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**A Further Analysis of  
Exchange Rate Targeting in Canada**

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Banque du Canada



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## CONTENTS

|   |    |
|---|----|
| ABSTRACT/RÉSUMÉ.....                                  | v  |
| 1 INTRODUCTION .....                                  | 1  |
| 2 THE LINEAR-QUADRATIC MODEL .....                    | 3  |
| 3 ESTIMATION STRATEGY OF THE EULER EQUATION .....     | 5  |
| 4 EMPIRICAL RESULTS .....                             | 8  |
| 4.1 Pre-tests for integration and cointegration ..... | 8  |
| 4.2 Results for the Euler equation .....              | 10 |
| 5 CONCLUDING REMARKS .....                            | 11 |
| DATA APPENDIX .....                                   | 13 |
| TABLES .....  | 14 |
| REFERENCES .....                                      | 18 |



## Abstract

In a recent paper Mercenier and Sekkat (1988) conclude that the Bank of Canada has followed a policy of exchange rate targeting using the money supply. We reexamine their results using a different estimation approach and with different assumptions about the forcing process of the exogenous variables. We also extend the sample period to include more recent observations. While we find some weak evidence to support their conclusion, the results, in general, suggest that the Bank of Canada has not used the money supply to target the Canada-U.S. exchange rate.

## Résumé

Dans une étude publiée en 1988, Mercenier et Sekkat ont conclu que la Banque du Canada avait des objectifs en matière de taux de change qu'elle cherchait à atteindre en influant sur la masse monétaire. Les auteurs de la présente étude réexaminent les résultats obtenus par Mercenier et Sekkat en mettant à contribution une méthode d'estimation différente et en faisant d'autres types d'hypothèses au sujet du processus de dynamisation des variables exogènes. Ils retiennent aussi une période d'estimation plus longue de manière à tenir compte d'observations plus récentes. Les résultats empiriques confirment dans une très faible mesure la thèse de Mercenier et Sekkat, mais ils indiquent, de manière générale, que la Banque du Canada ne s'est pas servie de la masse monétaire dans la poursuite d'un taux de change cible du dollar canadien par rapport au dollar américain.





## 1 INTRODUCTION

In a recent paper Mercenier and Sekkat (1988) use a rational expectations linear-quadratic (LQ) model framework to examine the importance that the Bank of Canada has assigned to targeting the Canada-U.S. exchange rate. Based on their empirical results, the authors conclude that the Bank has targeted the exchange rate since the early 1970s and followed a clear rule in its short-term management of the domestic money supply to achieve this objective. Though the Bank of Canada has been prepared to intervene in the foreign exchange market from time to time to maintain “orderly conditions,” this conclusion appears to be inconsistent with the Bank’s stated policies. The inconsistency lies in the fact that the stated purpose of the Bank’s (sterilized) intervention program is to reduce the short-term volatility of exchange rate movements rather than to fix the value of the exchange rate at a particular target level. While we have serious reservations about the model Mercenier and Sekkat (MS) use, the purpose of the present paper is to reexamine and update the MS study emphasizing a methodological approach that differs from theirs in a number of important ways.

First, unlike MS we estimate the parameters of the LQ model using a limited-information procedure that is based on the model’s Euler equation. MS, on the other hand, use a full-information approach that requires an explicit solution for the model’s control variables in terms of the exogenous forcing variables. It is well-known that under full-information estimation, the forcing process must be specified and estimated jointly with the law of motion and with certain cross-equation restrictions. Provided the model is correctly specified, the full-information procedure will be more efficient than the Euler equation approach.<sup>1</sup> In contrast, the limited-information approach adopted in this paper provides us with consistent parameter estimates under more general conditions than the full-

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1. In a Monte Carlo study (based on stationary forcing variables), West (1986) finds that even under the assumption of no misspecification, full-information estimation is only moderately more efficient than limited-information estimation.

information procedure.

An additional reason for focussing on the Euler equation is to determine the usefulness of the LQ model. If the LQ model is a reasonable specification, then the full- and limited-information procedures should give broadly similar results. Thus, in replicating MS's study using a limited-information procedure, we provide some additional evidence on the usefulness of the LQ specification for modelling central bank behaviour and on the robustness of their results.

