The Life-Cycle Effects of House Price Shocks

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ABSTRACT

We develop a life-cycle model to study the effects of house price shocks on household consumption and welfare. The model explicitly incorporates the dual feature of housing as both a consumption good and an investment asset and allows for costly housing adjustment and mortgage refinance. Our analysis suggests that the effects of house price changes on household consumption and welfare crucially depend on a household's age and house tenure status. In particular, the non-housing consumption of young and old homeowners are more sensitive to house price changes than that of middle-aged homeowners. More importantly, although house price appreciation increases consumption of all homeowners, it only improves the welfare of middle-aged and old homeowners. Young homeowners and renters are worse off. The consumption and welfare consequences of housing wealth gains contrast sharply with those obtained for liquid asset gains, where consumption and welfare increase for all households.

Key Words: Life-cycle Model, Consumption, Savings, Housing, Mortgage
JEL Classification Codes: E21, R21

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1. Introduction

U.S. house prices have appreciated significantly in recent years. At the national level, house prices have risen at an average annual rate of 7 percent since 1998, faster than household income and major equity indices.[1] The rapid growth in house prices, accompanied by widespread home equity extraction, has been credited for supporting consumer spending during the economic recession in 2001 and its slow recovery since then.[2] Thus, recent research has focused on the implications of house price appreciation on consumption and consumer welfare. Most of the studies, however, have relied on simple formulations of the life-cycle savings hypothesis to analyze empirically the sensitivity of consumption to changes in house wealth at either the aggregate or the household level.[3] In contrast, for a typical household, residential housing plays the inseparable dual role of both consumption and investment and serves as a collateral for borrowing.

In this paper, we develop a stochastic life-cycle model to investigate the effects of house price changes on consumption and consumer welfare. In the model, a household faces uninsurable labor income risk and house price risk. It makes housing decisions along both the extensive margin of owning versus renting and the intensive margin of house value. House purchases can be financed through a fixed-rate mortgage that requires a down payment. A homeowner can refinance his mortgage after paying a one-time refinancing charge. Changing house sizes also incurs an adjustment cost that is independent of the size of the adjustment. Besides home equity, a household can also save in a riskless liquid asset.

Our model generates life-cycle consumption and wealth profiles that are consistent with the empirical evidence. In particular, the home ownership rate initially increases with the household age and then decreases after the household retires. The mortgage loan-to-value ratio declines steadily with the household age. Thus, the household accumulates home equity as it ages. Liquid savings, however, grow faster than home equity prior to the household's

[1]During the same period, personal income grew an average annual rate of 5 percent, and S&P 500 stock index grew 5 percent.

[2]For example, Greenspan remarked at the 2003 Securities Industry Association annual meeting that "Fortunately, a vibrant housing market lifted construction activity and, by facilitating home equity extraction, provided extra support to consumer spending."

[3]See, for example, Case, Quigley, and Shiller (2003), Benjamin, Chinloy, and Jud (2004), Campbell and Cocco (2004), and Hurst and Stafford (2004).

retirement, and are drawn upon first after its retirement. As a result, the home equity-total net worth ratio demonstrates a U-shape over the life cycle. Finally, mortgage refinancing activities exhibit a bimodal pattern. While young households refinance to ease liquidity concerns, old households refinance to liquefy home equity so as to delay the payment of house selling cost and higher rental cost.

We show in our simulated economy that the effects of a permanent increase in house price on consumption and welfare crucially depend on the household's age and its home ownership status. Specifically, under the assumption of Cobb-Douglas preferences, while house price appreciation does not affect a renter's non-housing consumption, it increases that of a homeowner's. Furthermore, the non-housing consumption of a young or an old homeowner is more sensitive to house price changes than that of a middle-aged homeowner. The reason is as follows. A young homeowner has a higher marginal propensity to consume out of housing wealth due to liquid constraints. By relaxing collateral borrowing constraints, house price appreciation allows the household to better smooth consumption intertemporally. An old homeowner has a short expected consumption horizon. Hence, he increases his consumption significantly after a rise in his housing wealth. A middle-aged homeowner, by contrast, has accumulated enough savings to overcome temporary liquidity constraint. He also faces a long expected life horizon. His consumption is thus least responsive to changes in house prices.

Our welfare analysis indicates that house price appreciation unambiguously lowers a renter's lifetime welfare since he suffers from higher costs in acquiring housing services and yet does not receive any housing wealth gain. Even among homeowners, housing wealth gains do not always lead to welfare improvement. Young homeowners expect to upgrade their housing services as their income increases and as their families expand. Therefore, a positive house price shock incurs net welfare losses for them since the rise in the value of their existing houses is not large enough to compensate them for the rise in their lifetime housing costs. Only middle-aged and old homeowners receive net welfare gains from housing appreciation.

The effects of housing wealth gains contrast sharply with those of liquid wealth gains. Specifically, gains in liquid wealth improve both housing and non-housing consumption and lead to welfare improvement for all households. Gains in housing wealth, however, are accompanied by higher life-cycle housing consumption cost and have significant distributional effects. The net welfare consequence depends on the household's age and its house tenure. These differences stem from the dual role of housing as both a consumption good and an investment asset.

Our paper is closely related to the life-cycle consumption literature. The earlier works by Zeldes (1989), Deaton (1991), Blundell, Browning, and Meghir (1994), and Carroll (1997), among many others, have generally focused on nondurable good consumption. More recent studies such as Fernandez-Villaverde and Krueger (2002), and Lustig and Nieuwerburgh (2004) have introduced housing to the stochastic general equilibrium life-cycle models. Both papers incorporated the collateral role of durable housing as a real asset. While our framework shares many basic elements with these models, including collateral borrowing constraints and uninsurable labor income risk, the key innovation lies in the explicit the allowance of house price risks and the modeling of transaction costs in housing adjustment and mortgage refinancing.

There is also a recent literature that examines a household's life-cycle housing, mortgage, and asset allocation decisions. These studies, however, have either ignored house tenure or mortgage choice, or modeled them in a minimal way. These simplifications generate implications inconsistent with empirical observations and preclude a complete analysis of the effects of house price changes on consumption and consumer welfare. For example, Campbell and Cocco (2003) examine a household's mortgage choice between a fixed rate loan and an adjustable rate loan. In their model, the household's house tenure and housing services are held constant and the new mortgage is assumed to be of the same size as the loan paid off. The model setups in Cocco (2004), and Yao and Zhang (2004) are closest to ours. Yet both papers assume that homeowners can refinance their mortgages costlessly as long as the collateral constraints are satisfied. These two papers focus on the effects of housing positions and house price risk on household portfolio choices. [4]

The rest of the paper is organized as follows. Section 2 introduces the model economy. Section 3 characterizes the households' consumption, housing, and mortgage decisions. Section

[4]Bajari, Benkard, and Krainer (2003) also study the welfare effects of house price changes using an equilibrium approach. But their model abstracts from housing adjustment costs as well as collateral borrowing through mortgages. Other related studies include Gervais (2002), Flavin (2001), Flavin and Yamashita (2002), Ortalo-Magne and Rady (2003), Piazzesi, Schneider, and Tuzel (2004), and Chambers, Garriga, and Schlagenhauf (2004).

4 analyzes the effects of a permanent house price shock on household consumption and welfare and contrast the results with those derived from liquid wealth gains. Section 5 concludes.

2. The Model Economy

2.1. Preferences and Endowments

We consider an economy where a household lives for at most T ($T > 0$) periods. The probability that the household lives up to period t is given by the following survival function,

$$F(t) = \prod_{j=0}^{t} \lambda_j, \quad 0 \leq t \leq T, \tag{1}$$

where λ_j is the probability that the household is alive at time j conditional on being alive at time $j-1$, $j = 0, ..., T$. We assume $\lambda_0 = 1$, $\lambda_T = 0$, and $0 < \lambda_j < 1$ for all $0 < j < T$.

The household derives utility from consuming a numeraire good C_t and housing services H_t, as well as bequeathing wealth Q_t. The within-period utility takes the following modified Cobb-Douglas functional form,

$$U(C_t, H_t; N_t) = N_t \frac{[(\frac{C_t}{N_t})^{1-\omega}(\frac{H_t}{N_t})^{\omega}]^{1-\gamma}}{1-\gamma} = N_t^{\gamma} \frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma}, \tag{2}$$

where N_t denotes the exogenously given effective family size as in Laibson, Repetto, and Tobacman (2003). The effective family size captures the economies of scale in household consumption as argued in Lazear and Michael (1980), and many others.

In each period, the household receives income Y_t. Prior to the retirement age, which is set exogenously at $t = J$ ($0 < J < T$), Y_t represents labor income and is given by

$$Y_t = P_t^Y \varepsilon_t, \text{ where } P_t^Y = \exp\{f(t, Z_t)\} P_{t-1}^Y \nu_t \tag{3}$$

is the permanent labor income at time t, and ε_t is the transitory shock to P_t^Y. P_t^Y has a deterministic component $f(t, Z_t)$, which is a function of age and household characteristics Z_t. ν_t represents the shock to permanent labor income. We assume that $\ln \varepsilon_t$ and $\ln \nu_t$ are independently and identically normally distributed with mean $\{-0.5\sigma_\varepsilon^2, -0.5\sigma_\nu^2\}$, and variances $\{\sigma_\varepsilon^2, \sigma_\nu^2\}$, respectively. Thus, $\ln P_t^Y$ follows a random walk with a deterministic drift $f(t, Z_t)$.

After retirement, the household receives an income which constitutes a constant fraction θ $(0 < \theta < 1)$ of its pre-retirement permanent labor income,

$$Y_t = \theta P_J^Y, \quad \text{for } t = J, ..., T. \tag{4}$$

2.2. Housing and Mortgage Contracts

A household can acquire housing services through either renting or owning. A renter has a house tenure $D_t^o = 0$, and a homeowner has a house tenure $D_t^o = 1$. To rent, the household pays a fraction α $(0 < \alpha < 1)$ of the market value of the rental house. To become a homeowner, the household pays a proportion ρ $(0 < \rho < 1)$ of the house value as closing cost to secure the title and the mortgage. We denote the price per unit of housing service at time t as P_t^H. $P_t^H H_t$ is then the value of the house at time t. The house price has a gross return $1 + \tilde{r}_t^H$, which follows an i.i.d. normal process with mean $1 + \mu_H - 0.5\sigma_H^2$ and variance σ_H^2. The shock to house prices is thus permanent and exogenous in our model economy.[5]

A household can finance home purchases with a mortgage. We assume that a mortgage loan initiated at time t matures at T.[6] The mortgage balance denoted by M_t needs to satisfy the following collateral constraint,

$$0 \leq M_t \leq (1 - \delta) P_t^H H_t, \tag{5}$$

[5] Flavin and Yamashita (2002), Campbell and Cocco (2004), and Yao and Zhang (2004) also assume that house price shock is i.i.d. and permanent. Case, Quigley, and Shiller (2003) explore home prices dynamics using data between 1982 and 2003. They find that home buyers' expectations are substantially affected by recent experience. Even after a long boom, home buyers typically have expectations that prices over the next 10 years will show double-digit annual price growth with a modest level of risk. Since we do not model many important factors such as immigration waves that have had significant impact on recent house price appreciation, we treat house price shocks as exogenous in our economy.

