Commodity Currencies, Global Imbalances and Multilateral Adjustment

Jeannine Bailliu
Ali Dib
Takashi Kano
Lawrence Schembri

International Department
Bank of Canada

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Abstract

Rapid and significant appreciations of floating exchange rates, such as those experienced by the Australian, Canadian and New Zealand dollars in recent years, pose a number of challenges for central banks in formulating the optimal monetary policy response, if any. In particular, how the central bank should react critically depends on the underlying forces behind the appreciation. In this paper, we examine the recent exchange rate behavior of three countries – Australia, Canada and New Zealand – in an attempt to identify separately the magnitude of the impact of two factors that may explain their appreciations: the recent increase in commodity prices and the possible multilateral adjustment of these currencies to the large U.S. current account deficit. Although our findings should be viewed as preliminary at this stage, we do find evidence to suggest that during periods of large U.S. imbalances, fiscal and external, a bilateral exchange rate model for the ACNZ dollars should allow for multilateral adjustment effects. Moreover, the results for both the threshold and Markov switching models suggest that the adjustment of exchange rates to multilateral factors are best modelled as a non-linear process.

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1. Introduction

Since the beginning of 2001, the currencies of a number of small and industrialised open economies have appreciated significantly, especially relative to the U.S. dollar. For example, the Australian, Canadian and New Zealand dollars have appreciated by about 30, 25, and 55 per cent, respectively, with most of the appreciation occurring in 2003.[1] These rapid appreciations posed a challenge for monetary policy in these economies because they implied a dramatic change in the relative price of domestic goods and thus had a substantial impact on aggregate demand. In addition, these appreciations put downward pressure on inflation, to varying degrees depending on the extent of exchange rate pass-through.

Understanding the causes of these appreciations—and exchange rate movements, more generally—is, therefore, critical for determining the appropriate monetary policy response. This is particularly true for central banks that target inflation based on the expected future path of the output gap and various measures of core inflation. In theory, the monetary policy response to a sustained exchange rate movement would be muted if this relative price movement were driven largely by real fundamentals—that is, relative shifts in the demand for and supply of domestically-produced goods and services—because the exchange rate would be adjusting to stabilize output close to potential. In other circumstances, some monetary accommodation might be useful to offset the impact

[1] In 2003, the Canadian dollar appreciated by almost 25 per cent in less than twelve months; it was the most rapid increase in the currency's history.

of the exchange rate movement on the real economy or to facilitate the reallocation of resources between the traded and non-traded sectors.[2]

The purpose of this paper is to examine the recent exchange rate behavior of a subset of these open economies, Australia, Canada and New Zealand – often referred to as "commodity currencies" because of the importance of commodities in their exports and in the determination of their exchange rates.[3] This comparative analysis is useful because, as already noted, their currencies experienced appreciations of a similar order of magnitude and thus by studying them we may be able to discern the relative impact of a similar set of underlying fundamentals. In particular, we wish to identify separately the magnitude of the impact of two factors that may explain their appreciations: the recent increase in commodity prices and the possible multilateral adjustment of these currencies to the large U.S. current account deficit. Indeed, the focus of this paper will be on the issue of multilateral adjustment because cursory evidence indicates that the recent increase in commodity prices cannot fully explain the recent appreciations and that factors reflecting multilateral adjustment to U.S. imbalances may be an important—yet often overlooked—determinant of these commodity currencies.

This paper builds on earlier work by Djoudad, Murray, Chan and Daw (2001) and Chen and Rogoff (2003), which examined the impact of commodity prices on these three currencies.[4] It also extends recent work by Bailliu, Dib and Schembri (2004), which

[2] For a discussion of the monetary policy response to exchange rate movements in the Canadian context, see Dodge (2005) and Ragan (2005).
[3] Norway was initially also considered, but had to be dropped from our sample because the span of the data was too short for our purposes.
[4] Djoudad, Murray, Chan and Daw (2001) find that commodity prices play an economically and statistically significant role in explaining the behaviour of all three currencies. Chen and Rogoff (2003), on the other hand, find that although commodity prices were statistically significant in explaining exchange rate movements for Australia and New Zealand, they were not significant for Canada. Differences in the

estimated a nonlinear threshold model for the Canadian dollar and found evidence that factors related to multilateral adjustment are an important determinant of movements in this currency in periods where U.S. imbalances are significant. The exchange rate equations used in the studies by Djoudad *et al* (2001) and Bailliu *et al* (2004) are based on a terms of trade model for the bilateral exchange rate that was initially developed for Canada by Amano and van Norden (1993).[5] This error-correction model is built around a long-run relationship between the real exchange rate, real energy commodity prices and real non-energy commodity prices; short-run dynamics are captured by an interest rate differential. For most of the post-Bretton Woods period, it has been fairly successful at explaining broad movements in the Canadian dollar. There are episodes, however, when the equation has fared poorly. Notably, it failed to explain the large and rapid appreciation of the Canadian dollar starting in 2003—despite the concomitant rise in commodity prices—because there appeared to be other factors driving the exchange rate that were not included in the simple terms of trade model.

Multilateral adjustment is one possible candidate. Because the U.S. economy occupies a predominant position in the world economy, when it incurs—for example—a current account deficit that is viewed as potentially unsustainable at prevailing exchange rate levels, then all countries will see the value of their currencies appreciate relative to the U.S. dollar in order to facilitate global adjustment to these U.S. imbalances.[6] For instance, in 2003, the United States was running a current account deficit of roughly 5 per

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methodology used most likely explain the mixed results for Canada. Notably, Chen and Rogoff (2003) detrend the unit root variables in their equations.

[5] Researchers at the Bank of Canada have continued to work on improving the Amano-van Norden equation since it was first developed over ten years ago. Recent work has focused on the role of energy prices in the determination of the Canadian dollar (Issa, La france, and Murray (2005)) and on differences in Canadian and U.S. rates of productivity growth (Helliwell *et al*. (2005)).

[6] Obstfeld and Rogoff (2005) and Blanchard, Giavassi and Sa (2005) predict sizable depreciations of the U.S. dollar (i.e., 30 per cent or more).

cent of gross domestic product (GDP) that many observers felt was unsustainable at existing exchange rate levels. Consequently, all major currencies began to appreciate relative to the U.S. dollar. Table 1 shows that the rapid appreciation of the Australian, Canadian, and New Zealand (ACNZ) dollars compared to that experienced by other currencies.

Nominal Appreciation from Jan 2, 2003 vs. US$ (percentage)

	To 1 October 2003	To September 30 2005 (Can$ Peak)	To 18 November 2005
Canadian Dollar (Can$)	16.82	35.62	32.31
Euro	12.96	16.37	13.31
British Pound	4.30	10.74	7.32
Japanese Yen	8.34	5.80	0.54
Australian Dollar	21.31	35.67	29.71
New Zealand Dollar	14.42	32.61	31.33

Source: Daily recorded values at 12:00 p.m. E.S.T. by the Bank of Canada.

As discussed earlier, Bailliu *et al* (2004) examined the importance of these multilateral effects for the Canadian dollar by extending a simple terms of trade model to incorporate multilateral adjustment and to allow the specification of the empirical exchange rate model to change depending on the magnitude of U.S. imbalances. The key finding of this study is that the specification changes in periods when the United States is running a substantial fiscal deficit (i.e., more than 2.6 per cent of GDP). The result is intuitively appealing, because during episodes in the post-Bretton Woods period when the U.S. fiscal deficit was large, especially on a cyclically adjusted basis, the United States often had a substantial current account deficit. Yet the reverse was, in general, less often true, because current account deficits also occurred during investment booms when there was a fiscal surplus. Thus, current account deficits driven by fiscal deficits were seen as less sustainable thus warranting a substantial multilateral exchange rate adjustment.

This paper extends the work of Bailliu *et al* (2004) in two important dimensions. First, we study the importance of multilateral adjustment factors in the determination of the Australian and New Zealand dollars, in addition to the Canadian dollar. As already noted, this comparative analysis is useful because these currencies experienced similar appreciations in recent years and by studying this common episode, we may be able to discern the relative impact of a similar set of underlying fundamentals—notably commodity prices and multilateral adjustment factors. Second, we adopt two different estimation methodologies that allow the specification of the empirical model for the currencies in question to change when U.S. imbalances are significant. More specifically, the non-linear nature of this process is modeled by estimating both a threshold model and a Markov switching model. This enables us to check the consistency of our estimates across two econometric models that both treat the multilateral effects as threshold effects, but that model this process in different ways. Both models are suitable for this purpose, but rely on different assumptions, and as result, have different advantages in this context. Although our findings should be viewed as preliminary at this stage, we do find evidence to suggest that during periods of large U.S. imbalances, fiscal and external, an exchange rate model for the ACNZ dollars should allow for multilateral adjustment effects. Moreover, the results for both the threshold and Markov switching models suggest that the adjustment of exchange rates to multilateral factors are best modelled as a non-linear process.

The paper is organized into six sections. The next section examines large U.S. external imbalances in the post-Bretton Woods period and their implications for the adjustment of exchange rates, including the ACNZ dollars. Section 3provides the theoretical motivation for the inclusion of factors related to multilateral adjustment in a bilateral exchange rate equation as well as their treatment as threshold effects. The empirical framework and data required are explained in section 4, which is followed in section 5 by a description of the estimation procedure and a presentation and interpretation of the empirical results. Concluding remarks are made in the final section.

2. U.S. Imbalances in the Postwar Period and the ACNZ Dollars

Since 1960, the United States has run current account deficits on several occasions (as shown in Figure A1.1). And while the first two episodes in the 1970s (i.e., 1971-2 and 1977-80) were relatively small and short in duration, the two most recent episodes (i.e., 1984-89 and the ongoing episode, which started in 1992) have been much larger (exceeding 2 per cent of GDP for most of the period) and much more persistent. Indeed, over the four episodes, the size and persistence of the deficits have been increasing monotonically; the current deficit has lasted for over 13 years, with no clear signs of abating, and is now in excess of 6 per cent of GDP. It is worth noting that the increasing size and persistence of U.S. current account deficits is consistent with the Greenspan view that the increasing globalization of financial markets has made it easier for countries to finance external deficits. For the purpose of this paper, however, we wish to understand the implications of the increasing magnitude and persistence of U.S. current account deficits for the behavior of the exchange rates of the N-1 countries. In particular, it would be useful to determine the extent to which the value of these currencies is driven by the need for multilateral adjustment to these U.S. imbalances.

Figure A.1.1 displays the U.S. current account and the exchange rates of the ACNZ dollars relative to the U.S. dollar. There appears to be a strong positive, albeit slightly out-of-phase, correlation among these currencies and the U.S. current account deficit during the latter two episodes when the deficits were the largest. In particular, the figure shows that during the period of the U.S. current account deficit of the mid-1980s, all of these exchange rates depreciated slightly before the U.S. current account deficit increased and then appreciated, again slightly before, the decline of the U.S. current account deficit. During the most recent episode, these currencies again depreciated just before the U.S. deficit increased and since 2003 they have all appreciated. Thus, if the experience of the 1980s is to be repeated in this case, we should expect the U.S. current account to begin to close shortly. From Figure A.1.2, we see that this out-of-phase correlation between the ACNZ dollars and the U.S. current account deficit is not atypical because it is also true for the U.S. effective exchange rate expressed as the U.S.-dollar price of a trade-weighted basket of currencies.[7] Furthermore, this finding would also be true if the real effective exchange rate were considered because inflation rates between the United States and its major trading partners have not diverged greatly in recent years.

Hence, the stylized evidence seems to indicate that when U.S. external imbalances are large, multilateral exchange rate adjustment to these imbalances is underlying the movement of bilateral exchange rates. Given that the United States is the predominant economy, representing approximately 30 per cent of world GDP, it is reasonable to believe that when the United States runs a large external deficit, the relative value of the

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[7] While the currencies of most of the major trading partners of the United States have appreciated over the 2002-2005 period by a relatively large amount, two exceptions stand out: China and Mexico. China's exchange rate has remained almost fixed vis-à-vis the U.S. dollar over this period, although there was a small 2.1% revaluation in July 2005.

currencies of all other countries must depreciate to facilitate the adjustment of the world economy to this imbalance. Indeed, if we compare the simulated values of the nominal exchange rates for the ACNZ currencies based on the traditional *bilateral* specification of the Bank of Canada's exchange rate equation[8]—which is based on commodity prices and interest rate differentials—to the actual values of the exchange rates (shown in Figures A1.3a—c), we see that the model cannot explain adequately the slope and magnitude of the appreciation of these currencies since 2002. It is perhaps not surprising that the exchange rate equation fits the Canadian dollar the closest, since it was originally designed for this purpose; nonetheless it does not closely capture the timing and the extent of the appreciation. The deviations are much greater for the Australian and New Zealand exchange rates.

Figures A1.4a—b display the real energy and non-energy commodity prices that we will later use in modeling the ACNZ dollars. The differences across countries are due to the different weights used to construct the indices.[9] Both energy and non-energy commodity prices increased significantly over the period of the recent appreciation. In most cases, the increases were in the neighbourhood of 25-50%, but the clear outlier was the real energy price for Canada which increased by over 200%. It is clear that these increasing commodity prices undoubtedly are part of the story behind the appreciations of the ACNZ dollars over this period because these countries are large net exporters of these products. The outstanding question is how much of the appreciations of these currencies were to due to the concomitant increases of commodity prices and how much due to possible multilateral adjustment.

[8] This equation is discussed in more detail in Section 4.
[9] It should be noted that production weights are used for Canada whereas trade weights are used for Australia and New Zealand.

To examine these U.S. external imbalances further, it is useful to employ the national income accounting identity; this identity implies that current account imbalances occur when there are fiscal deficits or a deficit of private savings relative to domestic investment. Figure A1.5 plots the U.S. current account deficit and the U.S. federal government fiscal deficit. Also shown is the difference between private savings and investment, which is calculated as the residual. Figure A1.5 shows that over the periods 1984-88, and 2002-04, the large U.S. external imbalances coincided with large fiscal deficits. When this simultaneous occurrence was observed in the 1980s, it was labeled the "twin-deficits" phenomenon, and the argument was made that the significant reductions in taxes and the concomitant increase in military spending during the Reagan administration caused both the fiscal and current accounts deficits over this period. While there was much public debate over this causal argument at the time, the standard non-Ricardian, open-economy model would predict that a temporary increase in domestic (government) spending would increase the current account deficit. This increase in demand would fall partly on traded goods, but largely on non-traded goods and services. Thus, resources would be shifted out of the traded goods sector to meet this demand. As a result, there would be less domestic supply of traded goods to satisfy the increase in demand, and a current account deficit would ensue. Domestic interest rates would rise and foreign borrowing would be used to finance the higher level of absorption.

The second period of significant external imbalance (i.e., more than 2 per cent of GDP), from 1998 to the present, is different from the earlier period in the 1980s, because the current account deficit emerged several years before the fiscal deficits of 2002-04; indeed, the US current account deficit begins when there is a large fiscal surplus. The critical difference is that over this period (1998-2001), the current account deficit is caused by an investment boom and relatively low domestic savings. As a result of this investment-savings gap, foreign capital flowed into the United States in expectation of higher returns owing to the rapid increases in productivity, which were anticipated to continue for the foreseeable future. This expectation of higher productivity growth also increased domestic consumption and reduced savings as U.S. residents intertemporally shifted higher expected future outputs and incomes to the present.

It is also noteworthy that in the three recessionary periods (i.e., 1974-75, 1981-82, and 1991-92), there was a slowdown in economic activity, and consequently, the fiscal position went into deficit because of lower tax revenues and increased transfers, and the current account deficit declined as imports fell. In these situations, higher fiscal deficits did not coincide with current account deficits, because aggregate investment fell below savings as economic prospects turned negative. To measure the underlying degree of fiscal stimulus, it is sometimes useful to adjust the fiscal position for effects of the business cycle. In Figure A1.6, the cyclically adjusted fiscal position is included in addition to the fiscal and current account positions. This figure shows clearly that a large part of the fiscal deficits observed during these episodes were indeed cyclical.

In summary, we have tried to demonstrate in this section that multilateral exchange rate adjustment to large U.S. external imbalances may have played a substantial role in explaining bilateral exchange rate movements between the United States and its trading partners. Moreover, this multilateral exchange rate adjustment is more likely to occur when these imbalances are caused in part by fiscal imbalances, rather than investment, because it is less likely that these imbalances can be sustained. Although the focus of this analysis was on the possible impact of multilateral adjustment on the value of the ACNZ currencies, the findings are generally true for other currencies as well. Thus, incorporating multilateral adjustment to U.S. external imbalances into empirical exchange rate models may improve their explanatory power relative to traditional models based on differences in bilateral macroeconomic variables.

3. Theoretical Considerations

The discussion in the previous section suggests that there are periods in which movements in the ACNZ dollars that are not well explained by domestic fundamentals may be accounted for by factors related to the multilateral adjustment to large U.S. external imbalances, in turn caused partly by fiscal imbalances. Therefore, an analysis of our commodity currencies based on a bilateral exchange rate equation that does not account for these multilateral effects stemming from the United States may suffer from omitted variable bias. Furthermore, it also suggests that these effects should be modeled as threshold effects given that they likely only emerge as an important determinant of the ACNZ dollars in periods where there are significant imbalances. The determinants of our commodity currencies might thus be better modeled in the context of a model with two regimes: one in which domestic fundamentals are the main drivers of the bilateral exchange rate and a second in which factors related to the multilateral adjustment to large U.S. imbalances kick in to become an additional important determinant. This section presents the theoretical motivation for the inclusion of factors related to multilateral adjustment in a bilateral exchange rate equation as well as their treatment as threshold effects.

