Money and Capital*

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Abstract

We extend microfounded models of money with both centralized and decentralized markets to include capital. Although we consider several versions, in the baseline model, capital produced in the centralized market is an input in the decentralized market. We calibrate the model, and find the following. With bargaining in the decentralized market, inflation has virtually no impact on investment, but still affects consumption and welfare: going from 10% inflation to the Friedman rule is worth around 3.5% of consumption. With price taking the same policy works quite differently: now capital increases 12%, and although the steady state gain is also 3.5%, the transition cost cuts it to 1.9%. Although we also find big distortions from fiscal policy, even if we must make up the revenue with proportional taxes, eliminating inflation may still be desirable. Finally, we quantify the impact of holdup problems in bargaining models.

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1 Introduction

The goals of this paper are: (i) extend some recent work in monetary theory; (ii) quantify the resulting model; and (iii) use it to study policy, especially the effects of long-run changes in inflation and taxation on capital accumulation, consumption and welfare. Our approach builds on the model with periodic meetings of centralized and decentralized markets in Lagos and Wright (2005), although for the goals described above we need to generalize that framework along several dimensions. In particular, we need to introduce capital.

To motivate this endeavor, note that the long-run relation between money and growth – or inflation and capital formation – is one of the classic issues in macroeconomics, going back at least to Tobin (1965) and Sidrauski (1967a, 1967b), continuing through Stockman (1981), Cooley and Hansen (1989, 1991), Gomme (1993), Ireland (1994) and many others to the present day. All of these papers use 'reduced-form' models of money; e.g. they put it directly into the utility function or impose cash-in-advance constraints. Modern monetary theory proceeds without recourse to such shortcuts by taking seriously the frictions that generate a role for a medium of exchange in the first place, and shows that for many questions this can make a big difference. We want to know if the same is true for questions related to money and capital accumulation.

A previous attempt to integrate growth and microfounded monetary theory in Aruoba and Wright (2003) was at best partially successful, because that specification displays a strong *dichotomy*: one can solve independently for the equilibrium allocations in the centralized and decentralized markets. This has some undesirable implications – e.g. monetary policy can have no impact on investment, employment or consumption in the centralized market – and indeed, one might say that money and growth theory have really not been integrated at all (Howitt 2003; Waller 2003). In our baseline model, the dichotomy breaks down simply because capital produced in the centralized market is used as an input in the decentralized market. This generalization implies potentially rich feedback across markets, and from monetary policy to investment, employment and consumption.

Although we think this is worthwhile as a theoretical exercise, we also want to emphasize the quantitative analysis. It is because we are interested in quantitative analysis in the first place that we include capital, and also some other ingredients, such as government spending and taxation, that are staples in mainstream macroeconomics. These features are important for calibration purposes and also allow us to analyze an extended set of policy issues. In discussing policy, the analysis here is much more interesting than in models without capital, like most of those in monetary theory, because we need to take into account transition paths and hence solve for the decision rules rather than simply comparing steady states; we find that this leads to new qualitative insights and can make a big difference for the quantitative results.[1]

We also compare the model under two scenarios with respect to price formation in the decentralized market. In one version we assume bilateral bargaining, which is arguably natural given the kinds of frictions that make money essential, and is used in much of the microfoundations literature. In the other version we assume price taking. This allows us to isolate the effects in the bargaining model of holdup problems in both the demand for money and the demand for capital, and to try to quantify their importance.[2] It turns out that the two models generate very different predictions about the effects of monetary policy on investment. Due to the double holdup problem, under bargaining, inflation has very little impact on capital formation. Under price taking, which avoids holdup problems, inflation can have a big effect on investment.

We calibrate the model to standard observations and discuss the extent to which different specifications do a more or less reasonable job of capturing the key observations. We then

[1] There have been few previous attempts to take microfounded monetary theories to the data, especially versions with capital. Exceptions are Shi (1999) and Menner (2005), who start from the model in Shi (1997), and Molico and Zhang (2005), who start from Molico (1999). Those models are very different because they have *only* decentralized markets, so this is where all investment occurs. Since we start from the model in Lagos and Wright (2005), we can have investment occur either in centralized or decentralized markets. In our benchmark model we assume investment occurs in the centralized market, but for comparison we also study the alternative.

[2] The former has been discussed in the recent monetary literature but the latter has not, even though it has been suggested elsewhere that holdup problems generally are important for aggregate investment; see e.g. Caballero and Hamour (1998) and Caballero (1999). We discuss this further in Section 2.2.

perform several policy experiments. In the price-taking model, we find that going from 10% inflation to the Friedman rule can increase long-run capital by 12%. In the bargaining model, although it has virtually no effect on capital, inflation is still very costly because it directly affects decentralized market consumption, and due to the money holdup problem this is painful. In fact, both specifications generate about the same cost of inflation when one compares across steady states: going from 10% to the Friedman rule is worth around 3.5% of consumption. In the price-taking model, however, much of the gain accrues in the long run, and the costly transition reduces the net benefit to just below 2%.

Although the two versions of the model have very different channels through which monetary policy matters – in one case it works via decentralized market consumption, and in the other it works via the long-run capital stock and centralized market consumption – in either case our welfare numbers are bigger that those typically found in the 'reduced form' literature. We also find big effects from taxes. But even if we have to make up the lost revenue with distortionary taxation, we find that eliminating inflation can still be beneficial, which also differs from the previous literature. Finally, we show that the costs of the holdup problems can be quantitatively important, even though we have bargaining only in the decentralized market, and our calibration implies that this market accounts for only around 5% of aggregate output.

The rest of the paper is organized as follows. In Section 2 we describe the baseline model. In Section 3 we present several extensions. In Section 4 we discuss calibration. In Section 5 we present the quantitative results. In Section 6 we conclude.[3]

[3] A few more words are perhaps in order as to why we adopt the Lagos-Wright model, with two kinds of markets. First, having some decentralized trade is what makes a medium of exchange essential. Then, having a centralized market not only seems natural for discussing investment, it also generates a big gain in tractability over models without it because we do not have to keep track of the distribution of money as a state variable. While models where one does have to keep track of this distribution, including Green and Zhou (1998, 2002), Molico (1999), Zhou (1999), and Zhu (2003, 2005), are interesting, it is nice to have a benchmark that is relatively easy to analyze and understand, the way e.g. the complete-market, representative-agent, neoclassical growth model serves as a benchmark for nonmonetary macroeconomics. Previous search-based monetary models, including Kiyotaki and Wright (1993), Shi (1995), or Trejos and Wright (1995), are also easy to analyze, but mainly because they make some strong assumptions that preclude quantitative and policy analysis as it is usually practiced.

2 The Basic Model

2.1 General Assumptions

As in Lagos and Wright (2005), hereafter LW, a $[0, 1]$ continuum of agents live forever in discrete time. In each period there convene two distinct markets. One is a frictionless or centralized market, referred to as the CM, while the other is a decentralized market, referred to as the DM, with two main frictions. These frictions are: (i) a double-coincidence problem; and (ii) anonymity, which precludes credit. This means that some medium of exchange is essential (Kocherlakota 1998; Wallace 2001). The main issue in much of modern monetary theory (e.g. Kiyotaki and Wright 1989) is to determine endogenously which object will play this role. In order to focus on other questions, however, other papers avoid the issue by assuming there is a unique storable asset – perhaps fiat money, perhaps commodity money, perhaps something else – that qualifies as a potential medium of exchange.[4]

For the current project, we want to follow the latter approach and avoid the interesting but difficult problem of determining the medium of exchange endogenously. We cannot, however, assume there is a unique storable asset in a paper called "Money and Capital." Our strategy is to assume that physical capital is fixed in place in the CM, and thus cannot be traded in the DM. Then, to address the question of why claims to (rather than physical units of) capital do not circulate in the DM, we assume that agents can costlessly counterfeit such claims, but cannot so easily counterfeit currency. Given this, sellers will never accept claims to capital from anonymous buyers in the DM, any more than they would accept personal IOU's. But they may accept money.[5]

[4] For example, in Trejos and Wright (1995), it is assumed that "Agents consume *services* (or, equivalently, nonstorable goods)" to rule out commodity money and concentrate on fiat money as a medium of exchange. However, there is no *constraint* saying that agents have to use money to consume – e.g. they are free to try direct barter, and there typically is some barter in equilibrium.

[5] One need not interpret money here literally as cash. He, Huang and Wright (2005) study a related albeit much simpler model where agents can deposit money in bank accounts in the CM, and pay with either cash or checks in the DM (see also Berentsen et al. 2005 or Chiu and Meh 2006). It is feasible to do something similar here, but adding banks complicates the presentation without changing much else. The important thing to understand is that as long as banks hold some reserves in equilibrium, for whatever reason, and these reserves earn less than the market return, liquidity has a cost and this cost is influenced by policy that affects interest rates.

So money is the only object that can serve as a medium of exchange in this environment, while capital is merely a productive input. We emphasize that we do *not* regard this as a particularly interesting or elegant solution to the rate-of-return-dominance question – how can money coexist with other assets paying higher rates of return? It is rather a simple solution that allows us to study interactions between money and capital when one serves as medium of exchange and the other as a factor of production. Our position is that, even if we do not have a prize-winning answer to the rate-of-return-dominance puzzle, for now, we want to study other issues in models that include many of the ingredients from micro-based monetary economics, including double-coincidence problems, bargaining problems, and so on.

While we acknowledge that our assumptions about capital are crude, at the same time we insist that they are logically consistent assumptions about the physical environment, and not direct assumptions about agents' behavior or institutions. As a general principle it should be clear that it is better to be explicit about the assumptions leading to an outcome, other things being equal, rather than assuming the outcome as a 'reduced-form' for something left implicit. This is not (only) because some people may doubt that there exist logically consistent assumptions generating the outcome in question, but because one ought to want to know what *other* implications these assumptions may have. The only way to know this is to be explicit about the environment.[6]

To continue, in the CM there is a general good that can be used for consumption or investment. It is produced using labor H and capital K, hired by firms in perfectly competitive markets. As usual, profit maximization implies $r = F_K(K, H)$ and $w = F_H(K, H)$, where F is the technology, r is the rental rate and w the real wage, and by constant returns equilibrium profits will be 0. In the DM these firms do not operate, but an agent's own

[6] Of course it is interesting to think more about the coexistence of currency and other assets in these kinds of models, but this is left for other work. See Lagos and Rocheteau (2005) and Waller (2003) for some discussion. Devices that could potentially be used to capture why capital does not drive out money if we did allow it to circulate include government policies like those in Aiyagari et al. (1996), Shi (2005), and Lagos (2005), or private information as in Williamson and Wright (1994), Trejos (1997), and Berentsen and Rocheteau (2004).

effort e and capital k may be used with technology $f(e,k)$. Note that k appears as an input in DM production, even if it cannot be traded in the DM. So while perhaps k cannot be physically moved to the location where the DM convenes, it still may affect productivity at that location; think about logging on to your computer from a remote site, or more generally, think about being more efficient for owning a better computer even when you don't have it with you.

To generate a double-coincidence problem we adopt the following specification in the DM: with probability σ an agent wants to consume but cannot produce; with probability σ he can produce but does not want to consume; and with probability $1-2\sigma$ he can neither produce nor consume. This is equivalent for our purposes to the standard bilateral matching specification in the search literature, where there is a probability σ of wanting to consume a good produced by a random partner. We frame things here in terms of taste and technology shocks rather than matching because it facilitates some parts of the discussion, but otherwise very little hinges on this part of the specification.[7]

Instantaneous utility in the CM is $U(x)-Ah$, where x is consumption and h hours. In the DM, with probability σ an agent is a consumer and has utility $u(q)$, and with probability σ he is a producer and has utility $-\ell(e)$, where q is consumption and e effort. Functions $U(x)$, $u(q)$ and $\ell(e)$ have the usual properties. Linearity in h is not important in principle, but yields a big gain in tractability; alternatively, one can assume general utility and indivisible labor, as in Rogerson (1988), and get essentially the same results, as shown by Rocheteau et al. (2005). In any case, it is convenient to write disutility in the DM as follows: given k, solve $q=f(e,k)$ for $e=\xi(q,k)$ and let $c(q,k)=\ell[\xi(q,k)]$. As we show in the Appendix, $c_q>0$, $c_k<0$, $c_{qq}>0$, and $c_{kk}>0$ under the usual monotonicity and convexity assumptions on f and ℓ, and $c_{qk}<0$ if $f_k f_{ee}<f_e f_{ek}$, which always holds under the additional assumption that k is a normal input.

[7] As discussed in fn.4, random matching models typically have some direct barter. We can get this by having agents sometimes able to produce, but wanting to consume something other than what they produce. Given this is understood, for most of the presentation we will not discuss barter explicitly.

