A Market Microstructure Analysis of Foreign Exchange Intervention*

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ABSTRACT

We formulate a market microstructure model of exchange determination we employ to investigate the impact of foreign exchange intervention on exchange rates and on foreign exchange (FX) market conditions. With our formulation we show i) how foreign exchange intervention influences exchange rates via both a portfolio-balance and a signalling channel and ii) derive a series of testable implications which are coherent with a large body of empirical research. Our investigation also proposes some normative recommendations, as we show i) that in extreme circumstances large scale foreign exchange intervention can have destabilizing effects for the functioning of FX markets and ii) that the route chosen for the implementation of official intervention has important implications for its impact on exchange rates and on market conditions.

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Two important topics of research which have captured the attention of students of exchange rate economics have been the investigation of the effects of foreign exchange intervention and the analysis of the microstructure of foreign exchange (FX) markets.

In the 1980s a number of contributions have sighted to understand whether central bank intervention operations in the markets for foreign exchange have important effects on currency values, on exchange rate volatility and on market conditions. Indeed, theoretically it can be established that sterilized intervention affects the value of currencies and the level of activity in FX markets, either through a *portfolio-balance channel* or via a *signalling channel*.[1] Thus, several empirical investigations, e.g. Dominguez and Frankel (1993a,1993b), have analyzed the actual effectiveness of sterilized intervention.[2] These investigations have generally concluded that foreign exchange intervention alters exchange rates, even if it is still unclear what monetary authorities can actually achieve and how sterilized operations condition FX markets.

The inconclusive nature of this strand of research has been associated with the lack of adequate high frequency data and of a microstructural perspective.[3] Indeed, until the late 1990s no detailed data on FX transactions was available to researchers and it was not possible to conduct any empirical study of microstructural aspects of FX markets with detailed information on the trading activity of their participants. However, recently the increased competition between trading platforms and data vendors has given researchers and practitioners access to detailed information on individual transactions between FX traders. This has allowed researchers to investigate the microstructure of FX markets and the impact of trading activity on exchange rate dynamics.

The principal result of this new market microstructure approach to exchange rate determination is that order flow is an important determinant of exchange rate dynamics in the short term and possibly even in the medium term.[4] Theoretical underpinnings of this empirical result link the explanatory power of order flow to two different channels of transmission, due respectively to liquidity and information effects. With respect to the former channel, it has simply been suggested that trade innovations perturb the inventories of FX investors which need to be compensated with a shift in expected returns.

With respect to the latter, it has been claimed that the empirical failure of the asset market approach lies with the particular forward looking nature of the exchange rate and with the

[1]See Mussa (1981), Bhattacharya and Weller (1997), Montgomery and Popper (2001) and Vitale (1999,2003).
[2]Edison (1993) and Sarno and Taylor (2000) contain extensive reviews of this literature.
[3]See Payne and Vitale (2003).
[4]See Lyons (2001) and Vitale (2004) for presentations of this literature.

impact that news on exchange rate fundamentals, such as interest rates, employment levels and so on, has on the value of currencies. When news arrivals condition market expectations of future values of these fundamental variables, exchange rates immediately react anticipating the effect of these fundamental shifts. Since news is hard to observe, it is difficult to control for news effects in the empirical investigation of exchange rate dynamics and hence it is difficult to conduct any meaningful analysis of the asset market approach.

However, the analysis of the relation between fundamental variables and exchange rates can be bypassed by analyzing buying/selling pressure in FX markets, as the imbalance between buyer-initiated and seller-initiated trades in FX markets, i.e. signed order flow, represents the transmission link between information and exchange rates, in that it conveys information on deeper determinants of exchange rates, which FX markets need to aggregate and impound in currency values. More specifically, as it is typical of rational expectations (RE) models of asset pricing, FX traders collect from various sources information on the fundamental value of foreign currencies and trade accordingly. A general consensus and equilibrium exchange rates are then reached via the trading process, in that information contained in order flow is progressively shared among market participants and incorporated into exchange rates.

Empirical studies of the impact of order flow on exchange rates, notably Lyons (1995), Payne (2003), Biønnes and Rime (2005), and Evans and Lyons (2005), suggest that in FX markets order flow possesses an information content. Indeed, the impact of trade innovation on exchange rate is large, significant and persistent. There is also evidence that order flow anticipates shifts in foreign exchange fundamentals.

Following Mussa (1981), we can associate this empirical evidence with foreign exchange intervention, as the monetary authorities can employ this policy instrument to credibly signal their monetary policy. This implies that order flow presents an informative component, sterilized central bank operations, which anticipates shifts in short-term interest rates and monetary aggregates. Consequently, it becomes important to understand how the intervention operations of a central bank condition the functioning of FX markets, influencing their price formation process and their market characteristics.

We analyze this issue from a market microstructure perspective, formulating a detailed microstructure model of the spot FX markets. This framework allows to study the effects of sterilized intervention operations on market conditions and exchange rates via both the signalling and portfolio-balance channels.

Our framework presents two important features: i) it allows the identification of a clear *link* between the intervention operations of a central bank, traders' expectations and exchange rates; ii) it produces a series of testable implications which are consistent with a large body of empirical evidence concerning the statistical relations between intervention operations, exchange rates and monetary aggregates. This framework also sheds light on the impact of central bank activity on the liquidity and efficiency of FX markets. In addition, it permits studying the different effects of alternative intervention strategies, as central banks may decide to feed their intervention operations through different trading routes.

This paper is organized as follows. In Section I we briefly present the main features of the structure of the markets for foreign exchange. In Sections II to VI we develop our analytical framework. Thus, in Section II we describe how foreign exchange contracts are traded in FX markets and set out the corresponding equilibrium conditions; in Section III we discuss the dynamics of the fundamental process which governs the spot rate; in Section IV we introduce foreign exchange intervention; in the following Section we discuss the role of public information; and finally in Section VI we characterize the RE equilibria of the model.

In Section VII we analyze the characteristics of these equilibria in light of the recent market microstructure approach to exchange rate determination. In the following Section we employ our model to investigate the impact of foreign exchange intervention on currency values and on market conditions. We also debate the relevance of alternative routes of intervention. In the last Section we offer some final remarks.

I. The Functioning of the Spot FX Markets

Before we can introduce a model of exchange rate determination which attempts to replicate the structure of the spot FX markets we first need to describe how these markets operate.

Spot FX markets are dealership markets, where dealers trade spot contracts for foreign currencies among themselves and with their customers. Clients usually contact FX dealers which operate as market makers, ie. are ready to trade foreign currencies with their customers. When called they quote *bid* and *ask* prices at which they are respectively willing to buy and sell a specific foreign currency. Customers can complete transactions at the quoted prices and can search the best quotes contacting several FX dealers before completing an order. When a client's order is executed, the FX dealer may decide to trade in the inter-dealer market, ie.

in the market where transactions between FX dealers are completed, to unwind her customer trade.

Inter-dealer trading can be either mediated or be the result of private bilateral meetings. In the *direct* inter-dealer market trades are the results of bilateral negotiations, typically conducted via computerized communication systems, such as the *Reuters Dealing 3000 Spot Matching System*. In the *brokered* inter-dealer market transactions among FX dealers are mediated via a brokerage service. While in the past this was operated on the phone by a brokerage firm, nowadays the brokered market is dominated by electronic brokerage platforms, such as *Electronic Broking Services* (EBS) and *Reuters Dealing System 2000-2* (Reuters D2). On these platforms transactions are completed via a centralized *limit order book*, where subscribers can at any time either add/delete *limit orders* or hit outstanding limit orders with *market orders* of opposed sign.

Recently the share of trading volume completed on centralized electronic brokerage platforms has increased, as they deliver a great deal of liquidity, immediacy and transparency.[5] However, whereas these electronic brokerage platforms allow a great deal of information on the trading process, revealing direction, size and price for all transactions, traders' identity remain anonymous. Indeed, when a transaction is completed on either Reuters D2 or EBS platforms only the counter-parties know each other identity.

Customers have very limited access to the inter-dealer market. Only recently a small proportion of financial customers, mainly hedge and currency funds, has been allowed to complete transactions on the EBS trading platform. Typically, clients can only trade with FX dealers, which themselves can feed customer orders onto the electronic brokerage platforms. In fact, some FX dealers automatically transfer customer orders onto Reuters D2 and EBS's limit order books. In this way these dealers act as intermediaries between customers and the inter-dealer market.

II. Basic Set Up

We can now present a basic market microstructure model of exchange rate determination which replicates the specific features of the spot FX markets. This model is inspired by the analytical framework proposed by Bacchetta and van Wincoop (2006) and is based on the formulation

[5]See the BIS survey of FX markets (BIS (2004)) and Rime (2003).

put forward by Breedon and Vitale (2004). This formulation is a first attempt to formalize both the liquidity and information effects of order flow on exchange rates.

A. The FX Market

Given the complexity of the structure of the spot FX markets, in the formulation which we now set out we have to introduce some simplifying assumptions. Thus, we assume that a single foreign currency is traded for the currency of a large domestic economy in the spot FX market. Trading takes place in two different *sections* of this market: in the direct section of the market FX customers' trades are executed by a population of FX dealers; in the brokered section of the market inter-dealer trades are completed in a sequence of Walrasian auctions. Clearly, this sequence of auctions intends to represent the trading activity of FX dealers over the two centralized electronic trading platforms, Reuters D2 and EBS, which dominate the inter-dealer market.

We assume that in any day t trading takes place into two *stages*: firstly, customers' orders reach the FX dealers, which fill their clients' demand for (supply of) the foreign currency at the currently best prices; secondly, the FX dealers trade among themselves in the inter-dealer market, where a daily equilibrium value for spot rate is reached via a Walrasian auction. Thus, in day t, when an auction is called, all the FX dealers simultaneously enter either market or limit orders into the centralized platform and then a clearing price (exchange rate) for the foreign currency is established.

On Reuters D2 and EBS trading platforms some FX dealers *de facto* operate like brokers. These FX dealer-brokers (*brokers* henceforth) are simple *passive* traders, which receive customer orders that are then automatically unwound into the inter-dealer market. As brokers they do not use their inventories to satisfy their customer base, do not absorb any imbalance in the order flow and hence do not take any risk. They simply collect a fee from their clients for their brokerage service. On the contrary, the FX dealer-investors (*dealers* henceforth) are *active* traders which absorb all the imbalance in the customer order flow. These traders are rational risk averse investors, which select optimal portfolios of domestic and foreign assets.

As already mentioned, some sophisticated investors other than the traditional FX dealers can now access EBS electronic limit order book. These investors are mostly currency and hedge funds which trade foreign currencies for speculative motives. Therefore, we can assume that part of our population FX dealers is formed by this sort of traders.

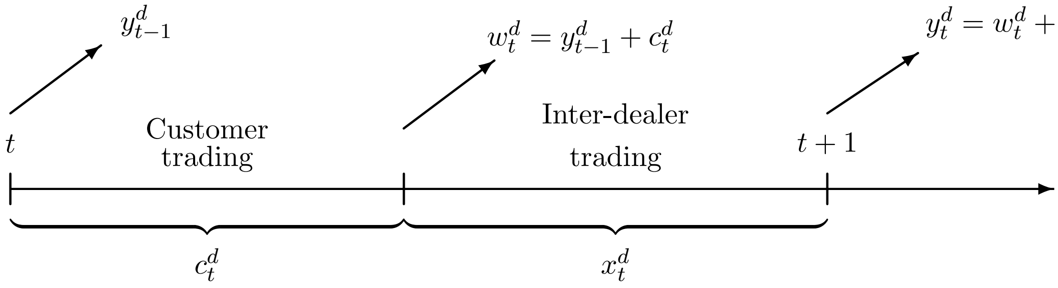


Figure 1. The time line representation of dealer d's trading in day t

Assume in particular that at the beginning of a trading day, t, a generic (active) FX dealer, dealer d, possesses δ_t^d units of a domestic bond and y_{t-1}^d units of a foreign bond. First she trades with her customer base. Her clients collectively place a market order for the foreign currency equal to $-c_t^d$, so that if c_t^d is positive (negative) dealer d purchases (sells) the foreign currency from (to) her clients. Thus, before trading starts on the centralized inter-dealer platform, she owns w_t^d units of the foreign bond, where $w_t^d = y_{t-1}^d + c_t^d$.

Dealer d can then liquidate her endowment and invest into a portfolio of domestic bonds, g_t^d, foreign bonds, y_t^d, and real balances, $\underline{M}_t^d \equiv (M_t^d/P_t)$. Domestic bonds pay day-by-day interest rates i_t, foreign bonds pay day-by-day interest rates i_t^*, whereas real balances return total output $f(\underline{M}_t^d)$ at the end of the day, as they are employed into a given production technology. This last assumption is introduced for tractability, as it is the simplest way to obtain demand functions for domestic real balances.[6]

Therefore, her budget constraint is

$$\delta_t^d + S_t w_t^d = g_t^d + S_t y_t^d + \underline{M}_t^d,$$

[6]Alternatively, one could formulate a micro founded model of exchange rate determination, in which money, alongside consumption, enters into the utility functions of our FX dealers. In this way one would need to keep track of aggregate consumption to pin down the equilibrium value of the exchange rate. Since this would greatly complicate the model, without much gain in terms of testable implications, we have chosen not to follow this route.

where S_t is the spot rate, ie. the number of units of domestic currency required to purchase one unit of the foreign one. A log-linearization of the end-of-day wealth for dealer d allows us to write her end-of-day wealth as follows

$$\tilde{W}_{t+1}^d \;=\; (1\,+\,i_t)\,\delta_t^d \,-\, (1\,+\,i_t)\,\underline{M}_t^d \,+\, f(\underline{M}_t^d) \,+\, (1\,+\,i_t\,+\,s_t)\,w_t^d \;+\, (\,i_t\,+\,\tilde{s}_{t+1}\,-\,s_t)\,x_t^d,$$

where $s_t = \ln(S_t)$, ie. the log of the spot rate, and $x_t^d = y_t^d - w_t^d$, ie. the actual quantity of the foreign currency dealer d will purchase in the spot FX market. If our dealer d acts strategically, she will face an inverted supply function when trading the foreign currency in the inter-dealer market,

$$s_t \;=\; s_t^d \,+\, \varphi_t^d\, x_t^d.$$

Our dealers are supposed to be short-sighted in that their investment horizon is just one day long. This assumption is introduced for tractability but also captures a quite well known feature of the behavior of FX dealers, which usually attempt to unwind their foreign exchange exposure by the end of any trading day.[7] Thus, dealer d selects her optimal portfolio of domestic and foreign assets in order to maximize the expected utility of her end-of-day wealth, where her utility is given by a CARA utility function with coefficient of absolute risk-aversion γ_d (and risk-tolerance $\tau_d = 1/\gamma_d$). In particular, the optimal quantity of foreign currency she will trade in the FX market corresponds to a linear excess demand function, ie. a *limit order*, in the log spot rate,

$$x_t^d\,(s_t) \;=\; \nu_t^d \left(E_t^d\left[\,\tilde{s}_{t+1}\,\right] \,-\, s_t \,+\, (i_t^* \,-\, i_t) \;-\; \gamma_d\,\sigma_d^2\,w_t^d \right),$$

where x_t^d is positive (negative) when dealer d buys (sells) the foreign currency in the spot market, $E_t^d[\tilde{s}_{t+1}]$ represents the conditional expectation of next day spot rate given the information dealer d possesses in day t, Ω_t^d,

$$E_t^d\left[\,\tilde{s}_{t+1}\,\right] \;\equiv\; E\left[\,\tilde{s}_{t+1} \mid \Omega_t^d\,\right],$$

$\sigma_{t+,d}^2$ represents the corresponding conditional variance,

$$\sigma_{t+,d}^2 \;\equiv\; \mathrm{Var}\left[\,\tilde{s}_{t+1} \mid \Omega_t^d\right],$$