A second important difference between the approach used here and that in MS is the assumed time-series properties of the data. Like most early empirical studies that used the LQ model, MS assume that the variables in the model contain deterministic trends and detrend their variables by projecting them on a time trend. In contrast, we assume that the variables are nonstationary due to the presence of stochastic trends or unit roots. The unit-root tests we present later in this paper suggest that it is important to examine the LQ model under the nonstationary assumption. This highlights one of the advantages of the LQ model over other intertemporal optimization models. That is, the LQ model admits a linear decision rule that allows for the statistical analysis of integrated forcing variables.

Recently, Dolado, Galbraith and Banerjee (1991) and Gregory, Pagan and Smith (1990) have examined the econometric issue of the LQ model in this context. More specifically, Dolado, Galbraith and Banerjee (DGB) suggest a two-step procedure for estimating the Euler equation similar to the Engle and Granger (1987) two-step approach for cointegration tests. However Gregory, Pagan and Smith (GPS) have shown that under some conditions, one of the parameters in the Euler equation may not be asymptotically identifiable. Moreover, even if the formal conditions for identification are satisfied, estimation of the discount rate may be difficult. GPS's findings thus provide an important theoretical justification for the common practice of fixing the discount rate in estimating LQ

models. In this paper we provide empirical evidence on this issue.

The organization of the paper is as follows. Section 2 describes the linear-quadratic model employed by Mercenier and Sekkat (1988). Our estimation strategy is outlined in Section 3, and the empirical results are given in Section 4. Section 5 concludes the paper.

## 2 THE LINEAR-QUADRATIC MODEL

In this section we briefly review the LQ model and the optimal money supply rule derived in MS.<sup>2</sup> The Bank of Canada is assumed by MS to have the stabilization of the exchange rate as its long-run objective. An optimal money supply rule to meet this objective is chosen by minimizing the following intertemporal optimization problem:

$$\min_{\{M_i\}} E_t \sum_{i=t}^{\infty} \beta^{i-t} [\gamma (M_i - M_i^*)^2 + (M_i - M_{i-1})^2] \quad (1)$$

for  $i \geq t$ , where  $\beta \in (0, 1)$  is the subjective discount rate,  $\gamma > 0$  is the cost to the central bank of being away from the target variable  $M_i^*$ , and  $E_t$  is the expectations operator conditional on the central bank's information at time  $t$ . The central bank is assumed to be facing a policy trade-off between a "constant-money-supply-flexible-exchange-rate" and a "fixed-exchange-rate-volatile-money-supply." If the central bank pursues a pure floating exchange rate then  $\gamma \rightarrow 0$ , whereas if it follows a strict peg, then we would have  $\gamma \rightarrow \infty$ . Hence, the central bank's willingness to stabilize the exchange rate using the money supply would be suggested by  $\gamma > 0$ . We note that under these assumptions, a central bank is said to pursue a pure float only if it fixes the domestic money supply at the same level as in the previous period and maintains it at that level without error. In a growing economy, this would seem to be a restrictive assumption that could bias the results in favour of the alternative hypothesis of exchange rate targeting.

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2. For a detailed description of this model, see Mercenier and Sekkat (1988).

The static equilibrium relationship describing the law of motion for the target stock may be specified as

$$M_t^* = X_t^T \alpha + e_t \quad (2)$$

where  $\alpha$  is a  $(k \times 1)$  parameter vector,  $X_t^T$  is a  $(k \times 1)$  row vector of forcing variables and  $e_t$  is the disturbance term, capturing short-run deviations known to the central bank, but unknown to the econometrician. This term is assumed to be independent and identically distributed with zero mean and constant variance,  $\sigma_e^2$ .

