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FRICION FACTOR MEASUREMENTS
ON NON-UNIFORM SURFACES:
SAMPLING FREQUENCIES REQUIRED

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16. Abstract <p>It is well known that surface conditions on roads in wintertime can vary over a wide range on a variety of distance scales, reflecting the effect of factors such as: (a) local variations in road conditions and vegetation; and (b) variations in structure (e.g., bridges vs. pavement; intersections and corners vs. straight sections).</p> <p>It is intuitively obvious that less frequent sampling is required to measure the average friction reliably on long, relatively uniform road sections than on short ones or on ones with more variability. Numerical analyses have been conducted for a wide range of potential road surfaces to investigate sampling requirements by comparing the friction factor that a device would be expected to measure with the actual friction factor.</p> <p>The analyses suggested that sampling intervals should be no more than about 20 to 30 percent of the segment length to keep sampling errors less than 1 to 5 percent. For sampling intervals that are in the range of about 20 to 30 percent of the segment length, the analyses suggested that randomness will introduce variations of about +/- 1 and +/- 2 percent at one and two standard deviations from the mean, respectively.</p>					
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16. Résumé <p>Il est bien connu qu'en hiver, l'état de la surface des routes peut varier grandement sur différentes échelles de distance, sous l'effet de facteurs comme : a) les variations locales de l'état de la route et de la végétation; et (b) les variations de la structure (p. ex., ponts vs chaussées; carrefours et virages vs tronçons rectilignes).</p> <p>Il tombe sous le sens qu'il suffit d'une fréquence d'échantillonnage moindre pour mesurer de manière fiable le frottement moyen sur des tronçons de route longs et relativement uniformes que sur des tronçons courts ou changeants. Des analyses numériques comparant le coefficient de frottement réel avec le coefficient de frottement attendu ont permis de déterminer les exigences d'échantillonnage associées à un large éventail de couches de roulement potentielles.</p> <p>Selon les résultats des analyses, les pas d'échantillonnage ne devraient être supérieurs à environ 20 à 30 p. cent de la longueur du segment d'intérêt, de façon à maintenir les erreurs d'échantillonnage en deçà de 1 à 5 p. cent. Pour des pas d'échantillonnage équivalant à environ 20 à 30 p. cent de la longueur du segment d'intérêt, les analyses laissent entrevoir des variations d'environ +/- 1 et de +/- 2 p. cent, à un écart-type et deux écarts-types de la moyenne, respectivement, attribuables au hasard.</p>					
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EXECUTIVE SUMMARY

Background and Objectives: The Ministry of Transportation of Ontario (MTO) is currently considering continuous friction measurements as one potential means for evaluating and monitoring the quality of winter maintenance operations. It is well known that surface conditions on roads in wintertime can vary over a wide range on a variety of distance scales reflecting the effect of factors such as: (a) local variations in road conditions and vegetation; and (b) variations in structure (e.g., bridges vs. pavement; intersections and corners vs. straight sections).

It is intuitively obvious that less frequent sampling is required to measure the average friction reliably on long, relatively uniform road sections than on short ones or on ones with more variability. Numerical analyses have been conducted for a wide range of potential road surfaces to investigate sampling requirements by comparing the friction factor that a device would be expected to measure with the actual friction factor.

Conclusions:

Measuring the Average Friction Factor Along the Length of a Runway or Road – The sampling interval should be selected based on the following:

- (a) the expected variability in surface conditions with respect to friction levels, and also the relative proportion of the overall road or runway length covered by each;
- (b) the minimum length over which friction factor variations (e.g., produced by differences in surface conditions) are of concern. This was termed the minimum segment length of interest;
- (c) the maximum error that is acceptable.

The analyses suggested that sampling intervals should be no more than about 20 to 30 percent of the segment length to keep sampling errors less than 1 to 5 percent.

Friction Factor Variability – Randomness in the road surfaces will introduce variations in the measured friction factor. The magnitude of the variations is governed by the following:

- (a) sampling interval;
- (b) segment length of interest;
- (c) total sampled distance; and
- (d) confidence level (e.g., one vs. two standard deviations from the mean).

For sampling intervals that are in the range of about 20 to 30 percent of the segment length, the analyses suggested that randomness will introduce variations of about +/-1 and +/-2 percent at one and two standard deviations from the mean, respectively.

Recommendations: Continued work would be useful in the following areas:

- (a) The ranges of cases analyzed should be reviewed and compared to field information, if available.
- (b) Maximum permissible errors should be specified so that the analyses could be focussed on that range.

SOMMAIRE

Contexte et objectif : Le ministère des Transports de l'Ontario (MTO) étudie présentement des moyens d'évaluer et surveiller la qualité des opérations d'entretien hivernal. Un de ces moyens est la mesure continue du frottement. Il est bien connu que l'état de la surface des routes, en hiver, peut varier grandement sur différentes échelles de distance, sous l'effet de facteurs comme: (a) les variations locales de l'état de la route et de la végétation; et (b) les variations de la structure (p.ex., ponts vs chaussées; carrefours et virages vs tronçons rectilignes).

Il tombe sous le sens qu'il suffit d'une fréquence d'échantillonnage moindre pour mesurer de manière fiable le frottement moyen sur des tronçons de route longs et relativement uniformes que sur des tronçons courts ou changeants. Des analyses numériques comparant le coefficient de frottement réel avec le coefficient de frottement attendu ont permis de déterminer les exigences d'échantillonnage associées à un large éventail de couches de roulement potentielles.

Conclusions :

Calcul du coefficient de frottement moyen sur un tronçon de route ou de piste –

Le pas d'échantillonnage doit être déterminé en fonction des critères suivants:

- (a) la variabilité attendue des états de surface, en ce qui a trait au degré de frottement, et la proportion de chaque état de surface par rapport à la longueur totale de la route ou de la piste;
- (b) la plus petite longueur, sur une route ou une piste aux états de surface variables, présentant un coefficient de frottement préoccupant; il s'agit de la «longueur du segment d'intérêt»;
- (c) l'erreur maximale admissible.

Selon les résultats des analyses, les pas d'échantillonnage ne devraient pas être supérieurs à environ 20 à 30 p. cent de la longueur du segment d'intérêt, afin de maintenir les erreurs d'échantillonnage en deçà de 1 à 5 p. cent.

Variabilité du coefficient de frottement – Le caractère aléatoire du choix des surfaces entraînera des variations du coefficient de frottement mesuré. L'ampleur des variations dépendra des facteurs suivants :

- (a) pas d'échantillonnage;
- (b) longueur du segment d'intérêt;
- (c) distance totale échantillonnée;
- (d) niveau de confiance (p. ex., un écart-type vs deux écarts-types de la moyenne).

Pour des pas d'échantillonnage d'environ 20 à 30 p. cent de la longueur du tronçon, les analyses laissent entrevoir des variations d'environ +/-1 et de +/-2 p. cent, à un écart-type et deux écarts-types de la moyenne, respectivement, attribuables au hasard.

Recommandations : Il est recommandé de mener d'autres travaux dans les domaines suivants :

- (a) Revoir les ensembles de cas analysés et les comparer aux données recueillies sur le terrain, le cas échéant.
- (b) Préciser les erreurs maximales admissibles, de façon que les analyses puissent en tenir compte.

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1. INTRODUCTION AND PROJECT OBJECTIVES

1.1 Background

The Ministry of Transportation of Ontario (MTO) is currently considering continuous friction measurements as one potential means for evaluating and monitoring the quality of winter maintenance operations.

Although no specific criteria have yet been developed (to our knowledge), it is reasonable to expect that a two-component criterion might be considered based on the following:

- (a) the average friction over a long distance;
- (b) the average friction over a short distance (e.g., at intersections, bridges, hills, etc.).

This follows the general approach currently used by Transport Canada's Aerodrome Safety Branch, the International Civil Aviation Organization (ICAO), and the Federal Aviation Administration (FAA) for specifying wet pavement friction levels at airport runways in summertime conditions. These organizations specify a higher value for the whole runway than for the minimum friction over any short section. A distance of 100 m is used for the "short section" criterion in Transport Canada's case.

It is well known that surface conditions on roads in wintertime can vary over a wide range on a variety of distance scales reflecting the effect of factors such as:

- (a) local variations in road conditions and vegetation; and
- (b) variations in structure (e.g., bridges vs. pavement; intersections and corners vs. straight sections).

It is intuitively obvious that less frequent sampling is required to measure the average friction reliably on long, relatively uniform road sections, than on short ones, or on ones with more variability.

1.2 Objectives

The objectives of the analyses were to investigate and quantify the sampling requirements for collecting friction data for the two general cases below:

- (a) measuring the average friction over long distances; and
- (b) identifying short sections that have variable friction.

1.3 Overview of Scope of Analyses

Several different analyses were carried out as summarized in Table 1.1.

Table 1.1: Overview of Analyses

Report Section	Parameters Varied (see note 1 for definition)				
	Surface Sequence		Segment Length (m)	Total Distance Sampled (m)	Sampling Interval (m)
	No. of Surfaces	Distribution			
Analysis Objective: Measuring the average friction over a long distance					
2	2	repeated regularly proportion varied	10; 100; 1000	5000	Varied from 1 to 1000
Analysis Objective: Measuring the average friction over a long distance					
3	4	varied randomly	10; 20; 100	100; 1000; 5000	18; 22; 36; 44; 90; 111 (note 2)
Analysis Objective: Measuring the average friction over a short distance					
4	4	repeated regularly	10; 20	100	Varied from 1 to 50

Notes:

1. Definition of Terms:
 - (a) Sampling Interval – the distance between measurement points.
 - (b) Surface Sequence: No. of Surfaces – the number of surface conditions (e.g., wet, bare and dry, ice, slush, packed snow, etc.) in the sequence.
 - (c) Surface Sequence: Distribution – the order in which the surface types occur in the sequence (i.e., repeat regularly vs. random), and the overall proportion of each surface type in the sequence in relation to the total sampled distance.
 - (d) Segment Length – the length of each surface condition in the surface sequence.
2. These sampling intervals are produced by sampling frequencies of 1.0, 0.5 and 0.2 Hz in combination with vehicle speeds of 65 and 80 km/h.

2. THE ABILITY TO MEASURE THE AVERAGE FRICTION OVER LONG DISTANCES: DETERMINISTIC ANALYSES

Relatively simple analyses are described here in which the road surface was considered to be either wet or ice-covered. These surfaces were presumed to be distributed along the length of the road in a regular, repetitive sequence.

More complex analyses were done as well in which the road was presumed to be covered with a variety of surfaces (wet pavement, ice, slush and packed snow) that were distributed randomly. These analyses are described in Section 3.

2.1 Analysis Scope and Approach

A 5-km long road section was analyzed for all cases.

The analyses were done by presuming that the road is either wet with a friction factor of 0.5, or ice-covered with a friction factor of 0.1. These friction factors are typical of those that have been measured for these respective surface conditions. A range of surface condition distributions were analyzed as summarized in Table 2.1. These two surface conditions were assumed to be distributed regularly along the length of the road in a repetitive sequence (Table 2.2).

Table 2.1: Surface Condition Distribution Cases Analyzed

Total Length of Wet:Ice-Covered Road	Segment Length (m)	Actual Friction Factor for a 5-km Long Road
50%:50%	10	0.30
	100	0.30
	1000	0.34
67%:33%	10	0.37
	100	0.37
	1000	0.42
75%:25%	10	0.40
	100	0.40
	1000	0.42
80%:20%	10	0.42
	100	0.42
	1000	0.42

The segment length was defined as the length of each wet ice-covered section within the total road track length that was analyzed. This was varied from 10 to 1000 m (Table 2.1), which spans a range of road surface non-uniformities, with the 10 m case being the most non-uniform one analyzed (Table 2.2).

