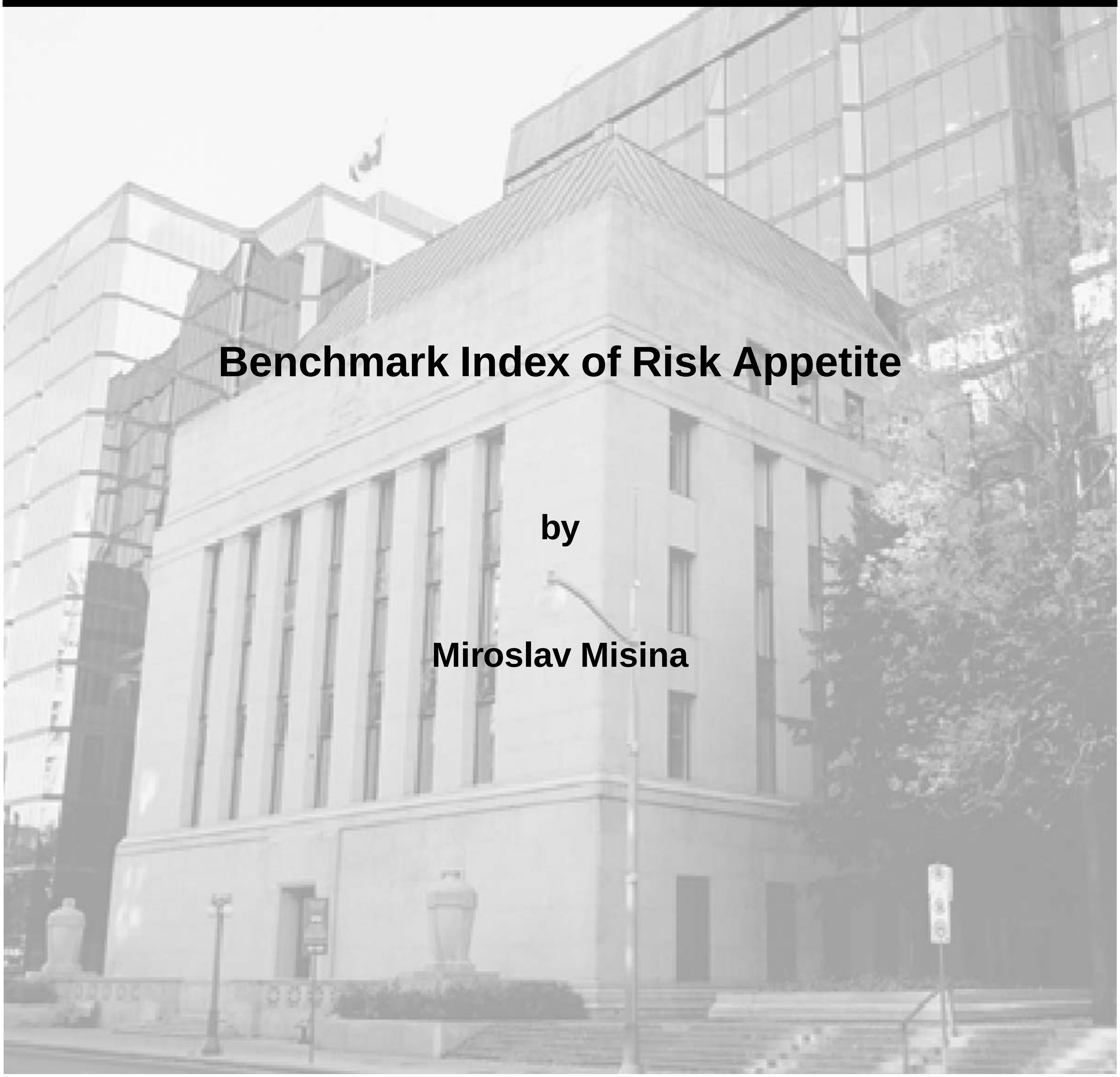
Bank of Canada Banque du Canada

Working Paper 2006-16 / Document de travail 2006-16



Benchmark Index of Risk Appetite

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2006-16

May 2006

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Acknowledgements

I would like to thank Mark Illing for valuable contributions to section 4; Gino Cateau, Céline Gauthier, Pierre St-Amant, and David Tessier for questions and detailed comments; and Glen Keenleyside for editorial assistance.

Abstract

Changes in investors' risk appetite have been used to explain a variety of phenomena in asset markets. And yet, popular indicators of changes in risk appetite typically have scant foundation in theory, and give contradictory signals in practice. The question is which popular indicator, if any, captures these changes. Kumar and Persaud (2002) offer an intuitively appealing argument regarding the effects of changes in risk appetite on asset prices in a portfolio, and Misina (2003) establishes the conditions under which these effects will be present. The author proposes a method that empirically implements these conditions and thus ensures that the resulting index can identify changes in risk appetite in the data. This index is then used to assess other risk appetite indexes used in practice. An example illustrates how the index can be used to help interpret price movements in foreign exchange markets.

JEL classification: G12
Bank classification: Economic models; Financial markets

Résumé

Les modifications de l'appétit des investisseurs pour le risque ont été invoquées pour expliquer toute une gamme de phénomènes observés sur les marchés d'actifs. Pourtant, les indicateurs les plus connus de l'appétit pour le risque ont souvent des assises théoriques peu étoffées et envoient dans les faits des signaux contradictoires. Dès lors, la question se pose de savoir lequel de ces indicateurs, s'il en est un, reflète vraiment les variations de cet appétit. Kumar et Persaud (2002) présentent un argument intuitif séduisant concernant les effets de celles-ci sur les prix des actifs d'un portefeuille. Misina (2003) établit de son côté les conditions dans lesquelles ces effets se manifestent. L'auteur propose maintenant une méthode qui permet de mettre en œuvre ces conditions et de bâtir un indice qui soit à même de déceler dans les données les modifications de l'appétit pour le risque. Il compare ensuite son indice à ceux employés dans la profession, puis illustre par un exemple l'usage que l'on peut en faire afin d'interpréter les mouvements du cours des devises.

