Working Paper 99-9/Document de travail 99-9

Measuring Potential Output within a State-Space Framework

by

Maral Kichian

Bank of Canada Banque du Canada

ISSN 1192-5434
ISBN 0-662-2799-6

Printed in Canada on recycled paper

Bank of Canada Working Paper 99-9

April 1999

Measuring Potential Output within a State-Space Framework

by

Maral Kichian

Research Department
Bank of Canada
Ottawa, Canada K1A 0G9
Tel: (613) 782-7460
Fax: (613) 782-7163
mkichian@bank-banque-canada.ca

Contents

Acknowledgements

The author would like to thank Dave Longworth, Tiff Macklem, Chantal Dupasquier, Alain Guay, Seamus Hogan, Douglas Hostland, Patrick Perrier, Gregor Smith, Pierre St-Amant and Simon van Norden, as well as seminar participants at the Bank of Canada and at UQAM for their comments and suggestions. Special thanks also to Nicholas Ricketts for his participation in writing the GAUSS programs used in the estimation. The author is solely responsible for any remaining errors.

Abstract

In this paper we measure potential output (and consequently the output gap) using state-space models. Given that the estimated output gap is used as an indicator to measure the extent of inflationary pressures in the economy, we evaluate the use of such models for the implementation of monetary policy. Our starting point is the Gerlach and Smets (1997) unobserved-components model, which they applied to the G7 countries. After subjecting this model to various diagnostic tests, we modify certain assumptions in it to reflect specific aspects of the Canadian economy. In particular, we focus on the specification of the permanent component of output and of inflation expectations, the issue of whether to use core or total inflation in the model, and the integration of appropriate supply shocks in the Phillips curve. In each case, the model is subjected to diagnostic tests and is examined for its out-of-sample forecasting performance. With the various modifications, we find that misspecification is somewhat alleviated and out-of-sample forecast performance is improved. Based on this performance, we feel that state-space models of the output gap can be quite useful in the formulation of monetary policy.

Résumé

Comme l’écart de production estimatif est un indicateur de l’intensité des pressions inflationnistes au sein de l’économie, l’auteure de l’étude cherche à évaluer si le recours aux modèles espace d’états pour estimer la production potentielle et, partant, l’écart de production peut être d’une quelconque utilité dans la mise en oeuvre de la politique monétaire. Le point de départ de son analyse est le modèle à composantes non observées que Gerlach et Smets (1997) appliquent aux pays du Groupe des Sept. Après avoir soumis le modèle à plusieurs tests de diagnostic, l’auteure modifie certaines des hypothèses initiales pour tenir compte d’aspects précis de l’économie canadienne. Elle s’attache en particulier à la spécification de la composante permanente de la production et des attentes relatives à l’inflation, à la question du choix de l’indice des prix servant à mesurer l’inflation (IPC global ou IPC hors alimentation, énergie et effet des impôts indirects) et à l’intégration de chocs d’offre appropriés à la courbe de Phillips. Dans chacun des cas, le modèle est soumis à un test de diagnostic et la qualité de ses prévisions en dehors de la période d’estimation est examinée. Les diverses modifications apportées au modèle permettent d’améliorer quelque peu la spécification de ce dernier ainsi que sa capacité de prévision en dehors de la période d’estimation. L’auteure conclut, à la lumière de ces résultats, que le recours aux modèles espace d’états pour estimer l’écart de production peut s’avérer très utile dans la formulation de la politique monétaire.

1. Introduction

The output gap, defined as the discrepancy between actual output and potential output, is an important variable in macroeconomic models of forecasting and policy analysis. It is considered to be a useful tool for gauging the extent of inflationary or disinflationary pressures present in goods and services markets in the economy. Being aware of such pressures is particularly important for the monetary authority when it is committed to an objective of stable inflation. Based on estimates of the output gap, the authority will tighten (loosen) monetary conditions in the face of higher (lower) inflation pressures.

Unfortunately, potential output is notoriously difficult to measure with accuracy. For the same country, various studies find significantly different estimates of potential and, consequently, the gap, with similar data spans and frequency. This lack of consistency stems from the fact that potential is unobserved and must be extrapolated from other macroeconomic variables. The question then becomes: which variables should be used for this purpose? A related question is: what method of extraction should be used?

In a recent study, St-Amant and van Norden (1997) discuss the properties of various methodologies of measuring potential. These techniques include univariate models, “hybrid” models, and multivariate filtering models. In the first category are models such as the well-known Hodrick-Prescott (HP) filter which are designed to extract business cycle frequencies from the data. Models of the second type combine univariate methods with information from assumed macroeconomic relationships such as Phillips curves. The extended multivariate filter (EMVF) used at the Bank of Canada in the quarterly projection model is such a model.[1] Finally, the authors examine filtering models based on vector autoregressions (VARs). Among these are the structural VAR methods of Blanchard and Quah (1989), the multivariate Beveridge-Nelson decomposition and Cochrane’s (1994) methodology.[2]

St-Amant and van Norden show that both univariate mechanical filters, as well as their hybrid extensions, may have problems in adequately separating output into trend and mean-reverting components.[3] These filters perform poorly in extracting business cycle frequencies of 6 to 32 quarters from series such as real GDP which have an important permanent component. In addition, they are subject to severe end-of-sample problems.

---

1. See Butler (1996).
2. An example of the application of these methods to U.S. data can be found in Dupasquier, Guay and St-Amant (1999).
3. See also Harvey and Jaeger (1993) and King and Rebelo (1993).

As one alternative to the above class of models, St-Amant and van Norden suggest the use of structural VAR (SVAR) methods for measuring trend output.[4] These models are favoured because they are not confronted with end-of-sample issues. In addition, they make use of limited but widely agreed upon restrictions on the data and provide forecasted values of the output gap. However, the authors also caution that confidence intervals around the estimated gap are generally wide and that results are sensitive to the variables included in the estimated system.

Other reasons for being cautious about the interpretation of the implied dynamics obtained from SVARs are given in Cooley and Dwyer (1998). These authors explain that, since the identification of SVARs is conditional on a set of atheoretical assumptions which cannot be tested, results from the SVAR are quite sensitive to misspecification in these assumptions. The authors illustrate this by applying the Blanchard and Quah (BQ) methodology to data obtained from a fully articulated artificial model which simultaneously imposes the BQ economic restriction. Furthermore, the authors show that structural conclusions from the SVAR may also be unreliable for an altogether different reason, which is that the identification strategy used generally relies on weak instruments.[5]

In this paper we focus on a different category of models for measuring potential output: state-space models. This framework of analysis is chosen for a number of reasons: (i) The relative flexibility of the modelling structure; that is, its ability to simultaneously include a fair amount of structural information directly while maintaining parsimony. In this case, even if the link is not explicit between taste and technology parameters on the one hand and the equations in the state-space system on the other, we can still analyze a well-defined macroeconomic system and interpret the parameters and residuals in a straightforward fashion.[6] (ii) The capacity of the model to provide us directly with confidence intervals around the measured gap or potential.[7] (iii) The

---

4. Another is the TOFU method proposed in van Norden (1995).
5. For more details, see Phillips (1989).
6. Ideally, it would be desirable to make use of a fully specified structural and stochastic model. Unfortunately, this also is problematic as it necessitates, among other things, making assumptions on the equilibrium level of the various components used in the production function, as well as its aggregated form. Studies employing these types of models frequently encounter difficulties in finding suitable data for their long-run values.
7. This is the mean square error associated with the estimated state vector and is obtained as a by-product of applying the Kalman filter. It is also known as the filter uncertainty of the model. However, as pointed out by Hamilton (1986), two types of uncertainties are associated with these models: filter uncertainty and parameter uncertainty. If one adopts the Bayesian perspective that the true value of the state vector is random, then our knowledge of it, based on observable variables, is reflected in a probability distribution. This distribution is a function of the observable variables as well as the true parameters of the data generating process. The uncertainty associated with the econometrician's best guess of the state vector at any time t is therefore the variance of this distribution at that time period. This is the so-called filter uncertainty. If, in addition, one takes into account the fact that model parameters are unknown and must be estimated, a second type of uncertainty, parameter uncertainty, must be added to the above.

fact that we can also directly obtain out-of-sample forecasts on the observable variables in the model, which provide an easy way of assessing the goodness of fit of the adopted specification and of ensuring that our estimates are useful in the formulation of policy.

The state-space framework was first used to estimate trend output on U.S. data by Kuttner (1994). It was subsequently extended by Gerlach and Smets (1997) and applied to the G7 countries. For our part, we examine the implications of using different versions and extensions of the Gerlach and Smets (1997) model (hereafter GS) for Canada. In all cases, the output gap is defined as the component of output which is consistent with change in the rate of inflation, all other things being equal.

The organization of the paper is as follows. In the next section we briefly present the general state-space methodology. In section 3 we present, re-estimate and test the GS model for Canada and discuss the results. In section 4 we propose some modifications to the model, along with estimation and test results. Section 5 concludes.

2. The state-space methodology

In this section we briefly describe the general state-space methodology. For this purpose, we adopt a slight modification of the set-up used by Harvey, Sentana and Ruiz (1992).This framework can also allow for autoregressive conditionally heteroskedastic (ARCH) error terms in the model equations. Consider the model

$$
\begin{aligned}
y_t &= Z\alpha_t + \beta X_t + \varepsilon_t + \varepsilon_t^* \qquad t = 1, 2, \ldots, T \\
\alpha_t &= T\alpha_{t-1} + \delta W_t + \eta_t + \eta_t^* \\
\\
\varepsilon_t &= \sqrt{h_t} \cdot \bar{\varepsilon}_t \qquad and \qquad \bar{\varepsilon}_t \sim NID(0, 1) \\
\eta_t &= \sqrt{q_t} \cdot \bar{\eta}_t \qquad and \qquad \bar{\eta}_t \sim NID(0, 1) \\
h_t &= \alpha_0 + \sum_{i=1}^{P} \alpha_i \varepsilon_{t-i}^2 \\
q_t &= \gamma_0 + \sum_{i=1}^{P} \gamma_i \eta_{t-i}^2 \\
\\
\varepsilon_t^* &\sim NID(0, H_t^*) \\
\eta_t^* &\sim NID(0, Q_t^*)
\end{aligned}
\tag{1}
$$

Here, y_t is an $Nx1$ vector of observed variables and α_t is an $Mx1$ vector of unobserved state variables. The first equation is called the measurement equation and links the observables to the non-observables through the parameter matrix Z which is of dimension NxM. It also includes a vector of observable exogeneous variables X_t, of dimension $Kx1$ with NxK coefficients β.

The second equation describes the dynamics of the state vector and is termed the transition equation. Exogeneous observed variables enter this equation through an $sx1$ vector W_t with coefficients in the Mxs matrix of parameters δ.

The disturbance vectors ε_t* and η_t* are normally and independently distributed with mean zero and variances H_t* and Q_t* respectively which are of dimensions NxN and MxM. The remaining error[8] terms, ε_t and η_t are ARCH errors[9] of lag P. Finally, all disturbances are assumed to be mutually uncorrelated.

Assuming, for the moment, that all the above parameters are known, one could then apply the Kalman filter to extract the values of the state-space variables. As a by-product of the above application, one also obtains all the necessary elements to construct a likelihood function. Since, in reality, model parameters are not known and need to be estimated, one can simply maximize the likelihood function with respect to these parameters.

In the special case of the presence of ARCH error terms in the model equations, the quasi-optimal Kalman filter is used instead. To estimate this system, one can re-write the model slightly differently to obtain:

$$y_t = Z^A \alpha_t^A + \beta X_t + \varepsilon_t*$$
$$\alpha_t^A = T^A \alpha_{t-1}^A + \delta^A W_t + G \cdot U_t$$

where the superscript A denotes the augmented vector. For example, in the case of ARCH errors of lag one, we have that $Z^A = \begin{bmatrix} Z & 0 & \Lambda \end{bmatrix}$ and

8. The reason for including two distinct error terms is expositional as well as technical since it allows computer programming of the code to be more tractable.
9. Many macroeconomic series exhibit thick-tailed distributions, specially at high frequencies. One explanation for this phenomenon is that these series have variances that are conditionally heteroskedastic. That is, the present period uncertainty depends on past periods' uncertainty. This is an important feature to consider in models where uncertainty varies with the different regimes. For instance, some authors argue that Canadian monetary policy has been subject to three different regimes with changing means and variances in inflation. Although we do not address this issue in the present version of the paper, we adopt a general enough framework to test for ARCH effects and allow them in the model subsequently when needed.

$$\alpha_t^A \equiv \begin{bmatrix} \alpha_t \\ \eta_t \\ \varepsilon_t \end{bmatrix} = \begin{bmatrix} T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_{t-1} \\ \eta_{t-1} \\ \varepsilon_{t-1} \end{bmatrix} + \begin{bmatrix} \delta \\ 0 \\ 0 \end{bmatrix} \cdot W_t + \begin{bmatrix} I_M & I_M & 0 \\ 0 & I_M & 0 \\ 0 & 0 & I_N \end{bmatrix} \cdot \begin{bmatrix} \eta_t^* \\ \eta_t \\ \varepsilon_t \end{bmatrix} \tag{2}$$

The quasi-optimal Kalman filter[10] can now be applied. This is given by

$$\begin{aligned} \varepsilon_t^{\alpha} &= T^A \alpha_{t|t-1}^A + \delta^A W_t \\ \varepsilon_t^{P} &= T^A P_{t|t-1}^A T^{A\prime} + G \cdot E_{t-1}(U_t U_t') \cdot G' \\ B_t &= Z^A \cdot \varepsilon_t^P \cdot Z^{A\prime} + H_t^* \\ \varepsilon_t^m &= y_t - Z^A \alpha_t^A - \beta X_t \\ K_t &= \varepsilon_t^P \cdot Z^{A\prime} \cdot B_t^{-1} \\ \alpha_{t|t}^A &= \varepsilon_t^{\alpha} + K_t \cdot \varepsilon_t^m \\ P_{t|t}^A &= (I - K_t \cdot Z^A) \cdot \varepsilon_t^P \end{aligned} \tag{3}$$

Above, the notation $a_{t|t-1}$ refers to the value of a at time t given the available information at time $t-1$. Similarly, $a_{t|t}$ is the value of a at time t given the available information at time t.

Given starting values for the conditional mean of the state vector $\alpha_{1|0}^A$ and its conditional variance $P_{1|0}^A$, the state residual terms ε_1^{α} and ε_1^{P} can be calculated. The respective terms are then substituted in the measurement equations to obtain the variance-covariance matrix B_1 as well as the mean residual term ε_1^m. Now, it is possible to calculate the Kalman gain term K_1. Substituting the last three terms in the state updating equations, it is possible to obtain $\alpha_{1|1}^A$ and $P_{1|1}^A$. At this point, the log likelihood function for that observation can be calculated. For the observation at time t this function is given by:

$$lf_t = -\frac{N}{2}\log(2\pi) - \frac{\log|B_t|}{2} - \frac{1}{2}(\varepsilon_t^{m\prime} \cdot B_t^{-1} \cdot \varepsilon_t^m) \tag{4}$$

10. If past values of the disturbances ε_t and η_t were directly observable, the model would have been conditionally Gaussian and the Kalman filter would have produced minimum mean square estimates of the state. In this sense, the filter would have been optimal. However, this is not necessarily the case here since past disturbances are not known. Nevertheless, we can treat the model as though it was conditionally Gaussian and, consequently, refer to the Kalman filter as being quasi-optimal.

