across maturities within each moneyness group, although this correlation is a bit lower for OTM options. Second, the correlations across moneyness groups is considerably lower, especially between OTM options and either ATM or ITM options. It appears that there is significant movement in the scaled bid-ask spreads, across maturities and strikes, but the OTM options seem to vary a bit differently from ATM and ITM options. The time-series plots of scaled bid-ask spreads presented in Figure 2 indicate similar patterns.

These correlations and time-series plots indicate that some part of the variation in the scaled bid-ask spreads appears to be systematic. From a market-wide perspective, it is important to understand if there is any systematic component to the liquidity shocks that have an impact on this market. This issue has strong implications for the pricing of liquidity risk in this market, as well as for hedging aggregate liquidity risk in interest rate options. If the liquidity shocks to this market are entirely unsystematic, then they do not create liquidity risk concerns, since they can be diversified away in a portfolio of options. However, if there is a systematic component to these liquidity shocks, then there may be liquidity risk concerns in this market, especially during periods of market stress.[16] The structure of such systematic liquidity shocks, and their macro-economic interpretation, can provide very important inputs for designing strategies to hedge aggregate liquidity risk in this market.

A. *Extracting the Common Liquidity Factor*

We use a panel regression framework to examine whether the time-series variation in the liquidity of individual options has any systematic market-wide component, after controlling for the changes in option specific information. We divide our options into 27 panels (9 maturities each for the 3 moneyness groups – OTM/ATM/ITM), and estimate the following regression model on daily changes: [17]

$$\begin{aligned}\Delta ScaledBAS_{it} = c1 + c2 * \Delta ScaledIV_{it} + c3 * \Delta LMR_{it} + c4 * \Delta LMR^2{}_{it} + \\ c5 * \Delta\left(1_{LMR<0}.LMR\right)_{it} + \varepsilon_{it}\end{aligned} \tag{4}$$

We include fixed effects for each panel to account for any panel-specific effects that may not be captured by the specification above. The intuition behind this regression is to examine the changes in liquidity, and remove the part of those changes that can be explained by changes in option specific variables, such as price (ScaledIV) and functions of LMR. Although the panels are formed based on three categories for moneyness, we still include the functions of LMR as

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[16] This issue has been explored in the equity market by Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), and in the bond market by Longstaff (2004) and Longstaff et al (2005).

[17] In the equity markets, Jameson and Wilhelm (1992) show that the bid-ask spreads for options are related to option Greeks. As explained earlier, our asymmetric quadratic function of LMR acts as an approximate control for these Greeks, in addition to controlling for the shape of the volatility smile.

controls, because even within a moneyness bucket, the LMR of an option will change every day. Hence, we must account for the part of the change in ScaledBAS that is due to changes in LMR. In addition, including the change in ScaledIV takes into account the change in any other option specific information.

We estimate this panel regression model using the Prais-Winsten Full FGLS estimator. The disturbances in this model are assumed to be heteroskedastic and potentially correlated across the 27 panels. In addition, we allow for first-order autocorrelation in the disturbances, within each panel, with the coefficient of the AR(1) process allowed to be different for each panel. Therefore, the standard errors are robust to the error structure specified in the model, and the parameter estimates are conditional on the estimates of the disturbance covariance matrix and the autocorrelation parameters estimated for each panel. For robustness, we estimate this panel regression using alternative error structures and estimation procedures (including maximum likelihood), and find similar results.

The results for this panel regression are presented in Table VII. As expected, based on the results in the previous section, positive changes in scaled implied volatilities are associated with positive changes in the scaled bid-ask spreads, suggesting that improvements in liquidity are associated with a *decrease* in prices in this market, controlling for strike rate effects. Since this regression is estimated as a panel over our entire dataset, we have a very large number of observations. Across all of these 49,731 observations, we are trying to explain the changes in liquidity in terms of changes in prices and changes in LMR controls, using only 5 parameters. The model is statistically significant and explains about 6% of the variation in liquidity changes. Therefore, even though some part of the liquidity changes are statistically significantly associated with changes in option specific parameters, there appears to be a large part of the change in liquidity that may have systematic components.

If there are no systematic components in the changes in liquidity in this market, then we should not see any structure in the residuals obtained from this regression. Any structure in these residuals would point towards a missing common systematic factor that affects liquidity changes. Therefore, we examine the principal components of the correlation matrix of these residuals across the panels. We use the correlation matrix, since principal components are sensitive to the units in which the underlying variables are measured. Using the correlation matrix instead of the covariance matrix avoids this potential error.

Since we have 27 panels, we obtain a 27x27 correlation matrix, which provides us with 27 principal components, each one of length 27. If the residuals were perfectly correlated, the first eigen value would be 27, and a single factor would explain all the variation. If the residuals were uncorrelated, all 27 eigen values would be 1. The results of this principal components analysis are presented in Table VIII. The first eigen value is 8.66, which implies that about 32% (8.66/27) of the variation in these residuals can be explained by first common factor. This is statistically significant, and indicates that about one-third of the variation in the changes in residual liquidity (those not explained by the changes in option specific variables) is accounted for by one common factor. This strongly suggests that there is a market-wide systematic component to the liquidity shocks that affect this market. The second principal component explains an additional 14% of the variation, while the third principal component and others are statistically insignificant.

The structure of these principal components (eigenvectors), especially the first one, shows some interesting patterns. The first principal component has a negative weight on all the OTM options, and a positive weight on the ATM and the ITM options. Further, the weight on the ITM options is greater than that on the ATM options. Within each moneyness bucket, the weights across maturities are relatively flat. This suggests that the market-wide systematic liquidity shock affects the OTM options differently from the ATM and the ITM options. In particular, the effect of this common liquidity shock is to widen the bid-ask spreads for the ATM and the ITM options, while at the same time narrowing the bid-ask spreads for the OTM options. This indicates a substitution effect, where the market demand, when hit by an adverse common liquidity shock, shifts away from the ATM and the ITM options to the OTM options. Since the OTM options are much cheaper than the ATM/ITM options, this finding is quite intuitive – adverse common liquidity shocks do not just dry up the liquidity of these options across the board. Instead, they shift the demand from expensive to cheaper options. The loading on the ITM options is even higher than that on the ATM options, which supports this explanation, since it implies that the reduction in demand is greater amongst the ITM options than in the ATM options. The second principal component is a parallel shock across all maturities and moneyness, while the third eigenvector and others have no significant structure.

B. Macro-economic Drivers of the Systematic Liquidity Shock

Our results, so far, indicate the presence of a significant common factor that drives liquidity changes in interest rate options across strikes and maturities. In this section, we shed light on the more primitive drivers of this systematic liquidity factor. If changes in macro-economic variables can be linked with contemporaneous or future changes in the systematic liquidity factor, this would have important implications for the measurement of liquidity risk in this market, as well as for hedging aggregate liquidity risk in a portfolio of interest rate options.