[6] This specification of mortgage loan term follows Campbell and Cocco (2003). It eliminates the time-to-maturity as a separable state variable and considerably simplifies the model solution.

where $0 \leq \delta \leq 1$.[7] The borrowing rate r is time-invariant. The mortgage balance for a newly initiated loan also needs to satisfy the following income constraint for underwriting:

$$M_t \leq \chi P_t^Y. \tag{6}$$

A homeowner is required to spend a fraction ψ ($0 \leq \psi \leq 1$) of the house value on repair and maintenance in order to keep housing quality constant.

At the beginning of each period, the household receives a moving shock, D_t^m, that takes a value of 1 if the household has to move for reasons that are absent in the model, and 0 otherwise. The moving shock does not affect a renter's housing choices since moving does not incur a cost for a renter. When a homeowner ($D_{t-1}^o = 1$) receives a moving shock ($D_t^m = 1$), he is forced to sell his house. The selling decision, D_t^s, is 1 if the homeowner sells and 0 otherwise. If the homeowner sells his house, he will incur a selling cost that is a fraction ϕ ($0 \leq \phi \leq 1$) of the market value of the existing house. Additionally, the full mortgage balance becomes due upon home sale. A homeowner can also sell his house voluntarily ($D_t^M = 0$). Following a house sale, a homeowner faces the same decisions as a renter coming into period t. We use $\bar{l}_t$ to denote the household's beginning-of-the-period mortgage loan-to-value ratio, and $\bar{l}_t = \frac{M_{t-1}(1+r)}{P_t^H H_{t-1}}$.

If the homeowner does not have to move and chooses to stay in his house, he has the option to convert some home equity to liquid wealth through a "cash-out" mortgage refinancing. D_t^r denotes the refinancing decision by the homeowner that takes a value of 1 if the homeowner refinances his mortgage, and 0 otherwise. Refinancing requires a cost that is a proportion τ ($0 < \tau < 1$) of the house price. If the household decides not to refinance, it will pay down its mortgage balance according to the fixed-rate mortgage amortization schedule set at the mortgage initiation:

$$M_t = M_{t-1}(1+r) - \frac{M_{t-1}}{\sum_{j=t}^{T}(1+r)^{t-j-1}} = \frac{1-(1+r)^{t-T}}{1-(1+r)^{t-T-1}} M_{t-1}. \tag{7}$$

The mortgage loan-to-value ratio upon mortgage initiation, mortgage payment, or refinancing is denoted as $l_t = \frac{M_t}{P_t^H H_t}$.

[7] By applying collateral constraint to both newly initiated and existing loans, we effectively rule out default on mortgages.

2.3. Liquid Assets

In addition to holding assets in home equity, a household can save in liquid assets which earn a constant riskfree rate r that is the same as the borrowing rate. By equating the lending rate and the saving rate, we thus eliminate the financial incentive for mortgage refinancing. It follows then all mortgage refinances are for consumption purposes only. Furthermore, a household who purchases a new home or refinances his mortgage will borrow the maximum amount allowed by the lender since the transaction cost is independent of the size of the loan initiated. Partial loan prepayment will not occur for the same reason. We denote the liquid savings as S_t. Non-collateralized borrowing is not allowed:

$$S_t \geq 0, \quad \text{for } t = 0, ..., T. \tag{8}$$

2.4. Wealth Accumulation and Budget Constraints

We denote the household's spendable resources or "wealth" upon house sale by Q_t.[8] It follows that for a renter ($D^o_{t-1} = 0$),

$$Q_t = S_{t-1}(1 + r) + P^Y_{t-1} \exp\{f(t, Z_t)\}\nu_t\varepsilon_t, \tag{9}$$

and for a homeowner ($D^o_{t-1} = 1$),

$$Q_t = S_{t-1}(1 + r) + P^Y_{t-1} \exp\{f(t, Z_t)\}\nu_t\varepsilon_t + P^H_{t-1}H_{t-1}(1 + \tilde{r}^H_t)(1 - \bar{l}_t - \phi). \tag{10}$$

The intertemporal budget constraint, therefore, can be written as follows:

1) For a renter at $t-1$ ($D^o_{t-1} = 0$), or a homeowner who decides to sell his house and rent in period t ($D^o_{t-1} = 1$ and $D^s_t = 1$):

$$Q_t = C_t + S_t + \alpha P^H_t H_t. \tag{11}$$

[8]Under this definition, conditional on selling his house, a homeowner's problem is identical to that of the renter, and independent of his housing and mortgage positions.

2) For a renter at $t-1$ ($D^o_{t-1}=0$), or a homeowner who decides to sell his house and buy another one in period t ($D^o_{t-1}=1$ and $D^s_t=1$):

$$Q_t = C_t + S_t + (1+\psi+\rho)P^H_t H_t - M_t. \tag{12}$$

3) For a homeowner at $t-1$ who decides to stay in the house without mortgage refinancing in period t ($D^o_{t-1}=D^o_t=1$, and $D^s_t=D^r_t=0$),

$$Q_t = C_t + S_t + \{1 - l_t + \psi - \phi\}P^H_t H_{t-1}. \tag{13}$$

4) For a homeowner at $t-1$ who decides to stay in the house for time t and refinance its mortgage ($D^o_{t-1}=D^o_t=1, D^s_t=0$, and $D^r_t=1$),

$$Q_t = C_t + S_t + \{1 - \delta + \psi + \tau - \phi\}P^H_t H_{t-1}. \tag{14}$$

2.5. The Optimization Problem

The household solves the following optimization problem at time $t=0$ given its house tenure status (D^o_{-1}), moving shock (D^m_0), after-labor income wealth (Q_0), permanent labor income (P^Y_0), house price (P^H_0), housing stock (H_{-1}), and mortgage balance (M_{-1}):

$$\max_{\{C_t,H_t,S_t,D^o_t,D^s_t,D^r_t\}} E \sum_{t=0}^{T} \beta^t \Big\{ F(t)\; U(C_t, H_t; N_t) + [F(t-1) - F(t)]\, B(Q_t; N_t) \Big\}, \tag{15}$$

subject to the mortgage collateral constraint and income constraint (equation 5 and 6, the mortgage amortization schedule (equation 7), the borrowing constraint on liquid asset (equation 8), and the intertemporal budget constraints (equations 11 to 14). β is the time discount factor, and $B(Q_t;N_t)$ denotes the bequest function.

Following Damon, Spatt, and Zhang (2001), we assume that upon death, the household's liquidated wealth is used to purchase a L-period annuity to pay for its beneficiary's numeraire good consumption and housing services. Parameter "L" controls the strength of bequest motives. We assume that the beneficiary has the same effective family size as that of the departing household and rents his residence. This results in the beneficiary's expenditure on numeraire good and housing service consumption at a fixed proportion ($\frac{1-\omega}{\omega}$). We define the annuity factor A_L as $A_L = \frac{r(1+r)^L}{(1+r)^L-1}$. Then the bequest function is defined by $B(Q_t, N_t) = N_t^\gamma \frac{\beta(1-\beta^L)[(A_l Q_t)\omega^\omega(1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha P_t^H)^{\omega(1-\gamma)}}$.

3. Model Calibration

In this section, we first present the model parameterization. Then we discuss the optimal decision rules for renters and homeowners under the parameterization, followed by the life-cycle profiles of household consumption and saving.

3.1. Model Parameterization

We parameterize our model economy according to the U.S. economy. The decision frequency is annual. A household enters the economy at age 20 ($t = 0$), and lives for a maximum of 80 years ($T = 60$). The mandatory retirement age is 65 ($J = 45$). The conditional survival rates are taken from the 1998 life tables of the US National Center for Health Statistics (Anderson 2001). We use the 1989-2001 Survey of Consumer Finances (SCF) to calibrate the effective household size at each age (N_t). Specifically, we first calculate the average effective household size by the age of household head using the equivalence scale from the US Department of Health and Human Services (Federal Register 2001). We then obtain a life-cycle profile of effective family size, using the synthetic cohort technique as described in Appendix A. Moving probabilities are calibrated to the average intercounty migration rate of high school graduates between March 2000 and March 2001 as reported in the Current Population Survey (CPS) by the U.S. Census Bureau (2003).[9]

For preferences, we set the annual time discount factor β at 0.96 and the relative risk aversion γ at 2. The housing preference parameter ω is set at 0.25, a value in between the average share of household housing expenditures as reported in the empirical survey and the

[9]In reality, moving can be caused by job relocation, or changes in family demographics, wealth and labor income. The latter has already been taken into account in our model. Since the data did not specify the reasons for moving, we assume that moving to a location in a different county is caused by exogenous reasons that are not explicitly modeled here.

weight of housing consumptions used in the Consumer Price Index (CPI).[10] We set the bequest strength parameter L at 10 to match the wealth decumulation at the terminal period ($T = 80$) in the data.

We use the parameters for income process as reported in Cocco, Gomes, and Maenhout (2004) for a high school graduate. The authors fit a third-order polynomial to the labor income of households using the Panel Study of Income Dynamics (PSID). In particular, we choose values of 0.1 for the standard deviation of the permanent shock σ_ν and 0.25 for the standard deviation of the transitory shock σ_ε prior to retirement. Income replacement ratio at retirement is set at 0.68.[11]

The riskfree rate r is set at 0.03. We assume that the housing appreciation rate $\tilde{r}_t^H$ is serially uncorrelated and has a mean μ_h of 0.016. The housing return volatility σ_H is set at 0.115. Both estimates are taken from the empirical estimation by Campbell and Cocco (2003) using the PSID data.[12] We assume there is no correlation between house returns and shocks to labor income.

