Co-Movement in Sticky Price Models with Durable Goods*

Charles T. Carlstrom[a] and Timothy S. Fuerst[b]

[a]Federal Reserve Bank of Cleveland, Cleveland, OH 44101, USA.
[b]Bowling Green State University, Bowling Green, OH 43403, USA; and Federal Reserve Bank of Cleveland.

August 29, 2006

Abstract:

JEL Classification: E31, E52

1. Introduction.

Recent work on the monetary transmission mechanism has emphasized some of the unique characteristics of durable consumption goods (including residential housing) in the monetary business cycle. Using a VAR approach, Erceg and Levin (2005) document the high degree of interest-sensitivity of the durable goods sector, eg., an exogenous increase in the interest rate leads to an estimated decline in durable goods investment of nearly five-times that of the decline in the remaining components of GDP. For residential housing investment the decline is nearly ten-times the size of non-durable consumption. This heightened interest sensitivity is likely a result of the fact that the stock of durables is large relative to the annual durable investment flow.

Erceg and Levin (2005) construct a two-sector general equilibrium model in which the durable and non-durable sectors are subject to price stickiness. They calibrate the model to match the sectoral responses to a monetary shock. They assume an equal degree of price stickiness across the two sectors. This assumption seems quite heroic, and there are reasons to think that the prices of durable goods are much more flexible than are non-durable goods. For example, housing prices are surely not set in advance, and Bils and Klenow (2004) report much greater frequency of price adjustment for consumer durables.

Using a general equilibrium model quite similar to Erceg and Levin (2005), Barsky et al. (2005) demonstrate that the behavior of aggregate output in the model in response to a one-time change in the money supply is largely determined by the relative degree of price stickiness in the durable goods sector. In particular, if durable goods prices are flexible, but non-durable goods prices are sticky, then a monetary contraction will lead to a *decline* in non-durable goods production but an *increase* in durable goods production, so that aggregate output is little changed. This lack of co-movement between non-durables and durables is wildly at odds with the data. We call this the "co-movement puzzle." The source of the puzzle is clear. If the only shock to the system is a monetary shock, then this shock will directly affect only the sector with sticky nominal prices (the non-durable sector). The other sector (durable goods) is only affected indirectly as the decline in demand for inputs in the non-durable sector leads to a decline in production costs for durable goods and thus an expansion of durable goods production.

This paper begins with the premise that durable goods prices are much more flexible than are non-durable goods prices. We interpret durable goods to be residential housing. It seems unlikely that there is much, if any, nominal price stickiness in housing. We therefore assume that durable goods prices are perfectly flexible. We assume that monetary policy is conducted by a Taylor-type interest rate operating procedure. This is in contrast to Barksy et al (2005) who assume that monetary policy is given by a random walk of the money supply. We first demonstrate that the co-movement puzzle arises under this alternative monetary operating procedure: an exogenous increase in the interest rate leads to a modest decline in the non-durable sector, a large expansion in the durable goods sector, and a near-zero response in aggregate activity.

Our sensitivity analysis includes an investigation of two key parameters. First, higher levels of complementarity between non-durables and durables will make co-movement more likely. However, the results below indicate that we need an implausibly high degree of complementarity (close to Leontief preferences) before co-movement arises. As a second form of sensitivity analysis, we follow Topel and Rosen's (1988) empirical evidence and impose firm-level adjustment costs on the change in housing construction. Using Topel and Rosen's (1988) estimate of the short-run elasticity of supply, we find that aggregate activity in the model does sharply decline in response to an increase in the interest rate but the co-movement puzzle remains.

Barsky et al (2003) suggest two possible solutions to the puzzle: nominal wage stickiness, and credit constraints. The former is a "supply" story. With nominal wage stickiness a monetary contraction will tend to increase real wages, leading to a reduction in desired output by the durable goods firms. The credit constraint is a "demand" story. If a monetary contraction makes it more difficult for consumers to purchase durable goods, then the resulting decline in labor demand will tend to decrease real wages. Similarly, the demand nature of the credit story implies that the relative price of durables will fall more sharply in response to a monetary contraction than in the sticky wage model. We will investigate both a sticky-wage model and a credit model below.

