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Deriving Agents' Inflation Forecasts from the Term Structure of Interest Rates

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Contents

Abstract

In this paper, the author uses the term structure of nominal interest rates to construct estimates of agents' expectations of inflation over several medium-term forecast horizons. The Expectations Hypothesis is imposed together with the assumption that expected future real interest rates are given by current real rates. Under these maintained assumptions, it is possible to compare the nominal yields on two assets of different maturities and attribute the difference in nominal yields to differences in expected inflation over the two horizons (assuming a constant term premium). The results for the United States and Canada over the past several years suggest that there is a significant static element to agents' inflation expectations.

Résumé

À l'aide de la structure par terme des taux d'intérêt nominaux, l'auteur établit sur plusieurs horizons de prévision à moyen terme des estimations des anticipations d'inflation que forment les agents économiques. L'hypothèse d'anticipations de la courbe de rendement est retenue conjointement avec celle selon laquelle les taux d'intérêt réels futurs anticipés sont donnés par les taux réels du moment. Sous ces deux hypothèses, il est possible de comparer les taux de rendement nominaux de deux actifs à échéances différentes et d'attribuer l'écart entre ces taux aux divergences entre les taux d'inflation anticipés sur les deux horizons (en supposant que la prime payable à l'échéance est constante). Les résultats obtenus pour les États-Unis et le Canada au cours des dernières années laissent croire que les anticipations d'inflation des agents économiques sont en grande partie statiques.

1 Introduction

Changes in expected inflation are usually thought to affect nominal interest rates; if expected real interest rates are constant, then any change in expected inflation should lead to one-for-one changes in nominal interest rates. Despite Fama's (1975) finding for the United States that expected real interest rates appeared to be constant over the 1953-71 period, there are obviously good reasons to expect real interest rates to change. Temporary productivity and taste shocks should affect real interest rates, as should monetary shocks in the short run. With variable real interest rates, it would be inappropriate to attribute every change (or even most changes) in nominal interest rates to changes in expected inflation.

A popular view is that the tilt of the nominal yield curve contains information about agents' expectations of inflation over short versus long horizons. For example, a tightening of monetary policy might be associated with an increase in short-term interest rates but a reduction in long-term rates, because agents believe that the tightening of policy will eventually reduce inflation; an "inverted" yield curve is often thought to reflect the belief that future inflation will be less than current inflation. Despite the intuitive appeal of this interpretation of movements in short-term and long-term interest rates, the nominal term structure has not been widely applied to the estimation of agents' implicit inflation forecasts.[1] The main reason for this is that the presence of any unobservable risk or term premiums driving a wedge between real rates of return on assets of different maturities confounds the problem of estimating agents' underlying inflation forecasts.

This paper uses the nominal term structure of interest rates in Canada and the United States to estimate agents' expectations of future inflation. By using

1. However, it has been widely applied to the related issue of examining whether the nominal term structure contains information helpful for predicting actual future inflation. This is discussed shortly.

various pairs of assets located along the short to medium end of the term structure – maturities ranging from 1 month to 5 years – I generate estimates of expected inflation over horizons up to 5 years. The basic approach is capable of generating an estimate of expected inflation over any horizon for which interest rate data on an equal-maturity asset is available. This differs from the basic approach of Frankel (1982) in which the nominal term structure is used only to generate an estimate of expected long-run (steady-state) inflation.

From the perspective of the monetary authority, knowing agents' expectations of inflation – over both short and long horizons – is of obvious importance. It is now accepted doctrine in macroeconomics that unanticipated changes in money or prices have more significant real effects than do fully anticipated changes. Thus, in order to accurately predict the immediate real effects of a given disinflation policy, for example, the monetary authority must have a reliable estimate of the level of inflation that is expected over the near future by market participants. A related issue, with perhaps more importance over the longer term, is that a reliable estimate of agents' inflation expectations over longer horizons provides the monetary authority with a convenient gauge of its own policy credibility; an announcement of significant monetary tightening that is not followed by a reduction in agents' longer-term inflation forecasts is suggestive of a credibility problem for the monetary authority.

It is important to note that this paper does not examine the issue of whether the nominal term structure contains useful information for predicting the actual path of future inflation. This issue is the focus of much research, including papers by Mishkin (1988, 1990a, 1990b), Fama (1984, 1990), Blough (1994) and Frankel and Lown (1994). This literature is directed at examining the success of the nominal term structure in predicting actual future inflation. In contrast, we make some identifying assumptions which impose the condition that the nominal term structure contains useful information about agents' expectations of future inflation.

The ex post success or failure of these expectations is not a focus of this paper. In this sense, this paper is similar in motivation to Frankel (1982) and Hamilton (1985).

The two central identifying assumptions used in this paper are first, that the Expectations Hypothesis holds for real returns on assets of different maturities and second, that expected future real interest rates are given by current real interest rates. With these two maintained assumptions, it is then possible to compare the nominal yields on any two bonds of different maturities and attribute any differences in nominal interest rates to differences in expected inflation over the two horizons (and to a time-invarying term premium). The method used here estimates this term premium and expected inflation simultaneously.

The layout of the paper is as follows. Section 2 discusses why estimation of expected inflation based on the observed term structure might be preferable to some other methods. Section 3 presents the basic estimation method and the central identifying assumptions. Section 4 discusses the selection of the sample and presents the results for the estimated term premiums. Section 5 presents the estimates of expected inflation. The estimates of expected inflation are shown to be relatively insensitive to the estimated term premiums and very closely related to the current inflation rate. Section 6 concludes the paper.

2 Why use the term structure to estimate expected inflation?

In principle, an ideal measure of agents' expectations of future inflation should be capable of reflecting any changes in the beliefs of forward-looking agents in response to newly acquired information. One way of satisfying this requirement is to use a measure that is based on observable market prices and/or quantities. In this case, as agents receive new information and act upon it, thus affecting either prices or quantities in the economy, the ideal measure of expected inflation will change in response to the change in beliefs.

Not all methods for estimating expected inflation satisfy this forward-looking property. For example, one simple approach is to estimate a regression equation in which the rate of inflation is the dependent variable and then interpret the predicted values from the estimated regression as agents' expectations of inflation. A notable example, in a setting where nominal money growth rather than inflation is the variable of interest, is Barro's (1977) classic paper examining the relationship between unemployment and unanticipated money growth. A slightly different approach also uses a simple regression equation in which inflation is the dependent variable but estimation proceeds recursively, so that the equation is re-estimated every period as new information becomes available. The measure of expected inflation is then the inflation forecast based on the current parameter estimates, which change every period. This method is similar in spirit to the approach used by Hamilton (1985).

