Testing the Link Between Inflation and Growth

*Steve Ambler and Emanuela Cardia**

Introduction

There is a large and growing body of empirical literature on the relationship between economic growth and inflation. The goals of this literature are twofold. The first is to identify a stylized fact and answer the following questions: What is the empirical relationship between growth and inflation? Is the relationship statistically significant? Is the relationship stable across countries and across time periods?

The second goal is to *interpret* the relationship and to answer these questions: Is the relationship structural? Does the empirical relationship show that there is an *exploitable trade-off* by monetary policymakers? If there is an exploitable trade-off, what are the welfare implications of that trade-off and what is the optimal rate of inflation?[1]

1. A body of theoretical literature has also developed, in tandem with the empirical literature, that uses dynamic general-equilibrium models such as ours to analyse the impact of the inflation tax on the *level* of income (in exogenous growth models) or on *growth* (in endogenous growth models) and the welfare effects of the inflation tax. See Aiyagari (1990); Ambler, Phaneuf, and Sauthier (1995); Bean (1993); Black, Macklem, and Poloz (1994); Chari, Jones, and Manuelli (1995); Cooley and Hansen (1989, 1991, 1995); De Gregorio (1993); Devereux and Love (1994); Dotsey and Ireland (1996); Gomme (1993, 1996); Love and Wen (1996); Rebelo (1991); Sidrauski (1967); and Stockman (1981), among others.

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In this paper we base our discussion of the empirical literature on a dynamic general-equilibrium model with money and endogenous growth. Our endogenous growth model has transitional dynamics. The inflation tax in the model affects the allocation of resources and the rate of growth in the long and the short run. The rate of growth is also affected by other exogenous shocks including technology shocks and government policy variables such as marginal tax rates and public spending. We use the model to analyse the following questions:

- What relationship does the model predict between inflation and growth?
- To what extent is this relationship exploitable by the monetary policy authorities in the model?
- If the model is used to generate artificial data, what relationship between inflation and growth is uncovered by estimating regressions similar to those used in the empirical literature?
- In the model, both inflation and growth are endogenous variables. What is the effect of different exogenous variables on the conditional correlation between inflation and growth?
- What does the model imply for the choice of instrumental variables in cross-section and time-series regressions?
- What does the model imply concerning the proper measurement of the underlying exogenous variables, especially the Solow residual?
- Which macroeconomic aggregates are non-stationary, and what are the cointegration relationships among variables in the model that could hold in the data?
- What does the model imply for the inclusions in growth regressions of variables designed to capture the convergence across countries of the per capita levels of income implied by the basic neoclassical exogenous growth model?

Our findings can be summarized as follows:

- The model predicts a negative relationship between inflation and growth. This relationship holds both for time series and in the long run for cross sections of countries with different long-run levels of the underlying exogenous variables.
- The slope of the relationship depends on the size of the shocks to the underlying exogenous variables. The rate of growth of the money supply leads to a weak negative link between inflation and growth. This is the exploitable trade-off from the point of view of the monetary authorities.

- Since inflation in the long run is just equal to the rate of growth of the money supply minus the real rate of growth (velocity is stationary in the long run), all the other exogenous variables in the model lead to a *strong* negative relationship between inflation and growth.
- The ability to obtain good empirical estimates of the impact of exogenous variables on inflation and growth is complicated by two main factors. First, some of the underlying exogenous variables, such as public spending, are non-stationary. Second, some of the underlying variables, such as the Solow residual, are unobservable, and our model implies that standard measures of the Solow residual are incorrect.
- It is difficult to interpret the equations estimated in the empirical literature either as semireduced-form estimates or as estimates of structural relationships. At best, the results of the empirical literature can be seen as uncovering the conditional correlation between inflation and growth, with no meaningful structural interpretation and little or no implication for monetary policy or welfare.
- Future empirical work should concentrate on approaches that are more structural, in which the overall adequacy of endogenous growth models can be assessed, and in which coefficient estimates have clearer interpretations and policy implications.

In the following sections we first present the model, then discuss its calibration, the main predictions of the model, and the implications of the model for how the empirical relationship between inflation and growth *should be* tested; finally we compare this with the methods that have actually been used in the literature. In our conclusions we also present some suggestions for further work.

1 The Model

The model is a relatively simple endogenous growth model that has transitional dynamics, in which the allocation of resources matters for growth, and in which money growth and the inflation tax can affect the allocation of resources. We use a model with a human capital accumulation externality (learning by doing). The crucial assumption is that productivity growth depends on the level of employment, so that exogenous changes that affect the allocation of time between leisure and work will affect the rate of growth. Individual agents do not take into account the effects of their actions on productivity growth, but changes in government policy, especially changes in rates of distortionary taxation (including the inflation tax) can affect the economy's growth rate. We outline the objectives and constraints facing private agents, firms, and the government, and describe the exact specification of the human capital externality before considering in detail the dynamic maximization problem of private agents.

1.1 Households

The representative household maximizes the following intertemporal utility function:

$$U = \sum_{i=0}^{\infty} \beta^i U_t(c_{t+i}, l_{t+i}), \tag{1}$$

where β is the subjective discount rate, c_t is the household's consumption expenditure at time t, and l_t is leisure, and where the period utility function is given by

$$U_t(c_t, l_t) = 1\text{n}c_t + \frac{\phi}{(1-\gamma)} l_t^{(1-\gamma)}. \tag{2}$$

The maximization problem is subject to the sequence of budget constraints

$$(1-\tau^n_{t+i})W_{t+i}n_{t+i} + (1-\tau^k_{t+i})R_{t+i}k_{t+i} + \tau^k_{t+i}\delta k_{t+i} + \frac{m_{t+i}}{P_{t+i}}$$
$$+\chi_{t+i} = (1+\tau^c_{t+i})c_{t+i} + i_{t+i} + \frac{m_{t+i+1}}{P_{t+i}} + T_{t+i} + \psi_{t+i}, \tag{3}$$

where W_{t+i} is the real wage rate; R_{t+i} is the real rental rate of capital; n_{t+i} is the number of hours worked by the household; k_{t+i} denotes the household's holdings of capital at time $t+i$, δ is the constant rate of depreciation of capital; m_{t+i} denotes the household's holdings of nominal money balances; P_{t+i} is the price level; i_{t+i} is the household's investment expenditure; T_{t+i} represents lump-sum taxes; τ^n_{t+i}, τ^k_{t+i}, and τ^c_{t+i} are proportional tax rates on, respectively, labour income, income from renting capital to firms, and consumption; ψ_{t+i} is a term that represents pecuniary transactions costs; and χ_{t+i} represents the real value of monetary balances transferred to households through the (costless) deposit expansion mechanism. Transactions costs at time t are given by

$$\psi_t = a_0((1+\tau^c_t)c_t)^{a_1}(m_t/P_t)^{(1-a_1)}, \tag{4}$$

with $a_0 > 0$, $a_1 > 1$. Increasing real money balances lead to a reduction in real pecuniary transactions costs. This specification is identical to that used by Black, Macklem, and Poloz (1994). They note that this transactions cost specification leads to a standard long-run demand for money function and that the standard cash-in-advance constraint emerges as a_1 approaches infinity. Maximization is also subject to the law of motion for the household's capital stock,

$$k_{t+i+1} = (1-\delta)k_{t+i} + i_{t+i}, \tag{5}$$

and to

$$n_{t+i} + l_{t+i} = 1, \tag{6}$$

where the total time endowment of the household is normalized to equal 1.

