The New-Keynesian Phillips Curve When Inflation Is Non-Stationary: The Case of Canada*

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April 2005

Abstract

The New-Keynesian Phillips curve (NPC) has conventionally been estimated under the assumption that inflation is stationary — an assumption that is questionable for Canadian data. Using a method by Johansen and Swensen (1999), we show that when appropriately casted in a system, Canadian inflation indeed seems to have a unit root— a finding that invalidates existing Maximum Likelihood and GMM estimates. Estimation of the NPC based on this method yields broadly similar estimates to previous studies, although the estimated coefficient of the forward-looking component for Canada tends to be higher than what found before. Contrary to much previous literature, our estimates also support the super-neutrality result for Canada. Importantly, estimation of the NPC using this method overcomes the problem of identification associated with GMM estimation and so we can discern empirically between pure forward-looking and hybrid versions of the NPCs.

1 Introduction

In its purest form, the New-Keynesian Phillips curve (NPC) relates inflation in period t to expected inflation in period $t+1$ and a cyclical indicator, and can be derived by assuming optimizing behavior on the side of firms that set their prices following a time-dependent rule as in Calvo (1983) (see Sbordone, 2005). Traditionally the NPC has been estimated under the assumption that inflation

*We thank Søren Johansen, Neil Ericsson, Anders Rygh Swensen and Anindya Banerjee for comments on an earlier draft using these techniques on U.S. data. All errors and ommissions are our own. The views expressed herein are those of the authors and do not necessarily reflect those of the International Monetary Fund.

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is stationary (see Gali, Gertler (1999), GG, for estimates on U.S. data; Gali, Gertler and Lopez-Salido, 2001, GGLS, for estimates on euro area data). Existing estimates of the NPC on Canadian data are also based on this assumption (Gagnon and Khan, 2001; Guay and others (2003); Khan, 2004).

The assumption that inflation is stationary, however, is questionable for Canadian data. Figure 1, plotting annualized quarterly changes in the GDP deflator for Canada since 1973 Q1, suggests that Canadian inflation over the last 30 years may be in fact integrated of order 1—a phenomenon that could stem from a variety of factors, including quasi-rational inflation expectations, inflation indexing, autocorrelation in the cyclical indicator, a shift in the mean of inflation over time (due, for example, to a change in the anti-inflation preferences of the monetary authorities), private sector learning about shifts in the policy target of inflation or a combination of any of the above. If true, this is problematic because non-stationarity invalidates many estimation techniques often used by the literature to estimated NPCs for Canada, including the General Method of Moments (and more refined versions of it, like the Continuous Updated Estimator by Newey and Smith, 2001) and the Full Information Maximum Likelihood (FIML) procedure.

In this paper we employ a method by Johansen and Swensen (1999) to examine whether non-stationarity in inflation is a property of the system underlying Canadian inflation dynamics. The method suggests that inflation in Canada is indeed non-stationary, even when shorter samples are considered to account for shifts in monetary regimes like the move to inflation targeting in 1991. The method provides a framework for both detecting the cointegration rank of the system and testing the cointegrating restrictions implied by rational expectations, starting from an unrestricted system. Following Batini, Jackson and Nickell (2005), throughout we specify the NPC assuming more general technologies than Cobb-Douglas and we account for the fact that Canada is an open economy.

This method offers three main advantages.

First, it advances on other methods used in the literature on other countries data that model inflation as $I(1)$, but assume the cointegration specification a priori (e.g. Sbordone, 2002; Kozicki and Tinsley, 2002).

Second, like the methods used by Rudd and Whelan (2001), Sbordone (2002, 2003), McAdam and Willman (2002) and Banerjee and Batini (2003), the method used here ensures that model-consistent expectations are tested and subsequently imposed in estimation, and is thus to be preferred to methods previously used in the literature, like for instance the General Method of Moments (GMM), that simply assume — without imposing and testing for — the rational expectations assumption implicit in the model.

Finally, this method gets rid of the identification problem raised by Ma (2002) and Mavroeidis (2002), who have pointed to the fact that empirical methods commonly used to estimate the NPC either cannot identify or can only weakly identify the parameters in the estimated regression. We show that when the NPC is estimated within the Johansen and Swensen (1999) framework, we do not need to make any ex ante assumption on the process of the forcing variable in order for the estimation method to identify the parameters and so we are spared the trouble of using exact analytical methods.

The main findings of the paper are:

(i) The NPC offers a good representation of inflation dynamics in Canada for some—but not all—measures of marginal costs;

(ii) The degree of forward-looking behavior in price-setting varies wildly over time. Split sample regressions indicate that Canadian price-setters have become slightly more forward-looking, especially after the move to inflation targeting in the early 1990s–a finding also arrived at by Batini (2005). In the recent past, Canadian firms exhibit a degree of forward-looking behavior that is comparable to that observed in the United States or in the euro area;

(iii) The empirical method used here is capable of identifying the forward and backward-looking terms in the NPC, and our results using this method indicate that both terms are important in determining Canadian inflation;

(iv) Real marginal cost is a significant determinant of Canadian inflation, especially when this is adjusted for the cost of imported intermediates—a finding that mirrors that for other open economies (in line with Banerjee and Batini , 2004, but contrary to results by Gagnon and Khan, 2004). However its importance varies greatly depending on how it is measured;

(v) The estimated weights on lagged and expected inflation generally sum to one. This suggests that Canadian inflation does not depend on real factors in the long run—the super-neutrality result.

The plan of the paper is as follows. Section 2 reviews the recent related literature on Canadian data and discusses estimation methods. Section 3 specifies a simple dynamic system that can be taken to characterize inflation dynamics under the NPC paradigm. Section 4 briefly describes the Johansen and Swensen (1999) method, and formulates the restrictions implied by the NPC on such statistical model. Section 5 estimates the NPC on Canadian data using this method and Section 6 offers concluding remarks.

2 Recent related literature

There are several empirical papers that estimate open-economy NPCs for Canada using GMM. Most of these papers employ a version of the NPC as in Gali and Gertler (1999) that allows for the fact that a fraction of firms set their prices in a myopic was ("hybrid") NPC.

For example, Gagnon and Khan (2004) follow Sbordone (2002) and fit various NPC specifications to Canadian data using alternative measures of marginal costs derived assuming different kinds of production technologies. They find that CES technology-based NPC fit well Canadian data over the period 1970-2000, and more so than an NPC derived assuming a Cobb-Douglas technology. They also find that, for this sample, the backward-looking component in the Canadian NPC is stronger than for the Unites States and the euro area.

Khan (2004) and Leith and Malley (2002) also estimate "hybrid" NPCs for Canada. In particular, Khan (2004) estimates a rolling NPC regression and shows that the NPC in Canada may have flattened over time. This is consistent with increasing competition among firms over part of the period, under the assumption that price contracts in Canada are set as in Calvo (1983). Estimating both CES and Cobb-Douglas based NPCs over the period 1960 Q1-1999 Q4, Leith and Malley (2002) find that these fit Canadian data, and show that, generally, Canada enjoys less price inertia than other G-7 countries but similar inertia to the United States and the United Kingdom.

Banerjee and Batini (2003) estimate NPCs for Canada and other open economies over the period 1970 Q1-2002 Q1 using the Maximum Likelihood estimator and assuming various contracting specifications. They find that NPC based on time-dependent contracts as in Dotsey, King and Wolman (1999) fit the Canadian data well and better than when Calvo-contracts-based or Taylor-contracts-based NPCs are used in estimation. They also find that Canadian firms are predominantly backward-looking when setting prices.