Empirical exchange rate models that incorporate multilateral effects—such as the ones used in this paper—view exchange rates as being interdependent. In this type of framework, a bilateral exchange rate can thus be driven either by domestic fundamentals or by fundamentals that are influencing another currency (i.e., multilateral effects).[10] This is in contrast to the bulk of the empirical literature on the macroeconomic determinants of exchange rate which emphasizes the importance of intercountry differences in fundamentals. There are several potential explanations for multilateral effects. For instance, news about the fundamentals of another currency may reveal information about the domestic economy. Consider the following example. News about U.S. fundamentals may reveal information about the Canadian economy. And given the economic significance of the United States, news about U.S. fundamentals may provide information regarding the direction of the world economy, both of which should influence the Canadian dollar. Multilateral effects could also arise due to trade and financial-market linkages across countries. In other words, given that the real and financial sectors of the world economies are linked, so will their respective exchange rates. This is the basic idea behind models of exchange rate determination based on a multi-country framework, such as those developed by the International Monetary Fund (IMF).[11]

In our paper, we focus on an alternative potential explanation that relies on the assumption of informational heterogeneity. We thus motivate the presence of multilateral factors in our bilateral exchange rate equations by drawing on exchange rate models that are based on informational heterogeneity, such as the "scapegoat" model of Bacchetta

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[10] For more on exchange rate models that incorporate multilateral effects, see Bailliu, Dib and Schembri (2004) and the references therein.
[11] The IMF's work on estimating equilibrium exchange rates using a multi-country approach is outlined in Isard and Faruqee (1998) and Faruqee, Isard and Masson (1999).

and van Wincoop (2004). This framework was inspired by survey data that suggests that foreign exchange traders change the weight they attach to different macroeconomic indicators over time. Based on this, a model is developed where foreign exchange traders give a certain fundamental excessive weight during a period of time. The basic mechanism they rely on in the model is confusion in the market as to the true source of exchange rate fluctuations because agents have heterogeneous information. As the market rationally searches for an explanation, it may attribute these fluctuations to some observed macroeconomic indicator; this indicator then becomes the scapegoat and influences trading strategies.

This type of model also provides a theoretical rationale for treating multilateral effects as threshold effects. It can be used to justify why variables such as the U.S. current and fiscal account balances might only drive the U.S. dollar (and hence initiate a multilateral adjustment) once they hit a certain threshold level and become scapegoat variables. Indeed, the scapegoat model was itself motivated by the important role that foreign exchange traders appear to have given to U.S. imbalances in driving the U.S. dollar in recent years.

4. Empirical Framework

4.1 A Terms of Trade Model of the Bilateral Exchange Rate for Commodity Currencies with Multilateral Adjustment Effects

As discussed earlier, we use a terms of trade model for the bilateral exchange rates of Australia, Canada, and New Zealand that is based on an equation that was initially developed for Canada by Amano and van Norden (1993) as a starting point for our analysis. This single-equation error-correction model is built around a long-run relationship between the real exchange rate, real energy commodity prices and real non-energy commodity prices.[12] Short-run dynamics are captured by an interest rate differential. Although parsimonious, this equation has been relatively successful at tracking most of the major movements in the Canadian dollar over the past few decades, has proven to be stable over time and has outperformed the random walk in out-of-sample forecasting exercises.[13]

For convenience, we focus on bilateral exchange rates in our paper. This is justified by the fact that the bilateral and effective series for our three countries are highly correlated (as shown in Figures A1.7a—A1.7c). This is hardly surprising for a country like Canada, where roughly 87% of exports go to the United States. This is a little more surprising for Australia and New Zealand, where the U.S. share of exports—at 9% and 15%, respectively—is much smaller.[14] As pointed out by Djoudad *et al.* (2000), this could be due to the number of their trading partners in Asia that peg their currencies to the U.S. dollar, either explicitly or implicitly.

The specifications for the terms of trade model for the Australian, Canadian, and New Zealand dollars, respectively, are as follows:

$$\begin{aligned}\Delta \ln(rfx_t^{a}) = \mathbf{a}^{a}\,(\ln(rfx^{a}{}_{t-1}) - \mathbf{b}^{a} - f^{a}\ln(comtot^{a}{}_{t-1}) - \mathbf{p}^{a}\ln(enetot^{a}{}_{t-1}))\\ + \mathbf{d}^{a}\,R\mathrm{int}\ dif^{\,a}{}_{t-1} + \mathbf{e}_t\end{aligned} \tag{1a}$$

[12] Under certain circumstances, a single-equation approach—as opposed to estimating the entire vector error-correction model—can be justified. Indeed, as discussed by Johansen (1992), estimation and inference based on the single-equation system will be equivalent to that of the full system if there is only one cointegrating vector and all the other cointegrating variables are weakly exogenous with respect to the first variable under consideration (in this case, the real exchange rate). As shown in Tables A3.2a--A3.2c and Tables A3.3a--A3.3c in Appendix 3, cointegration and weak exogeneity tests generally support this approach for our sample countries (the weak exogeneity tests suggest a potential problem with the Australian data – this issue needs to be explored further).

[13] For more information on this equation and its performance over time, see Murray *et al*. (2000).

[14] These figures are averages for the 2000-2004 period.

$$\Delta \ln(rfx_t^{\,b}) = a^b (\ln(rfx^b_{t-1}) - b^b - f^b \ln(comtot^b_{t-1}) - p^b \ln(enetot^b_{t-1})) \\ + d^b \operatorname{int} dif^{\,b}_{t-1} + h\, \Delta debtdif_{t-1} + e_t \qquad (1b)$$

$$\Delta \ln(rfx_t^{\,c}) = a^c (\ln(rfx^c_{t-1}) - b^c - f^c \ln(comtot^c_{t-1}) + d^c\, R \operatorname{int} dif^{\,c}_{t-1} + e_t \qquad (1c)$$

where rfx^a, rfx^b, *and* rfx^c are the real dollar exchange rates for the Australian, Canadian and New Zealand dollars, respectively[15]; $comtot^a$, $comtot^b$, $comtot^c$ are the real non-energy price indices for Australia, Canada, and New Zealand[16]; $enetot^a$, $enetot^b$ are the real energy price indices for Australia and Canada; $Rintdif^a$, $Rintdif^b$ are the real short-term interest rate differentials with the U.S. for Australia and New Zealand; *intdif* is the Canada-U.S. short-term interest rate differential; and *Ddebtdif* is the first difference of the Canada-U.S. relative public sector debt. Appendix 2 provides more details on the data.

Unit root tests were conducted on all the series in equations (1a)--(1c) using the DF-GLS test developed by Elliot, Rothenberg and Stock (1996). The results, as well as a description of this test, are provided in Tables A3.1a--A3.1c in Appendix 3. They suggest (for all three countries) that *rfx*, *comtot*, and *enetot* are non-stationary, as assumed.[17] Initial results (which are reported) for the interest rate and debt differential variables were mixed, and suggested in some cases that these variables may be non-nonstationary. By increasing the sample size (when possible), we were able to find support for our priors that these variables should indeed be stationary.[18]

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[15] In each case, the nominal exchange rate (which is expressed in local currency units) is deflated by the ratio of the GDP deflators for the two countries.

[16] The energy and non-energy price indices are each deflated by the U.S. GDP deflator to convert them into real terms.

[17] Johansen cointegration test results, shown in Table A3.2a--A3.2c, support the presence of one cointegrating vector between the real exchange rate, real non-energy commodity prices, and real energy commodity prices for Australia and Canada, and the presence of one cointegrating vector between the real exchange rate and real non-energy commodity prices for New Zealand.

[18] The only variables that are still problematic are the interest rate differential for New Zealand and the debt differential term for Canada. This issue needs to be explored further for these two variables.

There are some differences in the equations across the three countries. First, the terms of trade variables (i.e., *comtot* and *enetot*) are constructed using different weights for the three countries, to reflect the different basket of commodities that each country produces and imports. Second, the energy price index is not included as a variable in the equation for New Zealand.[19] Third, we use the nominal interest rate differential as an explanatory variable for Canada but the real interest rate differential for Australia and New Zealand. Finally, we include a debt differential term for Canada. This variable was not in the original Amano-van Norden equation, but was included in later versions following work suggesting that it added some explanatory power.[20] We had originally excluded it from the equation for Canada, for the sake of consistency across the three countries. In addition, this omission was justifiable given that the Canada-U.S. debt differential is not believed to be one of the key determinants of the Canadian dollar. However, we decided to include it as it proved to be an important variable to help identify the two regimes in both the threshold and Markov-switching models.

Next, we modify this basic framework by adding some variables that reflect multilateral exchange rate effects stemming from the United States. As discussed in Section 2, the two key variables that reflect U.S. imbalances and that are likely to instigate a multilateral adjustment of the U.S. dollar are the U.S. fiscal and current account balances. Thus, we consider these two variables. In addition, we construct a third variable to capture the phenomenon of twin deficits; this variable is simply the average of the U.S. fiscal and current account deficits.

[19] The model with energy prices for New Zealand performed poorly so we decided to exclude this variable from the analysis for New Zealand only.
[20] For example, see Djoudad and Tessier (2000).

Unit root tests were also conducted on these two variables and are reported in Table A3.1a. As shown, the DF-GLS unit root test suggests that both the fiscal balance to GDP ratio and the current account to GDP ratio contain a unit root. By increasing the time span used in the tests, we found evidence that the fiscal balance to GDP ratio follows a stationary process but that the current account ratio does not. The latter is contrary to what one would expect and suggests that the intertemporal budget constraint is violated and that the current account in on an explosive path. Christopoulos and Leon-Ledesma (2004) also find that traditional unit root tests for the U.S. current account to GDP ratio suggest that the series is non-stationary, even when the sample is extended back to 1960. However, they argue that these tests suffer from an important loss of power if the dynamics of the series being tested exhibit non-linearities, which they show is the case for the U.S current account. They address this issue by analyzing the stationarity of the US current account using new econometric tests based on a non-linear adjustment, and find evidence that the U.S. current account to GDP ratio is stationary when this non-linearity is taken into account. Given these results and our priors based on theoretical considerations, we decide to treat the U.S. current account to GDP ratio as a stationary variable in our analysis.

By making these modifications, we obtain the following specifications for the terms of trade model with multilateral effects for our three commodity currencies:

$$\begin{aligned} \Delta \ln(rfx_t^{\,a}) = {} & a^{a}\,(\ln(rfx^{a}{}_{t-1}) - b^{a} - f^{a}\ln(comtot^{a}{}_{t-1}) - p^{a}\ln(enetot^{a}{}_{t-1})) \\ & + d^{a}\,R\mathrm{int}\,dif^{a}{}_{t-1} + c^{a}\,US\_cabal^{a}{}_{t-1} + l^{a}\,US\_fisbal^{a}{}_{t-1} + e_t \end{aligned} \qquad (2a)$$

$$\begin{aligned} \Delta \ln(rfx_t^{\,b}) = {} & a^{b}\,(\ln(rfx^{b}{}_{t-1}) - b^{b} - f^{b}\ln(comtot\ {}_{t-1}) - p^{b}\ln(enetot^{b}{}_{t-1})) \\ & + d^{b}\,\mathrm{int}\,dif^{b}{}_{t-1} + h\,\Delta debtdif_{t-1} + c^{b}\,US\_cabal^{b}{}_{t-1} \\ & + l^{b}\,US\_fisbal^{b}{}_{t-1} + e_t \end{aligned} \qquad (2b)$$

$$\Delta \ln(rfx_t^c) = a^c (\ln(rfx^c{}_{t-1}) - b^c - f^c \ln(comtot^c{}_{t-1}) + d^c \, Rint\, dif^c{}_{t-1} \\ + c^c \, US\_cabal^c{}_{t-1} + l^c \, US\_fisbal^c{}_{t-1} + e_t \qquad (2c)$$

where *US_cabal* is the US current account balance as a proportion of GDP and *US_fisbal* is the US fiscal balance as a proportion of GDP. As mentioned, we also modify the specifications above by replacing these two multilateral variables with *twin_def*, a term that captures both the U.S. fiscal and current account deficits. The next two sections describe a threshold model and a Markov switching model of the bilateral exchange rate with multilateral effects.

Before turning to these econometric models, it may be useful to discuss the expected signs on the coefficients in equations (2a)--(2c). First, the energy and non-energy price indices in the cointegrating vector are proxies for the terms of trade and should play a role in the determination of the long-run value of our three commodity currencies. Since all three countries are major exporters of non-energy commodities, one would expect that an increase in their price would lead to an appreciation of all three currencies. As for energy commodities, only Australia and Canada are net exporters. Therefore, one would expect that an increase in the price of energy would cause an appreciation of both the Australian and Canadian dollars.

Second, the interest rate differential term captures the effect of relatively higher interest rates in our three countries—as a result of, for instance, relatively tighter monetary policy compared to the United States—on their bilateral exchange rates. One would expect that an increase in this variable would lead to an appreciation of the commodity currencies, as an increase in the rate of return of assets denominated in these currencies should increase the demand for such assets.

Finally, the expected effects of the U.S. current account and fiscal balances on the three commodity currencies are also ambiguous. One would expect that an increase in the U.S. fiscal deficit should lead to a depreciation of the U.S. dollar in the long-run—and hence to an appreciation of our three commodity currencies—since higher US government debt will likely lead to both higher domestic and foreign debt, which will eventually necessitate higher net exports to finance this excess absorption. It should be noted that in the short run, the effects of a deterioration of the fiscal balance on the exchange rate can be ambiguous. Indeed, the stimulative effects of higher government debt could put upward pressure on the currency in the short-run, but this could be partially or fully offset by risk considerations if the level of the debt increased beyond the level considered to be sustainable. Thus, these arguments for the effects of the fiscal balance on the national currency suggest that they depend on both the time horizon and the market's perception as to the sustainability of the level of national debt.

Similar arguments can also be made for the current account balance. Thus, if the U.S. government is running fiscal and current account deficits, this could put upward pressure on the U.S. dollar and hence lead to a depreciation of our commodity currencies, as long as the market perceived the twin deficits to be sustainable. Once the markets' perception changed, this could reverse the effects and lead to downward pressure on the U.S. dollar and hence an appreciation of the commodity currencies. This suggests that the effects of these U.S. variables on these three currencies might be best modelled in a framework with threshold effects, which is what we turn to in the next two sections.

4.2 A Threshold Model

In this paper, we use two different econometric models that allow us to treat these multilateral effects on our three commodity currencies as threshold effects. The first, which is described in this section, is a threshold (THR) model. The second, which we turn to in the next section, is a Markov switching model with a threshold variable.

Threshold regression models have a variety of applications in economics and have increased in popularity in recent years. This type of model splits the sample into "regimes" based on the threshold value of an observed variable, the so-called threshold variable. Given that the threshold value of the variable is typically unknown, it needs to be estimated along with the other parameters of the model. Several authors have contributed to developing a theory of estimation and inference of threshold models (also referred to as sample-splitting models) over the past decade or so, including Chan (1993), Hansen (1996), Hansen (1999), Caner (2002), and Caner and Hansen (2004).

Thus, our bilateral exchange rate equation with multilateral effects for each one of our countries (shown above) can be transformed into the following threshold model with two regimes:

$$\begin{aligned}\Delta \ln(rfx_t) = {} & a_1 (\ln(rfx_{t-1}) - b_1 - f_1 \ln(comtot_{t-1}) - p \ln(enetot_{t-1})) \\ & + d_1 \operatorname{int} dif_{t-1} + c_1 US\_cabal_{t-1} + l_1 US\_fisbal_{t-1} + e_t, \qquad q_{t-1} > q^*\end{aligned} \tag{3i}$$

$$\begin{aligned}\Delta \ln(rfx_t) = {} & a_1 (\ln(rfx_{t-1}) - b_1 - f_1 \ln(comtot_{t-1}) - p \ln(enetot_{t-1})) \\ & + d_1 \operatorname{int} dif_{t-1} + c_1 US\_cabal_{t-1} + l_1 US\_fisbal_{t-1} + e_t, \qquad q_{i-1} \leq q^*\end{aligned} \tag{3ii}$$

where q is the threshold variable and q^* is the estimated threshold value. It is worth pointing out that this model allows all the regression parameters to vary in the two regimes. We use two different threshold variables: the US fiscal balance as a proportion of GDP and our twin deficits measure. Both choices are motivated by the fact that current account deficits can occur during investment booms and such external deficits are likely to be viewed as more sustainable at existing exchange rate levels than ones caused by fiscal deficits (i.e., the twin deficits phenomenon). The estimation procedure for the threshold model is discussed in Section 5. Before turning to the estimation procedures, we first present our Markov switching model.

4.3 A Markov-Switching Model with a Threshold Variable

This section describes the second econometric model that we use to estimate our multilateral effects as threshold effects: a Markov switching model with a threshold variable (henceforth MSTV). More specifically, we develop an econometric model by augmenting a standard Markov switching framework (henceforth MS) with a threshold variable. Our MSTV model is characterized by an unobservable state of the economy and a time-varying transition probability matrix. The main feature of our model is that the transition probability matrix depends on a time-varying threshold variable. To the best of our knowledge, our paper is the first to develop a Markov switching model with a threshold variable.[21]

Our MSTV model is based on the following standard MS framework[22]:

[21] We are not the first, however, to use a Markov-switching model with a time-varying transition probability matrix. Ghysels, McCulloch, and Tsay (1998) developed such a model where the transition probability matrix of the state of the economy changes based on the seasonal characteristics of the data. Also, Filardo (1994) developed a MS model to examine properties of the U.S. business cycle in which the transition probability matrix is time-varying and given as a logistic function of an exogenous information variable. And in the context of PPP, Taylor (2004) constructed a MS model with a time-varying transition probability matrix which is a logistic function of the past duration of the state of the economy, the sample mean of the U.S.-German bilateral real exchange rate, and an index of intervention activity. In our model, on the other hand, the transition probability matrix changes depending on the position of the threshold variable relative to its threshold value.

[22] For more information on a standard Markov switching model such as this one, see Hamilton (1989, 1994).

$$y_t = \begin{cases} X_t q_1 + e_{1,t} \text{ if } S_t = 1, \\ X_t q_2 + e_{2,t} \text{ if } S_t = 2, \end{cases} \tag{4}$$

$$e_{1,t} \sim \text{i.i.d. } N(0, s^2_{e1}), e_{2,t} \sim \text{i.i.d. } N(0, s^2_{e2}),$$

where y_t is the dependent variable, X_t is a vector of explanatory variables, S_t is the unobservable state of the economy (where $S_t = \{1,2\}$) and $e_{1,t}$ and $e_{2,t}$ are Gaussian errors, respectively. This is thus a regime-switching model with a two-point unobservable state S_t where the coefficient vector on X_t varies with the state of the economy (i.e., $q_1 \neq q_2$). The transition from one state or regime to the other is determined by the transition probability matrix Γ_t, where

$$\Gamma_t = \begin{bmatrix} m & 1-n \\ 1-m & n \end{bmatrix}$$

which is assumed to be known.

