

*Issues in Inflation Targeting*  
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# Factor-Market Structure, Shifting Inflation Targets and the New Keynesian Phillips Curve

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# Summary of Presentation

1. Motivation of Paper
2. Description of Model
3. Identification of Historical Inflation Target
4. Estimation Strategy and Empirical Results
5. Conclusion and Discussion of Future Work

# 1 Motivation

- No clear consensus for Canada on the ability of the NKPC to capture the key features of inflation such as its
  - degree of persistence
  - procyclicality (does real marginal cost explain inflation??)

# 1 Motivation

- Khan and Gagnon (2005, 2002) find evidence supporting the NKPC for Canada using
  - Raw GDP price inflation
  - Firm-specific capital generates more realistic average price-contract durations
  - CD and CES-based Marginal Cost (CES is preferable for Canada)
  - Hybrid Calvo price model
  - Variations with overhead labour and open-economy extensions make little difference

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  - Hybrid Calvo price model
  - Variations with overhead labour and open-economy extensions make little difference
- Kozicki and Tinsley (2002) find some support for structural pricing models, in particular a Taylor-style setup for Canada
  - CPI prices
  - Estimate an inflation gap based on Moving Endpoints methodology
  - Output gap based on the HP filter

# 1 Motivation

- Guay, Luger and Zhu (2002) reject the NKPC for Canada using a bias-corrected continuous updating (CUE) estimator
  - Raw GDP inflation
  - CD-based marginal cost (labour share) with overhead labour and open-economy extensions
  - Hybrid Calvo model
  - Overhead labour and open-economy extensions make little difference to results

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  - Hybrid Calvo model
  - Overhead labour and open-economy extensions make little difference to results
- Nason and Smith (2005) find little evidence of forward-looking behaviour and inflation is essentially unrelated to labour's share of income
  - Raw GDP inflation
  - CD-based marginal cost (labour share)
  - Hybrid Calvo model

# 1 Motivation

What can we conclude from this?

1. Accounting for historical changes in the inflation objective is important for Canada, although some intrinsic persistence remains, Kozicki and Tinsley (2002)
2. Some evidence to support the use of CES production, Khan and Gagnon (2002)
3. The absence of real rigidity appears to result in average contract lengths that are at odds with survey data, Khan and Gagnon (2005, 2002)
4. Open-economy extensions make little difference when modeling the GDP deflator, Khan and Gagnon (2005, 2002), Guay, Luger and Zhu (2002)
5. No clear conclusion regarding the success of the NKPC for Canada



## 2 Model

### Our approach

1. Model the inflation gap as in Kozicki and Tinsley (2002), but
  - Replace detrended output with marginal cost
  - Replace CPI with CPIX (the operational measure of underlying inflation at the BOC)
2. Use CES production and firm-specific capital as in Gagnon and Khan (2005,2002) but
  - Modify the role of imports
3. Allow for the possibility that the inflation is partially pre-determined
  - Partial indexation
4. Allow for adjustment costs in employment following Sbordone (2002)

## 2 Model

Assume a continuum of monopolistically-competitive firms, indexed by  $i$ ,  $i \in [0, 1]$ , that each produce a differentiated final good using a CES production technology in labour,  $L_{it}$ , capital,  $K_{it}$ , and imported inputs,  $M_{it}$

$$Z_{it} = \left( \delta_1^{\frac{1}{\sigma}} (A_t L_{it})^{\frac{\sigma-1}{\sigma}} + \delta_2^{\frac{1}{\sigma}} (K_{it})^{\frac{\sigma-1}{\sigma}} + (1 - \delta_1 - \delta_2)^{\frac{1}{\sigma}} (M_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} - \Omega_{i,t},$$

with

$$\Omega_{i,t} = \frac{Z_t \chi}{2} \left( \frac{L_{it}}{L_{i,t-1}} - 1 \right)^2,$$

## 2 Model

$$\widehat{\lambda}_{i,t} = \Theta \left( \widehat{s}_t - \frac{\chi}{s_L} \left( \Delta^2 \mathbf{E}_t \widehat{L}_{i,t+1} \right) \right) + \Lambda \widehat{z}_{k,i,t} + (1 - \Theta) \widehat{p}_{m,t},$$