Finally, Guay, Luger and Zhu (2003), instead estimate NPCs on Canadian data using a biased-corrected estimator (Continuous Updating Estimator, CUE) as proposed by Newey and Smith (2001) in conjunction with an automatic lag-selection procedure proposed by Newey and West (1994) to calculate estimates of the variance-covariance matrix of the moment conditions. This empirical approach attenuates the potential bias of GMM estimates when there are many instruments and the low power of specification tests based on over-identifying restrictions (see Guay and others, 2003; and Gali, Gertler and Lopez-Salido, 2005). They find that contrary to estimates of the NPC on Canadian data obtained using standard GMM, the CUE-based estimates do not fit well the data and the NPC is statistically rejected.

In addition to usual modelling and data measurement issues, there are three main estimation issues that are particularly important when estimating NPCs. The empirical work on Canadian data discussed above has dealt with some of the issues but never all of them at once, from which the need to deal with them in a unified framework like the one suggested here. These issues are:

- Non-validity of estimates when inflation is non-stationary

Existing estimates of the NPC for Canada assume that inflation is a stationary variable. If Canadian inflation is non-stationary, this approach is not ideal. Modelling variables as stationary when these instead contain a unit root invalidates estimation result, like in the case of GMM (or CUE) and FIML. This is because if inflation is non-stationary, the asymptotic distributions of the GMM and FIML estimators are not necessarily Gaussian normal, implying that the estimated NPC's coefficients and standard errors are invalid: thus any inference about the parameter values is incorrect, and more efficient estimators can be obtained by taking into account the unit root. Pre-transforming the data to make it stationary on a priori assumptions about the source of non-stationarity of the data may lead to results that are not robust to alternative hypotheses about the exact nature of the common trend.

- Testing for model-consistent expectations

Some of the empirical work using GMM (notably Leith and Malley, 2002; Gagnon and Khan, 2004; Khan, 2004) characterize the NPC model based only on a very weak property of rational expectations: namely that the expectational error ($\pi_{t+1} - E_t\pi_{t+1}$) should be unforecastable by variables dated at time t or earlier. In the context of GMM, this boils down to choosing instruments for inflation expectations that are correlated with the portion of π_{t+1} that is orthogonal to π_{t-1} and the cyclical indicator at time t. Estimation is carried out on the assumption that the chosen instruments accomplish this requirement—in other words, rational expectations are simply assumed on the presumption that instrument orthogonality is indeed met. However, another feature of rational expectations is that they should be model consistent: expectations for next period's inflation rate should be consistent with the process for inflation described by the model. As shown by Fuhrer (1997), Sbordone (2002, 2003), Linde (2002), Rudd and Whelan (2001), McAdam and Willman (2002), Kozicki and Tinsley (2002) and Banerjee and Batini (2003), this additional prediction yields specific testable implications for how inflation expectations in the NPC are modeled. Typically, all these works address this issue by following the present value approach of Campbell and Shiller (1987) and Fuhrer (1997) and cast the NPC in a system of equations. This procedure essentially amounts to computing the expected present value of the driving variable (the cyclical factor in the NPC case) under the assumption that this follows a specific process, and then determining what fraction of inflation is accounted for by this present-value term. If the present value is well characterized as a function of lags of the driving variable, then this method will be equivalent to that originally suggested by Hansen and Sargent (1980).

- Parameter identification

Several recent papers have drawn attention to the problem of identifying the parameters in the NPC, and more specifically, to whether existing estimation methods can correctly distinguish between backward and forward-looking solutions. Mavroeidis (2002) demonstrates how identification of the parameters depends on the uniqueness of the solution to the system containing the NPC and the equations governing the exogenous variables. In the GMM framework, this involves making assumptions on the process of the forcing variable, which is largely ignored in the NPC literature using Canadian data.[1] Ma (2002) points out that GMM estimation relies upon a quadratic concentrated objective function, because GMM solves a locally quadratic minimization problem. For the hybrid NPC, the objective function is non-quadratic with respect to the share of firms that set prices in a backward-looking manner, and consequently the coefficients on the expected and lagged inflation are only weakly identified.

Nason and Smith (2005) examine this for a number of countries including Canada. They employ Anderson-Rubin (1949) exact analytic methods to examine the identification problem in several statistical environments: under strict

[1] See also Bårdsen, Eitrheim, Jansen and Nymoen (2005)

exogeneity, in a vector autoregression, and in the context of a small closed-economy AS-IS plus monetary policy rule equation system. They find that the NPC model is rejected on Canadian data for different set of instruments over the sample 1963 Q1-2000 Q4 when these methods are used.

In the next sections we re-estimate the NPC on Canadian data addressing these key issues simultaneously within the Johansen and Swensen (1999) unified framework.

3 The theoretical model

Consider the "hybrid" version of the NPC is given in GGLS by

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t(\pi_{t+1}) + \lambda z_t \tag{1}$$

where π_t is inflation at time t, z_t is the cyclical indicator—typically the output gap or real marginal cost, and E_t is the expectation operator indicating expectations formed at time t. The complete dynamic system also contains an equation for the real marginal cost; we assume that it is an auto-regressive process given by

$$z_t = \delta_1 z_{t-1} + \delta_2 \pi_{t-1} + \eta_t \tag{2}$$

where η_t is a white noise residual.

The parameters γ_b, γ_f and λdepend on "deep" or "structural" parameters, including the probability that firms reset prices at any given time, the discount factor, the fraction of rule-of-thumb firms that set their prices in a backward-looking, myopic way(as in Gali and Gertler, 1999) or, alternatively, the degree of indexation to past prices of the firms that are not allowed to re-optimize at time t (as in Christiano, Eichenbaum and Evans, 2002). Direct estimation of γ_b and γ_f as opposed to the structural equations expressed in terms of deep parameters facilitates the interpretation of the NPC under a broad class of sticky-price models and modelling assumptions. γ_b and γ_f must satisfy the following restrictions:

$$\begin{aligned} \gamma_b, \gamma_f &\geq 0 \\ \gamma_b + \gamma_f &\leq 1 \end{aligned} \tag{3}$$

Written in the closed form solution, the interpretation of equation (**??**) is that fundamental inflation equals the discounted stream of expected future real marginal costs, taking into account the backward-looking behavior:

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2} \right)^k E_t(z_{t+k}) \tag{4}$$

where δ_1 and δ_2 are respectively the stable and unstable roots of the dynamic system. Equations (1) and (4) are examples of an exact rational expectation hypothesis, in the sense that the econometric test of this hypothesis involves testing whether the only error present is the expectational error.

In the NPC literature (see Linde, 2002), as well as in the inflation dynamics model by Fuhrer and Moore (1995), it is commonly assumed that

$$\gamma_b + \gamma_f = 1 \tag{5}$$

a restriction often referred to as "dynamic price homogeneity" or "superneutrality". If equation (5) holds, inflation does not depend on real factors in the long run–legitimating the use of monetary policy for the exclusive pursuit of price stability in the long run.

4 Estimation method

The method used here and proposed by Johansen and Swensen (1999) is a Full Information Likelihood method. It exploits the idea that the mathematical expectation conditional on a theoretical model and the observed data can be used to substitute for the forward-looking term in an estimated model.

The method comprises three steps.

- Step 1: The first step requires specifying and estimating an unrestricted VAR containing the relevant variables, under the assumption that at least one variable has a unit root. In this step diagnostic tests are run to ensure that residuals are white noise and the number of stationary relations in the system are determined. This is done using the trace statistic of Johansen and Juselius (1990)—a test comparing the likelihood of the unrestricted VAR with the likelihood of a cointegrated VAR. If the test indicates that there are less stationary relations than variables, the assumption that at least one variable has a unit root is confirmed.

- Step 2: The second step requires estimating the parameters of the structural system via maximum likelihood. This implies re-parameterizing the cointegrated VAR model to account for any forward-looking term. The Technical Appendix explains how we carry out such re-parameterization.

- Step 3: The final and third step requires to test the restrictions implied by the rational expectation model (here the NPC).

