Second draft

Interpreting Euro Area Inflation at High and Low Frequencies

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Abstract

Several authors have recently interpreted the ECB's two-pillar framework as separate approaches to forecast and analyse inflation at different time horizons or frequency bands. The ECB has publicly supported this understanding of the framework. This paper presents further evidence on the behaviour of euro area inflation using band spectrum regressions, which allow for a natural definition of the short and long run in terms of specific frequency bands, and causality tests in the frequency domain. The main finding is that variations in inflation are well explained by low-frequency movements of money and real income growth and high-frequency fluctuations of the output gap.

Keywords: spectral regression, frequency domain, quantity theory, inflation, money growth

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1. Introduction

On October 13, 1998, the European Central Bank (ECB) announced that its monetary policy strategy would combine a *"prominent role for money with a reference value for the growth of a monetary aggregate"*, later defined to be 4.5% annual growth of M3, and *"a broadly-based assessment of the outlook for future price developments"*.[1] Interpreted by many observers as combining monetary and inflation targeting, the framework quickly became controversial.[2] In particular, it was not clear why the ECB deemed it necessary or even helpful to use "two pillars" – one incorporating "monetary analysis" and the other "economic analysis" – in assessing inflation developments and in setting interest rates. This did not necessarily indicate hostility to the reliance on money growth as an information variable for monetary policy purposes, but rather reflected the view that the determinants of inflation, whatever they are, should presumably be included in a single, composite analysis of price developments, as is the practice in central banks operating with an inflation-targeting strategy.

Recently, several authors have sought to formalise the ECB's policy strategy and to rationalise the two pillars by incorporating money growth in empirical Phillips curve models for inflation in the euro area. Gerlach (2003, 2004) interprets the two pillars as separate approaches to forecast inflation at different time horizons or frequency bands. Under this view, the monetary pillar is seen as a way to predict inflation at long time horizons and to account for gradual changes in the steady-state rate of inflation over time. Empirically, the monetary pillar is captured by a geometrically declining, one-sided moving average of M3 growth computed using the simple exponential filter employed by Cogley (2002) to study core inflation. Importantly, Gerlach (2004) finds that filtered money growth contains information useful for forecasting future prices that is not already embedded in a similarly filtered measure of inflation. Thus, including money growth in the inflation analysis adds to policy makers' information set.[3]

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1 See the ECB's press releases of October 13, 1998 and December 1, 1998, which are available at www.ecb.int.

2 See, for instance, the annual CEPR reports on *Monitoring the ECB* (which are available at www.cepr.org) or Svensson (1999 and 2002).

3 Stock and Watson (1999) find that for the US including money growth in a Phillips curve model does not improve inflation forecasts. Estrella and Mishkin (1997) examine the information content of monetary aggregates in the US and Germany and conclude that velocity is in both cases too unpredictable for money growth to serve as a guide to monetary policy. Interestingly, the authors use frequency domain techniques, as we do below.

Furthermore, the non-monetary pillar, the economic analysis, is understood as the ECB's method to predict short-run variations in inflation around the steady-state level. In the analysis the output gap is identified as the main factor explaining these temporary swings in inflation, but it is recognised that other factors – including oil prices, exchange rates, unit labour costs and tax changes – also play a critical role in the short run.

Neumann (2003) and Neumann and Greiber (2004) present a closely related model, but sharpen the analysis in several ways. In particular, they explicitly incorporate the role of real income growth in determining the trend, or "core", rate of money growth. This is important since it allows for changes in the growth rate of potential to impact on inflation. Furthermore, they use a number of filters to calculate the growth rates of potential and core money growth and investigate what frequency band of money growth has the closest correlation with inflation. The authors find that money growth and output gaps are significant in empirical inflation equations for the euro area, but that the exact choice of filter is of less importance (although the exponential filter used by Gerlach seems to perform less well than the alternatives considered). One interesting finding is that fluctuations in money growth of a periodicity of less than 8 years appear not to matter for inflation.

The importance of low-frequency variation in money growth for inflation in the euro area is also studied by Bruggeman et al. (2005), who employ frequency domain techniques and consider a number of different filters. They find that longer-term movements of money growth are strongly correlated with inflation, and that the output gap seems to be more important for short-term inflation dynamics. Jaeger (2003) also uses spectral analysis to study the comovements of money and inflation in Europe and notes that these are limited at high frequencies.[4]

One way to think of the papers by Gerlach (2003, 2004), Neumann (2003) and Neumann and Greiber (2004) is that they essentially augment a standard reduced-form, empirical Phillips curve with a measure of the low-frequency component of money growth which is obtained by filtering money growth in a preliminary step.[5] Sustained changes in money growth therefore shift the Phillips curve vertically, generating changes in the average rate of inflation. By

[4] Haug and Dewald (2004) use frequency domain techniques to study the relationship between money growth and inflation in a sample of eleven countries, using data spanning 1880-2001. Thoma (1994) studies the effect of money growth on inflation and interest rates across frequency bands for the United States.

[5] Gerlach (2004) estimates the long-run trend of money growth jointly with the parameters in the inflation equation.

contrast, movements in the output gap, which by construction are temporary, generate variations in inflation around that average. Interestingly, the ECB in its recent review of the monetary policy strategy attaches a very similar role to money growth in the inflation process. For instance, in an article in the June 2003 Monthly Bulletin on the outcome of its evaluation of the strategy, the ECB (2003, p. 87) writes:

> *"An important argument in favour of adopting the two-pillar approach relates to the difference in the time perspective for analysing price developments. The inflation process can be broadly decomposed into two components, one associated with the interplay between demand and supply factors at high frequency, and the other connected to more drawn-out and persistent trends. The latter is empirically closely associated with the medium-term trend growth of money."*

Furthermore, in commenting on recent studies on the link between money and inflation in the same article, the ECB writes (p. 90):

> *"On the basis of statistical methodologies suited to breaking down a time series into the relative contributions of components acting at different time horizons, it has been found that long-term variations in inflation are closely associated with long-term movements in money. Furthermore, it has been found that euro area inflation can be described by a Phillips-curve relationship – i.e. a relationship explaining inflation in terms of indices of economic slack – augmented by a term capturing low-frequency movements in money. This relationship has been interpreted as being indicative in that, whereas fluctuations in inflation in the euro area are driven by factors associated with the state of activity in relation to its long-term potential, the long-term average of inflation is highly correlated with money growth."*

The fact that the ECB has adopted the interpretation that the two pillars refer to the determinants of inflation at different time horizons or frequency bands suggests that further research on the inflation process in the euro area at different frequencies is well warranted.[6] In this paper we explore the hypothesis that the two pillars, the monetary and economic analysis, contain information useful for understanding inflation in the euro area at different time horizons using frequency domain methods.[7] We first use frequency domain techniques to obtain estimates of potential output and the output gap, and to deseasonalise inflation. Next,

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6 The Bundesbank (2005) also argues that low-frequency fluctuations in money growth impact on the long-term evolution of inflation, in contrast to high-frequency swings which are much less informative about price developments.

7 Interestingly, Jordan, Peytrignet and Rich (2001) describe the new monetary concept introduced by the Swiss National Bank in 2000 as relying on money as a useful indicator for long-run price developments, whereas the output gap is considered as one among other indicators of short-run inflation. Assenmacher-Wesche and Gerlach (2005) provide frequency domain estimates of inflation equations for Switzerland. It should be noted that in contrast to the data we study in this paper, the Swiss data are stationary, which facilitates the statistical analysis.

we apply the band spectrum regression approach pioneered by Engle (1974) and later extended by Phillips (1991) for non-stationary time series to estimate reduced-form inflation equations. This approach allows the filtering and estimation to be performed jointly, in contrast to the papers cited above. Finally, we investigate the patterns of (predictive) causality between inflation, money growth and the output gap at different frequencies.

Before proceeding, we highlight that we do not study the ability of New Keynesian Phillips curve (NKPC) models to explain movements in inflation in the euro area.[8] The reason for not doing so is that our primary interest is to explore how well the *ECB's view of the inflation process* fits the data. That view emphasises the role of low-frequency variations of money growth in explaining gradual changes over time in the steady-state rate inflation. In contrast, NKPC models characterise the behaviour of inflation around an assumed steady state, the determination of which is not studied explicitly. While we recognise that the NKPC has become the dominant theoretical model for analysing inflation, whether or not that model fits the data would seem to be of little relevance to our objective.[9]

The paper is organised as follows. In Section 2 we review the empirical model before we discuss the data in Section 3. Though inflation and money growth in the euro area seem to have changed their behaviour in the mid-1980s, we cannot reject that the long-run relation between both variables is stable over the entire sample period. In Section 4 we present Phillips' (1991) band spectral estimator for cointegrated time series and discuss in Section 5 estimation of inflation equations for different frequency bands. We show that there is a tight link between money and inflation at low frequencies, and that there is a similarly close relationship between inflation and the output gap at high frequencies. These results are thus compatible with the interpretation of the two-pillar framework as applying to different frequency bands. Section 6 investigates the causal relations between inflation, money growth and the output gap in the frequency domain, using the methodology proposed by Breitung and Candelon (2005). We find that money causes inflation at low frequencies whereas the output gap causes inflation at business cycle frequencies.

Section 7 contains our conclusions. Overall, the empirical findings are strikingly compatible with the notion that money growth is useful for predicting low-frequency, and the output gap

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8 See, for instance, Galí et al. (2001).