In contrast to this standard model, in our MSTV model the transition probability matrix is time-varying and depends on a threshold variable q_{t-1} as follows:

$$\Gamma_t = \begin{bmatrix} m_1 & 1-n_1 \\ 1-m_1 & n_1 \end{bmatrix} \text{ if } q_{t-1} > g \text{ and } \begin{bmatrix} m_2 & 1-n_2 \\ 1-m_2 & n_2 \end{bmatrix} \text{ if } q_{t-1} \leq g \tag{5}$$

where each (i, j) element of the transition matrix represents the probability of $S_t = i$ conditional on $S_{t-1} = j$ and either $q_{t-1} \leq g$ or $q_{t-1} > g$; for example, $m_1 = P[S_t = 1 | S_{t-1} = 1, q_{t-1} \leq g]$. Following Hansen (1996, 2000), we further assume that threshold variable q_{t-1} is distributed as follows:

$$q_{t-1} \sim \text{i.i.d. with cdf} : P[q_{t-1} \leq g] = F^q(g) \tag{6}$$

where $F^q(g)$ is the probability that the threshold variable, q_{t-1}, takes on a value of less than g .

Moreover, our MSTV model also differs from the standard MS model in the identification scheme used. In the latter case, restrictions are imposed on the coefficient vectors in (4) (i.e., q_1 and q_2) in order to identify the state of the economy that is realized in a given period. However, in our model, no such restrictions are imposed. It is both the threshold variable and the restrictions imposed on the time-varying transition probability matrix that enable us to identify the model.

It is worth pointing out that equation (4) is symmetric across the two states. This means that, in order to identify our model, we need to impose some restrictions on the transition probability matrices in (5). In this model, the identification restrictions are simply given as the following two inequalities: $m_1 > m_2$ and $n_2 > n_1$. To understand the economic intuition behind the two inequalities, it is useful to emphasize that both $P[S_t = 1 | S_{t-1} = 1, q_{t-1} > g] > P[S_t = 1 | S_{t-1} = 1, q_{t-1} \le g]$ and $P[S_t = 1 | S_{t-1} = 2, q_{t-1} > g] > P[S_t = 1 | S_{t-1} = 2, q_{t-1} \le g]$ will also hold. These inequalities with respect to the conditional probabilities imply that, if $q_{t-1} > g$, then State 1 has a higher probability of occurring in the current period than does State 2, regardless of the state of the economy realized in the previous period.

Similarly, both $P[S_t = 2 | S_{t-1} = 2, q_{t-1} \le g] > P[S_t = 2 | S_{t-1} = 2, q_{t-1} > g]$ and $P[S_t = 2 | S_{t-1} = 1, q_{t-1} \le g] > P[S_t = 2 | S_{t-1} = 1, q_{t-1} > g]$ will also hold. Therefore, if $q_{t-1} \le g$, State 2 has a higher probability of occurring in the current period than does State 1, regardless of the state of the economy that was realized in the previous period.

Hence, in this model, State 2 is identified as the state of the economy which has a probability of realization strictly higher than that of the other state when the threshold variable, q_{t-1}, is less than its threshold value g. Similarly, State 1 is identified as the state of the economy which has a probability of realization strictly higher than that of the other state when the threshold variable, q_{t-1}, is greater than its threshold value g.

As discussed earlier, we estimate our bilateral exchange rate equations using both the THR and MSTV models because this enables us to check the consistency of our estimates across two econometric models that both treat the multilateral effects as threshold effects, but that model this process in different ways. Both models are suitable for this purpose, but rely on different assumptions, and as result, have different advantages in this context. The main advantage of the THR model, compared to the MSTV model, is that the results are more intuitive because the link between the threshold variable and the two regimes is very clear. For example, in the THR model, one regime is directly associated with values of the threshold variable that are below the threshold value (i.e., regime 1). This thus enables us to associate periods in which the U.S. fiscal deficit is below its threshold value with a regime where global imbalances are significant and hence where multilateral effects could be an important determinant of the bilateral exchange rates of our three commodity currencies.

This link is not as clear with the MSTV model where the position of the threshold variable with respect to its threshold value *increases the probability* of being in a given regime but does not guarantee that this regime will occur. In other words, the MSTV model is a generalization of the THR model that allows for some positive probability (where this probability = 1) of switching regimes based on the position of the threshold variable relative to its threshold value. This generalization, however, can also be viewed as a strength of the MSTV model, relative to the THR model. Indeed, the former is a less restrictive model because it generalizes a very strong assumption that is imposed on the transitory probability matrix in the THR model, and could hence be more realistic. For example, in the THR model, it is assumed that the fundamental long-run cointegration relationship changes from one regime to the other based only on the position of the U.S. fiscal balance whereas in the MSTV model, the change in the cointegration relationship from one regime to another could be explained by many factors including the threshold variable.

5. Estimation Methodology and Results

5.1 Estimation Procedures

This section describes the estimation procedures that we use for the different bilateral exchange rate equations that we estimate in our paper. First, we estimate the bilateral exchange rate equations without multilateral effects for each one of our commodity currencies (equations (1a) -- (1c)) – this is our benchmark model. Second, we estimate the bilateral exchange rate equations with multilateral effects where theses effects are *not* modelled as threshold effects (equations (2a) – (2c)) – this is our multilateral effects (ME) model. This latter model assumes that the effects of multilateral adjustment factors on the exchange rate are constant across all periods (or in other words, it is assumed that there is only one regime). Both sets of equations described above are estimated using non-linear least squares (NLLS). Such a procedure is necessary given the presence of the long-term relationship between the real exchange rate and the terms of trade variables (i.e., the error correction terms).

Finally, we estimate our bilateral exchange rate equations using two different econometric models that treat the multilateral adjustment factors as threshold effects: the THR model and the MSTV model.

5.1.1 Estimation Procedure for the Threshold Model

The THR model (equations (3i)—(3ii)) is estimated for each one of our commodity currencies using a two-step procedure, as proposed by Hansen (1999). The first step involves estimating the threshold parameter, q^*, that splits the sample into two regimes. In the second step, the other parameters associated with each regime are then estimated.[23]

Using Hansen's (1999) notation, we can rewrite the THR model (equations (3i)-(3ii)) in the following form:

$$y_t = ?_2' X_t + e_t \quad , \qquad q_{t-1} > q^* \tag{7}$$

$$y_t = ?_1' X_t + e_t \quad , \qquad q_{t-1} = q^* \tag{8}$$

where $?_1$ and $?_2$ are two parameter vectors associated with regimes 1 and 2, the observed sample is $\{y_t, X_t, q_{t-1}\}^T_{t=1}$, y_t is the dependent variable (? ln(rfx_t)), X_t is a vector of exogenous variables, q_t is the threshold variable that is also included in X_t, and e_t is a random variable of errors. The threshold parameter q^*, which is a strict subset of support of q_t, is unknown and needs to be estimated.

The estimator of q^* minimizes the sum of squared errors from the regression of y_t on $X_{qt} = X_t \mathbf{1}(q_{t-1} = q^*)$. The non-linear least square estimator for q^* is the minimizer of the sum of squared errors, $S_T(q^*)$:

[23] This procedure is equivalent to jointly estimating the parameters in both regimes.

$q^*=\text{argmin}_{q^*?Q}\, S_T(q^*)$, (9)

where $Q=\{q_1 \dots q_T\}$. Since $S_T(q^*)$ may take T distinct values, the estimation of q^* requires T function evaluations (where T is the total number of observations).

5.1.2 Estimation Procedure for the Markov Switching Model with a Threshold Variable

To explain the estimation procedure that we use for the MSTV model, it is useful to first present another representation for the time-varying transition matrix (2) that is time invariant. In order to do so, we introduce a new state variable, S_t^*, that is defined as follows:

$$S_t^* = \begin{cases} 0 \text{ if } S_t = 1 \text{ and } \mathrm{q_t} \leq g \\ 1 \text{ if } S_t = 1 \text{ and } \mathrm{q_t} > g \\ 2 \text{ if } S_t = 2 \text{ and } \mathrm{q_t} \leq g \\ 3 \text{ if } S_t = 2 \text{ and } \mathrm{q_t} > g. \end{cases}$$

Given the transition probability matrix (5) and the cumulative density function of the threshold variable $q_{t-1}, F^q(g)$, it can be shown that this new state variable, S_t^*, has the following time-invariant transition matrix[24]:

$$\Gamma^* = \{P[S_t^* = i \mid S_{t-1}^* = j]\}$$

$$\begin{bmatrix} F^q(g)m_1 & F^q(g)m_2 & F^q(g)(1-n_1) & F^q(g)(1-n_2) \\ [1-F^q(g)]m_1 & [1-F^q(g)]m_2 & [1-F^q(g)](1-n_1) & [1-F^q(g)](1-n_2) \\ F^q(g)(1-m_1) & F^q(g)(1-m_2) & F^q(g)n_1 & F^q(g)n_2 \\ [1-F^q(g)](1-m_1) & [1-F^q(g)](1-m_2) & [1-F^q(g)]n_1 & [1-F^q(g)]n_2 \end{bmatrix}.$$

[24] This is shown in Appendix 5.

Therefore, in our model, the probability of the current state of the economy depends on the past state of the economy through the first-order Markov process with the above time-invariant transition matrix.

In Appendix 5, we show that, given the MSTV model in (4) with its time-invariant transition matrix Γ^*, the log-likelihood function of the series y^T, X^T, and q^T will be given as

$$\begin{aligned} \ln L(y^T, X^T, q^T \mid q_1, q_2, m_1, m_2, n_1, n_2, g, s_{e1}, s_{e2}) &= \sum_{t=1}^{T} \ln f(y_t \mid \Psi_{t-1}, X_t) \\ &= \sum_{t=1}^{T} \ln\{\sum_{S_t^*=0}^{3} f(y_t \mid S_t^*, \Psi_{t-1}, X_t) P[S_t^* \mid \Psi_{t-1}, X_t]\}, \end{aligned}$$

where

$$f(y_t \mid S_t^*, \Psi_{t-1}, X_t) = \begin{cases} \dfrac{1}{\sqrt{2p\, s_{e1}^2}} \exp\left\{ -\dfrac{(y_t - X_t q_1)^2}{2s_{e1}^2} \right\} \text{if } S_t^* = 0 \text{ or } S_t^* = 1, \\ \dfrac{1}{\sqrt{2p\, s_{e2}^2}} \exp\left\{ -\dfrac{(y_t - X_t q_2)^2}{2s_{e2}^2} \right\} \text{if } S_t^* = 2 \text{ or } S_t^* = 3. \end{cases}$$

In this paper, we exploit Hamilton's (1989) filter to construct the conditional probability of each state, $P[S_t^* \mid \Psi_{t-1}, X_t]$ for $t = 1, 2, \ldots, T$. By maximizing the log-likelihood function, $\ln L$, we then obtain estimates of the parameters of our MSTV model (i.e., $q_1, q_2, m_1, m_2, n_1, n_2, g, s_{e1}, s_{e2}$). And since our model satisfies the regularity conditions of the Maximum Likelihood (ML) estimation, our ML estimates have standard asymptotic properties.

5.2 Estimation Results

In this section, we present the estimation results for the bilateral exchange rate equations described above (all the relevant tables are found in Appendix 4). We first present and discuss our estimation results for Canada which are more complete than those for Australia and New Zealand. We then turn to the presentation of our results for the other two commodity currencies, which should be viewed as preliminary at this stage. Indeed, we experienced some problems in applying the different bilateral exchange rate equations models used for Canada to Australia and New Zealand; we elaborate on these problems below.

5.2.1 Estimation Results for Canada

The estimation results for Canada are shown in Tables A4.1a – A4.1c. Table A4.1a depicts the estimates for the benchmark model, the multilateral effects model, and the THR model using both the U.S. fiscal and current account balances as the multilateral variables. Table A4.1b shows the results for these same models using our twin deficits variable as the multilateral variable instead. Finally, the results for the Markov switching model are presented in Table A4.1c. All models for Canada are estimated using data for the period 1973Q1 to 2005Q1.[25]

For the THR model, we use the first lag of the U.S. fiscal balance, US_fisbal_{t-1}, or our measure of the U.S. twin deficit, $twin\_def_{t-1}$, as the threshold variable, q_{t-1} whereas in the MSTV model, we use only the U.S. fiscal balance as the threshold variable. The estimated values of the threshold parameters for the THR model for Canada are: -1.86% for US_fisbal_{t-1} and -1.45% for $twin\_def_{t-1}$.[26] Thus, in the THR model, the first regime is characterized by a situation where the U.S. is running a fiscal deficit that is smaller than 1.86% of GDP (i.e., $US\_fisbal_{t-1} > -1.86\%$) or a twin deficit that is smaller

[25] It is worth noting that Canada had a floating exchange rate regime over this entire period.
[26] Hansen's test—an LM test for no threshold—rejects the null hypothesis of no threshold at 2% significance level for both threshold variables.

than 1.45% of GDP (i.e., $twin\_def_{t-1} < 1.45\%$) or a surplus.[27] The second regime is then characterized by periods in which either the U.S. fiscal deficit is larger than 1.86% of GDP (i.e., $US\_fisbal_{t-1} = -1.86\%$) or the U.S. twin deficit is larger than 1.45% of GDP (i.e., $twin\_def_{t-1} = 1.45\%$). There are 51 (57) observations in the first regime and 78(72) in the second when using the U.S. fiscal balance (the U.S. twin deficit measure) as the threshold variable. Figures A4.1a—b plot the evolution of the three multilateral variables—the U.S. current and fiscal account balances and the twin deficits measure—across the two regimes identified for the THR model for Canada, where the shaded area represents the second regime. As expected, in each case the second regime contains all of the observations for the periods where there were large twin deficits in the U.S.. For the MSTV model, the estimated value of the threshold parameter is very similar, at -1.7%.[28]

The parameter estimates in the equations for Canada are generally statistically significant at conventional levels and are of the expected sign. The estimated long-run effects suggest that an increase in the non-energy commodity price index leads to an appreciation of the real exchange rate across all models, as expected. The coefficient on the energy commodity price term is not always statistically significant and when it is, it is positive, suggesting that an increase in energy prices leads to a depreciation in the Canadian dollar.

This counterintuitive result has been noted in previous research and has been the focus of some recent work by Issa, Lafrance, Murray (2005). More specifically, they examine whether the nature of the relationship between the exchange rate and energy

[27] Or alternatively, the budget could be balanced in this regime.
[28] For the specification that uses the U.S. twin deficit measure as the multilateral variable, the estimated value of the threshold parameter is not statistically different from zero. As shown in Table A4.1c, it is not estimated as precisely as the threshold parameter for the U.S. fiscal balance.

prices has changed over the past three decades given the increased importance of energy products in Canadian exports. They find evidence that this relationship changed in the early 1990s, which is consistent with major changes in Canada's energy policies and international trade. Thus, it appears that the benefits of higher energy prices (through higher export revenues, increased investment and greater net wealth) started to offset the costs in the early 1990s. They then re-estimate the Amano-van Norden model allowing the coefficient on the energy term to change in the second half of the sample, and find that it becomes positive. Moreover, the explanatory power of the model is improved with this modification.

The coefficients on the two variables that capture short-run dynamics are generally statistically significant and of the expected sign. Indeed, they suggest that an increase in the Canadian short-run interest rate spread results in an appreciation of the Canadian dollar whereas an increase in Canadian government debt relative to the U.S. will tend to depreciate the Canadian dollar.

The coefficients on the multilateral variables are also generally statistically significant and of the expected sign, suggesting that a deterioration of both the U.S. current and fiscal accounts leads to an appreciation of the Canadian dollar. In the model without threshold effects, the U.S. fiscal and current account balances are statistically significant regardless of whether they are entered separately (as shown in Table A4.1a) or combined in our twin deficits measure (shown in Table A4.1b). In the threshold model, both multilateral variables are only statistically significant in the second regime (i.e., when the fiscal deficit is larger than 1.86 per cent of GDP or when the U.S. twin deficit is larger than 1.45% of GDP).[29] This also holds both for the case where the U.S. fiscal and current account balances are entered separately and when they are combined in our twin deficits measure. We find a somewhat similar result for the Markov switching model. In the latter, the current account balance is statistically significant in the second regime (i.e., the regime that is more likely to occur when the fiscal deficit is larger than 1.7 per cent of GDP) but the fiscal account balance is not; our measure of twin deficits, however, is statistically significant in the first regime only (i.e., the regime that is more likely to occur when the fiscal deficit is smaller or when there is a fiscal surplus). This latter result is puzzling and inconsistent with the rest of the evidence found.[30] Notwithstanding this, our results are generally consistent with the view that multilateral adjustment factors are a determinant of the bilateral Canadian exchange rate, but only in periods where the United States is running significant current account and fiscal deficits. In the other regime (i.e., in periods not characterized by global imbalances), multilateral adjustment factors are not a statistically significant determinant of the Canadian dollar.

We conducted some specification and diagnostic tests on our models for Canada. First, we tested whether the coefficients on the two multilateral variables, *US_cabal* and *US_fisbal*, are equal to zero. For the ME model, we did this by constructing a likelihood ratio (LR) test using the maximum values of the log-likelihood functions for equations (1b) and (2b), which are reported in Table A4.1a. The LR ratio test rejected the restriction that the coefficients on the U.S. fiscal and current account balances in equation (2b) are

[29] It should be noted that the current account is lagged four quarters whereas the fiscal account is lagged one quarter in the THR model for Canada. We use the fourth lag of the current account because it is the only lag that is statistically significant.
[30] We have less confidence in this result given that the threshold parameter in this case in not estimated as precisely.

equal to zero.[31] For the MSTV model, the LR ratio test also rejected the joint null of no multilateral effects in both regimes (as shown in Table 4.1c). These test results, therefore, imply that the U.S. fiscal and current account balances play a statistically as well as an economically important role in the quarterly movements of the U.S.-Canada bilateral real exchange rate.