Nominal wage stickiness is added to the model in a manner similar to Erceg, Henderson and Levin (2000). Since labor is the key input in production, nominal wage stickiness induces a great deal of nominal stickiness in the durable good price. For plausible degrees of wage stickiness, and no adjustment costs, we find excessive volatility in the first quarter and negative co-movement in subsequent quarters (Barsky et al (2003) report similar results.) However, we demonstrate that by adding adjustment costs as in Topel-Rosen (1988), the sticky wage model solves the co-movement puzzle and delivers reasonable volatilities.

We also investigate the role of credit constraints. A traditional argument for the increased sensitivity of housing to the business cycle is that housing and durable goods purchases are subject to credit constraints that are not applicable to non-durable goods. We modify the basic model to incorporate a credit constraint that applies only to durable goods purchases. In contrast to the recent literature, we examine a "flow constraint" in that current durable goods purchases are constrained by the household's current labor income.[1] This constraint is motivated by a classic hold-up problem. We show that credit constraints can also solve the hold-up problem. However, the behavior of the real wage in this model appears to be counterfactual.

The paper proceeds as follows. Section 2 presents the baseline sticky-price model with durable goods and documents the co-movement puzzle in the model. Section 3 investigates the sticky wage model. Section 4 introduces the credit constraint and demonstrates the ability of this credit-constraint model to generate co-movement. Section 5 concludes and discusses whether the sticky wage solution or the credit constraint solution best matches the data.

2. A Benchmark Durable Goods Model.

The economy consists of numerous households and firms each of which we will discuss in turn.

2.a. Households.

[1] Iacoviello (2005) analyzes a model with borrowing constraints in which the stock of debt is constrained by the stock of housing.

Households are identical and infinitely-lived with preferences over consumption (C_t), durable goods (D_t), real money balances (M_{t+1}) and labor (L_t). The utility functional is given by

$$U(C_t, D_t, L_t, \frac{M_{t+1}}{P_t}) \equiv \frac{\left\{\left[bC_t^{1-1/\rho} + (1-b)D_t^{1-1/\rho}\right]^{\frac{\rho}{\rho-1}}\right\}^{1-\sigma}}{1-\sigma} - \varphi\frac{L_t^{1+\omega}}{1+\omega} + V\left(\frac{M_{t+1}}{P_t}\right)$$

where V is concave. The parameter $\rho > 0$ is the elasticity of substitution between durable and non-durable consumption, $b > 0$, $\sigma > 0$ is the intertemporal elasticity of substitution, and $\omega > 0$ is the inverse of the Frisch labor supply elasticity. Since money balances are separable and we are using an interest rate operating procedure, the form of V is irrelevant in what follows. The household's resource constraints include:

$$D_t = (1-\delta)D_{t-1} + X_t$$

$$P_t^x X_t + P_t^c C_t + M_{t+1} \leq W_t L_t + \pi_t + M_t + T_t$$

where X_t denotes the purchase of new housing, δ is the depreciation rate of housing, P_t^x is the nominal price of housing, P_t^c is the nominal price of non-durable goods, W_t is the nominal wage, π_t denotes the profits flow from firms, and T_t is the endogenous lump-sum monetary transfer needed to meet the interest rate target. The first order conditions to the household's problem include:

$$\frac{-U_L}{U_c} = \frac{W_t}{P_t^c} \tag{1}$$

$$U_D(t) + E_t\frac{P_{t+1}^x}{P_{t+1}^c}\beta(1-\delta)U_c(t+1) = \frac{P_t^x}{P_t^c}U_c(t) \tag{2}$$

$$U_c(t) = E_t\frac{P_t^c}{P_{t+1}^c}\beta R_t U_c(t+1) \tag{3}$$

$$\frac{U_m(t)}{U_c(t)} = \frac{R_t - 1}{R_t} \tag{4}$$

where we have also priced a one-period bond paying gross interest R_t.