For housing and mortgage expenditures, we set the annual rental cost α at 6% of the current house value, and the mortgage collateral constraint at 80%. The income constraint is set at 5 times of the household's permanent income. The annual maintenance and depreciation cost ψ is set at 2.5% of the house value (U.S. Census Bureau 1992). The selling cost of a house ϕ is 6% of the market value of the house, the conventional fee charged by real estate agents. House purchasing cost ρ is set at 1.2% of the house value, while the mortgage refinancing charge τ is set at a lower value of 0.2%.[13] Table 1 summarizes our model parameterization. Details on obtaining a numerical solution are provided in Appendix A.

[10] The 2001 Consumer Expenditure Survey (The U.S. Department of Labor 2001) reports housing expenditures are about 20% of total household consumption. The CPI-U index gives a weight of 32% for housing costs.

[11] Campbell and Cocco (2003), and Storesletten, Telmer, and Yaron (2003) use similar numbers in their analysis.

[12] This number falls within the empirical range calculated by Goetzmann and Spiegel (2000). Based on 80 quarters of housing index data between March 1980 and March 1999, they estimate that the real housing returns for the 12 largest Metropolitan Statistical Areas (MSA) vary from -1.0 percent to 3.46 percent, while the annualized arithmetic/geometric mean housing returns vary from 1.1 percent to 3.26 percent. Similar estimates are also reported in Flavin and Yamashita (2002).

[13] Benett, Peach, and Peristiani (2001) report that an industry standard for the transaction cost for a new mortgage, excluding any up front points paid to the lender, is between 1 percent and 1.5 percent of the mortgage amount. We use a lower number to take into account the lower costs in obtaining home equity loans or home equity lines of credit.

3.2. A Renter's Optimal Decisions

A household entering the current period as a renter is described by its age (t) and wealth-permanent labor income ratio ($\frac{Q_t}{P_t^Y}$). Figure 1 presents the renter's optimal housing tenure choice. The solid line represents the wealth-labor income ratio at which the household is indifferent between renting and owning. The household owns a home when its wealth-labor income ratio falls above this line, and rents otherwise. In our parameterization, on average, renting costs more per period than owning the same house, i.e. $\alpha > r + \psi - \mu_h$. However, due to house selling and purchasing costs, a household prefers a house that matches its life-cycle income and wealth profiles so that his expected tenure is sufficiently long. A household with a large amount of wealth on hand can afford the down payment for a house of desired size, and, therefore benefits from home ownership.

The wealth-income ratio that triggers home ownership first decreases with the household's age. This observation is driven by the household's life-cycle income and mobility profiles. Since a young household faces high income growth rates, its desired house is large relative to its *current* income level. Hence a higher wealth-labor income ratio is needed to satisfy house down payment requirement to trigger home ownership. Higher exogenous mobility rates also contribute to higher triggering level of normalized wealth for home ownership by raising the expected cost from house sales. As the household approaches the terminal period, the threshold wealth-income ratio for home ownership moves up sharply, reflecting the importance of bequeathed wealth, which is defined as net of house liquidation costs, for households at this stage of the life cycle. Under our parameterization, a renting household does not choose to become a homeowner during the last three years of its life irrespective of its wealth-income ratio.

A renter's consumption and savings functions are similar to those identified in the precautionary savings literature with liquidity constraints (figures 2 and 3). At a low wealth level, a renter continues to rent and spends all his wealth on numeraire goods and rent payments. At slightly higher wealth levels, a renter saves a fraction of the wealth in liquid assets for housing down payment, and intertemporal consumption smoothing. Upon making a down payment toward purchasing a home, the household's liquid savings drop substantially.

3.3. A Homeowner's Optimal Decisions

A household entering the current period as a homeowner is described by its age (t), wealth-income ratio ($\frac{Q_t}{P_t^Y}$), house value-income ratio ($\frac{P_t^H H_t}{P_t^Y}$), and mortgage leverage ratio ($\frac{M_{t-1}(1+r)}{P_t^H H_t}$). If a homeowner receives a positive moving shock ($D_t^m = 1$), then he has to sell the house. After the sale of the house, the homeowner behaves as if he were a renter and follows the decision rules discussed above.

For a homeowner who receives a negative moving shock ($D_t^m = 0$), figure 4 plots a its endogenous house tenure and mortgage decisions as a function of his beginning-of-the-period mortgage loan-to-value ratio and house value-income ratio.[14] The presence of housing and mortgage adjustment costs lead to four regions of actions: (1) the non-admissible region (N.A.)—the homeowner's mortgage loan-to-value ratio and house value-income ratio cannot take combinations of values in this region; (2) the stay region (STAY)—the homeowner stays in his existing house without mortgage refinancing; (3) the stay and refinance region (REFI)—the homeowner stays in his house and refinances the mortgage; and (4) the sell region (SELL)—the homeowner sells his house.

Since a homeowner cannot take on debt other than a mortgage, the value of his home equity cannot not exceed his total wealth. The boundary of the non-admissible region is defined by $(1 - \bar{l}_t - phi)P_t^H H_{t-1} = Q_t$. The homeowner stays in the house when his house value-labor income ratio is not too far from the optimal level he would chose if he sells the house. If he stays in the house, the homeowner can convert some home equity into liquid form through refinancing. This occurs when the homeowner has sufficient equity in the house, i.e., the mortgage loan-to-value ratio is low. Note that while the lower selling boundary does not move much with the mortgage loan-to-value ratio, the upper selling boundary increases. This results from the homeowner's willingness to preserve liquidity since a higher mortgage loan-to-value ratio is associated with more liquid asset and the full mortgage balance becomes due upon selling the house.

After a homeowner sells his house, his subsequent consumption and housing decisions are identical to those of a renter. For a homeowner who stays in his house, the composition of

[14]For figures 4 and 5, we hold household age and the wealth-income ratio at 50 and 1.5, respectively.

a his wealth affects his non-housing consumptions. More precisely, for a given house value-income ratio, as his leverage ratio decreases, the homeowner's liquid savings drop (figure 6), which in turn reduces his non-housing consumption. When the liquid asset becomes too low, the homeowner refinances his mortgage. The additional "cash" leads to immediate increases in both non-housing consumption and liquid savings. For a given mortgage loan-to-value ratio, the homeowner's numeraire good consumption falls as the house value-income ratio increases (figure 5), reflecting the non-separability of housing and non-housing consumption in the period utility function and the intratemporal substitution of the housing consumption for numeraire good consumption.[15]

3.4. Housing, Mortgage, and Consumption Choices over the Life Cycle

We now examine a household's average life-cycle consumption and wealth accumulation through simulation. To do so, we first simulate housing prices, permanent and transitory labor income, and moving shocks according to their respective governing stochastic process. Then, we update state variables each period according to the optimal decision rules discussed earlier. For all simulated paths, households start at age 20 without housing or liquid wealth. We generate the time-series profiles of the optimal decisions by taking the average of $50,000$ simulations from $t = 0$ (age 20) to $t = 60$ (age 80).

Our calibrated economy generates life-cycle profiles (figure 7) similar to those found in the data (figure 8).[16] Specifically, home ownership rate is hump-shaped over age (figure 7a), while mortgage leverage decreases steadily with age (figure 7b). However, compared to the data, the home ownership rate in our simulated economy increases more rapidly among young households. By age 40, virtually all households in our simulation own houses. In addition, the average mortgage loan-to-value ratio generated by our model decreases more slowly prior to retirement that that in the data. The sharp increase in home ownership rate and the slower reduction in mortgage loan-to-value ratio are mainly due to the long amortization schedule

[15] A homeowner's housing and consumption decisions also vary with his wealth-labor income ratio. We omit the discussion to save space.

[16] Appendix B provides details on the empirical estimations of the life-cycle profiles of home ownership rate, mortgage leverage ratio, house value, and home equity relative to household net worth using a pseudo-panel constructed from the 1995-2001 Survey of Consumer Finances.

assumed in the model, which substantially reduced the principal payments for young mortgage borrowers and made housing more affordable.

Simulated housing consumption demonstrates a hump shape over the life cycle, matching that obtained in the data (figure 7c). As in the consumption literature with liquidity constraint and precautionary savings motives, non-housing consumption also exhibits a hump shape. Due to adjustment cost, the housing consumption does not drop as quickly as non-housing consumption after peaking in the household's late 40s. The hump shape pattern in both housing and non-housing consumption reflects the impact of liquidity constraints and household demographics.

In our simulation, the refinancing rate demonstrates a bimodal pattern with the first peak reached in the household's early 30s and the second peak reached in the late 70s (figure 7d). A young household does not have a significant amount of liquid wealth and is more likely to be liquidity constrained. Therefore, it benefits most from mortgage refinancing. These households, however, will not refinance immediately after home purchase since it needs to accumulate a significant amount of home equity to make it worthwhile to pay the refinancing cost. As a young homeowner ages and its income increases, he accumulates a significant amount of wealth both in home equity—by paying down its mortgage balance—and in liquid savings (figure 7e). Hence a middle-aged household is able to absorb negative income shocks by temporarily depleting its liquid savings, lessening the need of costly refinancing. Indeed the refinancing rate falls to almost zero around retirement when the household has the most liquid wealth.[17]

After retirement, the household starts to run down its wealth to supplement its retirement income. In order to defer house liquidation costs and refinancing charges, the household first draws upon its liquid assets. Once the household has depleted its liquid savings, it resorts to its illiquid home equity. Through "cash-out" mortgage refinancing, a homeowner can further defer house selling costs and avoid the more expensive alternative means of acquiring housing

[17] Using micro data from the Panel Study of Income Dynamics, Hurst and Stafford (2002) find similar refinancing patterns. They show that households who have experienced a spell of unemployment and who have little liquid assets are much more likely to refinance and to remove equity during refinancing than otherwise similar households. These household also tend to be relatively young. Furthermore, the propensity to refinance and remove equity for households who experienced a negative income shock decline as the amount of liquid assets they hold increase.

services through renting.[18] If a household's relative housing value deviates too much from its consumption need, selling and then renting becomes a better alternative. By the terminal period, nearly 20% of all households in our simulation have sold their houses and moved to renting. [19]

The proportion of net worth tied up in home equity demonstrates a U-shape pattern over the life cycle, consistent with empirical evidence (figure 7f). Intuitively, when the household is young, most of its wealth is committed to its house. As the household ages, liquid assets surpass home equity as a primary vehicle of saving. After retirement, due to reduced income and increased mortality risk, the household begins to draw down its liquid assets so as to defer refinancing and selling charges. Eventually, as a last resort, the household will accesses home equity to finance its consumption.