In this fashion, the cycle repeats as above until all the observations are processed and parameters are estimated.

Notice that this is a very general state-space framework so that when no ARCH errors are present in the system, the above filter reduces to the well-known Kalman filter.

3. The Gerlach and Smets model

We now move to the presentation of the Gerlach and Smets (1997) model which is a special case of the general model presented above. As mentioned, it is a modification of the Kuttner (1994) model which was originally applied to the United States and where potential output and the slope of the Phillips curve were simultaneously estimated in an unobserved components framework.

The log of quarterly real output, y_t, is assumed to be the sum of log real potential output, y_t^P, and a log cyclical component, g_t. This cyclical component is assumed to be a stationary second-order autoregressive process[11] while the trend is assumed to follow a random walk with drift. The drift, in turn, is also assumed to follow a random walk. The output equations are thus given by:

$$\begin{aligned} y_t &= y_t^P + g_t \\ y_t^P &= \mu_t + y_{t-1}^P + \varepsilon_t^y \\ \mu_t &= \mu_{t-1} + \varepsilon_t^\mu \\ g_t &= \varphi_1 g_{t-1} + \varphi_2 g_{t-2} + \varepsilon_t^g \end{aligned} \tag{5}$$

An expectations-augmented Phillips curve is then added to the above to complete the system and to make the link between inflation and output as follows:

$$\pi_t = c + \lambda\pi_{t-1} + (1-\lambda)\pi_t^e + \beta_0 g_t + \beta_1 g_{t-1} + \gamma(L)\omega_t + \delta(L)\varepsilon_t^\pi \tag{6}$$

where c is a constant, π_t is the quarterly inflation rate, π_t^e is expected inflation and ω_t is a vector that includes log changes in the nominal trade-weighted exchange rate and nominal oil prices. These variables are included to capture the effects of temporary relative price shocks on inflation. Finally, the error terms ε_t^π are assumed to be mean zero and normally distributed. They enter the

11. This is the assumption made by Watson (1986). It has since been used in a standard fashion in various macroeconomic models.

inflation equation as a moving average process to capture any remaining inertia in supply shock variables. This also allows the model to be identified.

As the authors point out, despite its simplicity, this specification captures fairly well the conventional wisdom about general macroeconomic behaviour. In particular, it separates potential and gap outputs into variables with distinct dynamics. In addition, it allows short-run inflation to deviate from its long-run equilibrium because of sluggishness in the inflation process itself and because changes in the gap and various relative price shocks influence its level. To complete the model, all error terms in the model are assumed to be uncorrelated, mean zero and distributed normally.

Finally, a simplifying assumption is made by making expected inflation totally backward-looking and imposing $\lambda = 1$. Thus the Phillips curve becomes:

$$\Delta\pi_t = c + \beta_0 g_t + \beta_1 g_{t-1} + \gamma(L)\omega_t + \delta(L)\varepsilon_t^{\pi} \tag{7}$$

The chosen specification is somewhat justified on the grounds that one cannot reject the null of the presence of a unit root in the inflation process of various G7 countries. In addition, since expected inflation is unobserved, the authors find it convenient to assume it to be equal to past inflation.

3.1 Estimation results and discussion

Before turning our attention to the model assumptions and possible extensions, we re-estimate this model for Canada.[12] For this purpose, as usual, the vector of the initial values of the latent variables was assumed to have a diffuse distribution.

The model to be estimated can be written slightly differently in order to avoid identification problems. Substituting the expression for potential output in the first equation of the system and knowing that $y_{t-1}^P = y_{t-1} - g_{t-1}$, the system can be written as:

$$\begin{aligned}
\Delta y_t &= \mu_t + g_t - g_{t-1} + \varepsilon_t^y \\
\mu_t &= \mu_{t-1} + \varepsilon_t^{\mu} \\
g_t &= \varphi_1 g_{t-1} + \varphi_2 g_{t-2} + \varepsilon_t^g \\
\Delta\pi_t &= c + \beta_0 g_t + \beta_1 g_{t-1} + \gamma(L)\omega_t + \delta(L)\varepsilon_t^{\pi}
\end{aligned} \tag{8}$$

12. All estimations in this paper are carried out by applying the one-sided Kalman filter.

We refer to the first of these equations as the “output equation” and the last as the “inflation equation” although, strictly speaking, these are equations for the differences in output and inflation.

We estimate the above model with quarterly data for the period extending from 1961Q4 to 1997Q1. This adds an additional 10 observations to the data span used by GS. An additional four observations at the end of the sample are set aside for out-of-sample comparisons.[13] The results are tabulated in the fourth column of Table 1.

Results indicate that both the contemporaneous and the lag gap terms are significant and cumulatively positively affect inflation and that the sacrifice ratio is 8.33.[14] The coefficients on the assumed AR(2) process of the unobserved gap are also significant and sum to less than one, indicating stationarity for this series. Amongst the supply shock variables, the contemporaneous value of real oil price changes is found to be significant, with 90 per cent of any increase in this term being reflected in the acceleration of inflation. On the other hand, nominal exchange rate shocks are found to have no significant effect on the dependent variable. Finally, we find that the coefficient on the first moving average term is strongly significant.

Figure 1 shows the graphs of potential output and the gap measured with our data. According to this estimate, there have been three important periods of excess supply in Canada around the dates of 1977, 1982 and 1991, the second being the most pronounced. The average duration of these downturns has been a little above four years. The estimated model also suggests a modest amount of excess supply around 1971. As for periods of excess demand, the three major ones are mid to late 1960s, from 1972 to about 1974, and from around mid-1987 to about 1990. Finally, according to this estimated model, the Canadian economy was at or very close to potential from 1996 till the end of the sample, that is, 1997Q1.

At this point, we conduct some diagnostics tests on the model residuals. More specifically, we test the observable equation residuals for serial correlation, ARCH effects and normality. We use the LM statistic to test for first to fourth-order autocorrelation, Engle’s LM test to check for the existence of ARCH effects of order one to four, and the Jarque-Bera test for normality. Results are collected in column three of Table 2.

---

13. While it would have been better to have had more than four out-of-sample observations with which to compare model forecasts, given the number of parameters in some of the models, we simply could not afford to use less data for the estimation. Nonetheless, a one-year ahead forecast profile is still a fairly useful horizon.
14. GS obtain a similar sacrifice ratio when they include both the contemporaneous and the first lag of the gap in their Phillips equation.

It is clear from this table that there is some misspecification in this model. For instance, with regards to the output equation, we are unable to reject the null of no first order autocorrelation at the 10-per-cent level. Similarly, there seem to be important ARCH effects of up to the fourth order in the residuals. As for the inflation equation, there is strong evidence for the presence of up to fourth order autocorrelation although no ARCH effects are evident. Indeed, the Jarque-Bera test rejects the null of unconditional normality for the residuals of this equation reinforcing the LM test results.

This evidence of misspecification suggests that there is room to improve the model. Several possibilities come to mind. For instance, with regard to the output equation, we could examine the appropriateness of the random walk assumptions on the level and the drift term of potential.[15] Various studies have shown that permanent supply shocks feed gradually into output and, while it is possible to account for this with a unit root on the drift term and a coefficient of one on a single lag of potential, other sets of hypotheses may produce similar effects. For instance, we could assume a one-time exogenous change in the drift and more than one lag on potential. Having said this, the fact that we find ARCH effects in the output equation may instead be an indication of time-varying coefficients in this equation. It is therefore important to look at the assumptions made on output more carefully.

With respect to the inflation equation, one aspect to be examined is the specification of expectations. Backward-looking expectation modelled with a single lag of inflation may be inappropriate. In the literature, many models allow for richer dynamics by including more lags of inflation.[16] Other studies adopt forward-looking elements in expectations using, for example, survey data.[17] It is therefore important to verify whether different assumed expectations influence the gap estimate. A related issue is checking for the validity of the assumption of a single regime over the sample considered. Indeed, some studies have made the case for the presence of three distinct regimes for Canadian inflation data.[18]

Finally, it is assumed that there is no feedback from inflation to output. This is a difficult issue to settle as theoretical opinions diverge on it. However, it remains an important question and is left for future research.

---

15. In fact, if the true process is I(1), but instead an I(2) is assumed, the model may not be well-identified. This adversely affects any inference results from the model.
16. See, for instance, Duguay (1994).
17. One such study is by Laxton, Rose and Tetlow (1993).
18. An example is the study by Ricketts and Rose (1995) for Canadian inflation data. See also Fillion and Léonard (1997) for an estimated Phillips curve with various regimes.

4. Extensions and model applicability to Canada

4.1 Fine tuning

In Table 1 parameters significant at the 10-per-cent level are in bold. Looking at Column 4 we can see that many of the coefficients are not significant. On the other hand, moving average coefficients are strongly significant. It then becomes relevant to ask whether some variable is missing from the model.

Many studies have shown a role for real exchange rates and indirect taxes in the Phillips curve. Changes in these variables are indicative of relative price movements and have been found to influence the dynamics of inflation, specially in the short run. We therefore estimate the above model substituting real exchange rate changes in place of the nominal ones and adding changes in indirect taxes in the inflation equation. Outcomes are reported in the second column of Table 1 results and the corresponding graph is in Figure 2.

Once again, the contemporaneous oil price change variable is significant. In addition, we find that real exchange rate shocks and indirect taxes influence the short run dynamics of inflation. Thus, a more than usual increase in real exchange rates positively contributes to an acceleration of inflation in the next quarter, followed by a reverse effect, but to a lesser degree, in the quarter after. The dynamics of changes in indirect taxes is similar, except that the impact on the dependent variable is felt contemporaneously and in the next quarter. Notice that the remaining parameters in the model are estimated at values that are very similar to the previous case. As expected, the root mean square error around the gap term in the present case is lower at 1.005 than the value of 1.055 obtained with the basic model, and the maximized likelihood value is higher.

At this stage it is important to ask whether we are using the best measure for inflation. While the price level used in our models so far has been total CPI, the operational target of monetary policy in the recent past in Canada has been core inflation.[19] Remembering that our estimates for the gap are pinned down by the data on output and on inflation simultaneously, and that fluctuations in the real economy are more likely to influence trend inflation, it becomes interesting to examine the case where the gap measure in the Phillips curve is consistent with this particular definition of inflation. We therefore re-estimate restricted versions of the two models presented above using core inflation instead of total inflation.

19. Since there is a good deal of movement in the CPI caused by transitory fluctuations in the prices of food and energy as well as by changes in indirect taxes, the Bank of Canada focuses on a CPI measure excluding prices of food and energy and indirect taxes. This measure is referred to as core inflation.

Outcomes are tabulated in columns 3 and 4 of Table 1 results and the corresponding graphs are in Figures 3 and 4 respectively. From the third column we can see that, as expected, the estimated variance for inflation is lower than for total inflation and relative price shock variables, except for changes in indirect taxes, are no longer significant. In addition, the gap series is more volatile and the sacrifice ratio lower (4.17) than when total CPI inflation is used. Finally, with core inflation, only the contemporaneous gap is significant. The results in the next column are qualitatively similar except for the fact that the first lag of the change in the real exchange rate is significant.

Based on the maximized value of the likelihood function, it would seem that the model with core inflation and real exchange rate changes performs the best.[20] Yet the model in the second column is the one that exhibits the smallest filter uncertainty for the gap. In this case, the root mean square error quickly converges to a value of 1.005 compared with 1.063 and 1.069 obtained when using core inflation. At this stage we note that, as mentioned in the footnotes, this uncertainty is only part of the total uncertainty associated with these models. Strictly speaking, one should also account for the fact that model parameters are estimated at the same time as filtering is applied.

Next, we run diagnostic tests on the estimated models and tabulate the corresponding results in Table 2. From this table we can see that there is no improvement in the dynamics of the observed equation residuals. Indeed, with core inflation, misspecification is even exaccerbated in the case of the output equation residuals.

In light of the above evidence, more scrutiny is needed on the Phillips curve specification. For the moment, however, we focus our attention on the output equation specification. This is the topic of the next section.

4.2 The assumption of random walk for output

A plot of the log of actual output in Canada seems to indicate a change in slope some time in the mid to late seventies. Such a change may have occurred due to major negative exogenous shocks to the economy, such as oil price shocks, or to changes in government policy, or because of other structural changes.[21] Whatever caused the slowdown in productivity, in order to account for it, GS assume a unit root in the drift term of the output equation. This supposes that the observed

---

20. Since these models are non-nested, we do not use likelihood ratio tests to assess the overall goodness of fit. Instead, we rely on indirect criteria to select the best specification.
21. For more detail, see Stuber (1986).

slowdown took place very gradually. Yet, an explanation based on a one or two-time break in the drift term is equally likely.

Opinions diverge as to the right explanation, but, at this stage, both appear equally valid. Accordingly, in this section, we look at the implications of assuming that there has been a break in the drift term of trend output. The obvious difficulty with this approach, however, is determining the number of existing break points in a series and pin-pointing them precisely. This debate has been ongoing for a long time now and has spawned numerous studies on methods of testing for break points when the break time is difficult to pin down.

The latest in this category is the extensive work by Bai and Perron (1998). These authors discuss the properties of least squares estimators in a wide class of linear regression models in the presence of multiple structural breaks with unknown break points. These models include cases where the residuals are autocorrelated and heteroskedastic and where there are lagged dependent variables in the regressors. In addition, they develop a number of tests amongst which is a sequential testing strategy where one tests the null hypothesis of l changes against the alternative of $l+1$ changes in the series considered. Thus, their method successively estimates each break point in the data.

We apply the Bai and Perron sequential test to the output growth equation $\Delta y_t = \mu + \vartheta_t^y$, allowing for autocorrelation and heteroskedasticity in the ϑ_t^y term. The test detects a single break point corresponding to 1976Q2.

Having established that we cannot reject a one-time break in the drift term of trend output, we turn to the fact that technology shocks feed only gradually into the economy. Factors such as habit formation, adjustment costs, time to build and learning are some of the ideas behind this hypothesis. We therefore increase the number of autoregressive lags of potential output to two and impose that the sum of the coefficients on the lagged terms equal one. Based on the above discussion, we specify the following model for Canada:

$$
\begin{aligned}
y_t &= y_t^P + g_t \\
g_t &= \varphi_1 g_{t-1} + \varphi_2 g_{t-2} + \varepsilon_t^g \\
y_t^P &= \mu_1 + (\mu_2 - \mu_1) DU_t + \alpha y_{t-1}^P + (1-\alpha) y_{t-2}^P + \varepsilon_t^y \\
\Delta\pi_t &= c + \beta_0 g_t + \beta_1 g_{t-1} + \rho_0 do_t + \rho_1 do_{t-1} + \gamma_0 de_t + \gamma_1 de_{t-1} + \theta_0 dx_t + \theta_1 dx_{t-1} \\
&\quad + \varepsilon_t^\pi + \delta_1 \varepsilon_{t-1}^\pi
\end{aligned}
\tag{9}
$$

where do_t is the log change in the real oil price, de_t is the first difference in the log real exchange rate and dx_t is the change in indirect taxes. In addition, DU_t is a dummy variable which captures the break in the drift of potential output.