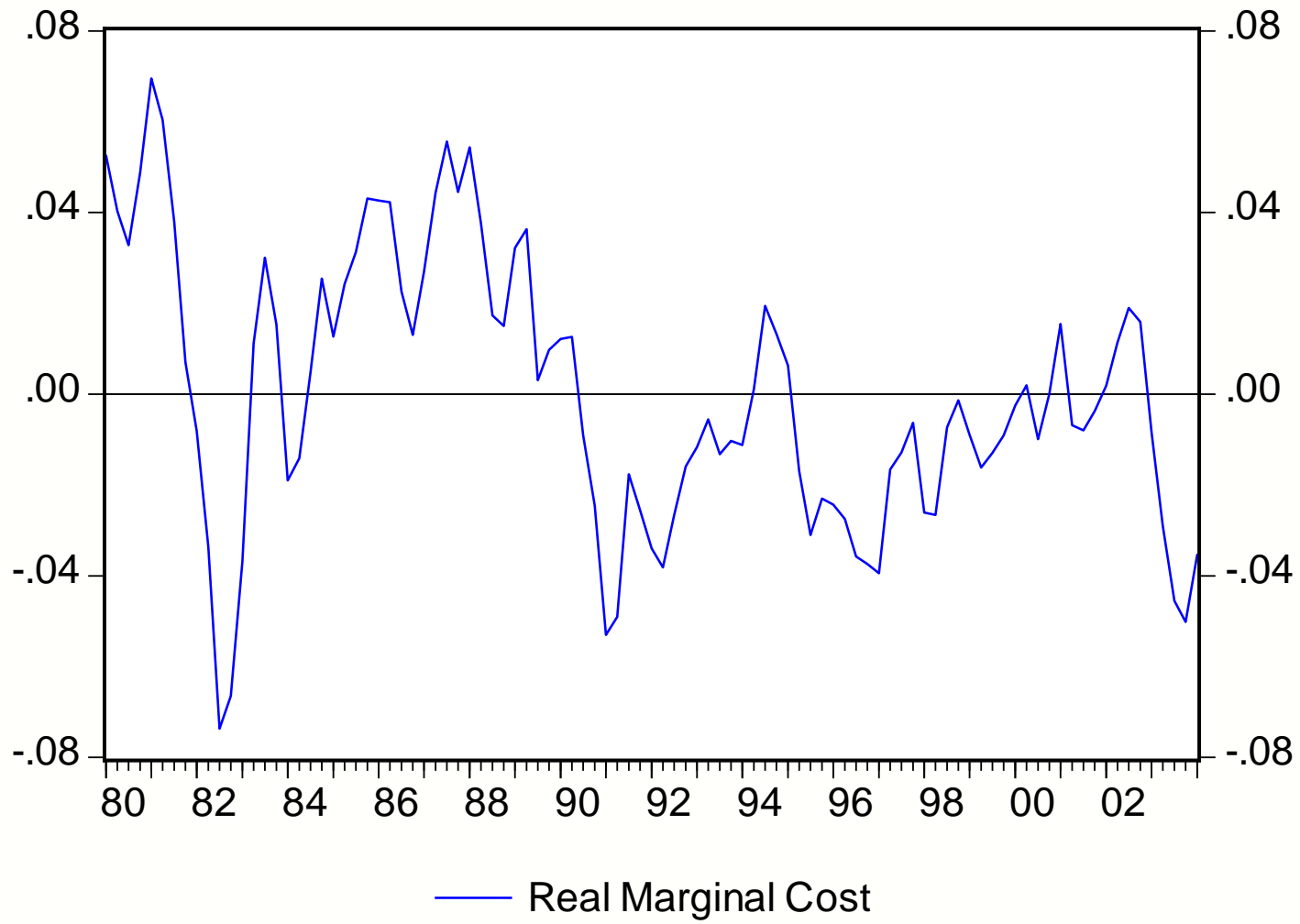
which includes the unobserved expectation of future employment

$$\Delta \widehat{L}_{t+1} = \rho_1 \Delta \widehat{L}_t + \rho_2 \Delta \widehat{L}_{t-1} + u_t,$$

which yields

$$\widehat{\lambda}_{i,t} = \Theta \left( \widehat{s}_{i,t} - \frac{(\rho_1 - 1)\chi}{s_L} \left( \Delta \widehat{L}_{i,t} + \frac{\rho_2}{\rho_1 - 1} \Delta \widehat{L}_{i,t-1} \right) \right) + \Lambda \widehat{z}_{k,i,t} + (1 - \Theta) \widehat{p}_{m,t}.$$

if  $\sigma = 1$  and  $\chi = 0$ , we obtain  $\widehat{\lambda}_{i,t} = \widehat{s}_{i,t}$