4. The Effects of House Price Appreciation on Household Consumption and Welfare

Using our calibrated economy, we now investigate the effects of exogenous permanent house price shocks on consumption and welfare for households at different stages of their life cycle. We then compare these effects of housing wealth gains with those of liquid wealth gains. Lastly, we explore the importance of housing and mortgage adjustment cost by examining an economy that is similar to the benchmark model but with zero adjustment costs.

4.1. The Effects on Consumption

To examine the effects of house price changes on consumption, we first simulate the economy for a large number of households and obtain a distribution of the households over all the state variables including home ownership status, wealth-income ratio, and house value-income ratio. Then we compare two economies. In the first economy, these households experience a positive house price shock after which house prices move up or down according to their governing

[18] Early empirical work finds that the elderly rarely refinance or take on reverse mortgages (Feinstein and McFadden 1989). Yet recent studies suggest that seniors do take money out of their homes through reduced expenditures on routine maintenance, alterations, and repairs. (Gyourko and Tracy 2003, and Davidoff 2004).

[19] This number is comparable to that documented in Venti and Wise (2000).

stochastic process. In the second economy, the same households experience a negative house price shock after which house prices move stochastically according their shock process. Our wealth and consumption gains are calculated as the mean difference in homeowners' wealth between those in the first economy and those in the second. The marginal propensity of consumption (MPC) out of housing wealth is measured as the ratio of the mean difference in consumption and the mean difference in housing wealth at various ages.

Since a renter does not hold any equity position in housing, house price shocks do not incur any wealth gains or losses for him. Under the assumption of Cobb-Douglas utility function, a renter responds to house price shocks by adjusting the level of housing service flows (H_t) while keeping housing expenditure ($\alpha P_t^H H_t$) unchanged. Hence house price appreciation does not change a renter's non-durable consumption or savings behavior. By contrast, a positive house price shock increases a homeowner's wealth-income ratio and house value-income ratio but decreases his mortgage loan-to-value ratio. We present housing wealth gains, housing and non-housing consumption changes, and MPC out of housing wealth for households that start the period as homeowners in figure 9.

Figure 9a presents housing positions after the realization of the house price shock, both before ("in") and after ("out") housing adjustments for the current period. Figure 9b presents the impact of house price shocks on house tenure status. As can be seen, homeowners younger than age 40 do not actively adjust their housing tenure status nor house sizes after a positive house price shock but tend to upgrade to bigger houses after a negative house price shock. A negative house price shock also forces a fraction of these young homeowners and some of the old homeowners to become renters since they can no longer satisfy the collateral constraints. By contrast, middle-aged and old homeowners tend to downgrade to smaller houses after a positive house price shock and do not change housing status after a negative house price shock. A small fraction of old homeowners also switch to renting after a positive house price shock to capture the house wealth gains.

These results are primarily driven by household life-cycle housing consumption profiles. Young households increase their housing expenditures as their income grows and as their families expand. After reaching middle age, homeowners tend to reduce housing expenditures. Thus, a house price appreciation substitutes for active housing up-sizing for young homeowners and accelerates down-sizing for old homeowners. The opposite is true for a house price depreciation.

Under the assumption of proportional house price appreciation across homes of all sizes, middle-aged and old homeowners receive more housing wealth gains than young homeowners (not shown). Figure 9d depicts the differences in non-housing consumption between two scenarios and figure 9e reports the MPC out of housing wealth. The average MPC out of housing wealth is around 3.7 percent, which is consistent with the conventional wisdom and within the range of the existing empirical estimates.[20] Interestingly, while house price appreciation increases all homeowners' non-housing consumption, young and old homeowners' non-housing consumption is particularly sensitive to changes in house prices. As discussed earlier, young households in their late 20s and early 30s are more likely to be liquidity constrained. Since a house can serve as a collateral, house price appreciation helps to relax young homeowners' borrowing constraints and consequently increase their non-housing consumption. Indeed, the "cash-out" refinancing rates are highest for households in this age group. Old homeowners have a short consumption horizon. They thus increase non-housing consumption and bequest significantly after a rise in their housing wealth. By contrast, middle-aged homeowners have accumulated enough liquid savings to overcome liquidity constraints. They also face a relatively long expected life span. Their consumption is, therefore, least responsive to house price changes.

4.2. The Effects on Household Welfare

We calculate welfare changes from house price shocks as the necessary proportional compensation in durable and non-durable consumptions and bequeathed wealth to the households experiencing negative housing shocks that will make them indifferent from receiving a negative house price shock and a positive house price shock. Specifically, we first calculate by age

[20] Case, Quigley, and Shiller (2003) find that an additional dollar of house wealth increases household consumption by 3 to 15 cents. Benjamin, Chinloy, and Judd (2004) find the effect of housing wealth on household consumption of similar magnitude – 8 cents out of a dollar. Campbell and Cocco (2004) estimate the largest effect of house prices on consumption for older homeowners with an elasticity of 1.7, and the smallest effect, insignificantly different from zero, for younger renters.

the sum of value functions for the households experiencing a positive shock and a negative shock, respectively:

$$\overline{V}_t^j = \sum_{i=1}^{N} V_{t\ i}^j = \sum_{i=1}^{N} \left[v_{t\ i}^j \left(P_{t\ i}^{Y\ j} / (P_{t\ i}^{H\ j})^{\omega} \right)^{1-\gamma} \right], \quad t = 0, ..., T, \quad and \quad j = up,\ dn, \tag{16}$$

where j is the index for the state of housing returns and i is the index for the heterogenous agents in state j. Our utility cost measure is then calculated as:

$$\Omega_t = \left(\frac{\overline{V}_t^{up}}{\overline{V}_t^{dn}} \right)^{\frac{1}{1-\gamma}} - 1. \tag{17}$$

Aiyagari and McGrattan (1998) use a similar measure in an infinite horizon economy.

Figure 9f presents the welfare consequences of house price changes for renters, homeowners, and households as a whole. Observe that house price appreciation unambiguously lowers renters' welfare since they bear the higher cost of acquiring housing services without receiving any housing wealth gains. According to our calculation, a positive house price shock of 11.5 percent leads to a welfare loss of around 5.5 percent of remaining lifetime consumption relative to the case of a negative house price shock of the same magnitude.[21]

Although house price appreciation raises the non-housing consumption for all homeowners as we have seen, these consumption gains do not translate into welfare gains for all homeowners. In particular, a positive housing shock incurs 1-2 percent utility costs for homeowners in their 20s and early to mid 30s. This result arises because young homeowners face a long horizon of future housing consumption, and on average, are expecting to move up in the housing ladder (see figure 7e). Thus their investment gains from existing housing positions are not sufficient to compensate for the increase in their lifetime housing consumption costs. According to our analysis, the break-even age for welfare is reached around age 40, earlier than the age where housing expenditure flattens out (age 50). Only households beyond the age of 65 receive a welfare gain exceeding 2 percent.

[21]For two renters identical before the house price shocks, their ratio of housing service flows in all future states after the current shocks will stay constant at $\frac{1+\mu_H-\sigma_H}{1+\mu_H+\sigma_H}$, while their other dimensions of consumption and wealth profiles will be the same. This leads to a theoretical utility cost of 1-$(\frac{1+\mu_H-\sigma_H}{1+\mu_H+\sigma_H})^{\omega}$ = 5.53 percent for a renter.

4.3. Comparison with the Effects of Liquid Asset Shocks

We now examine the effects of wealth gains from liquid asset and compare them to those through housing. Since the only liquid asset in our model is the riskless bond and it has a constant rate of return, we study the effects of gains in liquid asset through temporary income shock.[22] Since retired households no longer face any income risk, we restrict our discussion to households below age 65. The results are reported in figure 10.

Unlike housing wealth gains, wealth gains from liquid asset always lead to gains in both housing and non-housing consumption. The MPC out of liquid wealth ranges from 10 percent for young homeowners to around 5 percent for homeowners approaching retirement, much higher than the MPC out of housing wealth. This difference reflects the significant transaction costs in housing adjustment and home equity access. As the MPC out of housing wealth, the downward trend of MPC out of liquid wealth before retirement is a direct result of the liquidity constraints. In terms of welfare consequences, the effect of wealth gains through liquid wealth is always positive since it is not accompanied by a rise in future consumption costs.

4.4. Comparison with Models without Adjustment Costs

We examine the effects of housing and mortgage adjustment costs on household consumption and welfare in this subsection. Here we study an extreme case where these costs are set to zero. Households, however, still face the collateral and the income constraints. The new economy, therefore, resembles those of Fernandez-Villaverde and Krueger (2002), and Lustig and Nieuwerburgh (2004).

As seen in figure 11, households become homeowners much earlier and never switch back to renting. Their housing consumption closely follows that of non-housing consumption, and demonstrates a clear hump. Homeowners' consumption, housing or non-housing, is now much more responsive to changes in their housing wealth than the benchmark case for households of all ages. The MPC out of housing wealth is similar to the MPC out of liquid wealth. Despite that the consumption elasticity to housing wealth is now very similar to the consumption

[22] A household is indifferent between a one-dollar gain from liquid asset and a one-dollar gain in transitory income in our setup.

elasticity to liquid asset, the welfare consequences retain the same shape as in the benchmark case. Specifically, young homeowners are still worse off after the house price appreciation and middle-aged and old homeowners are better off. Quantitatively, the age at which homeowners are indifferent between the price changes is now much smaller since zero adjustment costs lead to less distortion in housing consumption. Overall, these results reveal that that households consumption and welfare are significantly affected by the presence of adjustment costs.

5. Conclusions

In this paper, we develop a life-cycle model to study the effects of house price shocks on household consumption and welfare. Several key features distinguish our framework from the existing literature. First, we model housing choices along both the extensive margin of owning versus renting and the intensive margin of house value. Second, we introduce a long-term fixed-rate mortgage contract for financing house purchases. Third, we distinguish between liquid savings and illiquid home equity by accounting for house liquidation cost and mortgage refinancing cost.

Numerical simulations of the model generate life-cycle consumption and wealth profiles that are consistent with the empirical findings. In particular, the home ownership rate is hump-shaped over the life cycle. The mortgage loan-to-value ratio declines steadily with the household age. The home equity-total net worth ratio exhibits a U-shaped pattern over the life cycle.