In order to carry out the estimation, the break in drift is imposed at 1976q2 and the system is re-written as:

$$\begin{aligned} g_t &= \varphi_1 g_{t-1} + \varphi_2 g_{t-2} + \varepsilon_t^g \\ \Delta y_t &= \mu_1 + (\mu_2 - \mu_1) DU_t + (\alpha - 1)\Delta y_{t-1} + g_t - \alpha g_{t-1} - (1-\alpha) g_{t-2} + \varepsilon_t^y \\ \Delta \pi_t &= c + \beta_0 g_t + \beta_1 g_{t-1} + \rho_0 do_t + \rho_1 do_{t-1} + \gamma_0 de_t + \gamma_1 de_{t-1} + \theta_0 dx_t + \theta_1 dx_{t-1} \\ &\quad + \varepsilon_t^\pi + \delta_1 \varepsilon_{t-1}^\pi \end{aligned} \tag{10}$$

Estimation results of the above model with total and core inflations are found in Table 3 and the corresponding graphs are in Figures 5 and 6. These results show that almost all of the model coefficients are highly significant. More specifically, the drift term after the break point is estimated at half its value before 1976Q2 which indicates a substantial slowdown in productivity. For the model with total inflation, we notice also that the coefficient on the first lag of potential output is highly significant and equal to 1.03 indicating that the persistence in trend output is adequately captured through its first lag. This indicates that one quarter is sufficient time for technology shocks to fully feed into real output.

In comparing the models with the different inflation measures, once again we see that the likelihood value is higher when core inflation is used in the Phillips curve. We also find that the root mean square error is lower in this case. It is also interesting to note that the sacrifice ratio is 6.25 in the model that uses total inflation and 5.00 in the other case. These are close to the numbers obtained from the corresponding models with a unit root drift. Added to the fact that the estimated variances in the two categories of models are also similar, we conclude that the two models are essentially observationally equivalent. Nevertheless, the diagnostic tests and the out-of-sample forecasts will show whether one set of assumptions is preferable over the other.

Once again, therefore, diagnostic tests are carried out and results are reported in Table 4. These basically show that, compared to the unit root drift category of models, there is a modest improvement in the diagnostics when a one-time break in drift is assumed. Thus, in the case of total inflation, we can no longer reject the null of first-order autocorrelation and ARCH in the output equation. Similarly, while ARCH effects are still present in the models using core inflation, the p-values for these are much higher when a break in drift is imposed. Nevertheless, in the latter case, we cannot reject the presence of low-order ARCH in the residuals of the model using total inflation.

Amongst the possible reasons for the observed ARCH effects above is time-variation in the model parameters. Given that the purpose of the output gap measure is consistency with short term inflation forecasting, and taking into account the relative complexity of our framework, we will not be addressing this issue in the present study. Of more importance is the observed autocorrelation in the inflation equation residuals in all of the models. These effects could be due to misspecified expectations. For instance, Ricketts and Rose (1995), using a Markov-switching model, found three distinct regimes for Canadian inflation between 1961 and 1993. In their model both means and variances change with the state of the economy. If agents know which inflation regime they are in when they make their forecasts, inflation expectations will be different in each state. This, in turn, will influence the level of inflation. Perhaps, then, accounting for regime shifts will diminish the observed autocorrelation in the residuals.

4.3 Accounting for regime shifts in inflation expectations

In this section, we examine the role of regime shifts in inflation expectations for the measurement of the output gap. This is accomplished in two steps.

First, in a separate model, we estimate a three-state Markov-switching model with state-varying means and variances on our core inflation series. In all these regimes, inflation is assumed to follow an autoregressive process of three lags, and in one case, it is imposed to have a unit root. Having obtained the means, variances, and ex-post probabilities of being in each regime, we then construct dummy variables corresponding to each of the three regimes.[22] Thus, regime one extends from 1960Q1 to 1963Q2 and from 1989Q2 to 1998Q1, regime two occurs in the 1963Q3 to 1970Q2 period as well as during 1980Q3 to 1989Q1, finally, the third regime falls between 1970Q3 and 1980Q2.

Notice that while the early sixties and the nineties are both found to be consistent with a low mean low variance regime, their inflation dynamics may have been somewhat different. In order to account for this fact, we assign different dummy variables to these two time periods. This amounts to specifying a total of four distinct regimes for inflation, and consequently, of inflation expectations.

22. The construction of the dummy variables is similar to the method followed by Fillion and Léonard (1997).

Second, we re-estimate the state-space model with a unit root drift and core inflation having imposed these four distinct regimes for inflation.[23] In all regimes, and to preserve parsimony, core inflation is assumed to follow a stationary autoregressive process of the second order.[24]

Estimation results for the models with nominal and real exchange rates are tabulated in Table 5. Results are very similar for both models, although the one with real exchange rates has the higher likelihood function while the other model has the lower root mean square error. Interestingly, most of the estimated coefficients are also quite similar to the case where no regime switching is allowed. Only the MA coefficient values seem to be substantially different. In addition, when allowing for regime switches, the sacrifice ratio increases. As for the constants and the autoregressive terms for inflation, only a few coefficients are found to be significant. Despite this fact, an examination of the diagnostic test results in Table 7 indicates that allowing for different inflation regimes may be useful in improving the model fit. Indeed, compared to the last two columns of Table 2, we can see that residual autocorrelation is reduced with regime switching. Overall, then, it seems that inflation regimes do affect inflation expectations. However, further scrutiny is needed to adequately specificy the dynamics of inflation in each of these regimes.[25]

4.4 Accounting for ARCH effects

Diagnostic testing of residuals of all of the models examined so far has revealed the presence of ARCH effects in the output equation of these models. While we do not attempt to determine the source of the conditional heteroskedasticity, in this section, we evaluate the role of accounting for these effects by allowing the output equation to have ARCH errors. Thus, we re-estimate the above model with a unit root drift, core inflation, regime changes and real exchange rate changes, imposing an ARCH error of lag two for output.

Estimation results are found in Table 6 and the diagnostic test outcomes in the last column of Table 7. Surprisingly, the autoregressive coefficients on the conditional variance of output are not significant and all other estimated values, except for the variance of the gap term, remain similar to those obtained in the previous model estimation. As for the diagnostics test results, we

---

23. We also tried to estimate the model with the breaking drift imposing different inflation regimes but found that the model did not converge.
24. The state-space model did not converge for the case where a unit root was imposed in one of the regimes indicating that a stationary process is probably more appropriate.
25. We should note, however, that as the number of model parameters increases, model uncertainty also increases.

see a marked decrease in the residual autocorrelation of the inflation equation, but the ARCH effects remain unchanged.

4.5 Out-of-sample forecasts

Given the misspecification issues that still need to be resolved, what is the usefulness of these models for the conduct of monetary policy? Given the fact that trend output and the gap are unobserved, a natural way of assessing this usefulness is to verify the extent to which the estimated gap measure can forecast inflation and output out-of-sample.

From the obtained parameter estimates and the values of the latent variables, we forecast the values of log output and inflation over the 1997Q2–1998Q1 period. These forecasts are then compared with the realized observations for that period which we had put aside at the beginning of the study. The comparisons are made in terms of root mean square errors and are reported in Table 8 in units of annual percentages.

On the whole, results are quite good. Average forecast errors on the log of output vary between 1.9 to 3.3 per cent per annum, while those on inflation lie only between 0.55 to 1.1 per cent per annum. More precisely, the average prediction error on the log of real output is smallest in the model that uses core inflation and which imposes regime changes and ARCH effects. In annual percentages this represents a less than 2-per-cent average error.

The model that has the smallest average annual out-of-sample prediction error of inflation is the one with core inflation and a break in drift in 1976Q2. In this case, the error is only 0.55 per cent per annum. In addition, this model performs quite well in predicting log output as well with an average error of only 2 per cent per annum.

4.6 Discussion

At this stage we would like to single out the most promising models among the ones considered based on all of the criteria available to us; that is, the value of the maximized likelihood ratio, diagnostic test results, estimated uncertainty around the gap measure, out-of-sample forecast performance, as well as plausibility of coefficient values. The difficulty is that, in general, these criteria do not all point in the same direction. Furthermore, models are non-nested rendering statistically significant comparisons amongst them even more problematic. Thus, identifying which models are most promising means selcting those that perform best in some overall sense and requires trading off some criteria against others. We proceed by first comparing different categories of models and then by choosing the best model within the retained category.

4.6.1 Core inflation models versus total inflation models

The examination of maximized log likelihood values provides for an overall goodness of fit of the model. From these we notice that, in general, models that use core inflation have higher log likelihood functions than those using total inflation. This means that these models exhibit a better fit than total inflation models. Interestingly, models using core inflation also yield more plausible sacrifice ratios with values that lie between 2.78 and 5. In contrast, models making use of total inflation produce sacrifice ratios between 6.25 and 8.33 which are notably higher than most previous estimates for Canada.[26]

As for forecasting performance, models using core inflation always have smaller average annual errors for inflation than models with total inflation. In addition, they also perform better than the latter with respect to the average prediction error on output, with the exception of the total inflation model with a breaking drift. It remains that, from the monetary authority's perspective, better predicting inflation should weigh more heavily than closely predicting output.

On the negative side, models using core inflation generally have higher root mean square errors around the gap estimate than the total inflation models. However, given that this is only the filter uncertainty around this estimate, models that have a higher parameter count are also expected to have a higher parameter uncertainty associated with them. Notice that most models in Table 1 require the estimation of two additional parameters than the models in Table 3. In this respect, the total uncertainty around the gap measure is probably more or less the same for all of the models in these two tables. However, in the case of the core inflation models that include regime changes and ARCH effects, not only is the filter uncertainty higher than for all the other models, but given the additional 12 estimated parameters, total uncertainty is expected to be higher as well.

As for the plausibility of model parameters, except for the core inflation models that incorporate regime changes, most of the estimated parameter values in the remaining models seem fairly plausible. For instance, the autoregressive parameters of the gap equation in all the models sum to less than one and exhibit hump-shaped dynamics. In addition, all the estimated variances are positive and significant, with the variance of the drift term being the smallest as expected. Finally, the relative price shock variable coefficients generally have the correct sign in either category of models. In the case of the core inflation models with regime changes, the autoregressive coefficients of inflation in the first regime are negative and insignificant. This outcome may be due to the lack of sufficient data for this particular regime.

26. See Black, Coletti, and Monnier (1997) for a survey.

As for diagnostic testing, core inflation models are found to exhibit more severe misspecification in the output equation than the total inflation models. However, with respect to the inflation equation, the first category of models are as well-specified as total inflation models in most cases, and perform even better when regime changes and ARCH effects are imposed.

Based on the above analysis, we conclude that the core inflation category of models offers more useful alternatives than those models using total inflation. Consequently, next, we will try to select the best model within this division.

4.6.2 Which core inflation model?

Based on the highest likelihood value, one would be tempted to select one of the core model with changing regimes as being the best. In addition, these models exhibit less specification problems than the others in this same category, specially in the inflation equation. However, their disadvantage is that they have somewhat higher uncertainty around the gap estimate than the others. Similarly, most of the regime-related parameters are insignificant, while some even have the wrong sign. This leads us to believe that these models are not as precisely estimated and robust as we would like. With the availability of more data in future, they could prove to be more useful.

For now, however, we have the most confidence in the core inflation model with the breaking drift. This model has a good overall fit, plausible parameter estimates, and the best forecast performance.

5. Conclusion

In this paper, we assess the implications of using state-space models for measuring potential output (and the output gap) in Canada. The models examined are different versions or extensions of the unobserved-components model of Gerlach and Smets (1997). These modifications were motivated partly in response to existing empirical findings on Canadian data, and partly because of the results of diagnostics checking on the basic model.

First we focus on the assumed specification for the output equation of the model and apply the Bai and Perron (1998) structural change test with unknown change point. Since we find that we cannot reject a one-time break in the drift of output growth, we estimate versions of the model with a breaking drift. We also estimate a version of the GS model with a modified error structure after detecting the presence of ARCH effects in this equation.

Similarly, we try to modify the specification of the inflation equation. We do so by examining the implications of using either total or core inflation in the Phillips curve, by integrating more relative price shock variables in this equation, and by examining the role of changing inflation regimes.

Through these various attempts, we find that we are able to improve the model fit somewhat. In particular, for some versions of the model, the uncertainty around the estimate of the gap is slightly reduced and the out-of-sample forecasts of inflation and output are improved. Yet we feel that further improvements could be obtained by allowing time-variation in certain parameters of the model and by a more systematic examination of inflation regime changes.

Overall, we conclude that state-space models are useful for estimating the output gap for the purpose of assessing inflationary pressures. In particular, based on various criteria, we favor the use of a model which uses core inflation in the Phillips curve and imposes a breaking drift in the output equation.

References

Bai, J. and P. Perron. 1998. "Estimating and Testing Linear Models with Multiple Srtuctural Changes." *Econometrica* 66:47-78.

Black, R., D. Coletti, and S. Monnier. 1997. "The Cost and Benefits of Price Stability." Bank of Canada Conference Proceedings Volume.

Blanchard, O.J. and D.Quah. 1989. "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review* 79(4):655-73.

Butler, L. 1996. The Bank of Canada's New Quarterly Projection Model, Part 4. A Semi-Structural Method to Estimate Potential Output: Combining Economic Theory with a Time-Series Filter. Technical Report. Ottawa:Bank of Canada.

Cooley, T.F. and M. Dwyer. 1998. "Business Cycle Analysis Without Much Theory. A Look at Structural VARs." *Journal of Econometrics* 83:57-88.

Duguay, P. 1994. "Empirical Evidence on the Strength of the Monetary Transmission Mechanism in Canada." *Journal of Monetary Economics* 33(1).

Dupasquier C. and N. Ricketts. 1998. "Non-linearities in the Output-Inflation Relationship." Conference Proceedings Volume, Bank of Canada.

Dupasquier C. and N. Ricketts. 1998. "Non-linearities in the Output-Inflation Relationship: Some Empirical Results for Canada" Working Paper 98-14. Bank of Canada, Ottawa.

Dupasquier C., A. Guay, and P. St-Amant. 1999. "A Survey of Alternative Methodologies for Estimating Potential Output and the Output Gap." forthcoming *Journal of Macroeconomics*.

Engle, R. 1983. "Estimates of the Variance of U.S. Inflation Based upon the ARCH Model." *Journal of Money, Credit and Banking* 15(3):286-301.

Fillion, J.F. and A. Léonard. 1997. "La courbe de Phillips au Canada: un examen de quelques hypothèses." Working Paper 97-3. Bank of Canada, Ottawa.

Gerlach, S. and F. Smets. 1997. "Output Gaps and Inflation. Unobservable-components Estimates for the G-7 Countries." Unpublished Paper, Bank for International Settlements, Basle.

Guay, A. and P. St-Amant. 1996. Do Mechanical Filters Provide a Good Approximation of Business Cycles? Technical Report No. 78. Ottawa:Bank of Canada.