The first-order conditions necessary for the minimization of (1) is given by

$$\Delta M_t = \beta E_t \Delta M_{t+1} - \gamma (M_t - M_t^*) \quad (3)$$

and the following transversality condition:

$$\lim_{T \rightarrow \infty} E_t [\beta^T \{ \gamma (M_T - M_T^*) + \Delta M_T \}] = 0 \quad (4)$$

To obtain an estimable form for equation (3), we adopt McCallum's (1976) substitution methodology to replace  $E_t \Delta M_{t+1}$  in (3) by its realization  $(\Delta M_{t+1} + \eta_{t+1})$  where  $\eta_{t+1}$  is the pure expectational error, such that  $E_t \eta_{t+1} = 0$ , and rewrite equation (3) as

$$\Delta M_t = \beta (\Delta M_{t+1} + \eta_{t+1}) - \gamma (M_t - X_t^T \alpha + e_t)$$

or

$$\Delta M_t = \beta \Delta M_{t+1} - \gamma (M_t - X_t^T \alpha) + v_t \quad (5)$$

where  $v_t = \beta \eta_{t+1} + \gamma e_t$ , such that  $E_t v_t = 0$ . Note that  $v_t$  is a composite error term that can be rewritten as a MA(1) process, given the assumption that the structural error term  $e_t$  is serially uncorrelated.

We also obtain a forward solution to (3), given by

$$M_t = \lambda M_{t-1} + (1 - \lambda) (1 - \beta\lambda) E \sum_{i=t}^{\infty} (\beta\lambda)^{i-t} M_i^* \quad (6)$$

where  $\lambda < 1$  is the stable root of the Euler equation (3). It follows from equation (6) that the control variable  $M_t$  will inherit the properties of the stochastic trends assumed for the forcing variables. As an example, consider the case where  $X_t$  is an independent random walk. Then

$$(1 - L) X_t = \varepsilon_t \quad (7)$$

where  $E_{t-1} \varepsilon_t = 0$ . Substituting equation (2) into (6) and (7) yields

$$(1 - \lambda L) M_t = (1 - \lambda) X_t^T \alpha + (1 - \lambda) v_t \quad (8)$$

As the root  $\lambda$  lies inside the unit circle, it follows from (8) that the endogenous variable  $M_t$  must be integrated of order one or I(1). Moreover, since the white-noise error term  $v_t$  is I(0),  $M_t$  and  $X_t$  must be cointegrated with cointegrating vector  $(1, \alpha)$ . GPS show that similar results also hold when the forcing variables follow more complicated I(1) processes. Thus, if the forcing variables are I(1) processes, then a testable cointegration restriction between  $M_t$  and  $X_t$  is implied by the LQ model.

### 3 ESTIMATION STRATEGY OF THE EULER EQUATION

In this section, we describe our estimation strategy in some detail. DGB have suggested a two-step procedure for estimating equation (5). In the first step, we obtain consistent estimates of the long-run coefficients ( $\alpha$ ) by estimating  $M_t = X_t^T \alpha + u_t$ . Since the LQ model implies that  $M_t$  and the forcing variables  $X_t$  are cointegrated, a consistent

estimate of  $\alpha$  can be obtained by ordinary least squares (OLS) (see Stock 1987 and Phillips and Durlauf 1986). Since any bias in the OLS estimates is  $O_p(T^{-1})$ , it is possible to substitute this estimate into equation (5) and ignore any sampling uncertainty in the estimate of  $\alpha$  when we estimate the other parameters ( $\beta$  and  $\gamma$ ); in other words, there is no “generated regressor” problem here.

However, the rate T-convergence result does not, by itself, ensure that the parameter estimates of  $\alpha$  will have good finite sample properties. This is true because the OLS estimates of  $\alpha$  are not asymptotically efficient, in the sense that they have an asymptotic distribution that depends on nuisance parameters due to serial correlation in the error term and the endogeneity of the regressor matrix  $X_t$  induced by Granger-causation from innovations in  $M_t$  to innovations in  $X_t$ . Hence, it is desirable to use an estimation procedure that is asymptotically optimal under more general conditions than simple OLS. One such procedure has recently been advocated by Stock and Watson (1993). They propose to eliminate the aforementioned nuisance parameter dependencies by including leads and lags of the first differences of the regressors in the estimated “test” regression.<sup>3</sup> In this paper we use the Stock and Watson (SW) method to estimate long-run parameters ( $\hat{\alpha}$ ) that are super-consistent as well as asymptotically efficient. We then use this estimate to form the cointegrating residuals  $\hat{u}_t = M_t - X_t^T \hat{\alpha}$ . This in turn allows us to rewrite equation (5) as

$$\Delta M_t = \beta \Delta M_{t+1} - \gamma \hat{u}_t + v_t \quad (9)$$

Since all the variables in (9) are stationary, DGB suggest estimating the discount rate  $\beta$  and the adjustment term  $\gamma$  by some type of generalized instrumental variables procedure. In the empirical analysis we use Hansen’s (1982) generalized method of moments (GMM)

