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**Tracy Chan, Ramdane Djoudad, and Jackson Loi**

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The views expressed in this paper are those of the authors.  
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## Abstract

Financial innovations and the removal of the reserve requirements in the early 1990s have made the distinction between demand and notice deposits arbitrary. This classification issue has affected those narrow monetary aggregates (gross and net M1) that rely on a proper distinction for their definition, and may have eroded their value as indicators. The authors examine whether the indicator properties of various narrow aggregates for the growth of real output have changed over time. They find evidence of a regime shift in the relationship between real and narrow monetary aggregates and the growth of real output, which seems to have occurred in 1992. More specifically, their results show that real M1+, the definition of which is not based on the distinction between demand and notice deposits, has become a more useful indicator in predicting the growth of real output over the more recent period.

*JEL classification: E40, E42, E50*

*Bank classification: Monetary aggregates*

## Résumé

Les innovations financières et l'élimination des réserves obligatoires au début des années 1990 ont rendu arbitraire la distinction entre les dépôts à vue et les dépôts à préavis. Étant donné que la définition des agrégats monétaires au sens étroit (M1 brut et M1 net) dépend d'une telle distinction, ce problème de classification pourrait avoir entraîné une diminution de leur utilité en tant qu'indicateurs. Les auteurs examinent si les propriétés de divers agrégats monétaires étroits comme indicateurs de la croissance de la production réelle se sont modifiées au fil des ans. D'après leurs résultats, un changement de régime se serait produit en 1992 dans la relation entre les agrégats étroits mesurés en termes réels et la croissance de la production réelle. Les résultats indiquent en particulier que M1+ réel, dont la définition ne repose pas sur la distinction entre les dépôts à vue et les dépôts à préavis, est devenu au cours de la période récente un indicateur plus utile pour la prévision de la croissance de la production réelle.

*Classification JEL : E40, E42, E50*

*Classification de la Banque : Agrégats monétaires*

## 1. Introduction

Financial innovations in banking products over the past years have made it increasingly difficult to differentiate between demand and notice deposit accounts. For example, both types of accounts currently offer similar interest rates and comparable accessibility to funds. In addition, the elimination of reserve requirements on all bank accounts between 1992 and 1994 in Canada has removed the need for banks to discriminate between demand and notice deposit accounts (Aubry and Nott 2000).<sup>1</sup> As a result, the classification of deposit accounts by financial institutions into “demand” and “notice” has become increasingly arbitrary.

The blurred distinction between demand and notice deposits raises questions regarding the quality of those monetary aggregates that rely on such a distinction for their definition. Specifically, net M1 and gross M1 (hereafter M1 and GM1), which include currency and demand deposit accounts, are directly affected by this classification issue. Since the classification has become artificial, it has led some researchers to assert that these narrower aggregates (M1 and GM1) may no longer contain additional information beyond those in M1+ and M1++. Moreover, this arbitrary categorization by financial institutions may have created more noise in M1 and GM1 and heightened their volatility. On the other hand, the broader measures of narrow aggregates, namely M1+ and M1++, capture both demand and notice deposits and hence are not affected by this taxonomy concern.

While it is widely believed in the literature that M1 and GM1 have had traditionally superior leading information for output growth, it is not clear whether the classification issue of demand and notice deposits has eroded the information content of M1 and GM1. It is therefore important to assess and compare the various narrow monetary aggregates (M1, GM1, M1+, and M1++), with respect to their leading-indicator properties for the future growth of real output. This paper aims to answer the following two questions. First, how do the indicator properties of M1 and GM1 for output growth compare with those of M1+ and M1++ since the early 1990s? And second, is there evidence of regime shifts in the information content of narrow monetary aggregates for output growth over the more recent period?

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1. The reserve requirements were 10 per cent on demand deposits and 3 per cent on notice deposits. These requirements were imposed only on the chartered banks.

The rest of this paper is organized as follows. In section 2 we describe how financial innovations have affected monetary aggregates in Canada. Section 3 provides a brief literature review, summarizing key results of existing empirical work on the relationship between narrow monetary aggregates and real output in Canada in the past two decades. Section 4 describes the data used in this paper. Section 5 reports correlation analysis between each of the narrow monetary aggregates and the growth of real output, summarizes a comparative analysis using rolling vector autoregressions (VARs), and describes the results using Stock and Watson's (2003) method on a reduced-form money indicator model. In section 6 we utilize a regime-switching model to test for changes in regime over the period from 1975 to 2005. Section 7 offers some conclusions.

## **2. Financial Innovations and Distortions in Canadian Monetary Aggregates**

Over the years, the quantity of the money stock has been aggregated in many different ways to represent the notion of "money." Economists generally aggregate money using two approaches (Laidler 1969). The first is to group monetary assets that most closely represent some underlying definition of money (as a medium of exchange or a store of value). The second approach is to define money as groupings of financial assets that have the most significant empirical relationship with certain macroeconomic variables, such as real output and consumer price inflation. Indeed, no single way of monetary aggregation has been universally accepted, because there is no simple "one size fits all" approach to dealing with the numerous economic concepts of money (Laidler 1999). As White (1976, 49) remarks, "the answer to . . . the related choice between alternative money definitions [is] based on the usefulness of the various aggregates for policy purposes." As the role of money in monetary policy has changed over the years, monetary aggregates have also evolved.

While the Bank of Canada began publishing monthly data for monetary components well before 1970, it was not until the 1970s that the monetary aggregate M1 was reported. This monetary aggregate was intended to provide information to the public about changes in the nation's money supply. During the 1980s, the Bank also began reporting M1A, which is defined as the sum of M1 plus daily interest chequing accounts and non-personal notice deposits. This aggregation comprised the most liquid monetary accounts and was intended to represent money for transactions purposes and purchasing power.



In the past 20 years, financial innovations have played a significant role in the way economic agents manage their money and financial assets. These innovations have caused important structural shifts between deposit accounts that ultimately blurred the distinction between various monetary aggregates. The first wave of innovations in banking products took place from 1978 to 1986, significantly reducing the demand for M1 in both the corporate and household sector in Canada (Aubry and Nott 2000). On the corporate side, a number of new cash-management packages allowed businesses to consolidate several accounts into one centralized account. As a result, firms were able to reduce their total working cash balances. On the household side, the introduction of daily interest savings accounts (chequable and non-chequable) boosted incentives to deposit and transfer money into these accounts, which were not included in the measurement of M1 because they were unlikely to be used for transactions purposes prior to such financial innovations. In the following years, new financial products introduced by deposit-taking institutions continued to offer households and firms increasing flexibility in the type of account in which they held deposits.

The second major wave of financial innovations occurred around 1993. Mutual fund products gained popularity relative to notice deposits as a savings vehicle, and free credit balances (cash or margin accounts intended for trading financial assets) grew rapidly. More importantly, as noted earlier, the removal of reserve requirements in the mid-1990s eliminated the need for banks to differentiate between demand (transactions) and notice (savings) deposits for reserve purposes. The innovations in business accounts also contributed largely to boosting GM1 growth. As a result, a sizable share of GM1 became more related to the sales and purchases of financial assets than to transactions for purchasing goods and services (Aubry and Nott 2000). Lastly, the development of Internet banking during the late 1990s enabled bank clients to easily transfer money between non-savings and savings accounts. This allowed bank clients to deposit money in accounts that yielded higher interest while being easily accessible for transactions purposes via transfers, without having to first give notice to the bank.

Financial innovations have made it increasingly difficult to differentiate between money held for transactions purposes and money held for savings purposes. This has led to concerns regarding whether the various monetary aggregates adequately represent their originally intended

definitions. Financial institutions are also experiencing difficulties in categorizing and reporting their deposit accounts as either demand or notice, raising concerns about the quality of M1 and GM1 data. In an effort to capture a broader notion of transactions money and to internalize the shifts occurring in some of the components, two alternative measures of narrow monetary aggregates, M1+ and M1++, have been published and monitored by the Bank since 1999. M1+ and M1++ are not affected by the requirement to distinguish between demand and notice deposits because they incorporate both account categories. They therefore capture the components related to transactions purposes as well as those related to savings purposes. In addition to these broader aggregations of narrow money, a model-based aggregate, adjusted-M1, was created to account for distortions in GM1. The procedures in estimating adjusted-M1, however, are not free of deficiencies, and the resulting measure could lead to a loss in valuable information. For these reasons, the Bank has been motivated to explore new ways to define measures of transactions money (Gilbert and Pichette 2003).

### **3. Literature Review**

While monetary targeting has been abandoned in many major countries over the past two decades, monetary aggregates still serve as part of the indicators that help to predict future economic activity. Although changes in the monetary aggregates may be subject to shifts in money demand during certain periods (thus modifying their indicator properties at times), money may nevertheless contain some leading information.

Research outside Canada (e.g., Beckett and Morris 1992; Feldstein and Stock 1994; Stock and Watson 1989) has provided strong evidence that narrow monetary aggregates are useful for predicting the growth of nominal and real output. Using VAR models on quarterly U.S. data, Hafer and Kutan (1997) find that both narrow and broad monetary aggregates, expressed in nominal terms, have predictive power for U.S. income. In a similar exercise using monthly U.S. data, Swanson (1998) draws the same conclusion as Hafer and Kutan (1997), that M1 and M2 are significant in predicting U.S. income throughout the period from February 1960 to March 1996. Amato and Swanson (2001), however, cannot come to the same conclusion when real-time data imperfection is taken into account. Utilizing monthly and quarterly U.S. real-time data, instead of

revised data, their VAR model and vector error-correction model (VECM) suggest that M1 is insignificant in predicting output growth.

At the Bank of Canada, narrow monetary aggregates expressed in real terms (i.e., deflated by a price index) continue to be monitored and analyzed in the context of their information content for real output. In particular, past studies find that narrow monetary aggregates, specifically real M1 and GM1, have some explanatory power for the growth of real output one to two quarters ahead. No study, however, compares how the leading-indicator properties of various narrow aggregates (GM1, M1, M1+, and M1++) for output growth have evolved over the recent period. In the research conducted, the growth in real M1 is concluded to be the best indicator of the future growth of real output in Canada, with most of the explanatory power coming from the first two lags of real M1 growth and with a sum of coefficients of about 0.35 (Hostland, Poloz, and Storer 1987; Cockerline and Murray 1981). Using quarterly data from 1968 to 2001 and a rolling VAR analysis, Longworth (1997) studies the explanatory power of narrow and broad monetary aggregates for key macroeconomic variables in Canada. The VAR systems consist of four lags and include M1 growth, real output, and inflation (CPI or GDP deflator) as endogenous variables. When using M1 growth and CPI inflation, M1 is significant at the 1 per cent level in explaining the future growth of real output from 1976 to 1994; the sum of coefficients on M1 is about 0.20.

The empirical relationship between real M1 and real output appears to be robust to the addition of other explanatory variables. The inclusion of other financial variables, such as the lag changes in short-term interest rates and the first difference in the Toronto Stock Exchange (TSX) index, does not seem to alter this empirical result (Muller 1992). Hassapis (2003) estimates the long-run covariance matrix of various financial variables and real output using kernel-based estimators. His results suggest that real M1, along with equity prices and bond yield spreads, are a useful predictor of output growth. On the other hand, Cozier and Tkacz (1994) show that the inclusion of the term spread, defined as the difference between the long-term interest rate and the short-term interest rate, tends to eliminate the forecasting power of real M1 on real output in an indicator model over the four- and six-quarter horizon.

A number of studies assess the information content of various monetary aggregates. Hostland, Poloz, and Storer (1987), for example, examine the predictive power of various monetary

aggregates on nominal GDP in Canada. After comparing the information content of those monetary aggregates, they conclude that M1 is the most informative monetary aggregate for the growth of nominal and real GDP. Results from Granger causality and VAR (marginal significance and forecast-error variance of output) in Serletis and Molik (2000) suggest that Divisia M1++ is the best leading indicator of real output among an extensive set of monetary aggregates (M1, Divisia M1, CE<sup>2</sup> M1, M2, Divisia M2, CE M2, M3, Divisia M3, CE M3, M1+, Divisia M1+, CE M1+, M1++, Divisia M1++, and CE M1++). On the other hand, using Akaike information criteria (AIC) and Davidson and MacKinnon's (1981) J-test on short-run indicator models, Longworth and Atta-Mensah (2000) reaffirm that M1<sup>3</sup> has a superior forecasting performance for real output compared with that for the weighted monetary aggregates (in particular, the Fisher ideal aggregates) suggested by Barnett (1980) and Barnett and Serletis (2000). Maclean (2001) addresses the changes in money demand that cause the intercept term of the indicator model to shift up during the period from 1991Q1 to 1998Q4. She finds that M1++ works best in its indicator model for real GDP growth with a dummy variable capturing the shift in money demand.

#### 4. Data

The sample used in this research spans the period from 1975Q1 to 2005Q1, with a total of 121 observations. The monthly observations on monetary aggregates are converted to quarterly frequency by averaging the monthly series. The series for the real gross domestic product ( $Y$ ), gross domestic product deflator ( $Y^p$ ), total consumer price index ( $P^t$ ), core consumer price index ( $P^c$ ), GM1 ( $GM1$ ), M1 ( $M1$ ), M1+ ( $M1P$ ), M1++ ( $M1PP$ ), and the 3-month prime corporate paper rate ( $I^{cp}$ ) are extracted from Statistics Canada's CANSIM database. Table 1 lists the Statistics Canada mnemonics for each series. All variables are expressed as quarterly percentage growth rates at annual rates unless otherwise indicated.<sup>4</sup>

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2. Currency-equivalent indexes (Serletis and Molik 2000).

3. It is not clear whether this is GM1 or M1.

4. For instance, the growth of real output is given by  $[\ln(Y_t) - \ln(Y_{t-1})]400$ .

## 4.1 Data properties

To develop a sense of the data features of the narrow monetary aggregates, we first investigate simple descriptive statistics such as the sample mean and standard deviations. Figure 1 plots the five-year moving average of the growth rate of each of the narrow monetary aggregates (GM1, M1, M1+, and M1++). It is clear that the moving averages of M1 and GM1 have tended to evolve closely together over history, while the moving averages of M1+ and M1++ have converged at times. Figure 2 shows the five-year moving standard deviation of the growth rate of these aggregates. No one uniform pattern emerges from these sample standard deviations. However, the moving standard deviation of all narrow measures (GM1, M1, and M1+) has generally declined over history, from relatively high levels in the mid-1980s to the early 1990s, while for M1++ it has remained relatively flat over time. It is interesting to note that during the most recent period there appears to be an increase in the volatility of all four aggregates.

