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Long-Run Demand for M1

by
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Abstract

The goal of this paper is to investigate and estimate long-run relationships among M1, prices, output and interest rates, with a view to determining if there is a stable relationship that can be interpreted as long-run money demand. The paper uses a maximum-likelihood multiple-equation cointegration technique, developed by Johansen, to fit a system of equations to the data.

One finding is that long-run, but not short-run, unitary price elasticity is easily accepted, while the income elasticity is close to one-half. The coefficients on the deviation of money from its long-run equilibrium in the vector error-correction model imply that when M1 is above its long-run demand, money will decrease and prices increase to restore long-run equilibrium. The effects of the deviation on output and interest rates are insignificant, pointing to the weak exogeneity of these variables.

The implication of the results is that all the adjustment to return the economy to monetary equilibrium comes from fluctuations in money and prices. However, this does not preclude the possibility that changes in the stock of money may have short-run real effects. Indeed, the results suggest that changes in M1 lead short-term changes in output.

Résumé

Dans cette étude, l'auteur procède à l'analyse et à l'estimation des relations à long terme entre M1, les prix, la production et les taux d'intérêt, en vue de déterminer s'il y a entre ces variables une relation stable qui peut être interprétée comme une fonction de demande de monnaie à long terme. À cette fin, l'auteur cherche à rattacher un système d'équations aux données en recourant à la technique de cointégration proposée par Johansen, en l'occurrence celle à équation multiple axée sur la méthode du maximum de vraisemblance.

L'une des conclusions que tire l'auteur est que, à long terme, une élasticité-prix égale à l'unité se vérifie aisément, ce qui n'est pas le cas à court terme, tandis que l'élasticité-revenu s'approche de un demi à long terme. Les coefficients de l'écart du stock de monnaie par rapport à sa valeur d'équilibre de long terme dans le modèle vectoriel de correction des erreurs montrent que, lorsque M1 est supérieur à la demande à long terme, ce dernier baisse, et les prix montent de manière à rétablir l'équilibre de long terme. Les effets de l'écart du stock de monnaie sur la production et les taux d'intérêt ne sont pas significatifs, ce qui témoigne de la faible exogénéité de ces variables.

Il ressort de ces résultats que l'ajustement nécessaire à la restauration de l'équilibre monétaire au sein de l'économie est entièrement attribuable à des fluctuations du stock de monnaie et des prix. Cela n'empêche pas toutefois que des variations du stock de monnaie puissent avoir des effets réels à court terme. En effet, les résultats de l'étude tendent à montrer que les variations de M1 précèdent les variations à court terme de la production.

CONTENTS

1 Introduction and Summary

The goal of this paper is to investigate long-run relationships among money, prices, output and interest rates, with a view to determining whether there is a stable long-run relationship that can be interpreted as money demand.

Much of the work involving M1 in the recent past has not been able to find stable and well-specified long-run money-demand equations for Canada and other industrialized countries over the period including the 1980s. Haug and Lucas (1994) found a stable cointegrating relationship but it had unsatisfactory parameter values. Otto (1990) also found a unique, stable cointegrating vector through the use of segmented time trends, but his final estimates were made using the Engle-Granger methodology, even though more efficient techniques are now available. This paper finds that, with the use of the maximum-likelihood *multiple-equation* cointegration techniques proposed recently by Johansen and Juselius (1990), it is possible to find a unique and stable long-run cointegrating vector among nominal M1, a price level, real income and a short-term nominal interest rate, that resembles a money-demand relationship.

There is strong evidence in the data of cointegration among M1, prices, income and interest rates, even when including the 1980s. However, with just those variables in the short-run dynamics of the Johansen vector error-correction model (VECM), the resulting cointegrating vectors were non-unique, unstable, and difficult to interpret as long-run money-demand functions. A number of exogenous variables were then added to help improve the estimates of the short-run dynamics of the system. Once better estimates of the short-run parameters were obtained, it was possible to estimate unique and stable cointegrating vectors that conformed to the usual expectations for a money-demand relationship. Included among the exogenous variables were a short-term U.S. interest rate, the U.S.-Canadian exchange rate, a simple measure of the output gap, and a permanent shift dummy for the early 1980s. This shift variable was interpreted as a proxy for the financial innovations that occurred at chartered banks at that time. Although much effort went into finding other sensible short-run exogenous variables with clear economic interpretation to substitute into the system to replace the shift dummy, success has not yet been achieved. Among the variables that were explored were daily interest savings account (DISA) and daily interest chequing account (DICA) rates and dollar values, the yield curve, and the volatility of long-term rates. If future research finds a set of exogenous variables that can explain the change in the short-run dynamics of the endogenous variables which occurred in the early 1980s, then it will be possible to eliminate the shift dummy.

Cointegrating vectors were estimated for a number of different data definitions, including raw and seasonally adjusted, nominal and real, and monthly and quarterly. The best results seemed to be found using raw, quarterly data with nominal gross M1 (i.e. M1 not adjusted for float) from 1956 to 1993. These final results, which are given in Tables 6a to 6c along with some hypothesis tests, follow below. System #10a finds a cointegrating relationship among the natural logarithm of nominal M1, the CPI, real GDP and the level of the 90-day commercial paper rate (CP90), while System #11a uses the GDP deflator (DEF) as a measure of the price level.

$$M1_t = -0.503 - 0.141 D80_t + 0.930 CPI_t + 0.597 GDP_t - 0.038 CP90_t \quad (10a)$$

$$M1_t = -0.986 - 0.115 D80_t + 1.020 DEF_t + 0.428 GDP_t - 0.023 CP90_t \quad (11a)$$

The variable D80 is the dummy that proxies for any financial innovations or other factors causing a permanent negative shift in the M1 demand function in the early 1980s. The hypothesis of long-run unitary price elasticity was easily accepted for these two vectors (see Table 6c) with little change in the other coefficients. The income elasticities were relatively close to one-half, as they would be in a simple Baumol-Tobin type model, and the hypothesis of unitary income elasticity was rejected. The income elasticities and interest rate semi-elasticities are comparable to those estimated in previous Bank of Canada work.

The coefficients on these cointegrating vectors in the VECM, i.e. the adjustment coefficients, also conformed with a priori expectations. There was a significant negative adjustment coefficient in the ΔM1 equation and a significantly positive adjustment coefficient in the price equation. This means that, when money is above its long-run demand, money will decrease and prices increase in order to restore long-run equilibrium. The output and interest rate equations had positive and negative loadings, respectively, but they were insignificant, implying the weak exogeneity of these two variables. The implication of these results is that all the adjustment to return the economy to monetary equilibrium comes from fluctuations in money and prices. However, this does not preclude the possibility that changes in the stock of money can have short-run real effects. In fact, from the short-run dynamic coefficients it is found that lagged values of $\Delta M1$ do affect $\Delta M1_t$, ΔGDP_t, and $\Delta CP90_t$ but not ΔCPI_t or ΔDEF_t. M1 thus seems to be important for the longer-term trend or movement in prices but less so for the short-run fluctuations of inflation.

Figures 12 and 13 plot the cointegrating vectors or money gaps for Systems #10a and #11a, respectively. These vectors imply that monetary disequilibriums are quite persistent, since there seems to have been excess supplies of M1 for most of the period since 1956. These money gaps are in fact highly positively correlated with inflation such that positive money gaps are associated with positive rates of inflation. Further work on the indicator properties of this money gap and its importance for predicting inflation is ongoing.

Section 2 below gives a brief summary of the estimation technique and its implications, while Section 3 gives a brief résumé of some recent empirical work on M1 in Canada and other countries. Section 4 describes the data set, while Sections 5 and 6 report the results of the empirical estimation for the base system as well as a number of alternative specifications. Section 6 also discusses the role of the constant and presents an estimate of the cointegrating vector with the constant restricted to appear in only the long-run relationship. Section 7 describes the money gap, that is, the estimate of the monetary disequilibrium for a number of systems. Section 8 concludes the paper.

2 An Overview of the Johansen-Juselius Technique

Until recently, much of the empirical work involving the estimation of cointegrating vectors utilized the single-equation error-correction technique proposed primarily by Engle and Granger (1987). While quite useful, this technique suffered from a number of problems. For instance, it allowed for the estimation and testing of only one cointegrating vector, even though there could be as many as the number of variables involved less one. As well, since this single-equation method estimates the cointegrating vector with the data in levels only, it has been criticized for ignoring potentially valuable information contained in the short-run fluctuations of the variables. In the past few years, there has been a movement towards estimating cointegrating relationships in a system of equations framework to make better use of all the information available in the long- and short-run fluctuations of each variable. Johansen (1988) outlined a method, which was later expanded by Johansen and Juselius (1990), that allowed for the testing of more than one cointegrating vector in the data and for the calculation of maximum-likelihood estimates of these vectors.

The Johansen-Juselius (JJ) methodology begins with a statistical model of the following form:

$$X_t = \Pi_1 X_{t-1} + \ldots + \Pi_k X_{t-k} + \mu + \Psi D_t + \varepsilon_t \qquad (1)$$

where X_t is a vector of p variables, ε_t is a vector of disturbances such that $\varepsilon_1,\ldots,\varepsilon_T$ are IINp(0,Λ), μ is a constant, and D_t is a vector of exogenous variables including seasonal dummies. This paper will use an X_t vector that contains M1, prices, output and an interest rate. Many economic variables are nonstationary, so a first difference operator is generally applied to equation (1) to ensure the variables are stationary. However, this can lead to a loss of valuable long-run information unless it is done properly. Instead, JJ use some simple algebra to rewrite equation (1) as follows:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-k} + \mu + \Psi D_t + \varepsilon_t \qquad (2)$$

where $\Gamma_i = -(I - \Pi_1 - \ldots - \Pi_i)$

and $\Pi = -(I - \Pi_1 - \ldots - \Pi_k)$.

The JJ technique decomposes the matrix Π (pxp) to discover information about the long-run relationships between the variables in X. In particular, if Π has a rank of $0<r<p$ then it can be written as $\Pi=\alpha\beta'$ where β' is an rxp matrix of r cointegrating vectors (so that the r variables $\beta_i'X_t$ are stationary) and α is a pxr matrix of loadings or speeds of adjustment. Johansen and Juselius design a maximum-likelihood estimator to obtain estimates of α and β'. This procedure also yields two test statistics of the number of statistically significant cointegrating vectors. One test is called the λ-max statistic and compares the null of $H_0(r)$ with an alternative of $H_1(r+1)$. The second is the trace test, which examines the same null of $H_0(r)$ versus an alternative of $H_1(p)$. These are tests with non-standard distributions, but a number of papers have derived asymptotic critical values (see Johansen 1988, Johansen and Juselius 1990, and Osterwald-Lenum 1992) and there is some work being done to examine their small sample properties.

The β vectors represent estimates of the long-run cointegrating relationships between the variables in question. The α parameters measure the speed at which the variables adjust to restore a long-run equilibrium. If $\beta_i'X_t$ measures this long-run disequilibrium for a particular vector i, then from equation (2) (given $\Pi=\alpha\beta'$) the α parameter will determine the size of the contemporaneous change as the economy tries to move back toward equilibrium. (For the purpose of this study, monetary theory is able to provide strong prior expectations about the proper signs for these speeds of adjustment and cointegrating vector coefficients.) One problem with the JJ methodology is that it is not able to identify exactly the parameters in the α and β matrices. This is easily seen when it is pointed out that $\Pi=\alpha\beta'=\alpha AA^{-1}\beta'$ for any rxr matrix A. Because of this, the JJ technique defines a cointegrating space for which β is simply a set of basis vectors. Only if there is just one cointegrating vector found can truly concrete conclusions be made about any unique long-run relationship between the variables.

It is possible to do hypothesis testing on both the loadings and the cointegrating vectors using well-known likelihood ratio tests. For instance, it is possible to test whether there is unitary price elasticity in a long-run money-demand function or whether the coefficient on output is 1.0 or 0.5.

Similarly, hypothesis tests on the speeds of adjustment can be performed to determine whether monetary disequilibria have any significant effect on such variables as money, output and prices.

3 Some Recent Studies

A number of recent studies have searched for a stable long-run money-demand function for M1 in Canada as well as in other countries. The goal of these papers has been primarily to explain the instability that has been found in money-demand relationships since the early 1980s. The cause of this instability has generally been attributed to the many financial innovations that occurred during that period.

Haug and Lucas (1994) are able to find a stable cointegrating relationship for real M1 in Canada from 1968 to 1990 but find that the stability tests fail when a longer sample from 1953 to 1990 is used. Although there was a change in the Bank Act in 1967, this seems unusual, since it is the generally accepted belief that there was greater evidence of instability in the early 1980s than in any previous period. Also, they estimate an implausibly low income elasticity (0.12 or 0.24, depending on the estimation method[1]) and an insignificant interest-rate elasticity (either -0.29 or -0.04). These results may have been the result of not properly accounting for the innovations mentioned above, even though the tests implied stability. Haug and Lucas also claimed to find better results using real M1 or even M1 velocity. The current paper, by contrast, finds more reasonable parameter values and stability results when a nominal M1 specification is employed. A long-run unitary price elasticity was discovered, but allowing for short-run non-neutralities (entering the price variable separately in the short-run dynamics) was useful in improving the fit of the model.

Another study by Glenn Otto (1990) finds a unique cointegrating vector for M1, the price level, real output and the nominal interest rate for quarterly data over the period from 1956 to 1978. Although Otto finds the

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1. Haug and Lucas (1990) used Johansen-Juselius and Philips-Hansen estimation techniques.

unique cointegrating vector using the Johansen methodology, all the subsequent analysis uses Engle-Granger estimates. No weak exogeneity tests were included to verify that single-equation methods were applicable. He finds evidence of a permanent downward shift in the steady-state demand for M1 at the end of 1981. He models this shift with segmented deterministic time trends (instead of the segmented constant terms utilized in this paper) and finds that there was no change in the long-run price, income, and interest rate elasticities during the innovation period of the early 1980s. Otto (1990) found a unitary price elasticity, as in this paper, but also discovered a unitary income elasticity in stark contrast to the values of near 0.5 estimated here. He found an interest rate semi-elasticity of about -0.01, slightly below the values discussed later.

In two recent papers by D. Hendry and N. Ericsson (1991) and by Y. Baba, D. Hendry, and R. Starr (1992), money-demand relationships for M1 in the U.S.A. and the U.K. were examined. The authors believe that instability in money-demand functions is more likely the result of model misspecifications than any fundamental behavioural shifts in demand. The latter paper lists four possible types of misspecifications that could arise: a) incomplete dynamic structure, b) inadequate treatment of M1's own yields and those of alternative monetary instruments in the presence of financial innovations, c) inappropriate exclusion of inflation from the model, and d) the omission of the yield and risk level of other assets such as long-term government bonds.

Both of these studies find that, once the short-run dynamics are "properly" modelled, it is possible to find stable long-run money-demand relationships for M1, even when the 1980s are included. The primary variables used to achieve this goal were interest rates on new financial innovations. Rates on new non-transaction-based M2 accounts as well as those on new M1 accounts (Now and SuperNow accounts in the U.S.) were learning-adjusted to model a slow introduction period for the innovations. Other important variables were the inflation rate, the yield curve and the volatility of long-term bond rates.

In a recent study, however, Hess, Jones, and Porter (1994) criticize the Baba, Hendry, and Starr (1992) paper on the grounds that, once data for the

1988-1993 period are added to the sample, most of the good results disappear. Many of the parameters became unstable or insignificant and the predictive power of the model was substantially reduced. The quality of the results for the Baba, Hendry, and Starr model were found to be overly sensitive to the specification of lag lengths and to minor changes in the learning functions.

The methodology of the current paper also attempts to find stable long-run M1 demand relationships by more properly accounting for the short-run dynamics of the relevant explanatory variables.

