The New-Keynesian Phillips Curve When Inflation Is Non-Stationary: The Case of Canada*

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Recent related Literature

- There are several empirical papers that estimate hybrid open-economy NPCs for Canada using GMM or FIML.

Authors	Specification	Method	Fit
Gagnon and Khan (2004)	Open-economy NPC & gen. tech.	GMM	✓
Leith and Malley (2002)	Open-economy NPC	GMM	✓
Banerjee and Batini (2003)	Open-economy NPC $\neq$ contracts & gen. tech.	ML	✓
Guay, Luger and Zhu (2003)	Open-economy NPC	CUE	$\times$
Kozicki and Tinsley (2002)	Closed-economy NPC	IV	✓
Khan (2004)	Closed-economy NPC (rolling regression)	GMM	✓

Drawbacks of related literature

(1) *Non-validity of estimates when inflation is non-stationary*

- Existing estimates of the NPC for Canada assume that inflation is **stationary**. **If Canadian inflation is non-stationary, GMM and FIML approaches are not ideal.**...

- ...because the asymptotic distributions of the GMM and FIML estimators are *not necessarily Gaussian normal*, implying that the **estimated NPC's coefficients and standard errors are invalid**.

- **Pre-transforming the data** to make it stationary on *a priori* assumptions may give unrobust results.

Fuhrer, J. C. and G. R. Moore (1995), Inflation Persistence, *Quarterly Journal of Economics*, Vol. 110, Issue 1 (Feb.1995), pp. 133 and 135

"**At conventional significance levels, we cannot reject the hypotheses that the inflation rate and the interest rates series are integrated of order one.** The log of per capita output, on the other hand, appears to be trend stationary over the same period."

"The maximum eigenvalue and trace statistics in Table IV are consistent with the univariate Dickey-Fuller tests. We can reject the hypothesis that the VAR contains three unit roots in favor of two unit roots at the 1 percent significance level. But we can reject two unit roots in favor of one, and unit root in favor of zero unit roots, only at the 20 percent significance level."

"**While we cannot reject that the data contain one or two unit roots, we choose a stationary representation of the data...**"

(2) *Testing for model-consistent expectations*

- Work using **GMM** assume that the expectational error $(\pi_{t+1} - E_t\pi_{t+1})$ should be unforecastable by variables dated at time t or earlier.

- This is a **very weak property of RE** and with GMM boils down to choosing instruments for inflation expectations that are correlated with inflation but uncorrelated with the expectational errors.

- In estimation RE **are simply assumed** on the presumption that instrument orthogonality is indeed met.

- **Popular solution:** present value approach of Campbell and Shiller (1987) and cast the NPC in a system of equations.

(3) *Parameter identification*

- There is a problem of **identifying the parameters in the NPC**, and more specifically, to whether existing estimation methods can correctly distinguish between backward and forward-looking solutions (Mavroedis, 2002; and Ma, 2002).

- In the GMM framework, this **involves making assumptions on the process of the forcing variable**

- Nason and Smith (2005) examine this employing Anderson-Rubin (1949) exact analytic methods and find that the **NPC model is rejected** on Canadian data for different set of instruments when these methods are used.

Aim of this paper

The aim of this paper is to re-estimate the NPC on Canadian data **addressing these key issues simultaneously** within the Johansen and Swensen (1999) unified framework.

Advantages of this method

- **First**, it advances on other methods used in the literature on other countries data that model inflation as $I(1)$, but assume the cointegration specification a priori (e.g. Sbordone, 2002; Kozicki and Tinsley, 2002).

- **Second**, the method used here ensures that model-consistent expectations are tested and subsequently imposed in estimation.

- **Third**, this method gets rid of the identification problem raised by Ma (2002) and Mavroeidis (2002), since it does not require to make any ex ante assumption on the process of the forcing variable in order for the estimation method to identify the parameters and spares us the trouble of using exact analytical methods.

Theoretical Model

- Consider the "hybrid" version of the NPC is given in GGLS by

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t(\pi_{t+1}) + \lambda z_t \tag{1}$$

$$z_t = \delta_1 z_{t-1} + \delta_2 \pi_{t-1} + \eta_t \tag{2}$$

γ_b, γ_f and λ depend on "deep" or "structural" parameters, including the probability that firms reset prices at any given time, the discount factor, the fraction of rule-of-thumb firms that set their prices in a backward-looking, myopic

Theoretical Model (cont.)