Our analysis indicates that the effects of a permanent house price increase on a household's consumption and welfare depend crucially on the household's age and its home ownership status. Specifically, the marginal propensity to consume out of housing wealth for homeowners is U-shaped over the life cycle with the non-housing consumption of young and old homeowners more responsive to house price changes than that of middle-aged homeowners. In terms of welfare, only middle-aged and old homeowners benefit from house price appreciation. Young homeowners and renters are strictly worse off.

These results contrast sharply with those obtained from appreciation in liquid assets, which generate uniform consumption and welfare gains across households of all ages and with different house tenure status. The substantial differences in consumption and welfare responses to housing versus liquid asset gains stem from the dual role of housing as both a consumption good and an investment asset. Therefore, any analysis that fails to incorporate these features can potentially be very misleading.

References

Attanasio, O. P., 1998, Cohort Analysis of Saving Behavior by US Households, *Journal of Human Resources*, 33, 575-609.

Aiyagari, R. S., and Ellen McGrattan, 1998, The Optimal Quantity of Debt, *Journal of Monetary Economics*, 42(3), 847-69.

Anderson, Robert N., 2001, United State Life Tables 1998, *National Vital Statistics Reports*, 48, 1-40.

Bajari, Patrick, C. Lanier Benkard, and John Krainer, 2003, House Prices and Consumer Welfare, NBER working paper No. W9782.

Benett, Paul, Richard Peach, and Stavros Peristiani, 2001, Structual Change in the Mortgage market and the Propensity to Refinance, *Journal of Money, Credit and Banking*, 33(4), 955-75.

Benjamin, John, Peter Chinloy, and Donald G. Jud, 2004, Real Estate Versus Financial Wealth in Consumption, *Journal of Real Estate Finance and Economics*, 29(3), 341-54.

Blundell, Richard, Martin Browning, and Costas Meghir, 1994, Consumer Demand and the Lif-Cycle Allocation of Household Expenditure, *Review of Economic Studies*, 61, 57-80.

Campbell, John, and Joao F. Cocco, 2003, Household Risk Management and Optimal Mortgage Choice, *Quarterly Journal of Economics*, 118, 1149-1194.

Campbell, John, and Joao F. Cocco, 2004, How do house prices affect consumption? Evidence from micro data, working paper, Harvard University and London Business School.

Carroll, Christopher D., 1997, Buffer Stock Saving and the Life Cycle/Permanent Income Hypothesis, *Quarterly Journal of Economics*, 112(1), 1-55.

Case, Karl E., and Robert T. Shiller, 2004, The Efficiency of the Market for Single-Family Homes, *American Economic Review*, 79(1), 125-37.

Case, Karl E., John M. Quigley, and Robert T. Shiller, 2003, Comparing Wealth Effects: The STock Market versus the Housing Market, working paper, University of California, Berkeley.

Chambers, Matthew, Carlos Garriga, and Don E. Schlagenhauf, 2004, Accounting for Changes in the Homeownership Rate, manuscript.

Cocco, Joao F., 2004, Portfolio Choice in the Presence of Housing, *Review of Financial Studies*, forthcoming.

Cocco, Joao F., Francisco J. Gomes, and Pascal J. Maenhout, 2004, Consumption and Portfolio Choice over the Life-Cycle, *Review of Financial Studies*, forthcoming.

Dammon, Robert M., Chester S. Spatt, and Harold H. Zhang, 2001, Optimal Consumption and Investment with Capital Gains Taxes, *Review of Financial Studies*, 14, 583-616.

Davidoff, Thomas, 2004, Maintenance and the Home equity of the Elderly, working Paper, University of California at Berkeley.

Deaton, Angus, 1991, Saving and Liquidity Constraints, *Econometrica*, 59 (5), 1221-48.

Deaton, Angus, 1997, The Analysis of Households Surveys: A Microeconomic Approach to Development Policy. Baltimore and London: Johns Hopkins University Press for the World Bank.

Feinstein, Jonathan, and Daniel McFadden, 1989, The Dynamics of Housing Demand by the Elderly: Wealth, Cash Flow, and Demographic Effects, The Economics of Aging, National Bureau of Economic Research Project Report series Chicago and London: University of Chicago Press, 55-86.

Fernandez-Villaverde, Jesus, and Dirk Krueger, 2002, Consumption and Saving over the Life Cycle: How Important Are Consumer Durables? *Proceedings of the 2002 North American Summer Meetings of the Econometeric Society.*

Flavin, Marjorie, 1981, The Adjustment of Consumption to Changing Expectations About Future Income, *Journal of Political Economy*, 89, 974-1091.

Flavin, Marjorie, and Takashi Yamashita, 2002, Owner-Occupied Housing and the Composition of the Household Portfolio over the Life-cycle, *American Economic Review*, 92, 345-362.

Gervais, Martin, 2002, Housing Taxation and Capital Accumulation, *Journal of Monetary Economics*, 49(7), 1461-89.

Goetzmann, William N., and Matthew Spiegel, 2000, The Policy Implication of Portfolio Choice in Underserved Mortgage Markets, working paper, Yale University.

Gyourko, Joseph, and Joseph Tracey, 2003, Using Home Maintenance and Repairs to Smooth Consumption, working Paper, Wharton School, University of Pennsylvania.

Hurst, Erik, and Frank Stafford, 2004, Home is Where the Equity is: Liquidity Constraints, Refinancing, and Consumption, forthcoming, *Journal of Money, Credit and Banking.*

Laibson, David, Andrea Repetto, and Jeremy Tobacman, 2002, A Debt Puzzle in eds. Philippe Aghion, Roman Frydman, Joseph Stiglitz, Michael Woodford Knowledge, Information, and Expectations in Modern Economics: In Honor of Edmund S. Phelps, forthcoming.

Lazear, Edward P., and Robert T. Michael, 1980, Family Size and the Distribution of Real Per Capita Income, *American Economic Review*, 70(1), 91-107.

Lustig, Hanno, and Stigin Van Nieuwerburgh, 2004, A Theory of Housing Collateral Consumption Insurance and Risk Premia, manuscript.

Ortalo-Magné, François, and Steven Rady, Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints, working paper, University of Wisonsin-Madison.

Schneider Martin, Monica Piazzesi, and Selale Tuzel, 2004, Housing, Consumption and Asset Pricing, working paper, New York University.

Speckman, P., 1988, Kernel Smoothing in Partial Linear Models, *Journal of the Royal Statistical Society*, 50, 413-36.

Storesletten, Kjetil, Chris Telmer, and Amir Yaron, 2004. Consumption and Risk Sharing over the Life Cycle, forthcoming, *Journal of Monetary Economics.*

U.S. Office of the Federal Register, National Archives and Records Administration, 2001 Federal Register.

U.S. Department of Labor, Bureau of Labor Statistics, 2003, Consumer Expenditures in 2001, Report 966.

U.S. Census Bureau, Population Division, 2003, Geographic Mobility: March 2000 to March 2001.

U.S. Census Bureau, Housing and Household Economic Statistics Division, 1992, 1991 Residential Finance Survey.

Venti, Steven, and David Wise, 2000, Aging and Housing Equity, NBER working paper 7882.

Yao, Rui, and Harold H. Zhang, 2004, Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints, forthcoming, *Review of Financial Studies*.

Zeldes, Stephen P., 1989, Optimal Consumption with Stochastic Income: Deviations from Certainty Equivalence, *Quarterly Journal of Economics*, 104(2), 275-98.

Appendix A: Model Simplifications and Calculations

An analytical solution for our problem does not exist. We thus derive numerical solutions through value function iterations. Given the recursive nature of the problem, we can rewrite the intertemporal consumption and investment problem as follows:

$$V_t(X_t) = \max_{A_t} \left\{ \lambda_t \left[N_t^{\gamma} \frac{(C_t^{1-\omega} H_t^{\omega})^{1-\gamma}}{1-\gamma} + \beta E_t[V_{t+1}(X_{t+1})] \right] + (1-\lambda_t) B(Q_t, N_t) \right\}, \tag{18}$$

where $X_t = \{D_{t-1}^o, D_t^m, P_t^Y, P_t^H, H_{t-1}, M_{t-1}, Q_t\}$ is the vector of state variables, and $A_t = \{C_t, H_t, S_t, D_t^o, D_t^s, D_t^r\}$ is the vector of choice variables.

We simplify the household's optimization problem by exploiting the scale-independence of the problem and normalizing the household's continuous choice variables by its permanent income P_t^Y or house value $P_t^H H_t$. Let $c_t = \frac{C_t}{P_t^Y}$ be the consumption-permanent income ratio, $h_t = \frac{P_t^H H_t}{P_t^Y}$ be the house value-permanent income ratio, $s_t = \frac{S_t}{P_t^Y}$ be the liquid asset-permanent income ratio, and $l_t = \frac{M_t}{P_t^H H_t}$ be the mortgage loan-to-value ratio. The vector of state variables is transformed similarly to $x_t = \{D_{t-1}^o, D_t^m, \overline{h}_t, \overline{l}_t, q_t\}$, where $\overline{h}_t = \frac{P_t^H H_{t-1}}{P_t^Y}$ is the beginning-of-period house value to permanent income ratio, $\overline{l}_t = \frac{M_{t-1}(1+r)}{P_t^H H_{t-1}}$ is the beginning-of-period mortgage loan–to–value ratio after the realization of house price shocks, and $q_t = \frac{Q_t}{P_t^Y}$ is the household's wealth-permanent labor income ratio. The evolution of normalized endogenous state variables is governed by:

$$\overline{h}_{t+1} = D_t^o h_t \left[\frac{1 + \tilde{r}_{t+1}^H}{\exp\{f(t+1, Z_{t+1})\}\nu_{t+1}} \right] \tag{19}$$

$$\overline{l}_{t+1} = l_t \left[\frac{1+r}{1+\tilde{r}_{t+1}^H} \right] \tag{20}$$

$$q_{t+1} = \frac{s_t(1+r) + D_t^o h_t (1+\tilde{r}_{t+1}^H)[1 - l_t(1+r)/(1+\tilde{r}_{t+1}^H) - \phi]}{\exp\{f(t+1, Z_{t+1})\}\nu_{t+1}} + \varepsilon_{t+1}. \tag{21}$$