One problem with both regression-based approaches is that they are inherently backward-looking; since forecasts at time *t* are based only on whatever relationship exists over historical data, it is difficult for "news," such as announcements regarding future policy changes, to be incorporated into the forecasts. A related problem with regression-based approaches is that the burden falls on the econometrician to choose the relevant independent variables, and typically a small set of such variables is chosen. In contrast, market participants may rely on a very different set of variables when forming their expectations.

A second basic approach involves conducting regular and frequent surveys of many "informed" market participants, such as professional forecasters for financial institutions, and then constructing an average inflation forecast from the survey responses. The Livingston Survey in the United States and the Conference Board of Canada, for example, both produce survey-based estimates of expected inflation. Such estimates are clearly capable of being forward-looking. But these survey-based estimates do not necessarily reflect economic behaviour – that is, the survey responses are not necessarily consistent with the movements in the relevant prices and quantities observed in the marketplace. Thus, a key identifying assumption required in order for survey-based estimates to accurately reflect actual inflation expectations is that the *stated* beliefs of the so-called informed individuals are representative of the *actual* beliefs on which countless other individuals base their day-to-day decisions. It is not clear why this should be true.

A third approach compares nominal rates on equal-maturity nominal and real (indexed) bonds. Such an approach is examined by Deacon and Derry (1994) in the United Kingdom. This approach has obvious merits, but because so few countries have active markets for indexed bonds, this estimation method is not widely available.

Given the general unavailability of indexed bonds, a natural alternative is to use observed nominal interest rates on assets of different maturities within a given country. By selecting pairs of assets located at various points along the nominal term structure and by imposing certain identifying assumptions it is possible to use the nominal interest rates associated with these assets to infer agents' expectations of future inflation. At any time t, this method will generate an estimate of expected inflation between time t and the date of maturity of the longest of the two assets. Such an estimate of expected inflation is capable of incorporating changes in agents' beliefs in response to "news" and is thus forward-looking. The identifying assumptions required to draw inferences about agents' expectations from the nominal term structure are then central to judging the reasonableness of the basic approach. In the next section, these identifying assumptions are discussed in detail.

3 Deriving inflation forecasts from the term structure

3.1 The basic approach

The method for generating an estimate of expected inflation begins with the simple assumption that the expected real rates of return between any two bonds denominated in the same currency must be due to some combination of a default premium, a liquidity premium and a term premium. The default premium needs no explanation; assets with higher probability of default require a higher expected return. The liquidity premium reflects the differential ease a potential holder of the assets might have in selling them before maturity. The term premium reflects the difference in the time to maturity of the assets. For example, two equal-maturity government bonds might exist side by side, but one might be traded only in less active markets for some reason. In this case, there would be no term premium between the two expected real returns, but there may be a liquidity premium. Conversely, two government bonds that are both actively traded may have different maturities. In this case, there would be no liquidity premium but there may be a term premium.

Now suppose that the two bonds can be thought of as lying at different points along the term structure. The two bonds are denoted "*S*" and "*L*," for short and long maturity, respectively. We now have

$$E\left[(1+R_t^L)/(1+\Pi_t^L)\right] - E\left[(1+R_t^S)/(1+\Pi_t^S)\right] = D_t^{S,L} + L_t^{S,L} + T_t^{S,L}, \qquad (1)$$

where R^S and R^L are the nominal yields to maturity observed on the two assets at time t, Π^S and Π^L are the rates of inflation from time t over the short and long horizons, and D, L and T are the default, liquidity and term premiums, respectively.

Given equation (1) as a starting point, the key identifying assumption used in this paper is that by carefully selecting countries, time periods and assets, we can choose various pairs of assets along the term structure that are characterized by a constant difference between the two expected real rates of return.

To examine the reasonableness of this identifying assumption, consider first the default premium. In stable countries, it is likely that government securities of different maturities – especially for maturities of less than 5 years – contain the same default risk. Furthermore, in countries like Canada and the United States this default risk is probably equal to zero. Note, however, that there is no requirement that the default risk be zero. Nor is there a requirement that any positive default risk be constant over time; the only requirement is that the two government securities of different maturities carry the *same* default risk at all times and thus that the default premium between them be zero. The first identifying assumption is therefore

- **A1: Short- to medium-term government securities (30 days to 5 years) in a stable country carry the same default risk, independent of their term to maturity.**

One can imagine specific countries in which assumption A1 would never be reasonable or may only be reasonable along the very short end of the term structure. Thus, it is important for the use of A1 that some care be used in choosing both the countries and the appropriate section of the term structure.

Now consider the liquidity premium. Two assets that are equally actively traded will likely have no liquidity premium driving a wedge between their expected real rates of return. Thus, if we restrict our attention to actively traded government securities, the liquidity premium should be zero. It is not important that the government securities be perfectly liquid in any meaningful sense; it is only important that however liquid or illiquid these government securities might be, the liquidity be the same across all maturities. The second identifying assumption is therefore

- **A2: Actively traded short- to medium-term government securities in a stable country are equally liquid, independent of their term to maturity.**

We discuss in Section 4 how choosing assets along the term structure to satisfy assumption A2 restricts the sample period used for this paper, especially in Canada.

Finally, consider the term premium between any pair of assets. This premium reflects the preferences of the asset holder for lending over different horizons. There are many reasons why one might prefer to lend over the short term rather than the long term and thus require a positive term premium for longer-maturity assets. From equation (1), however, the term premium is defined to be the premium required for factors *other* than changes in expected inflation, default risk and liquidity risk. Thus, the term premium reflects largely the lending preferences of the asset holder, and this leads to the final identifying assumption:

- **A3: The pure term premium between any pair of short- to medium-term government securities is constant over time.**

A3 is clearly a strong assumption, but note that it permits the term premium to be quite a general function of the term to maturity of the asset. Indeed, equation (1) allows a different term premium between every possible pair of assets, so that, for example, the term premium between a 1-year and a 2-year bond need not be equal to the term premium between a 2-year and a 3-year bond.

A constant term premium implies a constant pattern in the term structure of expected real interest rates. Pesando (1978) provides some evidence for time-invarying term premiums in Canada during the first half of the 1970s, though his evidence pertains more to the longer part of the term structure (a 10-year horizon). Mishkin's (1990b) results for the United States support the hypothesis of a constant term premium except at the shortest end of the term structure (less than or equal to six months). The apparent variability of the term premium only at the shortest end of the term structure may reflect the extent to which policy-induced changes in the overnight interest rate influence rates farther up the term structure. The presence of such high-frequency shocks suggests that assets located at the very short end of the term structure, with maturities perhaps as long as three or six months, may be unable to satisfy assumption A3. I return to this issue later in this section.

Applying assumptions A1, A2 and A3 to equation (1) yields the central equation of this paper:

$$E\left[(1+R_t^L)/(1+\Pi_t^L)\right]-E\left[(1+R_t^S)/(1+\Pi_t^S)\right] = T^{S,L}, \tag{2}$$

where $T^{S,L}$ is the time-invarying term premium between assets of S and L maturities.