Below we offer further details on key aspects of the procedure used in estimation. In particular, Section 4.1 sketches the numerical optimization methods that we employ in step 2 to derive the maximum likelihood estimators of the Canadian NPC when the parameters of the rational expectation model are unknown like in our case. Section 4.2 formulates the testable hypothesis in terms of the NPC—in which case restrictions on the expectations entail restrictions on the cointegration relationships—as required by step 3. The test of the rational expectations model compares the likelihood of the cointegrated VAR with the likelihood of the cointegrated VAR with the restrictions imposed. It then describes the associated maximum likelihood estimator and likelihood ratio tests. Section 4.3 describes the maximum likelihood ratio tests that we also use in step 3.

4.1 Numerical maximization methods

The Johansen and Swensen (1999) method provides a test of rational expectation for models in which coefficients are known. If coefficients are unknown, like in our case, with this method—and contrary to the method by Campbell and Shiller (1987)— it is still possible to derive maximum likelihood estimators and likelihood ratio tests by evaluating the likelihood at every fixed value of the coefficients.

To find the maximum value of the likelihood function we use the Broyden-Fletcher-Goldfarb-Shanno (BSFG) numerical optimization method for non-linear functions.[2] The BSFG optimization starts from initial values, and maximizes the function using a quasi-Newton method based on numerical derivatives. The convergence decision is based on the likelihood elasticities and the one-step ahead relative change in parameter values.[3] We also carry out a grid search for the parameters. This is a simple approach, where we calculate the value of the log-likelihood function for each possible combination of values, within given intervals, for the parameters $\left\{\gamma_b, \gamma_f, \lambda\right\}$.However, due to the dimension of the parameter matrix, with a grid search we cannot include a constant in the estimated NPC equation, and we therefore rely mainly on the BSFG numerical optimization results for estimation.

4.2 The testable rational expectations restrictions for the NPC

Consider the p-dimensional autoregressive process $X_t = (\pi_t, z_t)$ defined for $t = 1, .., T$ by the equations:

$$X_t = \Pi_1 X_{t-1} + .. + \Pi_k X_{t-k} + \mu_0 + \epsilon_t \tag{6}$$

for fixed values of $X_{-k+1}, .., X_0$ and $\epsilon_t \sim N_p(0, \Omega)$. The parameter space is given by the unrestricted parameters $(\Pi_1, .., \Pi_k, \Omega)$. μ_0 is the coefficient of the constant term. Equation (6) can equivalently be written in a vector error-correction form as:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \phi D_t + \epsilon_t \tag{7}$$

where $\Pi = \sum_{i=1}^{k} \Pi_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$. The first step of the procedure consists in testing the rank of the of the matrix Π. If there is at least one unit root in

[2] We maximise the log-likelihood controlling for the number of observations; -$\frac{T}{2}\left\{\log|S_{11}| + \log\left|\tilde{\Sigma}_{22}\right| - \log|a^{\prime}a| - \log|b^{\prime}b|\right\}$.This is because of the convergence criteria in the Broyden-Fletcher-Goldfarb-Shanno maximization method. This method is invariant to the scaling of parameters, but not invariant to sample sizes.

[3] See Fletcher (1987) for details.

the system, and if the NPC as specified in equation (1) holds, there must be one cointegration vector.[4] Hence the rank of the matrix must be 1.

Given the reduced rank of the matrix Π, the restrictions implied by the New-Phillips curve in equation (1) can be tested as a restriction on the parameters in the matrix Π. However, in order to able to find the maximum likelihood estimators with respect to freely varying parameters, rather than under constraints, we need to reformulate the restrictions in terms of the restrictions in Johansen and Swensen (1999). This is a simple reparametrization of the statistical model, and implies that the parameters of the statistical model are uniquely identified. The details of the reparameterization of the statistical model, as well as the complete formulation of the restrictions implied by rational expectations can be found in the Technical Appendix.. For $X_t = (\pi_t, z_t)$, the restrictions take the form

$$\begin{aligned} E_t\left(c_1^{\prime} X_{t+1} \mid \varphi_t\right) + c_0^{\prime} X_t + c_{-1}^{\prime} X_{t-1} + c &= 0 \qquad (8) \\ E_t\left(c_1^{\prime} \Delta X_{t+1} \mid \varphi_t\right) - d_1^{\prime} X_t + d_{-1}^{\prime} \Delta X_t + c &= 0 \qquad (9) \end{aligned}$$

In terms of the NPC, this is expressed as

$$\begin{aligned} \gamma_f E_t\left(\pi_{t+1}\right) - \pi_t + \lambda z_t + \gamma_b \pi_{t-1} &= 0 \qquad (10) \\ \gamma_f E_t\left(\Delta\pi_{t+1}\right) - \left(1 - \gamma_f - \gamma_b\right)\pi_t + \lambda z_t - \gamma_b \Delta\pi_t &= 0 \qquad (11) \end{aligned}$$

Hence, to test the NPC within the Johansen and Swensen (1999) framework, we can rewrite the coefficients of the statistical model in terms of the parameters in the NPC model

$$\begin{aligned} c_1 &= \left(\gamma_f, 0\right) \equiv b \qquad (12) \\ c_0 &= (-1, \lambda) \\ c_{-1} &= (\gamma_b, 0) \end{aligned}$$

with

$$\begin{aligned} d_1 &= \left(\left(1 - \gamma_f - \gamma_b\right), -\lambda\right) \equiv d \\ d_{-1}^{\prime} &= (-\gamma_b, 0) \end{aligned}$$

d is the cointegration vector. The second step of the method consists in testing the validity of the restrictions implied by the NPC using a maximum likelihood ratio test.

4.3 The maximum likelihood estimators and the maximum likelihood ratio test

The maximum likelihood ratio test compares the likelihood of the unrestricted cointegrated VAR with the likelihood of the cointegrated VAR under the rational

[4] The case where the number of cointegration vectors equals the number of rational expectations hypothesis to be tested is a special case, and simplifies the estimation procedure. Only this case is described below, see Johansen and Swensen (1999) for details.

expectations restrictions. Under the re-parametrization mentioned above, the likelihood of the cointegrated VAR under the rational expectations restrictions is the product of the conditional model for the cyclical factor and the marginal model for inflation.

In order to estimate the conditional model, one should regress $a^{\prime}\Delta X_t$ on $b^{\prime}\Delta X_t + d^{\prime}_{-1}\Delta X_{t-1}$, $d^{\prime}X_{t-1}$, ΔX_{t-1} and the constant term. In terms of the present model, we regress Δz_t on $\gamma_f \Delta\pi_t - \gamma_b \Delta\pi_{t-1}$, $\left(1-\gamma_f-\gamma_b\right)\pi_{t-1} - \lambda z_{t-1}$, ΔX_{t-1} and the constant term. Denote by R_{1t} the residuals in the regression and define S_{11} as be the sum of squared residuals

$$S_{11} \equiv \frac{1}{T}\sum_{t=1}^{T} R_{1t}R^{\prime}_{1t} \tag{13}$$

The part of the maximized likelihood function from the conditional model is then

$$L^{-2/T}_{1.2\max} = \frac{|S_{11}|}{|a^{\prime}a|} \tag{14}$$

The marginal model is given by

$$b^{\prime}\Delta X_t = d^{\prime}X_{t-1} - d^{\prime}_{-1}\Delta X_{t-1} \tag{15}$$

which for the specific model here is equivalent to

$$\gamma_f \Delta\pi_t = \left(1-\gamma_f-\gamma_b\right)\pi_{t-1} - \lambda z_{t-1} + \gamma_b \Delta\pi_{t-1} \tag{16}$$

Define $\tilde{\Sigma}_{22}$ as the sum of squared residuals in the marginal model.