Closed form solution indicates that inflation equals the discounted stream of expected future real marginal costs, taking into account the backward-looking behavior:

$$\pi_t = \delta_1 \pi_{t-1} + \frac{\lambda}{\delta_2 \gamma_f} \sum_{k=0}^{\infty} \left(\frac{1}{\delta_2} \right)^k E_t \left(z_{t+k} \right) \tag{3}$$

Estimation Method

- Johansen and Swensen (1999) is a Full Information Likelihood method.

- It exploits the idea that the mathematical expectation conditional on a theoretical model and the observed data can be used to substitute for the forward-looking term in an estimated model.

- Method has 3 steps

Estimation Method (cont.)

Step 1: Specify and estimate an unrestricted VAR, under the assumption that at least one variable has a unit root. Diagnostic tests to ensure that: (a) residuals are white noise and (b) the number of stationary relations in the system are determined.

Step 2: Estimate the parameters of the structural system via ML.

Step 3: Test the restrictions implied by the RE model (here the NPC). This implies re-parameterizing the cointegrated VAR model to account for any forward-looking term.

Data

- **Sample**: Canada from 1973:1 to 2003:4

- **Inflation** = log difference of the (officially seasonally adjusted) implicit price deflator of GDP at market prices

- **Marginal cost** = log deviation of the labor share from its sample mean.

We look at two measures of the share.

Data (cont.)

(1) unadjusted measure as in Gali and Gertler (1999) for the United States and Gagnon and Khan (2004) for Canada.

(2) adjusted measure (as suggested by Batini, Jackson and Nickell, 2005, and used on Canadian data by Guay and others, 2003)

- Both measures modified to allow for the **openness** of the Canadian economy as suggested in Batini, Jackson and Nickell (2005)

Results (Break in inflation?)

- Canadian inflation **seems to exhibit a break in early 1990s**—possibly in conjunction with the shift to inflation targeting in 1991—one question is whether we should do the analysis on the full sample or on split samples.

- Chow breakpoint test confirms a break in 1991 Q3, soon after Canada's shift to IT.

- Ravenna (2000) and Levin and Piger (2002) find similar shifts using a variety of methods.

- Testing for breaks complicated (see Stock, 2004). **We proceed both on full and split samples**.

Results (Stationarity)

- **For the full sample**, multivariate unit root tests using the trace test for cointegration, give **strong evidence of one stationary relation** and **one common trend in the system**

- **Results on the split sample are more mixed**. Specifically, for the first sub-sample, the multivariate unit root tests give strong indications of one stationary relation. However, in the second sub-sample we can reject no stationary vectors only at the 10 percent level.

Results (Estimates)

- Tables 1 and 2 show parameter estimates obtained maximizing the likelihood via BSFG numerical methods on full and split samples.

γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
$\mathbf{0.270}$	$\mathbf{0.729}$	$\mathbf{0.382}$	18.16	< 0.05

Table 1: Parameter estimates with BSFG, 1973-2003

Period	γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
1973-1990	$\mathbf{0.326}$	$\mathbf{0.714}$	$\mathbf{0.165}$	6.30	> 0.25
1991-2003	$\mathbf{0.269}$	$\mathbf{0.721}$	$\mathbf{0.415}$	5.52	> 0.25

Table 2: Parameter estimates with BSFG, 1973-1991 and 1992-2003

Results (Estimates) (cont.)

- Likelihood ratio tests indicate that on the split sample, **the NPC fits well Canadian data** using the unadjusted share. The fit, however, is less good for the full sample, as the $\mathcal{LR}$ test restrictions implied by the NPC can be rejected at the 5 percent significance level.

- Both on full and split samples, the **estimated weight on the lag of inflation**, γ_b, is **much lower than the estimated weight on the lead of inflation,** γ_f, in contrast with Gagnon and Khan, 2004, and Guay, Luger and Zhu (2003), but in line with Nason and Smith, 2002).

- Our estimates of the **coefficient on marginal cost are on the high side** of those in the existing literature.

Super-neutrality

When the model is estimated with the restriction

$$\gamma_b + \gamma_f = 1$$

the $\mathcal{LR}$ test statistic is 3.96 for the first sample and 3.32 for the second sample.

- Given the critical value for the $\mathcal{LR}$-test with 1 degree of freedom at the 5 percent significance level is 3.84, **the restriction cannot be rejected for the second sample**. This likely indicates that the Phillips curve in Canada is now vertical in the long run, with obvious implications for the objective of monetary policy.

What happens when we adjust the share for taxes, self-employment and public sector?

- We repeated the analysis using an adjusted measure of the share.

- The cointegration tests indicate one stationary relation on the full sample but none on the split samples.

