Analyzing and Forecasting Credit Ratings: Some Canadian Evidence

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Notice to the reader

One of the main objectives of this paper is to assess the usefulness of econometric models in forecasting credit ratings. Consequently, the forecasting approach is purely technical. The objective is not to get the most accurate forecast by province by taking into account information that is often difficult to quantify.

Concerning relative provincial indebtedness, Statistics Canada's Financial Management System provides the most comparable figures. However, these are not entirely comparable because of the divergent accounting of Crown Corporation self-financing debt.

ABSTRACT

Can econometric tools be used successfully to predict recent downgrades or upgrades? The paper first identifies -- in constructing a credit rating model -- a relatively small number of explanatory variables from an extensive list of highly subjective criteria. Having selected a potential list of variables, an estimation procedure is then outlined that respects the ordinal but not necessarily linear nature of credit ratings. Next, the success of ordered-response models is demonstrated by replicating 75 per cent of the credit ratings assigned to the provinces by Standard and Poor's from 1976 to 1995. In a major innovation, out-of-sample forecasts are conducted over the 1996-99 period. Results indicate that the models display considerable promise by successfully predicting rating changes in various provinces. Estimates indicate that a 25-percentage point deterioration in a province's net debt-to-GDP ratio -- starting from an initial net asset position -- will result in a rating downgrade of 3 or 4 notches depending on the province's credit quality. Moving across the rating categories, there does not appear to be a debt level that triggers sudden rating changes. Finally, when the provinces are grouped according to their credit quality, an increased sensitivity to rating downgrades is identified at relatively low levels of indebtedness.

RÉSUMÉ

Les outils économétriques permettent-ils de prévoir les changements récents, à la hausse ou à la baisse, des notations de crédit? Existe-t-il un seuil d'endettement qui entraîne des changements soudains de notation de crédit? L'auteur commence par identifier — afin de construire un modèle de notation de crédit — un nombre relativement restreint de variables économiques et financières à partir d'une liste exhaustive de critères subjectifs. Après avoir choisi une liste de variables potentielles, une méthode d'estimation est utilisée, respectant le caractère ordinal, mais pas nécessairement linéaire, des cotes de crédit. L'efficacité de la méthode des logits ordonnés est ensuite démontrée en reproduisant correctement 75 p. 100 des notations octroyées par Standard and Poor's aux provinces canadiennes entre 1976 et 1996. Comme innovation majeure, des prévisions sont ensuite effectuées à l'extérieur de la période échantillonnale pour les années 1996 à 1999. Les résultats se sont avérés extrêmement prometteurs, puisqu'il fut possible de prévoir de façon exacte plusieurs changements récents de notation de crédit dans diverses provinces. Partant d'une position initiale d'actif net, les estimations indiquent qu'une détérioration de 25 points de pourcentage du ratio dette nette au PIB d'une province entraîne une décote de trois ou quatre crans tout dépendant de la qualité de crédit de la province. Lorsqu'on passe d'une catégorie à une autre, il ne semble pas y avoir de niveau d'endettement qui déclenche des modifications soudaines de la cote d'un emprunteur public. Finalement, lorsqu'on regroupe les provinces d'après la qualité de leurs dossiers de crédit, on détecte une sensibilité accrue aux révisions de la cote de crédit à des niveaux d'endettement relativement faibles.

Introduction and summary

In recent years, the importance of bond ratings -- the risk assessments assigned by credit rating agencies to government bonds -- has increased significantly. This was the case particularly when the state of public finances was steadily deteriorating. In Canada, the high level of public sector indebtedness earlier this decade triggered a round of downgrades in several jurisdictions. Based on the Standard & Poor's (S&P) categorization, all provinces except New Brunswick currently have a lower rating than in the early 1980s. The worst case has been Saskatchewan, which was downgraded five notches by S&P and three times by Moody's Investors Service (Moody's) from 1986 to 1992.

This paper attempts to answer the following two questions: Can recent downgrades or upgrades be predicted using econometric tools and a limited number of key macroeconomic variables? What level of indebtedness should a province reach in order to maximize the probability of being upgraded?

This paper has three main objectives. First, by using ordered-response models, we tested the reliability of econometric models in predicting the long-term debt ratings of Canadian provinces as attributed by S&P. Within the sample period and under different pooling samples of provinces, **we successfully replicated three-quarters of the categories** assigned to the Canadian provinces by S&P between 1976 and 1995. In addition, for each risk category, the predicted classification never incurred a forecasting error of more than one increment, validating the use of a statistical model for analyzing and forecasting credit ratings.

Second, rating decisions are forward-looking by nature as the agency assesses the potential future risk of default. Consequently, we performed out-of-sample forecasts over the 1996-1999 period using information contained in the 1997-98 provincial budgets and more specifically medium-term plans. To our knowledge, this is the first time such an exercise has been performed. Ordered-response models appear to be very promising as it was possible, among other outcomes, to forecast the two successive upgrades of Saskatchewan in 1996 and 1997 and the upgrade of Alberta in October 1997.

Finally, we investigated the relationship between credit ratings and level of indebtedness. More precisely, we tested the possibility of debt level threshold effects. Our results suggest that non-linearity -- or increased sensitivity to changing levels of indebtedness -- occurred only at lower levels of indebtedness.

This paper is organized as follows: Section 1 discusses the selection of variables while Section 2 provides an overview of the econometric method used. Empirical results are presented in Section 3. To evaluate our models, in-sample and rolling out-of-sample forecasts are provided in Sections 4 and 5, respectively. Section 6 presents an analysis of the link between credit ratings and net debt. Finally, Section 7 suggests a number of key conclusions.

1. Considered explanatory variables

The selection of potential explanatory variables for inclusion in the model was guided by various S&P reports on Canadian provinces. Choosing a parsimonious set of indicators from these reports posed a major challenge. First, in its statement of rating criteria, the agency lists an extensive number of **financial**, **policy, economic** and **demographic** indicators underlying its decisions.[1] However, little guidance is provided as to the relative weights assigned to each indicator. In our initial specification, only key explanatory variables were retained from the four main categories listed above. Table 1 presents these variables. In addition, assigning bond ratings is highly subjective. Some of the criteria mentioned, such as degree of economic diversification and factors related to political uncertainty, are hardly quantifiable. To overcome this problem, the best available proxies have been used.

In the category of financial indicators, net direct debt on a Financial Management System (FMS) basis (expressed as a proportion to GDP) has been used to account for level of indebtedness. Debt charges as a proportion of total revenue, the deficit-to-GDP ratio, federal cash transfers as a percentage of total revenue and per capita program expenditures have also been used. The tax effort index produced by Finance Canada was used as a policy indicator to measure the various tax regimes.

As far as economic indicators are concerned, two provincial tax bases were retained: per capita nominal provincial GDP and the fiscal capacity index as measured by Finance Canada. GDP growth, employment growth and the unemployment rate were also used. To approximate the degree of economic diversification, the ratio of provincial to national GDP growth was tested as well as employment in the manufacturing sector as a share of total employment. Theoretically, we would expect that a fully diversified economy would grow in line with the national economy, implying that province-specific idiosyncratic risk is more diversified, reflecting the nature of individual provinces' industrial structure. Provinces strongly reliant on idiosyncratic production sectors such as natural resources will experience bigger swings in their growth rates. Finally, we included population growth as our demographic indicator.

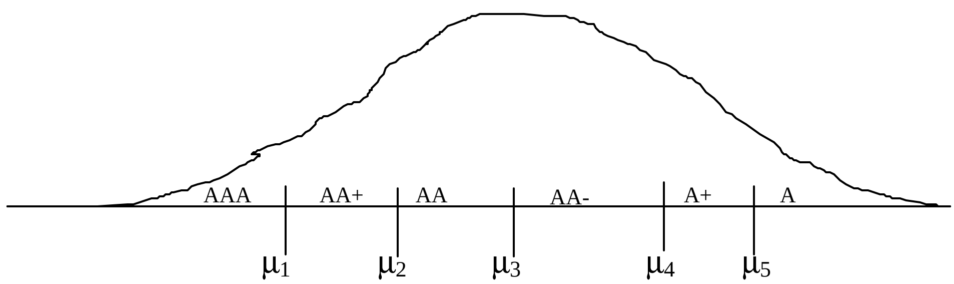
[1] See S&P, *Canadian Focus* (1997).

Table 1: List of potential variables

POTENTIAL VARIABLES	SOURCE
Financial Indicators	
Net direct debt (as a share of GDP)	Financial Management System
Deficit (as a share of GDP)	Financial Management System
Debt charges (per cent of total revenue)	Financial Management System
Federal cash transfers (per cent of total revenue)	Financial Management System
Program expenditures per capita	Financial Management System
Policy Indicator	
Tax effort index (proxy for tax regimes)	Finance Canada
Economic Indicators	
Nominal GDP per capita	Provincial Economic Accounts
Fiscal capacity index	Finance Canada
GDP growth	Provincial Economic Accounts
Employment growth	Provincial Economic Accounts
Unemployment rate	Provincial Economic Accounts
Employment in the manufacturing sector as a share of total employment	Provincial Economic Accounts
Ratio of provincial to national GDP growth	National and Provincial Economic Accounts
Demographic Indicator	
Population growth	Provincial Economic Accounts

2. Estimation methodology

To estimate a model with the credit rating assignment as the dependent variable, it is important to consider the ordinal nature of the variable. For estimation purposes, S&P credit ratings[2] were assigned numerical values to capture the ordinal relationships (e.g., AAA = 1, AA+ = 2, AA = 3, and so on). An estimation technique such as ordinary least-squares (OLS) implicitly assumes that the underlying dependent variable, in this case the degree of default risk, is ranked into equally-spaced discrete intervals. However, it is unlikely that the risk differential between two adjacent categories is distributed equally (e.g., that the difference between AA+ and AA is equivalent to the one between AA- and A+). The chart below is a hypothetical illustration of this point.



