A Medium-Term Forecasting Equation for the Canada-U.S. Real Exchange Rate

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2001-08

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*The ideas and opinions expressed in this paper are the author's, and any error or omission is the author's sole responsibility. This paper has greatly benefited from many helpful comments from Don Coletti, Stephen Murchison, Benoît Robidoux, Carolyn Wilkins, and Bing-Sun Wong.

JEL classification numbers: F53, C22, C53
Keywords: Real exchange rate; Cointegration; Error-correction model, Forecasting
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Abstract

That shifts in the fundamental factors determining real exchange rates in the long term have strongly affected exchange rate movements is an increasingly popular idea. In particular, Amano and van Norden (1993) suggest that movements in Canada's terms of trade were particularly important in explaining the evolution of the Canada-U.S. real exchange rate over the post–Bretton-Woods era. Building on that idea, this paper identifies additional economic factors as having significantly influenced that evolution over the past 25 years. It uses cointegration techniques to show the importance of net foreign asset holdings by Canadians in real exchange rate determination. It also breaks down the terms-of-trade effect identified by previous authors into two key components: real non-energy commodity prices and real computer prices. Finally, it develops a medium-term forecasting equation for the Canada-U.S real exchange rate. An out-of-sample forecasting simulation exercise shows that the new specification outperforms the existing alternatives.

Résumé

Que les changements des facteurs fondamentaux déterminant le taux de change réel de long terme aient fortement affecté les mouvements du taux de change est une idée de plus en plus populaire. En particulier, Amano et Van Norden (1993) suggèrent que les mouvements des termes de l'échange du Canada ont été particulièrement importants pour expliquer l'évolution du taux de change réel Canada-É.U. au cours de la période post-Bretton-Woods. Suivant cette idée, ce texte identifie des facteurs économiques additionnels qui ont influencé de façon significative cette évolution au cours des 25 dernières années. Les techniques de cointégration sont utilisées pour démontrer l'importance des avoirs nets financiers détenus par les Canadiens comme déterminant du taux de change réel. L'effet des termes de l'échange identifié précédemment par certains auteurs est divisé en deux composantes clés : le prix relatif des matières premières hors-énergie et le prix relatif des ordinateurs. Finalement, une équation de prévision de moyen terme pour le taux de change Canada-É.U. est développée. Un exercice de simulations hors-échantillon démontre que la nouvelle spécification fonctionne mieux que les alternatives existantes.

1. Introduction

Until recently, exchange rate models had a long history of producing unsatisfactory results. Surveys such as Meese 1990, Mussa 1990, and MacDonald and Taylor 1992 agree on this point. In the 1970s and 1980s, most exchange rate models were based on the monetary approach and failed to beat even a simple random walk model in out-of-sample forecasts. These results have helped popularize the view that the exchange rate is best characterized by a random walk. More recent papers argue, however, that trends in the real exchange rate can be explained by trends in underlying economic factors.

Recent papers that develop models based on long-term relationships between the real exchange rate and economic fundamentals have found some encouraging results. Cerisola, Swagel, and Keenan 1998, Clark and MacDonald 1998 and MacDonald 1997, developed models based on an error-correction framework that links the real exchange rate with economic fundamentals such as the terms of trade, government debt, the net foreign asset position, and country productivity differentials. Work done at the Bank of Canada by Amano and van Norden 1993 and Lafrance and van Norden 1995 demonstrates the strong link between the terms of trade and the real exchange rate. The Amano and van Norden 1993 model has the attractive feature of being one of the first models to outperform a random walk in out-of-sample forecasts.

The goal of this paper is to develop a reduced-form medium-term forecasting equation for the Canada-U.S. real exchange rate based on economic fundamentals. The approach taken is an extension of the papers mentioned above. The preferred equation is estimated in the familiar error-correction framework and is based on a long-term relationship between the Canada-U.S. real exchange rate, Canada's cumulated current account balance, real U.S. dollar non-energy commodity prices, and the real U.S. dollar price of computers. The Canada-U.S. short-term nominal interest rate differential influences the real exchange rate in the short run.

The next section of this paper briefly reviews the recent literature. The third section reports the data used to build the model, while the fourth and fifth sections present the estimation results and the equation's forecasting performance. Section six presents a decomposition of key factors underlying the evolution of the real exchange rate over the 1973 to 1999 period.

2. Literature Review

In the most recent literature, economists have tried to re-estimate exchange rate models using cointegration techniques. Early attempts using these methods were unsuccessful in repopularizing the monetary model of the 1970s (see, for example, Adams and Chadha 1991). The results of this work suggest that simple monetary models omit important factors that can affect the real exchange rate over the long term.

Recent papers have tried to identify these factors. One important real economic factor that has affected the Canadian dollar in particular is terms-of-trade shocks. For example, Amano and van Norden 1993 and Lafrance and van Norden 1995 show that shocks to the terms of trade can explain most of the historical evolution of the real exchange rate during the post–Bretton-Woods period.

McCallum 1998 extends the Lafrance and van Norden 1995 framework by adding the ratio of government debt to gross domestic product (GDP) as an explanatory variable. He finds that the government debt to GDP ratio plays an important role in real exchange rate determination. He suggests two reasons why a rising government debt might result in a weaker currency. First, rising government debt is likely to be associated with rising debt to foreigners. With rising interest payments to foreigners, the country will require a larger trade surplus to partly offset the negative impact of rising interest payments on the current account of the balance of payments. Second, a higher public debt might raise fears that the debt might one day be monetized. The consequent increase in inflationary expectations and the higher risk premium attached to Canadian interest rates would put downward pressure on the currency. McCallum argues that the Canada-U.S. inflation differential, the Canada-U.S. government debt differential, and global commodity prices are the three main factors behind the depreciation of the Canadian dollar over the last 25 years.

Cerisola, Swagel and Keenan 1998 develop a model for the Canada-U.S. real exchange rate based on work done by Clark and MacDonald 1998 and MacDonald 1997 for different OECD countries. They find that, in addition to the terms of trade and government debt to GDP ratio, the net foreign asset position, the Canada-U.S. productivity differential, and the Canada-U.S. real interest rate differential are also long-run determinants of the real exchange rate.

The theoretical link between net foreign indebtedness and the real exchange rate has been suggested in many papers, including Dornbush and Fischer 1980, Hooper and Morton 1982, and Gavin 1991. Hooper and Morton develop a model in which exogenous shocks to trade flows create a long-run positive correlation between net foreign assets and the real exchange rate. In a more complete theoretical model, Gavin shows that an exogenous shock to wealth creates a positive correlation between net foreign assets and the real exchange rate when the Marshall-Lerner condition is satisfied.[1] Hooper and Morton 1982, Faruquee 1994, and Obstfeld and Rogoff 1995 present empirical results confirming a positive relationship between net foreign assets and the real exchange rate.

The transmission to the real exchange rate is based on the conclusion that, in equilibrium, a country with negative net foreign assets must have a trade surplus to finance the stream of interest and dividend payments. The mechanism to generate this trade surplus is real exchange rate depreciation. Similarly, countries with positive net foreign assets must have a trade deficit in equilibrium. Thus, a shock to net foreign assets has a long-run effect on the real exchange rate. It is important to recognize that exchange rate dynamics may be the mechanism through which desired and actual net foreign assets converge. Thus, an increase in desired net foreign assets may cause an immediate depreciation of the exchange rate in order to generate a trade surplus, followed by a long-

[1] The same shock creates a negative correlation when the Marshall-Lerner condition is not satisfied. The Marshall-Lerner condition is that the price elasticity of demand for tradables should be sufficiently high

run real appreciation of the exchange rate above its initial level once the desired net foreign asset stock is achieved.