$$\begin{aligned} \tilde{\Sigma}_{22} \quad \equiv \quad & \frac{1}{T}\sum_{t=1}^{T}\left(\gamma_f \Delta\pi_t - \left(1-\gamma_f-\gamma_b\right)\pi_{t-1} + \lambda z_{t-1} - \gamma_b \Delta\pi_{t-1}\right) \\ & \times \left(\gamma_f \Delta\pi_t - \left(1-\gamma_f-\gamma_b\right)\pi_{t-1} + \lambda z_{t-1} - \gamma_b \Delta\pi_{t-1}\right)^{\prime} \end{aligned}$$

The part of the maximized likelihood function from the marginal model is then

$$L^{-2/T}_{2\max} = \frac{\left|\tilde{\Sigma}_{22}\right|}{|b^{\prime}b|} \tag{17}$$

Consequently, the maximum value of the likelihood function under the rational expectations hypothesis is given by

$$L^{-2/T}_{H\max} = \frac{\left|\hat{S}_{11}\right|}{|a^{\prime}a|}\frac{\left|\tilde{\Sigma}_{22}\right|}{|b^{\prime}b|} \tag{18}$$

The maximum likelihood ratio test compares the loglikelihood under the rational expectation hypothesis to the loglikelihood of the unrestricted cointegrated VAR, where the asymptotic distribution of the test statistic is χ^2 with degrees of freedom equal to the difference in the number of the parameters in the unrestricted case and under the hypothesis, corrected for the number of estimated parameters.

5 Data used in estimation

To estimate the NPC for Canada we use inflation, marginal cost and real import price data for Canada from 1973:1 to 2003:4. The inflation rate is measured as the log difference of the (officially seasonally adjusted) implicit price deflator of GDP at market prices.[5] The real marginal cost is measured by the deviation of the labor share from its sample mean. We look at two measures of the share. First, we use an unadjusted measure as in Gali and Gertler (1999) for the United States and Gagnon and Khan (2004) for Canada. Then, in line with the adjustment proposed by Batini, Jackson and Nickell (2005) and used on Canadian data by Guay and others (2003), we also use a measure of the share that is net of indirect taxes, it includes part of remuneration of the self-employed that constitutes a return to labor rather than to capital, and is adjusted to remove public sector inputs and outputs from the expression for the share. [6] Finally, to allow for the openness of the Canadian economy, we follow Batini, Jackson and Nickell (2005) and modify marginal cost to account for the role of imported material input prices under more general technologies than Cobb-Douglas. This implies adding to (the log of) marginal cost the log of the ratio of import prices to the GDP deflator, weighted by a time-varying indicator for the openness for the economy (export plus import volumes divided by GDP).[7] The time-series of the data used are plotted in the Data Appendix.

6 Results

We present results along four subsections. Subsection 6.1 describes stationarity results obtained using multivariate tests.[8]. Subsection 6.2 presents parameter estimates of the NPC system (equation (1)) and the likelihood ratio test for the NPC obtained using Canadian data. Subsection 6.3 considers the possibility that shifts in the mean of inflation have occurred, arising, for example, through shifts in anti-inflation preferences of the Bank of Canada, and thus presents estimates of the NPC system on two different samples–one pre and one post-inflation targeting. Finally, sub-section 6.4 repeats the analysis in 6.1-6.3 using a measure of the labor share adjusted for net indirect taxes, self-employment and the public sector.

[5] We thank Nicolas Raymond at the Bank of Canada for help with these data.

[6] These specific adjusted labor share data for Canada are those used by Guay and others, 2003. We thank Zhenhua Zhu for providing us with these data.

[7] Following Batini and others (2005), Gagnon and Khan (2004) first modified Canadian marginal cost in this way for estimates on Canadian data.

[8] We also conducted univariate tests. The results are plausible and largely consistent with results from multivariate tests, but on the whole they tend to be less reliable as explained in Johansen (1996). For space limitations, we thus only report results from multivariate tests.

6.1 Step 1

6.1.1 Unrestricted VAR and residual diagnostics

The first step requires estimating an unrestricted VAR on the variables of the system. Since Canadian inflation seems to exhibit a break around 1990—possibly in conjunction with the shift to inflation targeting in 1991—one question is whether we should do the analysis on the full sample or on split samples.

To ascertain this, we check for a possible break at the time of the regime shift using the Chow breakpoint test. In line with findings in Ravenna (2000), who documents a large post-1990 drop in inflation persistence, the break-point Chow test confirms a structural break in 1991 Q3, in the year when Canada shifted to inflation targeting.(The sequences of the Chow test statistics normalized by their critical values are plotted in Charts 2 and 3 in the Data Appendix.) Levin and Piger (2002) also find evidence of structural breaks for various measures of Canadian inflation around this time–but not GDP price inflation— using a variety of test for structural breaks.

Testing for structural breaks is complicated by a number of factors and results can differ sensibly given the test method and underlying assumptions (see Stock, 2004). So we do not take a stand here on whether a break has occurred or not and proceed looking both at results on the full sample and on the split sample. In line with the findings of our test and in the literature, we choose 1991 as the time of the break for our split sample.[9]

We estimate an unrestricted three-lag VAR in Canadian GDP price inflation and the unadjusted labor share modified for open economy considerations, and a constant term over the full sample and two similar VARs on the split samples. The lag length was determined using standard lag length information criteria (see Table 7 in the Data Appendix). Tests suggest that the residuals are not white noise for the full sample. However, similar tests indicate that residuals are no longer misspecified when the sample is split (see Table 8 in the Data Appendix), implying that it is sensible to proceed with the Johansen and Swensen method, at least on split samples.

6.1.2 Stationarity tests

Chart 1 in the Data Appendix plots Canadian GDP price inflation and our (unadjusted) measure of open-economy marginal cost. These look highly correlated contemporaneously, with inflation moving from high and more volatile in the 1970s and 1980s to a lower and more stable level post 1990, and so lend visual support to the hypothesis that inflation in Canada has a unit root. We test for the existence of a unit root in inflation both over the entire sample and on the split samples.

[9] Each step of the estimation of the parameters involves testing for the cointegrating rank, estimating the reduced rank VAR and then estimating the parameters in a reduced rank restricted VAR. To date no automated software exists to perform this task. Due to the complexity of the estimation method and the fact that the estimates do not change much when we consider one sample split, we decided to leave recursive estimation for future research.

For the full sample, multivariate unit root tests using the trace test for cointegration, give strong evidence of one stationary relation and one common trend in the system (see Table 1 below).[10] This is in line with inflation being integrated of order 1 over the full sample. For the full sample, we can thus proceed to the analysis of the cointegrated VAR model with one cointegrating relationship.

Results on the split sample are more mixed. Specifically, for the first sub-sample, the multivariate unit root tests give strong indications of one stationary relation. However, in the second sub-sample we can reject no stationary vectors only at the 10 percent level.

1973-2003		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	21.926 ($\star\star$)
0.149	$p \leq 1$	2.875
0.024		
1973-1990		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	13.487 ($\star\star$)
0.130	$p \leq 1$	4.287
0.063		
1991-2003		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	19.841
0.241	$p \leq 1$	5.47
0.099		

Table 1: Cointegration rank test statistics

6.2 Steps 2 and 3

6.2.1 Parameter estimates and restriction tests

Results in the previous section indicate that there is a unit root in the NPC system (both in the full and in the split sample), and that variables in the system are linked through one stationary relationship. Given this, we are interested in testing whether such stationary relationship satisfies the restrictions implied by the NPC (restrictions (3)) *for reasonable parameter values* of $\{\gamma_b, \gamma_f, \lambda\}$.[11] Under the assumption that the inflation has a unit root, there are three possible cases. First, the stationary relation is not the NPC. This would be the case where the maximum likelihood estimates of the parameters in the stationary relation do not correspond to plausible parameter estimates for the NPC.

[10] In Table 1, λ denotes the eigenvalues of the Π-matrix, and the test is for the number of non-zero eigenvalues. $**$ means that the null-hypothesis is rejected at the 99 percent significance level.