Equation (2) is equivalent to imposing the joint assumption of the validity of the Expectations Hypothesis of the real term structure and the constancy of expected real interest rates. The Expectations Hypothesis requires that the (annualized) expected long-term real rate equals the (annualized) expected real return on an equally long sequence of short-term assets, adjusted perhaps for a constant term premium. Given the Expectations Hypothesis, constant expected real returns then imply that expected future real returns on the sequence of short-term assets are given by the expected real return on the current short-term asset (Shiller 1990).

It is worthwhile to make the distinction between the assumption that real rates are *expected* to be constant and the more restrictive assumption that real rates are *actually* constant. Equation (2) does not impose the constancy of real interest rates; it only imposes the condition that expected real rates of return across assets of different maturities be equalized (up to the term premium) at all times. In other words, actual and expected real interest rates can be highly variable over time, but expected real rates on assets of different maturities move together.

3.2 Approximations

It is commonplace to use a log approximation so that the real interest rate is expressed as R-Π rather than $(1+R)/(1+\Pi)$. This is a good approximation only when inflation and nominal interest rates are low, as Patinkin (1993) has recently shown in his discussion of the 1985 Israeli stabilization. For example, the difference between R and $\log(1+R)$ when R=5% is about 12 basis points, whereas the equivalent error when R=20% is over 175 basis points.

Over the past two decades, inflation rates and nominal interest rates have often exceeded 10 per cent; in the early 1980s they approached 20 per cent. Using log approximations for inflation and interest rates during these periods would lead to large errors. More important to this paper, however, is the fact that the high variability of inflation and nominal interest rates over the past 20 years implies that the errors from the log approximation will also vary over time. Thus, the errors would be in the order of 200 basis points in the early 1980s but only about 10 basis points in the early 1990s. Given the assumption of a time-invarying term premium, which might reasonably be expected to take on values between 0 and 200 basis points (depending on S and L), such errors would be very damaging to the basic approach of extracting agents' inflation forecasts from the observed term structure. For this reason, we do not apply a log approximation to equation (2).

We do, however, use another approximation. We violate Jensen's Inequality by assuming that

$$E[1/(1+\Pi)] = 1/(1+E[\Pi]) . \tag{3}$$

In contrast to the log approximation, the approximation in equation (3) is more serious in low-inflation environments. This is because the function $\omega(\Pi) = 1/(1+\Pi)$ is the most convex around the point Π=0. Thus, if actual inflation is quite variable around a low mean, so that inflation is sometimes positive and sometimes negative, then violating Jensen's Inequality is potentially serious. But actual inflation in Canada and the United States over the past few decades has almost always been positive; a graph which plots $\omega(\Pi)=1/(1+\Pi)$ against $(1+\Pi)$ for Canada or the United States over the past few decades shows the function ω to be very close to linear, indicating that the violation of Jensen's Inequality is not serious.

When equation (3) is imposed, given that the nominal yields to maturity on the two assets are observed at the time inflation forecasts are being formed, equation (2) becomes

$$1 + E[\Pi_t^L] = \frac{(1 + R_t^L) \cdot (1 + E[\Pi_t^S])}{T^{S,L} \cdot (1 + E[\Pi_t^S]) + (1 + R_t^S)}, \tag{4}$$

where $E[\Pi_t^S]$ and $E[\Pi_t^L]$ are the agents' actual expectations about inflation over the S and L horizons, based on information available at the beginning of period t.

3.3 Expected inflation over the *L* horizon

Equation (4) expresses expected inflation over the long (L) horizon as a function of observed nominal interest rates, the unobserved expected inflation over the short (S) horizon and the unobserved term premium. Note that this relationship holds for each possible combination of S and L deemed to satisfy the central identifying assumptions. The approach taken in this paper is to simultaneously estimate the term premium and expected inflation over the L horizon, while taking as given an assumption regarding agents' expectations of inflation over the S horizon. We denote our estimate of agents' expectations of inflation over the L horizon as $\hat{\Pi}_t^L$, and our estimate of the term premium as $\hat{T}^{S,L}$.

Consider first the estimation of $E[\Pi_t^L]$ and $T^{S,L}$. For any given value of $E[\Pi_t^S]$, equation (4) is used to construct several different time-series for $\hat{\Pi}_t^L$, one for each of several values for $T^{S,L}$. An estimate of agents' expectations of inflation over the L horizon is therefore expressed as a non-linear function of the term premium, $\hat{\Pi}_t^L(\hat{T}^{S,L})$. We then choose the time-series for $\hat{\Pi}_t^L$ (and hence the associated value of the term premium), which minimizes over the sample period the mean square of the forecast error:

$$\varepsilon_t^L = \Pi_t^L - \hat{\Pi}_t^L(\hat{T}^{S,L}), \tag{5}$$

where Π_t^L is the actual inflation rate from period t to period $t+L$ and $\hat{\Pi}_t^L$ is the agents' L-period-ahead expectation of inflation made at time t. Thus, the forecast error ε_t^L is not observed until period $t+L$.

One way to think of this estimation method is that the series for expected inflation is chosen so that the average bias implicit in the forecast is minimized. Imagine using equation (4) to construct a series for expected inflation imposing $T^{S,L}=0$. Since the true value of $T^{S,L}$ is presumably positive, these expectations of inflation will be biased upwards. The method used here chooses the value of $T^{S,L}$ to minimize the mean square of this bias. Note that since we impose $\hat{T}^{S,L}$ to be constant over the sample, this method does not force the measure of expected inflation to track actual inflation; it only restricts inflation expectations to be unbiased.

3.4 Expected inflation over the *S* horizon

Now consider the value of $E[\Pi_t^S]$ that is necessary to construct the estimates of the term premium and expected inflation over the L horizon. The approach taken here is to impose a final identifying assumption regarding expected inflation over the S horizon:

- **A4: For the lowest value of *S* used in the sample, denoted $\underline{S}$, agents' expectations of inflation over the $\underline{S}$ horizon are given by current inflation.**

With assumption A4, it is then a straightforward application of equation (4) to construct estimates of expected inflation over any L horizon.

Reliance on assumption A4 makes the selection of $\underline{S}$ particularly important. The issue is not just determining which choice of $\underline{S}$ is consistent with a "reasonable" view of agents' expectations; if this were the only issue, then presumably the lowest possible value of $\underline{\mathrm{S}}$ would be appropriate. A simple choice of $\underline{S}$ is made difficult by the influence of monetary policy on overnight interest rates and the effect of such policy-induced changes on nominal rates further up the term structure. Thus, there is a clear trade-off involved in the selection of $\underline{S}$. As $\underline{S}$ falls, assumption A4 clearly becomes more reasonable. On the other hand, the lower is $\underline{S}$, the more likely it is that the gap between nominal yields on $\underline{S}$-term and L-term assets will reflect more than just changes in expected inflation, bringing into question the validity of assumption A3. We discuss the selection of $\underline{S}$ in the next section.