1.2 Firms

Competitive firms rent capital and labour from households. The aggregate production function at time *t* is given by

$$Y_t = Z_t(H_t N_t)^{\alpha} K_t^{(1-\alpha)}, \tag{7}$$

where Y_t is output, Z_t is the level of technology, H_t is the level of human capital in the economy, N_t is aggregate employment, and K_t is the aggregate capital stock. As is common in this literature, when variables appear in both upper- and lower-case form, the lower-case versions indicate variables that are choice variables or state variables from the individual household's point of view, and the upper-case versions are their aggregate or per capita equivalents. Technology follows the law of motion:

$$\ln(Z_t) = \rho \ln(Z_{t-1}) + (1-\rho)\ln(\bar{Z}) + \varepsilon_{zt}, \tag{8}$$

with $0 < \rho < 1$, where $\bar{Z}$ is the long-run average level of Z_t, and ε_{zt} is a shock to production technology. Since households hold the capital stock and rent it to firms, the profit maximization problems of firms are static. Profit maximization by competitive firms implies

$$W_t = \alpha Z_t H_t^{\alpha} N_t^{(\alpha-1)} K_t^{(1-\alpha)}, \tag{9}$$

$$R_t = (1-\alpha) Z_t (H_t N_t)^{\alpha} K_t^{-\alpha}. \tag{10}$$

1.3 The government

The government's behaviour is exogenous. It finances a stream of expenditures via distortionary taxation on labour income, capital income, and consumption via money creation and via lump-sum taxation. Its budget constraint is

$$G_t = \tau_t^k(R_t - \delta)K_t + \tau_t^n N_t + \tau_t^c C_t + \frac{MB_{t+1} - MB_t}{P_t} + T_t, \quad (11)$$

where MB_t is the monetary base at time t. Since agents have infinite horizons in our model, lump-sum taxation has the same effect as deficit financing (holding constant all distortionary tax rates and inflation), so we do not model debt financing explicitly. The law of motion for the monetary base is

$$MB_{t+1} - MB_t = \mu_t MB_t, \quad (12)$$

so that μ_t is the net rate of growth of the base money supply.

Government spending, tax rates, and the rate of growth of the money supply are determined by a multivariate stochastic process, which is also allowed to depend on the level of technology. Given the rate of creation of money and the rates of taxation on capital income, labour income, and consumption, the level of lump-sum taxes is determined residually to ensure that the government budget is balanced in each period.[2]

Money balances are related to the base money supply by

$$M_t = MB_t / rr, \quad (13)$$

where rr is the (constant) ratio of base money to nominal money balances. As do Love and Wen (1996), we use this relationship to distinguish between the monetary aggregate relevant for measuring seigniorage revenues and the aggregate that is used to reduce transactions costs. It can be shown that the transfer of resources to private households via deposit creation is given by

$$\chi_t P_t = (M_{t+1} - M_t) - (MB_{t+1} - MB_t)$$

$$\Rightarrow \chi_t = (1 - rr)\frac{M_{t+1} - M_t}{M_t}\frac{M_t}{P_t},$$

so that

$$\chi_t = (1 - rr)\mu_t \frac{M_t}{P_t}. \quad (14)$$

2. This assumption implies that a change in the inflation tax rate has no impact on other marginal tax rates in the model. We discuss the consequences of relaxing this assumption below.

1.4 Productivity growth

Productivity growth is assumed to be a by-product of the production process, and to arise from learning by doing. This is an externality that neither households nor firms take into account in their maximization problems. The law of motion for human capital is

$$H_{t+1} = (1-\delta_h)H_t + \theta_0 H_t \ln(1+N_t). \tag{15}$$

This is the simplest possible set-up, in which the allocation of resources in general, and employment in our particular set-up, matter for growth, and in which monetary policy may affect the economy's growth rate. The functional form matters for the details of growth accounting (see below), but the important aspect of the equation is that human capital growth depends on the allocation of time between leisure and work.[3]

1.5 Stationary transformations

We solve the model using stationary discounted dynamic programming. To do so, we must rewrite it using variables that do not contain stochastic trends. We normalize the trended variables of the model by dividing by H_{t+i}, the level of human capital at time $t+i$. After some algebraic transformations, the utility function of the representative agent can be written in terms of stationary variables as

$$U = \frac{1}{1-\beta}\ln H_t + \sum_{i=0}^{\infty}\beta^i\left\{\ln\tilde{c}_{t+i} + \frac{\phi}{(1-\gamma)}l_{t+i}^{(1-\gamma)} + \frac{\beta}{1-\beta}\ln\psi_{t+i}\right\}, \tag{16}$$

where

$$\psi_{t+\mathrm{i}} \equiv H_{t+i+1}/H_{t+i},$$

and where

$$\tilde{c}_{t+i} \equiv c_{t+i}/H_{t+i}.$$

3. Other specifications for human capital accumulation are possible. In the human capital growth model of Mankiw, Romer, and Weil (1992), an increase in the level of current output translates into an increase in the growth rate, since more savings are channelled into human capital accumulation. In their model, there is a trade-off between physical and human capital accumulation, since both are subject to a global resource constraint. In our model, human capital accumulation does not come at the expense of investment in physical capital or current consumption, but rather at the expense of leisure.

Before writing down the representative agent's value function, we develop some additional notation. The transactions cost constraint, equation (4), can be rewritten in terms of normalized variables as

$$\tilde{\psi}_t = a_0((1+\tau_t^c)\tilde{c}_t)^{a_1}\left(\frac{m_t}{M_t}\frac{M_t}{P_t}\frac{1}{H_t}\right)^{(1-a_1)}$$

$$= a_0((1+\tau_t^c)\tilde{c}_t)^{a_1}\left(\frac{\tilde{m}_t}{\tilde{P}_t}\right)^{(1-a_1)},$$

with

$$\tilde{m}_t \equiv \frac{m_t}{M_t}, \quad \tilde{P}_t \equiv H_t P_t / M_t .$$

For all other variables X_t , define

$$\tilde{X}_t \equiv X_t / H_t .$$

1.6 Private agent's program

Using notation similar to that of Hansen and Prescott (1995), we can write the representative agent's value function as

$$V(Z, S, s, m) = \max_{d, m'} \{r(Z, S, s, m, d, m', D, P) + EV(Z', S', s', m')\}, \tag{17}$$

where

$$Z = \left\{Z_t, \tilde{G}_t, \mu_t, \tau_t^n, \tau_t^k, \tau_t^c\right\},$$

$$Z' = \left\{Z_{t+1}, \tilde{G}_{t+1}, \mu_{t+1}, \tau_{t+1}^n, \tau_{t+1}^k, \tau_{t+1}^c\right\},$$

$$S = \left\{\tilde{K}_t\right\},$$

$$S' = \left\{\tilde{K}_{t+1}\right\},$$

$$s = \left\{\tilde{k}_t\right\},$$

$$s' = \left\{\tilde{k}_{t+1}\right\},$$

$$m = \left\{\tilde{m}_t\right\},$$

$$m' = \left\{\tilde{m}_{t+1}\right\},$$

$$P = \left\{\tilde{P}_t\right\},$$

$$D = \left\{\tilde{K}_{t+1}, \tilde{\Psi}_t\right\},$$

$$d = \left\{\tilde{k}_{t+1}, \tilde{\psi}_t\right\},$$

$$r(Z, S, s, m, d, m', D, P) = \ln(\tilde{c}_t) + \phi \ln(1 - n_t) + \frac{\beta}{1-\beta} \ln \theta_t ,$$

where

$$\tilde{c}_t = (\tilde{m}_t / \tilde{P}_t)^{(a_1 - 1)/a_1} (\tilde{\psi}_t / a_0)^{1/a_1} (1 + \tau_t^c)^{-1},$$

$$\begin{aligned} n_t = {} & ((1 - \tau_t^n)\tilde{W}_t)^{-1} \bullet [(1 + \tau_t^c)\tilde{c}_t + \tilde{k}_{t+1}\theta_t - (1 - \delta)\tilde{k}_t \\ & + \tilde{m}_{t+1}(1 + \mu_t)/\tilde{P}_t + \tilde{\psi}_t - \tilde{\chi}_t + \tilde{G}_t - \tau_t^k (R_t - \delta)\tilde{K}_t \\ & - \tau_t^n \tilde{W}_t N_t - \tau_t^c \tilde{C}_t - \tilde{m}_t / \tilde{P}_t - (1 - \tau_t^k) R_t \tilde{k}_t - \tau_t^k \delta \tilde{k}_t], \end{aligned}$$

and θ_t is the gross growth rate of human capital given by

$$\theta_t = 1 - \delta_h + \theta_0 \ln(1 + N_t).$$

We have used the transactions cost constraint and the private agent's budget constraint to substitute out $\tilde{c}_t$ and n_t as choice variables from the private agent's problem. The household's maximization is subject to the laws of motion for the exogenous state variables Z, the law of motion for the household's holdings of capital, the law of motion for the aggregate capital stock, and feedback rules for the aggregate equivalents of its decision variables.

