The Information Content of Canadian Dollar Futures Options

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Introduction

Central banks implement monetary policy through financial markets. As a result, they need to understand how the markets will perceive their actions. To that end, central bankers routinely examine forward rates and the prices of futures for both interest rates and exchange rates for signs of the market's expectations of future movements in interest rates and exchange rates. These forward rates are useful for discussing the central tendency of interest rates and exchange rates (i.e., expected rates), but they do not provide information about the degree of uncertainty surrounding those expectations.

In this paper, we examine the evolution of market sentiment over future Canadian dollar exchange rates, using risk-neutral probability density functions (PDFs) to synopsize the information contained in the prices of Canadian dollar futures options. The risk-neutral PDFs provide the probabilities attached by a risk-neutral agent to particular outcomes for future exchange rates. Thus, if market sentiment is "bearish" on the Canadian dollar, lower values of future exchange rates will be assigned greater risk-neutral probability than higher values of future exchange rates.

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The opposite will occur if market sentiment is "bullish" on the Canadian dollar.

The Bank of Canada could use risk-neutral PDFs to monitor the response of market sentiment over future Canadian dollar exchange rates to its policy statements or actions. For example, a change in the location or shape of the risk-neutral PDF following a policy statement or action might indicate a change in the tone of the market toward the future value of the Canadian dollar. Bahra (1996) mentions that risk-neutral PDFs may prove to be useful to monetary authorities, as tools to: assess monetary conditions; assess monetary credibility; assess the timing and effectiveness of monetary operations; and identify market anomalies.

Two methods for estimating risk-neutral PDFs are applied to case studies to examine the information content of Canadian dollar futures options prices. Our objective is to characterize market sentiment about the value of the Canadian dollar for some fixed point in the future, rather than explain the actual movement in the Canadian dollar. The first case study focuses on the response of risk-neutral PDFs following the release of Bank of Canada *Monetary Policy Reports*. The second case study focuses on the response of risk-neutral PDFs to changes in the operating band for overnight interest rates during periods of international financial crisis, specifically the Mexican currency crisis of early 1995 and the recent Asian financial crisis.

The organization of the paper is as follows. Section 1: introduces the valuation of European- and American-style currency futures options; discusses the link between currency futures, spot exchange rates, and domestic and foreign interest rates; and outlines the relationship between options prices and risk-neutral PDFs. Section 2 reviews the literature on the estimation of risk-neutral PDFs from options prices. Section 3 discusses the two techniques used to estimate risk-neutral PDFs. Section 4 presents the data and the case study methodology, and section 5 analyzes the results. The last section concludes and discusses the analyses' limitations.

1 Introducing Currency Futures Options

Currency futures options are options whose underlying asset is another derivative security, specifically a currency futures contract.[1] When an investor exercises a currency futures option, he or she either delivers (in the case of put options) or receives (in the case of call options) a currency futures contract. For example, the investor who exercises a Canadian dollar

1. Options give the holder the right but not the obligation to buy or sell the underlying asset on or by a certain time at a certain price. A call option gives the holder the right to buy the underlying asset at the exercise or strike price. A put option gives the holder the right to sell the underlying asset at the exercise or strike price.

call futures option receives a Canadian dollar futures contract plus an amount in cash (U.S. dollars) equal to the difference between the current futures price of one Canadian dollar and the exercise price of the option. By convention, Canadian dollar futures and futures options are quoted as the cost of one Canadian dollar in U.S. cents. As a result, for notational purposes we treat the U.S. dollar as the numeraire currency.

1.1 The valuation of currency futures options

To introduce the valuation of currency futures options, it is easiest to first illustrate the valuation of European-style options. These are options that can be exercised only on their maturity date, *T*, at which time the value of call and put options will be:

$$\hat{C}(T) = \max\{0, \hat{S}(T) - X\}$$

$$\hat{P}(T, X) = \max\{0, X - \hat{S}(T)\}, \tag{1}$$

where $\hat{C}(T, X)$ is the value of a call option with a term to maturity of T and an exercise price of X, and $\hat{S}(T)$ is the (random) value of the exchange rate on the maturity date of the option. A European currency call (put) option is *in-the-money* if the exchange rate is above (below) the exercise price, *out-of-the-money* if the exchange rate is below (above) the exercise price, and *at-the-money* if the exchange rate equals the exercise price.