9 Jansen (2004) compares four inflation models on euro area data, including a hybrid NKPC model, and finds that those that make use of a broader information set including monetary variables do better in forecasting exercises.

the high-frequency, variations of inflation in the euro area. Moreover, at the highest frequencies, variables that capture cost-push shocks can be expected to play an important role for inflation.[10] However, more work remains to be done. While the analysis suggests that money can be used as an information variable for policy purposes, we do not address the question of whether this is best done using a two-pillar framework or by integrating the pillars in a single analysis of inflation.

2. An empirical model for inflation

As noted in the introduction, the ECB has motivated its adoption of the two-pillar strategy by arguing that the determinants of inflation vary by frequency. Under this view, the monetary analysis of the first pillar is intended to help forecast and analyse low-frequency movements of inflation, while the economic analysis in the second pillar seeks to predict and interpret short-run swings in prices. To formalise this view, we first decompose "headline" inflation, π_t, into low-frequency (*LF*) and high-frequency (*HF*) components:

(1) $$\pi_t = \pi_t^{LF} + \pi_t^{HF}.$$

Following Gerlach (2003), we hypothesise that the high-frequency movements of inflation are related to movements in the output gap, g_t:

(2) $$\pi_t^{HF} = \alpha_g g_t + \varepsilon_t^{HF}.$$

The specification of the Phillips curve in equation (2) is simple. To better explain the data, a more elaborate model that controls for cost-push shocks arising from changes in exchange rates, import and fresh food prices, value-added taxes, etc. is necessary. In the econometric work presented below, these factors are all captured by the high-frequency part of the residual, ε_t^{HF}. Before proceeding, note that, by construction, the output gap is a zero mean stationary process and is therefore unlikely to impact on the low-frequency, trend component of inflation.

Next, we assume that the low-frequency variation of inflation can be understood in terms of the quantity theory of money,[11] which after taking rates of change and rearranging we can write as:

[10] Assenmacher-Wesche and Gerlach (2006) provide evidence that such shocks to the exchange rate, oil prices and import prices help explain inflation at higher frequencies than the output gap..

[11] Lucas (1980) presents frequency domain evidence for US data in support of this proposition.

(3) $\pi_t^{LF} = \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \alpha_\nu \nu_t^{LF}$,

where μ_t, γ_t and ν_t denote the growth rate of money and real output, and the rate of change of velocity.[12] Furthermore, we assume that the change in velocity depends on the change of the long-term interest rate, ρ_t:

(4) $\nu_t = \tilde{\alpha}_\rho \rho_t + \varepsilon_t^\nu$.

Equation (3) warrants three comments. First, since velocity is defined in terms of money, output and prices, equation (3) is in fact an identity with $\alpha_\mu = -\alpha_\gamma = \alpha_\nu = 1$. It is the assumption that changes in velocity depend on changes in the long-term interest rate in equation (4) that implies testable assumptions for equation (3). Second, at low frequencies, the growth rate of real output is identical to the growth rate of potential. There are several ways to deal with this in the empirical work that follows. One is to use the actual growth rate of real output in the band spectral regressions; another is to first construct a measure of the trend growth rate of output and use this in the subsequent analysis. Since estimates of potential output are not available for the euro area, we follow the first approach and define low-frequency output growth by the spectral band considered in the respective regression, analogously to the definition of low-frequency money growth. Third, under the quantity theory, and provided that money growth is uncorrelated with velocity shocks, ε_t^ν, at low frequencies (that is, μ_t^{LF} and $\varepsilon_t^{\nu,LF}$ are orthogonal), we expect that $\alpha_\mu = -\alpha_\gamma = 1$.

The full model is given by:

(5) $\pi_t = \alpha_g g_{t-1} + \left\{ \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \alpha_\rho \rho_t^{LF} \right\} + \varepsilon_t$,

where $\varepsilon_t = \alpha_\nu \varepsilon_t^{\nu,LF} + \varepsilon_t^{HF}$ and $\alpha_\rho \equiv \alpha_\nu \tilde{\alpha}_\rho$. According to this model, the local trend of inflation during some period is given by the term in curly brackets, { }, that is, by the low-frequency part of money growth relative to real output and changes in the interest rate, which we think of as the first pillar. Variation in inflation around that local trend is determined by movements in the output gap, which is our shorthand for the second pillar.[13] Under this interpretation of the ECB's monetary policy strategy, in analysing and forecasting inflation it

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[12] Reynard (2005) shows that accounting for changes in velocity is critical for understanding the relationship between money growth and inflation in the euro area and in the US since the 1970s.

[13] While we think of the residuals as being unforecastable, it is the task of the *"economic and monetary analysis"* conducted by the ECB to seek to understand these factors in real time.

is appropriate to consider low-frequency, as opposed to “headline”, movements in money growth.

The inflation equation proposed above is entirely an empirical model and it is important to understand what it says about the monetary transmission mechanism. Let us first consider the short-run correlation between money growth and inflation. Our view is that movements in money growth reflect movements in aggregate demand, which, in turn, lead to swings in the output gap and in inflation. However, since money growth partially reflects temporary shifts in money demand and changes in the financial system that may not matter for inflation, perhaps because they are not of sufficient duration to do so, it is an empirical question whether the short-run effects of money are best measured by data on money growth or measures of the output gap, as emphasised by Nelson (2003). An additional reason for why money growth at high frequencies need not be significant in the inflation equation is that there may be other factors impacting on aggregate demand. A finding that the output gap, but not money growth, impacts on high-frequency swings in inflation does not therefore imply that money growth does not trigger short-run swings in inflation.

By contrast, the effects of money growth on inflation are likely to be clearer at low frequencies. Economic theory suggests that monetary disturbances have at most temporary effects on real variables such as the output gap. The output gap can therefore not capture the long-run effects of a shift in the money growth rate. Moreover, the output gap is by construction stationary while inflation may display a unit root, perhaps arising from an accommodative reaction of monetary policy to exogenous shocks (see, for instance, Ireland 2005). This difference in the time series properties suggests that one would not expect the two variables to be closely related in the long run. Rather, shifts in the money growth rate, which should be tied to changes in the inflation regime, are likely to be informative about changes in the average level of inflation over time.[14]

[14] Stock and Watson (2005) model inflation as the sum of a permanent stochastic trend and a transitory component. They argue that the magnitude of the variance of the permanent component has exhibited large changes since the 1950s whereas the magnitude of the transitory component has been essentially constant.

3. The data

As preliminary step to the formal econometric analysis below we consider the raw data.[15] Since the rate of inflation using the original CPI data displayed quite complicated dynamics and a seasonal factor, perhaps because they are synthetic for a large part of the sample period, we first deseasonalised the series by removing a frequency band around the seasonal peaks.[16] This obviates the need to model the seasonal dynamics in the regressions below. Figure 1 presents a plot of the quarterly rate of inflation using the seasonally adjusted data, the quarterly rate of money growth as measured by M3, the quarterly change in the government bond yield and the quarterly rate of real income growth, all for the period 1970Q2 to 2004Q4.

The figure shows that inflation accelerated in the early 1970s and remained high and volatile before declining in the early 1980s. Since the mid-1980s, inflation appears to have fluctuated around a constant level. Before proceeding, we note that considerable short-run dynamics remains in the seasonally adjusted data. We return to this issue below when discussing how to handle the resulting serial correlation in the residuals of the inflation equation.

The fall in inflation was associated with a gradual decline in money growth over the sample as central banks took measures to disinflate after the sharp increase in inflation during the 1970s. The change in the long-term interest rate lies slightly above its mean in the 1970s and below thereafter, with quite persistent fluctuations. Finally, real income growth was quite volatile over the sample. However, there appears to be some evidence that the rate of growth of output has declined, as evidenced by the fact that output growth was below average in most quarters in the 1990s.

Next we turn to the output gap (defined as output relative to a smooth trend). While most researchers use the HP filter to construct a measure of the trend output, we do so by extracting all variation of frequencies of more than 48 quarters from the demeaned quarterly growth rate of real output.[17] Converting the resulting series to the time domain and accumulating (incorporating the information in the average growth rate), we obtain a measure of the growth

[15] The interest rate, output and the price level are from the ECB's area-wide model (see Fagan et al. 2005) and have been updated with data from the ECB's Monthly Bulletin. The monetary data were provided by the ECB.

[16] See Appendix A. The seasonal adjustment had no effect on the low-frequency results. For the high-frequency regressions, the seasonal adjustment tends to reduce the estimated standard errors. None of the conclusions changed, however, if the unadjusted series was included in the regressions instead of the adjusted one.

[17] We demean the data before computing the spectrum. This is mainly a technical issue, since not doing so merely requires us to include a constant in the band spectrum regressions.

rate of potential.[18] The resulting output gap, which is plotted in Figure 1, is very similar to the HP-filtered output gap – the correlation coefficient between the two gaps is 0.95. The main movements seem associated with the large recession around 1974 following the first oil shock, and again in 1992-3.