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3. In a Monte Carlo study, Inder (1993) finds that cointegration estimates that include dynamics are much more reliable, even if the dynamic structure is overspecified, relative to simple OLS estimates.

estimator, since it can control for the effect of the MA(1) process in the composite error term on the standard errors.

If the structural error term  $e_t$  is serially uncorrelated, then lags of  $\Delta M_t$  and  $\hat{u}_t$  at time  $t-1$  and earlier are valid instruments for GMM estimation. However, in order to allow for the possibility that  $e_t$  follows an MA(1) process due to effects of temporal aggregation, we also estimate (9) using lags of  $\Delta M_t$  and  $\hat{u}_t$  at time  $t-2$  and earlier. To the extent that there are more instruments than parameters to be estimated, the validity of the model can be tested using the J-test for over-identifying restrictions developed by Hansen (1982).

Gregory, Pagan and Smith (1990) have pointed out a case, however, where the above two-step method would fail. If the forcing variables are I(1), then the covariance matrix between the instruments and the regressors will be singular and consequently only one of the two parameters of the Euler equation will be identifiable. In contrast, if  $\Delta X_t$  follows a higher-order (stationary) AR or VAR process then the nonsingularity will be satisfied and both parameters will be identifiable. However, calculations by GPS indicate that even if  $\Delta X_t$  follows a stationary AR(1) process, a joint estimation of  $\beta$  and  $\gamma$  may be difficult. The source of this difficulty arises out of the estimation of  $\beta$ . This argument suggests that one should preset the value of  $\beta$  in the estimation procedure. In the empirical analysis we will test whether the individual series  $\Delta X_t$  are innovation sequences and we test the sensitivity of the estimates of  $\gamma$  to different choices of the discount rate parameter.

Finally, in order to implement the two-step procedure we follow MS and specify the long-run equilibrium equation as

$$M_t^* = X_t^T \alpha + e_t = \mu + \alpha_1 r_t^* + \alpha_2 p_t^* + \alpha_3 \tilde{p}_t^* + \alpha_4 \tilde{y}_t^* + e_t \quad (10)$$

where  $r_t^*$  is a short-term U.S. interest rate,  $p_t^*$  is a measure of U.S. export prices,  $\tilde{p}_t^*$  is the rest-of-world export price and  $\tilde{y}_t^*$  is aggregate foreign demand. The measures of money

that we consider initially are Canadian base money, M1 and M2.<sup>4</sup> As well, the following restrictions are placed on the coefficients:  $a_1 < 0$ ,  $\alpha_4 > 0$ ,  $\alpha_2 + \alpha_3 = 1$  and  $\alpha_2, \alpha_3 > 0$ . These restrictions imposed by their structural model on the reduced form coefficients help test the empirical relevance of their behavioural assumptions. The authors note that this particular formulation is a result of assuming that the Bank of Canada uses a monetary theory of exchange rate determination, as characterized by Frenkel (1976) and Bilson (1979), to determine the long-run supply of money compatible with the assumed Canada-U.S exchange rate target. However, (10) may also be regarded as a standard money demand equation, with U.S. variables proxying Canadian ones. This suggests that equation (10) is simply capturing the equilibrium money supply determined by domestic rather than foreign variables.

## **4 EMPIRICAL RESULTS**

### **4.1 Pre-tests for integration and cointegration**

We use quarterly data from 1971Q1 to 1992Q4, which we truncate as necessary to compensate for leads and lags. Prior to estimation of the Euler equations, the properties of each series are examined using the parametric augmented Dickey-Fuller (ADF) test as suggested by Dickey and Fuller (1979) and Said and Dickey (1984), and a non-parametric test proposed by Phillips and Perron (1988).<sup>5</sup> The results of the ADF and PP tests are reported in Table 1. Both tests fail to provide evidence for rejecting the unit-root null even at the 10 per cent level of significance for any of the variables.<sup>6</sup> This suggests that the variables under consideration are well-characterized as nonstationary processes. In turn this provides support for our assumption of integrated forcing variables.

