Redistributive Shocks and Productivity Shocks

José-Víctor Ríos-Rull*

University of Pennsylvania,
CAERP, CEPR, NBER

Raul Santaeulalia-Llopis

University of Pennsylvania

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Abstract

We pose and estimate a bivariate shock to the production function that under competition in factor markets simultaneously accounts for movements in the Solow residual and in the factor shares of production. We show how confronting agents in a standard RBC economy with these shocks entail a much smaller response (about 33%) of hours relative to the standard modelization of the shocks that identifies the Solow residual with a univariate shock. Our findings raise a flag against the optimism embedded in the literature that states that productivity shocks are responsible for most of the cyclical behavior of output and hours.

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1 Introduction

The structural interpretation of the Solow residual as productivity shocks is the hallmark of the real business cycle (RBC) research program. Not without controversy, productivity-driven business cycle models have been regarded by RBC modelers as successful in accounting for a broad set of business cycle phenomena, in particular, the cyclical volatility of output and its co-movement with hours, and (with indivisible labor also) the cyclical volatility of hours. An ingredient common to (almost all) RBC models is the assumption that the functional distribution of income is constant at all frequencies[1]. Implicit there is the premise of unimportant implications for the business cycle of the fluctuations that we observe in the factor shares of income (which move within a range of 5-6%, U.S. 1954.I-2004.IV). In this paper, we investigate whether the interaction of the Solow residual and the movements in the factor shares matters for business cycle. We find that it does, so much that it dampens the explanatory power of the Solow residual to one third in terms of the volatility of hours, two thirds in terms of the volatility of output, and the correlation of hours with output drops to one fifth with respect to the standard RBC model characterized by constant factors shares of income.

Most RBC models (with few exceptions that we will discuss below) ignore the fluctuations in factor shares of income. The theoretical constancy of the factor shares at all frequencies in these models results from assuming a Cobb-Douglas technology with constant coefficients and maintaining the connection between factor prices and their respective marginal productivity[2]. However, although the shares of GNP accruing to each factor do not show a secular trend, they do have sizeable high frequency movements. Over the period 1954.I-2004.IV, the labor share of income is around 30% as volatile as output, 55% as volatile as the Solow residual, it is countercyclical (a correlation of -.24) and highly persistent (first order autocorrelation of .77)[3]. The consequences of these cyclical movements of the labor share for the business cycle are yet to be explored.

To do so we pose a bivariate shock to a Cobb-Douglas production function that, when factors

[1]This assumption is also common to a second-generation of RBC models that introduces other shocks estimated from the data (e.g., energy prices, government spending, tax rates and exchange rates) in order to cope with original business cycle puzzles such as the volatility of hours and the correlation of hours with average labor productivity. See the exhaustive study in **?**, and for more recent reviews **?** and **?**.

[2]The use of a Cobb-Douglas production function with constant coefficients is precisely motivated by the fact that while the relative price of capital and labor has changed over time, *"the capital and labor shares of output have been approximately constant"*, see **?**.

[3]For these figures we use HP-filtered quarterly series of labor share and logged real output. Section 2.1 describes in detail the cyclical properties of the labor share.

markets behave competitively, it can reproduce the joint cyclical movements of the Solow residual and the labor share of income (in particular, volatility, persistence and co-movement). Moreover, we do it in such a way that we preserve the Solow residual as the main driving force of the business cycle and treat innovations in the functional shares as purely redistributive in nature, that is, without productivity level effects.

? stated that 75% of the fluctuations in output can be accounted for by a stochastic neoclassical growth model feeded solely with productivity shocks[4]. Such figure rapidly arose (and continuous to arise) a wide set of criticisms. These criticisms were originally concerned on the technological nature of the Solow residualwhile the more recent quarrel focuses on the response of hours to productivity innovations and it is discussed on methodological grounds[5]. In all cases, the debate on the role of the productivity innovations for the business cycle has disregarded the fluctuations in the functional distribution of income. Our paper deals explicitly with these fluctuations. We follow the advocates of the RBC experiment: We test the explanatory power of the productivity shocks by introducing a redistributive shock through the labor share of income; solve for a extended calibrated stochastic neoclassical growth model with a bivariate shock estimated from the data that statistically reproduces the joint behaviour of the Solow residual and the labor share; and evaluate the predictions of this bivariate shock model with respect to the standard RBC model that identifies the Solow residual with a univariate shock.

We find that our small departure from the standard RBC model implies striking differences in the cyclical behavior of the real allocations. In our bivariate shock economy the volatility of hours drops to 13.5% of the data (32.8% of the standard univariate model) and the volatility of output to 56.6% of the data (69.5% of the standard univariate model), and the co-movement of output with hours also falls to .21 while it is .98 in the univariate model and .88 in the data. In addition, the bivariate model also accounts for the phase-shift of the labor share with output observed in the data. We obtain that the response of hours and, in turn, output to productivity shocks is substantially mitigated by a wealth effect that neither wages nor intertemporal substitution effects are able to offset. Our results hold independently of the Frischian elasticity of labor supply: our bivariate shock modelization under Hansen-Rogerson preferences drops the volatility of hours and output with respect to its univariate counterpart to, in that order, one third and about one half, and the correlation between output and hours drops from .98 in the univariate model to .33 in the bivariate model.

[4]For the sample 1954.I-2004.IV we obtain that such a model accounts for 80% of the standard deviation of output.

[5]See the ongoing discussion in Gali and Rabanal (2004) and Chari, Kehoe and McGrattan (2005).

Few papers place the cyclicality of the factor shares in the RBC lexicon. Importantly, none of these previous exercises accounts for the cyclical volatility and persistence of the labor share and its co-movement with output at once as we do. A first set of these papers builds on optimal labor contracts and the cyclical allocation of risk. ? studies a complete markets economy with workers and entrepreneurs that insure against business cycle income losses through the structure of the firm. They use two different financial arrangements that yield the same real allocations: first, workers' Arrow securities are directly included in the wage bill, and second, workers buy bonds issued by the entrepreneurs and only the insurance component net of workers' savings is added to the wage bill. Either wedge counterbalances the procyclical marginal product of labor and generates a countercyclical labor share of income. The former yields a labor share that is highly negatively correlated with output, while the latter attains a correlation more in the line with the data but with a persistence of the labor share very close to zero. Also, in both cases the volatility of the labor share exceeds that of their observed data by a factor of 1.6 and 2 respectively[6]. Importantly, in this model the labor choice is not affected by movements in the labor share. ? uses contract theory in a model with workers and entrepreneurs where workers are not allowed to self-insure through savings and are more risk averse than entrepreneurs. The optimal contract trades a provision of insurance from entrepreneurs to workers for a more flexible labor supply. These authors find a negative correlation of the labor share with the GNP which is 2.75 times higher than what they observe in the data[7]. Notice that shutting the worker's ability to smooth consumption not only alters factors prices but also the equilibrium allocations. In particular, they find that hours tend to move more (by a factor of 1.08) in their model than in its complete markets counterpart. ? analyzes stylized financial business cycle facts with a risk-sharing model where risk averse workers can not trade financial assets and shareholders are risk-neutral. They introduce a distribution of risk calibrated to generate the cyclical variation of the factor shares observed in the data. However, the model rests silent about the allocation of hours because agents in this model supply labor inelastically.

Models with occasionally binding capacity constraints can also treat the cyclical fluctuations of the labor share. ? introduces variable capacity utilization in an RBC model to study asymmetries generated by binding capacity constraints. In this model small plants face decreasing returns to scale and operate if they satisfy a minimum labor input requirement[8]. Aggregate output is

[6] See Tables 1 and 2 in ?.

[7] The volatility of the labor share in this model is about half as what they observe in the data.

[8] With decreasing returns to scale increases in output are generated by new operating firms if the maximum capacity has not been reached. The labor requirement sets an upper bound for the number of operative plants.

then determined by labor, capital and 'location' capital (which, in equilibrium, is the number of operative plants - all using the same input mix). At full capacity the labor share of income is lower than when some plants remain idle because in the latter case the 'location' capital is not a scarce factor and does not earn income. Since the capacity constraint binds in expansions, the model obtains a countercyclical labor share of income (correlation of -.51). The changes in the cyclical behavior of the real variables is minor with respect to the standard model, in particular, hours are 90% of that of the standard (Hansen-Rogerson) RBC. At odds with our findings, where hours drop to 30% of that of its univariate counterpart.

A third strand of the literature that can deliver cyclical variations in the factor shares is that with an explicit role for markups. With increasing returns to scale, a fixed number of firms in monopolistic competition, and a constant markup, **?** obtains a labor share that is half as volatile as what is observed in the data and that is perfectly and negatively correlated with output. It is noteworthy that in his model the volatility of hours drops to 27% of that of the standard RBC model, see his Table 2 column 3. This is due to a positive overhead cost[9] that creates a negative relation between employment and productivity near the steady state (see his expression (24)). **?** allows for the (not simultaneous) entry and exit of firms and obtain a labor share that co-moves with output similarly to the data while its volatility is 28% that of the data[10].

Our paper is more closely related to **?** which introduces a sole univariate process for the coefficients in the Cobb-Douglas production function and abstracts from productivity shocks in an otherwise standard RBC model[11]. He obtains a countercyclical labor share of -.99. The cyclical behavior of the real variables in his model is, however, sensitive to the capital-labor ratio (and, in turn, to the definition of the labor share). As we discuss below, whenever the capital-labor ratio is not equal to one, shocks to the labor share introduce level effects whose magnitude depends on the units in which the labor input is defined[12]. Consequently, if we recover a structural Solow residual from the model series of output, capital and labor in **?** the properties of this residual do

[9]This overhead cost is a common feature of these models and sets the long-run pure profits to zero.

[10]To deliver cyclical movements of the labor share these models of imperfect competition require the equilibrium profits not to be zero in the short-run. This is achieved in **?** by completely preventing the entry and exit of firms and in **?** by building entries and exits that do not occur simultaneously.

[11]This formalizes a broad idea of biased technical change. Notice that the elasticity of substitution between capital and labor is one at all periods.

[12]We find that when the shocks to the labor share are invariant to scale, the cyclical properties of the real variables in an economy that is feeded only with univariate shocks to the labor share are far off those in the standard RBC model. For example, the output volatility generated is 12% that of the data (independently of the average labor share), and more importantly, the labor share turns highly procyclical (a correlation with output of .99).

not correspond to the measure of the Solow residual obtained from the data.

We begin in Section 2 by describing how we construct the shocks. In Section 3 we estimate these shocks. Section 4 feeds the standard RBC model with the bivariate shock to derive our results and discuss our findings. Section 5 concludes. In the Appendix we lay out in detail the construction of the labor share and explore the sensitivity of our results to alternative definitions of the labor share, preferences, and estimation procedures.

2 The Specification of the Shocks

We start by describing the properties of the Solow residual and its structural interpretation as a shock in Section 2.1. We then describe the properties of labor share in Section 2.2. Finally, we turn to our specification of a joint process that yields both a residual and labor share as a bivariate process in Section 2.3.

2.1 The Standard Specification: Solow residuals as shocks

2.1.1 Obtaining the Solow residual from the data

The Solow residual that we denote S_t^0 is computed from time series of real output, Y_t, real capital, K_t, and labor N_t, and from a specification of a relative input share parameter that we denote by ζ (see **?** or **?**)

$$\ln S_t^0 = \ln Y_t - \zeta \ \ln K_t - (1-\zeta) \ \ln N_t \tag{1}$$

But S_t^0 has trend and we want a trendless object. Consider now a detrending procedure that uses the following linear regression

$$\ln X_t = \chi_x + g_x t + \widetilde{x}_t. \tag{2}$$

where X_t is any economic variable, and where χ_0 and g_x are the regressors and $\widetilde{x}_t$ are the residuals.

Applying such detrending procedure to the Solow residual we obtain a series $\widetilde{s}_t^0$ that is the (detrended) Solow residual that we are interested in.

Alternatively, and for reasons that will be clear later, the Solow residual can be calculated in two steps.

1. Use the detrending procedure described in (2) to obtain $\{\widetilde{y}_t, \widetilde{k}_t, \widetilde{n}_t\}$.[13]

2. Then the Solow residual s_t^0 is defined to be

$$s_t^0 = \widetilde{y}_t - \zeta\, \widetilde{k}_t - (1-\zeta)\, \widetilde{n}_t \tag{3}$$

To see the equivalence between the two definitions note that substituting the residuals of the economics variables in (3) we get

$$\begin{aligned} s_t^0 &= (\ln Y_t - \chi_y - t\, g_y) - \zeta\, (\ln K_t - \chi_k - t\, g_k) - (1-\zeta)\, (\ln N_t - \chi_n - t\, g_n) && (4)\\ &= \ln Y_t - \zeta \ln K_t - (1-\zeta) \ln N_t - (\chi_y - \zeta\chi_k - (1-\zeta)\chi_n) - \\ &\qquad\qquad t(g_y - \zeta g_k - (1-\zeta) g_n) && (5)\\ &= \ln S_t^0 - [\chi_y - \zeta\chi_k - (1-\zeta)\chi_n] - t[g_y - \zeta g_k - (1-\zeta) g_n] && (6) \end{aligned}$$

But s_t^0 is a linear function of residuals so it has mean zero and no trend which implies that $[\chi_y - \zeta\chi_k - (1-\zeta)\chi_n]$- $[g_y - \zeta g_k - (1-\zeta) g_n]$ are indeed the mean and the trend of $\ln S_t$ so $s_t^0 = \widetilde{s}_t^0$.

2.1.2 Giving a structural interpretation to the Solow residual

In the standard RBC model we can also calculate a Solow residual from the model. The nice property is that with Cobb-Douglas technology and provided that we use the right share parameter, the Solow residual is the shock to productity. To see this, consider the following Cobb-Douglas technology with constant coefficients and multiplicative shocks to productivity,

$$Y_t = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, \mu\, N_t\right]^{1-\theta} = e^{z_t^0} A\, K_t^{\theta} \left[(1+\lambda)^t\, \mu\, (1+\eta)^t\, h_t\right]^{1-\theta} \tag{7}$$

where z_t^0 represents a shock that follows a univariate process, and λ is the rate of labor-augmenting (Harrod-neutral) technological change. The labor input, N_t, is the product of the number of agents in the economy, L_t, and the fraction of time that agents devote to market activities, $0 \leq h_t \leq 1$. Population grows deterministically according to $L_t = (1+\eta)^t$. Parameters A and μ just units parameters (it will be clear later why we are posing two different unit parameters).

Note that in the balanced growth path, output Y_t, capital K_t, grow at rate (approximately)

[13] Over the 1954-2004 period, the growth rate of real output and capital are very similar, in that order, 3.29% and 3.12% annually.

$\gamma = \lambda + \eta$, and that if preferences are CRRA, the model economy generates paths for capital and output that can be written as $K_t = (1+\eta)^t (1+\lambda)^t\, k_t$ and $Y_t = (1+\eta)^t(1+\lambda)^t\, y_t$ where both k_t and y_t are stationary. Denote by lower-case-hat log deviations of the variables, *i.e.* $\widehat{x}_t = \log(\frac{x_t}{X^*})$ and with a star the steady state value of the variable, then we obtain

$$\begin{aligned} Y_t &= (1+\eta)^t(1+\lambda)^t\, Y^*\, e^{\widehat{y}_t}, && (8)\\ K_t &= (1+\eta)^t(1+\lambda)^t\, K^*\, e^{\widehat{k}_t}, && (9)\\ N_t &= (1+\eta)^t\, h^*\, e^{\widehat{h}_t}. && (10) \end{aligned}$$

We can rewrite the production function (7) as

$$(1+\eta)^t(1+\lambda)^t\, Y^*\, e^{\widehat{y}_t} = e^{z_t^0}\; A\, [(1+\eta)^t(1+\lambda)^t\, K^*\, e^{\widehat{k}_t}]^\theta\, [(1+\eta)^t(1+\lambda)^t\, \mu\, h^*\, e^{\widehat{h}_t}]^{1-\theta}, \quad (11)$$

cancelling terms

$$Y^*\, e^{\widehat{y}_t} = e^{z_t^0}\; A\, [K^*\, e^{\widehat{k}_t}]^\theta\, [\mu\, h^*\, e^{\widehat{h}_t}]^{1-\theta} \quad (12)$$

and taking logs of (12) and rearranging yields

$$z_t^0 \;=\; \widehat{y}_t - \;\theta\widehat{k}_t - \;(1-\theta)\widehat{h}_t + \ln\frac{Y^*}{AK^{*\theta}\,(\mu h^*)^\theta} \;=\; \widehat{y}_t - \;\theta\widehat{k}_t - \;(1-\theta)\widehat{h}_t \quad (13)$$

where the second equality follows directly from the fact that the denominator of the third term is steady-state output.