[11] In the literature, the parameters are commonly restricted by $0 < \gamma_b, \gamma_f, \lambda < 1$.

Second, the stationary relation is the NPC, and there is a cointegration relation between the inflation rate and the real marginal cost. This implies that both the inflation rate and the real marginal cost are integrated of order 1. Third, it is possible that the stationary relation is the NPC, but can be represented by a relation between the change in inflation and the real marginal cost. In this case, there is no cointegration, since the inflation rate is integrated of order 1, but the real marginal cost is stationary. In the following, we use the numerical optimization methods described in section 4.1 to obtain the maximum likelihood estimates, the maximized value of the likelihood function and the likelihood ratio test statistic for the NPC.

Tables 2 and 3 below show parameter estimates obtained maximizing the likelihood via BSFG numerical methods on full and split samples. Likelihood ratio tests indicate that on the split samples, the NPC fits well Canadian data, when the share of labor accounts for openness considerations but is not adjusted for net indirect taxes or other data considerations. The fit, however, is less good for the full sample, as the restrictions implied by the NPC can be rejected at the 5 percent significance level in the $\mathcal{LR}$ test.

Both on full and split samples, the estimated weight on the lag of inflation, γ_b, is much lower than the estimated weight on the lead of inflation, γ_f, suggesting that price-setters in Canada are predominantly forward-looking. The estimated value of γ_b is in general smaller than in Gagnon and Khan (2001) and Guay, Luger and Zhu (2003), and the estimated value of γ_f is in general larger. However, Nason and Smith (2005) find very similar estimates when a large set of instruments is included, but in their estimations the estimated value of the coefficient on the real marginal cost is much smaller than in our case. For both the full sample and the later sample, our estimates of the coefficient on real marginal cost are generally on the higher side of those found in the literature for comparable measures of the share (see notably Gagnon and Khan, 2004). The analysis on split samples points to parameter instability of the estimates over time: in the later sample price-setters seem to have become more forward-looking—in line with estimates of NPC or the United States and the euro area on samples starting in the 1970s. Likewise inflation seems to have become almost three times more sensitive to the real marginal cost than in the earlier sample.

γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
0.271	0.729	0.376	23.36	< 0.05

Table 2: Parameter estimates with BSFG, 1973-2003

When the model is estimated with the restriction

$$\gamma_b + \gamma_f = 1$$

the $\mathcal{LR}$ test statistic is 3.96 for the first sample and 3.32 for the second sample. Given the critical value for the $\mathcal{LR}$-test with 1 degree of freedom at the 5 percent

Period	γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
1973-1990	0.326	0.714	0.165	6.30	> 0.25
1991-2003	0.269	0.721	0.415	5.52	> 0.25

Table 3: Parameter estimates with BSFG, 1973-1991 and 1992-2003

significance level is 3.84, the restriction cannot be rejected for the second sample. This likely indicates that the Phillips curve in Canada is vertical in the long run, with obvious implications for the objective of monetary policy.

6.2.2 $\mathcal{LR}$-test for variable exclusion

The null-hypothesis that the one of the variables can be excluded from the NPC can be tested as a test on the ratio of the likelihood of the NPC with the variable and the likelihood of the NPC without the variable, under the assumption that the restriction implied by the NPC is valid. This test is distributed as $\chi^2(1)$, given the difference in the numbers of estimated parameters. The $\mathcal{LR}$ test statistics are in Table 4 below together with approximate p-value. They imply that all the coefficients, except from the coefficient on the real marginal cost are highly significant.

Period	γ_b	γ_f	λ
1973-2003	7.42 [0.01]	4.68 [0.05]	4.00 [0.05]
1973-1990	16.04 [0.01]	10.78 [0.01]	0.42 [0.50]
1991-2003	12.42 [0.01]	13.78 [0.01]	3.62 [0.10]

Table 4: Likelihood ratio tests for variable exclusion, p-values in parenthesis

6.3 What happens when we adjust the labor share for data considerations?

As explained in Batini, Jackson and Nickell (2005), on the face of it, the labour share appears easy to compute: take the total compensation of employees in the economy and divide it by the national income. In practice, there are three issues to bear in mind when computing the labour share.

First, the share must be derived relative to a measure of value added that is *net of indirect taxes*. Conceptually, firms and workers can only lay claims on revenue (in terms of output per head) that actually accrues to the firm. By definition, this will be net of taxes on value added, because the latter go to the government and are not received by the firm.

Second, as Bentolila and Saint-Paul (1999) emphasize, because the share represents the remuneration of employees in value added, it ignores the part of remuneration of the self-employed that constitutes a return to labour rather than to capital. There are two ways of adjusting for this. We can either augment the numerator of the ratio defining the labor share to include the fraction of total compensation of self-employed that relates to labour; or we can subtract the amount of value added generated by self-employed from the denominator of that ratio.

A final consideration when deriving a measure of labour share regards the contribution of the public sector. It might be argued that the concept of labour and capital shares only really make sense with regard to the market sector of the economy. In this spirit, we may amend the labor share to remove the public sector's inputs from the numerator and the denominator of its expression. We do so by subtracting from the numerator of the self-employed adjusted share, the compensation of employees by the general government; and by removing from the denominator of that share the general government total resources, essentially a measure of general government gross value added. Chart 4 in the Data Appendix offers a plot of the labor share adjusted for open economy considerations alone vis-a-vis the labor share adjusted also for net indirect taxes, self-employment and public sector considerations.

In this section we thus repeat the analysis of the previous three subsections using this adjusted measure of the share. Misspecification tests and cointegration results are reported in Tables 9 and 10 in the Data Appendix. The cointegration tests indicate one stationary relation on the full sample. However, there is no evidence of a stationary relation in the first sample, and evidence of no stationary relation in the second sample. When this adjusted measure of the share is used for 1973-1990, the parameters that maximize the maximum likelihood function are reasonable, as shown in Table 5 below. However, when we attempt to estimate the NPC parameters during 1991 to 2003, the BSFG algorithm, convergence depends on the initial values of the parameters and for most initial values, the algorithm does not converge for a reasonable set of parameters.

Period	γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
1973-1990	0.681	0.318	0.743	10.26	> 0.05
1991-2003	-	-	-	-	-

Table 5: Parameter estimates with BSFG, 1973-1990 and 1991-2003, labor share adjusted for taxes, public sector and self-employment

One possible interpretation of these results is that the shift to inflation targeting in Canada has broken the link between inflation and marginal cost as specified in the "hybrid" NPC perhaps through altering permanently the nature of price-setting away from Calvo (1983) type set-ups to other set-ups, like, for example, those in Dotsey, King and Wolman (2000). Another possible interpretations–that would explain why the NPC fits Canadian data well when we use the unadjusted measure of the share but not when we use the measure adjuste for taxes, self-employment and the public sector—is that the adjustment is not the appropriate one. Additional factors may also play a role in the bad fit of the NPC over the later sample, notably the fact that desired mark-ups (which are assumed fixed in the analysis above for simplicity) may have become time-varying after the shift, or that the share of labor is no longer a good proxy of marginal costs in Canada given shifts in taxes, the fraction of self-employed in employment and/or the public sector post-1990. (See Batini, Jackson and Nickell, 2001, for a discussion of how to measure marginal cost vis a vis average cost).

7 Conclusions and future research

In this paper we have used a new method for estimating linear rational expectation models containing $I(1)$ variables to estimate the New-Keynesian Phillips curve on Canadian data (1973-2003). Our results strongly indicate the presence of a unit root in Canadian GDP price inflation rate over the full sample and give evidence of a unit root in inflation over the earlier period of the sample when we split the data into a pre and a post-inflation targeting period. We find that the NPC offers a good representation of inflation dynamics in Canada for some—but not all—measures of marginal cost. Accounting for open economy considerations seems particularly important for the fit. Contrary to much previous literature, estimates of the NPC based on this method and this assumption also support the super-neutrality result. In addition, estimation of the NPC in the Johansen and Swensen (1999) framework overcomes the problem of identification associated with GMM estimation. Hence it possible to discern empirically between forward-looking and backward-looking NPC specifications. We find that both terms are important in determining inflation.