3.5 Estimated term premiums for all *S*, *L* combinations

For the sake of argument, suppose that $\underline{S}$ is chosen to be equal to one (month). If assumption A4 is imposed with $\underline{S}$=1, equation (4) can then be used to generate estimates of inflation over each horizon, where L exceeds one. This provides independent estimates of agents' expected inflation over each L horizon, together with an estimate of the term premium between 1-month securities and each of the longer-term government securities.

When the estimates based on $\underline{S}$=1 are used, equation (4) can then be used to infer the value of the term premiums between any values of S and L used in the sample (where S>$\underline{\mathrm{S}}$). To see this, consider $\underline{S}$=1 and the estimates of expected inflation over the 2-month and 3-month horizons. The appropriate versions of equation (4), based on assumption A4, are

$$1 + \hat{\Pi}_t^{1,2} = \frac{(1 + R_t^2) \cdot (1 + \Pi_t)}{\hat{T}^{1,2} \cdot (1 + \Pi_t) + (1 + R_t^1)}, \tag{6}$$

$$1 + \hat{\Pi}_t^{1,3} = \frac{(1 + R_t^3) \cdot (1 + \Pi_t)}{\hat{T}^{1,3} \cdot (1 + \Pi_t) + (1 + R_t^1)}, \tag{7}$$

where the superscripts in $\hat{\Pi}^{i,j}$ indicate an estimate of expected inflation over the long *(j)* horizon constructed with short *(i)* maturity assets. With $\hat{\Pi}_t^{1,2}$ so constructed, equation (4) generates a second estimate of expected inflation over the 3-month horizon. The equation is

$$1 + \hat{\Pi}_t^{2,3} = \frac{(1 + R_t^3) \cdot (1 + \hat{\Pi}_t^{1,2})}{\hat{T}^{2,3} \cdot (1 + \hat{\Pi}_t^{1,2}) + (1 + R_t^1)}. \tag{8}$$

Using Equation (6) to substitute into equation (8), however, yields

$$1 + \hat{\Pi}_t^{2,3} = \frac{(1 + R_t^3) \cdot (1 + \Pi_t)}{(\hat{T}^{2,3} + \hat{T}^{1,2}) \cdot (1 + \Pi_t) + (1 + R_t^1)}. \tag{9}$$

If equations (7) and (9) are compared and the term premium for any (S, L) pair is chosen to minimize the mean squared forecast errors as seen previously, then it must follow that

$$\hat{T}^{2,3} = \hat{T}^{1,3} - \hat{T}^{1,2}. \tag{10}$$

The basic estimation method can therefore be summarized as follows: Choose the value of $\underline{S}$ to satisfy assumptions A3 and A4 and construct the estimates of expected inflation over each of the L horizons where L exceeds $\underline{S}$. Associated with the estimate of expected inflation over each L horizon will be an estimate of each ($\underline{S}$, L) term premium. Then, using equation (9) or the equivalent equation for different values of S and L, construct the implied value of the term premiums for all (S, L) pairs for which $S>\underline{S}$.

4 Data and estimation

4.1 Data choices

The basic collection of data is the monthly average nominal yields to maturity on treasury bills and government bonds from Canada and the United States. I use treasury bills of 1- 2- 3- and 6-month terms, and government bonds of 1- 2- 3- and 5-year terms.[2] For the United States, data on the 2-year bonds are only readily available from 1976:6, so this determines the beginning of the U.S. sample. For Canada, data on all government securities with maturities greater than or equal to one year are only readily available from 1982:1, and this determines the beginning of the Canadian sample. Though medium-term Canadian government securities were indeed issued and traded long before 1982, the only data published by the Bank of Canada for the pre-1982 period are *average* rates on different-term government bonds. This reflects the fact that assets with medium-term maturities, especially 1-year instruments, were issued so infrequently that continuous time-series data on yields are not possible before that time.

Recall that the central identifying assumption used in this paper requires a choice of assets such that the default and liquidity premiums between any two assets can reasonably be expected to be zero. The choice of U.S. and Canadian treasury bills and medium-term government bonds reflects the belief that these countries are sufficiently stable that the default risks on assets of different maturities are identical. This choice of assets also reflects the belief that the markets in which these securities were traded were sufficiently active over the entire sample that the liquidity premium between any two assets is zero.

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2. For the United States the actual data series for the T-bills (from the Bank of Canada) are TB.USA.30D.CY.MID, TB.USA.60D.CY.MID, TB.USA.90D.CY.MID and TB.USA.180D.CY.MID; for the bonds (from DRI) they are RMGFCM@INS, RMGFCM@2NS, RMGFCM@3NS and RMGFCM@5NS. The Bank's series converts the U.S. T-bill rates from 360-day discount yields to 365-day true yields (that is, yields to maturity). For the Canadian T-bill rates, the series are TB.CDN.30D.MID, TB.CDN.60D.MID, TB.CDN.90D.MID, TB.CDN.180D.MID and TB.CDN.1Y.MID; and for the bonds they are B113891, B113892 and B113893. For Canada, the 1-year security is a treasury bill.

4.2 Estimates of the term premiums

Tables 1 and 2 (pp. 17 and 18) show the results for the estimation of the term premiums for the United States and Canada. Part A of each table shows the results for the estimated term premiums between 3-month treasury bills and the longer maturity assets. To get these estimates, assumption A4 was imposed for $\underline{S}$=3, so that the expected rate of inflation over the 3-month horizon was taken to be equal to the current rate of inflation. The bold numbers are the estimated term premiums expressed in basis points, the numbers in parentheses are the associated standard errors (also in basis points) and the numbers in square brackets are the mean squared forecast errors associated with each estimated term premium.

The estimates shown in Tables 1 and 2 are those generated by a maximum likelihood procedure rather than by minimizing the sum of squared forecast errors, as discussed in the previous section. The two different approaches, however, generate estimates of the term premiums within one basis point of each other. Estimation was also carried out by imposing assumption A4 for $\underline{S}$=1, 2 and 3 months. For $\underline{S}$≤4 months, the four time series for expected inflation are so close as to be nearly indistinguishable on a graph. One possible interpretation of this finding is that the policy-induced high-frequency movements at the very short end of the term structure are largely smoothed away in the monthly data. For the sake of brevity, Tables 1 and 2 report only the results for $\underline{S}$=3.

Consider first Table 1, which shows the U.S. results. There are two aspects of the results that should be highlighted. First, in Part A of the table, the estimated term premiums increase with the time to maturity of the long-term asset. For $\underline{S}$=3, the term premium required on a 6-month T-bill is 38 basis points, whereas the term premium for a 5-year bond is 201 basis points. Second, the value of the mean squared forecast error generally increases with L; this reflects the intuitive notion that it is harder to successfully forecast inflation over longer horizons than over shorter ones.