The government sets the money supply so that $M_{+1} = (1+\pi)M$, where $+1$ denotes next period and π is a decision variable that need not be constant over time; because the Fisher equation holds, it is equivalent to set the nominal interest or inflation rate here. Government also consumes G, levies a lump-sum tax T, a labor income tax t_h, a capital income tax t_k, and a sales tax t_x in the CM. In principle, we could allow a sales tax in the DM, t_q, but setting $t_q = 0$ streamlines the presentation a fair amount, and when we did calibrate $t_q > 0$ it mattered very little for the quantitative results; hence it seems prudent to assume sales taxes are only levied in the CM. Letting δ be depreciation on capital, which is tax deductible, and p the CM price level, the government budget constraint is

$$G = T + t_h wH + (r - \delta)\, t_k K + t_x X + \pi M/p.$$

Agents discount between the CM and DM at rate β, but to reduce notation, not between the DM and CM (think about the DM convening first within each period e.g.). If $W(m,k)$ and $V(m,k)$ are the value functions in the CM and DM, then the DM problem is

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1},k_{+1}} \{U(x) - Ah + \beta V_{+1}(m_{+1},k_{+1})\} \qquad (1) \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [1 + (r-\delta)\,(1-t_k)]\,k - k_{+1} - T + \frac{m - m_{+1}}{p}. \end{aligned}$$

After eliminating h using the budget equation, the FOC are

$$\begin{aligned} x &: \; U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \; \frac{A}{pw\,(1-t_h)} = \beta V_{+1,m}(m_{+1},k_{+1}) \qquad (2) \\ k_{+1} &: \; \frac{A}{w\,(1-t_h)} = \beta V_{+1,k}(m_{+1},k_{+1}), \end{aligned}$$

assuming interiority.[8] This leads to two key results. First, since (m,k) does not appear in (2), for any distribution of (m,k) across agents entering the CM, the distribution of (m_{+1},k_{+1})

[8]See LW for assumptions to guarantee an interior solution in these kinds of models (one cannot impose standard curvature assumptions since utility is linear in h). Also, note that the SOC can be complicated in equilibrium models with bargaining, involving third derivatives of u and c. For now we simply assume that V is strictly concave, but we check this in our quantitative analysis.

is degenerate. Second, from the envelope conditions, W is linear in (m,k):

$$\begin{aligned} W_m(m,k) &= \frac{A}{pw\,(1-t_h)} && (3) \\ W_k(m,k) &= \frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)} && (4) \end{aligned}$$

Moving back to the DM market, we have

$$V(m,k) = \sigma V^b(m,k) + \sigma V^s(m,k) + (1-2\sigma)W(m,k), \tag{5}$$

where the values to being a buyer and a seller are

$$\begin{aligned} V^b(m,k) &= u(q_b) + W\,(m-d_b,k) && (6) \\ V^s(m,k) &= -c(q_s,k) + W\,(m+d_s,k)\,, && (7) \end{aligned}$$

with q_b and d_b (q_s and d_s) denoting output and money exchanged when buying (selling). Using (3) we have

$$V(m,k) = W(m,k) + \sigma\left[u(q_b) - \frac{d_b A}{pw\,(1-t_h)}\right] + \sigma\left[\frac{d_s A}{pw\,(1-t_h)} - c(q_s,k)\right]. \tag{8}$$

To solve (2) we need:

$$\begin{aligned} V_m(m,k) &= \frac{A}{pw\,(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial m} - \frac{A}{pw\,(1-t_h)}\frac{\partial d_b}{\partial m}\right] \\ &\quad +\sigma\left[\frac{A}{pw\,(1-t_h)}\frac{\partial d_s}{\partial m} - c_q\frac{\partial q_s}{\partial m}\right] && (9) \\ V_k(m,k) &= \frac{A\,[1+(r-\delta)\,(1-t_k)]}{w\,(1-t_h)} + \sigma\left[u'\frac{\partial q_b}{\partial k} - \frac{A+A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)}\frac{\partial d_b}{\partial k}\right] \\ &\quad +\sigma\left[\frac{A+A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)}\frac{\partial d_s}{\partial k} - c_q\frac{\partial q_s}{\partial k} - c_k\right] && (10) \end{aligned}$$

It remains to specify how the terms of trade (q,d) are determined, so that we can substitute for their derivatives in (9) and (10). This will differ across versions of the model considered below. Before pursuing equilibrium, however, consider the planner's problem *without anonymity*, so that money is not essential:

$$\begin{aligned} J(K) &= \max_{q,X,H,K_{+1}} \left\{U(X) - AH + \sigma\,[u(q) - c(q,K)] + \beta J_{+1}(K_{+1})\right\} && (11) \\ \text{s.t. } X &= F(K,H) + (1-\delta)K - K_{+1} - G \end{aligned}$$

Eliminating X, and again assuming interiority, we have the FOC:

$$\begin{aligned} q &: \; u'(q) = c_q(q,K) \\ H &: \; A = U'(X)F_H(K,H) \\ K_{+1} &: \; U'(X) = \beta J'_{+1}(K_{+1}) \end{aligned} \tag{12}$$

From the envelope condition $J'(K) = U'(X)[F_K(K,H) + 1 - \delta] - \sigma c_k(q,K)$, we get

$$U'(X) = \beta U'(X_{+1})[F_K(K_{+1}, H_{+1}) + 1 - \delta] - \sigma\beta c_k(q_{+1}, K_{+1}). \tag{13}$$

Using the first condition in (12), given K we have $q = q^*(K)$ where $q^*(K)$ solves $u'(q) = c_q(q,K)$. Then the paths for (K_{+1}, H, X) satisfy the the Euler equation (13), the second FOC in (12), and the constraint in (11). This characterizes what we call the *first best*, or FB for short.[9] Note the presence of the term $-\beta\sigma c_k(q_{+1}, K_{+1}) > 0$ in (13), which reflects the fact that in general investment not only affects CM but also DM productivity. If K did not appear in $c(q)$, this term would vanish and the system would dichotomize: we could first set $q = q^*$, where q^* solves $u'(q) = c'(q)$, and then solve the other conditions – which reduce to those from the standard growth model – independently for (K_{+1}, H, X). In general, however, we need to solve all of the conditions simultaneously.

2.2 Bargaining

Suppose each agent with a desire to consume in the DM is paired with one who can produce. Since buyers are anonymous trade must be quid pro quo – which here means money. If the buyer's and seller's states are (m_b, k_b) and (m_s, k_s), the terms of trade (q, d) solve the generalized Nash problem, with bargaining power for the buyer given by θ and threat points given by continuation values. The buyer's payoff from trade is $u(q) + W(m_b - d, k_b)$ and his threat point $W(m_b, k_b)$, so (3) implies his surplus is $u(q) - Ad/pw\,(1 - t_h)$. Similarly, the seller's surplus is $Ad/pw\,(1 - t_h) - c(q, k_s)$. Hence our bargaining solution is

$$\max_{q,d} \left[u(q) - \frac{Ad}{pw\,(1 - t_h)}\right]^{\theta} \left[\frac{Ad}{pw\,(1 - t_h)} - c(q, k_s)\right]^{1-\theta} \text{ s.t. } d \leq m_b.$$

[9] There is of course also a transversality condition; this will remain implicit in all that follows.

As in LW, one can show that in any equilibrium $d = m_b$, and this implies $q \leq q^*(k_s)$ where $q^*(k_s)$ is the solution to $u'(q) = c_q(q, k_s)$.[10] In any case, inserting $d = m_b$ and taking the FOC with respect to q, we get

$$\frac{m_b}{p} = \frac{g(q, k_s) w (1 - t_h)}{A}, \tag{14}$$

where

$$g(q, k_s) \equiv \frac{\theta c(q, k_s) u'(q) + (1 - \theta) u(q) c_q(q, k_s)}{\theta u'(q) + (1 - \theta) c_q(q, k_s)}. \tag{15}$$

We write $q = q(m_b, k_s)$, where $q(\cdot)$ is given by solving (14) for q as a function of (m_b, k_s). Now one can compute $\partial d / \partial m_b = 1$, $\partial q / \partial m_b = A / pw (1 - t_h) g_q > 0$ and $\partial q / \partial k_s = -g_k / g_q > 0$ where

$$\begin{aligned} g_q &= \frac{u' c_q [\theta u' + (1 - \theta) c_q] + \theta (1 - \theta)(u - c)[(u' c_{qq} - c_q u'')}{[\theta u' + (1 - \theta) c_q]^2} > 0 \\ g_k &= \frac{\theta u' c_k [\theta u' + (1 - \theta) c_q] + \theta (1 - \theta)(u - c) u' c_{qk}}{[\theta u' + (1 - \theta) c_q]^2} < 0, \end{aligned}$$

while the other derivatives in (9) and (10) are 0.

Inserting these results and imposing $(m, k) = (M, K)$, (9) and (10) reduce to

$$V_m(M, K) = \frac{(1 - \sigma) A}{pw (1 - t_h)} + \frac{\sigma A u'(q)}{pw (1 - t_h) g_q(q, K)} \tag{16}$$

$$V_k(M, K) = \frac{A + A (r - \delta)(1 - t_k)}{w (1 - t_h)} - \sigma \gamma(q, K), \tag{17}$$

where it is understood that $q = q(M, K)$, and

$$\gamma(q, K) \equiv c_k + c_q \frac{\partial q}{\partial K} = \frac{c_k(q, K) g_q(q, K) - c_q(q, K) g_k(q, K)}{g_q(q, K)} < 0. \tag{18}$$

Substituting (16) and (17), as well as prices $p = AM / w (1 - t_h) g(q, K)$, $r = F_K(K, H)$, and $w = F_H(K, H)$, into the FOC for m_{+1} and k_{+1} in (2), we get the equilibrium conditions

$$\frac{g(q, K)}{M} = \frac{\beta g(q_{+1}, K_{+1})}{M_{+1}} \left[1 - \sigma + \sigma \frac{u'(q_{+1})}{g_q(q_{+1}, K_{+1})} \right] \tag{19}$$

$$\begin{aligned} U'(X) = {} & \beta U'(X_{+1}) \{ 1 + [F_K(K_{+1}, H_{+1}) - \delta](1 - t_k) \} \\ & - \sigma \beta (1 + t_x) \gamma(q_{+1}, K_{+1}). \end{aligned} \tag{20}$$

[10] Typically the inequality is strict; in models without capital, e.g., it is well known that $q < q^*$ unless $\theta = 1$ and the nominal interest rate is $i = 0$.

The other equilibrium conditions come from the FOC for X and the resource constraint,

$$U'(X) = \frac{A(1+t_x)}{(1-t_h)F_H(K,H)} \tag{21}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1}. \tag{22}$$

An *equilibrium with bargaining* is defined as (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (19)-(22), given policy and an initial condition K_0. We mainly focus on monetary equilibria, where $q > 0$, and ignore the nonmonetary equilibrium where $q = 0$ and (K_{+1}, H, X) satisfy (20)-(22) with $\gamma = 0$, which are exactly the equilibrium conditions for a standard nonmonetary model (e.g. Hansen 1985). When $M_{+1} = (1+\pi)M$ for fixed π, a *steady state* is a constant solution (q, K, H, X) to (19)-(22). In steady state inflation equals the money growth rate π, the real interest rate is $i_R = \rho$ where $\beta = \frac{1}{1+\rho}$, and the nominal interest rate i comes from the Fisher equation $i = (1+i_R)(1+\pi) - 1$. Then the steady state versions of (19)-(20) simplify to

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q,K)} - 1 \tag{23}$$

$$\rho = [F_K(K,H) - \delta](1-t_k) - \sigma(1+t_x)\frac{\gamma(q,K)}{U'(X)}. \tag{24}$$

A special case of this model is the specification in Aruoba and Wright (2003), where capital is not used in the DM, so $c(q,K) = c(q)$ and $\gamma(q,K) = 0$. That version dichotomizes: (19) determines a path for q, while (20)-(22) determine paths for (K_{+1}, H, X), independently. Hence, money affects q but not (K_{+1}, H, X). When the dichotomy prevails, many properties of this model are similar to ones without capital. One can show $\partial q/\partial i < 0$. Since $q < q^*$ for $i > 0$, welfare is maximized at the Friedman rule $i = 0$, which we abbreviate FR; but if $\theta < 1$ then $q < q^*$ even at $i = 0$. Notice that (for all $i > 0$) q is lower when θ is lower due to a holdup problem in money demand: buyers bear the cost of acquiring liquidity in the CM, but when $\theta < 1$ they must share with sellers the surplus generated by this liquidity in the DM. This lowers the demand for money and hence q.[11]

[11] Since q is a real variable, obviously, even in this case money affects welfare – i.e. dichotomy does not mean neutrality. See Rocheteau and Waller (2005) for more discussion of what we call the money holdup problem.

The dichotomy does *not* prevail when capital enters the DM cost function, since then K and q both appear in (19) and (20), and there is no way to solve for q independently of the other variables. In this case investors take into account the fact that K affects productivity in the DM as well as the CM. Given that monetary policy affects q – basically, inflation is a tax on DM activity – it thereby affects the value of K and hence investment. This impacts on productivity, employment, output and consumption in the CM. Notice, however, if $\theta = 1$ then $\gamma(q, K) = 0$ even when K enters $c(q, K)$. In this case the model is recursive, if not dichotomous: first (20)-(22) can be solved for (K_{+1}, H, X), then (19) determines q. So when $\theta = 1$, money still cannot influence investment, employment or consumption in the CM, even though anything that affects the CM (e.g. fiscal policy) does feed back to q.

Intuitively, when $\theta = 1$, sellers get none of the DM surplus, so investment decisions are based solely on returns in the CM. This is an extreme version of a holdup problem in the demand for capital. More generally, for any $\theta > 0$, sellers underinvest since they do not get the full return. This holdup problem has perhaps been neglected, although as Caballero and Hamour (1998) say: "From a macroeconomic perspective, the prevalence of unprotected specific rents makes it a potentially central factor in determining the functioning of the aggregate economy." Caballero (1999) further says "the quintessential problem of investment is that is almost always sunk ... opening a vulnerable flank," and the problem is more serious "when the exposed flanks are largely controlled by economic agents with the will and freedom to behave opportunistically." This is exacty what happens here.[12]

This distortion due to the capital holdup problem is over and above the usual inefficiencies that arise when $i > 0$, the money holdup problem that arises when $\theta < 1$, and the obvious effects of $t_h, t_k, t_x > 0$. If we run the FR ($i = 0$) and use lump-sum taxes exclusively, we would be left with only the holdup problems. In many models, all such problems can be resolved simultaneously if one simply sets θ correctly (the classic treatment is Hosios 1990;

[12] Holdup problems are often attributed to incomplete contracting, which seems reasonable given the nature of our DM. More generally, holdup problems might make more sense in search than in other models, to the extent that it is not possible to contract with a person before you contact a person.

see Rogerson et al. 2005 for a recent update). In our model, this is not possible: $\theta = 1$ resolves the problem in the demand for money, but this is the worst case for investment; and $\theta = 0$ resolves the problem in the demand for capital, but this this is the worst case for money. Under bargaining there is no θ that can solve both problems entirely; we next show that some alternative pricing mechanisms can.

2.3 Price Taking

It is known from labor market theory with ex ante investments that the solution concept called *competitive search equilibrium*, based on price posting rather than bargaining, eliminates holdup problems (Shimer 1995; Moen 1997; Acemoglu and Shimer 1999). Rocheteau and Wright (2005) show the same is true in monetary theory. Moreover, based on these results, we claim that *competitive equilibrium* with Walrasian price taking also does the trick here, although this is not true in general – e.g., Rocheteau and Wright show Walrasian pricing does not do as well when there are search externalities. We now analyze the model with price taking in the DM, and since there are no search externalities, it can be either interpreted as *competitive equilibrium* or as *competitive search equilibrium*.[13]

With price taking, the DM value function has the same form as (5), but (6) and (7) change. For a buyer,

$$V^b(m,k) = \max_q \ \{u(q) + W(m - \tilde{p}q, k)\} \text{ s.t. } \tilde{p}q \le m, \tag{25}$$

and for a seller

$$V^s(m,k) = \max_q \{-c(q,k) + W(m + \tilde{p}q, k)\}, \tag{26}$$

where $\tilde{p}$ is the price level in the DM, now taken parametrically. Note that $\tilde{p}$ generally differs from the price level in the CM, p. Market clearing implies buyers and sellers choose the same q. Also, exactly as in the bargaining model, buyers necessarily spend all their money in equilibrium, so $q = M/\tilde{p}$.

[13]One reason to study this case is that, since it avoids holdup problems, we can use it to quantify their impact in the other model. Another advantage of price taking is that it can be shown analytically that the SOC hold, which is not the case with bargaining.