If we use model generated data variables to construct a Solow residual with share parameter θ, we obtain in the first step (abstracting from sampling error) that

$$\begin{aligned} \chi_y &= Y^* \qquad & g_y &= \lambda+\eta \qquad & \widetilde{y}_t &= \widehat{y}_t && (14)\\ \chi_k &= K^* \qquad & g_k &= \lambda+\eta \qquad & \widetilde{k}_t &= \widehat{k}_t && (15)\\ \chi_n &= h^* \qquad & g_n &= \eta \qquad & \widetilde{n}_t &= \widehat{h}_t && (16) \end{aligned}$$

The second step yields

$$s_t^0 = \widehat{y}_t - \theta\,\widehat{k}_t - (1-\theta)\,\widehat{h}_t \quad (17)$$

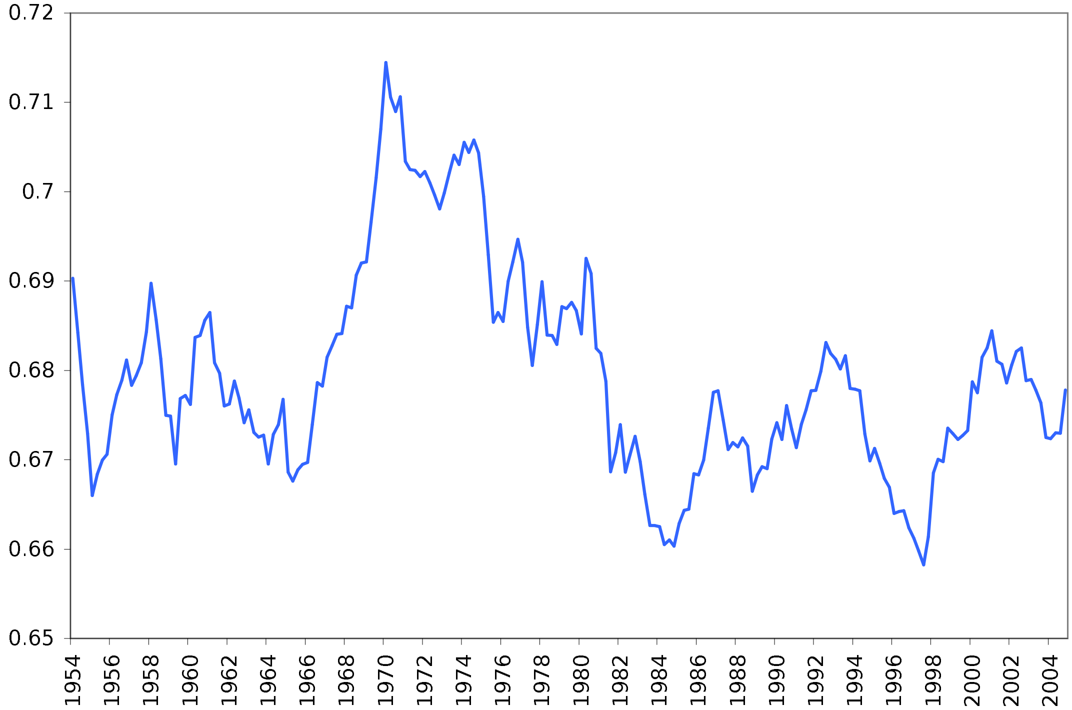

Figure 1: The Labor Share, U.S. 1954.I-2004.IV

but this expression is exactly z_t^0, by equation (13). Which means that we can interpret the Solow residual generated by the data as the multiplicative shock to the production function.

The Cobb-Douglas technology so defined implies under competitive factor markets that factor shares are constant at all frequencies.[14] But are they? We now turn to explore this issue.

2.2 The behavior of labor share

The ratio of all payments to labor relative to output is labor share. Its exact value depends on the details of the definition of output and its partition into payments to labor and payments to capital. Perhaps, the more standard definition of labor share, which is the one that we take as the benchmark, is that proposed by **?** that poses that the ratio of ambiguous labor income to ambiguous income is the same as the ratio of unambiguous labor income to unambiguous income. Another definitions that we explore expand the capital stock and capital services to include durables and government respectively, while a fourth definition sets labor share equal to the ratio of compensation of employees to Gross National Product, which renders all ambiguous income to capital.[15]

[14]See that $\frac{R\,K}{Y} = \frac{\frac{\partial F}{\partial K}K}{Y} = \theta$.

[15]A detailed analysis on the construction of these labor share of income data series is given in the Appendix.

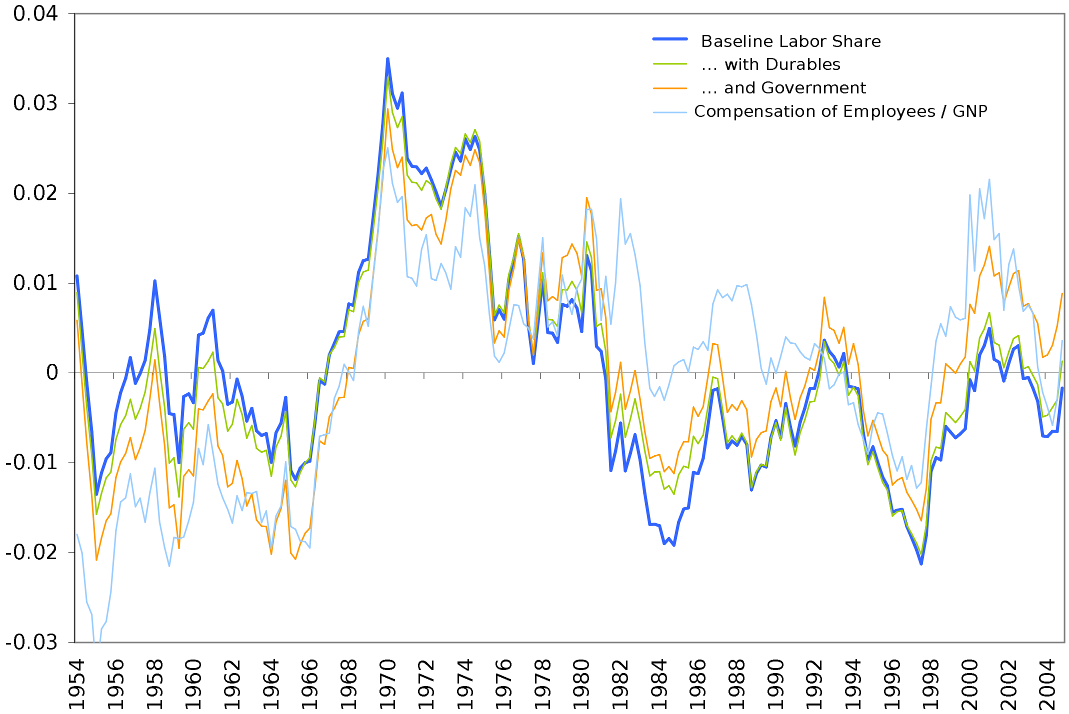

Figure 2: Demeaned Labor Share, U.S. 1954.I-2004.IV

The baseline definition of labor share for the period 1954.I-2002.IV is plotted in Figure 1. It oscillates between a minimum value of 0.66 and a maximum above 0.71. with no discernible trend. The other definitions, while differing on their average have very similar properties as can be seen in Figure 2 that plots their deviations with respect to the mean.

2.2.1 The cyclical behavior of labor share

From the point of view of the study of business cycles, what matters is not whether labor share moves but whether it does so in any systematic way with respect to the main macroeconomic aggregates. Table 1 displays some business cycle statistics (all variables are logged -except for the labor share- and hp-filtered): the standard deviations of output, the Solow residual and labor share are in the first column, these figures relative to output in the second column, the correlations of the Solow residual and labor share with output in the third column and the first autocorrelations in the fourth column. We see that labor share's volatility is a little bit more than half of that of the Solow residual, it is quite persistent, and, perhaps more importantly it is negatively correlated with output albeit not much [16].

[16]This is somewhat different from **?** and **?** because they log the labor share before filtering it. As in **?**, since the labor share is already a ratio we do not take logs.

	σ_x	σ_x/σ_{GNP}	$\rho(x, GNP)$	$\rho(x_t, x_{t-1})$
GNP	1.59	1.00	1.00	.85
Solow Residual: s^0	.85	.53	.74	.71
Baseline Labor Share	.47	.30	-.24	.78
... with Durables	.45	.28	-.21	.77
... and Government	.49	.31	-.26	.78
CE/GNP	.46	.29	-.23	.71

Table 1: Standard deviation and correlation with output of Labor Share, U.S. 1954.I-2004.IV.

	Cross-correlation of GNP_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
Baseline Labor Share	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
... with Durables	-.21	-.26	-.32	-.33	-.20	-.21	.07	.28	.41	.47	.42
... and Government	-.20	-.25	-.31	-.34	-.33	-.26	.03	.27	.42	.48	.44
CE/GNP	-.24	-.30	-.35	-.38	-.31	-.23	.09	.31	.47	.49	.46

Table 2: Phase-Shift of the Labor Share, U.S. 1954.I-2004.IV

Perhaps more important is the phase shift of these variables reported in Table 2. There is a clear pattern. Before the peak of an expansion, labor share is below average with the negative correlation being largest two period s before the peak of output. Subsequently, labor share starts to increase quite above its mean with its maximum value peaking one year after output peaked. In fewer words, labor share lags output by one year or so.

To explore the issue of whether this behavior of labor share has any implication for our understanding of business cycles, we specify a very simply real business cycle model that has a moving labor share. In such a model labor share is posed to be exogenous and stochastic. Given its specific cyclical properties, the process for productivity and for labor share cannot be independent. We now turn to describe how such a model can allow us to give structural interpretations as shocks to objects that can be constructed directly from the data in a very similar fashion to that specified in the previous section.

2.3 A bivariate process that determines factor shares and the Solow residual

We want to pose a stochastic process that simultaneously yields the movements in the labor share and in the Solow residual. Using time series of output Y_t, capital K_t, and labor L_t, and a measure of the labor income $W_t N_t$, we can construct a labor share data series ℓ_t, and in turn, a residual that is as closely related as possible to the Solow residual. As discussed above, the data definition of the labor share is a measure of the labor income divided by output, $\ell_t = \frac{W_t N_t}{Y_t}$, and the deviations of the labor share from its mean are

$$\widetilde{\ell}_t = \ell_t - \ell \tag{18}$$

with $\ell = \sum_t \frac{\ell_t}{T}$.

We now compute a residual as we did in Section 2.1, with one difference: that we use now the time-varying relative input share ℓ_t instead of a constant share parameter. We define the residual s_t^1 as

$$s_t^1(\ell_t) = \widetilde{y}_t - \ell_t\, \widetilde{k}_t - (1 - \ell_t)\, \widetilde{n}_t \tag{19}$$

where as before g_y, g_k and g_h are the slopes of a fitted linear trend to the **logged** original series of output, capital and labor and $\widetilde{y}_t$, $\widetilde{k}_t$ and $\widetilde{n}_t$ are the corresponding residuals.

We now pose a production function with stochastic factor shares but is otherwise a Cobb-Douglas production function

$$Y_t = e^{z_t^1}\ A\ K_t^{\theta - z_t^2}\ \left[\mu\ (1+\lambda)^t\ (1+\eta)^t\ h_t\right]^{1-\theta+z_t^2} \tag{20}$$

where z_t^1 and z_t^2 are the two elements of a bivariate stochastic process and we refer to them as the productivity and the redistributive shock respectfully. We use again parameters A and μ to determine the units of effective labor and to normalize output to one. However, unlike in the previous specification, μ plays now an important role.

Under competitive markets, labor share of income in the model is given by

$$\frac{W_t N_t}{Y_t} = \frac{\frac{\partial Y_t}{\partial N_t} N_t}{Y_t} = (1 - \theta) + z_t^2 \tag{21}$$

But this implies that the deviation from mean labor share is the redistributive shock: $\widetilde{\ell}_t = z_t^2$.

We now turn to the residual. First, divide both sides of (20) by $(1+\lambda)^t (1+\eta)^t$

which yields

$$Y^* \, e^{\widehat{y}_t} = e^{z_t^1} \; A \; \left(K^* \, e^{\widehat{k}_t}\right)^{\theta - z_t^2} \; \left(\mu \, h^* \, e^{\widehat{h}_t}\right)^{1-\theta+z_t^2}, \tag{22}$$

and taking logs we have

$$z_t^1 = \widehat{y}_t - (\theta - z_t^2)\widehat{k}_t - (1 - \theta + z_t^2)\widehat{h}_t + z_t^2 \ln\left(\frac{K^*}{\mu h^*}\right) \tag{23}$$

where we have used $Y^* = AK^{*\theta}\left(\mu h^*\right)^{\theta}$.

Using the equivalences in (14), note now that

$$z_t^1 = s_t^1 + z_t^2 \ln\left(\frac{K^*}{\mu h^*}\right) \tag{24}$$

which means that the units matter: If the units in the model are chosen so that the ratio of capital to effective labor is one then the residual s_t^1 coincides with the shock. This is what we do.

Another way of seeing the role of the choice of units is that if $K^* \neq \mu h^*$ then shocks to factor shares also have implications for productivity. We want to distinguish pure redistributive shocks, that we associate to z_t^2 from productivity shocks that we associate to z_t^1 and the suitable choice of units allows us to do so.

In addition, it turns out that the two residuals that we compute, s_t^0 and s_t^1 are extremely similar as can be in Figure 3. This can also be seen by noting that we can write an expression that links the two residuals s_t^0 and s_t^1 as follows,

$$s_t^1 \;\; = \;\; s_t^0 + \widehat{\ell}_t(\widehat{k}_t - \widehat{h}_t)$$

and that the last term, $\widehat{\ell}_t(\widehat{k}_t - \widehat{h}_t)$, is very small.

We now turn to estimate a parameterization to represent the univariate process z_t^0 and another one for the bivariate process $\{z_t^1, z_t^2\}$.