4 The Data

In order to be thorough in the search for a long-run money-demand relationship, a number of different variable specifications and time-period frequencies were employed. Both net (M1) and gross (GM1) measures of M1 were considered in both seasonally adjusted and non-seasonally adjusted formats. Net M1 (which is adjusted for float) is available over a slightly longer sample period and has often been used in previous work, but it is gross M1 (which is not adjusted for float) that currently receives the closest attention at the Bank of Canada. The analysis was performed on both monthly and quarterly data using, as a scale variable, real total-economy GDP at factor cost (TE). Real expenditure-based GDP was also used for the quarterly estimations. The opportunity cost of holding money was proxied by the 90-day commercial paper rate (CP90). The total economy CPI and the GDP deflator (DEF) were both used to measure the general price level. Most of the analysis was performed on both raw and seasonally adjusted data because conventional seasonal adjustment techniques can bias unit-root and cointegration tests. The money, price and income variables were always in natural log form, while the interest rate variable was in percentages.

Table 1 contains Sargan-Bhargava, augmented Dickey-Fuller, Phillips-Perron, and KPSS unit root tests for M1, the CPI, output and the interest rate. Monthly raw data were used for the tests, and the results were basically the same if quarterly or seasonally adjusted data was used. Alternative measures of the price level (i.e. the GDP deflator) and output (i.e. expenditure-based GDP) also yielded comparable results. M1, output and the 90-day commercial paper rate were easily accepted as I(1) variables according to all four tests. There was some evidence that output was trend-stationary according to the Phillips-Perron test. Somewhat more controversial is the question of whether the price level is an I(1) or an I(2) variable. The Phillips-Perron and Sargan-Bhargava tests reject the presence of a unit root in the first difference of the CPI, thereby accepting that it is an I(1) variable. However, the ADF statistic fails to reject the presence of a unit root in the first-differenced data, and the KPSS test rejects the null of stationarity. These latter two tests only accept stationarity for the second difference of the CPI, implying that the price level is an I(2) variable. It is also possible to test for the presence of I(2) variables within the framework of the Johansen-Juselius methodology. The results of these tests, shown in the appendix, strongly reject that there are I(2) trends in the data. The remainder of the analysis in this paper shall continue under the assumption that the price level, whether measured by the CPI or the GDP deflator, is an I(1) variable.

5 The Base System

We begin with the view that it is preferable to estimate a system with as many observations as possible. Therefore, other things equal, longer time series and monthly as opposed to quarterly data are preferred.[2] Net M1 (M1) and gross M1 (GM1) are available from 1953 and 1954,

2. There is some evidence that it is the length of the period of available data that is important and not the frequency of the data. However, even though quarterly data are available for a longer time period, monthly data will still be examined, because the Bank of Canada monitors money and inflation on a monthly basis.

respectively.[3] However, the Bank of Canada currently monitors gross M1 more closely, so this measure of money will be the primary focus of this paper. Results using net M1 are also calculated and are shown when they are found to be pertinent. The only available monthly aggregate income measure is total-economy GDP at factor cost, available since January 1961. The CPI data begins in 1914, and it is the only price variable reported on a monthly basis. Finally, the opportunity cost measure is the 90-day commercial paper rate (CP90), which has been available on a monthly basis since 1956. It was also decided to use raw data because of the potential bias that conventional seasonal adjustment techniques can introduce into unit root and cointegration tests. In sum, the work begins with raw monthly data on gross M1, the CPI, TE, and CP90 from 1961 to 1993.[4]

Expected inflation and wealth are also sometimes used in long-run M1 demand studies and were examined in earlier drafts of this paper as well. Expected inflation (whether measured as actual inflation, a moving average of past inflation, or as an interpolation of the Conference Board survey results) was necessary in any cointegrating system that also included real M1, output and CP90 to obtain vectors resembling money demand. However, this variable almost always had a positive sign, implying that agents hold more money as its opportunity cost rises. Because of this anomaly and the fact that nominal specifications generally yielded more satisfactory results, systems including expected inflation are not included here. Wealth is another variable that is sometimes proposed as a determinant of money demand in the long run. However, wealth was

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3. Bank of Canada data for gross M1 is actually available starting in 1968. However, currency and gross demand deposit data, M1's two components, are available back to 1946 and 1954, respectively. An adjustment variable is also added to currency and demand deposits for form M1. This variable measures continuity adjustments, which are added back to avoid breaks that result from the acquisition of a near-bank by a bank. This variable, which starts in 1968, is large in the 1990s but is very small in 1968 (only $7 million). For the first four years following 1968, this adjustment variable is basically just a time trend that can be extended back over the pre-1968 period until it reaches zero in February 1967. This yields a gross M1 variable beginning in 1954.

4. All the systems using raw data also included seasonal adjustment dummies as exogenous variables.

insignificant in any system also involving current output and could not improve the results when it replaced output.[5]

5.1 Results

Table 2a presents the results of the cointegration tests using the Johansen and Juselius (hereafter JJ) (1990) technique on raw monthly data. The lag lengths for the VARs were selected according to the Akaike Information Criteria and tests for normality and serial correlation in the residuals of each equation. It was found that 14 lags were sufficient to remove most of the residual autocorrelation, but no lag length was able to ensure that all the residuals passed a normality test. In Table 2a and the others that follow, when more than one cointegrating vector is shown, they are ordered according to the corresponding eigenvalue beginning with the largest. The columns marked trace and λ-max (maximal eigenvalue) represent the two JJ cointegration tests. The column labelled PGp represents a test statistic designed by Gonzalo and Pitarakis (1994) to correct for small sample bias in the trace test.[6] The JJ technique tends to over-estimate the number of cointegrating vectors when there are small samples with too many variables or lags (Cheung and Lai (1993) and Pitarakis and Gonzalo (1994)). As such, this paper will place more weight on the outcome of the PGp test. Similarly, as an approximation, it is possible to consider higher confidence levels than usual (say, 99 per cent instead of 95 per cent) when using the trace or maximum-eigenvalue test statistics. Cheung and Lai (1993) have also done some work to define finite sample critical values for the JJ tests.[7] These critical values are not yet available for a wide variety of examples but still may be of some use.

5. The wealth measure used here was insignificant in the cointegrating vector at least in part because it was created from current income and interest rates and thus was highly correlated with these variables. Such a measure of wealth is sometimes found to be significant in two-step Engle-Granger techniques, which are less efficient but possibly better able to handle the multicollinearity that arises in the construction of the data. See Macklem (1994) for an example.

6. The technical appendix to this paper provides a brief description of this test statistic, which has the same asymptotic distribution as the trace statistic.

7. The results in Cheung and Lai (1993) imply that the asymptotic critical values should be increased, given the sample sizes and number of lags used here, by approximately 15 per cent for the monthly data and 18 per cent for the quarterly data examples. Monte Carlo simulation for examples in this paper discovered similar results.

In the interest of minimizing space, only the significant test statistics are shown. Note that the cointegrating relationships are shown in vector format and are normalized on the money variable. Therefore, the coefficients in the cointegrating vector β will be the negative of what they would be if placed in an equation format with money on the left-hand side and all the other variables on the right-hand side.

The results for System #1 in Table 2a show that nominal money, the CPI, income and CP90 form two cointegrating relationships at the 5 per cent significance level, according to the λ-max and PGp statistics, and three according to the trace test. Note that the PGp statistic is always smaller than the trace test – which is an illustration of the trace test's tendency to over-estimate the number of cointegrating vectors present in the data. Accepting two cointegrating vectors, it is seen that the first vector displays most of the desired qualities of a long-run money-demand relationship. All the coefficients of the cointegrating vector and the speeds of adjustment had signs that conformed with expectations. The price elasticity was positive although small (0.690). The income elasticity was close to one (0.947) and the interest rate semi-elasticity was -0.055.

The loadings or speeds of adjustment (i.e. α) also had theoretically meaningful signs. For instance, given that a positive error-correction term ($\beta' Zk(t)$) would represent an excess supply of money, there should be a negative speed of adjustment on that variable in the equation for money. Similarly, there should be a positive α in the output and price equations, and a negative α in the interest rate equation, so that money demand would increase to offset any excess supply. Given these a priori beliefs, the first cointegrating vector for System #1 satisfied all the criteria for α. There was a significant negative loading in the money equation and a significant positive speed of adjustment in the price equation. The remaining two speeds of adjustment had the proper signs but were insignificant. These results imply that only money and prices adjust in order to restore a long-run monetary disequilibrium.

Table 2b shows the results of χ^2 tests of whether each variable should actually be included in the cointegrating space and of whether all the loadings for a particular variable are zero. The null hypothesis for the first test is H_0: $\beta_{1j} = 0, \beta_{2j} = 0, \ldots, \beta_{rj} = 0$ for a particular variable j in the set {GM1, CPI, TE, CP90} where r is the number of cointegrating vectors. The null hypothesis for the loadings is H_0: $\alpha_{1j} = 0, \alpha_{2j} = 0, \ldots, \alpha_{rj} = 0$ for each variable j in the system. The number of degrees of freedom for these tests is equal to the number of cointegrating vectors (r). None of the four variables can be excluded from the cointegrating vector at even the 1 per cent significance level. The tests that all the loadings are zero in the money and price equations are easily rejected, implying again that these variables adjust to restore long-run monetary equilibrium. In contrast, the loadings in the output and commercial paper rate equations are easily accepted as being insignificantly different from zero. Output and CP90 are therefore described as being weakly exogenous with respect to the other variables, because there is no long-run feedback through the cointegrating vectors that affects these variables. The rejection of weak-exogeneity for the price level implies that single-equation techniques of estimating long-run money-demand relationships may be invalid, since they assume all explanatory variables are exogenous. A method that utilizes all the available information from the short and long run in a system of equations (such as the JJ procedure) does not suffer from this criticism.

Table 2c contains estimates for the constant from each equation as well as some diagnostic tests on the residuals. The Ljung-Box (LB) test for autocorrelation is distributed χ^2 with 24 degrees of freedom. The null of no autocorrelation was accepted for each equation. In contrast, the Bera-Jarque (BJ) test of the normality of the residuals is distributed χ^2 with 2 degrees of freedom and was strongly rejected for each equation. The price, output and interest rate equation residuals were particularly non-normal, but an examination of a plot of the residuals revealed that it was primarily due to a few outliers in the data. For instance, a spike in the price level due to the introduction of the GST in January of 1991 was responsible for the non-normality of the CPI equation residuals. It is uncertain how serious are the implications of non-normal errors, but the modifications discussed below correct for at least some of the problem.

Table 2d contains some hypothesis tests on the cointegrating vectors and loadings of System #1 (see Johansen 1991, or Johansen and Juselius

1990, 1992). The first test imposes price and income elasticities of 1.0 and 0.5, respectively, on the first vector and zero coefficients for money and interest rates in the second vector.[8] The test statistic is 5.135 so the restriction cannot be rejected at the 5 per cent significance level. Thus the first cointegrating vector appears to be a money-demand relationship, while the second may be a long-run relationship between output and prices. Imposing unitary price elasticity always had the effect of lowering the estimated income elasticity, so that it was much closer to 0.5 than to 1.0. This is consistent with the predictions of a simple Baumol-Tobin type model. The second hypothesis test shown for System #1 adds weak exogeneity of the interest rate ($\alpha_{CP} = 0$ for each vector in the CP90 equation) to the first test. The restriction is again accepted at the 5 per cent significance level and the first cointegrating vector still conforms with expectations for a long-run money-demand relationship.

5.2 Stability

In addition to having theoretically feasible coefficient values, one of the desirable properties of a long-run money-demand relationship is the stability over time of the parameter estimates. The stability of the cointegrating vectors in System #1 was investigated using a rolling regression technique. Given the full sample estimates shown in Table 2a, an estimate of the same number of cointegrating vectors was computed for a subsample, and a χ^2 test was performed to determine if the cointegrating vectors of the full sample (shown in Table 2a) were also in the cointegrating space estimated for the subsample (see Johansen and Juselius 1992)[9]. Another observation was then added to the subsample and the procedure was repeated. Figure 1 plots these χ^2 test statistics from this procedure with the numbers normalized so that 1.0 represents the 5 per cent critical value.

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8. The degrees of freedom are (q-r1)r2, where q is the number of restrictions, r1 is the number of unrestricted vectors, and r2 is the number of vectors to which the current restrictions apply. Therefore, with two cointegrating vectors, at least two restrictions must be applied to a single vector in order to obtain an identifying restriction with positive degrees of freedom. Otherwise, it is simply a different type of normalization.

9. The degrees of freedom are (p-r)r where p is the number of variables in the cointegrating system and r is the number of vectors.

The χ^2 test indicates that, for most of the sample periods ending before the end of 1981, there was a statistically significant difference between the subsample cointegrating vectors and the full sample vectors.

The remainder of the analysis in this section is aimed at reducing the number of cointegrating vectors, improving the normality of the residuals and correcting the instability present in System #1. It is believed that these results may originate from the exclusion of relevant variables from the short-run dynamics of the system. As such, a number of exogenous variables will be included in an attempt to improve these dynamics and the overall results. Also, if the number of cointegrating relationships can be reduced to a single vector, then one can be much more confident that it actually is a money-demand relationship that has been discovered.

5.3 Exogenous Variables

Instability in estimated M1 demand functions prompted the Bank of Canada to abandon M1 as a target variable in the early 1980s. This instability is often attributed to financial innovations that occurred in the latter part of the 1970s and early 1980s. The advent of daily interest savings and chequing accounts and improved methods of cash management by firms are common examples of the financial innovations that are credited for shifts in money-demand functions (Freedman 1982). Exogenous variables were added to the short-run dynamics of System #1 in an attempt to account for these possible shifts in the short-run movements of the variables and thereby permit more efficient estimates of the long-run cointegrating vectors.

The JJ technique is able to estimate multiple cointegrating vectors, all of which may be economically feasible, but there is no guarantee that all or any of the vectors will represent money-demand relationships. For instance, it is possible that one of the vectors estimated in System #1 should actually be interpreted as an aggregate demand or supply relationship between output and prices or possibly an output gap that differences aggregate supply and demand. A simple method of (potentially) controlling for this possibility is to explicitly include the output gap between aggregate demand and supply as an exogenous variable.

Specifying such a cointegrating vector could reduce the number of vectors to be estimated so that only a money-demand relationship remains. Another long-run relationship that may be important in Canada is an uncovered interest rate parity condition (UIP) between U.S. and Canadian interest rates. Explicitly modelling the close ties between interest rates in the two countries should also be helpful in improving the estimated cointegrating vectors. The JJ technique uses a system of equations methodology to employ all the available information in each endogenous variable. Therefore, any improvement in the fit of a particular equation or in the explanation of the short-run dynamics should lead to better estimates of the cointegrating vectors. This means that, even if the output gap explains output only and the interest rate parity condition is important for only CP90, it is still possible that the addition of these exogenous variables can improve the estimates of money-demand cointegrating vectors.

The output gap (TEGAP) was defined as the residual from a regression of output against a linear and a quadratic time trend.[10] The contemporaneous changes in the Canadian-U.S. exchange rate (Δe_t) and the U.S. 90-day commercial paper rate ($\Delta USCP_t$) were also included as exogenous variables in order to improve the fit of the interest rate equation.[11] A dummy variable (DGST) with a value of one in January 1991 and zero otherwise was included to account for the introduction of the Goods and Services Tax in that period. A number of dummies were also included to control for the effects of postal strikes and data reporting

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10. Other work not reported here utilized an output gap defined as the difference between actual output and a Hodrick-Prescott filtered trend. This gap was highly correlated with (and worked about as well as) the series described above. However, its use always led to the conclusion that there were I(2) trends in the data, while any system without an HP-filtered gap concluded that there were no I(2) trends in the data.

11. These two variables were found to improve the fit in comparison with the first lag of an uncovered interest rate parity variable. This is likely because interest rates and exchange rates adjust more quickly in response to disequilibria in financial markets than can be modelled properly even with monthly data.

changes on the demand for M1.[12] Finally, a dummy (D80) was added to account for any permanent shift in the constant of the money equation in the early 1980s. This variable is a proxy for any shift in money demand occurring as a result of financial innovations introduced at that time. This dummy variable had a value of zero before January 1980 and of one after December 1982. Between those dates, it increased linearly so as to approximate the slow introduction and dissemination of financial innovations such as daily interest savings accounts. (A short discussion later in the paper will focus on an attempt to replace this dummy with variables having more economic or theoretical interpretation.)

