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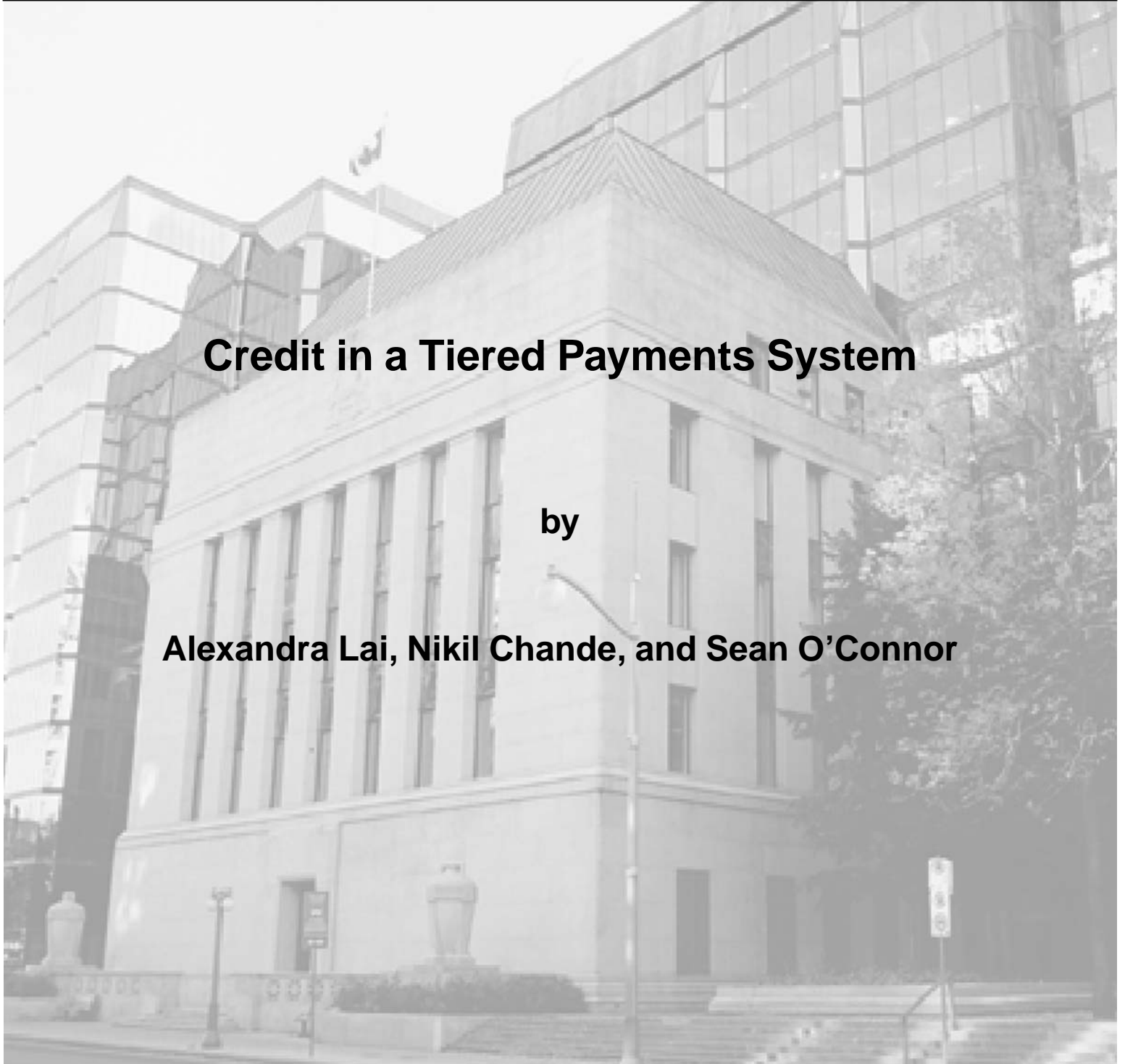
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Credit in a Tiered Payments System

by

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Contents

Acknowledgements.....	iv
Abstract/Résumé.....	v
1. Introduction.....	1
1.1 Importance of tiered payments systems.....	2
1.2 The modelling approach for the tiered system.....	4
1.3 Review of the academic literature.....	4
2. The Model.....	6
2.1 The set-up.....	7
3. Equilibrium.....	13
3.1 Equilibrium under no credit risk.....	14
3.2 Equilibrium with credit risk.....	15
4. Numerical Results.....	18
4.1 Comparing credit-risk and risk-free equilibria.....	20
4.2 Comparative statics.....	21
5. Conclusion.....	23
References.....	26
Appendix A: Proofs of Propositions.....	27
Appendix B: Graphs from Numerical Solutions.....	31

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Abstract

Payments systems are typically characterized by some degree of tiering, with upstream firms (clearing agents) providing settlement accounts to downstream institutions that wish to clear and settle payments indirectly in these systems (indirect clearers). Clearing agents provide their indirect clearers with an essential input (clearing and settlement services), while also competing directly with them in the retail market for payment services. The authors construct a model of a clearing agent with an indirect clearer to examine the clearing agent's incentives to lever off its upstream position to gain a competitive advantage in the retail payment services market. The model demonstrates that a clearing agent can attain this competitive advantage by raising the indirect clearer's costs, but that the incentive to raise these costs is mitigated by credit risk to the clearing agent from the provision of uncollateralized overdrafts to its indirect clearer. The results suggest that tiered payments systems, which require clearing agents to provide overdraft facilities to their indirect clearers, may result in a more competitive retail payment services market.

JEL classification: G21, L12, L13, L22

Bank classification: Financial institutions; Financial services; Market structure and pricing; Payment, clearing, and settlement systems

Résumé

Les systèmes de paiement sont généralement caractérisés à des degrés divers par le principe de la participation à plusieurs niveaux, où des firmes en amont (les agents de compensation) fournissent des comptes de règlement à des institutions en aval (les sous-adhérents) qui désirent faire compenser et régler indirectement des paiements par l'intermédiaire de ces systèmes. Les agents de compensation apportent une contribution essentielle aux sous-adhérents (des services de compensation et de règlement), tout en se trouvant en concurrence directe avec ces derniers sur le marché des services de paiement au détail. Les auteurs élaborent un modèle faisant intervenir un agent de compensation et un sous-adhérent, afin d'examiner la motivation du premier à mettre à profit sa position d'institution en amont pour se donner un avantage concurrentiel sur le marché des services de paiement au détail. Le modèle démontre qu'un agent de compensation peut acquérir cet avantage en augmentant les coûts imposés au sous-adhérent, mais que sa motivation à hausser ces coûts est restreinte par le risque de crédit auquel il s'expose du fait qu'il consent des découverts non garantis aux sous-adhérents. Les résultats de l'étude portent à croire que les systèmes de paiement à participation par paliers, qui obligent les agents de règlement à accorder

des découverts aux sous-adhérents avec lesquels ils traitent, sont susceptibles d'accroître la compétitivité du marché des services de paiement au détail.

Classification JEL : G21, L12, L13, L22

Classification de la Banque : Institutions financières; Services financiers; Structure de marché et fixation des prix; Système de paiement, de compensation et de règlement

1. Introduction

Participants in most payments systems around the world must choose between clearing and settling their payments directly in these systems, or clearing and settling their payments indirectly through institutions that act as clearing agents. In other words, payment arrangements typically involve various tiers of intermediation. At the top of the hierarchy are settlement institutions, generally central banks, which provide settlement accounts to participating banking institutions connecting directly to, and clearing directly in, this “first-tier” network. Of these direct clearers (DCs), some act as clearing agents (CAs) that operate a “second-tier” network, providing settlement accounts to downstream institutions that wish to clear and settle payments indirectly in the payments system (indirect clearers or ICs).

This study focuses on the suitability of the basic contractual terms between CAs and their ICs. In most payments systems, little is known about these contractual arrangements, except that the terms will likely be quite system-specific and non-standardized. For example, in the Canadian Payments Association’s Automated Clearing Settlement System (ACSS), the contracts between a CA and its ICs are bilaterally negotiated and can vary across ICs, since the clearing and settlement services are bundled with other IC-specific services (Tripartite Study Group 2006). Despite the lack of standardization, these contracts all feature a requirement that CAs provide settlement credit lines to their ICs. Some CAs in the ACSS indicate that, because of processing and information lags in payment accounting and monitoring technologies, the settlement credits extended by the CA to the borrowing IC are effectively uncollateralized overdrafts. For this study, the basic features of interest in the contract between a CA and its IC for wholesale payment services are the fee for clearing and settlement services and the overdraft facility provided to the IC.

In addition to providing wholesale payment services through second-tier networks, CAs compete directly with their ICs in the retail market for payment services. Consequently, a CA may face incentives to lever off its upstream position, using its wholesale payment fee charged to ICs or access to overdrafts, to gain a competitive advantage in the downstream market. The specific research question in this paper is whether the uncollateralized overdrafts extended by a CA to an IC mitigate the incentive for the CA to strategically price its clearing and settlement services to gain an advantage in the retail payment services market. Under the conditions specified in the following model, our analysis suggests that a CA can attain such a competitive advantage by raising the IC’s costs—its rival in the retail payment services market—but that the potential for credit loss from the provision of uncollateralized overdrafts may limit the incentive to do so.

1.1 Importance of tiered payments systems

A recent study by the Committee on Payment and Settlement Systems (CPSS 2003) on the role of central bank money measures the degree of tiering in selected payments systems, in a sample of developed countries. The study uses two measures for the degree of tiering: the proportion of financial institutions in the country that participate as DCs in the system; and the value of payments in the system accounted for by these DCs. In tiered systems, the large banking institutions are typically the major CAs in the system, while the smaller institutions participate as ICs. Mid-size banks may participate directly in the first-tier network, but not usually as CAs.

Table 1 describes the study's assessment of the degree of tiering in the selected payment, clearing, and settlement systems of the countries surveyed.

Table 1: Degree of tiering in various countries' payments systems

Country	Systems	Main transactions	Degree of tiering	
			By number	By value
Belgium	ELLIPS CEC	Large value Retail	High ¹ High ¹	Low* Low*
Canada	LVTS ACSS	Large value Retail	High High	N/A N/A
Europe	TARGET EURO1	Large value Large value	High Strong	Mixed* Mixed*
France	TBF PNS SIT	Large value Large value Retail	Mixed Strong Strong	Mixed* Mixed* Mixed*
Germany	RTGS ^{PLUS} RPS	Large value Retail	Strong Low	N/A N/A
Hong Kong	HKD RTGS USD RTGS Euro RTGS ²	Large value Large value Large value	None Mixed Mixed	None N/A N/A
Italy	BI-REL BI-COMP	Large value Retail	Low High	Low* High*
Japan	BOJ-NET Zengin	Large value Retail	High High	None* Low*
Netherlands	TOP Interpay	Large value Retail	Low None	Low None
United Kingdom	CHAPS Sterling CHAPS Euro BACS	Large value Large value Retail	Strong High Strong	Mixed* Mixed* Mixed*
United States	Fedwire CHIPS	Large value Large value	Mixed Strong	Mixed Mixed
Source: CPSS (2003). Situation at end of 2002, except where otherwise noted. N/A = not available; * estimate; ¹ end of 2001; ² system started operation in April 2003				
Degree of tiering	By number of institutions	By value of payments		
None	all, or virtually all, are DCs	all, or virtually all, value		
Low	at least 75% are DCs	at least 90% of value		
Mixed	25-75% are DCs	25%-90% of value		
High	5-25% are DCs	10-25% of value		
Strong	less than 5% are DCs	less than 10% of value		

The results indicate that most payment, clearing, and settlement systems in these countries are tiered systems, although the degree of tiering varies significantly across systems. Tiered clearing and settlement networks are, therefore, the norm rather than the exception in payments systems.

1.2 The modelling approach for the tiered system

To answer our basic research question, we construct a model of a vertically integrated monopolist competing against a downstream rival in the end-user market for payment services. To this standard industrial organization (IO) model, an element of the interbank relationship is introduced, arising from the provision of clearing and settlement services by the CA to its IC. The CA is obliged to provide daylight overdraft facilities to the IC by settling the IC's net payment flow before receiving those funds at the end of the day. Thus, the CA incurs credit risk, because the IC may default on its end-of-day payment obligations. This transforms a standard IO model, with a vertical structure and horizontal competition, into a model of banks in a tiered arrangement within a payments system

1.3 Review of the academic literature

Two major strands of the literature are relevant to this paper: the IO literature on vertically integrated firms and the settlement literature on credit and tiering.

1.3.1 IO literature on vertically integrated firms

It is well established that where there is imperfect competition and independent pricing in both the upstream (wholesale) and downstream (retail) markets, the retail price will encompass two stacked markups (called *double marginalization*) over combined production and delivery costs (Spengler 1950). Joint wholesale and retail profits are lower than they would be if wholesale and retail services were produced and priced by a single, vertically integrated firm, thus creating an incentive for vertical integration.

Even if the retail market were perfectly competitive and the retail price contained only the wholesale profit markup, an incentive for a firm to integrate vertically can still exist. As Salop (1998) explains, by raising downstream rivals' costs, a vertically integrated firm may be able to gain sufficient market power to price above the competitive level in the downstream market. These higher downstream prices harm consumers, and the decrease in purchases associated with the higher prices causes a deadweight loss. Salop states further that there

may be a reduction in economic efficiency if the increased input cost to downstream rivals causes them to use a less-efficient input mix.

When the input price is regulated, a vertically integrated firm can still raise its downstream rivals' costs by imposing some non-price discrimination, such as delaying the delivery or degrading the quality of the input. The imposition of non-price costs on downstream rivals is termed "sabotage" in the literature. Economides (1998), and others, demonstrates that, as long as the costs of sabotage and the upstream profit margin are sufficiently small, an incentive for cost-increasing sabotage exists.

Bustos and Galetovic (2003) show that, in the absence of wholesale price regulation, a vertically integrated firm with monopoly power in the wholesale market would prefer to increase the input price charged to downstream rivals instead of raising rivals' costs through non-price discrimination. Hence, non-price discrimination arises only where the input price is regulated. Thus, a vertically integrated monopolist will generally have an incentive to raise its rivals' costs, and furthermore it prefers to do so by increasing the input price it charges.

1.3.2 Settlement literature on credit and tiering

Kahn and Roberds (1998) examine the incentives for default by participants in a deferred net settlement system, where a participating bank's payment inflows and outflows are accumulated through the period and final settlement at the end of the period is on a net payable basis. In effect, the banks in a deferred net settlement system extend intraday credit and acquire intraday loans through the day. Although Kahn and Roberds analyze single-tiered networks, the credit relationship they articulate is relevant for tiered network structures, since second-tier networks are essentially deferred net settlement systems for intranetwork payments. From their paper, we take our model of a bank facing uncertain payment flows. However, unlike Kahn and Roberds, default in our model does not arise strategically, but rather from constraints on liquidity.