One interesting avenue of future research could include estimating on Canadian data the NPC jointly with a wage equation as in Sbordone (2004) and Sbordone and Cogley (2004) under the hypothesis of a unit root in price inflation or both on price and wage inflation.

8 Bibliography

Banerjee, R. and N. Batini (2003), Inflation Dynamics in Seven Industrialized Open Economies

Batini, N., B. Jackson and S. Nickell (2004), An Open-Economy New Keynesian Phillips Curve for the UK, forthcoming *Journal of Monetary Economics*

Batini, N, Jackson, B and Nickell, S (2001). "The Pricing behavior of UK Firms", External MPC Discussion Paper No. 9, Bank of England

Batini, N. and E. Nelson (2001), Optimal Horizons for Inflation Targeting, *Journal of Economic Dynamics and Control*, Vol. 25, pp. 891-910

Batini, N. (2002), Euro-Area Inflation Persistence, *ECB Working Papers*, No. 201

Bentolila, S. and Saint-Paul, G., (1999) "Explaining Movements in the Labor Share", CEMFI Working Paper No. 9905

Benigno, P. and D. Lopez-Salido (2002), Inflation Persistence and Optimal Monetary Policy in Europe, *ECB Working Papers*, No. 178

Beyer, A. and R. Farmer (2004), On the Indeterminacy of New-Keynesian Economics, *ECB Working Papers*, No. 323

Bårdsen, G., Ø. Eitrheim, E. Jansen and R. Nymoen (2005), The Econometrics of Macroeconomic Models, forthcoming *Oxford University Press*

Calvo, G. (1983), Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, Vol. 12, pp. 383-398

Campbell and Schiller (1987), Cointegration and tests of present value models, *Journal of Political Economy*, Vol. 93, pp. 1062-1087

Christiano, L. J., M. S. Eichenbaum and C. L. Evans (2001), Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *NBER Working Paper*, No. 8403

Clarida, R., J. Gali and M. Gertler (2000), Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory, *Quarterly Journal of Economics*, Vol. 37, pp. 1661-1707

Cogley, T. and T. J. Sargent (2001), Evolving Post-World War II U.S. Inflation Dynamics, *NBER Macroeconomics Manual*, Vol. 16, pp. 331-373

Erceg, C. and A. T. Levin (2003), Imperfect Credibility and Inflation Persistence, *Journal of Monetary Economics*, Vol. 50, pp. 889-914

Fletcher, R. (1987), Practical Methods of Optimization, Second Edition, New York: John Wiley and Sons.

Fuhrer, J. C. (1997), Comment, *NBER Macroeconomics Manual*, Vol. 12, pp. 346-355

Fuhrer, J. C. and G. R. Moore (1995), Inflation Persistence, *Quarterly Journal of Economics*, Vol. 110, Issue 1 (Feb.1995), pp. 127-159

Galí, J. and M. Gertler (1999), Inflation Dynamics: A Structural Econometric Analysis, *Journal of Monetary Economics*, Vol. 44, pp 195-222

Galí, J., M. Gertler and D. López-Salido (2001), European Inflation Dynamics, *European Economic Review*, Vol. 45, pp. 1237-1270

Galí, J., M. Gertler and D. López-Salido (2003), Roubstness of the Esimates of the Hybrid New Keynesian Phillips Curve

Giannoni, M. P. and M. Woodford (2003), Optimal Inflation Targeting Rules', in B. S. Bernanke and M. Woodford (eds), *Inflation targeting*, Chicago, University of Chicago Press

Guay, A., R. Luger and Z. Zhu (2004), The New Phillips Curve in Canada, forthcoming Bank of Canada conference proceeding

Hansen and Sargent (1980), Formulating and Estimating Dynamic Linear Rational Expectations Models, *Journal of Economic Dynamics and Control*, Vol. 2, pp. 7-46

Johansen, S. (1996), Likelihood Based Inference on Cointegration in the Vector Autoregressive Model. Oxford University Press, Oxford

Johansen, S. and K. Juselius (1990), Maximum likelihood estimation and inference on cointegration- with application to the demand for money, *Oxford Bulletin of Economics and Statistics*, Vol. 52, pp. 169-210

Johansen, S. and A. Swensen (1999), Testing rational expectations in cointegrated vector autoregressive models, *Journal of Econometrics*, Vol. 93, pp. 73-91

Johansen, S. and A. Swensen (2004), More on testing exact rational expectations in cointegrated vector autoregressive models: Resticted drift terms. Forthcoming *Econometrics Journal*

Jondeau, E. and H. Le Bihan (2001), Testing a Forward-Looking Phillips Curve: Additional Evidence from European and U.S. Data, *Notes d'Etudes et de Recherche*, No. 86, Bank of France

Kozicki, S. and P. A. Tinsley (1999), Vector rational error correction, *Journal of Economic Dynamics and Control*, Vol. 23, pp. 1299-1327

Kozicki, S. and P. A. Tinsley (2002), Alternative Sources of the Lag Dynamics of Inflation, Proceedings of a Conference on Price Adjustment and Monetary Policy held at the Bank of Canada, November 2002

Kozicki, S. and P. A. Tinsley (2002), Dynamic Specifications in Optimizing Trend-Deviation Macro Models, J*ournal of Economic Dynamics and Control*, Vol. 26, August 2002

Leith, C. and J. Malley (2002), Estimated Open Economy New Keynesian Phillips Curves for the G7, University of Glasgow, Department of Economics, Discussion Paper No. 6

Levin, A. T. and J. M. Piger (2002), Is Inflation Persistence Intrinsic in Industrial Economies, Federal Reserve Bank of St. Louis Working Paper, No. 23

Linde, J. (2002), Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach, *Bank of Sweden Working Paper*, No. 129

Ma, A. (2002), GMM Estimation of the New Phillips Curve, *Economics Letters*, Vol. 76, pp. 411-417

Mavroeidis, S. (2002), Identification and Misspecification Issues in Forward-Looking Models, Econometrics Discussions Papers, University of Amsterdam, No. 21

McAdam, P. and A. Willman (2002), New Keynesian Phillips Curves: a Reassessment Using Euro-Area Data, *ECB Working Papers*, No. 265

McConnell, M. and G. Perez-Quiroz (2000), Output Fluctuations in the United States: What has Changed Since the Early 1980s? *American Economic Review*, December 2000, pp. 1464-1476

Ravenna, F (2000) "The Impact of inflation Targeting in Canada: A Structural Analysis", mimeo, New York University

Rudd, J. and K. Whelan (2001), New tests of the New-Keynesian Phillips curve, forthcoming *Journal of Monetary Economics,* also Finance and Economics Discussion Series 2001-30

Sargent, T. J. (1999), The Conquest of American Inflation. *Princeton University Press*

Sbordone, A. M. (2002), Prices and Unit Labor Costs: A New Test of Price Stickiness, *Journal of Monetary Economics*, Vol. 49, pp. 265-292

Sbordone, A. M. (2003), Inflation Dynamics and Real Marginal Costs, unpublished, Rutgers University

Sbordone, A. M. (2004)**,** A limited Information Approach to the Simultaneous Estimation of Wage and Price dynamics, mimeo, Rutgers University

Sbordone and T. Cogley (2004), "A Search for a Structural Phillips Curve," Computing in Economics and Finance 2004 291, Society for Computational Economics.