Table 2.2: Road Surface Condition Profile for Selected Examples

Total Length of Wet:Ice Covered Road	Segment Length (m)	Road Surface Condition Profile	
		Distance from Start (m)	Surface Condition
75%:25%	1000	0 to 1000	wet
		>1000 to 2000	wet
		>2000 to 3000	wet
		> 3000 to 4000	ice
		>4000 to 5000	wet
75%:25%	100	0 to 100	wet
		>100 to 200	wet
		>200 to 300	wet
		> 300 to 400	ice
		>400 to 500	wet
		>500 to 600	wet
		>600 to 700	wet
		>700 to 800	ice
		Pattern repeats up to a distance of 5000 m	
75%:25%	10	0 to 10	wet
		>10 to 20	wet
		>20 to 30	wet
		> 30 to 40	ice
		>40 to 50	wet
		>50 to 60	wet
		>60 to 70	wet
		>70 to 80	ice
		Pattern repeats up to a distance of 5000 m	

The average actual and measured friction factors (which are defined below) were determined for each case in Table 2.1 for a 5-km long section of road.

- (a) Actual friction factor (μ_{actual}) – this is the real average friction factor for the whole 5-km long road track under consideration. These values are listed in Table 2.1.

- (b) Measured friction factor (μ_{measured}) – this represents the average friction factor that a device would measure for the whole 5-km long road track under consideration. This was calculated for sampling intervals ranging from 1 to 1000 m. It was presumed that:
- (i) the device had no measurement error and thus would measure a friction factor of either 0.1 or 0.5, depending on whether it was located on ice-covered or wet pavement, respectively;
 - (ii) the device did not average the friction at the boundaries between wet and ice-covered pavement sections. It was presumed that the friction factor measured by the device would always be that of the surface directly under the centre of the tire.

In practice, the tire would be in contact with both surface types at the boundaries, and thus some averaging might be expected. This was not accounted for in the analyses for simplicity. This errs conservatively, as this assumption will lead to an overestimation of the sampling error.

2.2 Results

The ratio between the measured and the actual friction factor ($\mu_{\text{measured}} / \mu_{\text{actual}}$) was used as an index for evaluating the sampling error. This varies with the sampling interval and the surface condition, as shown in the following figures:

- (a) Total length of wet:ice-covered road = 50%:50% – Figures 2.1, 2.2 and 2.3, for segment lengths of 10, 100 and 1000 m, respectively.
- (b) Total length of wet:ice-covered road = 67%:33% – Figures 2.4, 2.5 and 2.6 for segment lengths of 10, 100 and 1000 m, respectively.
- (c) Total length of wet:ice-covered road = 75%:25% – Figures 2.7, 2.8 and 2.9 for segment lengths of 10, 100 and 1000 m, respectively.
- (d) Total length of wet:ice-covered road = 80%:20% – Figures 2.10, 2.11 and 2.12 for segment lengths of 10, 100 and 1000 m, respectively.

2.3 Analyses

- (a) Effect of Sampling Interval – The analyses showed that the $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio (and thus the sampling error) varied greatly with the distance between measurement points for each case analyzed. The $\mu_{\text{measured}} / \mu_{\text{actual}}$ variation is much larger for sampling intervals of more than 10 m.

This result is due to the effects of aliasing. For some sampling intervals, the measurement spacings are favourably “tuned” to the road surface distribution with the result that there is very little sampling error. However, other cases are unfavourably “tuned” with the result that very large sampling errors are produced.

Of course, the sampling frequencies that produce favourable and unfavourable “tuning” will vary with the distribution and type of the various surface conditions on the road. Because these are highly variable, one cannot rely on the sampling program being “tuned” to the surface conditions on the road. Devices and sampling programs should be set up to avoid this range.

- (b) Actual Friction Factor Overestimated vs. Underestimated – In general, the analyses suggested that the actual friction factor will be overestimated by the measured values as the $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio tends to be larger than unity.

This is NOT a universal conclusion because it is specific to the wet:ice distributions that were used for analysis. The opposite conclusion (i.e., that the actual friction factor would have been underestimated) would have been obtained had cases where there was less wet pavement than ice-covered road been analyzed.

Thus, it is believed that the measured friction values are equally likely to overestimate or underestimate the actual friction factor. For this reason, subsequent analyses were done based on the maximum sampling error in either direction.

- (c) Combined Effect of Sampling Interval, Segment Length and Wet:Ice Distribution – The analyses showed that the sampling error tended to decrease as the sampling interval was decreased, and also as the segment length was increased (Figures 2.1 to 2.12). Both of these results follow the expected trend.

The combined effect of these two parameters was investigated by plotting the $\mu_{\text{measured}}/\mu_{\text{actual}}$ ratio against the sampling interval/segment length ratio. Figures 2.13, 2.14 and 2.15 show this for segment lengths of 10, 100 and 1000 m, respectively.

The error is clearly related to the sampling interval/segment length ratio because it reduces as this ratio is decreased. This follows the expected trend.

The sampling error is also related to the wet:ice proportion. It reduced as the wet:ice distribution was varied from 50%:50% to 80%:20% (Figures 2.13 to 2.15).

- (d) Maximum Sampling Errors – The maximum sampling errors for the four wet:ice surface condition distributions analyzed are plotted in Figures 2.16 to 2.19. They are also listed in Table 2.3. The results show that:

- (i) The maximum sampling error is highly sensitive to the Effect of Surface Distribution. The maximum error increased as the wet:ice surface distribution ratio was changed from 80%:20% to 50%:50%.
- (ii) The maximum sampling error is also highly sensitive to the Effect of Segment Length. The maximum error increased as the segment length was decreased from 1000 to 10 m. This follows the expected trend as the overall road track becomes more non-uniform with decreasing segment length.

Table 2.3: Maximum Sampling Errors

Distance Between Measurements (m)	Maximum Sampling Error (note 1) for Segment Lengths of:		
	10 m	100 m	1000 m
Surface Condition Distribution: 50%:50% Wet:Ice			
1-2	0.0%	0.0%	0.0%
3-5	13.3%	0.0%	0.0%
6-10	13.3%	2.7%	0.1%
11-20	66.7%	2.8%	0.5%
21-100	66.7%	22.2%	2.7%
101-1000	66.7%	66.7%	17.6%
Surface Condition Distribution: 67%:33% Wet:Ice			
1-2	0.0%	0.0%	0.0%
3-5	3.6%	0.4%	0.0%
6-10	14.5%	1.4%	0.1%
11-20	36.2%	3.5%	0.3%
21-100	36.2%	13.9%	1.4%
101-1000	36.2%	34.4%	8.5%
Surface Condition Distribution: 75%:25% Wet:Ice			
1-2	0.0%	0.0%	0.0%
3-5	5.0%	0.0%	0.0%
6-10	5.0%	1.0%	0.0%
11-20	25.0%	1.0%	0.2%
21-100	25.0%	7.9%	1.1%
101-1000	25.0%	23.8%	8.5%
Surface Condition Distribution: 80%:20% Wet:Ice			
1-2	0.0%	0.0%	0.0%
3-5	0.0%	0.0%	0.0%
6-10	0.1%	0.0%	0.0%
11-20	0.4%	0.3%	0.2%
21-100	19.0%	9.1%	1.4%
101-1000	19.0%	19.0%	8.5%

Notes:

- The maximum sampling error ($Error_{\max}$) was calculated as follows:

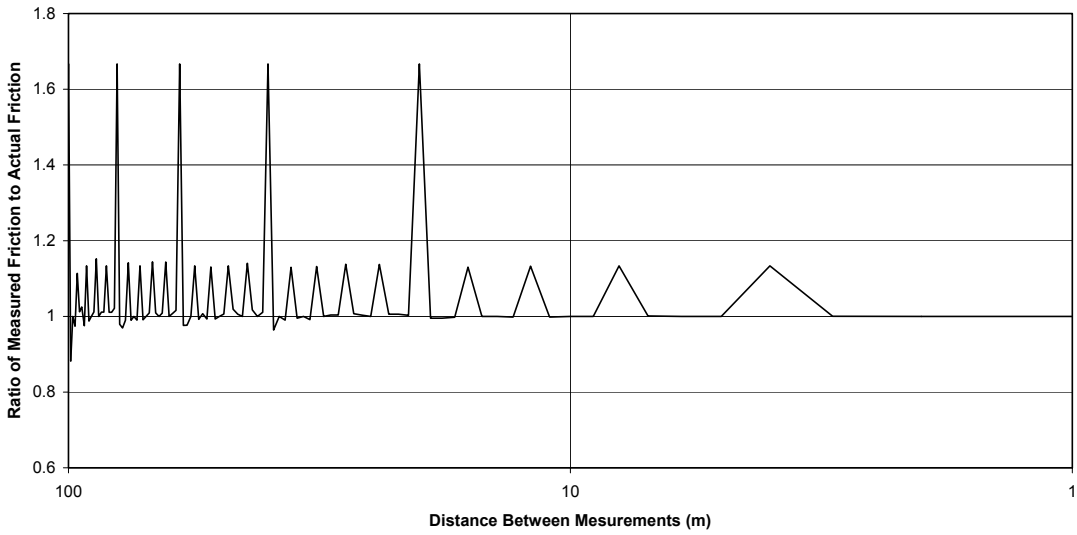
$$Error_{\max} = \text{Max} [\text{absolute value of } (\mu_{\text{measured}} - \mu_{\text{actual}}) / \mu_{\text{actual}}] \quad [2.1]$$

Equation 2.1 was evaluated for each case for the full range of sampling intervals analyzed.

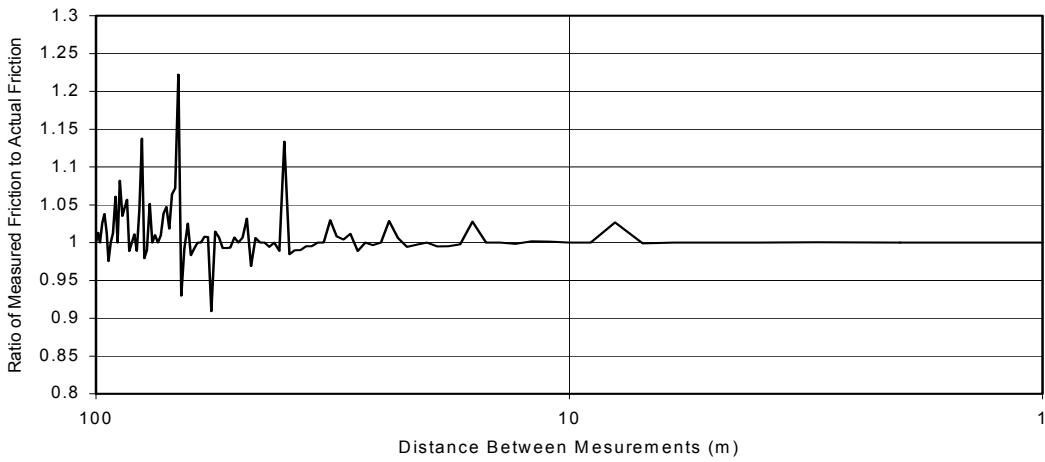
- (e) Sampling Intervals Required to Keep Sampling Errors Within Set Limits – This depends on the wet:ice ratio and the segment length (Figure 2.20 and Table 2.4). The implications of these results for field data collection are discussed further in section 3.3, which also presents the results of probabilistic analyses that were done.