System #2 in Table 2a shows the results from reestimating the first system when $TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $DGST_t$, $D80_t$, and the postal strike dummies are included as short-run explanatory variables. Both the λ-max and trace tests now accept only one cointegrating vector. The coefficients of this vector (β) and its loadings (α) all conform with what would be expected for a money-demand relationship. In comparison with the first vector in System #1, there has been an increase in the estimated price elasticity and a decline in the output elasticity and the interest rate semi-elasticity. The disequilibrium measured by this cointegrating vector still has a significant effect on only money and prices. Output and the interest rate were both weakly exogenous (see Table 2b).

The shift variable, D80, was negative and significant (see Table 2c) in the gross M1 equation, while the output gap had a significant positive effect on prices and a significant negative impact on output. The contemporaneous change in the exchange rate and the U.S. interest rate had significant positive coefficients in the CP90 equation, illustrating the close ties between the financial markets of Canada and the United States. The GST dummy was positive and significant in the CPI equation, as expected. The exogenous variables improved the fit of each equation and helped to reduce

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12. The following strike dummies had a value of one in the month corresponding to their label, a -1.0 in the subsequent month, and zero otherwise: Apr74, May74, Oct75, Nov75, Dec75, Oct78, Nov78, Dec78, Jul81, and Aug81. The same type of dummy for November 1981 is also often included in M1 demand studies to help explain an unusually large change in M1 in that month that was possibly related to a change in reporting which occurred that month as a result of the 1980 Bank Act.

the degree of non-normality in the money, price and interest rate equations. There was still a high degree of non-normality in the output and interest rate equations because of a number of large monthly changes in the data.

Table 2d shows the results of three hypothesis tests for System #2. The first tests the restriction of unitary price elasticity and rejects the null at even the 1 per cent significance level. As was mentioned above, increasing the price elasticity to unity (from an estimated range of 0.8 to 0.9) tends to reduce the output elasticity to about one-half (from an estimated value of about 0.7). The second hypothesis is a joint test of the weak exogeneity of output and the interest rate and is easily accepted with a p-value of 0.580. The final test combines the first two and once again rejects the null at the 5 per cent level. The importance of having only one cointegrating vector is illustrated by how much more difficult it is to accept unitary price elasticity in System #2 than in System #1 even though the unrestricted estimate is closer to one in System #2. Since the JJ technique identifies only a cointegrating space (instead of uniquely identifying vectors), restrictions on one vector can be offset by changes in the estimate of another vector thereby making acceptance of a null hypothesis that much easier.

Figure 2 plots the results of the stability test for System #2.[13] The exogenous variables succeeded in removing the instability in the cointegrating vector for the entire period following the end of 1977. There still seemed to be some instability present in 1976 and 1977. This instability could be evidence of either small sample problems or possibly another shift that occurred in the money-demand process. Some modernization of cash management techniques for large corporations did occur in the mid-1970s (Freedman 1982) and may be responsible for some of this remaining instability. Dummy variables for this earlier period were not able to remove the instability in the same manner as D80 was able to for the early 1980s.

13. It was necessary to modify the stability test somewhat in the presence of exogenous dummy variables. If the sample was shortened, then it was possible that certain variables would simply become fixed at zero. For instance, DGST could not be included in tests for samples ending before January 1991. For January 1991 to December 1993, the stability test compares the shortened sample results with the full sample results for System #2. For January 1980 to December 1990, the test compares the subsample results for a system without DGST to the full sample results for System #2. The test continues back in time in this manner, dropping dummy variables as they become fixed at zero.

Once again, however, this remaining instability may simply be the result of small sample bias, since there is only about 15 years of monthly data available before 1976. Quarterly estimates over longer samples did not exhibit evidence of this instability.

System #3 in Table 2a was estimated to investigate the effects of having a high degree of non-normality in the errors of some of the equations. Although including the U.S. interest rate and the exchange rate did help explain some of the large monthly changes in Canadian interest rates, they were not able account for all the large changes. System #3 adds a number of exogenous dummy variables to account for the primary outliers in the data.[14] Outliers in the output series all occurred in the early part of the sample and seem to be related to a shift in the seasonal pattern of the series that could not be modelled assuming fixed seasonal dummies. Outliers in the other series did not follow any apparent pattern but at least some, those in the spring of 1986 and the fall of 1992, were likely related to policy interventions that caused large interest rate changes during the exchange rate crises which occurred around those dates. (Using quarterly observations and seasonally adjusted data to smooth the series is another manner in which much of the non-normality can be removed: these results are discussed later in the paper.) Obviously, including explanatory variables with more economic content to explain outliers would be preferable, but no satisfactory results in this regard have been obtained.

The primary effect of the inclusion of the extra dummy variables was to increase the evidence in favour of a second cointegrating vector. However, appealing to the PGp statistic, which has better finite sample properties, only one cointegrating vector is accepted at the 5 per cent significance level.[15] The cointegrating vector and loadings did not change substantially in comparison with System #2, but the significance of the

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14. The following dummies were included with a value of one in the date shown and zero otherwise – for M1: Dec82; for output: Sep62, Sep63, Sep65, Sep66 Sep68, Sep69; for CP90: Jun62, Mar71, Jan74, Jan75, Oct79, May80, Oct80, Dec80, Jun82, Feb85, Apr86, Sep92, Oct92, and Nov92.

15. Subsequent work that calculated finite-sample critical values specific to the cases in this paper verified that the conclusion of a unique cointegrating vector was correct. All three test statistics reached this conclusion when using the appropriate critical values.

loading in the output equation did increase such that it was no longer insignificant at the 5 per cent level. Long-run unitary price elasticity was still rejected at the 1 per cent level as in System #2, and only the loading in the CP90 equation could be set to zero.[16]

Figure 3 plots the stability test statistics for System #3. Correcting for the non-normal errors allowed the period of instability to be pushed back to before the end of 1976. However, it did raise the average value of the test statistic for the 1978 to 1983 period, although no month actually exceeded the 5 per cent critical value.

A number of other exogenous variables were included in early specifications of the model but are not reported here because of their failure to yield significant or sensible results. The interest rates on daily interest chequing and savings accounts were insignificant in the money equation although they did have a significant effect on CP90 (probably because of their high correlation with treasury bill rates). Rates on these new innovations, adjusted for a slow learning period (see Hendry and Ericsson 1991), were also unable to estimate significant negative shifts in the money equation. The apparent lack of significance of these variables despite a strong belief in their importance may be the result of an incorrect lag structure or an improperly modelled learning curve. It may also be that any shift from M1 accounts to daily interest accounts occurred only during the first few years after the introduction of the new instruments. Following the phase-in period, the degree of substitution out of M1 was likely minimal and is therefore difficult to find in the data for the entire period after 1980. This was investigated with shift dummy variables that returned to zero in the latter part of the 1980s, but no real success was achieved.

The dollar value of daily interest accounts also had an insignificant effect on the demand for M1. The other major innovation thought to have influenced the demand for M1 was the introduction of new cash management techniques. However, no variable was found that could

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16. It is interesting to note that if a shorter sample from 1968 to 1993 (the period for which published gross M1 data exist) is used, then unitary price elasticity is easily accepted, as are zero loadings in the output and interest rate equations.

model this change in any meaningful manner. The yield curve and the volatility of long-term interest rates also had little explanatory power for the evolution of money on a monthly basis, even though they do have a significant effect on M1 in some other countries (see Hendry and Ericsson 1991, and Baba, Hendry and Starr 1992).

6 Alternative Data Definitions

6.1 Seasonally Adjusted Monthly Data

Tables 3a to 3d present the results for a system using seasonally adjusted data. System #4 still comprises the same four basic variables, except that gross M1, the CPI and output were expressed in seasonally adjusted form.[17] The exogenous variables were still required to reduce the number of cointegrating vectors, improve the stability of the cointegrating vector and achieve normal errors.[18] The second cointegrating vector was less significant than for System #3, but only the PGp test statistic said there was only one cointegrating vector.[19] The parameters of the cointegrating vector and loadings did not change substantially in the move to seasonally adjusted data. The loadings in the output and interest rate equation were once again insignificant, as in Systems #1 and #2, but in contrast to System #3, which had a significant loading in the output equation.

The exogenous variables were still all significant in the relevant equations and had values that conformed to expectations. The hypothesis tests in Table 3d show that, once again, unitary price elasticity is rejected at the 1 per cent significance level. However, the joint hypothesis that the

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17. The new gross M1 series constructed for this paper was seasonally adjusted over its entire sample (1954-1994) using RATS instead of the published adjusted series for the period after 1968. There did not seem to be a substantial difference between the two series for the overlapping period.

18. The dummies listed in footnote 12 that applied to output were not included since they modelled seasonal effects that have now been removed from the output series used. The remaining dummies were unchanged.

19. Slightly shorter samples were able to reject the second cointegrating vector more easily, even though the Johansen-Juselius technique is biased towards accepting more vectors in small samples.

loadings in the interest rate and output equations are both zero is easily accepted with a p-value of 0.310. Money and prices still seem to be the primary variables that adjust to restore a monetary disequilibrium.

6.2 Net M1

System #5 in Tables 3a to 3d uses net M1 and seasonally adjusted data. There are two cointegrating vectors accepted at the 5 per cent significance level, the first of which appears to represent a money-demand relationship with parameter values quite similar to those found using gross M1 in System #4. The unrestricted price and income elasticities were still both around 0.8, while the interest rate semi-elasticity was -0.03. The loadings for the first cointegrating vector still implied that long-run disequilibria are removed through adjustments in money and prices and not income or interest rates. The second cointegrating vector was not a money-demand relationship, since it had the wrong signs on the interest rate in the long-run vector and on the money-equation loading. Unitary price elasticity and an income elasticity of 0.5 could easily be imposed on the first vector (see Table 3d). However, this is not definitive evidence, since the JJ technique identifies only a cointegrating space, so that restrictions in one vector can often be offset by changes in the remaining vectors. When the first vector is restricted, the second changes to what appears to be a long-run relationship between output and the price level. The final hypothesis test in Table 3d shows, however, that money and the interest rate could not be removed from this second vector.[20]

In general, the differences between using gross or net M1 were not large. Systems with gross M1 could usually more easily reject a second cointegrating vector, although System #4 (with seasonally adjusted gross M1) did show evidence of another vector based on the λ-max and trace statistics. If the sample was restricted to begin in 1968 when published

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20. These results were common throughout all the specifications used. If a second cointegrating vector was found, then it appeared to be a relationship between output and prices. Many times money and the interest rate could be removed from this second vector, but this was not a certainty, as was found above. See System #1.

gross M1 numbers became available, then the differences in the results found for the two aggregates became somewhat larger but still were not substantial.

6.3 Quarterly Raw Data

The next set of tables (Tables 4a to 4d) present the results from estimating the cointegrating vectors using raw quarterly data on gross M1, the price level, output and CP90. The price level was measured using both the CPI and the GDP deflator. Output was either a quarterly average of monthly GDP at factor cost (TE) or quarterly GDP (GDP) calculated on an expenditure basis. The output gap, change in the exchange rate, change in the U.S. interest rate, and the 1980 shift dummy were still used as exogenous variables in each system shown. The two systems using the CPI also included a GST dummy, while the systems using the GDP deflator required a dummy for an outlier in the third quarter of 1974 (D74q3) in order to ensure normal errors. All the postal strike variables were insignificant in the quarterly specifications and therefore were omitted.

Each of the four systems in Table 4a accepted only one cointegrating vector. System #8 did find some evidence of a second vector according to the λ-max and trace tests but only one long-run relationship, judging by the PGp statistic. System #3 and System #6 use the same endogenous variables but different frequencies. With the quarterly data, there is an increase in the estimated income elasticity from 0.689 to 0.829 as well as an increase (in absolute value) in the interest rate semi-elasticity from -0.023 to -0.041. The loadings in the money and price equation were also larger, which is to be expected, given that quarterly data allow for a longer period over which an adjustment can occur. System #7 replaces factor cost output (TE) with expenditure-based output (GDP) and finds a smaller price elasticity but a larger income elasticity. Using GDP tended to yield higher estimates of the income elasticity in unrestricted examples. Once long-run unitary price elasticity was imposed, the differences between using TE and GDP were less pronounced (Table 4d).

Systems using the GDP deflator (#8 and #9) consistently had much higher price elasticities and lower income and interest rate elasticities in comparison with CPI-based systems. One unsatisfactory result from using the deflator was that the loading in the money equation was either insignificant or only had marginal significance (see Table 4b). In contrast, the loading in the price equation was much larger when using the deflator (0.173 vs. 0.042 using TE, and 0.154 vs. 0.037 using GDP).

Long-run unitary price elasticity was rejected at the 1 per cent significance level in both of the systems using the CPI (see Systems #6 and #7 in Table 4d). However, the restriction could not be rejected at the 10 per cent level when imposed on Systems #8 and #9 using the GDP deflator. None of the four systems could reject the joint hypothesis that income and the interest rate were weakly exogenous. The joint hypothesis of a unitary price elasticity and zero loadings in the income and interest rate equations was also accepted for the two deflator-based systems at the 5 per cent significance level.

Figures 4 to 7 show the results of the stability tests for these quarterly specifications. System #6 yielded the best stability results with only one quarter of instability at the 5 per cent significance level for the entire period from 1975 to 1993. The poorest results were found using the GDP and its deflator in System #9. There was evidence of instability until the end of 1980 and then again around 1983. The CPI gave more stable results than did the GDP deflator. Similarly, using output at factor cost (TE) improved stability in comparison with systems with expenditure-based GDP.

6.4 An Extended Sample for Quarterly Data

The quarterly systems using GDP can be extended back to 1956, which is the starting period for CP90.[21] Systems #10 and #11 in Tables 5a to

21. Unfortunately, the U.S. commercial paper rate is available only back to 1962. However, the 91-day U.S. treasury bill rate is available back to the mid-1940s and has a correlation coefficient of 0.986 with the commercial paper rate for their overlapping period. Results for the 1961 to 1993 period changed only marginally when the U.S. T-bill rate was used, so Systems #10 and #11 use this rate as an exogenous variable for 1956-1993.

5d show the results of this exercise. There was an increase in the long-run price elasticities, especially for the CPI-based system, as the sample was extended (see Table 5a). The estimated income elasticities declined to about 0.6 when using the CPI and to about 0.44 in the deflator-based system. Long-run unitary price elasticity could not be rejected for either price variable, even if combined with the joint test that income and the interest rate were weakly exogenous (see Table 5d). With the price coefficient in the cointegrating vector restricted to one, the income elasticity was just over 0.5 in the CPI-based system and just under 0.5 in the example using the GDP deflator. As well, under the (easily accepted) joint restriction of unitary price elasticity and the weak exogeneity of income and the interest rate, the loading in the money equation of the system using the GDP deflator became significant.

The primary benefit of using the longer sample from 1956 was that it removed virtually all evidence of instability from the post-1974 period for both systems. The CPI-based system (#10) was unstable at the 5 per cent significance level for only one subsample ending in 1976. Another subsample ending in 1982 had a test statistic that was near the critical value but did not exceed it. Similarly, except for one quarter in 1975, System #11 using the GDP deflator yielded subsample results that were insignificantly different from the full sample estimates (1956-1993).