## 5. Correlations and VAR Methodology

We use three different empirical approaches to assess the information content of the narrow monetary aggregates. In the first approach, we employ correlations. As Longworth (2003) suggests, there are many reasons to believe why money can have leading information for output growth. Money can play an important role in the transmission mechanism. The first channel would be through the wealth effects of the money stock. The second channel would be via the real-balances effects given money in the utility function. The third would be by way of disequilibrium effects that relate to the buffer-stock story for money. Besides these channels, in the presence of frictions in the economy, money may have leading-indicator properties for the future growth of output. This lag effect is shown in the literature to be the strongest between output growth and narrow monetary aggregates. Traditionally, GM1 and M1 exhibit the highest correlation with GDP growth. We want to assess how these correlations—(GM1, GDP), (M1, GDP), (M1+, GDP), (M1++, GDP)—have evolved over time, and whether they have settled towards a more common level in the recent period. In the second approach, we use a VAR model similar to Longworth (2003). We specify a VAR for each of the four narrow aggregates (GM1, M1, M1+, and M1++) and try to assess whether the response of output growth to money growth

became similar for the four aggregates over the more recent period. In the third approach, we use Stock and Watson's (2003) methodology to evaluate the leading-indicator properties of each of the narrow aggregates.

If the information coming from narrow monetary aggregates has become more comparable, it may suggest that, while M1 and GM1 traditionally were better indicators of the future growth of output, over the more recent period M1+ and M1++ have become the more relevant indicators. Furthermore, a regime shift may have occurred in their indicator properties. This possibility will be examined using regime-switching models.

## 5.1 Correlations

It is widely believed that there is a lag effect between the growth of narrow money and the growth of output. Typically, the lag effect is shown in the literature to be the strongest between output growth and the growth of GM1 and M1. In Figure 3, we plot the quarterly growth of real GDP and the two-quarter moving average of real M1+ and real M1++ (lagged one quarter), similar to a figure published in the Bank of Canada's semi-annual *Monetary Policy Report*. Figure 4 plots a similar graph with real GM1 and real M1 (lagged one quarter). There is a significant lagged correlation between the growth rate in narrow aggregates and output growth.

To quantitatively assess this lead-lag relationship, we use a simple empirical exercise to evaluate how the correlations between each of the narrow aggregates (M1, GM1, M1+, and M1++) and output growth have evolved over time. For each of the narrow monetary aggregates, we calculate rolling correlations (with starting point held fixed at 1975Q4) between its two-quarter moving average (lagged one quarter) and real GDP growth.

In Figure 5, we plot the correlations between output growth and the two-quarter moving average of the growth of real money for each of the narrow aggregates. The results show that, over the period from 1995 to 2005, the correlations using GM1, M1, and M1+ settled to a range of 0.43 to 0.57, while the correlation using M1++ moved to a slightly lower range of 0.37 to 0.42. In Figure 6, we plot the correlations of the growth of real output and the growth of nominal money lagged two periods. We see from the results that, over the period from 2000 to 2005, the correlations using GM1 and M1 trended to a closer level with the correlation using M1+. They all converged

within the range between 0.29 and 0.37 in 2005Q1. The correlation using M1++, however, does not converge to similar levels over the same period. When real monetary aggregates are used in the same exercise (Figure 7), the converging pattern of the correlations using M1, GM1, and M1+ is even more striking.

The purpose of this exercise is to test whether the correlations between M1, GM1, M1+, and M1++ with output growth have settled to a similar level over the period from 1995 to 2005. This converging pattern would suggest a comparable information content among the various narrow monetary aggregates. However, this is not observed for correlations using M1++.

## 5.2 Multivariate analysis (VAR)

Having assessed the simple correlation figures between the growth of real output and narrow monetary aggregates, we adopt a VAR approach to examine this relationship. As in Longworth (2003), we specify a VAR that has four variables: the growth of real output, the growth of money, short-term interest rates, and inflation. The aim is to estimate a separate VAR model for each of the narrow monetary aggregates: GM1, M1, M1+, and M1++. For each monetary aggregate we calculate two measures pertaining to the output-growth equation. First, the  $\bar{R}^2$  gives us an overall in-sample assessment of the explanatory power of the variables included. We then compare the  $\bar{R}^2$  from each of the monetary aggregates and observe how these  $\bar{R}^2$  have evolved over time. Second, we calculate the sum of coefficients of the lagged money variable and its significance level; this gives us a sense of the quantitative response of output growth to the money variable. The aim is to evaluate how each of the narrow aggregates impacts output growth and whether those impacts have changed over time.

We estimate a VAR that has four variables as follows:

$$\Delta \ln(Y_t) = \mu + \sum_{j=1}^2 \left( \phi_{(1,j)} \Delta \ln(M_{t-j}) + \phi_{(2,j)} \Delta \ln(Y_{t-j}) + \phi_{(3,j)} \Delta \ln(P^t_{t-j}) + \phi_{(4,j)} \Delta \left( \frac{I^{cp}_{t-j}}{400} \right) \right) + \varepsilon_t, \quad (1)$$

where:

$Y_t$  is the real output;

$M_t$  is the monetary aggregate considered (namely, M1, GM1, M1+, and M1++);

$P_t^t$  is the total consumer price index;

$I^{cp}_t$  is the short-term interest rate;

$\Delta$  is the first-difference operator; and

$\ln$  is the log operator.

The model is first estimated over the sample period from 1975Q4 through 1987Q3. For each monetary aggregate, we retrieve the corresponding  $\bar{R}^2$ , the sum of coefficients on the money variable, and its significance level. We then add one observation, in this case 1987Q4, to the original sample, re-estimate, and retrieve the corresponding  $\bar{R}^2$ , the sum of coefficients, and the statistical significance of those coefficients. We continue to roll forward the end date and retrieve the aforementioned statistics up to the most recent period in this sample: 2005Q1.

We plot the path of the sum of coefficients and their respective significance for GM1, M1, M1+, and M1++ over the period 1987Q3 to 2005Q1 (Figures 8 and 9). It is evident from these figures that the sums of coefficients have settled over time to a range between 0.15 and 0.20. The  $p$ -values of the sum of coefficients for all monetary aggregates have been significant at the 1 per cent level since the end of 1992.

In Figure 10, we report the path of the  $\bar{R}^2$  over the same period. It is interesting to note that GM1 and M1 tend to move in unison while M1+ and M1++ move together. These results suggest a stronger relationship between real output and (M1, GM1) compared with M1+ and M1++. As well, the  $\bar{R}^2$  of M1 and GM1 trend down and those of M1+ and M1++ trend up over this time horizon.

Consistent with the correlation exercise, the overall results indicate that the explanatory power of each of the narrow monetary aggregates for the future growth of output has changed over time. While the existing literature shows that M1 and GM1 traditionally had superior explanatory



power over M1+ and M1++ for the growth of GDP, our results suggest that, over the more recent period, their comparative performance has changed.

### 5.3 In-sample and out-of-sample forecasts

In this section, we compare the leading-indicator properties of the different narrow monetary aggregates, both in-sample and out-of-sample. The aim is to assess the extent to which the different narrow aggregates provide valuable information for the future growth of output. An alternative way to make this assessment is to use Stock and Watson's (2003) methodology,<sup>5</sup> which is based on an indicator model formalized by the following equation:

$$\Delta y_{t+h}^h = c + \alpha(L)\Delta y_t + \beta(L)\Delta x_t + \varepsilon_t^h, \quad (2)$$

where:

$t$  denotes time;

$\alpha(L)$  and  $\beta(L)$  are polynomial lag operators<sup>6</sup>;

$\Delta y_t = (y_t - y_{t-4})$ ;

$\Delta x_t = (x_t - x_{t-1})$ ;

$\Delta y_{t+h}^h = (y_{t+h} - y_t)$ , which predicts  $h$  periods ahead; and

$x_t$  is GM1, M1, M1+, or M1++.

The variable to forecast is the percentage growth of output four quarters ahead. It is expressed as

$\Delta y_{t+4}^4 = [\ln(GDP_{t+4}) - \ln(GDP_t)]100$ , calculated at an annualized percentage growth rate.

To remedy any potential misspecification problems in the residuals, we compute a consistent variance-covariance matrix allowing for heteroscedasticity and serial correlation. We apply White's (1976) adjustment to the variance-covariance matrix of the residuals.<sup>7</sup> The whole sample period runs from 1975Q1 through 2005Q1. We assess the information content for output growth

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5. Compared with the VAR methodology, the  $h$ -step ahead forecasts reduce the plausible effect of misspecification in the one-step-ahead model of Stock and Watson (2003).

6. We use one lag.

7. We use the command ROBUSTERRORS in RATS to compute a consistent estimate of the covariance matrix.

over a four-period horizon. This lag length is adopted since previous empirical work has shown that the main impact of money growth for output growth occurs during the first four quarters.

The in-sample analysis is based on the  $\bar{R}^2$ . For the out-of-sample exercise, the assessment hinges on the root mean squared error (RMSE) of the forecasts. Overall, the monetary aggregate that exhibits the lowest estimates for the RMSE indicator would be considered the best leading indicator. If necessary, Diebold and Mariano's (1995) loss-differential test could be applied. This test evaluates whether the estimated ratios are statistically different, based on predictive ability.

In Figures 11 and 12, respectively, we report the path of the  $\bar{R}^2$  over the period 1981Q1 to 2005Q1 for nominal and real monetary aggregates. Both figures show a convergence of the respective  $\bar{R}^2$  using M1, GM1, and M1+ over the period 1995 to 2005. While, for the nominal monetary aggregates, the  $\bar{R}^2$  settles in the range 0.20 to 0.23 in 2005, for the real monetary aggregates the respective  $\bar{R}^2$  for M1, GM1, and M1+ settles at 0.25 in 2005. In Table 2 we report the out-of-sample RMSE measures for all monetary aggregates under consideration. The table shows clearly that, over the period 1995Q1 to 2005Q1, M1+ exhibits the lowest RMSE, followed by GM1, M1++, and M1, whether the monetary aggregates enter the equation in nominal or real terms.

The results using Stock and Watson's (2003) methodology, both in- and out-of-sample, suggest that the leading-indicator properties of the various narrow monetary aggregates have become more comparable over the more recent period. This again is consistent with the results of both the correlation and the VAR analysis.

## **6. Regime-Switching Models**

From the previous analysis, we found evidence of changes in the relationship between output growth and various narrow aggregates. These exercises, however, do not indicate when these changes might have occurred, nor do they inform us which narrow aggregates have become more informative.

To examine whether there has been a regime shift for each of the narrow monetary aggregates in explaining the future growth of output, we specify the following two-state regime-switching model:

$$\Delta y_t = c_s + \alpha_s \Delta y_{t-1} + \beta_s \Delta M_{t-1} + \varepsilon_t^s \quad (3)$$

$$\varepsilon_t^s \sim N(0, \sigma^s),$$

where:

$t$  denotes time;

$s$  denotes state 1 and 2  $\{1,2\}$ ;

$\Delta$  is the first-difference operator;

$y_t$  is the real output;

$M_t$  is the real monetary aggregate  $\{M1, GM1, M1+, M1++\}$ ;

$\varepsilon_t^s$  is the error term in regime  $s$ ; and

$\sigma^s$  is the variance in regime  $s$ .

Given the specification of equation (3), the intercept—the coefficient on the autoregressive component of output growth, on lagged money growth, and on the variance of the error term—will vary, based on the unobserved state  $s \in \{1, 2\}$ . The states are governed by a discrete Markov chain with the following transition probability matrix:

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix},$$

where  $p_{ij}$  is the probability of being in regime  $j$  at  $t$  conditional on being in regime  $i$  at  $t-1$ . Thus, the model estimates the probability of being in regime 1 ( $p_{1t}$ ) or regime 2 ( $p_{2t}$ ), with  $p_{1t} + p_{2t} = 1$ , in each quarter.

Equation (3) is estimated for each of the four narrow monetary aggregates considered; we then compare  $\beta$  in states 1 and 2. If there is evidence of a regime shift for any individual aggregate,

we would find  $\beta_1 \neq \beta_2$ .

To investigate whether M1+ and M1++ have become more informative indicators for real output than M1 and GM1 over the more recent period, we specify the following equation:

$$\Delta y_t = c_s + \alpha_s \Delta y_{t-1} + \beta_s \Delta M_{t-1}^s + \varepsilon_t^s, \quad (4)$$

where:

$t$  denotes time;

$s$  denotes state 1 and 2 {1,2};

$\Delta$  is the first-difference operator;

$y_t$  is the real output;

$M_t^1$ : in the first regime, the real monetary aggregate is {M1 or GM1};

$M_t^2$ : in the second regime, the real monetary aggregate is {M1+ or M1++};

$\varepsilon_t^s$  is the residual in regime  $s$ ; and

$\sigma^s$  is the variance in regime  $s$ .

If M1+ and M1++ have become more informative than GM1 and M1 over the more recent period, we would find evidence of a higher probability of being in state 2 than in state 1 in this period, with  $\beta_2 > 0$ .

## 6.1 Estimation and results<sup>8</sup>

Both equations (3) and (4) are estimated using quarterly data from 1976Q1 to 2004Q3. The results for equation (3) are reported in Tables 3 to 6. All of the coefficients on money variables are significant at the 5 per cent level and different from one regime to the other, which suggests that the explanatory power of each of the monetary aggregates is different in the two states. As well, the estimated  $p_{11}$  and  $p_{22}$  are close to one, indicating that the states are absorbing and that each regime is highly persistent. This suggests that whenever the model is in one of the regimes, it is

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8. All estimations and tests performed for regime-switching models have been realized using programs written in Gauss by Frédéric Demers.

unlikely to switch to the other.

In Figures 13 to 16, for each observation the probability of being in either of the regimes is close to zero or one. Besides, the changes from regime 1 to 2 seem to have occurred in 1992. This transition period corresponds to the time when the reserve requirements were being phased out. Finally, while the coefficients of M1, GM1, and M1++ diminished from regime 1 to 2, the coefficient of M1+ for output growth increased significantly from regime 1 to 2.

We also perform specification tests to check for serial correlation in the residuals. The results, reported in Tables 3 to 6 as AR(1) (first-order autoregressive) and ARCH(1) (first-order autoregressive conditional heteroscedasticity) tests, indicate that no model shows any evidence of an AR or ARCH problem in the residuals.<sup>9</sup> Finally, to support the statistical departure from linearity for each of the aggregates, we test the non-linear model (two regimes) against the linear alternative (one regime) using Davies' (1987) statistical test. The results strongly reject the null hypothesis that there exists only one regime, since the *p*-values are smaller than 0.00 in all cases. According to Davies' test, there is evidence for each of the narrow aggregates that the explanatory power of money for output growth is subject to regime shifts.