Table 2.4: Sampling Intervals Required to Keep Sampling Errors Within Set Limits

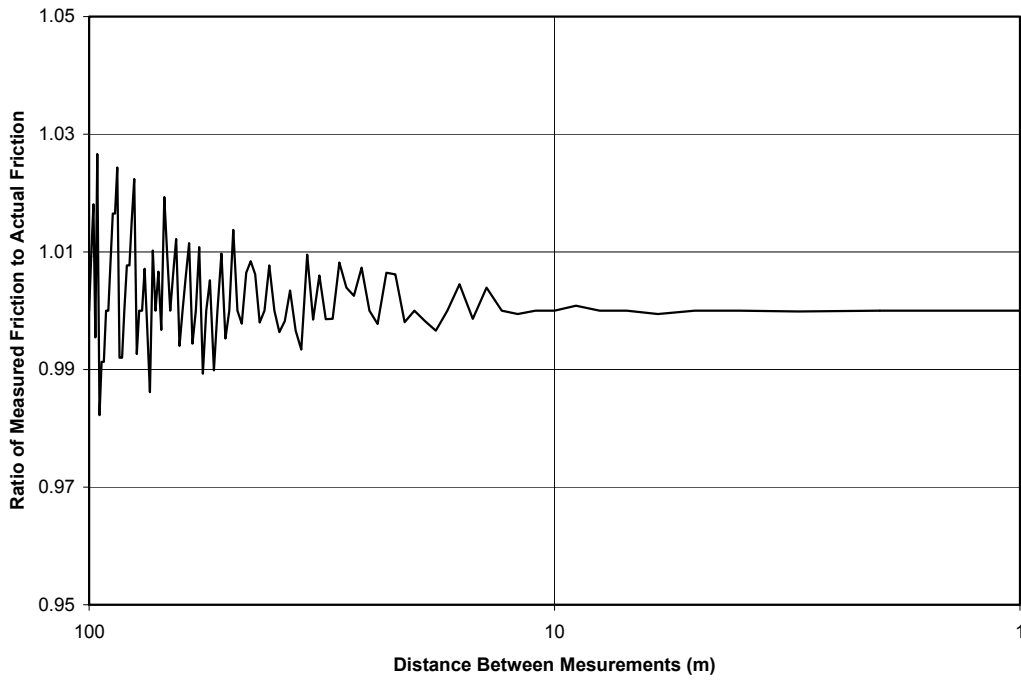
Segment Length (m)	Wet:Ice Distr'n	Maximum Sampling Interval for			
		Max. Sampling Error < 1%		Max. Sampling Error < 5 %	
		Sampling Interval (m)	Interval/Segment Length	Sampling Interval (m)	Interval/Segment Length
10	50%:50%	3	0.3	3	0.3
	67%:33%	5	0.5	5	0.5
	75%:25%	4	0.4	4	0.4
	80%:20%	24	2.4	25	2.5
100	50%:50%	7	0.07	40	0.40
	67%:33%	5	0.05	43	0.43
	75%:25%	7	0.07	39	0.39
	80%:20%	22	0.22	70	0.70
1000	50%:50%	21	.021	181	0.18
	67%:33%	38	.038	374	0.37
	75%:25%	43	.043	363	0.36
	80%:20%	35	.035	357	0.36



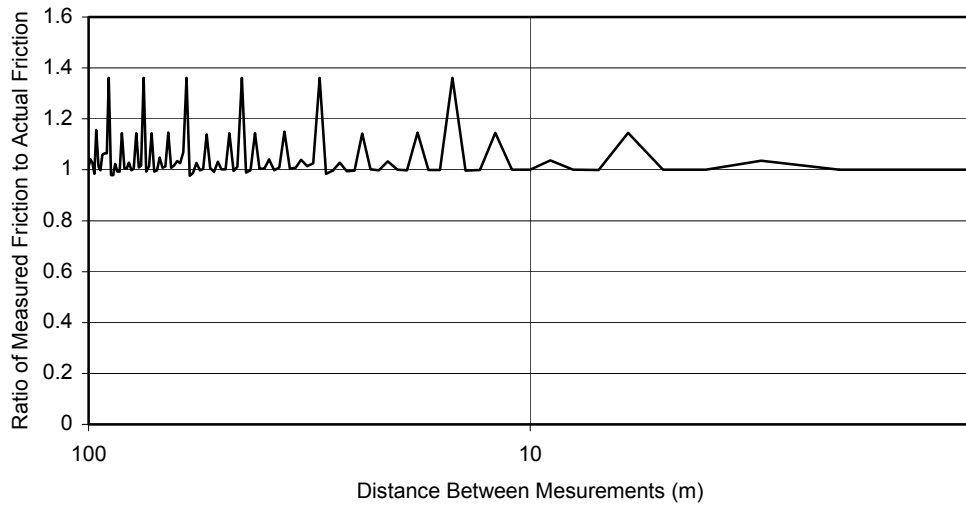
**Figure 2.1: Sampling Error for a 50%:50% Wet:Ice Surface Condition Distribution:
Segment Length = 10 m**



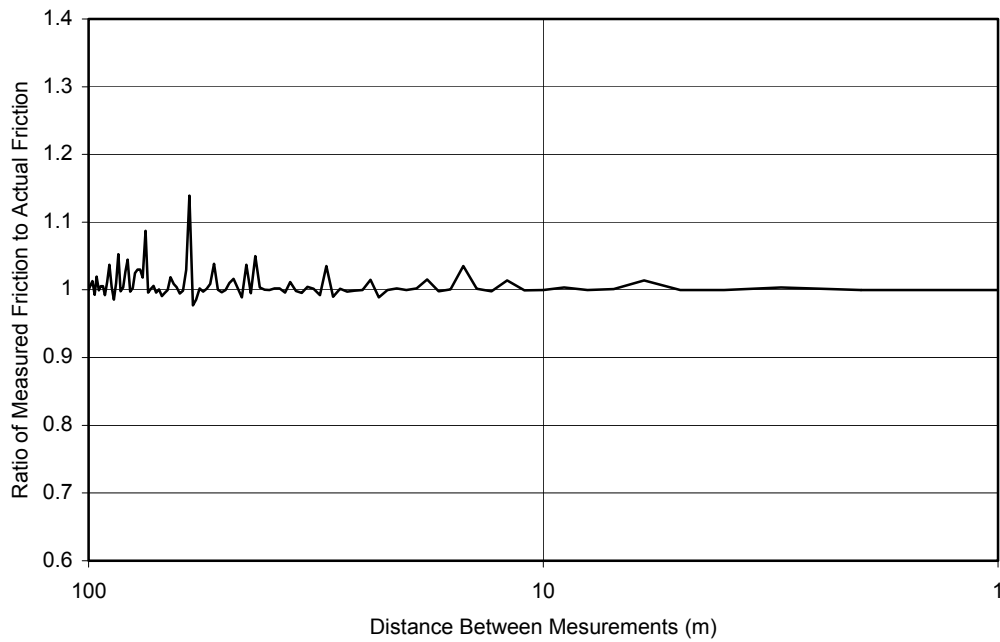
**Figure 2.2: Sampling Error for a 50%:50% Wet:Ice Surface Condition Distribution:
Segment Length = 100 m**



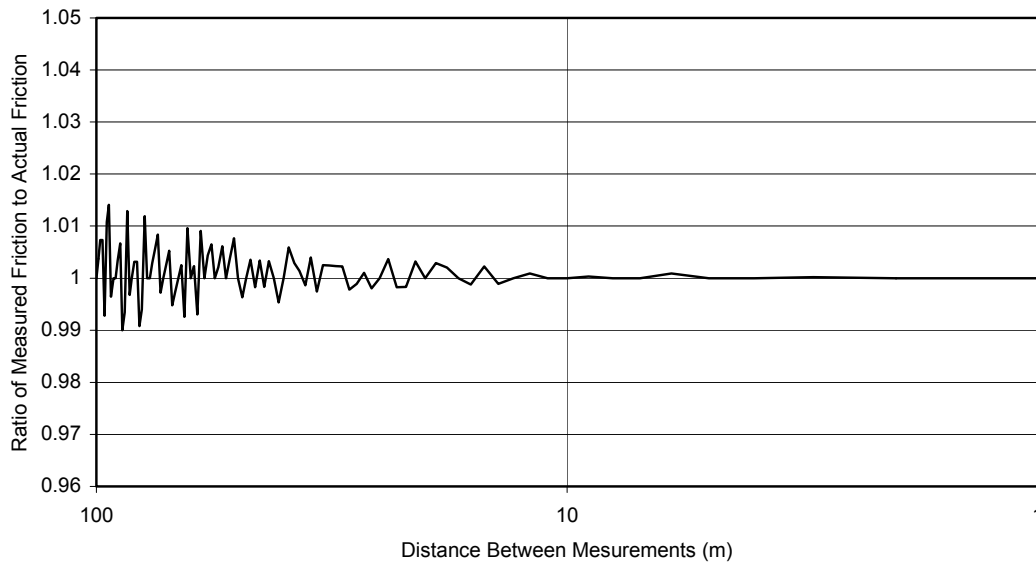
**Figure 2.3: Sampling Error for a 50%:50% Wet:Ice Surface Condition Distribution:
Segment Length = 1000 m**



**Figure 2.4: Sampling Error for a 67%:33% Wet:Ice Surface Condition Distribution:
Segment Length = 10 m**



**Figure 2.5: Sampling Error for a 67%:33% Wet:Ice Surface Condition Distribution:
Segment Length = 100 m**



**Figure 2.6: Sampling Error for a 67%:33% Wet:Ice Surface Condition Distribution:
Segment Length = 1000 m**

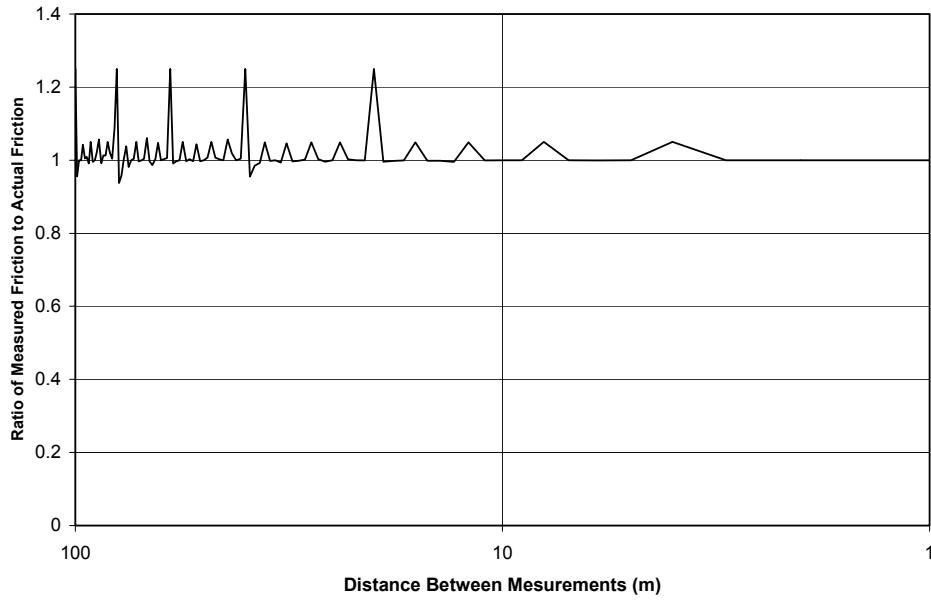


Figure 2.7: Sampling Error for a 75%:25% Wet:Ice Surface Condition Distribution: Segment Length = 10 m

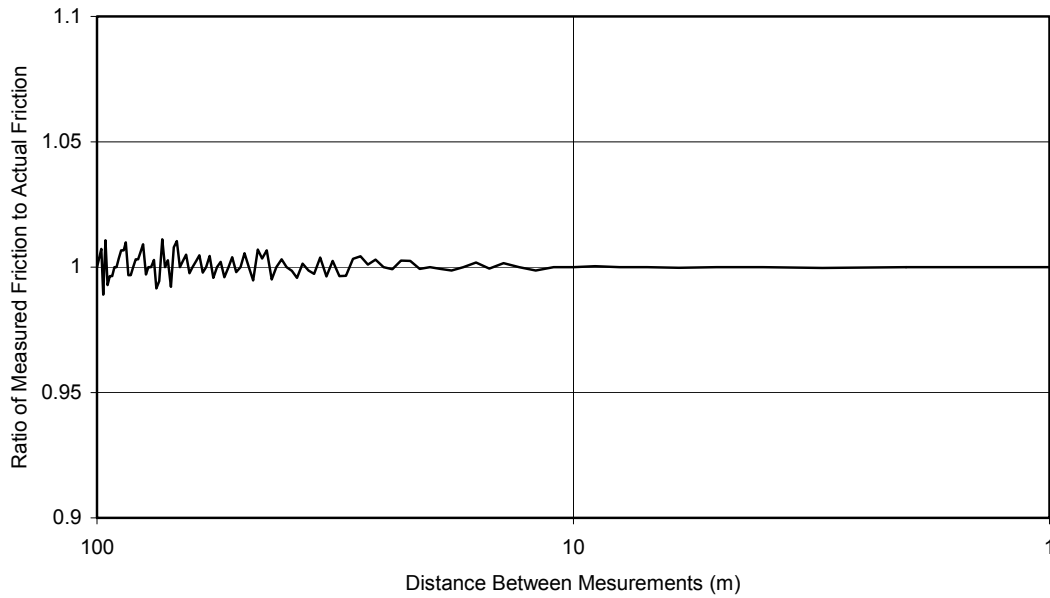


Figure 2.8: Sampling Error for a 75%:25% Wet:Ice Surface Condition Distribution: Segment Length = 100 m