The short-run exogenous variable coefficients shown in Table 5c basically conform with expectations. The shift dummy is significant and negative in the M1 equation, while the output gap is significantly positive in the CPI equation, although not in the deflator equation. The contemporaneous changes in the exchange rate and the U.S. T-bill rate were once again significant determinants of the change in CP90. For System #10, the first lag of ΔCPI and ΔCP90 and the third lag of ΔM1 were significant determinants of the current change in M1.[22] Surprisingly, only the first lags of ΔCPI and ΔCP90 were significant in the ΔCPI equation. Money was not an important short-run determinant, although the coefficients were positive. However, the first and the fourth quarterly lag of ΔM1 were

22. The estimates of these short-run dynamic parameters are not included in the tables but are available.

significant in the output equation, showing that money can have short-run real effects on the economy. For System #11 involving the GDP deflator, the M1 equation had estimated coefficients on the first and third lag of ΔM1 that were significantly positive and a coefficient on the first lag of ΔCP90 that was significantly negative. The current change in the GDP deflator was explained by four lags of itself, three lags of the ΔGDP, and two lags of ΔCP90. The first and fourth lags of ΔM1were significant positive determinants of the short-run dynamics of ΔGDP. It is quite interesting to note that none of the lagged differences of M1 were significant in either the CPI or GDP deflator equations, and yet the cointegrating vector itself (a measure of the excess supply of money) was a significant positive determinant of the change in the price level.

6.5 Real M1

Much of the theoretical and empirical work involving money demand focusses on real money only and does not bother with an examination of nominal money and elasticity of prices. Early work on this paper also began assuming that real M1 (RM1) was the relevant variable. However, there were a number of problems that led to the estimation of the model using nominal money. Assuming that real M1 is the relevant monetary variable imposes the neutrality of money in both the short run and long run (assuming prices or inflation are not also included in the model separate from real M1). Given that even long-run unitary price elasticity was rejected for some systems, particularly when using monthly data, it is not surprising that systems using real money were not entirely satisfactory.

One problem was that systems comprising only real M1, output and CP90 yielded cointegrating vectors with income elasticities that were unreasonably large. Only if inflation was assumed to be I(1) and was included as part of the cointegrating vector could more reasonable parameter values be found. However, inflation always had the wrong sign in the cointegrating vector (positive instead of the expected negative). These poor results were robust across specifications using monthly data, quarterly data, gross M1, net M1, GDP at factor cost or expenditure-based CPI, and the GDP deflator. Only systems that allowed for some form of non-neutrality of money (that is, a deviation from unitary price elasticity), whether in just the short run or in both the short and the long run, obtained cointegrating vector estimates that conformed with reasonable expectations for money-demand relationships.

6.6 The Constant

All the results discussed so far have included an unrestricted constant (μ) as part of the vector error-correction model shown in equation (2). This constant can be decomposed into two components,[23] as in equation (3),

$$\begin{aligned}\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \alpha[(\alpha'\alpha)^{-1}\alpha'\mu + \beta' X_{t-k}] \\ + \alpha_{\perp}(\alpha'_{\perp}\alpha_{\perp})^{-1}\alpha'_{\perp}\mu + \Psi D_t + \varepsilon_t\end{aligned} \tag{3}$$

so that $\mu = \alpha(\alpha'\alpha)^{-1}\alpha'\mu + \alpha_{\perp}(\alpha'_{\perp}\alpha_{\perp})^{-1}\alpha'_{\perp}\mu$, where $\alpha(\alpha'\alpha)^{-1}\alpha'\mu$ forms the intercept(s) in the cointegrating vector(s) and $\alpha_{\perp}(\alpha'_{\perp}\alpha_{\perp})^{-1}\alpha'_{\perp}\mu$ determines any linear trend in the level of the series. The matrix $\alpha_{\perp}$ represents the px(p-r) orthogonal complement of α such that $\alpha'_{\perp}\alpha = \alpha'\alpha_{\perp} = 0$ and $I = \alpha(\alpha'\alpha)^{-1}\alpha' + \alpha_{\perp}(\alpha'_{\perp}\alpha_{\perp})^{-1}\alpha'_{\perp}$. If the restriction is imposed that $\alpha'_{\perp}\mu = 0$ or, equivalently, that $\mu = \alpha\beta_0$, where $\beta_0 = (\alpha'\alpha)^{-1}\alpha'\mu$ is interpreted as an rx1 vector of intercepts of the r cointegrating vectors, then there will be no linear trend in the levels of the data. Johansen (1992b) describes a procedure for selecting between a model with an unrestricted constant and one with a restricted constant or no linear trend. The basic idea is to estimate the restricted and unrestricted versions of the model and accept the model with the fewest cointegrating vectors. If both models accept the same number of cointegrating vectors, then the

23. There are actually an infinite number of ways in which the constant can be separated between the cointegrating vectors and the short-run dynamics as long as the sum is still equal to μ. However, the methodology used here and in the literature allows the data to determine the split without imposing any qualifications or criteria.

restricted model should be used. Similarly, a χ^2 likelihood ratio statistic[24] can be calculated to test whether the restriction can be rejected when there is an equivalent number of vectors in each model.

Tables 6a to 6c show the results for System #3 using monthly data and for Systems #10 and #11 using quarterly data, when the constant and the shift dummy (D80) were restricted to being part of the cointegrating vector only. The monthly data strongly accepted two cointegrating vectors instead of just the one vector accepted when an unrestricted constant and shift dummy were assumed. Following Johansen (1992b) this outcome leads to the conclusion that the constant should not be restricted. Even if two vectors had been accepted as well for System #3 (there was some evidence of two vectors), the restriction would still be rejected with a χ^2 test statistic of 13.99 under four degrees of freedom.

However, there was much stronger evidence that the constant should be restricted to the cointegrating vector (i.e. no linear trend) using quarterly data from 1956 to 1993. As in the unrestricted systems, only one long-run vector was accepted for each of Systems #10a and #11a. Chi-squared test statistics of the restriction were 6.82 and 8.42, respectively. Both tests were easily accepted at the 5 per cent significance level with six degrees of freedom. There was little change in the parameters of the cointegrating vector and the loadings once the constant and D80 were restricted. The hypothesis tests in Table 6c show that unitary price elasticity was still easily accepted as were zero restrictions on the loadings in the income and interest rate equations. The exclusion tests for the constant and D80 in Table 6b revealed that, individually, these variables were not strongly significant. However, further testing concluded that joint tests of their exclusion from the cointegrating vector could easily be rejected.

The pattern of significance among the short-run parameters for Systems #10a and #11a was quite similar to their unrestricted versions, #10 and #1, respectively. Most importantly, lagged values of ΔM1 were still insignificant, although mainly positive, in both the ΔCPI and ΔDEF

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24. The statistic is $LR = Trace_{rest}(r) - Trace_{unrest}(r) \sim \chi^2(p-r)$, where p is the number of variables and r is the number of cointegrating vectors.

equations. The deviation of M1 from its long-run equilibrium is a significant determinant of inflation, but the short-run changes of M1 themselves do not seem to be significant.

7 The Money Gap

The money gap is defined as the current level of long-run disequilibrium in the money market. The cointegrating vectors estimated in this study are a measure of the level of this gap. Figures 12 and 13 plot the money gaps estimated by Systems #10a and #11a with restricted constants shown in Table 6a. These restricted systems had money gaps that were consistently positive (excess money supply) for virtually the entire sample. The size of the money gap is positively related to the inflation rate. This outcome has a strong monetarist flavour to it because it seems to imply that inflation has been strictly a monetary phenomenon. There has been inflation in virtually every period since the mid-1950s and the results suggest that there has been a corresponding excess supply of money. It is, however, difficult to understand how the money market can apparently be in long-run disequilibrium for such an extended period of time.

The money gaps for the other systems reported in this paper had basically the same shape as those shown here. However, differences in the estimated constants and shift dummies created substantial level differences among the measures of disequilibria. Except for the systems in which the constant was specifically restricted to appear in only the cointegrating vector, it was uncertain exactly how to isolate that portion of the estimated constants and shift dummies attributable to only the long-run relationship.[25] One method of decomposing the constant into its long-run and short-run components was given in equation (3). Another possibility is discussed below.

Even though the restriction of the constant to the cointegrating vector only was accepted for the quarterly data, the validity of the

25. Recall that there is a different constant in each equation of the VECM.

restriction might be criticized, since it implies that the average first difference of each of the endogenous variables is zero. This is a strong restriction for variables like money and prices, which have experienced positive growth throughout most of the sample. Alternatively, suppose that an unrestricted VECM, such as System #10 or #11, is dynamically simulated into the future, assuming no shocks and that the exogenous variables (the output gap and the changes in the U.S. T-bill rate and U.S.-Canadian exchange rate) are zero, as they would be in a balanced growth steady state. The money gap in this exercise converges to a constant (although a different constant in each quarter due to seasonal factors). However, under the assumption that the money gap is zero (monetary equilibrium) along a balanced growth path with no shocks, the result is a system of equations that can be solved for the cointegrating vector constant (and seasonal factors). Since the unrestricted VECM constants originally estimated are simply the sum of long-run and short-run components, it is then a simple matter to calculate the resulting short-run constants for each equation.

Table 7a shows some of the results of this method of decomposing the constant for Systems #10 and #11 using quarterly data. To conserve space, only the portion of the unrestricted constant allocated to the cointegrating vector is shown. In comparison with Systems #10a and #11a, this method estimated a larger shift in 1980 and a smaller overall long-run constant. Figures 14 and 15 plot the resulting money gaps for Systems #10 and #11, respectively. By construction, the gaps are now centred around zero. The overall shapes of the money gaps for Systems #10 and #11 are basically the same as for the comparable Systems #10a and #11a, which have the short-run constants restricted to being zero.

However, the gaps are now below zero over a much larger portion of the sample and indicate different periods during which there was monetary equilibrium. For example, Figure 14 for the CPI-based system shows a money gap of about zero from 1985 to 1987, while System #10a in Figure 12 had approximately a zero money gap from 1992 to 1993. A zero money gap implies either that inflation (and the other endogenous variables) is at its long-run equilibrium level or that, if it is not at this equilibrium value, then the cause of the deviation from steady state is not a monetary disequilibrium. The VECM for System #10 converged to a year-over-year inflation rate of 2.9 per cent (5.8 per cent for the period before the 1980 shift dummy), while the actual inflation rate was about 4 per cent for the period around 1986, when the gap was zero. This implies that something other than M1 disequilibrium was keeping inflation elevated during this period. Similarly, for System #10a, the money gap was zero from 1992 to 1993, when the average inflation rate was about 2 per cent and the equilibrium rate implied by the VECM was zero.

Further work on the nature of the relationship between the level of monetary disequilibrium and the rate of inflation is a topic for future research. That is, future work will address in detail how to best use the money gap to predict inflation.

8 Conclusion

The analysis of the demand for M1 in this paper examines the relationships among M1, output, prices and interest rates. In particular, relationships among different definitions of M1 (net and gross), output, prices and the 90-day paper rate are examined at monthly and quarterly frequencies and over different sample periods. The Johansen-Juselius multivariate approach used here allows for the specification of exogenous variables that improve the estimates of the short-run dynamics and help ensure the stability of the systems while enhancing the estimates of the cointegrating relationships themselves. By comparing the parameter estimates and dynamic behaviour of the estimated systems with theoretical priors, this work identifies unique, well-specified, and stable cointegrating relationships among M1, output, prices and an interest rate that has the properties of a well-behaved money-demand function. (However, the stability of these functions is assured only through the inclusion of a shift dummy in the early 1980s to account for the financial innovations introduced at that time.)

As well, the parameter estimates generally imply the weak exogeneity of output and interest rates, but reject this hypothesis for prices and inflation. The implication of this is that output and interest rates do not adjust to offset monetary disequilibria, that is, a deviation of actual M1 from money-demand. All the adjustment to return the economy to monetary equilibrium comes from fluctuations in money and prices. In particular, when M1 is above its long-run demand, money tends to fall and prices rise in order to restore equilibrium. However, this does not preclude the possibility that changes in the stock of money can still have short-run real effects.

Perhaps the most appealing system estimated in this work, based on parsimony of specification, system stability, and conformity of the results with theory, is the system on gross M1, GDP, the CPI and the 90-day paper rate, estimated over the 1956 to 1993 period, including the restriction of unitary price elasticity and with the constant and shift dummy restricted to the cointegrating vector (System #10a in Table 6a).

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Technical Appendix

This appendix contains a brief explanation of the Johansen-Juselius maximum-likelihood procedure for estimating cointegrating vectors. It is intended more as an overview than an exhaustive description. The data-generating process and statistical model are described in equation (A1)

$$X_t = \Pi_1 X_{t-1} + \dots + \Pi_k X_{t-k} + \mu + \Psi D_t + \varepsilon_t \qquad \text{(A1)}$$

where X_t is a vector of p variables, ε_t is a vector of disturbances such that $\varepsilon_1,\dots,\varepsilon_T$ are IINp(0,Λ), μ is a constant, and D_t is a vector of exogenous variables including centred seasonal dummies. Applying some simple algebra allows us to rewrite equation (A1) as either equation (2) shown in Section 2 or equivalently as

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \mu + \Psi D_t + \varepsilon_t \qquad \text{(A2)}$$

where $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$ and $\Pi = \sum_{i=1}^{k} \Pi_i - I$

The hypothesis of at most r cointegrating vectors is formulated as the restriction $H_0 : \Pi = \alpha\beta'$, where rank(Π)=r and α and β are pxr matrices. Both equations (2) and (A2) yield the same estimates for α and β, although the coefficients on the lagged first-difference terms will obviously be different. Rewrite equation (A2) as (A2′), where $Z_{0t}=\Delta X_t$, Z_{1t} denotes the stacked variables ΔX_{t-1}, ..., ΔX_{t-k-1}, D_t, and 1, $Z_{kt}=X_{t-1}$, and Φ is the matrix of parameters corresponding to Z_{1t}.

$$Z_{0t} = \Phi Z_{1t} + \Pi Z_{kt} + \varepsilon_t \qquad \text{(A2′)}$$

Define the product moment matrices as

$$M_{ij} = T^{-1} \sum_{t=1}^{T} Z_{it} Z'_{jt} \text{ with i,j=0,1,k.}$$

Next, combine these moment matrices to form

$$S_{ij} = M_{ij} - M_{i1} M_{11}^{-1} M_{ij} \text{ with i,j=0,k.}$$

Under the null hypothesis of $H_0{:}\Pi = \alpha\beta'$, the maximum-likelihood estimator of β is the set of eigenvectors associated with the eigenvalues (λ) solved from the following equation:

$$\left|\lambda S_{kk} - S_{k0}S_{00}^{-1}S_{0k}\right| = 0 \tag{A3}$$

To determine the value of r, Johansen (1988) constructed two LR-statistics to test the null hypothesis of r cointegrating vectors. The maximum eigenvalue statistic (λ-max) tests the null of H_0: $rank(\Pi) = r$ against the alternative H_a: $rank(\Pi) = r+1$ and is calculated as

$$\lambda max(r) = -T\log(1-\hat{\lambda}_{r+1}) \tag{A4}$$

where $\hat{\lambda}_{r+1}$ is the (r+1) largest estimated eigenvalue. The trace statistic tests the null of H_0: $rank(\Pi) = r$ against H_a: $rank(\Pi) = p$, where p is the number of variables and is calculated as

$$Trace(r) = -T\sum_{i=r+1}^{p}\log(1-\hat{\lambda}_i) \tag{A5}$$

where the $\hat{\lambda}_i$ are the (p-r-1) smallest estimated eigenvalues. Both test statistics have non-standard distributions that depend on the presence or absence of deterministic trends and have (p-r) degrees of freedom. Johansen and Juselius (1990) calculated tables of asymptotic critical values that were later expanded by Osterwald-Lenum (1992).

It has been pointed out by Gonzalo and Pitarakis (1994), Cheung and Lai (1993), and others that the λ-max and trace tests have a tendency to find too many cointegrating vectors when the size of the system (p) and/or the number of lags (k) is too large relative to the sample size (T). Cheung and Lai (1993) outlined a method for finding approximate finite sample critical values corresponding to the asymptotic values in Table A2 of Johansen and Juselius (1990). Paquet (1994) lists three corrected versions of the trace statistic that attempt to account for the size of the sample, the number of lags and the size of the system. One statistic, designed by Gonzalo and Pitarakis is given by

$$PGp = \frac{1}{2}\left[(T-pk+k)\sum_{i=r+1}^{p}\hat{\lambda}_i - \left(T-pk+\frac{p}{2}\right)\sum_{i=r+1}^{p}\log(1-\hat{\lambda}_i)\right] \tag{A6}$$

and has the same asymptotic distribution as the trace test but much better small sample properties.