## 2 Model

Pricing model is

$$\hat{\pi}_t = \frac{\gamma}{1 + \beta\gamma} \hat{\pi}_{t-1} + \frac{\beta}{1 + \beta\gamma} \hat{\pi}_{t+1} + \phi \hat{\lambda}_t,$$

with

$$\phi = \eta \cdot \frac{(1 - \theta)(1 - \beta\theta)}{(1 + \beta\gamma)\theta};$$

- With a rental market for capital, a firm's marginal cost is invariant to its relative price
- With a fixed or quasi-fixed capital stock, a firm's marginal cost increases in its output (constant technology)
- Firms must consider the effect on both their revenues and costs of a price change
- Reduces the sensitivity of price to marginal cost

### 3 Identification of the Historical Inflation Target

Approach 1 - Moving Endpoints Method (Kozicki and Tinsley (1998, 2002))

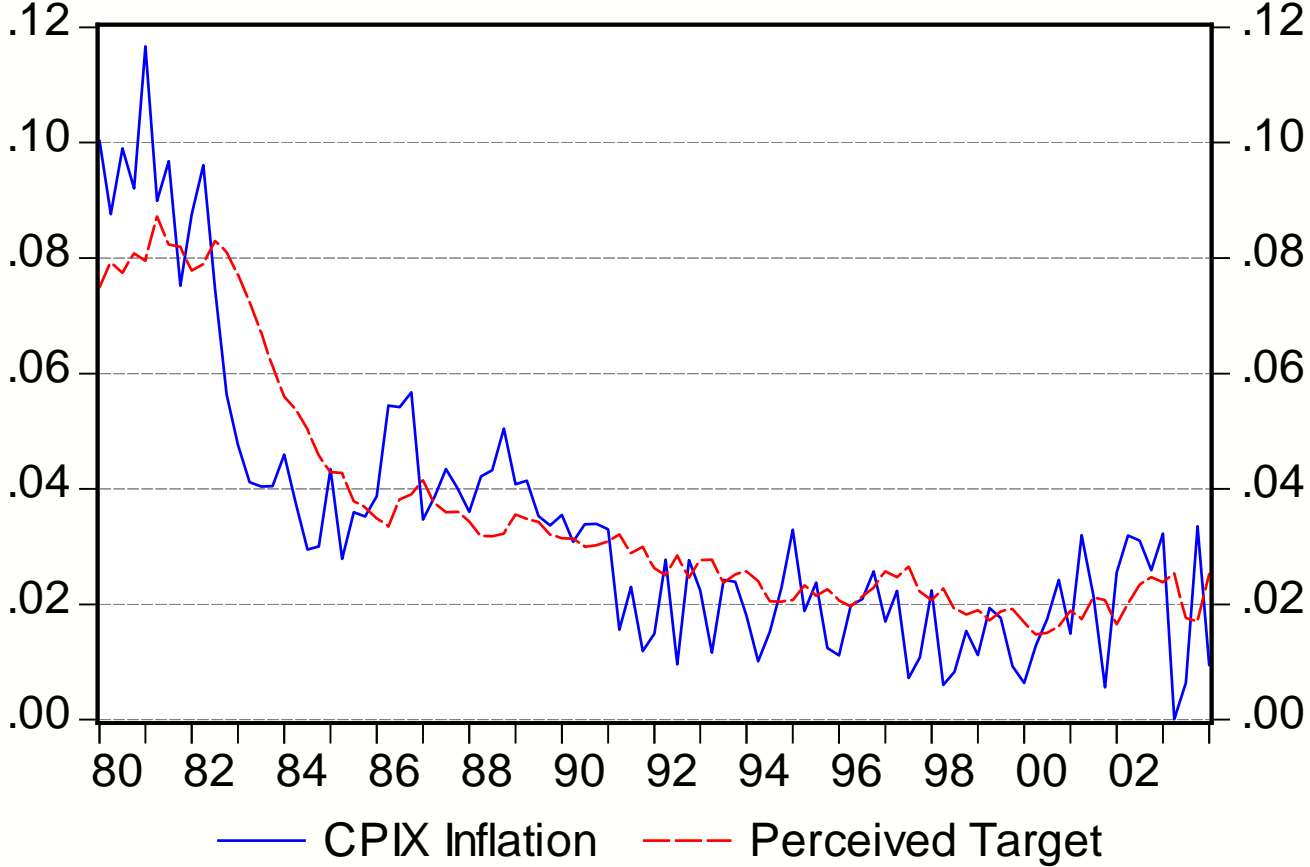
$$\begin{bmatrix} \pi_t \\ \lambda_t \\ R_t \end{bmatrix} = \sum_{i=1}^j \mathbf{A}_i \begin{bmatrix} \pi_{t-i} \\ \lambda_{t-i} \\ R_{t-i} \end{bmatrix} + \left( \mathbf{I} - \sum_{i=1}^j \mathbf{A}_i \right) \begin{bmatrix} \bar{\pi}_t \\ \bar{\lambda} \\ \bar{r} + \bar{\pi}_t \end{bmatrix} + \mathbf{u}_t.$$

