Liquidity, Asset Price, and Welfare

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Introduction

- Determinants of liquidity?
  - Need for liquidity (non-synchronization in trading).
  - Supply of liquidity.
- Importance of liquidity?
  - Asset prices.
  - Market stability.
  - Welfare.

Intuition

1. Two elements essential to liquidity: need to trade and cost to trade.

2. Costs affect liquidity provision and prices, taken liquidity needs as given.

3. But the same costs give rise to liquidity needs in the first place.

4. Without participation costs, there is no need for liquidity.
   - Trading needs come from idiosyncratic shocks, which sum to zero.
   - Trades are synchronized and do not move prices.
   - The market is "perfectly liquid" (only fundamentals move prices).

5. With participation costs, there is need for liquidity.
   - Not all traders are present in the market.
   - Traders with offsetting trading needs have different trading gains.
   - Trades are non-synchronized, which leads to need for liquidity.

Setup (1)

1. **Assets:** A riskless bond and a risky stock.
   - Bond pays constant (positive) interest rate r.
   - Stock pays dividend D_{t+1}, with mean $\bar{D}$ and volatility σ_D.
2. **Agents:** Homogenous preference, information, but heterogenous risks.
   - Same initial asset holdings: $\bar{\theta}$ shares of stock.
   - Different non-traded payoff N^i_{t+1} for agent i:

     $$N^i_{t+1} = (Y_t + \lambda^i X_t)\, n_{t+1}, \quad \lambda^i = 1 \text{ or } -1 \text{ with equal probabilities.}$$

   - Y_t gives aggregate non-traded risk.
   - Idiosyncratic risks $\lambda^i X_t$ sum to zero:

     $$\int_i N^i_{t+1} = Y_t\, n_{t+1}.$$

   - Denote agents with $\lambda^i X_t > 0$ and < 0 as a and b, respectively.

Setup (2)

3. Costs of Participation:

- Cost to be a market maker c_m (paid ex ante).
- Cost for spot participation c (paid before trading).

4. Simplifications for tractability and easy exposition:

- Constant absolute risk aversion α.
- Normal shocks $(Y_t, X_t, D_{t+1}, n_{t+1},\ t = 0, 1, \ldots)$.
- Stock and non-traded payoffs correlated.
- $Y_t = 0$ for simplicity.

Definition of Equilibrium

1. **Agents optimize** over
   - Participation decisions: $\eta_m^i = 0, 1$ and $\eta_t^i = 0, 1$.
     - Be a market maker ($\eta_m = 1$) and trade at all times
     - Be a trader and pay a cost to trade ($\eta = 1$) when needed.
   - Trading decisions: Stock holding $\theta_t^i(\eta_m^i, \eta_{t-1}^i)$.

Shocks		X_t	X_{t+1}	
		t	$t+1$	time
Choices	η_m	η_t^i	$\theta_{t+1}^i(\eta_m, \eta_t^i)$	

2. **Participation reaches equilibrium.**
   - A fraction μ of agents become market makers.
   - Among traders, fraction ω_t^i enter the market, $i = a, b$.

3. **Stock market clears among participating agents.**

Zero Participation Costs

- All agents are in the market at all times, $\mu = 1$ and/or $\omega_t^a = \omega_t^b = 1$.

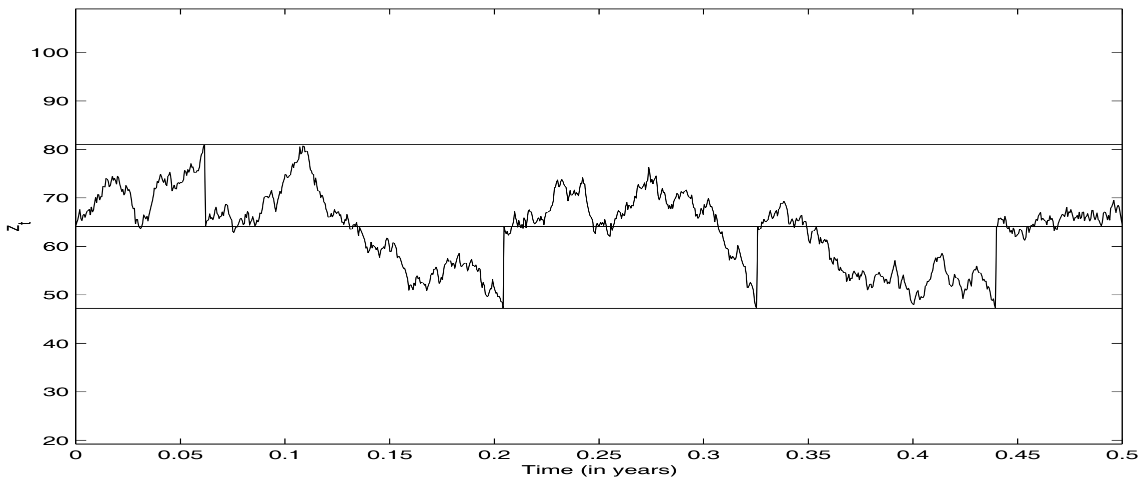
- The equilibrium price and agents' stock holdings are:

$$
\begin{aligned}
P_t &= \frac{\bar{D}}{r} - \alpha\sigma_D^2\bar{\theta} \\
\theta_t^i &= \bar{\theta} - \lambda^i X_t
\end{aligned}
$$

  - Agents with $\lambda^i X_t > 0$ are sellers (a) and $\lambda^i X_t < 0$ are buyers (b).
  - Trading needs are perfectly matched $(\lambda^a X_t = -\lambda^b X_t)$.
  - Trades are synchronized and there is no need for liquidity.
  - Prices depend only on "fundamentals" $(\bar{D}$ and $\bar{\theta})$, independent of individual trading needs (X_t).

Optimal Trading Policy Under Costly Participation

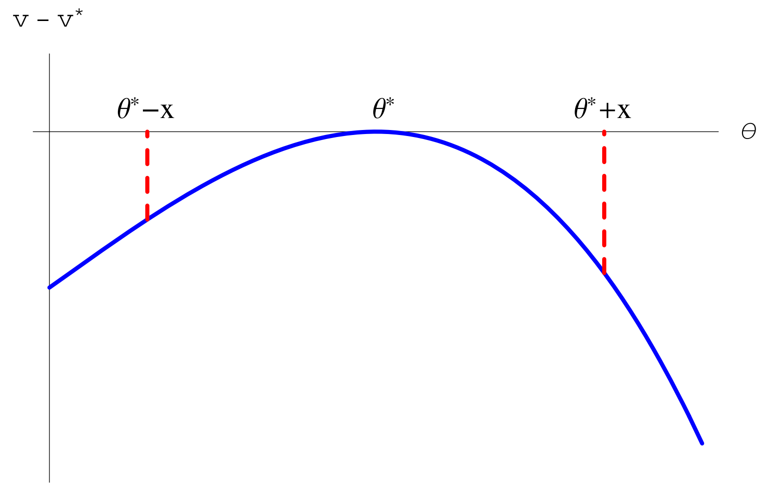
Trading becomes infrequent. A trader's net risk exposure is $\theta_t + \lambda^i X_t \equiv z_t$.



- Desirable exposure $\bar{z}$. (Without cost, $\bar{z} = \bar{\theta}$.)
- Trade occurs only when net risk exposure exceeds certain limits.
- Upper and lower limit, $\bar{z} + \delta_a$ and $\bar{z} - \delta_b$, respectively.
- In general, $\delta_a \neq \delta_b$.

Asymmetric Trading Gains

Let $v(\theta) \equiv \mathrm{E}[u(\theta, \cdot)]$ and θ^* be the optimal holding, i.e., $v'(\theta^*) = 0$.



- For small deviations from optimum, trading gains are symmetric:
$$v(\theta^*) - v(\theta^*+x) \simeq -\frac{1}{2}v''(\theta^*)(x)^2 = -\frac{1}{2}v''(\theta^*)(-x)^2 \simeq v(\theta^*) - v(\theta^*-x).$$
- With costs, traders trade only when they are far away from the optimum.
- Trading gains differ between traders with offsetting trading needs.

Sellers Expect Larger Trading Gains Than Buyers

- With trading needs only are partially met, risk sharing is not perfect.
- Having to bear idiosyncratic risks, traders become more risk averse.
  - True for "standard risk aversion" (DARA and DAP) (Kimball, 1993).
- Traders' stock demand decreases after new idiosyncratic shocks.
- Sellers become further away from desired holdings than buyers.
- Sellers enter market before buyers!