Classification JEL : G12
Classification de la Banque : Modèles économiques; Marchés financiers

1. Introduction

Investors' newsletters and daily reports are replete with stories of changing investors' "risk appetite" and suggestions as to the best way to benefit from those changes. Part of the difficulty with these stories is that it is often unclear what exactly is meant by "risk appetite." Broadly speaking, "risk appetite" seems to be a stand-in for market sentiment, but at this level of generality the concept is hard to operationalize. More precise meaning can be attached to the concept, but there are still several possibilities:

- risk appetite refers to investors' risk aversion,
- risk appetite simply means demand for risky assets,
- risk appetite refers to the quantity of risky assets demanded.

The second and third interpretations, while plausible, lead to non-informative statements about market developments, implying that asset prices have changed because demand for (quantity demanded of) risky assets has changed. Causes of changes in demand (quantity demanded) are not specified. From the point of view of mapping this concept into an asset-pricing model, the first interpretation seems to be the easiest. However, this interpretation implies that, if the stories of changes in risk appetite are to be taken seriously, agents' utility functions are non-constant.[1] Since constant preferences are thought of as safeguarding rigour in academic research, allusions to non-constant preferences are typically frowned upon in academic circles.

[1] Endogenously changing risk attitudes *can* be accommodated within the standard framework. Habit-persistence utility functions deliver risk attitude that depends on surplus consumption and changes over time as surplus consumption changes. This mechanism, however, is typically found unsatisfactory, given that practitioners use changing risk attitudes to explain *sudden* movements in asset prices, or a shorter-term phenomena.

Gai and Vause (2004) and Misina (2005) tackle this difficulty by distinguishing between risk aversion, which is assumed to be constant, and risk appetite, which is allowed to vary over time. Gai and Vause postulate that "risk aversion is part of the intrinsic make-up of the investor and is a parameter that does not change markedly, or frequently, over time." Risk appetite, on the other hand, is "somewhat more than the notion of risk aversion," and "shifts periodically as investors respond to episodes of financial distress and macroeconomic uncertainty"(Gai and Vause 2004, 127). Misina (2005) differentiates between investors' risk attitude as specified in theoretical models by the Arrow-Pratt coefficient of risk aversion, and the risk attitude implied by agents' actions. To describe the latter, the notion of implied risk aversion is introduced in the standard expected utility framework. Implied risk aversion can change over time. Moreover, this change can be characterized as a function of agents' future outlook. In this way, the requirement of constant risk attitudes is reconciled with observed behaviour that seems to indicate otherwise.

This may clarify conceptual issues, but there are practical problems. Identification of changes in risk appetite usually relies on some type of in-house index that is purported to capture this phenomenon. Practitioners use a wide variety of risk appetite indexes, and yet, as shown in the recent survey by Illing and Aaron (2005), these indexes give contradictory signals even though they are presumably capturing the same phenomenon. Depending on which indicator is used, it is possible to conclude that the same price change was due to either increasing or decreasing appetite for risk! These findings raise the question of which one of them, if any, captures changing risk appetite. More generally, is it possible to disentangle the effect of changes in risk and risk appetite?

Part of the answer to this question, at a theoretical level, is provided by Misina (2003). Starting from a broad class of asset-pricing models, Misina identifies the key condition needed to ensure that a particular index of changes in investors' risk appetite, introduced by Kumar and Persaud (2002), will distinguish between changes in risk appetite and asset riskiness. The key condition needed to break the 'observational equivalence' is that cross-correlations of asset returns be zero, which implies a diagonal variance-covariance matrix of asset returns. The condition is arguably unlikely to be satisfied in practice, especially if attention is limited to financial assets.[2] Moreover, even if one succeeds in finding two assets whose returns are uncorrelated, it would seem that the chances of finding an uncorrelated portfolio decrease significantly with each addition of a new asset.

The present work builds directly on the results of Misina (2003) and introduces an index of risk appetite that satisfies the key condition identified in that paper. The approach is based on the observation that although the requirement of zero-covariances among returns may be a strong one when the original returns data are used, one can reverse the procedure and transform the original data in such a way that the requirement of zero correlation is achieved. The transformed data can be viewed as a set of derivative assets, obtained by trades in the original assets. The rank correlation measure of risk appetite, based on the same method as in Kumar and Persaud (2002), is then computed on the transformed data set, rather than on the original one. Since the key condition is satisfied in transformed data, the index would be able to identify changes in prices in a portfolio of assets that are due to changes in investors' appetite for risk. The index can

[2] Broadening the horizon to real assets may improve the chances of success somewhat, but it is questionable how returns on these are to be measured. One might be tempted to argue that looking at both bonds and equity increases the chances of finding a portfolio with a diagonal variance-covariance matrix, but this need not be the case. Although returns on stocks and bonds should not be positively correlated, the same argument does not imply a zero correlation but rather a negative correlation.

be used as a benchmark against which other risk appetite indexes are to be assessed. As part of the assessment, I compare the index with the original global risk appetite index (GRAI) of Kumar and Persaud (2002), as well as with other risk appetite indexes.

This paper is organized as follows. In section 2 I briefly review the results given in Misina (2003) that motivate the rest of the paper. In section 3 I discuss the data transformation, interpret the transformed data, and relate the properties of the portfolio consisting of the transformed data to the original portfolio. Section 4 provides the results, including a comparison of the proposed index with other indexes in use and a description of the behaviour of the index during major financial episodes. Section 5 concludes.

2. Necessity of Independent Returns

Suppose that an analyst observes a change in prices of assets in a portfolio, and tries to infer whether it was due to a change in the riskiness of some assets, or to a change in investors' risk appetite. For this task to be feasible, one must assume that these two different causes of asset-price changes will not be observationally equivalent. In other words, it must be assumed that these two causes will result in different behaviour of asset prices. Kumar and Persaud (2002) introduce the following distinction: changes in investors' risk appetite should impact all assets in the portfolio in proportion to their degree of riskiness. On the other hand, changes in the riskiness of any particular asset would not have systemic effects on returns of other assets in the portfolio. Kumar and Persaud then propose a rank correlation of excess returns and asset riskiness as a measure that would capture these effects. In particular, a non-zero rank correlation would indicate that a change in prices is due to either an increase (positive) or a decrease (negative) in risk appetite, while a zero correlation would indicate a change in prices due to changes in the riskiness of a particular asset.