1.7 Equilibrium

Equilibrium in the model consists of a set of decision rules for the household, of the form

$$d = d(Z, S, s, m), \tag{18}$$

$$m' = m(Z, S, s, m), \tag{19}$$

which have the following properties:

- In equilibrium, households willingly hold the per capita money stock, so that $m = m' = 1$.
- In equilibrium, the household's decision rules are compatible with the feedback rules for the aggregate equivalents of its decision variables, which are constraints in its maximization problem, so that $D(Z, S) = d(Z, S, S, 1)$, and $1 = m(z, S, S, 1)$.

2 Calibration

Some of the model's parameter values are chosen on the basis of standard values in the literature. Other parameters are chosen on the basis of previous empirical studies using micro or macro data. The rest of the parameter values are chosen so that certain characteristics of the model's deterministic steady state match the data. The parameter values used in the model's base case are given in Table 1. The first row gives the values of structural parameters. The second, third, and fourth rows give parameter values related to the joint stochastic process generating the exogenous variables of the model. The last row of the table gives the values of certain variables and certain key ratios in the steady state.[4]

The discount rate β is set equal to 0.990. The depreciation rate δ is set equal to 0.021. We do not have good empirical evidence on the value of δ_h. In the absence of empirical evidence, we set its value equal to 0.021, the same as the depreciation rate of physical capital.

4. Note that the rental rate of capital R is gross of both depreciation and capital taxation. It corresponds to a risk-free after-tax rate of return of 0.015.

Table 1

Base-Case Parameter Values and Steady-State Values

Values of structural parameters									
θ_0	β	δ	δ_h	γ	ϕ	a_0	a_1	α	rr
0.077	0.990	0.021	0.021	1.350	1.115	0.138	3.220	0.640	0.107
Values of parameters in stochastic process									
		ρ_z	ρ_c	ρ_n	ρ_k	ρ_μ	ρ_g		
		0.950	0.950	0.950	0.950	0.950	0.950		
		σ_z	σ_{τ^c}	σ_{τ^n}	σ_{τ^k}	σ_μ	σ_g		
		0.010	0.0005	0.0019	0.0024	0.0009	0.020		
		τ^c	τ^n	τ^k	$\tilde{G}/\tilde{Y}$	z	μ		
		0.150	0.250	0.500	0.200	1.000	0.017		
Values in the steady state									
θ	$\tilde{\psi}/\tilde{Y}$	$\tilde{C}/\tilde{Y}$	$\tilde{I}/\tilde{Y}$	$\tilde{G}/\tilde{Y}$	$\tilde{K}/\tilde{Y}$	N	V	R	
1.0047	0.018	0.600	0.182	0.200	7.101	0.333	0.410	0.051	

The deterministic steady state of the model can be found as follows. The first-order conditions for the maximization of the household's problem are

$$\frac{\partial r}{\partial d} + \beta \frac{\partial V}{\partial ds'} \frac{\partial s'}{\partial d} = 0, \tag{20}$$

$$\frac{\partial r}{\partial m'} + \beta \frac{\partial V}{\partial m'} + \beta \frac{\partial V}{\partial s'} \frac{\partial s'}{\partial m'} = 0, \tag{21}$$

where the prime symbol, $'$, denotes next-period values. Imposing consistency between the private agent's choice variables and their aggregate equivalents, imposing the steady state, and using the envelope theorem, we can show that these conditions imply in the steady state that

$$\frac{\partial r}{\partial d} + \beta \frac{\partial r}{\partial s}\left(I - \beta \frac{\partial s'}{\partial s}\right)^{-1} \frac{\partial s'}{\partial d} = 0, \tag{22}$$

$$\frac{\partial r}{\partial m'} + \beta \frac{\partial r}{\partial m} + \beta \frac{\partial r}{\partial s}\left(I - \beta \frac{\partial s'}{\partial s}\right)^{-1}\left(\frac{\partial s'}{\partial m'} + \beta \frac{\partial s'}{\partial m'}\right) = 0. \tag{23}$$

This gives a system of three equations, which we can use to solve for three unknowns. We impose a steady-state quarterly growth rate of per capita income of 0.0047 (which gives an annual growth rate of per capita income equal to 1.9 per cent), so that $\tilde{\psi} = 1.0047$. We impose $N = 1/3$, which is close to the fraction of discretionary time (not including sleep) that households spend working. We choose the value of a_1 in the transactions costs function in order to reflect empirical studies on the long-run elasticity of the demand for money.[5] Then, given the value of a_1, we choose a_0 so that velocity is equal to its long-run average in the data.[6] The standard equation for velocity is

$$MV = PC,$$

which, after dividing non-stationary variables by the level of human capital (we assume that velocity itself is stationary), gives

$$V = \tilde{P}\tilde{C}.$$

The household's first-order condition with respect to capital implies in the steady state that

$$R = (\tilde{\psi}/\beta - (1-\delta) - \tau^k\delta)/(1-\tau^k),$$

where we have dropped time subscripts to denote the steady-state levels of variables. Then, the firm's first-order condition for profit maximization with respect to its choice of capital gives

$$R = (1-\alpha)Z(N/\tilde{K})^{\alpha},$$

which allows us to pin down the long-run level of the capital stock. The first-order condition for profit maximization with respect to labour then gives the equilibrium long-run real wage.

The optimal choice of hours worked by the household implies that the marginal benefit from working (the real wage net of labour taxes, weighted by the marginal utility of consumption) equals the marginal disutility of forgone leisure. Then, using this optimality condition, using the first-order condition with respect to transactions costs and next-period money balances, using the transactions cost constraint itself, and using the fundamental national accounting identity, we have the following system of equations:

$$-\phi(1-N)^{-\gamma} + \lambda(1-\tau^n)\tilde{W} = 0, \quad (24)$$

$$1 - \lambda(1+\tau^c)\tilde{C} - \lambda\tilde{\Psi} = 0, \quad (25)$$

$$\beta\frac{a_1-1}{a_1} - \beta\lambda(1+\tau^c)\tilde{C}\frac{a_1-1}{a_1} - \lambda(1+\mu)/\tilde{P} + \beta\lambda/\tilde{P} = 0, \quad (26)$$

5. See Black, Macklem, and Poloz (1994) and Love and Wen (1996) for more details.

6. Following Love and Wen (1996), we use M2 as the relevant monetary aggregate for calculating velocity.

$$\tilde{\Psi} = a_0((1+\tau^c)\tilde{C})^{a_1}(1/\tilde{P})^{1-a_1}, \tag{27}$$

$$\tilde{Y} = \tilde{C} + \tilde{I} + \tilde{G} + \tilde{\Psi}, \tag{28}$$

$$V = \tilde{P}\tilde{C}, \tag{29}$$

where λ is the marginal utility of consumption. We choose the value of γ so that the elasticity of labour supply is equal to 1.5.[7] We restrict the long-run level of public spending, $\tilde{G}$, to equal 20 per cent of steady-state output. We impose tax rates of $\tau^n = 0.25$, $\tau^k = 0.5$, and $\tau^c = 0.15$, and a value of $\mu = 0.017$. We impose the same value of *rr* as in Love and Wen (1996), obtained by estimating the average value of the ratio of the monetary base to M2 in the Canadian data. The quarterly rate of growth of the money supply is chosen to give an annual inflation rate of 5 per cent in the steady state. The value of *z* is normalized to equal 1. The system of equations then pins down the values of $\tilde{C}$, λ, $\tilde{\Psi}$, a_0, ϕ, and $\tilde{P}$. The long-run levels of the model's endogenous variables are also given in Table 1.