The simultaneous equation models estimated earlier in this paper indicate that changes in the uncertainty in equity and fixed income markets are associated with changes in liquidity for options, across strikes rates, for all maturities. In this section, we use the same five macro-financial variables to examine how much of the systematic liquidity factor they can explain.

We first construct a daily systematic liquidity factor based on the residuals analysis in the previous section. We use only the first principal component, which explains 32% of the residual variation in the scaled bid-ask spreads. Using the first eigenvector as weights, for each day, we estimate a weighted-average residual across the 27 maturity-strike buckets. This gives us a daily time-series of the unexplained first common factor of liquidity changes in this market. We regress this factor on contemporaneous and lagged daily changes in the five macro-financial variables (the short rate, slope of the term structure, swaption volatility, default spread, and equity market volatility). We use the Akaike information criterion to determine the appropriate number of lags to include in the regression. The results of this regression model are presented in Table IX.

We find that the incremental improvement in the explanatory power of the model is insignificant beyond the fourth lag in the macro-financial variables. These macro-financial variables together explain about 25% of the unexplained first common liquidity factor in this market. The short rate and the slope of the term structure do not appear to have much effect on contemporaneous and future values of this factor. This implies that the expectations about inflation, money supply, or general business conditions do not appear to have a significant effect on the liquidity of the options in this market. Nor do they appear to affect the prices of these options, beyond the effects dictated by the no-arbitrage conditions embedded in the option pricing models themselves.

Similarly, an increase in aggregate credit risk concerns in the economy, proxied by the default spread variable, do not appear to be related to the systematic liquidity shock that affects the fixed income options markets. In the case of option contracts, only the buyer of the option is exposed to the credit risk of the seller of the option. Since the market for these options is an OTC market, the buyers of these caps and floors are exposed to the risk of the dealers defaulting. However, there are two primary reasons why these credit risk effects do not appear to affect the liquidity in this market. First, most of the dealers in this market are investment grade institutions – in fact many of them are high investment grade firms, with very little credit risk.[18] Therefore, cap and floor buyers generally do not worry about the dealers defaulting on these contracts. Second, and more importantly, this is a dealer-driven market where most of the trades are in the form of dealers selling caps and floors to corporate clients, with the prices being set by the dealers, who have more market power than the typical buyer of these contracts. Therefore, it is not surprising to find that the dealers do not care as much about aggregate credit risk in the economy, since they are mostly on the sell side. It would be interesting to examine the impact of credit risk concerns on the liquidity of options where the buy side is as influential as the sell side in setting prices and bid-ask spreads, as would happen in exchange-traded option markets.

The uncertainty proxies, both in the fixed income and equity markets, appear to be significant drivers of this systematic liquidity shock in fixed income options markets. Lagged changes in the

[18] The institution that supplied our data, WestLB, was an AAA-rated institution, during the period of this study, since it was *de facto* guaranteed by the German Treasury.

DAX index volatility up to three days earlier, and lagged changes in the swaption volatility up to 4 days earlier, are significant in explaining the time variation in the systematic liquidity factor. However, for both of these variables, the coefficient on contemporaneous changes is either insignificant or weakly significant. The coefficients on lagged changes, with a lag of between one and four days, are mostly significant. These results indicate that the traders in fixed income option markets appear to use the levels of uncertainty in both the fixed income and equity markets to form their own expectations about future liquidity and the liquidity risk premium. When they observe a positive shock to the uncertainty in these markets, they appear to respond by increasing the price of caps and floors, while simultaneously widening the bid-ask spreads that they quote. It appears that these liquidity effects in the fixed income option markets appear between one and four days after volatility shocks are observed in the fixed income as well as equity markets.

As a robustness check, we return to the panel regression model of equation (4), and re-estimate the model, after including contemporaneous and four lags for each of the five macro-financial variables. The intuition behind this exercise is to check whether the macro-financial variables, identified as being related to the unexplained systematic variation in scaled bid-ask spreads, do indeed help in explaining the variation in the scaled bid-ask spreads of these options across strike rates and maturities. We find that this augmented panel regression model explains about 12.7% of the variation in the scaled bid-ask spreads, up from about 6% that was explained only by the changes in option specific variables. Therefore, introducing contemporaneous and lagged changes in these macro-financial variables more than doubles the explanatory power of the panel regression model, which attempts to jointly model the time-variation in the scaled bid-ask spreads across different strike rates and maturities using nearly 50,000 observations. Further, the first principal component of the correlation matrix of residuals from this augmented panel regression model accounts for about 14.5% of the variation in the residuals. While statistically significant, it is much lower than the 32% explained by the first principal component of residuals from the panel regression model without the macro-financial variables. Therefore, adding the macro-financial variables explains a large part of the common factor that drives liquidity in this market. In fact, the macro-financial variables, especially the volatilities in the equity and fixed income markets, remove most of the structure in the part of the variation in liquidity in caps and floor unexplained by changes in option-specific variables. The remaining unexplained variation in the liquidity of these options is largely unsystematic.

Our results in this section indicate a certain level of predictability in the systematic liquidity shock that affects the fixed income option markets. This has important implications for the measurement of liquidity risk in this market, as well as for the hedging of aggregate liquidity risk in portfolios of caps and floors. From a risk measurement perspective, since the liquidity factor in this market is related to lagged changes in volatilities, a GARCH-type model could be used for forecasting the volatility in equity and fixed income markets, and the systematic liquidity shock estimated based on these volatility shocks. From a risk-hedging perspective, institutions holding portfolios of caps and floors could construct macro hedges against the liquidity risk in these options by taking appropriate positions in the volatilities in equity and fixed income markets. For both of these objectives, it is crucial to understand the extent of commonality in liquidity in this market, and the primitive structure of this systematic liquidity factor.

IV. Conclusions

The liquidity of an asset has an important influence on its market price. This influence has been analyzed extensively in the U.S. equity markets, and, to a lesser extent, in the U.S. treasury, corporate bond, and some foreign exchange option markets, in recent years. Two important facts have emerged from these investigations – illiquidity suppresses the price of an asset, resulting in higher expected return, and there is a common factor in liquidity across various assets, covariation with which may be a systematic risk factor.

In contrast to this work on the underlying stock and bond markets, there is very little work on the influence of liquidity in the derivatives market. This gap is striking for three reasons. First, derivatives markets are an important segment of the global financial markets, and thus need to be taken into account, in assessing overall liquidity in financial markets. Second, as pointed out in the introduction, the effect of liquidity on the prices of derivatives is, by no means, clear cut. With zero net supply, both the buyers and sellers of derivatives are exposed to its illiquidity. Depending on the risk exposures and the hedging needs of either side, the prices of illiquid derivatives could be higher or lower, as compared to the prices of derivatives that are more liquid. Third, the interest rate derivatives market is an OTC market, where the counter-parties have to take into account the risk of default on the contract. In times of crisis, this risk becomes high and may have an influence on the liquidity in these markets.