There is a strong presumption that the dependent variable can be evaluated on an ordinal but not necessarily linear scale. For example, bonds rated AAA by S&P are considered more secure than bonds rated AA+, but we have no quantitative measure about their relative risk. As a result, the use of OLS could bias the estimates, imposing false restrictions on the data-generating process.[3] Ordered-response models provide a means to exploit the ordering information without imposing any of these restrictions. The ordered logit model is built upon the following latent regression:

$$Y_i^* = X_i b + e_i \qquad (1)$$

[2] We retained S&P credit ratings, as opposed to Moody's, for the following reason: both agencies introduced modifiers to provide a finer classification of ratings. S&P introduced "+ -" signs to their ratings in 1974 while Moody's introduced numerical modifiers"1,2,3" in 1982. The introduction of modifiers creates a break in the index of measurement and in the dependent variable. To enhance estimation powers, S&P ratings are used.

[3] For more details, see McKelvey and Zavoina (1975).

where, Y* is a continuous unobservable index of risk measurement and e_i is a vector of error terms assumed to be distributed according to a logistic function. Despite the fact that Y* is unobservable, we know in which bond rating category it belongs as each category is related to the index in the following way:

$Z_i = 1 = AAA$, if $Y^* \leq \mu_1$

$Z_i = 2 = AA+$, if $\mu_1 < Y^* \leq \mu_2$

$Z_i = 3 = AA$, if $\mu_2 < Y^* \leq \mu_3$

:

$Z_i = 8 = BBB+$, if $\mu_m < Y^*$

In the above definitions, the μ's represent measures of partition boundaries, or cut-off points, delimiting the range of S&P credit ratings assigned to the Canadian provinces over the sample period. By imposing that $\mu_1 = -\infty$, $\mu_m = \infty$ and, $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_m$, the cut-off point parameters can be estimated along with the vector of b coefficients. The probability of observing a specific rating depends on where the conditional mean ($X_i\beta$) lies in relation with partition boundaries. Using a logistic distribution,[4] the conditional probabilities of obtaining each rating category could be evaluated using the following formulae:

$$\Pr\left(X_i b + e \leq m_1\right) = \frac{1}{1 + e^{X_i b - m_1}} \tag{2}$$

$$\Pr\left(m_{j-1} \langle X_i b + e \leq m_j\right) = \frac{1}{1 + e^{X_i b - m_j}} - \frac{1}{1 + e^{X_i b - m_{j-1}}} \qquad \textit{for } j = 2,3,\ldots,m\text{-}1 \tag{3}$$

$$\Pr\left(m_m \langle X_i b + e\right) = 1 - \frac{1}{1 + e^{X_i b - m_m}} \tag{4}$$

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[4] There are no theoretical reasons for the selection of a logistic distribution. Maddala (1983) argued that in small samples, estimation results are less sensitive to distribution choice due to the lack of information at the end tails.

The log likelihood function of the model could be written as:

$$\log L = \sum_{i=1}^{n}\left[Z_{i1}\log\left(\frac{1}{1+e^{X_i b - m_1}}\right) + \sum_{j=2}^{m-1} Z_{ij}\log\left(\frac{1}{1+e^{X_i b - m_j}} - \frac{1}{1+e^{X_i b - m_{j-1}}}\right) + Z_{im}\log\left(\frac{e^{X_i b - m_m}}{1+e^{X_i b - m_1}}\right)\right] \quad (5)$$

Where i = 1,2,…,n represents the number of observations, and m the number of categories. Z_{ij} is an indicator variable taking the value 1 if observation i falls in the jth category and 0 otherwise. The estimation could then be performed by maximum likelihood, and the non-linear function (5) is solved iteratively by the Newton-Raphson procedure.

It should be noted that the use of ordered-response models requires a sufficient number of observations to ensure convergence. Furthermore, maximum likelihood properties are valid only asymptotically. The fact that several series could not be reconciled prior to 1976 at the provincial level (Labour Force statistics in particular) and that FMS data for the provincial-local sector are available up to 1994-95 has constrained the sample period from 1976 to 1995. This has prevented a by-province analysis on an equation-by-equation basis, leaving no other option but to pool provincial S&P ratings.[5] Consequently, the estimation results must be interpreted with care due to potential small sample biases.

3. Empirical results

As the objective is to find a parsimonious set of predictor variables, we used two interactive stepwise procedures to govern the selection of variables that appear in the final specification. First, each potential indicator from Table 1 was regressed solely against the dependent variable. Starting from the most significant contributor, all remaining series were tested again in a forward stepwise manner to look for any additional relevant information. Next, to prevent the mis-selection pitfall encountered in using this type of selection, a backward stepwise elimination procedure was also used starting from the complete list of variables. At each step, variables significant at a 90-per-cent level of confidence were retained. Finally, only variables common to the combined selection procedures were used in the analysis. The retained specification for the nine provinces is:

$$CRATING_{it} = ß_0 + ß_1 NDEBT_{it} + ß_2 FCAP_{it} + ß_3 FTRAN_{it} + ß_4 TAXEF_{it} + ß_5 POPG_{it} + ß_6 ED_{it} + e_{it}$$

[5] It should be noted that Prince Edward Island is not rated by S&P and is therefore not included in our analysis.

Where:

NDEBT:	net direct debt as a share of GDP
FCAP:	Finance Canada's fiscal capacity index
FTRAN:	federal transfers as a share of total revenue
TAXEF:	Finance Canada's tax effort index
POPG:	population growth
ED:	the proxy for economic diversification as measured by employment in the manufacturing sector over total employment.

Estimated coefficients of the ordered-response model must be interpreted with care[6] as the signs of the *ß's* show only the direction of the change in the probability of falling in the end-point rankings (only AAA and BBB+). For a positive variable, a plus sign implies that a marginal increase of the factor lowers the probability of being rated AAA and increases the probability of getting the BBB+ rating. It is important to note that the sign of the estimated coefficients says nothing about the direction of the change in the probability of falling in each in-between category. The impact on the probability of falling in any middle ranking is given by:

$$\frac{\partial \Pr\left(CRATING_{it}\right)}{\partial X_{it}} = \frac{\partial F\left(X_{it} b - m_j\right)}{\partial X_{it}} - \frac{\partial F\left(X_{it} b - m_{j-1}\right)}{\partial X_{it}} \qquad for \quad j = 2, 3, \ldots m-1$$

and thus depends on the difference between the two density functions.

Note that coefficient values cannot be interpreted as the marginal effect on credit ratings because the **ß**'s are weighted by the density functions which depend on the values of all the regressors and the distribution of the error term. Also, the magnitudes of the coefficients are not directly comparable across models since the logistic distributions have different variances.

Concerning the signs of the parameters, it is expected that net debt (NDEBT), being a negative variable, will have a negative sign. An increase in net debt, or decrease in net asset, increases the probability of being in the lowest category (BBB+) and lowers the probability of accessing the top category (AAA). A higher federal transfers-to-revenue ratio (FTRAN) means heavier reliance on the federal government and less fiscal flexibility. Accordingly, a positive sign should be associated with that variable. Negative signs are expected for the fiscal capacity index and the economic diversification indicator. The larger the potential tax base of a province, the greater its ability to repay debt. A more diversified economy implies more stable tax bases.

The a priori sign for population growth and the tax effort index is more difficult to assess.

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[6] See Greene (1997, chapter 19) for a discussion.

An increase in population affects transfer payments received under programs such as Equalization and Canada Health and Social Transfer. For "have-not" provinces, this could mean a revenue shortfall in the event of a sudden population decline. Population growth can also have an indirect incidence on both revenues and expenditures. An increase in population may imply growth in the provincial tax base, but also increasing costs for public services, especially if this results in an aging population. The combined effects imply an ambiguous sign. Concerning the tax index, a heavier tax burden can considerably limit fiscal capacities and may also have perverse effects on the economy. As a result, it should generally be associated with a positive sign. However, as taxes are related to level of indebtedness by the intertemporel budgetary constraint, a decrease in the tax effort may result in higher net debt and may well imply a negative sign.

Estimates were obtained under four different groupings. The estimation results are shown in Tables 2, 3, 4 and 5. We have first estimated the nine provinces pooled together. As there is no test of fixed or random effects to distinguish province-specific effects associated with ordered qualitative models, it is better to look at smaller poolings and relax the implicit assumption of the equality of coefficients among all provinces. Hence, we estimated sub-groups of provinces according to S&P's assessment of their relative "credit characteristics." The first group includes Newfoundland, Nova Scotia and Saskatchewan. These provinces are characterized by resource-based economies with higher vulnerability to market conditions and less fiscal flexibility, being more dependent on federal transfers. As these provinces have historically received lower ratings, they form the higher-risk group. The moderate-risk group includes New Brunswick, Manitoba and Quebec, while Ontario, Alberta and British Columbia form the lower-risk group.

