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Finding an empirically realistic model of pricing dynamics is of crucial importance for the conduct of monetary policy. One reason is obvious: central banks cannot be expected to maintain a reliable and stable macroeconomic environment if their deliberations are not guided by an accurate conception of the effects of their actions. But the questions addressed at this conference are significant for other reasons as well. For example, the way in which prices are set has important consequences for the central bank's stabilization objectives if it wishes to maximize social welfare. Price stability matters for welfare because of the way prices are adjusted in response to changing conditions. In the textbook world of perfectly flexible wages and prices, maintaining stable purchasing power of the unit of account would be of little significance. It then follows as well that the sense in which it is desirable for inflation to be stabilized will depend on how inflation dynamics are related to real distortions.

As we shall see, the nature of inflation dynamics matters for several aspects of the optimal conduct of monetary policy. First, alternative pricing models can lead to different views of the optimal long-run inflation target. Second, they can imply different perspectives of the optimal dynamic responses of inflation to disturbances, and hence to different views of the degree to which temporary departures from the long-run target should be allowed. And finally, alternative models can change the form of the targeting rule to which a central bank should commit itself to bring about those optimal responses to disturbances, while guaranteeing the desired long-run average rate of inflation. These aspects demonstrate that the way prices are set should affect the nature of a central bank's policy objectives and commitments, and not simply the actions that it must take to fulfill those commitments.

I shall illustrate these points in a simple example drawn from recent debates on the correct empirical specification of the aggregate-supply relationship. To do so, I shall compare specifications that differ in the degree of inflation inertia that they imply. A straightforward way of incorporating inertia into the dynamics of inflation has been proposed by Christiano, Eichenbaum, and Evans (2001) and used in the empirical models of Smets and Wouters (2002), Altig et al. (2002), Boivin and Giannoni (2003), Sbordone (2003), and Giannoni and Woodford (2003). In this extension of the standard Calvo model of staggered pricing, prices do not remain fixed between the random occasions on which they are reoptimized. The log price of a good i in period t can be expressed as

$$p_t(i) = v_t(i) + \gamma P_{t-1},$$

where $v_t(i)$ is a good-specific base price, and the coefficient $0 \le \gamma \le 1$ indicates the degree of indexation of prices to the (lagged) general price index P_{t-1} . The base price remains fixed between the random occasions on which it is reoptimized; thus, in periods when no reconsideration of the base price occurs, the price of good i increases by fraction γ of the lagged overall rate of inflation. (The lag is presumably attributed to lags in the availability of data on the aggregate price index.) The base price is reset at random intervals to the level that maximizes expected discounted profits at that time.

As shown in Woodford (2003, chapter 3), this leads to an aggregate-supply relationship of the form

$$\pi_t - \gamma \pi_{t-1} = \kappa x_t + \beta E_t (\pi_{t+1} - \gamma \pi_t) + u_t,$$
 (1)

where π_t is the inflation rate (change in the price index P_t), x_t is an output-gap measure, $0 < \beta < 1$ is the discount factor, the coefficient $0 < \kappa < 1$ depends on both the frequency with which prices are re-optimized and the degree of real rigidity, and u_t is an exogenous cost-push shock. Note that when $\gamma = 0$, this reduces to the familiar New Keynesian Phillips curve considered in several papers at this conference. Furthermore, the inertial specification that results when $\gamma = 0$ is closely related to the hybrid model of Galí and Gertler (1999), also considered in extensive recent empirical literature, though the microeconomic foundations proposed by these authors are slightly different. Finally, in the limiting case that $\gamma = 1$, equation (1) is essentially the form of aggregate-supply relationship proposed by Fuhrer and Moore (1995), on the basis of different micro-foundations and fit to U.S. data in a number of studies.