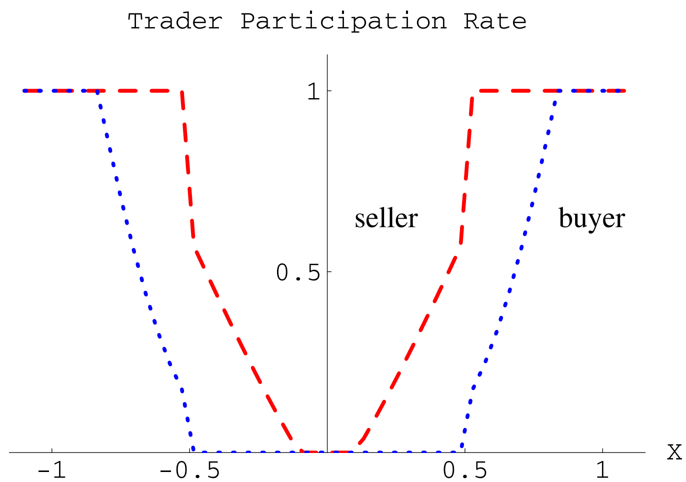
Non-synchronized Trading and Order Imbalances

- In equilibrium, sellers participate more than buyers: $\omega_t^a \geq \omega_t^b$.
- Order imbalance is usually negative:

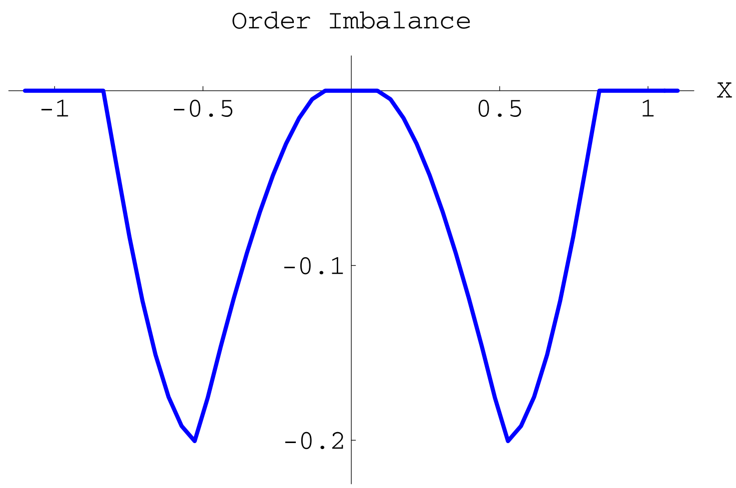
$$\Delta_{t+1} \equiv -\tfrac{1}{2}(1-\mu)(\omega_t^a-\omega_t^b)\lambda^a X_{t+1}, \quad \mathrm{E}[\Delta_{t+1}|X_t] \leq 0.$$

- Liquidity needs, when arise, are sell orders of large sizes.

(a) Equilibrium Participation Rate ω^a, ω^b



(b) Order Imbalance



Parameter values: $\sigma_D = 0.25$, $\bar{\theta} = 1$, $\sigma_X = 0.6$, $\sigma_{nD} = 0.0625$, $\alpha = 4$, $c = 0.1$, $\mu = \frac{1}{3}$.

Stock Price

The equilibrium stock price is

$$P_t = \frac{\bar{D}}{r} - \alpha\sigma_D^2\bar{\theta} - d + \tilde{p}_t$$

- Fundamental value $\bar{D}/r - \alpha\sigma_D^2\bar{\theta} \equiv \bar{P}$.
- Illiquidity discount d.
- Impact of order imbalance (liquidity need) $\tilde{p}_t$.
- $\tilde{p}_t$ consists of two components: $\tilde{p}_t = \widehat{p}_t + u_t$:
  - $\widehat{p}_t$ depends on expected future order imbalance $\mathrm{E}[\Delta_{t+1}|X_t] \leq 0$
  - u_t depends on unexpected current order imbalance.
- Liquidity need, when arises, influences stock price negatively, $\widehat{p} \leq 0$.