Recently, a number of studies have been performed relating to the integration of securities infrastructure networks operated by central securities depositories (CSDs). There are interesting similarities between tiered structures for securities systems and those of payments systems.

Tapking and Yang (2004) examine horizontal and vertical consolidation in a two-country model with a CSD and single trading system in each country. All securities traded in a given

country are settled in that country's CSD, but a cross-border trade requires the transfer of securities between CSDs through a horizontal service link. They find that either horizontal or vertical integration of CSDs leads to higher social welfare than a decentralized system. They also find that, under some conditions, vertical integration between a CSD and the trading system in a country is relatively more cost-efficient than horizontal integration between CSDs.

The social welfare gains from vertical and horizontal integration arise from externalities in demand for complementary goods or services. Economides and Salop (1992) consider multiple brands of compatible (complementary and substitute) products or services in network market arrangements under Cournot competition. They find that market prices are lower under joint ownership (i.e., vertical or horizontal integration) when cross-service complementarity in demand is sufficiently high (vertical integration) and substitutability in demand is sufficiently low (horizontal integration).

In a model with a single CSD and two custodian banks, one of which is vertically integrated with the CSD, Holthausen and Tapping (2004) demonstrate that, in equilibrium, the CSD will raise the costs of the rival custodian bank. The CSD can, therefore, offer a more attractive pricing scheme to customers than can the rival custodian bank. Thus, the rival custodian bank retains only those customers who have a strong preference for its services. However, the CSD's market share is not necessarily larger than the socially optimal one, since there is a greater potential for netting across the CSD's increased customer base.

Rochet (2005) examines the incentives for a CSD to vertically integrate with one of two custodian banks, each of which charges a fixed access fee and per-transaction fee in the downstream market. Through vertical integration, the CSD can increase its profit by refusing to provide the rival custodian bank with settlement services. If regulation prevents exclusion of the rival custodian bank, the vertically integrated CSD raises its rival's costs, and the result is a lower per-transaction fee but higher fixed access fee charged by the vertically integrated CSD than by its rival. The market share of the vertically integrated CSD increases, and that of the rival bank decreases, relative to the case without vertical integration. Moreover, social welfare increases, not taking into account the direct and indirect costs of regulation.

2. The Model

The basic theoretical model developed for this analysis builds on the work described above. There are two principal differences in the model structure in this paper from the models of

vertically structured securities systems: (i) only a single per-unit price scheme is considered for wholesale payment services (any fixed access fee to the clearing and settlement networks is exogenous and embedded in the fixed costs of the CA and IC), and (ii) the wholesale services provided by the CA to the IC include both clearing and settlement services and overdraft credit services. This is a crucial difference, since aggressive exploitation of even its limited market power in the wholesale settlement services market might, under some circumstances, reduce its profitability in providing overdraft loan services. The model specified below is designed to determine the circumstances under which this would be the case and its implications for retail market competitiveness.

2.1 The set-up

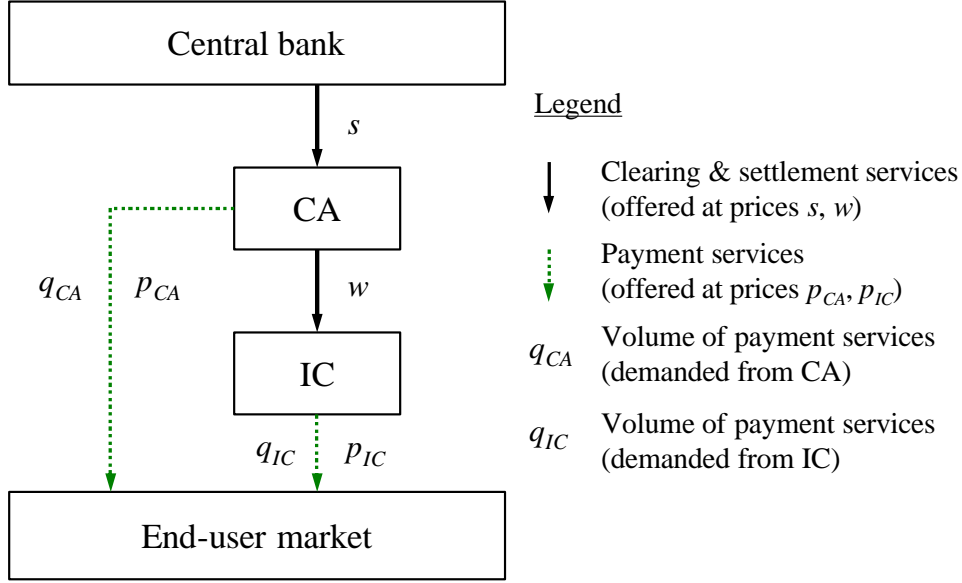
We model a representative tiered arrangement, in which there is a DC providing clearing and settlement services to an IC. In other words, the DC is acting as a CA for the IC. The CA provides payment services to end users at the price p_{CA} , and provides clearing and settlement services to an IC at the price w . The IC also provides payment services to end users, but at a different price, p_{IC} , and hence competes downstream with the CA in a Cournot game. Note that we abstract from competition between CAs for ICs.

For each bank, a proportion $\beta \in (0, 1]$ of payments have to be cleared and settled externally. In other words, a proportion $1 - \beta$ of payments are cleared and settled internally on the books of each institution (these are also referred to as “on-us payments”). Payments that are cleared and settled externally for the IC incur the clearing and settlement charge, w , chosen by the CA, and payments that are cleared and settled externally for the CA incur a clearing and settlement fee, s , imposed by the central bank. These externally cleared and settled payments are also referred to as “on-them payments.” Finally, let q_{CA} and q_{IC} denote the volume of payment instructions demanded by end users from the CA and IC, respectively, given their respective retail payment services fees. The model will, however, be solved in quantities with an inverse demand rather than prices with a demand function. By construction, the equilibrium prices and quantities are identical.

Figure 1 illustrates the structure of the representative tiered payments system that will be modelled.

Retail Demand for Payment Services The inverse demand functions facing the IC and the CA are $P(q, q') = 1 - q - \gamma q'$, where $0 < \gamma \leq 1$ is a parameter for the substitutability

Figure 1: The tiered payments system being modelled



between the CA's and the IC's payment services (less than one denotes imperfect substitutability), q denotes own quantities, and q' denotes the rival's quantities. There is some evidence of less-than-perfect market segmentation in the customer bases for CAs and ICs. Typically, this is reflected in differences in the range and type of banking service packages, which include payment services, offered to business and personal customers. This allows for imperfect substitutability in payment services demand.

Technology The processing cost of payments is $C(q)$, where $C' > 0$ and $C'' \leq 0$, implying increasing returns to scale in technology. Total payment processing costs consist of a fixed cost and a constant marginal cost, c .

Profits from Retail Payment Services Banks' profits from payment services can be written as follows.

$$\pi_{IC}(q_{IC}, q_{CA}) = (1 - q_{IC} - \gamma q_{CA}) q_{IC} - (c + \beta w) q_{IC} - F_{IC}, \quad (1)$$

$$\pi_{CA}(q_{CA}, q_{IC}) = (1 - q_{CA} - \gamma q_{IC}) q_{CA} + w\beta q_{IC} - (c + \beta s)(q_{CA} + \beta q_{IC}) - F_{CA}, \quad (2)$$

where F_{IC} is the fixed cost of being an IC and F_{CA} is the fixed cost of being a CA. Generally, it is assumed that $F_{CA} > F_{IC}$. CAs must set up internal processing and account management systems to both participate in the first-tier network and operate the second-tier network. CAs must also integrate a sophisticated liquidity management function with their payment services operations. ICs avoid many of these costs by outsourcing clearing and settlement to a CA. Both CAs and ICs have more similar fixed costs, however, for similar retail payment services.

The profit functions are concave with respect to their own outputs: $\partial^2\pi_{CA}/\partial q_{CA}^2 < 0$, $\partial^2\pi_{IC}/\partial q_{IC}^2 < 0$. Furthermore, $\partial\pi_{CA}/\partial q_{IC}^2 = -\gamma q_{IC} + \beta(w - c - \beta s)$ can be positive or negative, while $\partial\pi_{IC}/\partial q_{CA} < 0$ and $\partial^2\pi_{CA}/\partial q_{CA}\partial q_{IC} = \partial^2\pi_{IC}/\partial q_{CA}\partial q_{IC} < 0$.

Credit Risk in the Provision of Daylight Overdrafts The demand and cost functions for retail payments are defined over transaction volumes. Since financial risk will be introduced, arising from the overdraft facilities that the CA is required to provide the IC, the banks' problems need to involve (stochastic) payment flows. The net values of payment flows are denoted by a random variable, v , that is distributed according to a unimodal *cdf*, $F(\cdot)$, with zero mean and bounded supports $[-\bar{v}, \bar{v}]$. The *pdf* of v is denoted by $f(\cdot)$.¹ Since it is not evident how (and whether) the distribution of net payment positions is related to the volume of payments instructions submitted, we assume that this distribution is independent of payment volume and is identical across the CA and the IC. That is,

$$\bar{v}_{IC} = \bar{v}_{CA} = \bar{v}.$$

Letting the distributional supports differ across the CA and the IC would not affect the results qualitatively. Furthermore, letting the supports be monotonic functions of q would not change the results as long as the supports are such that probability densities at the lower and upper bounds, $f(\bar{v})$, are very small.

Let X_{IC} be the IC's realized net payment flow. $X_{IC} > 0$ denotes a net inflow (the IC is a net receiver of funds), and $X_{IC} < 0$ denotes a net outflow (the IC is a net sender of funds and hence incurs daylight overdraft provided by the CA). An overdraft charge, rX_{IC}^- , is incurred whenever $X_{IC} < 0$, where $X^- = \max\{-X, 0\}$ and r is an exogenous interest rate or predetermined overdraft fee. Hence, the net end-of-day payment obligation for an IC who

¹Data on net debit positions of DCs in ACSS are suggestive of a bounded unimodal distribution centred around zero.

is a net sender is $(1+r)X_{IC}^-$.

Let A_{IC} be the IC's end-of-day net asset position, before payment flows are considered. Recall that π_{IC} is the IC's profit from payment services, net of processing and clearing costs but gross of payment obligations to the CA. The IC's cash flow, before payment obligations to the CA are considered, is then $\pi_{IC} + X_{IC}^+$. Potential liquid assets are higher than cash flow because the IC can liquidate some of its net assets, A_{IC} .

There are three possible scenarios when the IC realizes a net outflow, $X_{IC}^- > 0$:

- (i) If $\pi_{IC} - (1+r)X_{IC}^- \geq 0$, then the IC can meet its payment obligations out of cash flow (profits) and no assets need to be liquidated.
- (ii) If $\pi_{IC} - (1+r)X_{IC}^- < 0$, then the IC has to liquidate some assets to meet its payment obligations. The cost of liquidating an amount y of assets is δy , where $\delta \in (0, 1)$. Therefore, in order to meet a shortfall of $-\left[\pi_{IC} - (1+r)X_{IC}^-\right] > 0$, the IC is required to liquidate the amount $-(1+\delta)\left[\pi_{IC} - (1+r)X_{IC}^-\right]$.
- (iii) If $\frac{A_{IC}}{1+\delta} + \pi^{IC} - (1+r)X_{IC}^- < 0$, then the IC cannot meet its payment obligation even if it liquidates all of its net assets. In this case, the IC defaults on its payment obligations and the CA recovers only $\frac{A_{IC}}{1+\delta} + \pi^{IC}(q)$.²

Timing Finally, the two institutions play a two-stage game. In the first stage, the CA chooses the clearing fee charged to the IC, w , and in the second stage the CA and the IC simultaneously choose quantities, q_{CA} and q_{IC} , respectively, to maximize their expected net worth subject to participation constraints, $\pi_i \geq 0$, $i = \{CA, IC\}$.

In a tiered system, the expected profit from operating as a clearing agent should be higher than it would be as a direct clearer only. Moreover, survey evidence (Tripartite Study Group 2006) indicates that, because of the cost conditions, the IC prefers to clear indirectly through a CA than to become a DC itself, which is reflected in the model set-up. That is, expected profits from being an IC are higher than the expected profits from a symmetric Cournot game between two DCs. Thus, the IC engages in a symmetric Cournot competition with the CA, in the retail services market.

²We allow only liquidity (or forced) defaults in this model. The model can be extended to study strategic default, but since this is not the focus of the problem, this approach is not considered to reveal more about the core behaviours.

The model is solved for the subgame-perfect Nash equilibrium (SPNE) to the two-stage game; that is, by backward induction, in which (i) the equilibrium Cournot quantities at the retail level are derived, for a given clearing fee, w , and (ii) the CA's equilibrium clearing fee is derived, taking into account the Cournot quantities from the retail-level competition.

2.1.1 The indirect clearer's problem

First, consider the IC's problem. The IC's net worth is as follows:

$$\text{NW}_{IC} = \begin{cases} A_{IC} + \pi_{IC} + v & \text{if } v \geq 0 \\ A_{IC} + \pi_{IC} + (1+r)v & \text{if } v < 0 \leq \pi_{IC} + (1+r)v \\ A_{IC} + (1+\delta)[\pi_{IC} + (1+r)v] & \text{if } -A_{IC} \leq (1+\delta)[\pi_{IC} + (1+r)v] < 0 \\ 0 & \text{if } (1+\delta)[\pi_{IC} + (1+r)v] < -A_{IC}, \end{cases}$$

where π_{IC} is given by equation (1).

When the net payment flow to the IC is positive ($v \geq 0$), the IC's net worth increases by the amount of the payment inflow. When the IC realizes a negative net payment flow, it takes a daylight overdraft from the DC and incurs an overdraft charge of $-rv$. Hence, its payment obligation is $-(1+r)v$. If this amount is greater than its cash flow from the profits earned by providing payment services, the IC is required to liquidate a portion of its net assets at a cost δ to cover the shortfall between profits and payment obligations. Finally, if the realized net payment flow is negative enough that the IC's net assets cannot cover the shortfall, the IC defaults and has zero net worth.

The IC's problem, then, is one of choosing q_{IC} to maximize expected net worth:

$$\begin{aligned} \text{E}(\text{NW}^{IC}) = & A_{IC} + \pi_{IC} + \int_0^{\bar{v}_{IC}} v dF(v) + (1+r) \int_{-\frac{\pi_{IC}}{1+r}}^0 v dF(v) \\ & + \int_{\max\{-\bar{v}_{IC}, -\frac{A_{IC} + (1+\delta)\pi_{IC}}{(1+\delta)(1+r)}\}}^{-\frac{\pi_{IC}}{(1+r)}} [\delta\pi_{IC} + (1+\delta)(1+r)v] dF(v), \end{aligned} \quad (3)$$

subject to $\pi_{IC} \geq 0$. In the above expression for the IC's expected net worth, default occurs if and only if $-\bar{v}_{IC} < -\frac{A_{IC} + (1+\delta)\pi_{IC}}{(1+\delta)(1+r)}$.