The results for equation (4) are reported in Tables 7 to 10.<sup>10</sup> For consistency, we estimate the model using the two-quarter moving average of money growth.<sup>11</sup> First, all coefficients on lagged output growth and money growth are significant at the 5 per cent level and have positive signs, as expected. Second, as Figures 17 to 20 show, the regime shifts seem to have occurred in 1992. While M1 and GM1 are better indicators for output growth from 1975 to 1992, M1+ is more informative after that point. This supports the hypothesis that there is a regime shift in the explanatory power of narrow money on output. As well, each point in time is distinctly in either

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9. We test up to a fourth-order autocorrelation and ARCH. In state 2, none of the models exhibits any autocorrelation or ARCH in the residuals. In state 1, however, there are some signs of autocorrelation and ARCH processes in the residuals at the third order. Nevertheless, these results do not have an impact on the estimates, but simply on the covariances of the residuals in the first state. We argue that GDP growth exhibits empirically an AR(1) process and GDP growth cannot be defined by an ARCH process.

10. We estimate the same model using the two-quarter moving averages of narrow money. The estimates are reported in Tables 11 to 14. They are qualitatively similar to the basic models.

11. Tables 11 to 14 and Figures 21 to 24 illustrate the results using one-quarter lagged money growth instead of the two-quarter moving average of money growth. The results of the two approaches are broadly similar.

regime 1 or 2 with a state probability close to zero or one. Finally, the estimated  $p_{11}$  and  $p_{22}$  are close to one, indicating that each state is absorbing (persistent).

The likelihood function for regime-switching models is known to have a number of local maximums. To reduce the probability that our results correspond to a local maximum, we calculate over 250,000 different points on the surface of the likelihood function, for some of the models of Tables 7 to 10, to examine whether there exists a higher likelihood value. The results fail to find a higher point of the likelihood function in the range of the values we test. While our results do not prove that we are at global maximums, they are at least reassuring.

## **7. Conclusions**

Financial innovations in general, and the removal of reserve requirements in particular, have made the distinction between demand and notice deposits somewhat artificial. As a result, financial institutions are experiencing increasing difficulties in sorting new accounts among these two categories. Also, there are growing concerns that a distinction that may no longer be relevant may have eroded the leading-indicator properties of M1 and GM1 for future GDP growth. Consequently, M1 and GM1 may no longer add any useful information over M1+ and M1++.

Our findings suggest that the leading-indicator properties of M1, GM1, M1+, and M1++ for output growth have shifted over time. Empirical results suggest that M1 and GM1 traditionally have been better indicators for future output growth. Over the more recent period, however, M1+ has become more informative than M1 and GM1. Moreover, we find evidence in favour of the existence of a regime shift in the indicator properties of narrow money for output growth. This regime change occurred in 1992 and is likely to be persistent.

The primary goal in constructing the narrow monetary aggregates was to capture the notion of transactions money. Given institutional changes and financial innovations, the concept of transactions money is no longer likely to be adequately captured by GM1 or M1. We argue that, currently, a broader measure, M1+, better defines transactions money. Indeed, there is less need for agents to consider carefully their holding of spendable cash, since many non-term assets are easily converted into cash. This likely renders the distinction between demand and notice deposits less relevant for money demand.

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**Table 1: Data Description**

Variable	Description	Statistics Canada
$(Y_t)$	Gross domestic product; chained 1997 dollars	v1992067
$(Y_t^p)$	Gross domestic product; implicit price indexes 1997=100	v1997756
$(P_t)$	Consumer price index, 2001 basket content - all items	v18702611
$(P_t^c)$	Consumer price index excluding the eight most volatile components and the effect of changes in indirect taxes (1992=100)	v36398 <sup>a</sup>
$(GM1_t)$	Gross M1 ( x 1,000,000)	v37141
$(M1_t)$	Total M1 ( x 1,000,000)	v37124
$(M1P_t)$	M1+ ( x 1,000,000)	v37151
$(M1PP_t)$	M1++ ( x 1,000,000)	v37152
$(I_t^{cp})$	Prime corporate paper rate: 3-month (per cent)	v122491

a. seasonal factors applied to v36398

**Table 2: RMSE 1995Q1–2005Q1, Real Monetary Aggregates**

Aggregate	GM1	M1	M1+	M1++
GM1	0.78	0.89	<b>0.73</b>	0.82

**Table 3: Parameter Estimates for the Markov-Switching Model for GM1 over 1976Q1–2004Q3**

		Constant	GDP <sub><i>t</i>-1</sub>	GM1 <sub><i>t</i>-1</sub>	σ	Prob
Regime 1 (GM1)	Estimates	2.1574	0.3387	0.1243	9.4580	0.9896
	<i>t</i> -stat	2.4724	2.2884	2.3860	5.1823	72.2685
Regime 2 (GM1)	Estimates	0.5072	0.5503	0.1075	2.6769	<b>0.9898</b>
	<i>t</i> -stat	1.3794	5.5429	3.1201	2.9302	<b>74.4770</b>
Residual analysis				Test for no regime change <i>P</i> -value	Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value		-265.5	
Regime 1 (GM1)		0.2501	0.2582			
Regime 2 (GM1)		0.2934	0.3017	0.00		

**Table 4: Parameter Estimates for the Markov-Switching Model for M1 over 1976Q1–2004Q3**

		Constant	GDP <sub><i>t</i>-1</sub>	M1 <sub><i>t</i>-1</sub>	σ	Prob
Regime 1 (M1)	Estimates	2.2204	0.3055	0.1470	9.2031	0.9890
	<i>t</i> -stat	3.0885	2.2154	2.8609	5.0253	66.7819
Regime 2 (M1)	Estimates	0.6309	0.6076	0.0689	2.9673	<b>0.9899</b>
	<i>t</i> -stat	1.7108	6.4767	2.5018	4.5010	<b>76.4666</b>
Residual analysis				Test for no regime change <i>P</i> -value	Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value		-266.1	
Regime 1 (M1)		0.1647	0.1719			
Regime 2 (M1)		0.1400	0.1467	0.00		

**Table 5: Parameter Estimates for the Markov-Switching Model for M1+ over 1976Q1–2004Q3**

		Constant	GDP <sub>t-1</sub>	M1+ <sub>t-1</sub>	σ	Prob
Regime 1 (M1+)	Estimates	1.4406	0.3992	0.1022	9.4616	0.9892
	t-stat	2.8533	3.5771	2.2514	5.6175	69.7480
Regime 2 (M1+)	Estimates	0.6628	0.5355	0.1540	2.0120	<b>0.9906</b>
	t-stat	1.5135	5.3052	3.6782	4.3196	<b>83.1980</b>
Residual analysis				Test for no regime change P-value	Likelihood value	
		AR(1) P-value	ARCH(1) P-value		-263.7	
Regime 1 (M1+)		0.5923	0.5992			
Regime 2 (M1+)		0.5807	0.5870	0.00		

**Table 6: Parameter Estimates for the Markov-Switching Model for M1++ over 1976Q1–2004Q3**

		Constant	GDP <sub>t-1</sub>	M1++ <sub>t-1</sub>	σ	Prob
Regime 1 (M1++)	Estimates	1.0890	0.4093	0.1624	9.4753	0.9869
	t-stat	2.1385	3.7882	2.3352	5.3200	49.5413
Regime 2 (M1++)	Estimates	1.1659	0.5562	0.0923	2.2318	<b>0.9857</b>
	t-stat	2.8889	5.3171	2.5301	4.9677	<b>46.1799</b>
Residual analysis				Test for no regime change P-value	Likelihood value	
		AR(1) P-value	ARCH(1) P-value		-266.4	
Regime 1 (M1++)		0.3757	0.3838			
Regime 2 (M1++)		0.7036	0.7083	0.00		

**Table 7: Parameter Estimates for the Markov-Switching Model for  
GM1 versus M1+ over 1975Q1–2004Q3  
(two-quarter moving average)**

		Constant	GDP <sub>t-1</sub>	GM1 <sub>t-1</sub>	M1+ <sub>t-1</sub>	σ	Prob
Regime 1 (GM1)	Estimates	2.8679	0.1847	0.2740		7.6285	0.9910
	t-stat	4.7158	1.5159	4.3347		5.1136	88.8775
Regime 2 (M1+)	Estimates	0.4874	0.4865		0.2007	2.7330	<b>0.9906</b>
	t-stat	1.3941	4.5098		3.1200	5.2171	<b>83.4577</b>
Residual analysis				Test for no regime change		Likelihood value	
		AR(1) P-value	ARCH(1) P-value	P-value		-259.4	
Regime 1 (GM1)		0.0018	0.0022	0.00			
Regime 2 (M1+)		0.7408	0.7451				

**Table 8: Parameter Estimates for the Markov-Switching Model for  
GM1 versus M1++ over 1975Q1–2004Q3  
(two-quarter moving average)**

		Constant	GDP <sub>t-1</sub>	GM1 <sub>t-1</sub>	M1++ <sub>t-1</sub>	σ	Prob
Regime 1 (GM1)	Estimates	2.3586	0.2494	0.2760		7.9060	0.9871
	t-stat	4.8017	2.3021	4.3585		5.4236	54.2268
Regime 2 (M1++)	Estimates	1.2837	0.5334		0.0771	2.3920	<b>0.9884</b>
	t-stat	3.2027	4.8228		1.8207	4.9348	<b>60.0169</b>
Residual analysis				Test for no regime change		Likelihood value	
		AR(1) P-value	ARCH(1) P-value	P-value		-262.3	
Regime 1 (GM1)		0.1627	0.1699	0.00			
Regime 2 (M1++)		0.5162	0.5231				

**Table 9: Parameter Estimates for the Markov-Switching Model for  
M1 versus M1+ over 1975Q1–2004Q3  
(two-quarter moving average)**

		Constant	GDP <sub>t-1</sub>	M1 <sub>t-1</sub>	M1+ <sub>t-1</sub>	σ	Prob
Regime 1 (M1)	Estimates	2.3140	0.2176	0.2876		7.3823	0.9900
	t-stat	3.7183	1.7675	4.9673		5.5208	76.8806
Regime 2 (M1+)	Estimates	0.7741	0.4795		0.1657	2.2900	<b>0.9910</b>
	t-stat	1.2399	4.0976		2.3378	3.0530	<b>88.2171</b>
Residual analysis				Test for no regime change		Likelihood value	
		AR(1) P-value	ARCH(1) P-value	P-value		-257.8	
Regime 1 (M1)		0.0036	0.0043	0.00			
Regime 2 (M1+)		0.3266	0.3353				

**Table 10: Parameter Estimates for the Markov-Switching Model for  
M1 versus M1++ over 1975Q1–2004Q3  
(two-quarter moving average)**

		Constant	GDP <sub>t-1</sub>	M1 <sub>t-1</sub>	M1++ <sub>t-1</sub>	σ	Prob
Regime 1 (M1)	Estimates	2.2330	0.2286	0.2878		7.4875	0.9881
	t-stat	4.7913	2.1664	4.9070		5.5259	61.5130
Regime 2 (M1++)	Estimates	1.2583	0.5395		0.0774	2.3748	<b>0.9893</b>
	t-stat	3.1913	4.9425		1.8340	4.9855	<b>74.6806</b>
Residual analysis				Test for no regime change		Likelihood value	
		AR(1) P-value	ARCH(1) P-value	P-value		-260.3	
Regime 1 (M1)		0.1457	0.1527	0.00			
Regime 2 (M1++)		0.6529	0.6579				

**Table 11: Parameter Estimates for the Markov-Switching Model for GM1 versus M1+ over 1976Q1–2004Q3**

		Constant	GDP <sub>t-1</sub>	M1 <sub>t-1</sub>	M1+ <sub>t-1</sub>	σ	Prob
Regime 1 (GM1)	Estimates	2.2513	0.3283	0.1229		9.3601	0.9902
	t-stat	2.8601	2.3358	2.3868		5.2512	78.5631
Regime 2 (M1+)	Estimates	0.4193	0.5530		0.1731	2.5136	<b>0.9903</b>
	t-stat	1.1481	6.0576		3.8208	3.8155	<b>80.8545</b>
Residual analysis				Test for no regime change <i>P</i> -value		Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value	0.00		-263.2	
Regime 1 (GM1)		0.9835	0.9839				
Regime 2 (M1+)		0.6115	0.6181				

**Table 12: Parameter Estimates for the Markov-Switching Model for GM1 versus M1++ over 1976Q1–2004Q3**

		Constant	GDP <sub>t-1</sub>	GM1 <sub>t-1</sub>	M1++ <sub>t-1</sub>	σ	Prob
Regime 1 (GM1)	Estimates	1.8474	0.3730	0.1245		9.4247	0.9874
	t-stat	3.6358	3.3383	2.3890		5.4007	53.2845
Regime 2 (M1++)	Estimates	1.1303	0.5617		0.0939	2.2405	<b>0.9862</b>
	t-stat	2.7942	5.3415		2.5888	4.9049	<b>49.6764</b>
Residual analysis				Test for no regime change <i>P</i> -value		Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value	0.00		-266.3	
Regime 1 (GM1)		0.9762	0.9766				
Regime 2 (M1++)		0.4890	0.4965				

**Table 13: Parameter Estimates for the Markov-Switching Model for M1 versus M1+ over 1976Q1–2004Q3**

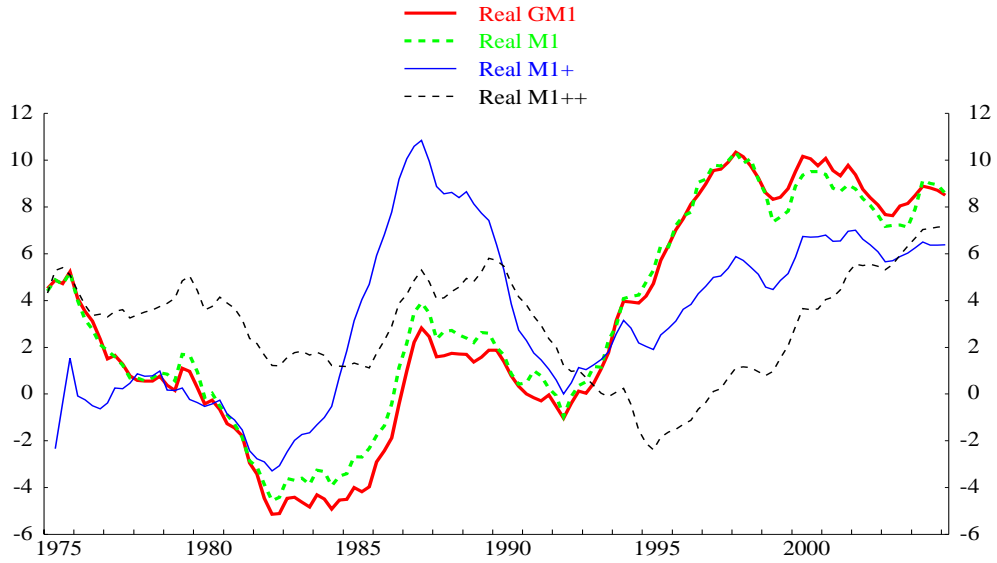
		Constant	GDP <sub><i>t</i>-1</sub>	M1 <sub><i>t</i>-1</sub>	M1+ <sub><i>t</i>-1</sub>	σ	Prob
Regime 1 (M1)	Estimates	2.2250	0.3115	0.1403		8.9740	0.9902
	<i>t</i> -stat	2.7767	2.1186	2.8705		5.2210	78.7251
Regime 2 (M1+)	Estimates	0.4051	0.5569		0.1730	2.5322	<b>0.9899</b>
	<i>t</i> -stat	1.1263	6.0438		3.8453	3.9968	<b>75.0899</b>
Residual analysis				Test for no regime change <i>P</i> -value		Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value			-262.0	
Regime 1 (M1)		0.5305	0.5367	0.00			
Regime 2 (M1+)		0.5574	0.5599				