Table 1. Results for the United States, 1976:6–1994:5

Part A. Estimated term premiums with $E[\prod_t^S] = \prod_t$, $\underline{S}$=3

	L=6	*L*=12	*L*=24	*L*=36	*L*=60
$\underline{S}$=3	**38.06**	**70.02**	**112.76**	**138.34**	**201.39**
	(9.74)	**(15.67)**	**(19.34)**	**(21.85)**	**(23.53)**
	[2.100]	[5.227]	[7.411]	[8.803]	[8.677]

Part B. Implied term premiums for (*S*, *L*), *S*>3

S=6	***T*=32**	***T*=75**	***T*=100**	***T*=163**
	S=12	***T*=43**	***T*=68**	***T*=131**
		S=24	***T*=25**	***T*=88**
			S=36	***T*=63**

Notes:

- Bold numbers are estimated term premiums in basis points.
- Numbers in parentheses are standard errors.
- Numbers in square brackets are mean squared forecast errors.

The estimates in Part A of Table 1 are then used to generate the structure of term premiums in Part B. Given the recursive method of substituting successive versions of equation (4) discussed in the previous section, the term premiums in Part B satisfy the condition that the term premium for any (*S, L*) pair is equal to the difference between the term premiums for ($\underline{S}$, *S*) and ($\underline{S}$, *L*). For example, the value of $\hat{T}^{12,24}$ in Part B of Table 1 is given by the difference between $\hat{T}^{3,12}$ and $\hat{T}^{3,24}$ from Part A. Note, however, that this does not impose the stronger condition that the term premium between any (*S, L*) pair depends only on the difference between *S* and *L*. An example occurs in Part B; the term premium between 1-year and 2-year bonds is 43 basis points, whereas the premium between 2-year and 3-year bonds is only 25 basis points.

Table 2. Results for Canada, 1982:1–1994:5

Part A. Estimated term premiums with $E[\Pi_t^S] = \Pi_t$, $\underline{S}$=3

	L=6	*L*=12	*L*=24	*L*=36	*L*=60
$\underline{S}$=3	**63.32** **(10.66)** [1.572]	**118.24** **(17.25)** [3.867]	**127.00** **(20.19)** [4.818]	**125.65** **(21.45)** [4.909]	**171.59** **(23.83)** [4.755]

Part B. Implied term premiums for (*S*, *L*), *S*>3

S=6	***T*=55**	***T*=64**	***T*=63**	***T*=108**
	S=12	***T*=9**	***T*=8**	***T*=53**
		S=24	***T*=-1**	***T*=44**
			S=36	***T*=45**

***Notes*:**
- Bold numbers are estimated term premiums in basis points.
- Numbers in parentheses are standard errors.
- Numbers in square brackets are mean squared forecast errors.

Table 2 shows the results for Canada. For the most part, the Canadian results are as sensible as the U.S. results, but there are some notable differences. First, while the estimated term premiums in Part A of Table 2 do tend to increase with *L*, 2-year and 3-year government bonds appear to be very close substitutes in the sense that the estimated term premiums for these assets are within one basis point of each other. A second point is that the estimated structure of term premiums is different in Canada than in the United States. For assets with terms of up to 24 months, the estimated term premiums are higher in Canada; but for 3- and 5-year bonds, the estimated term premiums are lower in Canada. As in the United States, the mean squared forecast error (MSE) associated with each estimated term premium tends to rise with L, but for L=60 the MSE is actually less than for L=36. This suggests that agents' inflation forecasts improve once the horizon lengthens beyond three years.

Finally, note that for each value of L, the MSE in Canada is considerably less than the value in the United States, suggesting a better ability to forecast inflation in Canada than in the United States. One possible explanation for this comes from the different sample periods. The Canadian sample, beginning in 1982, is a period of significant disinflation into the early 1990s; the U.S. sample, in contrast, contains the same disinflationary period but also the large run-up in inflation in the late 1970s. To the extent that the disinflation was for whatever reason largely anticipated, it is not surprising that the Canadian forecasts from 1982 to 1994 might outperform the U.S. forecasts over the longer and more volatile sample period.

5 Estimates of expected inflation

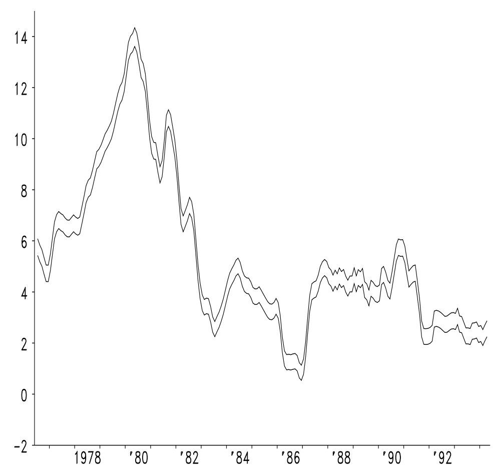
5.1 Sensitivity to estimated term premiums

The estimated term premium in each cell in Part A of Tables 1 and 2 corresponds to a time-series of estimated expected inflation. From equation (4), it is clear that the estimate of expected inflation over any L horizon has the potential of being very sensitive to the estimated term premium, $\hat{T}^{S,L}$. Furthermore, one strong identifying assumption used to generate the estimates of expected inflation is that such term premiums are constant for any (S, L) pair. It is only natural, therefore, to examine the sensitivity of the estimates of expected inflation to changes in the value of the estimated term premiums.

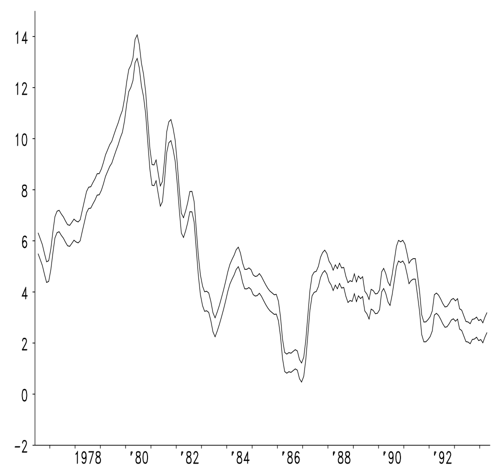
In Figures 1 and 2 (pp. 21-22) I show, for the United States and Canada respectively, the estimated inflation forecasts over the 1- 2- 3-, and 5-year horizons. But rather than graph the actual inflation forecasts implied by the estimates in Part A of Tables 1 and 2, I plot for each forecast horizon the 95 per cent confidence interval associated with the point estimate of the relevant term premium from Tables 1 and 2. For example, in the case of the 1-year inflation forecast in the United States, the point estimate of the term premium is 70 basis points, and two standard errors is approximately 31 basis points. I therefore construct inflation forecasts associated with a term premium equal to 39 basis points (the upper-bound inflation forecast) and with a term premium equal to 101 basis points (the lower-bound inflation forecast).