We shed light on this important question and find that more illiquid interest rate options are more *expensive*. Thus, this result is in sharp contrast to earlier findings in the stock/bond markets and some exchange traded currency option markets. As our results indicate, the relationship between illiquidity and asset prices cannot be generalized based on evidence from just the stock and the bond markets.

Our second result, on the commonality in liquidity across options, is similar to what has been found in other markets. We find that there is a significant common component to the liquidity of interest rate options at various strike prices and maturity. We also find that this common movement is explained by the shocks to the volatility in the equity and interest rate markets. An increase in uncertainty in the equity and interest rate markets appears to cause a negative liquidity shock in the interest rate options market. In terms of more primitive macro-economic factors, it is not the expectations about inflation or growth that seem to affect the liquidity in interest rate options – it is the uncertainty about these expectations that affects the liquidity in this market. Interestingly, this systematic liquidity shock causes shifts in the demand for these options towards cheaper OTM options, and away from relatively more expensive ATM and ITM options.

Our results have important implications for the role of liquidity in the prices of derivative instruments. It would be worthwhile to explore this effect in other derivatives markets and for derivative instruments other than options, to see if this influence is similar, especially in different market settings. It would also be interesting to focus on crisis periods, such as the aftermath of the Russian default in 1998 and the LTCM failure that followed thereafter, to examine the issue of liquidity in such an extreme scenario. A related question that has not been explored in the literature so far is the interplay between the liquidity effects in the underlying asset market versus the market for derivatives. The key question is whether and how the commonality in the liquidity factor affects the interactions and the lead-lag relationships between these two markets. We leave these questions for future research.

Appendix: Implied Volatility in the Black Model for Caps and Floors

The standard model used for dealer quotations for interest rate caps and floors is the Black (1976) model of pricing of options on futures/forward contracts. The model is a variant of the basic Black and Scholes (1973) option-pricing model. Applied to the interest rate option context, the model assumes that interest rates are lognormally distributed and relates the price of a European call option (*C*) and a put option (P), at time *0*, on an interest rate forward rate agreement (FRA) to the underlying variables as follows:[19]

$$
\begin{aligned}
C &= \left[fN(d_1) - kN(d_2) \right] \times m \times B_{0,t+m} \\
P &= \left[-kN(-d_2) - fN(-d_1) \right] \times m \times B_{0,t+m} \\
d_1 &= \frac{\ln f/k + t\sigma^2/2}{\sqrt{t}\sigma} \\
d_2 &= d_1 - \sqrt{t}\sigma
\end{aligned}
\tag{A.1}
$$

where

f = forward interest rate for the period t to $t+m$,

σ = annualized volatility of the forward interest rate t on the maturity date,

m= the maturity period of the underlying loan,

t = maturity date of the option,

k = strike rate of the option,

$B_{0,t+m}$= the zero bond price at time *0*, for the bond maturing at date $t+m$.

Of course, the key variable in the above equations, which is not observable, but about which market participants may have differing views, is the volatility. Interest rate option quotations are usually for the implied volatility that reflects the market price, rather than the price directly.

An interest rate cap (floor) is a collection of caplets (floorlets). A caplet (floorlet), in turn, is a single European call (put) option on a reference interest rate, expiring on a specific date. Hence, a cap (floor) can be regarded as a portfolio of European call (put) options on interest rates, or equivalently, put (call) options on discount bonds. Typically, an interest rate cap is an agreement between a cap writer and a buyer (for example, a borrower) to limit the latter's floating interest payments to a specific level for a given period of time. The cap is structured on a specific reference rate (usually the 3- or the 6- month Libor (London Interbank Offer Rate) or Euribor

[19] This formula is also consistent with the model proposed by Brace, Gatarek and Musiela (1997) [BGM] and Miltersen, Sandmann and Sondermann (1997), which is popular among practitioners. BGM derive the processes followed by market quoted rates within the HJM framework, and deduce the restrictions necessary to ensure that the distribution of market quoted rates of a *given* tenor under the risk-neutral forward measure is lognormal. With these restrictions, caplets of that tenor satisfy the Black (1976) formula for options on forward/futures contracts.

(Euro Interbank Offer Rate)) at a predetermined strike level. The reference rate is reset at periodic intervals (usually 3- or 6- months). In a similar manner, an interest rate floor contract sets a *minimum* interest rate level for a floating rate *lender*. The cap and floor contracts are defined on a pre-specified principal amount.[20]

A caplet with maturity t_i and strike rate k, pays at date t_i, an amount based on the difference between the rate (r_i) at time t_i and the strike rate, if this difference is positive, and zero otherwise. The amount paid is based on the notional amount and the reset period of the caplet and is paid on a discounted basis at time t_i. The payoff of this caplet at date t_i on a notional principal of €A is:

$$c_{t_i} = A(t_{i+1} - t_i)\max\left[\frac{r_i - k}{1 + r_i(t_{i+1} - t_i)}, 0\right] \quad \text{(A.2)}$$

The payoff from a floorlet can be described in a similar manner.

Since the interest rate over the first period is known, there is no caplet corresponding to the first period of the cap. For example, a 2-year cap on the 6-month Euribor rate, with 4 semiannual periods over its life, would consist of 3 caplets, the first one expiring in 6 months, and the last one in 1 year and 6 months. Thus, the underlying interest rate for the first period is the 6-month Euribor rate on the date 6 months from initiating the cap contract.

Each caplet or floorlet has to be valued separately, using a valuation model such as the Black or BGM model in equation (1), (the same model that is generally used by the market for quotation purposes), with the price of the cap or floor being the sum of these prices. The volatilities used for each caplet or floorlet, which are generally different, across strike rates and maturities, are sometimes called *spot volatilities*. The market quotation for interest rate caps and floors, however, is based on the *same* volatility for all the caplets in a particular cap (or the floorlets in a particular floor). In other words, the market price of a cap (or floor) can be derived by plugging in this constant volatility for all the component caplets (or floorlets) in the contract. This constant volatility is referred to as the *flat volatility* for the particular cap (or floor) and varies with the maturity of the contract. Since caps are portfolios of caplets, the implied flat volatilities of caps reflect some average of the implied spot volatilities of individual caplets. In this paper, our primary objective is to examine liquidity effects in interest rate options. For doing that, we need to focus on traded assets, which are caps and floors. Therefore, we use the flat volatilities of caps and floors, since spot volatilities would correspond to caplets and floorlets, which are, untraded assets. We also checked the prices of the individual caplets/floorlets, which are obtained by "bootstrapping" and found that our results are broadly similar.

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[20] Interest rate caps and floors for various maturities and reference rates in all the major currencies are traded in the over-the-counter (OTC) markets. The most common reference rate is the 3-month Libor for USD caps/floors, and the 6-month Euribor in the euro markets.