Harvey, A.C. and A. Jeager. 1993. "Detrending, Stylized Facts and the Business Cycle." *Journal of Applied Econometrics* 8(3):231-47.

Harvey, A.C., Ruiz, E., and E. Sentana. 1992. "Unobserved Component time Series Models with ARCH Disturbances." *Journal of Econometrics,* 52:129-57.

Hamilton, J. 1986. "A Standard Error for the Estimated State Vector of a State-Space Model". *ournal of Econometrics* 33:387-97.

Hodrick, R.J. and E.C. Prescott. 1997. "Postwar U.S. Business Cycles: An Empirical Investigation." *Journal of Money, Credit, and Banking* 29(1):1-16.

King, R.G. and S. Rebelo. 1993. "Low Frequency Filtering and Real Business Cycles." *Journal of Economic Dynamics and Control* 17(1-2):207-31.

Kuttner, K.N. 1994. "Estimating Potential Output as a Latent Variable." *Journal of Business and Economic statistics* 12(3):361-68.

Laxton, D. and R. Tetlow. 1992. A simple Multivariate Filter for the Measurement of Potential Output. Technical Report No. 59. Ottawa: Bank of Canada.

Laxton, D., D. Rose, and R. Tetlow. 1993. "Problems in Identifying Non-Linear Phillips Curves: Some Further Consequences of Mismeasuring Potential Output." Working Paper 93-6. Bank of Canada, Ottawa.

Perron, P. 1997. "L'estimation de modeles avec changements structurels multiples." *L'Actualite Economique* 73(1-3):472-505.

Perron, P. 1989. "The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57(6):1361-401.

Phillips, P.C.B. 1989. "Partially Identified Econometric Models." *Econometric Theory*, 5:181-240.

Ricketts, N. and D. Rose. 1995. "Inflation, Learning and Monetary Policy Regimes in the G-7 Economies." Working Paper 95-6. Bank of Canada, Ottawa.

St-Amant, P. and S. van Norden. 1997. Measurement of the Output Gap: A Discussion of Recent Research at the Bank of Canada. Technical Report No. 79. Ottawa:Bank of Canada.

Stuber, G. 1986. The Slowdown in Productivity Growth in the 1975-83 Period: A s

Survey of Possible Explanations. Technical Report No. 43. Ottawa:Bank of Canada.

van Norden, S. 1995. "Why Is It So Hard to Measure the Current Output Gap?" Mimeograph. Bank of Canada.

Watson, M.W. 1986. "Univariate Detrending Methods with Stochastic Trends." *Journal of Monetary Economics* 18(1):49-75.

Data Appendix

The data used in this study are derived from the sources that follow. The notation (s.a.) stands for seasonally-adjusted. The notation (1992=100) implies that the base year for the series in question is 1992.

Series	Source Matrix	Base Year	s.a.	Frequency	Converted	Data Span
Total CPI	b820600	1992=100	yes	monthly	quarterly	1961:1-98:1
Core CPI	b820655	1992=100	yes	monthly	quarterly	1961:1-98:1
Real GDP	d14872	1992=100	yes	quarterly		1961:1-98:1
Canada/U.S. bilateral nominal exchange rate	b100000			monthly	quarterly	1961:1-98:1
U.S. Total CPI	m.cusa0	1982-84 =100	yes	monthly	quarterly	1961:1-98:1
Oil price (WTI)	coil			monthly	quarterly	1961:1-98:1
Indirext taxes	txpcpife[a]			quarterly		1962:1-98:1

a. Series calculated at the Bank of Canada

Inflation rates are obtained by taking annual percentages on the appropriate CPI measures. The oil price series “coil” is deseasonalised by using the “seasonal” command in the FAME software. Real oil price is obtained by deflating the seasonally-adjusted series with U.S. CPI. The real exchange rate series is obtained by summing the logs of the nominal exchange rate and U.S. CPI and by subtracting the log of total CPI.

Table 1: Maximum Likelihood Estimation Results for GS model

Dep. variable	Regressor	Parameter	GS -tot infl. 1961:4 - 1997:1 nom. exch. rate	GS -tot infl. 1962:4 - 1997:1 real exch. rate	GS -core infl. 1962:4 - 1997:1 nom. exch. rate	GS -core infl. 1962:4 - 1997:1 real exch. rate
g_t	g_{t-1}	φ_1	**1.64** (0.19)	**1.73** (0.13)	**1.37** (0.22)	**1.34** (0.25)
g_t	g_{t-2}	φ_2	**-0.71** (0.18)	**-0.78** (0.13)	**-0.50** (0.21)	**-0.48** (0.23)
$\Delta\pi_t$	g_t	β_0	**0.23** (0.07)	**0.23** (0.09)	**0.09** (0.07)	**0.06** (0.03)
$\Delta\pi_t$	g_{t-1}	β_1	**-0.20** (0.09)	**-0.19** (0.08)	-0.03 (0.07)	
$\Delta\pi_t$	ε^{π}_{t-1}	δ_1	**-0.70** (0.34)	**-0.71** (0.33)	**-0.76** (0.40)	**-0.78** (0.45)
$\Delta\pi_t$	ε^{π}_{t-2}	δ_2	-0.13 (0.15)	-0.10 (0.16)	-0.12 (0.15)	-0.10 (0.14)
$\Delta\pi_t$	ε^{π}_{t-3}	δ_3	-0.03 (0.09)	-0.01 (0.13)	0.05 (0.14)	0.03 (0.15)
$\Delta\pi_t$	de_t	γ_0	1.80 (4.33)	0.13 (3.51)	-2.01 (3.43)	-4.19 (3.31)
$\Delta\pi_t$	de_{t-1}	γ_1	3.60 (6.46)	**8.62** (4.75)	4.86 (4.51)	**6.28** (3.96)
$\Delta\pi_t$	de_{t-2}	γ_2	-5.22 (4.94)	**-5.04** (3.69)	0.38 (3.63)	1.42 (3.11)
$\Delta\pi_t$	do_t	ρ_0	**0.90** (0.38)	**1.13** (0.41)	0.29 (0.35)	0.19 (0.31)
$\Delta\pi_t$	do_{t-1}	ρ_1	0.49 (0.52)	0.27 (0.43)	0.37 (0.43)	0.40 (0.41)
$\Delta\pi_t$	do_{t-2}	ρ_2	-0.28 (0.48)	-0.31 (0.37)	0.14 (0.49)	0.17 (0.44)
$\Delta\pi_t$	dx_t	θ_0		**0.45** (0.09)	**0.44** (0.07)	**0.38** (0.08)
$\Delta\pi_t$	dx_{t-1}	θ_1		**-0.28** (0.09)	**-0.37** (0.09)	**-0.29** (0.09)
$\Delta\pi_t$	constant	c	-0.005 (0.02)	-0.018 (0.02)	-0.010 (0.03)	-0.009 (0.03)
Δy_t	$var\left(\varepsilon^{y}_{t}\right)$		**0.77** (0.08)	**0.80** (0.07)	**0.64** (0.15)	**0.64** (0.17)
g_t	$var\left(\varepsilon^{g}_{t}\right)$		**0.32** (0.13)	**0.26** (0.11)	**0.53** (0.18)	**0.53** (0.20)
$\Delta\pi_t$	$var\left(\varepsilon^{\pi}_{t}\right)$		**0.43** (0.08)	**0.39** (0.07)	**0.35** (0.07)	**0.33** (0.08)
μ_t	$var\left(\varepsilon^{\mu}_{t}\right)$		**0.05** (0.03)	**0.04** (0.03)	**0.06** (0.03)	**0.07** (0.03)
RootMean	sq.err(gap)		1.0547	1.0046	1.0626	1.0687
likelihood	func. value		-2.114	-2.034	-1.831	-1.811

Notes on previous table: Coefficients in bold notation are significant at 10 per cent. Standard errors are in parenthesis. de_t is the change in the exchange rate, do_t is the change in the real oil price, and dx_t is the change in indirect taxes.

Table 2: Diagnostic Tests on GS model

	Test	GS -tot infl. 1961:4 - 1997:1 nom. exch. rate	GS -tot infl. 1962:4 - 1997:1 real exch. rate	GS -core infl. 1962:4 - 1997:1 nom. exch. rate	GS -core infl. 1962:4 - 1997:1 real exch. rate
Output Eqn.	LM (1)	0.071	0.060	0.002	0.002
	LM (2)	0.181	0.136	0.006	0.006
	LM (3)	0.251	0.198	0.006	0.007
	LM (4)	0.398	0.325	0.015	0.016
	ARCH(1)	0.037	0.018	0.001	0.002
	ARCH(2)	0.007	0.007	0.000	0.000
	ARCH(3)	0.001	0.000	0.000	0.001
	ARCH(4)	0.004	0.001	0.001	0.001
	J-B	0.706	0.802	0.223	0.227
Inflation Eqn.	LM (1)	0.001	0.001	0.001	0.001
	LM (2)	0.005	0.002	0.000	0.000
	LM (3)	0.008	0.005	0.001	0.001
	LM (4)	0.015	0.006	0.002	0.002
	ARCH(1)	0.149	0.178	0.100	0.143
	ARCH(2)	0.353	0.396	0.265	0.347
	ARCH(3)	0.543	0.374	0.431	0.500
	ARCH(4)	0.711	0.504	0.606	0.664
	J-B	0.000	0.000	0.000	0.000

Notes: The reported results are P-values

Table 3: ML Estimation: Models with Break in Drift in 1976Q2

Dependent variable	Regressor	Parameter	Total Inflation	Core Inflation
g_t	g_{t-1}	φ_1	**1.69** (0.15)	**1.38** (0.19)
g_t	g_{t-2}	φ_2	**-0.75** (0.14)	**-0.49** (0.19)
$\Delta\pi_t$	g_t	β_0	**0.18** (0.08)	**0.09** (0.06)
$\Delta\pi_t$	g_{t-1}	β_1	**-0.14** (0.07)	-0.04 (0.06)
$\Delta\pi_t$	ε^{π}_{t-1}	δ_1	**-0.72** (0.22)	**-0.76** (0.21)
$\Delta\pi_t$	de_t	γ_0	0.58 (3.37)	**-5.69** (2.99)
$\Delta\pi_t$	de_{t-1}	γ_1	**4.80** (3.67)	**9.23** (3.14)
$\Delta\pi_t$	do_t	ρ_0	**1.11** (0.35)	0.19 (0.31)
$\Delta\pi_t$	do_{t-1}	ρ_1	0.10 (0.27)	**0.57** (0.31)
$\Delta\pi_t$	dx_t	θ_0	**0.44** (0.09)	**0.39** (0.08)
$\Delta\pi_t$	dx_{t-1}	θ_1	**-0.27** (0.09)	**-0.29** (0.08)
$\Delta\pi_t$	constant	c	-0.017 (0.03)	-0.010 (0.03)
Δy_t	drift 1	μ_1	**1.21** (0.19)	**1.25** (0.21)
Δy_t	drift 2	μ_2	**0.59** (0.14)	**0.62** (0.14)
Δy_t	y^P_{t-1}	α	**1.03** (0.14)	**0.99** (0.17)
Δy_t	$var(\varepsilon^y_t)$		**0.76** (0.10)	**0.61** (0.16)
g_t	$var(\varepsilon^g_t)$		**0.41** (0.05)	**0.58** (0.17)
$\Delta\pi_t$	$var(\varepsilon^{\pi}_t)$		**0.33** (0.15)	**0.35** (0.03)
Root Mean	Sq. Error (gap)		1.1271	1.1159
likelihood	function value		-1.9088	-1.7541

Notes: Coefficients in bold are significant at 10 per cent. Standard errors are in parenthesis. de_t is the change in the exchange rate, do_t is the change in the real oil price, and dx_t is the change in indirect taxes.

Table 4: Diagnostic Tests: Model with Break in Drift in 1976Q2

	Test	Total Inflation	Core Inflation
Output Eqn.	LM (1)	0.402	0.004
	LM (2)	0.587	0.011
	LM (3)	0.584	0.010
	LM (4)	0.795	0.021
	ARCH(1)	0.113	0.028
	ARCH(2)	0.040	0.022
	ARCH(3)	0.015	0.033
	ARCH(4)	0.037	0.057
	J-B	0.435	0.724
Inflation Eqn.	LM (1)	0.001	0.000
	LM (2)	0.001	0.000
	LM (3)	0.004	0.001
	LM (4)	0.003	0.001
	ARCH(1)	0.055	0.164
	ARCH(2)	0.153	0.390
	ARCH(3)	0.190	0.571
	ARCH(4)	0.287	0.699
	J-B	0.000	0.000

Note: The reported results are P-values.

Table 5: Core Inflation Model with Regime Shifts; Drift in Potential is Unit Root

Dependent variable	Regressor	GS -core infl. 1962:4 - 1997:1 nom. exch. rate	GS -core infl. 1962:4 - 1997:1 real exch. rate	Regressor (cont'd)	GS -core infl. 1962:4 - 1997:1 nom. exch. rate	GS -core infl. 1962:4 - 1997:1 real exch. rate
g_t	g_{t-1}	**1.38** (0.24)	**1.38** (0.23)			
g_t	g_{t-2}	**-0.48** (0.23)	**-0.48** (0.23)			
π_t	g_t	**0.05** (0.04)	**0.05** (0.04)	const.1	**0.72** (0.56)	0.71 (0.57)
π_t	g_{t-1}			$\pi_{1,\,t-1}$	-0.32 (0.80)	-0.31 (0.80)
π_t	ε^{π}_{t-1}	-0.33 (0.43)	-0.31 (0.46)	$\pi_{1,\,t-2}$	-0.03 (0.92)	-0.01 (0.90)
π_t	ε^{π}_{t-2}	-0.08 (0.25)	-0.09 (0.27)	const.2	**0.72** (0.34)	**0.72** (0.34)
π_t	ε^{π}_{t-3}	0.05 (0.14)	0.06 (0.14)	$\pi_{2,\,t-1}$	0.33 (0.45)	0.33 (0.50)
π_t	de_t	-4.18 (3.20)	-4.26 (3.02)	$\pi_{2,\,t-2}$	-0.03 (0.27)	-0.02 (0.30)
π_t	de_{t-1}	**6.27** (4.86)	**6.29** (4.49)	const.3	**0.59** (0.32)	0.58 (0.32)
π_t	de_{t-2}	**1.29** (5.63)	**1.31** (5.37)	$\pi_{3,\,t-1}$	**0.61** (0.46)	0.60 (0.49)
π_t	do_t	0.16 (0.32)	0.16 (0.32)	$\pi_{3,\,t-2}$	0.10 (0.35)	0.12 (0.38)
π_t	do_{t-1}	0.37 (0.42)	0.39 (0.42)	const.4	0.26 (0.59)	0.24 (0.53)
π_t	do_{t-2}	0.46 (0.44)	0.38 (0.43)	$\pi_{4,\,t-1}$	0.39 (1.30)	0.39 (1.24)
π_t	dx_t	**0.46** (0.07)	**0.45** (0.08)	$\pi_{4,\,t-2}$	0.01 (0.70)	0.03 (0.72)
π_t	dx_{t-1}	-0.04 (0.23)	-0.01 (0.24)			
Δy_t	$var(\varepsilon^{y}_{t})$	**0.60** (0.21)	**0.60** (0.20)			
g_t	$var(\varepsilon^{g}_{t})$	**0.58** (0.23)	**0.58** (0.23)			
π_t	$var(\varepsilon^{\pi}_{t})$	**0.33** (0.03)	**0.32** (0.03)			
μ_t	$var(\varepsilon^{\mu}_{t})$	**0.05** (0.03)	**0.05** (0.03)			
Root Mean	Sq. err (gap)	1.3467	1.3620			
likelihood	funct. value	-1.6939	-1.6692			

Notes: See notes in Table 1. $\pi_{j,\,t-i}$ is the inflation rate of lag i in regime j. const.j is the constant in regime j.