The main reason why the use of spectral regression techniques is particularly attractive in the present context is that, as indicated above, the ECB has stated that the choice of a two-pillar framework arises from the fact that the determinants of euro area inflation vary across frequencies. Thus, at low frequencies money growth is important, while at high frequencies movements in inflation are *"associated with the state of activity in relation to its long-term potential"*, that is, the output gap. Exploring whether this description of the inflation process is accurate plainly requires us to estimate inflation equations for different frequencies. A further reason why estimation of the inflation equation in the frequency domain is appealing is that, in contrast to the Johansen (1995) estimator, Phillips' (1991) spectral estimator does not require us to specify the precise model for the short-run dynamics. Furthermore, it is compatible with different types of error processes.[19]

Preliminary evidence to assess the model laid out in the previous section is presented in Figures 2 and 3. The two panels in Figure 2 show the low- and high-frequency components of inflation and money growth. We define the long run as fluctuations with a periodicity of more than, and the short run as fluctuations with a periodicity of less than, 4 years.[20] The low-frequency components of both series are shown in the left panel. While the low-frequency component of money growth captures the inflation trend well, there is no apparent relation between the high-frequency components of the two series. By contrast, the output gap is by construction not able to account for the trend-wise decline in inflation since it does not have any trend. However, the left panel of Figure 3 shows that fluctuations in the output gap with a periodicity of more than 4 years are associated with movements in inflation. Moreover, the

[18] Using an HP-filter with the conventional smoothing parameter of 1600, one effectively filters out all fluctuations with a frequency of less than 40 quarters (Kaiser and Maravall, 2001). We chose 48 quarters because this value maximises the correlation of the spectral-filtered output gap with the HP-filtered output gap. The output gap coefficients in the regression remain unchanged when the output gap is defined as containing only fluctuations of less than 32 quarters, which is the frequency often used in business cycle analysis; see Baxter and King (1999).

[19] Two applications of the Phillips estimator are Hall and Trevor (1993), who estimate a consumption function on Australian data, and Corbae et al. (1994) who test the permanent income hypothesis.

[20] To assess whether this arbitrary, but not unreasonable, definition impacts materially on the results, we also show results below when the distinction between the long and the short run is drawn at a frequency corresponding to a periodicity of 2 and 8 years.

scatter plot suggests a positive relation also between the high-frequency components of the output gap and inflation in Figure 3.

The time series characteristics of the data are important for the empirical analysis that follows, and we therefore perform unit root tests for all variables used in the estimation, that is, inflation, money growth, output growth, changes in the interest rate and the output gap. Since different tests often lead to contradictory results, to get a fuller picture of the unit-root behaviour of the variables we perform Augmented Dickey-Fuller (ADF) tests, Elliot, Stock and Rotenberg (ERS) tests, Phillips and Perron (PP) tests, and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, which in contrast to the other tests considers stationarity as the null hypothesis.[21] We perform all tests with a constant as well as with a constant and a deterministic trend.[22] The optimal lag length is determined by the Akaike criterion (AIC), under the assumption that it is at most 8 lags.

The results, which are shown in Table 1, suggest that inflation and money growth are non-stationary, but that output growth, the change in the interest rate and the output gap are stationary.[23] Additional evidence on the unit-root behaviour of inflation and money growth can be obtained by testing for cointegration between inflation and money growth in a Johansen framework. The system is estimated with a restricted constant and 5 lags, which is the number of lags recommended by the Akaike Information Criterion.[24] The trace statistic indicates the existence of a single cointegrating relationship between money growth and inflation.[25] Moreover, the restriction of a unit coefficient on money growth cannot be rejected with a p-value of 0.81. Since Figure 1 suggests that money growth and inflation may have experienced a structural shift around the mid-1980s, we investigate the stability of the cointegrating relationship by testing the recursive eigenvalue for constancy (see Hansen and Johansen 1999). Changes in either the cointegration vector or the parameters that capture how disequilibria impact on money growth and inflation will lead to non-constancy of the

21 The unit root tests are discussed in Maddala and Kim (1998).

22 We do not perform tests with a break in the trend as in Corvoisier and Mojon (2005). In our view, the validity of the quantity theory is not restricted to a certain regime but should apply over the whole sample. In fact, the hypothesis tested here is that money growth (probably corrected for output growth) is able to explain the apparent change in the behaviour of inflation.

23 The one unexpected result is that the Phillips-Perron test suggests that money growth is stationary.

24 Maintaining other assumptions for the deterministic components of the system (that is, an unrestricted constant, a restricted trend or an unrestricted trend) does not change the results.

25 With π and μ being $I(1)$, money and prices are $I(2)$. Cointegration of inflation and money growth means that money and prices also cointegrate and real money is $I(1)$. Kugler and Kaufmann (2005) obtain the same results for euro area data.

estimated eigenvalue. Figure 4 shows that the test statistic never exceeds the critical value for a test at the 5% level, implying that the null hypothesis of stability cannot be rejected.[26] Thus, despite the apparent shift in the inflation and the money growth series, the relationship between them appears to have remained stable over the sample period, which justifies treating the data for the full period as coming from one regime.

4. Methodology

The Phillips spectral estimator is the frequency domain equivalent of the fully modified ordinary least squares (FM-OLS) estimator in the time domain (Phillips and Hansen 1990). To estimate the cointegrating relation, a correction for serial correlation and endogeneity of the regressors is applied to the estimator. In the frequency domain, serial correlation can be treated like heteroscedasticity in the time domain (Engle 1974) and can be corrected for by applying generalised least squares (GLS). To estimate the cointegrating relation, Phillips (1991) suggests computing the estimator over a band around frequency zero, which matters most for long-run estimation. The band spectral estimator for the cointegrating parameter at frequency zero, $\beta(0)$, is:

$$(6) \qquad \beta(0) = \left[\frac{1}{2M} \sum_{j=-M+1}^{M} e' \hat{f}_{vv}^{-1}(0) e \hat{f}_{22}'(0) \right]^{-1} \left[\frac{1}{2M} \sum_{j=-M+1}^{M} \hat{f}_{2*}'(0) \hat{f}_{vv}^{-1}(0) e \right],$$

with variance-covariance matrix:

$$(7) \qquad V(\beta(0)) = \frac{1}{T} \left[\frac{1}{2M} \sum_{j=-M+1}^{M} e' \hat{f}_{vv}^{-1}(0) e \hat{f}_{22}'(0) \right]^{-1},$$

where M denotes the bandwidth, T the sample size, and e' is the first unit vector, $e' = (1, \mathbf{0})$.

In the equations above, $\hat{f}_{22}(0)$ is the spectral density matrix of the regressors at frequency zero, and $\hat{f}_{2*}(0)$ the spectral density matrix of the dependent variable and the regressors in first differences.[27] The Phillips estimator corrects for serial correlation and endogeneity by using the inverse of the spectral density matrix of the residuals from the cointegrating relation and the regressors in first differences, $\hat{f}_{vv}(0)^{-1}$, as weighting function.

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[26] The cointegration analysis has been performed with the MATLAB-based program VAR by Anders Warne which is available at http://texlips.hypermart.net/svar/index.html.

[27] See Appendix B for a detailed description of the estimator.

Next, we discuss our approach to the estimation of the inflation equations for the euro area. We think of equation (3) as defining the long-run steady state of inflation since in the long run the output gap is zero and has no influence on the inflation rate. The analysis is complicated by the fact that inflation and money growth are non-stationary, while output growth, the change in the interest rate and the output gap are stationary. If two non-stationary time series are cointegrated in the time domain, they show a stochastic trend individually but their residuals are $I(0)$. In the frequency domain, non-stationary series are characterised by an unbounded spectrum at the origin. Cointegration in the frequency domain implies that the spectra at the zero frequency cancel out, so that the spectrum of the residuals is bounded at the origin.

Because of the different degrees of integration of inflation and money growth on the one hand, and output growth, the interest rate change and the output gap on the other, we follow a two-step approach to estimate the relation between the variables in the long run. In the low-frequency band we use Phillips' (1991) band spectral estimator, which is appropriate for $I(1)$ variables, to estimate the long-run effect of money growth on inflation. In a second step, we impose the estimated long-run coefficient and regress the stationary residual from the first-step regression, $\pi_t - \hat{\alpha}_\mu \mu_t$, on output growth, the interest rate change and the output gap.[28] Since in the second step all variables in the regression are stationary, we use Engle's (1974) band spectral estimator. The fact that $\hat{\alpha}_\mu$ is superconsistent ensures that the estimators in the second stage have the same asymptotic distribution as if α_μ were known (e.g., Maddala and Kim 1998, p. 157).

The estimators in equations (6) and (7) are appropriate for cointegrated time series if the zero frequency is included in the estimation. Restricting the spectral regression to high frequencies makes estimation of the parameters more difficult. By excluding the zero frequency, inflation and money growth become stationary. However, because they are correlated at the zero frequency (as evidenced by the finding of cointegration), and because of leakage from the zero frequency into all other frequencies, they are correlated also at high frequencies. Thus, this leakage introduces correlation between the regressor and the error term at high frequencies. To estimate β consistently over the high-frequency band, Corbae et al. (1994)

[28] This two-step approach is necessary because the $I(1)$ variables have unbounded spectra at the zero frequency, which dominate the spectra of the stationary variables in the low-frequency band. For frequency bands that exclude the zero frequency, all variables are stationary. In this high-frequency band, we can estimate the coefficients on all variables in a single step.

propose a frequency domain Generalised Instrumental Variable Estimator (GIVE). The idea is to instrument the regressors with a vector of variables, z_t, that are uncorrelated with the error term in the cointegrating regression but correlated with the endogenous explanatory variables (in the present case money growth). The resulting instrumental variable estimator for the high-frequency band, β^{GIVE}, is:

$$(8) \qquad \beta^{GIVE} = \frac{\left(\sum_{\theta_j \in B_2} f_{2z}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{z1}(\theta_j) f_{uu}^{-1}(\theta_j)\right)}{\left(\sum_{\theta_j \in B_2} f_{2z}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{z2}(\theta_j) f_{uu}^{-1}(\theta_j)\right)},$$

with variance-covariance matrix

$$V(\beta^{GIVE}) = \frac{2M}{T}\left[\left(\sum_{\theta_j \in B_2} f_{2z}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{z2}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\right]^{-1}.$$

Here, f_{uu} is the spectrum of residual of the cointegrating relation.[29] Analogous to equations (6) and (7), f_{2z} and f_{z2} are the cross-spectral matrices of the regressors and the instruments, and f_{z1} is a vector of cross-spectra of the instruments and the dependent variable.