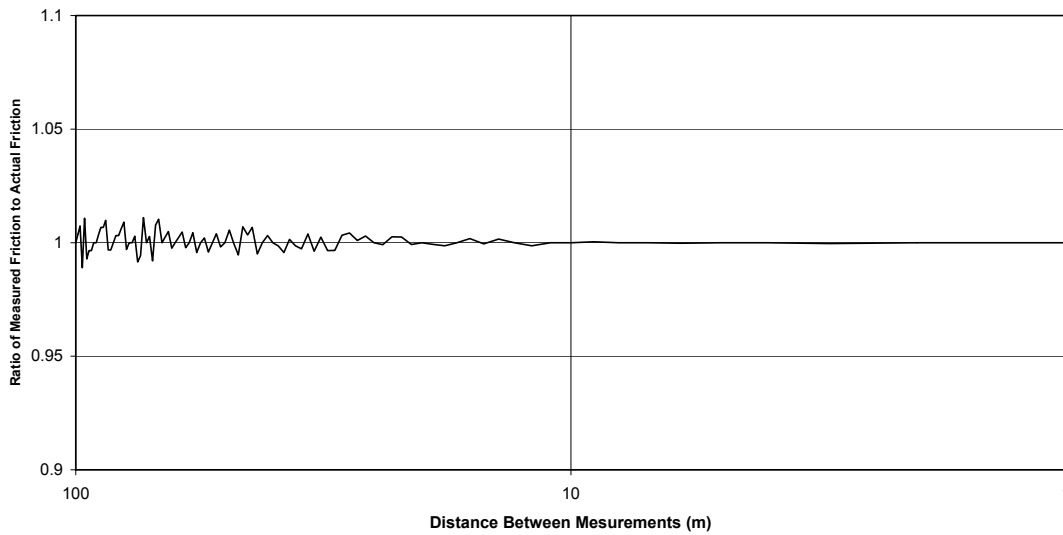


Figure 2.9: Sampling Error for a 75%:25% Wet:Ice Surface Condition Distribution: Segment Length = 1000 m

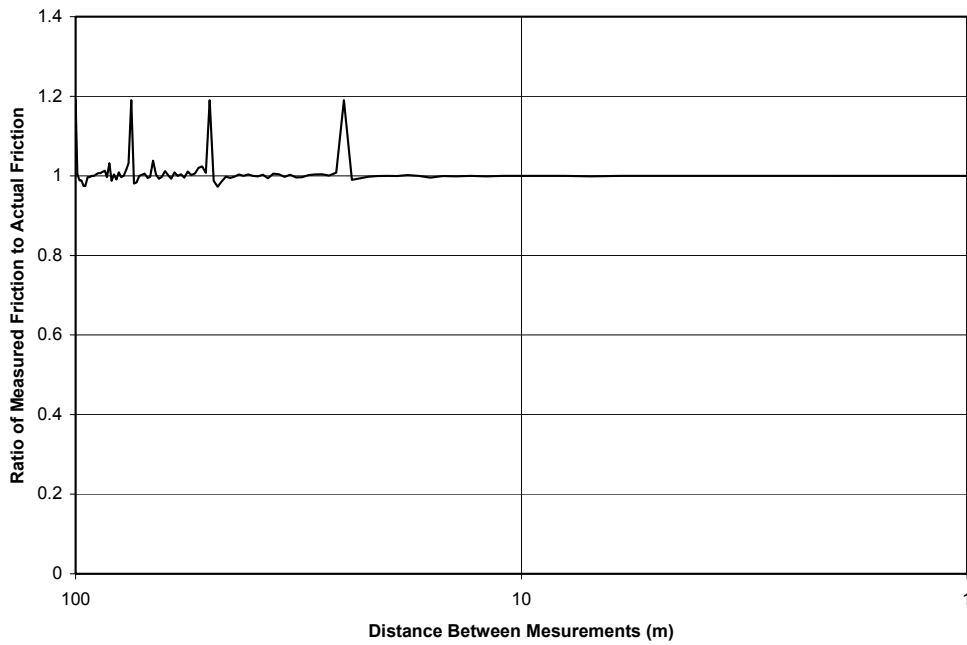


Figure 2.10: Sampling Error for an 80%:20% Wet:Ice Surface Condition Distribution: Segment Length = 10 m

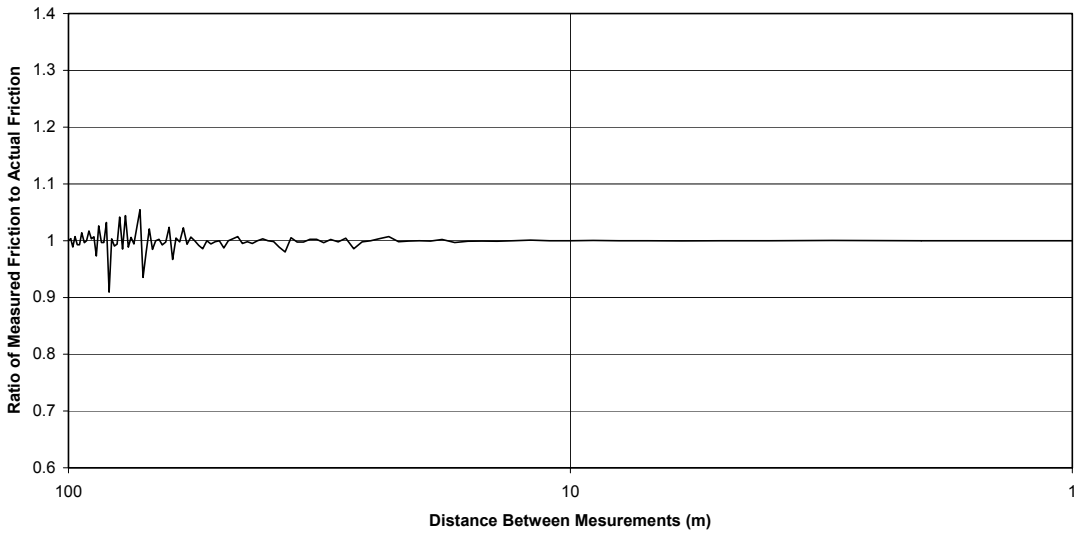
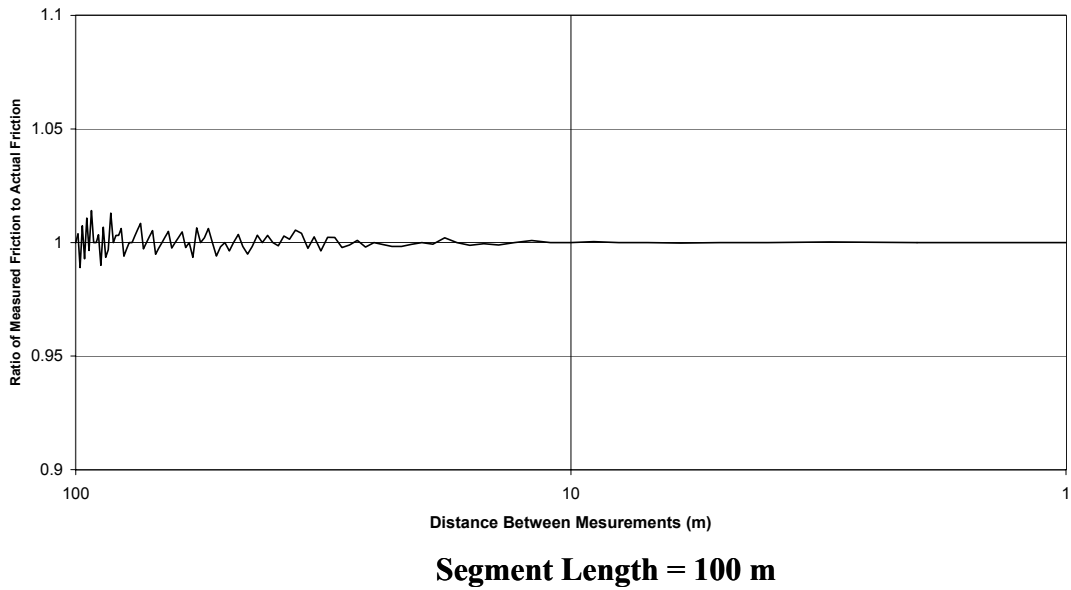
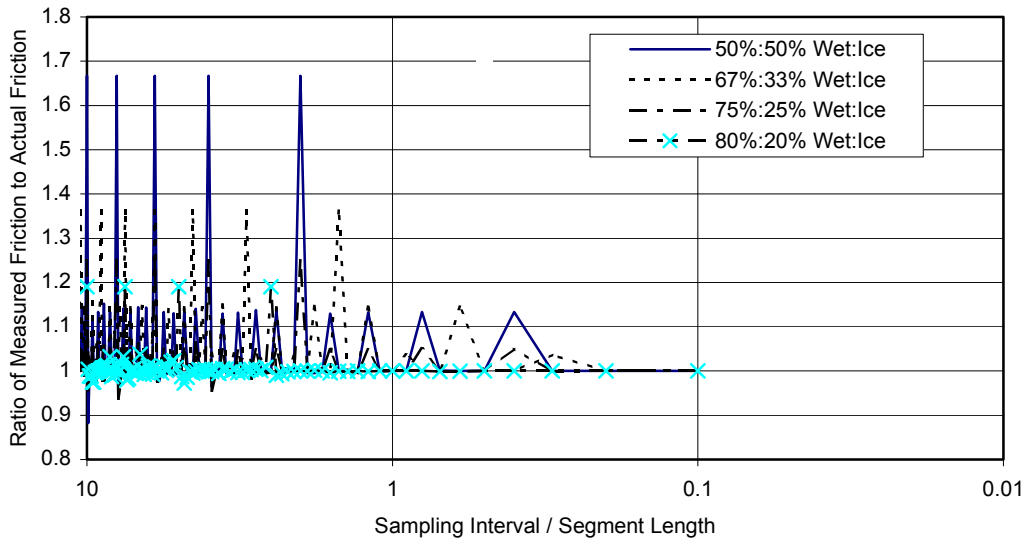