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4. See the data appendix for precise data definitions.

5. We use the normalized bias version of the Phillips and Perron test since it is more powerful than its t-statistic counterpart (see Campbell and Perron 1991 and Gregory 1991).

6. Due to the well-documented low power of these unit-root tests, in testing for integration and cointegration we focus on the 10 per cent critical values. In presenting the Euler equation results later in the paper, we use the (more conventional) 5 per cent critical values.



As we argued in the previous section, an implication of the LQ model is that if the forcing processes are all  $I(1)$ , then these variables should form a cointegrating relationship with the control variable  $M_t$ . We test whether this implication is supported by the data by applying tests for cointegration. The first test we use is the two-step approach proposed by Engle and Granger (1987) and Phillips and Ouliaris (1990). The test regressions include a constant, and a constant and a linear trend term. If we find cointegration in the demeaned specification, this corresponds to “deterministic cointegration,” which implies that the same cointegrating vector eliminates deterministic trends as well as stochastic trends. But if the linear stationary combinations of the  $I(1)$  variables have a non-zero linear trend, this then corresponds to “stochastic cointegration.”<sup>7</sup> Table 2 presents these cointegration test results. For base money and M1 we find evidence consistent with cointegration suggesting that the dummy variable incorporated in each of these equations by MS may have been unnecessary. Unfortunately we are unable to find evidence of cointegration for the M2 aggregate. However, the inability to reject the no-cointegration null may simply reflect the low power of these tests and may not be an indication of the absence of a long-run relationship. Therefore to control for type II error, we also use a recently developed test that has cointegration as its null hypothesis (see Shin 1992). The results of this test are reported in Table 3 and, in general, corroborate the conclusions from the cointegration tests. That is, we are unable to reject the null of cointegration for base money and M1 whereas for M2 we reject the null in favour of the no-cointegration alternative. The latter result implies significant evidence of no cointegration for the M2 aggregate. Since we are unable to find evidence of cointegration for M2 we, like MS, discard it as a potential candidate for further analysis.

Table 4 presents parameter estimates obtained from estimating the cointegrating system for both base money and M1 using the SW procedure over both the shorter sample

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7. See Ogaki and Park (1989) for a discussion of stochastic and deterministic cointegration.

used by MS (1971Q1-85Q3) and the extended sample.<sup>8</sup> In addition to providing parameter estimates that are T-consistent as well as efficient, the SW approach also allows us to perform hypothesis testing using conventional asymptotic methods. We note that all regressors have the expected signs and are statistically significant at conventional levels. However, the export prices' adding-up constraint is strongly rejected by the data. This result seems to contradict MS's theoretical model and casts some doubt on its empirical validity. Nevertheless, in the next section we proceed to use the SW parameter estimates reported in Table 4 to form a measure of  $\hat{u}_t$ , which in turn will be used to estimate equation (9).

## 4.2 Results for the Euler equation

In this section we determine whether the estimation results in MS are robust to different assumptions about the stochastic process of the forcing variables, estimation method and sample period. To this end, we use Hansen's (1982) GMM procedure to estimate the parameters in equation (9). The instruments include a constant and lags of both  $\Delta M_t$  and the estimated equilibrium quantity  $\hat{u}_t$ . Two different sets of instruments are used and are denoted  $I_4^1$  and  $I_5^2$  respectively, where  $I_j^i$  corresponds to the set  $\{\text{constant}; \Delta M_{t-i}, \dots, \Delta M_{t-j}; u_{t-i}, \dots, u_{t-j}\}$ . The instrument set lagged one period will yield consistent estimates of  $\beta$  and  $\gamma$  (subject to identification), given the assumption about the composite error term  $v_t$ , whereas the sets lagged two periods will yield consistent estimates even if the structural error  $e_t$  follows an MA(1) process, possibly due to the effects of temporal aggregation.