The effect of country productivity differentials on the real exchange rate, also known as the Balassa-Samuelson effect, has also been examined in several other studies.[2] Balassa 1964 and Samuelson 1964 argue that technological progress has historically been faster in the traded-goods sector than in the non-traded-goods sector, and that this traded-goods productivity bias is more pronounced in high-income countries. As a consequence, the price levels of non-traded-goods tend to be higher in wealthy countries. Because the exchange rate is determined by the prices of traded goods, the value of a currency will be higher in the high-income country. Generally speaking, movements in productivity differentials are sufficiently small and gradual that they explain little of the overall movements in real exchange rates over the last 20 years. Most papers that focus on the productivity differential as a determinant of exchange rates have relied on panel regressions rather than time-series analyses, but even so their results are mixed, reflecting perhaps the difficulty of constructing an appropriate measure of productivity. [3]

3. Data Description

The general form of the model estimate in this paper is:

$$rer_t = f(com_t, lrcomp_t, cca_t, nirst_t) \qquad (1)$$

The sample period used for the estimation is 1973Q1 to 1999Q4, consistent with the beginning of the floating exchange rate regime.

that a real depreciation leads to an increase in the trade balance. Nearly all econometric estimates of trade elasticities satisfy this condition, at least in the long run. See Marquez 1990.

[2] See Froot and Rogoff 1994 for a good presentation of the Balassa-Samuelson effect.

[3] See, for example, Chinn 1997.

The variable *rer* represents the logarithm of the nominal exchange rate expressed as U.S. dollars per Canadian dollar, adjusted for inflation by the ratio of Canada to U.S. GDP implicit price deflators (see appendix A, figure A1). An increase in the variable denotes an appreciation.

The variable *com* represents the logarithm of the ratio of the Bank of Canada U.S. dollar non-energy commodity price index to the U.S. GDP implicit price deflator.[4] Real U.S. dollar non-energy commodity prices have been drifting downward since 1973 (see figure A2). In 1999, non-energy commodities represented about 34 per cent of total Canadian exports and about 25 per cent of its imports, with the export value being 1.5 times the import value. For the most part, commodity prices are determined on world markets. Coletti and Murchison (1998) identify commodity prices as a key exogenous determinant of the price of Canadian exports and hence the terms of trade.

The variable *lrcomp* represents the logarithm of the ratio of the U.S. computer price index (U.S. National Accounts, computers and peripheral equipment investment deflator) to the U.S. GDP implicit price deflator (see figure A3).[5] Canada is a major importer of computer-related equipment, which represents about 2.3 per cent of Canada's total exports and 5 per cent of its imports, with the import value being twice the export value. Significant and sustained drops in the price of computers account for much of the downward trend in the real price of Canadian imports since the beginning of the 1980s (Coletti and Murchison 1998). As with commodity prices, computer prices are determined largely on world markets.

The variable *cca* represents the ratio of Canada's cumulated current account to GDP (see figure A4). It is calculated by cumulating the quarterly National Accounts current account balance from a starting value (1960Q4) taken from the National Balance Sheet. The current account measures the net flow of goods, services, and net investment income

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[4] The Bank of Canada index is preferred over other available indexes because it covers a larger number of commodities. This choice also allows us to compare our results with those of Lafrance and van Norden (1995), who use the Bank of Canada index.

[5] Since this particular index is from the U.S. National Accounts, there are numerous sources of forecast values.

between countries. The cumulated current account to GDP is used to proxy the country's level of net foreign indebtedness.[6]

Most monetary models of the real exchange rate suggest that the interest rate differential should reflect the deviation of the real exchange rate from its expected long-run value. Monetary models predict a relationship between the deviation of the real exchange rate from its expected long-run value and relative monetary policy. However, they also predict that monetary variables should have no permanent effect on the real exchange rate.

The variable *nirst* represents the nominal interest rate differential. The nominal interest rate is defined as the 90-day commercial paper rate (see figure A5). According to the non-covered interest rate parity condition, the real exchange rate should be equal to the interest rate differential between the domestic and the foreign country and the expectation of the exchange rate in period $t + 1$. This relationship can be viewed as follows:

$$rer_t = (nirst)_t + E_t(rer_{t+1}) \tag{2}$$

where rer_t is the value of the real exchange rate, *nirst* is the nominal interest rate differential and $E_t(rer_{t+1})$ is the expectation of the value of the real exchange rate in period $t + 1$ based on the information available at period t.

[6] Statistics Canada also published an "official" measure of Canada's net foreign assets. However, this measure is only available at an annual frequency, and our study uses quarterly data.

4. Empirical results

Pre test for Order of Integration

We begin our empirical analysis by examining the time series properties of each series. To this end we use the augmented Dickey-Fuller 1979, the Phillips and Perron 1988 and the Kwiatkowski, Phillips, Schmidt and Shin 1992 tests.[7]

The tests suggest that the real exchange rate, real non-energy commodity prices, real computer prices and the ratio of the cumulated current account to GDP are well characterized by I(1) processes (see table 1). Mixed information is available regarding the order of integration for the Canada-U.S. nominal interest rate differential. The augmented Dickey-Fuller (ADF) test cannot reject the hypothesis that this variable is I(1). However, evidence from the Phillips-Perron (PP) and the KPSS tests suggest the contrary. From prior theory, we assumed that the nominal interest rate differential is I(0).

Table 1: Tests for Unit Roots and Stationarity (sample period: 1973Q1 to 1999Q4)

Variable	ADF lag length	ADF	PP	KPSS
rer	4	-1.8233	-0.9803	1.0711***
com	5	-2.3283	-1.1544	1.7436***
lrcomp	4	-0.7792	-0.5401	2.2141***
cca	1	-1.2168	-0.7870	1.6360***
nirst	3	-2.8696*	-2.9828**	0.3026

Note: The asterisks, *, **, and *** indicate that the null hypothesis is rejected at the 10, 5, and 1 per cent level of significance respectively. The number of lags was determined by the Campbell–Perron procedure for the ADF tests and by the Newey-West automatic truncation lag selection for the PP tests.

Cointegration Tests

We use the Johansen 1988 procedure which estimates the system simultaneously, allowing for the possibility of endogenous regressors. The preferred system includes the

[7] It is well known that the two first tests may lack power against the alternative of stationarity if the data do not span a long enough time period. Therefore, we also use the Kwiatkowski, Phillips, Schmidt, and Shin 1992 test, which allows us to test the null hypothesis of stationarity against a unit root alternative.

log of the real exchange rate, the log of the Bank of Canada real U.S. dollar non-energy commodity price, the cumulated current account to GDP ratio, and the log of real U.S. dollar computer prices in the cointegrating vector.[8] The short-run dynamics include one lag of the nominal interest rate differential, three lags of the first difference of the real exchange rate, as well as the contemporaneous value and three lags of the first difference of non-energy commodity prices, the cumulated current account, and computer prices.[9]

Johansen 1992 demonstrates that estimation and inference on the single equation system will be equivalent to that of the full system only if all other cointegrating variables are weakly exogenous (in the sense of Engle, Hendry, and Richard 1983) with respect to the first variable in consideration (in our case, the real exchange rate), and if there is only one cointegrating vector. The Johansen procedure allows us to perform a weak exogeneity test on the full system using the likelihood ratio test described in Johansen and Juselius 1992. This is simply a test of whether the speed of adjustment α is significantly different from zero in the equations for the variables tested.

Table 2: Cointegration Test

Eigenvalue	λ_{max}	Trace	H_0: q =
0.2658	32.14**	66.04**	0
0.2076	24.20	33.90	1
0.0760	8.22	9.70	2
0.0142	1.49	1.49	3

Note: The asterisks * and ** indicate that the null hypothesis of *q* cointegration relationships is rejected at the 5 and 1 percent level of significance respectively.

[8] We also tried the productivity differential and the debt-to-GDP ratio. We find that the debt-to-GDP ratio does not play a statistically significant role in the model. Although we concur with the theory that productivity differentials should influence the real exchange rate, we find popular measures of productivity differentials to be unsatisfactory. These measures are based on the Balassa-Samuelson theory that links the ratio of non-traded to traded goods prices to the real exchange rate. Cerisola, Swagel and Keenan 1998, Clark and MacDonald 1998 and MacDonald 1997 approximate the traded goods price with the producer price index and the non-traded goods price with the consumer price index. Our causality tests suggest that causality runs from the exchange rate to the measure of the relative price of non-traded to traded goods. This result may stem from the construction of the proxy. In Canada, since 20 per cent of the Canadian CPI has import content, it may not be an appropriate proxy for the prices of non-traded goods.

