Bank of Canada Banque du Canada

Working Paper 2001-9 / Document de travail 2001-9

# Testing for a Structural Break in the Volatility of Real GDP Growth in Canada

by

**Alexandre Debs**

ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2001-9

June 2001

# Testing for a Structural Break in the Volatility of Real GDP Growth in Canada

**by**

**Alexandre Debs**

Research Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9

# Contents

# Acknowledgements

The subject of this paper was initially proposed by Allan Crawford and Dave Longworth. I would like to thank Maral Kichian for providing me with first versions of GAUSS programs and for many helpful comments; Gabriel Srour and Allan Crawford for supervision; and Bob Amano, Don Coletti, Jean Farès, Irene Ip, Hashmat Khan, Dave Longworth, and Brian O'Reilly for useful suggestions. Of course, the author is solely responsible for any errors.

## Abstract

This study tests for a structural break in the volatility of real GDP growth in Canada following the methodology of McConnell and Quiros (1998). A break is found in the first quarter of 1991. Based on disaggregated data, the tests indicate a break in the volatility of the rate of change of investment in residential structures and a break in the volatility of the rate of growth of personal expenditures on goods. Three possible explanations are given for the break in the data: a more service-oriented economy, improved inventory management, and a change in monetary policy.



*JEL classification: C12, E32*
*Bank classification: Business fluctuations and cycles; Econometric and statistical methods*



## Résumé

L'auteur cherche à déceler la présence d'un point de rupture structurel dans la volatilité de la croissance du PIB réel du Canada selon la méthode utilisée par McConnell et Quiros (1998). Il constate qu'une rupture structurelle s'est produite au premier trimestre de 1991. Les tests appliqués à des données désagrégées indiquent une rupture dans la volatilité du taux de variation de l'investissement dans le parc immobilier résidentiel et une rupture dans la volatilité du taux de croissance des dépenses des particuliers en biens. L'auteur propose trois explications possibles de cette rupture : le poids grandissant des services dans l'économie, l'amélioration de la gestion des stocks et le changement apporté à la conduite de la politique monétaire.



*Classification JEL : C12, E32*
*Classification de la Banque : Cycles et fluctuations économiques; Méthodes économétriques et statistiques*

# 1. Introduction

It has been widely observed that output growth has become less volatile in recent years in several industrialized countries, including the United States and Canada. Figures 1 and 2, which plot the quarterly growth rate of real GDP in the United States and Canada over the periods 1953Q2–2000Q1 and 1961Q2–2000Q1, respectively, confirm that fact.[1] Standard deviations over selected subperiods (Tables 1 and 2) show that fluctuations in quarterly output growth are significantly smaller in recent years: the standard deviation of GDP growth in the United States over the period ending in 1984 is more than twice as large as the standard deviation over the period since then, and the standard deviation of GDP growth in Canada has decreased by more than half since 1991. The selection of these subperiods is not arbitrary: McConnell and Quiros (1998) estimate a structural break in the volatility of U.S. GDP growth in the first quarter of 1984 and a break in the volatility of Canadian GDP growth in the second quarter of 1991. The reduced volatility of GDP growth can also be found in the smoother year-to-year growth-rate series (Figure 3).

A break in the volatility of GDP growth has important consequences. It can affect calibrations used in projection models, it can bias model estimation if not properly adjusted for, and it can affect the interpretation of data.

The presence of a break also raises important questions as to the causes of that break. One possible explanation is the new economy, or changes in production and organization techniques. McConnell and Quiros examine the disaggregated data in the United States and find that the break in the data for validity in aggregate output-growth stems from a reduction in the volatility in the growth of durable goods output. Moreover, they find that the break in data for durables is coincident with a break in the proportion of durables output accounted for by changes in inventories. Consequently, they argue that the break in volatility in the United States results from the implementation of just-in-time inventory-management techniques. This explanation is controversial, and alternatives have been suggested. Taylor (2000), for example, argues that the reduced volatility of real GDP growth in the United States is the result of a more-efficient monetary policy. In Canada, the adoption of inflation-control targets around the time of the break in data suggests that a change in monetary policy must be considered to be a possible explanation.

In this paper, I reproduce McConnell and Quiros' tests for a break in the data on the volatility of real GDP growth in Canada and find a break in the first quarter of 1991. Further, I examine

1. Data for the United States are chain-weighted GDP data in 1996 dollars, as provided by the Bureau of Economic Analysis (2000). Data for Canada are GDP at 1992 prices, seasonally adjusted, from Statistics Canada (2000).

whether McConnell and Quiros' findings for the United States regarding durables and inventories apply to Canada. This does not seem to be the case. In fact, the most obvious breaks concurrent with the break in data for overall production appear in investment in residential structures and in personal expenditures on goods.

This paper is organized as follows. Section 2 describes the tests and compares my results with those of McConnell and Quiros on aggregate data for Canada and the United States. Section 3 examines disaggregated data. Section 4 discusses possible explanations. Section 5 offers some conclusions.

# 2. Structural Break

## 2.1 The problem

Figures 1 and 2 show that the volatility of real GDP growth is lower at the end of the sample for both countries, but the timing of the break, if any, is not clear. The question is: Has there been a structural break in the volatility of real GDP growth? And if so, when?[2]

The GDP growth rate at quarter *t*, denoted $\dot{y}_t$, is characterized as an AR(1) process[3]:

$$\dot{y}_t = \mu + \phi \dot{y}_{t-1} + \varepsilon_t \tag{1}$$

If the residuals are assumed to follow a normal distribution, then $\sqrt{\frac{\pi}{2}}|\hat{\varepsilon}_\tau|$ is an unbiased estimator of their standard deviation. The task is therefore to look for a break in the constant of the equation:

$$\sqrt{\frac{\pi}{2}}|\hat{\varepsilon}_\tau| = \alpha + \nu_\tau \tag{2}$$

Following McConnell and Quiros, the break point, *T*, is estimated by jointly estimating (using the generalized method of moments (GMM)), equation (1) and equation (2), the latter being modified by means of added dummy variables to allow the constant to equal $\alpha_1$ if $\tau$ is less than *T*, and $\alpha_2$ otherwise.

---

2. This is not the only way to address the problem. One could instead use a Markov-switching analysis and attribute the higher stability of output growth to a narrowing gap between the mean growth rates during booms and recessions. McConnell and Quiros reject this possibility. Kim and Nelson (1999) disagree with McConnell and Quiros on this point.
3. McConnell and Quiros have found that the optimal level of the AR(*p*) is 1 for both U.S. GDP growth and Canadian GDP growth.

Letting $F_n(T)$ be the Wald, LM, or LR test of the equality of the coefficients $\alpha_1$ and $\alpha_2$ under the null hypothesis that there is no break, it is intuitive that the most probable break date, if any, is the period $T$ when $F_n(T)$ takes the highest value. (Notice that $T$ is a nuisance parameter that appears under the alternative hypothesis, but not under the null hypothesis.) Accordingly, Andrews (1993) proposes the statistic

$$\sup F_n = \sup_{T_1 \le T \le T_2} F_n(T)$$

to test for the presence of a break. He shows its asymptotic properties and derives the asymptotic critical values.[4] The $T$ that maximizes $F_n(T)$ will be the estimated date of the break point. (See McConnell and Quiros 1998 and Andrews 1993.)

Andrews and Ploberger (1994) show that this test is not optimal and propose the following alternative test statistics, whose *p*-values are computed as in Hansen (1997):

$$\exp F_n = \ln\left(\frac{1}{T_2 - T_1 + 1} * \sum_{T=T_1}^{T_2} \exp(1/2 * F_n(T))\right)$$

$$ave F_n = \frac{1}{T_2 - T_1 + 1} * \sum_{T=T_1}^{T_2} F_n(T)$$

This study focuses on the results obtained when $F_n(T)$ refers to the LM statistic.[5]

## 2.2 Results

Table 3 reports the results of the tests for a structural break in the residual variance for the United States over the sample period used by McConnell and Quiros. Like them, I find strong evidence for the rejection of the null hypothesis, and the estimated break date is the first quarter of 1984.[6]

---

4. To ensure convergence in distribution, I follow Andrews' suggestion of setting $T_1$=0.15**n* and $T_2$=0.85**n*. This is consistent with the work of McConnell and Quiros.
5. Wald statistics have also been computed, and differences can arise when one is used instead of the other. However, the LM statistics are invariant in the way the absence of break is parameterized, and they are more reliable.
6. Notice that my results are comparable to those of McConnell and Quiros, both in terms of the estimated break date and the *p*-values, but there seems to be a slight difference in the test statistics. We both use data from the Bureau of Economic Analysis (2000). However, the discrepancy could be attributed to data revision, since McConnell and Quiros run their analysis and alternative options in the GMM procedure (especially when computing the weighting matrix).