Figure 1 Confidence interval of inflation expectations, United States, 1976–94

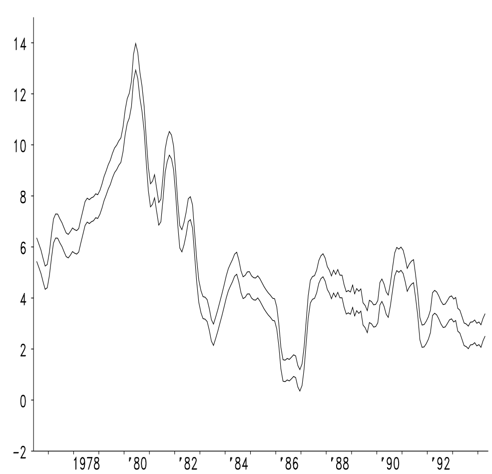
A. 1-year inflation expectations
(average annual rate)
(3- to 12-month term premium = 70 +/- 31 basis points)



B. 2-year inflation expectations
(average annual rate)
(3- to 24-month term premium = 113 +/- 38 basis points)



C. 3-year inflation expectations
(average annual rate)
(3- to 36-month term premium = 138 +/- 44 basis points)



D. 5-year inflation expectations
(average annual rate)
(3- to 60-month term premium = 201 +/- 47 basis points)

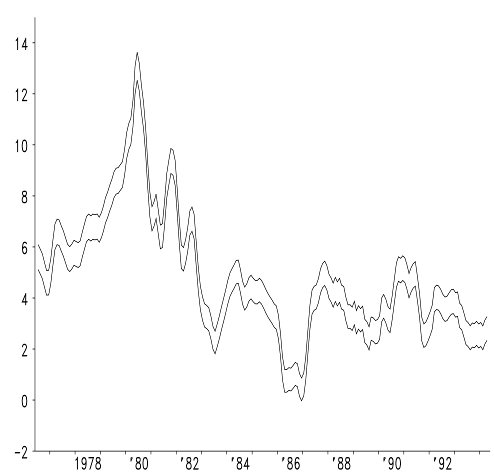
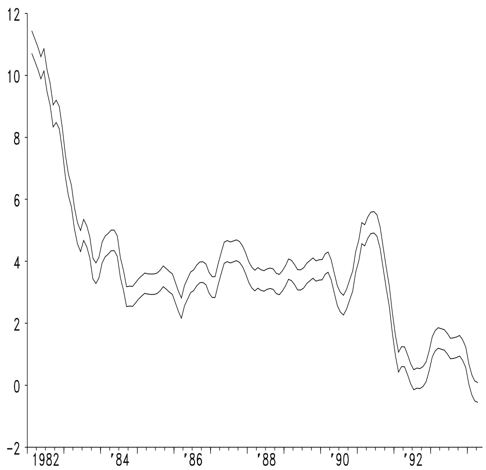
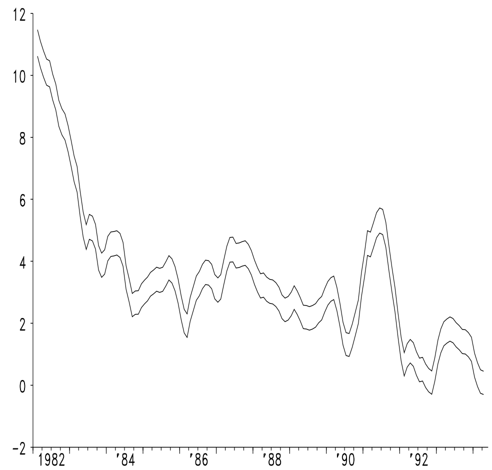


Figure 2 Confidence interval of inflation expectations, Canada, 1982–94

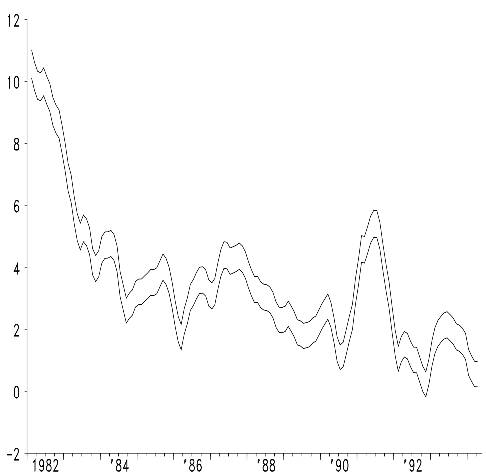
A. 1-year inflation expectations
(average annual rate)
(3- to 12-month term premium = 118 +/- 34 basis points)



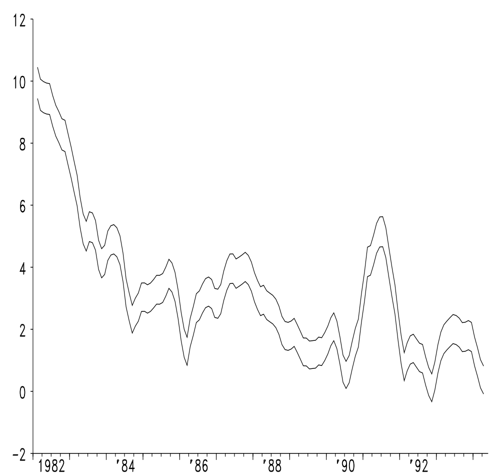
B. 2-year inflation expectations
(average annual rate)
(3- to 24-month term premium = 127 +/- 40 basis points)



C. 3-year inflation expectations
(average annual rate)
(3- to 36-month term premium = 126 +/- 43 basis points)



D. 5-year inflation expectations
(average annual rate)
(3- to 60-month term premium = 171 +/- 48 basis points)



Figures 1 and 2 make it quite clear that the estimates of expected inflation over these medium-term forecast horizons are relatively insensitive to the value of the term premium. Even over the 5-year forecast horizon, when the range of possible term premiums is almost 100 basis points, the range of inflation forecasts is relatively small. This insensitivity of the measure of expected inflation to changes in the estimated term premium is not surprising when one considers the form of equation (4), from which it is clear that economically significant changes in $T^{S,L}$ will lead to only small changes in the denominator and thus to only small changes in the estimate of expected inflation.