Liquidity and Asset Prices

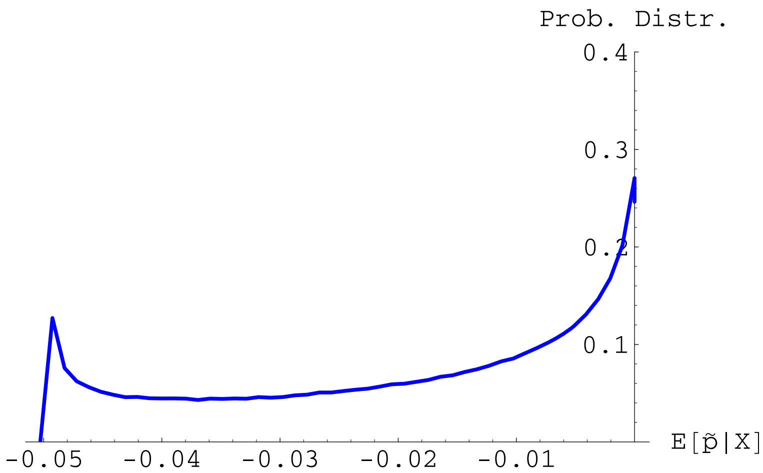
Calibrating the Impact of Illiquidity on Stock Prices

α	0.001	0.010	0.100	0.500	1.000	1.500	2.000	5.000
σ_X	10776.2	1077.62	107.762	21.552	10.776	7.184	5.388	2.155
$c/\bar{P}$ (%)	Cost as % of Average Trade Amount							
0.100	0.000	0.000	0.003	0.009	0.015	0.020	0.024	0.049
1.000	0.000	0.003	0.015	0.049	0.082	0.111	0.137	0.273
5.000	0.002	0.009	0.049	0.162	0.273	0.369	0.457	0.902
$c/\bar{P}$ (%)	Annual Turnover (%)							
0.100	8374.52	4708.68	2647.65	1770.29	1488.44	1344.83	1251.39	994.81
1.000	4708.68	2647.65	1488.44	994.81	836.14	755.25	702.59	557.79
5.000	3149.32	1770.29	994.81	664.26	557.79	503.38	467.87	369.24
$c/\bar{P}$ (%)	Illiquidity Discount (% of $\bar{P}$)							
0.100	0.054	0.172	0.546	1.233	1.756	2.161	2.507	4.042
1.000	0.172	0.546	1.756	4.042	5.847	7.287	8.542	14.443
5.000	0.575	1.233	4.042	9.678	14.443	18.462	22.123	41.509
$c/\bar{P}$ (%)	Return Premium (%)							
0.100	0.003	0.011	0.035	0.080	0.114	0.141	0.164	0.269
1.000	0.011	0.035	0.114	0.269	0.396	0.501	0.596	1.077
5.000	0.037	0.080	0.269	0.684	1.077	1.444	1.812	4.527

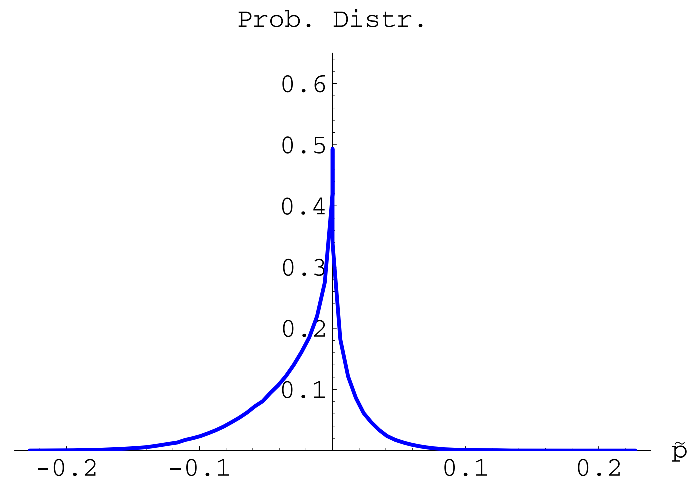
(Parameters: $\bar{D} = 0.050$, $r = 0.037$, $\bar{P} = 0.784$, $\sigma_{nD} = 0.0625$, $\alpha\sigma_Y = 1.347$, $\sigma_X = 8\,\sigma_Y$.)