The long-run levels of the elements of the *Z* vector influence the steady state of the model. Their values, given in the fourth row of Table 1, are chosen on the basis of long-run averages in the data.[8] For the stochastic simulations, we set parameters for the joint stochastic process generating the *Z*'s as simply as possible. We assume that each of the elements of the *Z* vector is determined by a scalar AR(1) process, with the persistence parameter given in the second row, and with the standard deviation of the innovation to the process given in the third row. We restrict the innovations to be mutually uncorrelated and the persistence parameters to be identical for all shocks. The standard deviations of the shocks are chosen so that the variances of the exogenous variables are close to the variances in the data.[9]

3 Predictions

The equilibrium solutions for inflation and for growth in the model depend on the model's state variables, and are of the form

7. See Black, Macklem, and Poloz (1994) for a discussion.
8. See Mendoza, Razin, and Tesar (1994) for a discussion.
9. For the technology shock, we use a value that is standard in the literature, and that yields a predicted variance of output close to that in the data. For money growth, we use a variance corresponding to the calibration of Cooley and Hansen (1995). For spending, we use the calibration of Ambler and Paquet (1996). For tax rates, we estimate the variances using quarterly interpolations of the data in Mendoza, Razin, and Tesar (1994).

$$\frac{P_t - P_{t\text{-}1}}{P_{t\text{-}1}} = \pi_t = \pi\Big(Z_t\,,\tau_t^k,\tau_t^n,\tau_t^c,\tilde{G}_t\,,\mu_t\,,\tilde{K}_t\Big), \tag{30}$$

$$\theta_t = \theta\Big(Z_t\,,\tau_t^k,\tau_t^n,\tau_t^c,\tilde{G}_t\,,\mu_t\,,\tilde{K}_t\Big). \tag{31}$$

The solutions for growth and inflation in the long run are of the same form and depend on the long-run levels of the exogenous variables. Figure 1 illustrates the trade-off between inflation and growth in the steady state in response to changes in the long-run levels of the elements of Z with respect to the base-case values given in Table 1. There is a very slight negative trade-off between inflation and growth induced by changes in the rate of monetary expansion μ. An increase in the rate of monetary expansion acts as a tax on consumption via the transactions cost equation. Agents substitute leisure for consumption, employment goes down in the steady state, and so does the rate of growth, which depends directly on employment because of equation (15). Because a reduction in monetary expansion leads to an increase in steady-state growth, the welfare of the representative household will increase. We have not attempted to quantify the increase in welfare in our model from a reduction in the inflation tax. To do so, we would need to calculate the effects on consumption and welfare along the transition path from the high-inflation steady state to the low-inflation steady state. More important, we assume in our model that the inflation tax can be replaced by lump-sum taxation, which would tend to exaggerate the welfare benefits of reducing inflation.[10]

Changes in *all* the other exogenous variables of the model except for the level of technology[11] induce a sharp negative trade-off between inflation and growth. The reason that all these variables lead to the same trade-off is that there is a simple identity that links inflation and growth in the long run if velocity is stationary. Since the normalized price level in the steady state is constant, we have that

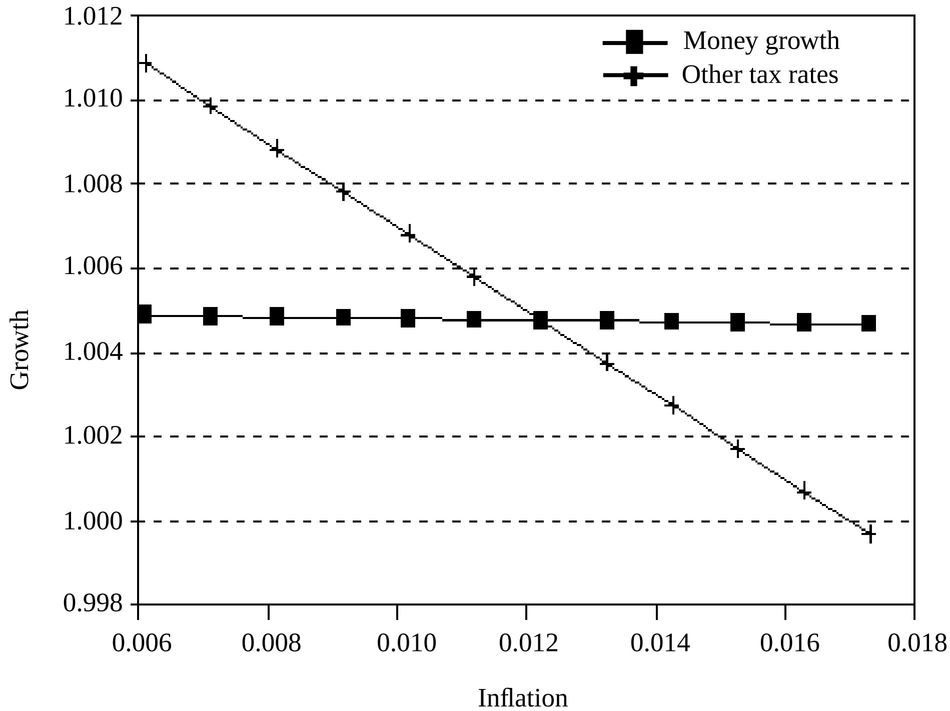
$$\frac{M'}{P'H'} = \frac{M}{PH}\,,$$

10. In a similar model, Love and Wen (1996) estimate welfare gains of 2 per cent of gross domestic product for reductions of inflation from 5 per cent to zero when the inflation tax is replaced with distortionary taxes.

11. Changes in z in the long run leave employment unaffected. Therefore, neither growth nor inflation depends on the level of z in the long run.

Figure 1

Growth-Inflation Trade-off



where the prime symbol, $'$, denotes next-period values, so that

$$\frac{M'}{M} = 1 + \mu = \frac{P'}{P}\frac{H'}{H}$$

$$\Rightarrow \pi = \mu - (\theta - 1). \tag{32}$$

In the steady state, the inflation rate is just the difference between the rate of money growth and the net real rate of growth of the economy. For a constant rate of monetary expansion, we have

$$\frac{\partial\pi}{\partial x} = -\frac{\partial\theta}{\partial x} \tag{33}$$

for any exogenous variable x other than μ. In rates of change, the trade-off has a slope of –1. In response to a change in the rate of monetary expansion, we have

$$\frac{\partial\pi}{\partial\mu} = 1 - \frac{\partial\theta}{\partial\mu}. \tag{34}$$

In our model, and in almost any endogenous growth model that one could imagine, the value of $\partial\theta/\partial\mu$ is quite small, so that changes in monetary policy have a large positive impact on inflation and a small impact on growth.[12]

Table 2 gives the results of stochastic simulation exercises.[13] In the first two columns, we report the co-movements among different macroeconomic aggregates in the U.S. and Canadian data. In the third column (case 1), we report the co-movements predicted by the model in response to technology shocks alone, when the technology process follows the calibration given in Table 2. All other forcing variables are kept constant at their steady-state levels. In column four (case 2), we report the model's predictions in response to both technology shocks and shocks to money growth. The co-movement statistics in the fifth column (case 3) are for the model's response to stochastic fluctuations of all six of the model's exogenous variables. The last column (case 4) reports co-movements from a version of the model with all six forcing variables allowed to vary, but in which the standard deviation of the innovations to money growth is 10 times higher than in the base-case calibration.