Table 4 reports the results of the tests for Canada over a sample subperiod considered by McConnell and Quiros. There is strong evidence in favour of the rejection of the null hypothesis in Canada, and the estimated break date is in the first half of 1991.[7] Finally, Table 5, which extends the period to 2000Q1, shows that the latter years confirm those results, the *p*-values being even lower. The break can also be found in the year-to-year growth rate series (Table 6).

Before examining disaggregated data, it is important to make sure that a structural break in the AR(1) process for output growth is not responsible for a break in the residual variance. Tables 7 and 8 report the results of joint and separate tests for a break in the constant and the coefficient of the lagged dependent variable in the AR(1) process, for the United States and Canada, respectively. For the United States, the null hypothesis of no break cannot be rejected. For Canada, the joint estimation indicates a possible break in 1973Q4, although the separate tests are not conclusive.

To determine whether the possible break in the AR(1) process could be responsible for the break in residual variance, I test for a structural break in the residual variance for the period starting in 1974Q1. Again, the test indicates 1991Q1 as a break date (Table 9).[8] Similarly, the test indicates a break in 1991Q1 when restricted to the subperiod beginning in 1981Q1, whereas it indicates no break when restricted to the earlier subperiod (Table 10). Table 11 reports the values of the parameters estimated over the subsamples determined by the break in the volatility of production and the break in the parameters of the AR(1) process: the constant $\alpha$ is significantly lower in the latter period, whereas the parameter on the lagged variable is higher.[9] The AR(1) process is stable for the year-to-year growth rate series.

## 3. Disaggregated Data

McConnell and Quiros examine whether an individual component of spending can explain the overall break. They find that the break in aggregate output growth volatility can be attributed to a break in the data for durable goods production. Moreover, they find that the break in durables production is coincident with a break in the data on the proportion of durables output accounted for by changes in inventories.

---

7. There are minor differences between my results and those of McConnell and Quiros (they present only the “exponential” and “average” statistics).
8. Note that the process is stable over that subsample (structural break tests on the parameters indicate no break).
9. Differences in the parameters before and after 1973Q4 seem to be important. If one imposes the restriction of different parameters before and after 1973Q4, the break date in the volatility of GDP would change (it would be 1987Q1).

What about Canada?

Table 12 shows that, except perhaps for investment, the standard deviation of every major GDP component has decreased significantly after 1991Q1.

One can try to assess the statistical importance of such declines by applying the structural break tests on each series.[10] Table 13 reports the results of the structural break tests in the residual variance on the growth rate of the major components of GDP. Table 13 also reports the results of the tests applied to the growth rate of major components weighted by their share of GDP. Such weighted series provide a measure of the component's contribution to GDP growth.[11]

Except for investment, all the components exhibit data breaks, but, surprisingly, those breaks do not coincide with those found in the aggregate data.[12]

The evidence of a break in consumption growth is especially strong. Why does it occur at the end of the 1970s and not at the beginning of the 1990s? Figure 4 shows that volatility did indeed drop at the beginning of the 1990s, but not as radically as at the end of the 1970s.[13] In fact, when restricted to the subperiod beginning in the mid-1970s, the test again indicates 1991Q2 as a break date for data on the volatility in the growth rate of consumption, suggesting that it would be useful to test for multiple breaks in this series.

Extending the level of disaggregation further, a data break is found in the consumption of goods at the same time as aggregate output (Table 16).[14] However, the breaks in the individual components of goods do not coincide with that found in the aggregate.[15]

---

10. The results of the tests on a particular series can be sensitive to the order of the AR process specified for the series. For disaggregated data in Canada, the "optimal" order is unfortunately not clear, as many of the models have poor explanatory power. Further work must be done on that aspect, since the breaks could come from a misspecified model.
11. Following McConnell and Quiros, the "contribution to growth" is defined as the product of the share of nominal GDP accounted for by a particular component in period $t$-1 and the real growth rate of that component in period $t$.
12. Some explanations are possible: misspecified models, multiple breaks, or changing cross-correlations between the components. If components do not move as much in the same direction after 1991Q2, that could cause a break in the volatility of overall GDP, even if no series experiences a break. In fact, cross-correlations between consumption and the other components, except government expenditures (a relatively stable series), go down after the break (Tables 14 and 15). Note that tests for a break in the parameters of the AR($p$) processes do not reveal any clear breaks (there is mixed evidence of a break for government expenditures and imports).
13. Its first break seems to be robust to the length of the AR process.
14. No break was indicated by the joint tests on the parameters of the AR($p$).
15. The break related to the consumption of goods seems to be robust to the length of the AR process. However, results for their components can vary.

It is also interesting to examine fixed investment, generally a highly volatile series. Recall from Table 12 that the two major components of fixed investment showed opposite patterns. Since the tests do not indicate a break in the volatility of overall investment, it seems that the higher volatility in business fixed investment offsets the lower volatility in investment in residential structures. Table 17 reports the results of the structural break tests applied on each separate component of investment. Investment in residential structures experiences a break around the same time as overall GDP (Figure 5).[16]

The last disaggregation follows a suggestion by McConnell and Quiros, and examines GDP expenditures by major type of product rather than by expenditure category. Unfortunately, I could only consider private spending.[17] Table 18 shows that “goods” and “services” experience no break, whereas “structures” experience a break with regard to their contribution to overall growth. This last break is especially interesting. The addition of non-residential structures to residential structures eliminates the break in the volatility of the growth rate. However, since the relative importance of residential structures in nominal GDP does not drop as much as that of non-residential structures, the break in the data for the contribution to growth still appears.[18]

Table 19 compares these results with those obtained using U.S. data. As in Canada, “structures” exhibit a break in the residual variance of their contribution to growth at the same time as aggregate output, and encounter no break in the volatility of their growth rate. However, contrary to evidence from Canadian data, “goods” experience a break in their growth rate and their contribution to growth. To assess the importance of sectoral shifts in production in explaining the break, McConnell and Quiros build a new U.S. GDP series in which they hold constant the proportion of total expenditures on goods, services, and structures for the post-break period to their pre-break average. A break in volatility is still found in the same period. They therefore conclude that “structures” cannot explain the break, since their declining contribution to growth results from the decline of their relative importance in nominal GDP.[19] Their attention turns to goods and they find that the output of durable goods does experience a break in 1985Q1 (Table 20).

16. Its break seems to be robust to the length of the AR process.
17. Canadian series for government expenditures are not directly available. Government expenditures are divided into two series: net expenditures on goods and services, and gross capital formation. Since I could not break down the first series into two, I focused on private expenditures.
18. Note that there is no change in the relative movements of the two series (their cross-correlations remain approximately 0.18).
19. Note that McConnell and Quiros do not examine investment in residential structures separately.

# 4. Possible Explanations

Assuming that the breaks in quarterly growth rates are not a "figment of the data,"[20] I review three possible explanations for the break in the volatility of real GDP growth: a more service-oriented economy, improved inventory management, and a change in monetary policy.[21]

## 4.1 A more service-oriented economy

Given that services are less volatile than goods, a shift in the composition of output from manufacturing to services could account for a drop in the volatility of GDP growth. In Canada, private spending on services increased remarkably as a share of nominal GDP, just before the data break in the volatility of the growth rate of GDP (Figure 6).[22] This explanation, however, is not fully satisfactory, since it would overlook striking features shown in Tables 16 and 17: an increased stability in personal expenditures on goods and in investment in residential structures. The first break was also found in U.S. data. Moreover, Taylor (2000) notes that the share of U.S. GDP accounted for by services experienced its biggest increase before the volatile period in the late 1960s and 1970s, rather than before or during the latest expansion period.[23]

## 4.2 Improved inventory management

As a recession unfolds, firms accumulate unwanted inventories if they are not able to cut production as soon as sales decline. Afterwards, they have to cut inventory investment dramatically, which increases swings in production and triggers a more severe recession. Considering that output is the sum of sales and changes in inventories, for a given volatility in sales improved inventory management should reduce the volatility of production. Therefore, techniques leading to improved inventory management should allow firms to smooth production over the business cycle and dampen recessions.