[7]See Lyons (1995) and Biønnes and Rime (2005).

and

$$\nu_t^d \;=\; \frac{1}{\varphi_t^d \,+\, \gamma_d\,\sigma^2_{t+,d}}.$$

Given the demands of the individual FX dealers, through aggregation we can obtain the total demand for the foreign currency on the part of the population of FX dealers. In particular, we assume that the FX dealers form a continuum of agents of mass 1, uniformly distributed in the interval $[0,1]$. This assumption implies that all the FX dealers will actually be price takers, so that in any day t and for any dealer d $\varphi_t^d = 0$ and hence that her trading intensity is equal to $\nu_t^d = \tau_d\,\pi^d_{s+,t}$, where $\pi^d_{s+,t}$ is her conditional precision of $\tilde{s}_{t+1}$ in day t, ie. $\pi^d_{s+,t} \equiv 1/\mathrm{Var}\,[\tilde{s}_{t+1} \mid \Omega_t^d\,]$. This assumption is reasonable given that according to the BIS (BIS (2004)) there are more than 2000 dealers operating in FX markets. Thus, in day t

$$x_t \;\;\equiv\;\; \int_0^1 x_t^{d'}\,dd' \;=\nu_t\left(\bar{E}_t^1\left[\,\tilde{s}_{t+1}\,\right] \,-\, s_t \,+\, (i_t^* \,-\, i_t)\right) \,-\, w_t, \tag{1}$$

where $\nu_t \;\equiv\; \int_0^1 \nu_t^{d'}\,dd'$ is the *aggregate* trading intensity of the population of FX dealers, $w_t \equiv \int_0^1 w_t^{d'}\,dd'$ is the corresponding *aggregate* initial endowment of the foreign bond they hold, and $\bar{E}_t^1[\tilde{s}_{t+1}]$ is the weighted *average* of the expected value of next day spot rate across all FX dealers, where the individual dealers' weights are given by their trading intensities,

$$\bar{E}_t^1\left[\tilde{s}_{t+1}\right] \;\;=\;\; \frac{1}{\nu_t}\int_0^1 \nu_t^{d'}\,E_t^{d'}\left[\,\tilde{s}_{t+1}\,\right]dd'.$$

In equilibrium the total demand for foreign currency on the part of the population of FX dealers, x_t, equals the total amount of foreign currency supplied in the inter-dealer market via the group of FX brokers, b_t,

$$x_t \;\;=\;\; b_t. \tag{2}$$

This *inter-dealer* order flow, b_t, corresponds to customer orders which reach the inter-dealer market via our group of FX brokers. Besides, clients trade directly with individual FX dealers. Thus, if c_t^d represents the value of dealer d's customer orders, the *customer* order flow, c_t, corresponds to

$$c_t \;=\; \int_0^1 c_t^{d'}\,dd'.$$

Then, denote with o_t the *total* order flow. This is given by the sum of the inter-dealer order flow, b_t, and of the customer order flow, c_t,

$$o_t \;=\; b_t \;+\; c_t,$$

and represents all orders submitted by the FX customers to the FX market.[8]

B. The Uncovered Interest Rate Parity

Considering equation (1) one finds that

$$i_t \;-\; i_t^* \;=\; \left(\bar{E}_t^1\left[\,\tilde{s}_{t+1}\,\right] \;-\; s_t\right) \;-\; \frac{1}{\nu_t}\,z_t, \tag{3}$$

where z_t corresponds to the *cumulative* order flow,

$$z_t \quad=\quad z_{t-1} \;+\; o_t.$$

Equation (3) implies that, thanks to the FX-dealers' risk-aversion, the *uncovered interest rate parity* does not hold. Indeed, the interest rate differential, $i_t - i_t^*$, is equal to the difference between the average expected devaluation of the domestic currency in day t and a risk-premium on the foreign currency the FX dealers collectively require to hold foreign assets. This is a *time-varying* risk-premium, given by the product of the total supply of foreign assets the FX dealers have to share and the inverse of their aggregate trading intensity, ν_t. This aggregate trading intensity is a measure of the FX dealers' capacity to hold risky assets in their portfolios. It is not difficult to show that this value is an increasing function of the weighted average of the FX dealers' risk-tolerances, $\bar{\tau}$, and of the weighted average of the FX dealers' conditional precisions, $\bar{\pi}_{s+,t}$. In fact,

$$\nu_t \;=\; \int_0^1 \tau^{d'}\,\pi_{s+,t}^{d'}\,dd' \quad=\quad \bar{\tau}\int_0^1 \pi_{s+,t}^{d'}\,dd' \;=\; \bar{\pi}_{s+,t}\int_0^1 \tau^{d'}\,dd',$$

where

$$\bar{\tau} \;=\; \frac{\int_0^1 \tau^{d'}\,\pi_{s+,t}^{d'}\,dd'}{\int_0^1 \pi_{s+,t}^{d'}\,dd'}, \qquad\qquad \bar{\pi}_{s+,t} \;=\; \frac{\int_0^1 \tau^{d'}\,\pi_{s+,t}^{d'}\,dd'}{\int_0^1 \tau^{d'}\,dd'}.$$

[8] Differently from the usual convention a positive o_t indicates a net sale of foreign currency. If instead o_t is negative, FX customers collectively place an order to purchase the foreign currency.

In other words, the larger the average risk-tolerance of our population of FX dealers, $\bar{\tau}$, the smaller the risk premium imposed on the foreign currency. Likewise, the smaller the perceived uncertainty on the currency return, measured by the inverse of the average precision $1/\bar{\pi}_{s+,t}$, the smaller the risk-ness of the foreign currency and the imposed risk premium.

Re-inserting the modified uncovered interest rate parity in the excess demand function of dealer d, we find that

$$y_t^d \;=\; \nu_t^d \left(E_t^d \left[\tilde{s}_{t+1} \right] \;-\; \bar{E}_t^1 \left[\tilde{s}_{t+1} \right] \right) \;+\; \frac{\nu_t^d}{\nu_t}\, z_t.$$

Thus, dealer d's holding of foreign currency in day t can be decomposed in two parts, one due to a speculative motive for investment, proportional to the difference between her individual expectation and the average expectation over the future spot rate, $\bar{E}_t^d[\tilde{s}_{t+1}] - \bar{E}_t^1[\tilde{s}_{t+1}]$, the other associated with risk-sharing. In particular, one should notice that if dealer d expects a larger value for next period spot rate than the rest of the population of FX dealers, she will be willing to bet on her *belief* and *ceteris paribus* purchase the foreign currency. Furthermore, if all FX dealers possess information of identical quality, so that $\forall\, d\; \pi_{s+,t}^d = \pi_{s+,t}$, then the amount of foreign currency absorbed by dealer d for risk-sharing is proportional to her risk-tolerance, as

$$\frac{\nu_t^d}{\nu_t} \;=\; \frac{\tau_d}{\int_0^1 \tau_{d'}\, dd'}.$$

This suggests that *ceteris paribus* investors with a larger degree of risk-tolerance will end-up holding a large proportion of the foreign currency. This is consistent with the normal presumption that moderately risk-averse end-users, such as currency and hedge funds, absorb most of the imbalance in the order flow in spot FX markets.

C. The Monetary Market

Assuming that the production function $h(\underline{M}_t^d)$ respects the following formulation,

$$h(\underline{M}_t^d) \;=\; \underline{M}_t^d \;-\; \frac{1}{\alpha}\, \underline{M}_t^d \left(\ln \underline{M}_t^d \;-\; 1 \right),$$

the optimal holding of domestic real balances on the part of dealer d turns out to be

$$\underline{M}_t^d \;=\; \exp\left(-\,\alpha\, i_t\right) \;\Longleftrightarrow\; m_t^d \;=\; p_t \;-\; \alpha\, i_t,$$

where $p_t = \ln(P_t)$, ie. the log of the price level, and $m_t^d = \ln(M_t^d)$, ie. the log of the demand for money of dealer d. As usual α, a positive coefficient, corresponds to the semi-elasticity of money demand to the interest rate.

Given the demands of the individual FX dealers, through aggregation we can obtain the total demand for domestic money on the part of the population of FX dealers. In particular, as we assume that the FX dealers form a continuum of agents of mass 1, uniformly distributed in the interval $[0, 1]$, we have that in day t

$$m_t \equiv \int_0^1 m_t^{d'} \, dd' = p_t - \alpha \, i_t. \tag{4}$$

Finally, we suppose that a demand function for foreign real balances, M_t^*/P_t^*, exists and presents an identical formulation to that for domestic real balances in equation (4), ie. we assume that

$$m_t^* = p_t^* - \alpha \, i_t^*. \tag{5}$$

D. The Spot Rate Fundamental Equation

By definition we can write the spot rate as follows

$$s_t = p_t - p_t^* + q_t, \tag{6}$$

where q_t is the natural log of the *real* exchange rate, $q_t \equiv \ln(S_t P_t^*/P_t)$. Combining (6) with (4) and (5) we conclude that

$$s_t = p_t - p_t^* + q_t = \alpha \, (i_t - i_t^*) + m_t - m_t^* + q_t,$$

an expression which we can also write as

$$s_t = \alpha \, (i_t - i_t^*) + f_t, \tag{7}$$

where f_t denotes the exchange rate "*fundamental*" variable,

$$f_t \equiv m_t - m_t^* + q_t.$$

Hence substituting the expression for the interest rate differential presented in equation (3) into equation (7), we obtain the following forward-looking equilibrium condition for the spot rate

$$s_t \;=\; f_t \,+\, \alpha \left(\bar{E}_t^1 \,[\, \tilde{s}_{t+1} \,] \,-\, s_t \right) \,-\, \frac{\alpha}{\nu_t}\, z_t. \tag{8}$$

In a stationary equilibrium the conditional precision of $\tilde{s}_{t+1}$ of our FX dealers is constant over time, ie. $\forall\, t\ \pi^d_{s,t} = \pi^d_s$, and so is the aggregate trading intensity of the FX dealers, ie. $\forall\, t$ $\nu_t = \nu = \int_0^1 \tau_{d'}\, \pi_s^d \, dd'$. Then, proceeding via iterated substitutions, we find from (8) that

$$s_t \quad = \quad \frac{1}{1+\alpha} \left(f_t \,-\, \frac{\alpha}{\nu}\, z_t \,+\, \sum_{k=1}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^k \left(\bar{E}_t^k \,[\, \tilde{f}_{t+k} \,] \,-\, \frac{\alpha}{\nu}\, \bar{E}_t^k \,[\, \tilde{z}_{t+k} \,] \right) \right), \tag{9}$$

where $\bar{E}_t^k[\tilde{f}_{t+k}]$ is the *order* k weighted average rational expectation across all the FX dealers of period $t+k$ "fundamental" value, f_{t+k}, i.e.

$$\bar{E}_t^k[\tilde{f}_{t+k}] \quad = \quad \bar{E}_t^1\, \bar{E}_{t+1}^1 \,\ldots\, \bar{E}_{t+k-2}^1\, \bar{E}_{t+k-1}^1 \,[\, \tilde{f}_{t+k} \,].$$

Similarly, $\bar{E}_t^k[\tilde{z}_{t+k}]$ is the *order* k weighted average rational expectation across all the FX dealers of period $t+k$ supply of foreign currency, $\tilde{z}_{t+k}$.

Equation (9) clarifies why we have put the term fundamental in quotes. Indeed, differently from traditional monetary models of exchange rate determination, this variable does not exhaust all the factors which underpin the equilibrium value for the spot rate. Thus, the term f_t denotes the "traditional" fundamental variable of the monetary approach, pertaining to nominal and real macroeconomic factors, such as $m_t - m_t^*$ and q_t, while the total supply z_t indicates a "new" fundamental value which captures the impact of portfolio-balance effects related to risk-aversion and exchange rate risk.

To determine a closed form solution for the equilibrium spot rate we need to investigate the laws of motion for the "fundamental" variable $\tilde{f}_t$ and the total supply $\tilde{z}_t$ alongside the information the FX dealers possess on these values.

III. The Fundamental Process

With respect to $\tilde{f}_t$ we assume that it follows a simple AR(1) process of coefficient ρ,

$$\tilde{f}_{t+1} \;=\; \rho\, \tilde{f}_t \,+\, \tilde{\epsilon}^f_{t+1},$$

where $|\rho| \in [0,1]$ and $\tilde{\epsilon}_t^f$ follows a white noise process, so that $\tilde{\epsilon}_t^f \sim \text{NID}(0, \sigma_f^2)$. The assumption that $|\rho| \in [0,1]$ encompasses both the scenario in which the stochastic process for the "fundamental" variable is non-stationary, $|\rho| = 1$, and that in which $\tilde{f}_t$ is a stationary variable, $|\rho| < 1$. Given the definition of $\tilde{f}_t$ the fundamental shock $\tilde{\epsilon}_{t+1}^f$ can be divided into two components, $\tilde{\epsilon}_{t+1}^m$ and $\tilde{\epsilon}_{t+1}^q$,

$$\tilde{\epsilon}_{t+1}^f = \tilde{\epsilon}_{t+1}^m + \tilde{\epsilon}_{t+1}^q,$$

where $\tilde{\epsilon}_{t+1}^m$ can be interpreted as a monetary innovation, directly influenced by the domestic and foreign monetary authorities, while $\tilde{\epsilon}_{t+1}^q$ pertains to real perturbations, associated with demand and supply shocks, such as technological innovations, fiscal stimuli, taste changes and so on. Indeed, studies of the impact of macro announcements on exchange rates, notably Hardouvelis (1985), Ito and Roley (1987), Goodhart (1992), and Andersen, Bollerslev, Diebold, and Vega (2003), indicate that news releases on several macro aggregates, such as money supply, industrial production, trade balance, employment level, etc., condition exchange rates, confirming that among the determinants of currency values we need to list both real and monetary factors.

For consistency we need to assume that both the monetary and the real components of the fundamental shock follow independent white noise processes, so that $\tilde{\epsilon}_t^m \sim \text{NID}(0, \sigma_m^2)$, $\tilde{\epsilon}_t^q \sim \text{NID}(0, \sigma_q^2)$, whereas $\forall\, s$ and t $\tilde{\epsilon}_t^m \perp \tilde{\epsilon}_s^q$. This clearly implies that $\tilde{\epsilon}_t^f \sim \text{NID}(0, \sigma_f^2)$ where $\sigma_f^2 = \sigma_m^2 + \sigma_q^2$.

IV. Order Flow and Central Bank Intervention

As we already mentioned, the FX customers provide all the supply of foreign currency. These customers are primarily formed by the financial arms of industrial corporations and by other unsophisticated commercial and financial traders, whose FX transactions are due to liquidity needs and are not motivated by movements in exchange rates. Customer and inter-dealer order flow may be associated with current account transactions, such as trade in goods and services, transfers of capital income, public and private unilateral transfers of funds, or with capital movements, such as foreign direct and portfolio investment. We suppose that these customers correspond to *liquidity* traders and that their orders are neither price-sensitive, nor linked to innovations in the fundamental variable. Therefore, their transactions will amount to pure *noise trading.*

Beside this population of unsophisticated customers, a central bank place orders with either FX brokers or FX dealers as part of its intervention activity.[9]

In the past foreign exchange intervention was typically operated in the direct section of the FX markets, in that a central bank would contact individual FX dealers and negotiate with them individual FX transactions. As a normal practice these transactions would be kept *secret*, in that the central bank would not announce its intervention activity. However, news of an intervention operation would quickly spread in the market through the inter-dealer section of the FX markets, as Chaboud and LeBaron (2001) find for the intervention activity of the Federal Reserve. Usually, the diffusion of news of foreign exchange intervention is so rapid that newswire services would report intervention activity in the space of few minutes.