In the first instance, we estimate both the discount factor and relative adjustment parameter for both base money and M1 over MS's original 1971Q1-85Q3 period and then

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8. We chose the number of leads and lags to equal 1 in the shorter sample and 3 in the extended sample, since this appears to be consistent with the simulation results in Stock and Watson (1993).

over the extended 1971Q1-92Q4 period. The larger information set should increase our ability to discern whether the Bank of Canada actually targets the Canada-U.S. exchange rate.

In general, the results are not encouraging (see Tables 5 and 6). Using the shorter sample period and the two different instrument sets we find that all parameter estimates are insignificant and in some cases of the “wrong” sign. In contrast, the estimation of the Euler equation corresponding to the longer sample period and base money aggregate produces discount factor estimates that are significantly different from zero at the 5 per cent level. However these discount factors appear too small to be consistent with economic theory and the relative adjustment parameters either are of the wrong sign or are insignificant. The parameter estimates for M1 are again insignificant.

Given the possible difficulties with identifying both  $\beta$  and  $\gamma$  from the data, we examine the sensitivity of the results by fixing the discount rates to a value ranging from 0.99 to 0.90 and reestimating the model. Table 7 provides the results for the 1971Q1-85Q3 period. The results for both base money and M1 show that all parameter estimates are statistically insignificant, regardless of the instrument set or discount factor at the 5 per cent level.<sup>9</sup> Although the J-tests are unable to reject the over-identifying restrictions, our results cast doubt on the robustness of MS’s conclusions and on the usefulness of the LQ model to describe the assumed exchange rate targeting of the Bank of Canada. The conclusions for the 1971Q1-92Q4 sample period are the same as those for the shorter sample and are therefore easily summarized (Table 8). The relative adjustment parameters, regardless of discount rate or instrument set, are never significant. Overall our results suggest that the Bank of Canada did not follow a policy of exchange rate targeting.

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9. However, we do note some support for the model at the 10 per cent level.

## 5 CONCLUDING REMARKS

In this paper we have reexamined Mercenier and Sekkat's conclusion that the Bank of Canada attempted to target its bilateral exchange rate with the United States using the money supply. As expected, we find MS's conclusions to be sensitive to different estimation methods and assumptions about the time-series properties of the forcing variables in the model. In contrast to MS, we find the relative adjustment parameter not to be different from zero (that is,  $\gamma = 0$ ) which suggests that the Bank of Canada did not follow a policy of exchange rate targeting over the sample periods we consider. This conclusion appears more consistent with the Bank of Canada's stated policy objective than MS's conclusion. Over the sample period, the Bank of Canada's policy objective has always been one of price stability. Even when the Bank was targeting the growth rate of M1 (November 1975 to November 1982), these targets were only intermediate ones set up to reach their long-run objective of price stability. Over the most recent period (26 February 1991 to the present) the Bank has set up explicit targets for reducing inflation and reaching price stability. In sum, our results support the Bank of Canada's contention that "... the objective of monetary policy is to promote domestic monetary, that is price, stability. Therefore, the Bank does not have in any strategic sense a target for the exchange rate." (Crow 1993, 52).

## DATA APPENDIX

The data definitions and their sources are as follows. The end-of-period base money, M1 and M2 aggregates, are taken from the *Bank of Canada Review*; all three are measured in billions of Canadian dollars. The end-of-period 3-month U.S. Treasury Bill rates are obtained from the *Federal Reserve Bulletin*. The U.S. and industrialized countries' export unit values (1985 = 100) are from the *International Financial Statistics* of the International Monetary Fund. Aggregate foreign demand is proxied by GDP at 85 exchange rates and prices are taken from the Organisation for Economic Co-operation and Development's *Main Economic Indicators*.