Second, we tested for the presence of serial correlation and heteroscedasticity in the residuals. We investigated this issue using a Lagrange Multiplier (LM) test—which is valid in a wider range of situations than the Durbin-Watson test and allows for autoregressive or moving-average errors of arbitrary order—and an autoregressive conditional heteroscedasticity (ARCH) test of first and second order. The LM (one and two quarters out) and ARCH tests suggested the presence of both autocorrelation and heteroscedasticity. To correct for this, we adjusted our standard errors using the Newey-West HAC procedure for the benchmark, ME and threshold models. In the Markov switching model, the standard errors are corrected for heteroscedasticity.

5.2.2 Estimation Results for Australia and New Zealand

The estimation results for Australia and New Zealand are shown in Tables A4.2a – A4.3c. Tables A4.2a and A4.3a depict the estimates for the benchmark and the multilateral effects model whereas the results for the Markov switching model are presented in Tables A4.2b and A4.3b. All estimations use the first lag of the U.S. fiscal

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[31] The likelihood ratio test statistic is equal to 15.8 (i.e., $-2(L^R - L^U)=15.8$ where L^R is the log-likelihood value for the restricted model and L^U is that of the unrestricted model) which is greater than the 5 percent critical value of 7.82. This test statistic is distributed as chi-square with degrees of freedom equal to the number of restrictions (i.e., 2 in this case).

account balance as the threshold variable. All models for Australia (New Zealand) are estimated using data for the period 1985Q1 to 2005Q1 (1986Q2 to 2005Q1).

The THR model could not be estimated for Australia and New Zealand because the regimes identified were such that almost all the observations ended up in the first regime (as shown in Figures A4.1c —A4.1d). This could be due to the shorter sample periods that we have for Australia and New Zealand, which is inevitable given the fact that these two countries did not move to a floating exchange rate regime until the mid-1980s. This highlights another limitation of the THR model, compared to the Markov switching model. Indeed, the latter performs better with a longer sample period and may be impossible to estimate when the data span is too short.

The estimated value of the threshold parameter for the Markov switching model is only statistically significant in the case of Australia, where it is equal to -2.6%.[32] Thus, in the Markov switching model for Australia, the first regime is characterized by a situation in which the U.S. is running a fiscal surplus or a deficit that is smaller than 2.6% of GDP (i.e., $US\_fisbal_{t-1} > -2.6\%$).[33] The second regime is then characterized by periods in which the U.S. fiscal deficit is larger than 2.6% of GDP (i.e., $US\_fisbal_{t-1} = -2.6\%$). In the case of New Zealand, the estimated value of the threshold parameter for the Markov switching model is statistically insignificant or in other words, it is not statistically different from zero. Thus, for New Zealand, the first regime is characterized by a situation in which the U.S. is running a surplus (i.e., $US\_fisbal_{t-1} > 0\%$). The second regime is then characterized by periods in which there is a deficit or a balanced budget (i.e., US_fisbal_{t-1}

[32] The estimated value of the threshold parameter is only statistically significant in the first specification reported in Table A4.2b where both the U.S. fiscal and current account balances are included as explanatory variables.

[33] Or alternatively, the budget could be balanced in this regime.

= 0%). Given that the estimated value for the threshold parameter is not estimated very precisely (as shown by the relatively large standard errors), our results for New Zealand should be interpreted with caution.

Overall, our results for Australia and New Zealand suggest that our models for these two countries are not estimated as precisely as those for Canada and should be viewed as preliminary at this stage.[34] Additional work is needed to attempt to improve the performance of the equations for these two countries. The parameters in the equations for Australia and New Zealand are not generally statistically significant at conventional levels but when they are, they are of the expected sign. The estimated long-run effects suggest that an increase in the non-energy commodity price index leads to an appreciation of the real exchange rate for both countries, as expected. The coefficient on the energy commodity price term is generally statistically insignificant for Australia. And the coefficient on the interest rate spread is of the expected sign when it is statistically significant. Indeed, it suggests that an increase in the interest rate spread with the U.S. results in an appreciation of the both the Australian and New Zealand dollars.

The coefficients on the multilateral variables, however, do tend to be generally statistically significant and mostly of the expected sign, suggesting that a deterioration of both the U.S. current and fiscal accounts leads to an appreciation of both the Australian and New Zealand dollars.

For Australia, in the model without threshold effects, the U.S. fiscal and current account balances are statistically significant regardless of whether they are entered separately or combined in our twin deficits measure (both shown in Table A4.2a). In the

[34] We conducted the same specification and diagnostic tests on our models for Australia and New Zealand as on our models for Canada, and the outcomes were similar as those discussed above.

Markov switching model, all three multilateral variables are statistically significant in both regimes (as shown in Table A4.2b). The coefficients on the U.S. fiscal and current account balances are negative in the first regime (i.e., when the fiscal deficit is small or there is a surplus) but positive in the second regime (i.e., when the fiscal deficit is large). This suggests that a deterioration of the U.S. fiscal and current account balances leads to an appreciation of the Australian dollar in the second regime (i.e., when the U.S. fiscal deficit is large), which is similar to our results for Canada. However, in contrast to our results for Canada, this also suggests that an *improvement* in the U.S. fiscal and current account balances *in regime 1* (i.e., in periods when the U.S. fiscal deficit is small or when there is a surplus) leads to a depreciation of the Australian dollar. The coefficient estimates for the specification with the U.S. twin deficits measure are negative across both regimes. This is similar to the result for Canada, which as noted above, is puzzling and inconsistent with the rest of the evidence found. However, as noted for Canada, we also have less confidence in this latter result given that the threshold parameter in this case in not estimated very precisely.

For New Zealand, in the model without threshold effects, the U.S. fiscal and current account balances are only statistically significant when they are entered separately (shown in Table A4.3a). In the Markov switching model, all three multilateral variables are statistically significant in both regimes (as shown in Table A4.3b). The coefficients on the U.S. fiscal and current account balances are positive in both regimes suggesting that a deterioration of the U.S. fiscal and current account balances leads to an appreciation of the New Zealand dollar regardless of the regime (i.e., regardless of the position of the U.S. fiscal deficit relative to its threshold value). The coefficient estimates for the specification with the U.S. twin deficits measure are negative across both regimes, as is the case for Canada and Australia.

5.3 Simulations and Forecasting Performance

Dynamic simulations of the different models are shown in Figures A4.2a—A4.3c in Appendix 4, using parameter estimates for the entire sample for each country. The dynamic simulations for Canada are shown in Figure A4.2a for the benchmark model, the ME model and the threshold model, and in Figure A4.3a for the Markov switching model.[35] All of the models are fairly successful at accounting for broad movements in the Canada-U.S. real exchange rate over the sample period. As shown, the correspondence between the simulated and actual values is quite close. There are, however, episodes of important deviations between the actual and simulated values—particularly in the mid-1980s, and in the early part of this decade—but they tend to disappear after a short period of time. There are also differences across the models. Indeed, the simulated values from the models with multilateral effects (with and without threshold effects) appear to match more closely the actual values than the benchmark model. In particular, they are more successful at accounting for the appreciation of the Canadian dollar over the 2003-05 period, especially the threshold and Markov switching models.

The dynamic simulations for Australia and New Zealand are shown in Figures A4.2b—2c (for the benchmark and ME models) and in Figures A4.3b—3c (for the MSTV model). In contrast to the simulations for Canada, the models are not as successful at tracking broad movements in the Australian and New Zealand dollars and there are important differences in the ability of the various models to track these movements.

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[35] The construction of the in-sample forecasts in the MSTV model is explained in Appendix 6.

Indeed, the simulated values from the models with multilateral effects seem to match the actual values better than those from the benchmark model.

We also examined the out-of-sample forecasting performance of some of our models, namely the benchmark, ME and threshold models. We are currently working on constructing out of sample forecasts for the Markov switching model, which is a more involved process that for the other models. For our out-of-sample forecasting exercise, we estimated all the models using dynamic rolling regressions starting with a period that covers the beginning of the sample (which varies by country) to 2003Q4 as the sample period, and moving up one quarter each time to generate a new forecast. This enables us to compare the forecasting performance of the competing models over the period from 2004Q1 to 2005Q1. Figures A4.4a—A4.4c in Appendix 4 depict the actual values of the real exchange rate as well as the forecasted values produced by the different models. The models with multilateral effects for Canada appear to do a better job with the out-of-sample forecasting over the 2004-05 period than the benchmark model. This is confirmed by the forecasting performance measures reported in Table A4.4a. Indeed, the reported values for the root mean squared error (RMSE) and Theil coefficient suggest that the models with multilateral effects outperform the benchmark specification over the period considered. For Australia and New Zealand, none of the models perform very well in the out of sample forecasting.

6. Concluding Remarks

Rapid and significant appreciations of floating exchange rates, such as those experienced by the ACNZ dollars over the past four years, pose a number of challenges for central banks in formulating the optimal monetary policy response, if any. In particular, how the central bank should react critically depends on the underlying forces behind the appreciation.

In this paper, we examined the recent exchange rate behavior of three countries – Australia, Canada and New Zealand – in an attempt to identify separately the magnitude of the impact of two factors that may explain their appreciations: the recent increase in commodity prices and the possible multilateral adjustment of these currencies to the large U.S. current account deficit. Although our findings should be viewed as preliminary at this stage, we do find evidence to suggest that during periods of large U.S. imbalances, fiscal and external, an exchange rate model for the ACNZ dollars should allow for multilateral adjustment effects. Moreover, the results for both the threshold and Markov switching models suggest that the adjustment of exchange rates to multilateral factors are best modelled as a non-linear process.

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Appendix 1
US Imbalances in the Postwar Period
and the Commodity Currencies: A Graphical Depiction

Figure A.1.1
The commodity currencies and the US current account balance

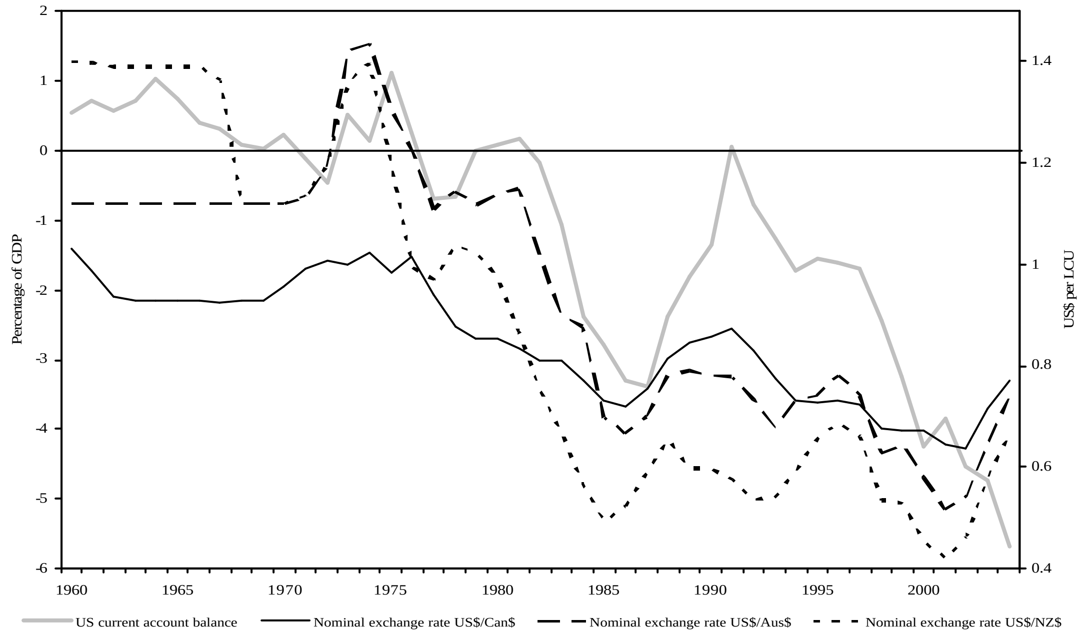


Figure A1.2
The US dollar and the US current account balance

Figure A1.3a
The US current account and the Canadian dollar
(dynamic simulation and actual values)

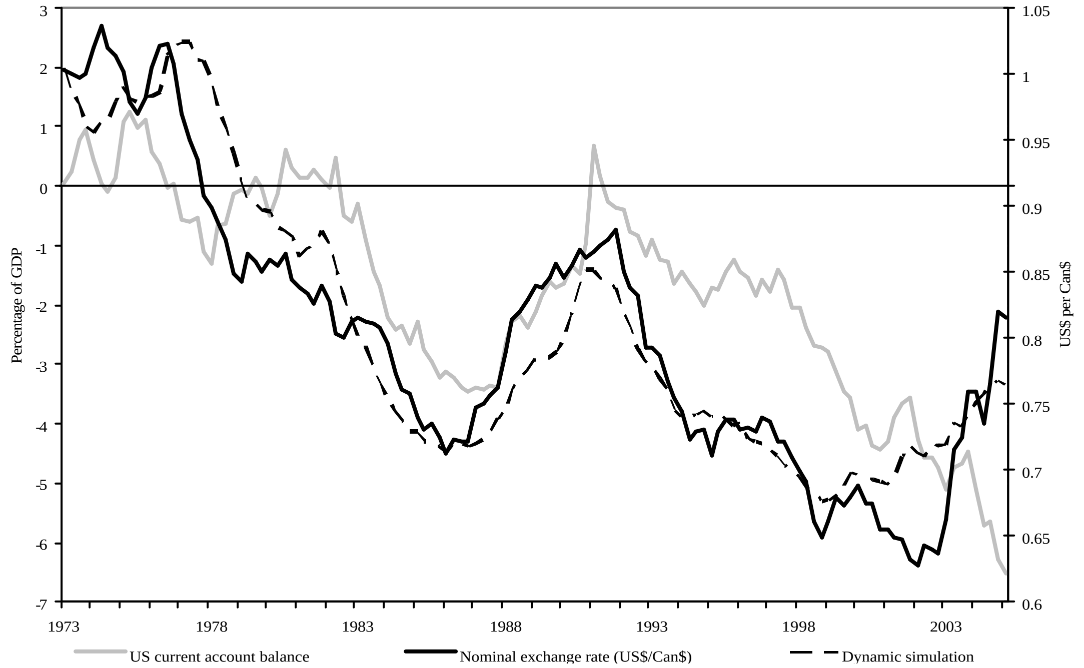


Figure A1.3b
The US current account and the Australian dollar
(dynamic simulation and actual values)

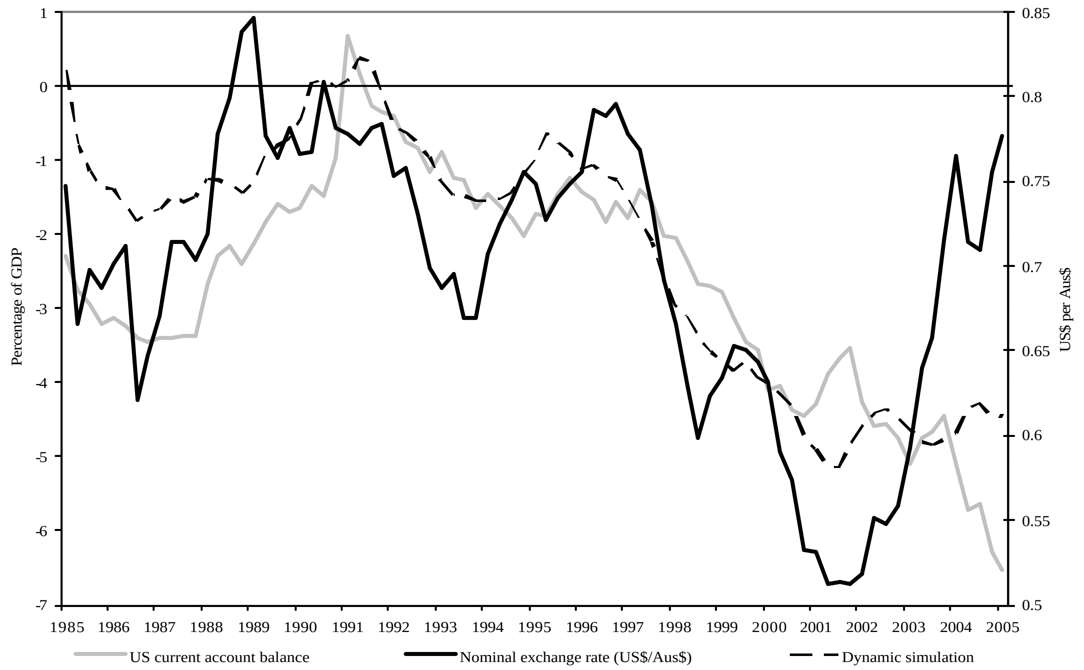


Figure A1.3c
The US current account and the New Zealand dollar
(dynamic simulation and actual values)