Our model captures the fragmented structure of the FX markets. So that, despite their massive trading volume, we will see how a central bank can trade with individual dealers and influence their beliefs and quotes with transactions of small size. Operating through different dealers a central bank typically provokes a wave of inter-dealer transactions that quickly spreads news of intervention in the market. Then, according to our formulation this wave influences market expectations and hence affects exchange rates. In other words, we establish a causal link between foreign exchange intervention, order flow and exchange rates.

Such a link is due to both a *signalling* and a *portfolio-balance* channel of transmission. With respect to the former, we assume that the central bank's intervention activity might present an informative content. In particular, we suppose that its orders might somehow be linked to impending fundamental innovations. Typically, monetary authorities have a better understanding of the economic environment and possess superior information on monetary policy. Thus, the central bank may anticipate shifts in monetary aggregates and other macroeconomic variables which affect currency values. Indeed, results by Klein and Rosengren (1991), Dominguez (1992), Watanabe (1992), Lewis (1995) and Kaminsky and Lewis (1996) generally suggest that central bank intervention is informative of future changes in the monetary policy, even if a clear link has not been established.

Then, considering that the FX dealers can extract some information on the future values of the exchange rate fundamentals from the order flow they observe, as this variable reflects the intervention activity of the central bank, a casual link between foreign exchange intervention,

[9] Notice, whereas intervention operations in FX markets are typically undertaken by national central banks, the jurisdiction over official intervention may lies with other institutions. Thus, in the United States and in Japan these institutions are respectively the Treasury and the Minister of Finance. With the term central bank we denote the governmental institution responsible for official intervention in FX markets.

order flow and exchange rates is easily established. Indeed, since the spot rate is a forward-looking variable, the impact on the FX dealers' expectations of the signal they extract from the order flow is immediately transferred on the current spot rate.

With respect to the latter channel of transmission, notice that even when they do not carry any information content intervention operations by a central bank influence exchange rates. In fact, as the FX dealers are risk-averse, they will be willing to accommodate a portfolio shifts associated with an intervention operation only if their are compensated with an adjustment in the value of the foreign currency.

Our formulation also allows to study the consequences of the evolution of the practice of foreign exchange intervention, which has followed recent developments in the organization of the FX markets. In particular, the partial consolidation of these markets brought about by the increasing dominance of electronic brokerage services such as EBS and Reuters D2 has induced some central banks to revise their intervention strategy. Thus, in their last operation in late 2000 the Federal Reserve split its activity between EBS, the main electronic brokerage service in the United States, and direct trading with many different dealers. Other central banks have used or could use a similar practice.

Consequently we consider two alternative scenarios: i) in the former, the *direct intervention scenario*, we assume that the central bank only operates via the direct market, trading small quantities of the foreign currency with a subset of the population of FX dealers; ii) in the latter, the *indirect intervention scenario*, the central bank employs a broker to enter large transactions directly into the inter-dealer market.

A. Direct Intervention Scenario

In the direct intervention scenario the *central bank* chooses to negotiate in day t with a subset of the FX dealers. Thus, any dealer $d \in [0, \zeta]$, with $0 < \zeta \ll 1$, trades the foreign currency with the central bank and her unsophisticated customers. On the contrary, any dealer $d \in (\zeta, 1]$ only receives market orders from her unsophisticated customers.

More precisely, any dealer d receives an uninformative collective market order $\tilde{c}_t^{n,d}$ from her base of unsophisticated customers, where we recall our convention that $\tilde{c}_t^{n,d} > 0$ ($\tilde{c}_t^{n,d} < 0$) if these clients sell (purchase) the foreign currency. As these transactions are motivated by liquidity needs, we assume that $\tilde{c}_t^{n,d}$ does not present any information content, it is neither price sensitive nor related to expected currency returns. By aggregation we calculate the *overall* unsophisticated customer order flow, $\tilde{c}_t^n$, as

$$\tilde{c}_t^n \;\equiv\; \int_0^1 \tilde{c}_t^{n,d'}\, dd'.$$

We suppose that $\tilde{c}_t^n$ follows a white noise process, so that $\tilde{c}_t^n \sim \mathrm{NID}(0, \sigma_c^2)$. A positive value for $\tilde{c}_t^n$ implies that commercial customers and/or non sophisticated financial traders collectively decide to sell the foreign currency in response to a current account surplus or to capital inflows. Clearly, an opposite interpretation applies to a negative value for $\tilde{c}_t^n$.

Beside market orders from her unsophisticated clients, any *privileged* dealer d, with $d \in [0, \zeta]$, receives a market order $\tilde{c}_t^{cb,d}$ from the central bank, where[10]

$$\tilde{c}_t^{cb,d} \;\equiv\; -\,\beta_D\, \tilde{\epsilon}_{t+1}^f \,+\, \tilde{\epsilon}_t^{cb,d}.$$

Here, β_D is a positive constant, which captures the intensity of the informative component of the central bank market order, whereas $\tilde{\epsilon}_t^{cb,d}$ represents a second component of the central bank operation not directly related to fundamentals.[11] Indeed, at times central banks purchase/sell foreign currencies as part of *market operations*. These are operations conducted on behalf of other governmental institutions, which are not aimed at influencing currency values, traders' expectations or market conditions. They are simply the consequence of the liquidity needs of some governmental institutions. In addition, central banks might purchase/sell foreign currency simply to "put a toe in the water", ie. to evaluate market conditions.

We assume that $\tilde{\epsilon}_t^{cb,d}$ follows a white noise process, so that $\tilde{\epsilon}_t^{cb,d} \sim \mathrm{NID}(0, \sigma_{cb}^2)$, ie. that the market operations of the central bank are random and not related to fundamentals and exchange rates. Then, aggregating across all the privileged FX dealers, we see that

$$\tilde{c}_t^{cb} \;\equiv\; \int_0^\zeta \tilde{c}_t^{cb,d}\, dd \;=\; -\,\zeta\, \beta_D\, \tilde{\epsilon}_{t+1}^f.$$

[10]Typically, all intervention operations conducted by a central bank are immediately sterilized via an open market operation, which neutralizes the impact of a purchase or sale of foreign currency on the monetary base. This implies that we do not have to worry about the possible influence of $\tilde{c}_t^{cb,d}$ on the money supply, m_t.

[11]Whereas we assume that β_D is a given constant, it would be possible to endogenize it by introducing some utility function for the central bank. In this formulation we abstract from this complication, as we intend to investigate foreign exchange intervention with respect to its impact and not its objectives.

Thus, our formulation entails that the central bank will buy, $\tilde{c}_t^{cb} < 0$, the foreign currency in anticipation of a positive shock to the fundamental variable, $\tilde{\epsilon}_{t+1}^{f} > 0$, which leads to a devaluation of the domestic currency. Such a shift may well be associated with an impending monetary expansion, ie. a positive $\tilde{\epsilon}_{t+1}^{m}$, which augments the expected excess return on the foreign currency. On the contrary, the central bank will sell, $\tilde{c}_t^{cb} > 0$, in anticipation of a negative shock to the fundamental variable, $\tilde{\epsilon}_{t+1}^{f} < 0$, which leads to a rise in the value of the domestic currency. In both cases, the central bank activity turns out to be profitable, as it will buy (sell) the domestic currency before its value increases (falls).

Our assumption is hence coherent with Friedman's prescription that foreign exchange intervention should be equivalent to stabilizing speculation (and hence profitable) and with some empirical results (Sweeney (1997, 2000)), which show that in the medium run the risk-adjusted profits central banks obtain when intervening in FX markets are positive.

Furthermore, as the central bank's market orders represent an important component of the total order flow, our formulation also implies that order flow is a good predictor of future values of the exchange rate fundamentals. This is in line with some evidence put forward by Evans and Lyons (2005) which shows that part of the order flow is a good predictor of *future* values of macro aggregates, such as money growth rates, output levels and inflation rates, which influence currency values. Then, consistently with several empirical studies of FX market microstructure, notably Evans and Lyons (2002), Payne (2003) and Froot and Ramadorai (2005), this result entails that order flow and exchange rate returns are strongly correlated.

In FX markets customers also place market orders with some FX brokers. These brokers usually unwind these trades into the inter-dealer market. Thus, we assume that a component of the total inter-dealer order flow, $\tilde{b}_t$, is not informative. Rather, it corresponds to orders completed on behalf of unsophisticated customers, $\tilde{b}_t^n$. Since these orders do not possess any information content, are neither price sensitive nor related to expected currency returns, we suppose that $\tilde{b}_t^n$ follows a white noise process, so that $\tilde{b}_t^n \sim \text{NID}(0, \sigma_b^2)$.

B. Indirect Intervention Scenario

In the indirect intervention scenario we maintain the assumption that any FX dealer receives uninformative market orders from a group of unsophisticated customers, we also maintain the assumption that some unsophisticated customer orders reach the inter-dealer market via the

FX brokers. However, differently from the direct intervention scenario, we suppose that the central bank now decides to conduct its intervention operation via the brokered section of the FX market. This means that in day t a broker places a market order, $\tilde{b}_t^{cb}$, on behalf of the central bank on the centralized trading platform, with

$$\tilde{b}_t^{cb} \equiv -\beta_B \, \tilde{\epsilon}_{t+1}^f,$$

where β_B is a positive constant, which captures the intensity of the informative component of the market order the central bank places on the centralized platform. The overall inter-dealer order flow, $\tilde{b}_t$, now comprises two components pertaining to unsophisticated customer trading, $\tilde{b}_t^n$, and to the central bank's informative intervention operations, $\tilde{b}_t^{cb}$. To preserve the overall intensity of informative trading from the central bank we impose the restriction that $\beta_B = \zeta \, \beta_D = \beta$. In this way, the total order flow, $\tilde{o}_t$, presents exactly the same formulation and statistical properties in the two scenarios. In fact, from consolidation we see that

$$\tilde{o}_t = \tilde{b}_t + \tilde{c}_t = \underbrace{\tilde{b}_t^n + \tilde{c}_t^n + \tilde{c}_t^{cb}}_{\text{Direct Intervention}} = \underbrace{\tilde{b}_t^n + \tilde{b}_t^{cb} + \tilde{c}_t^n}_{\text{Indirect Intervention}} = \tilde{b}_t^n - \beta \, \tilde{\epsilon}_{t+1}^f + \tilde{c}_t^n.$$

V. Public Information

News on macro aggregates continuously reaches financial markets. Rational investors can readily obtain from various official sources and publicly available data, such newswire services, newsletters, monetary authorities' watchers and so on, information on monetary aggregates, interest rates and other determinants of exchange rates. Hence we assume that our FX dealers observe in day t the current value of the fundamental process $\tilde{f}_t$. Thought, it could be argued that data on macro aggregates, notably price levels, are publicly released with some delay. Since this represents a complication and does not add any insight to our analysis we maintain the assumption that $\tilde{f}_t$ is observable in day t.

Furthermore, since news releases may influence the market evaluation of the future values of exchange rate fundamentals, we assume that our FX dealers observe in day t, before selecting their portfolios of assets, a public signal, $\tilde{\psi}_t^{pu}$, on future shocks to the fundamental variable. In particular, we assume that the common signal $\tilde{\psi}_t^{pu}$ respects the following formulation

$$\tilde{\psi}_t^{pu} = \tilde{\epsilon}_{t+1}^f + \tilde{\xi}_t^{pu},$$

where the signal error $\tilde{\xi}_t^{pu}$ follows a white noise process, ie. $\tilde{\xi}_t^{pu} \sim \mathrm{NID}(0, \sigma_{pu}^2)$, and is independent of the fundamental shock, $\tilde{\epsilon}_t^f$, so that $\tilde{\xi}_t^{pu} \perp \tilde{\epsilon}_{t'}^f$. In practice, our FX dealers collectively receive some information on impending changes in exchange rate fundamentals, such as shifts in interest rates or in monetary growth rates.

Public or private signals on the exchange rate fundamentals might concern values which materialize in the distant future. In similar cases some or all investors can observe in day t a signal on $\tilde{\epsilon}_{t+k}^f$ with $k > 1$. However, these sorts of signals increase dramatically the complexity of our formulation. In particular, assuming that such signals are heterogenous, as in the formulation we analyze, brings about an extremely difficult infinite regress problem to deal with. On the contrary, preserving the assumption that public and private signals on future realizations of the fundamental variable only concern next day values allows to identify an intuitive link between order flow and innovations in fundamentals.

VI. Rational Expectations Equilibria

Having set out our formulation, we can now isolate two different rational expectations (RE) equilibria, which identify the equilibrium spot rate which prevails in the FX market in the two scenarios. This allows to analyze the impact of foreign exchange intervention on traders' expectations, on exchange rate dynamics, and on market conditions.

A. The Equilibrium Spot Rate

We start from the the direct intervention scenario. In Appendix I we prove the following Proposition.

Proposition 1 (Direct Scenario) *When the central bank intervenes in the FX market by trading only with individual FX dealers, in a linear RE equilibrium the spot rate satisfies the following relation*

$$\tilde{s}_t \quad = \quad \lambda_b^D \, \tilde{b}_t \; + \; \lambda_c^D \, \tilde{c}_t^n \; + \; \lambda_\epsilon^D \, \tilde{\epsilon}_{t+1}^f \; + \; \lambda_f^D \, \tilde{f}_t \; + \; \lambda_{pu}^D \, \tilde{\psi}_t^{pu} \; + \; \lambda_z^D \, \tilde{z}_{t-1}. \tag{10}$$

Explicit expressions for the coefficients λ^D's are derived in Appendix I.

Likewise, in Appendix II we prove the following Proposition pertaining to the indirect intervention scenario.

Proposition 2 (Indirect Scenario) *When the central bank intervenes in the FX market only via the inter-dealer market, in a linear RE equilibrium the spot rate satisfies the following relation*

$$\tilde{s}_t \quad = \quad \lambda_b^B \, \tilde{b}_t \; + \; \lambda_c^B \, \tilde{c}_t^n \; + \; \lambda_f^B \, \tilde{f}_t \; + \; \lambda_{pu}^B \, \tilde{\psi}_t^{pu} \; + \; \lambda_z^B \, \tilde{z}_{t-1}. \tag{11}$$

Once again, explicit expressions for the coefficients λ^B's are derived in Appendix II.

B. The Conditional Variance of the Spot Rate

In Appendixes I and II we see that the values for the coefficients λ^D's and λ^B's in the two RE equilibria depend on the average trading intensity of the population of FX dealers, ν. This on turn depends on the conditional variance for the spot rate of any individual dealer. In day t the conditional variance of FX dealer d for next day spot rate is defined as follows $\sigma_{t+,d}^2 \equiv \text{Var}\,[\tilde{s}_{t+1} \mid \Omega_t^d]$. In a stationary equilibrium this conditional variance is time invariant, so that we can write $\text{Var}\,[\tilde{s}_{t+1} \mid \Omega_t^d] = \sigma_{+,d}^2$, $\forall t$.

Using the expressions for the coefficients λ^D's in Appendix I we prove that in the direct intervention scenario

$$\begin{aligned} \sigma_{+,d}^2 \quad = \quad & (\lambda_b^D)^2 \, \sigma_b^2 \; + \; (\lambda_c^D)^2 \, \sigma_c^2 \; + \; (\lambda_\epsilon^D \; + \; \lambda_{pu}^D)^2 \, \sigma_f^2 \; + \; (\lambda_{pu}^D)^2 \, \sigma_{pu}^2 \\ & + \; \left[\lambda_f^D \; - \; \lambda_z^D \, \left(\frac{\lambda_\epsilon^D}{\lambda_c^D} + \beta \right) \right]^2 \text{Var}\,[\, \tilde{\epsilon}_{t+1}^f \mid \Omega_t^d \,]. \end{aligned} \tag{12}$$

In this scenario we distinguish between privileged and non-privileged dealers. These dealers possess information of different quality on the fundamental shift, $\tilde{\epsilon}_{t+1}^f$, and hence present different values for the conditional variance of the spot rate. In practice, we have two different values for $\sigma_{+,d}^2$: $\sigma_{+,I}^2$ for the population of privileged dealers and $\sigma_{+,U}^2$ for the population of non-privileged dealers.