2.1.2 The clearing agent's problem

We next turn to the CA's problem. Let A_{CA} be the CA's end-of-day net assets. Recall that π_{CA} is the CA's profit from retail payment services and the clearing and settlement of retail payments submitted by the IC. This profit does not include payment obligations arising from net payment outflows or income earned from charges on overdrafts incurred by the IC.

Let X_{CA} be the CA's net payment flow arising from its own end-user demand, q_{CA} . The CA also clears X_{IC} on the IC's behalf. Hence, the CA's total net payment flow is $X_{CA} + X_{IC}$. As before, $X_{CA} + X_{IC} > 0$ denotes a net inflow, which is credited to the CA's settlement account at the central bank, and $X_{CA} + X_{IC} < 0$ denotes a net outflow, in which case the CA draws on credit lines supplied by other direct clearers in the system. We assume that these (bilateral) credit lines are uncollateralized and no interest charges are levied on them.

We assume that whenever the IC realizes a positive payment flow (net inflow), its account with the CA is credited, whereas a negative payment flow is settled at the end of the day (that is, the IC draws on the daylight overdraft facilities provided by the CA). Additionally, we assume that the CA has to settle its payment obligations with respect to other DCs in the payments system before the IC pays off its overdraft. As in the case of the IC, if the CA realizes a net outflow, $X_{CA} + X_{IC} < 0$, it can clear its end-of-day payment obligations without any asset liquidation only if its cash flow, π_{CA} , is large enough. Furthermore, we assume that the CA's end-of-day net asset position is large relative to its payment obligation, so that it never defaults on its payment obligations.³

In the course of the day, the CA sends payments on behalf of the IC worth X_{IC} and recovers $(1+r)X_{IC}^-$ when the IC does not default, or $\frac{A_{IC}}{1+\delta} + \pi_{IC}$ when the IC defaults. Thus, we can write the CA's net worth as follows:

$$\begin{aligned} \text{NW}^{CA} &= A_{CA} + \pi_{CA} + X_{CA} + X_{IC} \\ &+ \begin{cases} 0 & \text{if } \pi_{CA} + X_{CA} + X_{IC} \geq 0 \\ \delta[\pi_{CA} + X_{CA} + X_{IC}] & \text{if } \pi_{CA} + X_{CA} + X_{IC} < 0 \end{cases} \\ &+ \begin{cases} -X_{IC} & \text{if } X_{IC} \geq 0 \\ -(1+r)X_{IC} & \text{if } -\frac{A_{IC} + (1+\delta)\pi_{IC}}{(1+\delta)(1+r)} \leq X_{IC} < 0 \\ \frac{A_{IC}}{1+\delta} + \pi_{IC}(q_{IC}, q_{CA}) & \text{if } X_{IC} < -\frac{A_{IC} + (1+\delta)\pi_{IC}}{(1+\delta)(1+r)}, \end{cases} \end{aligned}$$

³Evidence shows that the largest net debit position of CAs in ACSS as a fraction of shareholders' equity ranges from 5 per cent to 12 per cent.

where π_{CA} is given by equation (2). The last term is just the CA's expected cash receipts from the IC, which refer to combined net (CA and IC) payment flows to its settlement account at the central bank and to any net returns from overdrafts to the IC.

Define $z = X_{CA} + X_{IC}$ and $(z - v)$ as its net impact on the CA's net worth. Note that the CA's net worth is unchanged for $X_{IC} > 0$, since this increases both its cash balances and its deposit liabilities, and increases for $X_{IC} < 0$, since it provides some net return to the CA. If $f(z - v)$ is the density function for this net worth variable, then z has a *pdf* given by

$$g(z) = \int_{-\bar{v}_{IC}}^{\bar{v}_{IC}} f(z - v) dF(v),$$

and has supports $[-\bar{v}_{CA} - \bar{v}_{IC}, \bar{v}_{CA} + \bar{v}_{IC}]$. We denote the corresponding *cdf* as $G(z)$.

Thus, the CA chooses the retail payment volume, q_{CA} , in the second stage and the clearing fee charged to the IC, w , in the first stage to maximize its expected net worth:

$$\begin{aligned} E(\text{NW}_{CA}) = & A_{CA} + \pi_{CA} + \int_{-\pi_{CA}}^{\bar{v}_{CA} + \bar{v}_{IC}} z dG(z) + \int_{-\bar{v}_{CA} - \bar{v}_{IC}}^{-\pi_{CA}} \left\{ z + \delta [\pi_{CA} + z] \right\} dG(z) \\ & - \int_0^{\bar{v}_{IC}} v dF(v) - \int_{\max\{-\bar{v}_{IC}, \chi\}}^0 (1 + r) v dF(v) \\ & + \int_{\min\{-\bar{v}_{IC}, \chi\}}^{\max\{-\bar{v}_{IC}, \chi\}} \left[\frac{A_{IC}}{1 + \delta} + \pi_{IC} \right] v dF(v), \end{aligned} \quad (4)$$

where

$$\chi = -\frac{A_{IC} + (1 + \delta)\pi_{IC}}{(1 + \delta)(1 + r)}. \quad (5)$$

Note that the final term in the above expression disappears if the IC imposes no credit risk on the CA.

3. Equilibrium

In this section, the equilibrium Cournot quantities are solved as functions of the clearing fee, and then, given these equilibrium quantities, we solve for the CA's equilibrium clearing fee. Recall that the support of the distribution, \bar{v} , is by assumption independent of the payment instruction volume, q . Under this assumption, the model solutions in the case with no credit risk (where the IC never defaults on its payment obligations) are compared with the model

solutions in the case where the IC imposes credit risk on the CA, given states of the world where the IC would be forced to default.

3.1 Equilibrium under no credit risk

In this case, the IC's available assets and expected profitability are assumed to be large enough by the CA to cover any potential overdraft that the IC might incur, and it is assumed that any net overdraft revenues are small enough to ignore. The CA perceives no credit risk.

Proposition 1 *Cournot equilibrium with no credit risk*

Assume that no credit risk is present in the relationship between the IC and the CA, or, $A_{IC} + (1 + \delta)\pi_{IC} - (1 + \delta)(1 + r)v \geq 0$, $\forall (q_{IC}, q_{CA}, v)$. Then, equilibrium quantities $[q_{IC}^{RF}(w), q_{CA}^{RF}(w)]$ solve

$$\frac{\partial \pi_{IC}}{\partial q_{IC}} = 0, \quad \frac{\partial \pi_{CA}}{\partial q_{CA}} = 0.$$

Proof. See Appendix A. ■

In the risk-free case, explicit solutions to the Cournot game are easy to obtain:

$$q_{IC}^{RF}(w) = \frac{(2 - \gamma)(1 - c) - 2\beta w + \beta\gamma s}{4 - \gamma^2}, \quad (6)$$

$$q_{CA}^{RF}(w) = \frac{(2 - \gamma)(1 - c) + \gamma\beta w - 2\beta s}{4 - \gamma^2}. \quad (7)$$

Prices charged by the IC and CA to retail customers are:

$$P_{IC}^{RF}(w) = 1 - \frac{(1 + \gamma)(2 - \gamma)(1 - c) - (2 - \gamma^2)\beta w - \beta\gamma s}{4 - \gamma^2}, \quad (8)$$

$$P_{CA}^{RF}(w) = 1 - \frac{(1 + \gamma)(2 - \gamma)(1 - c) - \beta\gamma w - (2 - \gamma^2)\beta s}{4 - \gamma^2}. \quad (9)$$

It is also clear from the explicit Cournot solutions that the CA charges a lower price, and hence has a higher retail market share, than the IC if and only if $w > s$ (i.e., the CA charges a higher wholesale service fee to the IC in the second-tier network than it is charged by the central bank in the first-tier network).

Proposition 2 *Equilibrium clearing fee with no credit risk*

When no credit risk is present, Cournot quantities are given by $[q_{IC}^{RF}(w), q_{CA}^{RF}(w)]$ and the equilibrium clearing fee w^{RF} solves

$$\frac{d\pi_{CA}}{dw} = \frac{\partial\pi_{CA}}{\partial q_{IC}} \frac{dq_{IC}^{RF}}{dw} + \frac{\partial\pi_{CA}}{\partial w} = 0.$$

Furthermore,

$$c + \beta s < w^{RF} < w_{pc} = \frac{(2 - \gamma)(1 - c) + \beta\gamma s}{2\beta}. \quad (10)$$

Proof. See Appendix A. ■

It is clear that, since $\frac{\partial\pi_{CA}}{\partial w} = \beta q_{IC} > 0$ and $\frac{dq_{IC}^{RF}}{dw} < 0$, $\frac{\partial\pi_{CA}}{\partial q_{IC}} = \beta(w - c - \beta s) - \gamma q_{CA} > 0$, or $c + \beta s < w^{RF}$. This first inequality in equation (10) ensures that the CA does not earn negative profit out of supplying the clearing service to the IC, ensuring that the CA's participation constraint is satisfied. The last inequality ensures that the IC's participation constraint in the second-tier network is satisfied so long as the fixed costs to being an indirect clearer are sufficiently small. That is, $\pi_{IC}(w^{RF}) \geq 0$.

Finally, we ensure that the IC has no incentive to withdraw from the tiering relationship and become a direct clearer (DC) itself (and compete symmetrically with the CA), by assuming that the difference $F_{DC} - F_{IC}$ is large enough and that F_{DC} approaches F_{CA} in value:

$$F_{DC} - F_{IC} \geq \left[\frac{1 - c - \beta s}{2 + \gamma} \right]^2 - \left[\frac{(2 - \gamma)(1 - c) - 2\beta w + \beta\gamma s}{4 - \gamma^2} \right]^2. \quad (11)$$

Note that the right-hand side of the above inequality is positive if and only if $w \geq s$.

3.2 Equilibrium with credit risk

In this section, we are interested in how the introduction of credit risk affects the CA's equilibrium wholesale clearing fee, w , and the corresponding effects on retail quantities and prices. In this case, the CA is unsure whether the IC's available assets and profits are sufficient to cover a potential overdraft, and whether it could absorb a potential default without significant profit loss.

Proposition 3 *Cournot equilibrium with credit risk*

Assume that credit risk is present in the relationship between the IC and the CA, or,

$A_{IC} + (1 + \delta)\pi_{IC} - (1 + \delta)(1 + r)\bar{v} < 0$. Then, equilibrium quantities $[q_{IC}^{CR}(w), q_{CA}^{CR}(w)]$ solve

$$\frac{\partial \pi_{IC}}{\partial q_{IC}} = 0, \quad \frac{\partial \pi_{CA}}{\partial q_{CA}} [1 + \delta G(-\pi_{CA})] + \frac{\partial \pi_{IC}}{\partial q_{CA}} F(\chi) = 0, \quad (12)$$

where $\chi = -\frac{A_{IC} + (1 + \delta)\pi_{IC}}{(1 + \delta)(1 + r)}$.

Totally differentiating the two first-order conditions with respect to w , we can show that $q_{IC}^{CR}(w)$ is decreasing in w while $q_{CA}^{CR}(w)$ is increasing in w , so long as credit risk, represented by $F(\chi) > 0$, is not too large.

Corollary For a given w , the CA lowers its quantities when credit risk is introduced, while the IC raises its quantities: $q_{CA}^{CR}(w) < q_{CA}^{RF}(w)$ and $q_{IC}^{CR}(w) > q_{IC}^{RF}(w)$. However, the retail prices of the CA and the IC both increase with the introduction of credit risk: $P_i^{CR}(w) > P_i^{RF}(w)$, $i \in \{CA, IC\}$.

Proof. The CA's first-order condition in q_{CA} demonstrates that, for a fixed w , the CA's quantities fall with credit risk, since $\partial \pi_{IC} / \partial q_{CA} = -\gamma q_{IC} < 0$ implies that $\partial \pi_{CA} / \partial q_{CA} > 0$ in equilibrium, or $q_{CA}^{CR}(w) < q_{CA}^{RF}$. Clearly, this implies an increase in q_{IC} in equilibrium.

From the IC's first-order condition in the Cournot game,

$$q_{IC} = \frac{1 - \gamma q_{CA} - c - \beta w}{2}.$$

Hence, any change in q_{CA} for a fixed w , say of size Δq_{CA} , results in a change of $-(\gamma/2)\Delta q_{CA}$. Hence,

$$P_{CA}^{CR}(w) - P_{CA}^{RF}(w) = -\frac{2 - \gamma^2}{2} \Delta q_{CA} > 0,$$

and

$$P_{IC}^{CR}(w) - P_{IC}^{RF}(w) = -\frac{1}{2} \Delta q_{CA} > 0,$$

since $\Delta q_{CA} < 0$, in response to the introduction of credit risk. ■

It is important to bear in mind that the above results are true only when the clearing fee, w , is fixed. The next proposition examines the effect of introducing credit risk into the equilibrium clearing fee.

Proposition 4 *Equilibrium clearing fee with credit risk*

With credit risk and Cournot quantities given by $[q_{IC}^{CR}(w), q_{CA}^{CR}(w)]$, the equilibrium clearing fee w^{CR} solves

$$[-\gamma q_{CA}^{CR} + \beta(w^{CR} - c - \beta s)][1 + \delta G(-\pi_{CA})] \frac{dq_{IC}^{CR}}{dw} + \beta q_{IC}^{CR}[1 + \delta G(-\pi_{CA}) - F(\chi)] = 0, \quad (13)$$

where Cournot quantities are evaluated at w^{CR} .

Proof. See Appendix A. ■

From the CA's first-order condition with respect to w with no credit risk, evaluated at w^{RF} , we know that

$$-\gamma q_{CA}^{RF}(w^{RF}) + \beta(w^{RF} - c - \beta s) > 0.$$

Also, $q_{CA}^{CR}(w^{RF}) < q_{CA}^{RF}(w^{RF})$ from the corollary. Hence,

$$-\gamma q_{CA}^{CR}(w^{RF}) + \beta(w^{RF} - c - \beta s) > 0,$$

given that $\frac{dq_{IC}^{CR}}{dw} < 0$, $\frac{dE(NW_{CA})}{dw}$ with credit risk, evaluated at w^{RF} , can be positive or negative.

In general, whenever

$$\left. \frac{dE(NW_{CA})}{dw} \right|_{w=w^{RF}} < 0,$$

we get the result that $w^{CR} < w^{RF}$; otherwise, $w^{CR} > w^{RF}$.