[9] This lag structure was based on Aikaike information criterion. It was also chosen to ensure white noise residuals.

A number of tests were conducted on this system of equations. The results presented in table 2 suggest that one cointegrating relationship exists among the four variables. Results reported in table 3 show that we cannot reject the hypothesis that non-energy commodity prices, the cumulated current account, and computer prices are all weakly exogenous, but we can reject weak exogeneity for the real exchange rate. This allows us to estimate the model as a single equation error-correction model (ECM) using non-linear least squares.

Table 3: Test for Weak Exogeneity: LR TEST CHISQ(q)

Rank (Π)	χ^2 (q) 5 %	*rer*	*com*	*cca*	*lrcomp*
1	3.84	6.63	0.30	1.43	0.01

Estimation Results

For model selection we follow the general to specific approach of Hendry 1980. In addition, we focus on parameter stability as the important criteria for model selection. To study the stability of the individual coefficients associated with the I(0) variables, we use the stability test developed in Hansen 1991.

The final specification of the equation is the following:

$$\Delta rer_t = \Phi_1 \Delta rer_{t-1} + \Phi_2 nirst_{t-1} + \alpha(rer_{t-1} - \xi_1 com_{t-1} - \xi_2 lrcomp_{t-1} - \xi_3 cca_{t-1}) \quad (3)$$

The estimated speed of adjustment parameter α is of the correct sign and has a low value, which suggests a slow adjustment of the real exchange rate toward its long-run equilibrium value (see table 4). The estimated value of the parameter of the real U.S. dollar non-energy commodity price in the error-correction term ξ_1 of 0.7 is similar to other studies. For example, ξ_1 is 0.5 in Lafrance and van Norden 1995, 0.8 in Amano and van Norden 1993, and 0.8 in Coletti and Murchison 1998.[10]

The estimated coefficient on the cumulated current account variable (ξ_3) of 1.3 is well within the wide range found in the literature. Faruqee 1994 estimates this coefficient to be 1.5 for the U.S. and Gagnon 1996 finds -0.1 for a panel of 20 industrialized countries. Coletti and Murchison 1998 estimates 0.9 for the Canada-G6 exchange rate, while Cerisola, Swagel and Keenan 1998 finds the same estimate for the Canada-U.S exchange rate. The estimated coefficient for the real U.S. dollar computer price ξ_2 is -0.07, similar to that in the Coletti and Murchison study. As for the estimated coefficient on the nominal interest rate differential $\Phi_{2,}$ we find a value of 0.43. On the basis of the nominal interest rate parity condition, we anticipate a one-to-one relationship between the nominal interest rate and the real exchange rate. On a quarterly basis, we expect a 0.25 coefficient for the nominal interest rate differential. We cannot reject this hypothesis at a standard significance level. This result supports short-term interest rate parity between Canada and the U.S.

Table 4: Estimation Results (Sample: 1973Q3 to 1999Q4)

Coefficient	Value	Standard error
Φ_1	0.2599	0.0907**
Φ_2	0.4332	0.1145**
α	-0.0944	0.0232**
ξ_1	0.6582	0.2227**
ξ_2	-0.0706	0.0304**
ξ_3	1.3096	0.1294**

Note: The asterisks * and ** indicate that the variable is statistically significant at the 10 and 5 per cent level of significance respectively.

Table A1 (appendix A) presents the results of various diagnostic tests performed on the equation; results show no evidence of serial correlation or heteroskedasticity at a 5 per cent significance level. Table A2 reports the results of the Hansen 1991 stability test for individual variables in the equation. That test shows that all coefficients are stable and

[10] Coletti and Murchison 1998 models the Canada-G6 real exchange rate.

that the equation is stable. The results of recursive estimation of a linear representation of the error-correction model show that each coefficient is fairly stable (figure A6).[11]

5. Forecasting Performance

To assess the equation's forecasting ability, we need to establish stronger exogeneity conditions than those used to this point. As Ericsson 1992 notes, valid prediction from a conditional model such as equation (3) requires strong, rather than weak exogeneity. In practical terms, we need to ensure that *rer* does not Granger-cause *com*, *lrcomp*, *cca*, and *nirst* in addition to the maintained hypothesis of weak exogeneity.

Our equation has a simple structure with a single cointegrating vector, so appropriate testing procedures have already been well defined.[12] In particular, we may use standard Granger-causality tests on a vector-autoregression level representation of our system, provided that we have a maintained hypothesis of one cointegrating vector and that we test the exclusion restrictions one variable at a time.[13]

The results of Granger-causality tests indicate (1) that we can reject the null hypothesis that non-energy commodity prices, computer prices, and the cumulated current account do not Granger-cause the real exchange rate at the usual level of confidence, and (2) that we cannot reject that hypothesis for the reverse cases (table A3). The results are the same for the interest rate differential: the nominal interest rate differential appears to Granger-cause the real exchange rate, while the real exchange rate does not Granger-cause the nominal interest rate differential. These results and the

[11] This finding seems to contradict the results of Laidler and Aba (2001). Using a specification similar to Amano and Van Norden (1993) they find that the coefficient of non-energy commodity prices has diminished in the 1990s. However, they do not perform formal stability test of their model. Their model also has autocorrelation in the residuals, which suggests that it would have required the first-difference of the lagged endogenous variable (like in Lafrance and Van Norden's and our models).

[12] See Sims, Stock, and Watson 1990.

[13] Note that the presence of cointegration implies that Granger-causality should exist in at least one direction between our long-run variables.

evidence of weak exogeneity allow us to conclude that the restriction of strong exogeneity is satisfied, so we can go on to perform valid conditional forecasting.

Figures A7 to A10 show the equation performs in forecasting the real exchange rate over time. Figure A7 presents an in-sample dynamic forecast. The equation tracks the actual series well and there is no constant bias. Figures A8 to A10 present three out-of-sample dynamic forecasts for 1992Q1 to 1999Q4, 1994Q1 to 1999Q4, and 1996Q1 to 1999Q4. These are based respectively on sample estimations of 1973Q1 to 1991Q4, 1973Q1 to 1993Q4, and 1973Q1 to 1996Q4 corresponding to an eight-, a six-, and a four-year forecast. We can see by the graphs that the equation does a good job of tracking the evolution of the real exchange rate.

Comparison with Other Forecasting Equations

This section compares the forecasting power of the equation against two viable alternatives: a random walk model and the specification suggested by Lafrance and van Norden 1995.

For a more precise look at the forecasting ability of the equation, we follow the method suggested by Meese and Rogoff 1983. Specifically, we begin by estimating the equation over the 1973Q1 to 1991Q4 period and then generating an out-of-sample dynamic forecast for all quarterly horizons up to 12 quarters. We then add another quarter of data, re-estimate the equation and generate new forecasts for all 12 quarters horizons. Repeat this process until the end of our data set. Finally, we calculate forecast errors for all estimation periods and horizons. The errors are summarized using four different criteria: mean error (ME), mean absolute error (MAE), root mean square error (RMSE), and the Theil’s U statistic, which is the ratio of the RMSE of our equation to the RMSE of a random walk forecast (this is no change). A Theil’s U with a value below 1 indicates that our equation performs better than a random walk.

Versus a random walk

Table A4 presents for our equation, these different forecast error statistical measures for forecast of 1, 4, 8, and 12 quarters ahead, starting at different periods. We can see that the results depend on the starting point. For most cases, the mean errors are positive, which indicates that the equation has the tendency to overpredict the exchange rate. Note that the values of the Theil's U statistics are less than 1 for all starting periods. That means that for all cases our equation beats a random walk forecast even for a one-quarter-ahead forecast. This result is in contrast to most other empirical models of the exchange rate, which cannot beat simple random walk forecasts. A noticeable exception is Lafrance and van Norden 1995.