5. Results

Tables 2 to 6 present the results. We define the long run as fluctuations with a periodicity between 4 years and infinity and the short run as fluctuations with a periodicity between 0 and 4 years, but we also show results when the distinction between the long and the short run is drawn at a frequency corresponding to a periodicity of 2 or 8 years. (For ease of exposition, henceforth we refer to frequencies in terms of their periodicity equivalents.) The first column of each table shows an estimate of the regression including all frequencies to help the reader see how the relation between inflation, money growth and the output gap varies between frequencies. The differences in results illustrate how band spectral regressions can be used to reach a more complete understanding of the inflation process in the euro area.

5.1 Band spectral estimates

Tables 2 to 5 present the band spectrum estimates of equation (5). We first study the relationship between money growth, interest rate changes, output growth, and inflation near

[29] See equation B.1 in Appendix B.

(including) the zero frequency at which we expect the dynamics of inflation to be governed by the quantity theory in equation (3). Thus, if the quantity theory held at all frequencies, the income elasticity of money demand, -α_γ, and the value of α_μ would be unity. Because of the different time series properties of the determinants of the inflation rate we first use Phillips' (1991) estimator to fit in a first stage the relationship between money growth and inflation, which are non-stationary, before we apply the Engle (1974) estimator to the first-stage residuals and the stationary variables.[30] The results in Table 2 indicate that the coefficient on money growth is not significantly different from unity, irrespective of whether the long run is defined as comprising periodicities from 2 years to infinity, 4 years to infinity or 8 years to infinity. Interestingly, the coefficient rises towards unity the more the high frequencies are excluded.

The lower part of Table 2 shows the results from the second-step regression. The coefficient on output growth falls towards minus unity the more high frequencies are excluded. The output coefficient is significant in all cases except in the frequency band of 8 years to infinity, presumably because of the reduction in degrees of freedom when only the lowest-frequency band is considered. As evidenced by the *t*-statistics, a coefficient of unity in absolute value on money and output growth can be rejected when all frequencies are included in the regressions, but we cannot do so when only the low-frequency bands are included. Turning to the parameter on the change in the interest rate, we note that it is only significant when the highest frequencies are included. Finally, we find that the R^2 is highest when the frequencies corresponding to 0.5 to 4 years are excluded from the regression.

Table 3 restricts the coefficient on output to minus unity as suggested by the quantity theory and uses the same two-step regression approach as above, but with money less output growth as an explanatory variable. Since output growth is stationary, money less output growth is *I*(1). The results in Table 3 broadly confirm those in Table 2 and we therefore do not discuss them in detail in the interest of brevity.

An important finding in Tables 2 and 3 is that the parameters on money and income growth are always smaller in absolute value when all the frequencies are included in the regression. This suggests that the quantity theory does not fit the data well at higher frequencies. To explore this issue, we next re-estimate the model, restricting our attention to the high-

[30] As expected, the low-frequency part of the output gap is always insignificant in the regressions in Table 2 and 3 and is therefore not included.

frequency band, using the generalised instrumental variable estimator in equation (6).[31] Table 4 shows the results. Money growth is never significant, irrespective of how the high-frequency band is defined. The parameter on output growth is however significant and negative in the frequency bands of 0.5 to 4 years, and 0.5 to 8 years. The coefficients on the change in the interest rate and the output gap are significant for the 0.5 to 8 years frequency band, though the output gap is marginally significant (with a p-value of 6%) in the 0.5 to 4 years frequency band.

The generally low significance of the variables in these regressions suggests that the high-frequency data contain mainly noise.[32] To explore this hypothesis further, we re-estimate the equations but exclude the 0.5 to 1 year frequency band. The results, which are shown in Table 5, differ from those in Table 4 only in that the output gap now is significant in the 1 to 4 year frequency band and output growth is always insignificant.

The band spectral regressions discussed above show that the relation between inflation, money, output growth, the interest rate change and the output gap varies by frequency. In particular, the quantity-theoretic variables seem to be important only at low frequencies, whereas the output gap contains information about inflation at high frequencies. Needless to say, these findings are strikingly compatible with the ECB's public statements on the inflation process in the euro area.

5.2 A two-pillar Phillips curve

To proceed, we follow Gerlach (2003, 2004), Neumann (2003) and Neumann and Greiber (2004) and estimate an equation with headline inflation as the dependent variable, and include the low-frequency components of money, output growth and the interest rate change and the high-frequency component of the output gap as the explanatory variables.[33] Since our results showed that fluctuations in the output gap in the 0.5 to 1 year frequency band are not informative for inflation, we exclude this band. Our final inflation equation, which Gerlach (2003, 2004) refers to as the "two-pillar Phillips curve", is thus:

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[31] We instrument money growth by its first lag. As output growth and the output gap are stationary, we do not use instruments for them.

[32] Such noise could be e.g. introduced by seasonal adjustment. Moreover, quarterly GDP data in general are difficult to measure and may therefore be subject to error.

[33] In a regression of an $I(1)$ variable on $I(1)$ and $I(0)$ variables, the asymptotic distribution of the coefficients on the $I(1)$ and $I(0)$ variables are independent (see Maddala and Kim 1998, p. 75). While the coefficient on the $I(1)$ variable follows a non-standard distribution, the coefficients on the $I(0)$ variable are normally distributed.

(9) $\pi_t = \beta_0 + \beta_g g_{t-1} + \left\{ \beta_\mu \mu_t^{LF} + \beta_\rho \rho_t^{LF} + \beta_\gamma \gamma_t^{LF} \right\} + \varepsilon_t .$

In contrast to the regressions in the last section the dependent variable is in this case not filtered, and there is therefore no loss in degrees of freedom.

As noted when discussing the data, there is significant short-run dynamics in the quarterly inflation rates. Since the regressors in equation (9) evolve slowly over time, that dynamics will induce autocorrelation of the residuals. There are three ways to deal with this issue in estimation. First, we could include variables that may explain temporary movements in inflation. Assenmacher-Wesche and Gerlach (2006) show that incorporating import price shocks in the model reduces this problem somewhat. Second, we could incorporate a sufficient number of lags of inflation among the regressors to render the residuals serially uncorrelated. Doing so, however, does not provide an economic explanation for the autocorrelation but does increase the number of parameters to be estimated. Third, we could let the residuals remain autocorrelated, but use Newey-West (1987) corrected standard errors for inference. Preliminary estimates indicated that the results were essentially unaffected if we used the second or third approach and in the interest of brevity we only present those using serially correlation consistent standard errors. [34]

The first column in Table 6 shows the results when all frequencies are included. While the coefficient on money growth has the right sign, it is significantly different from unity.[35] Furthermore, neither output growth nor the output gap are significant. Thus, a researcher attempting to model inflation using solely "headline" measures of the regressors would conclude that the ECB's notion of the inflation process is of little or no empirical validity.

In the other columns (disregarding for the moment the last column), the low-frequency part of money growth, output growth and the interest rate change and the high-frequency part of the output gap are included. Again, the cut-off point between the low and the high frequency is varied between 2, 4 and 8 years. The results are strikingly different from the all-frequencies regression. The coefficients on money and output growth are of the expected signs, highly significant, but not significantly different from unity in absolute value in the low-frequency

[34] The main difference in the results, and the reason we prefer the third approach, is that the significance of the parameter on low frequency real output growth declines as lags of inflation are included among the regressors. Since low frequency real output growth varies little in the sample, the difference appears to be due to a loss of power arising from the need to estimate additional parameters.

[35] As money growth is non-stationary, the coefficient follows a nonstandard distribution. We apply the critical value of a Dickey-Fuller test of -2.89 to test the hypothesis that the coefficient is unity.

band (except the coefficient on the output growth in the case of the 2 years to infinity frequency band). Except in the case of the 1 to 2 years frequency band, the output gap is highly significant. For the 4 years to infinity and the 8 years to infinity regressions, a Wald test of the coefficients on money growth and output growth, being 1 and -1, respectively, does not reject, whereas it rejects for the regressions for all frequencies and for the 2 years to infinity frequency band. The adjusted R^2 increases when the higher frequencies are excluded from money, output and interest rate growth. The largest differences occur when moving from the 2-year to the 4-year cut-off, whereas the choice between a cut-off of 4 and 8 years does not significantly change the fraction of the variance explained.