Similarly in the indirect intervention scenario, using the expressions for the coefficients λ^B's in Appendix II, we prove that

$$\sigma^2_{+,d} \;=\; (\lambda^B_b)^2\,\sigma^2_b \,+\, (\lambda^B_c)^2\,\sigma^2_c \,+\, (\beta\,\lambda^B_b \,-\, \lambda^B_{pu})^2\,\sigma^2_f \,+\, (\lambda^B_{pu})^2\,\sigma^2_{pu}$$

$$+\,(\lambda^B_f)^2\ \mathrm{Var}\,[\,\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t\,]. \tag{13}$$

Since in this scenario all the FX dealers share information of identical quality, they present the same value for the conditional variance of the spot rate, $\sigma^2_{+,B}$.

C. The Fixed Point Problem

The dependence of the conditional variances of the spot rate, $\sigma^2_{+,B}$, $\sigma^2_{+,I}$ and $\sigma^2_{+,U}$, on the coefficients λ^D's and λ^B's and the reverse dependence of these coefficients on the average trading intensity ν, which in turn depends on the conditional variances $\sigma^2_{+,B}$, $\sigma^2_{+,I}$ and $\sigma^2_{+,U}$, means that proving the existence of linear RE equilibria in the two scenarios entails solving two distinct fixed point problems.

To solve such fixed point problems we define two different mappings. Thus, in the direct intervention scenario one can conjecture that conditional precisions for the spot rate apply to the populations of privileged, π^I_{+s}, and non-privileged, π^U_{+s}, dealers, where $\pi^I_{+s} \equiv 1/\mathrm{Var}[\tilde{s}_{t+1} \mid \Omega^d_t]$ for $d \in [0, \zeta]$, and $\pi^U_{+s} \equiv 1/\mathrm{Var}[\tilde{s}_{t+1} \mid \Omega^d_t]$ for $d \in (\zeta, 1]$. Given these two values one can derive the corresponding average trading intensity ν and, using the formulae in Appendix I, calculate the coefficients λ^D's. These allow to derive, through equation (12), the conditional variances $\sigma^2_{+,I}$ and $\sigma^2_{+,U}$ and hence new values for the conditional precisions, $\pi^I_{+s}{}'$ and $\pi^U_{+s}{}'$, apply to the populations of privileged and non-privileged dealers. In synthesis, we have the following mapping

$$\Pi_{+s,D} \;\longrightarrow\; \Pi'_{+s,D} \;=\; G^D(\Pi_{+s,D}),$$

where $\Pi_{+s,D} \;= (\pi^I_{+s}, \pi^U_{+s})'$. Similarly, in the indirect intervention scenario one can conjecture that a conditional precision for the spot rate, π^B_{+s}, applies to all dealers, where $\pi^B_{+s} \equiv 1/\mathrm{Var}[\tilde{s}_{t+1} \mid \Omega^d_t]$ for all d, and derive an analogous mapping

$$\pi_{+s,B} \;\longrightarrow\; \pi'_{+s,B} \;=\; G^B(\pi_{+s,B}).$$

Fixed points for the two mappings identify RE equilibria for the two scenarios

$$\Pi^{\star}_{+s,D} \;=\; G^{D}(\Pi^{\star}_{+s,D}) \quad \text{and} \quad \pi^{\star}_{+s,B} \;=\; G^{B}(\pi^{\star}_{+s,B}).$$

However, analytical solutions of these fixed point problems are not available, nor can we rely on a theorem of existence, nor exclude the presence of multiple equilibria. Instead, we are forced to recur to a numerical procedure to find the roots of these two systems of equations. This numerical procedure indicates that these two systems of equations may have one, two or no solutions. We investigate this issue in Section VIII, when analyzing the impact of foreign exchange intervention on currency values and market characteristics.

VII. Discussion of the Equilibria and Related Literature

A. The Characteristics of the Equilibria

Inspection of equations (10) and (11) indicates that in both scenarios several factors enter into the equilibrium relation for the spot rate. They comprise the fundamental variable, $\tilde{f}_t$, the two components of the order flow, $\tilde{b}_t$ and $\tilde{c}^n_t$, the public signal, $\tilde{\psi}^{pu}_t$, and the lag value of the cumulative order flow, $\tilde{z}_{t-1}$. In addition, only for the direct intervention scenario, where the privileged dealers observe private signals, $\tilde{c}^{cb,d}_t$ for $d \in [0, \zeta]$, on the fundamental shock, $\tilde{\epsilon}^{f}_{t+1}$, this variable also enters into the specification of the equilibrium spot rate.

Since the cumulative order flow is non-stationary, even when the fundamental process, $\tilde{f}_t$, is stationary the exchange rate presents a unit root. This is a very appealing property of the two equilibria, as data often propose the apparent puzzle that traditional exchange rate fundamentals are stationary while the spot rate is not.

The signs of the coefficients λ^D's and λ^B's in equations (10) and (11) deserve some explanation. The fundamental coefficient, λ^{χ}_{f} with $\chi = B, D$, is positive. This is not surprising given that an increase in the fundamental value, $\tilde{f}_t$, typically corresponds to a rise in the relative money supply and in the interest rate differential, $\tilde{\imath}^*_t - \tilde{\imath}_t$. Then, an increase in $\tilde{f}_t$ corresponds to an increase in the excess return on the foreign currency, which determines its appreciation.

A positive public signal $\tilde{\psi}^{pu}_t$ increases the fundamental value perceived by the FX dealers and hence the corresponding coefficient, λ^{χ}_{pu}, is positive. Indeed, a positive value for the public signal, $\tilde{\psi}^{pu}_t$, induces the FX dealers to increase their expectations of current and future realizations of the fundamental process and hence presents an effect on the spot rate which is analogous to that of a positive value for $\tilde{f}_t$. Similarly, in the direct intervention scenario, the fundamental innovation, $\tilde{\epsilon}^f_{t+1}$, enters into the specification of the equilibrium spot rate with a positive coefficient.

The cumulative order flow coefficient, λ^χ_z, is negative because an increase in the supply of the foreign currency depresses its value via the portfolio-balance effect. In fact, the FX dealers will be willing to hold a larger quantity of the foreign currency only if they are compensated for the increased risk they bear. Thus, a larger $\tilde{z}_{t-1}$ forces a depreciation of the foreign currency as this corresponds to a larger excess return the rational FX dealers expect from holding foreign bonds.

The order flow coefficients, λ^χ_b and λ^χ_c, are negative as a positive trade innovation, via either the inter-dealer order flow, $\tilde{b}_t$, or the customer order flow, $\tilde{c}^n_t$, adds to the total supply of the foreign currency the FX dealers need to absorb. This provokes the portfolio-balance effect as well. Furthermore, in the indirect intervention scenario, the inter-dealer order flow, $\tilde{b}_t$, possesses an information content. When the central bank operates in the inter-dealer market, an excess of sell orders might indicate an impending negative fundamental shock, $\tilde{\epsilon}^f_{t+1} < 0$, and hence induces the FX dealers to expect an exchange rate depreciation. Consequently, they will be willing to hold the same amount of the foreign currency only if a reduction in $\tilde{s}_t$ re-establishes the expected excess return foreign bonds yield. This information effect entails that the impact of the inter-dealer order flow on the spot rate is larger in the indirect intervention scenario, as typically $|\lambda^B_b| > |\lambda^D_b|$.[12]

In both scenarios an interesting property of the equilibrium is that the spot rate is a weak predictor of the fundamental process. It is possible, in fact, to simulate the spot rate, $\tilde{s}_t$, and the fundamental process, $\tilde{f}_t$, and then employ the corresponding simulated series to regress leads of the variation in the fundamental variable, $\tilde{f}_{t+k} - \tilde{f}_t$, over the spot rate variation, $\tilde{s}_{t+1} - \tilde{s}_t$. Results of this exercise indicate a coefficient of multiple determination, R^2, in the 10-15% range for leads varying between 1 and 20 days. Such a property is also appealing, as recent empirical evidence (Engel and West (2005) and Froot and Ramadorai (2005)) shows how this is borne out by the data.

[12]See Figure 6, panel (a).

B. Review of Related Literature

As already mentioned, several authors, such as Bhattacharya and Weller (1997), Montgomery and Popper (2001), and Vitale (1999, 2003), have formulated theoretical models to analyze the signalling role of foreign exchange intervention. While developed from a market microstructure perspective, none of these models is fully dynamic, neither represents faithfully the structure of the spot FX markets. On the contrary, several structural models of exchange rate determination, based on the market microstructure approach, have recently been formulated.

Thus, Hau and Rey (2005) put forward a model in which exchange rate dynamics is linked to equity returns and portfolio flows. In particular, in the face of constant risk-free interest rates, dividend innovations influence the portfolio holdings of international investors and hence affect capital flows and exchange returns. Carlson and Osler (2005) on the other hand develop a model where exchange rate dynamics is linked to shifts in interest rates and current account flows. Thus, in their model the demand for foreign currency of risk averse speculators meets the supply of non-speculative traders. While the former reflects shifts in the interest rate differential, which modify expected currency returns, the latter is price sensitive and reflects current account transactions.

While these two models differ in various aspects, they share an important feature which differentiate them from our formulation. In these models there exists *no* asymmetric information between FX traders, so that, unlike our formulation, order flow cannot have any information content. In addition, in these formulations the microstructure of FX markets is minimal.

Evans and Lyons (2004) propose instead a very rich micro-founded model, which, combining a market microstructure component based on the analytical framework originally proposed by Lyons (1997) with a general equilibrium set up derived from the recent new open macroeconomics literature, assigns an informative role to order flow. In several respects, Evans and Lyons' formulation is close to ours, as their is a fully dynamic model, which properly takes into account the microstructure of FX markets and allows for some interplay between the micro and macro aspects of exchange rate determination.

However, two important features of their model are worth noticing. Firstly, for tractability Evans and Lyons assume that the FX dealers can at the end of any trading day unwind their foreign exchange exposure with the national central bank. This implies that, unlike our formulation, order flow cannot affect exchange rates via the portfolio-balance channel. Secondly, consumption enters into the specification of the fundamental process which governs exchange rate dynamics. This implies that at high frequency their model cannot be directly estimated. On the contrary, having access to the proper transaction data, our formulation can be directly estimated.

VIII. Analysis of Foreign Exchange Intervention

We are now in the position of investigating the impact of foreign exchange intervention. Given that distinct routes of intervention are possible, we analyze the differences in the impact of foreign exchange intervention on exchange rates and market conditions. Since we are not able to derive explicitly the coefficients λ^D's and λ^B's, which identify the equilibrium spot rate under the direct and indirect intervention scenarios, we rely on a numerical analysis of the impact of foreign exchange intervention on exchange rates and on market conditions. This requires that we choose appropriate values for the parameters of our formulation.

A. Benchmark Parametrization

These parameters are: the semi-elasticity of money demand, α; the intensity of the informative component of the intervention activity of the central bank, β; the coefficient of serial correlation in the fundamental variable, ρ; the variance of the noise trading component of inter-dealer order flow, σ_b^2; the variance of the noise trading component of customer order flow, σ_c^2; the variance of the market operations of the central bank, σ_d^2; the variance of the fundamental shock, σ_f^2; the variance of the noisy component of the public signal, σ_{pu}^2; the risk-tolerances of the groups of privileged and non-privileged dealers, τ_I and τ_U; and the percentage of privileged dealers in the population of all FX dealers, ζ.

Here we do not attempt to calibrate our model. Rather, we intend to choose reasonable values for the aforementioned parameters. Since the Bank of Japan has been the most active among the central banks of the major industrialized countries, we select values for these parameters that we deem appropriate for the Japanese case.

To choose the parameter α one could estimate the demand functions for money balances given in equations (4) and (5) for Japan and the United States. Breedon and Vitale (2004) have followed this route in their study of the impact of order flow on exchange rates using American and European data. In view of the lack of adequate data on transactions involving the Japanese currency we cannot replicate their approach. However, we can rely on Breedon and Vitale's estimates and choose a value of 1.47 for α.

Hence, considering that the real exchange rate cannot be observed at high frequencies, we use the difference between short-term interest rates in the United States and in Japan, $\tilde{\imath}_t^* - \tilde{\imath}_t$, as a proxy for the fundamental process, $\tilde{f}_t$. Indeed, we assume that $\tilde{q}_t \equiv 0$ and conclude that $f_t \sim \alpha(\tilde{\imath}_t^* - \tilde{\imath}_t)$. From daily observations of overnight interest rates in the United States and Japan between July 1993 and March 2004, we find that the interest rate differential presents a variance equal to 0.17 and a coefficient of serial correlation equal to -0.26. This suggests a value for ρ equal to -0.26 and for σ_f^2 equal to 0.343.[13]

From Breedon and Vitale (2004) we can derive plausible values for the risk-tolerance parameters, τ_I and τ_U. With reference to the FX dealers operating in the EUR/USD spot market Breedon and Vitale (2004) estimate a value for the average coefficient of risk-tolerance, τ, approximately equal to 0.25. Presuming that FX dealers operating in the USD/JPY market present a similar degree of risk-aversion, and given that we do not have reasons to differentiate between privileged and non-privileged dealers, we assume that $\tau_I = \tau_U = 0.25$.

To choose σ_b^2, the variance of the noise trading component of inter-dealer order flow, $\tilde{b}_t^n$, and σ_c^2, the variance of the noise trading component of customer order flow, $\tilde{c}_t^n$, we refer to the last survey of FX markets conducted by the Bank of Japan on behalf of the Bank of International Settlements (BIS (2004)). From this survey we conclude that in the USD/JPY spot market the average daily volume of brokered inter-dealer trading is equal to \$12.78 billions. In addition, from the same source, we conclude that the average daily volume of customer trading is equal to \$21.7 billions, while the average daily volume of bilateral inter-dealer trading is equal to \$10.21 billions.

Since more than 90% percent of brokered inter-dealer transactions are completed on a centralized limit order book, we take the average daily volume of brokered inter-dealer trading reported by the Bank of Japan, \$12.78 billions, as an estimate of $E[|\ \tilde{b}_t^n\ |]$. Similarly, we take the sum of average daily volume of customer trading, \$21.7 billions, and the average daily volume of bilateral inter-dealer trading, \$10.21 billions, as an estimate for $E[|\ \tilde{c}_t^n\ |]$. Under normality, and given that the expected values for $\tilde{b}_t^n$ and $\tilde{c}_t^n$ are zero, we know that

$$E[\,|\ \tilde{b}_t^n\ |\,] \;=\; \sqrt{\frac{2}{\pi}}\,\sigma_b \qquad \text{and} \qquad E[\,|\ \tilde{c}_t^n\ |\,] \;=\; \sqrt{\frac{2}{\pi}}\,\sigma_c.$$

[13]In fact, it is not difficult to check that $\mathrm{Var}(\tilde{\epsilon}_{t+1}^f) = \alpha^2(1-\rho^2)\,\mathrm{Var}(\tilde{\imath}_t^* - \tilde{\imath}_t)$.

From this result we conclude that reasonable estimates for σ_b^2 and σ_c^2 are respectively \$256 billions and \$587 billions, with a ratio of roughly 0.44.

To derive σ_{pu}^2, the variance of the noisy component of the public signal, $\tilde{\psi}_t^{pu}$, we refer once more to Breedon and Vitale (2004). They, in fact, obtain a ratio between the precision of the fundamental innovation, $\tilde{\epsilon}_{t+1}^f$, and the public signal, $\tilde{\psi}_t^{pu}$, of roughly 10. This suggests that $\sigma_{pu}^2 = 10\sigma_f^2$, ie. $\sigma_{pu}^2 = 3.43$. Finally, for the percentage of privileged dealers, considering that typically a central bank will contact a small number of FX dealers and that the Bank of Japan lists nearly 100 FX dealers operating in the USD/JPY spot market, we have chosen $\zeta = 0.05$.