2.b. Firms.

As for production, there are two sectors, housing and non-durables. Since the two sectors are entirely symmetric we will focus on a generic production sector. Within each sector there is a layer of perfectly competitive final goods firms. Final goods production is given by the CES production function:

$$Y_t = \{\int_0^1 [y_t(i)^{(\eta-1)/\eta}]di\}^{\eta/(\eta-1)}$$

where Y_t denotes the final good, and $y_t(i)$ denotes the continuum of intermediate goods, each indexed by i ∈ [0,1]. The implied demand for the intermediate good is thus given by

$$y_t(i) = Y_t\left[\frac{P_t(i)}{P_t}\right]^{-\eta}$$

where $P_t(i)$ is the dollar price of good i, and P_t is the final goods price. Perfect competition in the final goods market implies that the final goods price is given by

$$P_t = \{\int_0^1 [P_t(i)^{(1-\eta)}]di\}^{1/(1-\eta)} .$$

Intermediate goods firm i is a monopolist producer of intermediate good i. Each intermediate firm hires labor from households utilizing the linear production function $y_t = f(L_t) \equiv L_t$ where we have dropped the firm-specific subscript for simplicity. We assume that labor may freely flow across firms and sectors so that there is a common nominal wage. Imperfect competition implies that factor payments are distorted. In particular, we have

$$W_t = Z_t^j P_t^j$$

where Z_t^j denotes the real marginal cost of production in sector j (durables or non-durables). Since factor markets are competitive, the intermediate goods firms take W_t as given.

As for intermediate goods pricing, we follow Yun (1997) and utilize the assumption of Calvo staggered pricing. Each period fraction (1-ν) of firms get to set a new price, while the remaining fraction ν must charge the previous period's price times steady-state inflation (denoted by π). This probability of a price change is constant across time and is independent of how long it has been since any one firm has last adjusted its price. Suppose that firm i wins the Calvo lottery and can set a new price in time t. It's optimization problem is given by (we again have omitted the sectoral subscript for simplicity):

$$\max_{P_t(i)} \left\{ E_t \sum_{j=0}^{\infty} (\nu\beta)^j \frac{\Lambda_{t+j}}{\Lambda_t} \left[\left(\frac{P_t(i)}{P_t} \right)^{-\eta} Y_t \left(\frac{P_t(i)}{P_t} - Z_t \right) \right] \right\}$$

where $\Lambda_{t+j} \equiv U_c(t+j)/P_{t+j}$ denotes the marginal utility of a dollar. The optimization condition is given by

$$P_t(i) = \frac{\eta E_t \sum_{j=0}^{\infty} (\nu\beta)^j \Lambda_{t+j} P_{t+j}^{\eta} Y_{t+j} Z_{t+j}}{(\eta - 1) E_t \sum_{j=0}^{\infty} (\nu\beta\pi)^j \Lambda_{t+j} P_{t+j}^{\eta-1} Y_{t+j}}$$

If ν = 0 so that all prices are flexible each period, $Z_t = (\eta-1)/\eta < 1$. This latter term $\bar{Z} \equiv (\eta-1)/\eta$ is a measure of the steady-state distortion arising from monopolistic competition. In the case of sticky prices (ν > 0), Z_t will typically not equal $\bar{Z}$ and will reflect the time-varying monopoly distortion.

We will assume that prices are perfectly flexible in the housing sector ($\nu_x = 0$). Following Topel and Rosen (1988), we assume that there are firm-level adjustment costs in the housing industry, and that these costs are linked to the change in the level of production. In particular, the typical housing construction firm faces the following maximization problem:

$$Max\{P_t^x Y_t^x - W_t N_t^x - P_t^x g(Y_t^x - Y_{t-1}^x)\}$$

where we assume g(0) = g'(0) = 0, and g''(0) = φ > 0. This implies that the firm's short-run production elasticity is given by

$$ES \equiv \frac{\partial \ln Y_t^x}{\partial \ln P_t^x} = \frac{\eta}{\varphi(\eta - 1)}.$$

Below we will consider the sensitivity of the model to the presence of this adjustment cost.

