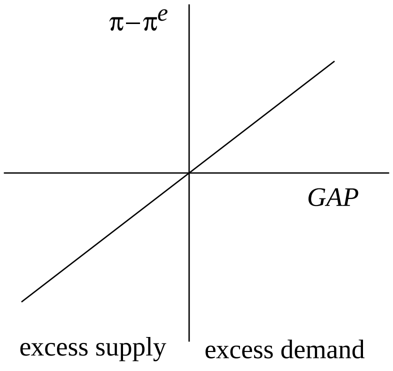
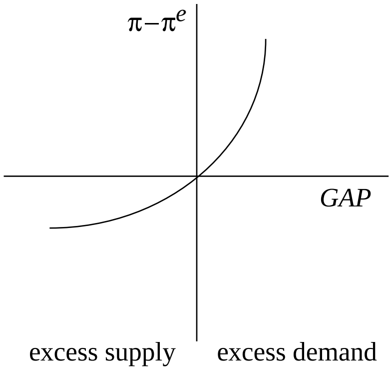
Appendix 1

Different Types of Output-Inflation Relationships

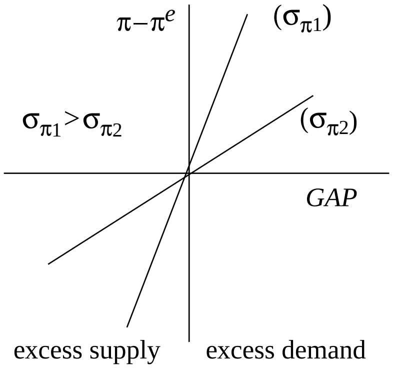
1. Linear model



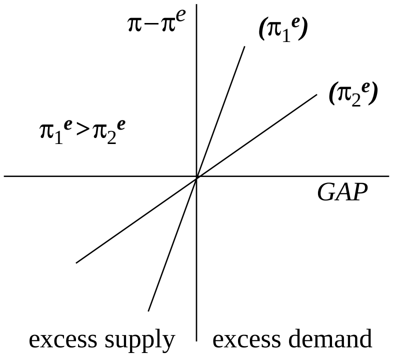
2. Capacity constraint model



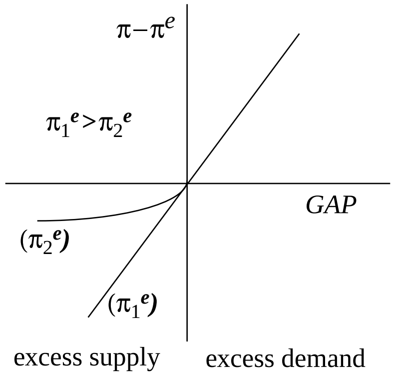
3. Misperception model



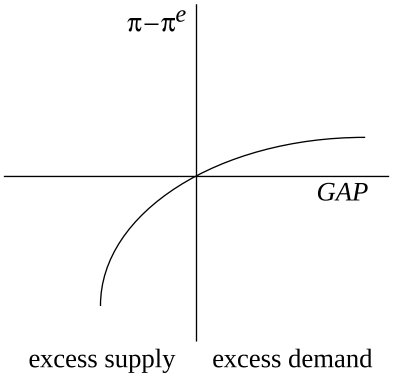
4. Costly adjustment model



5. Downward nominal wage rigidity model



6. Monopolistically competitive model



Appendix 2

Long-Run Restrictions Imposed on Output to Measure Potential Output

This appendix briefly presents the decomposition method based on long-run restrictions imposed on output (LRRO) to measure potential output.[1]

Let Z_t be an $n \times 1$ stationary vector including an n_1-vector of I(1) variables and an n_2-vector of I(0) variables such that $Z_t = (\Delta X_{1t}', X_{2t}')'$.[2] By the Wold decomposition theorem, Z_t can be expressed as the following reduced form:

$$Z_t = \delta(t) + C(L)\varepsilon_t, \tag{A2.1}$$

where $\delta(t)$ is deterministic; $C(L) = \Sigma_{i=0}^{\infty} C_i L^i$ is a matrix of polynomial lags; $C_0 = I_n$ is the identity matrix; the vector ε_t is the one-step-ahead forecast errors in Z_t, given information on lagged values of Z_t; $E(\varepsilon_t) = 0$; and $E(\varepsilon_t \varepsilon_t') = \Omega$ with Ω positive definite. We suppose that the polynomial det $|C(L)|$ has all its roots on or outside the unit circle, which rules out the non-fundamental representations emphasized by Lippi and Reichlin (1993).

Equation (A2.1) can be decomposed into a long-run component and a transitory component:

$$Z_t = \delta(t) + C(1)\varepsilon_t + C^*(L)\varepsilon_t, \tag{A2.2}$$

where $C(1) = \Sigma_{i=0}^{\infty} C_i$, and $C^*(L) = C(L) - C(1)$. We define $C_1(1)$ as the long-run multiplier of the vector X_{1t}. If the rank of $C_1(1)$ is less than n_1, there exists at least one linear combination of the elements in X_{1t} that is I(0).

The LRRO approach assumes that Z_t has the following structural representation:

$$Z_t = \delta(t) + \Gamma(L)\eta_t, \tag{A2.3}$$

where η_t is an *n*-vector of structural shocks, $E(\eta_t) = 0$, and $E(\eta_t \eta_t') = I_n$ (a simple normalization). From the estimated reduced form, we can retrieve

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[1] For a more detailed presentation of the LRRO approach see Watson (1994); Dupasquier, Guay, and St-Amant (1997); or St-Amant and van Norden (1997).