Two more parameters, β, the intensity of the informative component of the intervention activity of the central bank, and σ_d^2, the variance of the market operations of the central bank, need choosing. Since their ratio determines the noise-to-information ratio of the intervention activity of the central bank, we can freely fix one of the two parameters and then investigate the impact of foreign exchange intervention by considering different values for the other. We have arbitrary chosen to take a value for σ_d^2 equal to that selected for σ_c^2, the variance of the noise trading component of customer order flow. Since σ_d^2 depends on market operations dictated by the liquidity needs of other governmental institutions, we deemed more appropriate to fix this parameter.

B. Efficiency and Stability

As mentioned in Section VI, proving the existence of linear RE equilibria in the two scenarios requires solving two distinct fixed point problems.

In Figure 2 we plot for three different values of β, the intensity of the informative component of the intervention activity of the central bank, the mappings of the conditional standard deviation of the spot rate for: i) the population of the non-privileged FX dealers, $\sigma_{+,U}$, in the direct intervention scenario (panels (a), (b) and (c)); and ii) the entire population of FX dealers, $\sigma_{+,B}$, in the indirect intervention scenario (panels (d), (e), and (d)).

In panels (a), (d) and (e) two distinct intersections with the straight line identify two different fixed points for the conditional standard deviation of the spot rate in the two scenarios. This multiplicity brings about an indeterminacy issue, as two distinct sets of the coefficient λ^{χ}'s identify two different RE equilibria for the same parametric constellation. Notice, however, that any stationary equilibrium, where the coefficients λ^{χ}'s are time-invariant, is reached only in the long run via a convergence process. Therefore, we can legitimately focus only on dynamically stable stationary equilibria. Then, inspection of panels (a), (d) and (e) indicates that only the smaller standard deviations correspond to dynamically stable stationary equilibria.

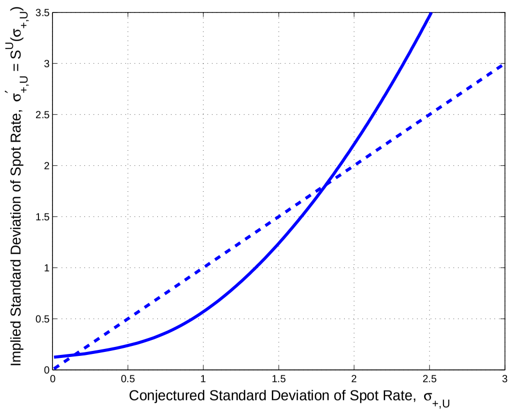
In panels (b), (c) and (f), on the other hand, no intersection exists between the continuous line, representing the conditional standard deviation of the spot rate, and the straight line, representing the corresponding conjectured conditional standard deviation of the spot rate. This means that for β large enough no RE equilibria exist in the two scenarios. This suggests that, whatever the route chosen for its intervention operations, a very aggressive central bank is destabilizing for the functioning of the market for foreign exchange.

However, the destabilizing effects of a very aggressive central bank are more acute for the direct intervention scenario, as shown by panels (b) and (e). In fact, for β equal to 0.50, in the former scenario no RE equilibrium exists, while in the latter two RE equilibria still exist.

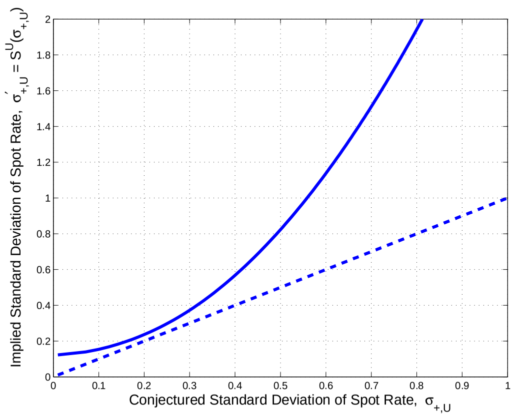
Figure 3 offers an explanation for the different destabilizing risks associated with foreign exchange intervention in the direct and indirect scenarios. In this Figure, in panel (a) we plot the relative conditional precision of the average dealer in the two scenarios against different values of the intensity of foreign exchange intervention, β. This ratio is given by $\pi^B_{+,\epsilon}/(\zeta\pi^I_{+,\epsilon} + (1-\zeta)\pi^U_{+,\epsilon})$. The coefficient $\pi^B_{+,\epsilon}$ denotes the average precision of the FX dealers in the indirect intervention scenario, given according to the following definition $\pi^B_{+,\epsilon} \equiv 1/\mathrm{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$. The coefficient $\pi^I_{+,\epsilon}$ denotes instead the precision for the population of privileged FX dealers in the direct intervention scenario, whereas $\pi^U_{+,\epsilon}$ denotes the corresponding value for the population of non-privileged FX dealers. These two values are obtained according to the following definitions, $\pi^I_{+,\epsilon} \equiv 1/\mathrm{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$ for $d \in [0, \zeta]$ and $\pi^U_{+,\epsilon} \equiv 1/\mathrm{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$ for $d \in (\zeta, 1]$.

Panel (a) clearly shows that the average FX dealer is less uncertain on next day fundamental shock when the central bank operates in the bilateral market. In the direct intervention scenario, in fact, the intervention operation of the central bank in day t is split in several transactions with individual FX dealers. Any of these transactions corresponds to a different signal. In equilibrium the spot rate aggregates the information contained in these signals, since $\tilde{s}_t$ is a function of the average signal the privileged FX dealers observe. This average signal reflects more precisely the information content of the intervention activity of the central bank and hence when foreign exchange intervention is carried out in the bilateral market more information on future movements in the fundamental variable is revealed. This means that the intervention operations of the central bank have also a larger impact on the currency value, and that the destabilizing effects of a very aggressive central bank are more severe in this scenario.

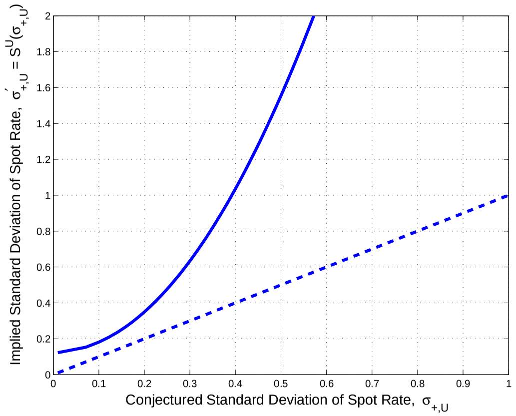
Figure 2: **Existence of Equilibria in the Direct and Indirect Intervention Scenarios**



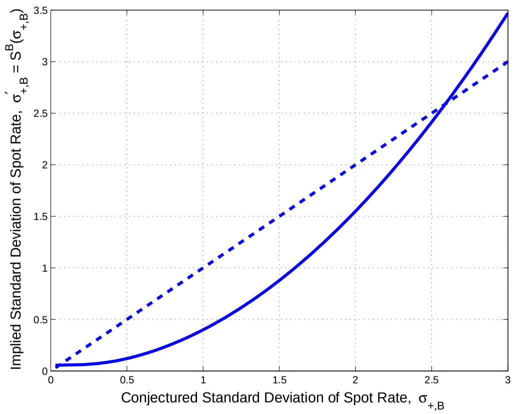
(a) Direct Intervention, $\beta = 0.10$



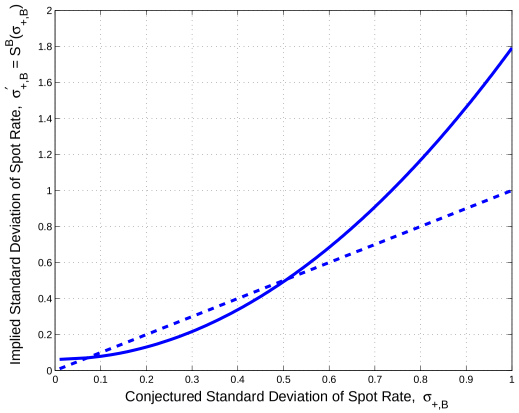
(b) Direct Intervention, $\beta = 0.50$



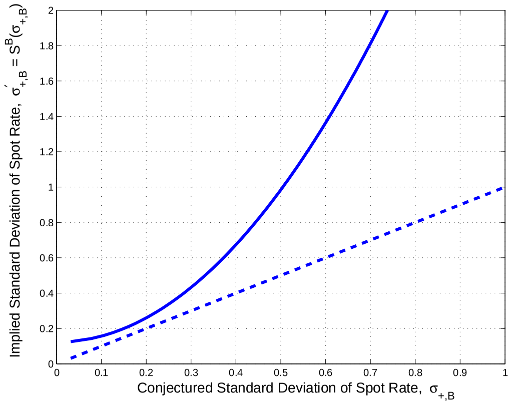
(c) Direct Intervention, $\beta = 1.00$



(d) Indirect Intervention, $\beta = 0.10$



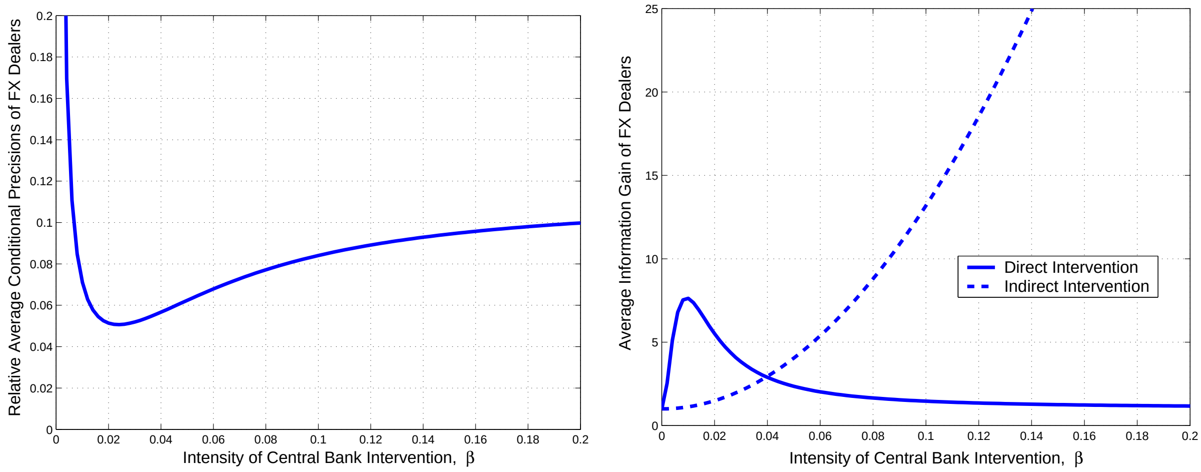
(e) Direct Intervention, $\beta = 0.50$



(f) Indirect Intervention, $\beta = 1.00$

Notes: The dashed line represents the conjectured conditional standard deviation for respectively: i) the population of the non-privileged dealers, $\sigma_{+,U}$, in the direct scenario (panels (a), (b) and (c)) and; ii) the entire population of dealers, $\sigma_{+,B}$, in the indirect scenario (panels (d), (e) and (f)). The continuous line represents instead the implied conditional standard deviation for two groups of dealers, i.e. $\sigma'_{+,U}$ and $\sigma'_{+,B}$ respectively. These implied conditional standard deviations are obtained from the mappings $\sigma'_{+,U} = S^U(\sigma_{+,U})$ and $\sigma'_{+,B} = S^B(\sigma_{+,B})$. The former is a reduced-form version of the mapping $\Pi'_{s+,D} = G^D(\Pi_{s+,D})$, where the conditional standard deviation of the privileged dealers has been substituted out through a routine which imposes the condition that $\sigma_{+,I} = S^I(\sigma_{+,I} \mid \sigma_{+,U})$. In the six plots any intersection between the continuous and dashed lines identifies a fixed point for the conditional standard deviation of the spot rate.

Figure 3. Information Acquisition of FX Dealers



(a) Conditional Precision

(b) FX dealers' Information Gain

Notes: In panel (a) we represent the ratio between the average conditional precisions of our FX dealers for the fundamental shock, $\tilde{\epsilon}^f_{t+1}$, in the two scenarios. This ratio is given by $\pi^B_{+,\epsilon}/(\zeta\pi^I_{+,\epsilon} + (1-\zeta)\pi^U_{+,\epsilon})$ where in the indirect intervention scenario $\pi^B_{+,\epsilon} \equiv 1/\text{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$, whereas in the direct intervention scenario $\pi^I_{+,\epsilon} \equiv 1/\text{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$ for $d \in [0, \zeta]$ and $\pi^U_{+,\epsilon} \equiv 1/\text{Var}[\tilde{\epsilon}^f_{t+1} \mid \Omega^d_t]$ for $d \in (\zeta, 1]$. In panel (b) we present the information gain our FX dealers obtain from operating in the foreign exchange market in the two scenarios. This information gain is measured by the ratio between the average conditional precision of our FX dealers when observing the equilibrium spot rate and when conditioning only on the private and public signals. In the indirect intervention scenario this ratio respects the following formulation $\pi^B_{+,\epsilon}/(\pi_f + \pi_{pu})$, whereas in the direct intervention one it is given by the following expression, $[\zeta\pi^I_{+,\epsilon} + (1-\zeta)\pi^U_{+,\epsilon}]/(\pi_f + \pi_{pu} + \zeta\pi_{cb,D})$. In panel (b) the continuous line pertains to the direct intervention scenario, where the central bank operates via the bilateral market, whereas the dashed line refers to the indirect intervention scenario, where instead the central operates via the brokered market.

Interestingly, panel (a) also shows that the relative conditional precision of the average dealer in the two scenarios is not a monotonic function of β, the trading intensity of the informative component of foreign exchange intervention. Whereas, this ratio is smaller than 1 for all values of β in the plot, it is first decreasing and then increasing in β. Thus, as the intervention activity of the central bank becomes more intensive, the extra information the average FX dealer obtains in the direct scenario with respect to the indirect one first rises and then falls.

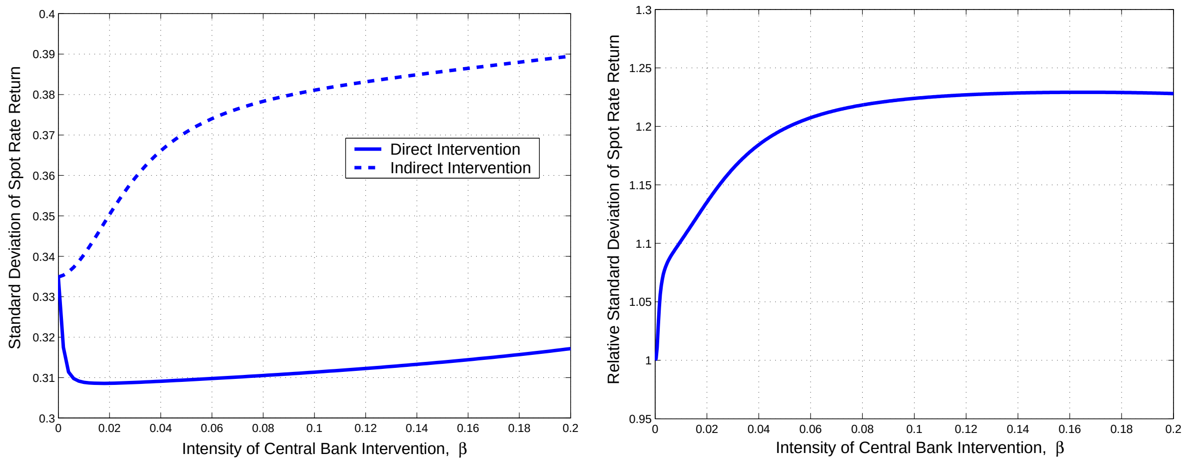
Panel (b) in Figure 3 explains such a non-monotonicity. In this panel, we plot the ratio between the average conditional precision of our FX dealers when observing the equilibrium spot rate and when conditioning only on the private and public signals against different values of the intensity of foreign exchange intervention in the two scenarios. We interpret such a ratio as representing the information gain our FX dealers obtain from observing the equilibrium spot rate in the market for foreign exchange.