2.c. Calibration.

We choose preference and production parameters consistent with empirical evidence and other studies. In their study of durable goods, Ogaki and Reinhart (1998) estimate $\sigma = 2$, and $\rho = 1.17$. We calibrate the model to residential housing, thus suggesting a lower value of ρ. We use $\rho = 1.0$. The Frisch labor supply elasticity is set to 1 ($\omega = 1$), and φ is chosen to imply a steady-state level of employment of 1/3. The preference parameter b is chosen to imply a steady-state with 82% non-durable consumption. Finally, $\beta = .995$ (quarterly) implies a 2% annual real interest rate, and $\delta =$ 2.5% is the annual durable deprecation rate.

As for firms, we assume a steady-state mark-up of 10% for both types of firms ($\bar{Z} = .9$). We assume that durable goods firms have perfectly flexible prices ($\nu_x = 0$), while non-durable goods prices are adjusted (on average) every three quarters ($\nu_c = .67$). The log-linearized Calvo price-adjustment equation is given by

$$\pi_t = \lambda z_t + \beta E_t \pi_{t+1}.$$

Our calibration of non-durable price adjustment implies $\lambda^c = 0.1715$.

In terms of production adjustment costs in the housing sector, our benchmark calibration assumes zero adjustment costs so that the short-run production elasticity is infinite (ES = ∞, $\varphi = 0$). Using housing data, Topel and Rosen (1988) estimate a short run production elasticity of unity (ES = 1.0, $\varphi = 1.1$), so we also report results for this elasticity.

To close the model we need to specify the central bank reaction function. In what follows we assume a reaction function where the current nominal interest rate is a function of inflation and the lagged interest rate. In log deviations, this rule is given by:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\tau\pi_t + \varepsilon_t^R$$

where τ = 1.5, and $\rho_R = 0.8$. The policy shock ε_t^R is assumed to be iid.

2.d. Results.

To develop some intuition for what follows, consider the simplest environment in which the utility functional is separable between housing and non-durables. Since housing durables are a stock with a relatively small depreciation rate, $U_D(t)$ will vary little in response to shocks. Define $\Delta_t \equiv \frac{P_t^x}{P_t^c} U_c(t)$. From (2) this implies that Δ_t responds very little to shocks. The labor supply equation (1) can be rewritten using this term as follows: $-U_L = \Delta_t \left(\frac{W_t}{P_t^x} \right)$.

Since durable goods have flexible prices, the real wage in terms of durables is constant. Hence, U_L does not vary much with shocks, that is, labor and thus production does not vary with monetary shocks.

The stickiness of the non-durable good price implies that the relative price of durables ($\frac{P_t^x}{P_t^c}$) will decrease with a monetary contraction. Since $\Delta_t \equiv \frac{P_t^x}{P_t^c} U_c(t)$ varies little with shocks, this implies that nondurable consumption must fall with a monetary contraction. Since output is nearly constant this implies that a monetary contraction will increase the production of durables.

Relatedly, we can rewrite the Fisher equation (3) as

$$\Delta_t = E_t \left\{ \beta \frac{P_t^x}{P_{t+1}^x} R_t \Delta_{t+1} \right\}$$

Using the same logic as before, we have that $\left(\frac{R_t P_t^x}{P_{t+1}^x} \right)$ does not vary much with shocks, that is, movements in the nominal rate are met with comparable movements in the inflation rate in durables. In particular, an increase in the nominal rate leads to a comparable increase in the expected price inflation of the durable good. The rational household chooses to purchase durables contemporaneously, before the durable price inflation.