Stock and Watson (2002), Has the business cycle changed and why, *NBER Macroeconomics Manual*, Vol. 17, pp. 159-218

Taylor, J. (2000), Low Inflation, Pass-Through and the Pricing Power of Firms, *European Economic Review*, Vol. 44, pp. 1389-1408

9 Technical Appendix: The statistical model and the rational expectation hypothesis

Assume that the p-dimensional vectors of observation are generated according to the VAR model[12]

$$X_t = A_1 X_{t-1} + ... + A_k X_{t-k} + \mu + \phi D_t + \varepsilon_t \qquad \text{for } t = 1, .., T \tag{19}$$

where $X_{-k+1}, ..., X_0$ are assumed to be fixed and $\varepsilon_1, ..., \varepsilon_T$ are independent identically Gaussian vector with mean zero and covariance matrix Σ. The matrices D_t, $t = 1, ..., T$, consist of deterministic series orthogonal to the constant term, μ. The VAR in equation (19) can be reparameterized as

$$\Delta X_t = \Pi X_{t-1} + \Pi_2 \Delta X_{t-1} + ... + \Pi_k \Delta X_{t-k} + \mu + \phi D_{t+1} + \varepsilon_t \qquad \text{for } t = 1, .., T \tag{20}$$

where

$$\Pi = A_1 + ... + A_k - I$$

[12] This part follows Johansen and Swensen (1999) closely.

and

$$\Pi_i = -\left(A_i + ... + A_k\right) \qquad \text{for } i = 2, .., k$$

For the process X_t to be $I(1)$, we assume that the matrix Π has reduced rank $0 < r < p$ and hence may be written as

$$\Pi = \alpha\beta^{|}$$

where α and β are $p \times r$ matrices of full column rank.

The formulation of the rational expectations hypothesis takes the form

$$E_t\left(c_1^{|}X_{t+1} \mid \varphi_t\right) + c_0^{|}X_t + c_{-1}^{|}X_{t-1} + ... + c_{-k+1}^{|}X_{t-k+1} + c = 0 \tag{21}$$

$E_t\left(c_1^{|}X_{t+1} \mid \varphi_t\right)$ denotes the conditional expectation given variables $X_1, .., X_t$. The $p \times q$ matrices c_i, $i = -k+1, .., 1$ are known matrices, possibly equal to zero. Assume that the two matrices c_1 and $c_{-k+1}, .., c_0, c_1$ are of full column rank. Defining

$$d_{-i+1} = -\sum_{j=i-1}^{k-1} c_{-j} \qquad \text{for } i = 0, .., k \tag{22}$$

the restriction in equation (21) may be reformulated as

$$E_t\left(c_1^{|}X_{t+1} \mid \varphi_t\right) - d_1^{|}X_t + d_{-1}^{|}\Delta X_t + ... + d_{-k+1}^{|}\Delta X_{t-k+2} + c = 0 \tag{23}$$

Reformulation of restrictions (21) and (23) as restrictions on the coefficients of the statistical model in equation (20). Taking the conditional expectations of ΔX_{t+1} given $X_t, .., X_0$, and multiplying equation (20) by $c_1^{|}$, we get

$$c_1^{|}E_t\left(\Delta X_{t+1} \mid \varphi_t\right) = c_1^{|}\Pi X_{t-1} + c_1^{|}\Pi_2\Delta X_{t-1} + ... + c_1^{|}\Pi_k\Delta X_{t-k} + c_1^{|}\mu + c_1^{|}\phi D_{t+1}$$

Inserting this expression into equation (23), implies that the following conditions must be satisfied:

$$\begin{aligned} c_1^{|}\Pi &= d_1^{|} \\ c_1^{|}\Pi_i &= -d_{-i+1}^{|} \quad \text{for } i = 2, .., k \\ c_1^{|}\mu &= -c \\ c_1^{|}\phi &= 0 \end{aligned}$$

Expressed in terms of the statistical model in equation (20)

$$\begin{aligned} \beta\alpha^{|}c_1 &= -\sum_{j=-1}^{k-1} c_{-j}^{|} = d_1 \\ c_1^{|}\Pi_i &= \sum_{j=i-1}^{k-1} c_{-j}^{|} = -d_{-i+1}^{|} \quad \text{for } i = 2, .., k \\ c_1^{|}\mu &= -H\omega \\ c_1^{|}\phi &= 0 \end{aligned}$$

Note that the first part of the restriction implies that the vector d_1 must belong to the space spanned by the columns of β, i.e. d_1 is a cointegration vector. Also, multiplying both sides by the matrix $\left(\beta'\beta\right)^{-1}\beta'$, one obtains the following restrictions on the adjustment parameters in α: $\alpha' c_1 = \left(\beta'\beta\right)^{-1}\beta' d_1$. Hence the restrictions implied by the rational expectations hypothesis are simultaneous restrictions on all parameters.

10 Data appendix

10.1 Tables

Series	Period	Level	Difference
π_t	1973:01-2003:04	-2.48	-12.48 ($\star\star$)
π_t	1973:01-1990:04	-1.60	-9.70 ($\star\star$)
π_t	1991:01-2003:04	-3.56 ($\star$)	-8.16 ($\star\star$)
z_t	1973:01-2003:04	-1.30	-4.67 ($\star\star$)
z_t	1973:01-1990:04	-2.27	-9.68 ($\star\star$)
z_t	1991:01-2003:04	-2.45	-7.18 ($\star\star$)

Table 6: Univariate unit root test, 1973-2003.

[13]

Model	Schwarz	Hannan-Quinn	Akaike
VAR(1)	$\mathbf{-6.69}$	-6.78	-6.84
VAR(2)	-6.59	-6.73	-6.82
VAR(3)	-6.60	$\mathbf{-6.79}$	$\mathbf{-6.93}$
VAR(4)	-6.46	-6.71	-6.88

Table 7: Information criteria for determination of lag length, 1973-2003.

	1973-2003		1974-1990		1991-2003	
AR 1-5	$F(20, 198)$	0.98 [0.48]	$F(20, 94)$	0.65 [0.87]	$F(16, 72)$	1.70 [0.07]
Normality	$\chi^2(4)$	69.48 [0.00]	$\chi^2(4)$	31.36 [0.00]	$\chi^2(4)$	8.00 [0.09]

Table 8: Misspecification tests.

[13] The sample is 1973:01-2003:04. The critical values of the ADF test are -2.89 at the 5 percent significance level and -3.49 at the 1 percent significance level.

	1973-2003		1974-1990		1991-2003	
AR 1-5	$F(20, 200)$	2.05 [0.01]	$F(20, 96)$	2.30 [0.00]	$F(16, 74)$	2.10 [0.027]
Normality	$\chi^2(4)$	37.68 [0.00]	$\chi^2(4)$	8.15 [0.06]	$\chi^2(4)$	7.44 [0.11]

Table 9: Misspecification tests, labor share adjusted for taxes, public sector and self-employment

1973-2003		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	28.599 ($\star$)
0.143	$p \leq 1$	10.404
0.084		
1973-1990		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	15.032
0.199	$p \leq 1$	3.296
0.060		
1991-2003		
λ	$H_0 : r = p$	Trace statistic
	$p = 0$	15.478 ($\star$)
0.147	$p \leq 1$	5.025 ($\star$)
0.073		

Table 10: Cointegration rank test statistics, labor share adjusted for taxes, public sector and self-employment