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Table I

Descriptive Statistics for Cap and Floor Prices

This table presents descriptive statistics on euro (€) interest rate cap and floor prices, across maturities and strike rates, over the sample period Jan 99 - May 01, obtained from the WestLB Global Derivatives and Fixed Income Group. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All prices are averages, reported in basis points, with the standard deviations of these prices in parenthesis.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	2.1	11.1	43.2	107.7	250.5	250.5	153.7	55.5	13.6	3.6
	(0.5)	(5.8)	(19.8)	(30.9)	(58.8)	(48.1)	(50.7)	(25.4)	(7.9)	(2.0)
3-year	10.7	37.7	91.9	209.6	481.3	529.1	285.3	111.3	32.7	6.9
	(10.0)	(20.0)	(33.8)	(52.3)	(133.4)	(114.2)	(74.7)	(44.6)	(18.0)	(4.6)
4-year	22.3	72.6	152.7	311.3	674.4	728.3	406.4	176.1	62.1	12.0
	(12.5)	(32.2)	(49.7)	(78.3)	(193.1)	(138.7)	(98.9)	(64.8)	(27.8)	(7.9)
5-year	42.7	119.4	221.7	409.1	872.3	910.8	519.5	244.7	94.3	19.2
	(16.3)	(48.6)	(67.2)	(95.4)	(252.2)	(161.2)	(122.5)	(84.5)	(35.2)	(13.9)
6-year	66.9	163.7	286.6	507.9	1,006.6	1,093.1	663.8	323.7	128.6	27.2
	(20.2)	(64.4)	(84.6)	(109.5)	(257.4)	(173.2)	(133.1)	(101.9)	(43.5)	(18.7)
7-year	93.7	210.9	355.8	610.8	1206.4	1,239.0	809.3	393.3	164.1	36.9
	(25.4)	(82.2)	(99.3)	(125.3)	(275.5)	(147.0)	(127.5)	(115.2)	(51.9)	(33.0)
8-year	123.9	264.2	433.2	706.8	1,248.2	1,284.7	924.7	425.2	199.2	46.8
	(31.4)	(98.1)	(115.9)	(162.8)	(253.4)	(120.8)	(139.3)	(108.3)	(59.6)	(32.8)
9-year	152.1	309.6	509.9	811.8	1,310.3	NA	997.1	482.3	235.0	58.9
	(35.6)	(103.2)	(128.7)	(172.2)	(205.3)		(150.2)	(120.9)	(69.6)	(41.5)
10-year	179.6	347.8	598.0	881.3	1,493.4	NA	815.5	541.7	242.9	71.3
	(39.8)	(106.7)	(140.0)	(153.4)	(275.3)		(31.1)	(139.6)	(61.9)	(50.1)

Table II

Scaled Bid-Ask Spreads for Caps and Floors

This table presents summary statistics on the bid-ask spreads for euro (€) interest rate caps and floors, scaled by the average of the bid and ask prices for the options, across strike rates, for different maturities. The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from the WestLB Global Derivatives and Fixed Income Group. The caps and floors are grouped together by moneyness into five categories. The moneyness for these options is expressed in terms of the Log Moneyness Ratio (LMR), defined as the log of the ratio of the par swap rate to the strike rate of the cap/floor. All the spreads are averages, reported as percentages, with the standard deviations of the scaled spreads in parenthesis.

Maturity	Caps					Floors				
	Deep OTM	OTM	ATM	ITM	Deep ITM	Deep ITM	ITM	ATM	OTM	Deep OTM
	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3	LMR < -0.3	-0.3 < LMR < -0.1	-0.1 < LMR < 0.1	0.1 < LMR < 0.3	LMR > 0.3
2-year	80.9%	32.4%	14.7%	7.1%	3.8%	2.5%	4.5%	13.3%	30.8%	77.2%
	(21.2%)	(14.3%)	(4.8%)	(2.4%)	(0.5%)	(1.3%)	(1.3%)	(7.9%)	(11.7%)	(24.1%)
3-year	44.2%	19.0%	11.4%	7.0%	3.8%	2.9%	4.7%	11.2%	31.6%	72.0%
	(22.9%)	(5.7%)	(3.2%)	(2.5%)	(0.6%)	(1.1%)	(1.1%)	(6.1%)	(18.1%)	(25.2%)
4-year	26.1%	14.4%	9.1%	6.2%	4.1%	2.9%	4.5%	8.4%	22.2%	59.9%
	(9.4%)	(4.7%)	(2.5%)	(2.2%)	(1.0%)	(1.0%)	(1.0%)	(2.5%)	(14.5%)	(28.7%)
5-year	20.0%	12.6%	8.6%	6.1%	4.1%	3.1%	4.7%	8.2%	19.8%	59.5%
	(5.5%)	(3.9%)	(2.3%)	(2.1%)	(0.9%)	(1.0%)	(1.1%)	(2.3%)	(13.2%)	(27.4%)
6-year	18.3%	12.1%	8.5%	5.7%	4.1%	3.3%	4.7%	7.9%	15.8%	50.2%
	(4.8%)	(3.6%)	(2.2%)	(1.4%)	(0.9%)	(0.9%)	(1.2%)	(2.0%)	(7.5%)	(24.6%)
7-year	17.6%	11.5%	8.4%	5.5%	4.1%	3.4%	4.6%	7.8%	14.0%	45.3%
	(4.4%)	(3.4%)	(2.1%)	(1.3%)	(3.9%)	(0.9%)	(1.1%)	(1.9%)	(5.0%)	(24.6%)
8-year	17.1%	11.1%	8.3%	5.6%	4.0%	3.2%	4.5%	8.1%	14.0%	42.3%
	(3.8%)	(3.3%)	(2.0%)	(1.1%)	(0.3%)	(1.0%)	(1.1%)	(2.0%)	(5.1%)	(21.9%)
9-year	17.1%	11.0%	8.3%	6.0%	4.2%	NA	4.8%	8.3%	14.0%	40.0%
	(3.4%)	(3.1%)	(1.9%)	(0.7%)	(0.3%)		(1.0%)	(2.0%)	(5.2%)	(20.8%)
10-year	17.1%	11.2%	7.9%	6.2%	4.1%	NA	4.7%	8.1%	14.9%	38.6%
	(2.9%)	(3.0%)	(1.8%)	(0.6%)	(0.3%)		(1.2%)	(2.2%)	(5.5%)	(20.6%)

Table III

Functional Form for Controlling Strike Rate Effects

This table presents regression results when the scaled implied flat volatility for euro (€) interest rate caps and floors, for various maturities, is regressed on a quadratic function of the Log Moneyness Ratio (LMR) with an asymmetric slope term, as follows:

$$Scaled\ IV = c1 + c2 * LMR + c3 * LMR^2 + c4 * 1_{LMR<0} * LMR$$

The statistics are presented for the entire sample period, Jan 99 - May 01, based on data obtained from the WestLB Global Derivatives and Fixed Income Group. The coefficients and regression statistics are presented based on data for caps and floors pooled together, for all maturities. The asterisk implies significance at the 5% level.