The basic methodology outlined so far is designed for use when the endogenous X variables are I(1). If it happens that one or more of the X variables are actually I(2), then there is no guarantee that $\beta' X_t$ will be I(0). However, Johansen (1992a, 1992c, 1992d, 1992e and 1994) has designed a procedure for the testing and estimation of the model in the presence of I(2) trends. Equation (A1) can once again be reparameterized in the following error-correction form:

$$\Delta^2 X_t = \Gamma \Delta X_{t-1} + \Pi X_{t-2} + \sum_{i=1}^{k-2} \Phi_i \Delta^2 X_{t-i} + \mu + \Psi D_t + \varepsilon_t \qquad \text{(A7)}$$

where $\Gamma = \sum_{i=1}^{k-1} \Gamma_i - I + \Pi$ and $\Phi_i = -\sum_{j=i+1}^{k} \Gamma_j$ for i=1,...,k-2. The expression Δ^2 represents the second difference of a series.

An I(2) model, under the null $H_{r,s}$ with s=0,1,...,p-r and r=0,...,p, is defined by equation (A7) together with the conditions

$$\Pi = \alpha\beta' \qquad \text{(A8)}$$

$$\text{and } \alpha'_{\perp}\Gamma\beta_{\perp} = \varphi\eta' \qquad \text{(A9)}$$

The matrices α and β are still pxr with rank r while φ and η have dimension (p-r)xs and rank s. The number of I(2) trends present in the data is given by p-r-s. The methodology proposed by Johansen (see Johansen 1992d, in particular) is a two-step procedure in which the first stage is simply the regular analysis of the I(1) model imposing condition (A8) but not (A9). This determines r, α and β. The next step takes these values as given and analyses the I(2) model.

Let $\alpha_{\perp}$ and $\beta_{\perp}$ represent orthogonal px(p-r) matrices such that $\alpha'\alpha_{\perp} = \beta'\beta_{\perp} = 0$. Now multiply equation (A7) by $\alpha'_{\perp}$ to remove the term in levels and then apply condition (A9) and the identity $\beta\bar{\beta}' + \beta_{\perp}\bar{\beta}_{\perp} = I$, where $\bar{\beta} = \beta(\beta'\beta)^{-1}$, to obtain equation (A10) as follows:

$$\alpha'_{\perp}\Delta^2 X_t = \alpha'_{\perp}\Gamma\beta(\bar{\beta}'\Delta X_{t-1}) + \varphi\eta'(\bar{\beta}'_{\perp}\Delta X_{t-1}) \qquad \text{(A10)}$$

$$+ \sum_{i=1}^{k-2} \alpha'_{\perp}\Phi_i\Delta^2 X_{t-i} + \alpha'_{\perp}\mu + \alpha'_{\perp}\Psi D_t + \alpha'_{\perp}\varepsilon_t$$

Therefore, the analysis for fixed r, α, and β is a reduced rank regression of $\alpha'_{\perp}\Delta^2 X_t$ on $\bar{\beta}'_{\perp}\Delta X_{t-1}$ corrected for lagged second differences, a constant, seasonal dummies and $\bar{\beta}'\Delta X_{t-1}$. This is exactly the same type of procedure as is done for equation (A2) with I(1) variables. Thus, φ and η have the same interpretation as α, and β only for the difference of the original series instead of the levels. The trace test applied to equation (A10) has the same asymptotic distribution (with p-r-s degrees of freedom) as it does when applied to equation (A2) (with p-r degrees of freedom) for I(1) variables.

Table A1 contains the trace test statistics for the determination of the cointegration ranks of r and s in System #3 in the I(2) model. The columns marked Trace(r) and PGp(r) represent the results of the first step of the analysis. They are not quite identical to the outcome reported in Table 2a because one observation had to be dropped to calculate the second difference of monthly output. For a given value of r, the associated row under Trace(r,s) shows the trace tests for step two, which examines the presence of I(2) trends. At the 5 per cent significance level, the hypothesis of one cointegrating vector (r=1) is rejected by the trace test (Trace(1)=32.78) but accepted by PGp (PGp(1)=28.43), as was discussed in the main text. The trace test fails to reject two cointegrating vectors (trace(2)=5.17). If it is concluded that r=2, then the Trace(r,s) test leads to the rejecttion of s=0 (Trace(2,0)=56.52>15.41) and s=1 (Trace(2,1)=8.87>3.76) in favour of s=2 or p-r-s=0, or in other words, that there are no I(2) trends present. If instead r=1 is accepted, as implied by the PGp test, then once again the presence of I(2) trends is rejected (Trace(1,2)=4.23>3.76). Even if a PGp test statistic is calculated for this second stage, the tests still reject that there are I(2) trends in the data.[26]

26. The test statistic for PGp(r=1,s=2) was about 3.83, which still rejects s=2 at the 5 per cent significance level. However, no work has been done on the small sample properties of the JJ methodology in the presence of I(2) trends, so it is uncertain whether the PGp statistic is applicable for the second stage of the analysis.

Table A1: I(2) Test Results

r	Trace(r,s)				Trace(r)	PGp(r)	p-r	C(p-r)
0	189.61 s=0	90.59 s=1	19.79 s=2	2.39 s=3	103.55‡	87.99‡	4	47.21
1		139.48 s=0	45.46 s=1	4.23 s=2	32.78†	28.43	3	29.68
2			56.52 s=0	8.87 s=1	5.17	4.54	2	15.41
3				19.39 s=0	1.49	1.31	1	3.76
p-r-s	4	3	2	1				
C(p-r)	47.21	29.68	15.41	3.76				

TABLES AND FIGURES

Table 1: Unit Root Tests[a]

	Sample: 1961 to 1993				
Variable	lags	SB[b]	ADF[b]	PP[b]	KPSS[c]
CPI_t	12	0.000	-1.323	0.590	3.147†
ΔCPI_t	11	1.389†	-1.874	-18.200†	0.690†
$\Delta^2 CPI_t$	10	3.036†	-11.909†	-74.728†	0.051
$M1_t$	12	0.001	-0.460	-0.238	3.140†
$\Delta M1_t$	11	2.220†	-4.074†	-25.334†	0.134
$GM1_t$	12	0.002	-0.615	-0.447	3.122†
$\Delta GM1_t$	11	1.841†	-3.958†	-20.988†	0.170
TE_t	13	0.042	-2.534	-2.028	2.852†
with trend	13	0.700†	-1.750	-11.392†	0.880†
ΔTE_t	12	2.775†	-3.868†	-64.942†	0.235
CP_t	2	0.038	-2.545	-2.251	5.208†
ΔCP_t	1	1.798†	-11.346†	-18.018†	0.154

a. SB=Sargan-Bhargava, ADF=Augmented Dickey-Fuller, PP=Phillips-Perron, KPSS=Kwiatowski, Phillips, Schmidt, and Shin. †=significant at the 5 per cent level.
b. H_0: Variable X has a unit root.
c. H_0: Variable X is stationary.

Table 2a: Nominal Gross M1 Monthly Cointegration Results from 1961 to 1993[a]

					Cointegrating Vectors: β				Loadings: α			
CI Variables	Exog. Variables	λ-max	Trace	PGp	GM1	CPI	TE	CP90	ΔGM1	ΔCPI	ΔTE	ΔCP90
1) GM1, CPI, TE, CP90		49.88‡	87.65‡	75.45‡	1.0	-0.690	-0.947	0.055	-0.058 (-5.261)	0.010 (3.738)	0.025 (1.785)	-0.640 (-1.068)
		21.86†	37.77‡	32.90†	1.0	-1.467	0.706	0.007	0.012 (2.124)	0.005 (3.868)	-0.014 (-2.029)	-0.200 (-0.660)
		12.84*	15.91†	13.91*								
		3.07*	3.07*	2.69*								
2) GM1, CPI, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$, strike	61.66‡	85.85‡	73.32‡	1.0	-0.825	-0.769	0.031	-0.076 (-4.613)	0.026 (6.182)	0.022 (1.030)	-0.226 (-0.301)
		17.20	24.19	21.11								
3) GM1, CPI, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$, strike, non-normality dummies	69.74‡	102.50‡	87.19‡	1.0	-0.867	-0.689	0.023	-0.072 (-3.859)	0.031 (6.371)	0.051 (2.678)	-0.872 (-1.423)
		27.41‡	32.76†	28.42*								
		3.67	5.36	4.71								

a. Each system uses 14 lags.

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 2b: Exclusion Tests

		Cointegrating Vectors: β				Loadings: α			
System	r *or* d.f.	GM1	CPI	TE	CP90	ΔGM1	ΔCPI	ΔTE	ΔCP90
1	2	20.43‡	18.57‡	18.92‡	35.89‡	21.64‡	16.61‡	4.12	1.45
2	1	25.54‡	18.35‡	28.46‡	37.97‡	15.42‡	28.45‡	0.95	0.09
3	1	35.22‡	28.96‡	35.44‡	27.38‡	10.01‡	36.13‡	5.99†	1.81

Table 2c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
1	ΔGM1	-0.265 (-3.999)	-	-	-	-	-	22.06	22.96‡	0.825
	ΔCPI	-0.035 (-2.123)	-	-	-	-	-	13.45	102.93‡	0.537
	ΔTE	0.218 (2.639)	-	-	-	-	-	21.71	177.90‡	0.966
	ΔCP90	0.326 (0.091)	-	-	-	-	-	5.59	305.57‡	0.259
2	ΔGM1	-0.086 (-4.415)	-0.011 (-4.818)	-0.020 (-0.778)	0.003 (0.051)	0.003 (3.884)	0.004 (0.398)	29.23	10.34‡	0.866

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 2c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
	ΔCPI	0.030 (6.093)	0.003 (4.756)	0.030 (4.565)	0.014 (0.956)	0.000 (0.835)	0.022 (8.374)	20.82	1.689	0.634
	ΔTE	0.035 (1.379)	0.000 (0.080)	-0.141 (-4.187)	-0.051 (-0.700)	0.004 (3.398)	-0.012 (-0.887)	22.00	257.53‡	0.972
	ΔCP90	-0.481 (-0.540)	0.015 (0.144)	0.653 (0.550)	11.367 (4.435)	0.488 (12.703)	-0.534 (-0.514)	27.66	147.49‡	0.595
3	ΔGM1	-0.040 (-3.605)	-0.012 (-4.126)	-0.001 (-0.045)	-0.022 (-0.395)	0.005 (4.437)	0.005 (0.549)	34.73	15.90‡	0.878
	ΔCPI	0.018 (6.128)	0.004 (5.419)	0.029 (4.396)	0.007 (0.486)	0.000 (1.684)	0.022 (8.609)	23.30	1.56	0.651
	ΔTE	0.031 (2.817)	0.008 (2.696)	-0.098 (-3.838)	-0.080 (-1.382)	0.002 (2.182)	-0.010 (-1.021)	48.70‡	10.85‡	0.984
	ΔCP90	-0.734 (-2.038)	-0.126 (-1.314)	1.910 (2.313)	6.655 (3.572)	0.281 (7.993)	-0.540 (-1.679)	28.39	11.43‡	0.812

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 2d: Hypothesis Tests for Nominal GM1 Specifications

System	Hypothesis Test on β	χ^2	d.f.	p-val.	Cointegrating Vectors: β GM1	CPI	TE	CP90	Loadings: α ΔGM1	ΔCPI	ΔTE	ΔCP90
1	H_0:[1,-1,-0.5,a] on 1st CI	5.135*	2	0.077	1.0	-1.0	-0.5	0.079	-0.041 (-5.200)	0.007 (3.543)	0.011 (1.126)	-0.637 (-1.502)
	H_0:[0,b,1,0] on 2nd CI				0	-0.537	1.0	0	0.034 (3.557)	0.004 (1.799)	-0.037 (-3.119)	-0.241 (-0.470)
1	H_0:[1,-1,-0.5,a] on 1st CI	8.060*	4	0.089	1.0	-1.0	-0.5	0.083	-0.038 (-5.144)	0.007 (3.545)	0.012 (1.279)	0
	H_0:[0,b,1,0] on 2nd CI and α_{CP}=0				0	-0.528	1.0	0	0.032 (3.506)	0.004 (1.824)	-0.036 (-3.151)	0
2	H_0: [1,-1,a,b] for all CI	9.078‡	1	0.003	1.0	-1.0	-0.537	0.030	-0.058 (-3.618)	0.025 (6.120)	0.019 (0.930)	-0.429 (-0.590)
2	H_0: α_{TE}=0, α_{CP}=0	1.089	2	0.580	1.0	-0.815	-0.781	0.034	-0.704 (-4.723)	0.025 (6.204)	0	0
2	H_0:[1,-1,a,b], α_{TE}=0, α_{CP}=0	10.254†	3	0.017	1.0	-1.0	-0.538	0.032	-0.057 (-3.795)	0.023 (6.135)	0	0
3	H_0: [1,-1,a,b] for all CI	7.544‡	1	0.006	1.0	-1.0	-0.514	0.021	-0.055 (-2.994)	0.030 (6.273)	0.054 (2.917)	-0.680 (-1.139)
3	H_0: α_{CP}=0	1.813	1	0.178	1.0	-0.870	-0.697	0.024	-0.076 (-4.101)	0.031 (6.391)	0.053 (2.831)	0
3	H_0:[1,-1,a,b], α_{CP}=0	8.698†	2	0.013	1.0	-1.0	-0.525	0.022	-0.059 (-3.236)	0.030 (6.289)	0.055 (3.017)	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Figure 1: Test of Full Sample β Equal to Subsample β(t): System #1

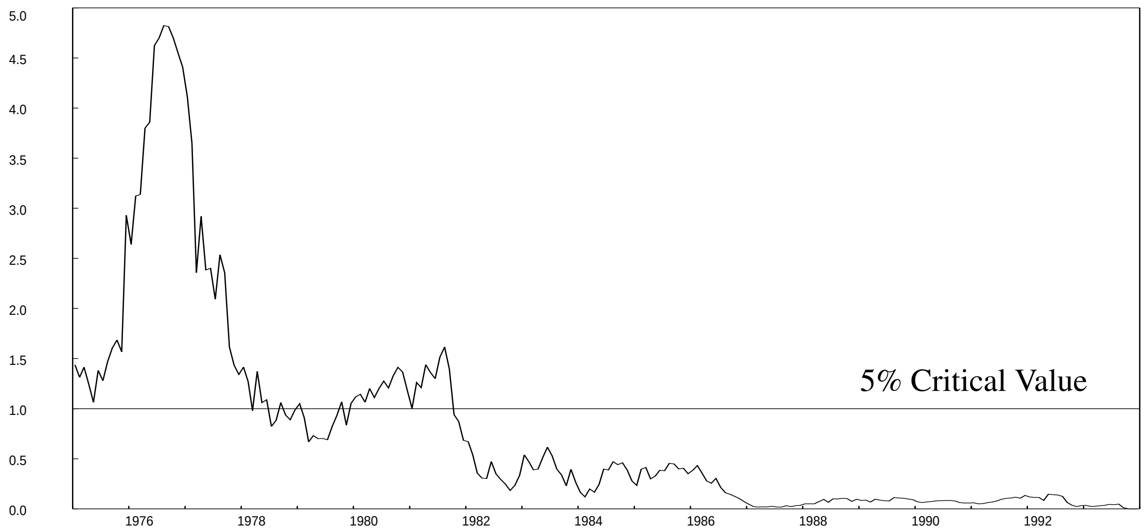


Figure 2: Test of Full Sample β Equal to Subsample β(t): System #2

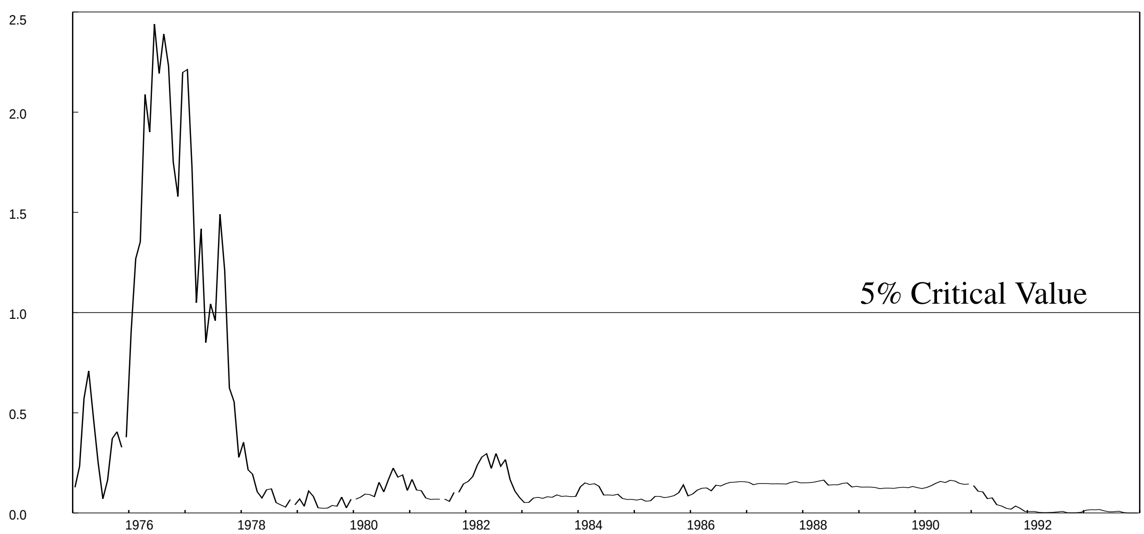


Figure 3: Test of Full Sample β Equal to Subsample β(t): System #3

Table 3a: Cointegration Results Using Monthly Seasonally Adjusted Data from 1961 to 1993[a]