Versus Lafrance and van Norden 1995

The Lafrance and van Norden 1995 equation is the following:

$$\Delta rer_t = \Phi_1 \Delta rer_{t-1} + \Phi_2 nirst_{t-1} + \alpha(rer_{t-1} - \xi_0 - \xi_1 com_{t-1} - \xi_2 oil_{t-1}) \qquad (4)$$

where the variable *nirst* is the nominal 90-day commercial paper rate differential between Canada and the U.S., and *oil* is the real U.S. dollar price of oil based on the West Texas Intermediate crude oil price. We re-estimate equation (4) over the 1973Q3 to 1999Q4 sample (see table 5).[14] This produces results very similar to those in the original study (estimated over a 1972Q2 to 1994Q3 sample), demonstrating the stability of the coefficient estimates.

[14] It is important to note that even after controlling for the sample we were not able to replicate exactly the results reported in Lafrance and van Norden 1995. The difference in the result can perhaps be explained by revisions to the data, in particular revisions to the GDP deflator.

Table 5: Lafrance and Van Norden Equation Estimation Results
(Sample: 1973Q3 to 1999Q4)

Coefficient	Value	Standard Error
Φ_1	0.3114	0.0833**
Φ_2	0.6383	0.1276**
α	-0.1494	0.0289**
ξ_0	-0.2064	0.0853**
ξ_1	0.4262	0.0594**
ξ_2	-0.0630	0.0291**

Note: The asterisks * and ** indicate that the variable is statistically significant at the 10 and 5 percent level of significance respectively.

We performed the same dynamic out-of-sample forecasting for the Lafrance and van Norden equation exercise as we had done for our preferred equation (see table A5). Our equation overpredicts the value of the exchange rate for most of the period tested, and that of Lafrance and van Norden underpredicts the value for every period. But, with almost all the different measures presented, our equation outperforms the Lafrance and van Norden specification. The superiority of our equation is clear for the first four starting periods, with MAE relatively smaller, especially in the longer-horizon forecasts. For the last two starting periods, the results are quite similar for both equations. Note also that the Theil's U coefficient for our equation is in almost every case smaller than for the Lafrance and van Norden equation.

6. Decomposition of the Exchange Rate Evolution

After establishing that the non-energy commodity prices, the computer prices, the cumulated current account, and the nominal interest rate differential provide a good stable model for the Canada-U.S. real exchange rate, we consider the relative importance of each of the variables in explaining the evolution of the real exchange rate over the last 25 years. This is done by dynamically simulating the estimated equation from 1973Q3 to 1999Q4 and then identifying the source of the changes in the simulated values of the real exchange rate.

The Canadian dollar depreciated from U.S. $1.00 in 1973Q3 to U.S. $0.68 in 1999Q4, a decrease of 32 per cent. Over the same period, real non-energy commodity prices decreased 54 per cent, real computer prices fell 99.7 per cent and the ratio of cumulated current account to GDP declined 48 per cent. We therefore anticipate that the decrease in real non-energy commodity prices and the ratio of cumulated current account to GDP had a negative effect on the Canadian dollar, while the decrease in real computer prices had a positive effect.

A dynamic simulation of the equation from 1973Q3 to 1999Q4 generates a value of U.S. $0.67 for 1999Q4. The simulation results suggest that, if non-energy commodity prices had remained at their 1973Q3 value, other things being equal, the Canadian dollar would have appreciated to U.S. $1.05 in 1999Q4. That suggests that the decrease in non-energy commodity prices over this period accounted for a depreciation of 38 cents U.S. Using the same method, we find that the decline in the ratio of cumulated current account to GDP contributed to for a further depreciation of 12 cents U.S. The real decrease in computer prices offsets part of these declines, accounting for an appreciation of 20 cents U.S. The remaining 5 cents U.S. comes from the inflation rate differential (- 9 cents) and the nominal interest rate differential (+ 4 cents). These results (shown in table 6A) suggest that the decrease in non-energy commodity prices is the main explanation for the depreciation in the value of the Canadian dollar over the last 25 years.

Table 6A: Decomposition of Exchange Rate Evolution (1973Q3 to 1999Q4)

	In U.S. dollars	
	Depreciation or appreciation	Value
Canadian dollar value in 1973Q3		**1.00**
Non-energy commodity price	– 0.38	
Ratio of the cumulated current account to GDP	– 0.12	
Computer price	+ 0.20	
Inflation differential	– 0.09	
Nominal interest rate differential	+ 0.04	
Residual	+ 0.03	
Total change	– 0.32	
Canadian dollar value in 1999Q4		**0.68**

A similar historical decomposition can be applied to the 1990's (see table 6B). The Canadian dollar depreciated from U.S. $0.86 in 1989Q4 to U.S. $0.68 in 1999Q4, a decrease of 18 cents U.S. in a decade. Again, important declines in real non-energy commodity prices and in the ratio of cumulated current account to GDP in the early 1990's accompanied the depreciation of the Canadian dollar. The decrease in real computer prices offsets part of these declines, accounting for an appreciation of the dollar. The results indicate that the relative importance of these three factors at explaining the movements of the Canadian dollar in the 1990s is qualitatively similar to that of the previous decade.[15] However, inflation and interest rate differentials have played a different role in the 1990's than over previous period.

Table 6B: Decomposition of Exchange Rate Evolution (1989Q4 to 1999Q4)

	In U.S. dollars	
	Depreciation or appreciation	Value
Canadian dollar value in 1989Q4		**0.86**
Non-energy commodity price	– 0.13	
Ratio of cumulated current account to GDP	– 0.08	
Computer price	+ 0.07	
Inflation differential	+ 0.04	
Nominal interest rate differential	–0.05	
Residual	–0.03	
Total change	– 0.18	
Canadian dollar value in 1999Q4		**0.68**

7. Conclusion

The goal of this research was to construct a real exchange rate equation for forecasting purposes. In this paper we document a cointegrating relationship between the real

[15] Our results show that the effect of non-energy commodity prices has remained strong in the 1990's. This seems to contradict the finding of a recent study by Laidler and Aba (2001) of the C.D. Howe Institute. Using a specification inspired by Amano and Van Norden (1993) they find that even if non-energy commodity prices continue to be an important determinant of the Canadian dollar, their quantitative impact

exchange rate, real U.S. dollar non-energy commodity prices, real U.S. dollar computer prices, and the cumulated current account-to-GDP ratio. Exogeneity tests show that all these variables are weakly exogenous with respect to the real exchange rate, allowing us to construct an equation based on a single equation framework (an error-correction model–ECM). We were able to find a parsimonious equation that shows a strong ability to forecast the evolution of the real exchange rate.

Our results show the important role played by non-energy commodity prices, computer prices, and the cumulated current account position in the determination of the real exchange rate. Our equation is thus a useful forecasting tool, exhibiting a better track record over most time horizons and forecasting criteria than either a naive model or the Lafrance and Van Norden 1995 equation.