Maturity	c1	c2	c3	c4	Adj R^2
2-year	1.06*	-1.27*	3.67*	0.99*	0.58
3-year	1.09*	-0.82*	2.12*	1.07*	0.51
4-year	1.08*	-0.61*	1.69*	0.91*	0.60
5-year	1.05*	-0.02*	0.77*	-0.05	0.58
6-year	1.01*	0.40*	0.08*	-0.69*	0.48
7-year	1.02*	0.55*	-0.10*	-0.83*	0.42
8-year	1.00*	0.38*	-0.11*	-0.44*	0.54
9-year	1.00*	0.32*	-0.05*	-0.36*	0.58
10-year	1.06*	0.34*	-0.04*	-0.27*	0.64

Table IV

Determinants of Implied Volatility and Bid-Ask Spreads in Caps and Floors

This table presents the results for a simultaneous equation model where scaled implied volatility of euro (€) interest rate caps and floors and scaled bid-ask spreads are determined endogenously as a function of each other and other exogenous variables, based on data obtained from the WestLB Global Derivatives and Fixed Income Group, for the entire sample period, Jan 99 - May 01:

$$ScaledIV = c1 + c2 * ScaledBAS + c3 * LMR + c4 * LMR^2 + c5 * \left(1_{LMR<0}.LMR\right) + c6 * 6Mrate + c7 * Slope + c8 * SwpnVol$$

$$ScaledBAS = d1 + d2 * ScaledIV + d3 * DefSprd + d4 * DAXVol$$

ScaledIV is implied volatility of the mid-price of the cap/floor scaled by ATM implied volatility. ScaledBAS is bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of swap rate to the strike rate of the option. 6Mrate is the 6 month Euribor rate. Slope is the difference between the 5-year and 6-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the 6-month Euribor and the 6-month Treasury rate. DAXVol is the volatility of the DAX index.

Panel A: ScaledIV as dependent variable

Maturity	c1	c2	c3	c4	c5	c6	c7	c8	Obs	R^2
2-year	0.625	0.025**	-0.826**	2.826**	0.037	1.095	1.838*	1.792**	3313	0.251
3-year	1.243**	0.093**	-0.194**	1.494**	0.030	1.728	1.582**	1.927**	6294	0.489
4-year	1.310**	0.376**	-0.026	1.216**	0.040**	0.941**	1.058**	1.896**	6592	0.691
5-year	1.392**	0.272**	0.171**	0.996**	0.022**	1.583**	-0.821	1.836**	6990	0.647
6-year	1.076**	0.422**	0.045**	0.249**	0.009**	0.588*	-0.676	0.427**	6758	0.611
7-year	1.156**	0.262**	0.295**	20123**	0.038**	1.452**	-11.276	2.563**	6195	0.615
8-year	1.029**	0.431**	0.080**	-0.009	0.011**	0.253	-0.273	0.254**	5341	0.435
9-year	1.034**	0.245**	0.129**	0.042	0.014**	1.115*	0.480	0.535**	4994	0.643
10-year	1.231**	0.281**	0.225**	0.409**	0.038**	1.049**	-5.812	0.958**	4147	0.433

Panel B: ScaledBAS as dependent variable

Maturity	d1	d2	d3	d4	Obs	R^2
2-year	0.203**	0.006*	0.000	0.001**	3313	0.060
3-year	0.066**	0.150**	0.000	0.003**	6294	0.103
4-year	-0.121**	0.297**	0.000	0.003**	6592	0.172
5-year	-0.501**	0.664**	0.000	0.004**	6990	0.310
6-year	-1.103**	1.141**	0.000**	0.001**	6758	0.533
7-year	-0.972**	1.029**	0.000	0.002**	6195	0.373
8-year	-1.713**	1.760**	0.000	0.001**	5341	0.196
9-year	-1.722**	1.771**	0.000	0.001*	4994	0.395
10-year	-1.215**	1.322**	0.001**	0.004**	4147	0.412

** implies significance at the 5% level; * implies significance at the 10% level.

Table V

Determinants of Changes in Implied Volatility and Bid-Ask Spreads

This table presents the results for a simultaneous equation model where daily changes in scaled implied volatility of euro (€) interest rate caps and floors and daily changes in scaled bid-ask spreads are determined endogenously as a function of each other and changes in other exogenous variables, based on data obtained from the WestLB Global Derivatives and Fixed Income Group, for the entire sample period, Jan 99 - May 01:

$$\Delta ScaledIV = c1 + c2 * \Delta ScaledBAS + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + c6 * \Delta 6Mrate + c7 * \Delta Slope + c8 * \Delta SwpnVol$$

$$\Delta ScaledBAS = d1 + d2 * \Delta ScaledIV + d3 * \Delta DefSprd + d4 * \Delta DAXVol$$

ScaledIV is implied volatility of the mid-price of the cap/floor scaled by ATM implied volatility. ScaledBAS is bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of swap rate to the strike rate of the option. 6Mrate is the 6 month Euribor rate. Slope is the difference between the 5-year and 6-month Euribor rates. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. DefSprd is the difference between the 6-month Euribor and the 6-month Treasury rate. DAXVol is the volatility of the DAX index.

Panel A: Changes in ScaledIV as dependent variable

Maturity	c1	c2	c3	c4	c5	c6	c7	c8	Obs	R^2
2-year	-0.001	1.194*	-0.474	1.979	0.001	-9.857	3.242	4.377*	3089	0.027
3-year	0.000	6.928*	0.056	2.788	0.010	-4.198	-1.605	7.550**	5902	0.081
4-year	0.002	6.176*	-0.061	1.394	0.005	-5.296	-0.848	6.385**	6213	0.089
5-year	0.000	1.421**	7.750*	-0.256	-0.014*	-6.575*	-1.764*	5.270**	6589	0.100
6-year	0.000	0.397**	0.276	0.359	-0.001	-5.724	-6.500	1.997*	6371	0.106
7-year	0.000	1.290**	2.384**	0.301*	-0.003	-4.791**	-3.187**	4.101*	5849	0.187
8-year	0.000	1.851*	1.843	-0.524	-0.001	-2.608	-6.640	1.802*	5027	0.177
9-year	-0.002	1.209**	1.972	-3.369	0.033**	-3.568	-1.842	1.618	4697	0.139
10-year	0.000	0.310**	0.928**	-0.079	-0.004	-6.818**	-9.038**	4.814**	3898	0.282