In all four cases, the model predicts the correct relative volatilities of most macroeconomic aggregates, although it underpredicts the relative volatility of employment. In all but the scenario with high money-growth variance, the model's prediction concerning the variance of inflation relative to that of real growth is close to that in both the U.S. and Canadian data. As could be expected from the above discussion, the model predicts a negative correlation between inflation and growth, but the predicted correlation is much higher in absolute value than what we see in the data. The weak negative correlation in the data probably reflects phenomena from which our model abstracts, such as nominal rigidities that would lead to a short-run Phillips curve with a positive trade-off between inflation and growth.

The model's predictions for the time-series relationship between inflation and growth are broadly compatible with the empirical literature. It is difficult to relate the model's predictions for time series from one artificial economy to the results from cross sections of data from many different countries. Barro (1995, 1996) obtains coefficient estimates in the range of

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12. One important assumption implied here is that the endogenous element of the government's budget constraint is the level of *lump-sum* taxation. If lump-sum taxation is not feasible and the government must adjust other marginal tax rates to satisfy its intertemporal budget constraint, monetary and fiscal policy are not independent, and the effects of a monetary policy change on growth could be stronger because of the induced effects on other tax rates.

13. The simulation results are averages from 20 replications using independent draws of the innovations to the forcing variables. For each replication, an artificial sample of 200 periods was generated, and the first 65 observations were dropped so that the results were not sensitive to initial conditions.

Table 2

Stochastic Properties of the Model

Moment	United States	Canada	Case 1	Case 2	Case 3	Case 4
σ_y	0.015	0.018	0.016	0.015	0.016	0.016
σ_c/σ_y	0.54	0.59	0.562	0.564	0.594	0.609
σ_i/σ_y	3.32	2.85	3.730	3.740	3.661	3.700
σ_n/σ_y	0.94	1.04	0.276	0.278	0.362	0.334
σ_g/σ_y	1.24	2.36	0.000	0.000	1.662	1.580
σ_π/σ_θ	0.65	0.84	0.589	0.723	0.756	1.476
$\sigma(c,y)$	0.88	0.59	0.983	0.978	0.879	0.898
$\sigma(i,y)$	0.93	0.63	0.989	0.989	0.950	0.954
$\sigma(n,y)$	0.79	0.86	0.974	0.966	0.829	0.822
$\sigma(g,y)$	0.25	0.20	0.000	0.000	0.139	0.068
$\sigma(\pi,\theta)$	−0.27	−0.22	−0.992	−0.800	−0.747	−0.402

Notes: The stylized facts are based on quarterly Hodrick-Prescott filtered data, except for inflation and real growth rates, which are unfiltered. Data for the United States are for the sample 1959Q1–1992Q3, from Ambler and Paquet (1996). Data for Canada are for the sample 1947Q1–1992Q4, from Phaneuf (1994). For the calculation of inflation and real growth, the sample for the United States is 1959Q1–1996Q4, and for Canada, 1947Q1–1996Q4.

−0.01 to −0.03 in cross-section regressions of average growth rates on average inflation rates and other explanatory variables. The relative variance of growth compared with inflation in our simulations is too high to be compatible with the low absolute value of this coefficient. However, our model would be compatible with Barro's results if the *cross-sectional* variance of money growth compared with the *cross-sectional* variance of tax rates and other explanatory variables were much higher than the relative variances we see in the U.S. and Canadian time series. This is quite likely, since Barro's sample includes countries with average inflation rates that exceed 100 per cent at annual rates.

4 Implications for Empirical Work

4.1 Implications for cross-section regressions

Many papers in the empirical literature on inflation and growth use data from a cross section of countries. The studies by Barro (1995, 1996) are a good example; Barro regresses average growth rates of the countries in his sample on average inflation rates and other explanatory variables, which include rates of male and female schooling, life expectancy, fertility rates, government consumption ratios, public education spending ratios, and democracy indexes. Barro finds that the average inflation rate variable has a coefficient that is negative and statistically significant. In the context of our model, taking averages of growth rates, inflation rates, and other variables over time can be thought of as comparing different steady states of countries that, because of different average levels of policy variables, have different allocations of resources on average and therefore different average growth rates and inflation rates.

Our model has the following implications for cross-section regressions:

1. Cross-section regressions of the type reported by Barro cannot be interpreted as reduced-form regressions. An immediate consequence of equations (30) and (31) is that both growth and inflation are endogenous in the steady state, and depend on underlying variables that affect the allocation of resources in the long run. At best, the signs of the coefficients in these reduced-form equations can tell us about the *conditional* correlations between inflation and growth—that is to say, conditional on the cross-sectional variations of the determinants of inflation and growth. Figure 1 shows that, in our model, permanent changes in all the explanatory variables lead to a negative trade-off between inflation and growth, so the negative sign is exactly what we would expect.

2. Our model predicts qualitatively that the absolute size of the coefficient in a regression of growth on inflation will depend on which explanatory variables are responsible for the cross-sectional variation of growth and inflation. If cross-sectional variations in money growth dominate the sample, we would expect that the absolute value of the coefficient on inflation would be relatively small. If cross-sectional variations in other variables dominate the sample, the coefficient on inflation should be close to −1.

We can use the model's quantitative predictions to say more. Because of the first term in equation (32) above, the partial derivative of inflation with respect to money growth is high. For our base-case parameter values, it is equal to 1.0187. In contrast, the partial derivative of θ with respect to μ is equal to −0.0187. The partial derivatives of growth and inflation with respect to the other explanatory variables are equal and opposite in sign. The largest partial derivatives are with respect to τ^n and are equal to 0.0189 in absolute value. With respect to τ^c, the partial derivatives are equal to 0.0121 in absolute value, and with respect to τ^k they are equal to 0.0019 in absolute value. This means that, unless the cross-sectional variations in tax rates and other explanatory variables are large compared with the cross-sectional variation of money growth, the latter variable will dominate in terms of determining the absolute size of the coefficient on inflation in growth regressions. It is interesting to note that the slope of the trade-off between growth and inflation in response to exogenous changes in μ (−0.0184) is about at the lower end of the range of coefficient estimates obtained by Barro (1995, 1996) in regressions of growth rates on inflation rates.

3. However, the effects of variables other than money growth on inflation and growth may be sufficient to change the sign of the conditional correlation induced by variations in money growth. Our model predicts that increases in μ reduce the equilibrium rate of growth because agents substitute leisure for consumption. Some endogenous growth models, with different underlying mechanisms for growth, may lead to the opposite result. For example, in models in which human capital accumulation is a separate activity, when the inflation tax is increased, agents may substitute time spent investing in human capital for time spent in the labour market. This would lead to a positive effect of money growth on inflation. This means that the coefficient estimates from the empirical literature tell us little or nothing about the trade-off between inflation and growth that is relevant from the point of view of the monetary policy authorities. Noise from other exogenous variables will always exaggerate the negative trade-off between inflation and growth, and would lead us to conclude that there is a negative trade-off *even if* an exogenous reduction in money growth had a mild negative effect on economic growth.