Whether a CA that is subject to credit risk imposed by the IC charges a lower or higher clearing fee to the IC compared with a CA that is not subject to credit risk depends on the relative strength of two effects. The first, which we call the *IO effect*, arises from the fact that, for a fixed w , the CA's quantities decrease and the IC's quantities increase with credit risk. This effect tends to raise equilibrium w , since it increases the left-hand side of equation (13) for a given w . The second effect, which we call the *credit-risk effect*, is explained by examining the second term in the CA's first-order condition with respect to w when credit risk is present. An increase in w lowers the IC's profit, $\pi_{IC}(w)$, which in turn increases the probability $F(\chi)$ of a default by the IC on its payment obligations. This tends to reduce the equilibrium w relative to the no-credit-risk case, where the second term is not present. Hence, with credit risk, the CA's equilibrium clearing fee can result in $w^{CR} < w^{RF}$ if the *credit-risk effect* outweighs the *IO effect*.

The CA charges a lower wholesale service fee to the IC than its risk-free fee, to avoid a default by the IC on any overdrafts it might incur. The CA recognizes that, if it charges a

high w , it will likely lower the IC's profits, and that lower IC profits will raise the likelihood of an IC default on any potential overdraft credit. Consequently, by charging a wholesale fee no lower than its risk-free fee, the CA might actually increase the probability of a reduction of its own net worth. Hence, the CA shaves w when significant credit risk exists from its overdraft services to ICs.

In the next section, we show that our numerical results generally yield $w^{CR} < w^{RF}$, for reasonable parameter ranges. This fall in the equilibrium clearing fee will exacerbate both the decrease in the CA's retail quantities and the increase in the IC's retail quantities. Furthermore, we will have to rely on our numerical analysis to determine whether retail prices rise or fall.

4. Numerical Results

A numerical analysis is needed to fully analyze the impact of credit risk on equilibrium variables, because of the difficulty associated with finding analytical solutions. One complication is that choice variables affect regions of analysis under the truncated density functions.

Johnson, Kotz, and Balakrishnan (1994) explain that the *cdf* of a truncated normal distribution, such as $H(\cdot)$, can be expressed in terms of the standard normal *cdf* (i.e., Φ), as follows:

$$H(x, a, b, \mu, \sigma) = \frac{\Phi\left(\frac{x-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)}{\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)},$$

where x is in the range $[a, b]$, a is the lower truncation point, b is the upper truncation point, μ is the mean, and σ is the standard deviation.

The relationship between a truncated normal *cdf* and that of a standard normal is used to define $F(x, -\bar{v}, \bar{v}, 0, 1)$ and $G(x, -2\bar{v}, 2\bar{v}, 0, \sqrt{2})$, from which the net value of payment flows for the IC and CA are, respectively, distributed.

The following base values are chosen for the model's parameters, in light of the justifications provided below:

A_{IC}	\bar{v}	c	s	r	β	γ	F_{IC}	F_{CA}
0.5	0.5	0.01	0.01	0.0001	0.9	0.9	0	0.19

To ensure that the IC imposes sufficient credit risk on the CA, base values of similar size are chosen for A_{IC} and \bar{v} . This is a reasonable assumption, since the IC may, on a given day, realize payment outflows comparable in size to its capital.

The base values chosen for c and s must be reasonably small relative to the prices charged by the IC and CA, and these prices cannot exceed 1, given the specific inverse demand facing these firms. The impact of changes in c and s is discussed in greater detail below.

The parameter r represents the rate of interest charged on daylight overdrafts provided by the CA to the IC. A rate of interest is selected that is consistent with the short time frame for such a loan. Moreover, the results of the numerical analysis are robust to changes in r .

The chosen base level of on-them transactions is 90 per cent; however, it is recognized that the proportion of on-them transactions is likely to vary across systems and institutions. Thus, a range of reasonable values are explored in the numerical analysis (i.e., $\beta \in [0.5, 1]$), and the results are discussed below.

The parameter of substitutability between the IC's and the CA's retail payment services is assigned a base level that reflects a significant degree of competition between the two firms. A reasonably high level of competition is needed to ensure the imposition of credit risk by the IC on the CA. The impact of changes in γ on the equilibrium variables will depend on the specific values for the other key parameters β , s , and c , as illustrated in Appendix B. The impact of changes in γ on the equilibria results is discussed below.

One of the driving forces in the decision to participate as a CA instead of an IC is the fixed cost differential between these two modes of participation. In the numerical analysis, the fixed cost of being an IC is effectively normalized to zero and the fixed cost of being a CA is set above zero, to reflect the extra cost associated with direct participation. The base level of F_{CA} is chosen to ensure that the IC has no incentive to withdraw from the tiering relationship.

Finally, numerical results are summarized in this section, but figures are relegated to Appendix B.

4.1 Comparing credit-risk and risk-free equilibria

Generally, relative to risk-free equilibrium values, the presence of credit risk ($-\chi < \bar{v}_{IC}$) leads to the following. For the CA, its (i) clearing fee decreases, (ii) retail payment services level decreases, (iii) retail price increases, and (iv) profits from the retail and wholesale payment services provision increase. For the IC, its (i) retail payment services level increases, (ii) retail price decreases, and (iii) profits from the retail payment services provision increase. This is summarized in the following result.

Result 5 *Comparing risk-free and credit-risk equilibria*

Assuming sufficient credit risk, $\gamma > \bar{\gamma}(\beta, s, c)$, the following is true:

- (i) *For the CA, $w^{CR} < w^{RF}$, $q_{CA}^{CR} < q_{CA}^{RF}$, $P_{CA}^{CR} > P_{CA}^{RF}$ and $\pi_{CA}^{CR} > \pi_{CA}^{RF}$.*
- (ii) *For the IC, $q_{IC}^{CR} > q_{IC}^{RF}$, $P_{IC}^{CR} < P_{IC}^{RF}$ and $\pi_{IC}^{CR} > \pi_{IC}^{RF}$.*

Furthermore, $\bar{\gamma}(\beta, s, c)$, the range of conditional γ values supporting these equilibria, is decreasing in β , s , and c .

The above result holds for sufficient levels of retail competition, and thus credit risk. Weaker competition leads to higher retail profits for the IC, and, as a consequence, less credit risk is imposed on the CA. Therefore, when retail competition is sufficiently weak, i.e., $\gamma < \bar{\gamma}(\beta, s, c)$, there is not enough credit risk for its effect to outweigh the IO effect, in which case the CA charges a higher equilibrium clearing fee relative to the risk-free level ($w^{CR} > w^{RF}$). This brings about a reversal of inequalities for the IC: (i) its retail payment services level decreases, (ii) retail price increases, and (iii) profits from the retail payment services provision decrease.

Result 6 *Comparing risk-free and credit-risk prices*

While the IC's retail price is always lower with credit risk compared with its risk-free price, the CA charges a lower retail price only if retail payment services by the CA and the IC are close enough substitutes:

$$\begin{aligned} P_{CA}^{CR} > P_{CA}^{RF}, & \quad \bar{\gamma}(\beta, s, c) < \gamma < \hat{\gamma}(\beta, s, c) \\ P_{CA}^{CR} < P_{CA}^{RF}, & \quad \gamma > \hat{\gamma}(\beta, s, c). \end{aligned}$$

Furthermore, $\hat{\gamma}(\beta, s, c)$, the critical value above which $P_{CA}^{CR} < P_{CA}^{RF}$, is decreasing in β and c , but is relatively unaffected by s .

When $\bar{\gamma}(\beta, s, c) < \gamma < \hat{\gamma}(\beta, s, c)$, the CA's retail price is higher under credit risk. Combined with the result that the IC lowers its retail prices in the credit-risk equilibrium, one cannot say whether consumers are made better or worse off. When $\gamma > \hat{\gamma}(\beta, s, c)$, the CA's retail price is lower with credit risk. Thus, for $\gamma > \hat{\gamma}(\beta, s, c)$, consumers are unambiguously better off when credit risk is imposed on the CA, or when the CA is required to include an overdraft facility in its contract arrangement with ICs for wholesale payment services. Since this critical value, $\hat{\gamma}(\beta, s, c)$ is decreasing in β and c , the higher these parameters, the more likely it is that consumers will unambiguously benefit from lower retail prices for payment services.

Finally, for γ too low, significant credit risk to the CA does not exist. This is due to a lack of competition at the retail level, which leads to IC expected profits that are lower than in the risk-free case but still sufficiently high enough, together with available assets, to avoid default. The CA's expected profits from both retail and wholesale payment services are sufficiently high to absorb losses with little effect.

Table 2 summarizes the discussion associated with results 5 and 6, assuming all other parameters are set at their base levels. These results indicate that, when credit risk exists, the CA's equilibrium pricing strategies in both wholesale and retail service markets, as well as the impact on the IC's retail market equilibrium, can vary considerably in relation to the risk-free equilibria with different degrees of retail market competition. The less competitive the retail market, the more able the CA is to profit in the retail service market from its vertical integration. Conversely, the more competitive the retail market, the more the CA would seem to rely on wholesale services for its profitability.

4.2 Comparative statics

The previous section outlined the comparative equilibrium results for a CA and an IC with and without credit-risk exposure for the CA from its wholesale payment services. This section indicates how the equilibrium results will change for both these cases, independently of each other, under different values for some of the key parameters of the model.

Table 2: Comparing credit-risk and risk-free equilibria

$0.4 < \gamma \leq 0.44$	$0.45 \leq \gamma \leq 0.9$	$0.91 \leq \gamma < 1$
$w^{CR} > w^{RF}$	$w^{CR} < w^{RF}$	
$q_{IC}^{CR} < q_{IC}^{RF}$ $P_{IC}^{CR} > P_{IC}^{RF}$ $\pi_{IC}^{CR} < \pi_{IC}^{RF}$	$q_{IC}^{CR} > q_{IC}^{RF}$ $P_{IC}^{CR} < P_{IC}^{RF}$ $\pi_{IC}^{CR} > \pi_{IC}^{RF}$	
$q_{CA}^{CR} < q_{CA}^{RF}$ $\pi_{CA}^{CR} > \pi_{CA}^{RF}$		
$P_{CA}^{CR} > P_{CA}^{RF}$		$P_{CA}^{CR} < P_{CA}^{RF}$

Result 7 *Regardless of whether there is credit risk,*

$$\frac{d w}{d \gamma} < 0 \text{ then } > 0, \quad \frac{d q_{CA}}{d \gamma} < 0 \text{ then } > 0, \quad \frac{d P_{CA}}{d \gamma} < 0 \text{ then } > 0, \quad \frac{d \pi_{CA}}{d \gamma} < 0,$$

$$\frac{d q_{IC}}{d \gamma} < 0, \quad \frac{d P_{IC}}{d \gamma} < 0, \quad \frac{d \pi_{IC}}{d \gamma} < 0.$$

With an increase in γ , the retail market becomes more competitive. Holding w fixed, this reduces quantities, prices, and retail profits (reaction functions pivot on the vertical axis and become steeper). However, the CA can partially offset the impact of greater competition by changing equilibrium w . Interestingly, there are ‘tipping points’ in CA pricing behaviour in the wholesale and retail service markets, which are related to the degree of competition with the IC in the retail market. For γ sufficiently low, the CA selects a slightly lower equilibrium w as γ increases, but only up to a point. Beyond a relatively high level of γ , the CA charges a higher w for a higher degree of IC competition, raising the IC’s marginal cost and enabling the CA to increase both its retail market share and retail price, p_{CA} . By doing so, the CA slows its expected profit declines. Not surprisingly, greater competition in the retail payment market lowers both the CA’s and the IC’s profitability, but, all else equal, greater competition results in lower retail prices to consumers only if the degree of competition does not rise beyond the CA’s tipping point.

Result 8 *Regardless of whether there is credit risk,*

$$\frac{d w}{d \beta} < 0, \quad \frac{d q_{CA}}{d \beta} < 0, \quad \frac{d P_{CA}}{d \beta} > 0, \quad \frac{d \pi_{CA}}{d \beta} < 0,$$

$$\frac{d q_{IC}}{d \beta} < 0, \quad \frac{d P_{IC}}{d \beta} > 0, \quad \frac{d \pi_{IC}}{d \beta} < 0.$$

An increase in β implies that more payments are external to both the CA and the IC and to the second-tier network. With lower internalization of payments, both banks incur higher clearing and settlement costs (s for the CA and w for the IC per payment). The higher marginal costs in the retail payment market, holding w fixed, not surprisingly raise prices and lower quantities and profits for both the IC and the CA. However, a higher β (along with the CA's higher retail price) also increases the volume of payments that the IC has to settle through the CA. This increase in wholesale service demand seems to outweigh the higher marginal cost effect on the CA, and the CA responds to the higher β with a lower wholesale fee, w . Consequently, the demand for wholesale payment services (βq_{IC}) increases even more with the lower w , mitigating the decline in q_{IC} .

Result 9 *Regardless of whether there is credit risk,*

$$\frac{d w}{d s} > 0, \quad \frac{d q_{CA}}{d s} < 0, \quad \frac{d P_{CA}}{d s} > 0, \quad \frac{d \pi_{CA}}{d s} < 0,$$

$$\frac{d P_{IC}}{d s} > 0, \quad \frac{d q_{IC}}{d s} \leq 0, \quad \frac{d \pi_{IC}}{d s} \leq 0.$$

Result 10 *Regardless of whether there is credit risk,*

$$\frac{d w}{d c} > 0, \quad \frac{d q_{CA}}{d c} < 0, \quad \frac{d P_{CA}}{d c} > 0, \quad \frac{d \pi_{CA}}{d c} < 0,$$

$$\frac{d P_{IC}}{d c} > 0, \quad \frac{d q_{IC}}{d c} < 0, \quad \frac{d \pi_{IC}}{d c} < 0.$$

The comparative statics with respect to c and s are similar and are as one would expect with changes in the marginal cost of providing payment services. However, while a change in c affects both the IC and CA to almost the same extent, a change in s affects the CA directly and the IC *indirectly* through changes in w . Hence, an increase in c clearly lowers the IC's quantities and profits, but an increase in s may not significantly change those variables in equilibrium, if at all.

Table 3 summarizes the comparative statics arising from the numerical analysis.

5. Conclusion

In tiered systems such as the one presented in the model, a CA provides its IC with an essential input (clearing and settlement services), but also competes against the IC in the

Table 3: Comparative statics

	w	q_{CA}	q_{IC}	P_{CA}	P_{IC}	π_{CA}	π_{IC}
β	-	-	-	+	+	-	-
γ	-/+	-/+	-	-/+	-	-	-
s	+	-	-	+	+	-	-
c	+	-	-	+	+	-	-

retail market for payment services. The CA takes advantage of its position as operator of the second-tier network by strategically pricing its wholesale clearing fee so as to raise its rival's costs. Consequently, the IC must offer its retail payment services at a higher price, which enables the CA to attract greater retail market share.