The FOC from (26) is $c_q(q,k) = \tilde{p}W_m = \tilde{p}A/pw\,(1-t_h)$. Inserting $\tilde{p} = M/q$, we get the analog to (14) from the previous specification:

$$\frac{M}{p} = \frac{qc_q(q,k)w\,(1-t_h)}{A} \tag{27}$$

Given this, the analogs to (16) and (17) are:

$$\begin{aligned} V_m(m,k) &= \frac{(1-\sigma)A}{pw\,(1-t_h)} + \frac{\sigma u'(q)}{\tilde{p}} \\ V_k(m,k) &= \frac{A + A\,(r-\delta)\,(1-t_k)}{w\,(1-t_h)} - \sigma c_k(q,k) \end{aligned}$$

Inserting these into (2) yields the analogs to (19) and (20):

$$\begin{aligned} \frac{c_q(q,K)q}{M} &= \frac{\beta c_q(q_{+1},K_{+1})q_{+1}}{M_{+1}}\left[1-\sigma+\sigma\frac{u'(q_{+1})}{c_q(q_{+1},K_{+1})}\right] && (28) \\ U'(X) &= \beta U'(X_{+1})\left\{1+\left[F_K(K_{+1},H_{+1})-\delta\right](1-t_k)\right\} && (29) \\ &\quad -\sigma\beta\,(1+t_x)\,c_k(q_{+1},K_{+1}) \end{aligned}$$

The other equilibrium conditions do not change, and are repeated here for convenience:

$$\begin{aligned} U'(X) &= \frac{A\,(1+t_x)}{F_H(K,H)(1-t_h)} && (30) \\ X+G &= F(K,H)+(1-\delta)K-K_{+1}. && (31) \end{aligned}$$

An *equilibrium with price taking* is given by (positive, bounded) paths for (q, K_{+1}, H, X) satisfying (28)-(31), given policy and K_0. The difference between the bargaining and price-taking models is in the difference between (19)-(20) and (28)-(29). The first pair of equations differ because, in general, we do not have $g(q,K) = c_q(q,K)q$ and $g_q(q,K) = c_q(q,K)$, except in the special case where $c(q,K)$ is linear in q and $\theta = 1$, which implies $g(q,K) = c(q,K)$. The second pair of equations differ because, in general, we do not have $\gamma(q,K) = c_k(q,K)$, unless $\theta = 0$. The fact that the equilibrium condition for q in this model looks like the one from the bargaining model when $\theta = 1$ and the equilibrium condition for K looks like the one from the bargaining model when $\theta = 0$ suggests that the price-taking model avoids both holdup problems. We now verify this.

First, set $t_k = t_h = t_x = 0$. Then (29)-(31) are exactly the conditions for (K_{+1}, H, X) from the planner's problem in Section 2.1. Hence the equilibrium coincides with the FB iff $q = q^*(K)$, or $u'(q) = c_q(q, K)$. From (28), this is equivalent to

$$\frac{c_q(q,K)q}{M} = \frac{\beta c_q(q_{+1}, K_{+1})q_{+1}}{M_{+1}}.$$

Using (27) this reduces to $1/pw = \beta/p_{+1}w_{+1}$. Since $w = A/U'(X)$ it further reduces to $p/p_{+1} = U'(x)/\beta U'(X_{+1})$. Since in any equilibrium (not only steady state) the slope of the indifference curve $U'(x)/\beta U'(X_{+1})$ equals the slope of the budget line $1+i_R$, with i_R the real interest rate, the relation in question finally reduces to $\frac{p_{+1}}{p} = \frac{1}{1+i_R}$. Obviously this holds, and hence $q = q^*(K)$ solves (28), iff $\frac{p_{+1}}{p}(1+i_R) = 1$, which is equivalent to setting the nominal rate to $i = 0$ by the Fisher equation $\frac{p_{+1}}{p}(1+i_R) = 1+i$. We conclude that when $i = 0$ and we use only lump-sum taxes, equilibrium under price taking coincides with the FB.

3 Extensions

3.1 Two Capital Goods

To show the main ideas are robust, we sketch some extensions. First, we relax the assumption that the same capital stock is used in both markets. Suppose now k is used in the CM and z in the DM; they depreciate at rates δ and ω. Although they are inputs in different markets, output of both k and z is produced in the CM. Neither k nor z can be used as a medium of exchange in the DM. For the sake of illustration there is no tax on z, and here we present only the bargaining model (price-taking is similar).

The problem in the CM is now

$$\begin{aligned} W(m,k,z) &= \max_{x,h,m_{+1},k_{+1},z_{+1}} \{U(x) - Ah + \beta V(m_{+1}, k_{+1}, z_{+1})\} \\ \text{s.t.} \;\; (1+t_x)\,x &= w\,(1-t_h)\,h + [1 + (r-\delta)\,(1-t_k)]\,k - k_{+1} - T + \frac{m - m_{+1}}{p} \\ &\quad + (1-\omega)\,z - z_{+1}. \end{aligned}$$

Eliminating h using the budget equation, we have the FOC:

$$\begin{aligned}
x &: \; U'(x) = \frac{A(1+t_x)}{w(1-t_h)} \\
m_{+1} &: \; \frac{A(1+t_x)}{pw(1-t_h)} = \beta V_m(m_{+1}, k_{+1}, z_{+1}) \\
k_{+1} &: \; \frac{A}{w(1-t_h)} = \beta V_k(m_{+1}, k_{+1}, z_{+1}) \\
z_{+1} &: \; \frac{A}{w(1-t_h)} = \beta V_z(m_{+1}, k_{+1}, z_{+1}).
\end{aligned}$$

The envelope conditions for W_m, W_k and W_z are derived in the obvious way. The usual logic implies the distribution of (m, k, z) is degenerate for agents leaving the CM.

The DM is exactly as before, except we replace $c(q, k)$ with $c(q, z)$ and $g(q, k)$ with $g(q, z)$. The value function in the DM and the envelope conditions for V_m, V_k and V_z are derived in the obvious way. This leads to:

$$\frac{g(q,Z)}{M} = \frac{\beta g(q_{+1}, Z_{+1})}{M_{+1}} \left[1 - \sigma + \sigma \frac{u'(q_{+1})}{g_q(q_{+1}, Z_{+1})}\right] \tag{32}$$

$$U'(X) = \beta U'(X_{+1}) \left\{1 + \left[F_K(K_{+1}, H_{+1}) - \delta\right](1 - t_k)\right\} \tag{33}$$

$$U'(X) = \beta U'(X_{+1}) \left[1 - \omega - \frac{(1+t_x)\,\sigma\gamma(q_{+1}, Z_{+1})}{U'(x_{+1})}\right] \tag{34}$$

$$U'(X) = \frac{A(1+t_x)}{F_H(K,H)(1-t_h)} \tag{35}$$

$$X + G = F(K,H) + (1-\delta)K - K_{+1} + (1-\omega)Z - Z_{+1} \tag{36}$$

where $\gamma(q, Z)$ is defined as in (18). An *equilibrium* is now given by (positive, bounded) paths for $(q, K_{+1}, Z_{+1}, H, X)$ satisfying (32)-(36).

Notice (32) is equivalent to (19), except Z replaces K. Also, (33) is the standard condition for K from the one-sector growth model: in contrast to (20), γ is not in (33), but now shows up in (34). In any event, this model obviously does not dichotomize: there is no way to solve for the CM and DM variables independently, since the Z used as an input in the DM is produced in the CM. Given monetary policy affects q, it affects Z, and hence it affects the CM. We return to this specification in Section 5.3, where although it gives similar quantitative results to the baseline model, it is useful in developing intuition.

3.2 Capital Produced in DM

So far all investment occurs in the CM. Since it has been known since Stockman (1981) that it makes a difference if money is needed for investment, we now consider a setup where new k is acquired in the DM. We follow Shi (1999) and assume agents do not consume DM output q, but use it as an intermediate input that is transformed one-for-one into k, which is then an input to CM production; this contrasts with the other models, where capital is acquired in the CM and used in the DM. Each period a fraction σ of agents in the DM can produce the intermediate input, and a fraction σ can transform it into capital. Although agents cannot acquire new capital in the CM, they are allowed to trade used capital.

Let k be the amount of capital held by an agent entering the CM and k'_{+1} the amount of capital taken out, and hence into the next DM. Then the CM problem is:

$$\begin{aligned} W(m,k) &= \max_{x,h,m_{+1},k'_{+1}} U(x) - Ah + \beta V_{+1}(m_{+1},k'_{+1}) \\ \text{s.t. } (1+t_x)\,x &= w\,(1-t_h)\,h + [r-(r-\delta)\,t_k]k + (1-\delta)\,\phi k - \phi k'_{+1} - T + \frac{m-m_{+1}}{p} \end{aligned}$$

where ϕ is the goods price of used capital in terms of x. The FOC are:

$$\begin{aligned} x &: \ U'(x) = \frac{A\,(1+t_x)}{w\,(1-t_h)} \\ m_{+1} &: \ \frac{A}{pw\,(1-t_h)} = \beta V_{+1,m}(m_{+1},k'_{+1}) \\ k'_{+1} &: \ \frac{A\phi}{w\,(1-t_h)} = \beta V_{+1,k}(m_{+1},k'_{+1}) \end{aligned} \tag{37}$$

The envelope conditions are obtained in the obvious way and again imply W is linear.

We now show how to construct equilibrium so the distribution of (m,k') coming out of the CM is degenerate, even though the distribution going in is not. Consider bargaining (price taking is similar). As always, a buyer in the DM gives up all his money in exchange for q, and now brings $k = k' + q$ to the CM. The usual LW logic implies the bargaining solution is independent of (m_s,k_b,k_s), and that $q = q(m_b)$ solves $m_b/p = g(q,r,w)$ where

$$g(q,r,w,\phi) \equiv \frac{(1-t_h)\,w\,[\theta c\,(q) + (1-\theta)\,c'\,(q)\,q]\,[r-(r-\delta)\,t_k + (1-\delta)\,\phi]}{\theta A[r-(r-\delta)\,t_k + (1-\delta)\,\phi] + (1-\theta)\,(1-t_h)\,wc'\,(q)}.$$

Also, upon entering the DM with any (m, k'), we have

$$\begin{aligned} V(m,k') &= W(m,k') + \sigma \left\{ \frac{A\left[r-(r-\delta)\,t_k+(1-\delta)\,\phi\right]q\,(m)}{w\,(1-t_h)} - \frac{Am}{pw\,(1-t_h)} \right\} \\ &\quad + \sigma E \left\{ \frac{A\tilde{m}}{pw\,(1-t_h)} - c\left[q\,(\tilde{m})\right] \right\}, \end{aligned}$$

where the expectation is with respect to the money holding $\tilde{m}$ of a random agent – which we are in the process of establishing, but have not yet established, is degenerate.[14]

Then we have

$$\begin{aligned} V_m(m,k') &= \frac{(1-\sigma)\,A}{pw\,(1-t_h)} + \frac{\sigma\left[r-(r-\delta)\,t_k+(1-\delta)\,\phi\right]}{pw\,(1-t_h)\,g_q(q,r,w,\phi)} \\ V_k(m,k') &= \frac{A\left[r-(r-\delta)\,t_k+(1-\delta)\,\phi\right]}{(1-t_h)\,w}. \end{aligned}$$

Since V_m is independent of k', the FOC for m_{+1} in (37) implies m_{+1} is independent of k'_{+1} and hence the same for all agents.[15] Proceeding as in the baseline model, the analog to (19) is

$$\frac{\hat{g}\,(q,K,H,\phi)}{F_H\,(K,H)\,M} = \frac{\beta\hat{g}\left(q_{+1},K_{+1},H_{+1},\phi_{+1}\right)}{F_H\,(K_{+1},H_{+1})\,M_{+1}} \left[1-\sigma+\sigma\frac{F_K(K_{+1},H_{+1})(1-t_k)+\delta t_k+(1-\delta)\phi_{+1}}{\hat{g}\left(q_{+1},K_{+1},H_{+1},\phi_{+1}\right)}\right] \tag{38}$$

where $\hat{g}(q,K,H,\phi) \equiv g\left[q, F_K(K,H), F_H(K,H), \phi\right]$. Using V_k, the FOC for k'_{+1} is

$$\frac{\phi}{F_H\,(K,H)} = \frac{\beta\left[F_K\,(K_{+1},H_{+1})\,(1-t_k)+\delta t_k+(1-\delta)\,\phi_{+1}\right]}{F_H\,(K_{+1},H_{+1})}. \tag{39}$$

This does not pin down k'_{+1}: it is simply an arbitrage condition that must hold in any equilibrium.

Given (39), the demand for k'_{+1} is indeterminate and we are free to set $k'_{+1} = (1-\delta)K$ for all agents. Hence, we have (m_{+1}, k'_{+1}) degenerate in equilibrium (of course, as always, assuming quasi-linearity and interiority). The other equilibrium conditions are derived in

[14] This expectation makes sense if agents are bilaterally matched at random in the DM, but the results also go through if we assume that they trade multilaterally.

[15] What we really should say is that the choice of m_{+1} is independent of k'_{+1}, and is unique if $V_{mm} < 0$; again, strict concavity is not obvious, but it can be checked numerically. Note that we actually proved something quite strong here: we are constructing equilibrium where (m_{+1}, k'_{+1}) is degenerate, but the logic implies (at least if $V_{mm} < 0$) that m_{+1} would be degenerate even if k'_{+1} were not.

the obvious way, and can be summarized as

$$\begin{aligned} K_{+1} &= (1-\delta)K + \sigma q_{+1} && (40) \\ U'(X) &= \frac{A(1+t_x)}{(1-t_h)\,F_K\,(K,H)} && (41) \\ X+G &= F(K,H) && (42) \end{aligned}$$

An *equilibrium* is now given by (positive, bounded) paths for (q, ϕ, K_{+1}, H, X) satisfying (38)-(42).[16] This system obviously does not dichotomize, because inflation is a direct tax on capital accumulation, as in Stockman (1981). We will return to this in Section 5.3

3.3 Nonseparable Utility

Finally, we show how to break the dichotomy with a more general but still quasi-linear utility function, $\hat{U}(x, q, e) - Ah$. Although one can do it in a variety of ways, suppose that x interacts with the (q, e) brought in from the previous DM, so the latter are state variables in the current CM. To isolate the effects of nonseparable utility, assume k does *not* appear in the DM technology; then we can write $e = \xi(q) \equiv f^{-1}(q)$. Again we assume bargaining, and here we set distorting taxes to 0 to keep the notation manageable.