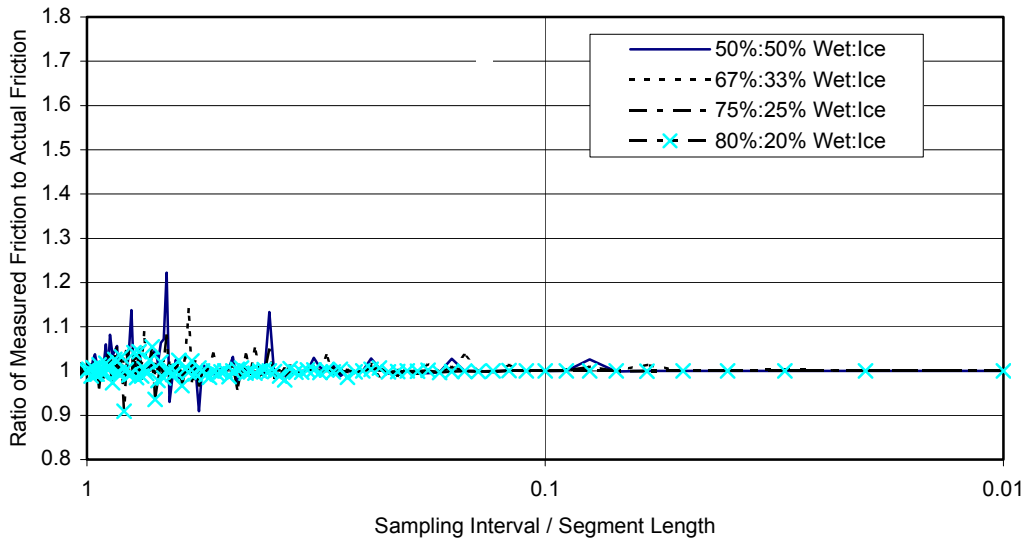
Figure 2.11: Sampling Error for an 80%:20% Wet:Ice Surface Condition Distribution:



**Figure 2.12: Sampling Error for an 80%:20% Wet:Ice Surface Condition Distribution:
Segment Length = 1000 m**



**Figure 2.13: Non-Dimensional Sampling Error for a 10 m Segment Length:
Maximum Sampling Interval = 100 m**



**Figure 2.14: Non-Dimensional Sampling Error for a 100 m Segment Length:
Maximum Sampling Interval = 100 m**

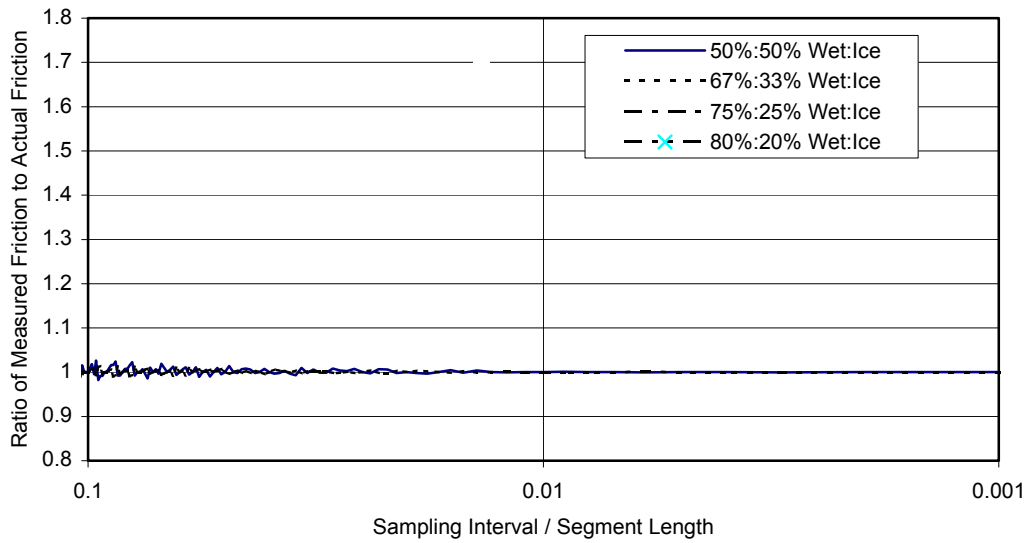


Figure 2.15: Non-Dimensional Sampling Error for a 1000 m Segment Length: Maximum Sampling Interval = 100 m