- Over the first sample the parameters that maximize the maximum likelihood function are reasonable

- However, when we attempt to estimate the NPC parameters over the second sample, the BSFG algorithm does not converge for any reasonable parameter.

Period	γ_b	γ_f	λ	$\mathcal{LR}(5)$	p-value
1973-1990	$\mathbf{0.681}$	$\mathbf{0.318}$	$\mathbf{0.743}$	10.26	> 0.05
1991-2003	-	-	-	-	-

Table 3: Parameter estimates with BSFG, 1973-1990 and 1991-2003, labor share adjusted for taxes, public sector and self-employment

To sum up

- We have used a new method for estimating linear rational expectation models containing $I(1)$ variables to estimate the New-Keynesian Phillips curve on Canadian data (1973-2003).

- Our results strongly indicate the presence of a unit root in Canadian GDP price inflation rate over the full sample and give evidence of a unit root in inflation over the earlier period of the sample when we split the data into a pre and a post-inflation targeting period.

To sum up (cont.)

- We find that the NPC offers a good representation of inflation dynamics in Canada for some—but not all—measures of marginal cost.

- Accounting for open economy considerations seems particularly important for the fit.

- Contrary to much previous literature, estimates of the NPC based on this method and this assumption also support the super-neutrality result.

- In addition, estimation of the NPC in the Johansen and Swensen (1999) framework overcomes the problem of identification associated with GMM estimation.

To sum up (cont.)

- Often it is hard to know where the non-stationarity in data comes from. (structural break versus unit roots)

- For example, in the case of Canada here, we have some indication of the presence of a unit root, although we cannot totally exclude the possibility that the non-stationarity is due to a structural break

- The shortness of the sample can make it harder to discern between these two possible sources

- However, inference errors made assuming stationarity when the true process has a unit root is worse than the inference error made assuming a unit root when the process is actually stationary.

Identification

In Johansen and Swensen (1999), under the reduced rank assumption, the model is reparametrized in the form:

$$\Delta x_{t+1} = \varepsilon \eta' x_t + \Gamma \Delta x_t + u_t \tag{4}$$

where the estimated parameters, ε, η, and Γ, are uniquely determined. From (4)

$$\begin{aligned}
E_t(\pi_{t+1}) - \pi_t &= \varepsilon_1(\eta_1 \pi_t + \eta_2 z_t) \\
&\quad + \gamma_{11}(\pi_t - \pi_{t-1}) + \gamma_{12}(z_t - z_{t-1}) \\
E_t(z_{t+1}) - z_t &= \varepsilon_2(\eta_1 \pi_t + \eta_2 z_t) \\
&\quad + \gamma_{21}(\pi_t - \pi_{t-1}) + \gamma_{22}(z_t - z_{t-1})
\end{aligned}$$

Inserting the expression for $E_t(\pi_{t+1})$ in the NPC gives

$$\begin{aligned}
\pi_t &= \gamma_b \pi_{t-1} \\
&+\gamma_f \{\pi_t + \varepsilon_1(\eta_1 \pi_t + \eta_2 z_t) + \gamma_{11}(\pi_t - \pi_{t-1}) + \gamma_{12}(z_t - z_{t-1})\} \\
&+\lambda z_t
\end{aligned}$$

so that **the parameters in the NPC must be uniquely determined** by

$$\begin{aligned}
\gamma_f &= \frac{1}{1+\varepsilon_1\eta_1+\gamma_{11}} \\
\lambda &= -\gamma_f(\varepsilon_1\eta_2+\gamma_{12}) \\
\gamma_b &= \gamma_f\gamma_{11}
\end{aligned}$$

Moreover, it must hold that

$$E_t(z_{t+1}) = \delta_1 z_t + \delta_2 \pi_t$$

so that

$$
\begin{aligned}
\delta_1 &= \varepsilon_2 \eta_1 \\
\delta_1 &= \mathbf{1} + \varepsilon_2 \eta_2
\end{aligned}
$$

Maximum likelihood test

Part of the maximized likelihood function from the conditional model

$$L_{1.2\,\max}^{-2/T} = \frac{|S_{11}|}{|a^{\prime}a|} \tag{5}$$

S_{11} : sum of squared residuals in regression of Δz_t on $\gamma_f \Delta\pi_t - \gamma_b \Delta\pi_{t-1}$, $\left(1-\gamma_f-\gamma_b\right)\pi_{t-1} - \lambda z_{t-1}$, ΔX_{t-1} and the constant term.