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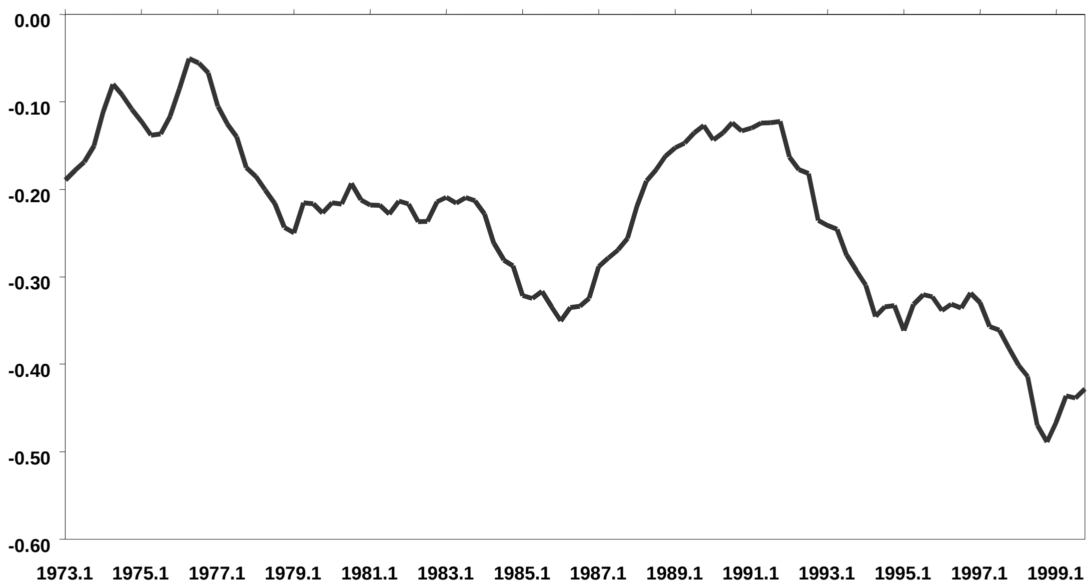
seems to have diminished in the 1990's. However, as we noted before (see footnote 11), they do not perform formal stability tests of their models.