For the nine-province model (Table 2), all the coefficients have the expected sign. For the higher-risk cohort (Table 3), the proxy for economic diversification has the wrong sign. This seems to reflect a multicollinearity problem. For example, removing the indebtedness variable changes the sign of the economic diversification coefficient to negative. Employment in the manufacturing sector encompasses an important cyclical component, significantly increasing the correlation with net debt. A better measure of economic diversification should preclude such a strong correlation. However, our main objective being the ability to predict credit ratings, the variable was retained as it improves significantly the overall fit.[7]

For the New Brunswick-Quebec-Manitoba group (Table 4), the signs of the parameters are as expected. However, four variables turned out to be insignificant. As a joint test of the null hypothesis (H_0:Aß=0~?^{2}(4)) for these indicators is rejected at 5 per cent critical values (P>|?^{2}|=0.049), they were all retained. Finally, as opposed to the other poolings, the tax effort index is negatively related to credit ratings for the end-point rankings in the lower-risk group.

[7] Excluding the variable results in a Pseudo-R^2 of 0.41 compared to 0.47 when it is included.

Table 2: Ordered logit estimates, nine-province model (1976-1995)

VARIABLES	COEFFICIENT ESTIMATES	STANDARD ERROR	P>|Z|
β_1 (NDEBT)	-0.22	0.02	0.00
β_2 (FCAP)	-0.04	0.01	0.00
β_3 (FTRAN)	0.10	0.03	0.00
β_4 (TAXEF)	0.04	0.02	0.04
β_5 (POPG)	-0.84	0.25	0.00
β_6 (ED)	-0.68	0.08	0.00
μ_1	5.01	2.79	
μ_2	7.01	2.81	
μ_3	10.02	2.91	
μ_4	12.71	2.96	
μ_5	15.66	3.04	
μ_6	17.16	3.09	
μ_7	20.31	3.20	
NUMBER OF OBSERVATIONS	180	H_0:Aμ=0~?^{2}(5)	P>| ?^{2} |
PSEUDO-R^2	0.47	12.33	0.031
LOG LIKELIHOOD	-189.05		

μ_j = cut-off point estimates.

Table 3: Estimation results for higher-risk provinces (Newfoundland, Nova Scotia, Saskatchewan)

VARIABLES	COEFFICIENT ESTIMATES	STANDARD ERROR	P>|Z|
β_1 (NETDEBT)	-0.43	0.07	0.00
β_2 (FCAP)	-0.19	0.09	0.04
β_3 (FTRAN)	0.54	0.15	0.00
β_4 (TAXEF)	0.10	0.05	0.04
β_5 (POPG)	-2.31	0.86	0.01
β_6 (ED)	6.38	1.68	0.00
μ_1	59.62	15.59	
μ_2	62.24	15.85	
μ_3	63.27	15.91	
μ_4	65.63	16.14	
μ_5	69.16	16.42	
μ_6	74.30	16.95	
NUMBER OF OBSERVATIONS	60	H_0:Aμ=0~?^{2}(4)	P>| ?^{2}|
PSEUDO-R^2	0.50	11.39	0.023
LOG LIKELIHOOD	-51.21		

Table 4: Estimation results for moderate-risk provinces (Manitoba, New Brunswick, Quebec)

VARIABLES	COEFFICIENT ESTIMATES	STANDARD ERROR	P>|Z|
β_1 (NETDEBT)	*-0.37*	*0.07*	*0.00*
β_2 (FCAP)	*-0.07*	*0.07*	*0.59*
β_3 (FTRAN)	*0.12*	*0.10*	*0.51*
β_4 (TAXEF)	*0.07*	*0.05*	*0.36*
β_5 (POPG)	*0.45*	*0.86*	*0.52*
β_6 (ED)	*-0.13*	*1.68*	*0.05*
μ_1	*8.62*	*21.83*	
μ_2	*12.55*	*21.91*	
NUMBER OF OBSERVATIONS	60	H_0:Aß=0~?^{2}(4)	P>| ?^{2}|
PSEUDO-R^2	0.48	9.53	0.049
LOG LIKELIHOOD	-32.87		

Table 5: Estimation results for lower-risk provinces (Ontario, Alberta, British Columbia)

VARIABLES	COEFFICIENT ESTIMATES	STANDARD ERROR	P>|Z|
β_1 (NETDEBT)	*-0.49*	*0.11*	*0.00*
β_2 (FCAP)	*-0.08*	*0.03*	*0.00*
β_3 (FTRAN)	*0.52*	*0.16*	*0.00*
β_4 (TAXEF)	*-0.09*	*0.05*	*0.08*
β_5 (POPG)	*-1.11*	*0.21*	*0.03*
β_6 (ED)	*-1.20*	*0.24*	*0.00*
μ_1	*1.01*	*6.24*	
μ_2	*3.98*	*6.35*	
μ_3	*9.75*	*6.51*	
NUMBER OF OBSERVATIONS	60	H_0:Aμ=0~?^{2}(1)	P>| ?^{2}|
PSEUDO-R^2	0.44	3.34	0.067
LOG LIKELIHOOD	-41.61		

Coefficients related to the partition boundaries **μ**'s correspond to values of the conditional means delimiting each category. A meaningful test with the **μ**'s is to verify if the creditworthiness index is ranked into equally spaced discrete intervals. This implies performing the following joint tests: $(\mu_1 - \mu_2) = (\mu_2 - \mu_3)$, $(\mu_2 - \mu_3) = (\mu_3 - \mu_4)$,.., $(\mu_{m-2} - \mu_{m-1}) = (\mu_{m-1} - \mu_m)$. This can also be rewritten as a linear hypothesis for the (m-1)×1 vector of μ's in the following way:

$$H_0 : A\mu = 0 = \begin{pmatrix} 1 & -2 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -2 & 1 & 0 & \dots & 0 \\ : & : & : & : & : & : & : \\ 0 & 0 & 0 & \dots & 1 & -2 & 1 \end{pmatrix} \qquad \begin{pmatrix} \mu_1 \\ \mu_2 \\ : \\ \mu_{m-1} \end{pmatrix}$$

The test is asymptotically distributed as a ?2 with (m-3) degrees of freedom, where m equals the number of rating categories within the pooling. Rejection of the null hypothesis means two things. First, it implies that discreteness matters so that the use of OLS may generate important bias. Second, it indicates that the relative risk-differential as measured by two adjacent categories would not necessarily be equal. This has important implications since threshold effects, or non-linearities, can then be found between ratings and the set of explanatory variables without specifying any a priori nonlinear relationship.[8]

?2 tests were performed for each risk-cohort except for the moderate-risk pooling. For this sub-group, no degrees of freedom are available as the pooling contains only three categories of rating. To compare for risk-differential, a minimum of two pairs of adjacent categories are needed. The null hypothesis (H_0:Aμ=0) is rejected at 5 per cent critical values in the case of the nine-province pooling and the higher-risk group with p-values of 0.031 and 0.023, respectively (Tables 2 and 3). For lower-risk provinces (Table 5), a p-value of 0.067 indicates rejection at the 10-per-cent level.

3.1 Misspecification tests

Evaluation of the overall model specification is achieved by examining the residual properties. For standard linear or non-linear models, diagnostic tests are straightforward as residuals are easily available. In the case of ordered-response models, it is impossible to calculate the residuals from the latent model as the real continuous index of risk measurement Y^* cannot be observed directly. In this context, Gourieroux, Monfort, Renault and Trognon (1987) suggest the use of generalized residuals. These residuals are orthogonal to the exogenous variables and can be used in a variety of diagnostic tests in

[8] Section 6 presents a detailed analysis applied to net debt.

the form of regression-based likelihood maximum (LM) or conditional moment tests.[9] These residuals are obtained by deriving the conditional expectation of residuals from the latent model, conditional on the observed endogenous variables and the maximum likelihood parameter estimates. Accordingly, generalized residuals were computed using the following formula:

$$e_g = \sum_{j=1}^{8} \frac{f\left(X_i b - m_{j-1}\right) - f\left(X_i b - m_j\right)}{F\left(X_i b - m_j\right) - F\left(X_i b - m_{j-1}\right)} \qquad (6)$$

Where $f(\cdot)$ and $F(\cdot)$ are the probability density function and the cumulative distribution function, respectively. Misspecification tests were then performed by estimating auxiliary regressions of the type e_G = **Xß + Z?** + **v** or e^2_G = **s**^{2}**+ Z? + v**, with ε_G being the generalized residuals while Z varies according to the test performed. In the case of the Reset test, Z corresponds to the square of fitted values of the dependent variables and the LM statistic follows a $?^2$ distribution with 1 degree of freedom. Other misspecification tests are also reported in Table 6.

The results presented in Table 6 reveal evidence of autocorrelation for each pooling whereas signs of heteroskedasticity emerged in the case of the nine-province pooling and the moderate-risk sub-group. As decisions regarding credit ratings are likely to include both backward- and forward-looking components, the autocorrelation problem may result from lack of dynamic specification in the model. In addition, pooling data involving regional cross-sectional data is likely to involve contemporaneous correlation because shocks affecting economic activity in one region may affect another region as well, reflecting close economic inter-linkages.

In order to account for the potential lack of dynamics, we added all explanatory variables, lagged one year. This approach was not successful as tests performed with the transformed specification were still showing the presence of autocorrelation. Accounting for the source of the problems could improve the efficiency of the forecasts. However, adopting a spurious specification to correct for the nonspherical problem could be even worse. In the absence of straightforward non-parametric solutions for the problem, we decided not to impose any parametric corrections. However, no evidence of omitting important variables or misspecified functional form was detected as indicated by the non-rejection of Reset tests.