When we consider the goals of monetary stabilization policy, what are the consequences of the estimated value of γ , which indicates the importance of lagged inflation as a determinant of current inflation? Woodford (2003,

chapter 6) shows that in a model with staggered pricing of the kind just described, maximization of the expected utility of the representative household is equivalent (up to a quadratic approximation) to minimization of the expected discounted value of a loss function of the form

$$L_{t} = (\pi_{t} - \gamma \pi_{t-1})^{2} + \lambda_{x} (x_{t} - x^{*})^{2},$$
(2)

if one abstracts from monetary frictions, where $\lambda_x > 0$, $x^* > 0$ are functions of model parameters.

Thus, the appropriate stabilization objective with regard to inflation is not stabilization of the rate of inflation, but rather of $\pi_t - \gamma \pi_{t-1}$. The reason is simple: in this model, inflation results in real distortions owing to the fact that the prices of different goods are not adjusted in a perfectly synchronized fashion, and this results in price misalignments. The distortion is minimized by a current inflation rate equal to $\gamma \pi_{t-1}$, which is the rate of automatic increases in the prices that are not reoptimized in period t. Inflation occurs at this rate when conditions are such that reoptimizing firms choose a rate of price increase equal to $\gamma \pi_{t-1}$, the rate at which their prices would have increased had they not reoptimized, and this is the case in which the fact that some firms reoptimize while others do not does not increase the misalignment of prices. Note in the Fuhrer-Moore case that it is only the rate of inflation acceleration that matters for welfare, and not the absolute rate of inflation at all (abstracting from monetary frictions).

If the model is extended to allow for monetary frictions (and hence a positive demand for base money despite its being dominated in rate of return), the appropriate welfare-theoretic loss function (2) takes the more general form

$$L_{t} = (\pi_{t} - \gamma \pi_{t-1})^{2} + \lambda_{x} (x_{t} - x^{*})^{2} + \lambda_{i} (i_{t} - i^{m})^{2},$$
(3)

where $\lambda_i > 0$ depends on the size of the monetary frictions, and i^m is the interest rate (if any) paid on base money. When $\lambda_i > 0$, the absolute rate of inflation is relevant to welfare even when $\gamma = 1$, since the long-run average nominal interest rate will depend on the long-run average rate of inflation. Suppose, for example, that interest rates are related to inflation and real activity through the log-linear Euler equation

$$x_{t} = E_{t}x_{t+1} - \sigma(i_{t} - E_{t}\pi_{t+1} - r_{t}^{n})$$
(4)

of the basic neo-Wicksellian model presented in Woodford (2003, chapter 4), where $\sigma > 0$ represents the intertemporal elasticity of substitution, and r_t^n is the (exogenously varying) natural rate of interest. The long-run

average rates of interest and inflation must therefore satisfy the Fisher relationship

$$E[i] = \bar{r} + E[\pi],$$

where $\bar{r} \equiv -\log \beta > 0$ is the long-run average value of the natural rate of interest.

I now turn from the appropriate loss function, describing the objectives of monetary policy, to the characterization of the optimal equilibrium paths of inflation and other variables. The first concern relates to the optimal long-run inflation target. Let us consider the problem of choosing a path for the inflation rate, under commitment, from some date t_0 onward in the case where there are no random disturbances ($u_t = 0$, $r_t^n = \bar{r}$ at all times). This amounts to choosing a sequence $\{\pi_t\}$ to minimize the discounted sum of losses (equation 3), if the associated paths of the output gap and the interest rate are determined by equations (1) and (4). One can show that the optimal sequence converges asymptotically to a constant long-run inflation target $\bar{\pi}$. To a first-order approximation (adequate in the case of small enough disturbances), this is also the long-run average value of inflation under an optimal state-contingent commitment in the presence of random disturbances.