To value options prior to maturity, one discounts the option's cash flow, which for European options may occur only on the maturity date. Thus, assuming risk-neutral investors, the value of European call and put options is:[2]

$$C(0, X) = \exp\{-rT\}\mathrm{E}_0[\max\{0, \hat{S}(T) - X\}]$$

$$P(0, X) = \exp\{-rT\}\mathrm{E}_0[\max\{0, X - \hat{S}(T)\}], \tag{2}$$

where E_0 represents the risk-neutral expectation, rather than the true or actual expectation, and r refers to the continuously compounded annualized U.S. nominal interest rate for the term of the option.

Formulae for pricing currency options have been derived by Garman and Kohlhagen (1983), who extended the basic options-pricing formulae of Black and Scholes (1973) and Merton (1973). The Black–Scholes/Garman–Kohlhagen (BSGK) currency options-pricing formulae assume that markets

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2. Under risk-neutrality all cash flows can be discounted at the risk-free interest rate. The assumption of risk-neutrality is for convenience, but will not affect the pricing formulae to be discussed. See Hull (1993) for a discussion of risk-neutral valuation techniques.

are perfect and impose no restrictions on short sales and continuous trading, and that the exchange rate follows a lognormal distribution derived from the following diffusion process:[3]

$$\frac{dS}{S} = (r - r_c)dt + \sigma dz, \tag{3}$$

where r_c refers to the continuously compounded annualized Canadian nominal interest rate, σ refers to the volatility of the spot exchange rate, and dz is a Wiener process. Interest rates and volatility are assumed to be constant.[4]

The BSGK currency options-pricing model can easily be extended to value currency futures options by using the covered interest parity relationship, which under risk-neutral valuation and constant interest rates is:[5]

$$F(0, T) = \exp\{(r - r_c)T\}S(0). \tag{4}$$

The resulting BSGK prices for European currency futures options are:[6,7]

$$\begin{aligned} C(0, X) &= \exp\{-rT\}[F(0, T)N(d_1) - XN(d_2)] \\ P(0, X) &= \exp\{-rT\}[XN(-d_2) - F(0, T)N(-d_1)] \end{aligned} \tag{5}$$

where

$$d_1 = \frac{\log\{F(0, T)/X\}}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},\ d_2 = d_1 - \sigma\sqrt{T}, \tag{6}$$

and $N(x)$ represents the standardized cumulative normal probability distribution function evaluated at x.

3. That is, the logarithm of the exchange rate is a normally distributed random variable with constant mean and constant variance. The diffusion process is also referred to as a geometric Brownian motion.

4. Later, a more general approach to pricing options that allows interest rates and volatility to be deterministic functions of time will be presented and discussed. Stochastic interest rates could also be admitted if the volatility parameter is redefined to be a function of the variances and covariances of the interest rates. An analysis of the correlations between the interest rates and the spot exchange rate would also be necessary, but is beyond the scope of this paper. See Grabbe (1983).

5. Forward exchange rates will equal the price of currency futures contracts for the same maturity under the assumptions of risk neutrality, so long as risk-free interest rates are constant and the same for all maturities.

6. These European futures options prices are exact only if the options and the currency futures have the same maturity.

7. The price of a currency futures contract is also assumed to follow a lognormal diffusion process.

Notice that, except for the volatility of the exchange rate, all the variables required to calculate the price of a futures option are readily available. Notice also that to value currency futures options neither Canadian interest rates nor the spot exchange rate are directly necessary; these rates are indirectly embedded in the prices for Canadian dollar futures contracts.