Figure 1 exhibits the model economy's behavior to a policy shock that causes the nominal interest rate to increase by 25 basis points (100 annual basis points). The model is labeled "Baseline." The endogeneity of the policy rule implies that the needed policy innovation is larger than this, $\varepsilon_t^R = 0.47$. Price stickiness in the non-durable sector leads to a sharp decline in non-durable production (-0.72). This implies a decline in demand for labor, and thus a decline in nominal marginal cost for the durable goods industry. These lower production costs lead to a sharp fall in the relative price of durable goods prices and a sharp *increase* in durable goods production. Durable good investment increases by 3.3%. Total employment and total production are essentially unchanged, with employment falling by only 0.007%. All of these effects are protracted because of the persistence in the interest rate change.

These sectoral implications are wildly at odds with the empirical evidence of co-movement across the durable and non-durable sectors, with the durable sector production falling by much more than non-durable production. We have conducted extensive sensitivity analysis. A key parameter is the degree of substitutability between non-durables and durables. For smaller values of ρ the two goods are complementary so that they are more likely to move together even with differing degrees of price rigidity. Figure 1 considers the extreme case of $\rho = 0.1$ ($\rho = 0$ is Leontief preferences). Note that even in this case we do not get co-movement. In fact, it takes a ridiculously low value of $\rho = 0.029$ (!) before durable purchases actually fall with an increase in interest rates.

Finally, Figure 1 considers the case with adjustment costs in the durable goods sector calibrated to ES = 1. The adjustment costs greatly dampens the increase in durable good production. Because of this, employment now decreases on impact instead of being essentially constant as in the baseline case. However, the co-movement puzzle remains.

3. Adding sticky nominal wages.

One interpretation of the co-movement puzzle is that the relative price of housing falls too sharply in the wake of a monetary contraction. One way of solving the puzzle is to add elements to the model that moderate this relative price movement. Since durable prices are assumed to be flexible, they are constant mark-ups over nominal wages. Hence, if we assume that nominal wages are sticky, then durable prices will inherit this stickiness. There is no reason to suppose that wage stickiness differs across durable vs. non-durable firms. We therefore consider the case of symmetric nominal wage rigidity.

Following Erceg, Henderson, and Levin (2000), we assume that households are monopolistic suppliers of labor and that firms employ a CES aggregator of household labor with an elasticity of substitution equal to $\theta_w > 1$. In particular, the labor aggregator is symmetric with (2):

$$L_t = \left[\left(\frac{1}{d} \right)^{\frac{1}{\theta_w}} \int_0^d \left(L_t(j) \right)^{\frac{\theta_w - 1}{\theta_w}} dj \right]^{\frac{\theta_w}{\theta_w - 1}} .$$

Nominal wages are adjusted as in Calvo (1983). In this case labor supply behavior is given by

$$\frac{-U_L}{U_c} = Zh_t \frac{W_t}{P_t^c} .$$

Following the logic from above we can write this as

$$-U_L = Zh_t \Delta_t \left(\frac{W_t}{P_t^x} \right) .$$

The variable Zh_t is the monopoly distortion as it measures how far the household's marginal rate of substitution is from the real wage. In the case of perfectly flexible but monopolistic wages, $Zh_t = Zh$ is constant and less than unity. The smaller is Zh, the greater is the monopoly power. In the case of sticky nominal wages, Zh_t is variable and moves in response to the real and nominal shocks hitting the economy. These fluctuations will necessarily imply fluctuations in employment. Erceg et al. (2000) demonstrate that in log deviations *nominal* wage adjustment is given by:

$$\pi_t^W = \lambda^W zh_t + \beta E_t \pi_{t+1}^W ,$$

where π_t^W is time-t net nominal wage growth, zh_t denotes the log deviation from steady-state, and $\lambda_w \equiv \frac{(1-\nu_w)(1-\nu_w\beta)}{\nu_w(1+\theta_w/\omega)}$, with ν_w denoting the fraction of households that cannot adjust their nominal wages in the current quarter.[2]

Figure 2 presents the results. We set θ_w = 11, implying a 10% labor supply distortion in the steady-state. We consider two different cases of ν_w, ν_w = 4% (96% of wages can be adjusted each period) and ν_w = 15% (85% of wages can be adjusted each period). The reason for these small degrees of nominal stickiness are apparent in Figure 2. Although in either case the model generates co-movement in the initial period, the size of the durable response becomes very large, very quickly as we increase ν_w. For example, if we imposed common stickiness across non-durables and wages (ν_w = 0.67), the response of durables to a monetary shock is more than 1000 times larger than the response of non-durables! Note also that the assumption of sticky wages leads to rapid over-shooting of durable spending, ie., durable investment moves above normal after only one period.[3]