and

$$\bar{\pi}_t = \bar{\pi}_{t-1} + \nu_t,$$

- Estimate the unobserved perceived target using the Kalman filter
- 1975Q1 to 2004Q1, drop first 5 years of data
- J=2

Figure 1: MEP-Based Perceived Inflation Target



### 3 Identification of the Historical Inflation Target

#### Approach 2 - Staff Projection Method

- Exploit the information contained in historical Staff projections of inflation to identify a time-varying target

Assumptions:

1. Bank of Canada behaviour can be well-captured by a linear policy rule of the form

$$R_t = \zeta_t R_{t-1} + \mathbf{E}_{t-1} (1 - \zeta_t) (\bar{r} + \pi_t + \omega_{1,t} (\pi_t - \bar{\pi}_t) + \omega_{2,t} \tilde{y}_t + \omega_{3,t} \Delta z_t) + \varepsilon_t,$$

with possible time variation in the parameters.



2. The Staff have access to time (t-1) information only at time (t).
3. Staff forecasts of inflation can well approximated by a small-dimension VAR model.

$$\mathbf{X}_t = \sum_{i=1}^p \mathbf{A}_{i,t} \mathbf{X}_{t-i} + \mathbf{u}_t$$

$$\mathbf{X}'_t = \{1, \pi_t, \tilde{y}_t, \Delta z_t\}$$

Augmented with the interest rate rule, the structural VAR is given as

$$\mathbf{E}_{t-1} \mathbf{B}_{0,t} \mathbf{X}_t = \sum_{i=1}^p \mathbf{B}_{i,t} \mathbf{X}_{t-i}$$

with

$$\mathbf{B}_{0,t} = \begin{bmatrix} \mathbf{I}_{4 \times 4} & \mathbf{0}_{4 \times 1} \\ \xi_{1,t} & \xi_{2,t} & \xi_{3,t} & \xi_{4,t} & 1 \end{bmatrix}; \quad \mathbf{B}_{1,t} = \begin{bmatrix} & & \mathbf{A}_1 & & \\ 0 & 0 & 0 & 0 & \zeta_t \end{bmatrix}.$$

$$\mathbf{X}'_t = \{1, \pi_t, \tilde{y}_t, \Delta z_t, R_t\}$$

- To identify the target in period (s):
  1. Estimate the reduced form VAR on data up to and including (s-1) with
  2. Form the vector