"Liquidity Crashes"

(a) Distribution of $\widehat{p}_t$



(b) Unconditional distribution of $\tilde{p}_t$



Parameters values: $\sigma_D = 0.25$, $\bar{\theta} = 1$, $\sigma_{nD} = 0.0625$, $\sigma_X = 0.5$, $\sigma_Y = 0$, $\alpha = 4$, $c = 0.1$, $\mu = \frac{1}{3}$.

The liquidity impact on prices has the following properties:

- Usually negative
- Large (of finite sizes), when occurs
- Leading to "fat-tails" and negative skewness in returns.

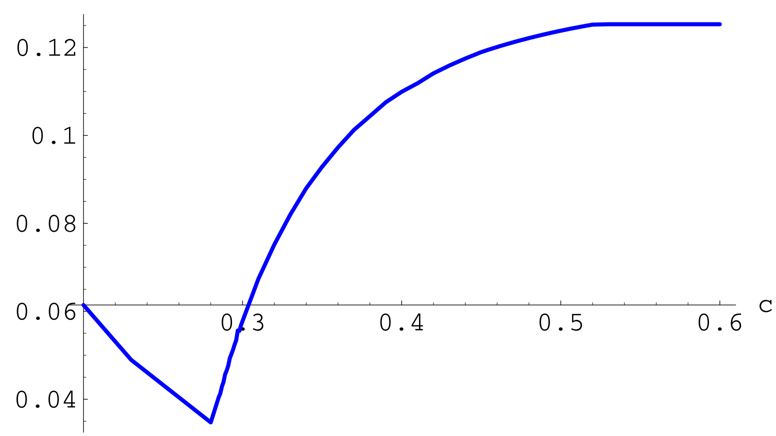
Welfare

- Trading enhances liquidity and generates positive externality.
- Market mechanism may fail to achieve efficient liquidity provision.

Use Certainty Equivalence (CE) as a welfare measure.

1. Decreasing cost of spot participation can decrease welfare.

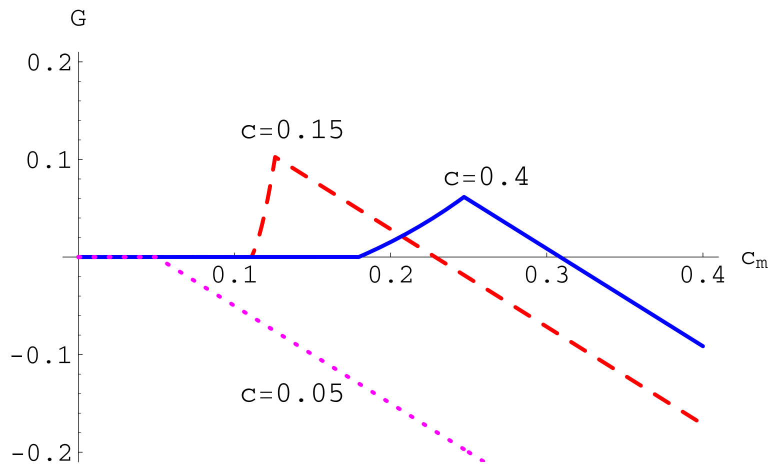
Welfare and Cost of Spot Participation c.



Parameters values: $\sigma_D = 0.25$, $\bar{\theta} = 0$, $\sigma_{nD} = 0.0625$, $\sigma_X = \sigma_Y = 0.6$, $\alpha = 4$, $c_m = 0.15$.

2. Liquidity provision by market can be suboptimal.

Welfare Gain Under Forced Participation $G = CE_{FP} - CE$.



Parameters values: $\sigma_D = 0.25$, $\bar{\theta} = 0$, $\sigma_{nD} = 0.0625$, $\sigma_X = \sigma_Y = 0.6$, $\alpha = 4$.

Conclusion

- Market frictions lead to endogenous liquidity needs.
- Liquidity affects prices.
  - "Liquidity crashes" without fundamental shocks.
  - "Fat-tails" and skewness in returns.
- Trading generates positive externality.
- Market forces may fail to lead to efficient liquidity provision.

- Origins of participation costs? Magnitudes?
- Policy implications?