The household's budget constraints (11) to (14) can then be written as

$$q_t = c_t + s_t + \alpha h_t, \tag{22}$$

$$q_t = c_t + s_t + (1 - l_t + \psi + \rho) h_t, \tag{23}$$

$$q_t = c_t + s_t + (1 - l_t + \psi - \phi)\overline{h}_t, \tag{24}$$

$$q_t = c_t + s_t + (1 - l_t + \psi + \tau - \phi)\overline{h}_t. \tag{25}$$

Define $v_t(x_t) = \frac{V_t(X_t)}{[P_t^Y/(P_t^H)^{\omega}]^{1-\gamma}}$ to be the normalized value function, then the recursive optimization problem (18) can be rewritten as,

$$v_t(x_t) = \max_{a_t}\left\{ \lambda_t \left[N_t^{\gamma} \frac{(c_t^{1-\omega} h_t^{\omega})^{1-\gamma}}{1-\gamma} + \beta E_t[v_{t+1}(x_{t+1})(\exp\{f(t+1, Z_{t+1})\}\nu_{t+1})^{1-\gamma}(1+\tilde{r}_{t+1}^H)^{\omega(\gamma-1)}] \right] \right.$$
$$\left. + (1-\lambda_t) N_t^{\gamma} \frac{\beta(1-\beta^L)[A_L q_t \omega^{\omega}(1-\omega)^{1-\omega}]^{1-\gamma}}{(1-\beta)(1-\gamma)\alpha^{\omega(1-\gamma)}} \right\},$$

subject to

$$c_t > 0, \quad h_t > 0, \quad s_t \geq 0, \quad l_t \leq 1-\delta, \quad l_t h_t \leq \chi, \tag{26}$$

and equations (19) to (25), where $a_t = \{c_t, h_t, s_t, D_t^o, D_t^s, D_t^r, D_t^d\}$ is the normalized vector of choice variables. Hence the normalization reduces the number of continuous state variables to three, i.e., P_t^Y no longer serves as a state variable, while P_t^H and H_t are combined to $P_t^H H_t$.

In terms of numerical solution, we discretize the house value–labor income ratio $(\overline{h}_t)$ and the mortgage loan-to-value ratio $(\bar{l}_t)$ into equally-spaced grids of 100, and the wealth–labor-income ratio (q_t) into 200 grids equally-spaced in the logarithm of the ratio. The boundaries for the grids are chosen to be wide enough so that our simulated time series path always falls within the defined state space.

At the terminal date T, $\lambda_T = 0$ and the investor's value function coincides with the bequest function:

$$v_T(x_T) = N_T^{\gamma} \frac{\beta(1-\beta^L)\left[A_L q_T \omega^{\omega}(1-\omega)^{1-\omega}\right]^{1-\gamma}}{(1-\beta)(1-\gamma)(\alpha)^{\omega(1-\gamma)}}. \tag{27}$$

The value function at date T is then used to solve for the optimal decision rules for all points on the state space grid at date $T-1$. We use two discrete states to approximate the realizations of each of the three exogenous state variables $(\varepsilon, \nu,$ and $\tilde{r}_t^H)$ by gaussian quadrature. The procedure results in eight discrete exogenous states for numerical integration. A three-dimension B-spline interpolation is used to approximate the value function for points that lie between grid points in the state space. The procedure is repeated recursively for each time period until the solution for date $t = 0$ is found.

Appendix B: Empirical Analysis

This appendix describes data sources and explains the nonparametric regressions used to construct our empirical stylized facts as summarized in Figures 1 and 2.

The Data

Our data comes from the Survey of Consumer Finances (SCF), collected by the Federal Reserve Board of Governors. The SCF is a triennial survey of the balance sheet, pension, income, and other demographic characteristics of US families. Our sample years include 1989, 1992, 1995, 1998, and 2001. The term "household" used in the paper corresponds to the term "family" used in the SCF.[23] The term "household age" used in the paper corresponds to the age of the family head in the SCF.

Following Fernandez-Villaverde and Krueger (2002), we exploit the repeated nature of the survey and build a pseudopanel. New households entering the survey are a large randomly-chosen sample of the US population and, consequently, they contain information about the means (home ownership, mortgage LTV, etc.) of the groups they belong to. This information can be exploited by interpreting the observed group means as a panel for estimation purposes. This method is known as the pseudopanel or synthetic cohort technique.

We define 55 cohorts according to the birth year of the household (1915 to 1970) and follow them through the sample, generating a balanced panel. The average size of cells for all households is 316, and the average size of cells for homeowners is 243.

Nonparametric Regressions

To relate age and household housing decisions, we estimate the partial linear model

$$y_{it} = constant + \beta_i cohort_i + \beta_t \gamma_t + m(age_{it}) + \epsilon_{it}, \tag{28}$$

where $cohort_i$ is a dummy for each cohort except the youngest one, and γ_t a dummy for each survey year except 1989, $m(age_{it}) = E(y_{it}|age_{it})$ is a smooth nonparametric function of age_{it}.

[23] "Household" is reserved by the SCF to denote the set of the "family" (technically known as the "primary economic unit") and any other individual that lives in the same household but it is economically independent.

To identify the separate effects of time, age, and cohort effects, we assume that time effects are orthogonal to a time trend and that their sum is normalized to zero, i.e., we attribute linear trends in the data to a combination of age and cohort effects (Deaton 1997, Attanasio 1998).

The partial linear model is estimated using the two-step estimator proposed by Speckman (1988). The nonlinear part is estimated using the Gaussian Kernel with a bandwidth of 5. We regress y_{it} on $m(age_{it})$ to obtain residuals and then we project the residuals on the time and cohort effects. The constructed new adjusted values for y_{it} are nonparametrically regressed on age.

Table 1
Baseline Model Parameters

Parameter	Symbol	Value
Demographics		
Maximum life-cycle period	T	61
Mandatory retirement period	J	45
Preferences		
Relative risk aversion	γ	2
Bequest strength	L	10
Discount rate	β	0.960
Housing preference	ω	0.250
Labor Income and House Price Processes		
Standard deviation of permanent income shock	σ_{υ}	0.100
Standard deviation of temporary income shock	σ_{ε}	0.248
Income replacement ratio after retirement	θ	0.682
Mean real house price growth	μ_H	0.016
Standard deviation of log real house price growth	σ_H	0.115
Correlation between house return and permanent labor income	$\rho_{H\upsilon}$	0.000
Liquid Savings		
Risk-free interest rate	r	0.030
Housing and Mortgage		
Rental cost	α	0.060
Down payment requirement	δ	0.200
Income requirement	χ	0.350
House selling cost	ϕ	0.060
Maintenance and depreciation cost	ψ	0.025
House purchasing cost	ρ	0.012
Mortgage refinancing cost	τ	0.002
Mortgage default cost	κ	0.200

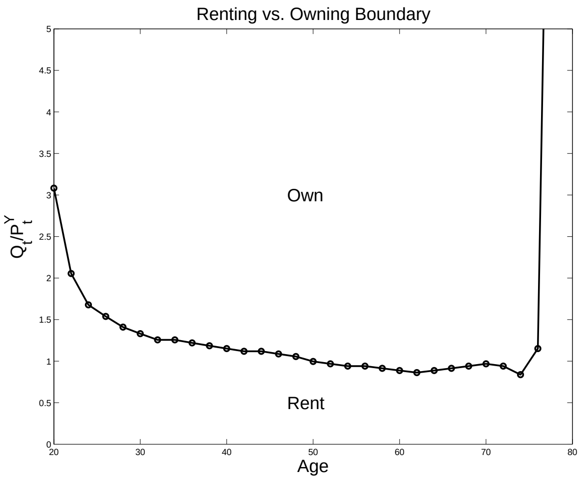

Figure 1. A Renter's House Tenure Decision as a Function of His Age and Net Worth-Permanent Labor Income Ratio