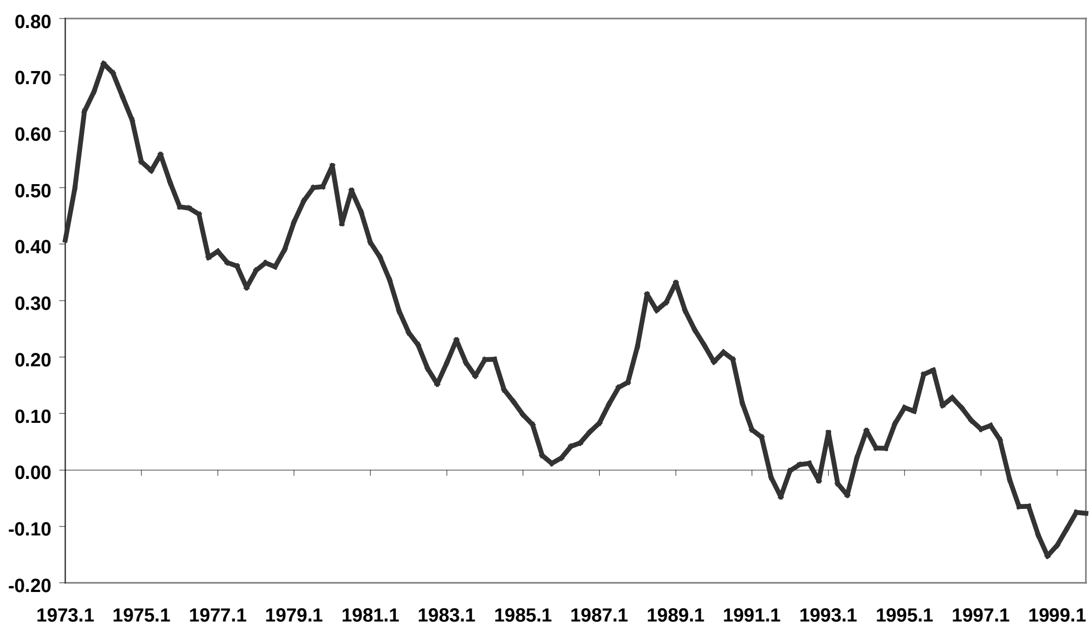
Appendix A: Graphs and Tables

Figure A1: Logarithm of the Canada-U.S. Real Exchange Rate



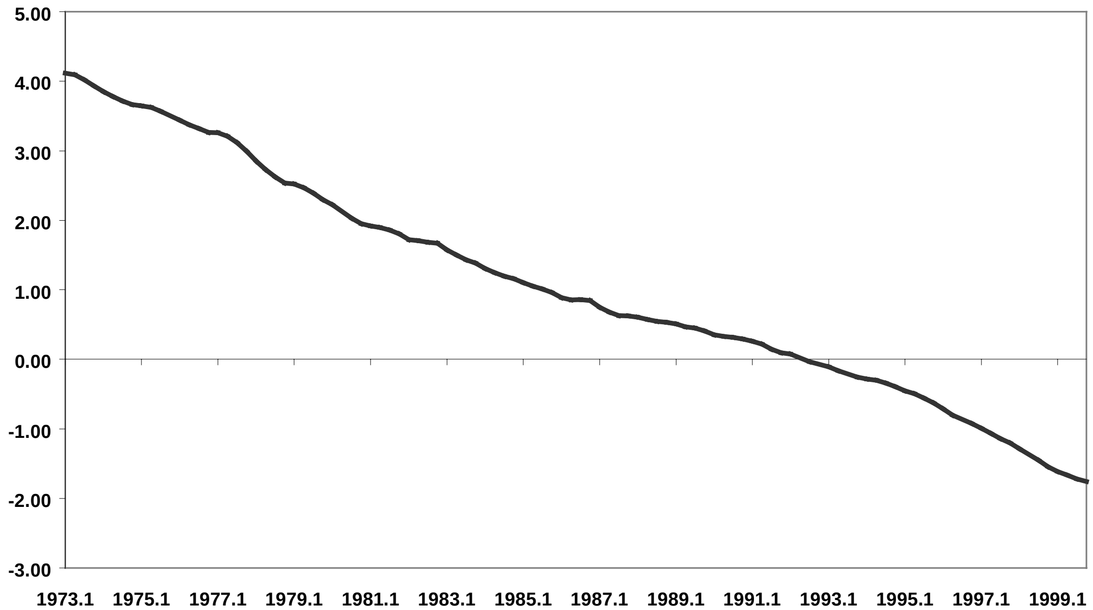
Source: Statistics Canada

Figure A2: Logarithm of the Real U.S. Dollar Non-Energy Commodity Price Index



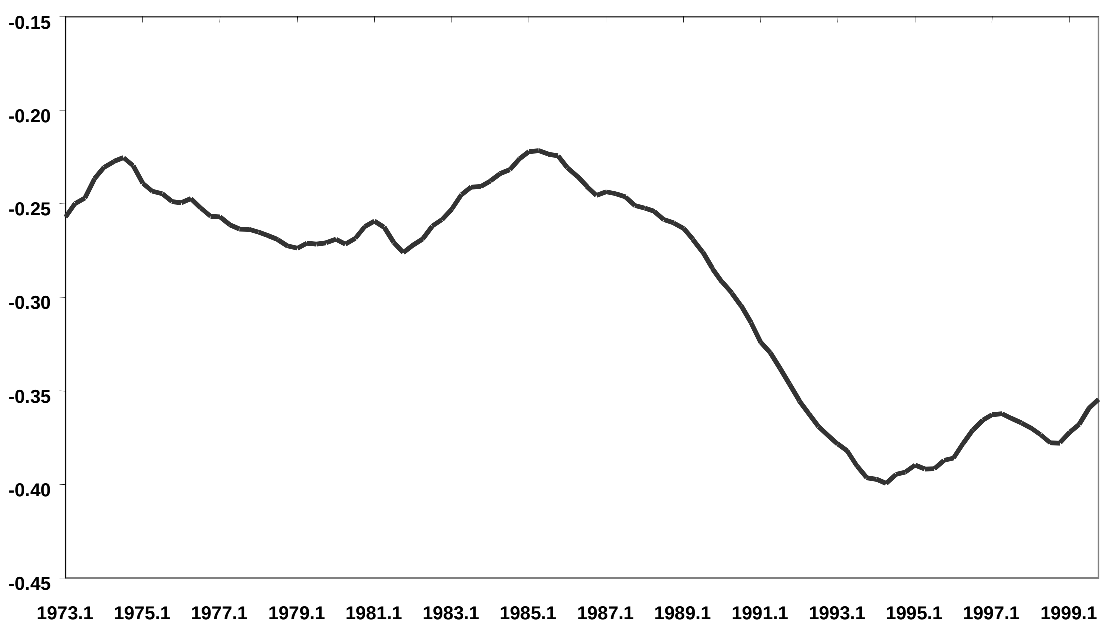
Source: Statistics Canada

Figure A3: Logarithm of the Real U.S. Dollar Computer Price Index



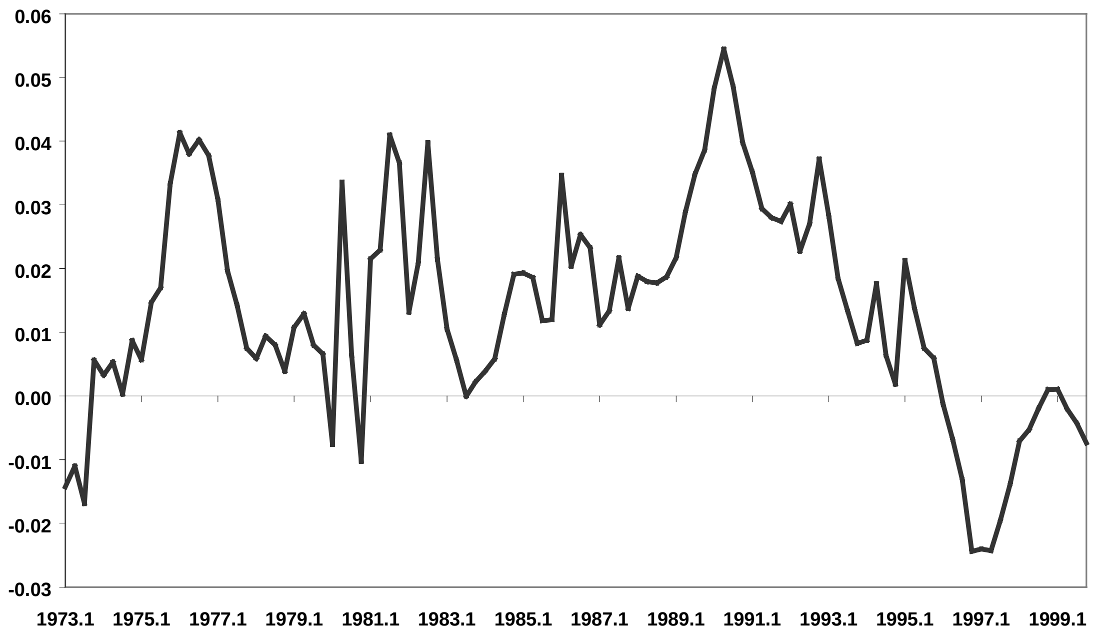
Source:Statistice Canada

Figure A4: Ratio Of The Cumulated Current Account to GDP



Source: Statistics Canada

Figure A5: Canada-U.S. Nominal Interest Rate Differential



Source: Statistics Canada

Table A1: Diagnostic Tests for Equation 3

Test	Test statistic	p-value
White	10.7997	0.5705
LM(4)	9.3752	0.0524
RESET (3)	2.4957	0.4761
ARCH(4)	4.4129	0.3530

Table A2: Hansen Test of Coefficient Stability for Equation 3

Variables	Test statistics	5% critical value
$nirst_{t-1}$	0.12472	0.470
Δrer_{t-1}	0.46475	0.470
rer_{t-1}	0.11445	0.470
com_{t-1}	0.19838	0.470
$lrcomp_{t-1}$	0.21428	0.470
cca_{t-1}	0.09950	0.470
Joint test	1.16856	1.900

Figure A6: Recursive Coefficient Estimates for Equation 3

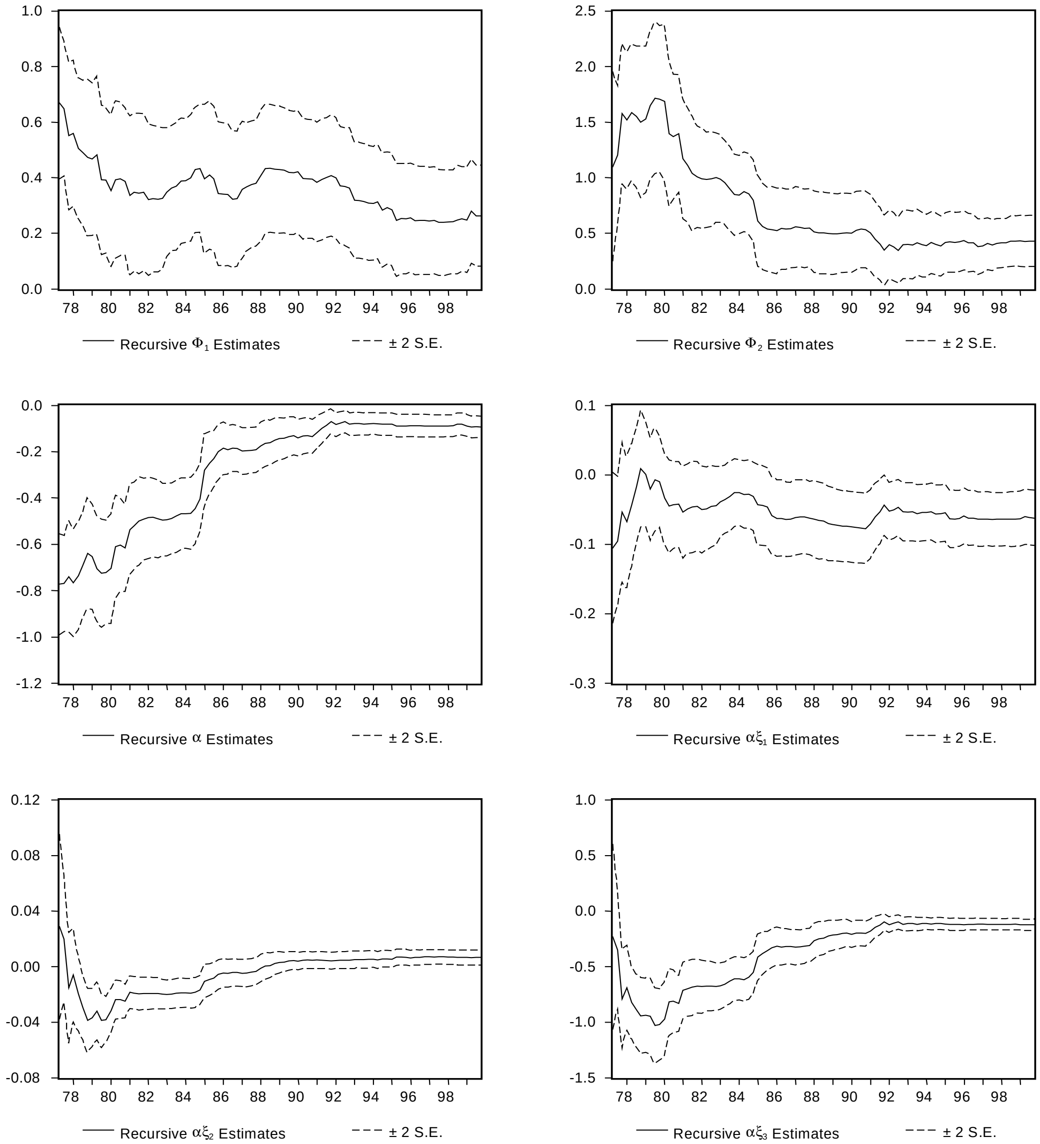


Table A3: Granger-Causality Tests for Equation 3

Null hypothesis	F-statistic	Probability
com does not Granger-cause *rer*	2.41042	0.05419
rer does not Granger-cause *com*	0.17774	0.94940
lrcomp does not Granger-cause *rer*	2.99764	0.02213
rer does not Granger-cause *lrcomp*	2.43946	0.05186
cca does not Granger-cause *rer*	2.17922	0.07680
rer does not Granger-cause *cca*	1.01490	0.40352
nirst does not Granger-cause *rer*	4.22829	0.00334
rer does not Granger-cause *nirst*	1.43585	0.22779