[9] See Murphy (1996) and Weiss (1997) for additional details.

TABLE 6: Misspecification LM tests*

	ε_G (NINE PROVINCES)	ε_G (HIGHER-RISK)	ε_G (MODERATE-RISK)	ε_G (LOWER-RISK)
WHITE HETEROSKEDASTICITY ?^{2}(13)	**0.01**	0.22	**0.02**	0.37
LM (RESET TYPE) ?^{2}(1)	0.24	0.81	0.17	0.62
ARCH ?^{2}(2)	0.11	0.69	0.13	0.41
SERIAL CORRELATION ?^{2}(2)	**0.00**	**0.00**	**0.00**	**0.02**

**Table 6 reports p-values from a c^2 distribution. P-values<.10 imply rejection of the null hypothesis at 90-per-cent level of confidence. Misspecification tests are derived from artificial LM-type regressions, using generalized residuals.*

4. In-sample performance

For evaluation purposes, we first compared the actual credit ratings against those predicted by the different models. As a summary statistic, we computed a Prediction Success Index (PSI) for each model.[10] When an ordered logit[11] was used for all provinces, the model was able to predict 63 per cent of the S&P ratings between 1976 and 1995. The PSI increased when smaller poolings were considered. For the higher-risk cohort, a PSI of 68 per cent was obtained while it reached 82 per cent for the moderate-risk group and 68 per cent for the lower-risk group, for a combined success ratio of 73 per cent. The higher success in predicting the second sub-group mainly reflects the lower overall variability of ratings for these provinces. Moreover, for each category predicted, the forecasting error never exceeded more than one notch when considering sub-groups of provinces, and only three errors of two notches occurred when using the nine-province pooling. These accuracy rates are comparable to those found in Cantor and Packer (1996) in their international analysis and are higher than those reported by Cheung (1996) for the Canadian provinces (56 per cent).

To evaluate the estimated models in a more comprehensive way, prediction success tables were calculated for the nine-province pooling (Table 7) and the aggregation of the three sub-groups of provinces (Table 8). Results show that the nine-province model predicts more accurately top categories (AAA to AA) than lower ones. Further, "over predictions" were proportional to "under predictions," reflecting the difficulty in distinguishing among adjacent categories. Better predictions for all categories, except

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[10] This can be written as $PSI_i = \frac{1}{N_{..}}\left(\sum_{i=1}^{m} N_{ii}\right)$ where $N_{..}$ equals the number of total predictions and N_{ii} refers to the number of correct predictions for alternative i.

[11] We found similar results using the normal distribution.

Table 7: Prediction success table (nine-province pooling)

		Predicted choice								
		AAA	AA+	AA	AA-	A+	A	A-	BBB+	*Observed count*
	AAA	17	4							21
Observed choice	AA+	2	14	7						23
	AA		7	25	5					37
	AA-		1	6	9	8				24
	A+				5	29				34
	A				1	8	2	8		14
	A-					5	2	14		21
	BBB+						1	3	2	6
Predicted count		19	26	38	20	45	5	25	2	

Table 8: Prediction success table (individual sub-groups, i.e., lower-risk, moderate-risk, higher-risk)

		Predicted choice								
		AAA	AA+	AA	AA-	A+	A	A-	BBB+	*Observed count*
	AAA	18	3							21
Observed choice	AA+	2	15	6						23
	AA		7	27	3					37
	AA-			4	13	7				24
	A+				5	28	1			34
	A						11	3		14
	A-						3	16	2	21
	BBB+							3	3	6
Predicted count		20	25	37	21	35	15	22	5	

A+, were obtained when using sub-groups. Consequently, with fewer categories, more observations are used for the identification of partition boundaries, and hence this facilitates the distinction of each category.

4.1 Evaluation of rating predictions by province[12]

The model reproduced Newfoundland's ratings fairly closely, with a success rate of 80 per cent (Table 9). It captures the downgrades that occurred in 1985 and 1994, although with a one-year lag in the latter case. Budgetary deficits and steadily rising debt led to downgrades for Nova Scotia in 1982 and 1985. Our results suggest downgrades for 1982 and 1986. They also indicate a downgrade to BBB+ in 1994, which never occurred, despite an increase from 21 to 45 per cent in the net debt-to-GDP ratio between 1985 and 1995. The percentage of accurate predictions for New Brunswick is 65 per cent. New Brunswick is the only province that did not face any downgrade since 1976. The province was actually upgraded in 1991 although its net debt ratio increased 9.6 percentage points from 1980 to 1991. The model did not predict any rating change in the 1990s.

Ratings for Quebec and Manitoba are well predicted with 19 and 17 correct category predictions out of 20, respectively. Ratings of the remaining provinces -- Ontario, Saskatchewan, Alberta and British Columbia -- were harder to predict, largely because of the greater historical fluctuations in their respective credit ratings. Although the success rate was 70 per cent for Ontario, the model persisted in predicting a rating of AA from 1993 on, instead of AA-. For Alberta, a downgrade to AA- was predicted in 1994, which never happened. Apart from a constant deviation between 1978 and 1980, the model tracked British Colombia's credit ratings quite well. Saskatchewan experienced the most frequent changes in its credit ratings as it was downgraded by six notches between 1985 and 1992. While it is possible to forecast the extent of the downgrades the province experienced (predictions show a cut from AA+ in 1984 to BBB+ in 1994), the model reproduced only 50 per cent of the categories over that period.

5. Out-of-sample performance

Given the highly persistent nature of credit ratings, a comparison with a benchmark such as a random walk (RW) model represents a better way to assess the relative predictive power of our estimated models. Accordingly, rolling out-of-sample forecasts were performed over one- to five-year horizons starting in 1990. To compute forecasts for the naïve model, we assumed no rating change regardless of the forecasting period. Note that the PSI does not represent an ideal measure of goodness of fit when comparing competing models, as it only gives information about the capacity of replicating the level of ratings. Forecasting a rating change before or after it occurs is different from predicting a temporary change in rating that never happened. The PSI does not make such distinctions over a short period. Also, replicating rating changes is more challenging and very important for the validation of such models. Consequently, we

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[12] All the results reported in this section are related to the sub-group models.

Table 9: In-sample prediction of Standard and Poor's ratings (by province)

	NEWFOUNDLAND			NOVA SCOTIA			NEW BRUNSWICK			QUEBEC			ONTARIO			MANITOBA			SASKATCHEWAN			ALBERTA			BRITISH COLUMBIA		
Year	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]	Actual	Pred.[1]	Prob.[1]
1976	A	A	(.49)	A+	A+	(.59)	A+	A+	(.67)	AA	AA	(.97)	AA	AAA	(.65)	AA	AA	(.86)	AA	AA	(.63)	AA	AA	(.85)	AA	AA+	(.56)
1977	A	A	(.72)	A+	A+	(.41)	A+	A+	(.59)	AA	AA	(.87)	AAA	**AA+**	**(.63)**	AA	AA	(.87)	AA	AA+	(.61)	AA	AA	(.92)	AA	AA	(.70)
1978	A	A	(.61)	A+	A+	(.59)	A+	A+	(.88)	AA	AA	(.82)	AAA	AAA	(.75)	AA	AA	(.57)	AA	AA	(.61)	AA	**AA+**	**(.64)**	AA	AA+	(.55)
1979	A	**A-**	**(.81)**	A+	A+	(.55)	A+	**AA-**	**(.51)**	AA	**AA**	(.85)	AAA	AAA	(.93)	AA	AA	(.62)	AA	**AA**	**(.49)**	AAA	AAA	(.64)	AA	**AA+**	**(.61)**
1980	A	**A-**	**(.81)**	A+	**A**	**(.61)**	A+	**A+**	**(.79)**	AA	**AA**	(.64)	AAA	AAA	(.86)	AA	AA	(.70)	AA	**AA**	**(.50)**	AAA	AAA	(.92)	AAA	**AA+**	**(.64)**
1981	A	A	(.65)	A+	A+	(.57)	A+	A+	(.54)	AA	**AA-**	(.65)	AAA	AAA	(.72)	AA	AA	(.85)	AA+	AA+	(.62)	AAA	AAA	(.93)	AAA	AAA	(.82)
1982	A	**A-**	**(.60)**	A	**A**	**(.74)**	A+	**AA-**	**(.50)**	AA-	**AA-**	**(.51)**	AAA	AAA	(.77)	AA	AA	(.72)	AA+	**AA**	**(.57)**	AAA	AAA	(.88)	AAA	**AA+**	**(.56)**
1983	A	A	(.69)	A	A	(.59)	A+	A+	(.90)	AA-	AA-	(.68)	AAA	AAA	(.71)	AA-	AA-	(.67)	AA+	AA+	(.62)	AAA	AAA	(.83)	AA+	AA+	(.59)
1984	A	A	(.65)	A	A	(.73)	A+	A+	(.94)	AA-	AA-	(.74)	AAA	AAA	(.68)	AA-	AA-	(.74)	AA+	AA+	(.59)	AAA	AAA	(.99)	AA+	**AA**	**(.82)**
1985	A-	A-	(.87)	A-	A	(.59)	A+	A+	(.97)	AA-	AA-	(.77)	AA+	AA+	(.60)	AA-	AA-	(.77)	AA+	**AA**	(.65)	AAA	**AA+**	**(.60)**	AA	AA	(.86)
1986	A-	A-	(.89)	A-	A-	(.74)	A+	**A+**	(.89)	AA-	AA-	(.76)	AA+	AA+	(.53)	A+	**AA-**	**(.64)**	AA	**AA+**	(.70)	AA+	**AAA**	**(.99)**	AA	AA	(.92)
1987	A-	A-	(.92)	A-	A-	(.78)	A+	A+	(.89)	AA-	AA-	(.77)	AA+	**AAA**	**(.48)**	A+	A+	(.90)	AA-	**AA**	(.41)	AA+	**AAA**	**(.53)**	AA	AA	(.91)
1988	A-	A-	(.82)	A-	A-	(.63)	A+	A+	(.94)	AA-	AA-	(.70)	AAA	AAA	(.64)	A+	A+	(.92)	AA-	**A+**	(.50)	AA+	AA+	(.62)	AA	AA	(.90)
1989	A-	A-	(.52)	A-	**A-**	(.63)	A+	A+	(.96)	AA-	AA-	(.56)	AAA	AAA	(.88)	A+	A+	(.73)	AA-	**A**	(.71)	AA+	AA+	(.62)	AA+	**AA**	**(.48)**
1990	A-	A-	(.49)	A-	A-	(.75)	A+	A+	(.90)	AA-	AA-	(.63)	AAA	**AA+**	**(.78)**	A+	**AA-**	**(.55)**	A	A	(.54)	AA	**AA+**	**(.56)**	AA+	AA+	(.64)
1991	A-	A-	(.77)	A-	A-	(.92)	AA-	**A+**	**(.95)**	AA-	AA-	(.77)	AA+	AA+	(.62)	A+	**AA-**	**(.52)**	A-	**A**	(.72)	AA	AA	(.52)	AA+	AA+	(.64)
1992	A-	A-	(.81)	A-	A-	(.91)	AA-	**A+**	**(.85)**	AA-	AA-	(.57)	AA	AA	(.72)	A+	A+	(.72)	BBB+	**A-**	**(.61)**	AA	AA	(.87)	AA+	AA+	(.54)
1993	A-	A-	(.81)	A-	A-	(.81)	AA-	**A+**	**(.94)**	A+	A+	(.72)	AA-	**AA**	**(.87)**	A+	A+	(.90)	BBB+	**A-**	**(.61)**	AA	AA	(.63)	AA+	AA+	(.49)
1994	BBB+	**A-**	**(.57)**	A-	**BBB+**	**(.72)**	AA-	**A+**	**(.96)**	A+	A+	(.83)	AA-	**AA**	**(.74)**	A+	A+	(.97)	BBB+	BBB+	(.69)	AA	**AA-**	**(.75)**	AA+	AA+	(.64)
1995	BBB+	BBB+	(.97)	A-	**BBB+**	**(.95)**	AA-	**A+**	**(.95)**	A+	A+	(.97)	AA-	**AA**	**(.71)**	A+	A+	(.96)	BBB+	BBB+	(.81)	AA	AA	(.61)	AA+	AA+	(.63)