The CM problem is:

$$\begin{aligned} W(m,k,q,e) &= \max_{x,h,m_{+1},k_{+1}} \left\{ \hat{U}(x,q,e) - Ah + \beta V(m_{+1}, k_{+1}) \right\} \\ \text{s.t. } x &= wh + (1+r-\delta)k - k_{+1} - T + \frac{m - m_{+1}}{p} \end{aligned}$$

The FOC are:

$$\begin{aligned} x &: \hat{U}_x(x,q,e) = \frac{A}{w} \\ m_{+1} &: \frac{A}{pw} = \beta V_m(m_{+1}, k_{+1}) && (43) \\ k_{+1} &: \frac{A}{w} = \beta V_k(m_{+1}, k_{+1}) \end{aligned}$$

[16]We cannot generally reduce this model to a system in (q, K_{+1}, H, X), because the price ϕ cannot be eliminated from the equilibrium conditions. However, we can use standard asset pricing methods to write $\phi_t = \sum_{s=0}^{\infty} \beta^{s+1} (1-\delta)^s \frac{U'(X_{t+s})}{U'(X_t)} \left[(1-t_k)\, r_{t+s+1} + \delta t_k\right]$ (see e.g. Ljungqvist and Sargent 2000, eq. 10.24). In steady state, we can solve for ϕ and insert it into (38) to yield four equations in (q, K_{+1}, H, X).

We again get a degenerate distribution of (m, k), but now there is a distribution of x in the CM, as this choice is affected by random events in the last DM. Let $x_s = x_s(q, w)$, $x_b = x_b(q, w)$ and $x_0 = x_0(w)$ be the choices of agents who were sellers, buyers and non-traders in the previous DM, determined by the first condition in (43).

By the usual logic, $d = m_b$, and q solves the analog of (14) with $g(q, w)$ replacing $g(q, k)$, where in this model $g(q, w)$ satisfies

$$\begin{aligned}\Upsilon(q,w)g(q,w) \equiv\ & (1-\theta)\left\{\hat{U}\left[x_0(w),0,0\right]-\hat{U}\left[x_b(q,w),q,0\right]\right\}\hat{U}_e\left[x_s(q,w),0,\xi(q)\right]\xi'(q)\\ & +\theta\left\{\hat{U}\left[x_0(w),0,0\right]-\hat{U}\left[x_s(q,w),0,\xi(q)\right]\right\}\hat{U}_q\left[x_b(q,w),q,0\right]\\ & +(1-\theta)\frac{A}{w}\left[x_b(q,w)-x_0(w)\right]\hat{U}_e\left[x_s(q,w),0,\xi(q)\right]\xi'(q)\\ & +\theta\frac{A}{w}\left[x_s(q,w)-x_0(w)\right]\hat{U}_q\left[x_b(q,w),q,0\right],\end{aligned} \tag{44}$$

with $\Upsilon(q, w) \equiv \theta U_q\left[x_b(q, w), q, 0\right] - (1-\theta) U_e\left[x_s(q, w), 0, \xi(q)\right]\xi'(q) > 0$. Although (44) is messy, it simplifies dramatically in some special cases.[17] In any case, the usual methods yield the analog to (19)

$$\frac{\tilde{g}(q,K,H)}{M} = \frac{\beta\tilde{g}(q_{+1},K_{+1},H_{+1})}{M_{+1}}\left\{1-\sigma+\sigma\frac{U_q\left[\tilde{x}_b(q_{+1},K_{+1},H_{+1}),q_{+1},0\right]}{\hat{g}_q(q_{+1},K_{+1},H_{+1})}\right\}, \tag{45}$$

where $\tilde{g}(q, K, H) \equiv g[q, F_K(K, H)]$ and $\tilde{x}_b(q_{+1}, K_{+1}, H_{+1}) \equiv x_b[q, F_H(K, H)]$.

The other equilibrium conditions, analogs to (20)-(22), are derived in the obvious way and are omitted. It is clear from (45) that this model does not dichotomize: q cannot be determined independently of (K, H), in general. This model is really quite flexible in terms of its predictions, and we will briefly return to it below, but for the most part we want to keep the discipline and parsimony of separable preferences, and break the dichotomy via technology.

[17] Of course, if $\hat{U} = U(x) + u(q) - \ell(e)$ is separable then $\hat{g}(q, w) = g(q)$ and we are back to a model that dichotomizes. In the intermediate case $\hat{U} = \tilde{U}(x, q) - \ell(e)$, where we can write $c(q) = \ell[\xi(q)]$, the RHS of (44) reduces to

$$\theta c(q)\tilde{U}_q\left[x_b(q,w),q\right]+(1-\theta)\left\{\tilde{U}\left[x_b(q,w),q\right]-\tilde{U}\left[x_0(w),0\right]+\frac{A}{w}\left[x_0(w)-x_b(q,w)\right]\right\}$$

and $\Upsilon(q, w) = \theta\tilde{U}_q\left[x_b(q, w), q\right] - (1-\theta)\, c'(q)$. Alternatively, for any $\hat{U}$, if $\theta = 1$ then $\hat{g}(q, w) = U\left[x_0(w), 0, 0\right] - U\left[x_s(q, w), 0, \xi(q)\right] + \frac{A}{w}\left[x_s(q, w) - x_0(w)\right]$.

4 Quantitative Analysis

4.1 Preliminaries

In order to calibrate the model we first need to do some accounting. The price levels in the CM and DM are p and $\tilde{p} = M/q$, respectively, where p satisfies

$$p = \frac{AM}{(1-t_h)\, g\,(q,K)\, F_H(K,H)} \tag{46}$$

in the bargaining version of the model by (14), and

$$p = \frac{AM}{(1-t_h)\, qc_q\,(q,K)\, F_H(K,H)} \tag{47}$$

in the price-taking version by (27). Nominal output is $pF\,(K,H)$ in the CM and σM in the DM. Using p as the unit of account, real output in each sector is $Y_C = F\,(K,H)$ and $Y_D = \sigma M/p$, and total output is $Y = Y_C + Y_D$. The *share* of output in the DM is $s_D = Y_D/Y$.

Define the *markup* μ by equating $1+\mu$ to the ratio of price to marginal cost. The markup in the CM market is always 0, since it is competitive. The markup in the DM under price taking is also 0. With bargaining, however, the markup in the DM is derived as follows. Marginal cost in terms of utility is $c_q\,(q,K)$. Due to quasi-linearity, a dollar is always worth $A/p\,(1-t_h)\,w$ utils, so marginal cost in dollars is $c_q\,(q,K)\,p\,(1-t_h)\,w/A$. Since $\tilde{p} = M/q$, the DC markup μ_D is given by

$$1 + \mu_D = \frac{M/q}{c_q\,(q,K)\,p\,(1-t_h)\,w/A} = \frac{g\,(q,K)}{qc_q\,(q,K)},$$

after eliminating M using (46). The aggregate markup is $\mu = s_D \mu_D$.

We also discuss certain elasticities, including what we call the interest elasticity of money demand $\xi = \frac{\partial(M/p)}{\partial i}\frac{i}{M/p}$, derived in usual fashion. Consider ξ under bargaining (price-taking is similar). Using (46) and differentiating, we get

$$\xi = \left(g_q \frac{\partial q}{\partial i} + g_k \frac{\partial K}{\partial i}\right)\frac{i}{g} + \left(F_{HH}\frac{\partial H}{\partial i} + F_{HK}\frac{\partial K}{\partial i}\right)\frac{i}{F_H}. \tag{48}$$

It is now a matter of substituting $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$, which we derive in the Appendix, to yield ξ as a function of the allocation and parameters.

4.2 Calibration

Consider the following functional forms for preferences and technology:

$$\text{CM:} \qquad U(x) = B\frac{x^{1-\varepsilon}-1}{1-\varepsilon} \text{ and } F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$\text{DM:} \qquad u(q) = C\frac{q^{1-\eta}-1}{1-\eta} \text{ and } c(q,k) = q^{\psi}k^{1-\psi}$$

The $c(q,k)$ function comes from $\ell(e) = e$ and $q = e^{\chi}k^{1-\chi}$ where $0 < \chi \leq 1$, so $\psi = 1/\chi \geq 1$; if $\psi = 1$ the model dichotomizes. We normalize $C = 1$ with no loss in generality, and assume B, ε, $\eta > 0$ and $0 < \alpha < 1$.[18] In the Appendix we prove analytically that under price-taking, for these functional forms, a monetary steady state always exists, and under a simple restriction it is unique. This is not so easy for bargaining models, but in the numerical work we always find a solution and it appears to be unique.

We now describe the calibration strategy. Beginning with preferences, first set β to match a real interest rate of $i_R = 0.035$, which comes from annual US data, 1951-2004, where the average nominal rate on Aaa-rated corporate bonds is 7.2% and the average inflation rate using the GDP deflator is 3.6%. Although in our baseline model the period is a year, we discuss other options in Section 5.3, and this turns out not to matter. As a benchmark we set $\varepsilon = \eta = 1$ mainly to facilitate comparison with previous studies, but also because this is required for balanced growth in a generalized version with technical change; we also check robustness on this dimension in Section 5.3 and it turns out not to matter too much. The remaining preference parameters A and B are determined simultaneously with several other parameters, as described below.

Moving to policy, as we said, inflation is $\pi = 0.036$. We also more or less directly observe taxes, and we use $t_h = 0.242$ and $t_k = 0.548$, the average effective marginal rates in McGrattan et al. (1997). We set $t_x = 0.069$, the average of excise plus sales tax over

[18] For DM utility, we actually use $u(q) = \frac{(q+b)^{1-\eta}-b^{1-\eta}}{1-\eta}$ to guarantee $u(0) = 0$ for all η, so that in the bargaining model the buyer's threat point is finite; then we set $b = 0.0001$. We can set $C = 1$ wlog because we already have the constants A and B weighting h and x, and we do not need to weight e since we do not use it in any of the calculations (a weight on e would merely pick units for effort).

consumption in the data. Moving to technology, we set $\delta = 0.070$ to match I/K, where K is residential and nonresidential structures plus producer equipment and software.[19] We then set the coefficient α in the CM production function to match labor's share, which is 0.712 in our data, using the method in Prescott (1986); hence $\alpha = 0.288$. This leaves us with the two remaining preference parameters A and B, the size of government G, the coefficient in the DM cost function ψ, the probability of being a consumer or a producer in the DM σ, and, in bargaining models, the buyers' share θ.

Table 1 partitions parameters into two groups: the ones we have already set based on 'obvious' observations, and six yet to be determined, along with six other observations that we now discuss (tables are at the end of the paper). First is average hours worked as a fraction of discretionary time, $H = 1/3$ (Juster and Stafford 1991). Second is average velocity, which is $v = M/pY = 5.29$ when we measure M by $M1$.[20] Third is $G/Y = 0.25$. Fourth is $K/Y = 2.32$. Fifth is the money demand elasticity, which we estimate to be $\xi = -0.226$, as discussed below. Last is the markup, $\mu = 0.10$ (Basu and Fernald 1997). Our method is to set the six remaining parameters simultaneously to minimize the distance between the targets in the data and the model. As the mapping between observations and parameters is nonlinear, there is no presumption that we can match things perfectly, but we can get close – how close depending on the specification.

This is all fairly standard, but two issues deserve comment. The first issue concerns the fact that we target K/Y even though we have already used labor's share to pin down α. In the standard one-sector growth model, given β, δ and t_k, the steady-state condition implies α and K/Y are proportional. This is not true here, however, because of the DM. When capital is used in the DM, the greater is ψ the greater are the returns to investing, and the idea is to set ψ to match K/Y. This is reasonable especially because we have taxes in the model, and

[19] We remove consumer durables, inventories and net exports from all measurements. Thus, consumption includes services plus nondurables, investment is private fixed investment, and output is the sum of these, plus government consumption and investment.

[20] Using $M1$ facilitates comparison with existing studies and makes sense in light of fn.5, but we consider alternatives in Section 5.3.

it is well known that if α is set to match labor's share and t_k is set realistically, K/Y tends to be too low (at least in models that ignore household production; see e.g. Greenwood et al. 1995 or Gomme and Rupert 2005).

The other issue concerns the elasticity ξ. Following a common specification in the literature (e.g. Goldfeld and Sichel 1990), we specify log real money demand $\tilde{m}_t$ as a linear function of log nominal interest $\tilde{\imath}_t$ and log real output $\tilde{y}_t$, allowing for first-order autocorrelation in the residuals. Due to nonstationarity we estimate this in first differences:[21]

$$\Delta\tilde{m}_t = \beta_y \Delta\tilde{y}_t + \beta_i \Delta\tilde{\imath}_t - \rho\beta_y\Delta\tilde{y}_{t-1} - \rho\beta_i\Delta\tilde{\imath}_{t-1} + \rho\Delta\tilde{m}_{t-1} + \nu_t \tag{49}$$

$$\beta_y = 0.369 \ \ (0.124)\,,\ \beta_i = -0.226 \ \ (0.045)\,,\ \rho = 0.347 \ \ (0.131)\,,\ R^2 = 0.423$$

Here ρ is the AR(1) coefficient for the residuals in the original equation in levels and the numbers in parentheses are standard errors. The estimated long-run interest elasticity is $\xi = -0.226$, with a relatively small standard error of 0.05. We match this to the theoretical long-run elasticity, as in (48).

4.3 Decision Rules

Although we calibrate to steady state, we need to go beyond this and solve for equilibrium decision rules in order to analyze transitions after a policy change. Here we briefly describe our method. As is standard, we begin by scaling all nominal variables by the aggregate money stock, so that $\hat{m} = m/M$, $\hat{p} = p/M$, etc. Then the individual state variable is $(\hat{m}, k, K)$. In equilibrium, $\hat{m} = 1$ and $k = K$. A recursive equilibrium is then described by time-invariant functions $[q(K), K_{+1}(K), H(K), X(K)]$, solving (19)-(22) for the bargaining version or (28)-(31) for the price-taking version, plus value functions $[W(K), V(K)]$ solving (1) and (8). We solve these equations numerically using a nonlinear global approximation, which can be important for accurate welfare computations.[22]

[21] There is a mapping between the regression coefficients and the underlying parameters whether the estimation is done using differences or levels. We did it both ways, and the relevant elasticity estimates were statistically identical, so we only report results for differences.

[22] Specifically, we use the Weighted Residual Method with Chebyshev Polynomials and Orthogonal Collocation. See Judd (1992) for details, and Aruoba et al. (2006) for a recent comparison of solution methods.

Figure 1 plots the decision rules and value function for two preferred parameterizations, Models 4 and 5, as described in the next section. We show these functions for four scenarios: the planner's problem; monetary equilibrium at the FR; monetary equilibrium at 10% inflation; and nonmonetary equilibrium. We will return to discuss the economic content of these graphs below. For now, we mainly want to point out that over the range shown, which allows K to vary $\pm 90\%$ of its steady state, the functions are fairly nonlinear. Although in most of the policy experiments we do the economy remains within roughly $\pm 20\%$ of steady state, where the nonlinearity is less important, it is good to know what happens when we are further from steady state.