Panel B: Changes in ScaledBAS as dependent variable

Maturity	d1	d2	d3	d4	Obs	R^2
2-year	0.000	0.033*	0.000	0.001	3089	0.004
3-year	0.000	0.123**	0.000	0.001*	5902	0.023
4-year	0.000	0.097**	0.000	0.001*	6213	0.049
5-year	0.000	0.055**	0.000	0.001**	6589	0.054
6-year	0.000	0.204**	0.000	0.001*	6371	0.049
7-year	0.000	0.122**	0.000	0.001**	5849	0.083
8-year	0.000	0.124**	0.000	0.001*	5027	0.060
9-year	0.000	0.201**	0.000*	0.000*	4697	0.043
10-year	0.000	0.122**	0.000**	0.001**	3898	0.088

** implies significance at the 5% level; * implies significance at the 10% level.

Table VI

Correlations Amongst Scaled Bid-Ask Spreads

This table presents average time-series correlations between scaled bid-ask spreads across moneyness buckets (at-the-money – ATM, in-the-money – ITM and out-of-the-money – OTM) of euro (€) interest rate caps and floors. The numbers below are averaged across the correlations between the nine maturities within each moneyness bucket. For example, the OTM/OTM value is the average correlation between the nine maturity buckets within OTM options (so it is an average of 9x8/2, i.e., 36 correlations). The OTM / ITM value is the average correlation between each maturity bucket within OTM options with the corresponding maturity within ITM options (so it is an average of 9x9, i.e. 81 correlations), and so on. The correlations are calculated for the entire sample period, Jan 99 - May 01, based on data obtained from the WestLB Global Derivatives and Fixed Income Group.

	Average Correlations		
	OTM	ATM	ITM
OTM	0.68		
ATM	0.34	0.86	
ITM	0.24	0.65	0.78

Table VII

Determinants of Changes in Bid-Ask Spreads

This table presents results of a panel regression of changes in scaled bid-ask spreads on changes in scaled implied volatility of mid-price and changes in the moneyness variables of euro (€) interest rate caps and floors.

$$\Delta ScaledBAS_{it} = c1 + c2 * \Delta ScaledIV_{it} + c3 * \Delta LMR_{it} + c4 * \Delta LMR^2{}_{it} + c5 * \Delta\left(1_{LMR<0}.LMR\right)_{it} + \varepsilon_{it}$$

Scaled IV is implied volatility of mid-price of cap / floor scaled by at-the-money implied volatility. Scaled BA is bid-ask spread scaled by the mid-price. LMR (Log Moneyness Ratio) is the log of the ratio of swap rate to the strike rate of cap / floor. Δ indicates first difference. There are 27 groups in the panel 9 maturities (2 year to 10 year) X 3 moneyness groups (at-the-money, out-of-the-money and in-the-money) for each maturity. The table presents GLS estimates for the entire sample period, Jan 99 - May 01, based on data obtained from the WestLB Global Derivatives and Fixed Income Group.

	c1	c2	c3	c4	c5	Obs	Adj R^2	p-value for F statistic
Coefficient	0.000	0.068	0.002	-0.001	0.000	49,731	0.06	0.005
t-stats	1.68	2.18	1.97	-0.66	1.07			

Table VIII

Commonality in Changes in Bid-Ask Spreads

This table presents the structure of the principal components of the correlation matrix of the residuals obtained from the panel regression:

$$\Delta ScaledBAS_{it} = c1 + c2 * \Delta ScaledIV_{it} + c3 * \Delta LMR + c4 * \Delta LMR^2 + c5 * \Delta\left(1_{LMR<0}.LMR\right) + \varepsilon_{it}$$

ScaledIV is the implied volatility of the mid-price of the option scaled by the ATM implied volatility. ScaledBAS is the bid-ask spread scaled by the mid-price. LMR is the logarithm of the ratio of swap rate to the strike rate of the option. Δ indicates first difference. There are 27 groups in the panel (i=1 to 27) consisting of 9 maturities (2 years to 10 years) for the 3 moneyness groups (OTM/ATM/ITM). The regression is estimated by for the entire sample period, Jan 99 - May 01, based on data on euro (€) interest rate caps and floors obtained from the WestLB Global Derivatives and Fixed Income Group.

		Principal Component		
		1	2	3
Eigen value		8.66	3.74	1.66
% explained		0.32	0.14	0.06
Moneyness	Maturity	Eigenvectors		
OTM	2-year	-0.04	0.05	0.23
OTM	3-year	-0.03	0.17	0.16
OTM	4-year	-0.04	0.24	0.08
OTM	5-year	0.03	0.19	0.16
OTM	6-year	-0.03	0.18	-0.04
OTM	7-year	-0.02	0.21	0.00
OTM	8-year	-0.05	0.17	0.07
OTM	9-year	-0.06	0.21	0.11
OTM	10-year	-0.07	0.15	-0.11
ATM	2-year	0.02	0.13	0.1
ATM	3-year	0.11	0.25	0.05
ATM	4-year	0.12	0.3	-0.04
ATM	5-year	0.14	0.3	0.14
ATM	6-year	0.07	0.26	-0.15
ATM	7-year	0.18	0.34	0.02
ATM	8-year	0.2	0.09	-0.25
ATM	9-year	0.19	0.12	-0.45
ATM	10-year	0.3	0.17	-0.24
ITM	2-year	0.2	0.03	0.16
ITM	3-year	0.27	0.05	0.33
ITM	4-year	0.34	0.08	0.28
ITM	5-year	0.33	0.1	0.3
ITM	6-year	0.2	0.31	0.07
ITM	7-year	0.36	0.11	0.11
ITM	8-year	0.25	0.24	-0.25
ITM	9-year	0.23	0.16	-0.27
ITM	10-year	0.33	0.11	-0.12

Table IX

Macro-Economic Determinants of the Systematic Liquidity Factor

This table presents the results of the regression of the first principal component of the correlation matrix of residuals (from the panel regression in Table 5) on contemporaneous and lagged changes in macro-financial variables. 6Mrate is the 6 month Euribor. Slope is the difference between the 5 year and 6 month Euribor rates. DefSprd is the difference between 6m Euribor and the 6 month Treasury rate. DAXVol is the volatility of DAX index. SwpnVol is the implied volatility of at-the-money swaption of comparable maturity. The regression is estimated by for the entire sample period, Jan 99 - May 01, based on data on euro (€) interest rate caps and floors obtained from the WestLB Global Derivatives and Fixed Income Group.