Table 6: Core Inflation Model with Regime Shifts & ARCH(2) in Output Equation; Drift in Potential is Unit root

Dep.variable	Regressor	real exch. rate	Regressor (cont'd)	real exch. rate
g_t	g_{t-1}	**1.49** (0.24)		
g_t	g_{t-2}	**-0.57** (0.23)		
π_t	g_t	**0.06** (0.04)	const.1	**0.79** (0.58)
π_t	ε^{π}_{t-1}	-0.25 (0.45)	$\pi_{1,\,t-1}$	-0.38 (0.81)
π_t	ε^{π}_{t-2}	-0.09 (0.26)	$\pi_{1,\,t-2}$	-0.09 (0.88)
π_t	ε^{π}_{t-3}	0.06 (0.14)	const.2	**0.75** (0.34)
π_t	de_t	-4.32 (3.10)	$\pi_{2,\,t-1}$	0.24 (0.49)
π_t	de_{t-1}	**6.05** (4.37)	$\pi_{2,\,t-2}$	0.03 (0.28)
π_t	de_{t-2}	**1.45** (5.21)	const.3	**0.66** (0.32)
π_t	do_t	0.20 (0.33)	$\pi_{3,\,t-1}$	0.57 (0.49)
π_t	do_{t-1}	0.33 (0.42)	$\pi_{3,\,t-2}$	0.12 (0.38)
π_t	do_{t-2}	0.50 (0.41)	const.4	0.26 (0.54)
π_t	dx_t	**0.45** (0.08)	$\pi_{4,\,t-1}$	0.34 (1.24)
π_t	dx_{t-1}	0.01 (0.23)	$\pi_{4,\,t-2}$	0.02 (0.81)
g_t	$var(\varepsilon^{g}_{t})$	**0.44** (0.21)		
π_t	$var(\varepsilon^{\pi}_{t})$	**0.32** (0.03)		
μ_t	$var(\varepsilon^{\mu}_{t})$	**0.04** (0.02)		
Δy_t	ARCH(0)	0.26 (0.25)		
Δy_t	ARCH(1)	0.18 (0.34)		
Δy_t	ARCH(2)	0.19 (0.25)		
Root Mean	Square err (gap)	1.1617		
likelihood	function value	-1.6528		

Notes: See notes in Table 1. $\pi_{j,\,t-i}$ is the inflation rate of lag i in regime j. const.j is the constant in regime j.

Table 7: Diagnostic Tests on GS Models with Regime Shifts and ARCH Effects

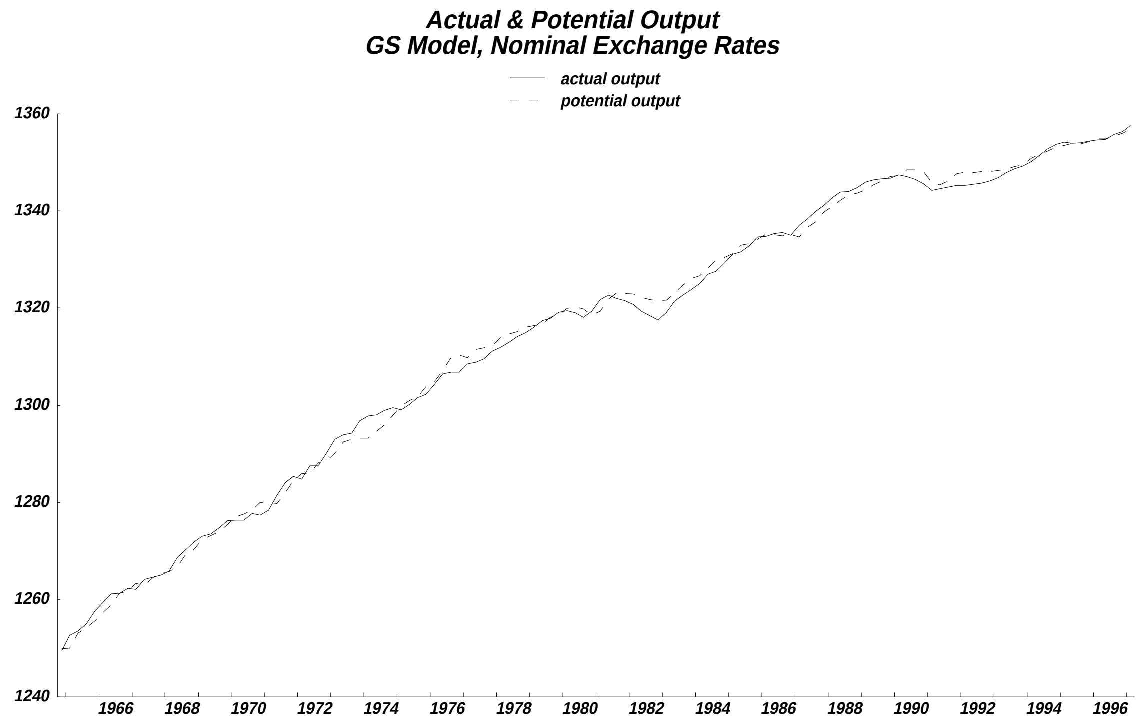
	Test	G&S -core infl. 1961:4 - 1997:1 nom. exch. rate	G&S -core infl. 1962:4 - 1997:1 real exch. rate	G&S -core infl. 1962:4 - 1997:1 real. exch. rate and ARCH
Output Eqn.	LM (1)	0.011	0.010	0.007
	LM (2)	0.043	0.041	0.028
	LM (3)	0.067	0.062	0.032
	LM (4)	0.128	0.120	0.063
	ARCH(1)	0.037	0.007	0.004
	ARCH(2)	0.007	0.000	0.000
	ARCH(3)	0.001	0.000	0.000
	ARCH(4)	0.004	0.001	0.000
	J-B	0.115	0.091	0.094
Inflation Eqn.	LM (1)	0.064	0.096	0.187
	LM (2)	0.046	0.064	0.097
	LM (3)	0.069	0.095	0.126
	LM (4)	0.129	0.171	0.209
	ARCH(1)	0.075	0.113	0.310
	ARCH(2)	0.195	0.176	0.567
	ARCH(3)	0.180	0.216	0.731
	ARCH(4)	0.661	0.684	0.868
	J-B	0.000	0.000	0.000

Notes: The reported results are P-values

Table 8: Actual and Forecasted Quarterly Values 1997Q2–1998Q1; Root Mean Square Errors in Annual Percentages

Models	Log Actual Output	Log Forecasted Output	Actual Inflation	Forecasted inflation
GS Model - Total Inflation, Nominal Exchange Rate	1358.56 1359.61 1360.31 1361.22	1358.14 1358.79 1359.47 1360.10	0.22 0.34 0.12 0.34	0.41 0.52 0.54 0.61
Root Mean Sq. Error (per cent p.a.)		3.35		1.13
GS Model - Total Inflation, Real Exchange Rate	1358.56 1359.61 1360.31 1361.22	1358.15 1358.80 1359.48 1360.12	0.22 0.34 0.12 0.34	0.32 0.49 0.51 0.61
Root Mean Sq. Error (per cent p.a.)		3.30		1.01
GS Model - Total Inflation; Break in Drift in 1976q2	1358.56 1359.61 1360.31 1361.22	1358.31 1359.08 1359.88 1360.56	0.22 0.34 0.12 0.34	0.32 0.43 0.50 0.53
Root Mean Sq. Error (per cent p.a.)		1.96		0.89
GS Model - Core Inflation, Nominal Exchange Rates, Indirect Taxes	1358.56 1359.61 1360.31 1361.22	1358.23 1358.86 1359.53 1360.13	0.47 0.16 0.16 0.47	0.32 0.30 0.41 0.51
Root Mean Sq. Error (per cent p.a.)		3.14		0.69
GS Model - Core Inflation; Break in Drift in 1976q2	1358.56 1359.61 1360.31 1361.22	1358.36 1359.12 1359.85 1360.50	0.47 0.16 0.16 0.47	0.23 0.20 0.26 0.39
Root Mean Sq. Error (per cent p.a.)		2.01		0.55
Core Inflation, Changing Regimes, Nominal Exchange Rate	1358.56 1359.61 1360.31 1361.22	1358.28 1358.98 1359.70 1360.35	0.47 0.16 0.16 0.47	0.35 0.48 0.29 0.40
Root Mean Sq. Error (per cent p.a.)		2.53		0.74
Core Inflation, Changing Regimes, Real Exchange Rates, ARCH	1358.56 1359.61 1360.31 1361.22	1358.36 1359.12 1359.90 1360.53	0.47 0.16 0.16 0.47	0.37 0.49 0.29 0.43
Root Mean Sq. Error (per cent p.a.)		1.92		0.74

FIGURE 1: GS Model - Nominal Exchange Rate



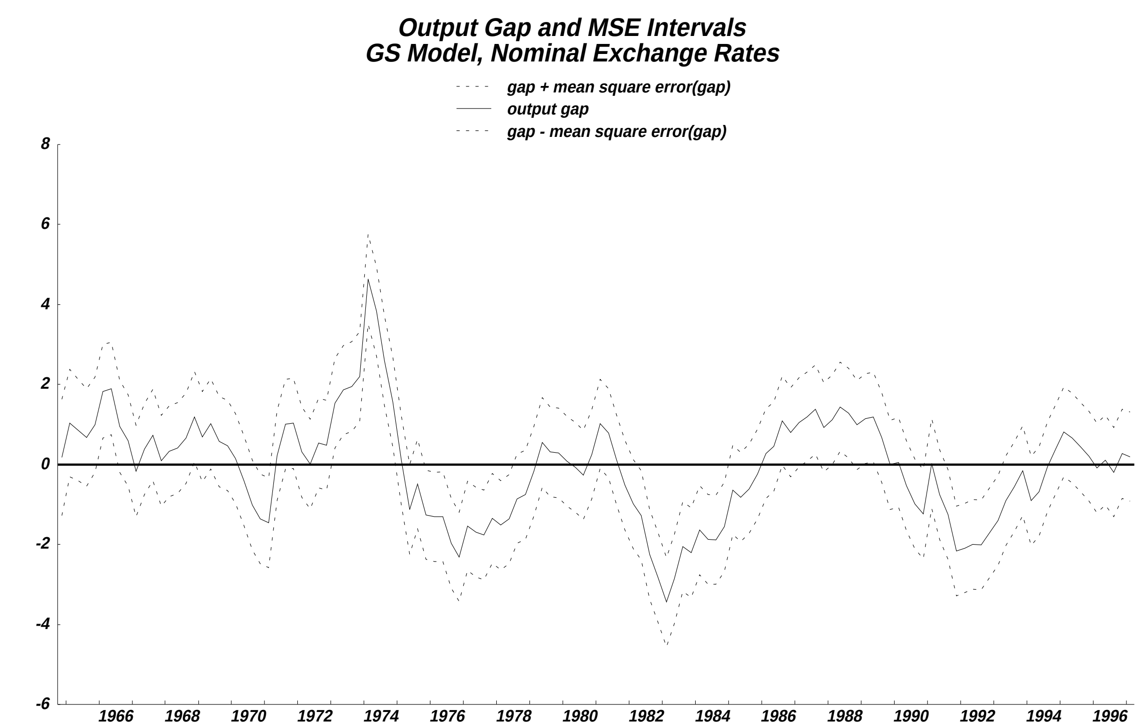
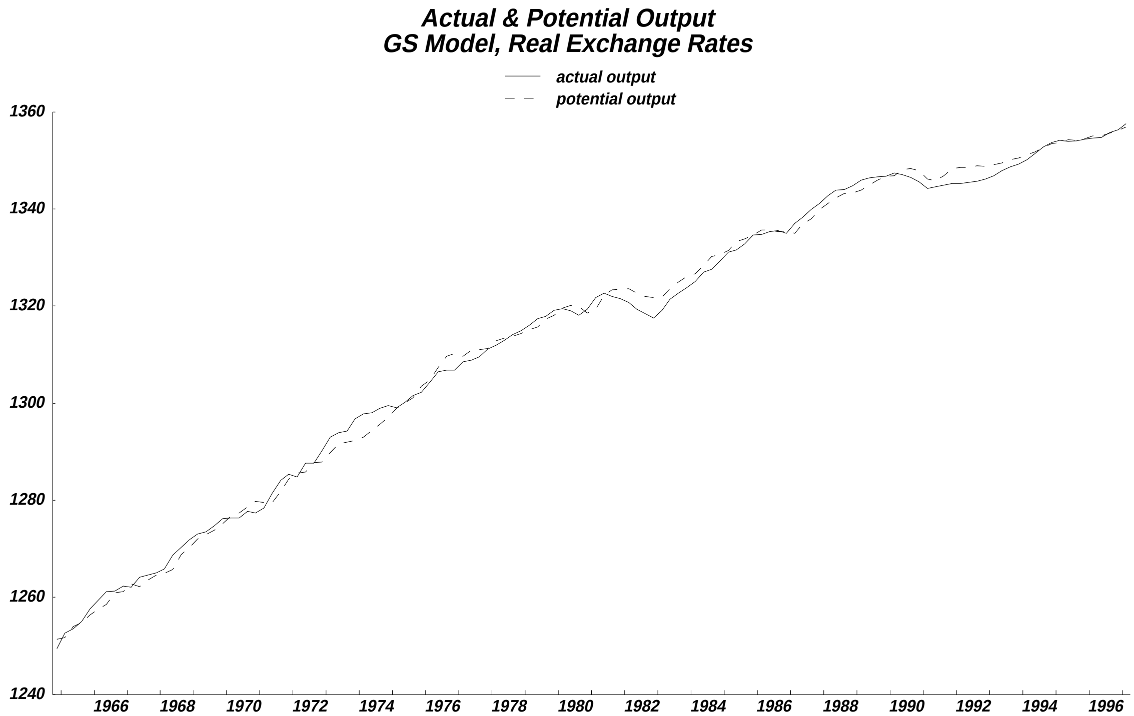