5 Results

5.1 Model 'Fit'

The basic calibration results are given in Table 2, where the first column lists the relevant moments in the data and the other columns list the moments from five different versions of the model. Model 1 fixes $\psi = \theta = 1$, giving up on K/Y and μ as targets; this model has no holdup problems, and in fact $\theta = 1$ is equivalent to price taking when $\psi = 1$ (but *not* when $\psi > 1$). Model 2 keeps $\psi = 1$ but calibrates θ. Model 3 fixes $\theta = 1$ and calibrates ψ. Model 4 calibrates θ and ψ. Model 5 calibrates ψ and assumes price taking, so there is no θ. In all cases we do well on the targets, with two exceptions. First, we match the markup μ only if we assume bargaining and calibrate θ (to around around 3/4) rather than assuming price taking or fixing $\theta = 1$, for reasons that should be clear.[23] Second, we do a good job matching K/Y only in the price-taking model, for reasons that we now explain.

As discussed above, our method is to calibrate α to labor's share and then try to match K/Y using ψ. When $\psi = 1$, capital does not affect DM production and we pin down K/Y exactly as in the standard model – meaning that it is too low compared to the data. When $\psi > 1$, K/Y is higher, but with bargaining this effect is small because the holdup problem

[23]Notice that in bargaining models with an arbitrary θ, the markup can actually be negative; e.g. $\theta = 1$ implies price equals average cost, which is less than marginal cost when $\psi > 1$.

erodes investors' returns from having capital in the DM. Hence, $\psi > 1$ does not help K/Y much in a bargaining model. With price taking, there is no holdup problem, and we can set ψ to generate a big enough return on capital to match K/Y exactly. So the price-taking model is better on this dimension, but again, it misses the markup.

We also compared the model to several statistics to which we do not calibrate. Here we focus on the elasticity of investment with respect to inflation, say ζ, since after all a main goal is to study the effects of money on capital accumulation. Using quarterly data, we estimate the long-run elasticity to be $\zeta = -0.023$ and statistically significant. Although this may appear small, it is economically relevant: raising inflation from our benchmark value to 7% e.g. reduces investment by arounf 2.3%, which is nothing to scoff at. As seen in Table 2, in Models 1–3 this elasticity is 0, since as we know from theory there is no feedback from money to the CM. In Models 4 and 5 we have $\zeta < 0$, although the effect is rather weak under bargaining (-0.001) and too strong under price taking (-0.060).

The weak effect in Model 4 is again due to the holdup problem: the extra return from DM production does not increase K much, so monetary policy, although it affects this return, does not have a big impact on total investment. Now obviously this depends on the value of ψ, but it turns out that the model cannot do much better if we choose ψ to make ζ as big as possible. Even if we eliminate μ as a target, freeing up θ to help match the other targets, a model with bargaining cannot do very well matching the elasticity of investment. If θ is big the capital holdup problem makes the DM return to K small; if θ is small the money holdup problem makes q small; and in either case investment is insensitive to inflation. With price taking, by contrast, in Model 5 we actually could match ζ by recalibrating, with relatively little sacrifice in other targets; we return to this in Section 5.3.

We also report the share of the DM in output, s_D, which varies between 4.6% and 5.2% across Models 1-5.[24] While we do not want to take a stand on the size of the different sectors in the real world, we think these numbers are reasonable, in the sense that we would be

[24]It is easy to see where this comes from. Velocity is $v = PY/M$ where $Y = Y_C + Y_D$. Since $Y_D = \sigma M/P$, $v = \sigma Y/Y_D$ and hence $s_D = Y_D/Y = \sigma/v$. Calibration implies $\sigma \approx 1/4$, so given $v \approx 5$, $s_D \approx 5\%$.

uncomfortable if the model predicted that anonymous monetary trade was too big a share of GDP. Because s_D is relatively small, we need a big markup in the DM – in the neighborhood of 200% – to match the economy-wide markup. Although we do not want to go into too much detail here, this prediction does not seem overly problematic: it would be no surprise if some anonymous decentralized trade did occur at very high markups.[25]

Finally, one might ask how we match the empirical money demand curve, compared to Lucas (2000), say. Figure 2 shows the relationship on which Lucas focuses, i versus M/pY, in the data and in a typical version of the model (Models 1-5 all look very similar in this regard). As is true of other approaches to these data, including those discussed by Lucas, it is not easy to fit the observations in the northwest part of the scatter plot. In our sample these are all from 1951-1960. Ignoring this decade, we think our money demand curve looks reasonable. In any case, one should not put too much weight this diagram, since the specification assumes the income elasticity of money demand is 1, which is rejected by the data. Alternatively, one can look at regression equation (49), which evidently fits quite well, and we match the velocity and interest elasticity in the data almost exactly.

5.2 Experiments

Table 3 contains the results in each of the five models when we perform the standard experiment of changing the inflation rate from $\pi^1 = 0.1$ to the FR, which is $\pi^2 = -0.0338$ for this calibration. For now, when we change π we make up any change in government revenue using lump-sum taxes, but we consider other fiscal policies below. The table gives ratios of equilibrium values of several variables at the two inflation rates. When a *1* appears in italics, the true number is not exactly unity, but shows up this way due to rounding: this is

[25] Although $\mu = 0.1$ seems a reasonable aggregate target, as Faig and Jerez (2005) point out, markups vary a lot. Data on retailers at http://www.census.gov/svsd/www/artstbl.html, e.g., indicate the following: at the low end, warehouse clubs, superstores, car dealers and gas stations have μ between 17% and 20%; and at the high end, specialty food, clothing, footwear and furniture stores are over 40%. Of course, retail does not mean anonymous or monetary trade. More work needs to be done on both theory and measurement in this area, but we are aware that our calibration strategy for Model 4 is in some sense extreme: since $\mu_C = 0$, we need a big μ_D to match aggregate μ. Model 3 is the other extreme, $\mu_D = 0$, and if we target μ between 0 and 10%, the results naturally are between Models 3 and 4. In any case, we want to point out that if we had $\mu_C > 0$, for whatever reason, or a bigger s_D, we could get a away with a smaller μ_D.

to distinguish results where we know as a matter of theory that inflation has no effect from results where there is an effect but it is too small to show up numerically.

The first thing to note is that in all models q^1/q^2 is considerably less than 1, varying between 2/3 and 4/5. Again, π is a tax on DM activity, and this shows it is quantitatively important. In Models 1-3 this is the only effect. In Model 4, in theory, π does affect the CM, but the impact is tiny (as might be expected from the discussion in Section 5.1). Models 1-4 predict that going from $\pi = 0.1$ to the FR increases aggregate output Y by 2%, essentially all due to the change in q. In Model 5 the effects of this policy are very different: q increases by about the same percentage, but now this raises K by 12%, X by 3% and Y by 6%. Hence, we conclude that inflation can have a very big impact on the CM through capital accumulation if we assume price taking, but not if we assume bargaining.[26]

Turning to welfare, we solve for Δ such that agents are indifferent between changing π and increasing total consumption by factor Δ We report the answer comparing across steady states – i.e. jumping instantly from π^1 and K^1 to π^2 and K^2 – as well as the cost of transition from K^1 to K^2 and the net gain to changing π starting at K^1. This last number gives the true benefit of a policy change, although the steady state comparison is still interesting: it tells us how much an agent having π^1 and K^1 would pay to trade places with an agent agent having π^2 and K^2. Of course, in Models 1-3 there is no transition and in Model 4 we expect the transition to be unimportant, since π does not affect K much, but in Model 5 it could be quite important. We also report the net gain to reducing π to 0, to see how much of the gain comes from eliminating inflation and how much comes from deflation.

In Models 1 and 3, with $\theta = 1$, going from 10% inflation to the FR is worth around 3/4 of 1% of consumption, commensurate with most previous findings such as Lucas (2000) or Cooley and Hansen (1991). In Models 2 and 4, with $\theta \approx 3/4$, this policy is worth over 3% of consumption. Intuitively, at $\theta \approx 3/4$ the money holdup problem makes q inefficiently

[26] Intuitively, the reason inflation affects K a lot in Model 5 is that our calibration implies DM production is relatively capital intensive: $\psi \approx 2.5$ implies $\chi \approx 0.4$, compared to $\alpha \approx 0.3$. Hence, when q falls there is a big impact on the overall return to K. This does not work in Model 4, even though it implies $\chi \approx 0.5$, because of holdup.

low, with or without other distortions, so any additional reduction is very costly. In Model 5 the steady state gain from this policy is about the same as in Models 2 and 4, but the economics is completely different. With price taking, π has a big impact on K, and hence on X. However, since much of the gain accrues only in the long run, and agents must work more and consume less during the transition in order to accumulate the additional capital, the net gain is only 1.87% – smaller, but still sizable.

We find it interesting that there are two distinct ways to get big numbers for the welfare cost: assume bargaining with $\theta \approx 3/4$, whence the money holdup problem makes the reduction in q due to inflation very costly; or assume price taking with $\psi \approx 2.5$, whence inflation has a big impact on K and X. We emphasize that while the first effect can be seen in the basic LW money model, the latter cannot: without capital, under either price taking or $\theta = 1$, the cost of inflation predicted by that model is about the same as in the 'reduced form' literature, around 3/4 of 1%. One gets a bigger number without capital if and only if there is a money holdup problem (see Rocheteau and Wright 2006 for more discussion). Here, because of the effect on K, inflation is costly even under price taking.

Both of these stories about why inflation is costly lead to similar steady state welfare comparisons, but as we said, for the investment story the gains are tempered by a costly transition. Figure 3 plots the transition paths following the reduction from 10% inflation to the FR for Models 4 and 5. As one can see, in Model 5, in the short run H increases by over 4% and X falls by around 1% before settling down to their new steady state levels. And q jumps on impact by around 16% before increasing 25% in the long run when K increases. The paths are qualitatively similar in Model 4, but the magnitudes are very different. The only quantitatively important effect in Model 4 is the increase in q, which is slightly bigger than predicted by Model 5 in the long run, and moreover occurs very quickly.

Table 4 compares the FR and FB allocations. The differences are big, but mainly due to taxation, and one finds similar results in nonmonetary models with proportional taxes (McGrattan et al. 1997). To check this, we also report the gain to moving from the FR to the FB after setting $t_h = t_k = t_x = 0$ and recalibrating other parameters. In Models 1 and 5, the gain in this case is 0 because the FR implements the FB. In Model 3, with capital holdup but not money holdup, the steady state gain is around 3%, although much is lost in transition. In Model 4, with both, it is around 5%. These calculations provide measures of the immpact of holdup: based on the steady state comparisons, e.g., one could say that 3% of consumption is the cost of capital holdup and an additional 2% the cost of money holdup, and although there is no single 'correct' way to decompose the effects, this suggests they may be quantitatively important even though s_D is only around 5%.[27]

Table 5 reports the actual allocations, not just the ratios of the allocations at different π, to facilitate comparisons across models for a given π. Notice q is considerably lower in Models 2 and 4 than in other models, due to the money holdup problem. Also, comparing Models 4 and 5, note that the latter has considerably bigger K and K/Y ratio for moderate inflation rates, although K/Y is basically the same at the FB. In other words, K is much more sensitive to π in the price-taking model, as we already discussed. The table also reports the allocation in the nonmonetary equilibrium, where $q = 0$, which is the limit of the monetary equilibrium as $\pi \to \infty$.[28]

At the risk of being redundant, one can also see what is happening from the decision rules in Figure 1. In Model 5, as we lower π the decision rule for K_{+1} shifts up and steady state K increases, although it is still far from the FB even at the FR (the FB steady state $K = 2.18$ is off the chart). Also, the decision rule for q shifts up, increasing q in the short

[27] One issue is that the calibrated parameters differ across the columns in Table 4. Suppose we instead fix the parameters at the ones calibrated to Model 4, and consider three cases: (i) $\theta = 1$; (ii) θ calibrated; and (iii) price taking. With taxes, going from the FR to the FB in these three scenarios is worth, in terms of steady state (net) comparisons: (i) 32.87 (16.33); (ii) 43.90 (26.06); and (iii) 14.31 (6.58). Recalculating with taxes set to 0, we get: (i) 5.57 (1.06); (ii) 14.80 (9.93); and (iii) 0 (0). Looking at the results with no taxes, one could say the cost of capital holdup in terms of steady state is 5.57, or 1.06 including transition, and the cost of money holdup is 9.23, or 8.87 including transition. With taxes the cost of capital and money holdup including transitions are almost identical, 9.75 and 9.73. Again there is no single 'correct' way to measure these costs.

[28] We can compute the welfare cost of large inflations – e.g. going from $\pi = 100\%$ to the FR is worth around 8% in Model 4 and 14% in Model 5 – but one has to take these calculations with a grain of salt, for at least two reasons. First, agents might devise other ways to trade in the DM at such high values of π (e.g. foreign currency). Second, when π is very high and hence q is very low, the results are sensitive to the value of b mentioned in fn. 19 (for moderate π the results are more robust; see Section 5.3).

run and more in the longer run as we move along the decision rule with the growth in K. The latter effect is important, in Model 5, since K grows a lot. In Model 4 the decision rule for K_{+1} and hence steady state K change little. The decision rule for q shifts, giving a short run effect, but there is very little additional growth effect. Still, inflation is very costly in Model 4 because the decision rule for q at the FR is far from the decision rule at the FB, so any change in q matters a lot; by contrast, in Model 5 the decision rules for q at the FR and FB are virtually coincident.

So far we computed the cost of inflation when any change in revenue is offset by changing T. One can also consider changing proportional taxes. Cooley and Hansen (1991) e.g. find that if one or more proportional taxes are used to make up the revenue shortfall, eliminating inflation is *not* beneficial. It seems of interest to revisit the issue in our model. The results are reported in Table 6 for the case where we make up the revenue with lump-sum taxes, with labor taxes, and with consumption taxes.[29] The first panel reproduces the results from Table 3, of course, and shows that with lump sum taxation it is always desirable to eliminate inflation. In the other panels there are two effects, a beneficial one due to the change in π, which is equal to the effect from the first panel, plus a detrimental one due to changes in the distortionary tax.

Going to the FR and making up the revenue by increasing t_h (from 25% to around 30%, depending on the specification) reduces Y by around 4% in Models 1-4, and increases Y by 1% in Model 5. The net welfare gain in the bargaining models is negative when $\theta = 1$ but positive when θ is calibrated; it is negative in the price-taking model. The net gain is positive if we make up the revenue by increasing t_x (from 7% to 14%) in the price-taking model and in the bargaining models with calibrated θ. We think these results are interesting because they imply that one does not necessarily need access to lump-sum taxation to argue that lower inflation may be desirable. More work could be done on this issue; e.g. it would be

[29]We could not solve the case where we make up the revenue with t_k, since increasing t_k lowered K by so much that sufficient revenue was not forthcoming (this is also true in Cooley and Hansen 1991). For these experiments we allow the government to issue a bond paying interest equal to the discount rate, simply so that we do not have to adjust taxes each period during the transition.

interesting to study the optimal monetary-fiscal policy mix, but this goes beyond the scope of the present paper.