Even if the CA faces an incentive to raise its rival's costs by charging a high wholesale clearing fee to its IC, the general conclusion of this study is that the CA will lower the wholesale service fee that it charges if the IC can impose sufficient credit risk on the CA. The CA recognizes that an increase in the IC's profits implies that the IC is less likely to default on credit provided by the CA. Therefore, when a CA incurs sufficient credit risk through the provision of overdraft settlement loans to an IC, this mitigates the CA's incentive to raise the IC's costs. As a consequence, the CA does not pursue the competitive advantage in the retail payment services market as aggressively as it might otherwise, and thus it loses some market share relative to the risk-free case.

Despite the CA's loss of market share, the analysis indicates that the CA earns higher profits in a contractual arrangement combining wholesale payment services with overdraft credit, compared with an arrangement that does not have such a credit facility. The IC also earns higher profits, except where the degree of competition between the IC and CA is so low that the credit risk imposed on the CA is insufficient to result in a lower wholesale clearing fee.

While the price of retail payment services charged by the IC is always lower when it imposes sufficient credit risk on the CA, the price charged by the CA falls only in the presence of credit risk when the degree of competition between the CA and IC is high. Hence, one cannot make a general statement about the impact of credit risk on the welfare of consumers. However, when a greater proportion of the banks' payments are on-them, less competition is required for credit risk to result in a decline in the CA's retail payment services fee, and in such a case consumers are unambiguously better off.

In some tiered arrangements—the ACSS, for example—the CAs are required to include an overdraft facility in their contract arrangements with ICs for wholesale payment services. There are some circumstances under which a tiered settlement system that requires the CA to supply overdraft credit to the IC could potentially be welfare-superior, from a consumer perspective, to those that do not. An obvious extension of this model is to define, more rigorously, the consumer welfare implications of an overdraft facility requirement.

The main findings of this paper pertain to the impact on equilibrium variables of credit risk imposed by the IC on the CA. However, the model also highlights the impact of competition, whether or not credit risk is present. In both the credit-risk and risk-free cases, the retail price charged by the IC declines with a higher degree of retail competition, yet the same is not true of the CA’s retail or wholesale price. For lower levels of retail competition, the wholesale price charged by the CA to the IC decreases as competition increases, but only up to a point. In other words, with or without credit risk, there is a level of retail competition beyond which the wholesale price charged by the CA actually increases with increases in competition, raising the IC’s marginal cost and enabling the CA to increase its retail market share and retail price.

As indicated earlier, the model that we describe abstracts from competition between second-tier networks. Developing a model in future work that reflects network competition may contribute insights into other policy concerns pertaining to tiered systems. There are, for example, two policy questions about tiered networks that are particularly relevant to payments system efficiency: (i) will more ICs, and the resulting internalization of payments in the second-tier network, influence the CA’s pricing strategies significantly; and (ii) will competition among CAs lower wholesale contract fees for all ICs, even if some contract discrimination persists? In terms of financial sector stability, the main question is whether tiered networks can propagate significant systemic risk from the second-tier to the first-tier network. Future work to incorporate relevant risk controls into the contract arrangements between CAs and ICs would help to clarify this issue and its effect on service pricing in both wholesale and retail markets.

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Appendix A: Proofs of Propositions

Proof of Proposition 1 Differentiating $E(NW_{IC})$ with respect to q_{IC} yields

$$\frac{dE(NW_{IC})}{dq_{IC}} = \frac{\partial \pi_{IC}}{\partial q_{IC}} \left[1 + \delta F \left(\frac{-\pi_{IC}}{1+r} \right) \right].$$

Hence, the IC's first-order condition, assuming an interior solution, is obtained at $\frac{\partial \pi_{IC}}{\partial q_{IC}} = 0$.

Furthermore, the second derivative of $E(NW_{IC})$ with respect to q_{IC} is negative:

$$\frac{d^2 E(NW_{IC})}{dq_{IC}^2} = \frac{\partial^2 \pi_{IC}}{\partial q_{IC}^2} \left[1 + \delta F \left(\frac{-\pi_{IC}}{1+r} \right) \right] - \frac{\partial \pi_{IC}}{\partial q_{IC}} \frac{\delta}{1+r} f \left(\frac{-\pi_{IC}}{1+r} \right) < 0,$$

since $\frac{\partial^2 \pi_{IC}}{\partial q_{IC}^2} < 0$.

Differentiating $E(NW_{CA})$ with respect to q_{CA} yields

$$\frac{dE(NW_{CA})}{dq_{CA}} = \frac{\partial \pi_{CA}}{\partial q_{CA}} [1 + \delta G(\pi_{CA})].$$

Hence, the CA's first-order condition, assuming an interior solution, is obtained at $\frac{\partial \pi_{CA}}{\partial q_{CA}} = 0$.

Likewise, the second derivative of $E(NW_{CA})$ with respect to q_{CA} is negative:

$$\frac{d^2 E(NW_{CA})}{dq_{CA}^2} = \frac{\partial^2 \pi_{CA}}{\partial q_{CA}^2} [1 + \delta G(-\pi_{CA})] + \left[\frac{\partial \pi_{CA}}{\partial q_{CA}} \right]^2 \delta g(-\pi_{CA}) < 0.$$

■

Proof of Proposition 2 First, we derive $\frac{dE(NW_{CA})}{dw}$ and show that the second-order condition is satisfied. The CA's first-order condition is

$$\frac{dE(NW_{CA})}{dw} = \frac{d\pi_{CA}}{dw} [1 + \delta G(-\pi_{CA})] = 0,$$

where, given Cournot quantities under no credit risk,

$$\begin{aligned} \frac{d\pi_{CA}}{dw} &= \frac{\partial \pi_{CA}}{\partial q_{IC}} \cdot \frac{dq_{IC}^{RF}}{dw} + \frac{\partial \pi_{CA}}{\partial w} \\ &= [-\gamma q_{CA}(w) + \beta(w - c - \beta s)] \left[-\frac{2\beta}{4 - \gamma^2} \right] + \beta q_{IC}(w). \end{aligned}$$

The second-order condition is satisfied because $E(NW_{CA})$ is strictly concave in w :

$$\frac{d^2 E(NW_{CA})}{dw^2} = \frac{d^2 \pi_{CA}}{dw^2} [1 + \delta G(-\pi_{CA})] - \left[\frac{d\pi_{CA}(w)}{dw} \right]^2 \delta g(-\pi_{CA}) < 0,$$

due to the fact that

$$\begin{aligned}\frac{d^2\pi_{CA}(w)}{dw^2} &= \left(\gamma\frac{\partial q_{CA}}{\partial w} - \beta\right)\frac{2\beta}{4-\gamma^2} + \beta\frac{\partial q_{IC}}{\partial w} \\ &= -\frac{8-\gamma^2}{4-\gamma^2}\cdot\frac{2\beta}{4-\gamma^2} < 0.\end{aligned}$$

Evaluating $\frac{d\pi_{CA}(w)}{dw}$ at the marginal cost of providing clearing services to the IC, the derivative is positive:

$$\left.\frac{d\pi_{CA}(w)}{dw}\right|_{w=c+\beta s} = \frac{2\beta\gamma}{4-\gamma^2}q_{CA}(w=c+\beta s) + \beta q_{IC}(w=c+\beta s) > 0.$$

To ensure that this is an equilibrium, we show that the IC's participation constraint is not violated. Consider the IC's participation constraint:

$$\pi_{IC}(w) = [1 - q_{IC}(w) - \gamma q_{CA}(w) - c - \beta w]q_{IC}(w) - F_{IC} \geq 0.$$

For small fixed costs, $F_{IC} \approx 0$, this is equivalent to

$$P_{IC}(w) = 1 - q_{IC}(w) - \gamma q_{CA}(w) - c - \beta w \geq 0,$$

and

$$q_{IC} \geq 0.$$

Both these conditions reduce to exactly the same inequality:

$$w \leq \frac{(2-\gamma)(1-c) + \beta\gamma s}{2\beta}.$$

Let w_{pc} denote the w that just satisfies the above condition,

$$w_{pc} = \frac{(2-\gamma)(1-c) + \beta\gamma s}{2\beta}.$$

We then can show that

$$\left.\frac{d\pi_{CA}(w)}{dw}\right|_{w=w_{pc}} = \{[(4-\gamma^2)\beta + (1-\gamma)(2-\gamma)]c - (1-\gamma)(2-\gamma) + (4-\gamma^2)(\beta-\gamma)\beta s\} \frac{2\beta}{4-\gamma^2}.$$

For c and s small enough, this expression is negative. That is, $w^{RF} < w_{pc}$.

Finally, we consider the case of a symmetric duopoly with two DCs who clear their own payment instructions. The equilibrium quantity and profit are functions of s :

$$q_{DC}^s = \frac{1-c-\beta s}{2+\gamma}, \tag{A1}$$

$$\pi_{DC}^s = \left[\frac{1-c-\beta s}{2+\gamma}\right]^2 - F_{DC}. \tag{A2}$$

Thus, the IC has no incentive to quit the tiering relationship and become a DC itself if and only if

$$\pi_{IC}(w^{RF}) - \pi_{DC}^s \geq 0,$$

which can be expressed as equation (11). ■

Proof of Proposition 3 With credit risk, differentiating $E(NW_{IC})$ with respect to q_{IC} yields

$$\frac{dE(NW_{IC})}{dq_{IC}} = \frac{\partial \pi_{IC}}{\partial q_{IC}} \left[1 - (1 + \delta)F(\chi) + \delta F\left(\frac{-\pi_{IC}}{1+r}\right) \right],$$

where

$$\chi = -\frac{A_{IC} + (1 + \delta)\pi_{IC}}{(1 + \delta)(1 + r)}.$$

Hence, the IC's first-order condition, assuming an interior solution, is obtained at $\frac{\partial \pi_{IC}}{\partial q_{IC}} = 0$. This is the same as in the no-credit-risk case. As in the no-credit-risk case, the second derivative is negative.

Differentiating $E(NW_{CA})$ with respect to q_{CA} yields

$$\frac{dE(NW_{CA})}{dq_{CA}} = \frac{\partial \pi_{CA}}{\partial q_{CA}} [1 + \delta G(\pi_{CA})] + \frac{\partial \pi_{CA}}{\partial q_{IC}} F(\chi).$$

Hence, the CA's first-order condition, assuming an interior solution, is obtained by setting the above to zero.

The second derivative of $E(NW_{CA})$ with respect to q_{CA} is

$$\begin{aligned} \frac{d^2 E(NW_{CA})}{dq_{CA}^2} &= \frac{\partial^2 \pi_{CA}}{\partial q_{CA}^2} [1 + \delta G(-\pi_{CA})] + \left[\frac{\partial \pi_{CA}}{\partial q_{CA}} \right]^2 \delta g(-\pi_{CA}) \\ &\quad + \frac{\partial^2 \pi_{CA}}{\partial q_{IC} \partial q_{CA}} F(\chi) - \frac{\partial \pi_{CA}}{\partial q_{IC}} \frac{\partial \pi_{IC}}{\partial q_{CA}} \frac{1}{1+r} f(\chi) < 0, \end{aligned}$$

because $\frac{\partial^2 \pi_{CA}}{\partial q_{IC} \partial q_{CA}} = 0$, $\frac{\partial \pi_{CA}}{\partial q_{IC}} < 0$ and $\frac{\partial \pi_{IC}}{\partial q_{CA}} < 0$. ■

Proof of Proposition 4 With credit risk, $-\bar{v} < \chi(q_{IC}, q_{CA})$. The CA's first-order condition is

$$\frac{dE(NW_{CA})}{dw} = \frac{d\pi_{CA}}{dw} [1 + \delta G(-\pi_{CA})] + \frac{d\pi_{IC}}{dw} F(\chi) = 0,$$

where

$$\frac{d\pi_{CA}}{dw} = \frac{\partial \pi_{CA}}{\partial q_{CA}} \frac{dq_{CA}}{dw} + \frac{\partial \pi_{CA}}{\partial q_{IC}} \frac{dq_{IC}}{dw} + \frac{\partial \pi_{CA}}{\partial w},$$

and

$$\frac{d\pi_{IC}}{dw} = \frac{\partial \pi_{IC}}{\partial q_{CA}} \frac{dq_{CA}}{dw} + \frac{\partial \pi_{IC}}{\partial w}.$$

Using the first-order conditions with respect to Cournot quantities, (12),

$$\frac{dE(NW_{CA})}{dw} = \left[\frac{\partial \pi_{CA}}{\partial q_{IC}} \frac{dq_{IC}}{dw} + \frac{\partial \pi_{CA}}{\partial w} \right] [1 + \delta G(-\pi_{CA})] + \frac{\partial \pi_{IC}}{\partial w} F(\chi) = 0.$$

Substituting for functional forms, we obtain (13).

Next, we show that the second-order condition is satisfied as long as $\frac{d^2 \pi_{IC}}{dw^2}$ is not too large:

$$\begin{aligned} \frac{d^2 E(NW_{CA})}{dw^2} &= \frac{d^2 \pi_{CA}}{dw^2} [1 + \delta G(-\pi_{CA})] - \left[\frac{d\pi_{CA}}{dw} \right]^2 \delta g(-\pi_{CA}) \\ &\quad + \frac{d^2 \pi_{IC}}{dw^2} F(\chi) - \left[\frac{d\pi_{IC}}{dw} \right]^2 \frac{1}{1+r} F(\chi), \end{aligned}$$

which is negative if $\frac{d^2 \pi_{IC}}{dw^2}$ is not too large, due to the fact that $\frac{d^2 \pi_{CA}}{dw^2} < 0$. ■

Appendix B: Graphs from Numerical Solutions

Figures B1, B2, and B3 correspond to Result 5. The first set of four graphs (Figures B1 and B2) show how equilibrium values differ across credit-risk and risk-free cases. Each of the graphs represents the difference of credit-risk minus risk-free equilibrium values. Hence, when a curve lies above the x-axis, it indicates that credit-risk values are higher than risk-free values and vice versa. The next set of three graphs (Figure B3) show the difference $w^{CR} - w^{RF}$ against γ for different levels of β , s , and c , and show that $\bar{\gamma}$ is decreasing in all these variables. Figure B4 corresponds to Result 6 and shows that the critical value of γ above which $P_{CA}^{CR} < P_{CA}^{RF}$ decreases in β and c , but is relatively unaffected by s . Figures B5-B9 correspond to Results 7-10, which present comparative statics.