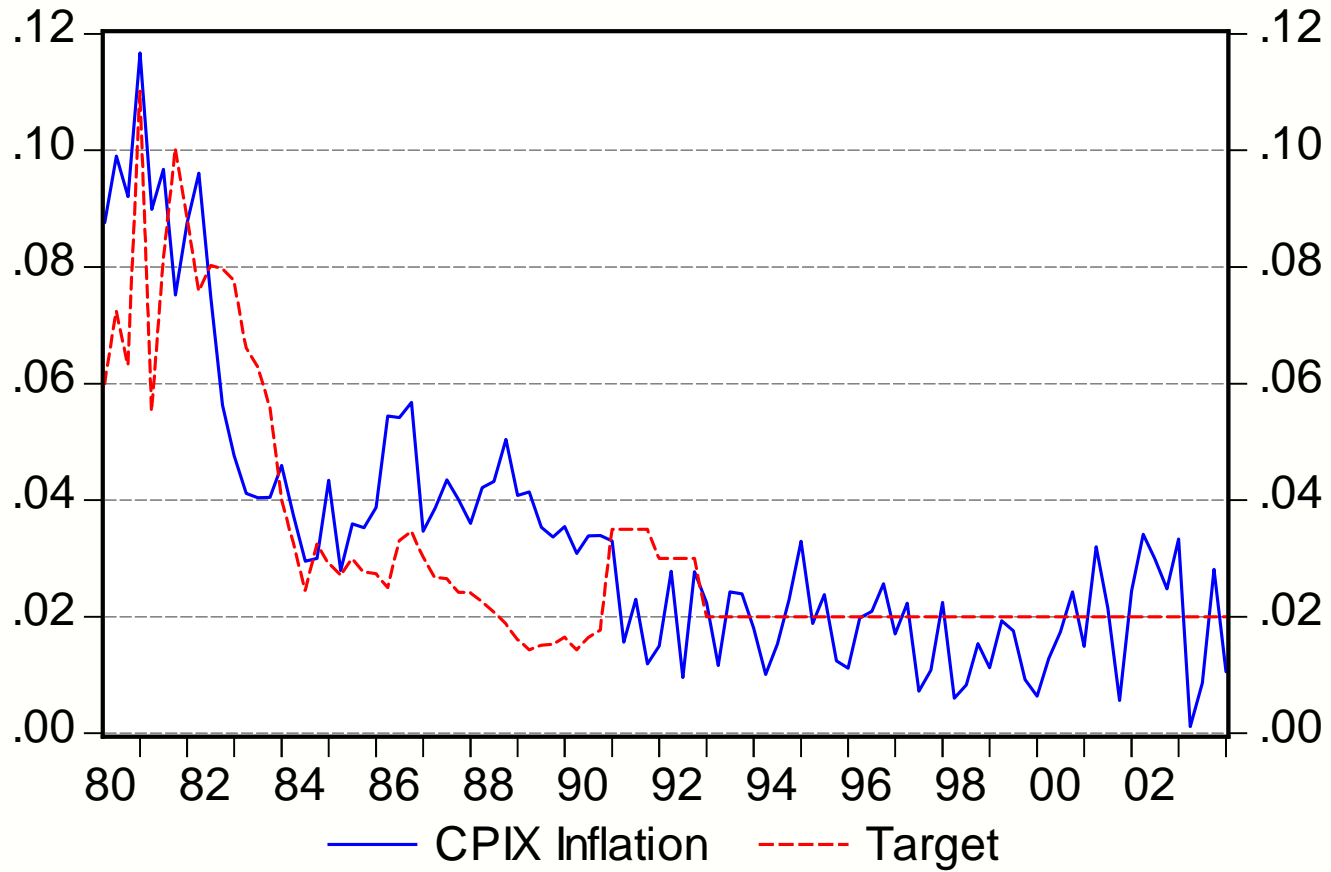
$$\mathbf{Q}_{s-1} = \mathbf{E}_{s-1} \begin{bmatrix} \pi_{s+4} - \pi_{s+4}^* \\ \pi_{s+6} - \pi_{s+6}^* \\ \pi_{s+8} - \pi_{s+8}^* \\ \pi_{s+20} - \pi_{s+20}^* \end{bmatrix},$$

3. Choose the constant in the interest rate equation to minimize the quadratic

$$\mathbf{Q}'_{s-1} \mathbf{W} \mathbf{Q}_{s-1}$$

4. Compute the parameters of the rule, including the target from
5. Repeat for (s+1), (s+2), ....

Figure 2: SEP-Based Inflation Target



### 3 Identification of the Historical Inflation Target

Table 1: Summary Statistics for Main Variables of Interest

Variable	1980Q1-2004Q1			1993Q1-2004Q1		
	Std. Dev. p-p.	Persistence		Std. Dev. p-p.	Persistence	
		$\rho$	AR(1)		$\rho$	AR(1)
Raw CPIX	2.4	0.93*	0.87*	0.8	0.14	0.14
MEP Gap	1.3	0.69*	0.45*	0.9	0.02	0.02
SEP Gap	1.5	0.63*	0.63*	0.8	0.14	0.14
Marginal Cost	3.0	0.79*	0.86*	1.7	0.92*	0.78*