The soundness of the proposed measure hinges on the validity of the distinction. Is the proposed distinction valid? Misina (2003) identifies the conditions under which the answer to the question is positive. The intuition offered by Kumar and Persaud is summarized in two propositions (Misina 2003, 5,6):

Proposition 2.1 *A change in investors' risk appetite will have monotonic effects on assets in different risk classes: the impact on returns will depend on the riskiness of a particular asset.*

Proposition 2.2 *A change in the riskiness of an asset will not have monotonic effects on excess returns across different asset classes. The impact on returns will not depend on the riskiness of a particular asset.*

Letting R_k^{ex} denote the excess return on a risky asset, and μ_k a measure of the riskiness of an asset in class k, Proposition 2.1 states that, when there is a change in risk appetite, returns on all assets will be affected, and there will be a *rank effect* - the magnitude of the impact will depend on the riskiness of each asset:

$$\mu_j > \mu_l \Rightarrow \triangle\ R_j^{ex} > \triangle\ R_l^{ex},\ \ \forall j > l. \tag{1}$$

Quantitatively, this effect can be captured by computing the correlation between assets' riskiness and excess returns after price changes. For this purpose one could use either the standard Pearson's correlation or Spearman's rank correlation.[3] Proposition 2.2 states that this effect will not emerge when the riskiness of assets changes.

[3] For the relationship between these two coefficients, see Zimmermann, Zumbo, and Williams (2003). Intuitively, one can think of Pearson's correlation as answering the question 'How well can the relationhsip in the data be represented by a **linear** function?' Spearman's correlation answers the question, 'How well can the relationship in the data be represented by a **monotonic** but otherwise arbitrary function?' When the underlying relationship is linear, the two measures will coincide. In capturing the rank effect, Pearson's correlation is overly restrictive, and Spearman's correlation is preferred.

The question is whether these propositions can be derived within a well-specified asset-pricing model. The answer is positive. Using a simple consumption-based asset-pricing model, the following expression is obtained[4]:

$$\left[\begin{array}{c} R_1^{ex} \\ \\ R_K^{ex} \end{array}\right] = \rho \left[\begin{array}{c} \sigma_{1,W} \\ \ldots \\ \sigma_{K,W} \end{array}\right] \equiv \rho \left[\begin{array}{ccc} \sigma_1^2 & \ldots & \sigma_{1,K} \\ & \ldots & \\ \sigma_{K,1} & & \sigma_K^2 \end{array}\right] \left[\begin{array}{c} \alpha_1 \\ .. \\ \alpha_K \end{array}\right], \tag{2}$$

where ρ is the coefficient of risk aversion; $\sigma_{i,W} \equiv \alpha_i \sigma_i^2 + \sum_{j \neq i} \alpha_j \sigma_{i,j}, \forall i$ is the riskiness of asset i, as part of the portfolio under consideration; and α_i represents the weight of asset i in the portfolio.

In this setting, Proposition 2.1 can be proved without imposing any further restrictions. From (2) it follows that

$$\frac{\partial R_k^{ex}}{\partial \rho} = \sigma_{k,W}, \ \forall k. \tag{3}$$

If portfolio assets are ordered in such a way that $\sigma_{j,W} > \sigma_{l,W}, \ \forall j > l$, one gets

$$\triangle \ R_j^{ex} > \triangle \ R_l^{ex}, \ \ \forall j > l.$$

This establishes Proposition 2.1.

To prove Proposition 2.2, further restrictions are needed. The key condition for rank correlation to be an indicator of changes in investors' risk appetite is that the variance-covariance matrix of asset returns be diagonal. Moreover, even with the diagonal variance-covariance matrix, the presence of common shocks such that $d\sigma_k^2 > 0$, or $d\sigma_k^2 < 0, \forall k$, may lead to a rank effect even when risk aversion is held constant.[5]

[4] See Misina (2003, 9), or Cochrane (2001, 154) for details.

[5] See Misina (2003, 10–16) for details and derivations. Note that the criterion for establishing whether a shock is common is the direction of the impact on assets' riskiness, rather than whether it can be traced to a single cause, as is common in the macroeconomic literature.

To implement this measure, one has to empirically satisfy the assumption of zero cross-correlations of returns, and find a way to assess whether assets in the portfolio have been subject to common shocks at any given time. The first issue is dealt with in the following section. The second issue is addressed in section 4.

3. Orthogonalization of Returns

It is clear that the requirement of independent returns is a strong one and unlikely to be satisfied empirically. As any practitioner can attest, one can perhaps find a couple of assets whose returns are uncorrelated. Finding a large number of assets whose returns are uncorrelated is extremely unlikely.

I propose to circumvent this problem by orthogonalizing the set of returns on the assets comprising a given portfolio. Suppose that the portfolio under consideration consists of K assets, and let R_i denote a $T \times 1$ vector of returns on asset k. The return matrix for the portfolio is

$$R = \left[\begin{array}{ccc} R_1 & ... & R_K \end{array} \right].$$

The transformation proposed here is based on the fact that if the space of returns is $K-$dimensional, there will be K orthogonal linearly independent vectors spanning it. Denote these vectors by F_i. The basis vectors will be linearly independent and as such satisfy the zero cross-correlation condition.

More specifically, the procedure is implemented as follows[6]:

(i) compute the variance-covariance matrix, Σ_R, associated with the return matrix R.

(ii) compute the eigenvalues and eigenvectors associated with Σ_R. The eigenvectors

[6] The technical background of this procedure is described in the appendix.

form an orthogonal basis. The loadings matrix is the matrix of eigenvectors. Denote it by B.

(iii) Obtain factor loadings using the fact that at each point in time t,

$$\underbrace{R\left(t\right)}_{Kx1}=\underbrace{B}_{KxK}*\underbrace{f\left(t\right)}_{Kx1}, \tag{4}$$

so that

$$f\left(t\right)=B^{-1}R(t),\ \forall t.$$

Each factor's value at time t is its return, which is a linear combination of returns on existing assets. The transformed data are then given by a $T\times K$ matrix:

$$F=\left[\begin{array}{ccc} F_1 & ... & F_K \end{array}\right],$$

where $F_i=\left[\begin{array}{cc} f(1)\ ... & f(T) \end{array}\right]'$, $\forall i$.