[2] I(d) denotes a variable that is integrated of order d.

the structural form (A2.3) using the following relationships: $\Gamma_0\Gamma_0' = \Omega$, $\varepsilon_t = \Gamma_0\eta_t$, and $C(L) = \Gamma(L)\Gamma_0^{-1}$.

The long-run covariance matrix of the reduced form is equal to $C(1)\Omega C(1)'$. From (A2.2) and (A2.3) we have:

$$C(1)\Omega C(1)' = \Gamma(1)\Gamma(1)'. \tag{A2.4}$$

This relationship suggests that we can identify matrix Γ_0 with an appropriate number of restrictions on the long-run covariance matrix of the structural form.

Let us assume that the log of output is the first variable in the vector Z_{1t}. It is then equal to:

$$\Delta y_t = \mu_y + \Gamma_1^p(L)\eta_t^p + \Gamma_1^c(L)\eta_t^c, \tag{A2.5}$$

where η_t^p is the vector of permanent shocks affecting output, and η_t^c is the vector containing shocks having only a transitory effect on output. Potential output based on the LRRO method is then:

$$\Delta y_t^p = \mu_y + \Gamma_1^p(L)\eta_t^p. \tag{A2.6}$$

Thus, “potential output” corresponds to the permanent component of output. The part of output due to transitory shocks is defined as the “output gap.”

Appendix 3

Description of Data

We have used quarterly gross domestic product (GDP) as the measure of real output in Canada and the United States from 1964 to 1995. Canadian and U.S. inflation are measured by the total consumer price index (CPI) (excluding GST, QST, and tobacco tax, in the case of Canada) and CPIXFE, the CPI excluding food and energy. For the Canadian data, the seasonal adjustment is made at the Bank of Canada, while for the U.S. data it is done by Data Resources INC. Interest rates are defined as the overnight rate (RON) for Canada (for a description of RON, see Armour, Engert, and Fung 1996), and the federal funds rate for the United States.

We test for unit roots using augmented Dickey-Fuller statistics. On the basis of our tests, we cannot reject the hypothesis that production, inflation rates, and interest rates are first-order integrated.

Appendix 4

Maximum-Likelihood Estimation of the State-Space Model

The parameters of the state-space model are estimated using maximum likelihood (ML). A Kalman filter generates the prediction error decomposition form of the likelihood function as in Harvey (1993). Numerical maximization is implemented with GAUSS software.

The state-space model is defined by equations (2) and (3) in the text as follows:

$$\pi_t = a \cdot \pi_t^e + (1-a) \cdot \pi_{t-1} + \beta_t \cdot GAP_t + \varepsilon_t \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \qquad \text{(A4.1)}$$

$$\beta_t = \alpha + \rho \cdot \beta_{t-1} + \gamma \cdot X_{t-1} + \mu_t \qquad \mu_t \sim N(0, \sigma_\mu^2). \qquad \text{(A4.2)}$$

The parameters to be estimated by ML are $\{a, \alpha, \rho, \gamma, \sigma_\varepsilon, \sigma_\mu\}$. These are called the hyper parameters of the model. The Kalman filter takes these parameters as given and produces time-series estimates of β_t and ε_t. Let $\beta_{t|s}$ denote the prediction of β_t given information up to period *s*, and let $P_{t|s}$ be the associated conditional variance. Then, given starting values for the elements of the distribution of β_0, denoted by $\beta_{0|0}$ and $\mathrm{P}_{0|0}$, the Kalman filter proceeds iteratively for $t = 1$ to $t = T$ as follows:

$$\beta_{t|t-1} = \alpha + \rho \cdot \beta_{t-1|t-1} + \gamma \cdot X_{t-1} \qquad \text{(A4.3)}$$

$$P_{t|t-1} = \rho^2 \cdot P_{t-1|t-1} \qquad \text{(A4.4)}$$

$$\varepsilon_{t|t-1} = \pi_t - a \cdot \pi_t^e - (1-a) \cdot \pi_{t-1} - \beta_{t|t-1} \cdot GAP_t \qquad \text{(A4.5)}$$

$$H_t = P_{t|t} \cdot GAP_t^2 + \sigma_\varepsilon^2 \qquad \text{(A4.6)}$$

$$K_{t|t-1} = P_{t|t-1} \cdot GAP_t \cdot H_t^{-1} \qquad \text{(A4.7)}$$

$$\beta_{t|t} = \beta_{t|t-1} + \mathrm{K}_{t|t-1} \cdot \varepsilon_{t|t-1} \qquad \text{(A4.8)}$$

$$P_{t|t} = (I - \mathrm{K}_{t|t-1} \cdot GAP_t) \cdot P_{t|t-1}\,. \qquad \text{(A4.9)}$$

H_t in equation (A4.6) is the conditional variance of the prediction errors, $\varepsilon_{t|t-1}$. It incorporates parameter uncertainty about the slope of the Phillips curve in addition to uncertainty about the supply shocks. The prediction error decomposition form of the likelihood function for observation *t* is therefore:

$$\log(l_t) = -\frac{\log 2.\mathrm{pi}}{2} - \frac{\log H_t}{2} - \frac{\varepsilon_{t|t-1}^2}{2H_t}. \tag{A4.10}$$

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