- Inflation persistence and volatility have fallen significantly since 1993
- Only the volatility of real marginal cost has declined since 1993

## 4 Estimation and Results

- Following Coenen and Levin (2004), and based on Smith (1993), we choose the structural parameters to match the parameters of a bivariate VAR

$$\min_{\{\gamma, \phi\}} (\mathbf{\Gamma} - \mathbf{\Gamma}(\gamma, \phi))' \mathbf{W} (\mathbf{\Gamma} - \mathbf{\Gamma}(\gamma, \phi)).$$

- The structural model admits a VAR(1) representation
- The data prefer a VAR(2)

## 4 Estimation and Results

Table 2: Calibrated Parameters

Parameter	Value
$\sigma$	0.5
$\beta$	0.99
$\epsilon$	11
$\chi$	6.0
$s_L$	0.46
$s_K$	0.37
Functions of Calibrated Parameters	Value
$\mu$	1.1
$\eta$	0.045
$\Theta$	0.84
$\Lambda$	0.66

- Inflation will be about 20 times less sensitive to real marginal cost relative to the rental-market-for-capital setup

## 4 Estimation and Results

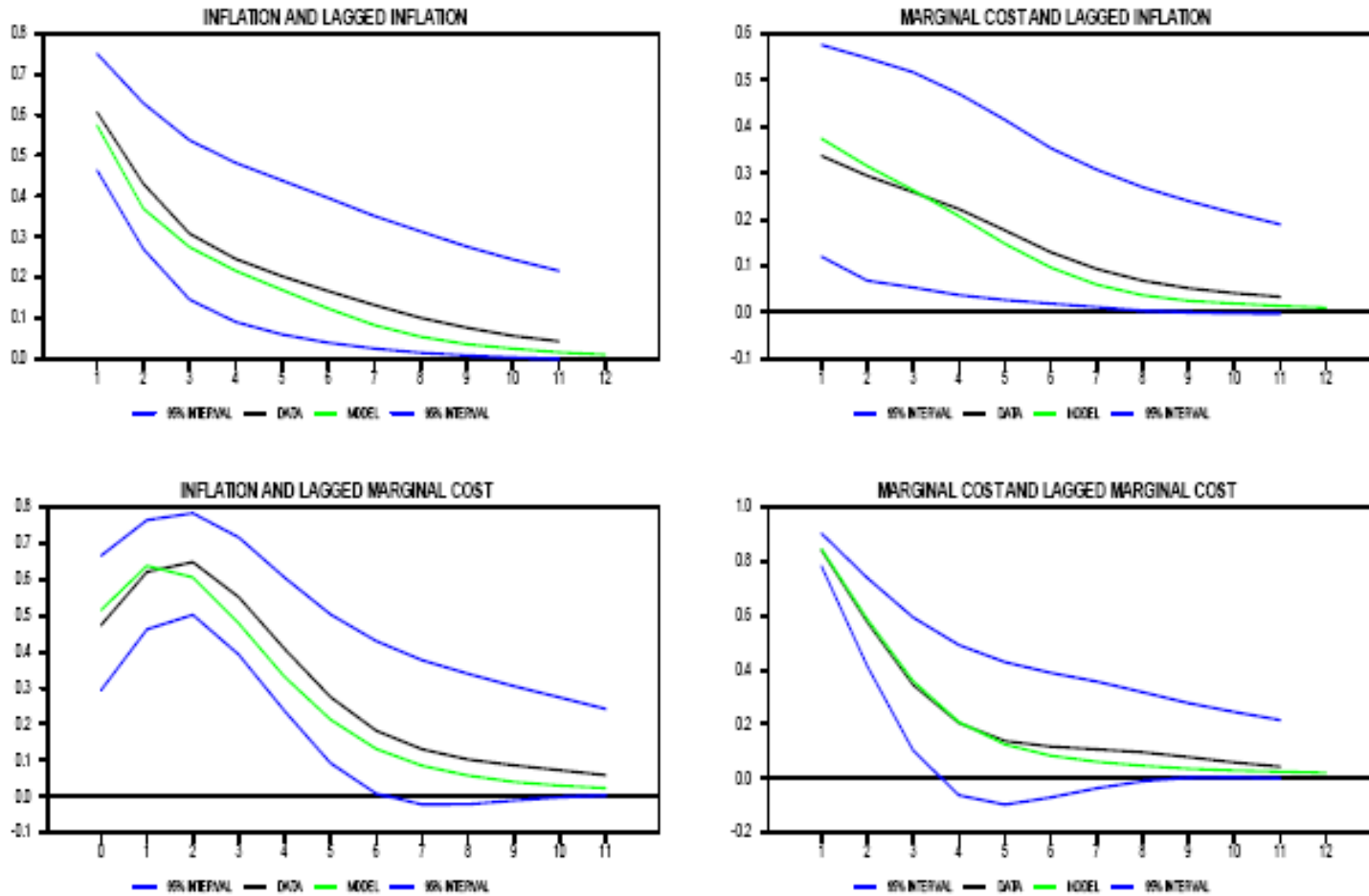
Table 3: Estimation Results - Preferred Model