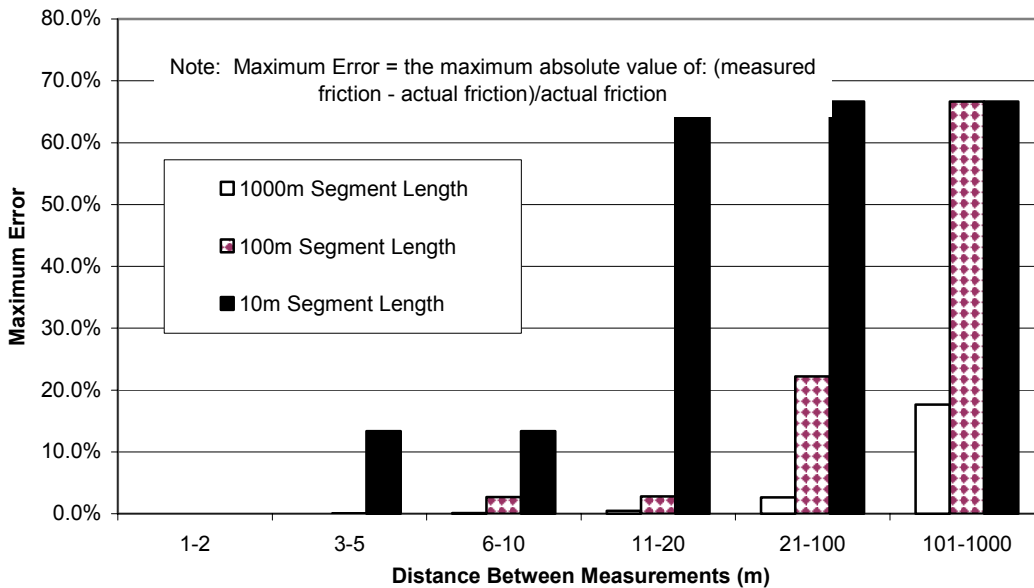


Figure 2.16: Maximum Sampling Error for a 50%:50% Wet:Ice Surface Distribution

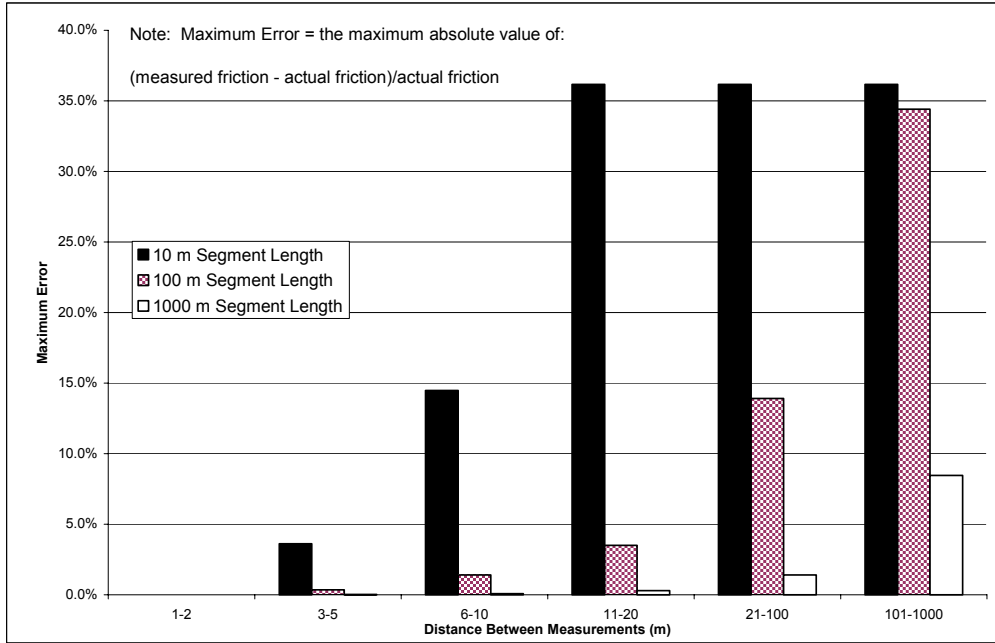


Figure 2.17: Maximum Sampling Error for a 67%:33% Wet:Ice Surface Distribution

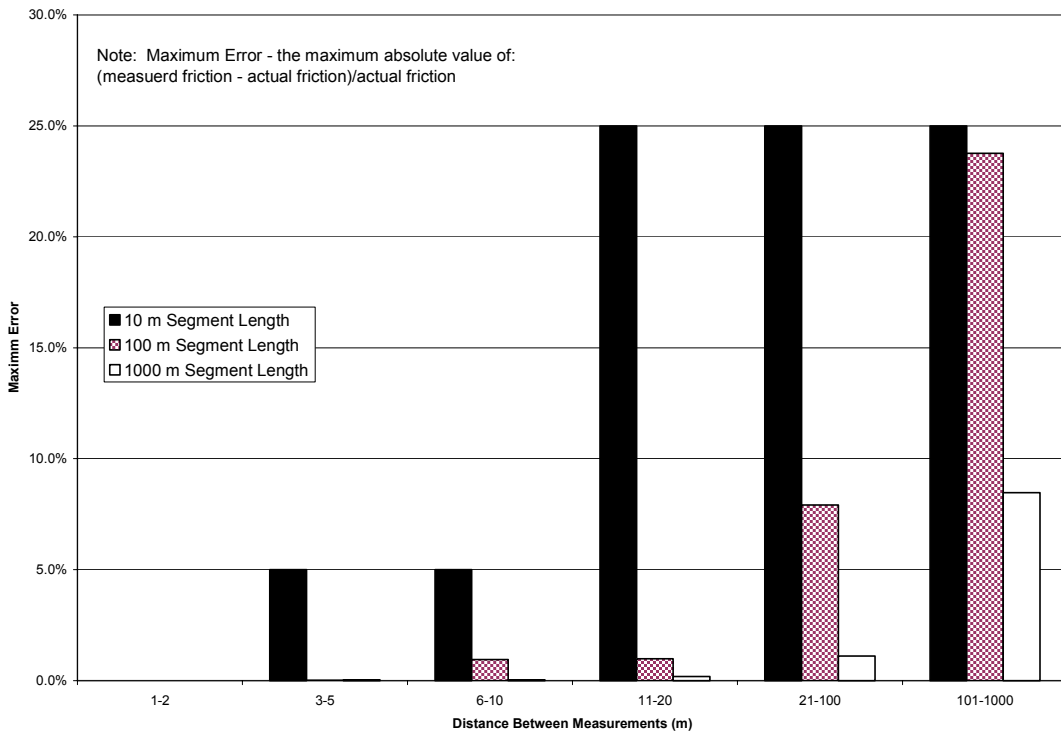


Figure 2.18: Maximum Sampling Error for a 75%:25% Wet:Ice Surface Distribution

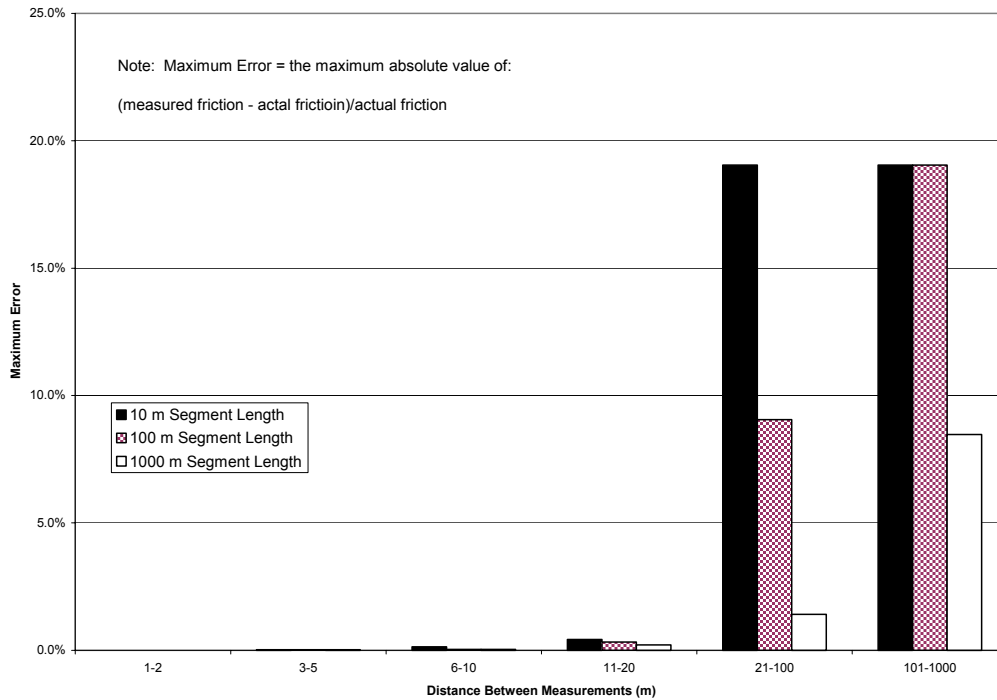


Figure 2.19: Maximum Sampling Error for an 80%:20% Wet:Ice Surface Distribution

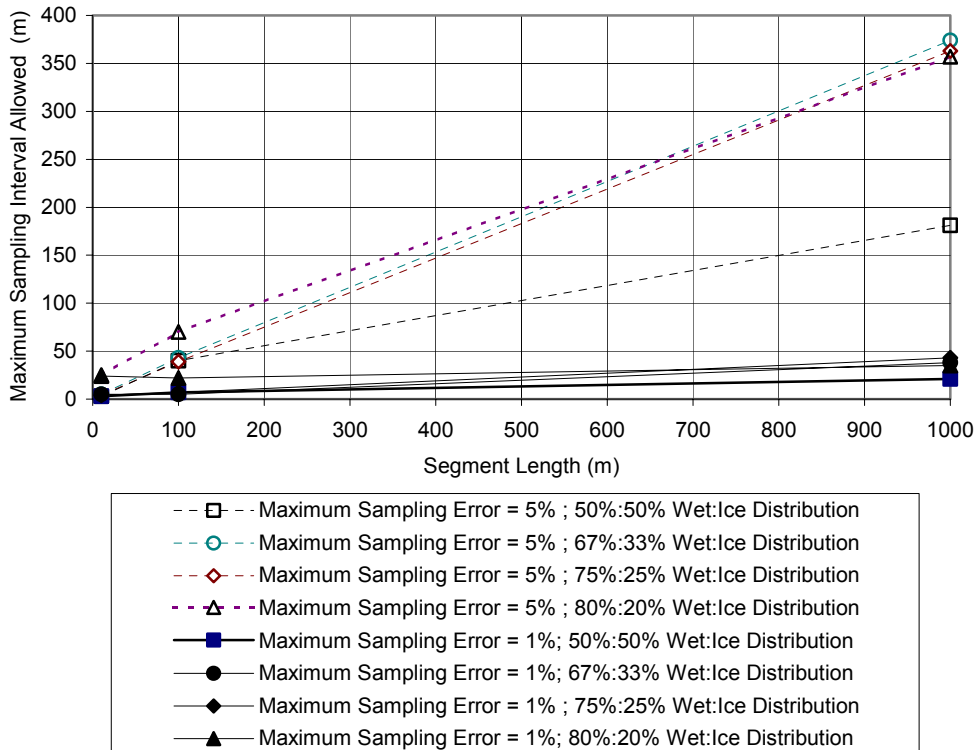


Figure 2.20: Maximum Sampling Intervals Allowed for Specified Sampling Errors

3. THE ABILITY TO MEASURE THE AVERAGE FRICTION OVER LONG DISTANCES: PROBABILISTIC ANALYSES

These analyses were done to expand upon those presented in section 2 by:

- (a) considering a more diverse mixture of surface conditions. A mixture of surface conditions (wet pavement, slush, ice and packed snow) was analyzed whereas only two surfaces (either wet pavement or ice) were considered in Section 2.
- (b) introducing randomness into the road sections being analyzed. The surface sequences analyzed here were selected randomly, whereas in Section 2 they were presumed to repeat regularly along the length of the road.
- (c) considering a shorter total distance. The analyses described here were done for total sampled distances of 1000 and 5000 m.

3.1 Analysis Approach and Scope

The analyses were done for a mixture of surface conditions that was assumed to cover the total road length with the distribution shown below. Each of these surface conditions was assigned a friction factor that is a representative value (Table 3.1). The cases analyzed are summarized in Table 3.2.

Table 3.1: Surface Condition Distribution

Surface Condition	Assigned Friction Factor	Prob. of Occurrence
Ice	0.1	0.2
Slush	0.3	0.2
Packed Snow	0.25	0.2
Wet Pavement	0.5	0.4

Table 3.2: Probabilistic Analyses: Summary of Cases Analyzed

Surface Condition Dist'n	Segment Length (m) (note 2)	Total Distance Sampled (m)	Sampling Interval (m) (note 1)
as per Table 3.1	10; 20; 100	1000; 5000	18; 22; 36; 44; 90; 111

Notes:

1. These sampling intervals are produced by sampling frequencies of 1.0, 0.5 and 0.2 Hz in combination with vehicle speeds of 65 and 80 km/h.
2. The segment length was defined as described in section 2 (Table 2.2). It is the length of each individual surface in the total sampled distance.

Monte-Carlo analyses were carried out for each case. Each run was done as follows:

- (a) The surface profile was defined along the total distance using the surfaces (i.e., wet pavement, slush, ice, and packed snow) and the friction factors assigned to them. The surface and friction factor profile was established for each run with the following steps:
 - (i) select the surface condition randomly for the first segment, which was either 10, 20 or 100 m long (Table 3.2). The appropriate friction factor value (given in Table 3.1) was then assigned to that segment.
 - (ii) repeat step (i) for each segment in the profile until the total length of the built-up track equalled the total target distance (i.e., either 1000 or 5000 m – Table 3.2).
- (b) The actual average friction factor was calculated for that surface and friction profile.
- (c) The measured friction factor was calculated for that surface and friction profile.

3.2 Results

The results were evaluated with respect to two general issues:

- (a) the ability to measure the friction factor correctly on average; and
- (b) the variability to be expected for the measured friction factor.

3.2.1 Average Friction Factor

The ratio of the measured to the actual friction factor ($\mu_{\text{measured}} / \mu_{\text{actual}}$) was used as an index for evaluating the sampling error. The results showed that, on average, the actual friction factor will be measured to better than within 0.01 percent for all cases.

The reader should note that this cannot be taken as a general rule because the analyses were only done for discrete sampling intervals (Table 3.2). The previous analyses (in Section 2) clearly showed that aliasing causes some sampling frequencies to be favourably “tuned” to the road conditions while large sampling errors are produced for other unfavourable sampling intervals.

These results are discussed further in Section 3.3.

3.2.2 Friction Factor Variability: Probability Density Function

In general, the ratio of the measured to the actual friction factor ($\mu_{\text{measured}} / \mu_{\text{actual}}$) can be described by a normal distribution (Figure 3.1).

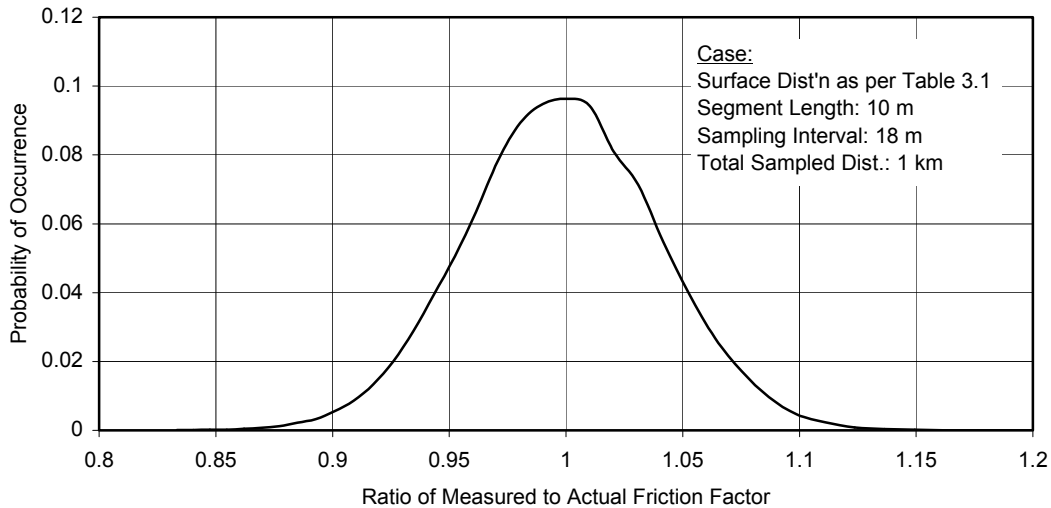


Figure 3.1: Sample Probability Density Function Predicted by the Analyses

3.2.3 Sampling Error at One Standard Deviation from the Mean

The results are plotted in Figures 3.2 and 3.3 for total sampled distances of 5 and 1 km respectively. The results are summarized below and discussed further in Section 3.3.

- (a) Effect of Sampling Interval – The $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio increased (showing that the sampling error was increasing) as the sampling interval was increased. This follows the trend that is to be expected in general.

Figures 3.2 and 3.3 appear to indicate that the sampling error increases steadily as the sampling interval is increased. This cannot be taken as a general rule because the analyses were only done for discrete sampling intervals (Table 3.2). The previous analyses (in Section 2) showed clearly that aliasing causes some sampling frequencies to be favourably “tuned” to the road conditions while large sampling errors are produced for other unfavourable sampling intervals.

- (b) Effect of Segment Length – The $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio increased (showing that the sampling error was increasing) as the segment length was decreased. This follows the expected trend.
- (c) Effect of Total Distance Sampled – The $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio was much higher (showing that the sampling error was increased) for a total sampled distance of 1 km compared to 5 km. This follows the expected trend.

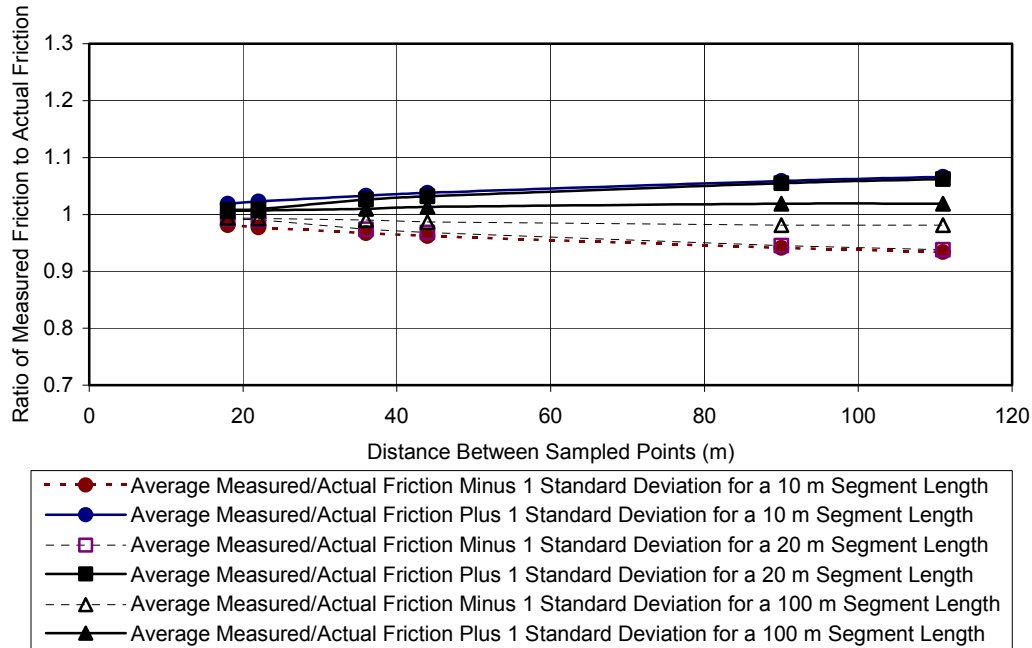


Figure 3.2: Variation in the Measured/Actual Friction Factor Ratio for a Total Sampled Distance of 5 km: Plus and Minus One Standard Deviation from the Mean