10.2 Charts

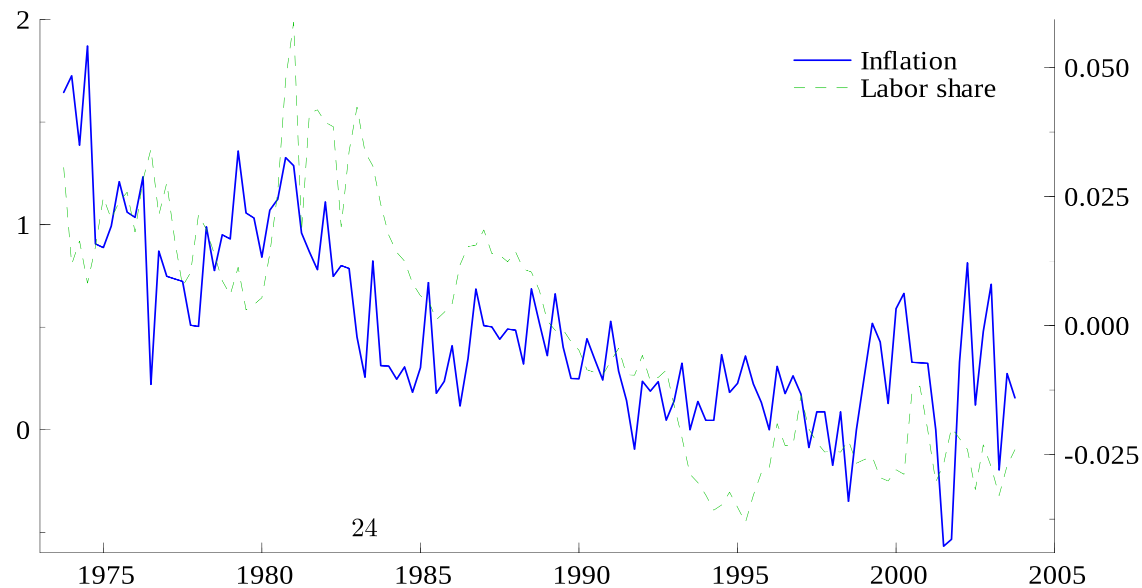

Chart 1: Inflation and labor share adjusted only for open economy considerations, 1973-2003.

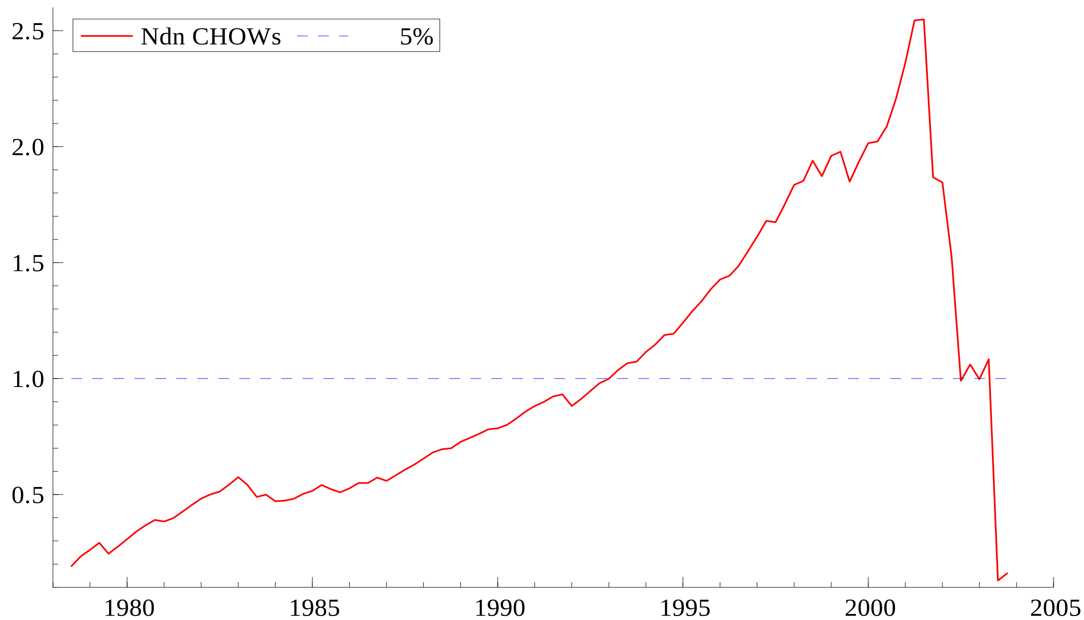

Chart 2: Sequence of the Chow test statistics for the AR inflation process, normalized by critical values.

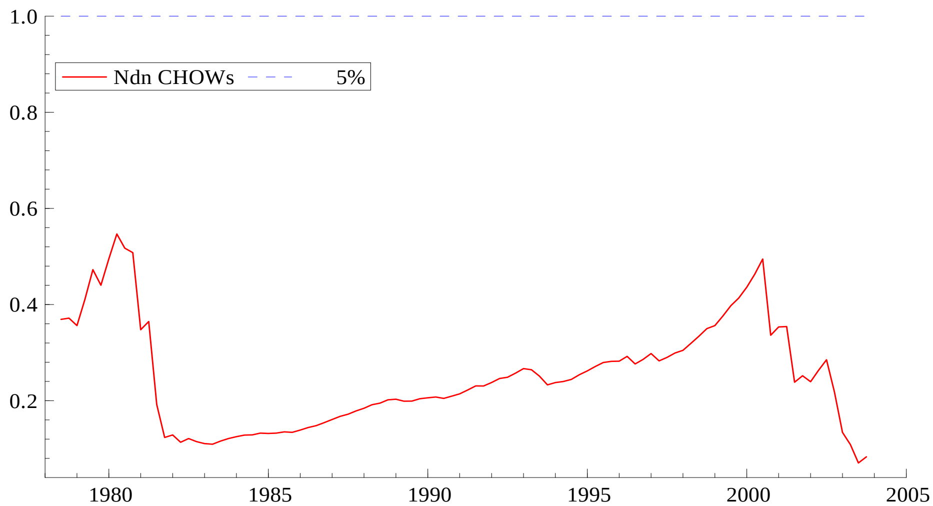

Chart 3: Sequence of the Chow test statistics for the AR labor share process, normalized by critical values.

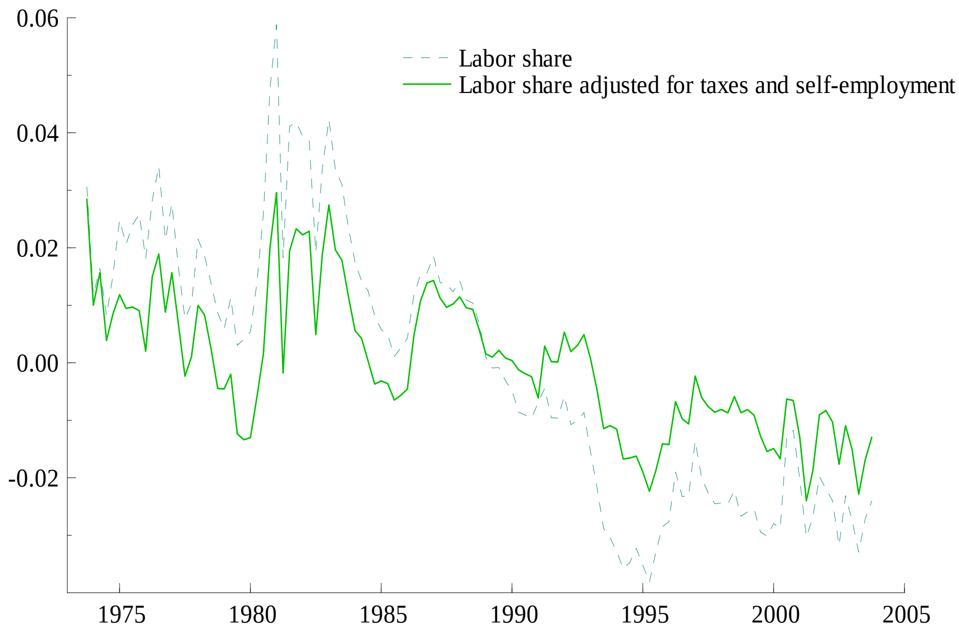

Chart 4: Labor share, adjusted and un-adjusted for net indirect taxes, self-employment and public sector considerations.