Since each factor is a linear combination of the original asset returns, one can interpret each of the basis vectors as a derivative asset formed from the original assets. The returns on these derivative assets are a linear combination of returns on given assets.[7] For example, a derivative asset with a return profile $F_1=-R_1+R_2$ would be obtained by going short on an asset with return R_1 and long on an asset with return R_2.

Inspection of (4) suggests that there might be a close relationship between portfolios of original assets, R, and the portfolio of factors, F. Letting $\alpha_t=\left[\begin{array}{ccc} \alpha_1 & ... & \alpha_K \end{array}\right]$ denote holdings of each asset at time t, it follows that the expected returns on the port-

[7] One could interpret these vectors as Arrow securities as well, but one need not. Arrow securities do form the 'usual' basis of the returns space. It is not necessary to use this usual basis, but the orthogonality property of vectors in the new basis is preserved.

folios are related by

$$E\left[\alpha R\left(t\right)\right] = E\left[\alpha B f(t)\right],$$

so that the expected returns on a portfolio of original assets can be replicated by a portfolio of factors with weights αB. The same result holds for the riskiness of these portfolios:

$$var\left[\alpha R\left(t\right)\right] = var\left[\alpha B f(t)\right].$$

Since the risk/return profile of these two portfolios is identical, it follows that an investor would be indifferent between holding one or the other.

An issue of potential concern is the uniqueness of the transformation. Whereas the eigenvalues associated with Σ_R are unique, eigenvectors are not. This non-uniqueness introduces the possibility of generating multiple sets of factors, F, and the issue of choice among these sets arises. To deal with this problem, I suggest the use of *normalized* eigenvectors. Normalized eigenvectors, represented by B, have a natural interpretation as weights of individual factors needed to replicate returns on each original asset.

4. Benchmark Index of Risk Appetite

Kumar and Persaud (2002) compute their rank-correlation index of risk appetite using a portfolio of currencies in the foreign exchange market. Whereas the above results can be applied to any portfolio, in the empirical exercise I follow Kumar and Persaud and focus on identifying changes in risk appetite in the foreign exchange market. This will facilitate comparison of results.

The data used are daily exchange rates spanning the period between 1981 and 2005. The portfolio analyzed consists of thirteen currencies. The constituent currencies for the period 1981–1999 are those of Austria, Belgium, Canada, Denmark, France, Germany, Great Britain, Italy, Japan, the Netherlands, Norway, Sweden, and Switzerland. After 1999, the currencies that were replaced by the euro have been dropped and the currencies of the following countries have been added: Australia, Mexico, New Zealand, the Philippines, and Thailand.

Following Kumar and Persaud (2002), excess returns to holding each currency are computed as the difference between the forward rate at $t-1$ for delivery at $t,$ and the matching spot rate at time $t.$ In the exercise, three-month forward rates are used. Riskiness is computed as the volatility of the currencies over the twelve months prior to the forward contract. The key difference between Kumar and Persaud's index and the one proposed in this paper is that, in this paper, the rank correlation between riskiness and excess returns is computed on the *transformed* data, which satisfies the key condition needed to identify changes in prices due to a change in investors' risk appetite.

Figure 1 shows the rank correlation between risk and excess returns for the portfolios of the currencies described above. I label this index as RAI-MI, to distinguish it from the index computed by Kumar and Persaud. The values of the index range from -0.89 in November 1991 to 0.8 in February 1998. Positive values of the index are interpreted as identifying price changes due to an increase in investors' risk appetite, whereas the negative values signal changes due to a decrease in risk appetite. Note that the value of the index in June 2005 is very close to an all-time high, having a value of 0.75.

4.1 RAI-MI and GRAI

How does the above index compare with the GRAI of Kumar and Persaud (2002)? Figure 2 shows the values of both indexes using the same underlying assets.

Clearly, GRAI is much more volatile, with $\sigma^2_{GRAI} = 0.34$, and $\sigma^2_{RAI-MI} = 0.11$ over the sample period. The contemporaneous correlation is 0.38. The indexes seem to be displaying somewhat similar patterns, with ups in the mid-80s, lows in the early 90s, and close to all-time highs in 1998. To show these patterns more clearly, Figure 3 depicts the smoothed values of the two indexes.

Although the general direction of movement of both indexes is similar, there are some notable exceptions. For example, the GRAI indicates that, in the mid-80s, a series of price changes was due to a dramatic decrease in investors' risk appetite, whereas RAI-MI is in the neutral territory. In late 1980s, RAI-MI moves from neutral territory towards negative values. GRAI follows eventually, but indicates that, before a decrease, there was a price change driven by a sharp increase in risk appetite.

Recall that the two indexes use the same methodology, the only difference being that the RAI-MI is computed on transformed data, so that the key condition needed to break observational equivalence holds. The difference in results gives a sense of the sensitivity of the index to a violation of the assumption of the independence of returns.

4.2 Identification of common shocks

To use the RAI-MI as an indicator of price changes due to changes in risk appetite, the above analysis needs to be complemented by a method to identify common shocks. This is accomplished by computing changes in the riskiness of assets in the portfolio at a given point in time. Periods when the riskiness of all assets changes in the same direction are interpreted as being due to a common shock. For example, an increase in the riskiness of all assets in the portfolio would be interpreted as being due to a common negative shock.

Figure 4 illustrates how this method is applied. For purposes of illustration, a two-year subsample is selected. For this subsample, RAI-MI and changes in the riskiness of derivative assets are computed at each point in time. In this figure, these changes are represented by bars, each of which denotes the number of assets whose riskiness in a given period has increased/decreased. For example, in March 1999, out of thirteen assets in the portfolio, the riskiness of eight of them has increased, whereas the riskiness of the remaining five has decreased.[8]

In the subsample analyzed here, there have been no points in which the riskiness of *all* derivative assets has changed in the same direction. Strictly speaking, this would indicate that there are no common shocks in this subsample. This conclusion, however, depends on the way changes are counted. In the counting procedure used to generate the above results, any change that is not identically equal to zero has been classified as either positive or negative. For example, a change in riskiness of the 10^{-7} order of magnitude would be counted as a decrease in riskiness, even though for all practical purposes there has been no change in the riskiness of this asset. In light of this consideration, and numerous experiments performed, unidirectional changes in the riskiness of ten or more

[8] In applying the above procedure to identify common shocks, one has to bear in mind that it is valid *only* when cross-correlations of asset returns are zero, since only in that case does asset volatility coincide with a measure of the riskiness of this asset as part of a portfolio. Furthermore, although the number of factors in the portfolio corresponds to the number of original assets, factors should not be interpreted as representing individual assets. As stated earlier, each factor is a derivative asset, a linear combination of original assets comprising a portfolio.

assets in the portfolio of thirteen assets should be taken as an indicator of common shocks.