Figure 3 presents the case with nominal wage stickiness and adjustment costs in durable production set to ES = 1. With adjustment costs the response of durables is greatly diminished so we report results with wage stickiness of the same order of magnitude as non-durable stickiness (ν_c = ν_w = 0.67). The adjustment costs smooth production over time so that durable production falls, and continues to fall for a sustained

[2] See page 224 of Woodford (2003) for details.
[3] We also considered the case of sticky durable goods prices. The results in this case are almost identical to the case of sticky nominal wages. However, for the case of residential housing we find the assumption of sticky prices implausible.

period of time (durable production reaches its trough one year after the monetary shock). The decline in production implies a decline in labor income so that non-durable consumption falls more sharply than in the baseline case.

4. A Durable Goods Model with Credit Constraints.

Because of the size of the transaction, durable good purchases are inherently linked to credit markets. To the extent that future labor income is illiquid, current durable good purchases are likely to be constrained by current income. This is the basic logic of the credit constraint that we consider. In particular, we assume that durable good investment is constrained by some portion of current labor income. An example of such a constraint is the familiar rule-of-thumb that the total amount that a household can spend on housing should not exceed 28% of their income.

An alternative way of thinking about such a constraint is that there is a hold-up problem in the durable goods market. Suppose that each period the representative household must re-purchase its entire stock of durables. This is of course an extreme assumption, but it magnifies the effect of the credit constraint. The household makes this durable purchase before receiving its labor income. At the end of the period, the durable good firm can ex post seize a fraction ($\mu < 1$) of the household's labor income along with the un-depreciated value of the durable stock. Because of this inability to seize all income, the household can always ex-post re-negotiate the durable good's selling price to the detriment of the firm. To entirely avoid this hold-up problem, the firm simply limits the purchases of the household so that the household has no incentive to re-negotiate. In particular we have:

$$P_t^x D_t \leq \mu W_t L_t + P_t^x (1-\delta) D_{t-1}$$

or

$$P_t^x X_t \leq \mu W_t L_t \qquad (5)$$

We call (5) the "hold-up" constraint. Note that (5) is a flow constraint: current durable investment is constrained by the current flow of income.

There are other motivations for a constraint like (5). For example, suppose again that the purchase of the durable good occurs before current labor income is earned. After the transaction the firm and household separate, with the firm anticipating full payment for the durable by the end of the period. If the household does not repay, the firm can seize the household's durable stock, and find the household with probability μ. In this case, the firm can seize all of the household's labor income, but there is a fixed bankruptcy cost ($F > 0$) of seizing household income expressed in terms of time. In this case we have the constraint:

$$P_t^x X_t \leq \mu (W_t L_t - W_t F) \qquad (6)$$

The risk averse household will want to avoid the uncertainty of losing all of their current income so that the existence of (6) implies that bankruptcy will not be observed in equilibrium.

We will use version (6) of the hold-up constraint in what follows. The fixed cost has an intuitive effect. Declines in labor income lead to a disproportionate decline in the household's ability to purchase durable goods. This mechanism implies that hold-up problems become disproportionately more severe in times of low income, and vice versa. It is this link that breaks the one-to-one relationship above and will cause durable goods to be more volatile than labor income.

The hold-up constraint (6) has the flavor of US individual bankruptcy law. The bankruptcy law is designed to allow the individual to keep his or her house. The court orders an individual's income net of living expenses to be seized in order to pay off the secured creditors. Living expenses likely contain a fixed "subsistence" level of income as well as one that varies with respect to the individual's income. We have priced this fixed cost in units of time.