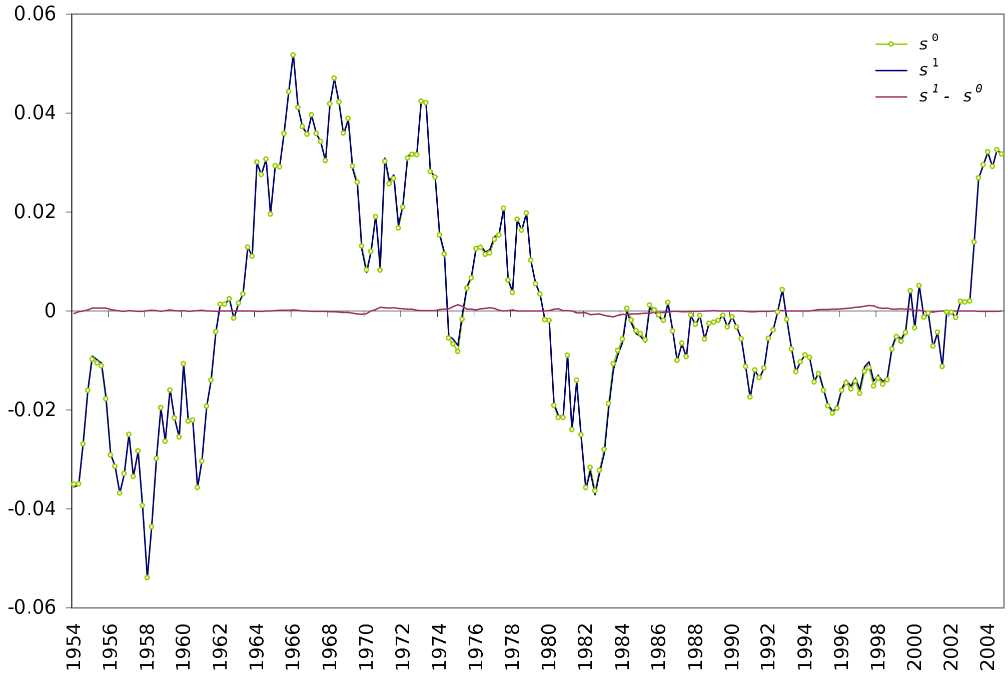

Figure 3: The two sets of productivity residuals s_t^0 and s_t^1, U.S. 1954.I-2004.IV

3 Estimation of a process for the shocks

We start discussing a univariate process for the Solow residual in Section 3.1 and then we move to a bivariate process for the Solow residual and labor share in Section 3.2.

3.1 A univariate process for the Solow residual

While a univariate representation of the Solow residual z_t^0 is one of the most widely used processes, there are very few actual estimations of it, and most authors just use ? calculations. We assume the Solow residual follows an AR(1) process with normally distributed innovations. For the whole sample 1954.I-2004.IV the *full* maximum-likelihood estimation delivers,[17]

$$z_t^0 = \underset{(.020)}{.954}\, z_{t-1}^0 \;+\; \epsilon_t^0, \qquad\qquad \epsilon_t^0 \sim N\,(0, \underset{(.000)}{.00668})$$

Notice that the volatility of the innovations is lower than the value of .00763 originally estimated in ? or the value of .007 used in ?. This is due to the sample period. There has been a reduction

[17]The OLS estimation yields a (biased) regressor coefficient of .947 and a standard deviation of .00667. Despite the high persistence of the process we do not find substantial differences between these estimates and the full maximum likelihood estimates in terms of fluctuations in Y_t, σ_y.

in volatility recently.[18]

3.2 A bivariate process for the Solow residual and labor share

We now pose a statistical model to find an underlying stochastic process that generates the joint distribution of z_t^1 and z_t^2 described in Section 2 using the residuals obtained. In particular, we aim at capturing the volatility and persistence of each series and their observed contemporaneous correlation. We assume the processes to be weakly covariance stationary so that classical estimation and inference procedures apply.

For estimation purposes we specify a vector autoregression model or VAR(n). Thus, we express each variable z_t^1 and z_t^2 as a linear combination of n-lags of itself and n-lags of the other variable.

Lags	Akaike's	Schwartz's Bayesian	Hannan and Quinn
1	-16.207*	-16.167*	-16.108*
2	-16.204	-16.137	-16.039
3	-16.197	-16.104	-15.966
4	-16.190	-16.070	-15.893

Table 3: Lag Selection Order Criteria

Information criteria (Akaike's, Schwartz's Bayesian and Hannan and Quinn, reported in Table 3) suggest that the correct specification is a VAR(1), which we write compactly as

$$z_t = \Gamma \; z_{t-1} \; + \; \epsilon_t, \qquad \epsilon_t \sim N\left(0, \Sigma\right) \tag{25}$$

where $z_t = (z_t^1, z_t^2)'$ and Γ is a 2-by-2 square matrix with generic element γ_{ij}. The innovations $\epsilon_t = (\epsilon_t^1, \epsilon_t^2)'$ are serially uncorrelated and follow a bivariate Gaussian distribution with unconditional mean zero and a symmetric positive definite variance-covariance matrix Σ. Thus, this especification has seven parameters: the four coefficient regressors in Γ, and the variances and covariance in Σ.

The endogenous variables z_t^1 and z_t^2 share the same set of regressors. Thus, we can separately apply the OLS method to each VAR equation and yield consistent and efficient estimates.

[18]For instance, using a similar sample (1955.III-2003.II), **?** obtains an autocorrelation coefficient of 0.95 and a volatility of the innovations of .0065.

Also, with normally distributed innovations, the OLS estimates are equivalent to the *conditional* maximum likelihood estimates. Using the whole quarterly 1954.I-2004.IV sample, the estimated parameters associated with the baseline labor share are

$$\widehat{\Gamma} = \begin{pmatrix} .946 & .001 \\ (.023) & (.042) \\ .050 & .930 \\ (.010) & (.019) \end{pmatrix}, \qquad \widehat{\Sigma} = \begin{pmatrix} .00668^2 & -.1045E-04 \\ -.1045E-04 & .00304^2 \end{pmatrix}$$

Notice that all parameters except γ_{12} are statistically significant. If we restrict the model with $\gamma_{12} = 0$, we obtain a set of constrained estimates similar to those originally unconstrained because the original estimate γ_{12} is already close to zero. We will use the unrestricted statistical model to feed our economic model. The Appendix explores the behavior of the model economy when we use the constrained estimates. The findings reported in Section 4 remain unchanged.

To get a better idea of dynamics of the VAR system we use impulse response functions and forecast error variance decompositions. First, we check that the estimated VAR is stable with eigenvalues .951 and .925 so that we can have a moving average representation of it. Second, since our innovations ϵ_t are contemporaneously correlated, we transform ϵ_t to a set of uncorrelated components u_t according to $\epsilon_t = \Omega\ u_t$, where Ω is an invertible square matrix with generic element ω_{ij}, such that

$$\widehat{\Sigma} = \frac{1}{n}\sum_t \epsilon_t \epsilon_t^{'} = \Omega \left(\frac{1}{n}\sum_t u_t u_t^{'} \right) \Omega^{'} = \Omega\Omega' \tag{26}$$

and we have normalized u_t to have unit variance. Notice that while $\widehat{\Sigma}$ has three parameters, the matrix Ω has four: there are many such matrices. We further impose the constraint that u_t^2 to have a contemporaneous effect on z_t^2 but not on z_t^1, that is, we set Ω to be a lower triangular matrix[19]. This choice follows from the fact that we aim to treat z_t^2 as purely redistributive shocks with no influence on productivity. Our factorization of $\widehat{\Sigma}$ results in

$$\begin{pmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{pmatrix} = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{12} & \omega_{22} \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix} = \begin{pmatrix} .00668 & .0 \\ -.00156 & .00260 \end{pmatrix} \begin{pmatrix} u_t^1 \\ u_t^2 \end{pmatrix}$$

[19]Because $\widehat{\Sigma}$ is positive definite symmetric, it has a unique representation of the form $\widehat{\Sigma} = ADA'$ where A is a lower triangular matrix with diagonal elements equal to one and D is a diagonal matrix. A particularization of this is to set $\Omega = AD^{1/2}$, as we do, which is the Cholesky factorization.

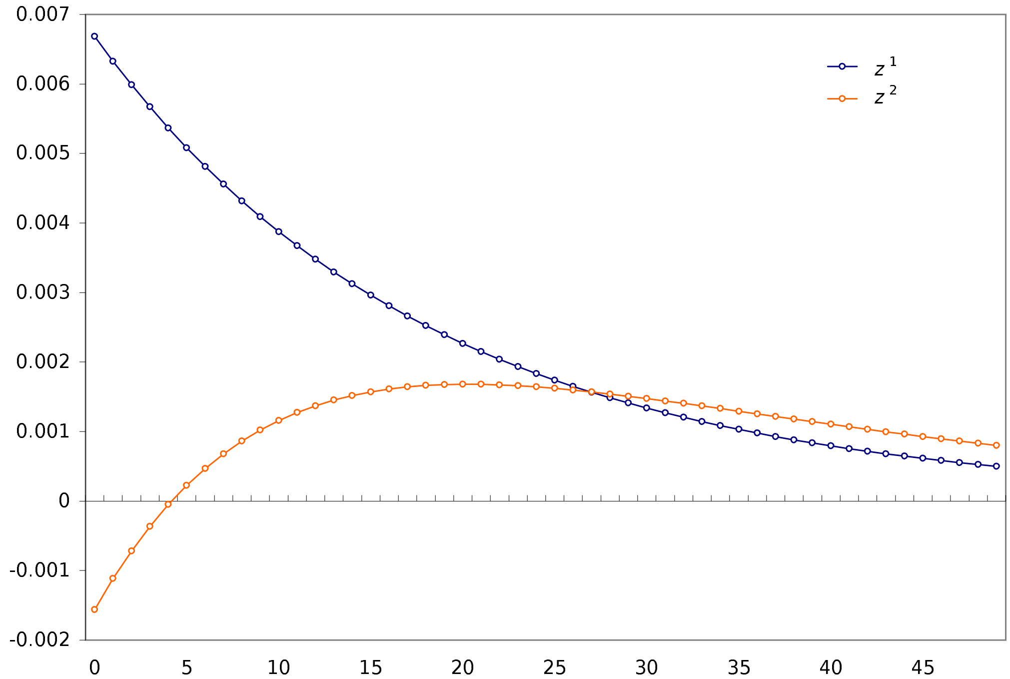

Figure 4: Impulse response functions to Orthogonalized Productivity Innovations ϵ^1.

where $\omega_{11} = \sigma_{\epsilon^1}$, $\omega_{21} = E[\epsilon_t^2|\epsilon_t^1]$, and ω_{22} is the standard error of the regression of ϵ_t^2 on ϵ_t^1.

In Figure 4 we observe the consequences for z_t^1 and z_t^2 if u_t^1 were to increase by one at $t = 0$ and be set to zero afterwards . We find that z_t^1 reacts promptly and positively to this perturbation in its own innovations and that it dies slowly out afterwards, very similarly (if not exactly) as the univariate process z_t^0 does in response to a one-time one-standard-deviation of ϵ_t^0. More interestingly, we find that the labor share of income immediately drops at $t = 0$ by -.156%, from where it raises to be above average after the fifth quarter, reaching a maximum in the after 5 years and approaching monotonically to its unconditional mean afterwards.

We learn the time-path of z_t^1 and z_t^2 derived from a one-time shock $u_0^2 = 1$ in Figure 5. This perturbation results in a labor share above average that monotonically decreases from a maximum attained at $t = 0$. The assumptions made on the purely redistributive nature of z_t^2 and u_t^2 make the response of z_t^1 to redistributive innovations negligible.

Finally, we decompose the variance of z_t^1 and z_t^2 and find that the fluctuations in z_t^1 are 100% due to its own innovations, u_t^1, while 64.6% of the variation in z_t^2 is due to innovations in u_t^1 and 36.4% to its own innovations u_t^2.

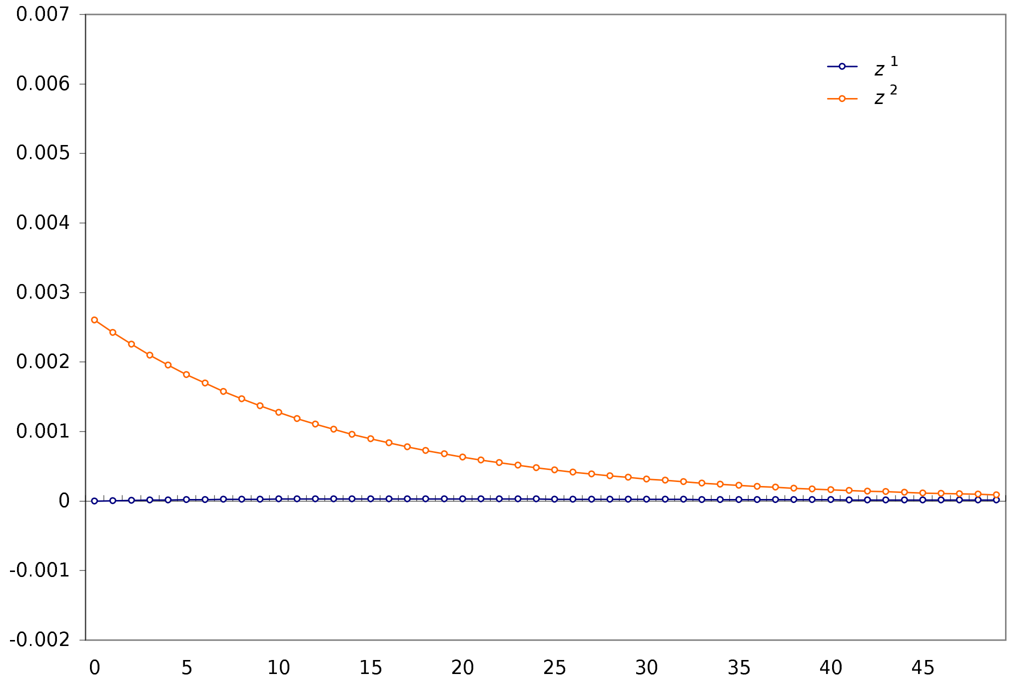

Figure 5: Impulse response functions to Orthogonalized Distributive Innovations ϵ^2.

4 The implications of the specification of the shocks for output and employment fluctuations

In this section we explore the implications of the two alternative specifications of shocks to the production function for the behavior of standard RBC models. Since it is well known that the answer to how important are productivity shocks in generating business cycle fluctuations depends on the labor elasticity, we explore two different sets of preferences with different values for this elasticity. We start specifying the model economies in Section 4.1

4.1 The Model Economies

The economy is populated by a large number of identical infinitely-lived households with the following preferences

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \; L_t \; u(c_t, 1 - h_t) \right\} \tag{27}$$

where c_t is per capita consumption and h_t denotes the proportion of time devoted to work. We choose standard momentary utility functions $u(.,.)$ that imply balanced growth paths. One parametrization that fulfills this requirement is the log-log utility function used in ?.

$$U(c_t, 1-h_t) = (1-\alpha)\log(c_t) + \alpha\ \log(1-h_t) \tag{28}$$

This specification has a Frisch labor elasticity of 2.

The other utility function that we use is the ? log-linear utility function popularized by ? where the linearity in leisure arises from nondivisibilities and the use of lotteries and it generates a very high aggregate labor elasticity (in fact, its Frisch labor elasticity is infinity).

$$U(c_t, 1-h_t) = \log(c_t) + \kappa\ (1-h_t) \tag{29}$$

Population grows at rate η, $L_t = L_0(1+\eta)^t$. Output, Y_t, is used either for consumption or for investment, I_t, and the aggregate stock of capital K_t evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t = (1-\delta)K_t + Y_t - C_t \tag{30}$$

where δ is the geometric depreciation rate.

The production function is as described in Section 2 Cobb-Douglas with labor augmenting technical progress where we consider model economies with univariate shocks z_t^0 and model economies with bivariate shocks z_t^1 and z_t^2. The specification that we posed to obtain the Solow residual and pose it as a univariate process with both productivity and population growth was

$$Y_t = e^{z_t^0} A\ K_t^{\theta} \left[(1+\lambda)^t\ (1+\eta)^t\ \mu\ h_t\right]^{1-\theta} \tag{31}$$

In this model economy the units are irrelevant. Still for consistency across models we choose the so that steady state output is one and the ratio of steady state capital K^* to steady state efficient labor μh^* is also set to one.

The production that we posed to model the bivariate process with productivity and redistributive shocks is

$$Y_t = e^{z_t^1} A K_t^{\theta - z_t^2} \left[(1+\lambda)^t (1+\eta)^t\ \mu\ h_t\right]^{1-\theta+z_t^2} \tag{32}$$

As we saw in Section 2.3 the units matter for this specification. We set again A and μ so that both steady state output and the capital to efficient labor ratio are one. In this fashion, z_t^2 do not have implications for productivity as they are pure redistributive shocks.