FIGURE 2: GS Model -Real Exchange Rates and Indirect Taxes



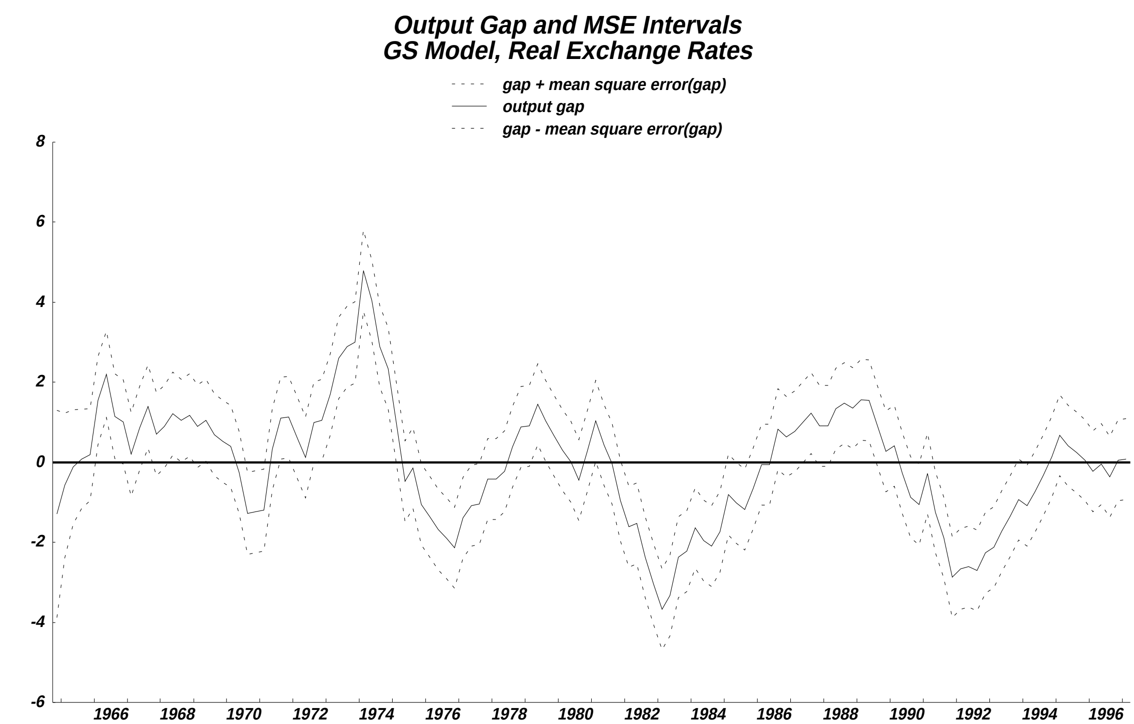
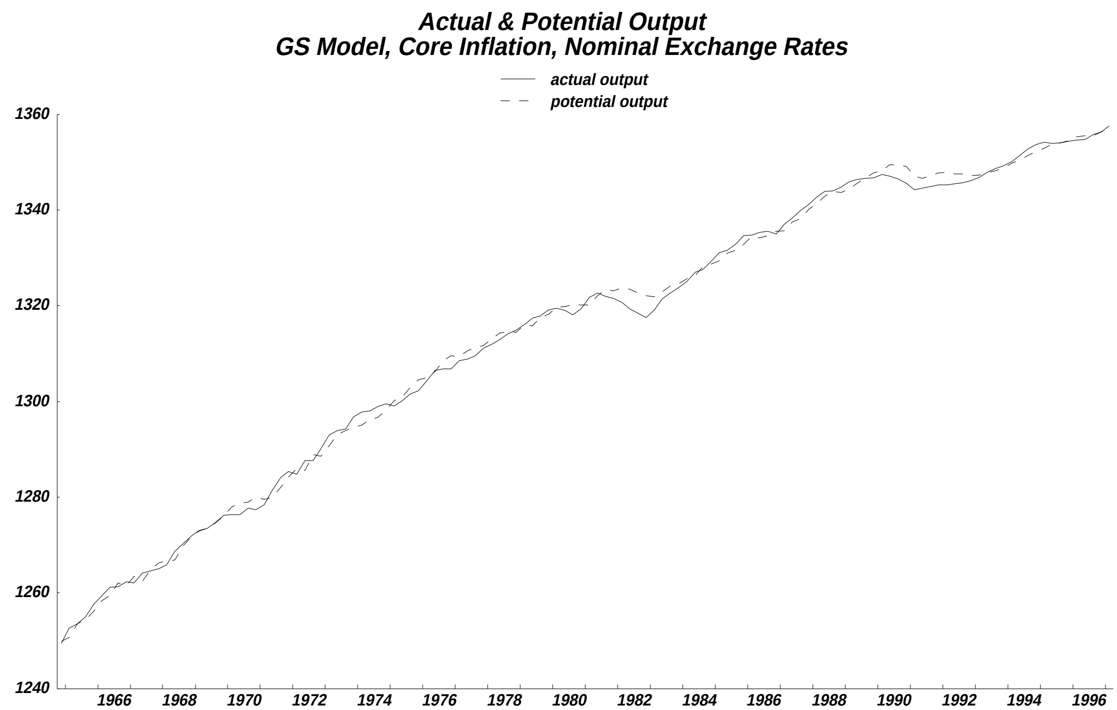


FIGURE 3: GS Model - Core Inflation, Nominal Exchange Rates, Indirect Taxes



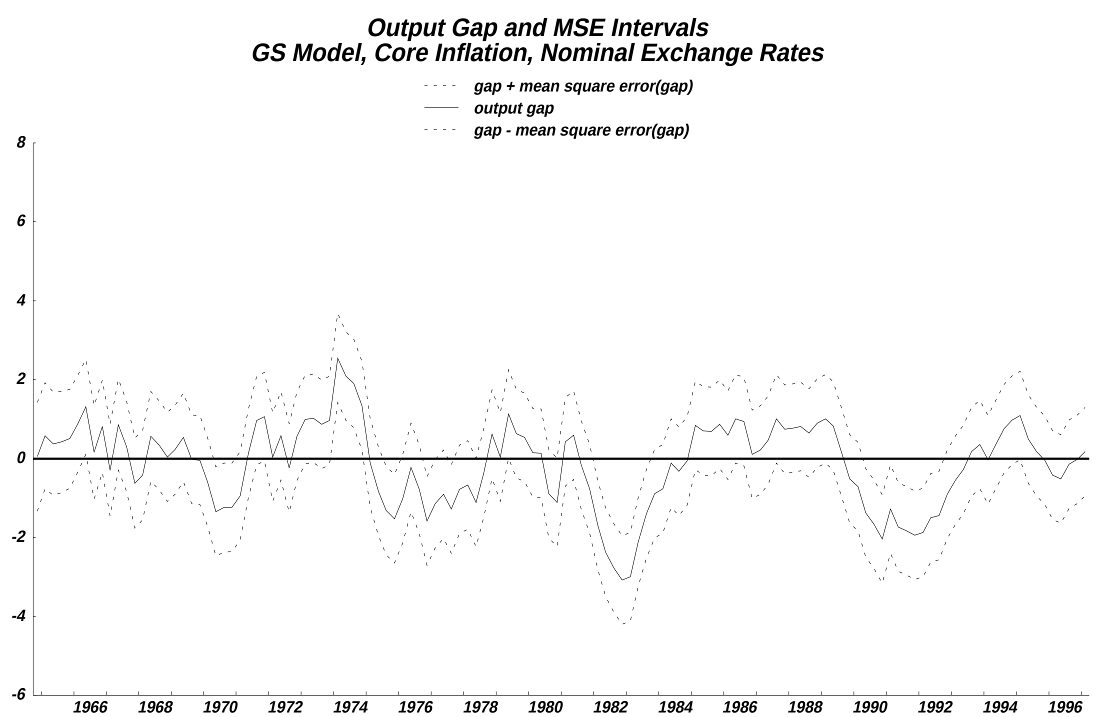
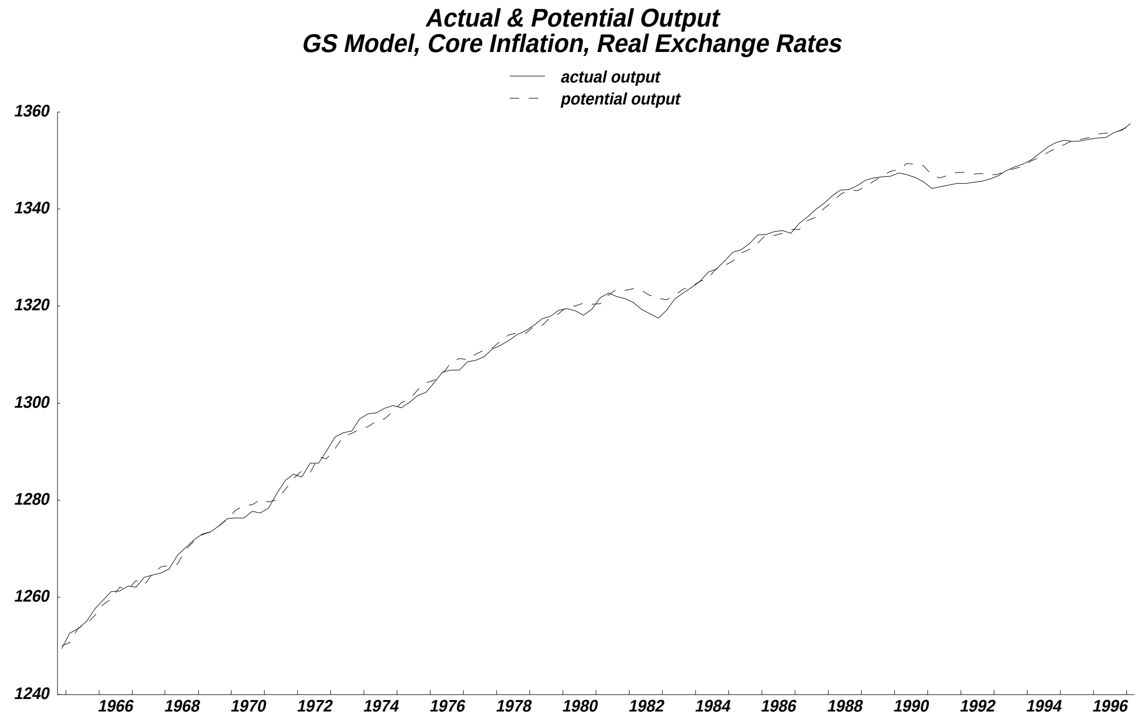


FIGURE 4: GS Model - Core Inflation, Real Exchange Rates, Indirext Taxes



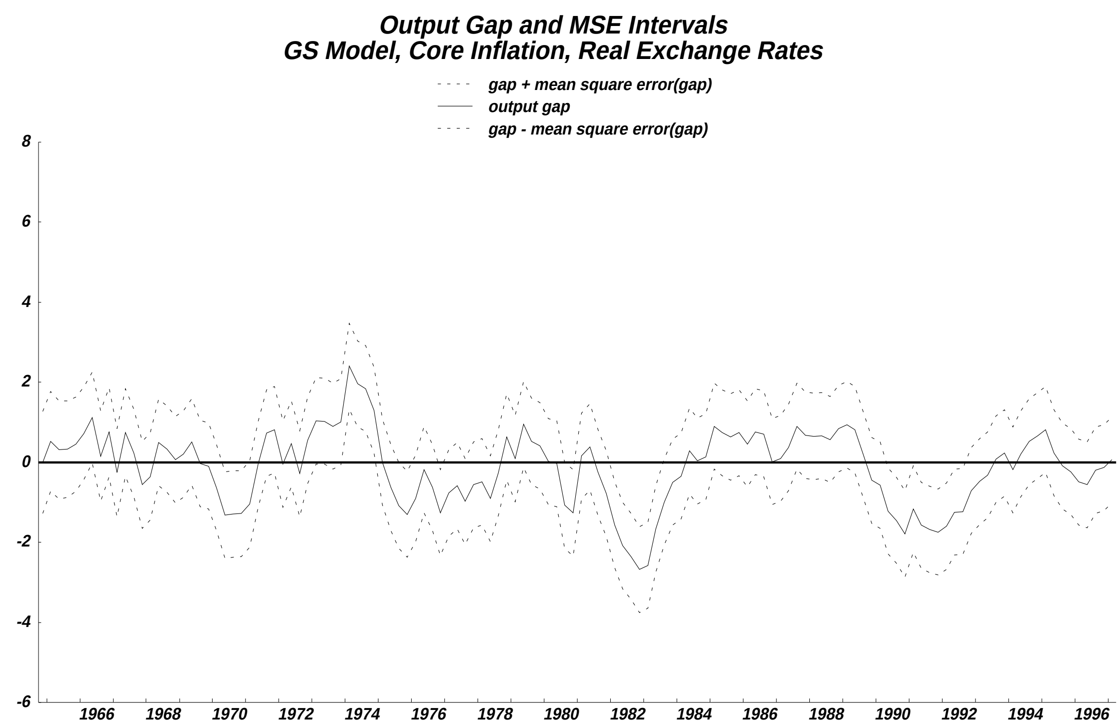
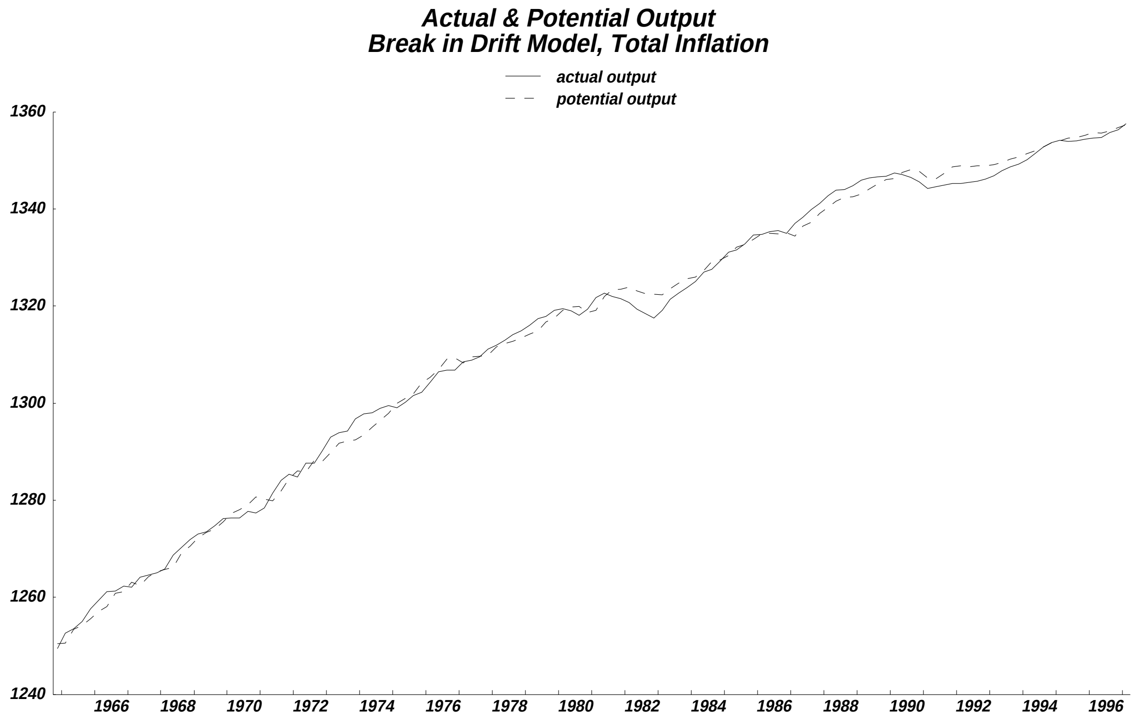