1. Pred. is the predicted category and Prob. corresponds to the related probability. Note that, for each observation, probabilities associated with each category always sum up to one.

calculated the ratio of predicted rating changes (PRCs) and the number of false signals (FSs) over the forecasting period, with an FS corresponding to a wrongly predicted change in the rating.

Results from Table 10 demonstrate that ordered-response (OR) models tend to produce better forecasts as the forecasting horizon is lengthened. When forecasts are performed one period ahead, the RW hypothesis is superior to any models as a result of the extremely high persistence in credit ratings. When the horizon is extended beyond a year, the OR approach yields better results in terms of overall criteria. For two- and three-year forecasts, the average PSI amounts to 68 and 67 per cent, respectively, for both models. The main difference is that when the forecasts are cumulated, four rating changes out of eight are correctly predicted using the OR approach. For longer horizons, the advantage of using the OR approach becomes clearer with a PSI averaging 64 per cent and 63 per cent over four- and five-year horizons, compared to 61 and 55 per cent for the RW approach, respectively. Moreover, the OR model consistently predicts 50 per cent of the rating changes for horizons of two to five years while generating only one FS.

Another important issue is that FMS data have always been released with considerable lags, raising doubts about the relevance of using such information for forecasting purposes. Given that rating agencies use information from provincial budgets, we tested the relevance of this information in predicting the evolution of credit ratings. The reason for using information from provincial budgets is that it represents the only possible way of performing an actual ex-ante out-of-sample forecast. Before extending the simulation period, we first tested whether Public Accounts (PA) data were good predictors over a five-year horizon (1991-95). PA data do not have to be very close proxies for FMS data as long as they can reproduce credit rating decisions. In fact, net debt as defined in the FMS is itself a proxy for tax-supported debt,[13] the measure mainly used by S&P. Accordingly, an alternative simulation was conducted, substituting FMS data for PA data for the out-of-sample period only. Net debt forecast values were calculated by adding budgetary balances. The federal transfers-to-total revenue ratio was adjusted by the differences in the ratios between the two accounting systems.

Overall, similar results were found when forecasts with PA data are compared to the base case forecasts (Table 10). These results are consistent with the fact that trends in FMS data closely follow those of public accounts. With a PSI of 62 per cent over a five-year horizon and four rating changes correctly forecast, the OR approach remains a better approach in predicting credit ratings than the RW hypothesis, even with PA information.

[13] This is defined as total public sector debt minus debt of provincially- or municipally- owned or supported entities, such as electricity or telephone utilities. In addition, S&P makes adjustments to financial accounts to improve comparability among provinces, adjustments that may differ from Statistics Canada's adjustments in deriving FMS data.

Table 10: Comparison of rolling out-of-sample forecasts (1991-1995)

	1 year			2 years			3 years			4 years			5 years		
	PSI	PRC	FS	PSI	PRC	FS	PSI	PRC	FS	PSI	PRC	FS	PSI	PRC	FS
OR	70	3/8	1	**68**	**4/8**	1	**67**	**4/8**	1	**64**	4/8	1	**63**	**4/8**	1
RW	**82**	0/8	0	**68**	0/8	0	**67**	0/8	0	61	0/8	0	55	0/8	0
OR (Public Accounts data)													62	4/8	2

PSI = Average Prediction Success Index over all dynamic forecasts; PRC = the cumulative ratio of correct Predicted Rating Changes over all dynamic forecasts; FS = the cumulative False Signals where a false signal corresponds to a predicted rating change that did not occur.

5.1 Out-of-sample forecast over the 1996-99 period using provincial budget information

In order to perform a real time forecast, out-of-sample forecasts were performed over the 1996-99 period, using PA data for 1996 and medium-term fiscal plans pertaining to the 1997-98 provincial budgets. Moreover, Finance Canada's projections were used for the remaining variables. For this exercise, the estimation period ended in 1995 and simulation results, reported for each province (Charts 1A to 1E), were calculated for the 1990-99 period.

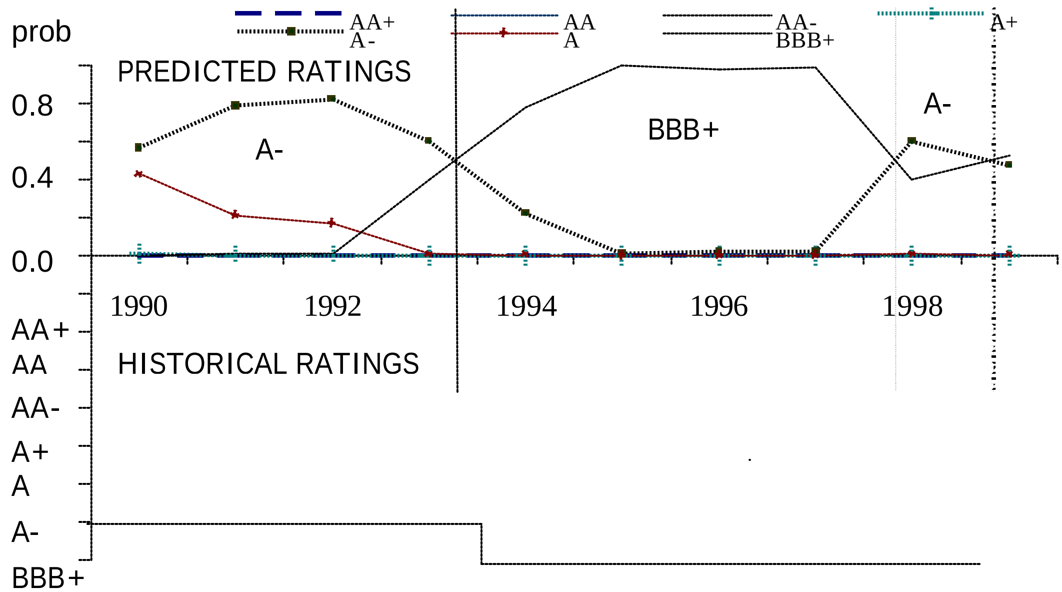
Each chart shows the predicted ratings in the upper part with the historical ratings in the lower part. More specifically, the upper part shows the conditional probabilities of obtaining each category within a sub-group. The sum of these probabilities equals one for each observation. Note that the use of OR models limits the number of predictable categories to those observed historically. Between 1976 and 1995, ratings assessed to the Newfoundland-Nova Scotia-Saskatchewan group were bound by the following range: BBB+ to AA+. The range was respectively A to AA for the New Brunswick-Quebec-Manitoba group and AA- to AAA for the pooling which includes Ontario, Alberta and British Columbia. Vertical lines are used to signal a forecast of a credit rating change.