We can stationarize the model economies by taking into account population and technological growth. As before, we use small case letters to denote detrended variables and we use small-case hat variables to denote detrended log deviations. With log-log utiliy, in the transformed economy the planner's problem is to solve[20]

$$\max_{\{c_t,k_{t+1},h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \, (1+\eta)^t \left[(1-\alpha)\log\left(c_t\right) + \alpha \log\left(1-h_t\right)\right] \tag{33}$$

subject to

$$c_t + k_{t+1}(1+\eta)(1+\lambda) \;\; = \;\; y_t + (1-\delta)k_t \tag{34}$$

and either

$$y_t \;\; = \;\; e^{z_t^0} \, A \, k_t^{\theta} \, h_t^{1-\theta} \tag{35}$$

or

$$y_t \;\; = \;\; e^{z_t^1} \, A \, k_t^{\theta - z_t^2} \, \left(\mu h_t\right)^{1-\theta+z_t^2} \tag{36}$$

The aggregate shocks, either z_t^0 or $\{z_t^1, z_t^2\}$ follow the processes described in Section 3.

4.2 Calibration

Calibration is very simple in this model since there are only four parameters, θ, δ, β, and α, in addition to the productivity growth rate λ and the population growth rate η, that we choose according to the estimated trends g_y and g_h. Denoting again with by x^* the steady state value of x (with the shocks set to zero–their unconditional mean) we have a system of four equations

[20] In our economies the welfare theorems hold so we can use the planner's problem in lieu of solving for the competitive equilibrium.

	θ	δ	β	α	A	μ	κ	λ
Baseline Labor Share	.320	.019	.988	.668	.109	29.45	2.92	.00367
... with Durables	.375	.019	.982	.649	.108	29.61	2.68	.00363
... with Government	.420	.015	.980	.632	.090	35.78	2.49	.00351
CE/GNP	.430	.019	.976	.628	.109	29.45	2.44	.00367
Cooley-Prescott (1995)	.40	.012	.987	.64	-	-	-	.00387
Hansen (1985)	.36	.025	.990	-	-	-	2.84	-

Table 4: Calibrated Parameters

that when solved yield the value of the four parameters for four targets of the steady state values.

$$(1-\theta)\frac{y^*}{c^*} = \frac{\alpha}{1-\alpha}\frac{h^*}{1-h^*} \tag{37}$$

$$(1+\lambda) = \beta\left[\left(1-\delta+\theta\frac{y_*}{k_*}\right)\right] \tag{38}$$

$$\delta = \frac{i^*}{k^*} - (1+\eta)(1+\lambda) + 1 \tag{39}$$

$$1-\theta = \text{Labor Share}^* \tag{40}$$

The targets that we choose are

1. The fraction of time devoted to market activities: $h^* = 0.31$.
2. The steady-state consumption-output ratio: $c^*/y^* = 0.75$.
3. The capital-output ratio in yearly terms $k^*/y^* = 2.28$.[21]
4. Labor share = 0.679.[22]

For the Hansen-Rogerson version of the model (with indivisible labor), the only equilibrium condition that changes is (37) that is substituted with

$$(1-\theta)\frac{y^*}{c^*} = \kappa h^* \tag{41}$$

[21] This is the target only for the benchmark model economy; it only includes fixed private capital. When we extend measured output with durables this ratio goes to 2.30, and adding government capital we get 2.77.

[22] This is the target only for the benchmark model economy. When we extend measured output with durables this share is 0.624, and 0.579 when we also consider the stock of government capital. It is 0.569 when we use the narrowest definition of labor share that only includes Compensation to Employees in Labor share.

The implied value of the parameters is reported in Table 4. We report both the discount rate and the depreciation rates in quarterly terms and we report for the sake of completion the values of A and μ and the values used in the original sources.

4.3 Findings

We now turn to discuss the main finding of the paper, that posing the productivity shocks as a bivariate process that affects factor shares implies a striking reduction in the volatility of the cycle: Aggregate hours worked are less volatile by a factor of 3.

We start by looking at the business cycle properties of the the U.S. and of the standard and the Hansen-Rogerson preferences RBC economies with both specifications of the shocks in Section 4.3.1. Next, we discuss the reasons for the small cyclical fluctuations of aggregate hours in the bivariate shocks economies in Section 4.3.2.

4.3.1 Business Cycle Properties of the Model Economies

Table 5 reports the business cycle statistics for the main economic variables and factor prices 1954.I-2004.IV in the U.S. and in the model economies with standard log-log preferences. The first thing to note is that in the univariate model economy, productivity shocks account for 81.76% of the standard deviation (66.84% of the variance) of output in the data. In the bivariate model economy shocks account for 56.60% (32.03% of the variance).

However, the most important statistic to measure the ability of the model to generate fluctuations is the standard deviation of hours since output moves both because of hours and because of the shocks. In this respect, the univariate model accounts for 41.02% of the standard deviation of the data (16.83% of the variance). The striking finding is that the bivariate model accounts for 13.46% of the standard deviation of hours in the data (1.81% of the variance). The differential behavior of hours in the bivariate economy also shows up in the correlation between hours and output. While it is very high in the data (.88) and in the univariate shock economy (.98), it is much lower in the bivariate shock economy (.29).

With respect to the other aggregate variables the behavior of consumption is quite surprising: in the economy with bivariate shocks its standard deviation is higher than in the economy with univariate shocks despite having a lower standard deviation of output, a feature that we discuss below. Consequently, the univariate shock economy displays much higher volatility of investment than the bivariate shock economy. Both factor prices are strongly correlated with output in the

	U.S. Data			Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.59**	1.00	.85	**1.30**	1.00	.72	**.90**	1.00	.73
h	**1.56**	.88	.89	**.64**	.98	.71	**.21**	.29	.73
c	1.25	.87	.86	.44	.91	.80	.71	.91	.77
i	7.23	.91	.80	4.05	.99	.71	1.91	.88	.69
r	.08	.74	.78	.05	.96	.71	.06	.68	.70
w	.76	.08	.70	.69	.98	.75	.78	.87	.77
z^0, z^1	.85	.74	.70	.87	.99	.71	.87	.98	.71
z^2	.47	-.24	.78	-	-	-	.42	-.27	.72

Notes: Data are obtained from NIPA-BEA: real GNP from Table (1.7.6) and real personal consumption expenditures and real gross private domestic investment from Table (1.1.6). The series of hours uses CES data, see Appendix A. The series of factor prices are constructed as w = Labor Share $\times$ Output/Hours and r = (1 $-$ Labor Share) $\times$ Output/Capital. All variables have been logged (except the rate of return and labor share, which are already in percentages) and hp-filtered.

Table 5: Cyclical Behavior of the U.S. Data 1954.I-2004.IV, and log-log Utility RBC Models with Univariate and Bivariate Shocks

univariate model economy and less so in the bivariate model economy. Finally, the behavior of both residuals is very similar and they are very correlated with output (recall that the residuals are virtually identical across economies, but output is not). While the univariate model economy does not display movements in labor share, the bivariate economy does and like in the data they are negatively correlated with output.

Table 6 shows the phase-shift of the variables [[in the data and]] in both economies. The behavior of hours is quite different between the two economies: While in the univariate economy hours are very procyclical and they have a slight lead in the cycle, in the bivariate economy hours are quite flat and they lag the cycle. In both economies, consumption lags the cycle and investment leads it, although not by much.

The behavior of rates of return is also quite different. In the univariate economy they are quite strongly correlated with output, they lead the cycle and they do not become negative until a year after output peaks. In the bivariate economy they are less correlated, the lead the cycle and they become negative three quarters after output peaks. Wages are very correlated with output in the univariate economy, and they lagged somewhat while in the bivariate economy they are less correlated with output and they slightly lag the cycle. Overall, the behavior of wages is more similar across the two economies than that of rates of return.

	Cross-correlation of y_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
	U.S. Data 1954.I-2004.IV										
y	-.04	.14	.37	.63	.85	1.00	.85	.63	.37	.14	-.04
h	-.22	-.06	.15	.40	.67	.88	.91	.81	.63	.41	.21
c	.14	.33	.51	.70	.84	.87	.71	.50	.26	.03	-.14
i	.05	.20	.39	.60	.79	.91	.75	.51	.24	-.01	-.22
r	.16	.31	.48	.63	.73	.74	.47	.17	-.09	-.29	-.40
w	.18	.19	.19	.17	.10	.08	-.06	-.11	-.15	-.12	-.12
s^0, s^1	.25	.39	.54	.68	.73	.74	.39	.08	-.18	-.33	-.43
ℓ	-.20	-.26	-.32	-.34	-.33	-.24	.03	.25	.40	.47	.44
	Univariate Model $\{z_t^0\}$										
y	-.01	.11	.27	.46	.70	1.00	.70	.46	.27	.11	-.01
h	.08	.20	.34	.52	.73	.98	.63	.35	.14	-.03	-.15
c	-.21	-.09	.07	.29	.56	.91	.77	.63	.50	.37	.26
i	.05	.17	.32	.50	.72	.99	.65	.39	.18	.02	-.10
r	.12	.24	.37	.54	.73	.96	.58	.30	.08	-.09	-.20
w	-.10	.02	.19	.40	.66	.98	.75	.55	.37	.23	.10
z_t^1	.01	.13	.28	.48	.71	1.00	.69	.44	.24	.08	-.04
	Bivariate Model $\{z_t^1, z_t^2\}$										
y	-.01	.12	.28	.47	.72	1.00	.72	.47	.28	.12	-.01
h	-.13	-.09	-.03	.05	.16	.29	.29	.27	.24	.20	.16
c	-.12	.00	.16	.36	.61	.91	.74	.57	.42	.28	.16
i	.11	.22	.35	.50	.68	.88	.53	.26	.05	-.10	-.21
r	.16	.25	.34	.44	.55	.68	.35	.10	-.07	-.20	-.28
w	-.13	-.01	.14	.33	.58	.87	.71	.56	.41	.28	.17
z_t^1	.03	.16	.31	.50	.72	.98	.67	.41	.20	.04	-.08
z_t^2	-.19	-.22	-.24	-.26	-.27	-.27	-.05	.10	.20	.26	.29

Table 6: Phase-Shift of the Model Economies

	U.S. Data			Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.59**	1.00	.85	**1.74**	1.00	.71	**.92**	1.00	.73
h	**1.56**	.88	.89	**1.28**	.98	.70	**.41**	.33	.72
c	1.25	.87	.86	.54	.88	.81	.74	.94	.77
i	7.23	.91	.80	5.58	.99	.70	1.74	.91	.69
r	.08	.74	.78	.06	.95	.70	.06	.61	.70
w	.76	.08	.70	.54	.88	.81	.74	.94	.77
z^0, z^1	.85	.74	.70	.87	.99	.71	.87	.94	.71
z^2	.47	-.24	.78	-	-	-	.42	-.13	.72

Table 7: Cyclical Behavior of the U.S. Data 1954.I-2004.IV, and of the Hansen-Rogerson RBC Models with Univariate and Bivariate Shocks

The Rogerson-Hansen Economies Table 7 reports the business cycle statistics for data and the Hansen-Rogerson log-linear preferences with univariate and bivariate shocks. As it is well-known, the higher elasticity of hours of this model generates a larger response to the shocks. The economy with univariate shocks displays 82.05% of the standard deviation of hours and 109.43% of output [[observed in the data]] (67.32% and 119.75% of the variance respectively) . However when we turn to the cyclicality of the bivariate model economy, the reduction is spectacular. The standard deviation of hours is now 26.28% of that in the data (6.90% of the variance), [[that is,]] the bivariate process generates a 32.03% of the standard deviation of the univariate process (10.26% of the variance). As in the log-log economy, consumption is more volatile ((than investment)) in the bivariate shock than in the univariate shocks, [[and investment less volatile]].

We avoid the cumbersome reporting of all the features of the Hansen-Rogerson economy, but the picture is clear. As it is well known, the higher elasticity of hours of these preferences translate in a much higher volatility of hours worked. However, posing the productivity shocks in the bivariate way that we are exploring in this paper dramatically dampens the volatility of hours worked. It does so in a similar or more dramatic fashion than it does to the economy with a lower elasticity of hours worked (the standard deviation of hours is less than a third than that of the univariate shock) and for similar reasons that we will explore next.

4.3.2 Why do hours move so little in the bivariate economies?

The key question now is why does such a seemingly small departure from the standard model generates such a large change in the behavior of aggregate hours.

	y	h	c	i	r	w	z^1	z^2
u^1	98.9	54.3	95.6	94.1	72.3	93.2	100.0	63.6
u^2	1.1	45.6	4.5	5.9	27.7	6.8	.0	36.4

Table 8: Forecast Error Variance Decomposition (%)

We find it useful to decompose the exploration of what happens into three parts:

((first is how the different behavior of wages and interest rates in both economies vary and how they imply different allocations, and second, how the two sets of shocks yield different paths for wages and interest rates)).

[[first is how the two sets of shocks yield different paths for hours; second, how wages and interest rates in the bivariate and univariate economies vary and how they imply different allocations of hours; and third, how robust the univariate economy is to the introduction of the bivariate factor prices.]]

[[Our discussion ends with the complementary analysis of consumption.]]

Hours response to productivity and redistributive innovations. Figure 6 shows the impulse response of hours to innovations to all three shocks in percentage deviations from the steady state. We see that a one standard deviation innovation to the only shock, e^0, in the univariate model increases hours by .48%. In addition, the response of hours dies out pretty rapidly. In the bivariate shock economy the situation is quite different. There is barely any immediate response of hours to a current innovation in the productivity shock, u^1, and the response is delayed dramatically as it increases for about 18 quarters (still not to a very high level, .09%) before coming up down. A redistributive shock u^2 towards labor increases hours initially by .16% (about a third of that of the level of a productivity shock in the univariate economy), and it dies out quite slowly.

Table 8 displays a variance decomposition of the main variables by the source of the innovation. We see that while for most variables most of the variance is due to the innovation to productivity (98.9% for output and 95.6% for consumption), the variance of hours is due in almost equal measure to both innovations. Innovations to the redistributive shock also have important effects on interests rates and less so to wages. In addition, ((note that given the orthogonalization of the innovations that we chose, 36.4% of the variance of the redistributive shock itself is due to the redistributive innovation.))

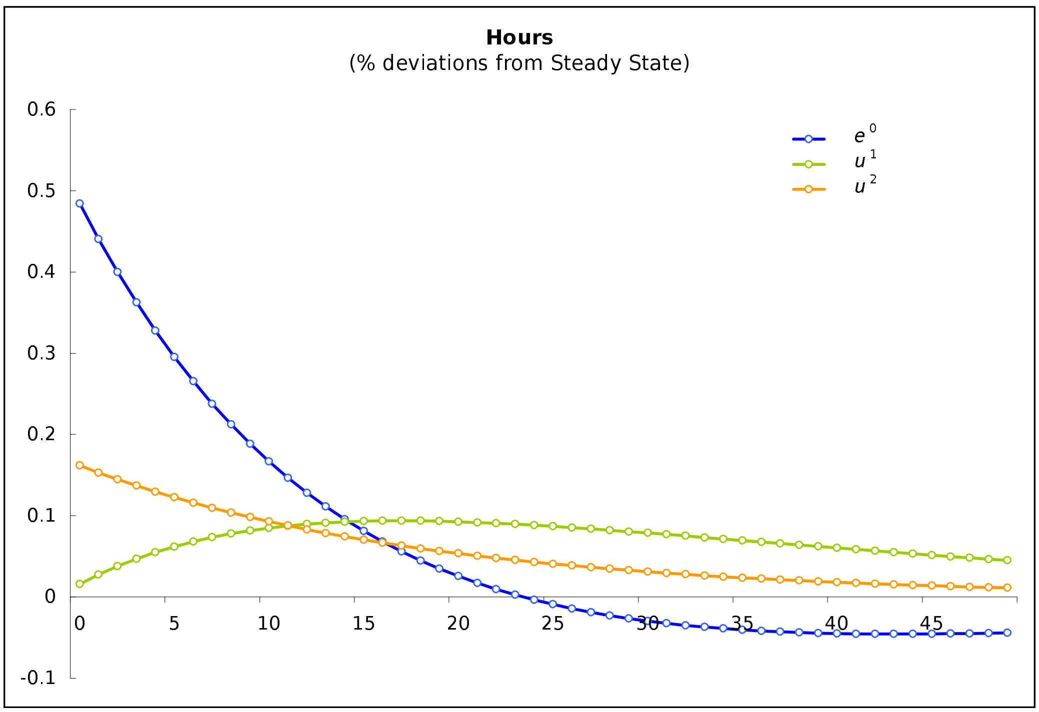

Figure 6: Hours impulse response functions to innovations to all shocks.