Figure 2. A Renter's Optimal Housing Consumption

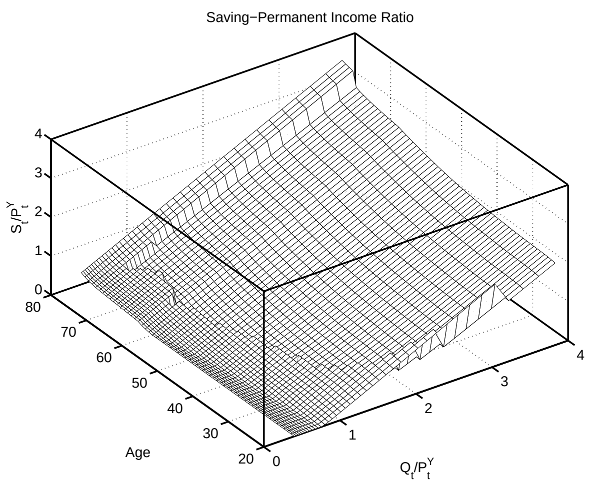

Figure 3. A Renter's Optimal Liquid Saving Decision

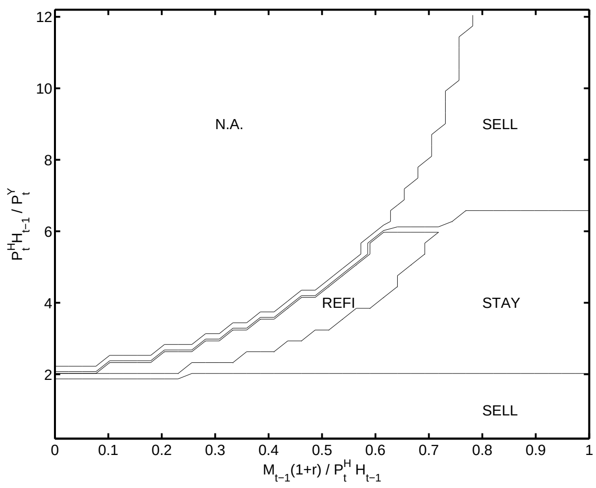

Figure 4. A Homeowner's Optimal House Tenure Choice

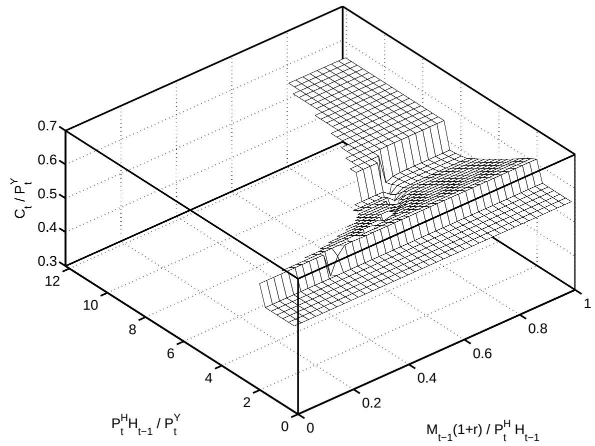

Figure 5. A Homeowner's Optimal Numeraire Good Consumption

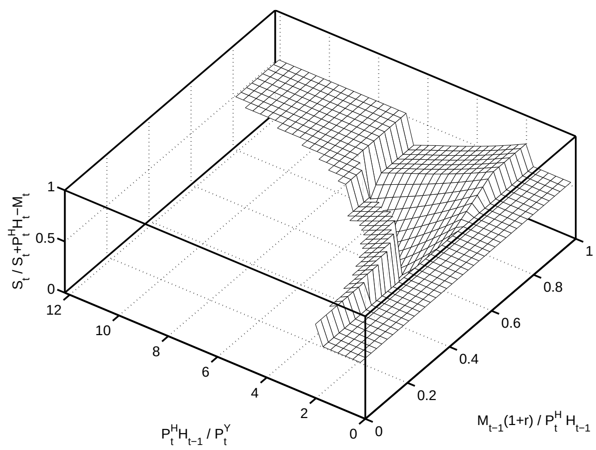

Figure 6. A Homeowner's Liquid Savings

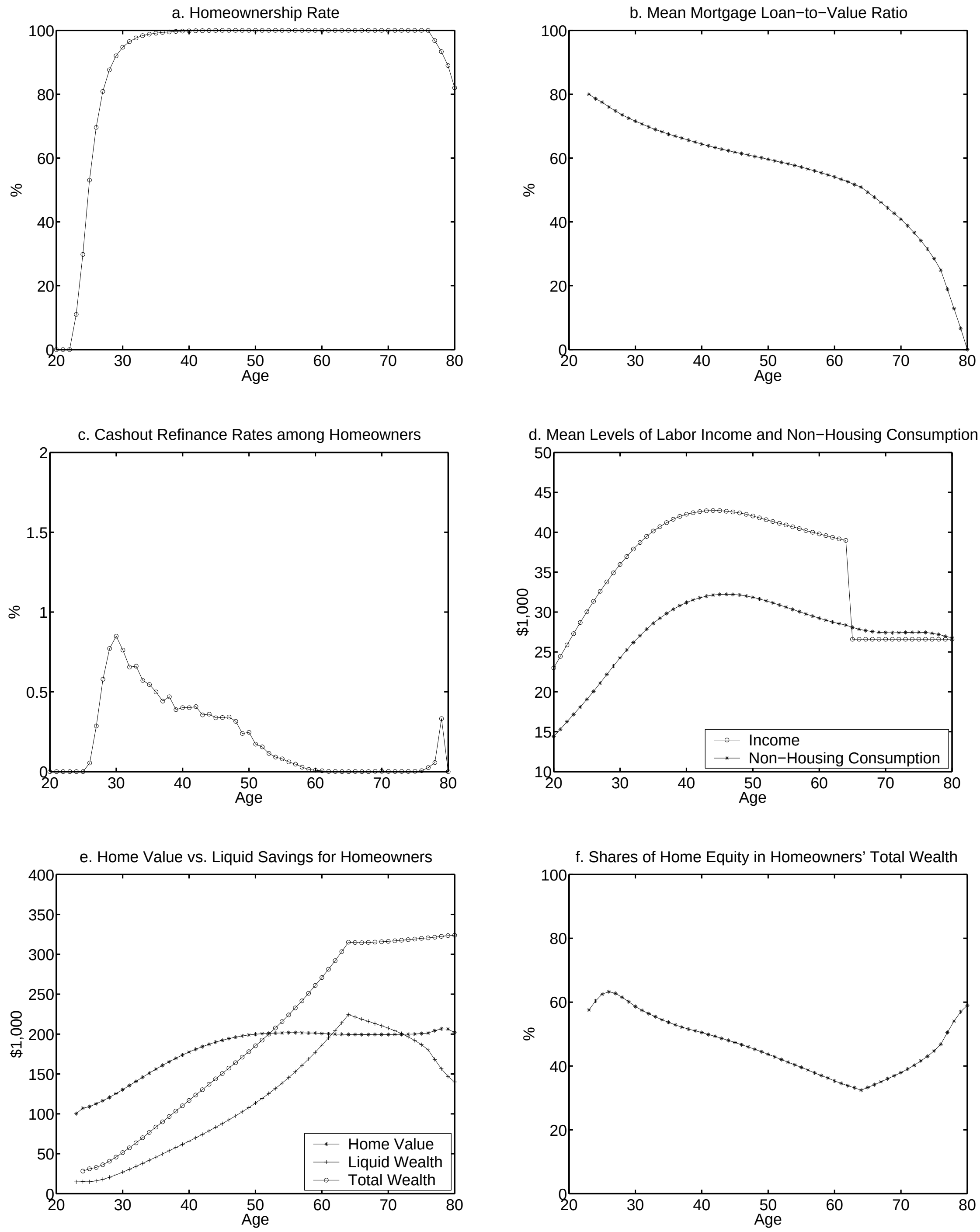

Figure 7. Optimal Life-cycle Housing and Consumption Decisions—Baseline Case

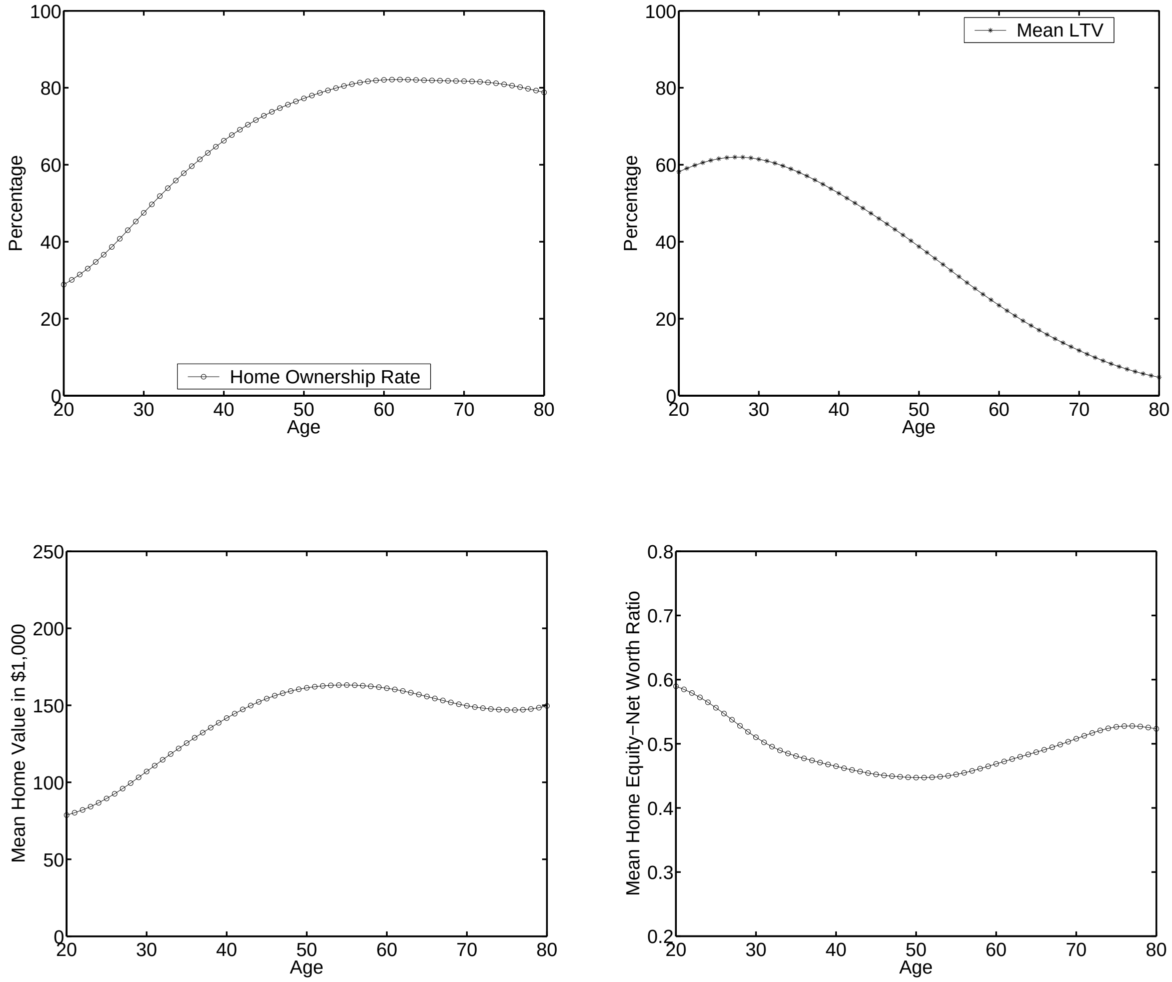

Figure 8. Empirical Life-Cycle Housing and Mortgage Choices Based on SCF Data from 1995 to 2001