1.2 American-style currency futures options

Currency futures options are typically American-style options, which can be exercised at any time up to and including their maturity date. As such, formulae for pricing European-style options are not directly applicable. Unfortunately, explicit formula for American-style options prices are not generally available, though Melick and Thomas (1996), Leahy and Thomas (1996), and Söderlind (1997) have shown that the following bounds can be placed on the prices of American-style currency futures options:[8]

$$
\begin{aligned}
\bar{C}_A(0, X) &= \mathrm{E}_0[\max\{0, \tilde{S}(T) - X\}] \\
\underline{C}_A(0, X) &= \max\{\mathrm{E}_0[\tilde{S}(T)] - X, \exp(-rT)\mathrm{E}_0[\max\{0, \tilde{S}(T) - X\}]\} \\
\bar{P}_A(0, X) &= \mathrm{E}_0[\max\{0, X - \tilde{S}(T)\}] \\
\underline{P}_A(0, X) &= \max\{X - \mathrm{E}_0[\tilde{S}(T)] \\
&\quad \exp(-rT)\mathrm{E}_0[\max\{0, X - \tilde{S}(T)\}]\}\,. \qquad (7)
\end{aligned}
$$

The upper bounds on the American-style currency futures options prices are simply the undiscounted European-style currency futures options prices from (1). The lower bounds are at least equal to the European-style currency futures options prices. Therefore, American-style currency futures options have a value at least as great as their European-style counterparts, and the premium arises because of the added flexibility of early exercise. The premium on American-style options is therefore called the early exercise premium.

1.3 Spot exchange rates, interest rates, and currency futures

Though Canadian nominal interest rates do not directly enter the pricing of Canadian dollar futures options, the differential between

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8. The Barone-Adesi and Whaley (1987) formulae for American options can be used assuming that the futures price on the maturity date of the option is drawn from a single lognormal distribution. However, to maintain consistency in the empirical analysis, this approximation will not be used.

Canadian and U.S. short-term interest rates is the main determinant of the relationship between Canadian dollar futures prices and the spot exchange rate for the Canadian dollar. The interest rate differential will prove to be important in the analysis of the case studies presented later.

Over the period analyzed, the spread between Canadian short-term interest rates and U.S. short-term interest rates has changed sign. Specifically, prior to early 1996, Canadian short-term interest rates were generally above U.S. short-term interest rates. Since then they have been below U.S. short-term interest rates. Thus, from the U.S. perspective, the interest rate differential has gone from being negative to positive.

From equation (4), if the interest rate differential is positive, the price of a Canadian dollar futures contract will exceed the spot exchange value of the Canadian dollar, and the Canadian dollar will be expected to appreciate over the term of the futures contract.[9] Thus, if $r > r_c$, then $F(0, T) > S(0)$ and the Canadian dollar will be expected to appreciate. However, if $r_c > r$, then $F(0, T) < S(0)$ and the Canadian dollar will be expected to depreciate.

1.4 Risk-Neutral PDFs

To illustrate how options prices and risk-neutral PDFs are related, we can rewrite the options prices in (2) as follows:

$$C(0, X) = \exp\{-rT\}\int_X^{\infty}\{\tilde{S}T - X\}q[\tilde{S}(T)]d\tilde{S}(T)$$

$$P(0, X) = \exp\{-rT\}\int_0^{X}\{X - \tilde{S}(T)\}q[\tilde{S}(T)]d\tilde{S}(T). \tag{8}$$

The function $q[\tilde{S}(T)]$ is the risk-neutral PDF for the exchange rate on the maturity date of the option. It provides the probabilities attached by a risk-neutral agent to particular outcomes for future Canadian exchange rates. Options prices thus depend on the degree to which an option is in-the-money, multiplied by the risk-neutral probability of being that deep in-the-money at expiration.

To analyze the evolution of market perceptions over the future value of the Canadian dollar, we will present a graphical depiction of the risk-neutral PDF plus summary statistics calculated with respect to the logarithm

9. Bodurtha and Courtadon (1987) discuss the impact of interest differentials on the pricing of American-style currency options and American-style currency futures options.

of the futures price, such as the mean (median, or mode), the annualized volatility, skewness, and kurtosis.