According to this criterion, changes in asset prices in November and December 1999 were most likely driven by common shocks. In November, eleven out of thirteen assets experienced a decrease in riskiness; the number in December was twelve out of thirteen. For these two periods, the value of RAI-MI is 0.31 and 0.37, respectively. Even though the rank correlation is positive, I cannot conclude that a change in prices in these two periods was driven by an increased risk appetite.[9] The situation is different in August 2000. In that period, nine assets experienced a decrease in riskiness, and four an increase, and the value of RAI-MI is 0.65.[10]

These examples illustrate a general procedure in using the index to identify sources of price changes in a given portfolio. The information about the value of the index has to be complemented by an assessment of the presence of common shocks, and a statistical assessment of the significance of a given value of the index. In the former assessment, current market and economic developments may give further insights into the nature of shocks, identified by the suggested procedure.

4.3 Comparison with other indexes

Table 1 reports the values of cross-correlations for a range of indexes in current use: the Goldman Sachs risk aversion index (GS), Credit Suisse First Boston (CSFB), a BIS index proposed by Tarashev, Tsatsaronis, and Karampatos (2003), the Bank of England index (BE) proposed by Gai and Vause (2004), and the investor confidence index (ICI)

[9] The p-values associated with these values of RAI-MI are 0.3 and 0.21, respectively, which would not lead to rejection of the null hypothesis of zero rank correlation.

[10] The associated p-value is 0.018, which leads to rejection of the null hypothesis of zero correlation.

proposed by Froot and O'Connell (2003). Bold-faced values are significant at the 5 per cent level.[11]

	RAI-MI	GS	CSFB	BIS	BE	ICI
RAI-MI	**1.00**	**0.66** (0.48)	-0.07 (-0.1)	**-0.18** (-0.06)	**0.57** (0.41)	**-0.28** (-0.06)
GS		**1.00**	**0.31** (0.34)	**0.33** (0.33)	**0.38** (0.38)	0.06 (0.18)
CSFB			**1.00**	**0.37** (0.36)	0.10 (0.10)	0.06 (0.01)
BIS				**1.00**	**-0.25** (-0.27)	-0.03 (0.01)
BE					**1.00**	**-0.25** (-0.26)
ICI						**1.00**

Table 1: Cross-correlations of various indexes (values in brackets are cross-correlations based on unsmoothed data)

The Goldman Sachs risk aversion index (GS) and the Bank of England index (BE) are both highly correlated with RAI-MI, with correlations of 0.66 and 0.57, respectively. GS behaves similarly to RAI-MI even though it employs an entirely different framework for measuring risk appetite.[12] One notable difference between the two series is the much sharper downward spike of the GS in August 1998, the month in which Russia defaulted on its sovereign debt (Figure 5).

The BE index, suggested by Gai and Vause (2004), behaves similarly to RAI-MI (Figure 6). This index extends the approach of Tarashev, Tsatsaronis, and Karampatos (2003). Perhaps surprisingly, the results of this index differ in a marked way from the BIS results.

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[11] Cf. Illing and Aaron (2005), Table 2. The authors provide details on computation of these indexes.

[12] The GS use a standard consumption capital asset-pricing model, where the Arrow-Pratt coefficient of risk aversion is allowed to vary over time (Goldman Sachs 2003). The model incorporates monthly real U.S. per capita consumption, the real 3-month U.S. Treasury bill rate, and the inflation-adjusted S&P 500 index. To convert the GS into a risk appetite index, one simply multiplies by -1.

Overall, the Bank of England index and the Goldman Sachs index are likely identifying periods of price changes associated with changes in investors' risk appetite, whereas the situation with the remaining three indexes is unclear.

5. Conclusions

The profusion of indexes purporting to capture changes in investors' risk appetite and the contradictory signals they offer to investors raises the question of which one of them, if any, captures changes in risk appetite. Kumar and Persaud (2002) offer an intuitively appealing argument regarding the effects of changes in risk appetite on asset prices in a portfolio, and Misina (2003) establishes the conditions under which these effects will be present. The contribution of this paper is to propose a method that can be applied to any portfolio that would empirically implement the key condition of independent return, and thus validate the interpretation of rank correlation as capturing changes in risk appetite.

The empirical part of the paper describes how the key ideas can be applied by examining price movements in a portfolio of currencies. The advantage of focusing on the foreign exchange market is that it is relatively straightforward to identify the 'market portfolio,' given that trades in a small number of currencies account for most daily trades. A potential disadvantage is that, with a relatively small number of currencies, rank correlation cannot be estimated precisely and the confidence intervals are large.[13] Nothing, however, in the above analysis precludes application to other assets, where much larger portfolios can be considered. That, in itself, would be an interesting exercise, since it could provide insight into the extent of integration of different asset markets.

[13] Another issue is data availability. One can construct portfolios of up to 25 currencies, for samples starting in 1998.

Other issues that deserve further consideration include measurement of asset riskiness, and identification of common shocks. Asset riskiness in this paper is computed as the standard deviation of returns over the twelve months preceding the month under consideration. Standard deviation is an appropriate measure of asset riskiness in portfolios of derivative assets with zero cross-covariances. Nonetheless, a forward-looking measure of riskiness might be preferable, and, if so, use of implied volatilities is an option.

One might also wish to consider alternative ways to identify common shocks. The methodology proposed is straightforward, but not mechanical. When using the index to monitor current developments, the procedure for identifying common shocks should be complemented by other information relevant to the markets in question. The fact that the proposed approach, in assessing the current situation, uses four different pieces of information—patterns in the data, statistical tests, identification of common shocks, and broader market information—is likely one of its key strengths.