The patterns observed for this ratio in the two scenarios are quite revealing. In the indirect intervention scenario, this ratio is an increasing function of the intensity of the intervention operations of the central bank. Indeed, the more aggressive its intervention activity, the larger the amount of information on the fundamental shocks our FX dealers can extract from the equilibrium spot rate and hence the larger their information gain. Thus, when β is very large the average dealer obtains a great deal of information on the fundamental shock, $\tilde{\epsilon}^f_{t+1}$. On the contrary, in the direct intervention scenario, when the central bank activity is tenuous, the privileged dealers learn more from the public signal buried into the equilibrium spot rate, whilst when the intervention activity of the central bank is more intense, they already obtain a great deal of information from their private signals.

C. Volatility, Trading Volume and Liquidity Conditions

In Figure 4 we plot the *unconditional* standard deviation of the spot rare variation in the two scenarios. This plot indicates that foreign exchange intervention may either increase or reduce exchange rate volatility depending on the route chosen by the central bank to implement its intervention operations. In fact, when intervening in the bilateral market, the central bank actually reduces the volatility of the spot rate. The opposite holds if its intervention operations are carried out via the brokered market. In particular, we see that while in the direct intervention scenario foreign exchange intervention may reduce exchange rate instability

Figure 4. Impact of Foreign Exchange Intervention on Volatility



(a) Absolute Values

(b) Relative Values

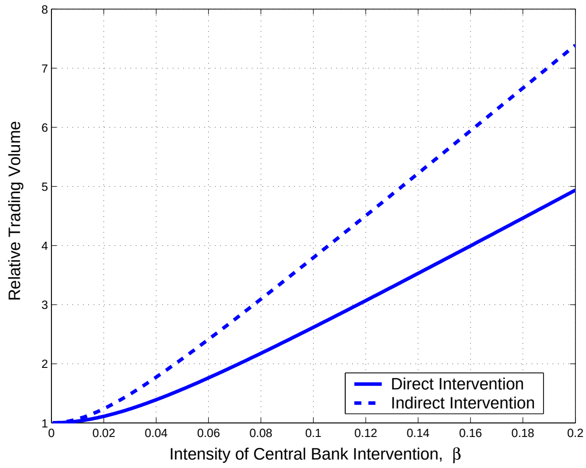
Notes: In panel (a) we plot the unconditional standard deviation of the spot rate variation, $\tilde{s}_{t+1} - \tilde{s}_t$, in the two scenarios. Thus, the continuous line refers to $\sigma_D(\tilde{s}_{t+1} - \tilde{s}_t])$, the standard deviation of the spot rate variation in the direct intervention scenario, whereas the dashed line corresponds to $\sigma_B(\tilde{s}_{t+1} - \tilde{s}_t)$, ie. the standard deviation of the spot rate variation in the direct intervention scenario. In panel (b) we plot the ratio between these two unconditional standard deviations, $\sigma_B(\tilde{s}_{t+1} - \tilde{s}_t)/\sigma_D(\tilde{s}_{t+1} - \tilde{s}_t)$.

by nearly 10 percent, in the indirect intervention scenario, the intervention activity of the central bank may increase the unconditional standard deviation of the spot rare variation by almost 20 percent.

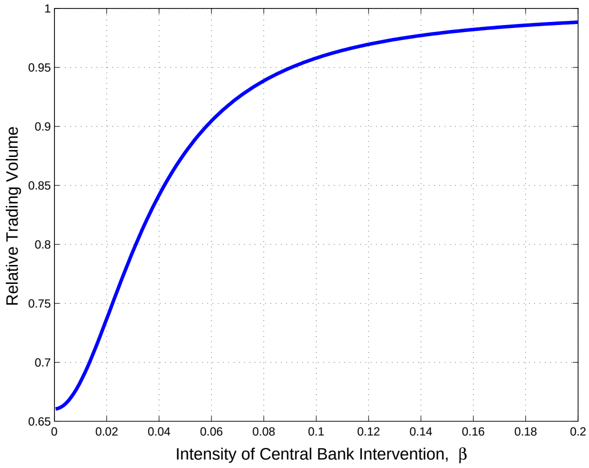
In synthesis, in panel (b) we see that the choice of the route of intervention is very important for the central bank. In particular, if the objectives of the authorities governing foreign exchange intervention are that of revealing fundamental information while maintaining exchange rate stability, then these authorities should conduct their operations in the bilateral market. We see in fact, that the difference in the unconditional volatility of the spot rate between the two scenarios can be close to 25 percent. This is an important conclusion, as an open question in the empirical investigation of foreign exchange intervention is its impact on exchange rate volatility.

From Figure 4 we also conclude that dependence of exchange rate volatility on foreign exchange intervention is not necessarily monotonic. In particular, whereas in the indirect intervention scenario a more aggressive central bank makes the currency more volatile, when the

Figure 5. Impact of Foreign Exchange Intervention on Trading Volume



(a) Intervention versus No Intervention



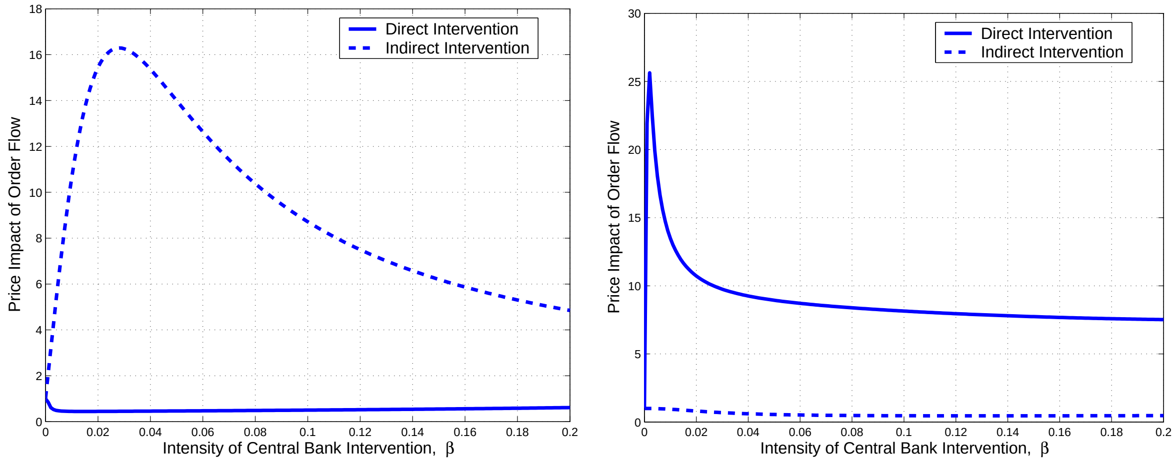
(b) Bilateral versus Inter-dealer Market

Notes: In panel (a) the continuous line represents the ratio between the expected volume of transactions in the customer market, $E|\tilde{c}_t|$, in the direct intervention scenario and its value in the absence of foreign exchange intervention, ie. for $\beta = 0$. Thus, the continuous line plots the ratio $E|\tilde{c}_t^n + \tilde{c}_t^{cb}|/E|\tilde{c}_t^n|$. The dashed line instead corresponds to the ratio between the expected volume of transactions in the inter-dealer market, $E|\tilde{b}_t|$, in the indirect intervention scenario and its value in the absence of foreign exchange intervention, ie. for $\beta = 0$. In other words, the dashed line plots the ratio $E|\tilde{b}_t^n + \tilde{b}_t^{cb}|/E|\tilde{b}_t^n|$. These two ratios can be taken as measures of the impact of foreign exchange intervention on the trading activity in the two scenarios. In panel (b) instead we plot the ratio between the expected trading volume in the inter-dealer and customer markets, $E|\tilde{b}_t|/E|\tilde{c}_t|$, where the two expected trading volumes, $E|\tilde{b}_t|$ and $E|\tilde{c}_t|$, are calculated under the assumptions of respectively the indirect and direct intervention scenarios, ie. are equal to $E|\tilde{b}_t^n + \tilde{b}_t^{cb}|$ and $E|\tilde{c}_t^n + \tilde{c}_t^{cb}|$.

central bank operates in the bilateral market a larger β may either decrease or increase the unconditional standard deviation of the spot rate. Interestingly, in the direct intervention scenario the minimum in this standard deviation is obtained for that value of β in correspondence of which the average FX dealer learns the most from the equilibrium spot rate (see Figure 3, panel (b)).

Finally notice, that whatever the route of intervention or its intensity, the unconditional standard deviation our model assigns to the spot rate variation is not far from actual values. Thus, in Figure 4 this standard deviation varies in the 0.30-0.40 range, whereas our reference data for the USD/JPY rate indicate a value of 0.88.

Figure 6. Impact of Foreign Exchange Intervention on Liquidity Conditions



(a) Inter-dealer Market

(b) Bilateral Market

Notes: In panel (a) we plot the ratios between the liquidity coefficients λ_b^B and λ_b^D and their respective values for $\beta = 0$ as measures of the impact of foreign exchange intervention on the liquidity conditions in the inter-dealer market in the two scenarios. Thus, the continuous line represents the ratio λ_b^D/λ_b^N, where λ_b^N indicates the liquidity coefficient in the inter-dealer market in the absence of any intervention activity, while the dashed line represents the ratio λ_b^B/λ_b^N. Similarly, in panel (b) we plot the ratios between the liquidity coefficients λ_c^B and λ_c^D and their respective values for $\beta = 0$ as measures of the impact of foreign exchange intervention on the liquidity conditions in the direct market in the two scenarios. Here, the continuous line represents the ratio λ_c^D/λ_c^N, where λ_c^N indicates the liquidity coefficient of the direct market in the absence of any intervention activity, whereas now the dashed line represents the ratio λ_c^B/λ_c^N.

In Figure 5 we plot the expected volume of transactions in the two scenarios. In particular, in panel (a) we represent the increase in the volume of trading, relative to the no intervention case, for the bilateral market in the direct intervention scenario and for the inter-dealer market in the indirect intervention scenario. This plot clearly indicates that given our benchmark parametrization trading volume increases very rapidly with the intensity of foreign exchange intervention, β. This result suggests that realistic values for this parameter are only the very small ones and that the possibility of a destabilizing effect of foreign exchange intervention on the FX market is indeed very remote.

Furthermore, we see that in relative terms foreign exchange intervention increases trading volumes more in the indirect intervention scenario. This is consequence of the larger depth of the bilateral market. Indeed, we have reported estimates of average daily volumes of trading of roughly \$21 and \$13 billions respectively in the bilateral and inter-dealer USD/JPY markets. This means that the same intensity of foreign exchange intervention will be more significative in the brokered market. On the other hand, panel (b) shows that for increasing values of β, the difference in the volume of trading in the bilateral and brokered markets tends to vanish.

In Figure 6 we represent the impact of foreign exchange intervention on the liquidity conditions of the bilateral and inter-dealer markets according to our formulation in the direct and indirect intervention scenarios. We measure transaction costs in the two markets with the liquidity coefficients λ_c and λ_b, which respectively indicate the price impact of customer and inter-dealer order flow.

In panel (a) we plot the impact of foreign exchange intervention on the transaction costs prevailing in the inter-dealer market in the two scenarios. Likewise, in panel (b) we have the impact on the transaction costs of the bilateral market. We see that when the central bank chooses to intervene in the brokered market, the impact of foreign exchange intervention on the liquidity conditions of the FX markets is mostly felt in the inter-dealer one. Indeed, the effect of foreign exchange intervention on the liquidity coefficient λ_c^D is negligible when the intervention operations of the central bank are conducted in the inter-dealer market. Likewise, in the direct intervention scenario, foreign exchange intervention presents virtually no impact on the liquidity conditions of the brokered market.

In both scenarios, the impact of foreign exchange intervention on the liquidity conditions of the FX markets is very large, as transaction costs increase by an order of magnitude. Plot (b) shows that this is particularly true for the bilateral market in the direct intervention scenario, as the coefficient λ_c^D can be nearly 25 times larger than its no intervention value. Thus, in spite of its larger volume of trading of the bilateral market, foreign exchange intervention presents a larger impact on the liquidity conditions of this market, in that, as shown in Figure 3, in the direct intervention scenario the intervention operations of the central bank convey more information on the impending fundamental shocks.

Notice that the size of the impact of foreign exchange intervention on the liquidity conditions of the FX markets represented in Figure 6 is consistent with some evidence provided by Payne and Vitale (2003). In fact, using transaction data for the spot USD/CHF market between 1986 and 1995, they estimate in 50 basis point the price impact on the spot USD/CHF rate of an intervention operation of \$100 millions on the part of the Swiss central bank. This figure is roughly 10 times larger than the price impact estimated for customer trades in the same market.

IX. Concluding Remarks

In a relatively recent survey Neely (2000) found that most central bankers agreed with the thesis that foreign exchange intervention possesses an impact on FX markets. However, these central bankers could not agree on which results foreign exchange intervention achieve. Thus, in this paper we have offered a formal analysis of the impact of foreign exchange intervention on FX markets, formulating a detailed market microstructure model where the intervention operations of the central bank plays an important signalling role.

From our analysis we conclude that when the intervention operations of the central bank present an informative content on future shifts in exchange rate fundamentals their impact on currency values and market conditions is significative. In particular, we see that sterilized intervention conditions exchange rate returns and volatility as it reduces market uncertainty on future exchange rate fundamentals. This also implies that trading volume and transaction costs in FX markets augment.

Our analysis offers some normative recommendations. Thus, we show that in extreme circumstances foreign exchange intervention may be destabilizing for the functioning of FX markets, in that extremely large intervention operations may precipitate a market crash. However, according to our analysis, a market crash requires a magnitude of intervention which is much larger than that typically selected by central banks.

Our analysis also shows how the choice of the route of intervention is not without consequences. Indeed, intervention operations channelled via the direct market convey more information than those routed via the inter-dealer one. Thus, whereas the former reduce exchange rate volatility, the latter present the opposite effect. Furthermore, given that foreign exchange intervention is more informative if conducted via the direct market, the impact of foreign exchange intervention on transaction costs is larger when the central bank trades with individual dealers.

Appendix I

In this Appendix we prove that in the direct intervention scenario in a RE equilibrium the spot rate satisfies the condition set in Proposition 1.

A. FX Dealers' Information

To identify the RE equilibrium under the conditions of the direct intervention scenario we need to study in more details the price formation process and the aggregation of information operated by the market participants. In this respect we need to define the information our rational investors, the FX dealers, possess when choosing their portfolios of assets.

Beside the information she can extract from the public signal, $\tilde{\psi}_t^{pu}$, any dealer d observes: i) the inter-dealer order flow, $\tilde{b}_t$; ii) her customer order flow, $\tilde{c}_t^d$; and iii) the current spot rate, $\tilde{s}_t$, given that she submits a limit order to the centralized trading platform. In fact, when dealer d places a limit order for the foreign currency, she *de facto* passes to the auctioneer a demand/supply schedule for the foreign currency, $x_t^d(\tilde{s}_t)$, which depends on the current spot rate, $\tilde{s}_t$. Then, in selecting her portfolio of assets she can condition on the observation of $\tilde{s}_t$. Moreover, in the direct intervention scenario, a privileged dealer d also observes the central bank's market order, $\tilde{c}_t^{cb,d}$.