We end this section by trying to find the optimal frequency bands for the quantity-theoretic variables and for the output gap in the regression by varying the frequency bands that are included in the regression. For the quantity-theoretic variables, we start with the 8 years to infinity band and increase the width of the band by adding one frequency ordinate at a time until we have included all frequencies from 2 years to infinity. For the output gap, we start by including frequencies between 1 and 8 years and then reduce the band until it includes only the 1 to 2 years frequency band.[36] For each of these different definitions of the long and the short run we run a regression and calculate the R^2. The frequency bands for both groups of variables are varied independently, so that we get a matrix of R^2s that is shown in Figure 5. The maximum R^2 is obtained with a regression where the quantity-theoretic variables enter in a frequency band of 5.6 years to infinity and the output gap with a frequency band between 1 and 5.4 years. We tabulate the parameter estimates of this regression in the last column of Table 6. Figure 5 shows that the R^2 drops considerably if the quantity-theoretic variables are considered at a frequency band of 7 years and above, which contradicts the result by Neumann and Greiber (2004) that money with a wavelength of more than 8 years has the closest relation to inflation.[37] By contrast, the information content in the output gap is not greatly reduced as long as the frequency band of 1 to 3 years is included. This confirms that the output gap is indeed informative for inflation fluctuations at business cycle frequencies.

[36] If the 0.5 to 1 year frequency band is also included, results are similar though the R^2 is slightly lower.

[37] In contrast to Neumann and Greiber (2004), who use wavelet analysis and thus are only able to investigate cycles of 0.5 to 1 year, 1 to 2 years, 2 to 4 years, 4 to 8 years and more than 8 years, we can vary the frequency band in much finer intervals.

6. Causality between money growth and inflation

While our results indicate that money growth is strongly correlated with inflation, we have not directly tested the hypothesis that low-frequency movements in money growth Granger cause inflation. While the existence of a stable money demand function suggests that inflation and money growth are related, that does not imply that money growth causes inflation. To properly understand the inflation process, an understanding of the patterns of causality is consequently needed.

We employ the notion of causality introduced by Granger (1969, 1980). Money growth is said to cause inflation if it contains information about future inflation that is not contained in some information set which includes past values of π. The extent and direction of causality can differ between frequency bands (Granger and Lin, 1995).[38] In a cointegrating system there must be causality at least in one direction between the series (Granger 1988).

The frequency-wise measure of causality is based on a bivariate vector autoregression (VAR) containing the variables of interest, in our case inflation and money growth.[39] The starting point is the moving average representation of the system,

$$(10) \qquad \begin{bmatrix} \pi_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix}$$

where the $\Psi_{ij}(L)$, $i, j = 1, 2$ are polynomials in the lag operator, L, and η_1, η_2 are the orthogonalised shocks.[40] Money growth Granger-causes inflation if $\Psi_{12}(L)$ is non-zero. The frequency-wise measure of causality suggested by Geweke (1982) and Hosoya (1991) is defined as:

$$(11) \qquad M_{\mu \to \pi}(\omega) = \log\left[1 + \frac{\left|\Psi_{12}(e^{-i\omega})\right|^2}{\left|\Psi_{11}(e^{-i\omega})\right|^2}\right].$$

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[38] Though the component of a series in a certain frequency band cannot be estimated without the use of a two-sided filter which destroys the chronological aspect of the causal definition, it is possible to deduce causal relationships at different frequencies without estimation of the series' components, as is done in the band spectrum regressions.

[39] See also Granger and Lin (1995) and Breitung and Candelon (2005).

[40] That is, the VAR reduced-form errors are transformed into the orthogonalised errors by multiplying them with the lower triangular matrix from a Choleski decomposition of the reduced-form covariance matrix.

This measure is zero if $\left|\Psi_{12}(e^{-i\omega})\right| = 0$, which implies that μ_t does not cause π_t. In assessing causality it is important to account for possible feedback from other variables to the variables of interest. In this case, it is necessary to base the causality measure in equation (11) on the partial periodograms and cross periodograms by conditioning on the information contained in the other variables.[41] Since the causal relation between money and inflation could be influenced by the output gap and interest rate changes, we condition our causality tests on these two variables. Instead of measuring causality between π_t and μ_t directly, we compute the causality measure for the projection residuals u_t and v_t, which are obtained by regressing money growth and inflation on the residuals from a regression of the output gap and the interest rate change on lagged values of π_t and μ_t.[42] Hosoya (2000) shows that the causality measure from money growth to inflation, given the output gap and the interest rate change, is equal to the bivariate causality measure between the projection residuals u_t and v_t.

For causality tests the lag length should neither be too short, since this possibly cuts off significant coefficients, nor too large, since in this case the tests may lack power. We perform the tests with a lag length of 12, under the hypothesis that the transmission from money to prices is completed in this time span. We compute the moving average (MA) representation of the VAR and apply a Fourier transformation to the resulting MA coefficients.[43] Figure 6 shows the causality measure over frequencies from 0 to π.

Breitung and Candelon (2005) show that the hypothesis $M_{\mu\to\pi}(\omega) = 0$ is equivalent to a linear restriction on the VAR coefficients and that its significance can be tested by a conventional F-test, which in our case yields a critical value at the 5% level of $F_{2,115} = 3.08$. Though Breitung and Candelon (2005) argue that in bivariate cointegrated systems the test for significance of the frequency-wise causal relation follows a standard limiting distribution, it is not clear whether this result extends to a multivariate system where some variables are $I(1)$ and others are $I(0)$. We therefore do not include a critical value in Figure 6 but focus on the peaks in the causal measure. We find a peak in causality from money growth to inflation at low frequencies. At a frequency of 0.1π, which corresponds to 20 quarters, the causality

41 See Hosoya (2000) and Breitung and Candelon (2005).

42 In this regression we include the contemporaneous values of inflation and money growth, since Hosoya (2000) argues that omitting them may lead to a finding of spurious causality. Excluding π_t and μ_t, however, does not alter the results.

43 In the case of nonstationary variables the VAR can be written in error-correction form and the resulting causality measure can be defined analogously to the stationary case, see Breitung and Candelon (2005).

measure drops to zero and remains low thereafter. In contrast, the causality measure from inflation to money growth peaks at a much lower level at a frequency of approximately 0.6π (corresponding to three quarters).

We also test causality from the output gap to inflation and conversely. The causal relationship from the output gap to inflation shows a clear peak at the business cycle frequency of 10 quarters. We thus find that output gaps predict inflation at higher frequencies than money growth. By contrast, causality from inflation to the output gap also peaks at a frequency of 0.6π, which – given the evidence from the two-pillar Phillips curve regressions that the high frequencies may contain mainly noise – may well be spurious.

7. Conclusions

In this paper we have analysed the behaviour of inflation in the euro area across frequency bands, using data for the period 1970-2004. The main findings – that money is useful for understanding the low-frequency variation, and that the output gap contains information about the high-frequency variation, of inflation in the euro area, and that these correlations reflect Granger causality – appear fully compatible with the ECB's statements regarding the inflation process in the euro area.

In concluding we emphasise that the fact that fluctuations in money growth have played an important role in accounting for inflation developments suggests that money can be used as an information variable for policy purposes, but does not necessarily imply that the use of a separate pillar for money is necessary. Whether money growth is best incorporated into the inflation analysis by using a two-pillar framework or by integrating the pillars in a single analysis of inflation is an important question that goes beyond the scope of this paper. While at one level this question appears largely semantic, in terms of the internal organisation of work at the ECB, the difference may be of importance.

Appendix A. Filtering in the frequency domain

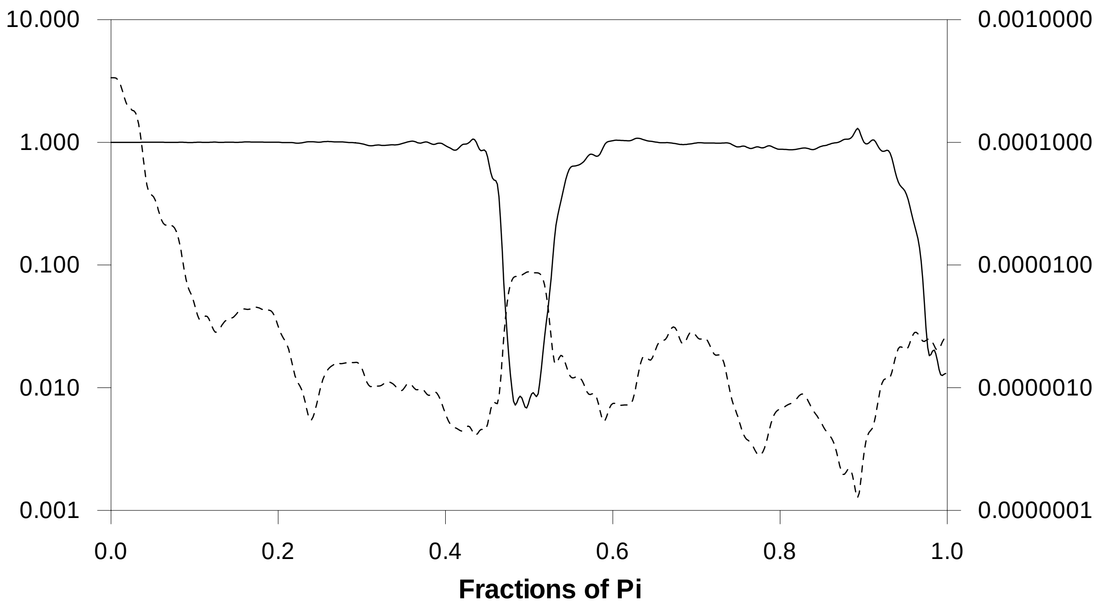
To filter in the frequency domain, we apply a Fourier transformation to the series. The series is then multiplied by a matrix that has zeros on the frequency ordinates that correspond to the frequencies that are to be filtered out and ones elsewhere. Finally, the series is converted back to the time domain by applying an inverse Fourier transformation. As Fourier transformations work faster for powers of 2, we use 1024 frequency ordinates and pad our series of 139 observations with zeros.