	Bivariate $\{z^0\}$			Bivariate with u_t^1 alone			Bivariate with u_t^2 alone		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**.92**	1.00	.73	**.89**	1.00	.72	**.14**	1.00	.69
h	**.41**	.33	.72	**.06**	.47	.95	**.21**	.99	.70
c	.74	.94	.77	.63	.96	.78	.33	.99	.69
i	1.74	.91	.69	1.86	.95	.68	.43	-.99	.69
r	.06	.61	.70	.06	.88	.69	.03	-.99	.69
w	.74	.94	.77	.65	.94	.79	.42	.99	.70
z^1	.87	.94	.71	.87	.99	.70	.00	.00	.95
z^2	.42	-.13	.72	.24	-.70	.74	.33	.99	.69

Table 9: Cyclical Behavior of log-log Utility RBC Models Bivariate Shocks, u1 alone and u2 alone

[[We further investigate the contribution of each shock to the cyclical behavior of each series computing a bivariate economy with productivity innovations alone and a bivariate economy with redistributive innovations alone. In practice, we consider only productivity innovations in the bivariate economy by setting $\omega_{22} = 0$, while for a bivariate economy in which only redistributive innovations are at play we set $\omega_{11} = \omega_{21} = 0$. The business cycle statistics of these economies are reported in Table 9 and the corresponding phase-shifts in Table 10.]]

[[When the bivariate economy is driven solely by productivity innovations we find that the volatility of hours falls to .06% about one third that of the bivariate model that receives both innovations (and one sixth of the univariate model), and the correlation of hours with output is of .47. Note that with productivity innovations alone we still yield movements in the labor share through ω_{21}. We find that the labor share is less volatile than in the data, and it is highly countercyclical, -.70. In this case, the effect of $\omega_{21} (< 0)$ is not counterbalanced by positive redistributive innovations, what strengthens the mechanisms that dampen the volatility of hours. The volatility of the rest of the variables resembles the bivariate model, though they present higher correlation with output. When only redistributive innovations are present in the bivariate economy, the volatility all real allocations is largely dampened except that of hours and the labor share[23], and all variables display a high (either positive or negative) correlation with output. In particular, it is noteworthy that the labor share turns highly procyclical.]]

[23] Notice that the volatilities of bivariate economies with u^1 alone and u^2 alone do not add up to volatilities in bivariate economy where both innovations are present. This is so because although u^1 and u^2 are orthogonal, the responses of the variables to each shock are not.

	Cross-correlation of y_t with										
	x_{t-5}	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}	x_{t+5}
	Bivariate Model $\{z_t^1, z_t^2\}$										
y	-.01	.12	.28	.47	.72	1.00	.72	.47	.28	.12	-.01
h	-.13	-.09	-.03	.05	.16	.29	.29	.27	.24	.20	.16
c	-.12	.00	.16	.36	.61	.91	.74	.57	.42	.28	.16
i	.11	.22	.35	.50	.68	.88	.53	.26	.05	-.10	-.21
r	.16	.25	.34	.44	.55	.68	.35	.10	-.07	-.20	-.28
w	-.13	-.01	.14	.33	.58	.87	.71	.56	.41	.28	.17
z_t^1	.03	.16	.31	.50	.72	.98	.67	.41	.20	.04	-.08
z_t^2	-.19	-.22	-.24	-.26	-.27	-.27	-.05	.10	.20	.26	.29
	Bivariate Model $\{z_t^1, z_t^2\}$ with u_t^1 alone										
y	.00	.13	.29	.48	.72	1.00	.72	.48	.29	.13	.00
h	-.41	-.33	-.22	-.05	.18	.47	.62	.70	.71	.68	.61
c	-.13	.00	.17	.37	.64	.96	.79	.62	.47	.34	.21
i	.13	.25	.38	.54	.73	.95	.58	.29	.07	-.09	-.21
r	.20	.31	.42	.56	.71	.89	.48	.17	-.06	-.21	-.32
w	-.14	-.01	.15	.36	.62	.95	.79	.63	.49	.35	.23
z_t^1	.05	.17	.32	.51	.73	.99	.68	.42	.22	.06	-.07
z_t^2	-.32	-.39	-.47	-.54	-.62	-.70	-.25	.07	.28	.43	.51
	Bivariate Model $\{z_t^1, z_t^2\}$ with u_t^2 alone										
y	-.02	.09	.25	.45	.70	1.00	.70	.45	.25	.09	-.02
h	-.06	.06	.22	.42	.68	.99	.72	.49	.30	.14	.02
c	-.01	.11	.27	.46	.70	.99	.69	.44	.23	.07	-.04
i	-.01	-.13	-.28	-.48	-.71	- .99	-.67	-.41	-.21	-.05	.07
r	.02	-.10	-.26	-.46	-.70	- .99	-.69	-.44	-.24	-.08	.03
w	-.02	.10	.26	.46	.70	.99	.69	.45	.25	.09	-.03
z_t^1	-.51	-.50	-.45	-.36	-.21	.00	.30	.49	.60	.65	.64
z_t^2	-.04	.08	.24	.44	.69	.99	.70	.46	.27	.11	-.01

Table 10: Phase-Shift of Bivariate Economies

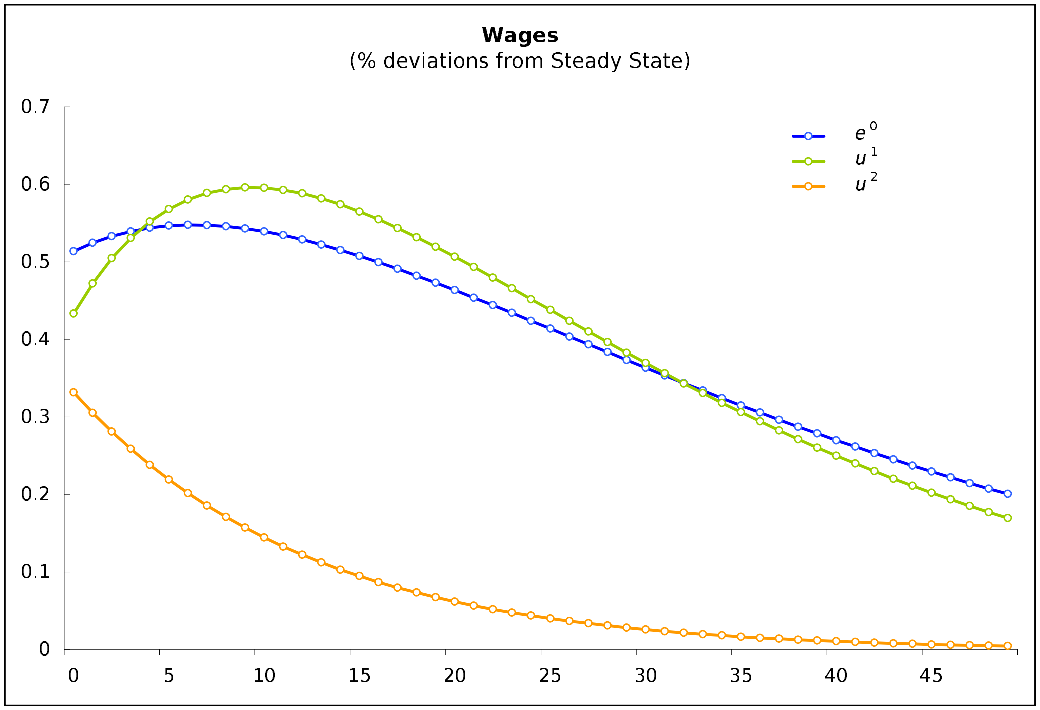

Figure 7: Wage impulse response functions to innovations to all shocks.

[[Implications of wages and interest rates for the allocation of hours]]((Hours and wages in the univariate and bivariate economies)). Agents have the same preferences in both the univariate and bivariate economies which means that if they do different things it is due to the fact that they face different wages and interest rates.

Figures 7 and 8 respectively plot the impulse response functions of the real wages and the interest rate (actually, tomorrow's rate of return) to productivity innovations [[and redistributive innovations in percentage deviations from the steady state. What is important to notice is that the response of wages to productivity innovations displays a clear hump-shaped pattern in the bivariate economy, not so in the univariate economy. After a productivity innovation agents in the bivariate economy face a large and continuous raise in wages for the following 9 quarters from an initial deviation of .43% at $t = 0$ to .60% (that is, 1.4 times the original deviation) after two years and one quarter. In the univariate economy, however, wages remain almost flat for the first three years, they respond initially deviating by .51% and barely increase to .55% after one year and a half. When wages respond to redistributive innovations they do so positively, initially they deviate from the steady state by .33% and die out monotonically afterwards. The rate of return increases initially in response to productivity innovations by .03% in the bivariate economy and .033% in the univariate economy, but it declines more steeply in the former. This way, while it falls below its steady state after 1 years in the bivariate economy, it does so one year later

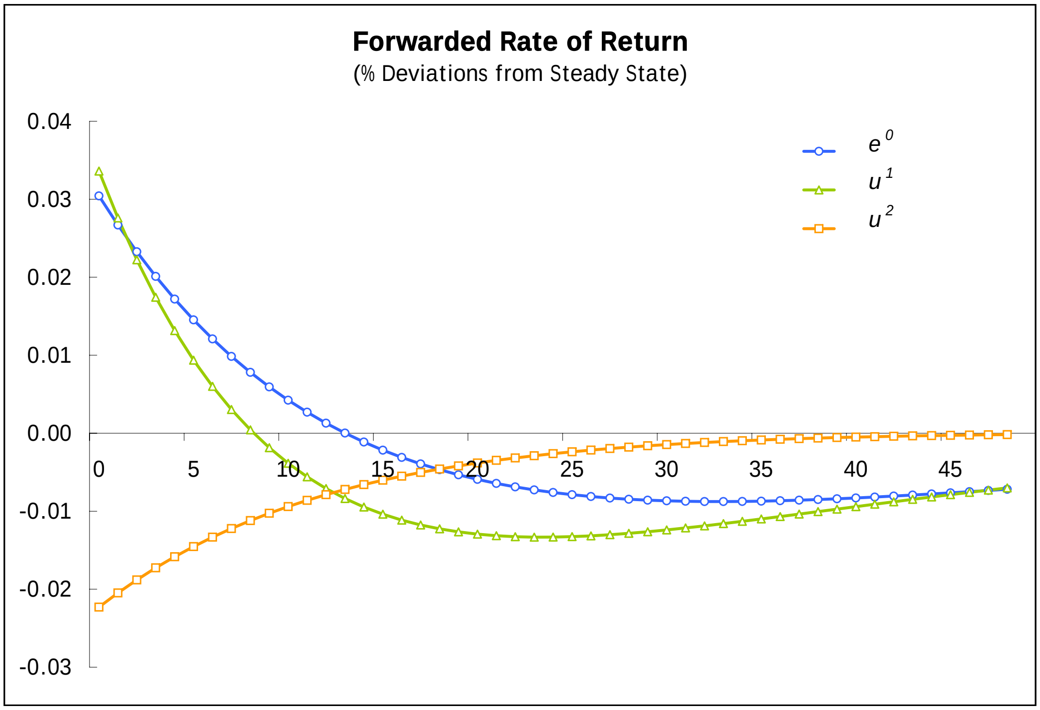

Figure 8: Rate of return impulse response functions to innovations to all shocks.

in the univariate economy. In response to redistributive innovations the rate of return remains always below its steady state, it drops to -.022% at prompt and increases monotonically towards its long-run value afterwards.]]

These movements in the price of factors alter the relative reward of factor inputs (substitution effects) and also alter the total resources of the agents (wealth effects). Furthermore, agents consider the relative importance of present and future by looking at the whole time-path of factor prices what introduces intertemporal substitution effects through the (inverse of the) rate of return that we use to discount the future. To investigate these effects we find convenient to write out the labor supply function explicitly in terms of present and future wages and interest rates. Then we isolate the contribution of each of these effects by means of 'Slutsky decomposition' of hours. This involves a lump-sum transfer to agents at $t = 0$ in order to control for the wealth effects by keeping the original equilibrium allocations just feasible at the new prices[24].

To derive the labor supply function we first consolidate the budget constraint at $t = 0$,

$$\sum_{t=0}^{\infty} \frac{(1+\gamma)^t c_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t(1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \sum_{t=0}^{\infty} \frac{(1+\gamma)^t w_t}{\prod_{s=1}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k_0 \quad (42)$$

[24] Alternatively, **?** and **?** use a 'Hicksian decomposition' that compensates agents by placing them back to their original indifference curve.

where we have used the transversality condition, $\lim_{T\to\infty} \frac{k_T}{\prod_{s=t}^{T}(1+r_s-\delta)} = 0$. The left hand side is the present value of all future expenditures on consumption and leisure and the right hand side is the present value of total resources (wealth) accumulated from period $t = 0$ onwards. Total resources are composed by the sum of the human wealth, that is, the first term in RHS(42) that we denote by HW_0, and the initial capital income evaluated in units of $t = 0$ consumption. We use the first order condition for labor to substitute out consumption c_t in LHS(42), and then we use the euler equation to rewrite the present value of expenditures as

$$\frac{1}{\alpha}\sum_{t=0}^{\infty}\frac{(1+\gamma)^t w_t(1-h_t)}{\prod_{s=1}^{t}(1+r_s-\delta)} = \frac{1}{\alpha}\sum_{t=0}^{\infty}\beta^{t-1}w_0(1-h_0) = \frac{w_0(1-h_0)}{\alpha(1-\beta)} \tag{43}$$

Now, we can plug (43) into (42) and rearrange to find the initial response of leisure, $w_0(1-h_0) = \alpha\ (1-\beta)\ \ (HW_0 + (1+r_0-\delta)k_0)$, and using the euler equation we can recursively find

$$\frac{(1+\gamma)^t w_t(1-h_t)}{\beta^t\prod_{s=1}^{t}(1+r_s-\delta)} = \alpha(1-\beta)\left(HW_0 + (1+r_0-\delta)k_0\right) \tag{44}$$

That is, the present value of the expenditure on leisure at period t is a constant share of the present value of total resources. This constant share is the marginal propensity to consume leisure, α, and per period, $1-\beta$.

If we log-linearize (44) around the steady state we find that the deviation of period-t hours from the steady state can be decomposed as a linear combination of the deviations of period-t wages, the present value of one unit of period-t consumption and the present value of total resources[25]:

$$\widehat{h}_t = \left(\frac{1-h^*}{h^*}\right)\left[\widehat{w}_t + \left(\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}\right) - (HW_0 + \widehat{(1+r_0}-\delta)k_0)\right] \tag{45}$$

where the constant $\frac{1-h^*}{h^*} = 2.2$ is the Frischian elasticity of labor supply. As we discuss next, the identity (45) decomposes the overall response of hours to all innovations through wage effects (intratemporal price-substitution effects), rate of return effects (intertemporal substitution effects), and total resources effects (wealth effects).