To verify whether better control of inventories may have caused the break in output-growth volatility, one can apply the structural break tests on the constant of the optimal AR($p$) process for the changes in absolute value of inventories. Unfortunately, data on changes in inventories for

---

20. Further study is needed to assess whether methodological changes in data treatment can explain the reduction in measured volatility in Canada.
21. This section is inspired by Taylor (2000).
22. The series shown is that computed for Table 18. Consumption expenditures on services exhibit the same pattern.
23. The experiment by McConnell and Quiros, referred to in section 3, seems to downplay the importance of sectoral shifts.

durable and non-durable goods are available only from 1981Q1. However, Table 10 shows that the break in overall production is still found if the sample starts at that time. Figure 7, which plots the ratio of changes in inventories of durable goods over private expenditures on durable goods, suggests that there has been no break. Table 21 confirms this impression. Therefore, there does not seem to be any evidence in favour of this explanation for Canada.

For the U.S. economy, recall that McConnell and Quiros find a break in the data on volatility in the production of durable goods. They also find a break in the proportion of durable goods accounted for by inventories. Finally, they subtract inventories from domestic purchases of goods and services and find no break in the growth rate of this series. Therefore, they consider improved inventory-management techniques to be responsible for the decreased volatility of GDP growth. They stress the importance of just-in-time inventory-management techniques and point out that Japan, where those techniques were first introduced, saw the volatility in its GDP growth rate break before the United States.

### 4.3 A change in monetary policy

In Canada and the United States, the timing of the break coincides with a structural change (not a gradual one) in the way monetary policy has been conducted. In the first quarter of 1991, Canada adopted explicit inflation-control targets with the objective of achieving and maintaining price stability. In the United States, some studies have shown that monetary policy was more accommodating in the late 1960s and 1970s than in the 1980s and 1990s during the Volcker–Greenspan era (e.g., see Clarida, Gali, and Gertler 1997).

One can argue that, by focusing on price stability, the new monetary policy regime has reduced the magnitude of the boom-bust cycles that characterized earlier periods and this has resulted in lower volatility in output growth. One objection to this theory is that structural breaks in other components should appear as well: in Canada, the volatility of business fixed investment has actually increased, while in the United States there is no break in the volatility in the rate of change of investment in structures and the break in the durables sector stems from changes in inventories, not from sales.

## 5. Conclusion

In this paper, the author has reported on structural break tests on the residual variance of the quarterly growth rate of real GDP in Canada, following the methodology of McConnell and Quiros. A significant break was found at the beginning of 1991. When disaggregated data were

examined, breaks were also found to occur at the beginning of 1991 in investment in residential structures and in consumption of goods. Three possible explanations were given for the break in the volatility of GDP growth rate: a more service-oriented economy, improved inventory management, and a change in monetary policy.

Further work could be done on this subject. For example, it would be interesting to test for multiple breaks, especially for consumption. It would also be a good idea to extend the class of models estimated; investigate the peculiar patterns of the components of investment; and, complete the analysis of McConnell and Quiros (1998) and Kim and Nelson (1999) by estimating a Markov-switching model. In addition, an explicit general equilibrium model might help reconcile the various results obtained.

# References

Andrews, D.W.K. 1993. “Test for Parameter Instability and Structural Change with Unknown Change Point.” *Econometrica* 61 (4): 821–56.

Andrews, D.W.K. and W. Ploberger. 1994. “Optimal Tests when a Nuisance Parameter is Present only under the Alternative.” *Econometrica* 62 (6): 1383–1414.

Bureau of Economic Analysis. 2000. *National Accounts Data.* United States Department of Commerce. <URL: http://www.bea.doc.gov/bea/dn1.htm> (July 2000).

Clarida, R., J. Gali, and M. Gertler. 1997. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” Mimeo.

Hansen, B.E. 1997. “Approximate Asymptotic *P* Values for Structural-Change Tests.” *Journal of Business and Economics Statistics* 15 (1): 60–67.

Kim, C.-J. and C.R. Nelson. 1999. “Has the U.S. Economy Become More Stable? A Bayesian Approach on a Markov-Switching Model of the Business Cycle.” *The Review of Economics and Statistics* (November).

McConnell, M.M. and G. Perez Quiros. 1998. “Output Fluctuations in the United States: What Has Changed since the Early 1980s?” *Staff Reports* (41, June). Federal Reserve Bank of New York.

Statistics Canada. 2000. *National Income and Expenditure Account*. Catalogue 13-001-DDB (second quarter 2000).

Taylor, J.B. 2000. “Remarks for the Panel Discussion on ‘Recent Changes in Trend and Cycle’.” Federal Bank of San Francisco Conference. <URL: http://www.sf.frb.org/conf2000/agenda.html>.

**Table 1: Quarterly growth rate of real U.S. GDP on selected subperiods**

| Full sample | | Mean | Standard deviation |
|---|---|---|---|
| 1953Q2–2000Q1 | | 3.39 | 3.98 |
| Two subperiods | | | |
| 1953Q2–1984Q1 | | 3.40 | 4.67 |
| 1984Q2–2000Q1 | | 3.37 | 2.10 |

**Table 2: Quarterly growth rate of real GDP in Canada on selected subperiods**

| Full sample | Mean | Standard deviation |
|---|---|---|
| 1961Q2–2000Q1 | 3.85 | 3.88 |
| Two subperiods | | |
| 1961Q2–1991Q1 | 4.07 | 4.27 |
| 1991Q2–2000Q1 | 3.13 | 1.98 |
| Three subperiods | | |
| 1961Q2–1984Q1 | 4.44 | 4.37 |
| 1984Q2–1991Q1 | 2.84 | 3.71 |
| 1991Q2–2000Q1 | 3.13 | 1.98 |

**Table 3: Residual variance break test: U.S. real GDP growth rate, 1953Q2–1997Q2[a]**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2), \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| McConnell and Quiros | | | Our estimations | | |
|---|---|---|---|---|---|
| SupLM | ExpLM | AveLM | SupLM | ExpLM | AveLM |
| 15.43 | 5.12 | 4.96 | 18.47 | 6.21 | 7.15 |
| (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| Break date: 1984Q1 | | | Break date: 1984Q1 | | |

a. *P*-values appear in parentheses below test statistics.

**Table 4: Residual variance break test: Canadian real GDP growth rate, 1961Q2–1997Q1[a]**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2), \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| McConnell and Quiros | | Our estimations | |
|---|---|---|---|
| ExpLM | AveLM | ExpLM | AveLM |
| 0.0012 | 0.0115 | 0.0152 | 0.0116 |
| Break date: 1991Q2 | | Break date: 1991Q1 | |

a. Only *p*-values are reported.

**Table 5: Residual variance break test: Canadian real GDP growth rate (quarter-to-quarter), 1961Q2–2000Q1[a]**

$$\text{Model: } y_t = \mu + \phi_1 y_{t-1} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2), \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| SupLM | ExpLM | AveLM |
|---|---|---|
| 13.649<br>(0.005) | 4.799<br>(0.001) | 7.292<br>(0.000) |
| | Break date: 1991Q1 | |

a. *P*-values appear in parentheses below test statistics.

**Table 6: Residual variance break test: Canadian real GDP-year-to-year growth rate, AR(2), 1961Q2–2000Q1[a]**

$$\text{Model: } y_t = \mu + \sum_{k=1}^{p} \phi_k y_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2) \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| SupLM | ExpLM | AveLM |
|---|---|---|
| 13.021<br>(0.006) | 4.363<br>(0.002) | 6.523<br>(0.001) |
| | Break date: 1991Q1 | |

a. *P*-values appear in parentheses below test statistics.

**Table 7: Structural break test in the AR(1) coefficients: U.S. real GDP growth rate, 1953Q2–1997Q2[a]**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

| | McConnell and Quiros | | | Our estimations | | |
|---|---|---|---|---|---|---|
| Null hypothesis | SupLM | ExpLM | AveLM | SupLM | ExpLM | AveLM |
| $\mu_1 = \mu_2, \phi_1 = \phi_2$ | 2.77<br>(0.99) | 0.64<br>(0.85) | 1.20<br>(0.75) | 3.01<br>(0.89) | 0.44<br>(0.90) | 0.79<br>(0.88) |
| Break date: | | none | | | none | |
| $\mu_1 = \mu_2$ | 2.09<br>(0.93) | 0.40<br>(0.61) | 0.72<br>(0.53) | 1.28<br>(0.96) | 0.20<br>(0.79) | 0.38<br>(0.76) |
| Break date: | | none | | | none | |
| $\phi_1 = \phi_2$ | 2.73<br>(0.82) | 0.46<br>(0.55) | 0.81<br>(0.47) | 2.98<br>(0.56) | 0.31<br>(0.64) | 0.51<br>(0.64) |
| Break date | | none | | | none | |

a. *P*-values appear in parentheses below test statistics.