5.3 Robustness

In order to consider robustness, we redo the calculations after changing several aspects of the specification. In the interest of space, Table 7 reports the results in terms of one key statistic: the net welfare gain of going from 10% inflation to the FR, including any transition. The first row is the benchmark model discussed above.

The first robustness check involves shutting down distorting taxes. We report results when parameters other than taxes are kept at benchmark values, and when they are recalibrated, which seems more reasonable. In either case, for Models 1-4 the results are very similar to the benchmark calibration, but in Model 5 the cost of inflation is lower without distorting taxes. This is no surprise. Without taxes the FR achieves the FB in Model 5, and hence the cost of a small inflation is low by the envelope theorem. With taxes the FR does not achieve the FB, so the envelope theorem does not apply and the cost is higher. We do not think this is worrisome: the results predicted by Model 5 *should* depend on what one assumes about taxation. Since taxes are a fact of life, we trust the benchmark calibration.

The next several rows vary the curvature parameters ε and η in the CM and DM utility functions, as well as the parameter b mentioned in fn. 19 that is used to guarantee $u(0) = 0$. One can look at the numbers for oneself, but our conclusion is that the results are not overly sensitive to these choices (although as we mentioned in fn. 28, b does matter more when we move to very high inflation rates). We think this is comforting, especially since the overall 'fit' of the model does not depend too much on these parameters. One can also vary other baseline parameters (e.g. β or δ) over a reasonable range without affecting the results too much; in the interest of space, we omit these results.

We tried several different measures of money, $M0$, $M2$ and $M3$, in addition to $M1$. As one can see, this does make a difference. The main reason is that these different measures imply different values for velocity, with $v = PY/M$ decreasing as we use broader measures. As v changes, the calibrated size of the DM changes, and so does the estimated cost of inflation. Some intuition for this comes from the traditional method of computing the welfare cost of inflation as the area under a money demand curve such as the one in Figure 2: using a narrower notion of money shifts the curve down and reduces the estimated welfare cost.[30] While it is perhaps unfortunate that results depend on the definition of M, this is bound to be true for *any* theory of money, not only ones that try to incorporate frictions explicitly. Still, one needs to think carefully about the 'best' empirical measure of money, since it does make a difference.

Table 7 also shows the impact of using different frequencies and time periods (recall the baseline sample is yearly, 1951-2004). Again one can look at the numbers for oneself, but we conclude that the results are not sensitive to these choices. The fact that using different frequencies does not change the results is especially comforting since this is not the case in typical 'reduced-form' models. To change the frequency here, we only have to adjust inflation, velocity, interest rates, K/Y, and I/K by the relevant factor. The calibrated σ declines as we increase the frequency, because shortening the period reduces the probability of consuming or producing each period in the DM, but the welfare conclusions are basically the same.

One can go beyond parameter values and check robustness with respect to larger modeling choices. Recall the extension in Section 3 with two capital stocks, K in the CM and Z in the DM. Tables 8 and 9 report calibration results and the effects of π for two versions of the model, one with bargaining and one with price taking, called Models 6 and 7, which are two-capital analogs of Models 4 and 5.[31] Models 6 and 7 do about as well as Models 4 and

[30] At least this is the case as we move from *M0* to *M1* to *M2*; as we move from *M2* to *M3*, although v goes down, the calibrated elasticity ξ also changes, and the net effect is to reduce the cost. In any case, this story is only meant to help with intuition, and is not an endorsement of the traditional way of measuring the cost of inflation – the different versions of our model all generate basically the same money demand curve, but predict different welfare costs. See also Craig and Rocheteau (2005).

[31] For Model 7 the calibration tried to make ψ very big – i.e. to make the underlying technology $q = e^{\chi} z^{1-\chi}$ almost linear in z – which led to numerical problems in solving for decision rules. We report results when we restrict $\psi \leq 10$, which avoids these problems with little sacrifice in terms of matching the steady state.

5 in terms of matching the targets. In Model 6, Z is very low due to the capital holdup problem; since both versions have similar K/Y, not surprisingly, Model 7 does better on $(K+Z)/Y$. Comparing Models 6 and 7 to Models 4 and 5, q actually increases by more in the two-capital models when we reduce π, which tends to make inflation more costly.

However, in Model 7 there is another effect, and the net cost of π is actually slightly lower than in Model 5. This effect is the following. As always, a decrease in π raises q, and with price taking this leads to a sizable increase in the demand for the capital used to produce q. In Model 5 this same capital is used to produce CM consumption, so in the long run X increases too. In Model 7, however, different capital stocks are used to produce q and X; hence, when lower π causes Z to grow, this does not spill over to X. Although K does grow when we reduce π, this is only because more K is needed to produce the additional Z; there is no effect on X. Despite this detail, the overall picture from the two-capital models is fairly similar to the base case.[32]

Tables 8 and 9 also report results from the extension where K is produced in the DM, for both bargaining and price-taking versions, called Models 8 and 9, which is interesting because now inflation taxes capital accumulation directly and not only indirectly via DM consumption.[33] In this model, π has a sizable effect on K under bargaining as well as price taking. Overall the results are not so different from the base case, although the welfare cost is somewhat higher than in the other models with bargaining. While there may be reasons to prefer models where investment occurs in the CM, it may also be worth studing this case in more detail, although one might want to rethink the calibration a little. We presented these results mainly to show that the basic ideas carry over under various alternative assumptions, and the results do not hinge too critically on some details of the specification.

[32]Model 7 actually has an interesting transition. After reducing π, Z jumps by nearly 50%, partly from increasing H and reducing X, and partly from capital flowing from the CM to the DM. Thus, K falls 4% in the short run. In the long run, Z grows even more as K goes up to around 2% above its initial level.

[33]In Model 8, when we try to match μ, we get a very low θ, a bad fit for money demand, and an excessively high inflation - investment elasticity. Because of this we did not have much faith in those results, and instead simply set $\theta = 3/4$, close to the calibration in the other bargaining models. Also, notice that in Models 8 and 9 s_D is bigger, because now all investment occurs in the DM.

We report one more robustness calculation. In terms of the relation between money and capital, Model 5 is in some sense our preferred specification – it has something interesting to say because it avoids the holdup problems that effectively kill this relation in bargaining models. However, one might worry that our strategy for picking parameters overestimates the effect, because as we mentioned in Section 5.1, it implies an elasticity of investment with respect to inflation of $\zeta = -0.060$ while the data indicate $\zeta = -0.023$. We reiterate that we did not try to match ζ; rather, solidly in the tradition of the business cycle literature going back to the early days of calibration, we target long-run averages and let the elasticities fall where they may (the one exception being the money demand elasticity).

To pursue the analogy, when in the textbook business-cycle model we set A so that H is $1/3$ we are also implicitly setting the labor supply elasticity; and when we set ψ to match K/Y we are implicitly setting the investment elasticity, given other parameters. Some people worry the implied labor supply elasticity is too high in the standard model, and may also worry that our investment elasticity is too high. If we recalibrate with ζ added to the list of targets, we do fairly well except that K/Y is a little low. See Model 10. The calibrated ψ is smaller, and hence the effect of π on K and the welfare numbers go down somewhat. It is not clear which calibration strategy and hence which results one should prefer is better, since Model 10 matches ζ but misses K/Y. We like the idea of targeting first moments, but even if we match the elasticity directly, the results are not so different.

We close by mentioning one final issue. All of the models considered so far predict inflation that has either a negative impact or no impact on investment. What about the so-called Tobin effect? Tobin (1965) said that inflation increases investment by tilting the relative return away from money and toward capital. This does not work in simple models. Imagine higher inflation leading to more investment, as agents increase savings when consumption becomes more expensive due to inflation. What do they do with the additional capital? They make more consumption goods. The inflation tax cannot be avoided by saving. This is why in the basic cash-in-advance model π has no effect on K. And in more sophisticated versions, like Stockman (1981), or the textbook model with both cash and credit goods, π tends to reduce K.

Now consider our setup with nonseparable preferences, as in Section 3.3. In fact, all we need is utility $U(x, q) - \ell(e) - Ah$ to be nonseparable in x and q. Here is the intuition.[34] As always, inflation reduces q. If q and X are substitutes, this increases X. Then as long as CM production is more capital intensive than DM production, inflation can increase K. That's it. So it is easy to get $\partial K/\partial\pi > 0$ with nothing fancy, other than two sectors that are asymmetric in terms of their need for money and their capital intensity, plus nonseparable utility. Now, to be sure, we are not trying to argue here that there is a positive relation between inflation and investment in the data – only that it is not difficult in this kind of model to get one in theory.

6 Conclusions

We presented a framework that combines elements from the microfoundations of money and from mainstream macroeconomics. We analyzed several models, and used them to study monetary and fiscal policy quantitatively. Although our benchmark model has feedback between the CM and DM because capital produced in the former is used as an input in both markets, several alternative specifications were also analyzed, including versions with different capital goods in the two markets, and versions with capital used in the CM produced in the DM. While the exact results depend on some details, there is no doubt that the idea works – this idea being that, given money affects decentralized trade, in economies with capital, money can also affects investment, consumption and employment.

One interesting finding is that the DM does not have to be very big – say, only around 5% of GDP – for it to matter quantitatively. Another finding that we believe is especially interesting is the following: although inflation has important welfare consequences in all

[34] The idea here comes from Rocheteau et al. (2005), where a similar explanation of the Phillips curve is developed with all the details made explicit. In discussing these kinds of results, it is useful to bear in mind that certain comparative static results always hold in this framework because of quasi-linear utility.

versions of the model, the transmission mechanism differs substantially across specifications. In particular, with price taking it has a big impact on investment and CM consumption, and with bargaining inflation has only a small impact on investment but it is still costly because it reduces DM consumption. These results were not obvious to us ex ante.

There is much left to do, including refinement of the quantification. As we said above, this paper is in the business-cycle-calibration tradition of trying to match mainly first moments in the data, and then asking what happens when something changes (e.g. when there is a technology shock or a policy shift). But there is no reason in principle why one could not bring to bear more sophisticated econometrics. We thought it would be good to start with calibration methods, which have some advantages in terms of simplicity and clarity.

Moreover, given what we have so far, it should be easy and might be interesting to add stochastic shocks and study the business-cycle properties of the model. Here we concentrated on the long-run implications of monetary policy, since this is a classic issue and one on which we shed new light. For business-cycle analyses one may want to add various embellishments; for instance, we see no reason in principle why the framework would not accommodate anything from sticky prices to job search to whatever else one finds interesting. Since it has been shown that taking money seriously matters for interesting questions in simple models, one might reasonably want to know how it matters in more complicated or realistic models. We leave this for future work.

A Appendix

A.1 The Cost Function

Here we verify the properties of the DM cost function $c(q,k)$ stated in Section 2. This function comes from a production function $q = f(k,e)$ that is strictly increasing and concave, and a disutility of effort function $\ell(e)$ that is is strictly increasing and convex. By definition, saying k is a normal input means that in the problem $\min\{we + rk\}$ s.t. $f(k,e) \geq q$, the solution satisfies $\partial k/\partial q = f_e f_{ek} - f_k f_{ee} > 0$.

To proceed, first rewrite $q = f(k,e)$ as $e = \xi(q,k)$. Then $\partial e/\partial q = \xi_q = 1/f_e > 0$ and $\partial e/\partial k = \xi_k = -f_k/f_e < 0$. Also $\xi_{qq} = -f_{ee}/f_e^3 > 0$, $\xi_{kk} = -\left(f_e^2 f_{kk} - 2 f_e f_k f_{ke} + f_k^2 f_{ee}\right)/f_e^3 > 0$, and $\xi_{kq} = -\left(f_{ek} f_e - f_{ee} f_k\right)/f_e^3$. Hence, $c_q = \ell'/f_e > 0$, $c_k = -\ell' f_k/f_e < 0$, $c_{qq} = \left[\ell'' \ell'^2 f_e - \ell' f_{ee}\right]/f_e^3 > 0$, $c_{kk} = -\left[\ell'\left(f_e f_{kk} - 2 f_e f_k f_{ke} + f_k^2 f_{ee}\right) - f_e f_k^2 \ell''\right]/f_e^3 > 0$ and $c_{qk} = -\left[\ell'' f_e f_k - \ell'\left(f_k f_{ee} - f_e f_{ek}\right)\right]/f_e^3$. These results establish that c is increasing and convex in q and decreasing and convex in k, and that $c_{qk} < 0$ if k is a normal input, as claimed.

A.2 Money Demand Elasticity

The interest elasticity of money demand is $\xi = \frac{\partial(M/P)}{\partial i}\frac{i}{M/P}$. To compute this in the bargaining model (price taking is similar) we need to determine $\partial q/\partial i$, $\partial K/\partial i$ and $\partial H/\partial i$ and substitute them into (48). Eliminating X, we can write the steady state as 3 equations in (q, K, H):

$$\frac{i}{\sigma} = \frac{u'(q)}{g_q(q,K)} - 1$$
$$\rho = \left[F_K(K,H) - \delta\right](1 - t_k) - \frac{\sigma\,(1+t_x)\,\gamma(q,K)}{U'\left[F(K,H) - \delta K - G\right]}$$
$$U'\left[F(K,H) - \delta K - G\right] F_H(K,H) = \frac{A\,(1+t_x)}{(1-t_h)}$$

We take the total derivative of this system to obtain

$$B \begin{bmatrix} dq \\ dK \\ qH \end{bmatrix} = \begin{bmatrix} di \\ 0 \\ 0 \end{bmatrix}$$

where

$$B = \begin{bmatrix} \frac{\sigma(g_q u'' - u' g_{qq})}{g_q^2} & -\frac{\sigma u' g_{qk}}{g_q^2} & 0 \\ -\frac{\sigma(1+t_x)\gamma_q U'}{U'^2} & \Theta & \frac{(1-t_k)U'^2 F_{KH} + \sigma(1+t_x)\gamma U'' F_H}{U'^2} \\ 0 & (F_K - \delta) F_H U'' + F_{KH} U' & F_H^2 U'' + F_{HH} U' \end{bmatrix}$$

and $\Theta = (1-t_k) F_{KK} - \frac{\sigma(1+t_x)}{(U')^2}\left[\gamma_k U' - (F_K - \delta)\gamma U''\right]$. We can now compute the partials as

$$\frac{\partial q}{\partial i} = B_{11}^{-1} \; \frac{\partial K}{\partial i} = B_{21}^{-1} \; \frac{\partial H}{\partial i} = B_{31}^{-1}$$

where B_{ij}^{-1} refers to the (i,j) element of B^{-1}.