Figure B1: Differences in equilibrium values against γ and β

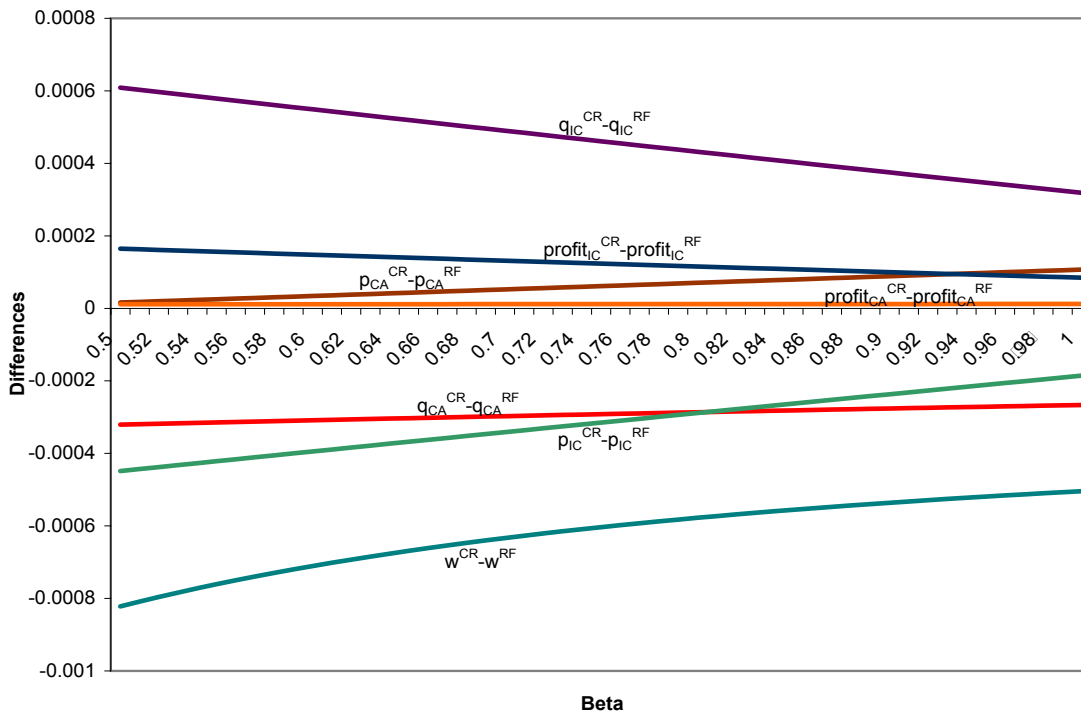
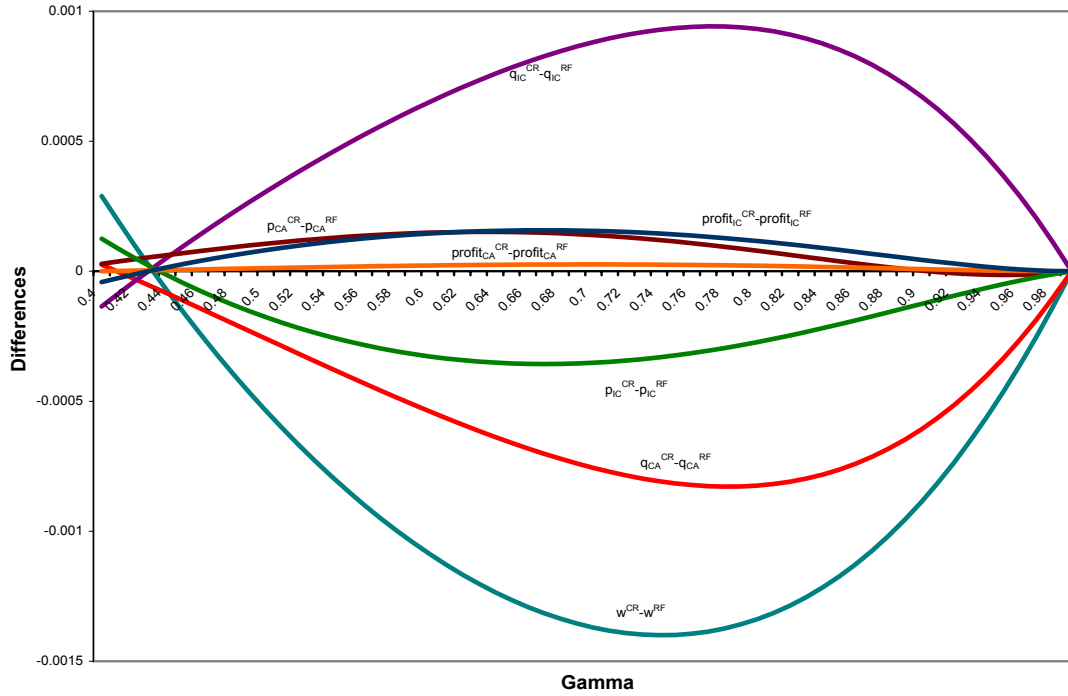


Figure B2: Differences in equilibrium values against s and c

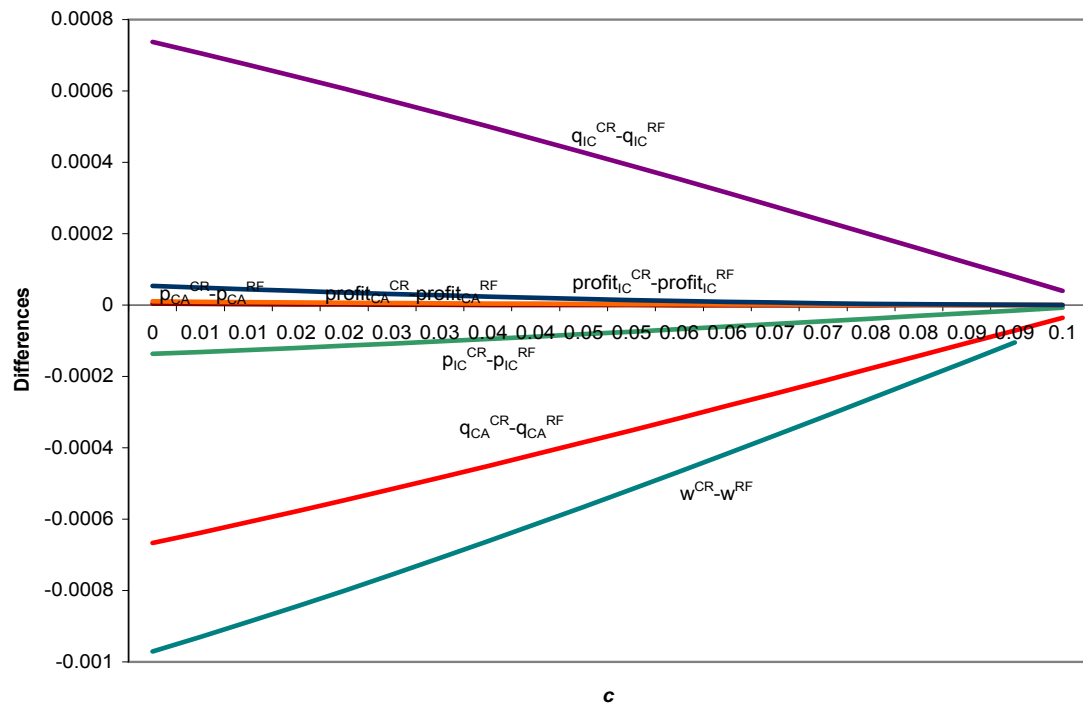
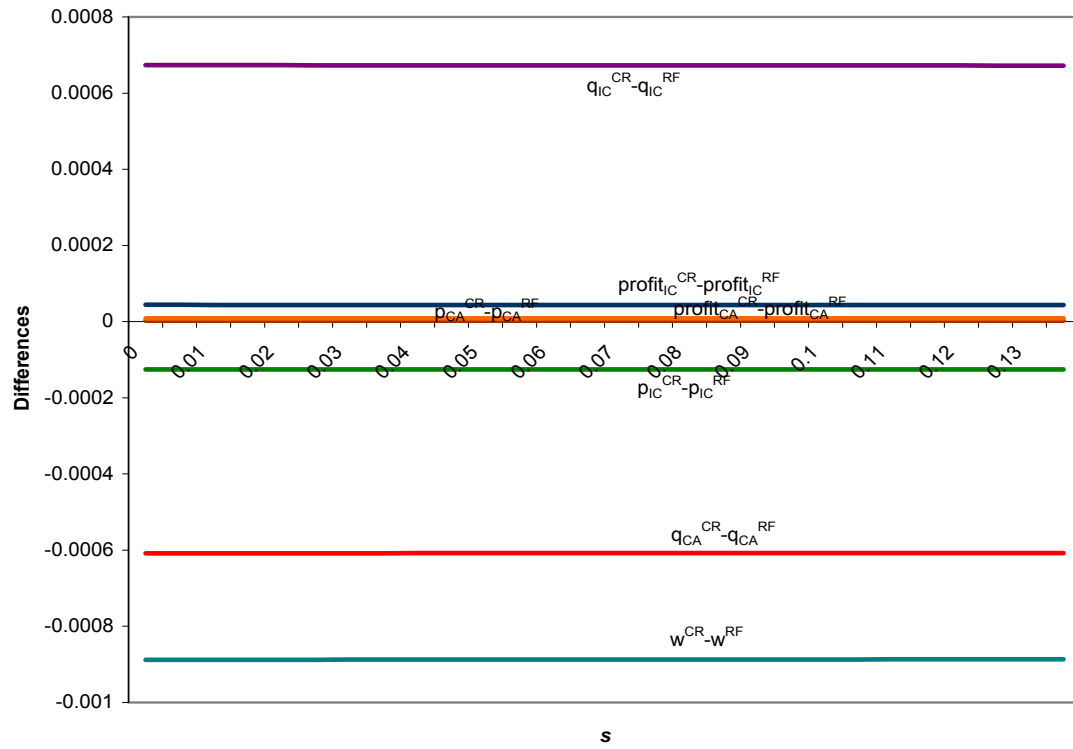


Figure B3: Differences in equilibrium w against γ for different β , s , and c

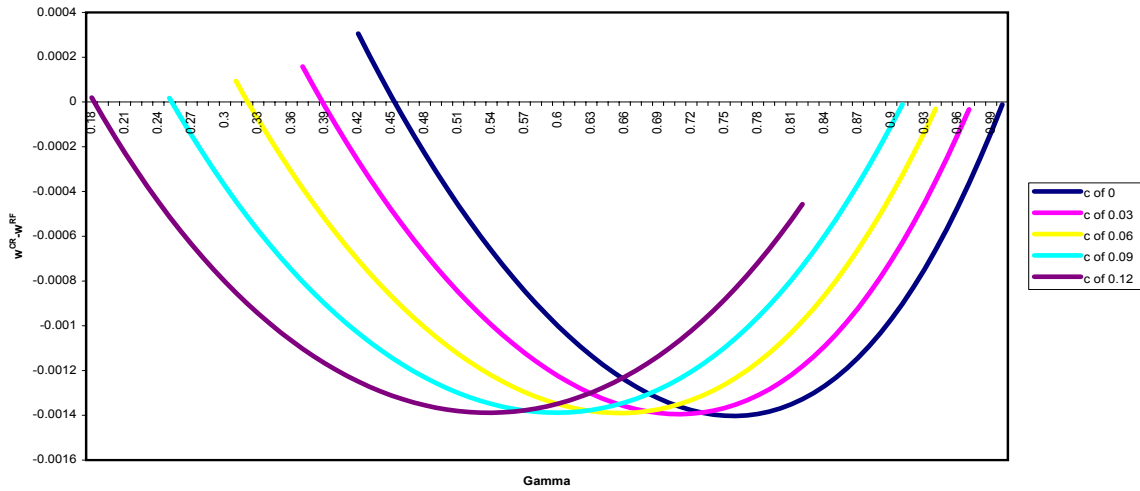
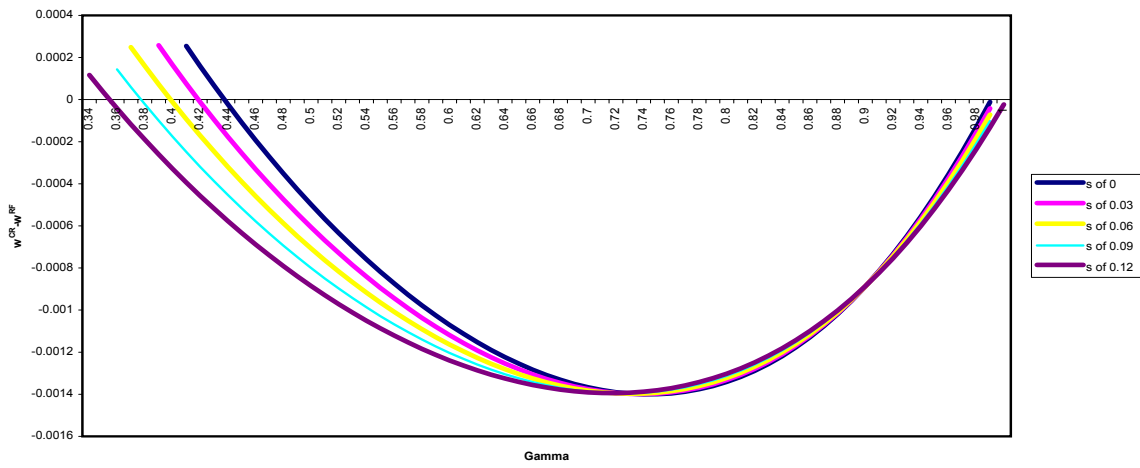
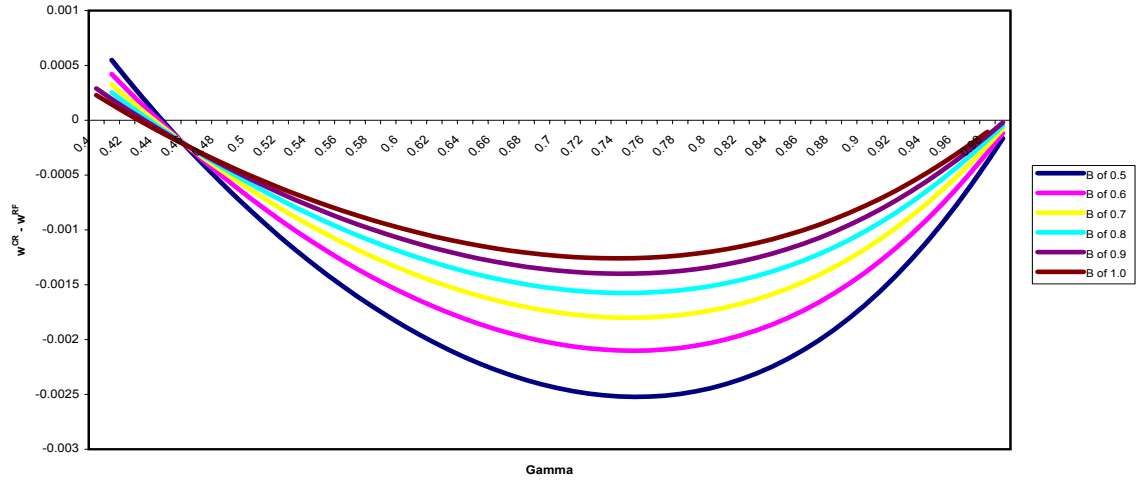