Figure A.1 shows a clear seasonal peak for inflation at the frequency of 0.5π, which corresponds to the annual frequency. To remove the seasonal pattern, we follow Sims (1974) and apply a filter with a bandwidth of $\pi/24$ around the annual frequency. Figure A.1 also shows the gain of the filter, which is defined as the spectrum of the filtered series, $g_s(\omega)$, relative to the spectrum of the unfiltered series, $g_x(\omega)$,

(A.1) $\quad \nu(\omega) = g_s(\omega)/g_x(\omega)$,

see e.g. Gómez and Maravall (1998). The gain shows that the filter performs well in removing the seasonal pattern at 0.5π and π (corresponding to cycles with a periodicity of 4 and 2 quarters) without introducing distortions at other frequencies. The seasonal adjustment leads to a loss of 9 degrees of freedom of the total of 139 observations.

Figure A1. Gain of spectral filter (solid line, left-hand scale) and spectrum of inflation (dashed line, right-hand scale)



Appendix B. The band spectral estimator

The Phillips (1991) spectral estimator is a single equation method that applies a correction for endogeneity of the regressors and serial correlation of the errors. The starting point is a cointegrating system in triangular form; see Phillips (1991):

(B.1) $$y_{1t} = \beta' y_{2t} + u_{1t}$$

(B.2) $$\Delta y_{2t} = u_{2t}$$

where y_{1t} is the dependent variable and y_{2t} is a vector of independent variables, all of which have a unit root. The error terms u_{1t} and u_{2t} are assumed to be stationary.

The time series are transferred into the frequency domain by calculating the finite Fourier transforms,

$$w_{\Delta}(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} \Delta y_t e^{i\lambda t} ,$$

(B.3) $$w_y(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_{t-1} e^{i\lambda t} ,$$

$$w_*(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_{*t} e^{i\lambda t} ,$$

for $\lambda \in [-\pi, \pi]$, $y_t = (y_{1t}, y_{2t})$ and $y'_{*t} = (y_t, \Delta y_{2t})$, with T denoting the sample length. The Fourier transform of y_{t-1}, $w_y(\lambda)' = (w_1(\lambda), w_2(\lambda)')$, is partitioned into the Fourier transforms of the regressand, $w_1(\lambda)$, and the regressors, $w_2(\lambda)'$. Next we compute the smoothed periodogram estimates,

(B.4) $$\hat{f}_{22}(\omega_j) = \frac{1}{m} \sum_{B_j} w_2(\lambda_s) w_2(\lambda_s)^* ,$$

(B.5) $$\hat{f}_{2*}(\omega_j) = \frac{1}{m} \sum_{B_j} w_2(\lambda_s) w_*(\lambda_s)^* ,$$

where the summation is over

(B.6) $$\lambda_s \in B_j = \left(\omega_j - \frac{\pi}{2M} < \lambda \leq \omega_j + \frac{\pi}{2M} \right),$$

such that if $\lambda \in B_j$, then $-\lambda \in B_j$ also. This ensures that the resulting estimator β is real valued, see Engle (1974). In effect, the spectra are computed by averaging over m = $T/2M$

neighbouring periodogram ordinates, where M is the total number of frequency ordinates divided by 2. Defining ν_t as

$$\text{(B.7)} \quad \nu_t = \begin{bmatrix} 1 & \beta' \\ 0 & I \end{bmatrix} \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

the correction for serial correlation and endogeneity of the regressors is obtained by using the inverse of the spectrum of the residuals from an OLS regression of equation (B.1), $\hat{f}_{\nu\nu}(\omega_j)^{-1}$, as weighting function in the estimator. In the frequency domain this can be written as

$$\text{(B.8)} \quad \hat{f}_{\nu\nu}(\omega_j) = \frac{1}{m} \sum_{\mathrm{B}_j} \left[w_\Delta(\lambda_s) - e\hat{\kappa}' w_y(\lambda_s) \right] \left[w_\Delta(\lambda_s) - e\hat{\kappa}' w_y(\lambda_s) \right]' ,$$

where $e' = (1, \mathbf{0})$ and $\hat{\kappa}' = (-1, \boldsymbol{\beta})$. As e is known by construction, non-linear estimation techniques are not required. Applying this weighting function to a system estimator of β in equation (B.1) and (B.2) in the frequency domain gives the zero-frequency spectral estimator for the cointegration parameters, $\beta(0)$, in equation (6) in the text.

For the generalised instrumental variable estimator, β^{GIVE}, the Fourier transform of the regressors, $w_2(\lambda)$, is instrumented to get a consistent estimate of β. If the generating mechanism for Δy_2 is $w_{\Delta 2}(\lambda) = \delta\, w_z(\lambda) + w_\zeta(\lambda)$ and z_t is a vector of variables that are independent of ζ_t and the error term u_{1t}, $w_z(\lambda)$ can be used as instrument in a spectral regression. The resulting estimator is given in equation (8) and its covariance matrix in equation (9) in the text.

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Tables and Figures

Table 1. Unit root tests

	ADF	*PP*	*ERS*	*KPSS*	*AIC lag*
Test with constant					
Inflation	-1.03	-1.82	-1.09	1.76*	5
Money growth	-1.59	-3.16*	-1.57	1.92*	5
Output growth	-5.53*	-7.93*	-2.30*	0.72*	1
Interest rate change	-6.23*	-6.84*	-6.23*	0.41	1
Output gap	-4.07*	-3.04*	-3.92*	0.05	4
Test with trend and constant					
Inflation	-2.75	-4.04*	-1.80	0.16*	5
Money growth	-2.62	-6.70*	-2.05	0.20*	5
Output growth	-5.77*	-8.25*	-4.31*	0.11	1
Interest rate change	-6.41*	-6.99*	-6.41*	0.07	1
Output gap	-4.09*	-3.07	-4.08*	0.05	4
Test of first differences					
Inflation	-6.90*	-17.81*	-6.41*	0.08	4
Money growth	-6.09*	-23.71*	-2.29*	0.08	8
Output growth	-6.59*	-24.90*	-6.51*	0.05	7
Interest rate change	-8.40*	-16.64*	-8.21*	0.02	5
Output gap	-8.85*	-8.85*	-4.70*	0.07	0

Note: The last column indicates the number of lags included in the test, which were chosen by the AIC criterion. The 5% critical values for the tests including a constant only are -2.89 for the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test, -1.95 for the Elliot, Stock and Rotenberg (ERS) test and 0.46 for the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test. The 5% critical values for the test including a constant and a trend are -3.45 for the ADF and the PP test, -2.89 for the ERS and 0.15 for the KPSS test. The tests of the first differences include a constant but no trend. The sample period is 1970Q2 to 2004Q4. An asterisk (*) indicates the rejection of the null hypothesis.

Table 2. Band spectrum regressions: low-frequency band

Frequency band	*0.5 years to ∞*	*2 years to ∞*	*4 years to ∞*	*8 years to ∞*
		I. Dependent variable: π_t		
Money growth	0.70**	0.81**	0.88**	0.96**
	(0.06)	(0.12)	(0.15)	(0.19)
R^2	0.45	0.57	0.67	0.79
		II. Dependent variable: $\pi_t - \hat{\alpha}_\mu \mu_t$		
Output growth	-0.23*	-0.56*	-0.82*	-0.98
	(0.08)	(0.23)	(0.42)	(0.97)
Interest rate change	2.41**	3.90**	5.09	3.01
	(0.52)	(1.46)	(2.87)	(6.92)
R^2	0.15	0.24	0.26	0.15
Degrees of freedom	137	32	15	6

Note: All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 3. Band spectrum regressions: low-frequency band

Frequency band	*0.5 years to ∞*	*2 years to ∞*	*4 years to ∞*	*8 years to ∞*
I. Dependent variable: π_t				
Money less output growth	0.48^{**}	0.76^{**}	0.98^{**}	1.15^{**}
	(0.06)	(0.13)	(0.19)	(0.26)
R^2	0.33	0.55	0.72	0.89
II. Dependent variable: $\pi_t - \hat{\alpha}_{\mu\gamma}(\mu_t - \gamma_t)$				
Interest rate change	3.16^{**}	4.53^{**}	4.95	1.41
	(0.54)	(1.62)	(3.64)	(8.96)
R^2	0.20	0.19	0.10	0.01
Degrees of freedom	137	32	15	6

Note: All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 4. Band spectrum regressions: high-frequency band

Frequency band	*0.5 to ∞ years*	*0.5 to 2 years*	*0.5 to 4 years*	*0.5 to 8 years*
Money growth	0.53^{**}	0.02	0.17	-0.02
	(0.06)	(0.67)	(0.66)	(0.30)
Output growth	0.01	-0.08	-0.15^{*}	-0.16^{*}
	(0.08)	(0.09)	(0.07)	(0.07)
Interest rate change	1.57^{**}	1.03^{*}	0.67	1.10^{*}
	(0.52)	(0.52)	(0.45)	(0.46)
Output gap	0.10^{**}	0.09	0.22	0.12^{**}
	(0.03)	(0.18)	(0.12)	(0.04)
R^2	0.51	0.13	0.20	0.27

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2004Q4. The non-stationary variable, money growth, is instrumented by its first lag.