	Coefficient.	t-stats
Constant	0.03	0.19
6Mrate		
Contemporaneous	0.22	0.04
Lag 1	0.85	0.24
Lag 2	4.15	0.87
Lag 3	3.65	0.98
Lag 4	6.17	1.47
Slope		
Contemporaneous	0.36	0.09
Lag 1	0.97	2.37
Lag 2	1.99	1.67
Lag 3	5.92	1.01
Lag 4	1.35	1.20
DefSprd		
Contemporaneous	-3.41	-1.31
Lag 1	-2.79	-1.01
Lag 2	1.57	0.52
Lag 3	2.65	0.87
Lag 4	1.17	1.50
DAXVol		
Contemporaneous	0.21	1.82
Lag 1	0.10	2.66
Lag 2	0.12	2.84
Lag 3	0.14	2.09
Lag 4	0.02	1.18
SwpnVol		
Contemporaneous	0.08	0.55
Lag 1	0.26	1.79
Lag 2	0.38	1.98
Lag 3	0.11	2.60
Lag 4	0.10	2.30
R^2		0.25
p-value for F-stat		0.0034

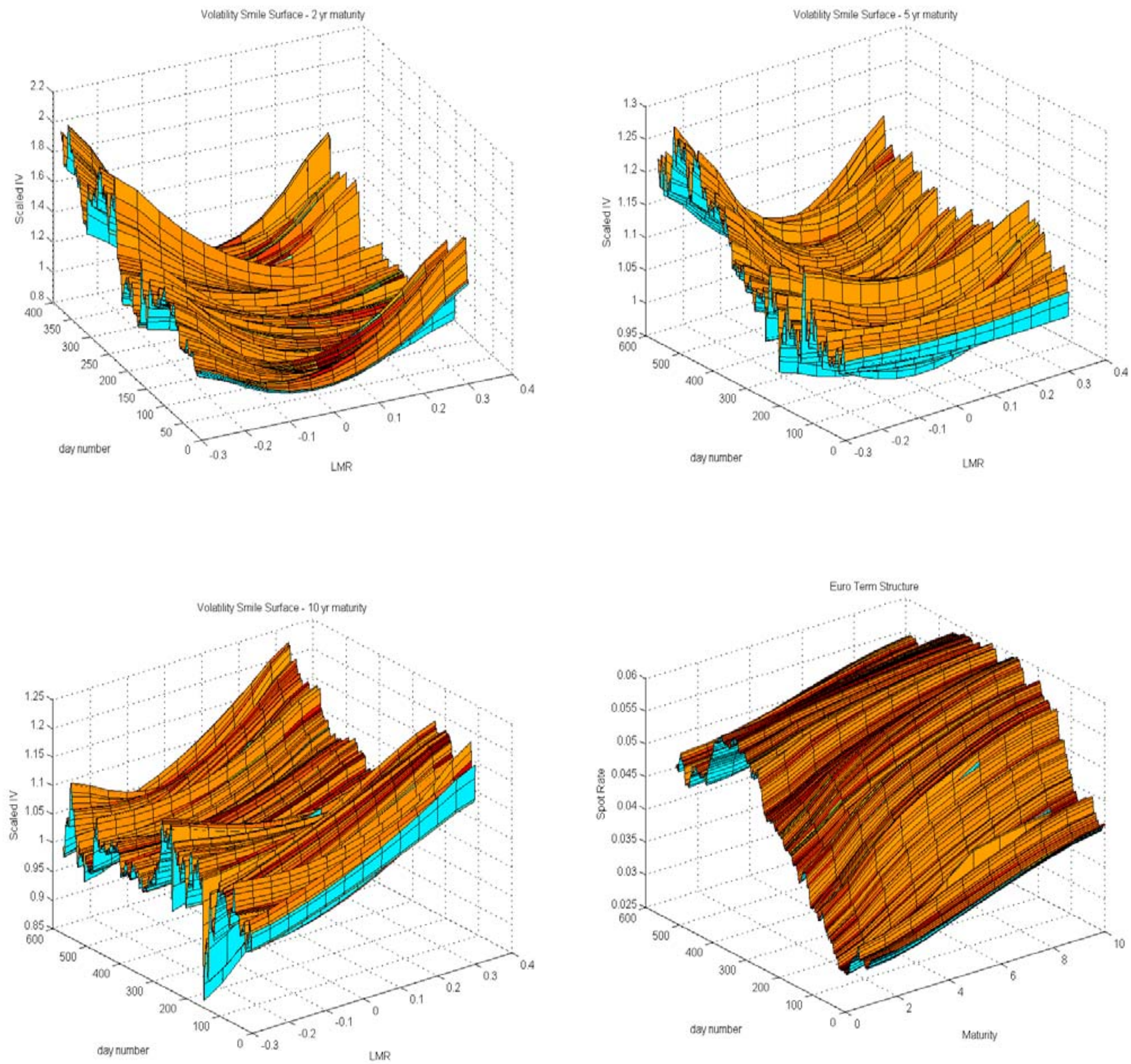

Figure 1. Time variation in volatility smiles and the Euro term structure. This figure presents surface plots showing the time variation in the implied flat volatilities of euro (€) interest rate caps and floors as well as the Euro term structure over our sample period (Jan 99 - May 01), using data obtained from the WestLB Global Derivatives and Fixed Income Group. The first three plots (for three representative maturities - 2-year, 5-year, and 10-year), the vertical axis corresponds to the implied volatility of the mid-price (average of bid and ask price) of the option, scaled by the at-the-money volatility for the option of similar maturity. The horizontal axes in these plots correspond to the logarithm of the moneyness ratio (defined as the ratio of the par swap rate to the strike rate of the option), and time. The fourth plot depicts the Euro spot rate surface by maturity (in years) over time (daily).