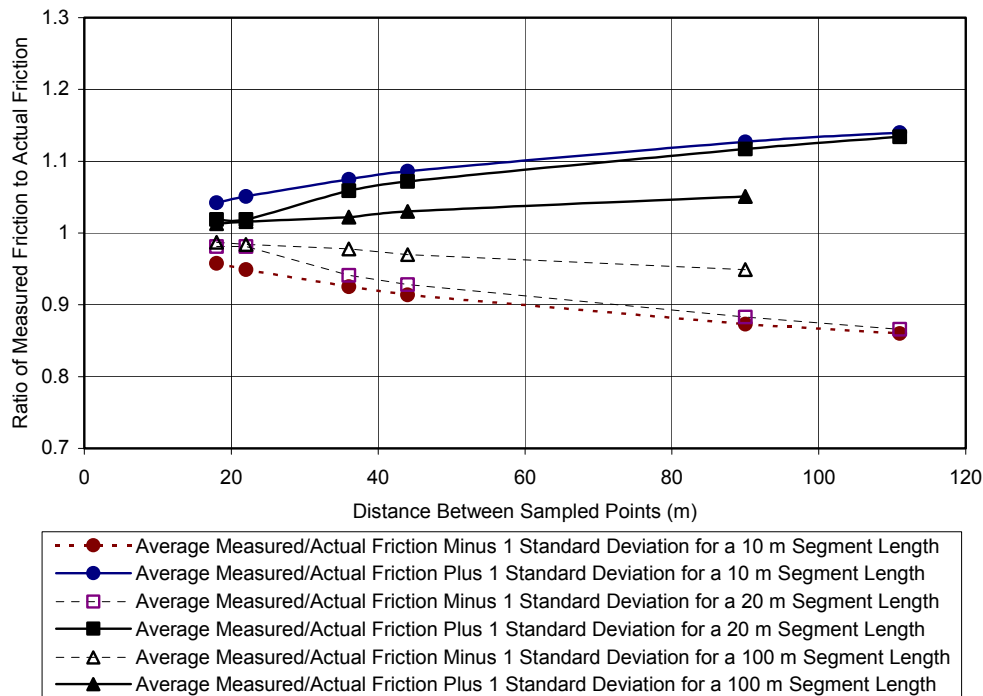


Figure 3.3: Variation in the Measured/Actual Friction Factor Ratio for a Total Sampled Distance of 1 km: Plus and Minus One Standard Deviation from the Mean

3.2.4 Sampling Error at Two Standard Deviations from the Mean

The results are plotted in Figures 3.4 and 3.5 for total sampled distances of 5 and 1 km respectively. The same trends described in section 3.2.3 with respect to the effect of sampling interval, segment length and total sampled distance are evident in these results.

As expected, the $\mu_{\text{measured}} / \mu_{\text{actual}}$ ratio was increased (showing that the sampling error was increased) at two standard deviations from the mean, compared to one standard deviation from the mean.

These results are discussed further in section 3.3.

3.3 Implications for Field Measurements

3.3.1 Measuring the Friction Factor on Average

Aliasing will occur, which has the potential to introduce significant errors. The sampling interval should be selected based on the following:

- (a) minimum segment length of interest;
- (b) surface condition variability that is expected; and
- (c) maximum error that is permissible.

The analyses suggested that sampling intervals should be no more than about 20 to 30 percent of the segment length to keep sampling errors less than 1 to 5 percent.

3.3.2 Friction Factor Variability

Randomness in the road surfaces will introduce variations in the measured friction factor. The magnitude of the variations is governed by the following:

- (a) sampling interval;
- (b) segment length of interest;
- (c) total sampled distance; and
- (d) confidence level (e.g., one vs. two standard deviations from the mean).

For sampling intervals that are in the range of about 20 to 30 percent of the segment length, the analyses suggested that randomness will introduce variations of about +/- 1 and +/- 2 percent at one and two standard deviations from the mean, respectively.

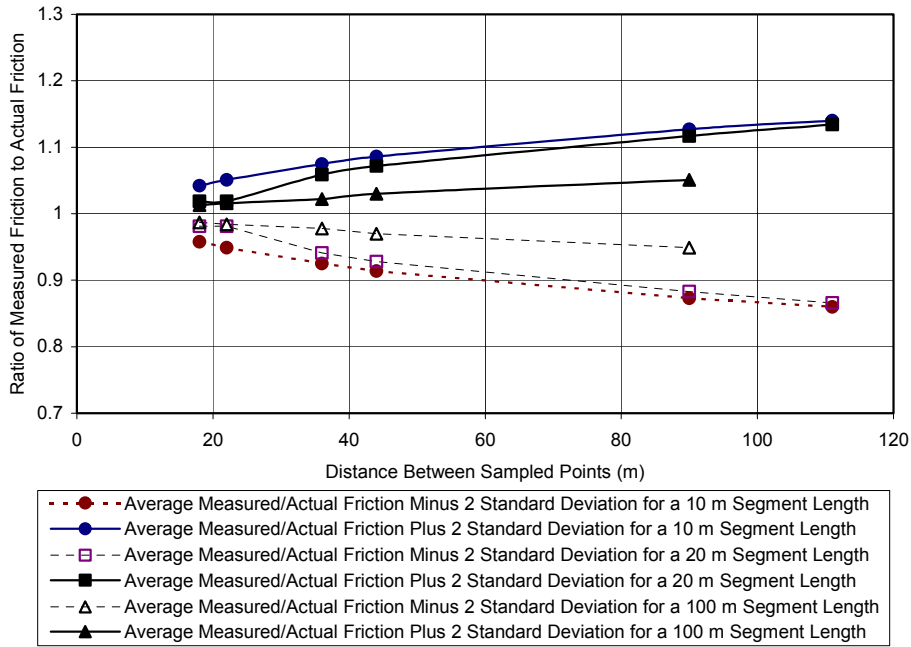


Figure 3.4: Variation in the Measured/Actual Friction Factor Ratio for a Total Sampled Distance of 5 km: Plus and Minus Two Standard Deviations from the Mean

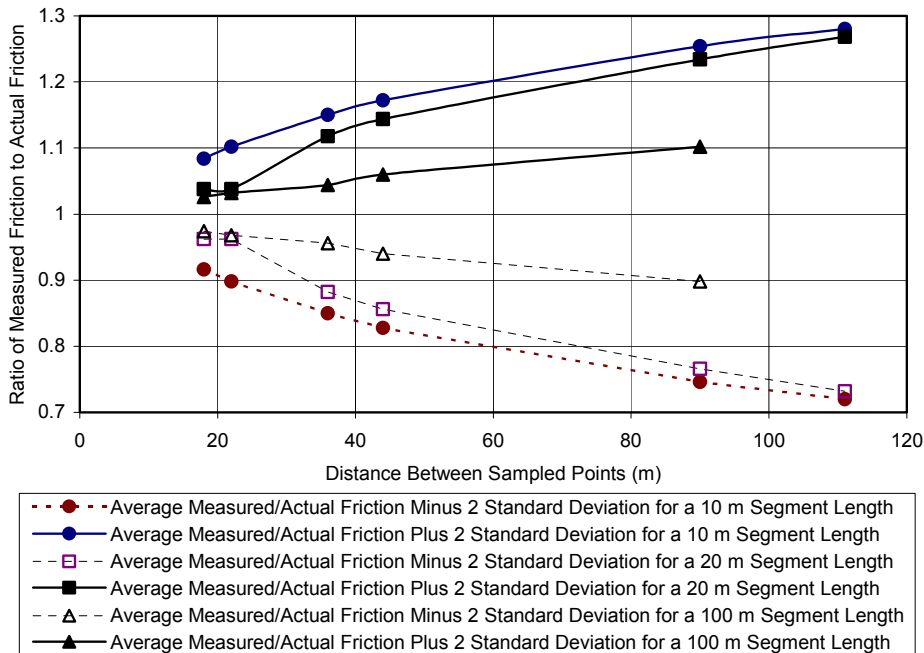


Figure 3.5: Variation in the Measured/Actual Friction Factor Ratio for a Total Sampled Distance of 1 km: Plus and Minus Two Standard Deviations from the Mean

4. MEASURING FRICTION OVER SHORT TRACK LENGTHS WITH VARIABLE CONDITIONS

These analyses were done to investigate potential sampling errors for short distances. These are of principal interest for evaluating suitable sampling techniques for a criterion based on the minimum friction of a short section of pavement (section 1).

All analyses were done for a total sampled distance of 100 m.

4.1 Analysis Approach and Scope

Deterministic analyses were done by comparing the average friction factor that a device would measure (μ_{measured}) over a 100 m long section, with the actual friction factor (μ_{actual}) for that section. As was done for the previously described analyses, it was assumed that the device would:

- (a) measure the friction level of each surface condition (wet pavement, dry pavement, slush, ice, and packed snow) perfectly; and
- (b) measure the friction factor of the surface condition that was directly under the centre of the tire in cases where the tire was at the boundary between two different surface conditions.

Two surface condition sequences were analyzed (Table 4.1). Both sequences were presumed to repeat regularly (Table 4.2). The friction factors assigned to the surface conditions in the sequences (Table 4.1) are representative values.

Table 4.1: Surface Condition Sequences Analyzed

Sequence No. 1		Sequence No. 2	
Surface Condition	Friction Factor	Surface Condition	Friction Factor
Slush	0.3	Slush	0.3
Wet Pavement	0.5	Dry Pavement	0.8
Ice	0.1	Ice	0.1
Wet Pavement	0.5	Dry Pavement	0.8
Packed Snow	0.25	Packed Snow	0.25

Table 4.2: Road Surface Condition Profile for the 100 m Distance Analyzed

Surface Sequence No. (Table 4.1)	Segment Length (m)	Road Surface Condition Profile	
		Distance from Start (m)	Surface Condition
1	10	0 to 10	slush
		>10 to 20	wet pavement
		>20 to 30	ice
		> 30 to 40	wet pavement
		>40 to 50	packed snow
		Pattern repeats for the next 50 m	
1	20	0 to 20	slush
		>20 to 40	wet pavement
		>40 to 60	ice
		> 60 to 80	wet pavement
		>80 to 100	packed snow
2	10	0 to 10	slush
		>10 to 20	wet pavement
		>20 to 30	ice
		> 30 to 40	wet pavement
		>40 to 50	packed snow
		Pattern repeats for the next 50 m	
2	20	0 to 20	slush
		>20 to 40	wet pavement
		>40 to 60	ice
		> 60 to 80	wet pavement
		>80 to 100	packed snow

The cases analyzed are summarized in Table 4.3.

Table 4.3: Short Distance Sampling: Summary of Cases Analyzed

Sequence No.	Segment Length (m)	Total Distance Sampled (m)	Sampling Interval (m)
1	10	100	1 to 25 (continuous variation)
	20	100	1 to 50 (continuous variation)
2	10	100	1 to 25 (continuous variation)

4.2 Results

4.2.1 Effect of Sampling Interval for Surface Sequences 1 and 2

Figures 4.1 and 4.2 show the effect of the sampling interval on the measured friction factor for surface sequences 1 and 2, respectively. The analyses showed that:

- (a) Effect of Sampling Interval – the effect of the sampling interval was quite variable, which made it difficult to draw general conclusions.

This is due to the effects of aliasing. Infrequent sampling (i.e., more infrequent than about 5 m) is likely to produce large errors in the mean friction value (Figures 4.1 and 4.2) unless the sampling frequency happens to be “tuned” to the frequency of the segment lengths for the different surfaces.

Of course, the sampling intervals that are “favourable” and “unfavourable” will depend on the road conditions, which are likely to vary significantly during wintertime. Hence, one cannot rely on the sampling program being “tuned” to the surface conditions on the road. Devices and sampling programs should be set up to avoid this range.

For both surface sequences, errors of more than 10 percent will be introduced if the distance between the measurement points is more than 5 m, which represents 50 percent of the segment length (i.e., 10 m) for these analyses.

- (b) Effect of Surface Sequence – although there were variations between the results for Sequences 1 and 2, these differences were relatively small. For both surface sequences, the analyses suggested that the sampling interval should be less than at least 5 m (for the 10 m segment length that was analyzed) to obtain the true friction factor within about +/- 10 percent.