					Cointegrating Vectors: β				Loadings: α			
CI Variables	Exog. Variables	λ-max	Trace	PGp	GM1 *or* M1	CPI	TE	CP90	ΔGM1 *or* ΔM1	ΔCPI	ΔTE	ΔCP90
4) GM1, CPI, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$, strike, non-normality dummies	53.34‡	83.90‡	72.03‡	1.0	-0.799	-0.818	0.032	-0.078 (-4.827)	0.021 (4.928)	-0.006 (-0.767)	-0.849 (-1.523)
		20.99†	30.55†	26.63								
		7.26	9.56	8.39								
5) M1, CPI, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$, strike, non-normality dummies	43.82‡	80.14‡	69.10‡	1.0	-0.842	-0.850	0.035	-0.074 (-4.754)	0.017 (3.970)	0.000 (0.013)	-0.563 (-0.996)
		25.92‡	36.32‡	31.58†	1.0	-1.079	-0.245	-0.010	0.038 (3.324)	0.008 (2.504)	0.009 (1.466)	-1.238 (-2.963)
		7.08	10.40	9.12								

a. Each system uses 14 lags.

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 3b: Exclusion Tests

		Cointegrating Vectors: β				Loadings: α			
System	r *or* d.f.	GM1 *or* M1	CPI	TE	CP90	ΔGM1 *or* ΔM1	ΔCPI	ΔTE	ΔCP90
4	1	19.12‡	13.85‡	24.29‡	27.82‡	16.00‡	19.65‡	0.48	1.79
5	2	20.83‡	17.08‡	20.55‡	33.99‡	27.77‡	19.47‡	1.59	8.03†

Table 3c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
4	Δ GM1	-0.276 (-4.770)	-0.011 (-5.082)	-0.009 (-0.311)	-0.032 (-0.586)	0.004 (4.159)	0.006 (0.632)	23.22	1.13	0.492
	Δ CPI	0.075 (4.890)	0.002 (3.783)	0.030 (4.077)	-0.002 (-0.132)	0.000 (1.489)	0.019 (7.870)	20.23	6.17†	0.601
	Δ TE	-0.019 (-0.651)	-0.002 (-1.710)	-0.055 (-3.866)	-0.010 (-0.346)	0.001 (1.985)	-0.008 (-1.696)	43.13‡	5.95*	0.365
	Δ CP90	-3.343 (-1.666)	-0.087 (-1.133)	3.174 (3.297)	6.776 (3.601)	0.282 (7.950)	-0.449 (-1.394)	26.23	23.34‡	0.799

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 3c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
5	ΔM1	-0.393 (-5.592)	-0.002 (-0.998)	-0.057 (-1.983)	0.027 (0.526)	-0.000 (-0.074)	-0.004 (-0.480)	29.76	7.61†	0.535
	ΔCPI	0.053 (2.752)	0.002 (3.660)	0.016 (2.026)	-0.001 (-0.071)	0.000 (1.257)	0.020 (7.878)	18.06	9.24‡	0.603
	ΔTE	-0.016 (-0.442)	0.000 (0.103)	-0.060 (-3.956)	-0.009 (-0.328)	0.001 (1.703)	-0.011 (-2.213)	48.79‡	3.88	0.399
	ΔCP90	0.091 (0.036)	-0.215 (-2.428)	4.997 (4.825)	6.863 (3.676)	0.315 (8.882)	-0.601 (-1.829)	30.49	15.01‡	0.802

Table 3d: Hypothesis Tests for Monthly Seasonally Adjusted Specifications

					Cointegrating Vectors: β				Loadings: α			
System	Hypothesis Test on β	χ^2	d.f.	p-val.	GM1 *or* M1	CPI	TE	CP90	ΔGM1 *or* ΔM1	ΔCPI	ΔTE	ΔCP90
4	H_0: [1,-1,a,b] for all CI	9.879‡	1	0.002	1.0	-1.0	-0.562	0.031	-0.059 (-3.918)	0.019 (4.953)	-0.005 (-0.659)	-0.552 (-1.067)
4	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	2.344	2	0.310	1.0	-0.807	-0.831	0.032	-0.080 (-5.007)	0.020 (4.811)	0	0
4	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	11.120†	3	0.011	1.0	-1.0	-0.584	0.031	-0.062 (-4.078)	0.019 (4.827)	0	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 3d: Hypothesis Tests for Monthly Seasonally Adjusted Specifications

					Cointegrating Vectors: β				Loadings: α			
System	Hypothesis Test on β	χ^2	d.f.	p-val.	GM1 *or* M1	CPI	TE	CP90	ΔGM1 *or* ΔM1	ΔCPI	ΔTE	ΔCP90
5	H_0:[1,-1,-0.5,a] on 1st CI and α_{TE}=0	2.233	3	0.526	1.0	-1.0	-0.5	0.015	-0.043 (-2.256)	0.025 (4.875)	0	-1.561 (-2.272)
					0.042	-0.459	1.0	-0.061	0.038 (5.804)	-0.002 (-1.365)	0	-0.268 (-1.137)
5	H_0:[1,-1,-0.5,a] on 1st CI H_0:[0,b,1,0] on 2nd CI	9.786‡	2	0.008	1.0	-1.0	-0.5	0.033	-0.075 (-4.606)	0.018 (4.085)	0.001 (0.146)	-0.477 (-0.808)
					0	-0.357	1.0	0	0.040 (5.036)	-0.001 (-0.478)	-0.002 (-0.432)	-0.407 (-1.412)

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 4a: Cointegration Results Using Quarterly Non-seasonally Adjusted Data from 1961 to 1993[a]

					Cointegrating Vectors: β				Loadings: α			
CI Variables	Exog. Variables	λ-max	Trace	PGp	GM1	CPI *or* DEF	TE *or* GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
6) GM1, CPI, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$	71.89‡	97.04‡	76.44‡	1.0	-0.833	-0.829	0.041	-0.163 (-5.322)	0.042 (5.370)	0.012 (0.488)	1.797 (1.197)
		19.93	25.15	21.24								
7) GM1, CPI, GDP, CP90	$GDPGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $DGST_t$	64.76‡	87.85‡	69.92‡	1.0	-0.733	-0.977	0.035	-0.154 (-4.995)	0.037 (4.885)	0.034 (1.196)	1.486 (1.017)
		16.08	23.09	19.64								
8) GM1, DEF, TE, CP90	$TEGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$, $D74q3_t$	61.48‡	91.24‡	72.92‡	1.0	-0.998	-0.521	0.023	-0.070 (-1.418)	0.173 (6.982)	-0.079 (-2.203)	-2.750 (-1.333)
		24.73†	29.76†	24.93								
9) GM1, DEF, GDP, CP90	$GDPGAP_{t-1}$, Δe_t, $\Delta USCP_t$, $D80_t$ $D74q3_t$	58.43‡	83.57‡	67.19‡	1.0	-0.909	-0.630	0.021	-0.114 (-2.312)	0.154 (6.220)	-0.062 (-1.585)	-3.503 (-1.708)
		16.96	25.14	21.35								

a. Each system uses 5 lags.

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 4b: Exclusion Tests

		Cointegrating Vectors: β				Loadings: α			
System	r *or* d.f.	GM1	CPI *or* DEF	TE *or* GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
6	1	21.79‡	15.93‡	28.31‡	50.15‡	19.99‡	20.50‡	0.23	1.33
7	1	18.63‡	11.22‡	27.75‡	39.97‡	18.65‡	19.05‡	1.34	0.91
8	1	24.69‡	20.76‡	20.68‡	31.14‡	1.33	28.35‡	4.47†	1.49
9	1	22.24‡	16.91‡	21.62‡	40.12‡	4.01†	26.78‡	2.39	2.41

Table 4c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$ *or* $GDPGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
6	Δ GM1	-0.276 (-5.160)	-0.027 (-5.479)	-0.097 (-1.419)	-0.130 (-1.728)	0.000 (0.356)	0.011 (0.815)	23.71	1.77	0.868
	Δ CPI	0.072 (5.269)	0.005 (4.195)	0.078 (4.446)	0.076 (3.955)	0.000 (1.464)	0.019 (5.393)	27.96	0.32	0.850
	Δ TE	0.033 (0.733)	-0.002 (-0.447)	-0.222 (-3.879)	-0.039 (-0.625)	0.003 (3.583)	-0.015 (-1.330)	26.26	1.64	0.944
	Δ CP90	2.753 (1.046)	0.316 (1.291)	6.699 (1.998)	9.943 (2.699)	0.652 (11.725)	-0.753 (-1.115)	22.77	2.96	0.712

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 4c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$ *or* $GDPGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
7	Δ GM1	-0.610 (-4.883)	-0.030 (-5.399)	-0.066 (-1.121)	-0.128 (-1.649)	0.001 (0.999)	0.008 (0.574)	25.36	1.56	0.861
	Δ CPI	0.152 (4.986)	0.004 (3.126)	0.081 (5.641)	0.079 (4.162)	0.000 (1.669)	0.020 (5.729)	32.79	1.62	0.856
	Δ GDP	0.144 (1.247)	0.002 (0.333)	-0.086 (-1.570)	-0.052 (-0.724)	0.004 (3.555)	-0.015 (-1.156)	16.33	0.55	0.959
	Δ CP90	5.874 (0.989)	0.246 (0.928)	4.189 (1.491)	9.340 (2.532)	0.653 (12.169)	-0.577 (-0.861)	25.92	2.65	0.716
8	Δ GM1	0.056 (1.722)	-0.015 (-2.068)	0.046 (0.665)	-0.095 (-1.122)	0.001 (0.828)	-	18.00	0.30	0.834
	Δ DEF	-0.107 (-6.510)	0.021 (5.677)	0.058 (1.651)	-0.108 (-2.531)	0.001 (1.438)	-	28.14	3.70	0.827
	Δ TE	0.058 (2.447)	-0.014 (-2.694)	-0.263 (-5.168)	-0.033 (-0.537)	0.004 (3.938)	-	25.90	4.97*	0.946
	Δ CP90	1.237 (0.902)	-0.239 (-0.776)	3.823 (1.308)	10.573 (2.981)	0.649 (12.083)	-	13.90	3.41	0.736
9	Δ GM1	-0.082 (-1.915)	-0.023 (-3.112)	0.072 (1.357)	-0.069 (-0.812)	0.001 (1.192)	-	16.40	0.61	0.833
	Δ DEF	0.143 (6.625)	0.016 (4.452)	0.081 (3.010)	-0.111 (-2.582)	0.001 (1.652)	-	31.05	2.71	0.827
	Δ GDP	-0.055 (-1.609)	-0.009 (-1.572)	-0.155 (-3.641)	-0.048 (-0.708)	0.004 (4.297)	-	23.14	1.89	0.964

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 4c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$TEGAP_{t-1}$ *or* $GDPGAP_{t-1}$	Δe_t	$\Delta USCP_t$	$DGST_t$	LB(24)	BJ(2)	R^2
	Δ CP90	-3.579 (-2.011)	-0.322 (-1.060)	0.036 (0.016)	9.787 (2.748)	0.656 (12.666)	-	18.95	3.52	0.739

Table 4d: Hypothesis Tests for Quarterly Nominal GM1 Specifications

System	Hypothesis Test on β	χ^2	d.f.	p-val.	Cointegrating Vectors: β				Loadings: α			
					GM1	CPI *or* DEF	TE *or* GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
6	H_0:[1,-1,a,b]	7.002‡	1	0.008	1.0	-1.0	-0.620	0.041	-0.134 (-4.538)	0.041 (5.675)	0.006 (0.235)	1.473 (1.038)
6	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	1.353	2	0.509	1.0	-0.840	-0.810	0.040	-0.156 (-4.977)	0.044 (5.587)	0	0
6	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	7.998†	3	0.046	1.0	-1.0	-0.607	0.039	-0.129 (-4.218)	0.044 (5.851)	0	0
7	H_0:[1,-1,a,b]	10.476‡	1	0.001	1.0	-1.0	-0.649	0.038	-0.114 (-4.088)	0.033 (5.018)	0.026 (1.056)	1.076 (0.834)
7	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	1.832	2	0.400	1.0	-0.732	-0.964	0.037	-0.149 (-4.903)	0.037 (5.003)	0	0
7	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	11.810‡	3	0.008	1.0	-1.0	-0.634	0.040	-0.110 (-4.016)	0.033 (5.103)	0	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 4d: Hypothesis Tests for Quarterly Nominal GM1 Specifications

					Cointegrating Vectors: β				Loadings: α			
System	Hypothesis Test on β	χ^2	d.f.	p-val.	GM1	CPI *or* DEF	TE *or* GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
8	H_0:[1,-1,a,b]	0.002	1	0.967	1.0	-1.0	-0.518	0.023	-0.069 (-1.403)	0.173 (6.991)	-0.079 (-2.208)	-2.765 (-1.338)
8	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	4.787*	2	0.091	1.0	-0.983	-0.559	0.023	-0.112 (-2.237)	0.170 (6.717)	0	0
8	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	4.893	3	0.180	1.0	-1.0	-0.535	0.022	-0.108 (-2.132)	0.175 (6.812)	0	0
9	H_0:[1,-1,a,b]	2.405	1	0.121	1.0	-1.0	-0.507	0.021	-0.096 (-1.880)	0.163 (6.405)	-0.073 (-1.795)	-3.793 (-1.789)
9	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	3.817	2	0.148	1.0	-0.891	-0.680	0.022	-0.145 (-3.030)	0.140 (5.732)	0	0
9	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	7.022*	3	0.071	1.0	-1.0	-0.535	0.021	-0.130 (-2.567)	0.151 (5.959)	0	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Figure 4: Test of Full Sample β Equal to Subsample β(t): System #6