The mean of the estimated risk-neutral PDF is equal to the Canadian dollar futures price, and represents the expected future Canadian exchange rate under risk-neutral valuation. A change in the mean of the risk-neutral PDF represents a change in the expected future Canadian exchange rate. However, so long as the interest rate differential with respect to the U.S. is positive (negative), the Canadian dollar will still be expected to appreciate (depreciate) over the term of the option in a risk-neutral sense.

The annualized volatility provides an indication of the dispersion of opinion in the market over future Canadian exchange rates. One potential effect of monetary policy statements or actions by the Bank of Canada might be to reduce the uncertainty surrounding expected exchange rates. A decline in the annualized volatility might be evidence of this.

The skewness statistic will show whether the market is, in a risk-neutral sense, "neutral," "bearish," or "bullish" on the Canadian dollar; the risk-neutral PDF will be either symmetric, skewed left, or skewed right. A skewed-left distribution suggests that the market is "bearish" on the Canadian dollar, as greater risk-neutral weight is placed on the likelihood that the Canadian exchange rate will be far below, as opposed to far above, the current futures price on the maturity date of the option. This is so even though skewed-left distributions have a greater-than-50-per-cent probability that the Canadian exchange rate will be above the current futures price on the maturity of the option.[10]

It is important to realize that, regardless of the skew of the risk-neutral PDF, the market expects the Canadian exchange rate on the maturity date of the futures option to equal the current Canadian dollar futures price. For example, consider a risk-neutral PDF for future Canadian exchange rates that is skewed left when the interest differential is positive. In this case, the Canadian dollar would still be expected to appreciate towards the futures price despite the negative skew or the "bearish" tone in the market towards the future Canadian dollar exchange rate.

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10. These comments follow from the fact that for non-symmetric distributions, the mean, median, and mode differ. The mode is the most frequently observed value, and basically corresponds to the peak of the distribution. Skewed-left distributions have mean < median < mode, and skewed-right distributions have the reverse ordering. See Kenkel (1989).

The fourth summary statistic is kurtosis, which shows the possibility of large changes in exchange rates (leptokurtosis) prior to the maturity of the futures options.[11]

In discussing the summary statistics from the risk-neutral PDF, one must consider the declining term to maturity of the futures option and of the underlying futures contract. Over the term of the futures contract, the futures price does not generally equal the spot exchange rate, but on the maturity date the price will equal the spot exchange rate. Thus, the futures price will deviate less and less from the spot exchange rate as the maturity date approaches. Therefore, as the futures contract nears maturity the price of Canadian dollar futures contracts becomes less and less volatile. This suggests that the most useful information is likely to come from futures options that are farther from maturity.[12]

2 Literature Survey

Breeden and Litzenberger (1978) were the first to derive the relationship between call options prices and the risk-neutral PDF. They found that the risk-neutral PDF was proportional to the second derivative of the call options price with respect to the exercise price:

$$\frac{\partial^2 C(0, X)}{\partial X^2} = \exp\{-rT\} q[\tilde{S}(T)] .$$

This result, together with violations of various assumptions underlying the BSGK options-pricing model, has spawned an active field of research on the extraction of risk-neutral PDFs from options prices.

To illustrate the deficiencies of the BSGK options-pricing model, let us consider the volatility of the exchange rate. As noted earlier, volatility is the only unknown in calculating BSGK options prices. For estimating volatility, the usual practice is to take the observed options price and numerically invert the pricing formula to obtain the “implied volatility” of the exchange rate over the term of the option. Implied volatility can also be

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11. Risk reversals and strangles are commonly used measures of skewness and kurtosis, respectively. In the over-the-counter market, risk reversals consist of the simultaneous purchase of an out-of-the-money call and sale of equally out-of-the-money put and yield information on the likelihood of a unidirectional movement in the currency. Strangles consist of buying equally out-of-the-money call and put options and yield information on the market sentiment of the likelihood of large movements in the exchange rate. See Malz (1997).