Figure A1.4a
Real non-energy commodity prices

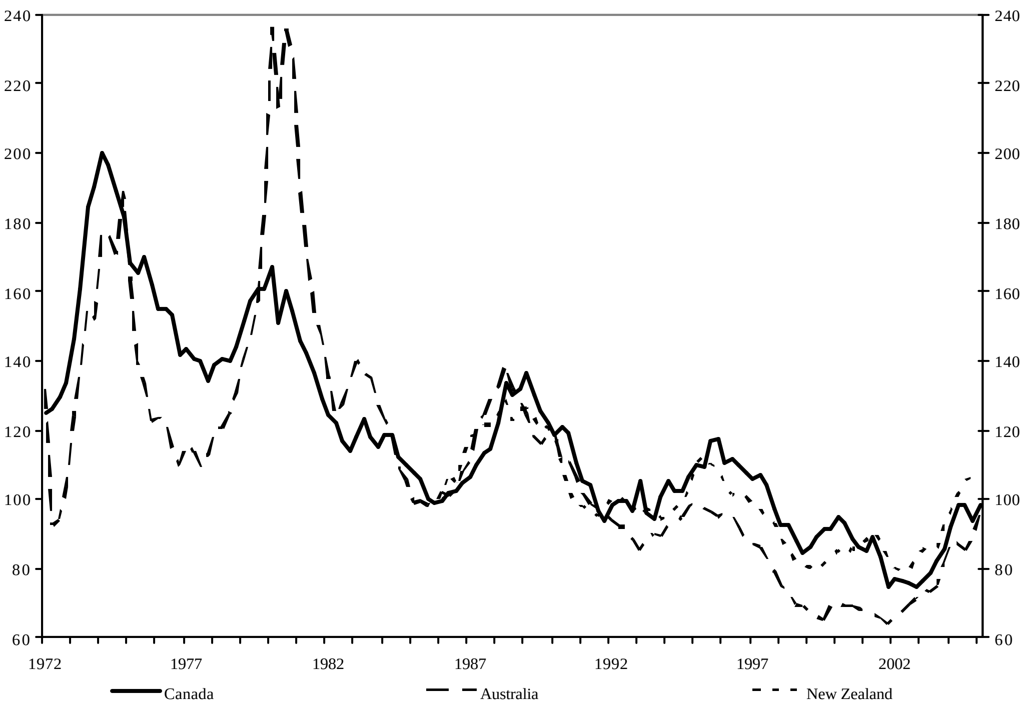


Figure A1.4b
Real energy commodity prices

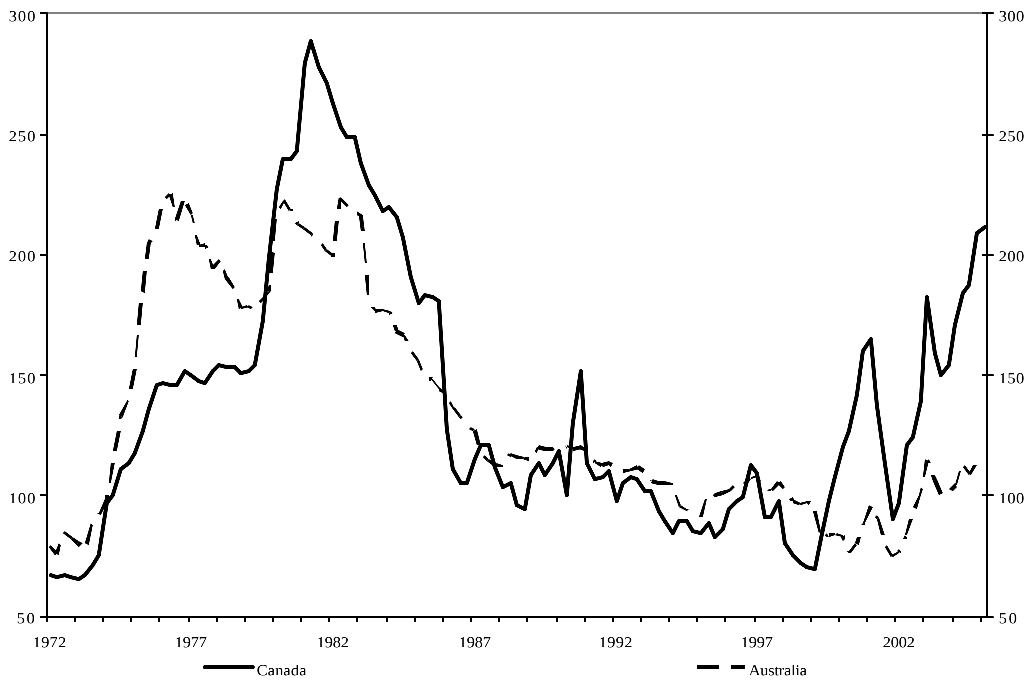


Figure A1.5
The US current and fiscal account balances

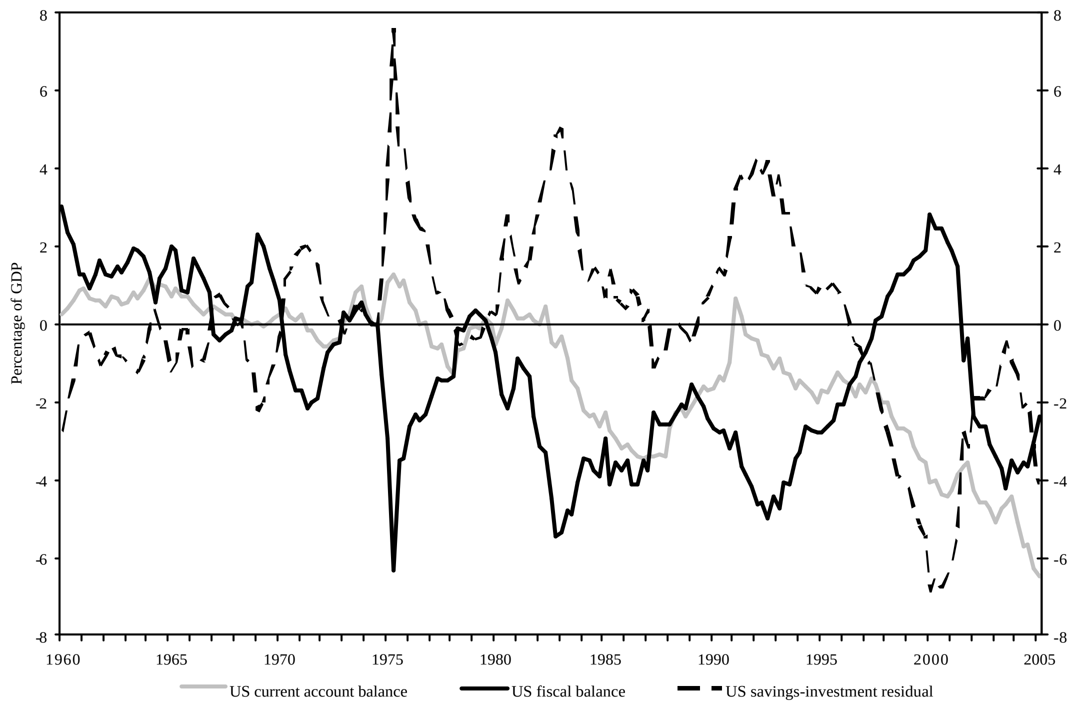


Figure A1.6
The cyclically adjusted US fiscal balance, the US fiscal balance and the US current account

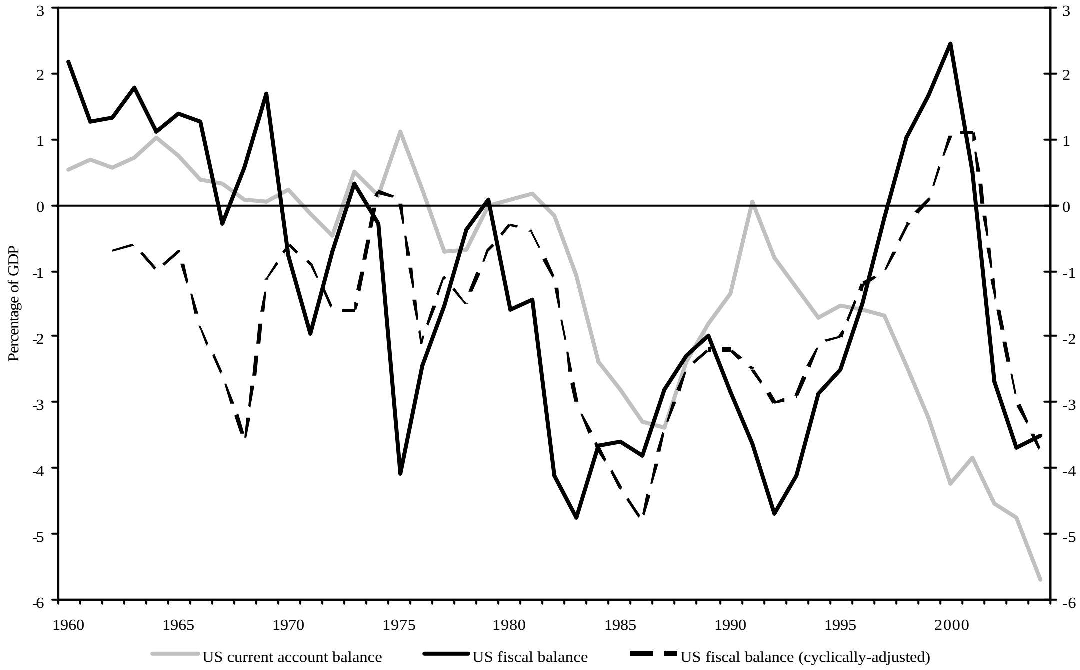


Figure A1.7a
Comparison of the Canadian real effective exchange rate and the bilateral US$/Can$ exchange rate

Figure A1.7b
Comparison of the Australian real effective exchange rate and the bilateral US$/Aus$ exchange rate

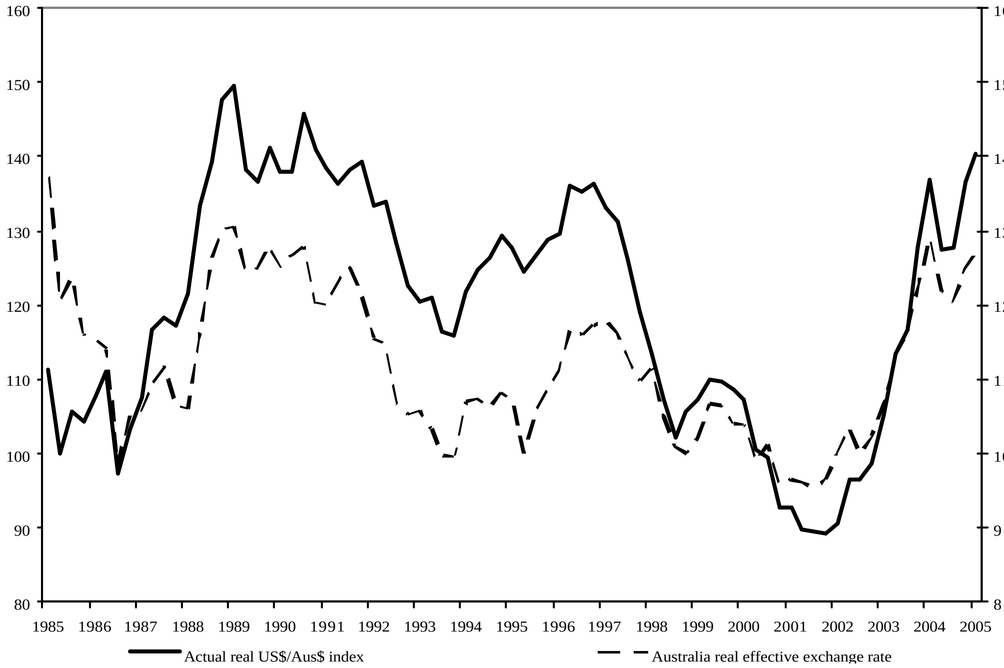
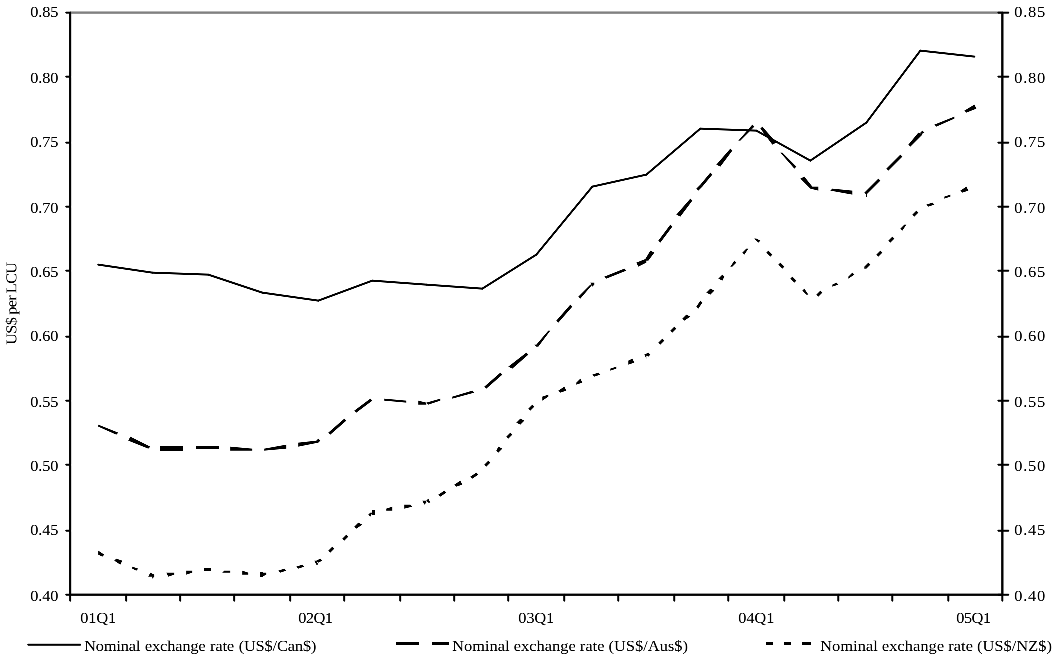


Figure A1.7c
Comparison of the New Zealand real effective exchange rate and the bilateral US$/NZ$ exchange rate

Figure A1.8
Recent appreciation of commodity currencies



Appendix 2
Sources and Definitions of Variables

Dependant variable

1. $\Delta\ln(rfx)$

- log difference in the real quarterly (Can/Aus/Nzl)-US bilateral exchange rate constructed using the nominal exchange rate, deflated by the ratio of the (Can/Aus/Nzl) and U.S. GDP deflators. Both deflators are indexed to 1997=1.0.

A. Canada
– Nominal exchange rate; Source: Bank of Canada internal database
– GDP deflator; Source: Statistics Canada series *V1997756*

B. Australia
– Nominal exchange rate; Source: International Financial Statistics (IFS) series *Q.193..RF.ZF...H*
– GDP deflator; Source: OECD Main Economic Indicators series *Q.AUS.EXPGDP.DNBSA*

C. New Zealand
– Nominal exchange rate; Source: IFS series *Q.196..RF.ZF...H*
– GDP deflator; Source: OECD Economic Outlook series *Q.NZL.PGDP*

D. US GDP deflator; Source: US Department of Commerce – Bureau of Economic Analysis (series *pdigdp)*

Explanatory variables

2. $\ln(comtot)$

- log of the real non-energy commodity price index constructed as the nominal non-energy commodity price index in US dollar terms, deflated by the US GDP deflator.

A. Canada
– Nominal non-energy commodity price index; Source: Bank of Canada

B. Australia
– Nominal non-energy commodity price index; Source: Used weights from the Reserve Bank of Australia's Index of Commodity Prices and constructed a non-energy index by reweighting. Price data used for commodities was obtained from the IFS, Datastream (alumina), and the Bank of Canada's internal database.

C. New Zealand
– Nominal non-energy commodity price index; Source: Australia and New Zealand Banking Group Limited (ANZ) Commodity Price Index.

3. ln(*enetot*)

- log of the real energy commodity price index computed as the nominal energy commodity price index in US dollar terms, deflated by the US GDP deflator.

A. Canada
– Nominal energy commodity price index; Source: Bank of Canada

B. Australia
– Nominal energy commodity price index; Source: Used weights from the Reserve Bank of Australia's Index of Commodity Prices and constructed an energy index by reweighting. Price data for commodities was obtained from the IFS and the Bank of Canada's internal database.

C. New Zealand
– Nominal energy commodity price index; Source: IFS (crude oil world price index series)

4. *intdif*

- short-term interest rate spread constructed as the difference between Canadian and U.S. rates.

A. Canada
– Three-month prime corporate paper rate; Source: Statistics Canada series *v122491*

B. US 90-day AA non-financial commercial paper closing rate; Source: Federal Reserve Board

5. *Rintdif*

- short-term real interest rate spread constructed as the difference between (Aus/Nzl) and US real rates.

▪ Australia
– Yield on 90-day bank-accepted bills; Source: OECD Main Economic Indicators series *Q.AUS.IR3TBB01.ST*

B. New Zealand
– 90-day bank bill rate; Source: OECD Main Economic Indicators series *Q.NZL.IR3TBB01.ST*

6. *debtdif*

- Canada-U.S. total government debt to GDP ratio.

A. Canada
– Total government debt; Source: Sum of Statistics Canada series *v34422, v34460, v34584*
– GDP; Source: Statistics Canada series *v498086*

B. US total government debt as a proportion of GDP; Source: US Congressional Budget Office

7. *US_cabal*

- Balance on US current account as a proportion of GDP.

  – Current account balance; Source: US Department of Commerce, Bureau of Economic Analysis series *bopcrnt*
  – GDP; Source: US Department of Commerce, Bureau of Economic Analysis series g*dp*

8. *US_fisbal*

- US total government fiscal balance as a proportion of GDP.

  – Fiscal balance; Source: US Department of Commerce, Bureau of Economic Analysis series *netsavg*

9. *twin_def*

- Arithmetic average of the US total government fiscal deficit and the US current account deficit, as a proportion of GDP.

Appendix 3
Unit-Root, Cointegration, and Weak Exogeneity Test Results

Table A3.1a
DF-GLS unit-root tests
(Canada, Sample period: 1973Q1 to 2005Q1)

Variable	**Trend**	**No Trend**
ln(*rfx*)	-2.04	-1.62
ln(*comtot*)	-1.99	-0.52
ln(*enetot*)	-1.30	-0.75
debtdif	-0.96	-0.93
?*debtdif*	-1.60	-1.35
intdif	-1.64	-1.24
US_cabal	-1.39	0.96
US_fisbal	-1.82	-1.46

Notes:
(i) The Dickey-Fuller Generalized Least Squares (DF-GLS) test is based on Elliott, Rothenberg, and Stock's (1996) modification to the Augmented Dickey-Fuller (ADF) test. Under this test, the variable is first locally detrended/demeaned and then tested for the presence of a unit root in the usual ADF manner. The power of the DF-GLS is substantially improved over the original version of ADF, particularly for finite samples. As with the ADF test, the null hypothesis states that the variable contains a unit root.
(ii) The number of lags used in the test was selected based on the Modified Schwarz Information Criterion, developed by Ng and Perron (2001).
(iii) Bolded values exceed the 5 per cent critical value.

Table A3.1b
DF-GLS unit-root tests
(Australia, Sample period: 1985Q1 to 2005Q1)

Variable	**Trend**	**No Trend**
ln(*rfx*)	-1.68	-1.55
ln(*comtot*)	-1.20	-1.15
ln(*enetot*)	-1.29	-0.81
Rintdif	-2.48	**-2.23**

Notes: See notes for Table A3.1a.

Table A3.1c
DF-GLS unit-root tests
(New Zealand, Sample period: 1986Q1 to 2005Q1)

Variable	**Trend**	**No Trend**
ln(*rfx*)	-1.64	-1.14
ln(*comtot*)	-0.59	-1.77
Rintdiff	-1.79	-0.89

Notes: See notes for Table A3.1a.