Variable	SMM (1980Q1-2004Q1)			
<i>NKPC</i>	MEP Inf. Gap		SEP Inf. Gap	
	VAR(1)	VAR(2)	VAR(1)	VAR(2)
$\gamma$	0.37 [0.1 0.65]		0.37 [0.23 0.7]	
Av. Duration	2.8 quarters [2.0 4.0]		2.6 quarters [1.8 3.5]	
$\bar{R}^2$	0.36	0.36	0.54	0.54
<i>LB Q - stat</i>	2.18	2.18	1.99	1.99
<i>VAR</i> ( $\varrho$ )				
$\bar{R}^2$	0.35	0.38	0.53	0.52
<i>LB Q - stat</i>	2.18	2.04	1.99	2.00
$H_0: NKPC = VAR$	<b>1.00</b>	<b>0.051</b>	<b>1.00</b>	<b>0.3</b>





Figure 5: Cross Correlations: VAR(2) with SEP-Based Inf. Gap



## 4 Estimation and Results

Table 4: Estimation Results - Variations on the Preferred Model

Variable <i>NKPC</i>	Raw CPIX Inflation		SEP Inf. Gap		
	$\eta = 1, \gamma = 0$	$\eta = 1$	$\eta = 1$	$\eta < 1$	$\tilde{\lambda}_t = \hat{s}_t$
$\gamma$	0	0.97	0.37	0.37	0.53
<b>Av. Duration</b>	8.9 quar.	16.1	10.8	2.6	$\infty$
$\bar{R}^2$	0.06	0.80	0.54	0.54	0.38
<i>LB Q - stat</i>	0.00	0.00	0.4	0.4	0.28
<i>VAR(2)</i>					
$\bar{R}^2$	0.83	0.83	0.52	0.52	0.52
<b><i>NKPC = VAR</i></b>	<b>0.00</b>	<b>0.00</b>	0.3	0.3	<b>0.00</b>

## 4 Estimation and Results

Table 5: Forecast Results (1985Q1 - 2004Q1)

Model	Forecast RMSE Tests	
	MEP Inf. Gap	SEP Inf. Gap
<i>NKPC</i>	<b>0.23</b>	<b>0.21</b>
<i>YGAP Phil. Curve</i>	0.26	0.25
<i>AR(2)</i>	0.25	0.24
Diebold-Mariano Test	Prob. Value under null of equal RMSE ( $\sigma$ )	
$\sigma_{NKPC} < \sigma_{YGAP}$	0.00	0.00
$\sigma_{NKPC} > \sigma_{YGAP}$	1.00	1.00
$\sigma_{NKPC} < \sigma_{AR(2)}$	0.01	0.02
$\sigma_{NKPC} > \sigma_{AR(2)}$	0.99	0.98

## 5 Conclusions and Directions for Future Work

- Estimated model implies an average frequency of price re-optimization that we view as reasonable
- Model captures the dynamics of inflation very well relative to a VAR from 1980 to 2004, provided partial indexation is permitted
- Marginal cost based on CES production with adjustment costs on labour is more successful at explaining inflation over this period
- Performs better out-of-sample relative to two common benchmarks

## 5 Conclusions and Directions for Future Work

- Apply SEP-based approach to other countries
- Modify the interpretation of partial indexation to yield a measure of nominal contract duration
- Better understand the source(s) of the decline in inflation persistence since the adoption of explicit targeting