Figure A7-A10: Forecasting Performance of Equation 3

Figure A7: Actual and In-Sample (1973Q3 to 1999Q4) Forecast Values of the Real Exchange Rate

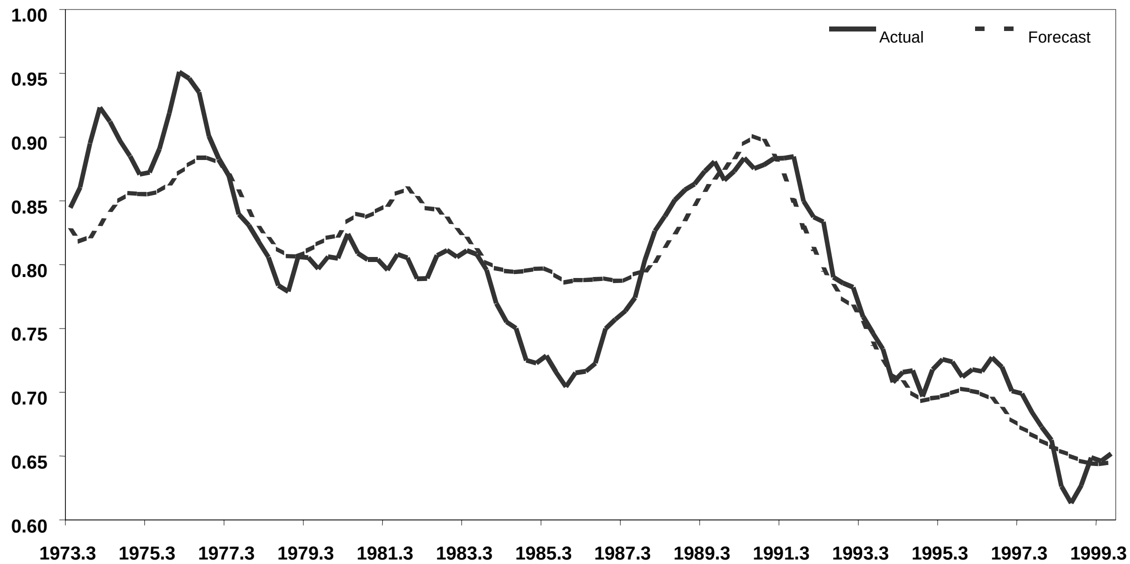


Figure A8: Actual and Out-Of-Sample (1992Q1 to 1999Q4) Forecast Values of the Real Exchange Rate

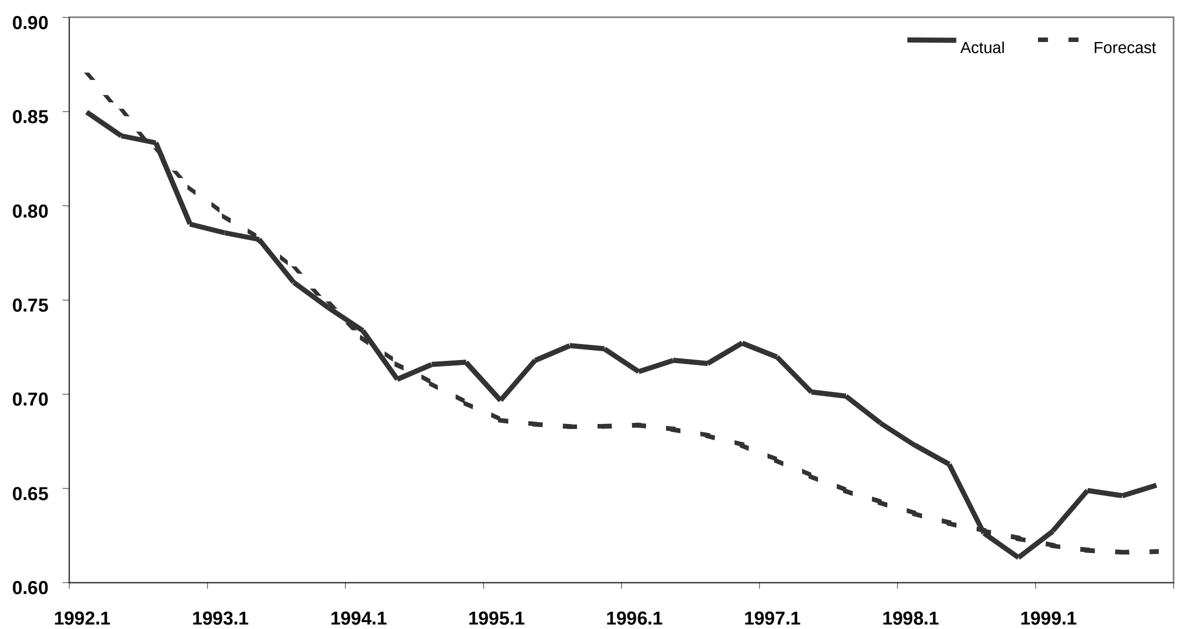


Figure A9: Actual and Out-Of-Sample (1994Q1 to 1999Q4) Forecast Values of the Real Exchange Rate

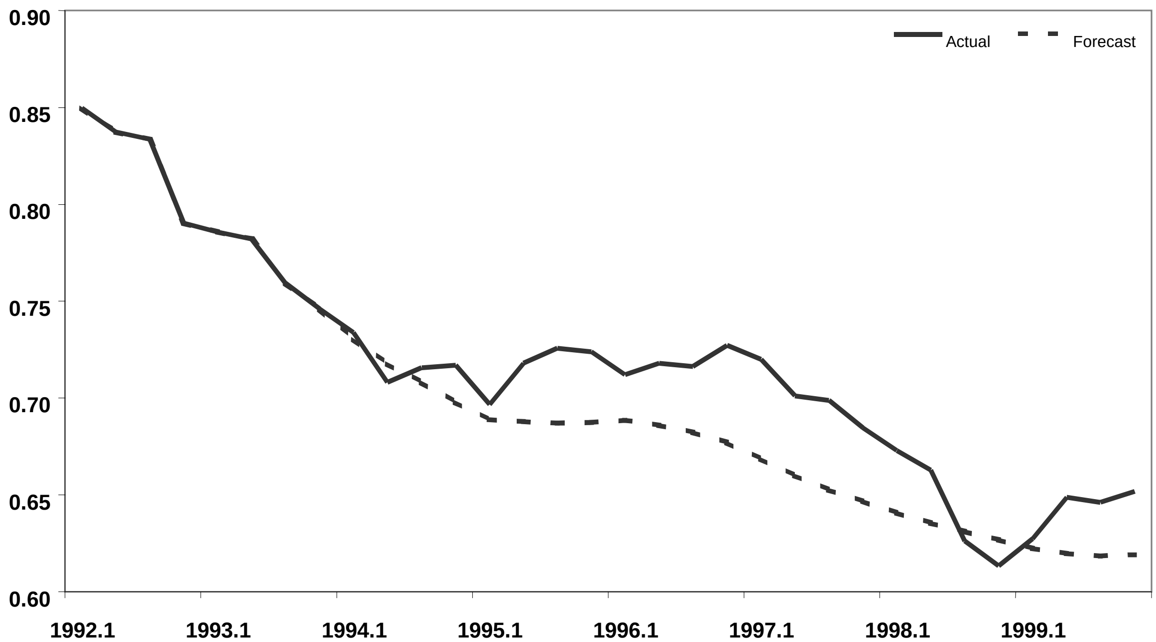


Figure A10: Actual and Out-Of-Sample (1996Q1 to 1999Q4) Forecast Values of the Real Exchange Rate