Table 5. Band spectrum regressions: high-frequency band excluding fluctuations of less than 1 year

Frequency band	*1 year to ∞*	*1 to 2 years*	*1 to 4 years*	*1 to 8 years*
Money growth	0.63**	0.09	0.11	0.02
	(0.07)	(0.28)	(0.20)	(0.17)
Output growth	-0.17	0.05	-0.14	-0.19
	(0.12)	(0.13)	(0.11)	(0.10)
Interest rate change	2.87**	1.08	0.55	1.25*
	(0.78)	(0.67)	(0.63)	(0.64)
Output gap	0.07*	0.07	0.22**	0.12**
	(0.04)	(0.14)	(0.07)	(0.04)
R^2	0.63	0.26	0.40	0.34

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2004Q4. The non-stationary variable, money growth, is instrumented by its first lag. Band excludes frequencies higher than 1 year.

Table 6. Two-pillar Phillips curve

	Dependent variable: π				
Frequency band	*0.5 to ∞ years*	*2 to ∞ years*	*4 to ∞ years*	*8 to ∞ years*	*5.6 to ∞ years*
Money growth	0.65^{**}	0.82^{**}	0.91^{**}	1.08^{**}	1.05^{**}
	(0.08)	(0.08)	(0.07)	(0.9)	(0.08)
Output growth	-0.18	-0.57^{**}	-0.83^{**}	-1.01^{**}	-1.06^{**}
	(0.11)	(0.18)	(0.21)	(0.27)	(0.20)
Interest rate change	2.13^{**}	3.90^{**}	4.96^{**}	1.87	2.98
	(0.87)	(1.12)	(1.48)	(1.86)	(1.59)
Frequency band	*0.5 to ∞ years*	*1 to 2 years*	*1 to 4 years*	*1 to 8 years*	*1 to 5.4 years*
Output gap (lagged 1 quarter)	0.07	0.05	0.20^{**}	0.13^{**}	0.18^{**}
	(0.05)	(0.13)	(0.08)	(0.05)	(0.06)
$\overline{R}^2$	0.55	0.67	0.77	0.79	0.82
$\chi^2(2)$	71.12	8.85	1.92	0.93	0.48

Note: All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2004Q4. Columns 2 to 4 include the low-frequency part of money growth, output growth and the interest rate change and the high-frequency part of the output gap. Newey-West corrected standard errors are reported. The last line reports the test statistic from a test that the coefficients on money and output growth are 1 and -1. The critical value at the 5% level is 5.99.

Figure 1. The data

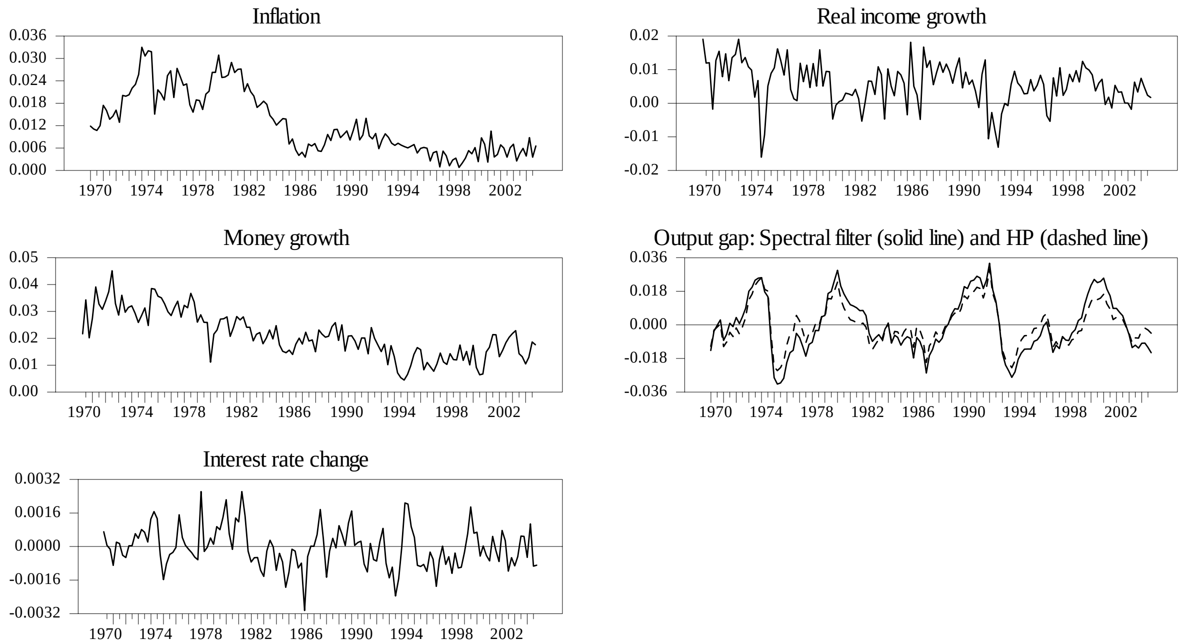


Figure 2. Inflation and money growth: low- (left panel) and high-frequency (right panel)

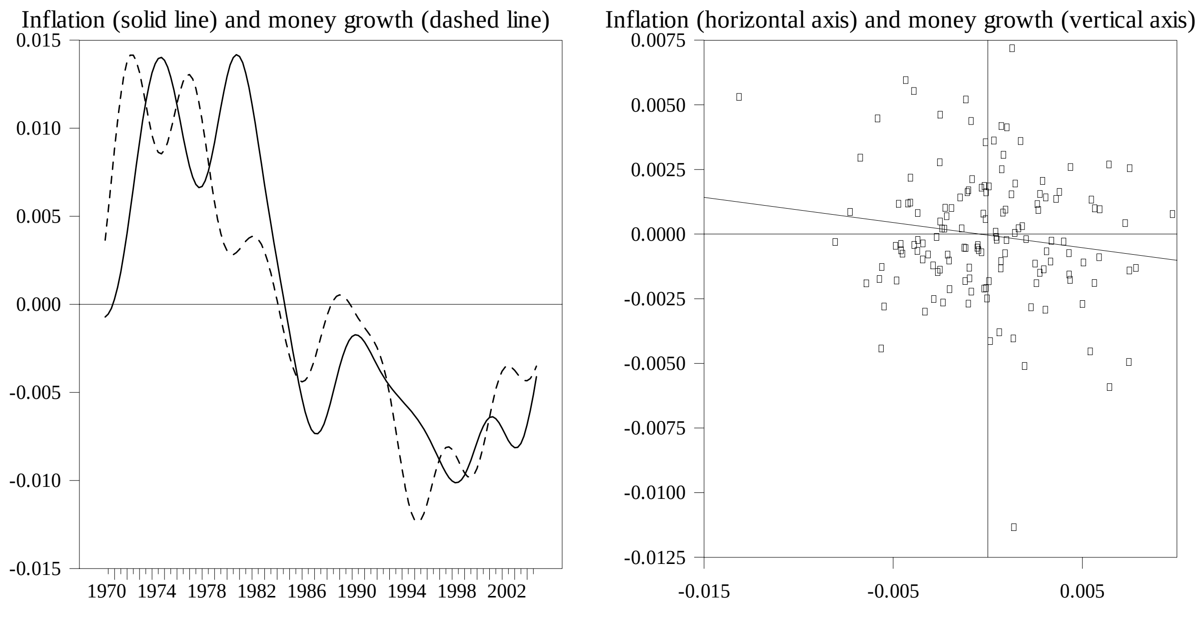


Figure 3. Inflation and output gap: low- (left panel) and high-frequency (right panel)