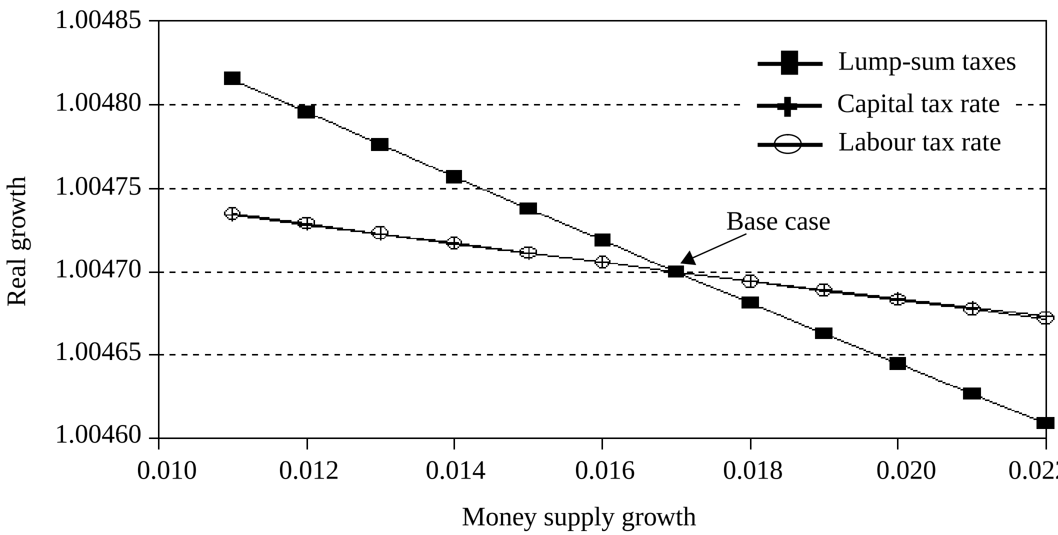
4. The explanatory variables that cause cross-sectional variations in inflation and growth are specific to our particular model, but in any structural endogenous growth model that one can imagine, it is likely that the other determinants of growth will have effects on inflation (and growth) that are small compared with the direct impact of money growth on inflation.

5. Our model predicts that variables such as average tax rates, differentiated according to type (consumption, labour, and capital) may have important qualitative effects on inflation and growth. Barro includes only the government spending ratio in his regressions. Higher public spending in general goes hand in hand with higher tax rates, but our model shows that the *mix* of tax rates may be important in determining growth rates in the long run. This means that the cross-section regressions typically encountered in the literature suffer from a systematic problem of omitted variable bias.

6. Our model's prediction that higher money growth reduces growth in the long run depends on our assumption that changes in the inflation tax are made up for by changes in the level of lump-sum taxation. If lump-sum taxation is not an available policy instrument, which is a more realistic assumption, the *policy trade-off* between inflation and growth may be very different. Figure 2 shows three possible scenarios. The first curve illustrates the effect of changes in money supply growth on the rate of growth of the economy when other tax rates are held constant and the level of lump-sum taxation varies to make up for changes in the inflation tax. The second and third curves (which lie almost on top of one another) illustrate the growth effects of variations in the inflation tax when lump-sum taxation is held

Figure 2

Growth Effects of Money Creation



constant. The second curve shows the response of the rate of growth when changes in the inflation tax are compensated for by changes in the rate of capital taxation, τ^k. The third curve illustrates the effects on growth when the labour tax rate, τ^n, is varied in response to changes in the inflation tax. This exercise shows that the trade-off between inflation and output in response to monetary policy changes, which was already very flat when other tax rates were held constant, becomes even more flat when other tax rates change endogenously.

7. Growth regressions of this type are not legitimate reduced forms, nor can they be interpreted as structural equations. Sims (1996) shows how Barro's basic regression equation is related to the transformation of an aggregate production function that depends on human and physical capital. Let us start with the production function

$$Y_t = Z_t H_t^{\alpha} K_t^{\beta},$$

where the variable definitions are the same as in the model presented in this paper. Using dots over variables to denote rates of change and dropping time subscripts, we have

$$\dot{Y} = \dot{Z} H^{\alpha} K^{\beta} + \alpha Z H^{\alpha-1} K^{\beta} \dot{H} + \beta Z H^{\alpha} K^{\beta-1} \dot{K},$$

so that

$$\frac{\dot{Y}}{Y} = \theta_1 \frac{\dot{K}}{Y} + \theta_2 \frac{\dot{H}}{Y} + \frac{\dot{Z}}{Z}, \tag{35}$$

with

$$\theta_1 = \beta \frac{K}{Y}, \quad \theta_2 = \alpha \frac{H}{Y}.$$

Equation (35) with $\dot{Z}/Z$ as the error term is similar to the equation estimated by Barro, with the investment ratio proxying for the rate of growth of capital, and with variables such as rates of schooling as proxies for the rate of growth of human capital. Sims shows how the equation can be transformed so that it includes the *levels* of income and of human capital, but that in the steady state the true coefficient on human capital is zero and the coefficient on the level of income is very small and different from zero only if $\alpha + \beta > 1$ (that is, with increasing returns to reproducible factors). Barro includes beginning-of-sample per capita income as an explanatory variable, and interprets the coefficients on this variable as capturing convergence effects. If the estimated equation is an aggregate production function, the coefficient is different from zero *only if there is no convergence*. The additional explanatory variables used by Barro, such as the rate of inflation, do not strictly belong in the equation, unless they are thought to be related to the $\dot{Z}/Z$ variable, which is unobserved. Finally, if the equation is an aggregate production function, the estimated impact of inflation on growth cannot be interpreted as the effect of a policy-generated change of inflation on output growth, since it holds fixed every endogenous variable in the equation but output growth.

8. As we have just noted, Barro includes the beginning-of-sample *level* of per capita income as an explanatory variable in some of his regressions, with the justification that it captures the *convergence* of income levels that is a consequence of the neoclassical exogenous growth model.[14] Quah (1993, 1995) has shown elsewhere that, even when growth rates are drawn from a time-invariant stationary distribution, the coefficient associated with the initial level of income will typically have a negative value because of mean reversion.

Our model implies an even stronger result. Because it has transitional dynamics, panel data from our model will show that per capita income is negatively correlated with growth rates. This is true even though, since growth rates depend on policy variables in the long run, there is *divergence*

14. See also Barro (1991), Barro and Sala-i-Martin (1992, 1994), and Coulombe and Lee (1995).

of per capita income levels across time unless all countries are following identical policies.

This result can be seen as follows. Consider Figure 3, which illustrates hypothetical data points that are compatible with our model. The data points are for hypothetical averages across time of inflation rates and real growth rates. We assume that there are four groups of countries. The group of countries close to A on the graph have low initial levels of per capita income and, because of their economic policies and other exogenous factors, have low steady-state rates of growth. The group of countries near B have low initial levels of per capita income and high steady-state growth rates. The countries near C and D have high initial levels of per capita income and, respectively, low and high steady-state growth rates. Within groups of countries, the only difference is, by assumption, the initial level of per capita income. Since our model implies that economies above their steady-state growth paths have lower rates of income growth, this will lead, all else being equal, to the negative relationship between initial income level and the average rate of growth in a finite sample. The raw correlation coefficient between initial income levels and growth will be negative. If growth rates are regressed on levels of per capita income, the coefficient will be negative and may well be significant.

To summarize, our model in particular and endogenous growth theory in general predict that cross-section regressions of growth on inflation should yield coefficient estimates of the effects of inflation that are small in absolute value and that may change in sign. Because the regressions are neither true reduced-form equations nor true structural equations, the results of such estimates have no meaningful implications for the relevant trade-off between inflation and growth available to monetary policy authorities.