Figure B4: Differences in equilibrium P_{CA} against γ for different β , s , and c

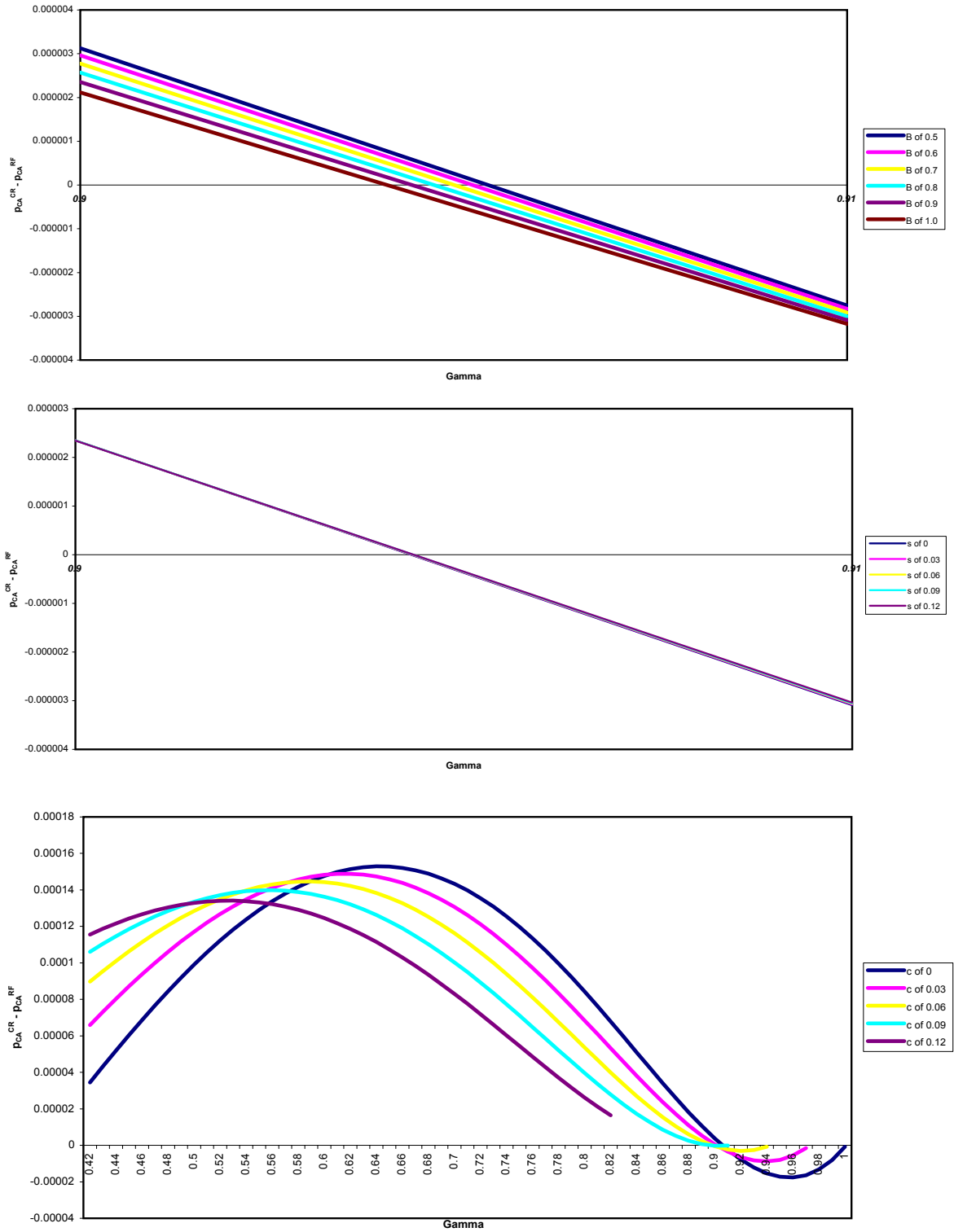


Figure B5: Comparative statics for γ

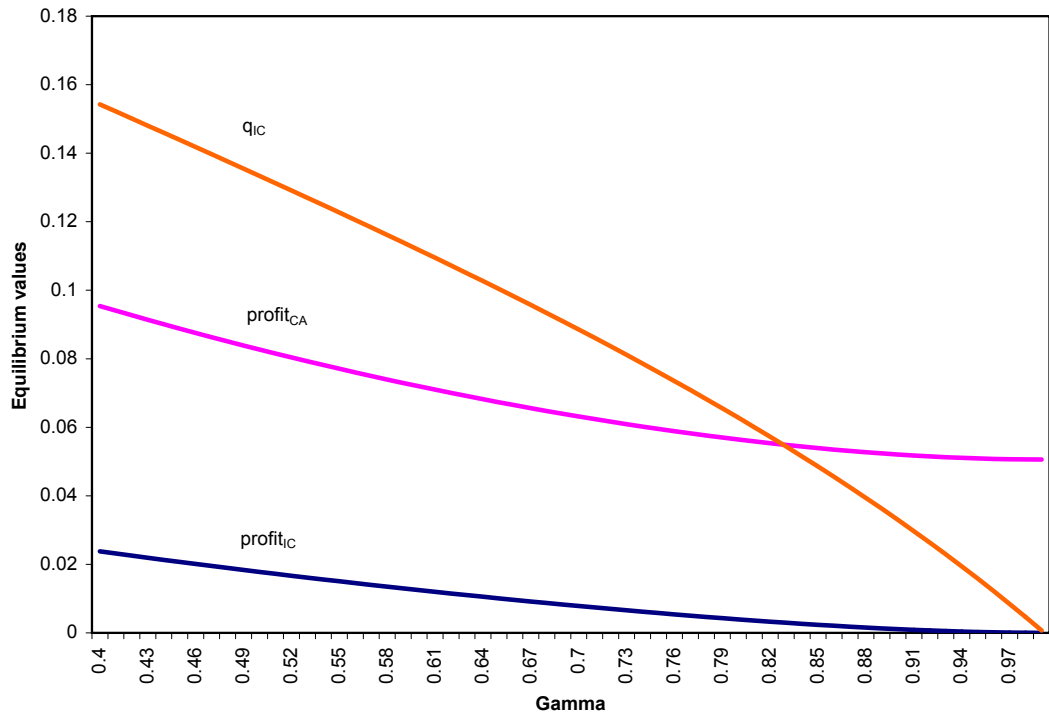
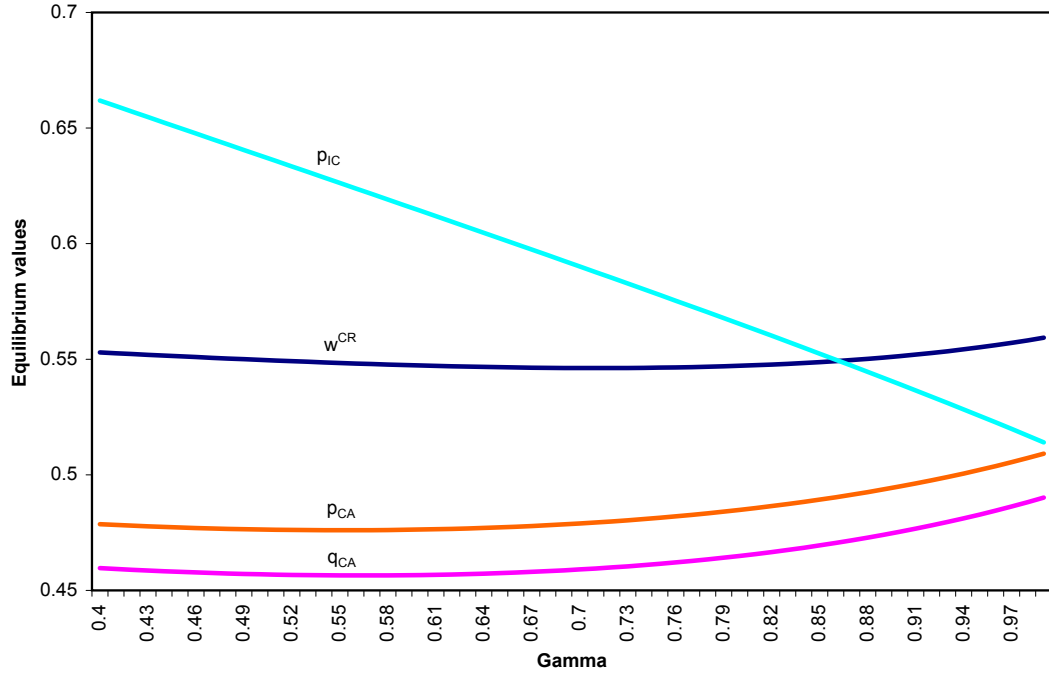


Figure B6: Comparative statics for β

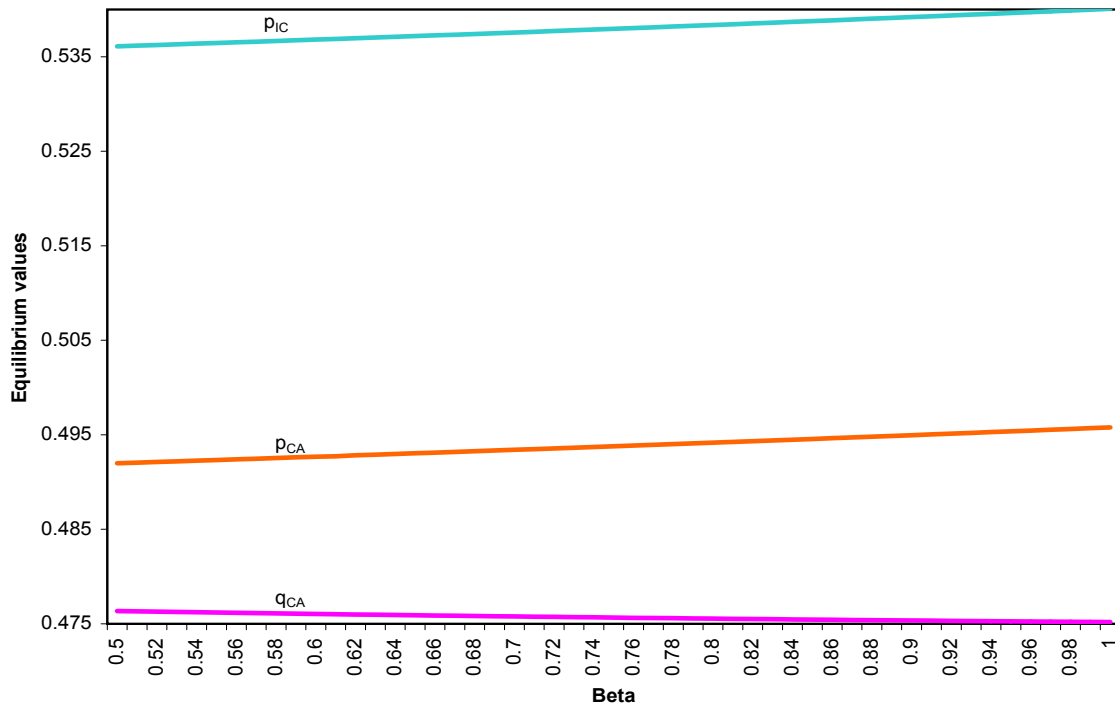
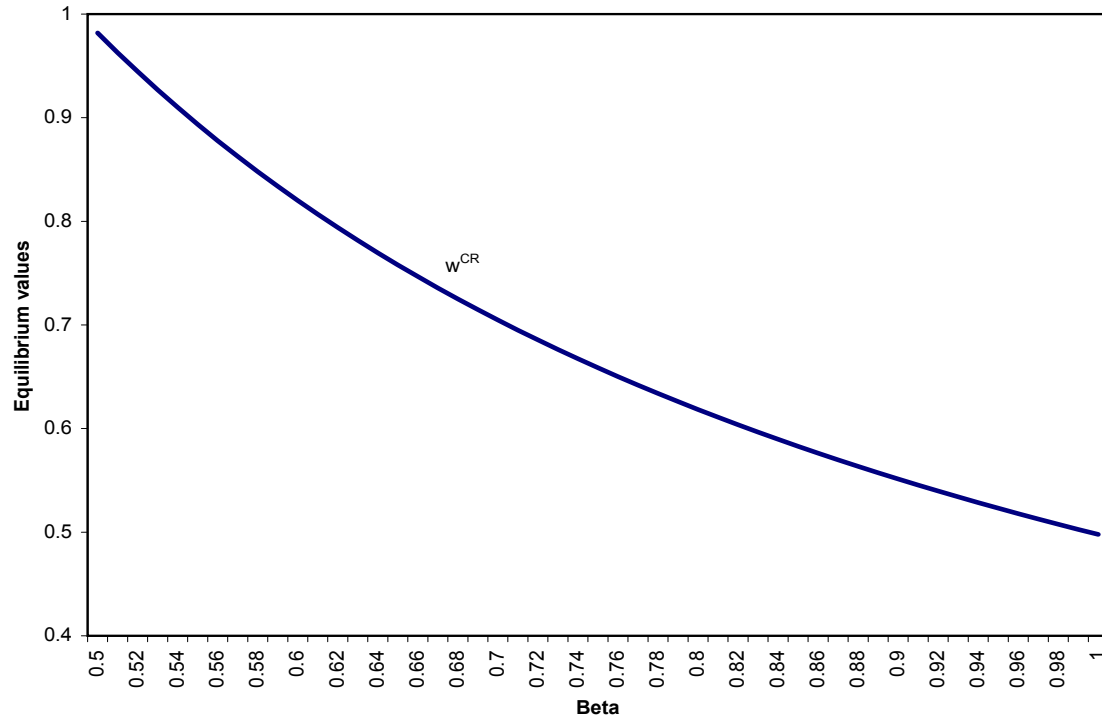