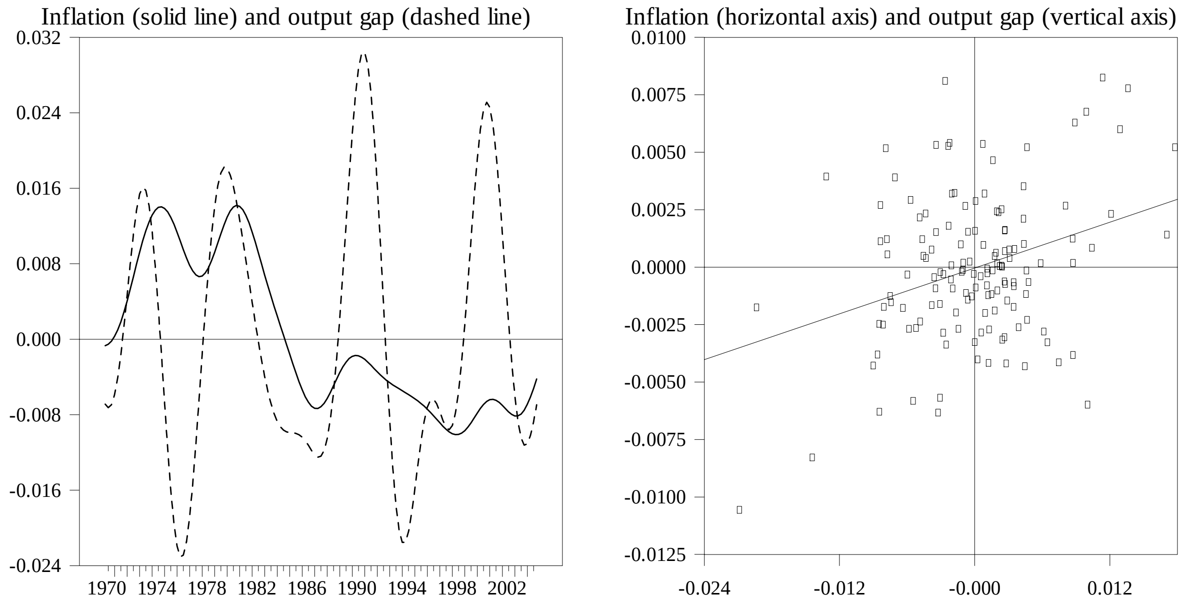


Figure 4. Recursive eigenvalue and critical value

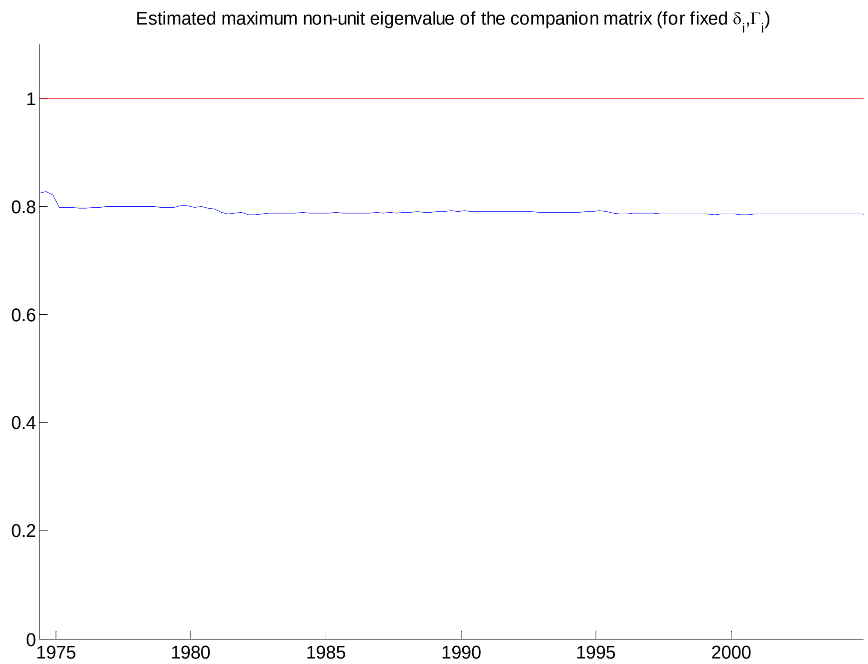


Figure 5. R^2 for two-pillar Phillips curve regressions

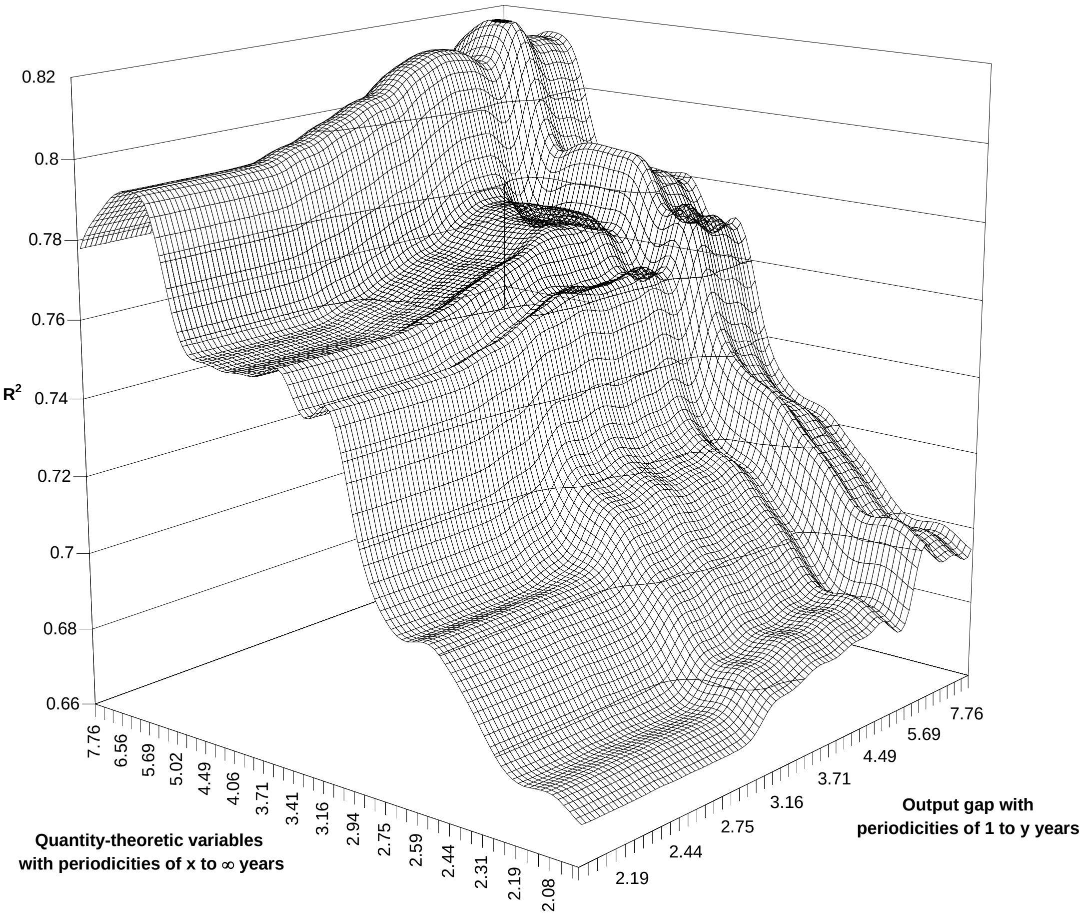
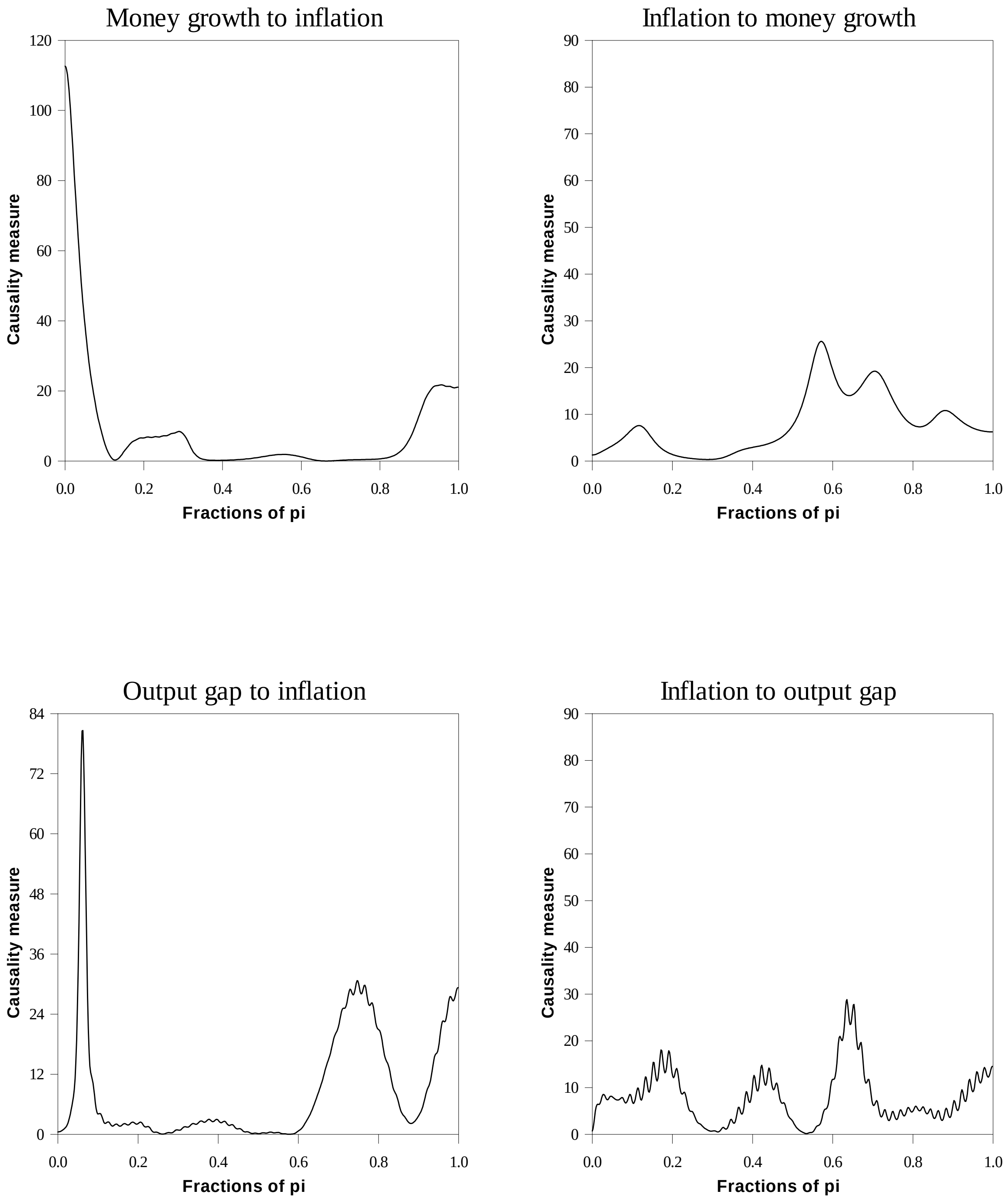


Figure 6. Causality measures



Note: Causality measures for a system containing inflation, money growth, the interest rate change and the output gap. The sample period is 1970Q2 to 2004Q4.