In the case of standard Calvo pricing $(\gamma=0)$, the optimal long-run inflation target is given by

$$\bar{\pi} = -\frac{\lambda_i}{\lambda_i + \beta} (\bar{r} - i^m), \qquad (5)$$

as shown in Woodford (2003, chapter 7). Note that in the flexible-price limit $(\lambda_i \to \infty)$, ¹

this reduces to

$$\bar{\pi} = -(\bar{r} - i^m) < 0, \tag{6}$$

which is the usual Friedman rule: bring about a rate of deflation where the interest rate differential between the monetary base and other riskless short-term nominal assets is eliminated. Instead, in the "cashless" limit of an economy with negligible monetary frictions $\lambda_i \rightarrow 0$, the optimal inflation target reduces to

^{1.} In the case of flexible prices, the direct distortions associated with inflation disappear. Because I have normalized the π_t^2 term in equation (3) with a coefficient of 1, this corresponds to infinitely large values for λ_x and λ_i .

$$\bar{\pi} = 0$$
.

When monetary frictions and sticky prices coexist (the case of finite positive λ_i), the optimal inflation target lies between these extremes.²

When we allow for inflation inertia $(\gamma > 0)$, matters are more complex. The optimal inflation target is given by

$$\bar{\pi} = -\frac{\lambda_i}{\lambda_i + \beta(1-\gamma)(1-\beta\gamma)}(\bar{r} - i^m),$$

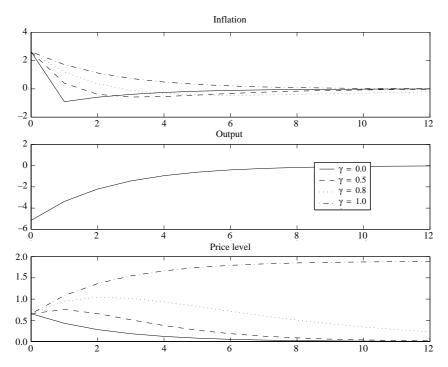
generalizing equation (5). Note that a larger γ means a lower optimal inflation target. Indeed, in the limiting case that $\gamma=1$, the Friedman rule (equation 6) is optimal regardless of the size of λ_i . As noted above, the reason is that distortions resulting from imperfect synchronization of price changes depend on the rate of inflation acceleration, and not on the absolute rate of inflation. Any constant long-run inflation rate is therefore equally good from that point of view, and it is only the minimization of the distortions associated with a positive opportunity cost of holding money that matters in the determination of $\bar{\pi}$.

One's belief about the empirically realistic value of γ also has important consequences for the optimal responses to random shocks. Consider the optimal response to a transitory, unforecastable cost-push shock u_t , under an optimal state-contingent commitment. In the case of a small enough shock, the optimal responses (per unit unexpected in u_t at the date corresponding to zero on the horizontal axis in the figure) of inflation, output, and the price level are of the form shown in Figure 1. See Woodford (2003, chapter 7) for further discussion of the kind of calculations involved in this figure. For the sake of simplicity, I abstract from monetary frictions, and the numerical results assume calibrated parameter values $\beta=0.99$, $\kappa=0.024$, as described in detail in the book manuscript.

The optimal dynamic response to such a disturbance depends in important ways on the value of γ . When $\gamma=0$ (standard Calvo pricing), it is optimal to immediately begin to undo the price increase resulting from such a shock as soon as the effects of the shock on the aggregate-supply relationship have dissipated (see analysis in Clarida, Galí, and Gertler 1999). The commitment to do so restrains price increases in the period of the shock, and as a result allows a reduction in the distortions resulting from inflation in that period without requiring a severe contraction of output relative to its

I have abstracted from a variety of reasons for the desirability of a slightly positive inflation target, such as measurement bias in the price index or downward nominal-wage rigidity.