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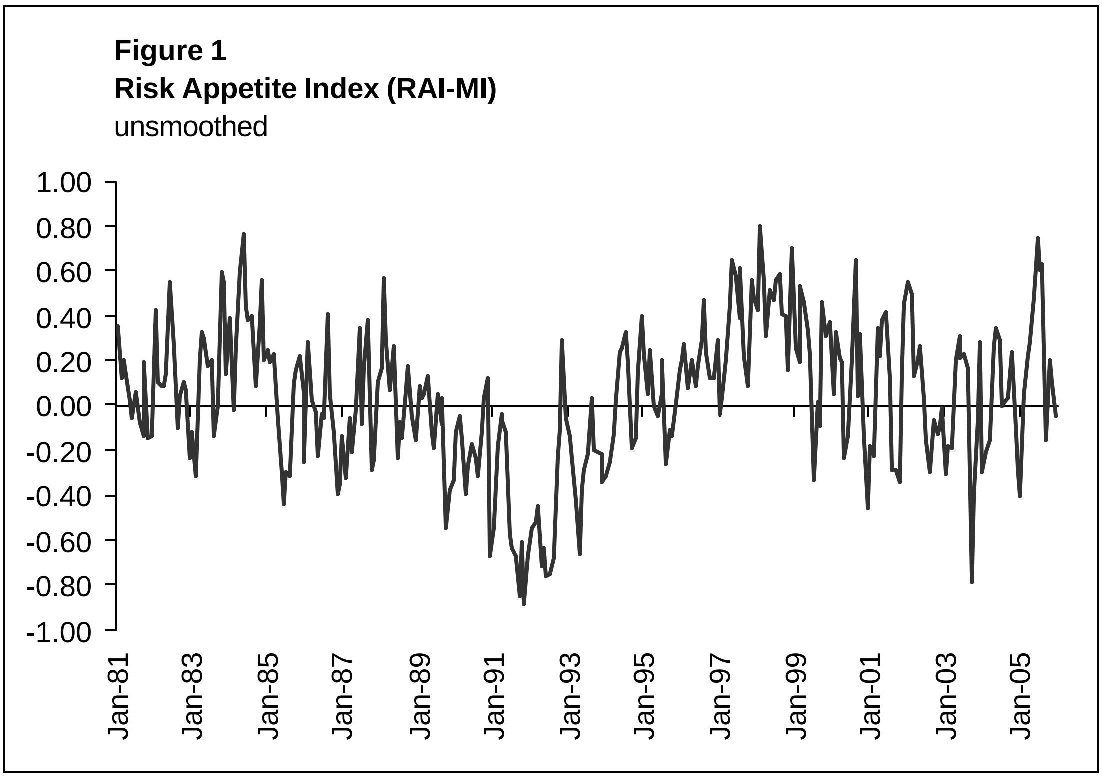

Figure 1
Risk Appetite Index (RAI-MI)
unsmoothed
1.00
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
Jan-81
Jan-83
Jan-85
Jan-87
Jan-89
Jan-91
Jan-93
Jan-95
Jan-97
Jan-99
Jan-01
Jan-03
Jan-05

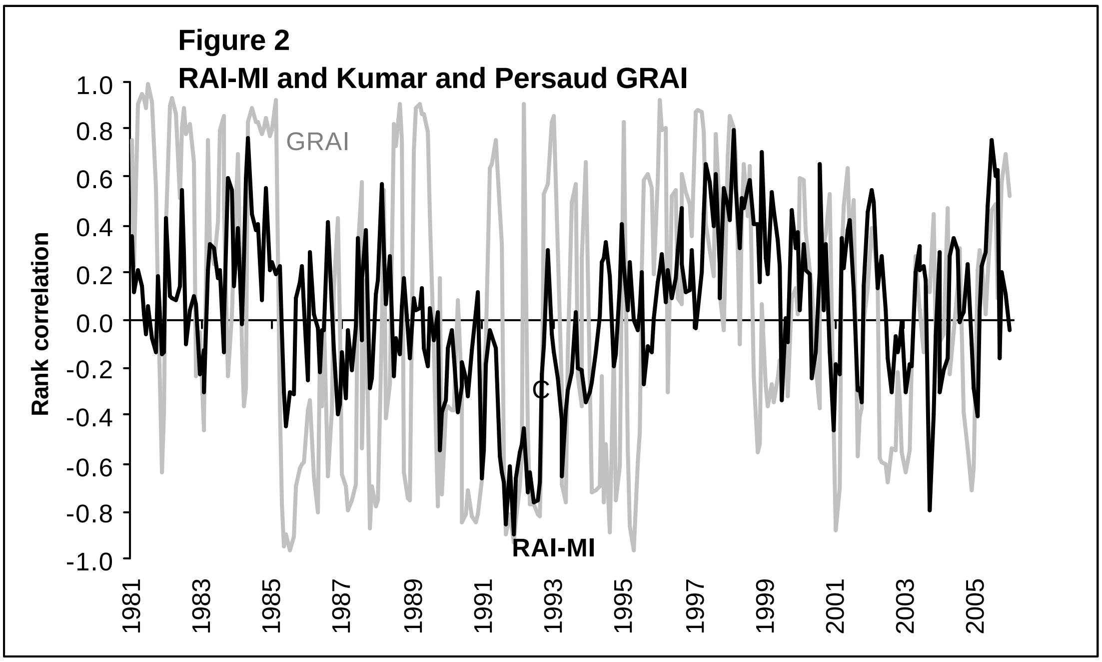

Figure 2
RAI-MI and Kumar and Persaud GRAI
Rank correlation
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
GRAI
C
RAI-MI
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005