4.2.2 Effect of Sampling Interval for Segment Lengths of 10 and 20 m

Figure 4.3 shows the effect of sampling interval on the measured friction factor for segment lengths of 10 and 20 m. Because the previous analyses showed that the results did not depend strongly on whether Sequence 1 or 2 was considered, only Sequence 1 was evaluated. The results show that:

- (a) Effect of Sampling Interval – as for the previous results (described in Section 4.2.1), the effect of the sampling interval is quite variable, which reflects the effects of aliasing.
- (b) Effect of Segment Length – segment length is a very important parameter. More frequent sampling (as reflected by smaller sampling intervals) is required for a shorter segment length. This follows the expected trend.

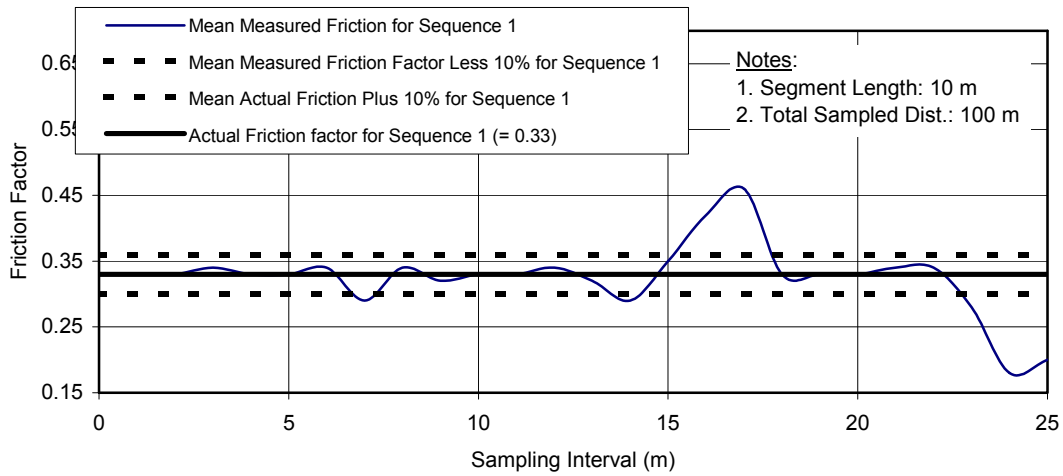


Figure 4.1: Effect of Sampling Interval on the Friction Factor: Surface Sequence No. 1

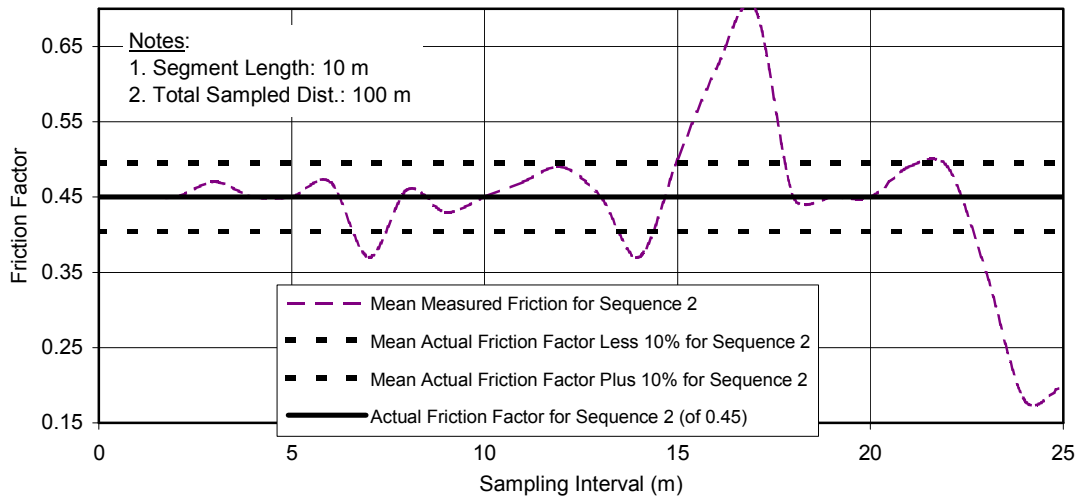


Figure 4.2: Effect of Sampling Interval on the Friction Factor: Surface Sequence No. 2

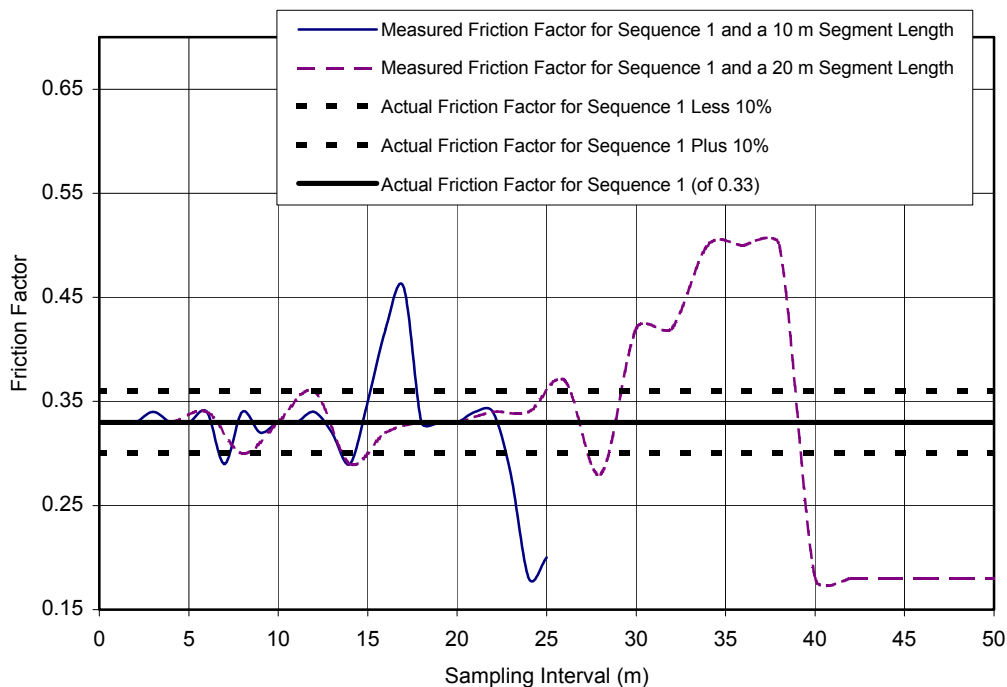


Figure 4.3: Effect of Sampling Interval and Segment Length for Sequence 1

4.2.3 Non-Dimensional Results

The measured friction factor has been shown to be related to both the sampling interval and the segment length.

To make the analyses more general, the sampling interval/segment length ratio was used as an index to non-dimensionalize the results. The analyses clearly showed that the measured friction factor was related to this ratio for both surface sequences and both segment lengths investigated (Figure 4.4).

Although minor variations were predicted for the 10 and 20 m segment lengths, and Surface Sequence 1 versus 2, the analyses showed that in general, the sampling interval should be less than about 50 percent of the segment length in the surface sequence to measure the true friction factor within about +/- 10 percent.

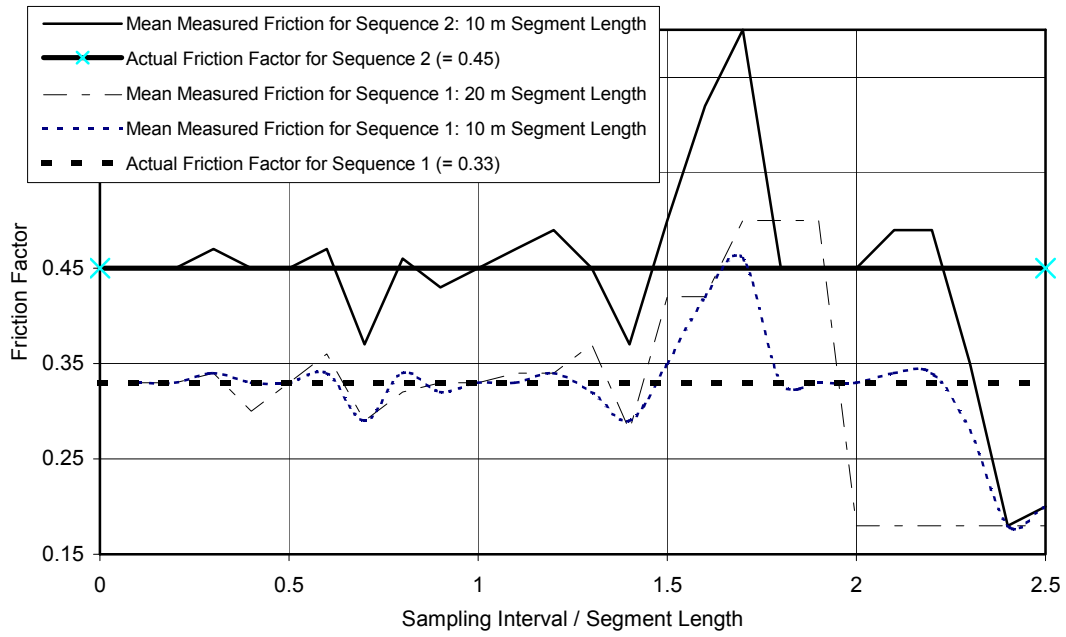


Figure 4.4: Non-Dimensional Results

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

5.1.1 Basis for Conclusions

Numerical analyses were conducted for a wide range of potential road surfaces to compare the friction factor that a device would be expected to measure with the actual friction factor.

5.1.2 Measuring the Average Friction Factor Along the Length of a Runway or Road

The sampling interval should be selected based on the following:

- (a) the expected variability in surface conditions with respect to friction levels, and also the relative proportion of the overall road or runway length covered by each.
- (b) the minimum length over which friction factor variations (e.g., produced by differences in surface conditions) are of concern. This was termed the minimum segment length of interest in this project.

This is related to the expected variability in surface conditions to some extent. As an example, let us consider a 1 km long road that is 20% ice-covered. The overall total length of ice is thus 200 m. However, this 200 m may be distributed in various ways. There could be one 200 m long continuous ice strip; this would have a “segment length” of 200 m. Alternatively, there could be twenty 10 m long ice strips; these would have a “segment length” of 10 m.

- (c) the maximum error that is acceptable (with error being defined as the variation between the average friction factor obtained from the measurements vs. the actual average friction factor for the whole length of road or runway).

A number of cases were analyzed in this project as follows:

- (a) minimum length – lengths of 10, 20, and 100 m were analyzed
- (b) expected variability – the following surface condition distributions were analyzed:
 - (i) Wet pavement with a friction factor of 0.5 – presumed to cover 40% of total runway or road length
 - (ii) Packed snow with a friction factor of 0.25 – presumed to cover 20% of total runway or road
 - (iii) Slush with a friction factor of 0.3 – presumed to cover 20% of total runway or road
 - (iv) Ice with a friction factor of 0.1 – presumed to cover 20% of total runway or road

For the above ranges, the analyses suggested that sampling intervals (defined as the distance between individual friction factor measurements) should be no more than about 20 to 30 percent of the minimum length of concern (item (a) above) to keep sampling errors less than 1 to 5 percent.

5.1.3 Friction Factor Variability

Randomness in the road surfaces will introduce variations in the measured friction factor. The magnitude of the variations is governed by the following:

- (a) sampling interval;
- (b) segment length of interest;
- (c) total sampled distance; and
- (d) confidence level (e.g., one vs. two standard deviations from the mean).

For sampling intervals that are in the range of about 20 to 30 percent of the segment length, the analyses suggested that randomness will introduce variations of about +/- 1 and +/- 2 percent at one and two standard deviations from the mean, respectively.

5.2 Recommendations

Continued work would be useful in the following areas:

- (a) The ranges of cases analyzed should be reviewed and compared to field information, if available.
- (b) Maximum permissible errors should be specified so that the analyses could be focused on that range.