Table A3.2a
Johansen cointegration test results for ln(*rfx*), ln(*comtot*) and ln(*enetot*)
(Canada, Sample period: 1973Q1 to 2005Q1)

	No trend			Trend		
No. of cointegrating vectors under H_0	**Trace statistic**	**5% critical value**	***P*-Value**	**Trace statistic**	**5% critical value**	***P*-Value**
Fewer than 1	17.84	29.80	0.578	44.22	42.92	0.036
Fewer than 2	9.82	15.49	0.29	12.93	25.87	0.74
No. of cointegrating vectors under H_0	**l-max statistic**	**5% critical value**	***P*-Value**	**l-max statistic**	**5% critical value**	***P*-Value**
Fewer than 1	8.01	21.13	0.90	31.29	25.82	0.008
Fewer than 2	5.22	14.26	0.71	7.95	19.39	0.825

Notes:
(i) The values reported under the column labeled ‘No trend’ assume a constant in the cointegration space and a linear deterministic trend in the data.
(ii) The values reported under the column labeled ‘Trend’ assume a constant and a linear deterministic trend in the cointegration space, as well as a linear deterministic trend in the data.
(iii) The critical values are based on MacKinnon, Haug, and Michelis (1999)

Table A3.2b
Johansen cointegration test results for ln(*rfx*), ln(*comtot*) and ln(*enetot*)
(Australia, Sample period: 1985Q1 to 2005Q1)

	No Trend			Trend		
No. of cointegrating vectors under H_0	**Trace statistic**	**5% critical Value**	**P-Value**	**Trace statistic**	**5% critical Value**	**P-Value**
Fewer than 1	34.45	29.80	0.013	44.51	42.92	0.0034
Fewer than 2	10.79	15.49	0.22	14.28	25.87	0.63
No. of cointegrating vectors under H_0	**l-max statistic**	**5% critical Value**	**P-Value**	**l-max statistic**	**5% critical value**	**P-Value**
Fewer than 1	23.65	21.13	0.022	30.23	25.82	0.012
Fewer than 2	6.12	14.26	0.597	9.41	19.39	0.784

Notes: See notes for Table A3.2a.

Table A3.2c
Johansen cointegration test results for ln(*rfx*) and ln(*comtot*)
(New Zealand, Sample period: 1986Q2 to 2005Q1)

	No Trend			Trend		
No. of cointegrating vectors under H_0	**Trace statistic**	**5% critical Value**	**P-Value**	**Trace statistic**	**5% critical Value**	**P-Value**
Fewer than 1	19.27	15.49	0.013	27.92	25.87	0.027
Fewer than 2	8.07	3.84	0.005	8.46	12.52	0.22
No. of cointegrating vectors under H_0	**λ-max statistic**	**5% critical Value**	**P-Value**	**λ-max statistic**	**5% critical value**	**P-Value**
Fewer than 1	11.19	14.96	0.145	19.46	19.38	0.048
Fewer than 2	8.07	3.84	0.004	8.60	12.51	0.216

Notes: See notes for Table A3.2a.

Table A3.3a
Weak exogeneity tests for ln(*rfx*), ln(*comtot*) and ln(*enetot*)
(Canada, Sample period: 1973Q1 to 2005Q1)

LR test statistic (no trend)	P-Value	LR test statistic (trend)	P-Value
3.34	0.19	10.00	0.01

Notes:
(i) Based on the benchmark model specification. The number of lags used in the test was selected based on a sequential modified likelihood-ratio test.
(ii) The likelihood-ratio (LR) test statistic follows a χ^2 distribution.

Table A3.3b
Weak exogeneity tests for ln(*rfx*), ln(*comtot*) and ln(*enetot*)
(Australia, Sample period: 1985Q1 to 2005Q1)

LR test statistic (no trend)	P-Value	LR test statistic (trend)	P-Value
15.69	0.00	14.22	0.00

Notes: See notes for Table A3.3a.

Table A3.3c
Weak exogeneity tests for ln(*rfx*) and ln(*comtot*)
(New Zealand, Sample period: 1986Q1 to 2005Q1)

LR test statistic (no trend)	P-Value	LR test statistic (trend)	P-Value
0.02	0.89	3.41	0.06

Notes: See notes for Table A3.3a.

Appendix 4
Estimation and Forecasting Results

Table A4.1a
Estimates for bilateral exchange rate equation
Benchmark, Multilateral Effects (ME) and Threshold (THR) Models
Threshold variable: $q = US\_fisbal_{t-1}$
(Canada, Sample period: 1973Q1 to 2005Q1)

	NLLS							
	Benchmark Model		**ME Model**		**Threshold Model**			
	All observations				*q > -1.861%*		*q = -1.861%*	
Variable	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**
speed of adj.	-0.102***	0.034	-0.126***	0.035	-0.235**	0.093	-0.120***	0.036
constant	2.246***	0.507	2.812***	0.528	2.556***	0.433	3.050***	0.622
Ln($comtot$)$_{t-1}$	-0.470***	0.124	-0.598***	0.124	-0.567***	0.107	-0.527***	0.117
Ln($enetot$)$_{t-1}$	0.060	0.055	0.085*	0.045	0.101***	0.031	-0.018	0.044
? $debtdif_{t-1}$	0.011***	0.002	0.011***	0.002	0.011**	0.004	0.010***	0.003
$intdif_{t-1}$	-0.694***	0.136	-0.554***	0.125	-0.421	0.267	-0.863***	0.182
US_cabal_{t-4}	--	--	0.298*	0.163	0.252	0466	0.371**	0.173
US_fisbal_{t-1}	--	--	0.380***	0.120	0.281	0.386	0.418**	0.180
R^2	0.287		0.370		0.483		0.451	
Adj. R^2	0.258		0.333		0.399		0.396	
Log likelihood	332.4		340.3		146.2		207.5	
No. of obs.	129		129		51		78	
Durbin Watson	1.43		1.62		1.73		1.86	

Notes:
(i) (***), (**), and (*) indicate statistical significance at the 1, 5 and 10 per cent levels, respectively.
(ii) Standard errors are adjusted using the Newey-West HAC procedure.
(iii) The dependant variable is expressed in local currency units.

Table A4.1b
Estimates for bilateral exchange rate equation
Benchmark, Multilateral Effects (ME) and Threshold (THR) Models
Threshold variable: *q*= *twin_def$_{t-1}$*
(Canada, Sample period: 1973Q1 to 2005Q1)

	NLLS							
	Benchmark Model		**ME Model**		**Threshold Model**			
	All observations				*q > -1.445%*		*q = -1.445%*	
Variable	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**
speed of adj.	-0.102***	0.034	-0.122***	0.037	-0.199*	0.104	-0.091**	0.036
constant	2.246***	0.507	2.928***	0.546	2.258***	0.355	3.809***	1.177
ln(*comtot*)$_{t-1}$	-0.470***	0.125	-0.632***	0.114	-0.491***	0.072	-0.764***	0.282
ln(*enetot*)$_{t-1}$	0.060	0.055	0.095*	0.050	0.081**	0.033	0.057	0.097
? *debtdif*$_{t-1}$	0.011***	0.002	0.011***	0.002	0.009*	0.005	0.012***	0.002
intdif$_{t-1}$	-0.694***	0.143	-0.589***	0.122	-0.762***	0.223	-0.672***	0.230
twin_def$_{t-1}$	--	--	0.645**	0.208	-0.123	0.138	0.697**	0.370
R^2	0.287		0.352		0.368		0.414	
Adj. R^2	0.258		0.32		0.292		0.359	
Log likelihood	332.4		338.6		157.0		187.76	
No. of obs.	129		129		57		72	
Durbin Watson	1.43		1.58		1.52		1.88	

Notes: See notes for Table A4.1a.

Table A4.1c
Estimates for bilateral exchange rate equation
Markov Switching model
Threshold variable: *q*=*US_fisbal*$_{t-1}$
(Canada, Sample period: 1973Q1 to 2005Q1)

State of Economy	$S_t = 1$		$S_t = 2$		$S_t = 1$		$S_t = 2$	
Variables	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**
Speed of adj.	-0.336***	0.042	-0.096***	0.032	-0.089**	0.038	-0.284***	0.024
constant	1.719***	0.227	3.517***	1.084	4.313***	1.157	1.981***	0.168
ln(*comtot*)$_{t-1}$	-0.364***	0.044	-0.696***	0.228	-0.737***	0.202	-0.442***	0.042
ln(*enetot*)$_{t-1}$	0.061***	0.016	0.046	0.078	-0.052	0.087	0.087***	0.018
? *debtdif*$_{t-1}$	0.001	0.003	0.015***	0.003	0.014***	0.003	0.004**	0.002
intdif$_{t-1}$	-0.763***	0.194	-0.602***	0.160	-1.238***	0.267	-0.784***	0.134
US_cabal$_{t-1}$	-0.035	0.139	0.598***	0.155	--	--	--	--
US_fisbal$_{t-1}$	-0.211	0.234	0.206	0.196	--	--	--	--
twin_def$_{t-1}$	--	--	--	--	-0.931***	0.313	-0.174	0.183
Heteroscedastity s^2	0.009***	0.001	0.015***	0.001	0.014***	0.002	0.011***	0.001
Threshold		-1.705*	0.991			-1.224	0.952	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} \le g]$		0.757***	0.114			0.709***	0.108	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} > g]$		0.779***	0.104			0.875***	0.077	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} \le g]$		0.926***	0.053			0.871***	0.074	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} > g]$		0.729***	0.112			0.691***	0.138	
Log Likelihood		350.616				349.660		
LR for the same cointegration vector		19.996	[0.000]			13.883	[0.007]	
LR for no multilateral effects		22.206	[0.000]			19.043	[0.000]	
RMSE		0.020				0.035		
No. of obs.		129				129		

Notes:

(i) Under the null, the LR statistic for the same cointegration vector is distributed as a c^2 with five degrees of freedom. The corresponding p-value is shown in parentheses.

(ii) Under the null, the LR statistic for no multilateral effects is distributed as a c^2 with four degrees of freedom. The corresponding p-value is shown in parentheses.

(iii) See notes (i) and (iii) for Table A4.1a.

Table A4.2a
Estimates for bilateral exchange rate equation
Benchmark, Multilateral Effects (ME) and Threshold (THR) Models
Threshold variable: $q = US\_fisbal_{t-1}$
(Australia, Sample period: 1985Q1 to 2005Q1)

	Benchmark Model		Multilateral Effects Model			
Variable	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**
speed of adj.	-0.133***	0.043	-0.099**	0.041	-0.105***	0.036
constant	0.308	1.604	1.012	2.192	1.106	2.107
ln(*comtot*)$_{t-1}$	-0.809**	0.336	-1.068**	1.463	-0.994**	0.383
ln(*enetot*)$_{t-1}$	0.818*	0.458	1.434**	0.715	1.373**	0.628
$Rintdif_{t-1}$	-1.746*	0.930	-1.337	0.933	-1.240	0.875
US_cabal_{t-1}	--	--	0.836**	0.348	--	--
US_fisbal_{t-1}	--	--	0.670**	0.331	--	--
$twin\_def_{t-1}$	--	--	--	--	1.478***	0.502
R^2	0.154		0.257		0.256	
Adj. R^2	0.109		0.197		0.206	
Log likelihood	142.6		147.8		147.8	
No. of obs.	81		81		81	
Durbin Watson	1.42		1.67		1.66	

Notes: See notes for Table A4.1a.

Table A4.2b
Estimates for bilateral exchange rate equation
Markov Switching model
Threshold variable: $q = US\_fisbal_{t-1}$
(Australia, Sample period: 1985Q1 to 2005Q1)

State of Economy	$S_t = 1$		$S_t = 2$		$S_t = 1$		$S_t = 2$	
Variables	**Estimate**	**S.E.**	**Estimate**	**S.E.**	**Estimate**	**S.E.**	**Estimate**	**S.E**
speed of adj.	0.015	0.068	-0.065*	0.034	-0.073*	0.038	0.126***	0.024
constant	7.784	33.250	-0.646	2.584	1.619	1.595	9.513***	1.417
ln(*comtot*)$_{t-1}$	17.700	82.368	-0.184	0.485	-1.021**	0.434	-1.639***	0.233
ln(*enetot*)$_{t-1}$	-17.888	82.697	0.570	0.552	0.852	0.551	-0.384**	0.182
$Rintdif_{t-1}$	-1.924*	1.012	-0.962	0.676	-0.703	0.515	-1.033**	0.430
US_cabal_{t-1}	-2.489***	0.437	1.171***	0.248	--	--	--	--
US_fisbal_{t-1}	-1.556***	0.426	0.993***	0.289	--	--	--	--
$twin\_def_{t-1}$	--	--	–	--	-2.004***	0.367	-1.972***	0.212
Heteroscedastity s^2	0.018***	0.003	0.026***	0.003	0.028***	0.003	0.002**	0.001
Threshold		-2.531**	0.996			-1.318	1.020	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} \le g]$		0.268	0.185			0.869***	0.046	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} > g]$		0.675***	0.187			0.869***	0.047	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} \le g]$		0.829***	0.062			0.322	0.216	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} > g]$		0.826***	0.063			0.068	0.065	
Log Likelihood			158.718				167.620	
LR for the same cointegration vector		9.939	[0.042]			24.117	[0.000]	
LR for no multilateral effect		22.309	[0.000]			30.163	[0.000]	
RMSE			0.036				0.045	
No. of obs.			81				81	

Table A4.3a
Estimates for bilateral exchange rate equation
Benchmark, Multilateral Effects (ME) and Threshold (THR) Models
Threshold variable: *q=US_fisbal*$_{t-1}$
(New Zealand, Sample period: 1986Q2 to 2005Q1)

	Benchmark Model		Multilateral Effects Model			
Variable	**Estimate**	**S.E**	**Estimate**	**S.E**	**Estimate**	**S.E**
speed of adj.	-0.099*	0.053	-0.074**	0.035	-0.076**	0.036
constant	4.132*	2.389	3.400	2.219	3.063*	2.107
ln(*comtot*)$_{t-1}$	-0.765	0.507	-0.473	0.476	-0.408	0.383
Rintdif$_{t-1}$	-0.705*	0.433	-0.444	0.435	-0.422	0.875
US_cabal$_{t-1}$	--	--	1130	0.291	--	--
US_fisbal$_{t-1}$	--	--	1.004	0.239	--	--
twin_def$_{t-1}$	--	--	--	--	2.097***	0.502
R^2	0.095		0.369		0.368	
Adj. R^2	0.057		0.324		0.332	
Log likelihood	133.5		147.3		147.2	
No. of obs.	76		76		76	
Durbin Watson	1.21		1.79		1.78	

Notes: See notes for Table A4.1a.

Table A4.3b
Estimates for bilateral exchange rate equation
Markov Switching model
Threshold variable: *q=US_fisbal*$_{t-1}$
(New Zealand, Sample period: 1986Q2 to 2005Q1)

State of Economy	$S_t = 1$		$S_t = 2$		$S_t = 1$		$S_t = 2$	
Variables	**Estimate**	**S.E.**	**Estimate**	**S.E.**	**Estimate**	**S.E.**	**Estimate**	**S.E**
speed of adj.	-0.141***	0.024	-0.065*	0.037	-0.133***	0.013	-0.066*	0.034
constant	0.901***	0.266	1.067*	0.539	1.319***	0.066	1.916	1.588
ln(*comtot*)$_{t-1}$	-0.077	0.049	0.090	0.161	-0.163***	0.011	-0.086	0.380
Rintdif$_{t-1}$	-0.191	0.391	-0.370	2.421	-0.191	0.239	-0.471**	0.215
US_cabal$_{t-1}$	1.109***	0.050	1.135***	0.506	--	--	--	--
US_fisbal$_{t-1}$	0.787***	0.217	0.952***	0.387	--	--	--	--
Twin_def$_{t-1}$	--	--	--	--	-1.980***	0.195	-2.183***	0.413
Heteroscedastity s^2	0.009**	0.004	0.028	0.310	0.009***	0.002	0.028***	0.003
Threshold		-2.781	1.677			-0.692	2.527	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} \le g]$		0.321	0.470			0.572***	0.156	
$P[S_t = 1 \mid S_{t-1} = 1, q_{t-1} > g]$		0.769	0.477			0.703***	0.247	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} \le g]$		0.814***	0.275			0.848***	0.105	
$P[S_t = 2 \mid S_{t-1} = 2, q_{t-1} > g]$		0.814***	0.249			0.643***	0.242	
Log Likelihood		157.971				157.458		
LR for the same cointegration vector						9.705	[0.021]	
LR for no multilateral effect		23.229	[0.000]			22.203	[0.000]	
RMSE		0.051				0.046		
No. of obs.		76				76		

Notes: See notes for Table A4.2a

Table A4.4a
Out-of-sample forecasting (dynamic)
using actual values for the explanatory variables
Canada
Estimation period: 1973Q1 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1

Model	RMSE	Theil Coefficient
Benchmark model	0.0440	0.0856
Multilateral effects model	0.0361	0.0747
Threshold model	0.0375	0.0782

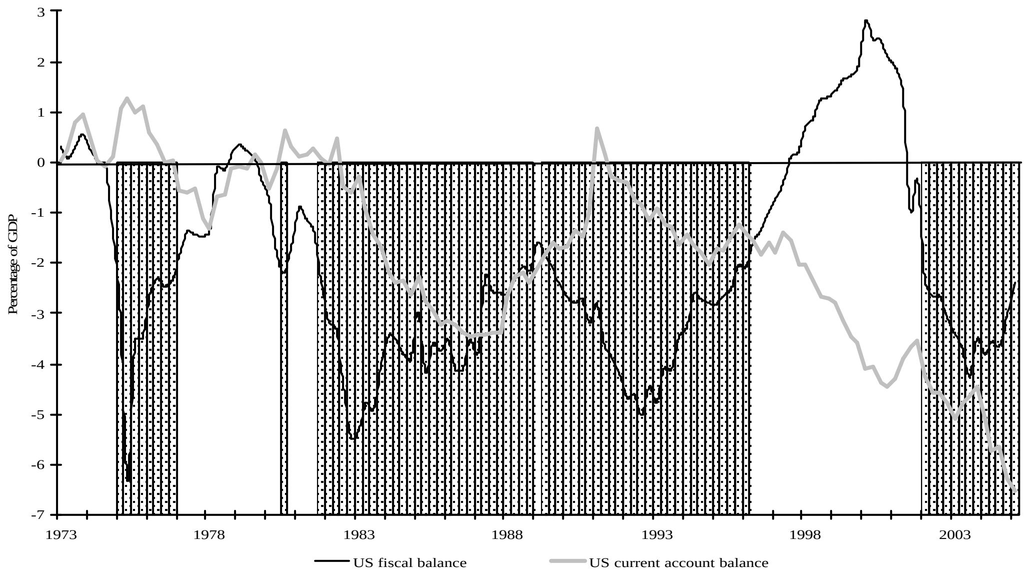
Table A4.4b
Out-of-sample forecasting (dynamic)
using actual values for the explanatory variables
Australia
Estimation period: 1985Q1 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1