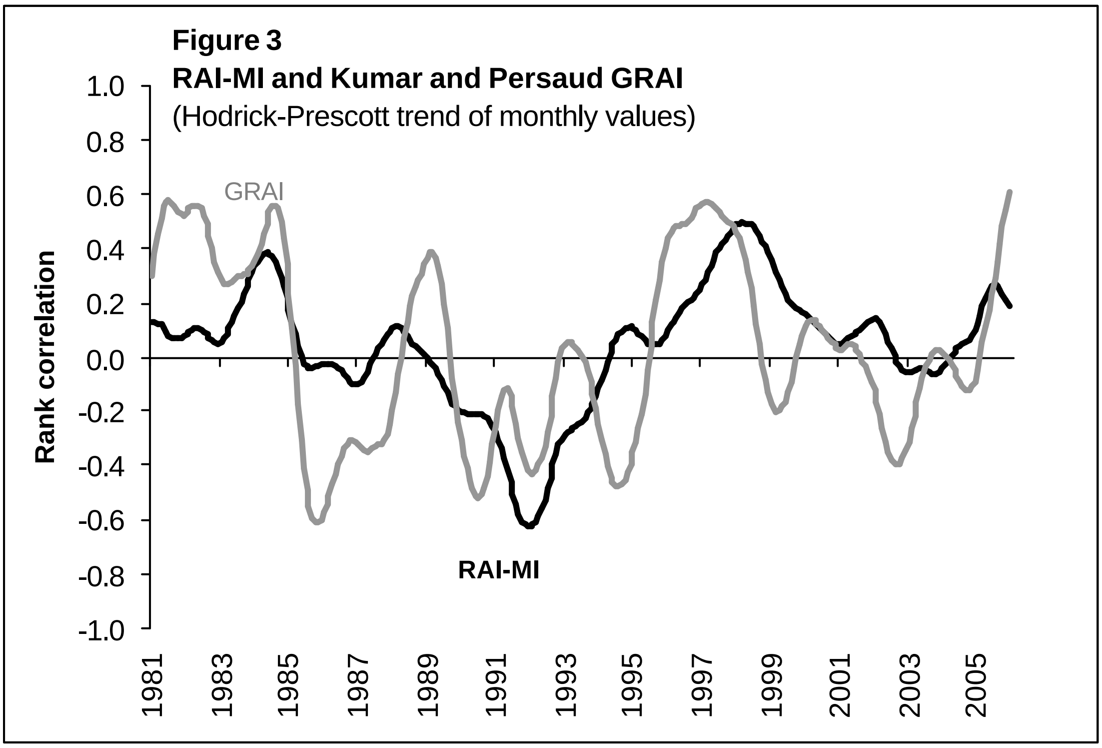
Figure 3
RAI-MI and Kumar and Persaud GRAI
(Hodrick-Prescott trend of monthly values)
Rank correlation
GRAI
RAI-MI
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005

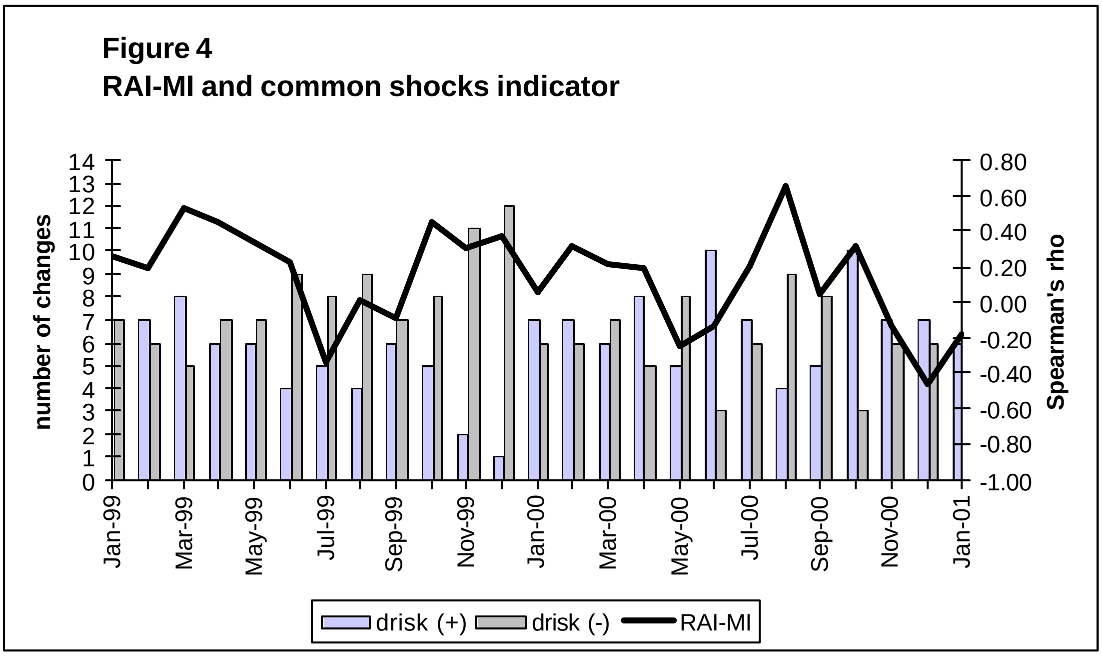
Figure 4
RAI-MI and common shocks indicator
number of changes
Spearman's rho
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
Jan-99
Mar-99
May-99
Jul-99
Sep-99
Nov-99
Jan-00
Mar-00
May-00
Jul-00
Sep-00
Nov-00
Jan-01
drisk (+)
drisk (-)
RAI-MI