Part of the maximized likelihood function from the marginal model

$$L_{2\,\max}^{-2/T} = \frac{\left|\tilde{\Sigma}_{22}\right|}{|b^{\prime}b|} \tag{6}$$

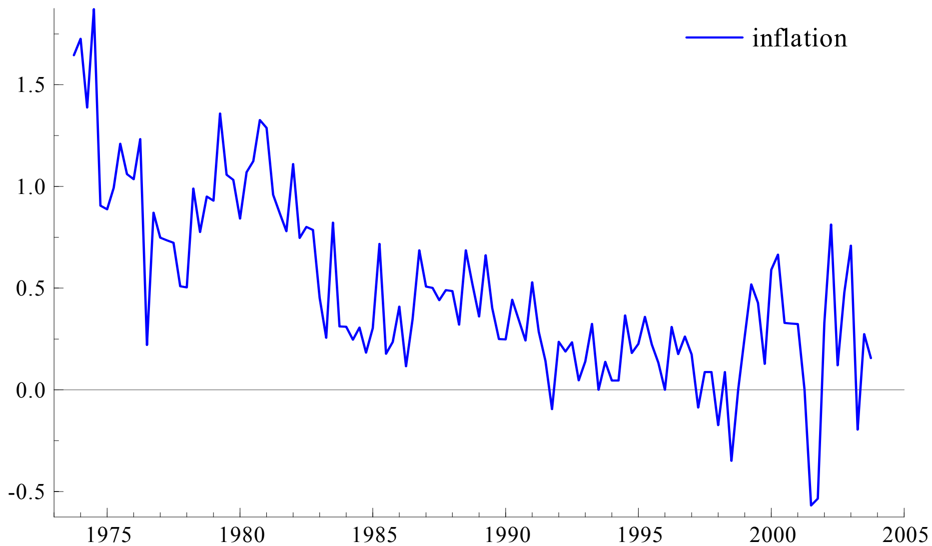
$\tilde{\Sigma}_{22}$: sum of squared residuals in marginal model, $\gamma_f \Delta\pi_t = \left(1-\gamma_f-\gamma_b\right)\pi_{t-1} - \lambda z_{t-1} + \gamma_b \Delta\pi_{t-1}$

Maximum value of the likelihood function, under the NPC hypothesis:

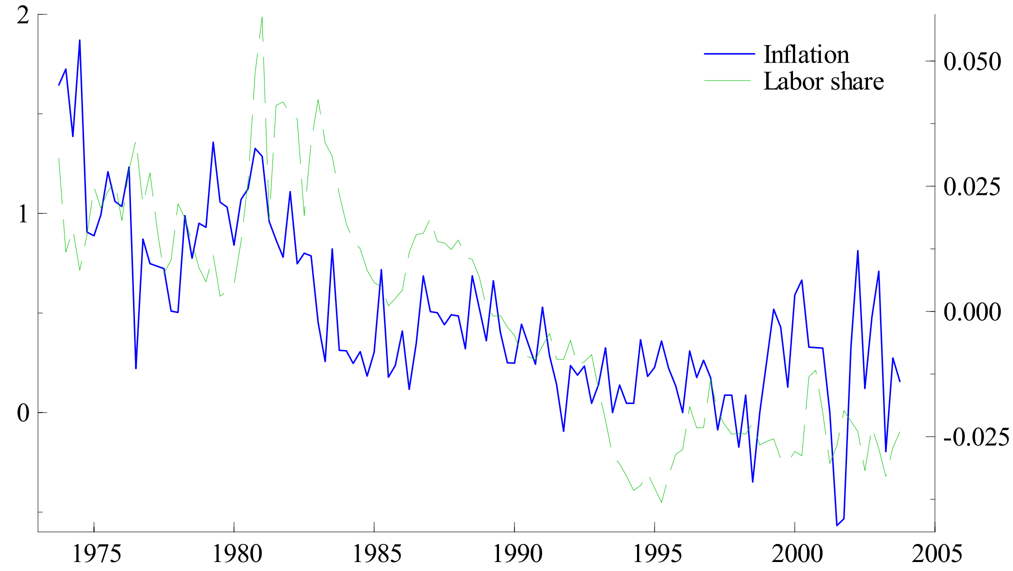
$$L_{H\,\mathsf{max}}^{-2/T} = \frac{\left|\hat{S}_{11}\right|}{|a^{\shortmid}a|}\frac{\left|\tilde{\Sigma}_{22}\right|}{|b^{\shortmid}b|} \tag{7}$$

- Step 2: Use BSFG alogrithm to find the parameters that maximize $L_{H\,\mathsf{max}}^{-2/T}$