Wage effect: To see how wages alone contribute to the response of hours we set the rate of return to the steady state and allow only for movements in wages in the bivariate and univariate

[25]Notice that $(\widehat{1-h_t}) = -\left(\frac{h^*}{1-h^*}\right)\widehat{h}_t$.

economies. However, wages not only affect hours directly but also act through the amount of total resources. One way to disentangle these two effects of wages is to add a 'Slutsky' transfer compensation that constraints agents to purchase at most the original bundle with the new wages. Since the original bundle is that of the steady state, the compensated wage effect calls for a transfer compensation that sets the total resources equal to the steady state[26]:

$$T(w_t, r^*) = \sum_{t=0}^{T} \frac{(1+\gamma)^t (w_t - w^*)}{(1 + r^* - \delta)^t}$$

If we provide agents with this transfer at $t = 0$ the 'compensated wage effect' on hours is given solely by $\frac{1-h^*}{h^*}\widehat{w}_t$, that is, the response of wages amplified by the elasticity of labor supply, which we plot for all innovations in the top panel of Figure 9. We find that in the bivariate economy the wage effect of productivity innovations generates an initial deviation of hours from steady state of .96% which keeps raising until 1.32% in the 9th quarter and slowly dies out to the the steady state afterwards. In the univariate economy the wage effect raises hours initially more to 1.14% of the steady state value but remains practically flat to start decreasing after having reached 1.21% in the 7th quarter. A distributive innovation raises hours to .74% at prompt and dies monotonically out afterwards. Overall, the size of the wage effects in the bivariate and univariate economies (with a maximum difference for productivity innovations of .17% reached at $t = 0$, that is, 15% of the univariate initial deviation) suggest that the wage effect can not account alone for the full drop in the volatility of hours in the bivariate economy.

Rate of Return Effect: Agents also care about when they consume and work. In doing so, agents compare allocations at different periods by transforming them into the same units through the market discount factor, which, if deviating from the steady state, introduces intertemporal substitution effects. In addition, changes in the rate of return also alter the present value of the human wealth and capital gains at $t = 0$, for which we introduce a transfer compensation that keeps total resources unchanged.

$$T(w^*, r_t) = \sum_{t=0}^{T} \frac{(1+\gamma)^t w^*}{\prod_{s=0}^{t}(1 + r_s - \delta)} + (1 + r_0 - \delta)k^* - \sum_{t=0}^{T} \frac{(1+\gamma)^t w^*}{(1 + r^* - \delta)^t} - (1 + r^* - \delta)k^*$$

With this transfer, if we fix wages to the steady state, the compensated rate of return effect on hours is given by $\frac{1-h^*}{h^*}\widehat{\frac{1}{\prod_{s=1}^{t}(1+r_s-\delta)}}$. This effect is plotted in the center panel of figure 9 for

[26]The horizon of the human wealth is set to $T = 200$, large enough to ensure the economy has come up back to the steady state.

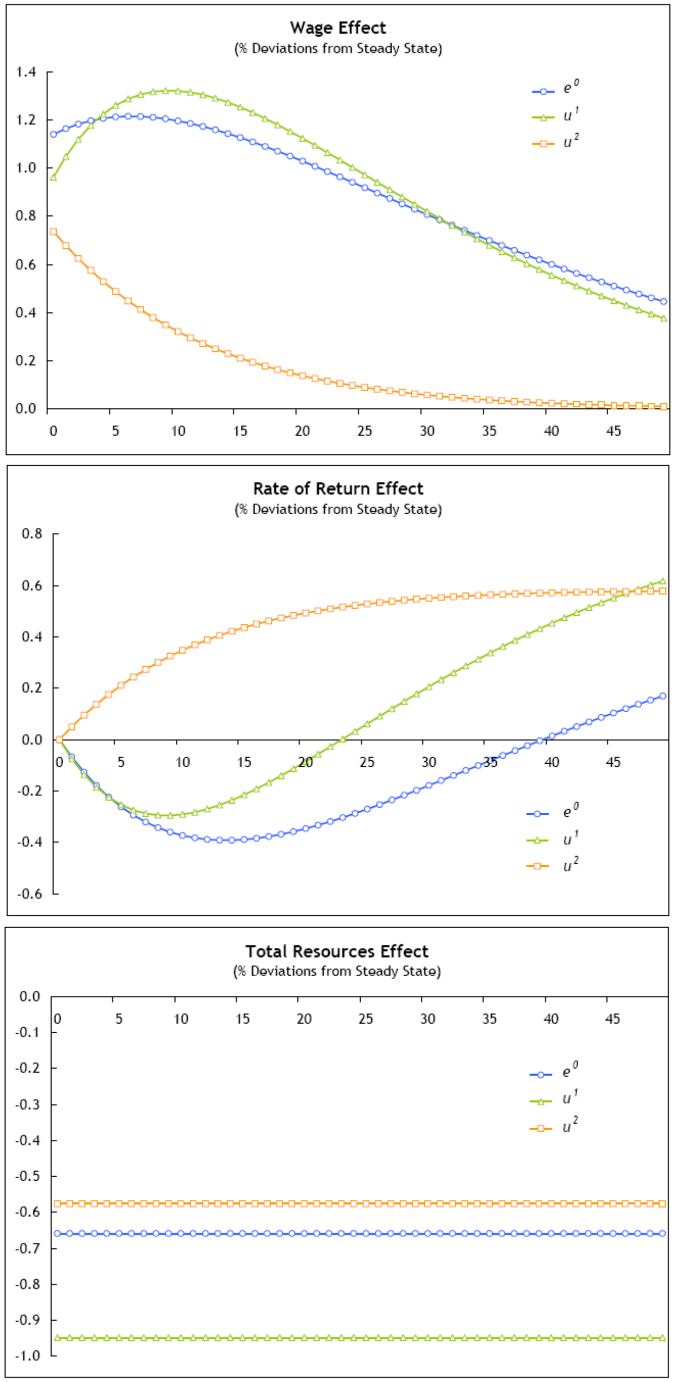

Figure 9: Slutsky Decomposition of Hours: Wage, Rate of Return and Total Resources Effect

all innovations. We find that productivity innovations in the univariate and bivariate economy initially cut the present value of future units of consumption (and therefore leisure) what reduces the incentive to work at these early periods. The supply of hours remains below its steady state value for the first 6 years in the bivariate economy and for the first 10 years in the univariate economy. It is only after the first year that in the bivariate economy it becomes more expensive to place future consumption into present than in the univariate economy, and therefore the incentive to work due to the rate of return effect remains higher from then on in the bivariate economy than in the univariate economy. The rate of return effect of a distributive innovation sets the supply of hours above its steady state from $t = 0$ and onwards, this is so because the rate of return falls below its steady state value at all periods in response to redistributive innovations and therefore it puts up the price of one unit of consumption (and leisure) (measured in $t = 0$ units) above the steady state.

Importantly, notice that the intertemporal substitution effect sets the long-run supply of hours above its steady state value. This is so because although the rate of return does converge to the steady state, the market discount factor accumulates all deviations of the rate of return. That is, whenever the rate of return deviates from the steady state the present value of one unit of future consumption will always differ of that of an economy had the rate of return not deviated. For all our innovations, the response of the rate of return is such that the long-run present value of one unit of consumption is higher than in the original steady state. Consequently, the rate of return effect sets long-run hours above the steady state. What brings back the limit of hours to the steady state value is the total resources effect that we discuss next.

Total Resources Effect: We compute the realized change in the total resources from the steady state as,

$$T(w_t, r_t) = \sum_{t=0}^{T} \frac{(1+\gamma)^t w_t}{\prod_{s=0}^{t}(1+r_s-\delta)} + (1+r_0-\delta)k^* - \sum_{t=0}^{T} \frac{(1+\gamma)^t w^*}{(1+r^*-\delta)^t} - (1+r^*-\delta)k^*$$

To measure the effect of this change alone on hours we hold constant wages and the rate of return and add this transfer $T(w_t, r_t)$ to the agents at $t = 0$. This is given by the term $\frac{1-h^*}{h^*}(HW_0 + \widehat{(1+r_0} - \delta)k_0)$. We obtain that the present value of total resources that agents have at their disposal raises for all innovations. Agents with log-log preferences optimally deplete this extra amount of wealth on consumption and leisure and they do so in equal (present value) terms per period and for all periods what generates constant deviations of hours from steady state and that we depict in the bottom panel in figure 9. What is important to notice is the size

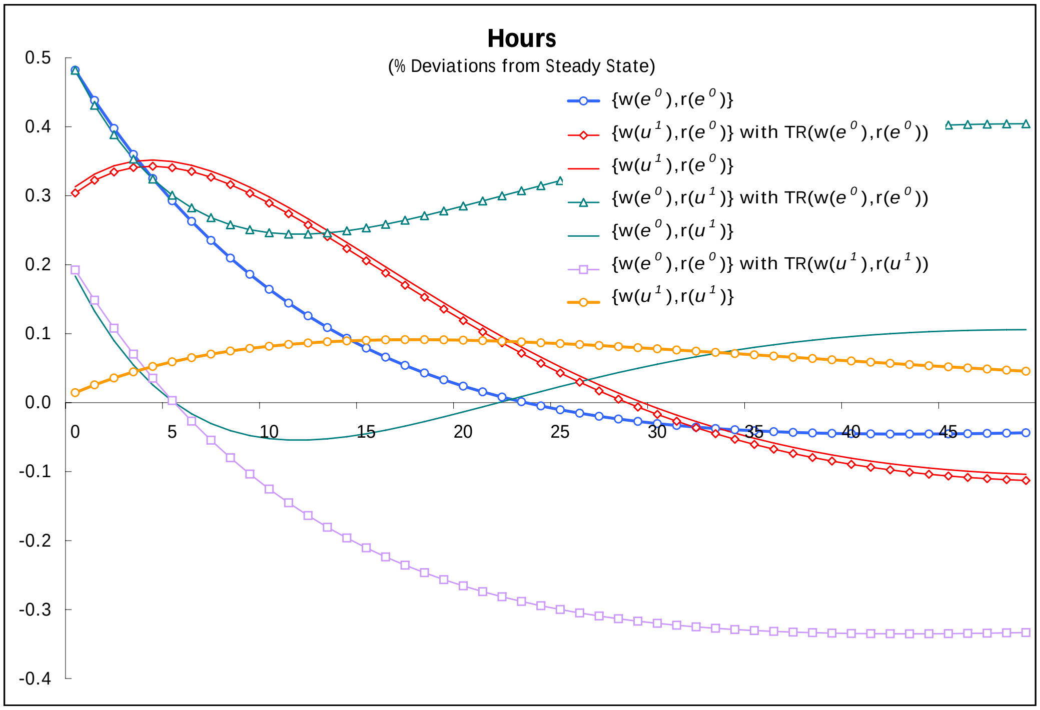

Figure 10: Hours choices for crossed factor prices that respond to productivity innovations.

of the wealth effect: under productivity innovations hours deviate in the bivariate economy by -.95%, which is 1.44 times the deviation of hours generated by the wealth effect in the univariate economy, -.66%. For distributive innovations the total resources effect on hours is -.57%.

Robustness of the univariate economy to the bivariate factor prices. To gain insight on the reduction in the volatility of hours in the bivariate economy with respect to the univariate economy we compute how the choice of hours changes when we introduce into the univariate economy the factor prices that respond to productivity innovations in the bivariate economy. In this case, we control for the total resources effect with a lump-sum transfer that sets as base for comparison the univariate model.

The choice of hours for all combinations of wages and interest rates that respond to productivity innovations in both economies is depicted in Figure 10 as percentage deviations from the steady state, and in Figure 11 as the difference between the bivariate and univariate responses as percentage deviations from the univariate economy. The response of hours to productivity innovations in the univariate economy is given by the univariate factor prices, $\{w(e^0), r(e^0)\}$, and in the bivariate economy by the bivariate factor prices, $\{w(u^1), r(u^1)\}$.

To study the effect of wages we hold the interest rate time-path of the univariate economy, we introduce the wages of the bivariate economy, $\{w(u^1), r(e^0)\}$, and we add a transfer such

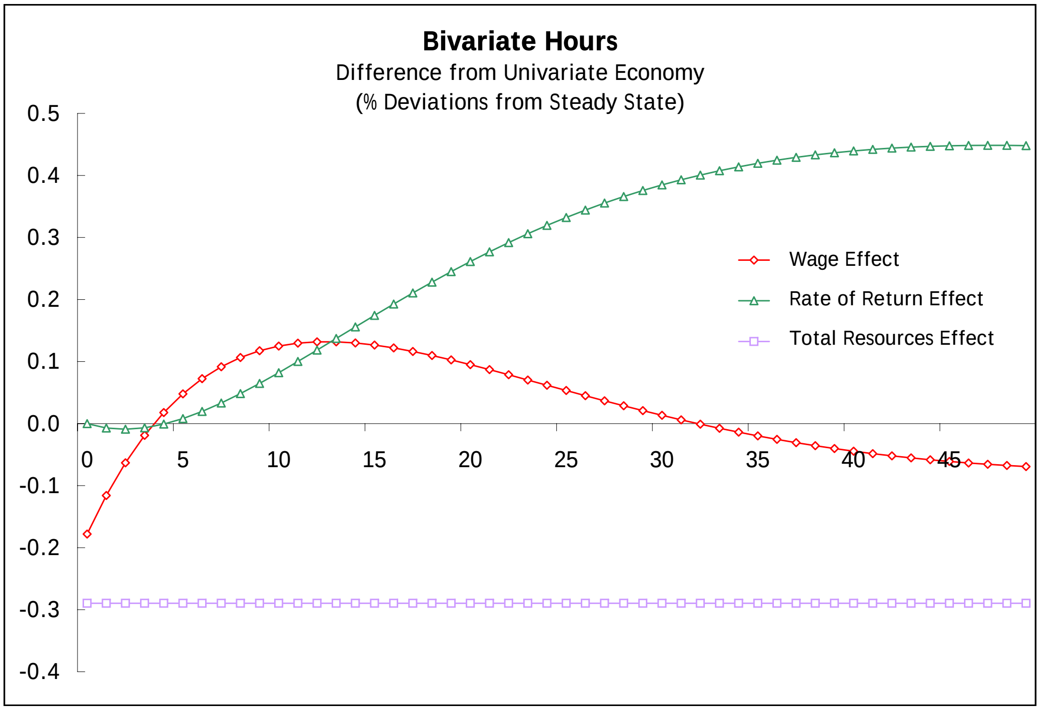

Figure 11: Crossed Slutsky Decomposition of Hours

that the present value of the total resources is equal to that of the univariate economy which we denote by $TR_0(w(e^0), r(e^0))$. We find that the initial response of hours dampens to .30%, that is, the bivariate wages drop the initial response of hours by -.18% (in percentage deviations from the steady state) the response in the univariate economy. In this economy, agents decide to postpone the supply of labor because they anticipate a raise in wages for the next 9 quarters. It is after the 5th quarter, exactly when $w(u^1)$ lies above $w(e^0)$, that the supply of hours is larger in this economy than in the univariate economy, and hours fall below those in the univariate model after eight years, as wages do. Without transfer we find very similar figures - that is, the change in wages has little effect in terms of human wealth.

We observe the impact of the bivariate rate of return on the hours of the univariate economy when we introduce the interest rate of the bivariate economy into the univariate economy while we hold the univariate wages and univariate total resources, that is, $\{w(e^0), r(u^1)\}$ and $TR_0(w(e^0), r(e^0))$. This economy displays not much difference from the univariate model for the first 5 quarters, after which the bivariate rate of return starts to generate a larger response of hours than the univariate model. Moreover, the intertemporal substitution effects are such that the deviations of hours remain above the steady state in the long run. Without the transfer the rate of return effect is not net out from the wealth effect and hours drop initially to .18%.