**Table 8: Structural break test in the AR(1) coefficients: Canadian real GDP growth rate, 1961Q2–2000Q1[a]**

Model: $y_t = \mu + \phi_1 y_{t-1} + \varepsilon_t$

| Null hypothesis | SupLM | ExpLM | AveLM |
|---|---|---|---|
| $\mu_1 = \mu_2, \phi_1 = \phi_2$ | 9.423<br>(0.119) | 3.341<br>(0.043) | 5.018<br>(0.037) |
| Break date: | | 1973:4 | |
| $\mu_1 = \mu_2$ | 5.812<br>(0.175) | 1.373<br>(0.117) | 2.250<br>(0.088) |
| Break date: | | none | |
| $\phi_1 = \phi_2$ | 3.419<br>(0.472) | 0.353<br>(0.580) | 0.561<br>(0.597) |
| Break date: | | none | |

a. *P*-values appear in parentheses below tests statistics.

**Table 9: Residual variance break test: Canadian real GDP growth rate, 1974Q1–2000Q1**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2), \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| SupLM | ExpLM | AveLM |
|---|---|---|
| 11.466 | 4.011 | 5.929 |
| (0.013) | (0.003) | (0.003) |
| | Break date: 1991Q1 | |

**Table 10: Residual variance break tests on a AR(1): Canadian GDP subsamples**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N(0, \sigma_t^2), \text{ where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| 1961Q2–1980Q4 | | | 1981Q1–2000Q1 | | |
|---|---|---|---|---|---|
| SupLM | ExpLM | AveLM | SupLM | ExpLM | AveLM |
| 1.620 | 0.287 | 0.528 | 11.582 | 4.179 | 6.689 |
| (0.879) | (0.661) | (0.623) | (0.013) | (0.003) | (0.001) |
| Break date: none | | | Break date: 1991Q1 | | |

**Table 11: Canadian real GDP-estimated values of the parameters in the AR(1)[a]**

$$\text{Model: } \dot{y}_t = \mu + \phi_1 \dot{y}_{t-1} + \varepsilon_t$$

$$\sqrt{\frac{\pi}{2}}\,\hat{|\varepsilon_t|} = \alpha + \nu_t$$

| Sample period | $\mu$ | $\phi$ | $\alpha$ |
|---|---|---|---|
| 1961Q2–2000Q1 | 2.65<br>(0.61) | 0.30<br>(0.11) | 3.59<br>(0.29) |
| 1961Q2–1991Q1 | 2.89<br>(0.75) | 0.27<br>(0.12) | 4.16<br>(0.32) |
| 1991Q2–2000Q1 | 1.58<br>(0.59) | 0.54<br>(0.11) | 1.45<br>(0.65) |
| | | | |
| 1961Q2–1973Q4 | 5.30<br>(1.00) | 0.038<br>(0.12) | 3.94<br>(0.48) |
| 1974Q1–2000Q1 | 2.09<br>(0.71) | 0.34<br>(0.14) | 3.29<br>(0.40) |

a. Standard errors appear in parentheses below coefficient estimates.

**Table 12: Volatility of growth in Canadian real GDP and its components: Standard deviations of annualized quarterly growth rates**

| | Standard deviation | | | |
|---|---|---|---|---|
| | 61Q2–91Q1 | 91Q2–00Q1 | Difference | Difference (%) |
| GDP | 4.27 | 1.98 | -2.29 | -54 |
| Consumer spending | 4.22 | 2.07 | -2.15 | -51 |
| Goods | 5.35 | 3.34 | -2.01 | -38 |
| Durables | 17.27 | 10.78 | -6.49 | -38 |
| Semidurables | 7.47 | 4.27 | -3.20 | -43 |
| Non-durables | 4.85 | 2.26 | -2.59 | -53 |
| Services | 3.23 | 1.80 | -1.43 | -44 |
| Government spending | 5.70 | 2.60 | -3.10 | -54 |
| Investment | 11.39 | 10.78 | -0.61 | -5 |
| Residential | 18.95 | 12.18 | -6.77 | -36 |
| Business fixed | 12.01 | 13.24 | +1.23 | +10 |
| Exports | 13.23 | 8.20 | -5.03 | -38 |
| Imports | 14.28 | 8.15 | -6.13 | -43 |

**Table 13: Residual variance break tests: Canadian GDP components, 1961Q2–2000Q1[a]**

$$\text{Model: } \dot{x}_t = \mu + \sum_{k=1}^{p} \phi_k \dot{x}_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N\left(0, \sigma_t^2\right), \text{where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| $x_t$= Growth rate | | | | | |
|---|---|---|---|---|---|
| Component | *p* | Break date | SupLM | ExpLM | AveLM |
| Consumer spending | 3 | 77Q2 | 0.005 | 0.001 | 0.001 |
| Investment | 4 | none | 0.779 | 0.673 | 0.666 |
| Government | 4 | 72Q1 | 0.007 | 0.007 | 0.012 |
| Exports | 4 | 84Q4 | 0.027 | 0.020 | 0.018 |
| Imports | 4 | 87Q4 | 0.017 | 0.005 | 0.006 |
| $x_t$= Contribution to growth | | | | | |
| Component | *p* | Break date | SupLM | ExpLM | AveLM |
| Consumer spending | 3 | 77Q2 | 0.002 | 0.000 | 0.000 |
| Investment | 4 | none | 0.403 | 0.290 | 0.299 |
| Government | 4 | none | 0.074 | 0.088 | 0.085 |
| Exports | 4 | none | 0.274 | 0.303 | 0.347 |
| Imports | 4 | none | 0.364 | 0.306 | 0.297 |

a. Only *p*-values are reported.

**Table 14: Cross-correlations between the growth rates of the major components of GDP: 1961Q2–1991Q1**

| | Consumer spending | Investment | Government | Exports | Imports |
|---|---|---|---|---|---|
| Consumer spending | 1 | 0.345 | 0.116 | 0.231 | 0.346 |
| Investment | | 1 | -0.031 | 0.063 | 0.334 |
| Government | | | 1 | 0.026 | -0.011 |
| Exports | | | | 1 | 0.494 |
| Imports | | | | | 1 |

**Table 15: Cross-correlations between the growth rates of the major components of GDP: 1991Q2–2000Q1**

| | Consumer spending | Investment | Government | Exports | Imports |
|---|---|---|---|---|---|
| Consumer spending | 1 | 0.267 | 0.134 | 0.094 | 0.242 |
| Investment | | 1 | -0.128 | -0.123 | 0.589 |
| Government | | | 1 | 0.217 | 0.128 |
| Exports | | | | 1 | 0.360 |
| Imports | | | | | 1 |

**Table 16: Residual variance break tests: Components of consumption in Canada, 1961Q2–2000Q1[a]**

$$\text{Model: } x_t = \mu + \sum_{k=1}^{p} \phi_k x_{t-k} + \varepsilon_t$$

| $x_t$=Growth rate | | | | | |
|---|---|---|---|---|---|
| Component | *p* | Break date | SupLM | ExpLM | AveLM |
| Consumer spending on goods | 3 | 91Q2 | 0.0480 | 0.027 | 0.018 |
| Durables | 4 | 82Q2 | 0.007 | 0.001 | 0.000 |
| Semidurables | 3 | 75Q3 | 0.002 | 0.000 | 0.000 |
| Non-durables | 6 | 86Q1 | 0.013 | 0.004 | 0.005 |
| Consumer spending on services | 3 | none | 0.250 | 0.347 | 0.542 |
| $x_t$=Contribution to growth | | | | | |
| Component | *p* | Break date | SupLM | ExpLM | AveLM |
| Consumer spending on goods | 3 | 91Q2 | 0.014 | 0.003 | 0.001 |
| Durables | 4 | 76Q2 | 0.005 | 0.000 | 0.000 |
| Semidurables | 3 | 75Q3 | 0.001 | 0.000 | 0.000 |
| Non-durables | 6 | 86Q1 | 0.005 | 0.001 | 0.001 |
| Consumer spending on services | 3 | none | 0.602 | 0.549 | 0.556 |

a. Only *p*-values are reported.

**Table 17: Residual variance break tests: Components of investment in Canada, 1961Q2–2000Q1[a]**

$$\text{Model: } \dot{x}_t = \mu + \sum_{k=1}^{p} \phi_k \dot{x}_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N\left(0, \sigma_t^2\right), \text{where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

$x_t$=Growth rate

| Component | *p* | Break date | SupLM | ExpLM | AveLM |
|---|---|---|---|---|---|
| Residential | 4 | 91Q1 | 0.049 | 0.028 | 0.015 |
| Business fixed | 2 | 79Q1 | 0.080 | 0.045 | 0.036 |

$x_t$=Contribution to growth

| Component | *p* | Break date | SupLM | ExpLM | AveLM |
|---|---|---|---|---|---|
| Residential | 4 | 91Q1 | 0.014 | 0.016 | 0.016 |
| Business fixed | 2 | none | 0.190 | 0.104 | 0.100 |

a. Only *p*-values are reported.