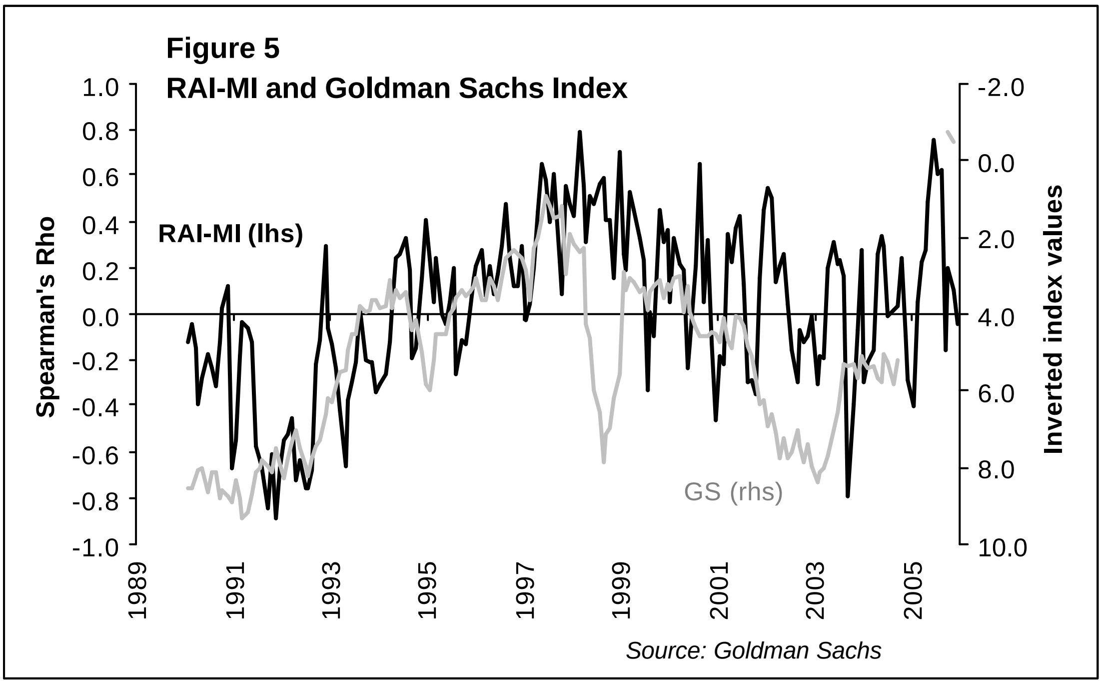
Figure 5
RAI-MI and Goldman Sachs Index
Spearman's Rho
Inverted index values
RAI-MI (lhs)
GS (rhs)
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-2.0
0.0
2.0
4.0
6.0
8.0
10.0
1989
1991
1993
1995
1997
1999
2001
2003
2005
Source: Goldman Sachs

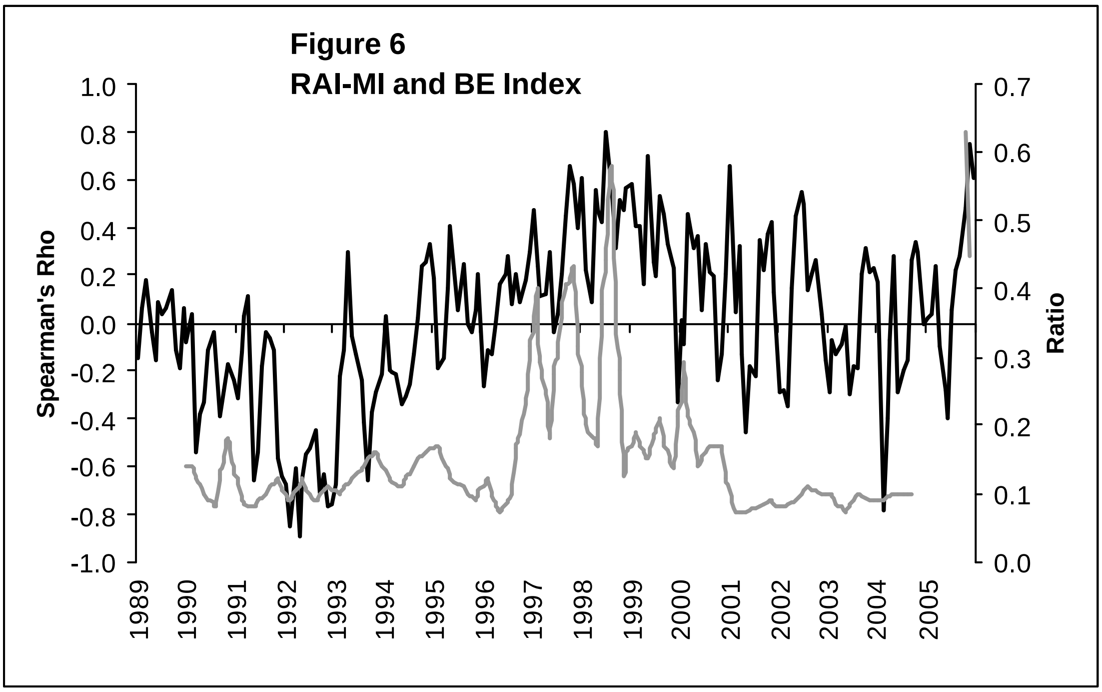
Figure 6
RAI-MI and BE Index
Spearman's Rho
Ratio
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1989
1990
1991
1992
1993
1994
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005

Appendix: Technical Details of Factor Analysis

The starting point of factor analysis is the variance-covariance matrix $\mathbf{V}$, associated with the returns matrix $\mathbf{R}$. The problem is to decompose the information about covariances into its components. This is done by diagonalization of $\mathbf{V}$.[14] Since $\mathbf{V}$ is a real symmetric matrix by construction, this task is easy.

Proposition 5.1 (Lipschutz, 8.14) *Let* $\mathbf{V}$ *be a real symmetric matrix. Then there exists an orthogonal matrix* $\mathbf{P}$ *such that the matrix* $\mathbf{D}$*,*

$$\mathbf{D} = \mathbf{P}^{-1}\mathbf{V}\mathbf{P},$$

is diagonal.

If the normalized orthogonal vectors of $\mathbf{V}$ are chosen for the columns of $\mathbf{P}$, the diagonal entries of $\mathbf{D}$ will be the eigenvalues of $\mathbf{V}$. It also follows that matrices $\mathbf{D}$ and $\mathbf{V}$ are similar.

The next step is to generate factors that correspond to $\mathbf{D}$. This is achieved by a change of coordinates: returns are represented in the new coordinate system associated with $\mathbf{D}$. From

$$\mathbf{R}_t = \mathbf{P}\mathbf{F}_t$$

it follows that

$$\mathbf{F}_t = \mathbf{P}^{-1}\mathbf{R}_t, \quad \forall t,$$

where $\mathbf{F}_t = \begin{bmatrix} F_{1t} & \ldots & F_{Kt} \end{bmatrix}$. This generates a set of factors associated with $\mathbf{D}$.

[14] Note that from the point of view of this procedure it does not matter what the interpretation of the matrix elements is. In the present case, it is the covariances, but the analysis is quite general, irrespective of the interpretation of the matrix.

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