Simulation results show that **Newfoundland's** rating should have changed from BBB+ to A- as of 1998 (Chart 1A). The forecast upgrade is explained by the decline (from 47.5 to 41.2 per cent) in the net debt-to-GDP ratio. This decline essentially reflects sustained GDP growth, as the level of debt is projected to remain relatively stable. Given the difficulties that Newfoundland recently faced in balancing its budget and the decline of its population, the predicted rating change could turn out to be a false signal as the probability of returning to a BBB+ rating exceeds the probability of remaining at A- for 1999. After predicting a BBB+ rating from 1994 to 1997, our model forecasts an increase in **Nova Scotia's** rating to A- as of 1998, which represents its current rating (Chart 1A). The probability of remaining at A- for 1999 is 78 per cent. As a consequence, it is expected that Nova Scotia's rating will remain unchanged in the near future.

For **New Brunswick** (Chart 1B), the probability of staying at A+ according to the model is hovering around 98 per cent. Upgrades are predicted for the provinces of **Saskatchewan** and **Alberta** (Chart 1D). For Saskatchewan, our model successfully caught the upgrade of 1996 as well as the change of 1997, although with a two-year lag. Similarly, it was possible to predict the upgrade of Alberta to AA+ in 1997, although with a one-year lag. Lags observed in predicting credit ratings reflect, in part, the dynamic inconsistency between the continuous frequency of credit ratings and the annual frequency of the data retained for modelling purposes. However, it is noteworthy that these rating changes were predicted with information dated at least six months prior to the events. Finally, no rating change is predicted for **Quebec** (Chart 1B)**, Ontario, Manitoba** (Chart 1C) or **British Columbia** (Chart 1E) over the 1996-99 out-of-sample period. The model overpredicted the provinces of Ontario and British Columbia by one notch. Unfortunately, the model failed to reproduce the 1997 downgrade for British Columbia. One reason might be that recent fiscal slippage for British Columbia has had only a modest impact on its debt-to-GDP ratio.

CHART 1A: OUT-OF-SAMPLE FORECASTS (1996-99)

NEWFOUNDLAND



NOVA SCOTIA

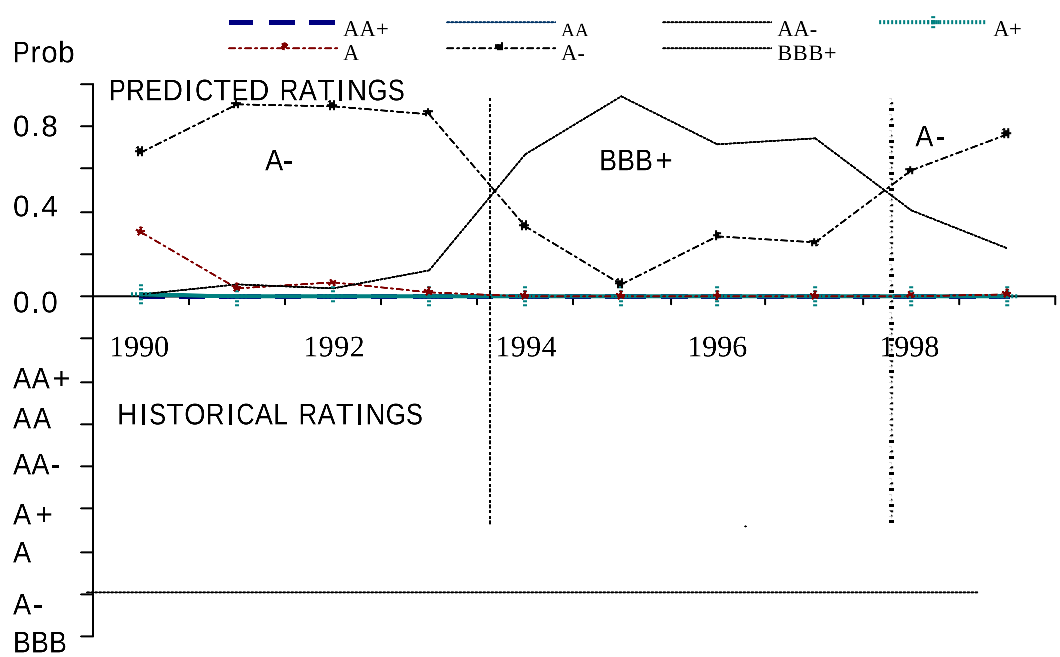
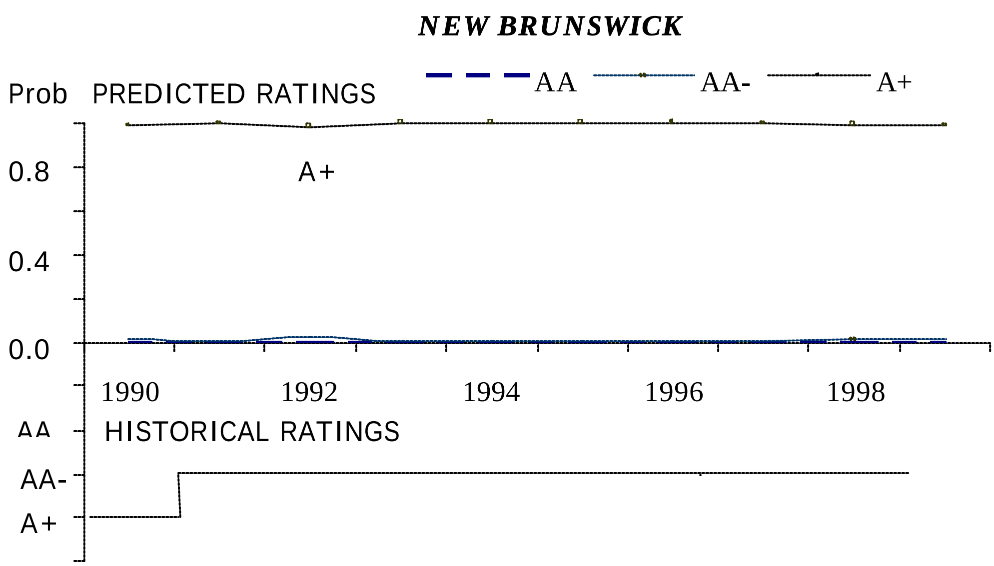


CHART 1B: OUT-OF-SAMPLE FORECASTS (1996-99)



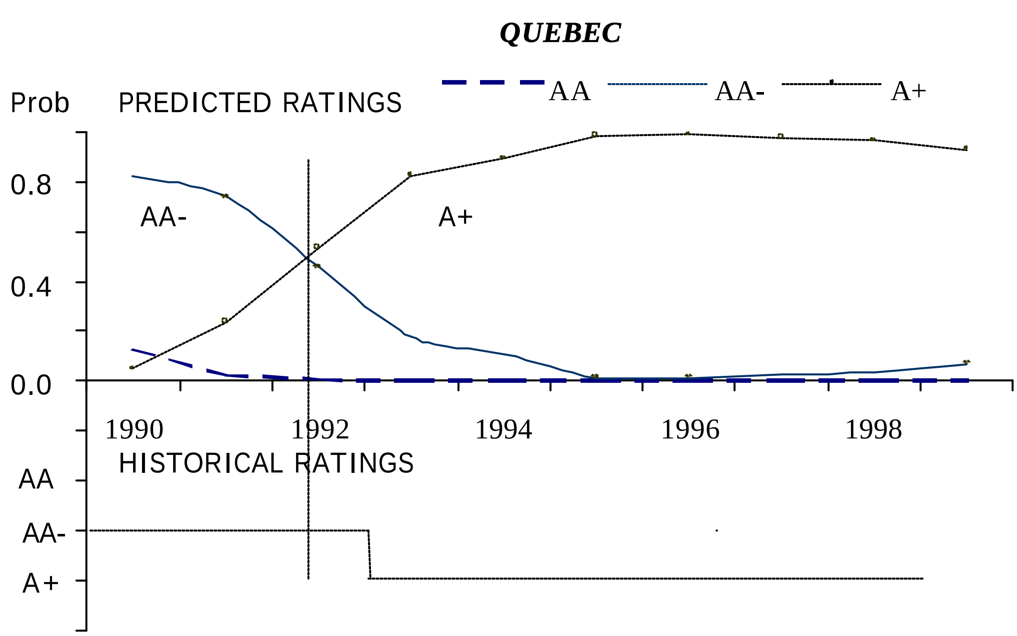
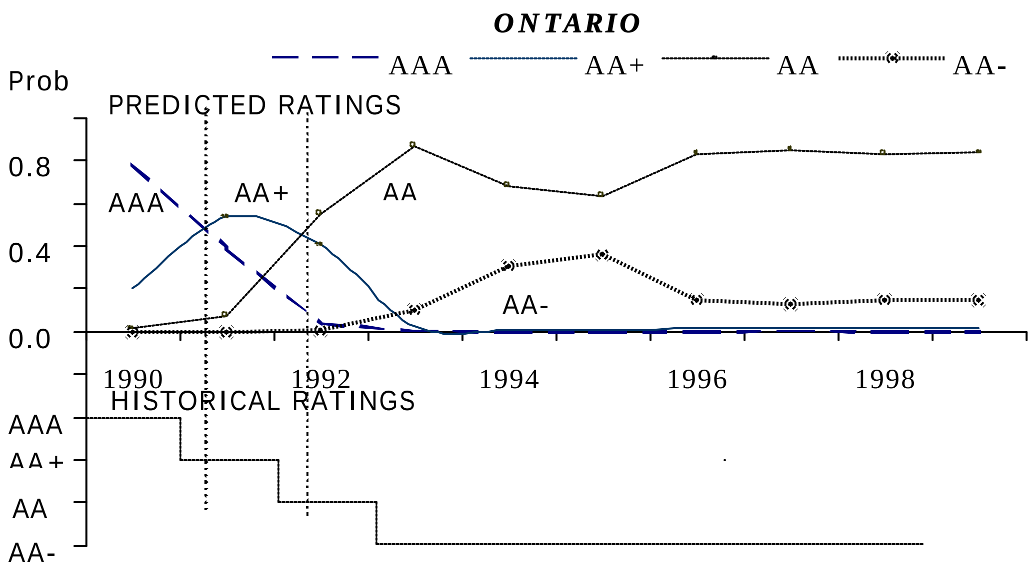