One interpretation of Figures 1 and 2 is that assumption A3 – the time-invariant term premium – is not a particularly strong assumption in the sense that even significant changes in the term premium do not lead to large changes in the estimate of expected inflation. Consider the 5-year U.S. inflation forecast. If the actual term premium between 3-month and 5-year securities is varying over time, but always between 154 and 248 basis points, then agents' 5-year inflation forecasts will always lie between the two lines shown in Figure 1D. Such a range in the term premium represents enormous economic variation in that variable, but the variation in the 5-year inflation forecast is quite small.

A second notable feature of Figures 1 and 2 is that there is clearly a high correlation across the inflation forecasts over different horizons. This is discussed more fully below.

5.2 How static are agents' expectations of inflation?

Figures 3 and 4 (pp. 25–26) plot the estimates of the inflation forecasts for the United States and Canada, respectively. In each case, the inflation forecast shown is the one associated with the point estimate of the term premium from Tables 1 and 2. Each figure also shows the current 12-month inflation rate.

Note that the difference at any time between the current inflation rate and the measure of expected inflation is not the forecast error discussed in the previous section; rather, the plotting of current inflation and expected future inflation provides some indication of the degree to which expectations of future inflation are influenced by the current inflation rate. In other words, a closeness of the two series in Figures 3 and 4 suggests some element of "static" expectations.

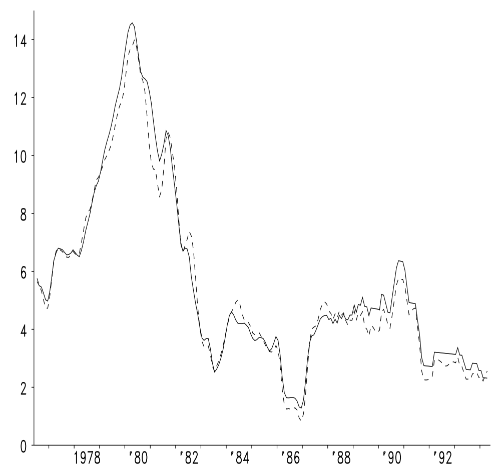
Consider first the U.S. results. Figure 3 shows the medium-term inflation forecasts to be very closely related to the current rate of inflation. For the 1-year inflation forecasts, there are no significant differences over the entire sample period between the forecasts and the current inflation rate; this may simply reflect the belief that inflation is unable to change quickly, and so the current rate is a reasonable indication of what to expect over the next year. As the forecast horizon increases to 3 and 5 years, however, one notable difference emerges. For essentially the entire 1978–82 period, when inflation rises from 6 per cent to over 14 per cent and then falls again to about 9 per cent, future inflation is expected to be less than current inflation. In the disinflation dating from 1982, however, the inflation forecasts are much closer to current inflation, and for the remainder of the sample period the only significant departure of the forecasts from current inflation occurs in 1989–90. This is especially apparent in the 5-year forecasts in Figure 3D.

The Canadian results are shown in Figure 4 (p. 26). The sample period begins in 1982 and thus represents a period of significant disinflation. In general, the Canadian inflation forecasts are less closely related to current inflation than are the American forecasts, but the pattern is broadly similar to that in the United States. First, there is still a high correlation between expectations and current inflation. Second, expectations of future inflation are less than current inflation during the early 1980s. Third, expectations diverge from current inflation in the late 1980s and very early 1990s.

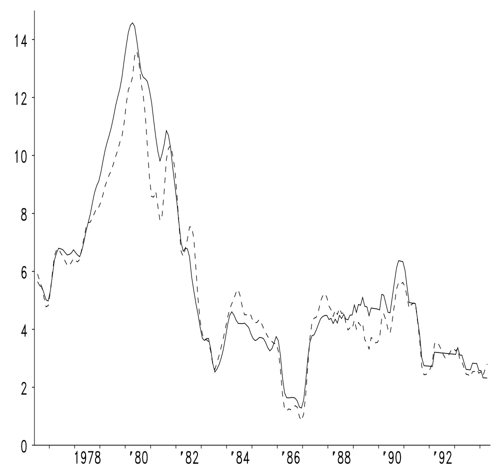
Figure 3 Current and expected inflation, United States, 1976–94

legend: ———————— current inflation
------------------ inflation forecast

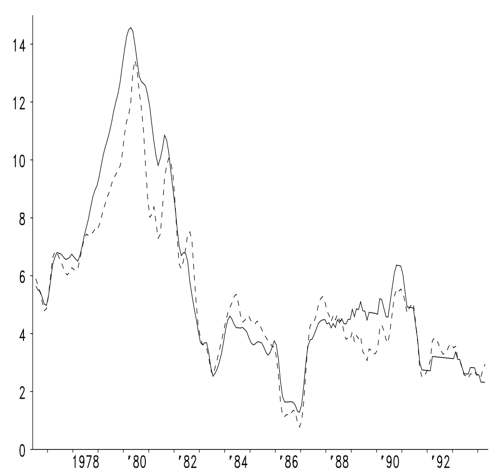
A. Current inflation and 1-year inflation forecast
(average annual rate)



B. Current inflation and 2-year inflation forecast
(average annual rate)



C. Current inflation and 3-year inflation forecast
(average annual rate)



D. Current inflation and 5-year inflation forecast
(average annual rate)

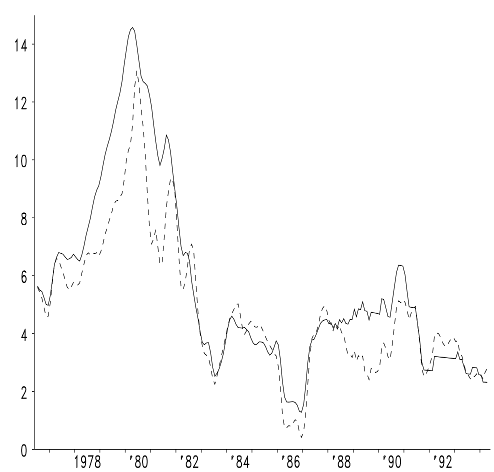


Figure 4 Current and expected inflation, Canada, 1982–94

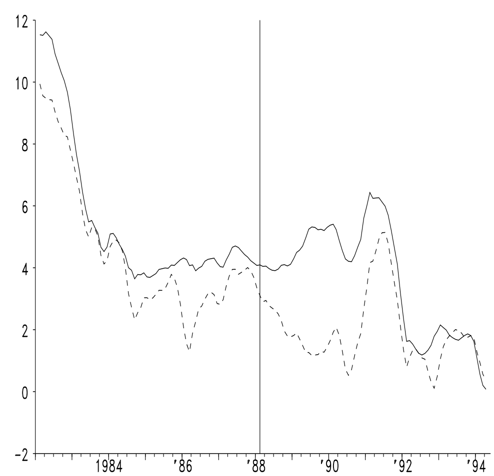
legend: ———————— current inflation
------------------ inflation forecast

A. Current inflation and 1-year inflation forecast
(average annual rate)

B. Current inflation and 2-year inflation forecast
(average annual rate)

C. Current inflation and 3-year inflation forecast
(average annual rate)

D. Current inflation and 5-year inflation forecast
(average annual rate)



Overall, it is clear that the estimates of expected future inflation are very closely related to the current inflation rates. To the extent that the procedure in this paper can produce estimates of expected inflation that accurately portray the actual expectations held by agents in the economy, one interpretation of Figures 3 and 4 is that there is a significant degree to which expectations of future inflation are static. Moreover, if expectations have a significant static component, for whatever reason, then the similarity across horizons of the inflation forecasts shown in Figures 1 and 2 is hardly surprising.