With this hold-up constraint the household's decision-making is now summarized by the following optimization conditions:

$$U_c(t) = P_t^c \lambda_{1t} \tag{7}$$

$$-U_L(t) = W_t(\lambda_{1t} + \mu\lambda_{2t}) \tag{8}$$

$$U_D(t) + P_{t+1}^x \beta(1-\delta)[\lambda_{1t+1} + \lambda_{2t+1}] = P_t^x[\lambda_{1t} + \lambda_{2t}] \tag{9}$$

where λ_{1t} is the multiplier on the budget constraint and λ_{2t} is the multiplier on the hold-up constraint. Let us define the hold-up distortion as $m_t \equiv \frac{\lambda_{2t}}{\lambda_{1t}} \geq 0$. We can re-write (9) as

$$U_D(t) + \frac{P_{t+1}^x(1+m_{t+1})}{P_{t+1}^c}\beta(1-\delta)U_c(t+1) = \frac{P_t^x(1+m_t)}{P_t^c}U_c(t) \tag{10}$$

The distortion acts as a tax on durable good purchases as durables are now more difficult to purchase because of the hold-up constraint.

T

he household's employment choice can be expressed as

$$\frac{-U_L}{U_c} = \frac{W_t}{P_t^c}(1+\mu m_t)\,. \tag{11}$$

The labor condition is also affected by the hold-up constraint. Since labor income relaxes the constraint, the distortion acts as a subsidy to employment. This subsidy tends to encourage employment partially offsetting the tax on durable goods purchases, but since μ < 1, the tax effect wins out. To see this formally let

$$rp_t \equiv \frac{P_t^x(1+m_t)}{P_t^c}\,. \tag{12}$$

Using this expression, employment is given by

$$-U_L = \left(\frac{W_t}{P_t^x}\right)\left(rp_t U_c\right)\left(\frac{(1+\mu m_t)}{(1+m_t)}\right). \tag{13}$$

Note first that since the durable good sector has flexible prices, the mark-up of prices over wages is invariant to monetary shocks. Second, equation (10) implies that $rp_t U_c(t)$ will vary only slightly with shocks. This is because the stock of durable goods will vary little with respect to shocks. A monetary contraction will thus have little effect on the first two terms on the right-hand side of (13). However, it will tighten the credit constraint, ie., an increase in m_t. Since μ < 1, this will cause a decline in the return to working and thus a decline in employment. Note that if μ = 1 labor does not respond to monetary shocks.

The size of the fixed cost is also very important. Log-linearizing the hold-up constraint yields

$$s_c c_t + (1-s_c)x_t = x_t(1-fmc) + fmc(p_t^c - p_t^x)$$

where $l_t = s_c c_t + (1 - s_c) x_t$,

$$s_c = \frac{C_{ss}}{Y_{ss}}$$

$$fmc = \frac{P^c_{ss} F}{W_{ss} L_{ss}} = \frac{(1 + markup) F}{Y_{ss}}$$

Solving we have:

$$x_t = \frac{s_c}{(s_c - fmc)} c_t + \frac{fmc}{(sc - fmc)} (p^x_t - p^c_t)$$

If fmc = 0 we have that durables and non-durables will move one for one. If fmc > 0, however, durable production must move more than non-durables. The fact that the relative price of durables falls with an increases in interest rates reinforces that effect. This illustrates the importance of the fixed cost.

4.a. Calibration.

For ease of comparison, we use the same calibration as in Section 2. For example, the preference parameter b is again chosen to imply a steady-state with 82% non-durable consumption (s_c = 0.82). Because of the credit constraint, this implies a lower value of b in comparison to the baseline model.

As for the credit constraint parameters, we interpret the F in (6) as the cost of bankruptcy. Estimates of these costs vary from 15% to 36%. We consider two calibrations, F = 20% of household income, and F = 30% of household income. The size of μ is then chosen endogenously to match the 18% durable share in consumption:

$$\mu = \left(\frac{P_{ss}^{x}}{W_{ss}}\right)\left(\frac{X_{ss}}{Y_{ss}}\right)\left(\frac{1}{1-\frac{F}{Y_{ss}}}\right) = \frac{(1.1)(0.18)}{(1-\frac{F}{Y_{ss}})}.$$

Hence there is only one free parameter. For F = 20% we have μ = 0.25; for F = 30% we have μ = 0.28.