If we endow the univariate economy with the total resources available in the bivariate economy while we keep the univariate factor prices, that is, $\{w(e^0), r(e^0)\}$ with $TR_0(w(u^1), r(u^1))$, we find that the total resources effect alone generates a substantial drop, -.29%, in the supply of hours.

The sum of all these effects is what accounts for the little response of hours to productivity innovations in the bivariate model. First, the bivariate economy shows a substantial rise in the present value of total resources available to the agents that mitigates the incentive to work by -.29% at all periods with respect to the univariate economy (recall that the maximum univariate deviation of hours is .48% at $t = 0$). This wealth effect dominates the wage and rate of return effects in size. Second, the wage effect presents a hump-shape pattern that initially contributes to farther dampen the response of hour but that later on helps to bring hours a little above the univariate model from the second year until the eighth year. Thirdly, the rate of return effect barely contributes to alter the choice of hours during the first year, but then this effect continuously raises hours above the univariate model until convergence to a limit deviation of hours that offsets the wealth effect in the long-run. It is the combination of the wage effect, first alone, and then together with the rate of return effect, what generates the hump-shape dynamics of hours in the bivariate model. This way, while wages peak around the third year, hours peaks after 8 years or so because although wages had already started to decline in the 3rd year the present value of one unit of consumption still keeps raising enough to counterbalance this decline in wages. However, the effect of total resources maintains the supply of hours low, it is so that the peak of hours barely reaches a .09% deviation from the steady state (less than one fifth of the peak attained in the univariate model).

Consumption. Using the labor supply function and the log-linearization around the steady state of the first order condition for labor we can derive the consumption function as

$$\widehat{c}_t = -\left(\widehat{\frac{1}{\prod_{s=1}^{t}(1 + r_s - \delta)}}\right) + (HW_0 + \widehat{(1 + r}_t - \delta)\, k_t) \tag{46}$$

Notice that wages enter the consumption function only through the present value of total resources. The deviations in consumption are driven by the price of future consumption evaluated in present units and the change in the present value of total resources. The top panel in Figure ?? displays the impulse response functions to all innovations, the center panel the rate of return effect, and the bottom panel the total resources effect. Productivity innovations cut the price of consumption in the bivariate and univariate economies very similarly during the first year.

However, although future units of consumption become more rapidly expensive in the bivariate economy (which would favor a higher consumption in the univariate economy), the important wealth effect in the bivariate economy more than offsets the previous intertemporal substitution effect and sets consumption in the bivariate model above that of the univariate model. With distributive innovations we find the wealth effect is reinforced by the rate of return effect, still, in a lesser magnitude that under productivity innovations.

[[In the Appendix, we report in some detail the results for alternative calibrations of labor share as well as some additional information about the economy with Hansen-Rogerson preferences and alternative estimation procedures of the bivariate shock. These results confirm the findings already discussed.]]

5 Conclusion

We pose and estimate a bivariate shock to the production function that under competition in factor markets simultaneously accounts for movements in the Solow residual and in the factor shares of production. We show how confronting agents in a standard RBC economy with these shocks entail a much smaller response (about 40%) of hours relative to the standard modelization of the shocks that identifies the Solow residual with a univariate shock. Our findings raise a flag against the optimism embedded in the literature that states that productivity shocks are responsible for most of the cyclical behavior of output and hours.

Our results cast serious doubt on the explanatory power of the Solow residual as an important source of business cycle fluctuations.

Appendix A. Data Construction

Raw Data Series:

All raw data series were retrieved from the the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1954.I-2004.IV. To save on notation we drop the period subindex in all series.

National Income Product Accounts (NIPA-BEA).

1. Table 1.7.5: Gross National Product (GNP), Consumption of Fixed Capital (DEP)[27] , Statistical Discrepancy (SDis)[28]
2. Table 1.12: Compensation of Employees (CE), Proprietor's Income (PI), Rental Income (RI), Corporate Profits (CP), Net Interests (NI), Taxes on Production (Tax), Subsidies (Sub), Business Current Transfer Payments (BCTP), Current Surplus of Government Enterprises (GE).
3. Table 5.7.5: Private Inventories (Inv)

Fixed Asset Tables (FAT-BEA).

1. Tables 1.1 and 1.2: Private Fixed Assets (KP), Government Fixed Assets (KG), Consumer Durable Goods (KD).
2. Tables 1.3: Depreciation of Private Fixed Assets (DepKP),Depreciation of Government Fixed Assets (DepKG), Depreciation of Consumer Durable Goods (DepKD).

Current Establishment Survey[29] (CES-BLS).

1. Employment (E) : Series ID CES0000000081
2. Average Weekly Hours (AWH): Series ID CES0500000082, Series ID EEU00500005

[27] This amounts for the difference between Gross National Product and Net National Product.

[28] The Statistical Discrepancy corrects the difference between Net National Product and National Income.

[29] The primary sources of employment and average weekly hours series are the Current Establishment Survey (CES) and Current Population Survey (CPS) which have been in existence in some form since 1947. Our choice of the CES data set is driven from comparison purposes with **?**.

Constructed Data Series:

Labor Share. The labor share of income is defined as one minus capital income divided by output. Several sources of income, mainly proprietor's income, can not be unambiguously allocated to labor or capital income. To deal with this we proceed similar to **?** by assuming that the proportion of ambiguous capital income to ambiguous income is the same as the proportion of unambiguous capital income to unambiguous income, and we compute these series as follows[30]:

1. Unambiguous Capital Income (UCI) = RI + CP + NI + GE
2. Unambiguous Income (UI) = UCI + DEP + CE
3. Proportion of Unambiguous Capital Income to Unambiguous Income: $\theta_P = \frac{\text{UCI+DEP}}{\text{UI}}$

   Then we can use θ_P to compute the amount of ambiguous capital income in ambiguous income,
4. Ambiguous Income (AI) = PI + Tax - Sub + BCTP + SDis
5. Ambiguous Capital Income (ACI) = $\theta_P \times$ AI

Then, capital income (service flows of private fixed capital), Y_{KP}, is computed as the sum of unambiguous capital income, depreciation, and ambiguous capital income, that is,

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI} \tag{47}$$

which we use to construct our baseline labor share[31] as

$$\text{Labor Share} = 1 - \frac{\text{UCI} + \text{DEP} + \text{ACI}}{\text{GNP}} = 1 - \frac{Y_{KP}}{\text{GNP}} = 1 - \theta_P \tag{48}$$

To see the equivalence with **?** notice that

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI} = \theta_P\text{UI} + \theta_P\text{AI} = \theta_P\text{GNP} \tag{49}$$

[30]The labor share is a ratio and we use nominal series to compute it. Notice that unless the same price index is applied to all nominal variables the use of real variables will not yield identical results.

[31]Our computation of the labor share differs from **?** in three regards: we add GE to UCI and Tax - Sub + BCTP to AI, so that UI + AI = GNP; we do not include the stock of land as private fixed assets; and we compute the depreciation rates of consumer durables and government stock differently as we discuss below

Assuming that the return on capital is the same for fixed private capital, consumer durables and government stock we can extend the measure of output, capital income and the labor share to include service flows from consumer durables and government stock as follows:

First, we determine the return on capital, i, by solving the following equation that relates the capital income to the capital stock[32]

$$Y_{KP} = i \times (\mathsf{KP} + \mathsf{Inv}) + \mathsf{DEP} \tag{50}$$

Second, the depreciation rates of consumer durables and government stock are computed as[33]

$$\delta_D = \frac{\mathsf{DepKD}}{\mathsf{KD}} \qquad\qquad \delta_G = \frac{\mathsf{DepKG}}{\mathsf{KG}} \tag{51}$$

This way, the flow of services from consumer durable goods and government capital can be derived as

$$Y_{KD} = (i + \delta_D) \times \mathsf{KD} \qquad\qquad Y_{KG} = (i + \delta_G) \times \mathsf{KG} \tag{52}$$

Finally, the labor share with durables that extends measured output and capital income with flow services from consumer durables is

$$1 - \frac{Y_{KP} + Y_{KD}}{\mathsf{GNP} + Y_{KD}} \tag{53}$$

and the labor share with durables and government that also includes flow services of government stock is

$$1 - \frac{Y_{KP} + Y_{KD} + Y_{KG}}{\mathsf{GNP} + Y_{KD} + Y_{KG}} \tag{54}$$

Our last measure of the labor share is defined as the compensation of employees divided by GNP, that is, we consider as labor income the only source that we can unambiguously allocate to labor and add all ambiguous income to capital income.

[32] We transform the annual capital series provided by FAT-BEA to a quarterly series by interpolation.

[33] **?** uses the perpetual inventory method and investment series to pin down δ_D and δ_G. Instead, we use the depreciation series for consumer durables and government stock reported in FAT, Table 1.3, and operate following (51). We find that our values for $\delta_D = .19$ and $\delta_G = .04$. are similar to those reported in **?**, respectively, .21 and .05 - here notice that we also have a different sample period, theirs runs from 1954 to 1992.

Aggregate Hours. We construct the series of aggregate hours by multiplying the series of employment and average weekly hours [34] : Hours = E $\times$ AWH [35].

Appendix B. Sensitivity to the Labor Share Definition

We explore the sensitivity of our results to alternative definitions of labor share in model economies with log-log preferences and Hansen-Rogerson preferences. We also present the results attained under the constrained estimation of z_t^1 and z_t^2 where we restrict past deviations of the labor share from affecting productivity, that is, we set $\gamma_{12} = 0$. The results herein confirms our findings discussed in Section 4.3.

Univariate and Bivariate Estimation

To be consistent in our computations of the Solow residual under each definition of the labor share, we take the corresponding extended measures of (deflated) output, and extend the measure of the real capital stock series accordingly. This way, when the labor share includes consumer durables (and government stock) the real output and real capital series[36] used to compute the Solow residual are respectively defined as (deflated) GNP + Y_{KD} (+ Y_{KG}) and KP + KD (+ KG). The series of the labor input remains the same in all computations. Table (11) reports the univariate estimation of the Solow residual for the four definitions of the labor share and Table (12) the bivariate estimation[37] of the modified Solow residual and the labor share.

Our estimations show a high persistence of the Solow residual and the labor share, larger volatility of the productivity innovations when government stock is included, larger volatility of the redistributive innovations in our narrowest definition of the labor share, a negative covariance between the productivity and redistributive innovations which is largest under our narrowest

[34] The series of average weekly hours CES0500000082 is available from 1964.I onwards. For the period before 1964 we retrieve the annual observations from the series EEU00500005 which we use as quarterly observations. This way, we attribute all quarterly variation in hours before 1964 to employment.

[35] Alternatively, the Productivity and Costs program office at the BLS also provides a quarterly index of aggregate hours since 1947, series ID PRS85006033, which is composed from CES and CPS data and has cyclical properties that are very similar to those of our constructed series of hours in terms of correlation with output (.88) but slightly more volatile (1.77). When we use PRS85006033 to construct the Solow residuals s_t^0 and s_t^1 with our baseline labor share the volatility of hours obtained in the bivariate model is 34% that of the univariate model.

[36] To construct the series of real capital we use the chain-type quantity index from Table 1.2 in FAT-BEA and the current-cost net stock in year 2000 from Table 1.1 in FAT-BEA.

[37] Although we do not report it here information criteria suggest the use of a VAR(1) for the bivariate estimation under all definitions of the labor share, as in our baseline case.

definition of the labor share, and negligible (statistically non-significant) marginal effects of z_{t-1}^2 on z_t^1 under all labor share definitions. The IRFs depicted in Figures (12) and (13) show very similar properties to our baseline labor share studied in Section (3.2).

Cyclical Behaviour

In Tables (13)-(15) we report the business cycle statistics of a RBC model with log-log preferences when we extend the labor share to include durable goods, and government stock, and also when we define the labor share as compensation of employees divided by GNP. With the baseline labor share aggregate hours in the bivariate model are 32% less volatile than in its univariate counterpart. When we include durable goods hours move 48% less in the bivariate model, and when we include government 53% less. Averaging over these three definitions of the labor share we yield a reduction of 44% in the volatility of hours. When we use CE/GNP the drop in σ_h is 59%. A variance decomposition exercise shows that this largest value of σ_h under CE/GNP is associated with an influence of u^1 on z^2, and in turn, on hours higher than in previous definitions, see Table (16). At the same time, under all definitions of the labor share the correlation of hours with output decreases with respect to the univariate case. This is best seen with the IRFs of output and hours in Figures (14) and (15). While hours display a clear hump-shape response to u^1, output does not.

Under Hansen-Rogerson preferences we find a very similar reduction in the volatility of hours. With these preferences the bivariate model displays an average σ_h that is 47% less than its standard univariate counterpart, see Tables (17)-(20).

Constrained Estimation

The use of constrained estimation under $\gamma_{12} = 0$ (slightly) strengthens the distributive nature of z_t^2. The estimation results are reported in Table (21). Our findings do not differ from our previous results, see Tables (22) and (23). With the baseline case the reduction in the volatility of hours is 36%, with durable goods 43%, with government stock 42%, and with CE/GNP 54%. This averages the decrease in the cyclical movements of hours to 44%.

	ρ	σ
Baseline Labor Share	.954	.00668
	(.020)	(.000)
... with Durables	.951	.00667
	(.022)	(.000)
... and Government	.937	.00726
	(.019)	(.000)
CE/GNP	.951	.00685
	(.021)	(.000)

Table 11: Univariate Estimation of the Solow Residual, z_t^0

	γ_{11}	γ_{12}	γ_{21}	γ_{22}	σ_1	σ_2	σ_{12}
Baseline Labor Share	.946	.001	.050	.930	.00668	.00303	-.1045E-04
	(.023)	(.042)	(.010)	(.019)			
... with Durables	.941	-.012	.055	.930	.00665	.00287	-.1001E-04
	(.024)	(.043)	(.010)	(.019)			
... and Government	.927	-.041	.058	.953	.00723	.00313	-.139E-04
	(.025)	(.044)	(.011)	(.019)			
CE/GNP	.948	-.025	.051	.937	.00685	.00345	-.1696E-04
	(.023)	(.040)	(.011)	(.020)			

Table 12: Bivariate Estimation of the Solow Residual, z_t^1, and Labor Share Deviations, z_t^2

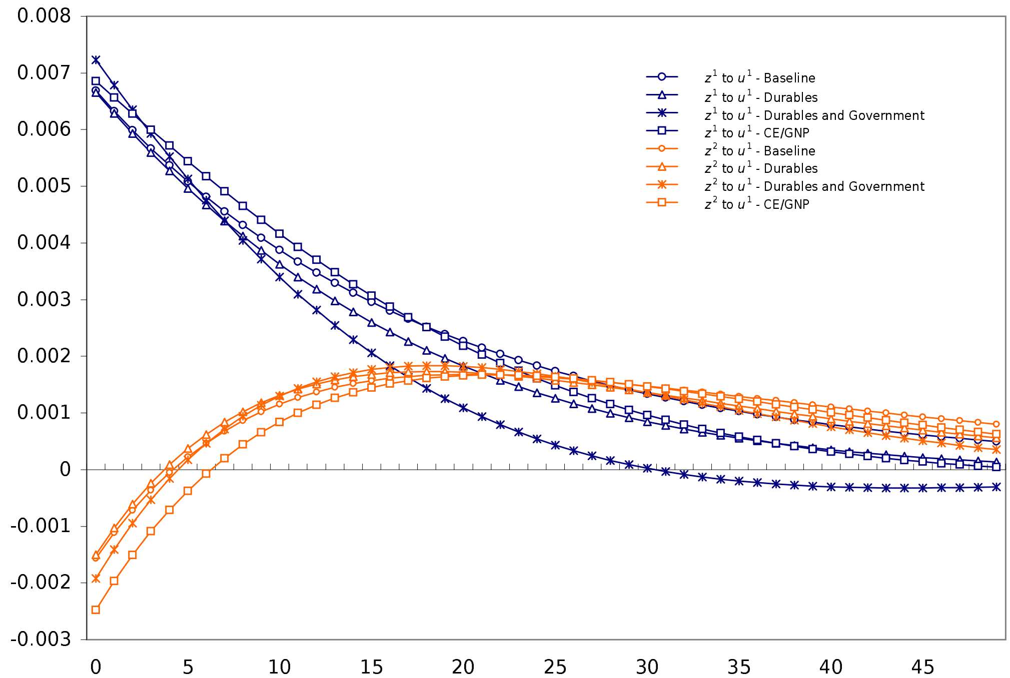

Figure 12: IRFs to productivity innovations u^1, All Labor Share Definitions.