**Table 18: Residual variance break tests: Private spending in Canada, 1961Q2–2000Q1[a]**

$$\text{Model: } \dot{x}_t = \mu + \sum_{k=1}^{p} \phi_k \dot{x}_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N\left(0, \sigma_t^2\right), \text{where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| $x_t$=Growth rate | | | | | |
|---|---|---|---|---|---|
| Component of private spending | *p* | Break date | SupLM | ExpLM | AveLM |
| Total goods[b] | 4 | none | 0.926 | 0.724 | 0.688 |
| Total services | 4 | none | 0.193 | 0.249 | 0.281 |
| Total structures | 4 | none | 0.150 | 0.111 | 0.123 |
| $x_t$=Contribution to growth | | | | | |
| Component of private spending | *p* | Break date | SupLM | ExpLM | AveLM |
| Total goods | 4 | none | 0.413 | 0.222 | 0.178 |
| Total services | 4 | none | 0.675 | 0.823 | 0.799 |
| Total structures | 4 | 90Q4 | 0.049 | 0.023 | 0.035 |

a. Only *p*-values are reported.

b. "Total goods" is the sum of consumption in goods, net exports of goods, and expenditures on machines and equipment; "Total structures" is the sum of residential structures (new housing construction, renovations, and ownership transfer costs) and non-residential structures (building and engineering); "Total services" is the sum of consumption in services and net exports of services.

**Table 19: Residual variance break tests: U.S. GDP by major type of products, 1953Q2–1997Q2 (taken from McConnell and Quiros 1998)**

$$\text{Model: } \dot{x}_t = \mu + \sum_{k=1}^{p} \phi_k \dot{x}_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N\left(0, \sigma_t^2\right), \text{where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| $x_t$= | | Growth rate | | | Contribution to growth | | |
|---|---|---|---|---|---|---|---|
| Component | *p* | Break date | Exp | Ave | Break date | Exp | Ave |
| Total output of goods | 1 | 84Q1 | 0.00 | 0.02 | 84Q1 | 0.00 | 0.00 |
| Total output of services | 1 | 67Q1 | 0.02 | 0.00 | none | 0.13 | 0.10 |
| Total output of structures | 1 | none | 0.37 | 0.37 | 84Q2 | 0.03 | 0.09 |

**Table 20: Residual variance break tests: U.S. goods 1953Q2–1997Q2 (taken from McConnell and Quiros 1998)**

$$\text{Model: } \dot{x}_t = \mu + \sum_{k=1}^{p} \phi_k \dot{x}_{t-k} + \varepsilon_t$$

$$\text{with } \varepsilon_t \sim N\left(0, \sigma_t^2\right), \text{where } \sigma_t^2 = \sigma_1^2 \text{ if } t \leq T \text{ and } \sigma_t^2 = \sigma_2^2 \text{ if } t > T$$

| Treatment | | Growth rate | | | Contribution to growth | | |
|---|---|---|---|---|---|---|---|
| Component | *p* | Break date | Exp | Ave | Break date | Exp | Ave |
| Output of durables | 1 | 85Q1 | 0.01 | 0.01 | 85Q1 | 0.00 | 0.00 |
| Output of non-durables | 1 | none | 0.41 | 0.34 | none | 0.19 | 0.26 |

**Table 21: Structural breaks in real changes in inventories of durable goods in Canada: 1981Q1–2000Q1**

$$\text{Model: } x_t = \mu + \phi_1 x_{t-1} + \varepsilon_t$$

| \|Δinv.\| | | | \|Δinv.\|/GDP | | | \|Δinv.\|/Cons. of durable | | |
|---|---|---|---|---|---|---|---|---|
| Break date | Exp | Ave | Break date | Exp | Ave | Break date | Exp | Ave |
| None | 0.263 | 0.292 | none | 0.511 | 0.496 | none | 0.292 | 0.350 |