Figure 1 Optimal responses to a positive cost-push shock under commitment, for alternative degrees of inflation inertia



natural rate. If, instead, γ is substantially positive, it is optimal to allow prices to continue to increase, albeit at a slower rate, for a time following the shock, even though it is still optimal to commit to an eventual period in which the inflation rate will undershoot its long-run target value, so that the price level eventually returns to the path that would have been expected in the absence of the shock. In the limiting case that $\gamma=1$, the period of undershooting never occurs under an optimal commitment; instead, inflation is only gradually reduced to its long-run target value, and the price level remains permanently higher. Results with some of the same flavour are also

^{3.} It is worth noting, however, that inflation does not remain permanently higher as a result of such a disturbance under an optimal commitment. It might be thought that our conclusion above—that only the rate of acceleration of inflation matters for welfare when $\gamma=1$ —would imply that cost-push disturbances should be allowed to permanently affect the rate of inflation as long as inflation is expected to stabilize again at *some* rate. But this is not true under the optimal state-contingent commitment, even when we abstract from monetary frictions. Allowing for monetary frictions provides another reason for the long-run average inflation rate not to be changed by a temporary disturbance.

obtained by Steinsson (2002) in the context of a model with backward-looking rule-of-thumb price-setters similar to the model of Galí and Gertler 1999.

Thus far, I have considered the consequences of the value of γ for the nature of the equilibrium path of inflation that one should seek to bring about through monetary policy. One can also consider the consequences of this parameter estimate for the form of an optimal policy rule to which a central bank might commit itself to achieve an equilibrium of the kind just characterized. A particularly appealing way in which to specify such a policy is to use a target criterion as part of a forecast targeting procedure. This is a policy rule under which the central bank is committed to use its policy instrument in whatever way is needed to ensure that the projected evolution of the economy continues to satisfy the target criterion. As discussed in Woodford (2003, chapter 8), one important aspect of this way of specifying the policy rule is that a characterization of optimal policy can be given in these terms that is robust to alternative specifications of disturbance processes.

As shown in Giannoni and Woodford (2003), the optimal target criterion takes a simple form in the cashless limit of the model discussed above; the central bank should ensure that in each period,

$$\pi_t - \gamma \pi_{t-1} + \frac{\lambda_x}{\kappa} (x_t - x_{t-1}) = 0.$$
 (7)

Remember that in this case, the optimal long-run inflation target is given by $\bar{\pi} = 0$. This indicates that a short-run departure of the projected inflation rate from the long-run target should be acceptable to the extent that it (i) reflects automatic inflation as a result of the indexation of non-yetreoptimized prices to the lagged price index; or (ii) is offset by a projected change in the output gap. Note that (as long as $\gamma < 1$) neither factor would justify a projected *permanent* departure from the long-run inflation target (here equal to zero). The amount of inflation in the future that can be justified as a result of inflation that has already occurred should fall to zero as one looks far enough ahead, and changes in the output gap should similarly not be projected to continue indefinitely. The value of γ matters for the formulation of the target criterion and not simply in determining what the feasible inflation/output trade-offs should be at various future horizons. A larger γ justifies a larger weight on recent past inflation in the calculation of the acceptable short-run inflation projection (or alternatively, a slower projected rate of return to the long-run inflation target).

Consequently, we see that a judgment of the value of γ should have important consequences for the characterization of optimal policy in several

respects. Which of the cases examined here should be considered as a basis for policy deliberations? Recent empirical literature tends to favour specifications with large values of γ . Several authors have even argued for the realism of assuming full indexation ($\gamma=1$). The papers presented at this conference have also tended to support the view that lagged inflation is an important determinant of current inflation, and that this effect should be considered structural, as in the model outlined above with $\gamma>0$.

Some important questions remain, however, before we can be comfortable making policy on the basis of the conclusions regarding the character of optimal policy when γ is large. These questions represent important caveats to the conclusions that I have just described. But they are also reasons to question whether the current empirical literature can answer the questions about the nature of price-setting that must be settled in order to reach firm conclusions about optimal policy.

Many of the studies that argue for the importance of inflation inertia use data from the 1960s through the 1980s, when policy in both the United States and Canada allowed inflation to drift without an apparent anchor for quite some time. (Axel Leijonhufvud once characterized this regime as a "random-walk monetary standard.") But should we really expect inflation inertia to continue to characterize price-setting behaviour to the same extent in an environment of more credible commitment to an inflation target? Even if a model of staggered pricing with a large value of γ accurately describes pricing dynamics in the period just mentioned, it is not clear that such indexation to lagged prices is immutable, rather than something that might well change under an alternative policy commitment—indeed, as something that may well be changing already, given the policy changes in the two countries over the past 15 years. After all, it hardly makes sense to think that indexation to a lagged price index should be an institutional necessity, as opposed to an adaptation to particular conditions.