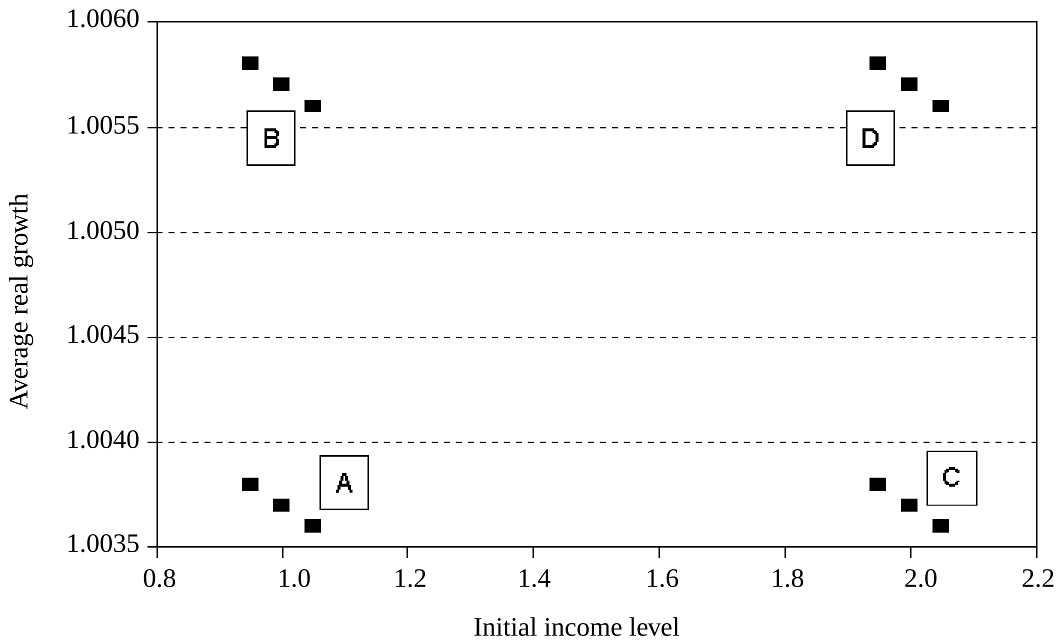
4.2 Implications for time-series regressions

Researchers have also used time-series techniques to address the question of the relationship between growth and inflation, both in the short run and in the long run.

1. Our model implies that both inflation and real growth rates are stationary variables, as long as their underlying determinants (tax rates, public spending ratios, and money supply growth) follow stationary stochastic processes. At first, this would appear to be at odds with some recent time-series studies, such as Bullard and Keating (1995). Using observations from 58 different countries, they find that real growth rates are stationary but inflation rates are I(1) so that they contain a unit root. The fundamental relationship in equation (32) shows that, if real growth is

Figure 3

"Convergence"



stationary, the only way that inflation can be I(1) is if money growth is non-stationary. We feel that it is highly implausible that money growth is literally I(1) for most or all the countries in Bullard and Keating's sample. This would mean, for example, that we should predict that German money growth would attain *any* arbitrarily high positive level within a finite amount of time, and that money growth in all countries could be predicted to become significantly negative within a finite amount of time.

In our view, a more likely interpretation of Bullard and Keating's results is that most countries in the sample considered by the authors have shifted between periods of high money growth and high inflation and periods of low money growth and low inflation. The unconditional mean and variance of inflation in these countries is constant, so that these series are stationary. With a small number of large shifts in conditional mean, tests of the null hypothesis of a unit root will have low power and will not be able to reject. Note that our model's predictions are perfectly compatible with the empirical finding that growth rates are stationary and that inflation is I(1). With our base-case calibration, shifts in the mean of the money supply process lead to large changes in the mean of the inflation rate and only small changes in the mean growth rate. Furthermore, shifts in the mean of the other determinants of inflation and growth lead to only small changes in inflation and growth. Tests of the unit root hypothesis on growth rates are less likely to suffer from a lack of power due to a small sample size than tests of the same hypothesis on inflation rates.

2. Time-series studies of the relationship between inflation and growth often use vector autoregressions (VARs) with restrictions on the long-run impact of different types of shocks in order to identify different “structural” shocks. Bullard and Keating (1995) estimate a bivariate VAR with output growth and inflation. They manage to identify one of their shocks as a “monetary” shock since inflation enters the model in *first differences*. The monetary shock affects neither output growth nor the *change* in inflation in the long run. The other shock, which has the interpretation of a real shock, also is not allowed to affect the change in inflation in the long run, while if it has a permanent component it may affect real growth rates in the long run. Our model suggests that the conclusion that inflation is I(1), which is theoretically implausible, may be wrong because of a small-sample problem. If both inflation and growth are stationary, our model says that it is impossible to identify shocks in a bivariate VAR with inflation and output growth. Both shocks to money growth and shocks to real variables such as tax rates can have permanent effects on both inflation and growth.

3. Our model implies that money, output, and prices should be cointegrated if there is balanced growth in the long run. This follows since equation (4) implies that there is a long-run demand-for-money function in the level of real balances, and depends on nominal interest rates being I(0), which we feel is a reasonable assumption. This relationship has not been thoroughly tested or exploited in the time-series studies.

4. Our model has implications for the variables that can be treated as exogenous in time-series regressions. Time-series regressions with other variables on the right-hand side should use these variables as instruments. Finding the right instruments is complicated by the fact that the true instruments in the model are unobservable, since they are in many cases variables that have been normalized by the (unobservable) level of human capital. This problem can in part be circumvented by a different choice of normalization. The most convenient normalization from the point of view of simulating the model involves dividing non-stationary variables by the level of human capital. While the algebra becomes more complicated, normalizing variables by an observable variable such as the capital stock or the per capita level of output should also lead to a tractable formulation of the household’s stationary dynamic programming problem.

5. Another problem related to unobservable state variables is that of the technology shocks. In models with perfect competition and no externalities, the rate of exogenous technological change can be measured

by calculating Solow residuals. This is no longer the case, as we show below. Taking logs and then first differences of the production function, we have

$$\Delta \ln Y_t = \Delta \ln Z_t + \alpha \Delta \ln H_t + \alpha \Delta \ln N_t + (1 - \alpha) \Delta \ln K_t \,. \tag{36}$$

Taking the law of motion for human capital and dividing by the level of human capital, we have

$$\frac{H_{t+1} - H_t}{H_t} = -\delta_h + \psi \ln N_t \,.$$

This relates the rate of growth of human capital to an observable variable, the level of employment. The particular variable depends on our specification, but in general with endogenous growth the rate of growth of human capital or of technology will be related to an observable aspect of the allocation of resources in the economy. Using the human capital accumulation process gives

$$\Delta \ln Y_t = \Delta \ln Z_t + \alpha(-\delta_h + \psi \ln N_{t-1}) + \alpha \Delta \ln N_t + (1 - \alpha) \Delta \ln K_t$$

$$\Rightarrow \Delta \ln Y_t - \alpha \Delta \ln N_t - (1 - \alpha) \Delta \ln K_t$$

$$= \Delta \ln z_t + \alpha(-\delta_h + \psi \ln N_{t-1}) \,. \tag{37}$$

The traditional measure of the rate of change of the Solow residual is contaminated with a term that depends on the lagged *level* of per capita employment (or, more generally, on the allocation of resources in the economy), which is of course an endogenous variable. If one believes that growth rate regressions should include as explanatory variables the true state variables of the model, our theoretical model suggests that there may be a problem. The traditional measure of the Solow residual may be contaminated with a term that depends on the allocation of resources. In our model, with learning by doing, this variable is just the level of employment. In other models, this term could include things like the relative importance of spending on research and development. Since the proper measure of resource allocation depends on the underlying causes of growth, it may be difficult to find a true measure of the Solow residual to use as an instrument in the growth regressions. The exact correction also depends on the specification of the mechanisms underlying growth, and will depend in general, as in our simple case, on unobserved structural parameters. It may be necessary to obtain parameter estimates from the full-blown structural model in order to measure correctly the rate of exogenous technological change.

To summarize, it seems to us that the simple qualitative consequences of basic theory have not been sufficiently exploited by the time-series literature. Endogenous growth theory in general says that money may not be superneutral (permanent changes in money growth may affect the level of output in the long run). It implies that changes in money growth may have permanent effects on real growth rates. This invalidates many of the identifying restrictions used in VAR studies, not only in the money and growth literature, but elsewhere.