A.3 Existence and Uniqueness

Here we show that for the functional forms we use in the calibrated model, under pricing taking, a steady state exists and under certain conditions is unique. With the functional forms in question, (28)-(31) can be written:

$$\begin{aligned}
\frac{K^{1-\psi}}{q^{-\psi}} &= \frac{\beta}{1+\pi}\left[(1-\sigma)\frac{K_{+1}^{1-\psi}}{q_{+1}^{-\psi}} + \sigma\psi(q_{+1}+b)^{-\eta}q_{+1}\right] && (50)\\
\frac{X_{+1}^{\varepsilon}}{X^{\varepsilon}} &= \beta(1-t_k)\left[\alpha\left(\frac{K_{+1}}{H_{+1}}\right)^{\alpha-1} + 1 - \delta\right] - \frac{\sigma\beta(1+\pi_x)(1-\psi)}{B}\frac{X_{+1}^{\varepsilon}K_{+1}^{-\psi}}{q_{+1}^{-\psi}} && (51)\\
X &= \left[\frac{B(1-\alpha)(1-t_h)}{A(1+t_x)}\frac{K^{\alpha}}{H^{\alpha}}\right]^{1/\varepsilon} && (52)\\
X &= K^{\alpha}H^{1-\alpha} + (1-\delta)K - K_{+1} - G && (53)
\end{aligned}$$

Let $\mathbb{k} = K/H$, and combine (53) and (52) to get

$$\frac{\mathbb{k}}{K}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} = \mathbb{k}^{\alpha} + (1-\delta)\mathbb{k} + \frac{H_{+1}}{H}\mathbb{k}_{+1} - \frac{G}{K}\mathbb{k}.$$

Hence, in steady state,

$$K = \frac{\mathbb{k}^{1-\alpha}\left[\frac{(1-\alpha)(1-t_h)}{A(1+t_x)}B\mathbb{k}^{\alpha}\right]^{1/\varepsilon}}{1-(\delta+\frac{G}{K})\mathbb{k}^{1-\alpha}}. \tag{54}$$

Given $b \approx 0$, (50)-(52) reduce to:

$$\begin{aligned}
q &= \left[\frac{\sigma}{\psi(i+\sigma)}\right]^{\frac{1}{\psi+\eta-1}} K^{\frac{\psi-1}{\psi+\eta-1}} && (55)\\
X &= \left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}^{\alpha}\right]^{1/\varepsilon} && (56)\\
1 &= \beta\left[1+(\alpha\mathbb{k}^{\alpha-1}-\delta)(1-t_k)\right] && (57)\\
&\quad + \tfrac{(\psi-1)\sigma\beta(1-\alpha)(1-t_h)}{A}\left[\tfrac{\sigma}{\psi(i+\sigma)}\right]^{\frac{\psi}{\psi+\eta-1}}\mathbb{k}^{\frac{\alpha(\psi+\eta-1)-(1-\alpha)\psi\eta}{\psi+\eta-1}}\left\{\frac{1-(\delta+G/K)\mathbb{k}^{1-\alpha}}{\left[\frac{(1-\alpha)(1-t_h)B}{A(1+t_x)}\mathbb{k}\right]^{1/\varepsilon}}\right\}^{\frac{\psi\eta}{\psi+\eta-1}}
\end{aligned}$$

Notice (57) is one equation in $\mathbb{k}$. The RHS approaches ∞ as $\mathbb{k} \to 0$ and approaches a value less than 1 as $\mathbb{k} \to (\delta + G/K)^{1/(\alpha-1)}$. Hence it has a solution. The solution is unique if we assume $\alpha(\psi+\eta-1) < (1-\alpha)\,\psi\eta$, since then the RHS is strictly decreasing. Given $\mathbb{k}$, (54) yields K, (55) yields q, (56) yields X, and $H = \mathbb{k}/K$. So we have existence, and uniqueness under a simple restriction.

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Table 1 - Calibration Parameters and Targets

(a) 'Obvious' Parameters

Parameters	β	ε	η	t_h	t_k	t_x	δ	α
Targets	0.966	1	1	0.242	0.548	0.069	0.070	0.288

(b) Remaining Parameters

Parameters	A	B	G	ψ	σ	θ
Targets	H	v	G/Y	K/Y	$-\xi$	μ
Target Values	0.33	5.29	0.25	2.32	0.23	0.10

Table 2 - Calibration Results

	Data	Model 1 $\psi = 1$ $\theta = 1$	Model 2 $\psi = 1$ θ free	Model 3 ψ free $\theta = 1$	Model 4 ψ free θ free	Model 5 ψ free p taking
Calibrated Parameters						
σ		0.24	0.27	0.24	0.26	0.27
B		2.52	2.50	1.53	1.30	2.39
ψ		–	–	1.65	1.93	2.51
A		6.42	6.39	3.89	3.31	6.49
G		0.11	0.11	0.11	0.11	0.13
θ		–	0.78	–	0.74	–
Calibration Targets						
μ	10.00	0.00	10.00	-1.81	10.00	0.00
K/Y	2.32	1.86	1.86	1.86	1.87	2.32
G/Y	0.25	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33	0.33
v	5.29	5.33	5.33	5.33	5.33	5.29
ξ	-0.23	-0.23	-0.23	-0.23	-0.23	-0.23
Miscellaneous						
s_D		4.59	5.01	4.59	4.87	5.16
μ_D		0.00	199.38	-39.36	205.28	0.00
ζ	-0.023	0.000	0.000	0.000	-0.001	-0.061

Table 3 - $\pi = 0.1$ vs. FR

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	0.64	0.64	0.76	0.80	0.79
K^1/K^2	1.00	1.00	1.00	*1.00*	0.88
H^1/H^2	1.00	1.00	1.00	*1.00*	0.99
X^1/X^2	1.00	1.00	1.00	*1.00*	0.97
Y_C^1/Y_C^2	1.00	1.00	1.00	*1.00*	0.96
Y^1/Y^2	0.98	0.98	0.98	0.98	0.94
Welfare					
ss gain	0.78	3.16	0.73	3.42	3.62
transition	0.00	0.00	0.00	-0.03	-1.75
net gain	0.78	3.16	0.73	3.40	1.87
net gain to 0	0.70	2.26	0.66	2.39	1.35

Table 4 - FR vs. FB

	Model 1	Model 2	Model 3	Model 4	Model 5
Allocation					
q^1/q^2	1.00	0.33	0.69	0.24	0.72
K^1/K^2	0.47	0.47	0.38	0.35	0.57
H^1/H^2	0.75	0.75	0.74	0.73	0.76
X^1/X^2	0.62	0.62	0.59	0.58	0.66
Y_C^1/Y_C^2	0.66	0.66	0.61	0.59	0.70
Y^1/Y^2	0.65	0.66	0.59	0.56	0.70
Welfare					
ss gain	20.25	25.19	28.37	43.90	16.92
transition	-10.23	-10.57	-14.35	-17.84	-9.08
net gain	10.02	14.62	14.02	26.06	7.84
Welfare with no Taxes					
ss gain	0.00	4.91	2.82	4.95	0.00
transition	0.00	0.00	-2.53	-0.02	0.00
net gain	0.00	4.91	0.31	4.93	0.00

Table 5 - Allocations

	Model 1	Model 2	Model 3	Model 4	Model 5
First Best					
q	1.00	1.00	1.01	1.09	1.11
Y_C	0.66	0.66	0.71	0.74	0.70
Y	0.70	0.71	0.78	0.83	0.74
K	1.81	1.81	2.22	2.43	2.18
H	0.44	0.44	0.45	0.45	0.44
X	0.42	0.42	0.44	0.45	0.42
K/Y	2.58	2.57	2.83	2.94	2.94
Equilibrium at FR					
q	1.00	0.33	0.69	0.26	0.79
Y_C	0.43	0.43	0.43	0.43	0.49
Y	0.46	0.46	0.46	0.46	0.52
K	0.85	0.85	0.85	0.85	1.25
H	0.33	0.33	0.33	0.33	0.33
X	0.26	0.26	0.26	0.26	0.27
K/Y	1.84	1.83	1.84	1.84	2.41
Equilibrium at $\pi = 0$					
q	0.88	0.29	0.64	0.25	0.74
Y_C	0.43	0.43	0.43	0.43	0.48
Y	0.46	0.46	0.46	0.46	0.51
K	0.85	0.85	0.85	0.85	1.20
H	0.33	0.33	0.33	0.33	0.33
X	0.26	0.26	0.26	0.26	0.27
K/Y	1.85	1.84	1.85	1.86	2.36
Equilibrium at $\pi = 0.1$					
q	0.64	0.21	0.53	0.21	0.62
Y_C	0.43	0.43	0.43	0.43	0.47
Y	0.45	0.45	0.45	0.45	0.49
K	0.85	0.85	0.85	0.85	1.10
H	0.33	0.33	0.33	0.33	0.33
X	0.26	0.26	0.26	0.26	0.26
K/Y	1.88	1.87	1.88	1.88	2.26
Nonmonetary Equilibrium					
q	0.00	0.00	0.00	0.00	0.00
Y_C	0.43	0.43	0.43	0.43	0.43
Y	0.43	0.43	0.43	0.43	0.43
K	0.85	0.85	0.85	0.85	0.84
H	0.33	0.33	0.33	0.33	0.33
X	0.26	0.26	0.26	0.26	0.24
K/Y	1.95	1.95	1.95	1.95	1.95

Table 6 - $\pi = 0.1$ vs FR and...

	Model 1	Model 2	Model 3	Model 4	Model 5
Making up Revenue by T					
q^1/q^2	0.64	0.64	0.76	0.80	0.79
K^1/K^2	1.00	1.00	1.00	*1.00*	0.88
H^1/H^2	1.00	1.00	1.00	*1.00*	0.99
X^1/X^2	1.00	1.00	1.00	*1.00*	0.97
Y^1/Y^2	0.98	0.98	0.98	0.98	0.94
T^1/Y^1	-4.79	-4.65	-4.79	-4.67	-2.52
T^2/Y^2	-2.35	-2.21	-2.35	-2.23	-0.33
ss gain	0.78	3.16	0.73	3.42	3.62
transition	0.00	0.00	0.00	-0.03	-1.75
net gain	0.78	3.16	0.73	3.40	1.87
Making up Revenue by t_h					
q^1/q^2	0.64	0.64	0.78	0.82	0.81
K^1/K^2	1.06	1.06	1.06	1.06	0.93
H^1/H^2	1.06	1.06	1.06	1.06	1.04
X^1/X^2	1.09	1.09	1.09	1.09	1.04
Y^1/Y^2	1.04	1.04	1.04	1.04	0.99
New t_h	0.31	0.31	0.31	0.31	0.29
ss gain	-1.43	0.93	-1.54	1.04	-1.29
transition	-0.66	-0.68	-0.69	-0.69	0.98
net gain	-2.09	0.25	-2.23	0.35	-0.31
Making up Revenue by t_x					
q^1/q^2	0.64	0.64	0.77	0.81	0.80
K^1/K^2	1.04	1.04	1.04	1.04	0.90
H^1/H^2	1.04	1.04	1.04	1.04	1.03
X^1/X^2	1.06	1.06	1.06	1.06	1.01
Y^1/Y^2	1.02	1.02	1.02	1.02	0.97
New t_x	0.14	0.14	0.14	0.14	0.13
ss gain	-0.70	1.66	-0.79	1.80	-1.00
transition	-0.46	-0.47	-0.48	-0.47	1.41
net gain	-1.16	1.19	-1.27	1.33	0.41

Table 7 - Robustness

	Model 1	Model 2	Model 3	Model 4	Model 5
Benchmark	0.78	3.16	0.73	3.40	1.87
Only Lump-sum Tax					
Recalibrated	0.90	3.68	0.85	3.68	0.90
Not	0.78	3.16	0.73	3.38	0.36
Utility Parameters ε and η					
$\varepsilon = 2$, $\eta = 1$	0.78	3.21	0.78	3.44	1.49
$\varepsilon = 5$, $\eta = 1$	0.79	3.36	0.79	3.59	1.27
$\varepsilon = 1$, $\eta = 1/2$	0.87	5.17	0.44	3.98	2.07
$\varepsilon = 2$, $\eta = 1/2$	0.87	5.28	0.48	4.23	1.68
$\varepsilon = 5$, $\eta = 1/2$	0.88	5.66	0.48	4.42	1.46
$\varepsilon = 1$, $\eta = 2$	0.76	2.24	0.76	2.45	1.68
$\varepsilon = 2$, $\eta = 2$	0.77	2.26	0.77	2.41	1.32
$\varepsilon = 5$, $\eta = 2$	0.77	2.33	0.77	2.44	1.10
Utility Parameter b					
$b = 0.00001$	0.78	3.33	0.73	3.47	1.87
$b = 0.0001$	0.78	3.16	0.73	3.40	1.87
$b = 0.001$	0.78	2.92	0.73	3.29	1.87
$b = 0.01$	0.78	2.53	0.73	3.08	1.86
$b = 0.1$	0.79	1.85	0.70	2.56	1.82
Measures of Money					
$M0$	0.14	0.14	0.12	0.46	0.14
$M1$	0.78	3.16	0.73	3.40	1.87
$M2$	2.08	5.95	2.08	7.21	3.04
$M3$	1.46	4.28	1.46	6.26	2.34
Frequency					
Quarterly	0.73	3.05	0.73	3.25	1.82
Monthly	0.72	3.02	0.72	3.12	1.87
Period					
1961-2004	0.69	2.94	0.65	3.16	1.96
1951-1998	0.78	3.24	0.74	3.53	1.72
1986-2004	0.90	3.06	0.87	3.11	1.96