Model	RMSE	Theil Coefficient
Benchmark model	0.1066	0.1809
Multilateral effects model	0.0550	0.1056

Table A4.4c
Out-of-sample forecasting (dynamic)
using actual values for the explanatory variables
New Zealand
Estimation period: 1986Q2 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1

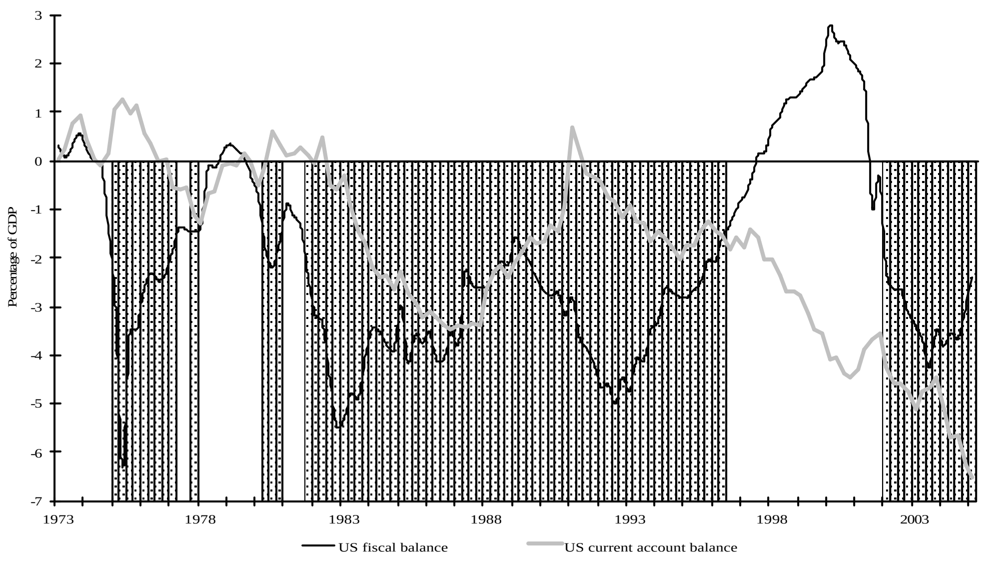
Model	RMSE	Theil Coefficient
Benchmark model	0.1178	0.1348
Multilateral effects model	0.1174	0.1715

Figure A4.1a
Multilateral variables in the two regimes as identified by the THR model using the U.S. fiscal deficit as the threshold variable
(Canada, Sample period 1973Q1 to 2005Q1)



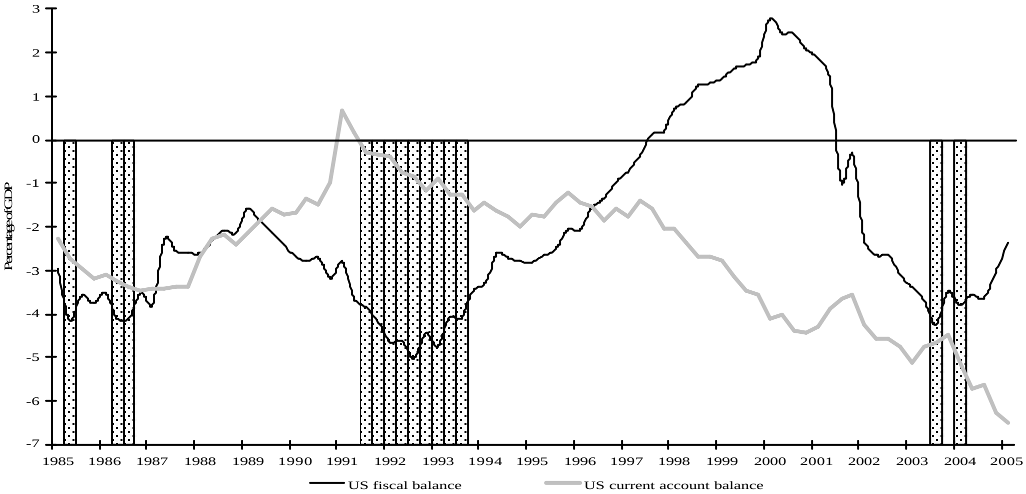
Note: Shaded regions indicate periods of total government fiscal balance less than the -1.86 per cent threshold.

Figure A4.1b
Multilateral variables in the two regimes as identified by the THR model using the U.S. twin deficits measure as the threshold variable
(Canada, Sample period 1973Q1 to 2005Q1)



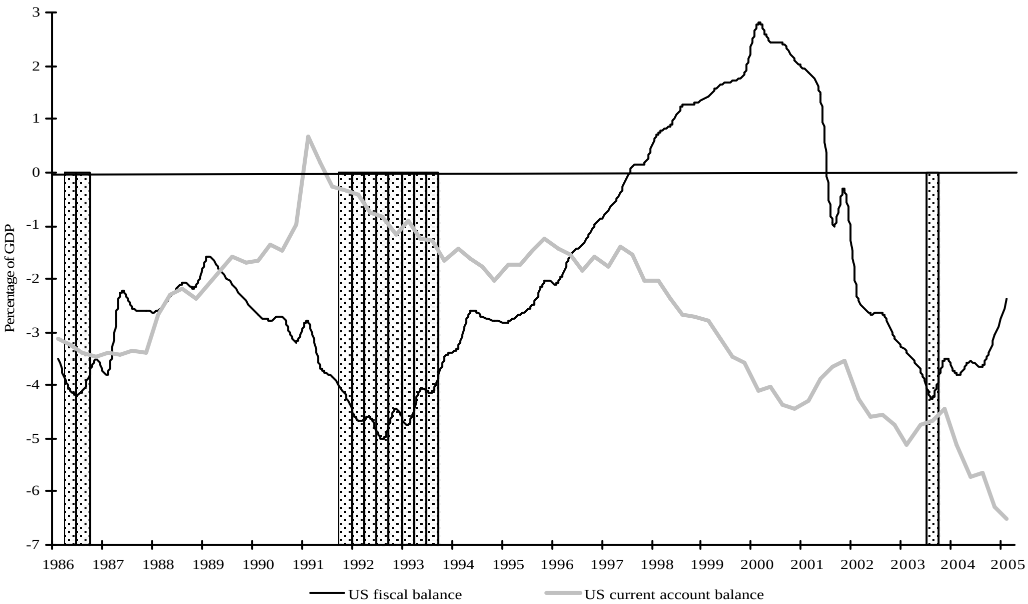
Note: Shaded regions indicate periods of total government fiscal balance less than the -1.445 per cent threshold.

Figure A4.1c
Multilateral variables in the two regimes as identified by the THR model using the U.S. fiscal deficit as the threshold variable
(Australia, Sample period 1985Q1 to 2005Q1)



Note: Shaded regions indicate periods of total government fiscal balance less than the -3.8 per cent threshold.

Figure A4.1d
Multilateral variables in the two regimes as identified by the THR model using the U.S. fiscal deficit as the threshold variable
(New Zealand, Sample period 1986Q2 to 2005Q1)



Note: Shaded regions indicate periods of total government fiscal balance less than the -3.86 per cent threshold.

Figure A4.2a
Dynamic simulations of competing models
(Canada, Sample period: 1973Q1 to 2005Q1)

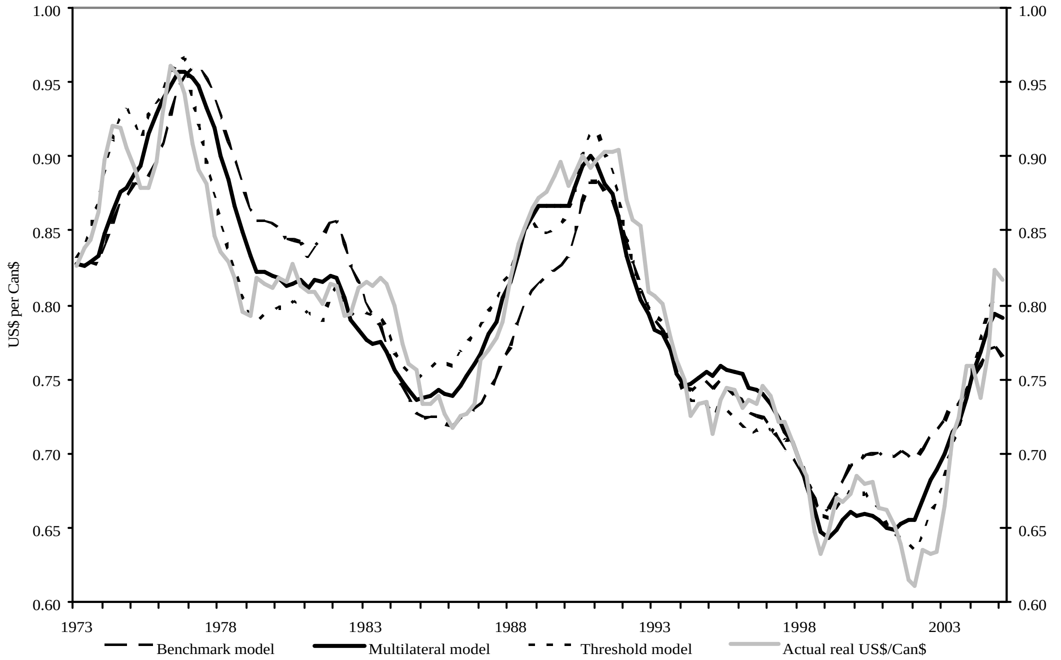


Figure A4.2b
Dynamic simulations of competing models
(Australia, Sample period: 1985Q1 to 2005Q1)

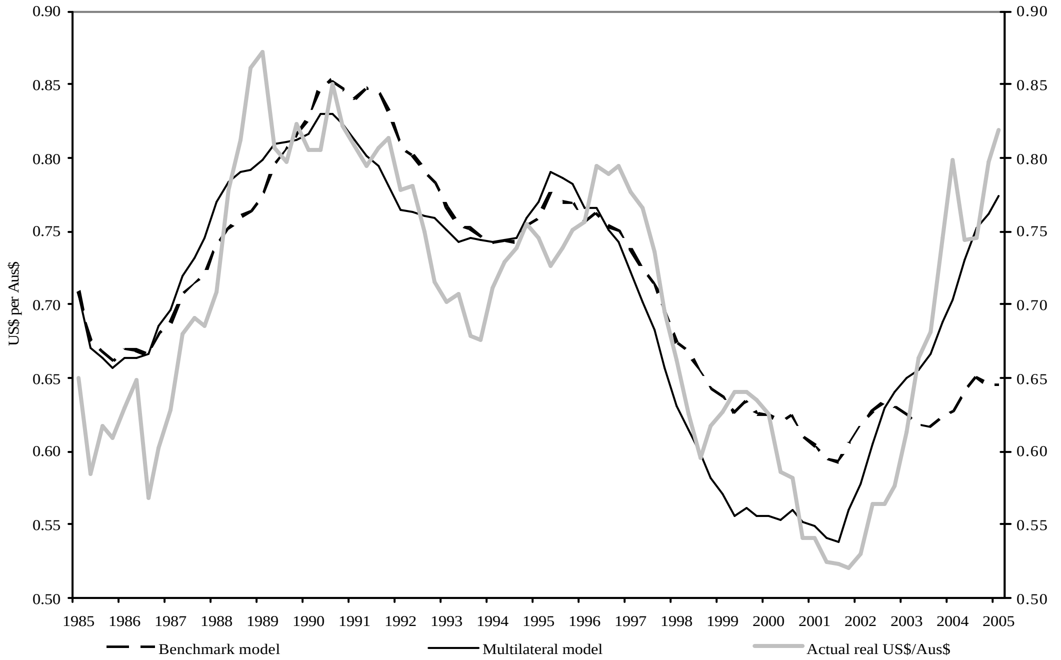


Figure A4.2c
Dynamic simulations of competing models
(New Zealand, Sample period: 1986Q2 to 2005Q1)

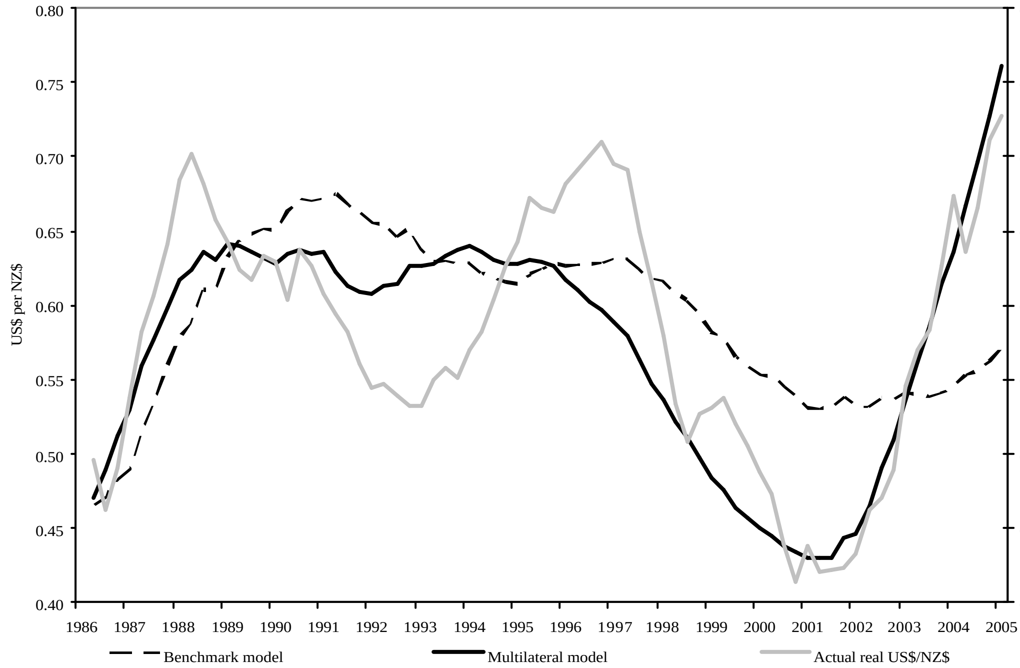


Figure A4.3a
Dynamic simulations of competing models
Markov Switching model
(Canada, Sample period: 1973Q1 to 2005Q1)

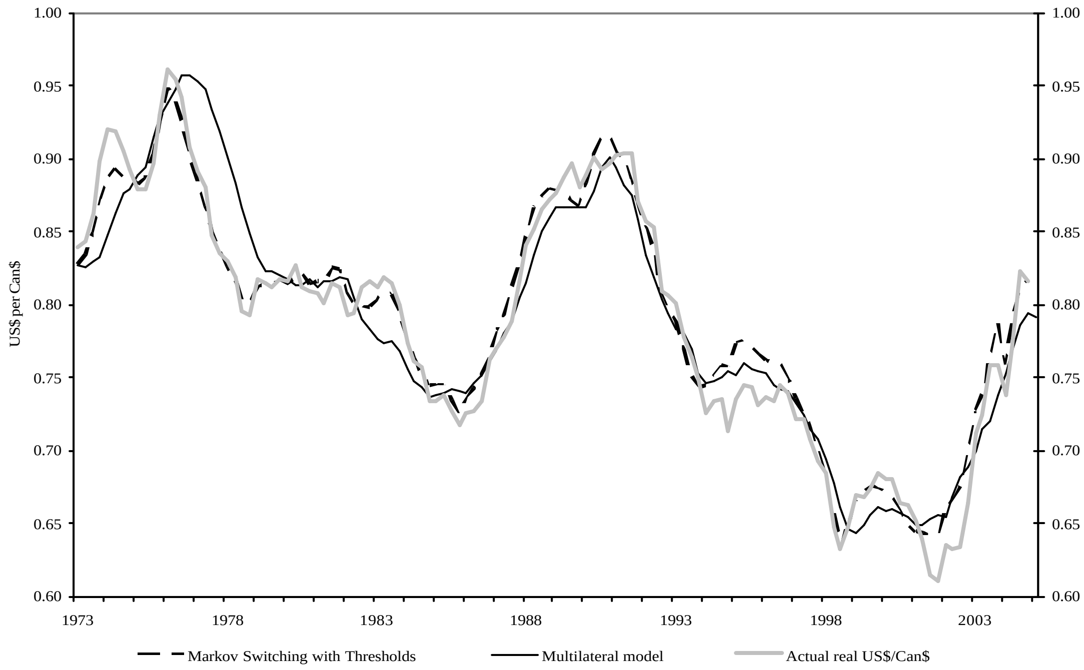


Figure A4.3b
Dynamic simulations of competing models
Markov Switching model
(Australia, Sample period: 1985Q1 to 2005Q1)