In synthesis, the information set, $\Omega_t^{d'}$, of any non-privileged dealer d', with $d' \in (\zeta, 1]$, satisfies the following recursion,

$$\Omega_t^{d'} \;\equiv\; \{\tilde{b}_t, \tilde{c}_t^{d'}, \tilde{f}_t, \tilde{s}_t, \tilde{\psi}_t^{pu}, \Omega_{t-1}^{d'}\}.$$

Analogously the information set, Ω_t^d, of a privileged dealer $d \in [0, \zeta]$, satisfies a similar recursion,

$$\Omega_t^d \;\equiv\; \{\tilde{b}_t, \tilde{c}_t^d, \tilde{c}_t^{cb,d}\tilde{f}_t, \tilde{s}_t, \tilde{\psi}_t^{pu}, \Omega_{t-1}^d\}.$$

According to the scenario we investigate, a non-privileged dealer d' extracts a particular set of heterogeneous signals on next day innovation in the fundamental variable, $\tilde{\epsilon}_{t+1}^f$, from the variables $\tilde{b}_t, \tilde{c}_t^{d'}, \tilde{f}_t, \tilde{s}_t, \tilde{\psi}_t^{pu}$, while a privileged dealer d extracts an ampler set of heterogeneous signals on next day innovation in the fundamental variable, $\tilde{\epsilon}_{t+1}^f$, from the variables $\tilde{b}_t, \tilde{c}_t^d, \tilde{c}_t^{cb,d}, \tilde{f}_t, \tilde{s}_t, \tilde{\psi}_t^{pu}$.

Thus, a privileged dealer d, with $d \in [0, \zeta]$, receives from the central bank an order $\tilde{c}_t^{cb,d}$, which contains a private signal on $\tilde{\epsilon}_{t+1}^f$,

$$-\beta_D \tilde{\epsilon}_{t+1}^f \;+\; \tilde{\epsilon}_t^{cb,d}.$$

We can write more conveniently this signal as follows,

$$\tilde{\psi}_t^{cb,d} \;\equiv\; -\frac{1}{\beta_D}\,\tilde{c}_t^{cb,d} \;=\; \tilde{\epsilon}_{t+1}^f \;+\; \tilde{\xi}_t^{cb,d},$$

where $\tilde{\xi}_t^{cb,d} = -\tilde{\epsilon}_t^{cb,d}/\beta_D$. Given the properties of $\tilde{\epsilon}_t^{cb,d}$, this means that $\tilde{\xi}_t^{cb,d} \sim N(0, \sigma_{cb,D}^2)$, where $\sigma_{cb,D}^2 = \sigma_{cb}^2/\beta_D^2$, with $\tilde{\xi}_t^{cb,d} \perp \tilde{\xi}_s^{cb,d}$ for $s \neq t$. Notice that only a small proportions of dealers receive private signals from the central bank.

In a RE equilibrium the exchange rate will aggregate information. Then, let us conjecture that observing the spot rate, $\tilde{s}_t$, is equivalent to observing a signal $\tilde{\psi}_t^s$, where

$$\tilde{\psi}_t^s \quad = \quad \tilde{\epsilon}_{t+1}^f \; + \; \tilde{\xi}_t^s$$

and the error term $\tilde{\xi}_t^s$ is normally distributed with mean zero and variance σ_v^2. Clearly, the error terms are uncorrelated over time (ie. $\tilde{\xi}_t^s \perp \tilde{\xi}_{t'}^s$) and with the fundamental shock (ie. $\tilde{\epsilon}_t^s \perp \tilde{\epsilon}_{t'}^f$). Notice that this assumption is consistent with the equilibrium condition set in Proposition 1, as if the spot rate satisfies equation (10), then observing the spot rate is equivalent to observing the signal $\lambda_\epsilon^D \tilde{\epsilon}_{t+1}^f + \lambda_c^D \, \tilde{c}_t^n$, which entails that $\tilde{\xi}_t^s = (\lambda_c^D/\lambda_\epsilon^D)\tilde{c}_t^n$.

In synthesis, all non-privileged dealers extract information from the vector of common signals, $\tilde{\Psi}_t^U$, where $\tilde{\Psi}_t^U \equiv (\tilde{\psi}_t^{pu}, \tilde{\psi}_t^s)'$; while a privileged dealer, d, extracts information from the vector of signals, $\tilde{\Psi}_t^d$, where $\tilde{\Psi}_t^d \equiv (\tilde{\Psi}_t^{U\prime}, \tilde{\psi}_t^{cb,d})'$. Given our assumptions, the unconditional distribution for $\tilde{\Psi}_t^U$ is as follows

$$\tilde{\Psi}_t^U \;\sim\; \mathbf{N}\,(\mathbf{0}, \Sigma_D^U), \quad \text{with} \quad \Sigma_D^U \;\equiv\; \begin{pmatrix} \sigma_f^2 + \sigma_{pu}^2 & \sigma_f^2 \\ \sigma_f^2 & \sigma_f^2 + \sigma_{s,D}^2 \end{pmatrix}.$$

Likewise, the unconditional distribution for $\tilde{\Psi}_t^d$ is as follows,

$$\tilde{\Psi}_t^d \;\sim\; \mathbf{N}\,(\mathbf{0}, \Sigma_D^I), \quad \text{with} \quad \Sigma_D^I \;\equiv\; \begin{pmatrix} \sigma_f^2 + \sigma_{pu}^2 & \sigma_f^2 & \sigma_f^2 \\ \sigma_f^2 & \sigma_f^2 + \sigma_{s,D}^2 & \sigma_f^2 \\ \sigma_f^2 & \sigma_f^2 & \sigma_f^2 + \sigma_{cb,D}^2 \end{pmatrix}.$$

B. FX Dealers' Expectations

Since our dealers can only receive signals on next day value of the fundamental process, $\tilde{f}_{t+1}$ and on today total order flow, $\tilde{o}_t$, we have that for $k > 1$ $E_t^k[\tilde{f}_{t+k}] = 0$ and for $k \geq 1$ $E_t^k[\tilde{o}_{t+k}] = 0$. Then, the equilibrium spot rate in equation (9) boils down to

$$\tilde{s}_t \quad = \quad \frac{1}{1+\alpha\,(1-\rho)} \left(\tilde{f}_t \; + \; \frac{\alpha}{1+\alpha}\, \bar{E}_t^1 \left[\, \tilde{\epsilon}_{t+1}^f \, \right] \right) \; - \; \frac{\alpha}{\nu} \left(z_t \; - \; \frac{\alpha}{1+\alpha} \left(\tilde{c}_t \; - \; \bar{E}_t^1 \left[\, \tilde{c}_t \, \right] \right) \right). \qquad (14)$$

Therefore, let us turn to FX dealers' expectations of next day value of the fundamental shift, $\tilde{\epsilon}_{t+1}^f$, and on today customer order flow, $\tilde{c}_t$. Firstly, we calculate the individual forecasts; secondly, we aggregate them.

B.1. FX Dealers' Expectations of Fundamental Shifts

From the information set of our FX dealers we can determine their expectations in day t of the impending shift in the exchange rate fundamental, $\tilde{\epsilon}^f_{t+1}$. Thus, employing the projection theorem for Normal distributions we see that any non-privileged dealer presents the following conditional distribution for $\tilde{\epsilon}^f_{t+1}$,

$$\left(\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^U_t\right) \quad \sim \quad N\left(E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^U_t\right], \mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^U_t\right]\right),$$

with

$$E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^U_t\right] \quad = \quad \theta^U_{pu,D}\,\tilde{\psi}^{pu}_t \;+\; \theta^U_{s,D}\,\tilde{\psi}^s_t,$$

$$\mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^U_t\right] \quad = \quad \left[\,\pi_{pu} \;+\; \pi_{s,D}\,\right]^{-1},$$

where π_{pu} and $\pi_{s,D}$ are the precisions of the public signal and the spot rate signal,

$$\pi_{pu} \;=\; \frac{1}{\sigma^2_{pu}} \qquad \text{and} \qquad \pi_{s,D} \;=\; \frac{1}{\sigma^2_{s,D}},$$

while the coefficients θ^U_D's are given by

$$\theta^U_{pu,D} \quad = \quad \frac{\pi_{pu}}{\pi_f \;+\; \pi_{pu} \;+\; \pi_{s,D}},$$

$$\theta^U_{s,D} \quad = \quad \frac{\pi_{s,D}}{\pi_f \;+\; \pi_{pu} \;+\; \pi_{s,D}},$$

where $\pi_f \;= 1/\sigma^2_f$. In a similar fashion one finds that for a privileged dealer d, with $d \in [0,\zeta]$, the conditional distribution of $\tilde{\epsilon}^f_{t+1}$ is the following

$$\left(\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^d_t\right) \quad \sim \quad N\left(E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^d_t\right], \mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^d_t\right]\right),$$

with

$$E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^d_t\right] \quad = \quad \theta^I_{pu,D}\,\tilde{\psi}^{pu}_t \;+\; \theta^I_{s,D}\,\tilde{\psi}^s_t \;+\; \theta^I_{cb,D}\,\tilde{\psi}^{cb,d}_t$$

$$\mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^d_t\right] \quad = \quad \left[\,\pi_{pu} \;+\; \pi_{s,D} \;+\; \pi_{cb,D}\,\right]^{-1},$$

where π^D_{cb} is the precision of the private signal dealer d extracts from the central bank market order, $\tilde{c}^{cb,d}_t$,

$$\pi_{cb,D} \;=\; \frac{1}{\sigma^2_{cb,D}},$$

while the coefficients θ^I_D's are given by

$$\theta^I_{pu,D} = \frac{\pi_{pu}}{\pi_f + \pi_{cb,D} + \pi_{pu} + \pi_{s,D}},$$

$$\theta^I_{s,D} = \frac{\pi_{s,D}}{\pi_f + \pi_{cb,D} + \pi_{pu} + \pi_{s,D}},$$

$$\theta^I_{cb,D} = \frac{\pi_{cb,D}}{\pi_f + \pi_{cb,D} + \pi_{pu} + \pi_{s,D}}.$$

B.2. FX Dealers' Expectations of Total Order Flow

The total order flow, $\tilde{o}_t$, is equal to the sum of customer order flow, $\tilde{c}_t$ and inter-dealer order flow, $\tilde{b}_t$. The latter is observable, so that for any generic dealer the expectation of this variable corresponds to its realization. Therefore, let us concentrate on the former component of $\tilde{o}_t$. In the direct intervention scenario,

$$\tilde{c}_t = \tilde{c}^{cb}_t + \tilde{c}^n_t = -\beta\,\epsilon^f_{t+1} + \tilde{c}^n_t.$$

As already suggested, we conjecture that the spot rate our dealers observe when placing their limit orders contains some signal on the noise trading component of customer order flow, $\tilde{c}^n_t$, ie. on those market orders collectively placed by the population of unsophisticated clients to our FX dealers. In particular, we conjecture that there exists a constant μ_D such that

$$\tilde{c}^n_t = \mu_D\,\tilde{\xi}^s_t.$$

Thus, employing the projection theorem for Normal distributions, we see that any non-privileged dealer presents the following conditional distribution for $\tilde{c}^n_t$,

$$\left(\tilde{c}^n_t \mid \tilde{\Psi}^U_t\right) \sim N\left(E\left[\tilde{c}^n_t \mid \tilde{\Psi}^U_t\right], \mathrm{Var}\left[\tilde{c}^n_t \mid \tilde{\Psi}^U_t\right]\right),$$

with

$$E\left[\tilde{c}^n_t \mid \tilde{\Psi}^U_t\right] = \kappa^U_{pu,D}\,\tilde{\psi}^{pu}_t + \kappa^U_{s,D}\,\tilde{\psi}^s_t,$$

$$\mathrm{Var}\left[\tilde{c}^n_t \mid \tilde{\Psi}^U_t\right] = \mu^2_D\,\left[\pi_{pu} + \pi_{s,D}\right]^{-1},$$

where the coefficients κ^U_D's are given by

$$\kappa^U_{pu,D} = -\mu_D\,\frac{\pi_{pu}}{\pi_f + \pi_{pu} + \pi_{s,D}},$$

$$\kappa^U_{s,D} = \mu_D\,\frac{\pi_{s,D}}{\pi_f + \pi_{pu} + \pi_{s,D}}.$$

Similarly, for a privileged dealer d, with $d \in [0, \zeta]$, the conditional distribution of $\tilde{\epsilon}^f_{t+1}$ is

$$\left(\tilde{c}^n_t \mid \tilde{\Psi}^d_t\right) \quad \sim \quad N\left(E\left[\tilde{c}^n_t \mid \tilde{\Psi}^d_t\right], \mathrm{Var}\left[\tilde{c}^n_t \mid \tilde{\Psi}^d_t\right]\right),$$

with

$$\begin{aligned} E\left[\tilde{c}^n_t \mid \tilde{\Psi}^d_t\right] &= \kappa^I_{pu,D}\,\tilde{\psi}^{pu}_t \;+\; \kappa^I_{s,D}\,\tilde{\psi}^s_t \;+\; \kappa^I_{cb,D}\,\tilde{\psi}^{cb,d}_t, \\ \mathrm{Var}\left[\tilde{c}^n_t \mid \tilde{\Psi}^d_t\right] &= \mu^2_D\;\left[\pi_{pu} \;+\; \pi_{s,D} \;+\; \pi_{cb,D}\right]^{-1}, \end{aligned}$$

where the coefficients κ^I_D's are given by

$$\begin{aligned} \kappa^I_{pu,D} &= -\,\mu_D\,\frac{\pi_{pu}}{\pi_f \;+\; \pi_{cb,D} \;+\; \pi_{pu} \;+\; \pi_{s,D}}, \\ \kappa^I_{s,D} &= \mu_D\,\frac{\pi_{s,D}}{\pi_f \;+\; \pi_{cb,D} \;+\; \pi_{pu} \;+\; \pi_{s,D}}, \\ \kappa^I_{cb,D} &= -\,\mu_D\,\frac{\pi_{cb,D}}{\pi_f \;+\; \pi_{cb,D} \;+\; \pi_{pu} \;+\; \pi_{s,D}}. \end{aligned}$$

C. FX Dealers' Average Expectations

Now, we need to aggregate our dealers' forecasts for $\tilde{\epsilon}^f_{t+1}$ and $\tilde{c}^n_t$. Considering that $\bar{E}^1[\cdot]$ is a weighted average, we have that

$$\begin{aligned} \bar{E}^1_{t,D}\left[\tilde{\chi}_t\right] &= \frac{1}{\nu}\left(\int_0^\zeta \nu_{d'}\,E^{d'}_t\left[\tilde{\chi}_t\right]dd' \;+\; \int_\zeta^1 \nu_{d''}\,E^{d''}_t\left[\tilde{\chi}_t\right]dd''\right) \\ &= \frac{1}{\nu}\left(\zeta\,\nu_I\,\bar{E}^I_t\left[\tilde{\chi}_t\right] \;+\; (1\;-\;\zeta)\,\nu_U\,E^U_t\left[\tilde{\chi}_t\right]\right) \\ &= \left(\phi_I\,\bar{E}^I_t\left[\tilde{\chi}_t\right] \;+\; (1\;-\;\phi_I)\,E^U_t\left[\tilde{\chi}_t\right]\right), \end{aligned}$$

where

$$\bar{E}^I_t\left[\tilde{\chi}_t\right] \;=\; \frac{1}{\int_0^\zeta \nu_{d'}\,dd'}\left(\int_0^\zeta \nu_{d'}\,E^{d'}_t\left[\tilde{\chi}_t\right]dd'\right) \quad \text{and} \quad \phi_I \;=\; \frac{\zeta\,\nu_I}{\zeta\,\nu_I \;+\; (1-\zeta)\,\nu_U},$$

with

$$\nu_I \;=\; \frac{1}{\zeta}\left(\int_0^\zeta \nu_{d'}\,dd'\right) \quad \text{and} \quad \nu_U \;=\; \frac{1}{1-\zeta}\left(\int_\zeta^1 \nu_{d''}\,dd''\right).$$