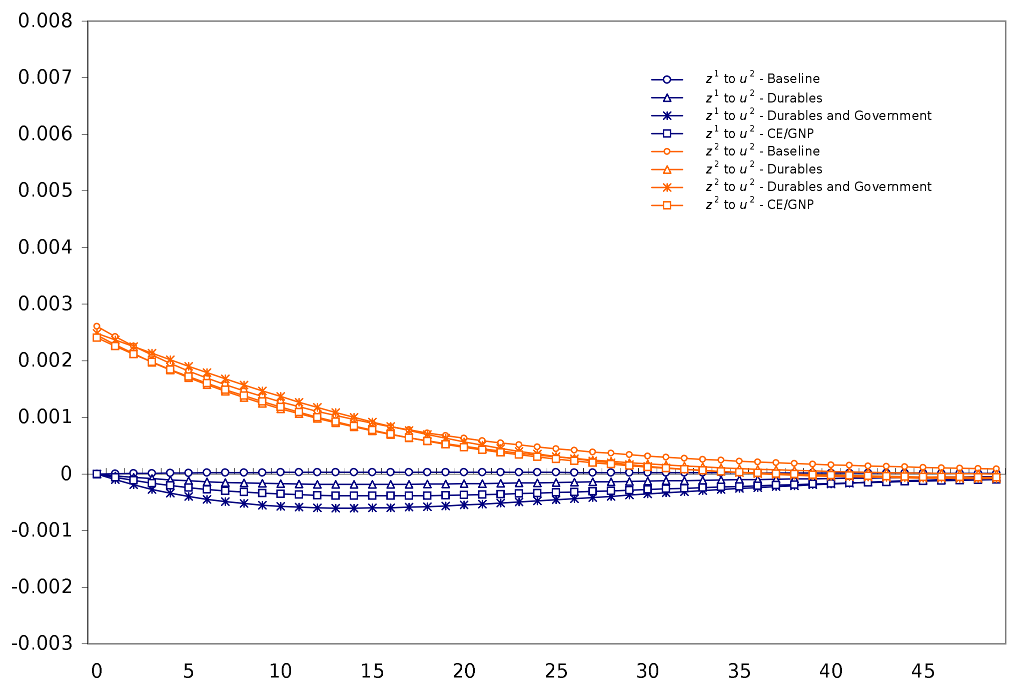

Figure 13: IRFs to distributive innovations u^2, All Labor Share Definitions.

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.26**	1.00	.72	**.93**	1.00	.73
h	**.62**	.98	.71	**.30**	.38	.74
c	.43	.89	.81	.67	.92	.79
i	3.94	.99	.71	2.01	.93	.71
r	.05	.96	.71	.07	.70	.71
w	.67	.98	.75	.77	.88	.79
y/h	.67	.98	.75	.87	.95	.72
z^0, z^1	.87	.99	.71	.87	.97	.72
z^2	-	-	-	.41	-.22	.73

Table 13: Labor Share with Durables, and Log-Log Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.37**	1.00	.72	**1.05**	1.00	.74
h	**.73**	.98	.70	**.39**	.45	.76
c	.39	.85	.83	.61	.91	.80
i	4.52	.99	.71	2.65	.95	.72
r	.05	.97	.70	.07	.76	.72
w	.67	.98	.74	.74	.84	.80
y/h	.67	.98	.74	1.04	.93	.72
z^0, z^1	.94	.99	.71	.96	.96	.72
z^2	-	-	-	.45	-.31	.74

Table 14: Labor Share with Durables and Government, and Log-Log Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	1.25	1.00	.72	.90	1.00	.75
h	.61	.97	.71	.36	.03	.74
c	.44	.88	.83	.60	.90	.82
i	3.89	.99	.71	2.12	.93	.72
r	.06	.96	.71	.08	.75	.71
w	.67	.98	.76	.68	.79	.82
y/h	.67	.98	.76	.96	.92	.72
z^0, z^1	.89	.99	.71	.91	.96	.72
z^2	-	-	-	.48	-.40	.72

Table 15: Compensation of Employees divided by GNP, and Log-Log Preferences

		y	h	c	i	r	w	y/h	z^1	z^2
Baseline Labor Share	u^1	97.5	7.8	78.1	94.8	74.8	69.8	99.4	100.0	35.1
	u^2	2.5	92.2	21.9	5.2	25.2	30.2	.6	.0	64.9
.... with Durables	u^1	96.5	14.6	82.0	99.4	82.2	71.5	98.5	100.0	37.9
	u^2	3.5	85.4	18.0	.6	17.8	28.5	1.5	.0	62.1
... and Government	u^1	96.0	18.6	80.6	99.7	88.5	67.5	97.3	99.8	46.1
	u^2	4.0	81.4	19.4	.3	11.5	32.5	2.7	.2	53.9
CE/GNP	u^1	95.5	20.7	79.2	99.8	89.7	62.5	97.8	99.9	56.2
	u^2	4.5	79.3	20.8	.2	10.3	37.5	2.2	.1	43.8

Table 16: Forecast Error Variance Decomposition (%), Log-Log Preferences

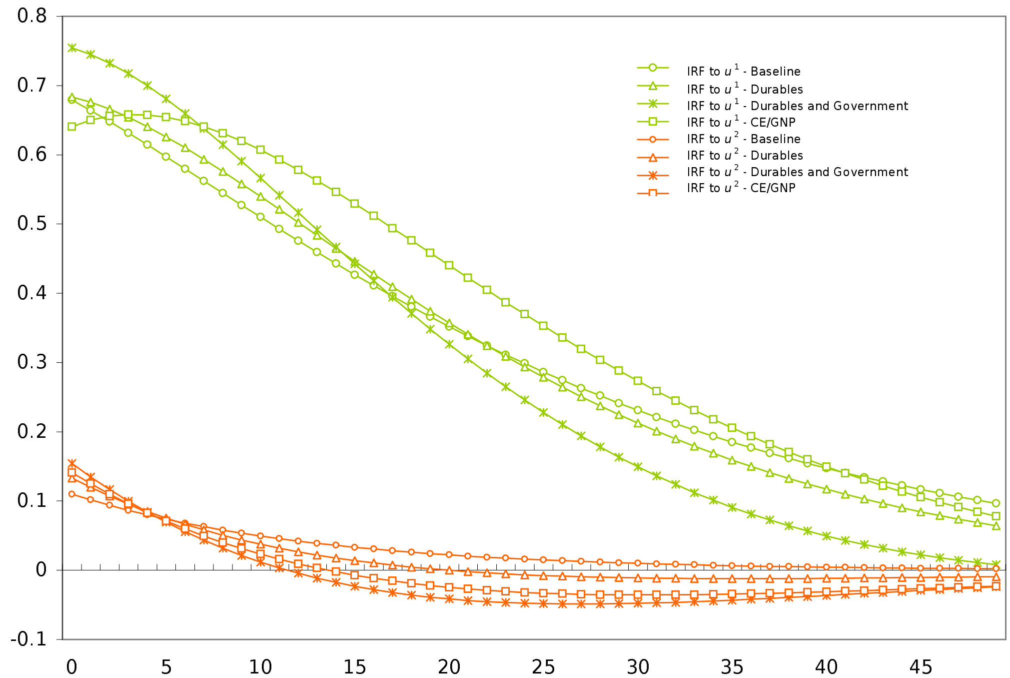

Figure 14: IRFs of Output, Log-Log Preferences and All Labor Share Definitions.

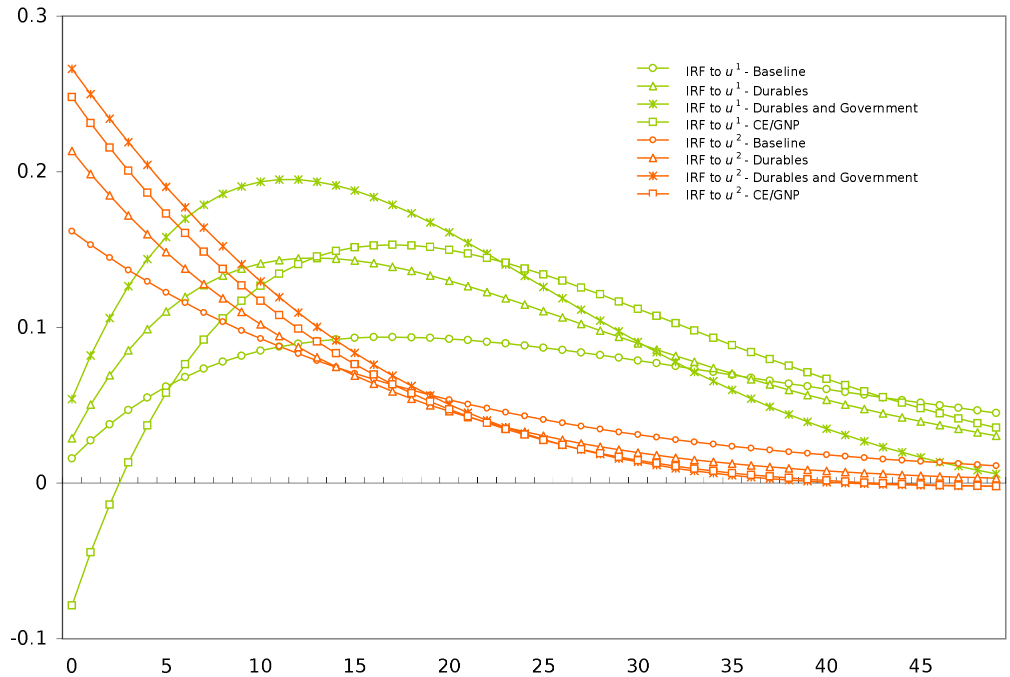

Figure 15: IRFs of Hours, Log-Log Preferences and All Labor Share Definitions.

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.61**	1.00	.72	**.97**	1.00	.75
h	**1.19**	.97	.70	**.55**	.45	.74
c	.51	.87	.82	.70	.96	.78
i	5.17	.99	.70	1.94	.96	.72
r	.07	.96	.70	.06	.64	.71
w	.51	.87	.82	.70	.96	.78
y/h	.51	.87	.82	.87	.82	.71
z^0, z^1	.87	.99	.71	.87	.92	.72
z^2	-	-	-	.41	-.06	.73

Table 17: Labor Share with Durables, and Hansen-Rogerson Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	**1.74**	1.00	.71	**1.11**	1.00	.75
h	**1.38**	.98	.70	**.71**	.53	.76
c	.47	.83	.84	.65	.94	.80
i	5.85	.99	.70	2.69	.97	.73
r	.07	.97	.70	.07	.72	.72
w	.47	.83	.84	.65	.94	.80
y/h	.47	.83	.84	.95	.77	.71
z^0, z^1	.94	.99	.71	.96	.91	.72
z^2	-	-	-	.45	-.16	.74

Table 18: Labor Share with Durables and Government, and Hansen-Rogerson Preferences

	Univariate $\{z^0\}$			Bivariate $\{z^1, z^2\}$		
	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$	σ_x	$\rho(y,x)$	$\rho(x_{t-1},x_t)$
y	1.53	1.00	.72	.89	1.00	.76
h	1.12	.97	.70	.67	.17	.73
c	.51	.86	.83	.61	.96	.81
i	4.89	.99	.71	1.87	.96	.73
r	.07	.96	.70	.08	.68	.72
w	.51	.86	.83	.62	.95	.81
y/h	.51	.86	.83	1.02	.76	.71
z^0, z^1	.89	.99	.71	.91	.88	.72
z^2	-	-	-	.48	-.22	.72

Table 19: Compensation of Employees divided by GNP, and Hansen-Rogerson Preferences

		y	h	c	i	r	w	y/h	z^1	z^2
Baseline Labor Share	u^1	91.2	7.2	75.6	100.0	79.8	75.6	97.8	100.0	35.1
	u^2	8.8	92.8	24.4	.0	20.2	24.4	2.2	.0	64.9
.... with Durables	u^1	88.9	13.4	79.5	96.7	88.1	79.5	95.0	100.0	37.9
	u^2	11.1	86.6	20.5	3.3	11.9	20.5	5.0	.0	62.1
... and Government	u^1	88.4	17.0	78.2	94.5	93.5	78.2	91.8	99.8	46.1
	u^2	11.6	83.0	21.8	5.5	6.5	21.8	8.2	.2	53.9
CE/GNP	u^1	85.7	24.3	75.2	94.5	93.4	75.2	94.0	99.9	56.2
	u^2	14.3	75.7	24.8	5.5	6.6	24.8	6.0	.1	43.8

Table 20: Forecast Error Variance Decomposition (%), Hansen-Rogerson Preferences

	γ_{11}	γ_{12}	γ_{21}	γ_{22}	σ_1	σ_2	σ_{12}
Baseline Labor Share	.947	-	.050	.930	.00666	.00302	-.10E-04
	(.022)		(.010)	(.016)			
... with Durables	.939	-	.055	.927	.00663	.00287	-.10E-04
	(.023)		(.011)	(.016)			
... and Government	.926	-	.058	.942	.00722	.00313	-.14E-04
	(.025)		(.011)	(.015)			
CE/GNP	.946	-	.052	.928	.00683	.00344	-.17E-04
	(.023)		(.011)	(.014)			

Table 21: Bivariate Constrained Estimation under $\gamma_{12} = 0$

	Baseline		with Durables		and Government		CE/GNP	
	σ_x	$\rho(y,x)$	σ_x	$\rho(y,x)$	σ_x	$\rho(y,x)$	σ_x	$\rho(y,x)$
y	**.91**	1.00	**.92**	1.00	**.99**	1.00	.87	1.00
h	**.23**	.30	**.27**	.36	**.31**	.36	.33	-.02
c	.72	.91	.67	.92	.63	.88	.60	.88
i	1.90	.88	1.98	.92	2.46	.93	2.09	.92
r	.06	.67	.07	.70	.06	.78	.08	.76
w	.79	.87	.76	.87	.74	.82	.67	.78
y/h	.87	.97	.86	.95	.93	.95	.94	.94
z^0, z^1	.87	.98	.86	.98	.94	.98	.89	.96
z^2	.42	-.25	.40	-.24	.44	-.35	.47	-.43

Table 22: Bivariate Shocks, Constrained Estimation and Log-Log Preferences

		y	h	c	i	r	w	y/h	z^1	z^2
Baseline Labor Share	u^1	97.4	8.1	78.2	95.0	73.9	69.8	99.3	100.0	33.5
	u^2	2.6	91.9	21.8	5.0	26.1	30.2	.7	.0	66.5
.... with Durables	u^1	96.9	13.6	80.8	98.5	81.9	70.9	98.7	100.0	37.9
	u^2	3.1	86.4	19.2	1.5	18.1	29.1	1.3	.0	62.1
... and Government	u^1	97.1	14.8	77.3	99.1	88.0	66.0	98.3	100.0	46.5
	u^2	2.9	85.2	22.7	.9	12.0	34.0	1.7	.0	53.5
CE/GNP	u^1	96.3	21.0	77.4	99.3	89.6	61.5	98.2	100.0	56.8
	u^2	4.7	79.0	22.6	.7	10.4	38.5	1.8	.0	43.2

Table 23: Forecast Error Variance Decomposition (%), Constrained Estimation and Log-Log Preferences