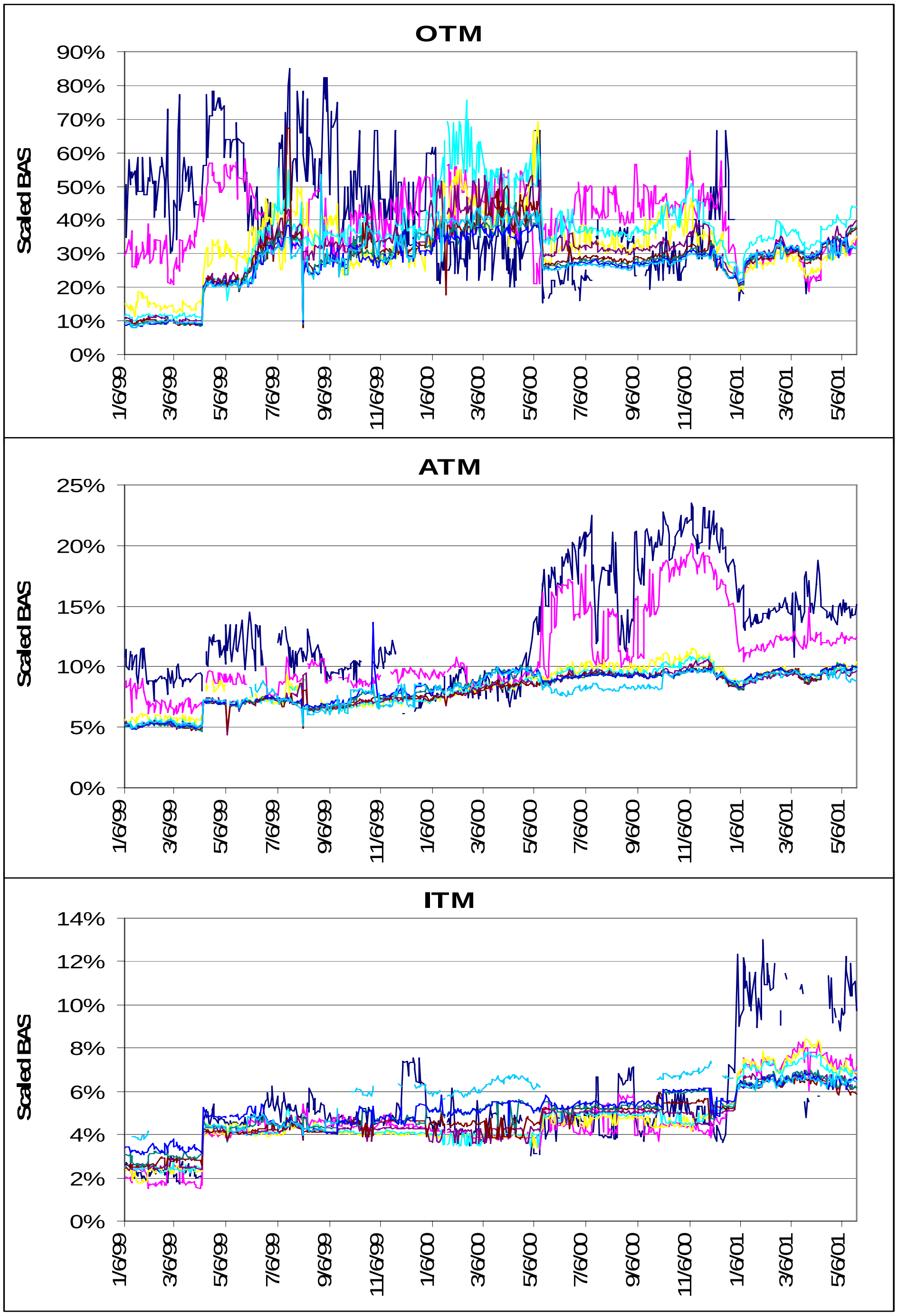

Figure 2. Time variation in scaled bid-ask spreads. This figure presents the time-series plots of the scaled bid-ask spreads of euro (€) caps and floors for each of the nine maturities (from 2 years to 10 years), separately by moneyness, over our sample period (Jan 99 - May 01) using data obtained from the WestLB Global Derivatives and Fixed Income Group. Each plot has nine time series representing the nine option maturities. The OTM bucket contains caps with LMR less than -0.1 and floors with LMR greater than 0.1. The ITM bucket contains caps with LMR greater than 0.1 and floors with LMR less than -0.1. The ATM bucket contains caps and floors with LMR between -0.1 and 0.1.

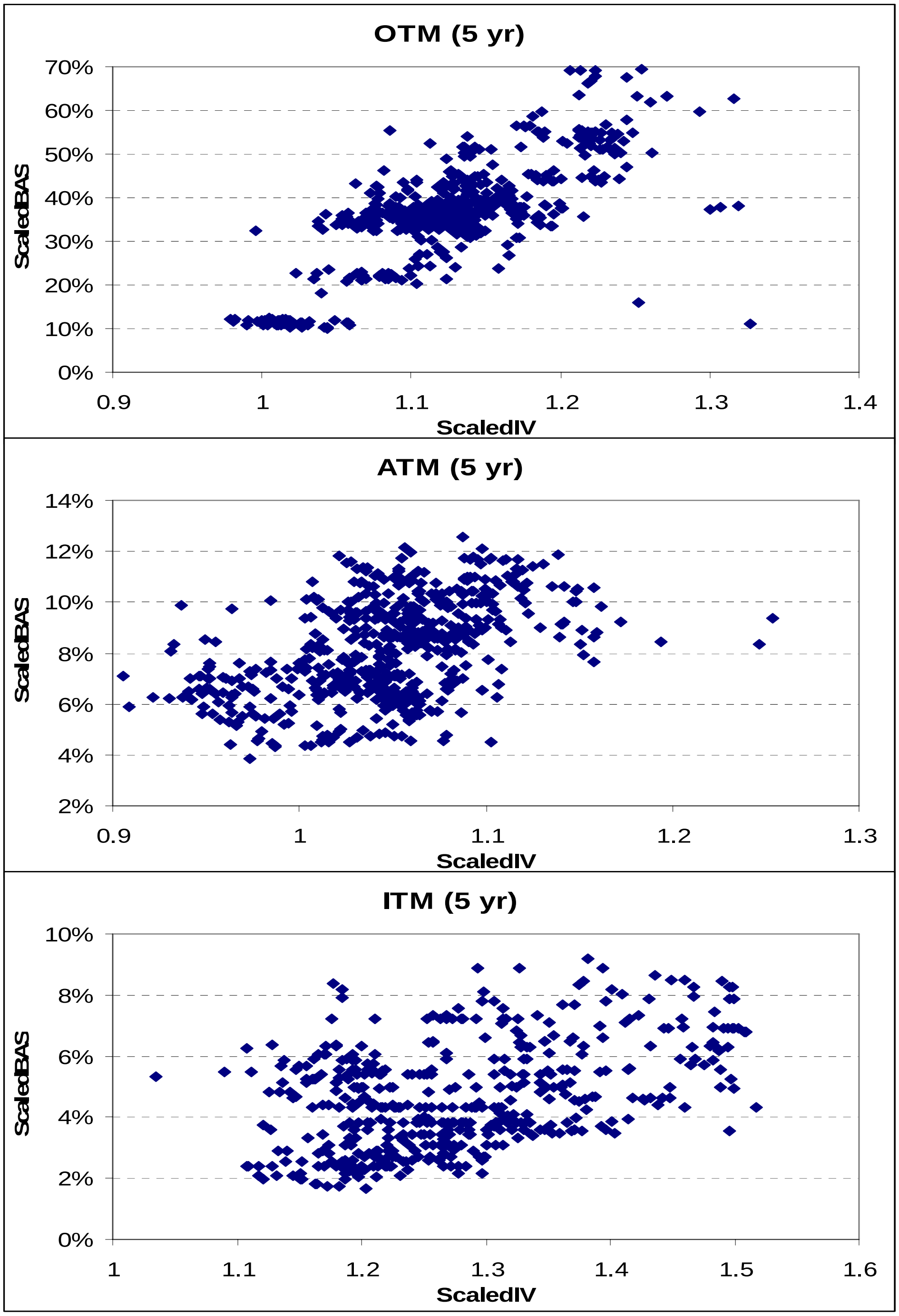

Figure 3. Plots of liquidity versus price. This figure presents three sample scatter plots showing the relationship between the ScaledBAS and the ScaledIV for 5-year maturity caps and floors, separately for OTM, ATM and ITM options. The plots for other maturities are similar. The plots are constructed using data for euro (€) interest rate caps and floors over our sample period (Jan 99 - May 01), obtained from the WestLB Global Derivatives and Fixed Income Group.