FIGURE 5: GS Model - Total Inflation; Break in Drift in 1976q2



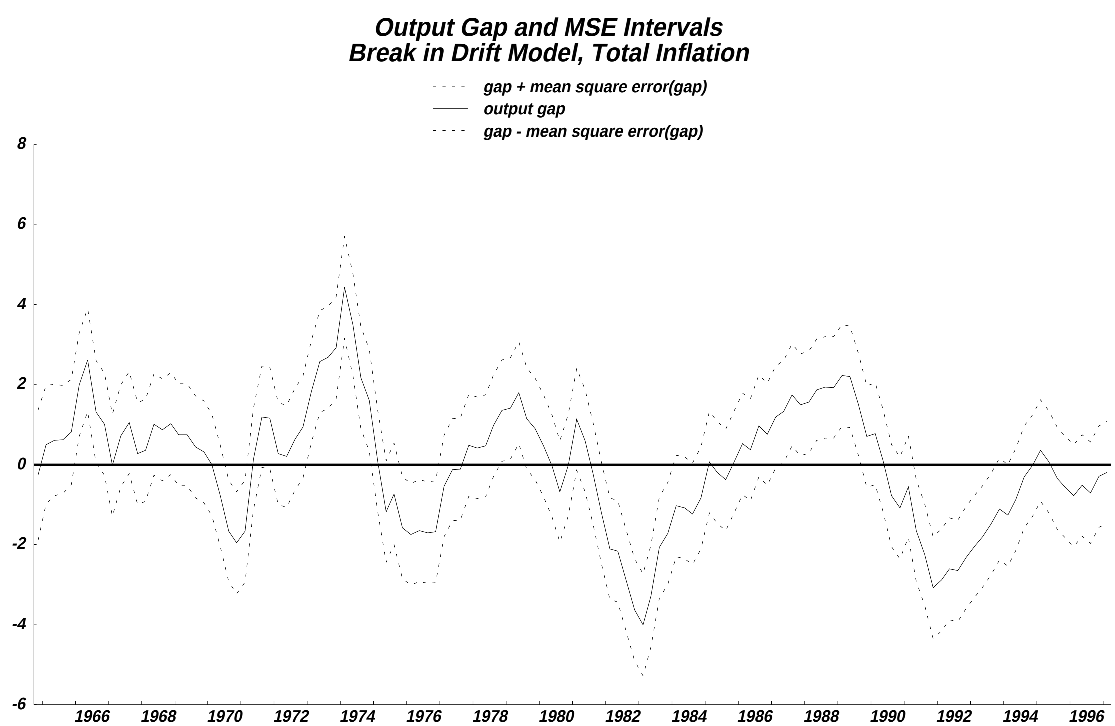
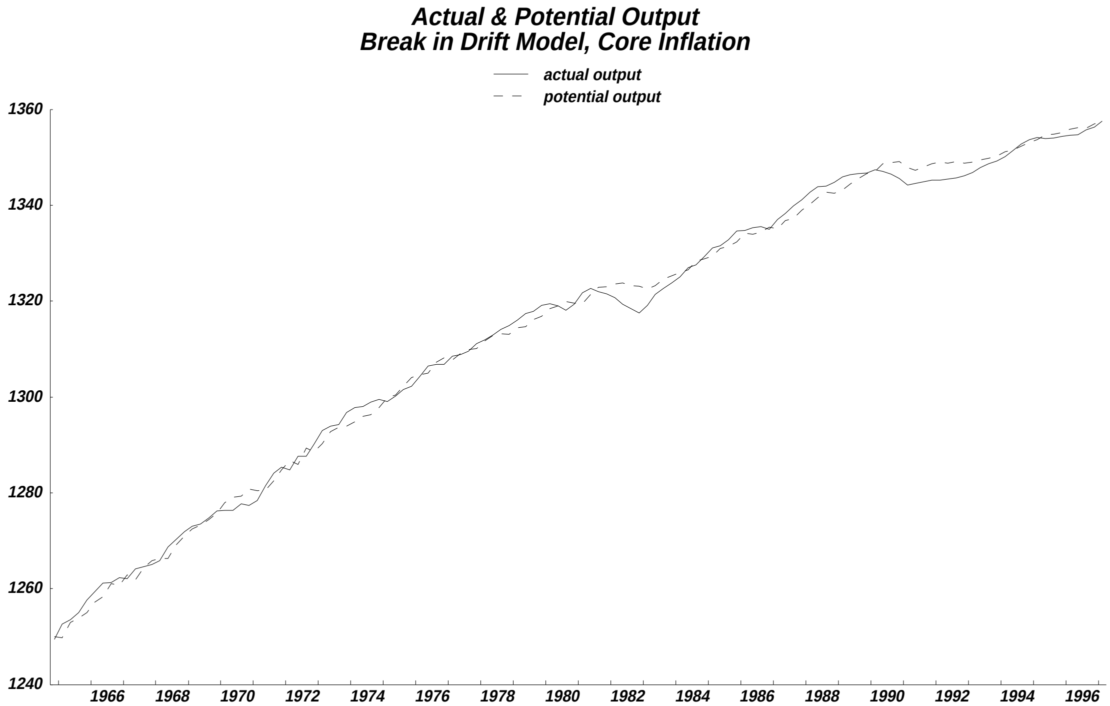


FIGURE 6: GS Model - Core Inflation; Break in Drift in 1976q2.



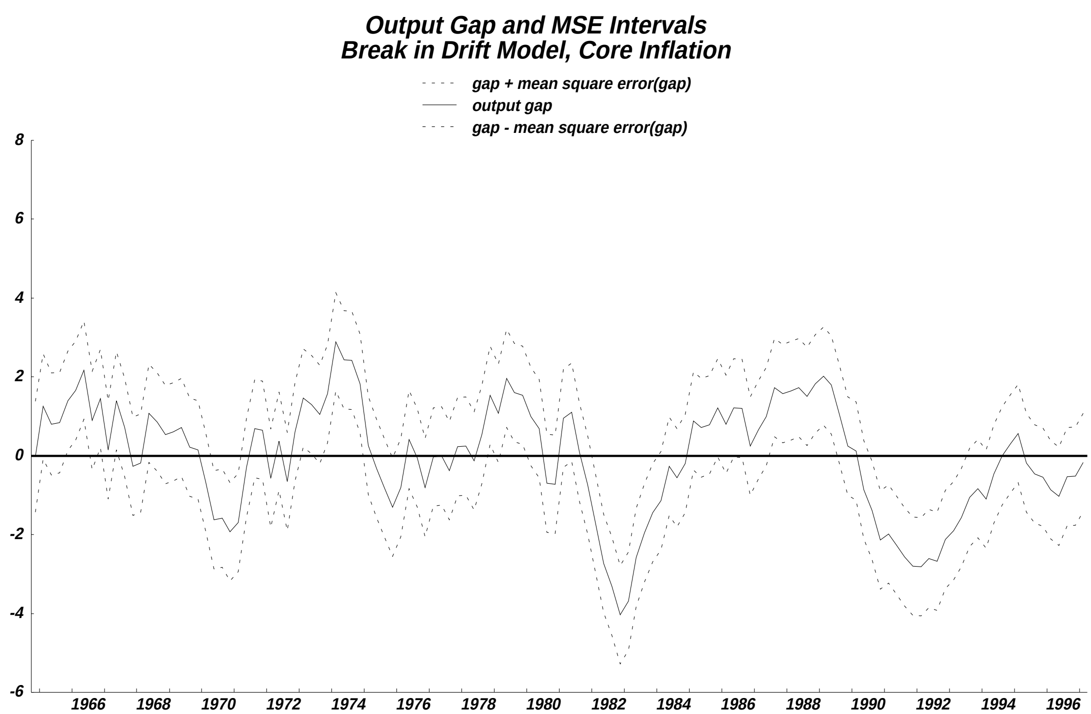
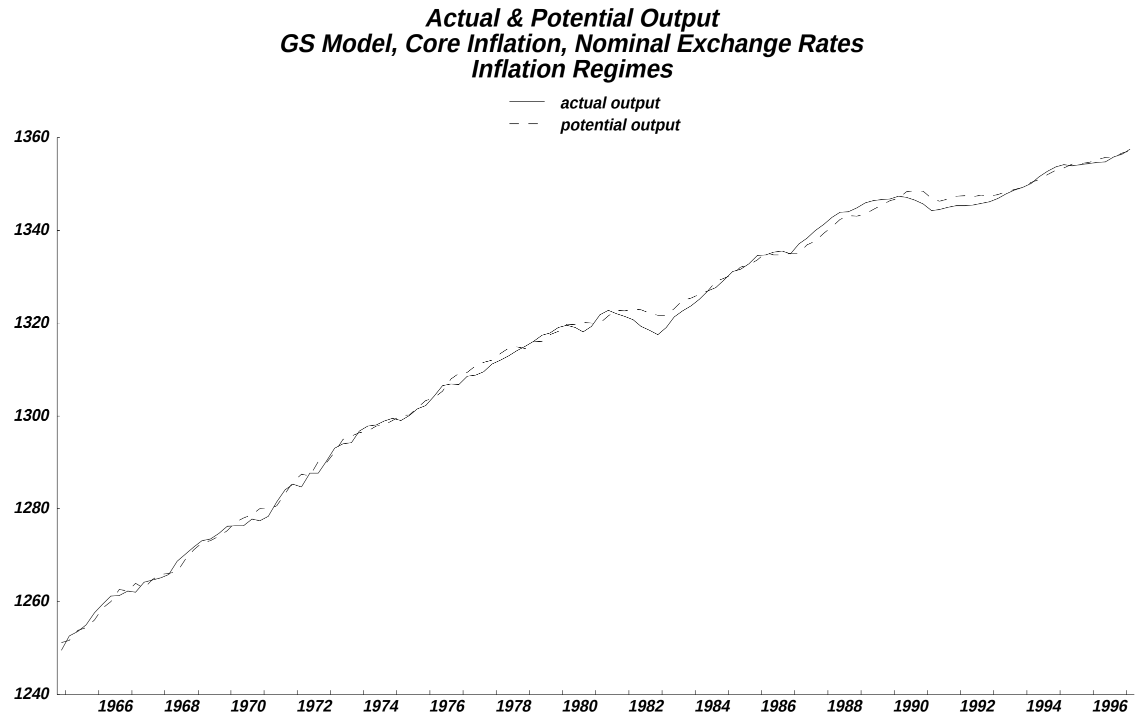


FIGURE 7: Core Inflation, Changing Regimes, Nominal Exchange Rate



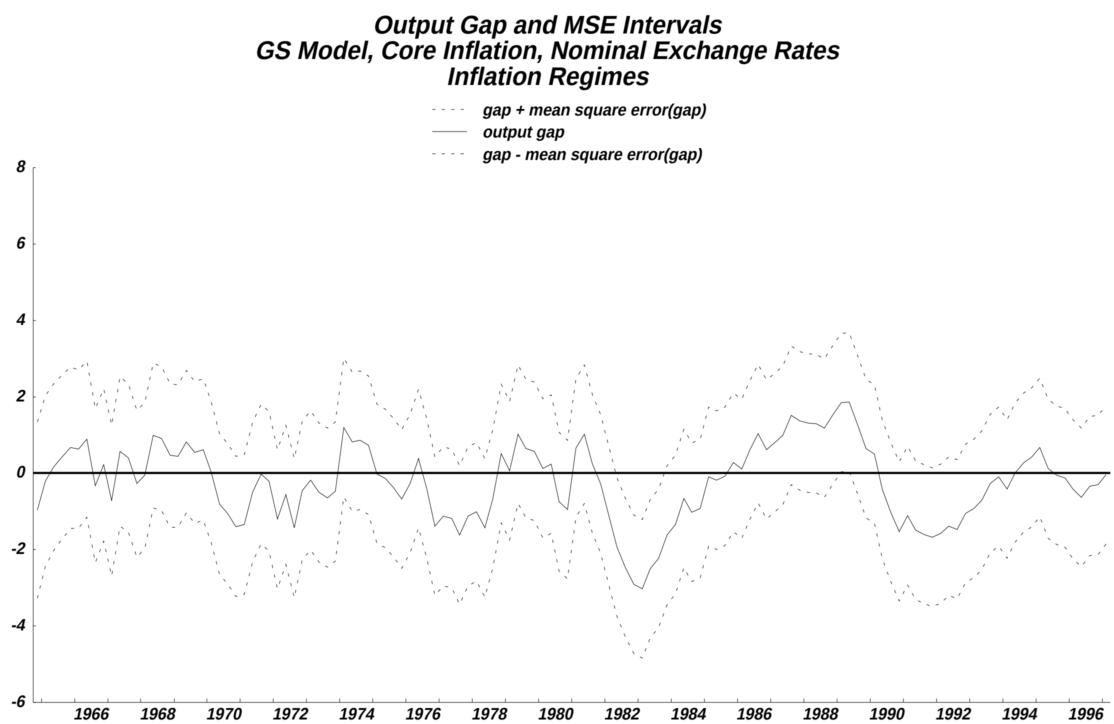
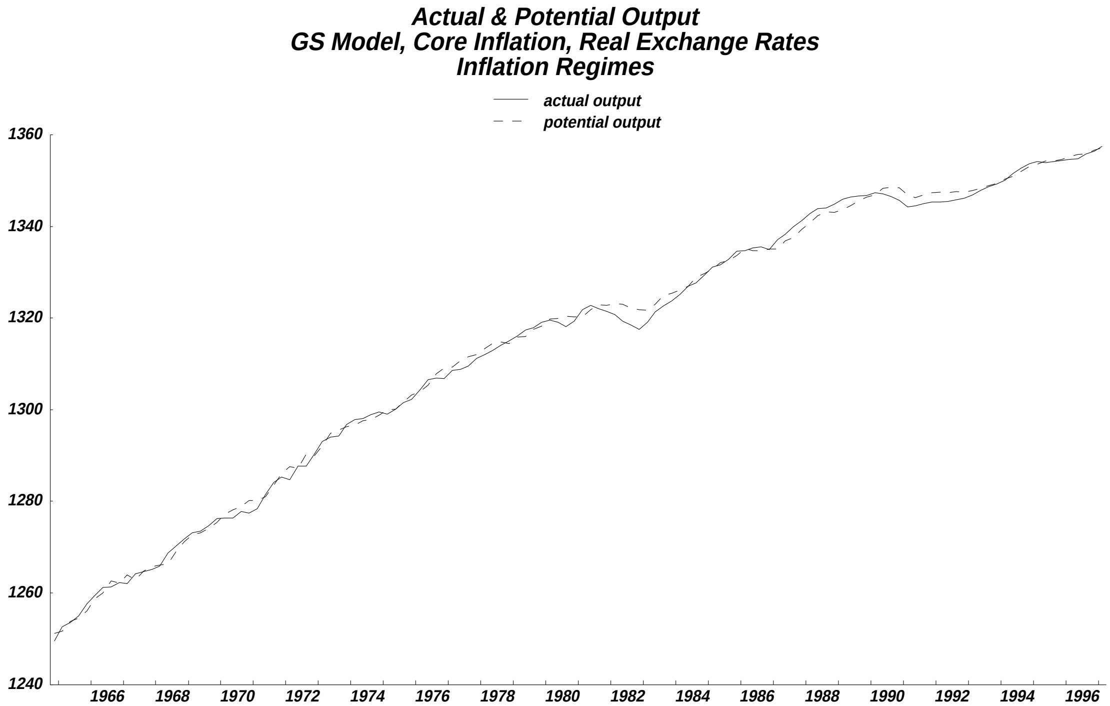