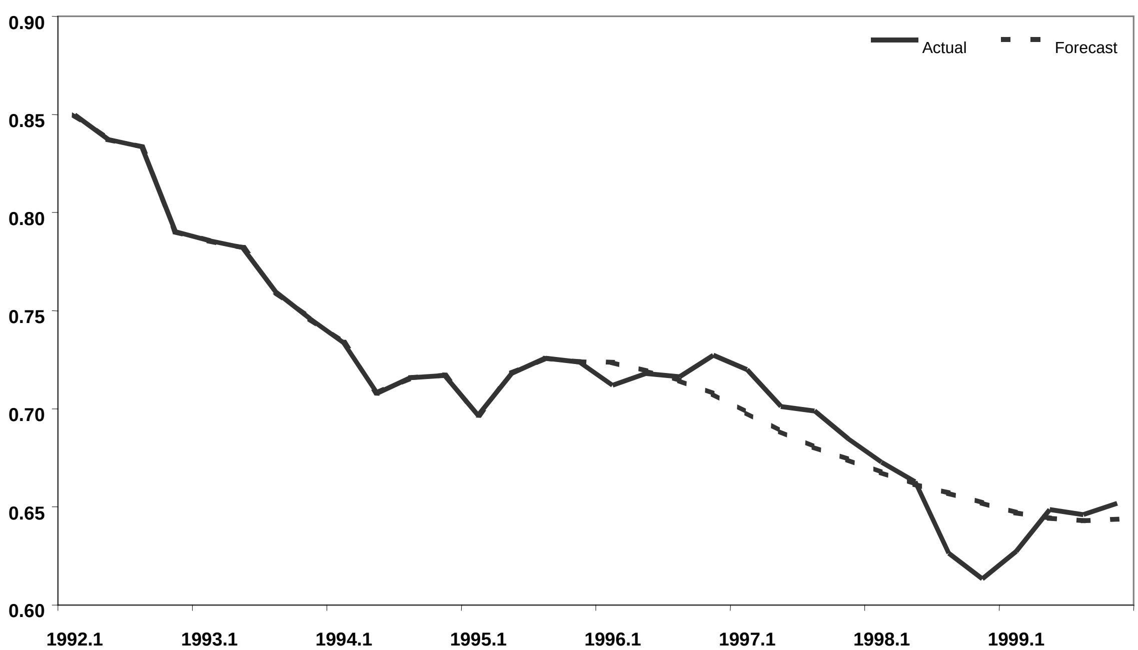


Table A4: Out-of-Sample Forecasts* for Equation 3

Horizon (quarters)	Mean error	MAE	RMSE	Theil's U	Number of observations
Starting period: 1992Q1					
1	0.0031	0.0159	0.0198	0.8609	32
4	0.0161	0.0286	0.0357	0.5908	29
8	0.0286	0.0364	0.0414	0.4270	25
12	0.0417	0.0458	0.0495	0.4387	21
Starting period: 1993Q1					
1	0.0069	0.0151	0.0193	0.9222	28
4	0.0300	0.0367	0.0427	0.7886	25
8	0.0498	0.0532	0.0573	0.7204	21
12	0.0610	0.0606	0.0669	0.7361	17
Starting period: 1994Q1					
1	0.0044	0.0146	0.0191	0.8915	24
4	0.0196	0.0311	0.0378	0.7595	21
8	0.0271	0.0384	0.0411	0.5358	17
12	0.0339	0.0420	0.0463	0.4978	13
Starting period: 1995Q1					
1	0.0019	0.0143	0.0189	0.8753	20
4	0.0068	0.0273	0.0347	0.6568	17
8	-0.0045	0.0276	0.0342	0.3946	13
12	-0.0030	0.0228	0.0274	0.2473	9
Starting period: 1996Q1					
1	-0.0007	0.0139	0.0184	0.8507	16
4	-0.0020	0.0314	0.0376	0.6291	13
8	-0.0142	0.0230	0.0331	0.3230	9
12	-0.0131	0.0203	0.0290	0.2332	5
Starting period: 1997Q1					
1	-0.0038	0.0147	0.0193	0.8061	12
4	-0.0200	0.0387	0.0451	0.6375	9
8	-0.0385	0.0385	0.0505	0.4528	5
12	-0.0146	0.0146	0.0146	0.1327	1

Note: The ME, MAE and RMSE should be viewed in terms of cents. For example, a 0.05 ME means an average error of 5 cents.

Table A5: Out-of-Sample Forecasts Based on Lafrance and van Norden equation

Horizon (quarters)	Mean error	MAE	RMSE	Theil's U	Number of observations
Starting period: 1992Q1					
1	-0.0125	0.0177	0.0224	0.9749	32
4	-0.0551	0.0554	0.0634	1.0494	29
8	-0.0812	0.0812	0.0873	0.8998	25
12	-0.0855	0.0855	0.0915	0.8111	21
Starting period: 1993Q1					
1	-0.0119	0.0165	0.0214	1.0248	28
4	-0.0514	0.0523	0.0607	1.1195	25
8	-0.0698	0.0698	0.0774	0.9719	21
12	-0.0716	0.0716	0.0798	0.8781	17
Starting period: 1994Q1					
1	-0.0111	0.0165	0.0215	1.0051	24
4	-0.0431	0.0456	0.0549	1.1033	21
8	-0.0577	0.0577	0.0680	0.8870	17
12	-0.0563	0.0563	0.0680	0.7319	13
Starting period: 1995Q1					
1	-0.0064	0.0154	0.0198	0.9146	19
4	-0.0252	0.0343	0.0441	0.8354	17
8	-0.0305	0.0355	0.0481	0.5542	13
12	-0.0463	0.0473	0.0566	0.5112	9
Starting period: 1996Q1					
1	-0.0019	0.0141	0.0187	0.8663	16
4	-0.0061	0.0311	0.0401	0.6694	13
8	-0.0249	0.0364	0.0423	0.4139	9
12	-0.0411	0.0411	0.0459	0.3692	5
Starting period: 1997Q1					
1	-0.0051	0.0137	0.0190	0.7929	12
4	-0.0289	0.0326	0.0458	0.6475	9
8	-0.0527	0.0527	0.0572	0.5124	5
12	-0.0266	0.0266	0.0266	0.2421	1

Note: The ME, MAE and RMSE should be viewed in terms of cents. For example, a 0.05 ME means an average error of 5 cents.

Appendix B: Econometric Method

The econometric method we used to estimate our equation is the full-system maximum likelihood estimation technique of Johansen (1988, 1995) and Johansen and Juselius (1992). This method has important advantages. It not only provides a test for cointegration, but it also gives us information on the number of cointegrating relationships. In addition, it allows us to test for weak exogeneity and for exclusion of the variables contained in the long-run relationship.

This approach is based on a vector error-correction model (VECM) represented as:

$$\Delta x = \eta + \sum_{i=1}^{p-1} \Phi_i \Delta x_{t-i} - \Pi x_{t-1} + \varepsilon_t \qquad \text{(C1)}$$

where η is an $(n \times 1)$ vector of deterministic variables, and ε is an $(n \times 1)$ vector of white noise disturbances, with mean zero. Φ is an $(n \times n)$ matrix of short-term effect coefficients, Π is an $(n \times n)$ matrix whose rank determines the number of cointegrating vectors. If Π is of either full rank, *n*, or zero rank, there will be no cointegration among the elements in the long-run relationships. If Π is of reduced rank, *r* (where $r < n$), then there will exist $(n \times r)$ matrices α and β such that $\Pi = \alpha\beta'$ where β is the matrix whose columns are the linear independent cointegrating vectors. The α matrix is interpreted as the adjustment matrix, indicating the speed with which the system responds to the last period's deviation from the equilibrium level of the real exchange rate.

We test for the existence of cointegration using two tests proposed by Johansen. The likelihood ratio, or Trace, test statistic for the hypothesis that there are at most *r* distinct cointegrating vectors is:

$$TR = T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i) \qquad \text{(C2)}$$

Additionally, the likelihood ratio statistic tests for at most r cointegrating vectors against the alternative of r + 1 cointegrating vectors – the maximum eigenvalue statistic – is given by:

$$LR = T \ln(1 - \lambda_{r+1}) \qquad \text{(C3)}$$

Both statistics will then be compared with the approximate critical values generated by Osterwald-Lenum 1993.

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