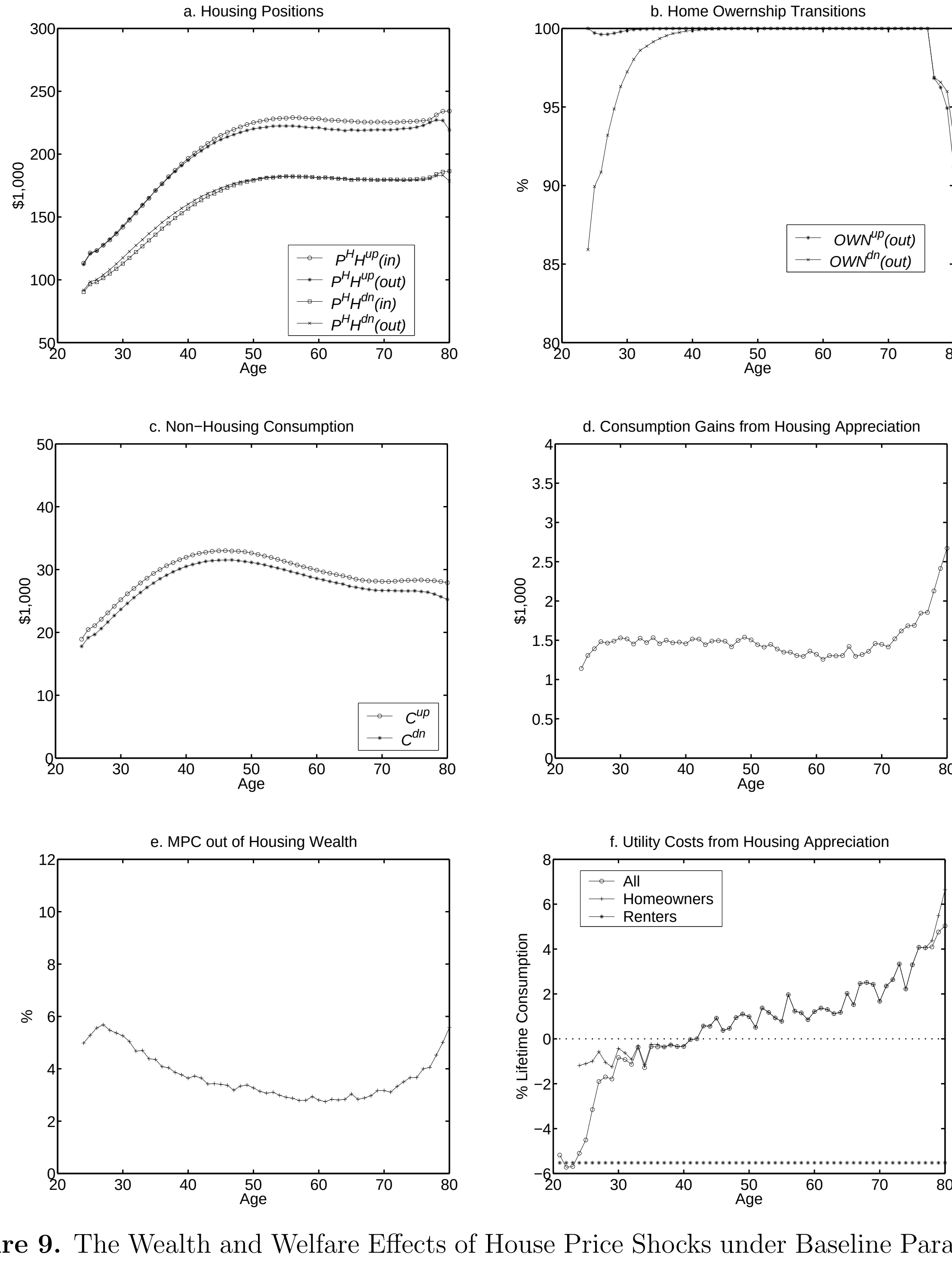

Figure 9. The Wealth and Welfare Effects of House Price Shocks under Baseline Parameterizations

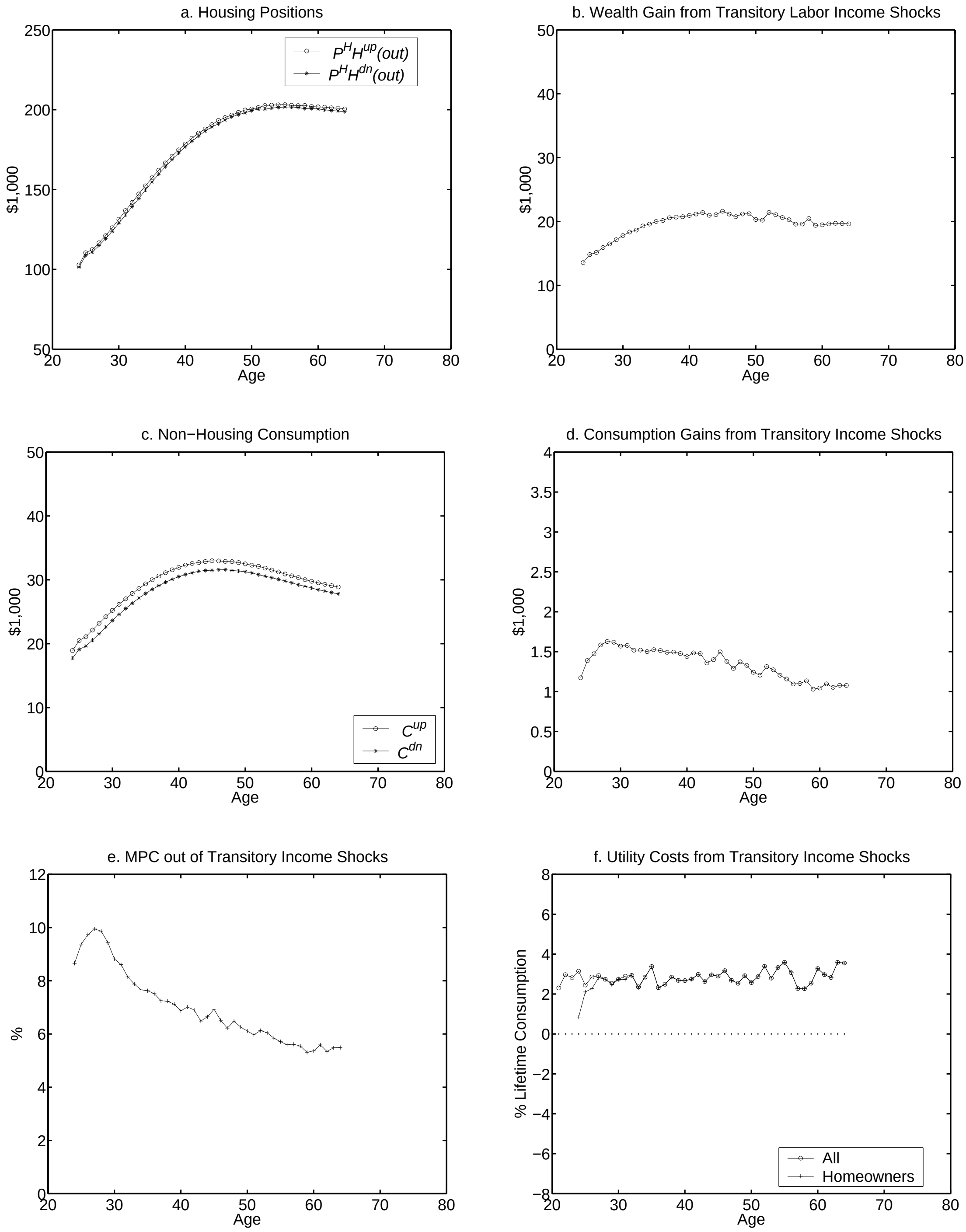

Figure 10. The Wealth/Welfare Effects of Transitory Income Shocks under Baseline Parameterizations

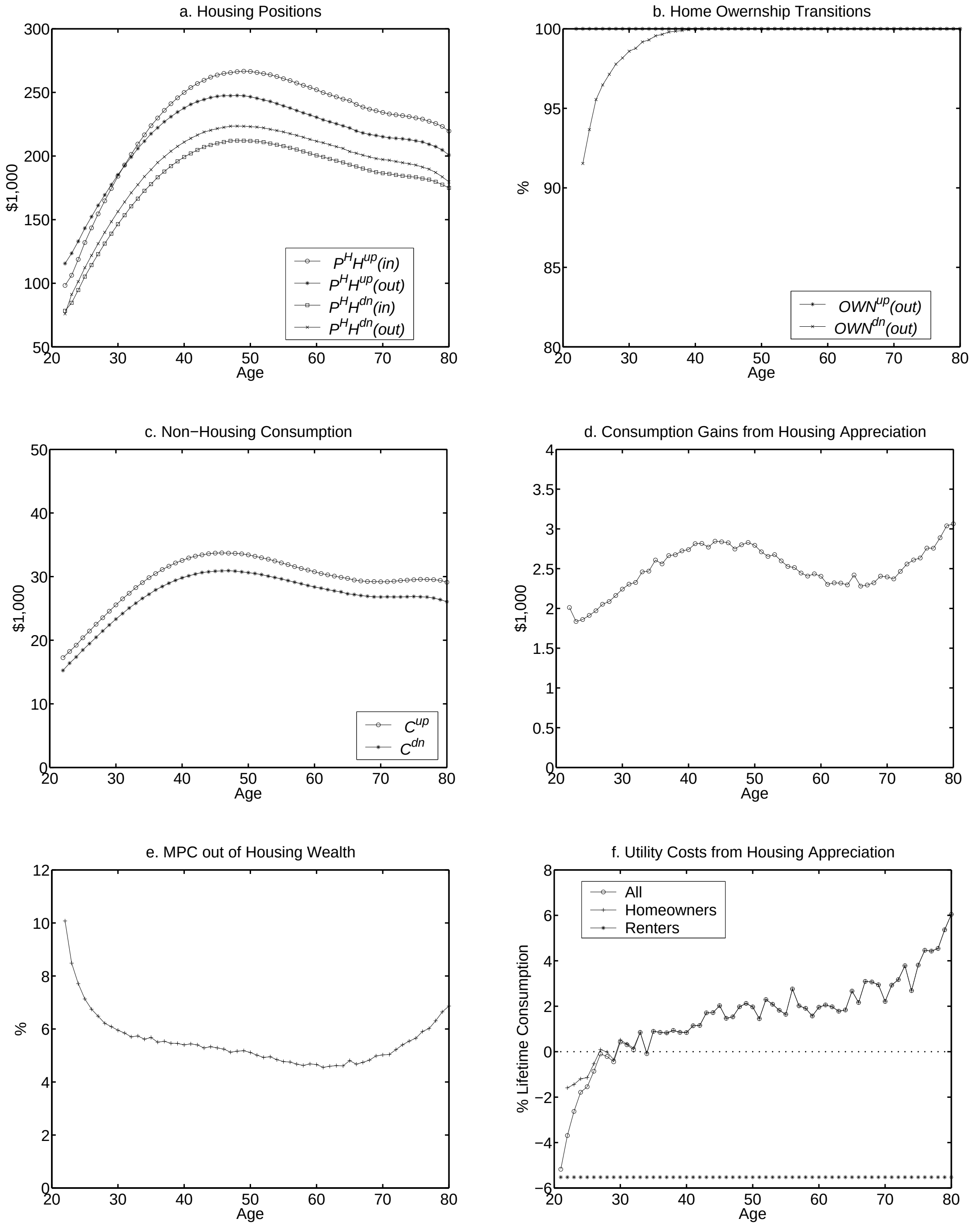

Figure 11. The Wealth and Welfare Effects of House Price Shocks with Zero Adjustment Costs