**Figure 1: U.S. Real GDP Growth**
**(annualized one-quarter rates)**

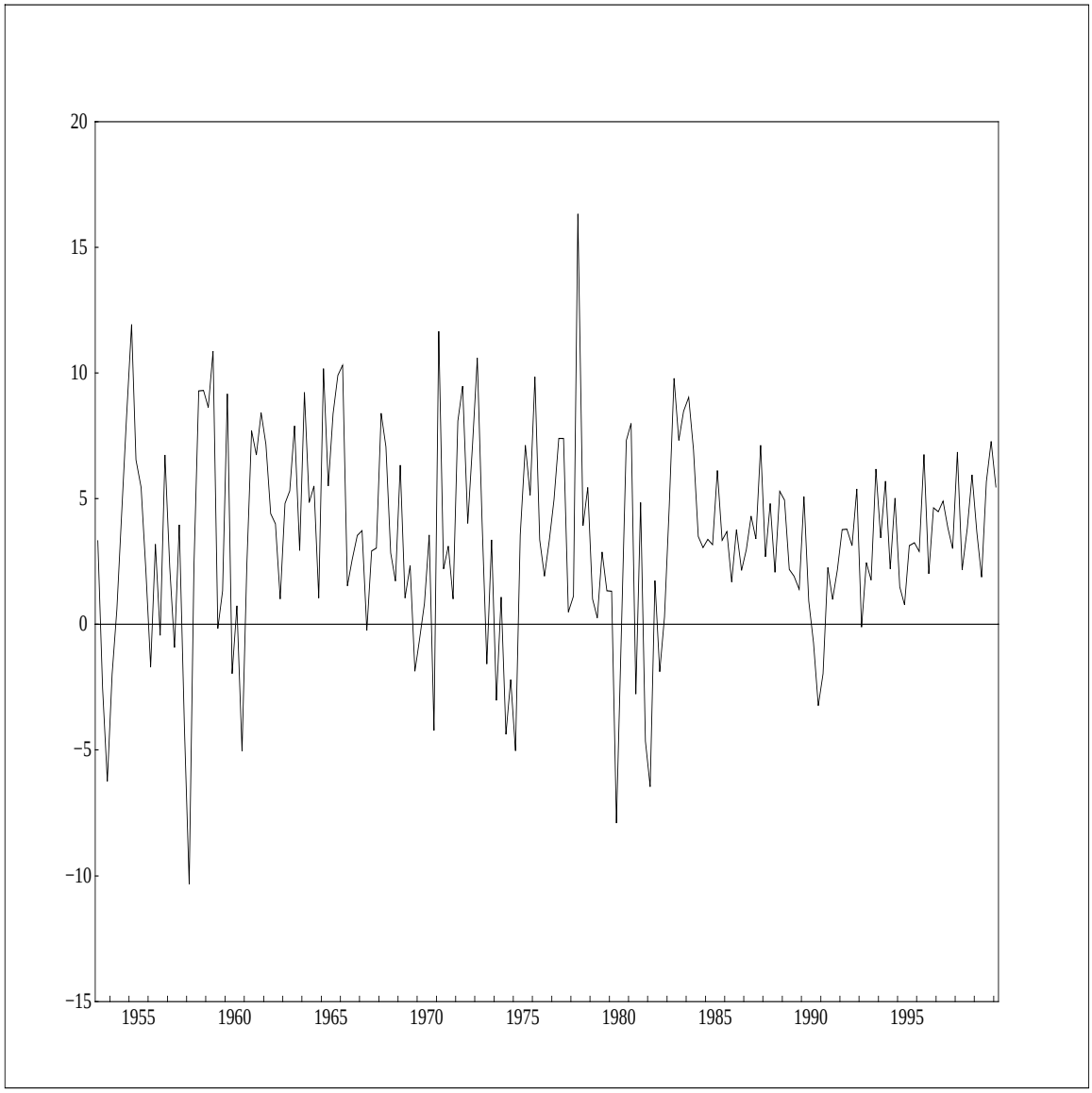


**Figure 2: Real GDP Growth in Canada**
**(annualized one-quarter rates)**

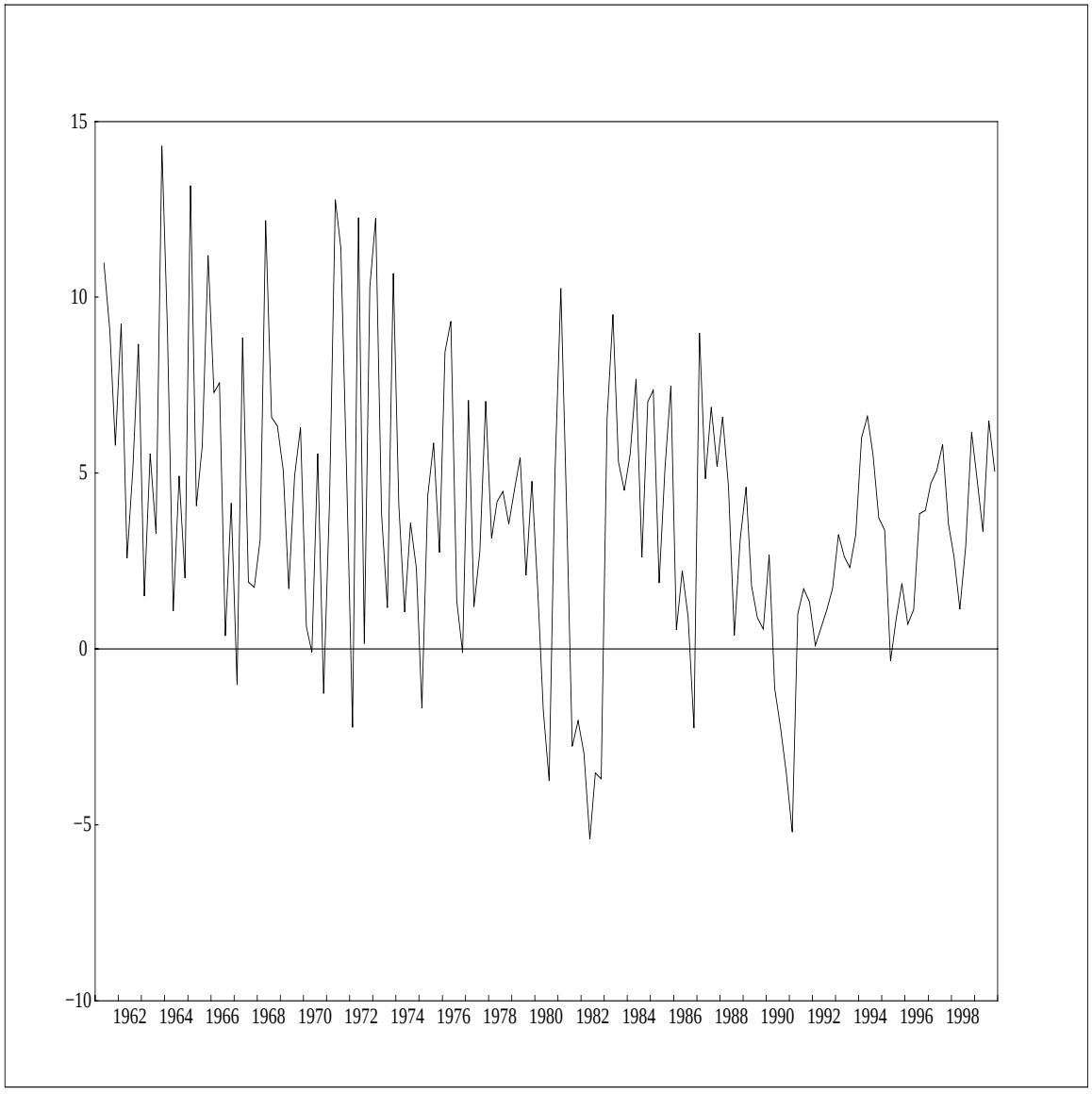


**Figure 3: Real GDP Growth in Canada**
**(4-quarter growth rates)**

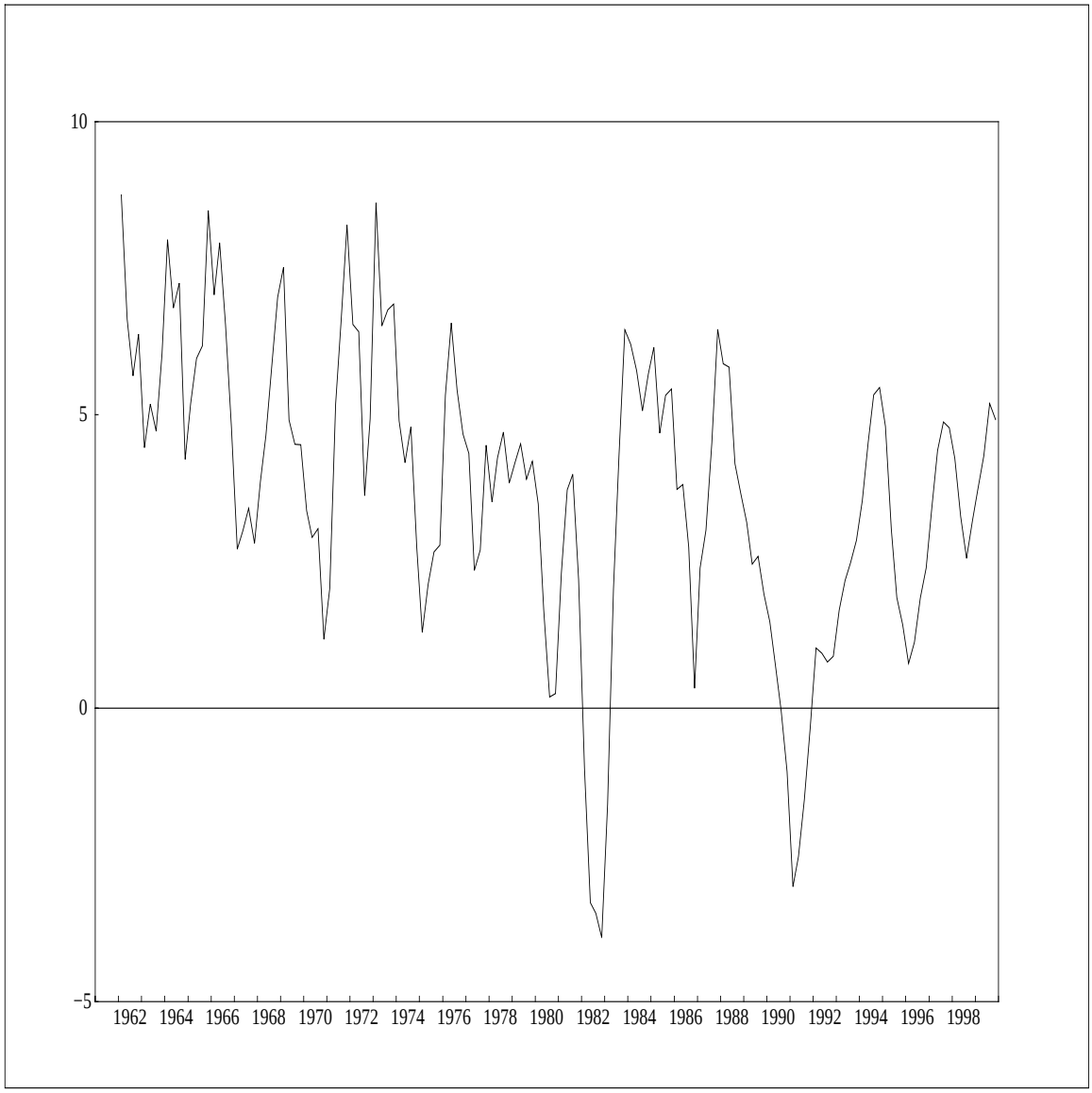


**Figure 4: Consumption Growth in Canada**
**(annualized one-quarter rates)**

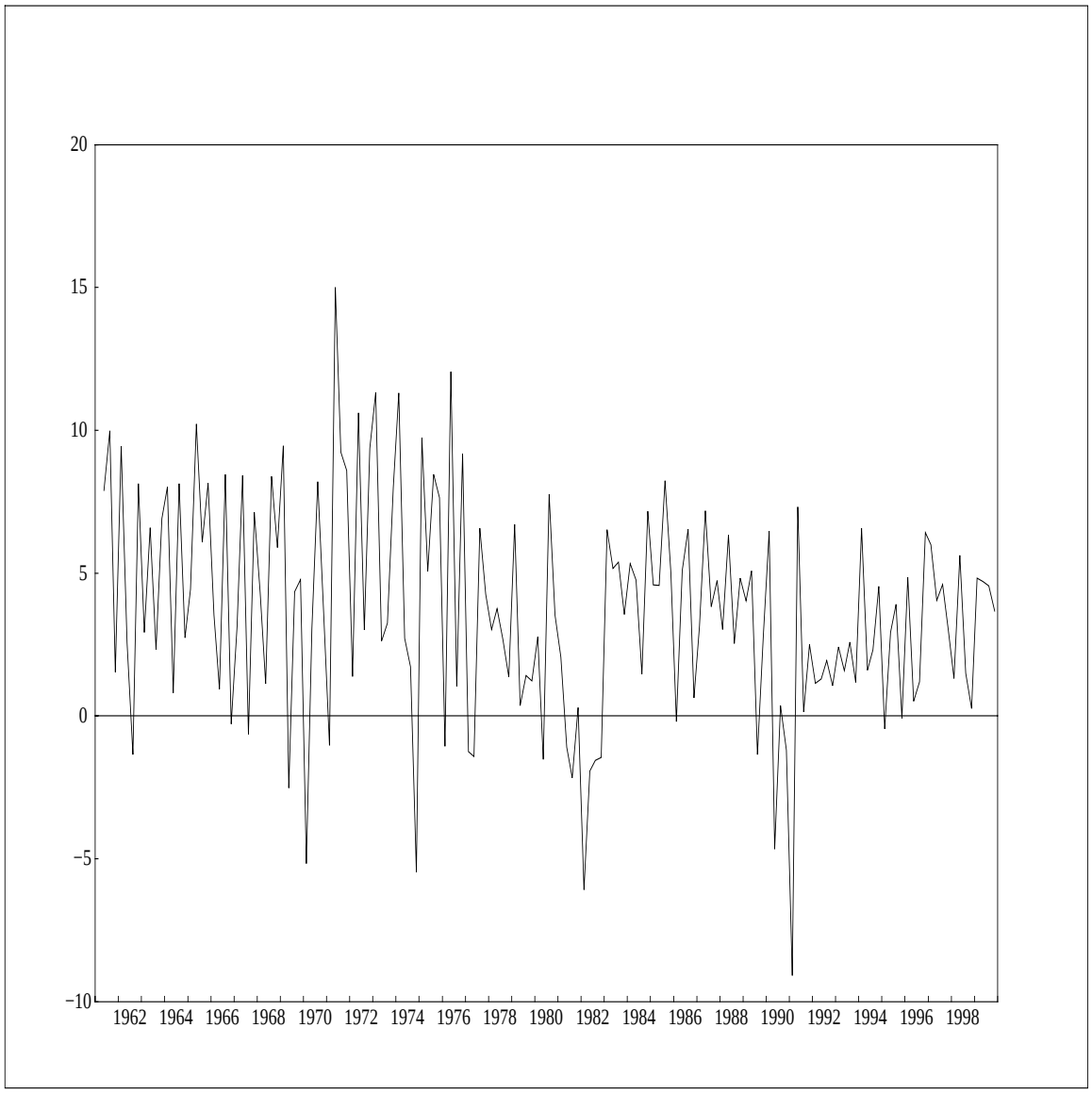


**Figure 5: Growth in Investment in Real Residential Structures in Canada (annualized one-quarter rates)**

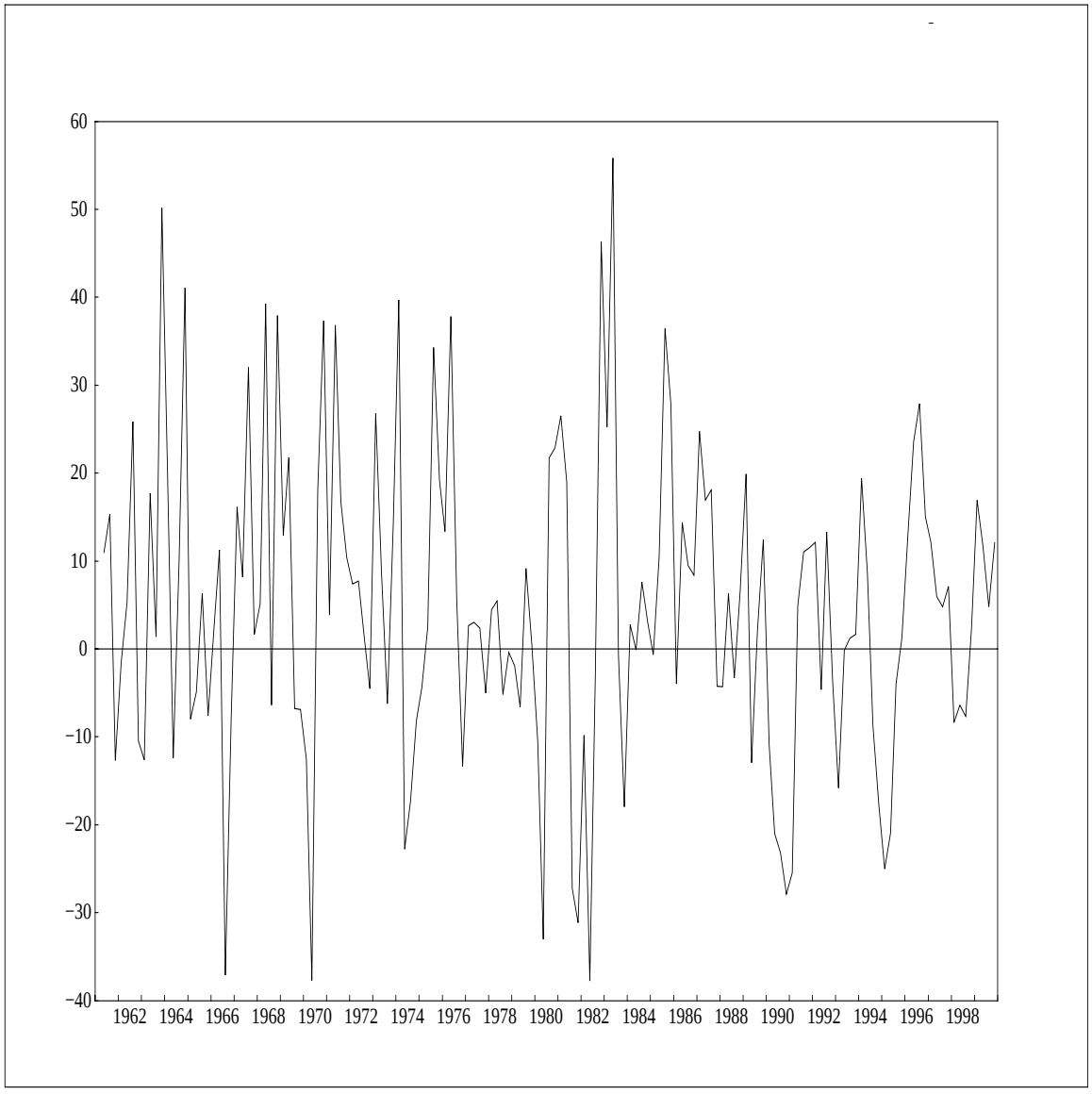


**Figure 6: Share of Services in Canadian Private Spending (current dollars)**

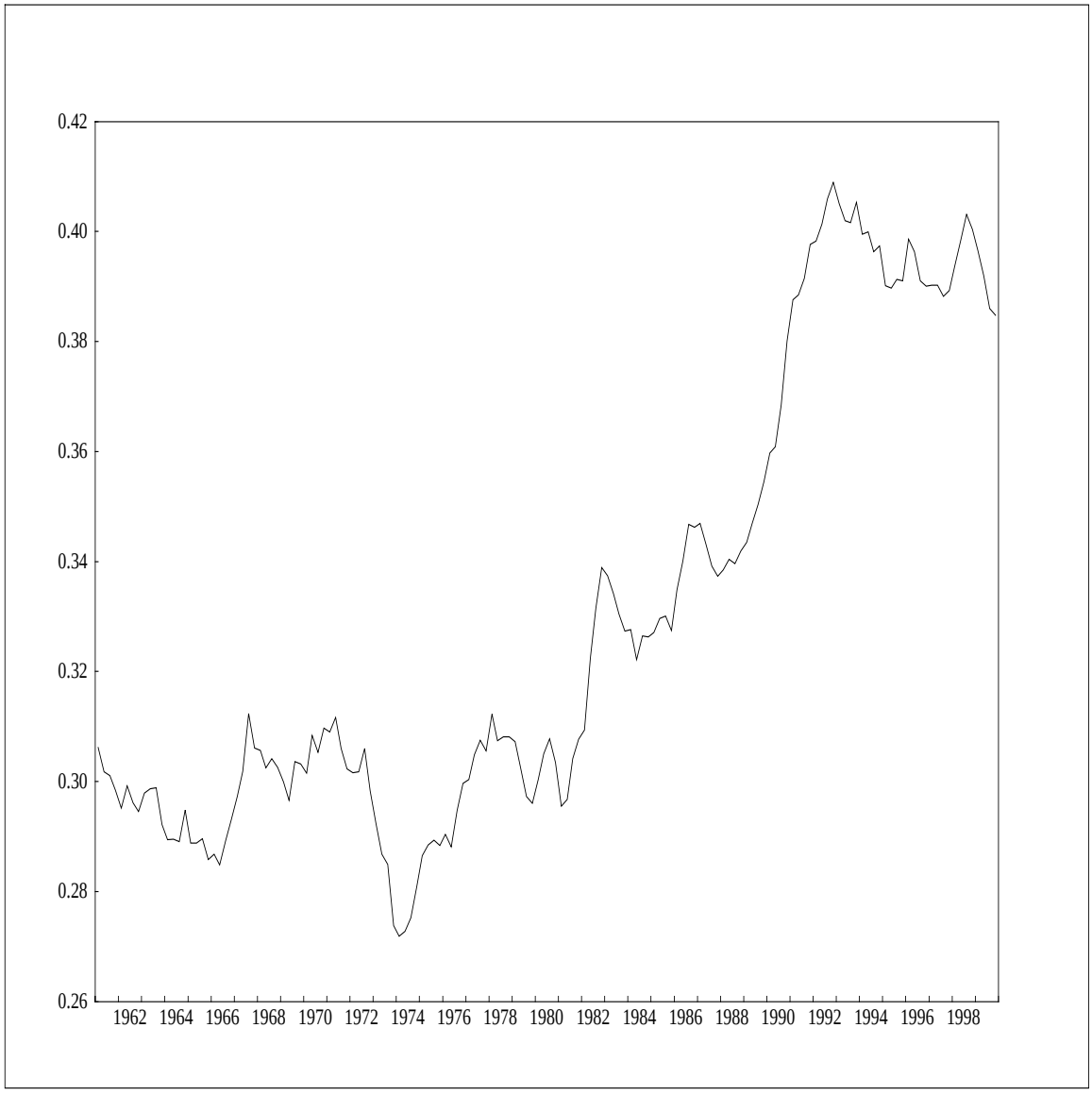
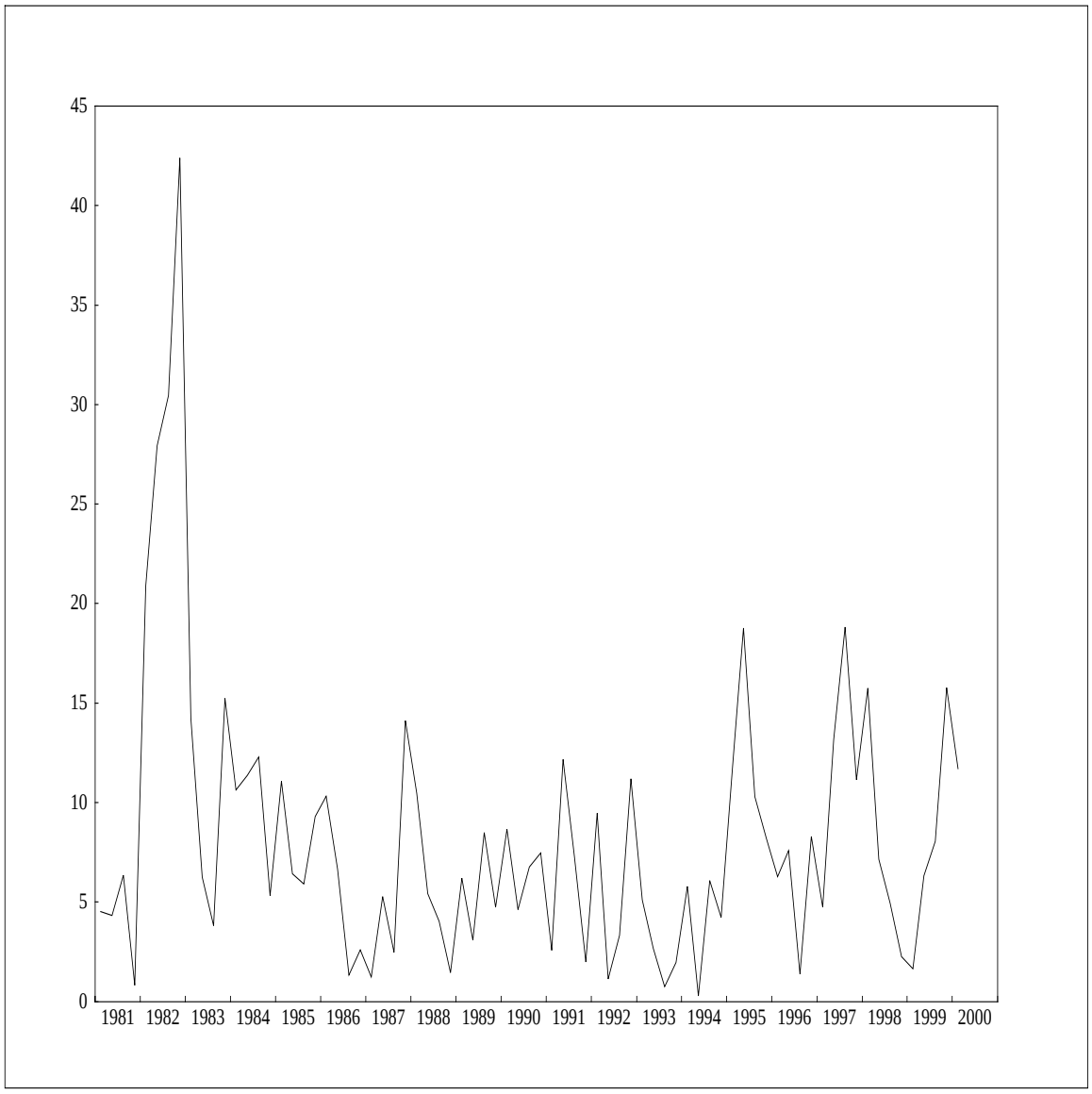


**Figure 7: Ratio of Changes of Inventories in Durable Goods over Consumption of Durable Goods (in absolute value)**



# Bank of Canada Working Papers
## *Documents de travail de la Banque du Canada*

**Working papers are generally published in the language of the author, with an abstract in both official languages.** ***Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue.***

### 2001

| | | |