**Table 1:**  
**Unit-root tests**  
**Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests<sup>a</sup>**

| Variable            | ADF lags <sup>b</sup> | ADF t-statistic | PP $Z_{\alpha}$ -statistic <sup>c</sup> |
|---------------------|-----------------------|-----------------|---|
| Base Money          | 0                     | -2.715          | -7.423                                  |
| M1                  | 0                     | -2.672          | -6.328                                  |
| M2                  | 2                     | -0.770          | -0.124                                  |
| World activity      | 1                     | -2.746          | -5.675                                  |
| ROW export prices   | 1                     | -2.208          | -4.523                                  |
| U.S. export prices  | 1                     | -1.625          | -1.016                                  |
| U.S. interest rates | 2                     | -1.285          | -9.809                                  |

a. Henceforth the symbols “\*\*\*\*”, “\*\*\*” and “\*\*” indicates significance at the 1, 5 and 10 per cent levels, respectively. The ADF critical values are calculated from MacKinnon (1991) while the PP critical values are taken from Haug (1992). All test regressions include a trend term.

b. We use the lag length selection procedure advocated by Hall (1989) using a 5 per cent critical value. The initial number of AR lags is set equal to the seasonal frequency plus one or 5.

c. The long-run variance is estimated using a VAR prewhitened procedure suggested by Andrews and Monahan (1992).

**Table 2:**  
**Residual-based no cointegration tests**  
**Augmented Engle-Granger (ADF) and Phillips-Ouliaris (PO) tests**

|            | AEG t-statistic          |             | PO $Z_{\alpha}$ -statistic <sup>a</sup> |           |
|------------|--------------------------|-------------|---|-----------|
|            | Demeaned                 | Detrended   | Demeaned                                | Detrended |
| Base money | -4.443* (2) <sup>b</sup> | -4.495 (2)  | -35.573**                               | -35.573*  |
| M1         | -4.209 (2)               | -4.868* (3) | -31.420*                                | -36.042*  |
| M2         | -4.161 (4)               | -3.678 (3)  | -12.346                                 | -18.440   |

a. See footnote c, Table 1.

b. AEG lag lengths are in parentheses. We use the lag length selection procedure advocated by Hall (1989) for a 5 per cent critical value. The initial number of AR lags is set equal to the seasonal frequency plus 1 or 5.

**Table 3:**  
**Shin cointegration test<sup>a</sup>**

|            | Demeaned | Detrended |
|------------|----------|-----------|
| Base money | 0.039    | 0.038     |
| M1         | 0.049    | 0.041     |
| M2         | 0.114**  | 0.068**   |

a. The residuals are taken from a Stock and Watson (1993) dynamic OLS test regression. The truncation parameter for the Newey and West (1987) long-run variance estimator is set equal to  $\text{INT}(T^{1/3})$  or 9.

**Table 4:**  
**Stock and Watson parameter estimates**

| Variable           | 1971Q1-1985Q3                  |                   | 1971Q1-1992Q4     |                   |
|--------------------|--------------------------------|-------------------|-------------------|-------------------|
|                    | Base money                     | M1                | Base money        | M1                |
| Constant           | -3.908<br>(1.042) <sup>a</sup> | -4.920<br>(1.240) | -4.411<br>(0.366) | -3.147<br>(0.790) |
| World activity     | 1.244<br>(0.135)               | 1.391<br>(0.167)  | 1.300<br>(0.051)  | 1.152<br>(0.123)  |
| ROW export prices  | 0.214<br>(0.050)               | 0.303<br>(0.057)  | 0.183<br>(0.050)  | 0.171<br>(0.050)  |
| U.S. export prices | 0.386<br>(0.085)               | 0.221<br>(0.108)  | 0.418<br>(0.048)  | 0.440<br>(0.071)  |
| U.S. interest rate | -0.088<br>(0.018)              | -0.079<br>(0.026) | -0.101<br>(0.018) | -0.064<br>(0.028) |

a. The values in parentheses are Newey and West's (1987) standard errors with the truncation parameter set equal to the seasonal frequency or 4.

**Table 5:**  
**Estimates of the Euler equation**  
**1971Q1 to 1985Q3**

| Instruments         | Base money                     |                  | M1                |                  |
|---------------------|--------------------------------|------------------|-------------------|------------------|
|                     | $I_4^1$                        | $I_5^2$          | $I_4^1$           | $I_5^2$          |
| $\beta$             | -0.010<br>(0.436) <sup>a</sup> | 0.591<br>(0.478) | 0.212<br>(0.390)  | 0.636<br>(0.417) |
| $\gamma$            | -0.005<br>(0.164)              | 0.226<br>(0.212) | -0.016<br>(0.165) | 0.202<br>(0.212) |
| constant            | 0.018<br>(0.010)               | 0.002<br>(0.013) | 0.014<br>(0.009)  | 0.002<br>(0.011) |
| J-test <sup>b</sup> | 7.622                          | 6.197            | 7.543             | 5.555            |

a. Henceforth, the values in parentheses corresponding to the parameter estimates are standard errors.

b. The degree of freedom for all J-statistics is 4.