Table 8 - More Robustness : Calibration Results

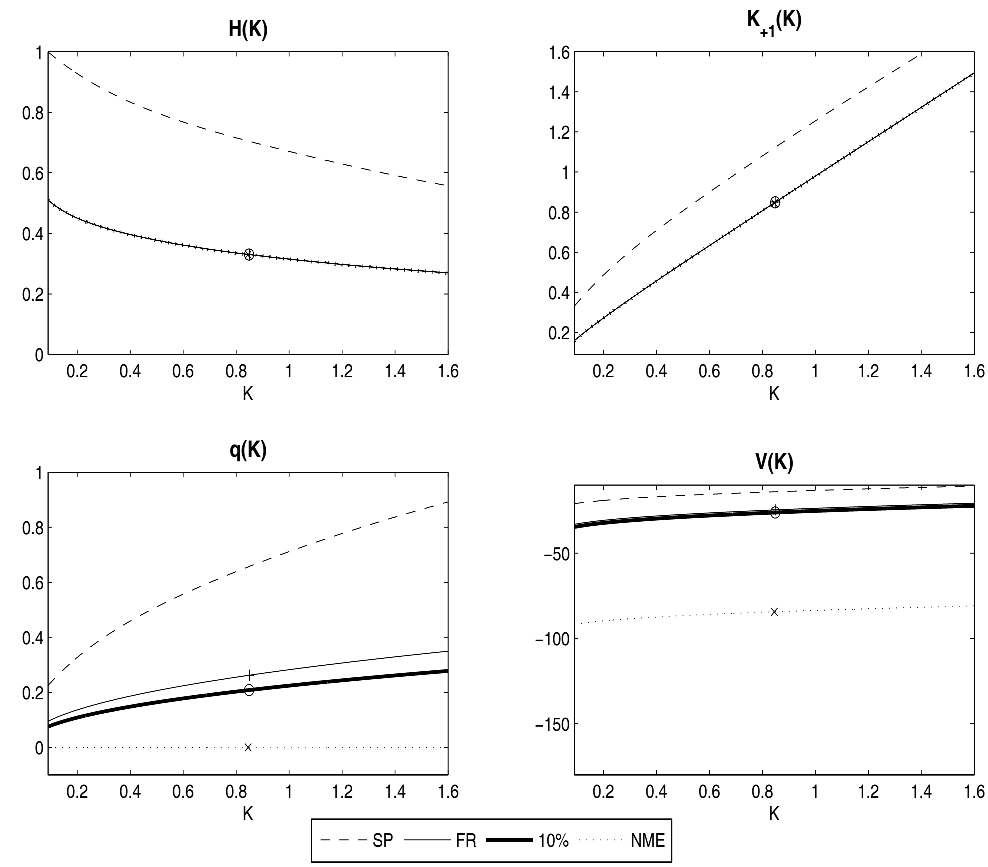
	Data	Model 6	Model 7	Model 8	Model 9	Model 10
Calibrated Parameters						
σ		0.27	0.25	0.50	0.50	0.26
B		1.76	2.37	0.47	0.10	2.49
ψ		1.42	10.00	1.13	2.29	1.25
A		4.49	6.36	1.01	0.22	6.47
G		0.11	0.11	0.14	0.14	0.12
θ		0.70	-	0.75	-	-
Calibration Targets						
μ	10.00	10.00	0.00	-0.53	0.00	0.00
K/Y	2.32	1.87	2.27	2.34	2.16	2.01
G/Y	0.25	0.25	0.25	0.25	0.25	0.25
H	0.33	0.33	0.33	0.33	0.33	0.33
v	5.29	5.33	5.27	4.45	4.20	5.32
ξ	-0.23	-0.23	-0.23	-0.12	-0.23	-0.23
Miscellaneous						
s_D		5.06	4.72	11.23	11.91	4.84
μ_D		197.53	0.00	-4.72	0.00	0.00
ζ	-0.023	0.000	-0.008	-0.094	-0.087	-0.023

Table 9 - More Robustness : $\pi = 0.1$ **vs. FR**

	Model 6	Model 7	Model 8	Model 9	Model 10
Allocation					
q^1/q^2	0.66	0.64	0.69	0.89	0.70
K^1/K^2	*1.00*	0.98	0.69	0.89	0.95
Z^1/Z^2	0.66	0.64	–	–	–
ϕ^1/ϕ^2	–	–	1.17	1.06	–
H^1/H^2	*1.00*	0.98	1.03	1.01	*1.00*
X^1/X^2	1.00	1.00	0.89	0.96	0.99
Y_C^1/Y_C^2	*1.00*	0.98	0.92	0.97	0.99
Y^1/Y^2	0.98	0.96	0.88	0.94	0.96
Welfare					
ss gain	4.95	2.63	7.87	2.46	1.77
transition	-0.07	-1.26	-1.98	-0.83	-0.57
net gain	4.88	1.37	5.89	1.63	1.20
net gain to 0	3.42	1.08	4.38	1.22	0.95

Figure 1 - Decision Rules and Value Functions

(a) Model 4



(b) Model 5

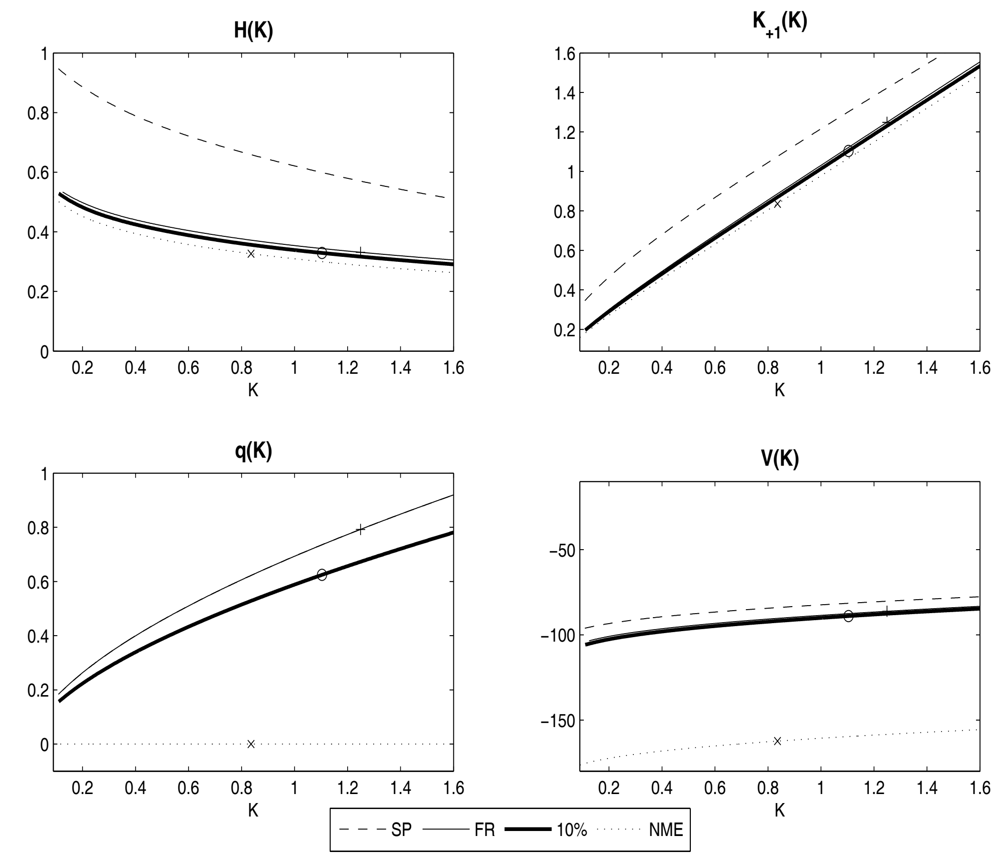


Figure 2 - Money Demand Curve

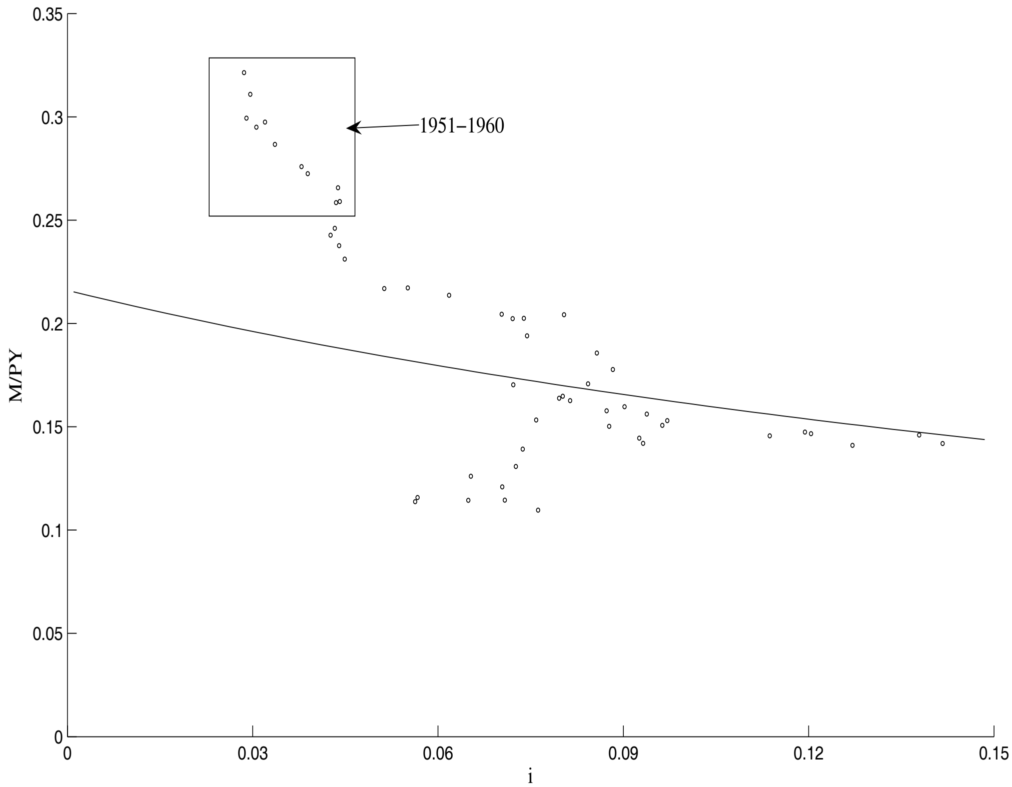
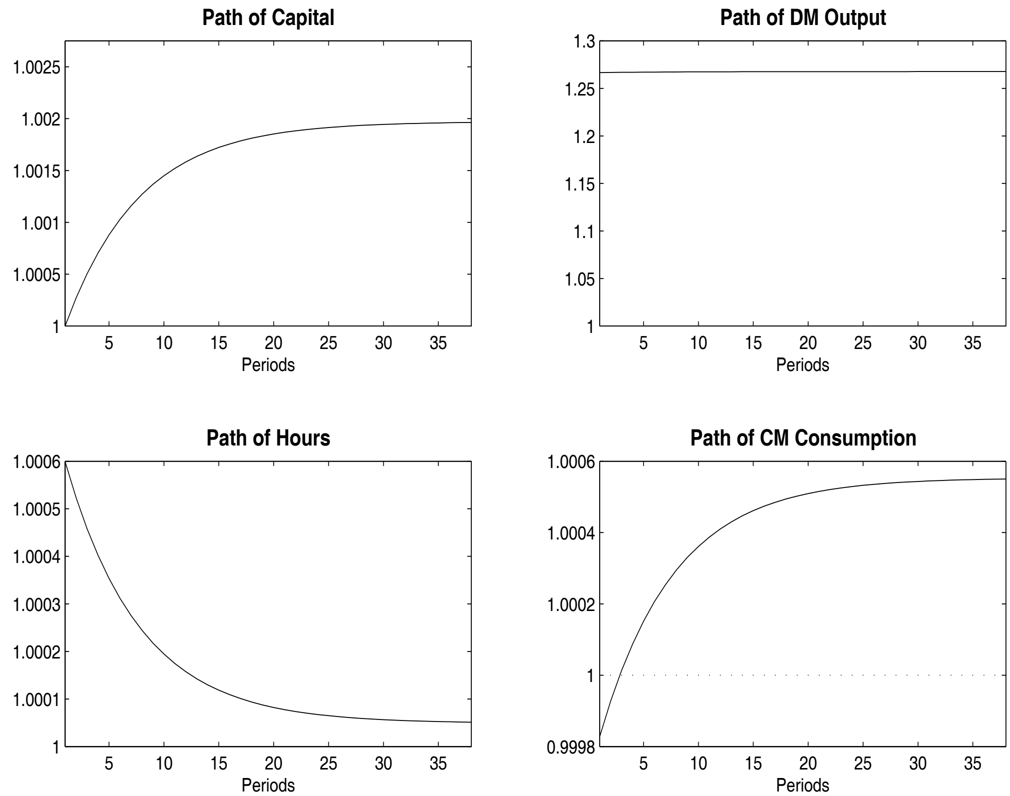
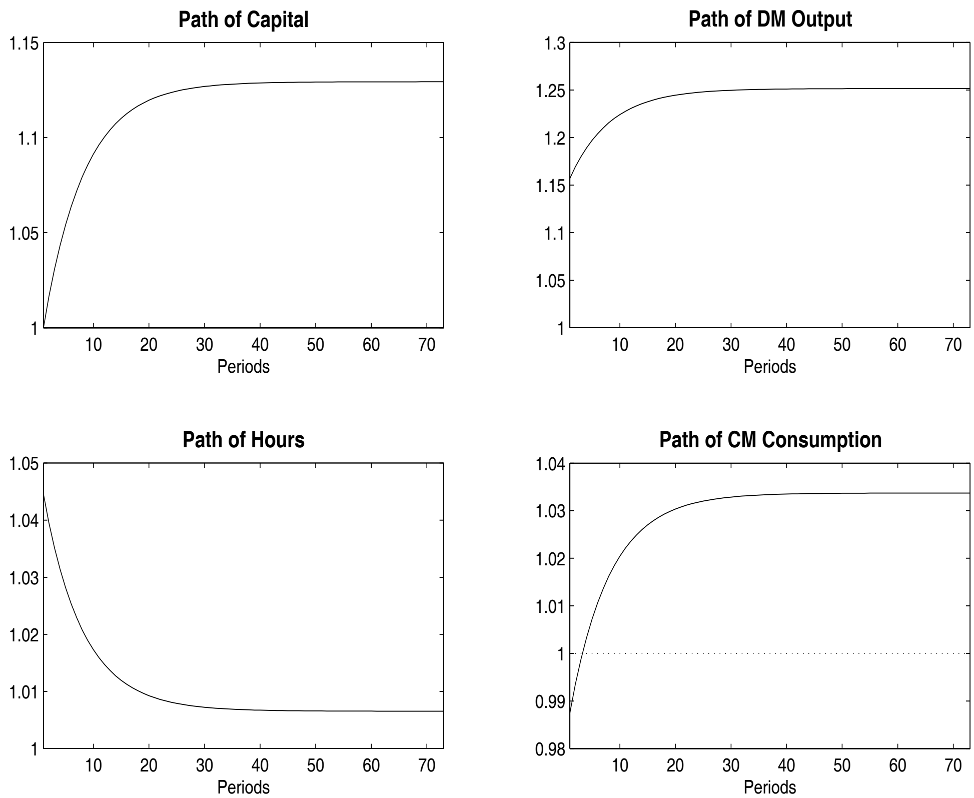


Figure 3 -10% to FR: Transitions

(a) Model 4



(b) Model 5



Path of Capital
1.15
1.1
1.05
1
10
20
30
40
50
60
70
Periods
Path of DM Output
1.3
1.25
1.2
1.15
1.1
1.05
1
10
20
30
40
50
60
70
Periods
Path of Hours
1.05
1.04
1.03
1.02
1.01
1
10
20
30
40
50
60
70
Periods
Path of CM Consumption
1.04
1.03
1.02
1.01
1
0.99
0.98
10
20
30
40
50
60
70
Periods