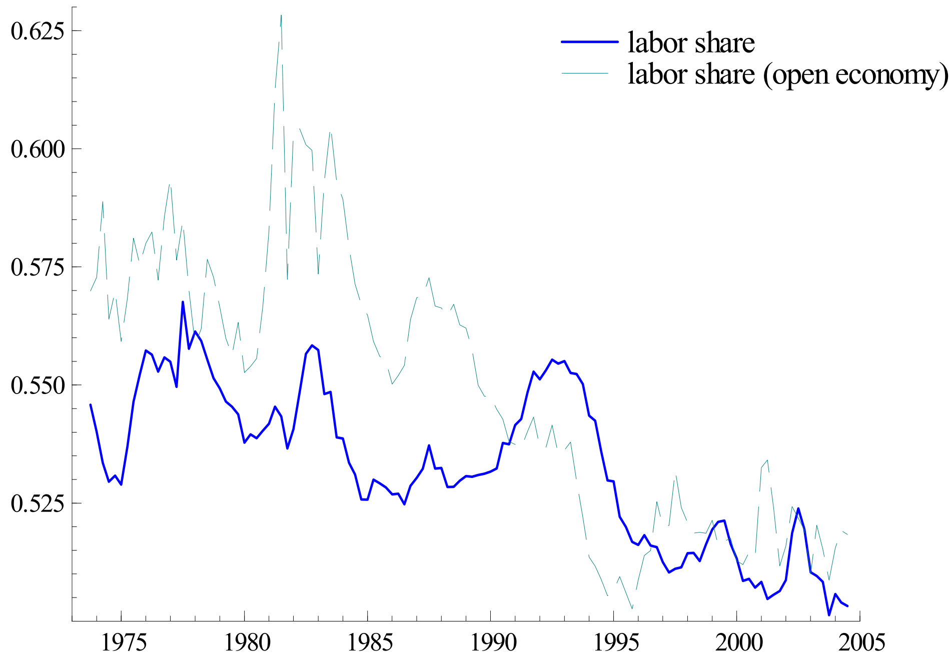
- Step 3: If the parameters obtained in Step 2 are reasonable, compare maximized $L_{H\,\mathsf{max}}^{-2/T}$ with the likelihood of the unrestricted model to test if the restrictions implied by the NPC are satisfied



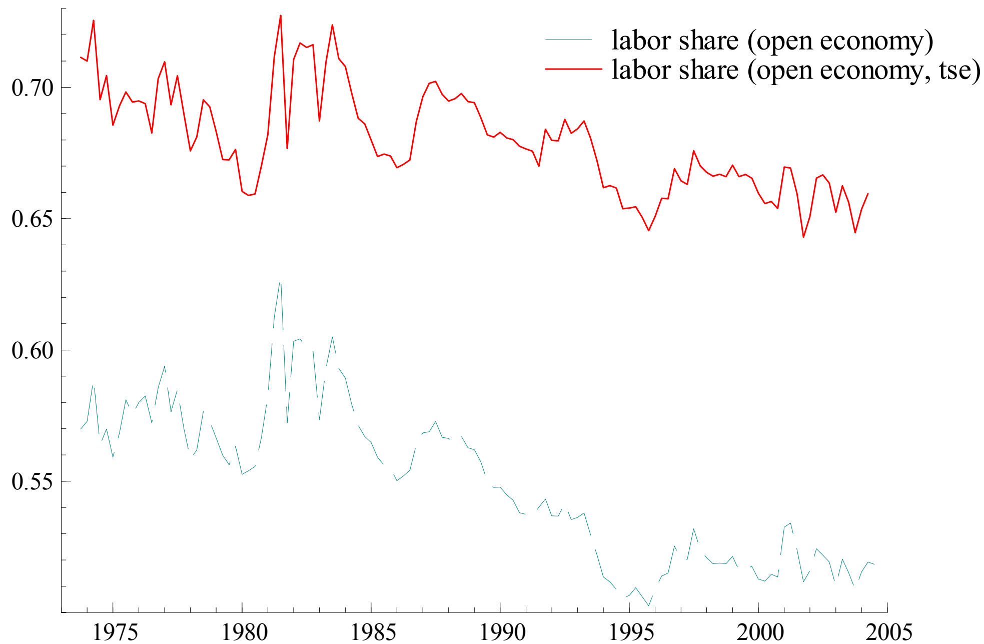
GDP inflation (log difference), 1973-2003



Inflation and demeaned labor share (adjusted for open economy), logarithms, 1973-2003



Labor shares, with and without adjustment for open economy, 1973-2003



Labor shares, with and without adjustment for self-employment and taxes, 1973-2003