**Table 6:**  
**Estimates of the Euler equation**  
**1971Q1 to 1992Q4**

| Instruments         | Base money                     |                   | M1               |                  |
|---------------------|--------------------------------|-------------------|------------------|------------------|
|                     | $I_4^1$                        | $I_5^2$           | $I_4^1$          | $I_5^2$          |
| $\beta$             | 0.849*<br>(0.339) <sup>a</sup> | 0.770*<br>(0.305) | 0.601<br>(0.313) | 0.587<br>(0.336) |
| $\gamma$            | -0.073<br>(0.094)              | 0.018<br>(0.172)  | 0.018<br>(0.064) | 0.054<br>(0.075) |
| constant            | 0.005<br>(0.006)               | 0.004<br>(0.009)  | 0.005<br>(0.005) | 0.005<br>(0.006) |
| J-test <sup>b</sup> | 2.568                          | 2.125             | 5.393            | 4.310            |

a. Henceforth, the values in parentheses corresponding to the parameter estimates are standard errors.

b. The degree of freedom for all J-statistics is 4.



**Table 7:**  
**Estimates of the adjustment term for pre-set values of the discount factor**  
**1971Q1 to 1985Q3**

|                 | Base money  |                        | M1                      |                        |
|-----------------|---|------------------------|-------------------------|------------------------|
| Instruments     | $I_4^1$   | $I_5^2$                | $I_4^1$                 | $I_5^2$                |
| $\beta = 0.990$ | -0.036 (0.161) <sup>a</sup><br>4.613 <sup>b</sup> | 0.276 (0.159)<br>3.971 | -0.040 (0.164)<br>4.260 | 0.250 (0.174)<br>3.256 |
| $\beta = 0.975$ | -0.035 (0.160)<br>4.606                           | 0.272 (0.158)<br>3.988 | -0.039 (0.163)<br>4.264 | 0.247 (0.173)<br>3.281 |
| $\beta = 0.950$ | -0.034 (0.159)<br>4.598                           | 0.268 (0.157)<br>4.021 | -0.038 (0.162)<br>4.277 | 0.242 (0.172)<br>3.326 |
| $\beta = 0.900$ | -0.032 (0.157)<br>4.603                           | 0.258 (0.154)<br>4.104 | -0.036 (0.160)<br>4.330 | 0.230 (0.170)<br>3.435 |

a. The figures in parentheses are standard errors.

b. This value is the test statistic from the J-test.

**Table 8:**  
**Estimates of the adjustment term for pre-set values of the discount factor**  
**1971Q1 to 1992Q4**

|                 | Base money  |                        | M1                      |                        |
|-----------------|---|------------------------|-------------------------|------------------------|
| Instruments     | $I_4^1$   | $I_5^2$                | $I_4^1$                 | $I_5^2$                |
| $\beta = 0.990$ | -0.075 (0.096) <sup>a</sup><br>2.274 <sup>b</sup> | 0.049 (0.124)<br>2.146 | -0.020 (0.064)<br>4.132 | 0.058 (0.087)<br>3.721 |
| $\beta = 0.975$ | -0.075 (0.096)<br>2.274                           | 0.048 (0.123)<br>2.106 | -0.019 (0.064)<br>4.125 | 0.058 (0.086)<br>3.700 |
| $\beta = 0.950$ | -0.075 (0.095)<br>2.280                           | 0.043 (0.122)<br>2.045 | -0.017 (0.064)<br>4.116 | 0.057 (0.085)<br>3.669 |
| $\beta = 0.900$ | -0.075 (0.094)<br>2.320                           | 0.036 (0.119)<br>1.959 | -0.013 (0.063)<br>4.122 | 0.055 (0.083)<br>3.623 |

a. The figures in parentheses are standard errors.

b. The value below the relative adjustment term estimate is the test statistic from the J-test.

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