FIGURE 8: Core Inflation, Changing Regimes, Real Exchange Rates



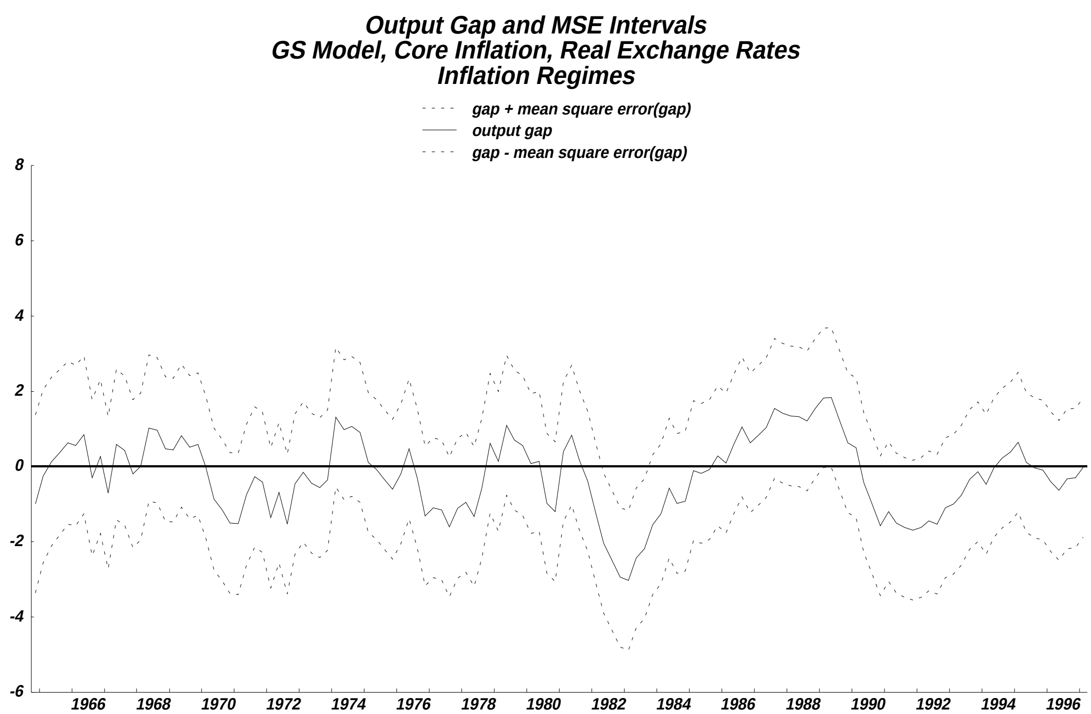
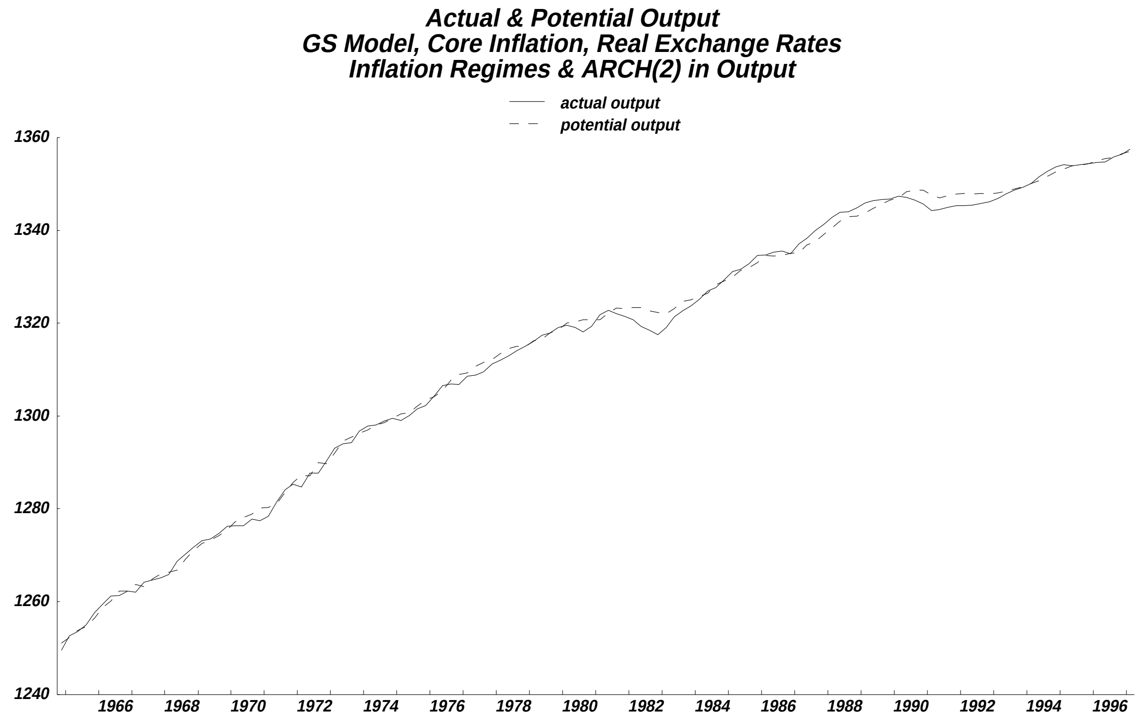


FIGURE 9: Core Inflation, Changing Regimes, ARCH(2) in Output Equation, Real Exchange Rates.



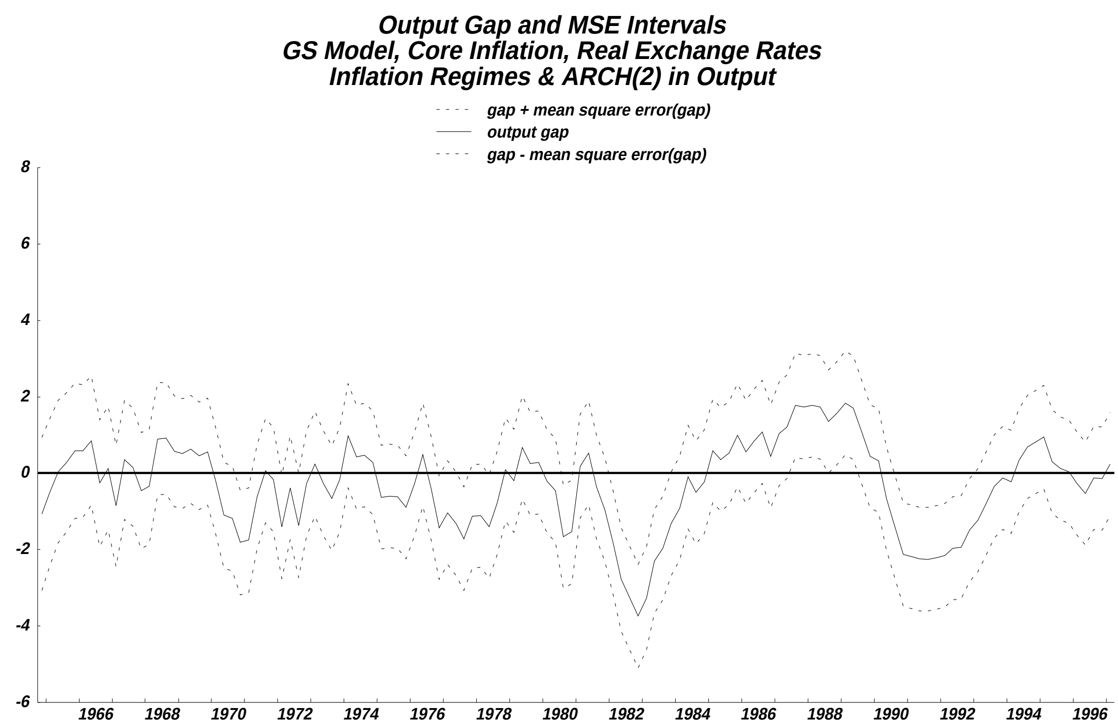
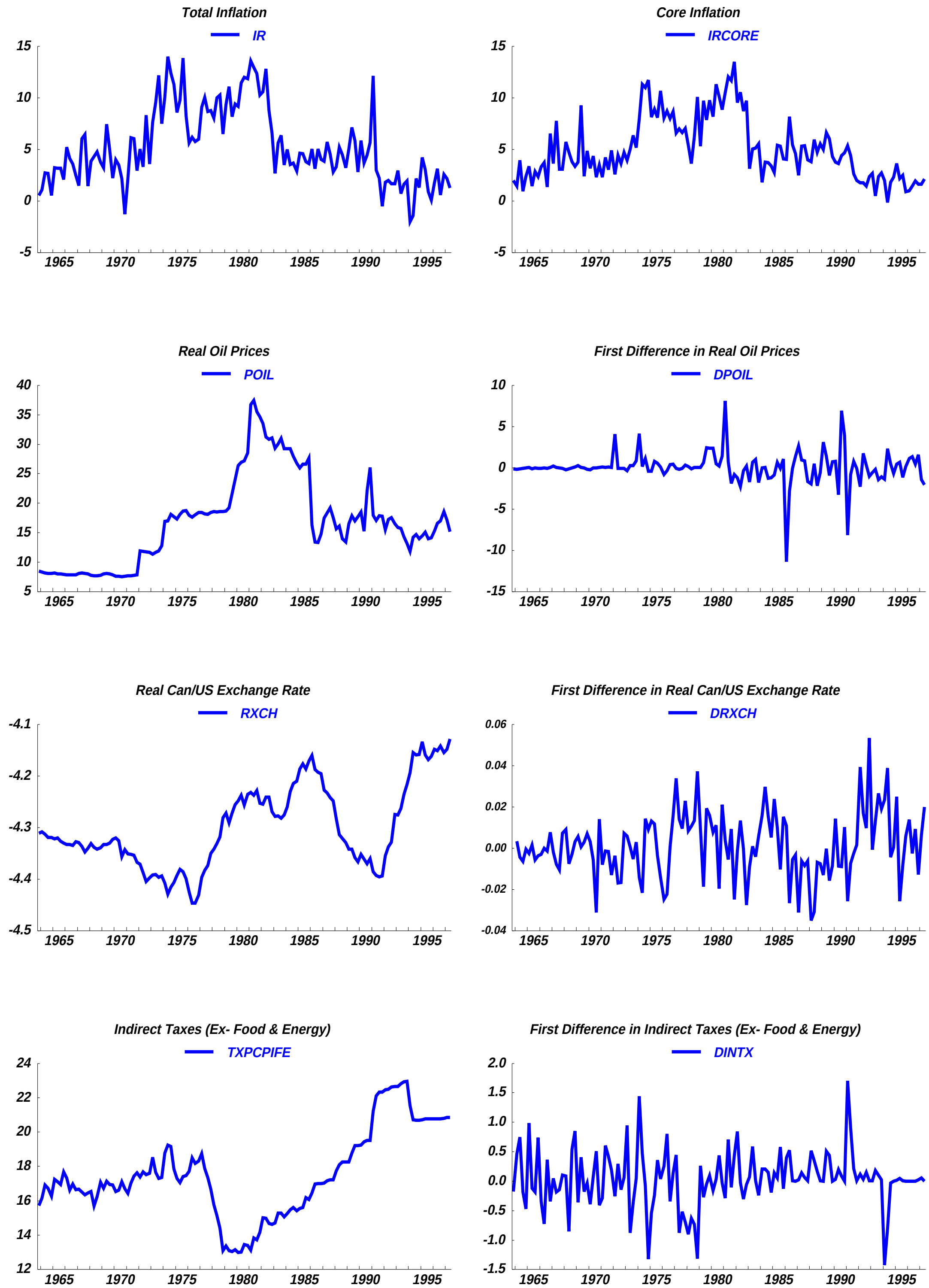


FIGURE 10: Total Inflation, Core Inflation & Relative Price Shock Variables. ‘



Bank of Canada Working Papers
Documents de travail de la Banque du Canada

Working papers are generally published in the language of the author, with an abstract in both official languages. ***Les documents de travail sont publiés généralement dans la langue utilisée par les auteurs; ils sont cependant précédés d'un résumé bilingue.***

1999

99-9	Measuring Potential Output within a State-Space Framework	M. Kichian
99-8	Monetary Rules When Economic Behaviour Changes	R. Amano, D. Coletti, and T. Macklem
99-7	The Exchange Rate Regime and Canada's Monetary Order	D. Laidler
99-6	Uncovering Inflation Expectations and Risk Premiums from Internationally Integrated Financial Markets	B.S.C. Fung, S. Mitnick, and E. Remolona
99-5	The Quantity of Money and Monetary Policy	D. Laidler
99-4	An Intraday Analysis of the Effectiveness of Foreign Exchange Intervention	N. Beattie and J-F. Fillion
99-3	Forecasting GDP Growth Using Artificial Neural Networks	G. Tkacz and S. Hu
99-2	Capital Gains and Inflation Taxes in a Life-cycle Model	C. Leung and G.-J. Zhang
99-1	Dynamic Employment and Hours Effects of Government Spending Shocks	M. Yuan and W. Li

1998

98-23	Résultats empiriques multi-pays relatifs à l'impact des cibles d'inflation sur la crédibilité de la politique monétaire	P. St-Amant et D. Tessier
98-22	A Non-Paradoxical Interpretation of the Gibson Paradox	S. Coulombe
98-21	Une nouvelle méthode d'estimation de l'écart de production et son application aux États-unis, au Canada et à l'Allemagne	R. Lalonde, J. Page et P. St-Amant
98-20	Evaluating Alternative Measures of the Real Effective Exchange Rate	R. Lafrance, P. Osakwe, and P. St-Amant
98-19	Can a Matching Model Explain the Long-Run Increase in Canada's Unemployment Rate?	A. Hornstein and M. Yuan
98-18	The Sale of Durable Goods by a Monopolist in a Stochastic Environment	G. Srour
98-17	La politique monétaire a-t-elle des effets asymétriques sur l'emploi	L. Pichette
98-16	Consumer Attitudes, Uncertainty, and Consumer Spending	D. Côté and M. Johnson
98-15	On the Believable Benefits of Low Inflation	C. Ragan
	Empirical Results for Canada	C. Dupasquier and N. Ricketts

Copies and a complete list of working papers are available from:
Pour obtenir des exemplaires et une liste complète des documents de travail, prière de s'adresser à:

Publications Distribution, Bank of Canada
234 Wellington Street Ottawa, Ontario K1A 0G9

Diffusion des publications, Banque du Canada
234, rue Wellington, Ottawa (Ontario) K1A 0G9

E-mail / Adresse électronique: publications@bank-banque-canada.ca
WWW: http://www.bank-banque-canada.ca/
FTP: ftp.bank-banque-canada.ca (login: anonymous, to subdirectory/pub/publications/working.papers/)