**Table 14: Parameter Estimates for the Markov-Switching Model for M1 versus M1++ over 1976Q1–2004Q3**

		Constant	GDP <sub><i>t</i>-1</sub>	M1 <sub><i>t</i>-1</sub>	M1++ <sub><i>t</i>-1</sub>	σ	Prob
Regime 1 (M1)	Estimates	1.7708	0.3709	0.1342		9.0563	0.9891
	<i>t</i> -stat	3.6235	3.3985	2.8580		5.5421	67.5476
Regime 2 (M1++)	Estimates	1.1078	0.5667		0.09475	2.2533	<b>0.9879</b>
	<i>t</i> -stat	2.7236	5.3592		2.6323	4.8723	<b>60.6003</b>
Residual analysis				Test for no regime change <i>P</i> -value		Likelihood value	
		AR(1) <i>P</i> -value	ARCH(1) <i>P</i> -value			-265.2	
Regime 1 (M1)		0.9764	0.9768	0.00			
Regime 2 (M1++)		0.3680	0.3761				

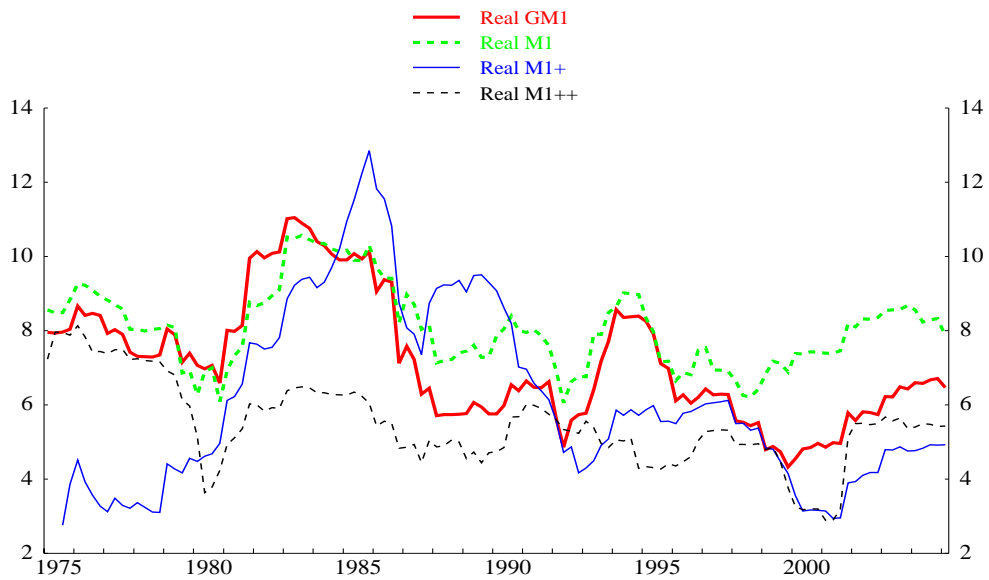
# Figure 1 Moving Average of Growth Rates (5 Years)

Quarter-over-quarter percentage change, at annual rates



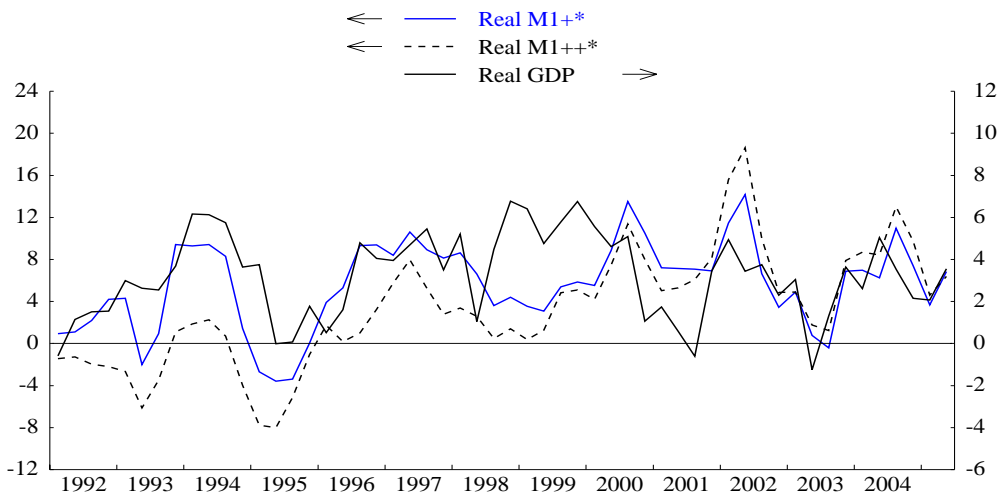
# Figure 2 Moving Standard Deviation of Growth Rates (5 Years)

Quarter-over-quarter percentage change, at annual rates



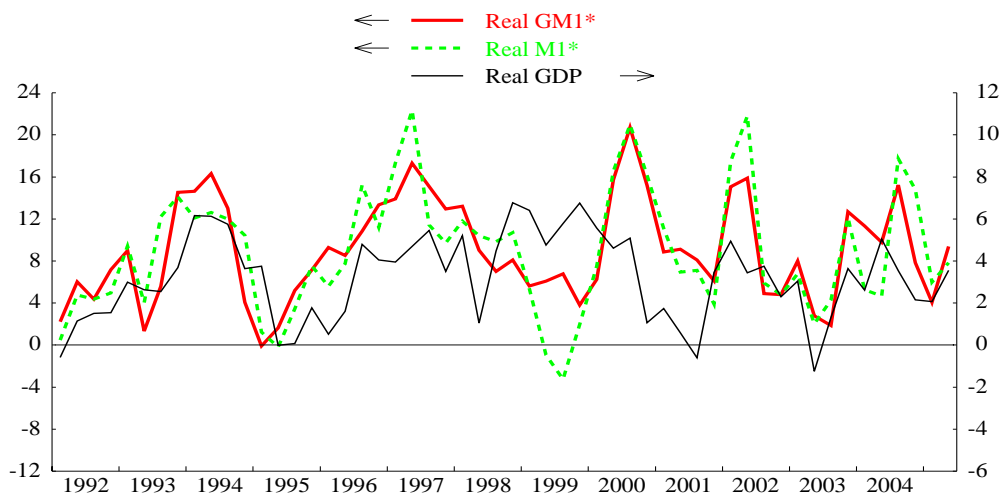


**Figure 3**  
**Growth of Real GDP, Real M1+, and Real M1++**  
 Quarter-over-quarter percentage change, at annual rates



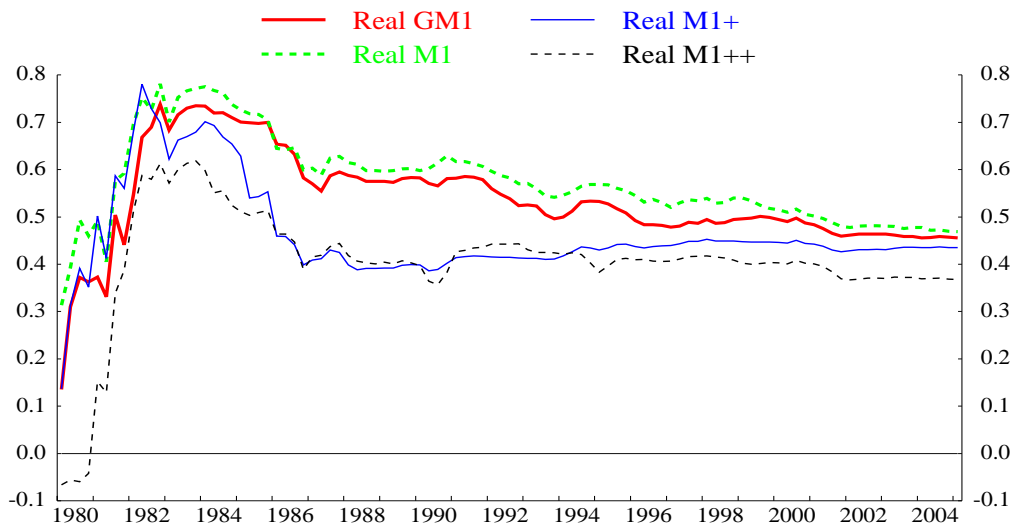
\* Two-quarter moving average of growth in M1+ and M1++ (deflated by core CPI), one quarter earlier. Core CPI is the consumer price index excluding the eight most volatile components and the effect of changes in indirect taxes on the remaining components.

**Figure 4**  
**Growth of Real GDP, Real GM1, and Real M1**  
 Quarter-over-quarter percentage change, at annual rates

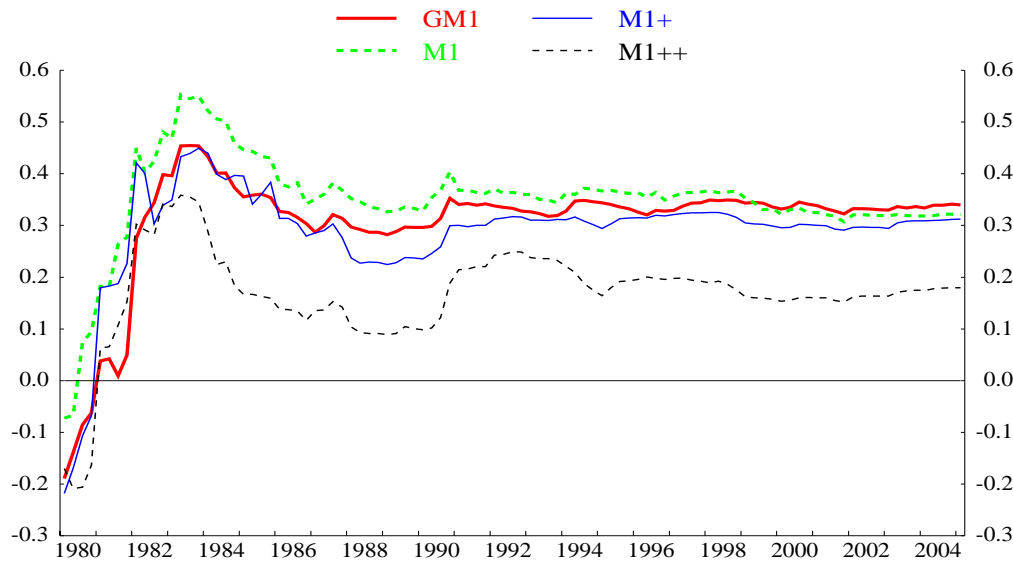


\* Two-quarter moving average of growth in GM1 and M1 (deflated by core CPI), one quarter earlier. Core CPI is the consumer price index excluding the eight most volatile components and the effect of changes in indirect taxes on the remaining components.

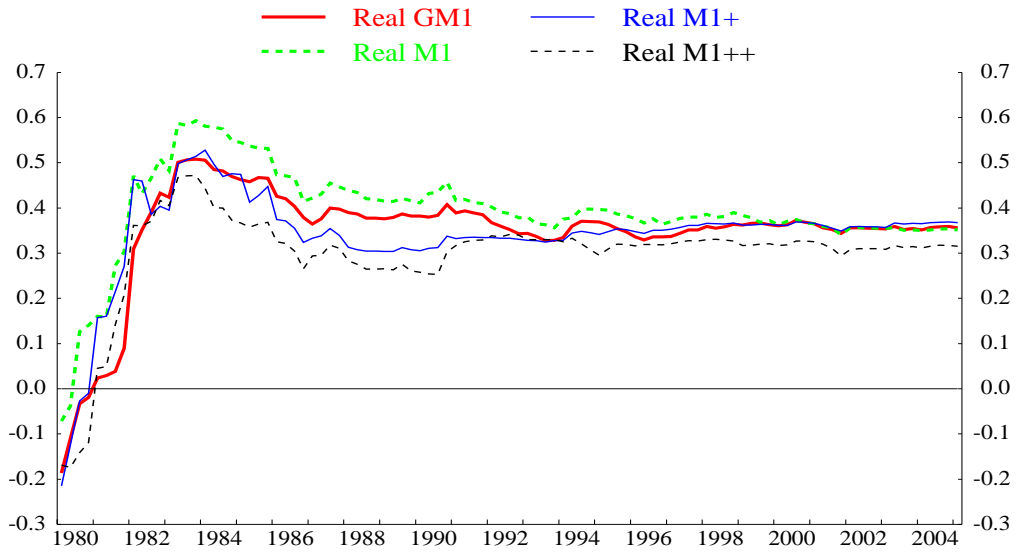
**Figure 5**  
**Correlation (End date rolling forward)**  
**Growth of Real GDP and Real Lagged Monetary Aggregates**  
 Quarter-over-quarter percentage change, at annual rates  
 (Two-quarter moving average)