|---|---|---|
| 2001-8 | How Rigid Are Nominal-Wage Rates? | A. Crawford |
| 2001-7 | Downward Nominal-Wage Rigidity: Micro Evidence from Tobit Models | A. Crawford and G. Wright |
| 2001-6 | The Zero Bound on Nominal Interest Rates: How Important Is It? | D. Amirault and B. O'Reilly |
| 2001-5 | Reactions of Canadian Interest Rates to Macroeconomic Announcements: Implications for Monetary Policy Transparency | T. Gravelle and R. Moessner |
| 2001-4 | On the Nature and the Stability of the Canadian Phillips Curve | M. Kichian |
| 2001-3 | On Commodity-Sensitive Currencies and Inflation Targeting | K. Clinton |
| 2001-2 | Exact Non-Parametric Tests for a Random Walk with Unknown Drift under Conditional Heteroscedasticity | R. Luger |
| 2001-1 | The Elements of the Global Network for Large-Value Funds Transfers | J.F. Dingle |

### 2000

| | | |
|---|---|---|
| 2000-23 | The Application of Artificial Neural Networks to Exchange Rate Forecasting: The Role of Market Microstructure Variables | N. Gradojevic and J. Yang |
| 2000-22 | Une analyse empirique du lien entre la productivité et le taux de change réel Canada-É-U | D. Dupuis et D. Tessier |
| 2000-21 | Les effets réels du cours des actions sur la consommation | L. Pichette |
| 2000-20 | Steps in Applying Extreme Value Theory to Finance: A Review | Y. Bensalah |
| 2000-19 | Le modèle USM d'analyse et de projection de l'économie américaine | R. Lalonde |
| 2000-18 | Inflation and the Tax System in Canada: An Exploratory Partial-Equilibrium Analysis | B. O'Reilly and M. Levac |
| 2000-17 | A Practical Guide to Swap Curve Construction | U. Ron |
| 2000-16 | Volatility Transmission Between Foreign Exchange and Money Markets | S.K. Ebrahim |

Copies and a complete list of working papers are available from:
*Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à* :


Publications Distribution, Bank of Canada
234 Wellington Street, Ottawa, Ontario K1A 0G9
E-mail: publications@bankofcanada.ca
Web site: http://www.bankofcanada.ca

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9
Adresse électronique : publications@banqueducanada.ca
Site Web : http://www.banqueducanada.ca