Figure B7: Comparative statics for β

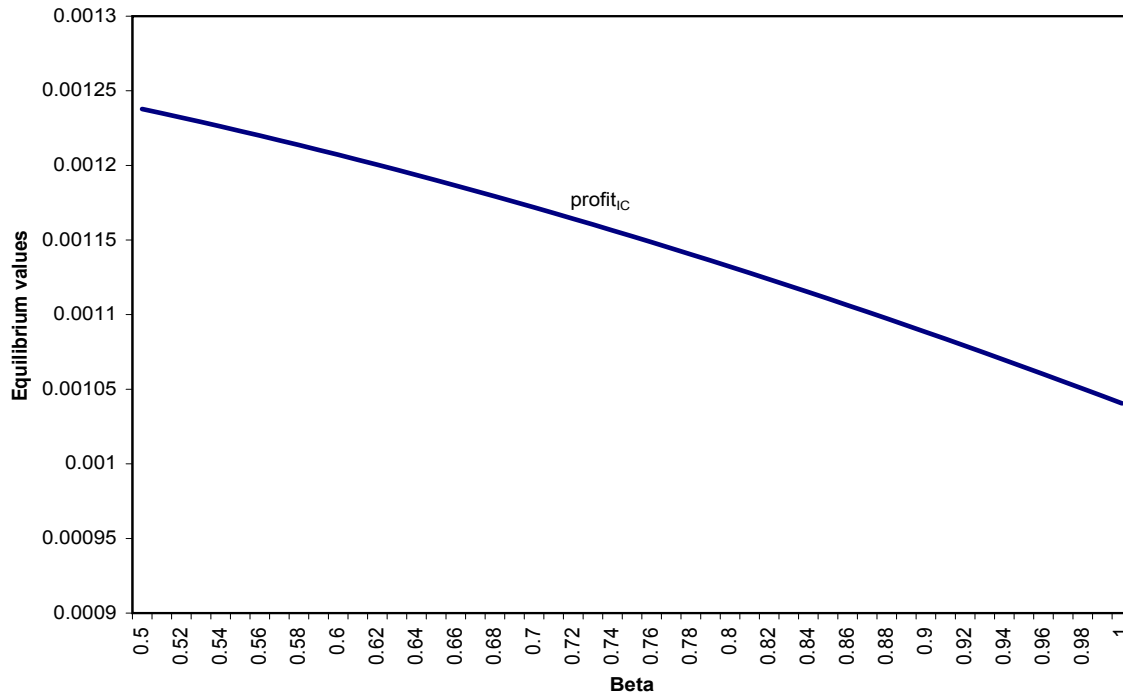
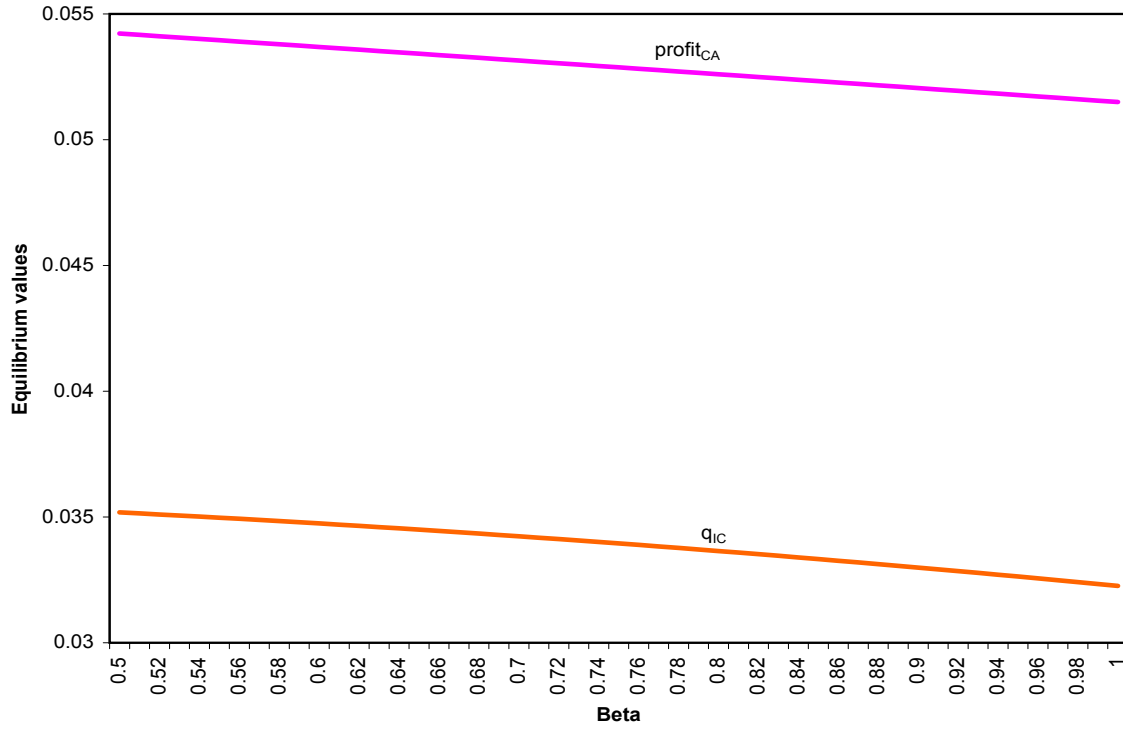


Figure B8: Comparative statics for s

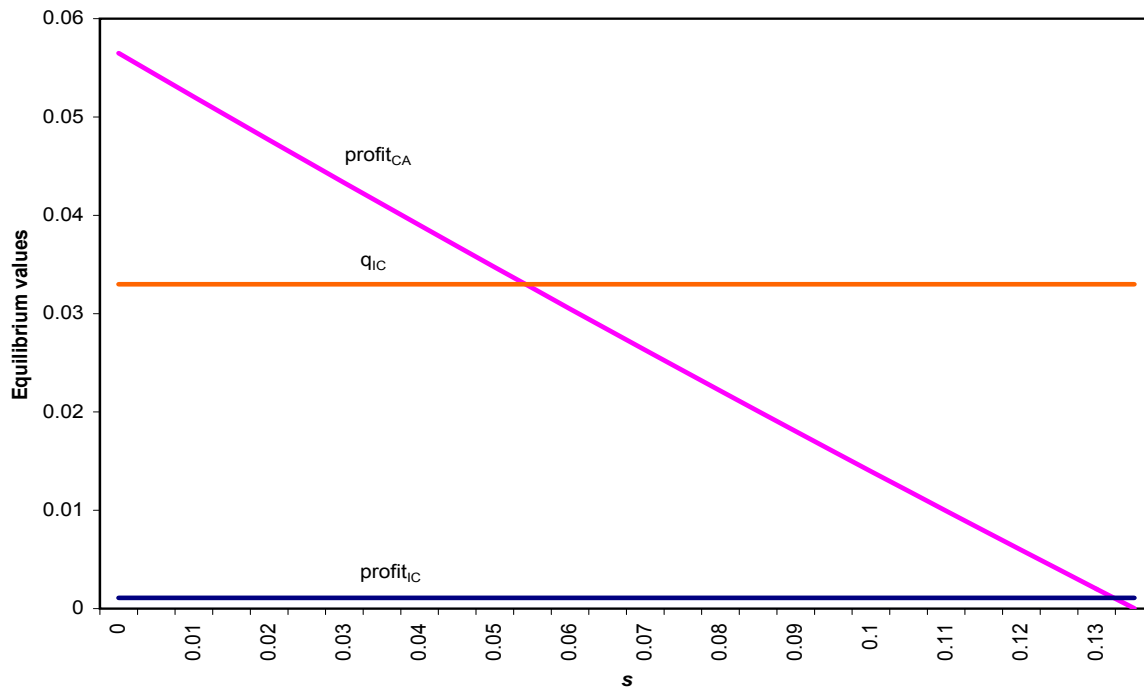
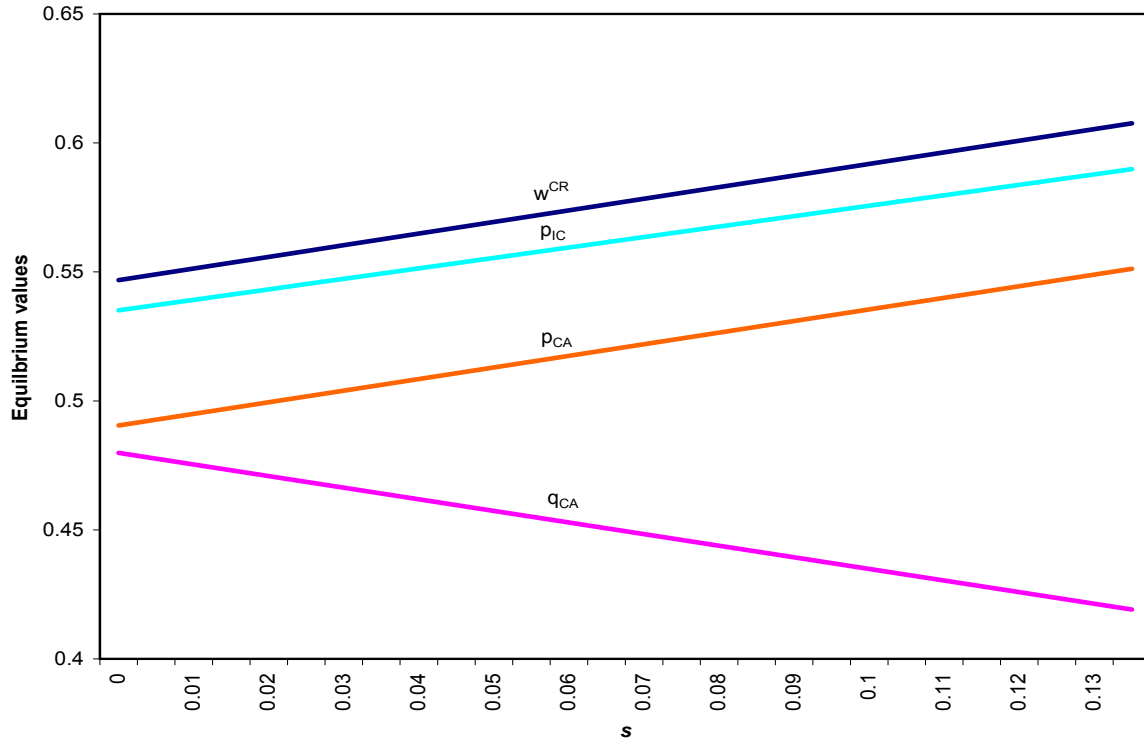
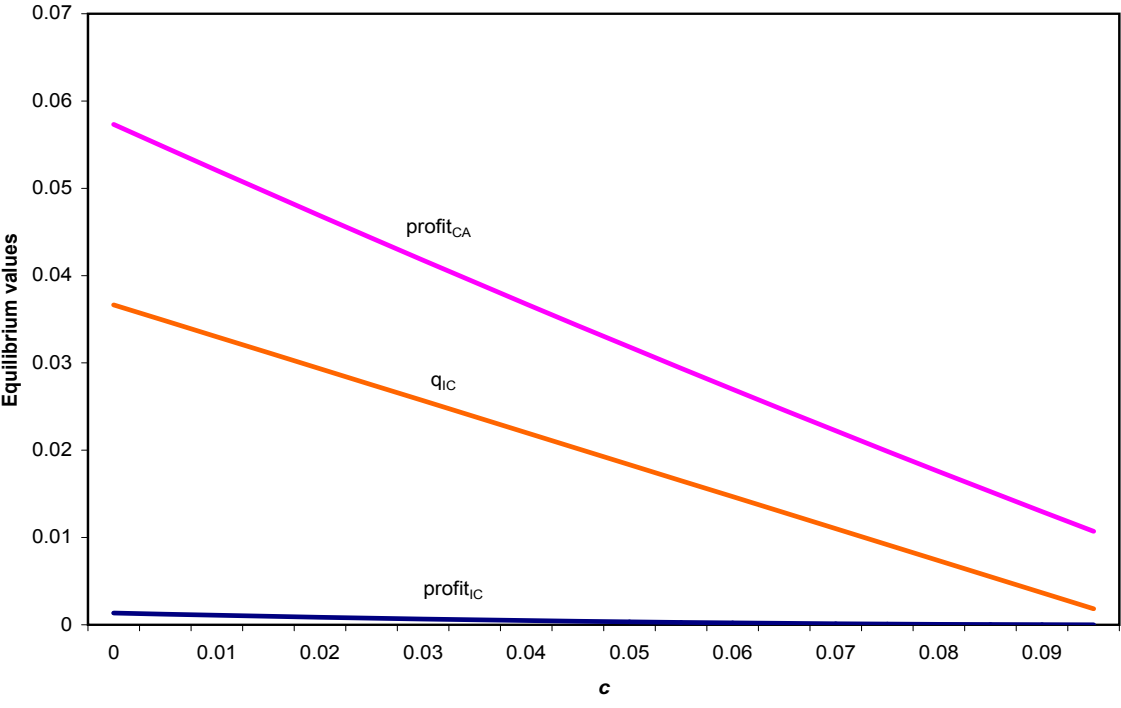
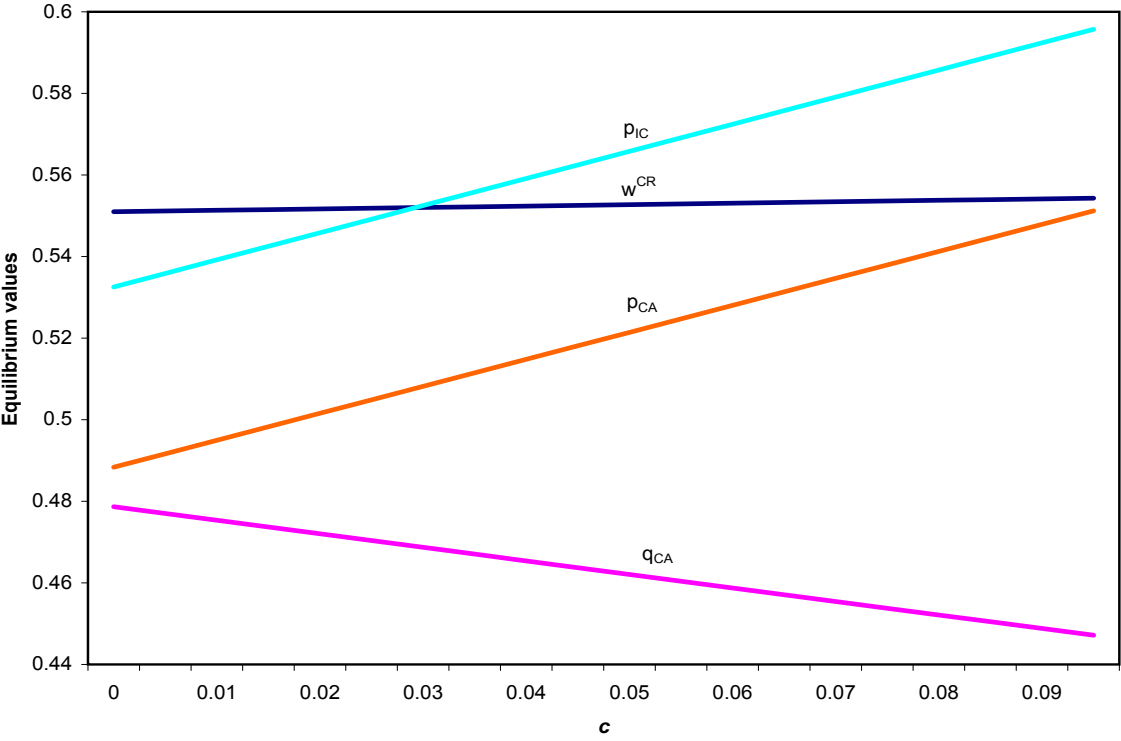


Figure B9: Comparative statics for c



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