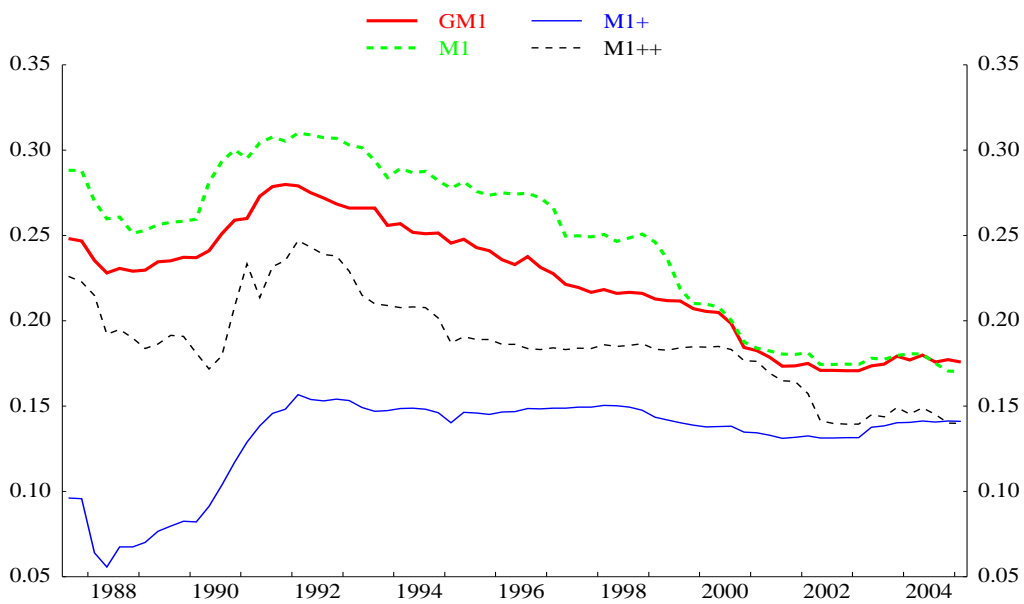
**Figure 6**  
**Correlation (End date rolling forward)**  
**Growth of Real GDP and Nominal Lagged Monetary Aggregates**  
 Quarter-over-quarter percentage change, at annual rates



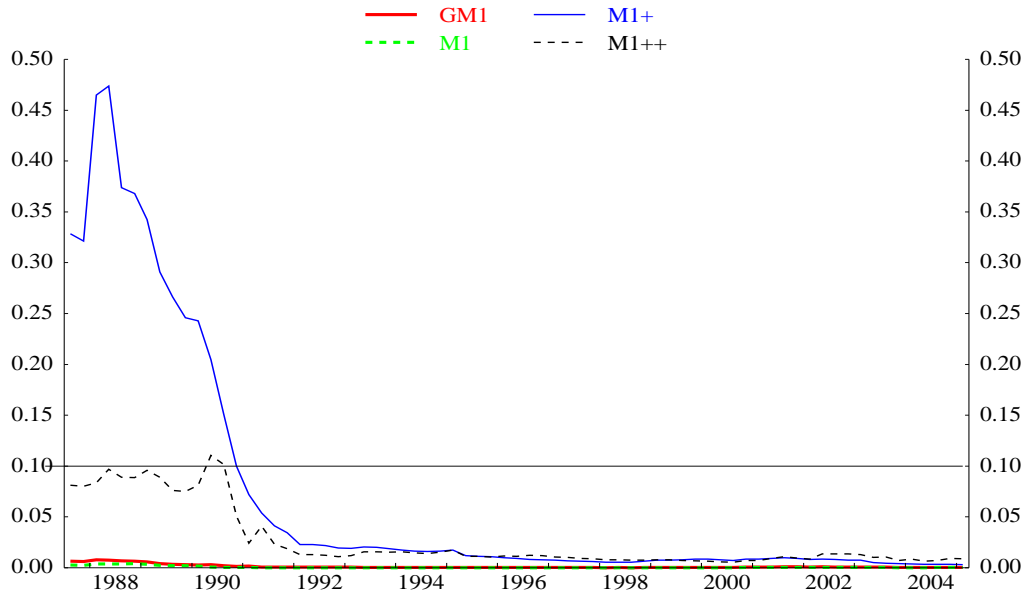
**Figure 7**  
**Correlation (End date rolling forward)**  
**Growth of Real GDP and Real Lagged Monetary Aggregates**  
 Quarter-over-quarter percentage change, at annual rates



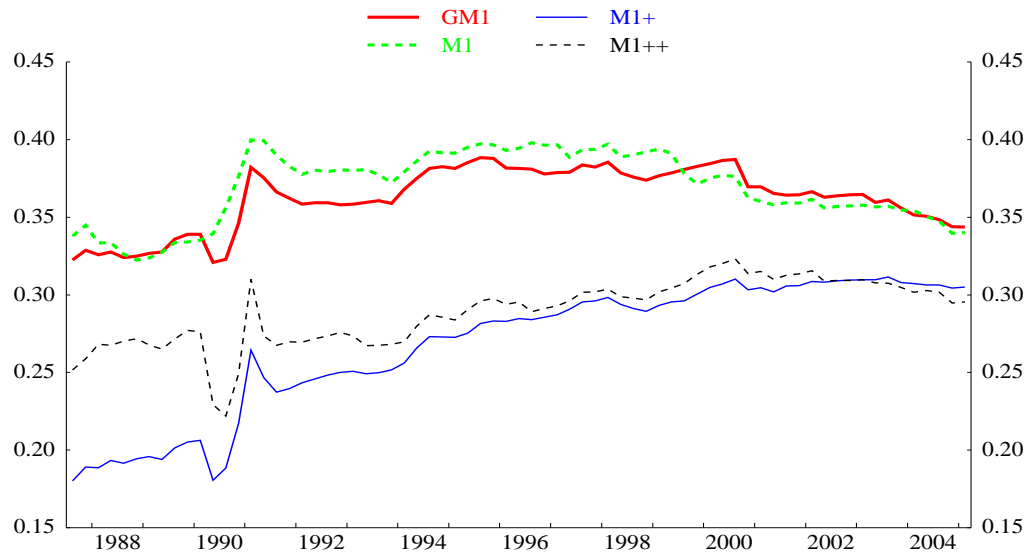
**Figure 8**  
**Sum of Coefficients - Monetary Aggregates (VAR)**



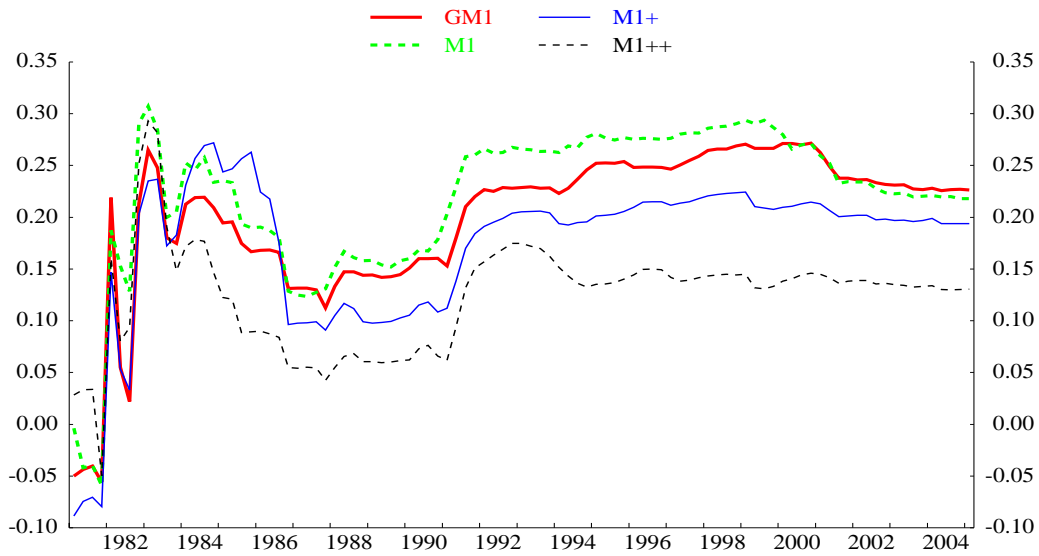
**Figure 9**  
**Significance - Monetary Aggregates (VAR)**



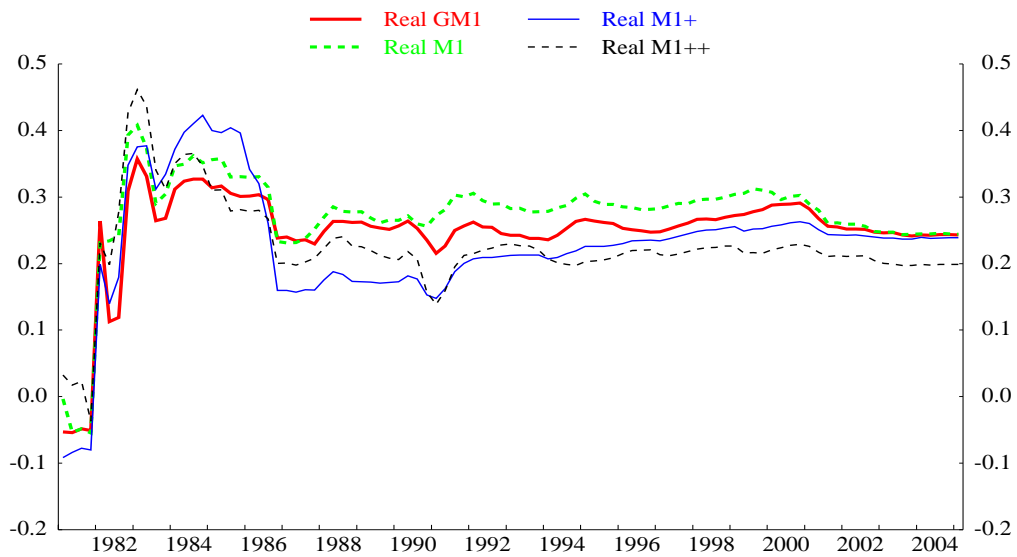
**Figure 10**  
**Adjusted-R-Squared**  
**Monetary Aggregates VAR**