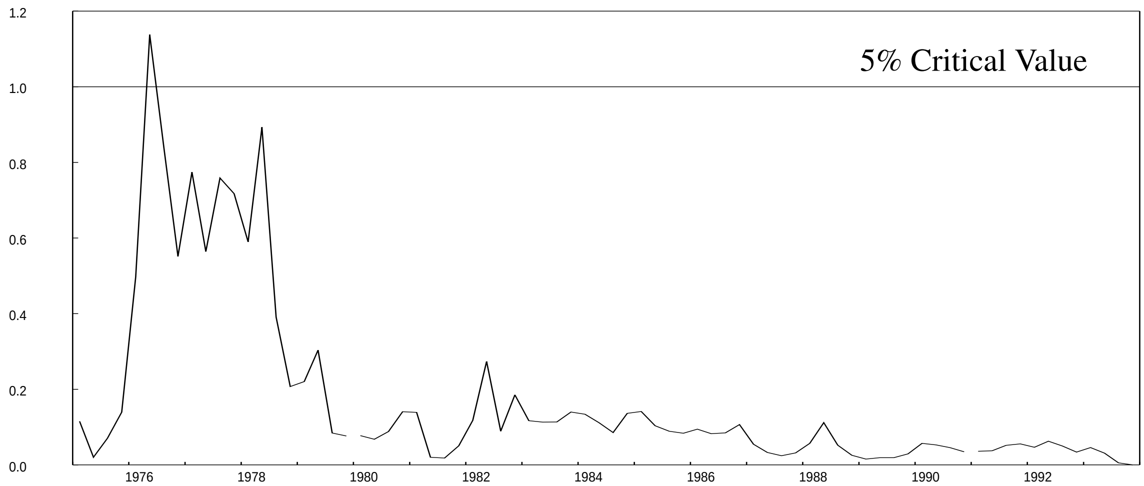


Figure 5: Test of Full Sample β Equal to Subsample β(t): System #7

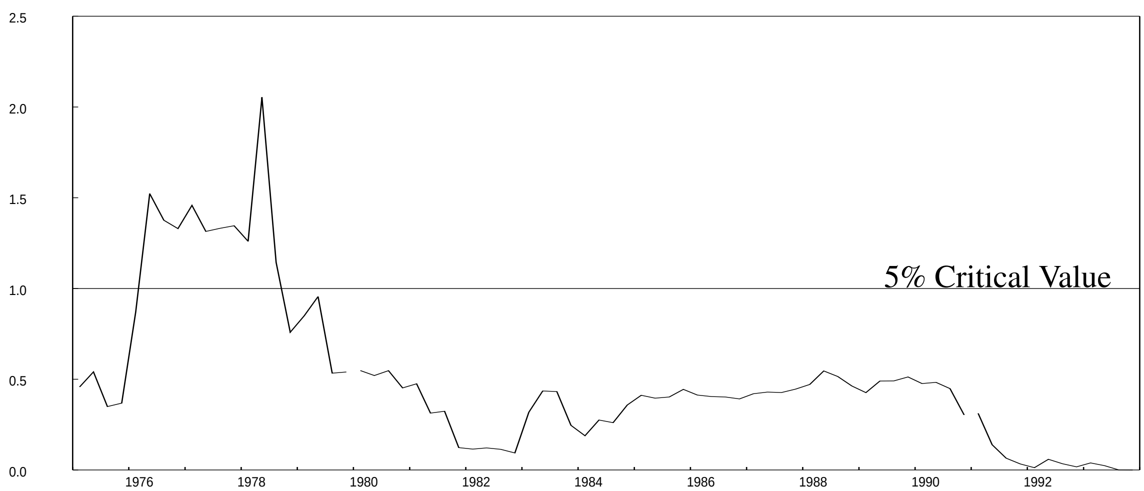


Figure 6: Test of Full Sample β Equal to Subsample β(t): System #8

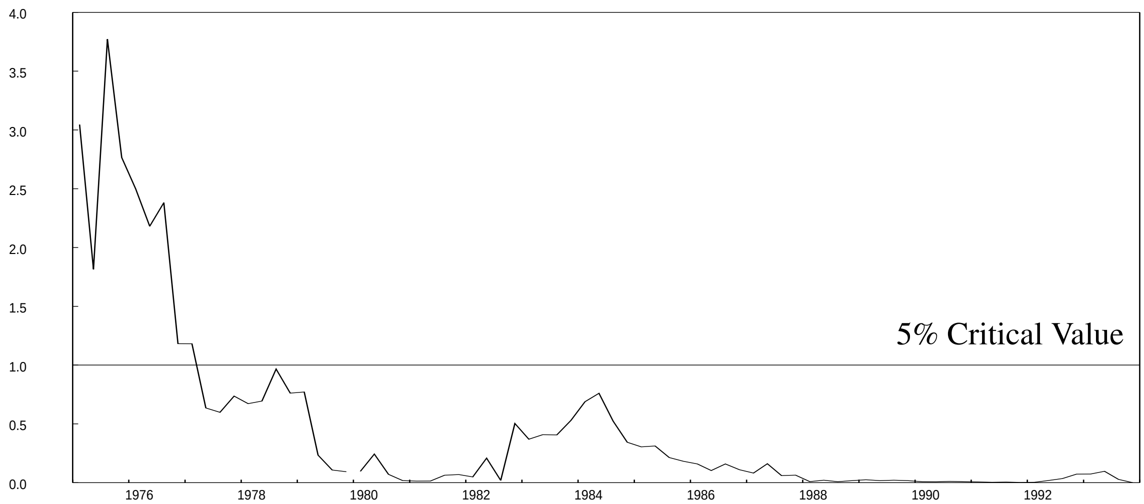


Figure 7: Test of Full Sample β Equal to Subsample β(t): System #9

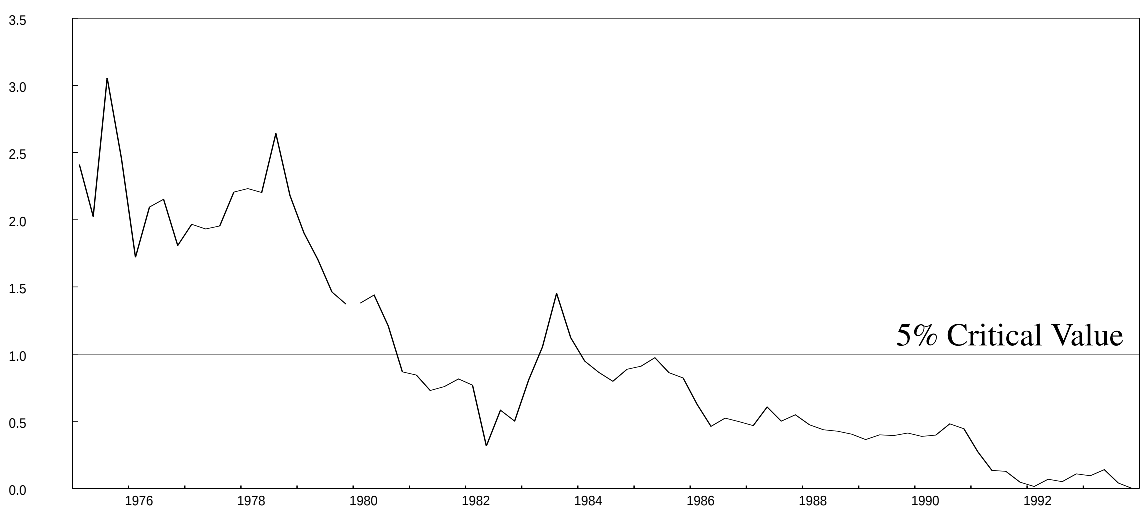


Table 5a: Cointegration Results Using Quarterly Non-seasonally Adjusted Data from 1956 to 1993[a]

					Cointegrating Vectors: β				Loadings: α			
CI Variables	Exog. Variables	λ-max	Trace	PGp	GM1	CPI *or* DEF	GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔGDP	ΔCP90
10) GM1, CPI, GDP, CP90	$GDPGAP_{t-1}$, Δe_t, $\Delta USTB_t$, $D80_t$, $DGST_t$	53.38‡	79.51‡	66.12‡	1.0	-0.931	-0.596	0.038	-0.122 (-4.103)	0.038 (4.991)	0.015 (0.547)	0.118 (0.084)
		22.12†	26.13	22.51								
11) GM1, DEF, GDP, CP90	$GDPGAP_{t-1}$, Δe_t, $\Delta USTB_t$, $D80_t$ $D74q3_t$	63.78‡	87.49‡	71.89‡	1.0	-1.017	-0.435	0.024	-0.079 (-1.826)	0.149 (6.980)	-0.059 (-1.615)	-2.805 (-1.480)
		18.16	23.71	20.56								

a. Each uses 5 lags.

Table 5b: Exclusion Tests

		Cointegrating Vectors: β				Loadings: α			
System	r *or* d.f.	GM1	CPI *or* DEF	GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔGDP	ΔCP90
10	1	15.46‡	12.05‡	19.57‡	26.30‡	10.23‡	16.85‡	0.28	0.01
11	1	25.61‡	21.18‡	24.02‡	39.88‡	2.50	32.68‡	2.56	1.96

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 5c: Some Exogenous Variable Coefficients and Residual Diagnostics

System	Equation	Constant	$D80_t$	$GDPGAP_{t-1}$	Δe_t	$\Delta USTB_t$	$DGST_t$	LB(24)	BJ(2)	R^2
10	ΔGM1	-0.050 (-3.251)	-0.022 (-4.430)	0.057 (1.123)	-0.087 (-1.131)	0.001 (0.448)	0.012 (0.789)	17.48	0.08	0.849
	ΔCPI	0.021 (5.282)	0.004 (3.487)	0.047 (3.621)	0.070 (3.538)	0.001 (1.686)	0.019 (5.058)	23.97	1.81	0.836
	ΔGDP	0.011 (0.738)	0.000 (0.022)	-0.159 (-3.367)	-0.120 (-1.673)	0.006 (4.163)	-0.018 (-1.350)	18.99	0.56	0.963
	ΔCP90	-0.064 (-0.088)	0.027 (0.113)	3.386 (1.401)	9.705 (2.646)	0.806 (11.502)	-0.585 (-0.837)	15.08	10.28‡	0.653
11	ΔGM1	0.086 (2.099)	-0.015 (-2.542)	0.098 (1.933)	-0.054 (-0.650)	0.001 (0.474)	-	16.14	0.23	0.833
	ΔDEF	-0.135 (-6.687)	0.016 (5.482)	0.023 (0.909)	-0.098 (-2.400)	0.001 (1.205)	-	26.11	2.07	0.813
	ΔGDP	0.055 (1.585)	-0.008 (-1.550)	-0.168 (-3.929)	-0.087 (-1.246)	0.006 (4.409)	-	22.10	1.70	0.966
	ΔCP90	2.289 (1.280)	-0.280 (-1.102)	2.169 (0.977)	10.537 (2.916)	0.794 (11.658)	-	12.65	7.42†	0.672

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Table 5d: Hypothesis Tests for Quarterly Nominal GM1 Specifications from 1956 to 1993

System	Hypothesis Test on β	χ^2	d.f.	p-val.	Cointegrating Vectors: β				Loadings: α			
					GM1	CPI *or* DEF	GDP	CP90	ΔGM1	ΔCPI *or* ΔDEF	ΔGDP	ΔCP90
10	H_0:[1,-1,a,b]	0.775	1	0.379	1.0	-1.0	-0.526	0.035	-0.114 (-3.751)	0.040 (5.238)	0.019 (0.678)	0.142 (0.099)
10	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	0.278	2	0.870	1.0	-0.923	-0.607	0.039	-0.121 (-4.192)	0.037 (4.961)	0	0
10	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	1.212	3	0.750	1.0	-1.0	-0.530	0.037	-0.113 (-3.833)	0.039 (5.238)	0	0
11	H_0:[1,-1,a,b]	0.109	1	0.741	1.0	-1.0	-0.451	0.024	-0.081 (-1.905)	0.145 (6.888)	-0.057 (-1.593)	-2.808 (-1.512)
11	H_0: $\alpha_{TE}=0$, $\alpha_{CP}=0$	3.990	2	0.136	1.0	-1.019	-0.442	0.024	-0.100 (-2.365)	0.140 (6.639)	0	0
11	H_0:[1,-1,a,b], $\alpha_{TE}=0$, $\alpha_{CP}=0$	4.111	3	0.250	1.0	-1.0	-0.460	0.025	-0.102 (-2.469)	0.135 (6.537)	0	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Figure 8: Test of Full Sample β Equal to Subsample β(t): System #10

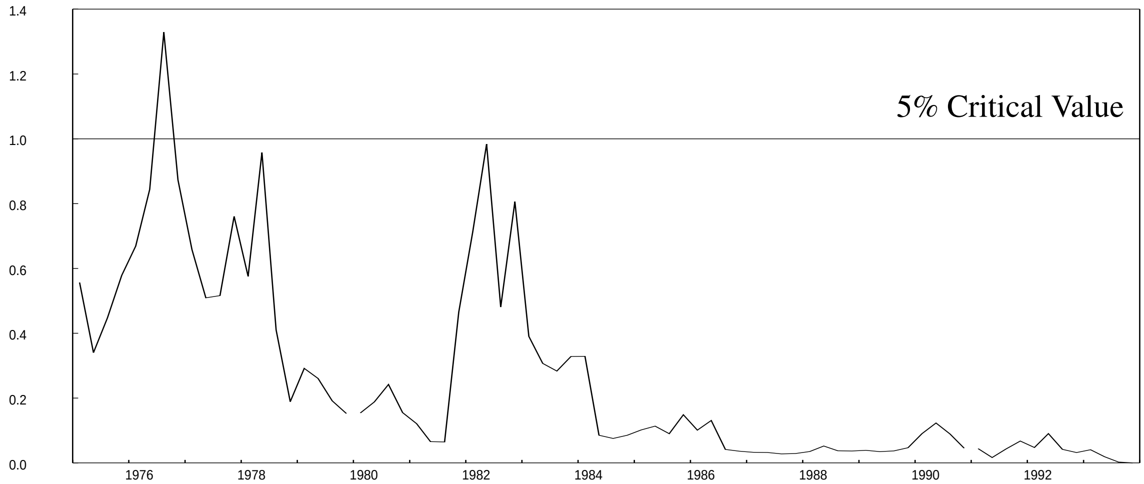


Figure 9: Test of Full Sample β Equal to Subsample β(t): System #11

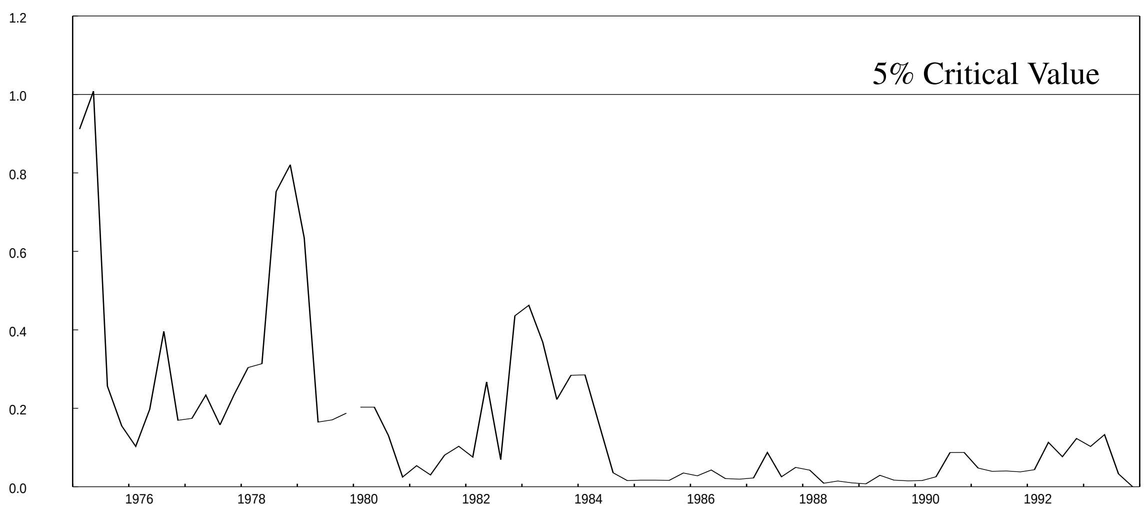


Table 6a: Results with Constant and Shift Dummy Restricted to Cointegrating Vector[a]

Endogenous Variables	λ-max	Trace	PGp	Cointegrating Vectors: β						Loadings: α			
				GM1	CPI *or* DEF	TE *or* GDP	CP90	Const.	D80	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
Monthly Raw Data: 1961-1993													
3a) GM1, CPI, TE, CP90	70.77‡	129.52‡	110.33‡	1.0	-0.867	-0.682	0.022	0.524	0.145	-0.070 (-3.758)	0.030 (6.012)	0.058 (3.038)	-1.300 (-2.121)
	39.41‡	58.76‡	50.74‡	1.0	-1.308	0.760	-0.116	-10.965	0.184	0.009 (3.522)	-0.001 (-1.330)	0.006 (2.500)	-0.319 (-3.970)
	15.21*	19.35*	16.90										
Quarterly Raw Data: 1956-1993													
10a) GM1, CPI, GDP, CP90	53.73‡	86.68‡	72.01‡	1.0	-0.930	-0.597	0.038	0.503	0.141	-0.122 (-4.069)	0.037 (4.943)	0.014 (0.519)	0.077 (0.055)
	28.28‡	32.95*	28.14										
11a) GM1, DEF, GDP, CP90	65.67‡	97.80‡	80.27‡	1.0	-1.020	-0.428	0.023	-0.986	0.115	-0.059 (-1.351)	0.154 (7.260)	-0.061 (-1.680)	-3.489 (-1.845)
	25.76†	32.13*	27.57										

a. Monthly systems use 14 lags while quarterly use 5 lags. Except for D80, the set of exogenous variables is the same as in the comparable systems with an unrestricted constant: System #3 for the monthly data and Systems #10 and #11 for the quarterly data.