The appearance of static expectations does not necessarily imply that agents are in any sense naïve or irrational. The close relationship between expected future inflation and current inflation may simply reflect the agents' beliefs about the ability or the credibility of the central bank. For example, in a model that emphasizes the difficulties faced by the monetary authority in establishing a reputation for inflation intolerance, Backus and Driffill (1985) show that the public will come to believe that the central bank dislikes inflation only if actual inflation has been low for some considerable amount of time. The longer the history of low inflation, the more the public believes that the central bank dislikes inflation. Thus, one interpretation of Figures 3 and 4 is that policy announcements of disinflation – made at a time of moderate inflation – are rarely fully credible and thus will not have a large effect on expectations. This absence of credibility need not reflect some perception of malevolence of the central bank, as in Barro and Gordon (1983); it may simply reflect the belief by agents that the central bank lacks the *ability* to follow the announced policy. Under this interpretation, the only way that agents will come to expect low inflation in the future is if they have been exposed to low inflation in the recent past.

The Canadian data show one significant episode that may be consistent with the possibility that central bankers' pronouncements to reduce inflation can be credible, even though inflation has not fallen for several years. By 1984, the annual rate of inflation was down to about 4 per cent in Canada, and it remained more or less constant for four years. In late 1987, John Crow became the new governor of the Bank of Canada. In January 1988, in his much-cited Hanson Lecture, Governor Crow outlined the Bank's objective of price stability (Crow 1988). This event is marked in Figure 4 with the vertical line. As is clear in Figure 4, the Hanson Lecture coincides very closely with the beginning of a very significant three-year decline in expectations of future inflation, even though actual inflation rises over this period. This decline in inflationary expectations is apparent for all four forecast horizons shown but is especially clear in the 3-year and 5-year expectations. Note that inflation expectations only rise again, temporarily, for the imposition of the Goods and Services Tax (GST) in January 1991, a policy that was expected to temporarily increase the measured rate of inflation by up to 2 percentage points. By 1992, inflation expectations were once again falling. One clear interpretation of this episode, with the danger of ascribing too much influence to the stated intentions of the Bank of Canada, is that agents believed the Bank's determination to reduce inflation and they lowered their inflation forecasts in *anticipation* of a successful policy. In this case, it was not necessary for the Bank of Canada to produce inflation rates close to zero in order for market participants to expect near-zero inflation to emerge over the medium term.

6 Final remarks

This paper has used the term structure of nominal interest rates to construct estimates of agents' expectations of inflation over a medium-term horizon. This paper *does not* test the Expectations Hypothesis of the term structure; nor does it test the hypothesis that the term structure contains useful information for predicting the future path of inflation or real activity. In contrast, the estimation approach begins by *imposing* the Expectations Hypothesis together with the assumption that expected future real interest rates are given by current real rates. It is then possible to compare the nominal yields on two assets of different maturities and attribute the difference in nominal yields to differences in expected inflation over the two horizons (assuming a constant term premium). The basic approach is therefore capable of generating an estimate of expected inflation over any horizon for which interest rate data are available. This differs from Frankel's (1982) approach in which he imposes the different assumption that real interest rates converge to a constant in the long run but is then only able to generate an estimate of long-run (steady-state) inflation.

The method is used for the United States from 1976 and for Canada from 1982. For both samples, the estimates suggest that agents' expectations of future inflation are heavily influenced by the current inflation rate, which points to a strong element of static expectations. One interpretation of this finding is that agents are naïve in the manner with which they generate their inflation forecasts. Another interpretation is that the central bank's credibility is closely linked to its past performance, so that agents come to expect low inflation only when inflation has recently been low.

References

Backus, D. and J. Driffill. 1985. "Inflation and Reputation." *American Economic Review* 75:530-38.

Barro, R. 1977. "Unanticipated Money Growth and Unemployment in the United States." *American Economic Review* 67:101-15.

Barro, R. and D. Gordon. 1983. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy* 91:589-610.

Blough, S. 1994. "Yield Curve Forecasts of Inflation: A Cautionary Tale." *New England Economic Review* (May/June): 3-15.

Crow, J. 1988. "The Work of Canadian Monetary Policy." *Bank of Canada Review* (February): 3-17.

Deacon, M. and A. Derry. 1994. "Estimating Market Interest Rate and Inflation Expectations from the Prices of UK Government Bonds." *Bank of England Quarterly Bulletin* 34:232-40.

Fama, E. 1975. "Short-Term Interest Rates as Predictors of Inflation." *American Economic Review* 65:269-82.

————————. 1984. "The Information in the Term Structure." *Journal of Financial Economics* 13:500-28.

————————. 1990. "Term-Structure Forecasts of Interest Rates, Inflation, and Real Returns." *Journal of Monetary Economics* 25:59-76.

Frankel, J. 1982. "A Technique for Extracting a Measure of Expected Inflation from the Interest Rate Term Structure." *Review of Economics and Statistics* 64:135-42.

Frankel, J. and C. Lown. 1994. "An Indicator of Future Inflation Extracted from the Steepness of the Interest Rate Yield Curve Along its Entire Length." *Quarterly Journal of Economics* 109:517-30.

Hamilton, J. 1985. "Uncovering Financial Market Expectations of Inflation." *Journal of Political Economy* 93:1224-41.

Mishkin, F. 1988. "The Information in the Term Structure: Some Further Results." *Journal of Applied Econometrics* 3:307-13.

————————. 1990a. "The Information in the Longer Maturity Term Structure About Future Inflation." *Quarterly Journal of Economics* 105:815-28.

————————. 1990b. "What Does the Term Structure Tell Us About Future Inflation?" *Journal of Monetary Economics* 25:77-95.

Patinkin, D. 1993. "Israel's Stabilization Program of 1985, or Some Simple Truths of Monetary Theory." *Journal of Economic Perspectives* 7:103-28.

Pesando, J. 1978. "On the Efficiency of the Bond Market: Some Canadian Evidence." *Journal of Political Economy* 86:1057-76.

Shiller, R. 1990. "The Term Structure of Interest Rates." In *Handbook of Monetary Economics*, edited by B. Friedman and F. Hahn. Amsterdam: North-Holland.

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