**Figure 11**  
**Adjusted-R-Squared**  
**Nominal Monetary Aggregates**

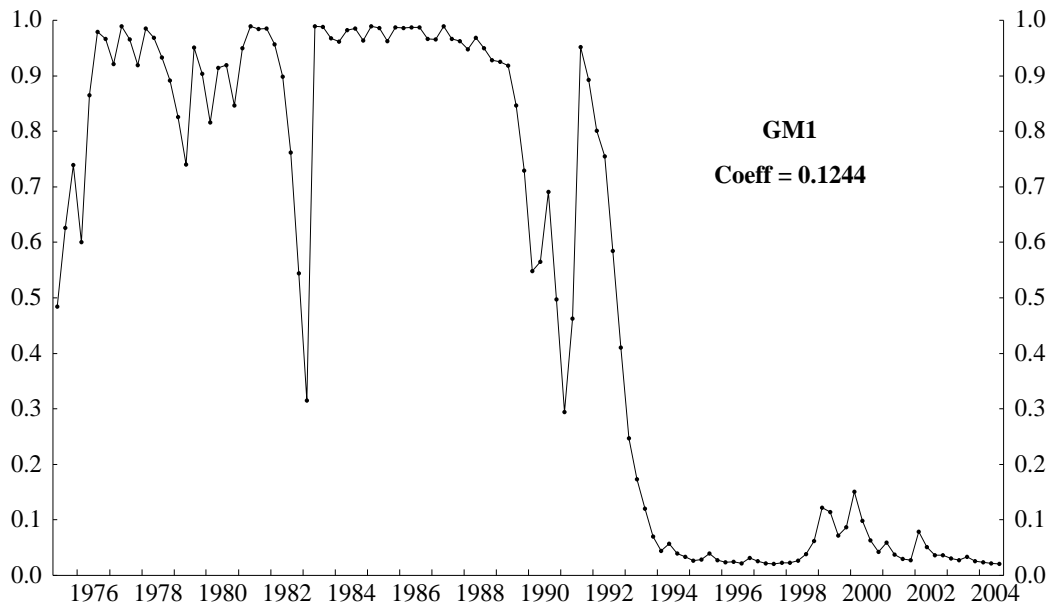


**Figure 12**  
**Adjusted-R-Squared**  
**Real Monetary Aggregates**

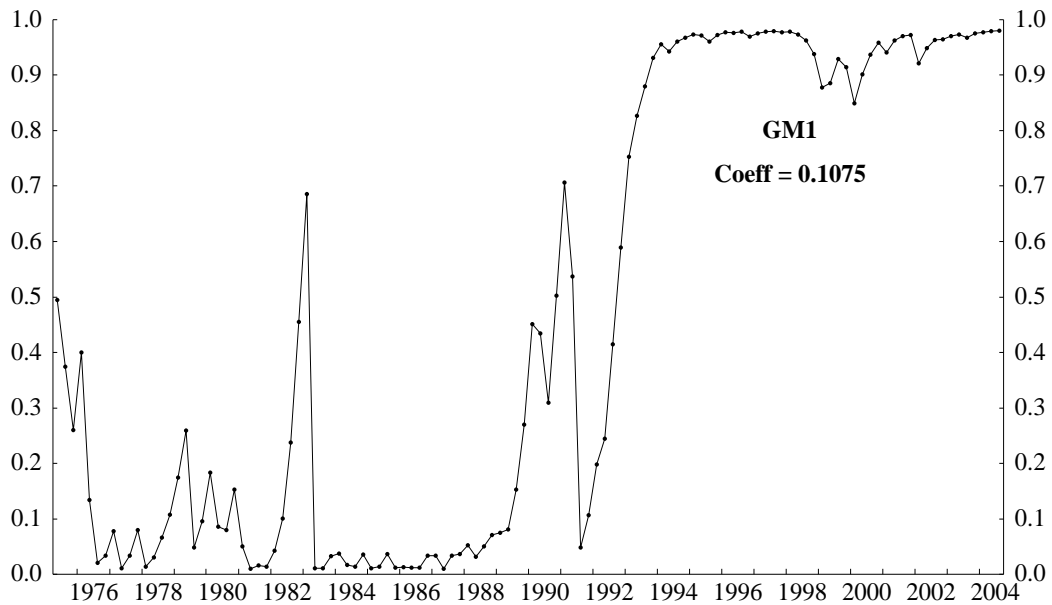


# Figure 13 Gross M1

## Regime 1

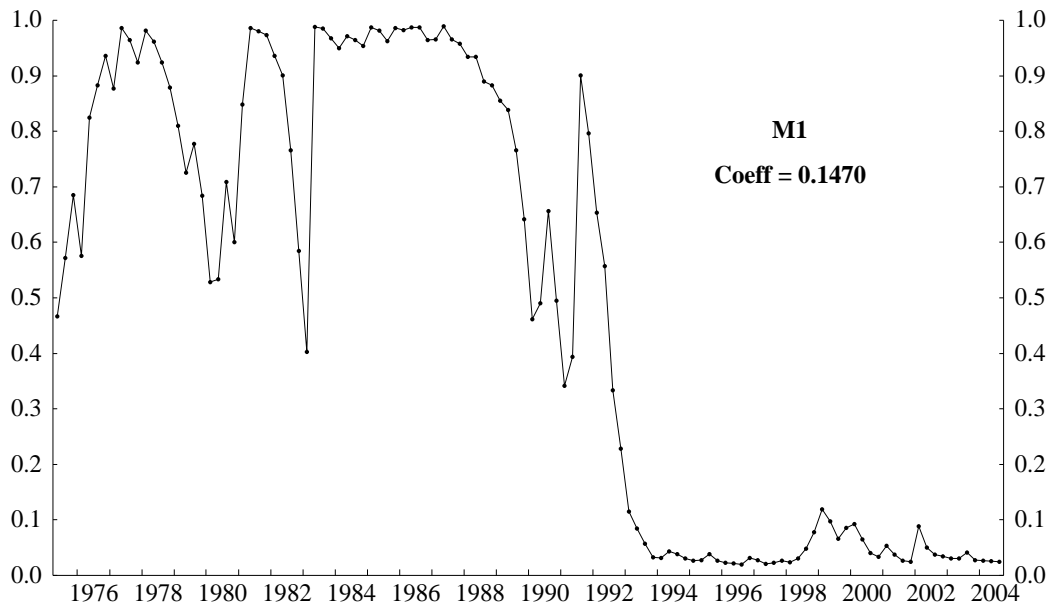


## Regime 2

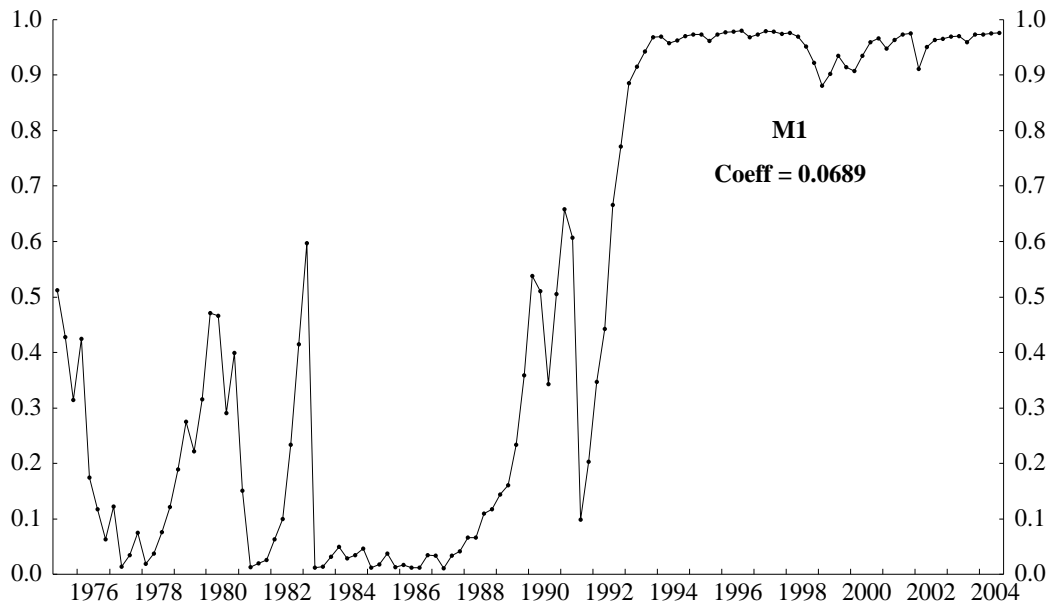


**Figure 14**  
**M1**

**Regime 1**



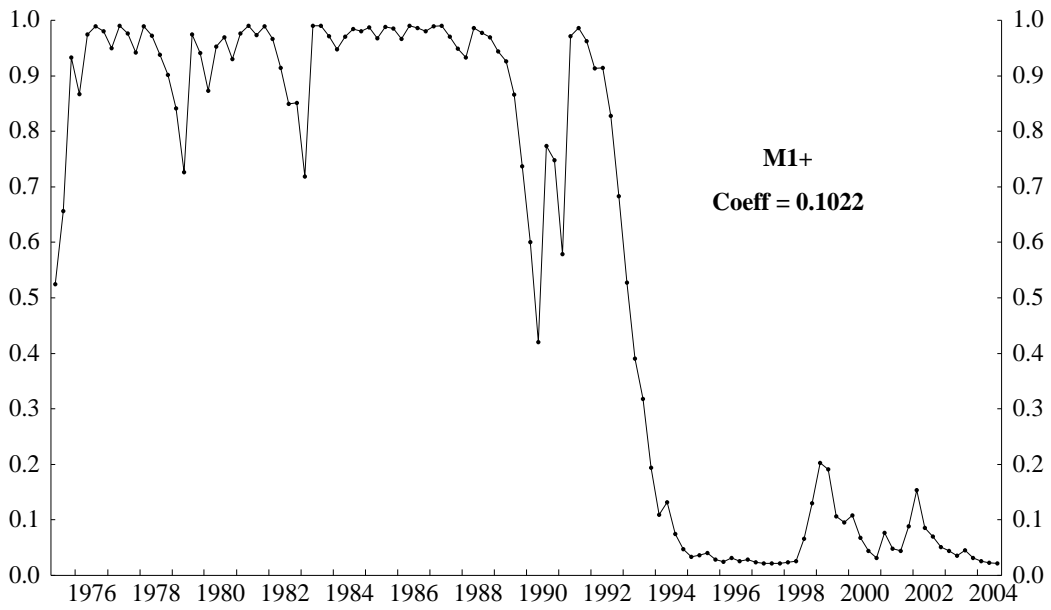
**Regime 2**



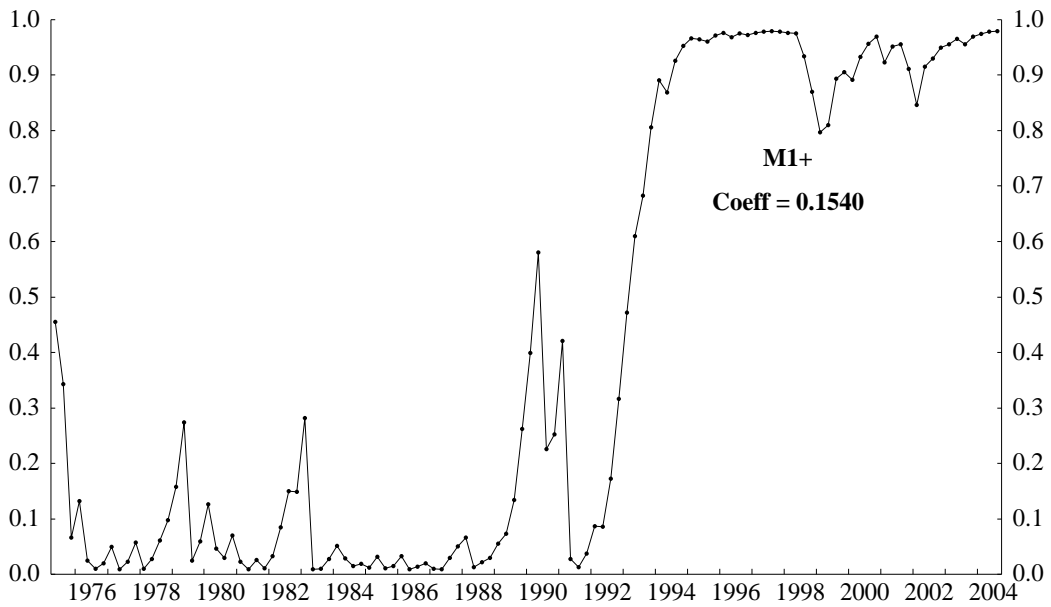
# Figure 15

## M1+

### Regime 1



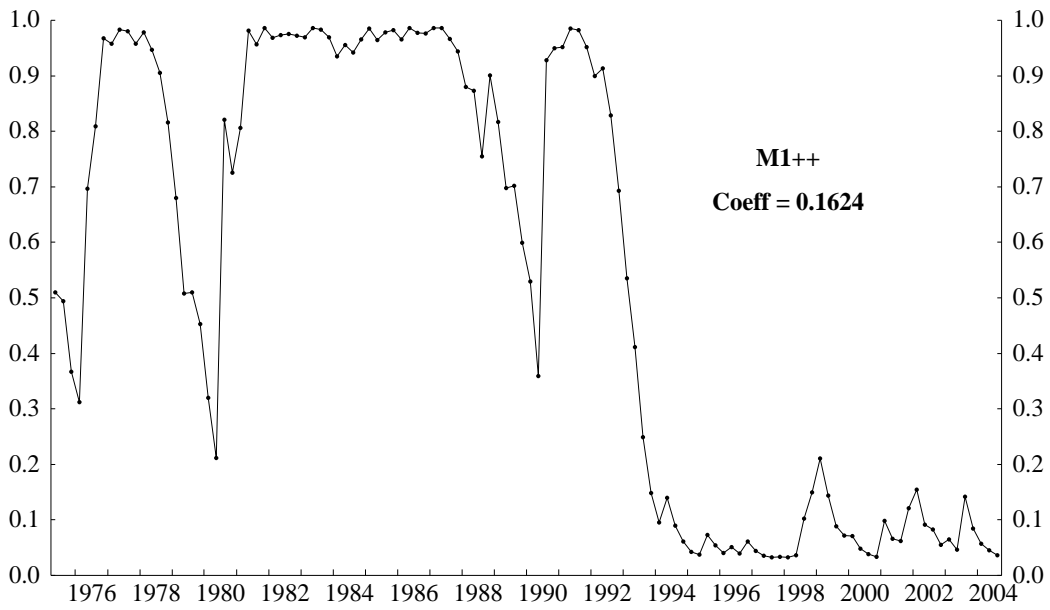
### Regime 2



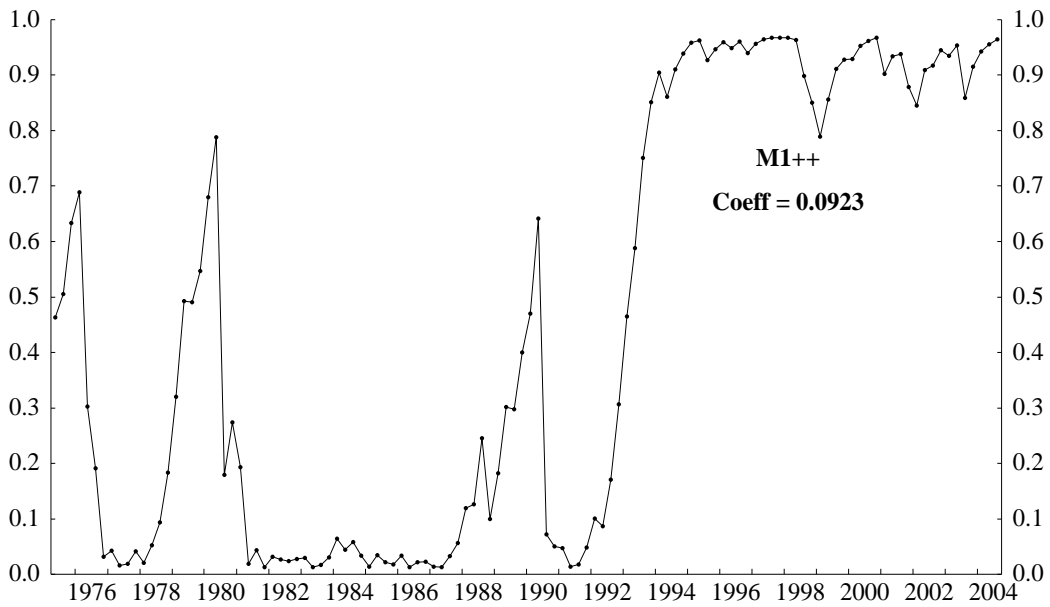


**Figure 16**  
**M1++**

**Regime 1**

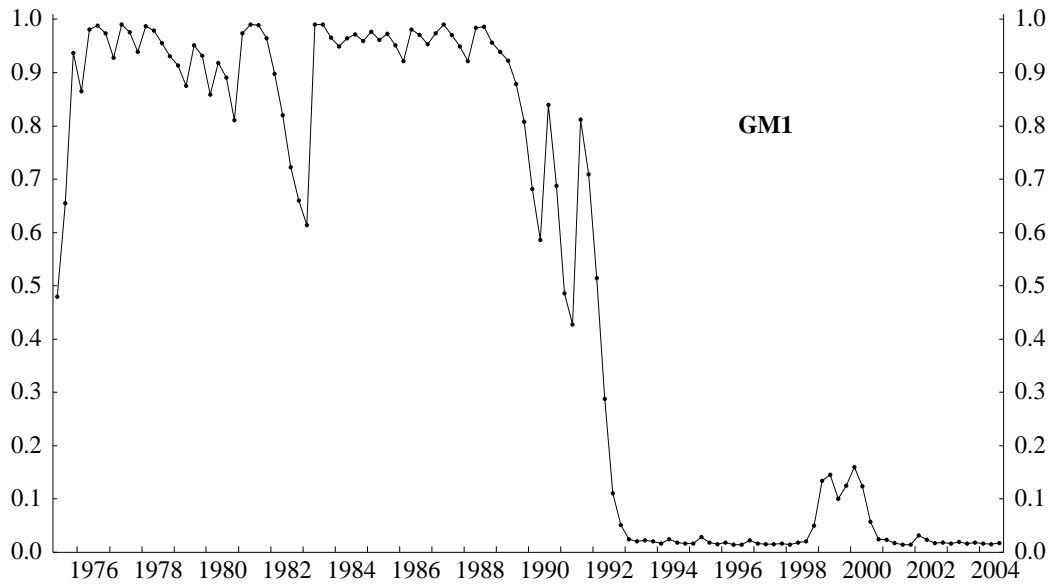