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level. t-statistics are in parentheses.

Table 6b: Exclusion Tests

		Cointegrating Vectors: β						Loadings: α			
System	r *or* d.f.	GM1	CPI *or* DEF	TE *or* GDP	CP90	Const.	D80	ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
3a	2	30.50‡	28.22‡	29.54‡	20.42‡	1.01	10.56‡	8.23†	24.41‡	6.96†	2.71
10a	1	14.06‡	10.90‡	19.60‡	22.23‡	0.44	3.46*	8.23‡	13.64‡	0.24	0.00
11a	1	26.35‡	21.85‡	23.10‡	33.76‡	3.93†	5.15†	1.20	31.05‡	2.77*	2.88*

Table 6c: Hypothesis Tests

System	Hypothesis Test on β	χ^2	d.f.	p-val.	Cointegrating Vectors: β GM1	CPI *or* DEF	TE *or* GDP	CP90	Const.	D80	Loadings: α ΔGM1	ΔCPI *or* ΔDEF	ΔTE *or* ΔGDP	ΔCP90
10a	H_0: [1,-1,a,b,c,d] for all CI	0.771	1	0.380	1.0	-1.0	-0.524	0.035	-0.031	0.178	-0.112 (-3.638)	0.040 (5.252)	0.019 (0.679)	0.064 (0.045)
10a	H_0:[1,-1,a,b,c,d], $\alpha_{TE}=0$, $\alpha_{CP}=0$	1.195	3	0.754	1.0	-1.0	-0.530	0.037	0.024	0.178	-0.113 (-3.792)	0.039 (5.205)	0	0
11a	H_0: [1,-1,a,b,c,d] for all CI	0.144	1	0.704	1.0	-1.0	-0.446	0.024	-0.853	0.104	-0.062 (-1.451)	0.149 (7.148)	-0.059 (-1.657)	-3.468 (-1.875)
11a	H_0:[1,-1,a,b,c,d], $\alpha_{TE}=0$, $\alpha_{CP}=0$	5.140	3	0.162	1.0	-1.0	-0.459	0.025	-0.730	0.110	-0.096 (-2.286)	0.138 (6.679)	0	0

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Figure 10: Test of Full Sample β Equal to Subsample β(t): System #10a

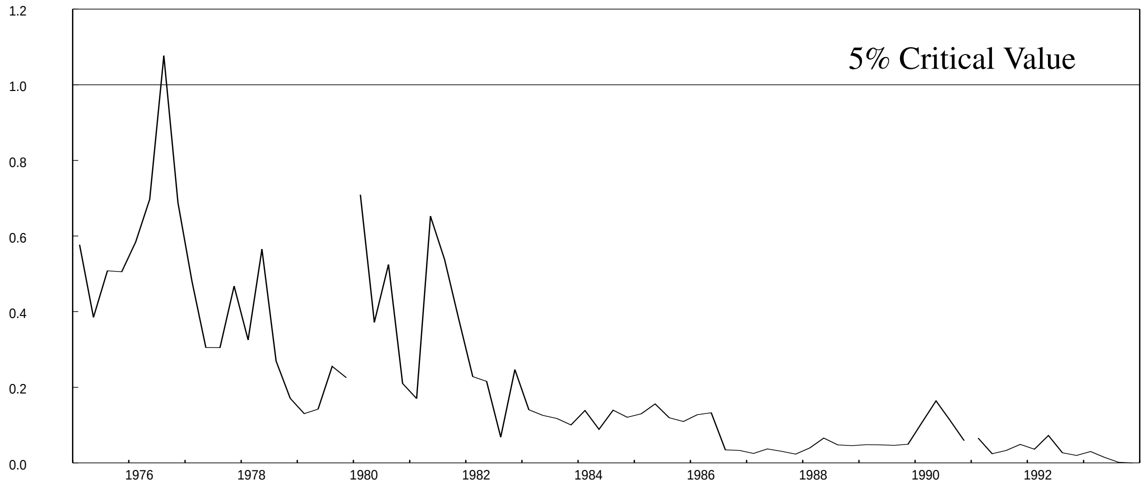


Figure 11: Test of Full Sample β Equal to Subsample β(t): System #11a

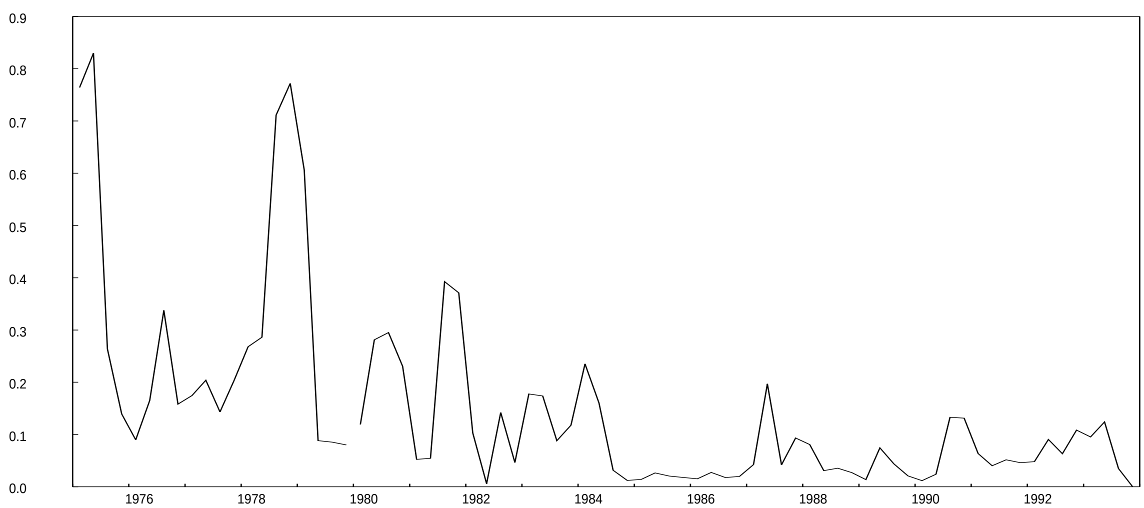


Table 7a: Results with a Portion of the Constant and the Shift Dummy Allocated to the Cointegrating Vector

Endogenous Variables	Cointegrating Vectors: β								
	GM1	CPI *or* DEF	TE *or* GDP	CP90	Const. Q1	Const. Q2	Const. Q3	Const. Q4	D80
Quarterly Raw Data: 1956-1993									
10) GM1, CPI, GDP, CP90	1.0	-0.931	-0.596	0.038	0.290	0.313	0.327	0.343	0.251
11) GM1, DEF, GDP, CP90	1.0	-1.017	-0.435	0.024	-1.064	-1.037	-1.025	-1.019	0.172

‡ Significant at the 1% level. † Significant at the 5% level. * Significant at the 10% level.

Figure 12: Money Gap from System #10a

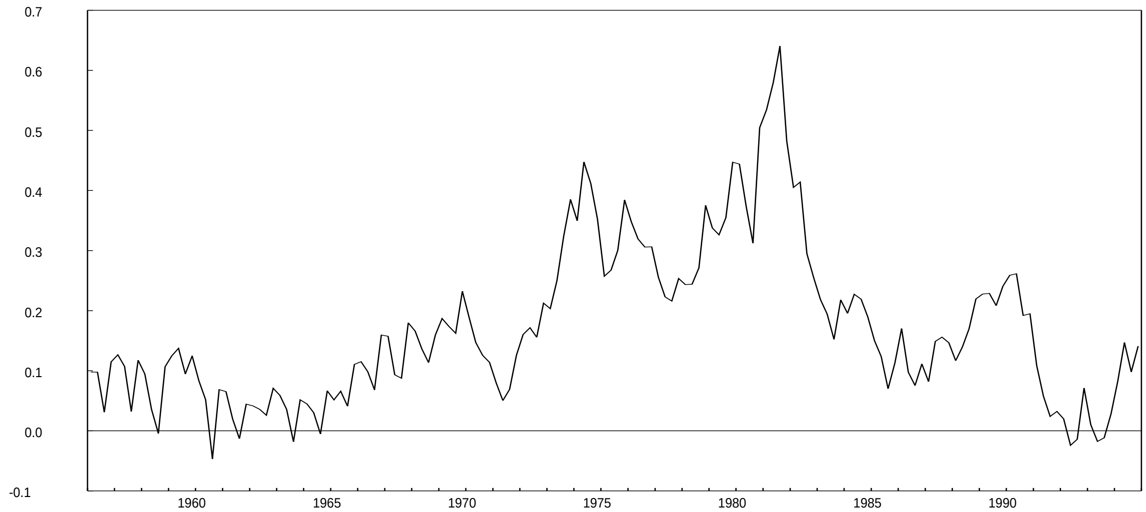


Figure 13: Money Gap from System #11a

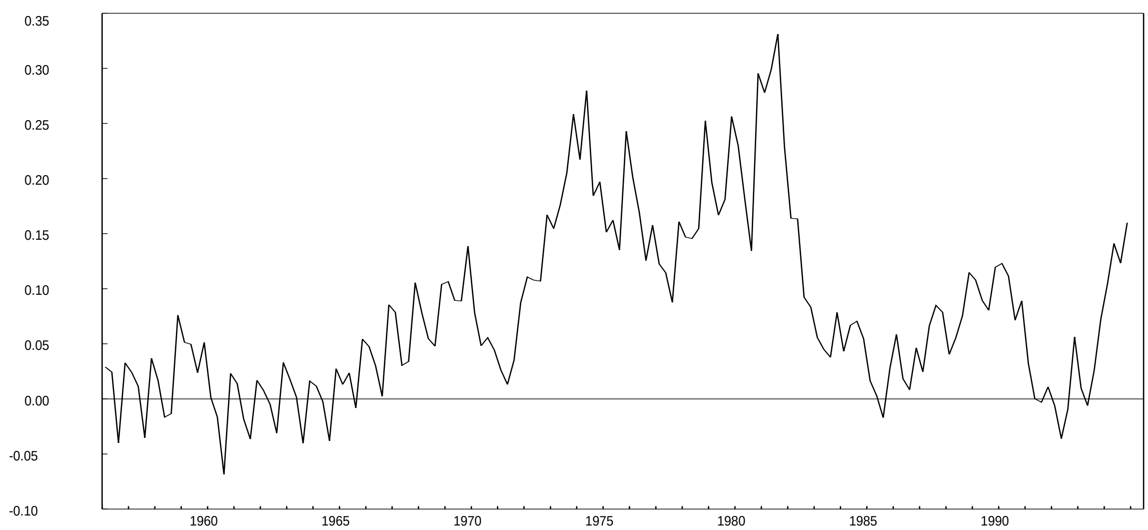


Figure 14: Money Gap for System #10

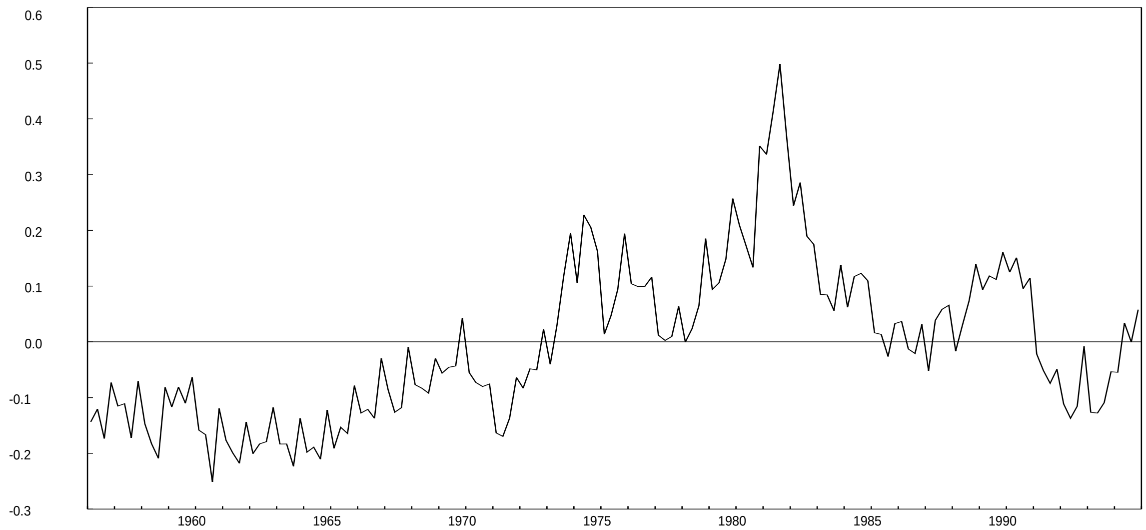
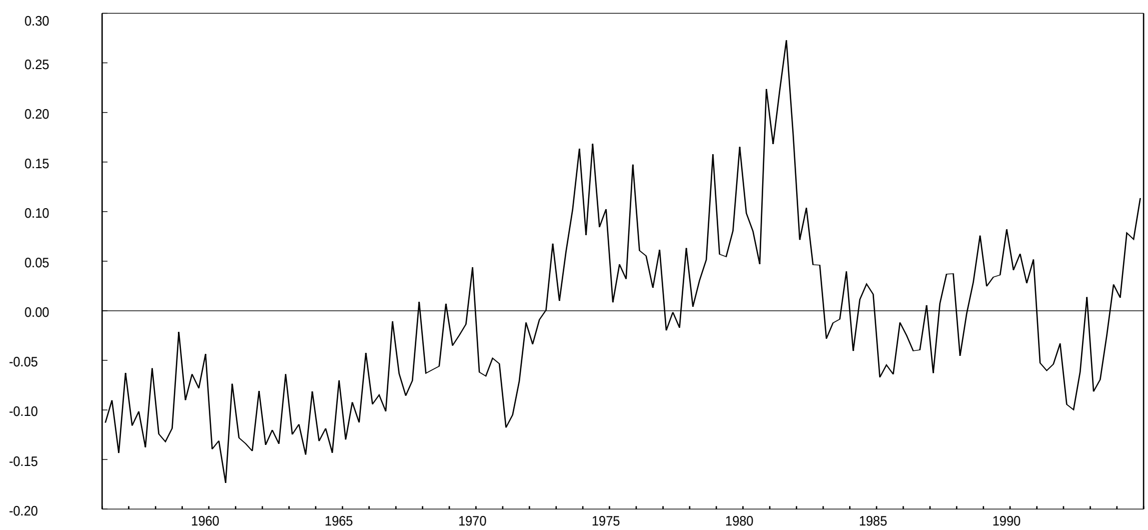


Figure 15: Money Gap for System #11



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