Moreover, it is not clear that the evidence offered in favour of the hypothesis of indexation price to actual past inflation could not be equally well explained by indexation of prices to past expectations regarding the current rate of inflation. This is particularly so in the period from 1965 to 1985, when there was substantial serial correlation in inflation, and past expectations of current inflation would reasonably have been highly correlated with the past rate of inflation. While a large number of recent empirical studies have estimated the degree of inflation inertia under the assumption that it is the actual past rate of inflation that should affect current inflation with a greater or lesser coefficient, none have sought to distinguish the effect of actual past inflation from past inflation expectations.

But these alternative interpretations of the evidence for inflation inertia would have significantly different consequences for the character of optimal policy. For example, if one supposes that there has recently been considerable backward-looking indexation, but that it would disappear in an environment of consistently low and stable inflation (or deflation), then one might argue that it would be more reasonable to commit to a monetary policy rule that leads to optimal outcomes in a world without indexation, given that such a commitment should lead to an abandonment of indexation. On the other hand, commitment to an optimal rule for an economy with large γ would be self-defeating, since it would tend to bring about an end to the conditions under which the rule had been expected to be optimal.

Similarly, if one assumes that prices are automatically indexed between occasions of reoptimization, but that the indexation is to past expectations of inflation (perhaps averaged over several prior quarters), one should not expect large distortions to result from a policy of committing to a rapid return of inflation to its normal level (or even to undershoot it) following a cost-push shock. When an increase in inflation associated with such disturbances is not expected to persist, such an increase should not have much effect on the automatic component of inflation in subsequent quarters. Thus, it might be optimal to commit to return inflation to its normal level much more rapidly than the simulations in Figure 1 would indicate in the case of a high value of γ .

Determining which of these interpretations is more realistic should be a topic for further empirical research on inflation dynamics. More theoretical analysis of the way in which alternative specifications matter for the conclusions that one would draw about optimal policy will also help, by clarifying which differences between the specifications are of greatest practical significance.

References

- Altig, D., L.J. Christiano, M.S. Eichenbaum, and J. Linde. 2002. "Technology Shocks and Aggregate Fluctuations." Federal Reserve Bank of Cleveland. Manuscript.
- Boivin, J. and M.P. Giannoni. 2003. "Has Monetary Policy Become More Effective?" NBER Working Paper No. 9459.
- Christiano, L.J., M.S. Eichenbaum, and C.L. Evans. 2001. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." NBER Working Paper No. 8403.

- Clarida, R., J. Galí, and M. Gertler. 1999. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature* 37 (4): 1661–707.
- Fuhrer, J.C. and G.R. Moore. 1995. "Inflation Persistence." *The Quarterly Journal of Economics* 110 (1): 127–59.
- Galí, J. and M. Gertler. 1999. "Inflation Dynamics: A Structural Econometric Analysis." *Journal of Monetary Economics* 44 (2): 195–222.
- Giannoni, M.P. and M. Woodford. 2003. "Optimal Inflation Targeting Rules." In *Inflation Targeting*, edited by B.S. Bernanke and M. Woodford. Chicago: University of Chicago Press. Forthcoming.
- Sbordone, A.M. 2003. "Inflation Dynamics and Real Marginal Costs." Rutgers University. Manuscript.
- Smets, F. and R. Wouters. 2002. "Sources of Business Cycle Fluctuations in the U.S.: A Bayesian DSGE Approach." Seminar presentation. Princeton University. November.
- Steinsson, J. 2002. "Optimal Monetary Policy in an Economy with Inflation Persistence." Manuscript.
- Woodford, M. 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.