**Regime 2**

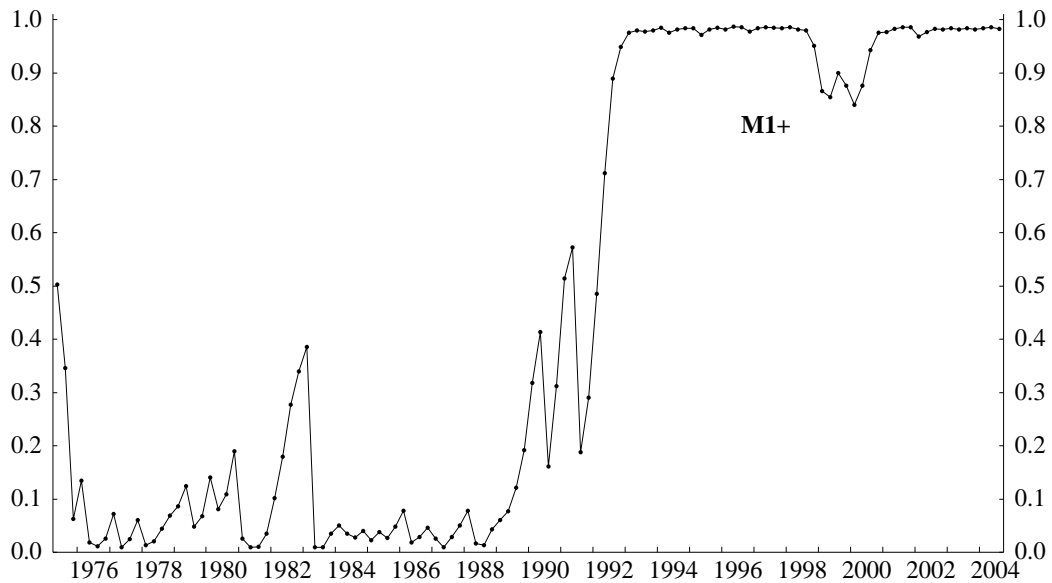


**Figure 17**  
**Gross M1 versus M1+**  
(two-quarter moving average)

**Regime 1**

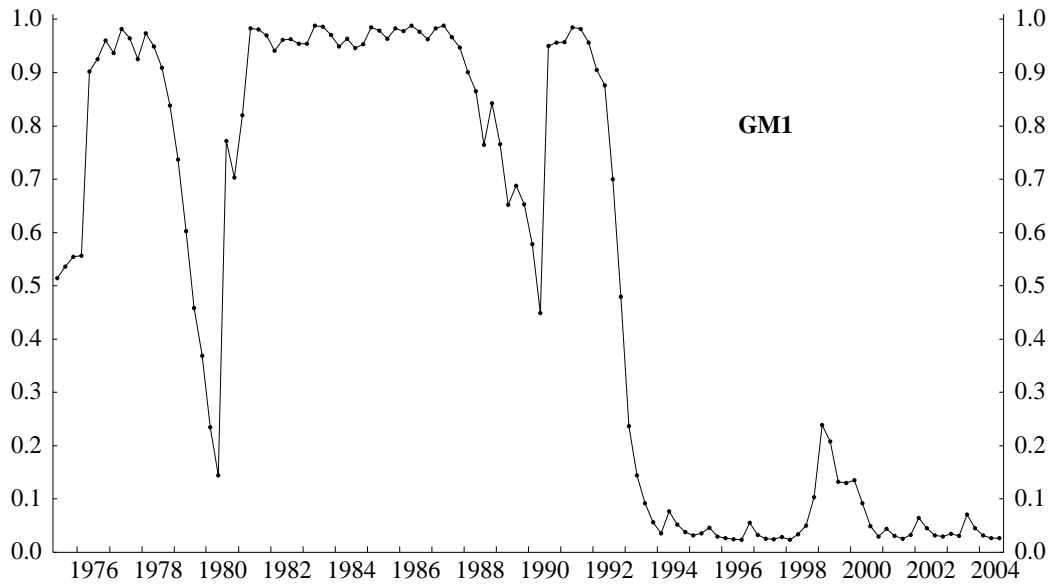


**Regime 2**

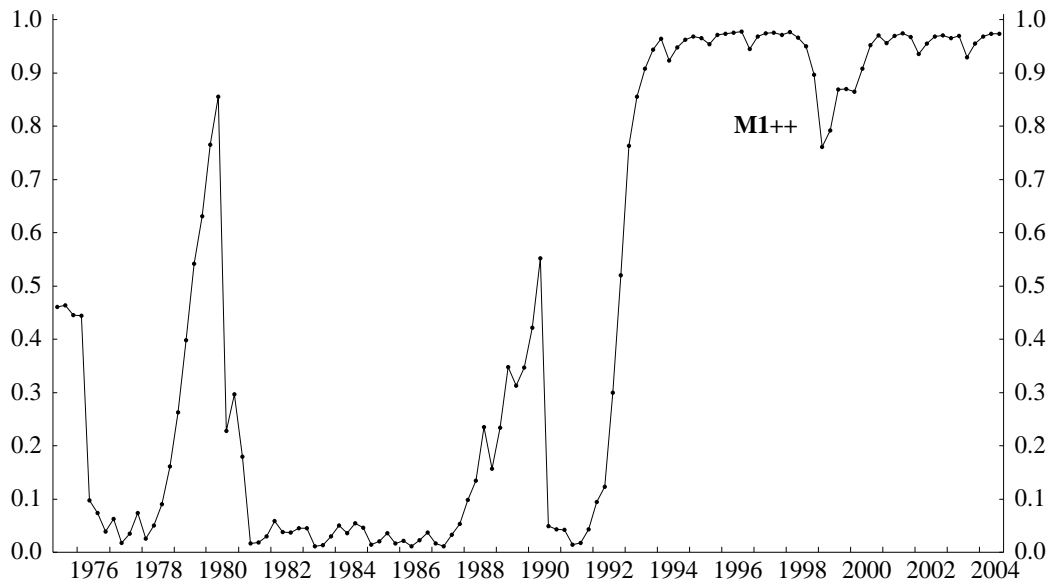


**Figure 18**  
**Gross M1 versus M1++**  
(two-quarter moving average)

**Regime 1**

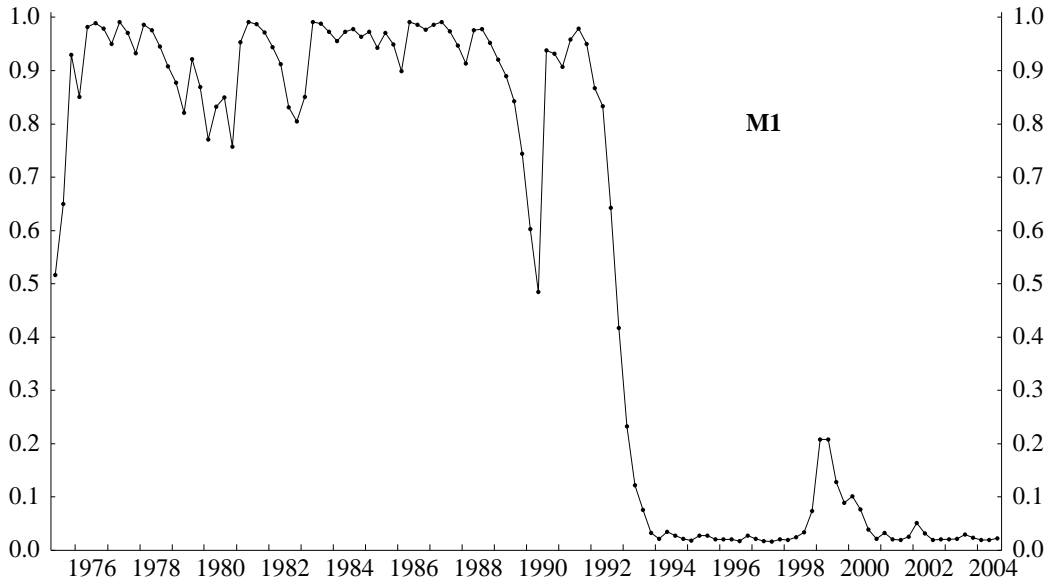


**Regime 2**

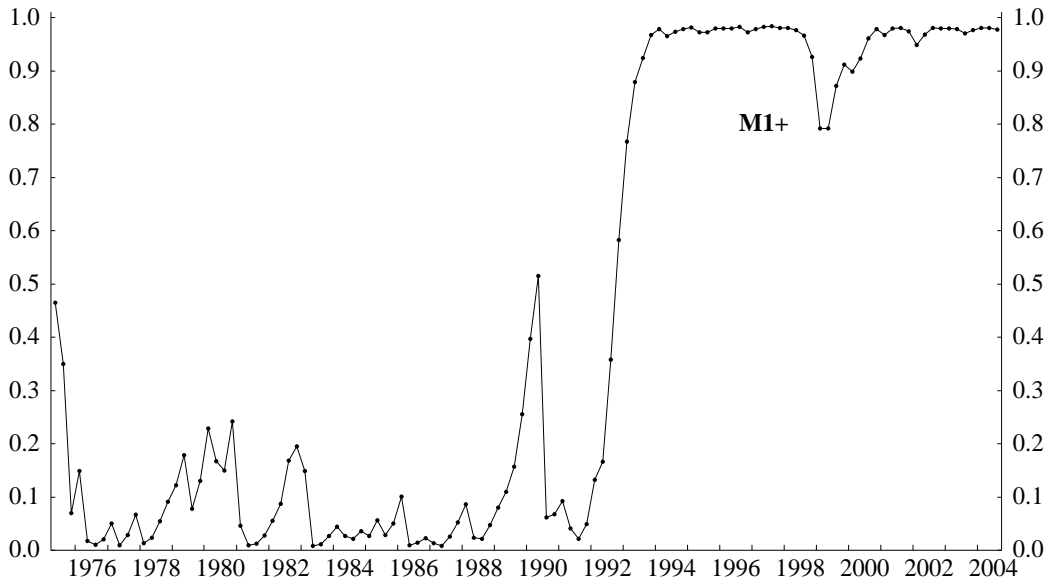


**Figure 19**  
**M1 versus M1+**  
(two-quarter moving average)

**Regime 1**

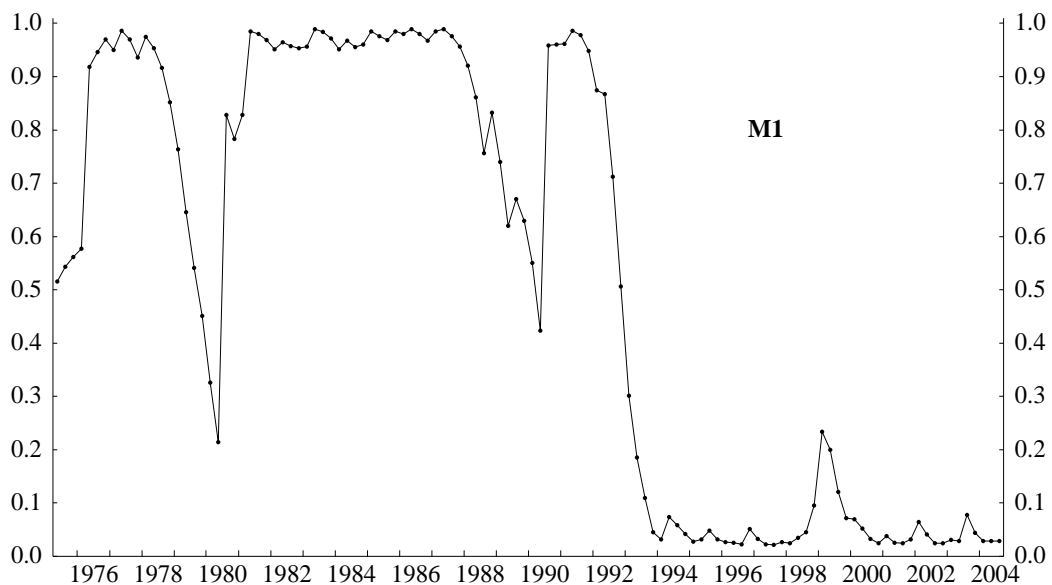


**Regime 2**

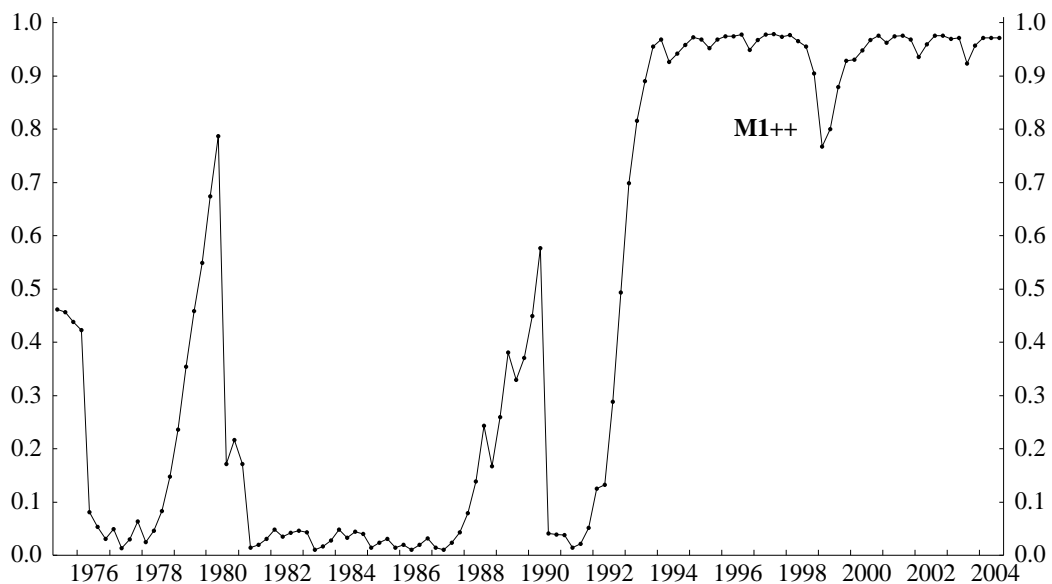


**Figure 20**  
**M1 versus M1++**  
(two-quarter moving average)