Conclusions

Our results have important implications for how empirical studies relating inflation and growth should be conducted, and help explain the results researchers have obtained in previous studies.

Reduced-form estimates of the effect of inflation on growth of the type seen in the empirical literature are likely to find a negative relationship. Furthermore, this relationship is likely to be far stronger than the trade-off between inflation and growth available to policymakers.

In one respect, our conclusions are pessimistic. We join Sims (1996) and others in concluding that interpreting the results of single-equation regressions, with their inherent endogeneity and misspecification problems, is fraught with difficulty. We have shown that the conditional correlation between inflation and growth depends on the relative importance of different types of exogenous shocks over the sample period. In addition, even for a given type of exogenous shock, the conditional correlation may depend on the structure of the true model, in particular the mechanism responsible for the presence of endogenous growth.

If one is interested in the question of what is the trade-off between growth and inflation that is *exploitable* by policymakers, then the nature of endogenous growth becomes crucial. In our model, there is a simple relationship between employment and growth. If we examine different categories of growth models (see Macklem 1993 for a survey), then the effect of money creation on growth may be different depending on the type of model. In a simple model such as that considered by Kocherlakota (1996) there is, by construction, no effect of exogenous changes in money creation on growth. In models like ours, monetary expansion that leads to increased inflation acts as a tax on consumption, and promotes a substitution away from work and consumption and towards leisure. In models in which agents must devote some of their own time not spent at work to the accumulation of human capital, a tax on consumption may lead agents to substitute away from time spent in the labour market towards time spent accumulating human capital, so that the policy trade-off between inflation and growth is actually positive.

Our recommendation concerning the proper way to proceed is to concentrate on the estimation of full-blown structural models using techniques such as the simulated method of moments, SMM (see, for example, Lee and Ingram 1991, and Jonsson and Klein 1996) that allow the researcher to test the adequacy of the model's overall specification, and perhaps to discriminate between competing specifications for endogenous growth.

References

Aiyagari, S. R. 1990. "Deflating the Case for Zero Inflation." *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Summer): 2-11.

Ambler, S., and A. Paquet. 1996. "Fiscal Spending Shocks, Endogenous Government Spending, and Real Business Cycles." *Journal of Economic Dynamics and Control* 20 (January–March): 237-56.

Ambler, S., L. Phaneuf, and M. Sauthier. 1995. "Money, Taxes and Business Fluctuations." Unpublished. Université du Québec à Montréal.

Barro, R. J. 1991. "Economic Growth in a Cross Section of Countries." *Quarterly Journal of Economics* 106 (May): 407-43.

———. 1995. "Inflation and Economic Growth." *Bank of England, Quarterly Bulletin* 35 (May): 166-76.

———. 1996. "Inflation and Growth." *Federal Reserve Bank of St. Louis Review* 78 (May–June): 153-69.

Barro, R. J., and X. Sala-i-Martin. 1992. "Convergence." *Journal of Political Economy* 100 (April): 223-51.

———. 1994. *Economic Growth*. New York: McGraw Hill.

Bean, C. R. 1993. "Government Policy and Economic Growth." In *Advanced Lectures in Quantitative Economics II*, edited by A. de Zeeuw, 83-112. New York: Academic Press.

Black, R., T. Macklem, and S. Poloz. 1994. "Non-Superneutralities and Some Benefits of Disinflation: A Quantitative General-Equilibrium Analysis." In *Economic Behaviour and Policy Choice Under Price Stability*, 477-516. Proceedings of a conference held at the Bank of Canada, October 1993. Ottawa: Bank of Canada.

Bullard, J., and J. W. Keating. 1995. "The Long-Run Relationship between Inflation and Output in Postwar Economies." *Journal of Monetary Economics* 36 (December): 477-96.

Chari, V. V., L. E. Jones, and R. E. Manuelli. 1995. "The Growth Effects of Monetary Policy." *Federal Reserve Bank of Minneapolis Quarterly Review* 19 (Fall): 18-32.

Cooley, T. F., and G. D. Hansen. 1989. "The Inflation Tax in a Real Business Cycle Model." *American Economic Review* 79 (September): 733-48.

———. 1991. "The Welfare Costs of Moderate Inflations." *Journal of Money, Credit, and Banking* 23 (August): 483-503.

———. 1995. "Money and the Business Cycle." In *Frontiers of Business Cycle Research*, edited by T. Cooley, 175-216. Princeton: Princeton University Press.

Coulombe, S., and F. C. Lee. 1995. "Convergence Across Canadian Provinces, 1961 to 1991." *Canadian Journal of Economics* 28 (November): 886-98.

De Gregorio, J. 1993. "Inflation, Taxation, and Long-Run Growth." *Journal of Monetary Economics* 31 (June): 271-98.

Devereux, M. B., and D. R. F. Love. 1994. "The Effects of Factor Taxation in a Two-Sector Model of Endogenous Growth." *Canadian Journal of Economics* 27 (August): 509-36.

Dotsey, M., and P. N. Ireland. 1996. "The Welfare Cost of Inflation in General Equilibrium." *Journal of Monetary Economics* 37 (February): 29-47.

Gomme, P. 1993. "Money and Growth Revisited: Measuring the Costs of Inflation in an Endogenous Growth Model." *Journal of Monetary Economics* 32 (August): 51-77.

———. 1996. "Anticipated Inflation in a Neoclassical Growth Model with a Cash-in-Advance Constraint." Unpublished. Simon Fraser University.

Hansen, G. D., and E. C. Prescott. 1995. "Recursive Methods for Computing Equilibria of Business Cycle Models." In *Frontiers of Business Cycle Research*, edited by T. F. Cooley, 39-64. Princeton: Princeton University Press.

Jonsson, G., and P. Klein. 1996. "Stochastic Fiscal Policy and the Swedish Business Cycle." *Journal of Monetary Economics* 38 (October): 245-68.

Kocherlakota, N. 1996. "Commentary." *Federal Reserve Bank of St. Louis Review* 78 (May–June): 170-72.

Lee, B-S., and B. F. Ingram. 1991. "Simulation Estimation of Time-Series Models." *Journal of Econometrics* 47 (February–March): 197-205.

Love, D., and J-F. Wen. 1996. "Inflation, Welfare, and the Time-Costs of Transacting." Unpublished. Brock University.

Macklem, T. 1993-94. "Recent Advances in Growth Theory: Perspective and Policy Implications." *Bank of Canada Review* (Winter): 37-55.

Mankiw, N. G., D. Romer, and D. N. Weil. 1992. "A Contribution to the Empirics of Economic Growth." *Quarterly Journal of Economics* 107 (May): 407-37.

Mendoza, E. G., A. Razin, and L. L. Tesar. 1994. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34 (December): 297-323.

Phaneuf, L. 1994. "Marché du travail et cycle économique : de la réalité aux modèles." *Interface* 15 (May–June): 42-50.

Quah, D. 1993. "Galton's Fallacy and Tests of the Convergence Hypothesis." *Scandinavian Journal of Economics* 95 (4): 427-43.

———. 1995. "Empirics for Economic Growth and Convergence." Discussion Paper No. 1140. Centre for Economic Policy Research, London.

Rebelo, S. 1991. "Long-Run Policy Analysis and Long-Run Growth." *Journal of Political Economy* 99 (June): 500-21.

Sidrauski, M. 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 57 (May): 534-44.

Sims, C. A. 1996. "Inflation and Growth: Commentary." *Federal Reserve Bank of St. Louis Review* 78 (May–June): 173-78.

Stockman, A. C. 1981. "Anticipated Inflation and the Capital Stock in a Cash-in-Advance Economy." *Journal of Monetary Economics* 8 (November): 387-93.