CHART 1C: OUT-OF-SAMPLE FORECASTS (1996-99)



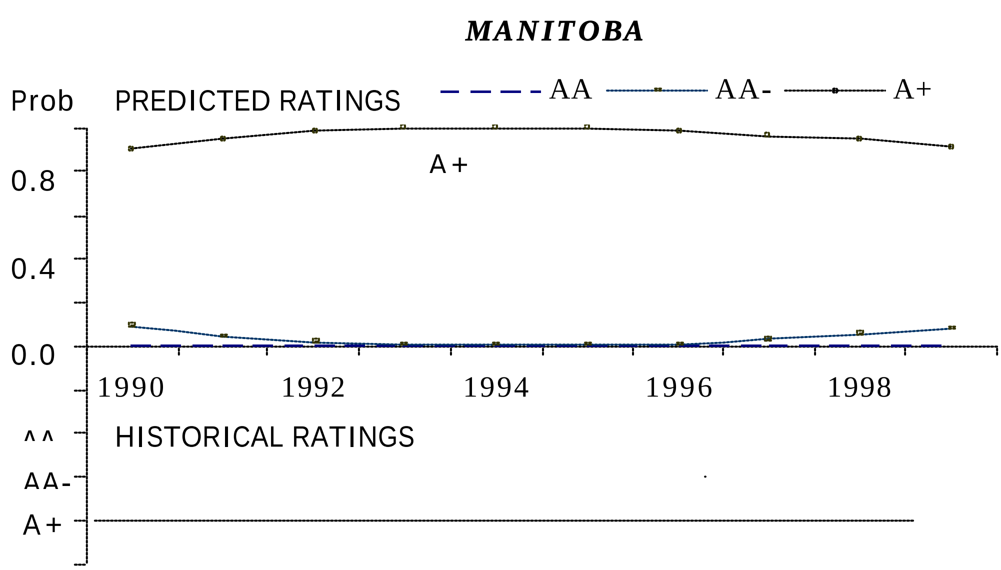
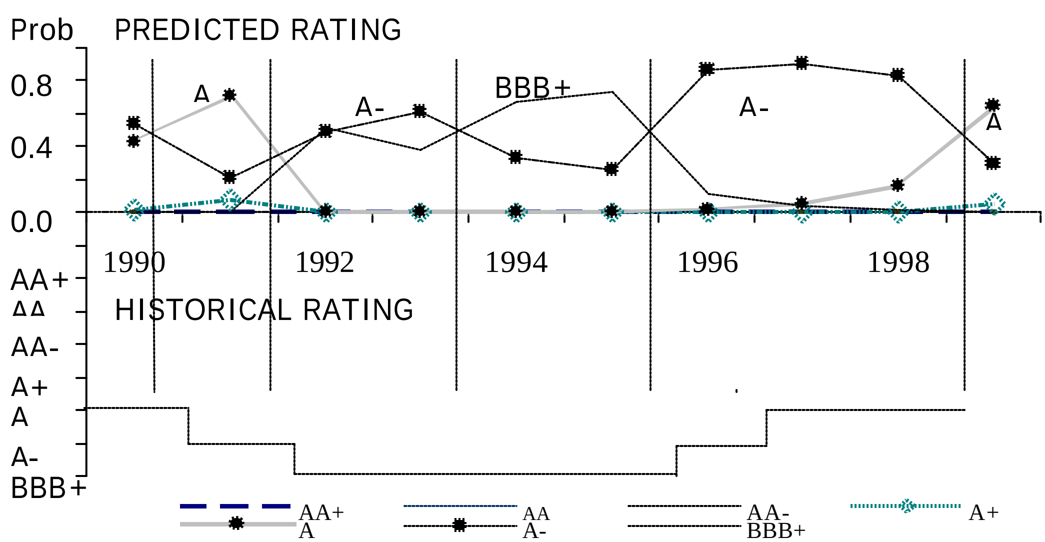


CHART 1D: OUT-OF-SAMPLE FORECASTS (1996-99)

SASKATCHEWAN



ALBERTA

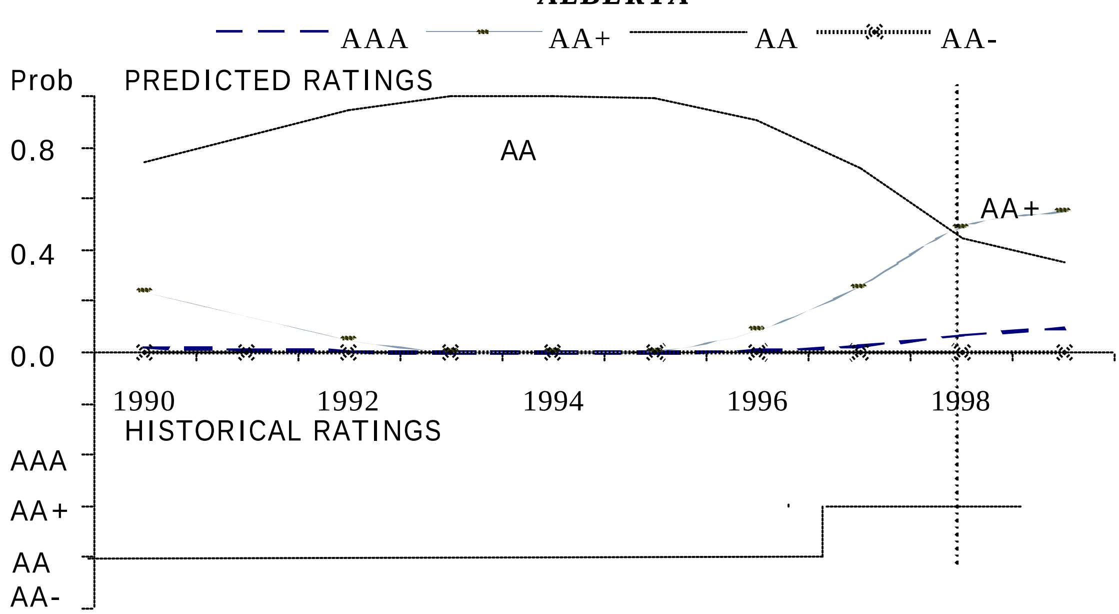
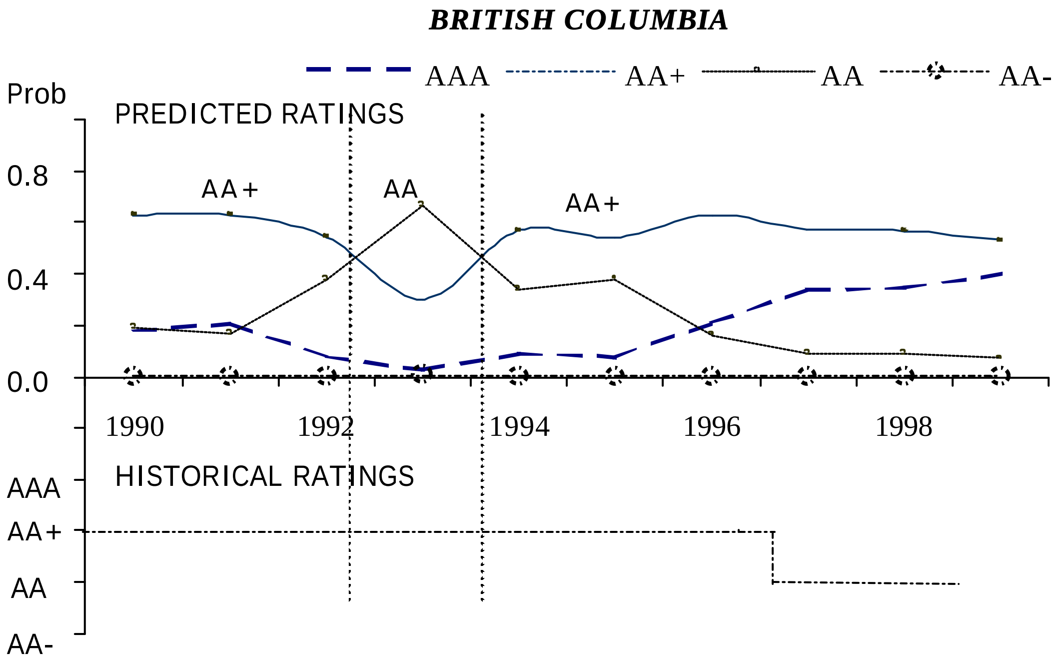


CHART 1E: OUT-OF-SAMPLE FORECASTS (1996-99)



6. Relationship between credit ratings and net debt

This section investigates further the link between level of indebtedness and credit ratings. The existence of a potential threshold effect triggered by a particular debt level is explored. Chart A1 illustrates the relation between the conditional probability distribution, calculated for each category, and the conditional means (i.e., the fit of the regression) ordered on the horizontal axis.[14] We have reproduced in the lower panel the levels of indebtedness at which the probability of switching from one credit rating to another is maximized. Accordingly, conditional means from regressions are mapped to the value of corresponding net debt/asset.