4.b. Results.

For the case of F = 20%, Figure 4 exhibits the model economy's behavior to a policy shock that causes the nominal interest rate to increase by 25 basis points (100 annual basis points). Price stickiness in the non-durable sector leads to a decline in non-durable production of 0.93%. The decline in labor income leads, via the credit constraint, to a decline in durable spending (-1.27%) and a sharp decline in the relative price of durables (-2.96%). Total production falls by 1%. All of these effects are protracted because the persistent interest rate change leads to a persistent decline in labor income.

Figure 4 also presents sensitivity analysis on the key credit parameter F with the value of F = 30% reported. Similarly, we report results in which there are housing adjustment costs, ES = 1. The sensitivity results in Figure 4 are as anticipated: a higher FC magnifies the effects, while ES = 1 dampens the effects.

5. Conclusions.

This paper has demonstrated two possible solutions to the co-movement puzzle. As a way of assessing the two solutions, we begin with a stylized review of the facts. Erceg and Levin's (2005) VAR evidence suggests that a 100 basis point (annualized) monetary contraction is followed by a 0.27% decline in non-durables and a 2.7% decline in residential investment (a 10-1 ratio). The responses are hump-shaped, with the peak response about four quarters out. Erceg and Levin (2005) report only a small decline in the relative price of the composite durable-goods/residential-housing in the wake of the shock.

Figure 4 reports the sticky wage model as well as the credit model for ease of comparison. The sticky wage model (with ES = 1 and $\nu_w = 0.67$) implies a peak decline of 0.81% in non-durables and 5.14% in housing in the wake of a 100 basis point monetary shock. The relative price of durables falls by 1.27%. The credit model (with ES = ∞ and FC = 20%) implies a peak decline of 0.98% in non-durables and 1.6% in durables in response to the 100 basic point contraction. Relative prices fall much more sharply here, by 3.14%, as this is a demand-side story. These figures suggest that the sticky wage story more successfully explains the co-movement puzzle, roughly matching the relative production volatilities in the data, and implying a modest decline in relative prices. This assessment is reinforced by the predictions about the real wage. The empirical evidence suggests very modest movements in real wages in the wake of a monetary shock. The wage model is consistent with this evidence, but the credit model's prediction on wage behavior seems counterfactual.

6. References

Barsky, R., C.L. House, and M. Kimball, September 9, 2005, "Sticky Price Models and Durable Goods," manuscript, U. of Michigan.

Barsky, R., C.L. House, and M. Kimball, June 17, 2003, "Do Flexible Durable Goods Prices Undermine Sticky Price Models," manuscript, U. of Michigan.

Bils, Mark, and Peter J. Klenow, "Some evidence on the importance of sticky prices," *Journal of Political Economy* 112 (5), 2004, 947-985.

Carlstrom, C., and T. Fuerst, 2004. "Investment and Interest Rate Policy: A Discrete Time Analysis," forthcoming, *Journal of Economic Theory.*

Erceg, C.J., and A.T. Levin, 2005. "Optimal Monetary Policy with Durable Consumption Goods," forthcoming, *Journal of Monetary Economics*.

Erceg, C.J., D.W. Henderson, A.T. Levin, October 2000. "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics* 46(2), 281-313.

Iacoviello, Matteo, "House prices, borrowing constraints, and monetary policy in the business cycle," *American Economic Review* 95(3), June 2005, 739-764.

Ogaki, M. and C.M. Reinhart, "Measuring intertemporal substitution: the role of durable goods," *Journal of Political Economy* 106(5), October 1998, 1078-1098.

Topel, Robert and Sherwin Rosen, "Housing Investment in the United States," *Journal of Political Economy* 96(4), August 1988, 718-740.

Woodford, Michael, (2003), *Interest and Prices*, Princeton University Press.