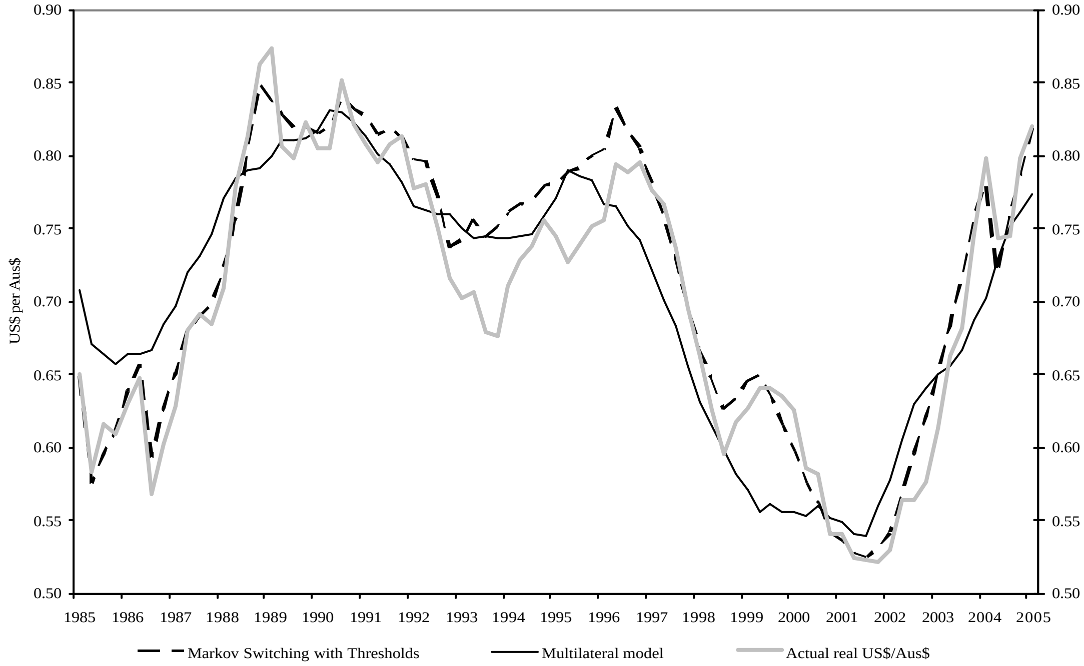


Figure A4.3c
Dynamic simulations of competing mo dels
Markov Switching model
(New Zealand, Sample period: 1986Q2 to 2005Q1)

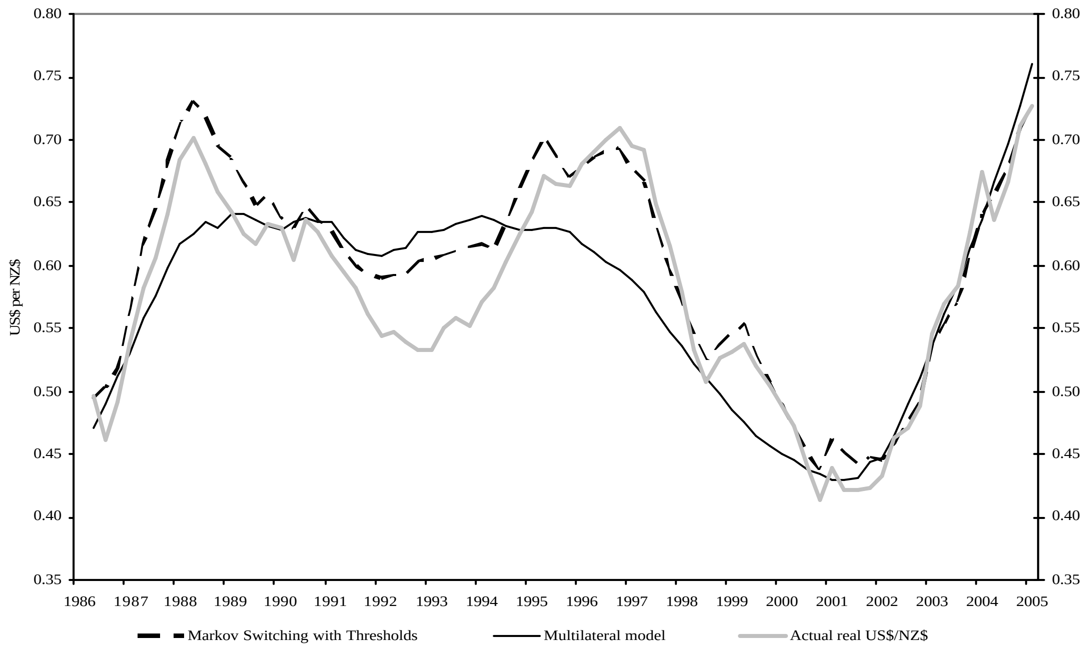


Figure A4.4a
Out-of-sample performance of competing models using actual values for explanatory variables
Canada
Estimation period: 1973Q1 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1

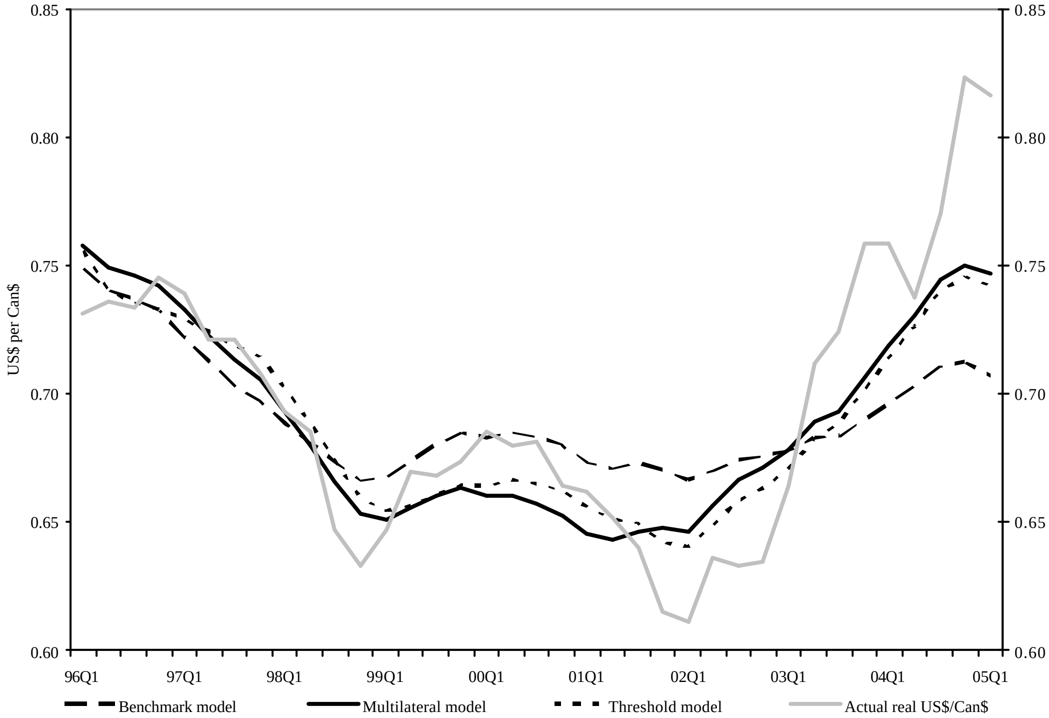


Figure A4.4b
Out-of-sample performance of competing models using actual values for explanatory variables
Australia
Estimation period: 1985Q1 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1

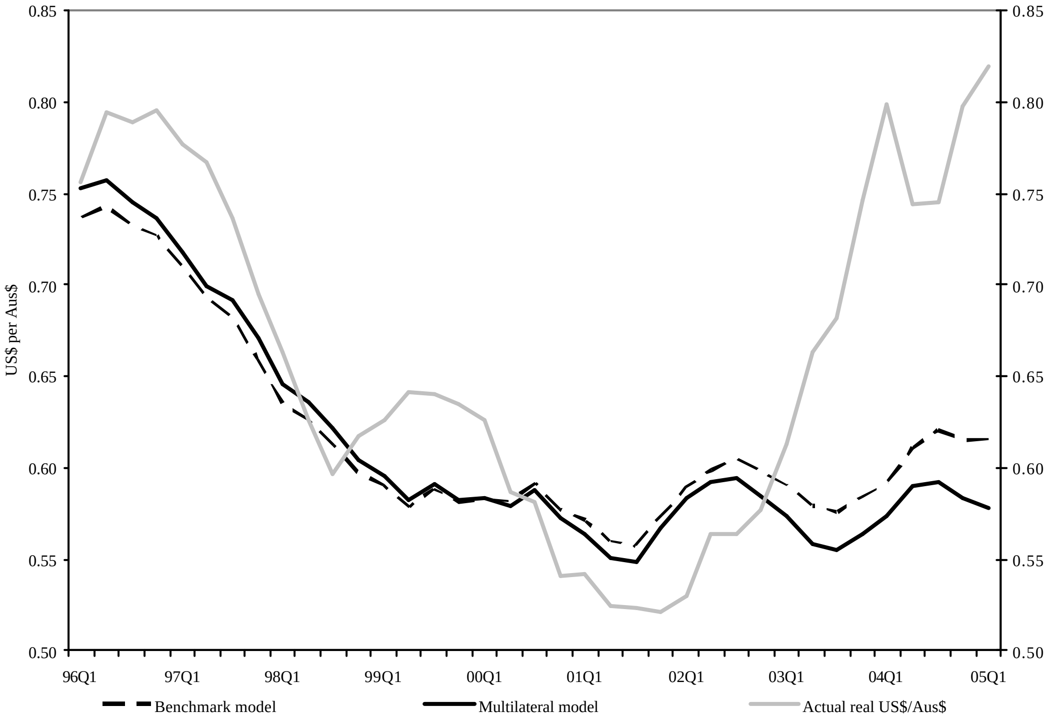
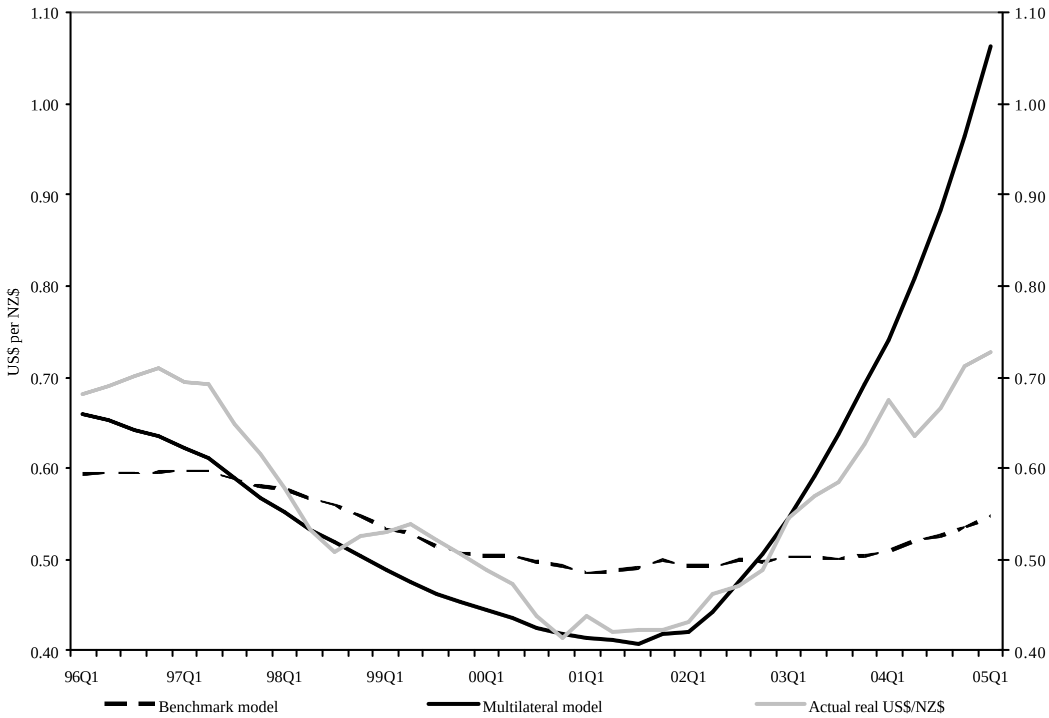


Figure A4.4c
Out-of-sample performance of competing models using actual values for explanatory variables
New Zealand
Estimation period: 1986Q2 to 2003Q4
Forecasting period: 2004Q1 to 2005Q1



Appendix 5
Construction of the likelihood function in the MSTV Model

In order to construct the likelihood function in the MSTV model, we first need to derive the non-time-varying transition probability matrix Γ^*. Using both the time-varying transition probability matrix from equation (5) and the definition of the state variable S_t^*, we can derive the following expression for the probability of $S_t^* = 2$ conditional on $S_{t-1}^* = 2$:

$$\begin{aligned}
P[S_t^* = 2 \mid S_{t-1}^* = 2] &= P[S_t^* = 2, q_t \le g \mid S_{t-1}^* = 2, q_{t-1} \le g] \\
&= P[q_t \le g \mid S_{t-1}^* = 2, q_{t-1} \le g]P[S_t^* = 2 \mid q_t \le g, S_{t-1}^* = 2, q_{t-1} \le g] \\
&= F^q(g)P[S_t^* = 2 \mid S_{t-1}^* = 2, q_{t-1} \le g] \\
&= F^q(g)n_2.
\end{aligned}$$

Similar expressions can also be derived for all the other relevant conditional probabilities (i.e., $P[S_t^* = i \mid S_{t-1}^* = j]$ for $i, j = \{0,1,2,3\}$.

In order to construct the likelihood function in the MSTV, we draw on Hamilton's (1989) filter to construct $P[S_t^* \mid \Psi_{t-1}, X_t]$ for $t = 1, 2, \ldots, T$. Note that:

$$\begin{aligned}
P[S_t^* \mid \Psi_{t-1}, X_t] &= \sum_{S_{t-1}^*=0}^{3} P[S_t^*, S_{t-1}^* \mid \Psi_{t-1}, X_t] \\
&= \sum_{S_{t-1}^*=0}^{3} P[S_{t-1}^* \mid \Psi_{t-1}, X_t] P[S_t^* \mid S_{t-1}^*, \Psi_{t-1}, X_t] \quad \text{(A.5.1)} \\
&= \sum_{S_{t-1}^*=0}^{3} P[S_{t-1}^* \mid \Psi_{t-1}] P[S_t^* \mid S_{t-1}^*],
\end{aligned}$$

where the third equality holds due to the definition of the non-time-varying transition probability matrix Γ^* and Bayes' law. The latter implies that:

$$P[S_{t-1}^* \mid \Psi_{t-1}] = P[S_{t-1}^* \mid \Psi_{t-1}, X_{t-1}]P[X_{t-1} \mid \Psi_{t-1}] / P[X_{t-1} \mid S_{t-1}^*, \Psi_{t-1}] = P[S_{t-1}^* \mid \Psi_{t-1}, X_{t-1}].$$

And given $P[S_{t-1}^* \mid \Psi_{t-1}]$ and Γ^*, eq.(A.5.1) therefore yields $P[S_t^* \mid \Psi_{t-1}, X_t]$. Then, given the data in y_t, we can update our inference on the current state of the economy according to the following updating formula:

$$\begin{aligned} P[S_t^* \mid \Psi_t] &= P[S_t^* \mid \Psi_{t-1}, y_t, X_t] \\ &= \frac{f(y_t, S_t^* \mid \Psi_{t-1}, X_t)}{f(y_t \mid \Psi_{t-1}, X_t)} \qquad \text{(A.5.2)} \\ &= \frac{P[S_t^* \mid \Psi_{t-1}, X_t] f(y_t \mid S_t^*, \Psi_{t-1}, X_t)}{\sum_{S_t=0}^{3} P[S_t^* \mid \Psi_{t-1}, X_t] f(y_t \mid S_t^*, \Psi_{t-1}, X_t)}. \end{aligned}$$

Using $P[S_t^* \mid \Psi_{t-1}, X_t]$, $f(y_t \mid S_t^*, \Psi_{t-1}, X_t)$, and y_t as inputs, the formula in (A.5.2) updates our inference on the current state of the economy, $P[S_t^* \mid \Psi_t]$. Iterating eqs. (A.5.1) and (A.5.2) from $t = 1$ to T then generates the sequence of $P[S_t^* \mid \Psi_t]$ for $t = 1, 2, \dots, T$.

Maximizing the log-likelihood function, we can then estimate the parameters of the MSTV model (i.e., $q_1, q_2, m_1, m_2, n_1, n_2, g, s_{e1}, s_{e2}$). Before implementing the Maximum Likelihood (ML) estimation, however, we need to ensure that we have a consistent estimate of the cdf of the threshold variable q_t, $F^q(g)$.[36] Since our model satisfies the regularity conditions of the ML estimation, our ML estimates have standard asymptotic properties.[37]

[36] In this paper, $F^q(g)$ is approximated by the cumulative normal distribution with the mean and standard deviation of q_{t-1}. One suitable way to estimate F^q is to use a nonparametric kernel density estimation.

[37] Several caveats should be noted. First, we cannot test the null of no threshold effects. This is because, under the null, the model cannot be identified. One way to make it possible to test for the null might be to construct the LM-based test statistics developed by Hansen (1996). Second, the Markov-switching model developed in this paper nests a simple threshold model if we set $m_1 = n_2 = 1$ and $m_2 = n_1 = 0$. However, we cannot test these restrictions with the LR statistics because these restrictions are on the boundary of the admissible range of variables m_1, n_1, m_2, and n_2 (i.e., [0,1]).

Appendix 6
Construction of the in-sample forecasts in the MSTV Model

Notice that $P[S_t^* \mid \Psi_t]$ is the probability of state S_t^* conditional on the information up to period t. Using Kim (1994)'s smoothed inferences, we can also derive the probability of state S_t^* conditional on the whole sample, $P[S_t^* \mid \Psi^T]$. Given the maximum likelihood estimates, q_1^{ML} and q_2^{ML}, and the smoothed probability of the state of the economy, $P[S_t^* \mid \Psi^T]$, we can construct the in-sample forecast of the dependent variable, y_t^f using the following expression:

$$
\begin{aligned}
y_t^f &= X_t \left\{ P[S_t^* = 0, S_t^* = 1 \mid \Psi^T] q_1^{ML} + P[S_t^* = 2, S_t^* = 3 \mid \Psi^T] q_2^{ML} \right\} \\
&= X_t \left\{ \sum_{i=0}^{1} P[S_t^* = i \mid \Psi^T] q_1^{ML} + \sum_{i=2}^{3} P[S_t^* = 2, S_t^* = 3 \mid \Psi^T] q_2^{ML} \right\}
\end{aligned}
\quad (A.6.1)
$$

Eq. (A.6.1) implies that the forecast of the dependent variable, y_t^f, is given by multiplying the deterministic explanatory variable X_t by the weighted average of the estimated coefficients in the two regimes, q_1^{ML} and q_2^{ML}, with the smoothed probabilities of the states of the economy as time-varying weights.