Now, $\bar{E}_t^I[\tilde{\epsilon}_{t+1}^f]$ is given by

$$\begin{aligned}
\bar{E}_t^I\left[\tilde{\epsilon}_{t+1}^f\right] &= \frac{1}{\int_0^\zeta \nu_{d'}\, dd'}\left(\int_0^\zeta \nu_{d'}\, E_t^{d'}\left[\tilde{\epsilon}_{t+1}^f\right] dd'\right) \\
&= \frac{1}{\int_0^\zeta \nu_{d'}\, dd'}\left(\int_0^\zeta \nu_{d'}\left(\theta_{pu,D}^I\, \tilde{\psi}_t^{pu} + \theta_{s,D}^I\, \tilde{\psi}_t^s + \theta_{cb,D}^I\, \tilde{\psi}_t^{cb,d'}\right) dd'\right) \\
&= \theta_{pu,D}^I\, \tilde{\psi}_t^{pu} + \theta_{s,D}^I\, \tilde{\psi}_t^s + \theta_{cb,D}^I\, \tilde{\epsilon}_{t+1}^f.
\end{aligned}$$

Likewise, for $\bar{E}_t^I[\tilde{c}_t^n]$ we have that

$$\begin{aligned}
\bar{E}_t^I\left[\tilde{c}_t^n\right] &= \kappa_{pu,D}^I\, \tilde{\psi}_t^{pu} + \kappa_{s,D}^I\, \tilde{\psi}_t^s + \kappa_{cb,D}^I\, \tilde{\epsilon}_{t+1}^f \\
&= \kappa_{pu,D}^I\, \tilde{\psi}_t^{pu} + \kappa_{s,D}^I\, \tilde{\xi}_t^s + (\kappa_{s,D}^I + \kappa_{cb,D}^I)\, \tilde{\epsilon}_{t+1}^f.
\end{aligned}$$

Given the conditional expectations of the non-privileged dealers in the direct intervention scenario,

$$\begin{aligned}
E_t^U\left[\tilde{\epsilon}_{t+1}^f\right] &= \theta_{pu,D}^U\, \tilde{\psi}_t^{pu} + \theta_{s,D}^U\, \tilde{\xi}_t^s + \theta_{s,D}^U\, \tilde{\epsilon}_{t+1}^f, \\
E_t^U\left[\tilde{c}_t^n\right] &= \kappa_{pu,D}^U\, \tilde{\psi}_t^{pu} + \kappa_{s,D}^U\, \tilde{\xi}_t^s + \kappa_{s,D}^U\, \tilde{\epsilon}_{t+1}^f,
\end{aligned}$$

we conclude that when the central bank trades with individual FX dealers

$$\bar{E}_{t,D}^1\left[\tilde{\epsilon}_{t+1}^f\right] = \eta_\theta^{pu,D}\, \tilde{\psi}_t^{pu} + \eta_\theta^{s,D}\, \tilde{\xi}_t^s + \eta_\theta^{\epsilon,D}\, \tilde{\epsilon}_{t+1}^f, \tag{15}$$

$$\bar{E}_{t,D}^1\left[\tilde{c}_t^n\right] = \eta_\kappa^{pu,D}\, \tilde{\psi}_t^{pu} + \eta_\kappa^{s,D}\, \tilde{\xi}_t^s + \eta_\kappa^{\epsilon,D}\, \tilde{\epsilon}_{t+1}^f, \tag{16}$$

where

$$\begin{aligned}
\eta_\theta^{pu,D} &= \phi_I\, \theta_{pu,D}^I + (1-\phi_I)\, \theta_{pu,D}^U, & \eta_\kappa^{pu,D} &= \phi_I\, \kappa_{pu,D}^I + (1-\phi_I)\, \kappa_{pu,D}^U, \\
\eta_\theta^{s,D} &= \phi_I\, \theta_{s,D}^I + (1-\phi_I)\, \theta_{s,D}^U, & \eta_\kappa^{s,D} &= \phi_I\, \kappa_{s,D}^I + (1-\phi_I)\, \kappa_{s,D}^U, \\
\eta_\theta^{\epsilon,D} &= \phi_I\left(\theta_{cb,D}^I + \theta_{s,D}^I\right) + (1-\phi_I)\, \theta_{s,D}^U, & \eta_\kappa^{\epsilon,D} &= \phi_I\left(\kappa_{cb,D}^I + \kappa_{s,D}^I\right) + (1-\phi_I)\, \kappa_{s,D}^U.
\end{aligned}$$

In this way, we find that

$$\begin{aligned}
\tilde{c}_t - \bar{E}_{t,D}^1\left[\tilde{c}_t\right] = {}& \left(\beta\, \eta_\theta^{pu,D} - \eta_\kappa^{pu,D}\right) \tilde{\psi}_t^{pu} - \left(\beta\,(1-\eta_\theta^{\epsilon,D}) + \eta_\kappa^{\epsilon,D}\right) \tilde{\epsilon}_{t+1}^f \\
& + \left(\beta \frac{\eta_\theta^{s,D}}{\mu_D} + 1 - \frac{\eta_\kappa^{s,D}}{\mu_D}\right) \tilde{c}_t^n.
\end{aligned} \tag{17}$$

D. The Equilibrium Spot Rate

Inserting the FX dealers' average forecast for $\tilde{\epsilon}^f_{t+1}$ and $\tilde{c}_t$, reported in equations (15) and (17) into the expression for the exchange rate in equation (14) we find that the spot rate respects equation (10),

$$\tilde{s}_t \;=\; \lambda^D_b\,\tilde{b}_t \;+\; \lambda^D_c\,\tilde{c}^n_t \;+\; \lambda^D_\epsilon\,\tilde{\epsilon}^f_{t+1} \;+\; \lambda^D_f\,\tilde{f}_t \;+\; \lambda^D_{pu}\,\tilde{\psi}^{pu}_t \;+\; \lambda^D_z\,\tilde{z}_{t-1},$$

where the coefficients λ^D's are given by the following expressions

$$\lambda^D_b \;=\; -\frac{\alpha}{\nu}, \qquad \lambda^D_c \;=\; \frac{\alpha}{1\,+\,\alpha}\,\frac{1}{\mu_D}\left[\left(\frac{1}{1+\alpha\,(1-\rho)} \;+\; \frac{\alpha}{\nu}\,\beta\right)\eta^{s,D}_\theta \;-\; \frac{\alpha}{\nu}\,\eta^{s,D}_\kappa \;-\; \frac{\mu_D}{\nu}\right],$$

$$\lambda^D_f \;=\; \frac{1}{1+\alpha\,(1-\rho)}, \qquad \lambda^D_\epsilon \;=\; \frac{\alpha}{1\,+\,\alpha}\left[\left(\frac{1}{1+\alpha\,(1-\rho)} \;+\; \frac{\alpha}{\nu}\,\beta\right)\eta^{\epsilon,D}_\theta \;-\; \frac{\alpha}{\nu}\,\eta^{\epsilon,D}_\kappa \;+\; \frac{1}{\nu}\,\beta\right],$$

$$\lambda^D_z \;=\; -\frac{\alpha}{\nu}, \qquad \lambda^D_{pu} \;=\; \frac{\alpha}{1\,+\,\alpha}\left[\left(\frac{1}{1+\alpha\,(1-\rho)} \;+\; \frac{\alpha}{\nu}\,\beta\right)\eta^{pu,D}_\theta \;-\; \frac{\alpha}{\nu}\,\eta^{pu,D}_\kappa\right].$$

E. Conjectured and Actual Equilibria

The FX dealers can condition on the value of the current spot rate, $\tilde{s}_t$, when selecting their demand schedules for the foreign currency. As mentioned, this implies that they can condition on the information revealed by the spot rate. Indeed, in equilibrium all rational investors derive a signal on $\tilde{\epsilon}^f_{t+1}$ from the spot rate in equation (10). In a RE equilibrium all rational FX dealers will realize that observing $\tilde{s}_t$ is equivalent to observing the signal

$$\tilde{\epsilon}^f_{t+1} \;+\; \frac{\lambda^D_c}{\lambda^\epsilon_D}\,\tilde{c}^n_t.$$

On the other hand, to derive equation (10) we have assumed that $\tilde{c}^n_t \;=\; \mu_D\,\tilde{\xi}^s_t$. We conclude that $\tilde{\xi}^s_t = (\lambda^D_c/\lambda^D_\epsilon)\tilde{c}^n_t$. We will have a proper equilibrium only if

$$1 \;=\; \frac{\lambda^D_c}{\lambda^D_\epsilon}\,\mu_D.$$

This allows to pin down the coefficient μ_D, which turns out to be equal to

$$\mu_D \;=\; -\,\beta \;-\; \frac{1}{1+\alpha\,(1-\rho)}\,\frac{\phi_I\,\theta^I_{cb,D}}{1\,+\,\alpha\,\phi_I\,\theta^I_{cb,D}}\;\nu.$$

Then, we can re-write the coefficients λ_c^D, λ_ϵ^D and λ_{pu}^D as follows

$$\begin{aligned}
\lambda_c^D &= \frac{\alpha}{1+\alpha}\,\frac{1}{\mu_D}\left[\left(\frac{1}{1+\alpha\,(1-\rho)}\,\frac{1\,+\,2\,\alpha\,\phi_I\,\theta_{cb,D}^I}{1\,+\,\alpha\,\phi_I\,\theta_{cb,D}^I}\,+\,2\,\frac{\alpha}{\nu}\,\beta\right)\eta_\theta^{s,D}\,-\,\frac{\mu_D}{\nu}\right],\\
\lambda_\epsilon^D &= \frac{\alpha}{1+\alpha}\left[\left(\frac{1}{1+\alpha\,(1-\rho)}\,\frac{1\,+\,2\,\alpha\,\phi_I\,\theta_{cb,D}^I}{1\,+\,\alpha\,\phi_I\,\theta_{cb,D}^I}\,+\,2\,\frac{\alpha}{\nu}\,\beta\right)\eta_\theta^{s,D}\,-\,\frac{\mu_D}{\nu}\right],\\
\lambda_{pu}^D &= \frac{\alpha}{1+\alpha}\,\frac{1}{1+\alpha\,(1-\rho)}\,\frac{1}{1\,+\,\alpha\,\phi_I\,\theta_{cb,D}^I}\,\eta_\theta^{pu,D}.
\end{aligned}$$

One can easily check that $\lambda_c^D < 0$, $\lambda_\epsilon^D > 0$ and $\lambda_{pu}^D > 0$.

Appendix II

In this Appendix we prove that in the indirect intervention scenario in a RE equilibrium the spot rate satisfies the condition set in Proposition 2. This proof is relatively simpler as all FX dealers possess symmetric information. Specifically, now the spot rate does not represents an extra signal on the fundamental shift $\tilde{\epsilon}_{t+1}^f$ our FX dealers observe when placing their limit orders. Rather, in equilibrium the FX dealers can recoup the customer order flow $\tilde{c}_t$. This means that equation (14) boils down to

$$\tilde{s}_t \;=\; \frac{1}{1+\alpha\,(1-\rho)}\left(\tilde{f}_t\,+\,\frac{\alpha}{1+\alpha}\,\bar{E}_t^1\left[\tilde{\epsilon}_{t+1}^f\right]\right)\,-\,\frac{\alpha}{\nu}\,z_t. \tag{18}$$

A. FX Dealers's Information

To derive the FX dealers' forecast for $\tilde{\epsilon}_{t+1}^f$ consider that in the indirect intervention case, any dealer observes the public signal $\tilde{\psi}_t^{pu}$ and the same common signal from the inter-dealer order flow, $\tilde{b}_t$,

$$\tilde{b}_t \;\equiv\; -\beta\,\tilde{\epsilon}_{t+1}^f\,+\,\tilde{b}_t^n,$$

which we can more easily write as

$$\tilde{\psi}_t^b \;\equiv\; -\frac{1}{\beta}\,\tilde{b}_t \;=\; \tilde{\epsilon}_{t+1}^f\,+\,\tilde{\xi}_t^b,$$

where $\tilde{\xi}_t^b = -\tilde{b}_t^n/\beta$. Given the properties of $\tilde{b}_t^n$, this means that $\tilde{\xi}_t^b \sim N(0, \sigma_{b,B}^2)$, where $\sigma_{b,B}^2 = \sigma_b^2/\beta^2$, with $\tilde{\xi}_t^b \perp \tilde{\xi}_s^b$ for $s \neq t$. In brief, all dealers extract information from the common vector of signals, $\tilde{\Psi}_t^B$, where $\tilde{\Psi}_t^B \equiv (\tilde{\psi}_t^{pu}, \tilde{\psi}_t^b)'$. Given our assumptions, the unconditional distribution of $\tilde{\Psi}_t^B$ is

$$\tilde{\Psi}_t^B \;\sim\; \mathbf{N}\,(\mathbf{0}, \Sigma_B), \quad \text{with} \quad \Sigma_B \;\equiv\; \begin{pmatrix} \sigma_f^2+\sigma_{pu}^2 & \sigma_f^2 \\ \sigma_f^2 & \sigma_f^2+\sigma_{b,B}^2 \end{pmatrix},$$

B. FX Dealers' Expectations

All dealers formulate the same forecasts of the fundamental innovation, $\tilde{\epsilon}^f_{t+1}$. This implies that in this scenario the individual expectations of our FX dealers and the market wide ones will coincide. Then, considering a generic dealer her conditional distribution for $\tilde{\epsilon}^f_{t+1}$ is

$$\left(\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^B_t\right) \quad \sim \quad N\left(E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^B_t\right], \mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^B_t\right]\right),$$

with

$$\begin{aligned} E\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^B_t\right] &= \theta_{pu,B}\,\tilde{\psi}^{pu}_t \;+\; \theta_{b,B}\,\tilde{\psi}^b_t, \\ \mathrm{Var}\left[\tilde{\epsilon}^f_{t+1} \mid \tilde{\Psi}^B_t\right] &= \left[\pi_{pu} \;+\; \pi_{b,B}\right]^{-1}, \end{aligned} \tag{19}$$

where $\pi_{b,B}$ is the precision of the signal all FX dealers extracts from the inter-dealer order flow, $\tilde{b}_t$,

$$\pi_{b,B} = \frac{1}{\sigma^2_{b,B}},$$

while the coefficients θ_B's are given by

$$\begin{aligned} \theta_{pu,B} &= \frac{\pi_{pu}}{\pi_f \;+\; \pi_{b,B} \;+\; \pi_{pu}}, \\ \theta_{b,B} &= \frac{\pi_{b,B}}{\pi_f + \pi_{b,B} \;+\; \pi_{pu}}. \end{aligned}$$

C. The Equilibrium Spot Rate

Finally, inserting the FX dealers' average forecasts reported in equations (19) into the expression for the exchange rate in equation (18) we find that

$$\tilde{s}_t \quad = \quad \lambda^B_b\,\tilde{b}_t \;+\; \lambda^B_c\,\tilde{c}_t \;+\; \lambda^B_f\,\tilde{f}_t \;+\; \lambda^B_{pu}\,\tilde{\psi}^{pu}_t \;+\; \lambda^D_z\,\tilde{z}_{t-1},$$

where

$$\lambda^B_b = -\frac{1}{\beta}\frac{\alpha}{1+\alpha}\frac{1}{1+\alpha(1-\rho)}\theta_{b,B} - \frac{\alpha}{\nu}, \qquad \lambda^B_c = -\frac{\alpha}{\nu},$$

$$\lambda^B_f = \frac{1}{1+\alpha(1-\rho)}, \qquad \lambda^B_{pu} = \frac{\alpha}{1\,+\,\alpha}\frac{1}{1+\alpha\,(1-\rho)}\,\theta_{pu,B},$$

$$\lambda^B_z = -\frac{\alpha}{\nu}.$$

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