**Regime 1**

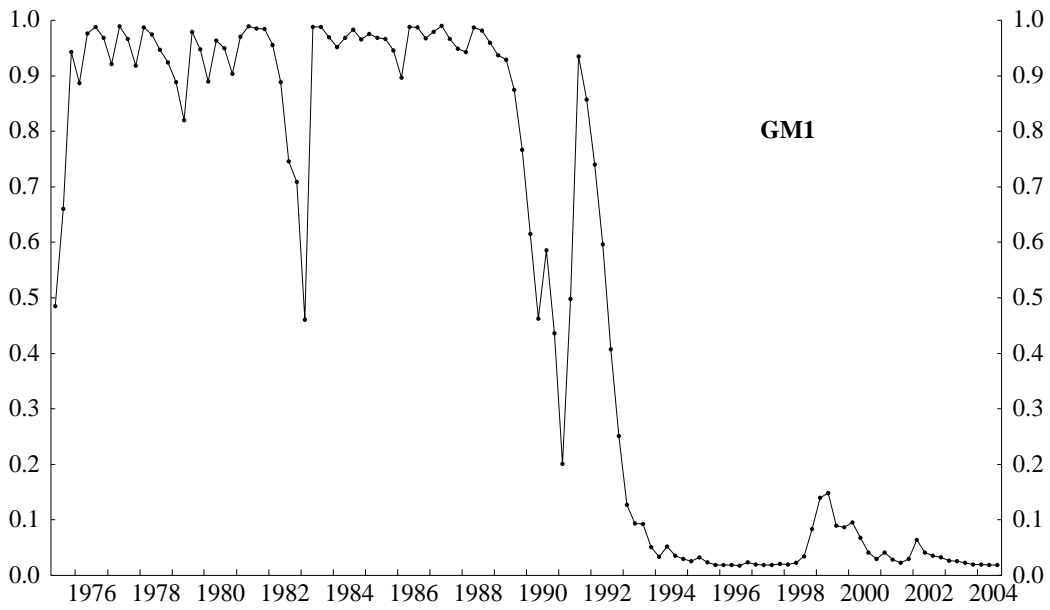


**Regime 2**

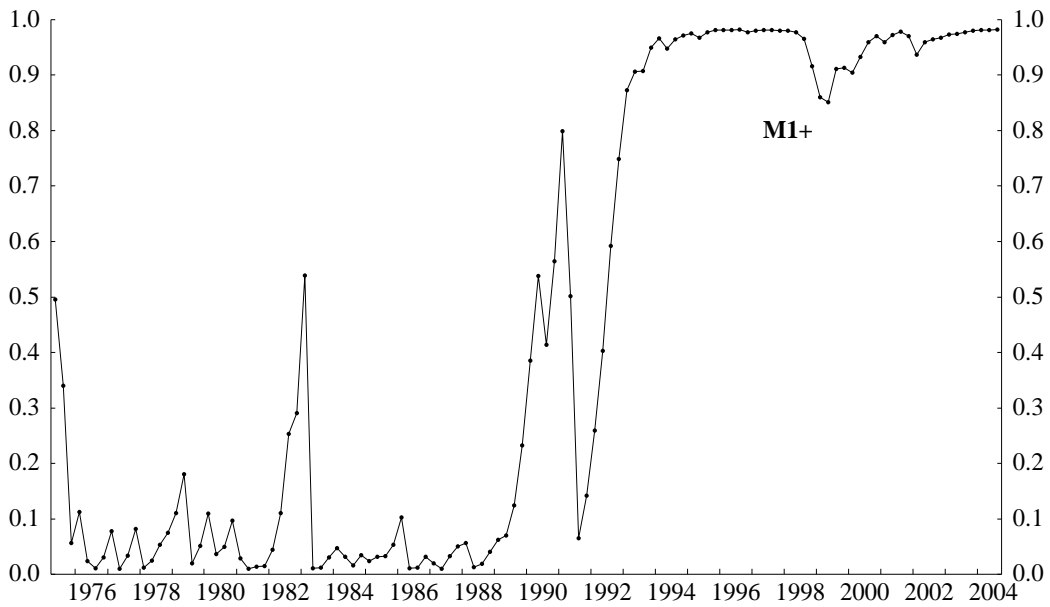


**Figure 21**  
**Gross M1 versus M1+**

**Regime 1**

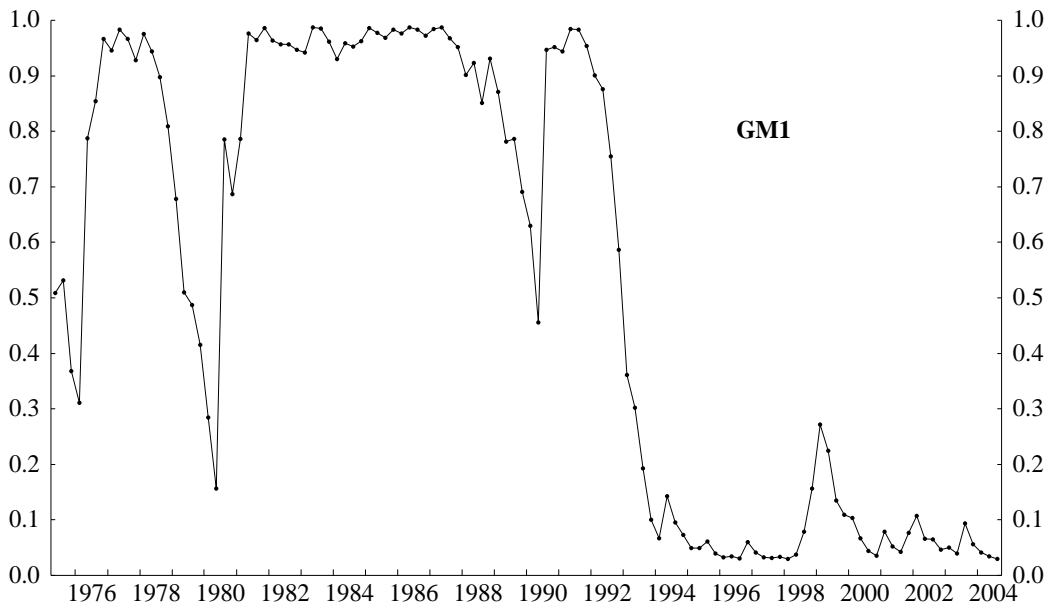


**Regime 2**

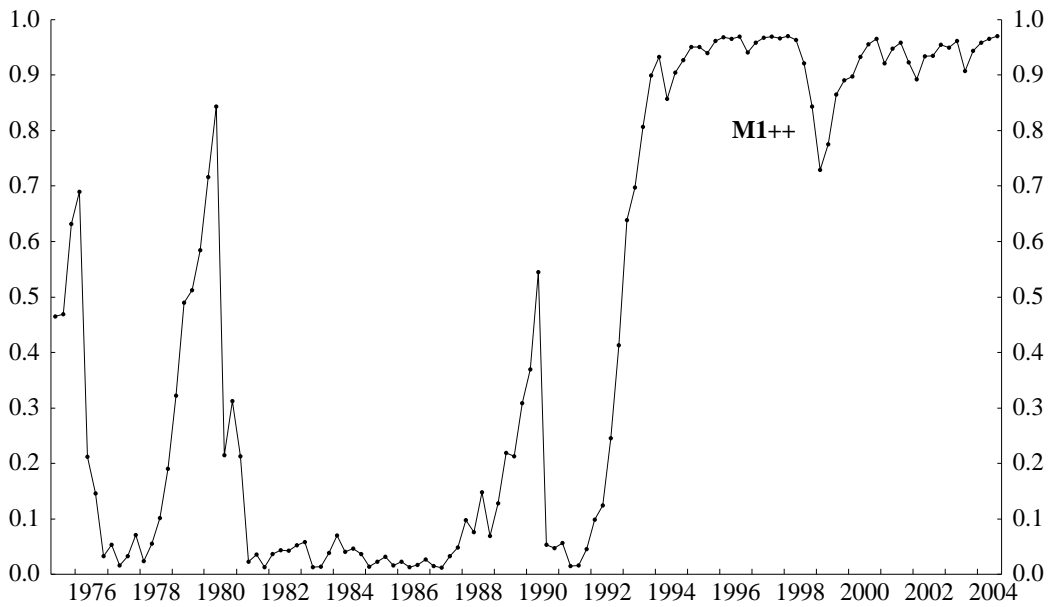


**Figure 22**  
**Gross M1 versus M1++**

**Regime 1**

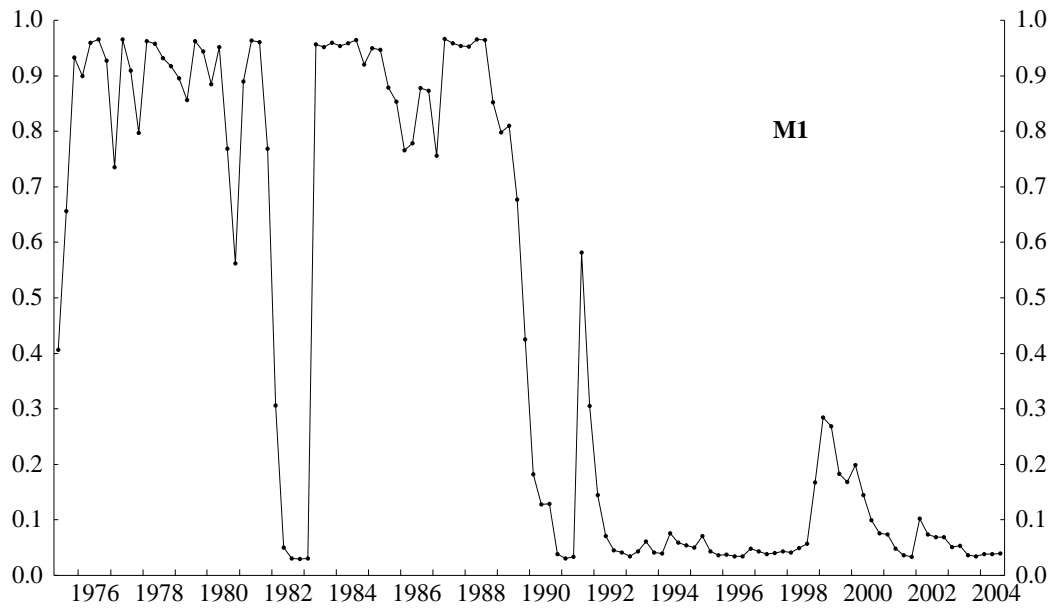


**Regime 2**

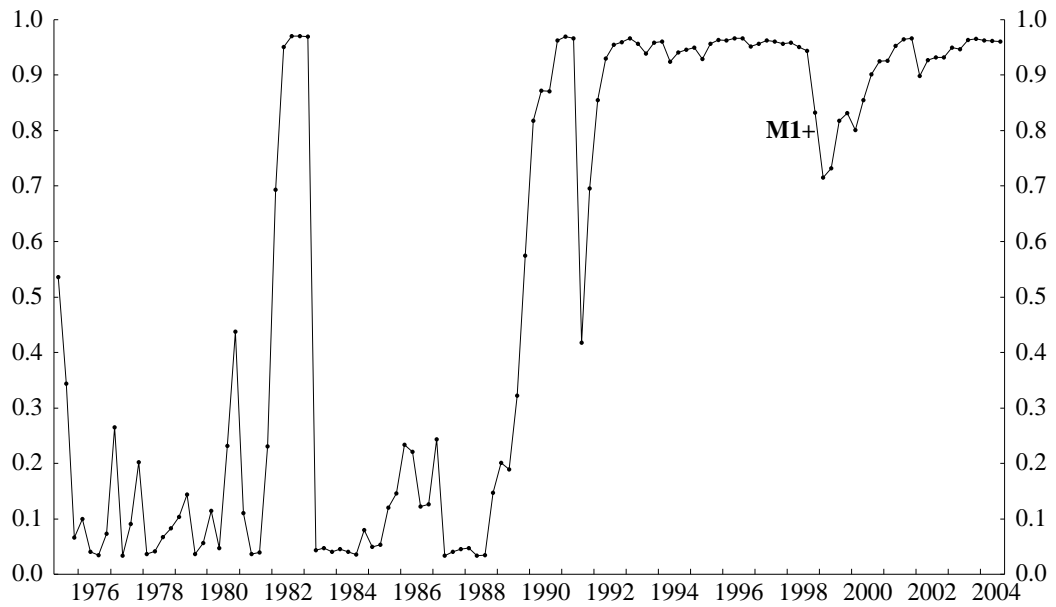


**Figure 23**  
**M1 versus M1+**

**Regime 1**



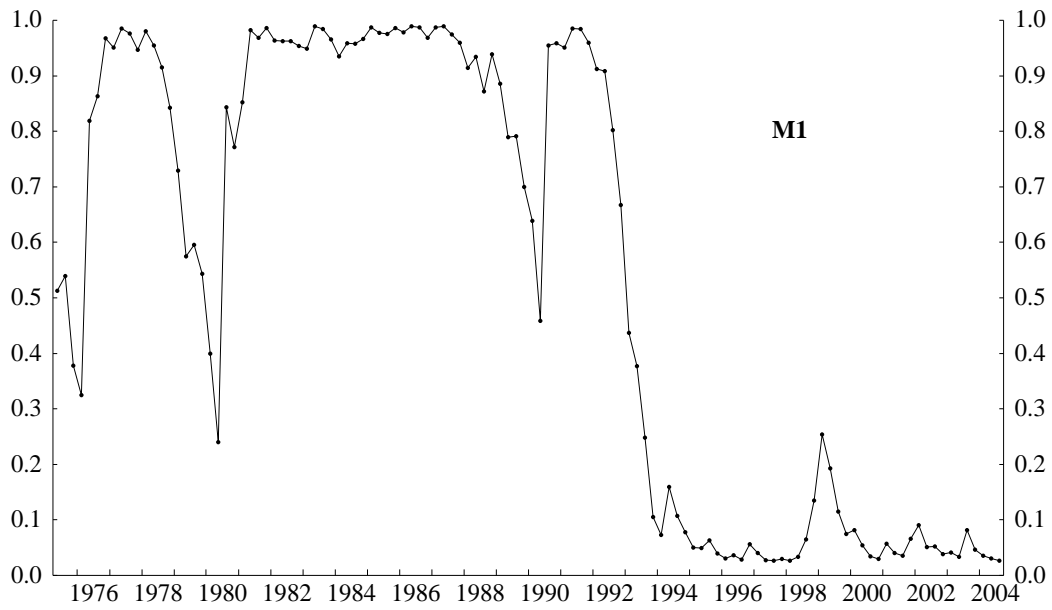
**Regime 2**



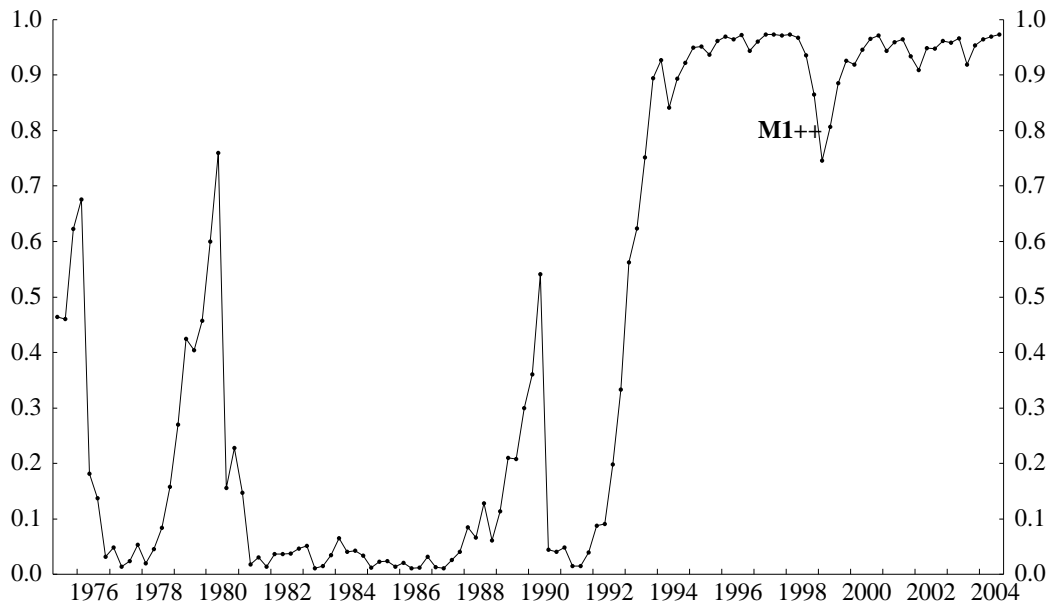


**Figure 24**  
**M1 versus M1++**

**Regime 1**



**Regime 2**



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