As the bottom part of Chart A1 shows, *ceteris paribus,* a net asset level of at least 12.0 per cent of GDP is needed for the average Canadian province to obtain the highest rating (AAA), while a net debt-to-GDP ratio of more than 45 per cent is associated with a BBB+ rating.[15] Moreover, any positive level of indebtedness would prevent the representative average province from obtaining a rating higher than AA. One plausible explanation justifying the requirement of such important buffer stocks to reach AAA is that provincial ratings are upwardly bounded by the federal rating. As it is impossible for a province to get a higher grade than the sovereign entity, a downgrade of the federal credit rating to AA+ would prevent a province from reaching AAA regardless of its level of net asset. Also, as can be seen from the vertical line distances, while there is clearly a lack of uniformity among the estimated ranges for the different categories, there does not seem to be any debt level that triggers sudden rating changes.

It was expected that the better-off provinces would move into the higher categories with less stringent fiscal conditions. To examine whether this assertion is true, we repeated the same exercise for the sub-groups of provinces. Chart A2 demonstrates that for the higher-risk provinces, a debt threshold of some 37 per cent of GDP is associated with a BBB+ rating. This contrasts with a debt ratio of about 45 per cent of GDP for the provinces overall. The results also suggest that a more intense fiscal effort is required for the higher-risk provinces to achieve the A- rating (indebtedness level of 23 per cent compared to 33.5 per cent of GDP). The debt ratio varies between 16 and 23 per cent for category A, compared to a range of 30.5 to 33.5 per cent for the nine-province model.

Chart A2 makes clear that, for riskier provinces, downgrades from AA+ to A+ take place at very narrow intervals. This phenomenon seems to correspond to a threshold effect described in Boothe (1993a,b), where a rise in indebtedness can provoke a series of downgrades. However, this would not occur if the debt level was perceived as a major problem but at a much earlier stage (i.e., when the rating is higher). In fact, this phenomenon seems to occur when the initial rating is relatively high. For the weaker categories, i.e., A to BBB+, no threshold effect has been identified and downgrades occur at much wider intervals.

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[14] The probabilities are conditional on the estimation of the parameters ß and µ, the distribution of the error term and the values associated with the explanatory variables. To isolate the effect of a change in the level of public debt, we have set the other exogenous variables at their respective average.

[15] As these net debt/asset thresholds are calculated from conditional distributions, they should serve only as examples of requirements to the attainment of a specific rating.

For New Brunswick, Quebec and Manitoba (Chart A3), it is found that a net debt-to-GDP ratio between 9.3 and 1.2 per cent is needed to reach a AA rating; between 19.4 and 9.3 per cent for a AA- rating; and between 39.1 and 19.4 per cent for an A+ rating. These requirements are in the neighbourhood of those of the nine-province model.

For the sub-group including Alberta, British Columbia and Ontario, the results suggest that a net asset-to-GDP position of 1.7 per cent is sufficient to reach the top rating, compared to about 12 per cent for the provinces all together. However, only a 4-percentage-point drop in the net debt/asset-to-GDP ratio is sufficient for a 2-notch decline from the AAA category (Chart A4). The probability of switching from an AA- to an AA rating is maximized at a debt-to-GDP ratio of 15.4 per cent; the probability of moving further to an AA+ rating is maximized at a debt-to-GDP ratio of 2.4 per cent.

Our results show that, for both higher-risk and lower-risk provinces, the non-linearity or the sensitivity to changing levels of indebtedness increases at lower levels of indebtedness. Note that Cheung (1996) reached a similar conclusion for the Atlantic provinces, using the gross direct debt-to-GDP ratio and a different set of explanatory variables. But contrary to Cheung's results, we found threshold effects without specifying a nonlinear relationship between the credit rating index and the set of explanatory variables. As our tests indicate, nonlinearity could arise because of the inequality of risk differential as measured by adjacent categories. Furthermore, other characteristics such as the fragility of the economy and the lack of fiscal flexibility could also influence the speed of rating changes and not just the rating level. The model predicts that starting initially with a net asset position, a 25-percentage-point reduction in the debt/asset-to-GDP ratio will initiate a 4-notch decline in the case of the higher-risk cohort compared to three notches for the lower-risk cohort. By comparing higher-risk with lower-risk provinces (Charts A2 and A4), we found that, in order to maximize the probability of obtaining the AA+ grade, a higher-risk province's net debt/asset must be lower/higher by about 3 percentage points. To get an AA rating, the difference widens to about 12 percentage points and reaches to 15 percentage points for AA-.

This apparent reduced sensitivity at lower ratings seems to suggest two things. First, risk-differential as measured by two adjacent categories may widen as provinces reached lower ratings. Second, as pointed out by Boothe (1993), lower ratings restrain the availability of credit and the possibility of higher indebtedness. This may induce provinces to implement corrective actions such as deficit reduction plans and legislated fiscal rules to maintain their creditworthiness.

7. Conclusions

In this paper we had three main objectives. First, we wanted to assess the usefulness of using econometric models to replicate long-term debt ratings attributed by S&P. Our results demonstrate that ordered-response models with a relatively limited set of economic and financial variables can replicate from two-thirds to three-quarters of the actual credit ratings.

To us, the paper innovates by forecasting out-of-sample provincial government rating changes. Using information from provincial medium-term plans, we have been able to forecast three of the four rating decisions made by S&P over the last two years. However, further investigation is required in order to test the reliability and forecasting ability of these models. The use of ordered-response models limits the number of predictable categories to those observed historically. Probabilities that are close to the limits of conditional distributions must therefore be interpreted with some caution. This can also represent a non-negligible constraint when conducting the forecasting exercise.

Our third objective was to investigate further the relationship between credit ratings and the level of indebtedness within a multivariate framework. By grouping provinces according to their credit quality, we found that nonlinearities (triggering downgrades) occurred at reasonably low levels of indebtedness (when the rating is relatively high), but not at relatively higher levels of net debt. These findings corroborate the results put forward by Cheung (1996) for the Atlantic Provinces using a similar framework. Because of the existence of redistribution mechanisms in the Canadian federation, a rating corresponding to the last investment grade (BBB-) may constitute a lower boundary beyond which further downgrades are extremely unlikely to occur. Saskatchewan's experience, however, seems to reflect the major impact other characteristics, such as fiscal flexibility and economic diversification, may have on the speed of rating changes.

References

Boothe, P. 1993a. "Provincial Government Debt, Bond Ratings and the Availability of Credit." In R.G. Harris, ed., Debt and Deficits in Canada. Policy Forum Series 29. Kingston, Ont: Queen's University, John Deutsch Institute for the Study of Economic Policy.

Boothe, P. 1993b. "Bond Ratings and Provincial Government Debt (A Tale of Two Provinces)." PEAP Policy Study 93-3, Institute for Policy Analysis, University of Toronto.

Cantor, R. and F. Packer. 1994. "The Credit Rating Industry." Federal Reserve Bank of New York *Quarterly Review* (Summer/Fall): 1281-94.

Cantor, R. and F. Packer. 1996. "Determinants and Impacts of Sovereign Credit Ratings." Federal Reserve Bank of New York *Research Paper No. 9608* (April).

Cheung, S. 1996. "Provincial Credit Ratings in Canada: An Ordered Probit Analysis." Working Paper 96-6, Bank of Canada.

Ederington, Louis, and Jess Yawitz. 1987. "The Bond Rating Process." In Edward Altman, ed., *Handbook of Financial Markets.* New York: John Wiley & Sons: 23-57.

Gourieroux, C., A. Monfort, E. Renault, and A. Trognon. 1987. "Generalised Residuals." *Journal of Econometrics* 34, 5-32.

Greene, William H. 1997. *Econometric Analysis*, 3rd Edition, Prentice Hall.

Hausman, Jerry A., Andrew W. Lo, and A. Craig MacKinlay. 1992. "An Ordered Probit Analysis of Transaction Stock Prices." *Journal of Financial Economics* 31, 319-379.

Kaplan, Robert, and Gabriel Urwitz. "Statistical Models of Bond Ratings. A Methodological Inquiry." *Journal of Business,* 52 (2): 231-261.

Maddala, G. *Limited Depend and Qualitative Variables in Econometrics.* New York: Cambridge University Press, 1983.

McKelvey, R., and W. Zavoina 1975. "A Statistical Model for the Analysis of Ordinal Level Dependent Variables." *Journal of Mathematical Sociology* 4, 103-20.

Murphy, A. "Simple LM tests of Mis-specification for Ordered Logit Models." *Economics Letters 52 (1996): 137-141*.

Standard and Poor's 1997 reports, *Canadian Focus*.

Weiss, A. "Specification Tests in Ordered Logit and Probit Models." *Econometric Reviews 16(4) (1997): 361-391*.

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