12. No correction to the summary statistics for the term-to-maturity effects have been done in this analysis, though Melick and Thomas (this volume) do propose such adjustments.

described as the estimate of the volatility parameter that equates the observed market price of an option to the theoretical BSGK price of the option.

If the assumptions underlying the BSGK model were correct, then the implied volatility would be the same regardless of the exercise price. Instead, when the estimated implied volatilities are plotted against the exercise prices, one typically sees a "smile" or "smirk." That is, the estimated implied volatility is higher for options that are deep in- or deep out-of-the-money. This empirical observation seems to refute the BSGK assumption that exchange rates and currency futures prices are lognormal diffusion processes. For example, if the implied volatility curve is a symmetric smile then it suggests the presence of leptokurtosis, or "fat tails," in the distribution, while a smirk to one side or the other suggests a skewed distribution.

If exchange rates and currency futures prices do not follow lognormal distributions then there might be interest in finding a risk-neutral PDF that can produce observed options prices. As a result, four main methods have been developed to obtain the risk-neutral PDF, $q[\tilde{S}(T)]$:

- specifying a generalized stochastic process for the price of the underlying asset;
- specifying a parametric form for the risk-neutral PDF;
- smoothing the implied volatility smile;
- estimating using non-parametric techniques.

In the first method, a stochastic process is specified for the exchange rate, from which the risk-neutral PDF can be derived and theoretical options prices calculated. Examples of stochastic processes are jump diffusions, stochastic volatility processes, and diffusions with non-constant coefficients. The parameters of the stochastic process are then estimated by minimizing a loss function defined over the pricing errors between the theoretical and actual options prices for a given set of exercise prices. This method has been used by Bates (1996b) and Malz (1996, 1997) to price foreign currency options, where the exchange rate is assumed to be driven by a jump-diffusion process. The derived options prices are weighted sums of BSGK options prices, where the weights depend on the parameters of the jump-diffusion process.[13] In the case of Malz (1997), which is a special case of Bates (1996b), the options prices are essentially weighted sums of two BSGK options prices.

13. See Bates (1996b) for a more complete discussion of options pricing when the underlying asset follows a jump-diffusion process.

A variant of this method specifies the volatility, and possibly interest rates, as deterministic functions of time. There are two benefits to specifying volatility as a deterministic function of time rather than as a stochastic process. First, the price of call options under deterministic volatility will satisfy the Black–Scholes partial differential equation, though the prices are not Black–Scholes options prices. Second, one does not have to consider the market price of risk that appears in stochastic volatility and jump-diffusion options-pricing models. Deterministic volatility processes have been discussed by Rubinstein (1994), Derman and Kani (1994), Dupire (1994), Jackwerth and Rubinstein (1996), Bodurtha and Jermakyan (1996), Lagnado and Osher (1997a, 1997b), Bouchouev and Isakov (1997), and Levin (1998).

The second method for characterizing the risk-neutral PDF is to forgo discussing the diffusion process and instead directly specify a parametric form for the risk-neutral PDF over the exchange value of the Canadian dollar on the option's maturity date.

A popular choice for the risk-neutral PDF is that of a weighted sum of independent lognormal density functions, called a mixture of lognormal distributions (MLN). Ritchey (1990) has demonstrated that, in this case, European options prices can be written as weighted sums of Black–Scholes options prices. This method has been used by Melick and Thomas (1996) to discuss oil prices during the Persian Gulf crisis of 1991, by Leahy and Thomas (1996) to discuss the Canadian dollar during the Quebec Referendum of 1995, and by Mizrach (1996) to investigate the British pound relative to the U.S. dollar during the Exchange Rate Mechanism crisis.

Mixtures of lognormal density functions have also been used by Bahra (1996), Söderlind and Svensson (1997), Söderlind (1997), and Butler and Davies (1998) to extract markets' expectations and views about monetary policy from options prices.

The third method for characterizing the risk-neutral PDF, one of the earliest approaches, was developed by Shimko (1993) as a way to smooth the volatility smile with a quadratic approximation. The result was a smooth continuous function relating implied volatilities to exercise prices. These implied volatilities were then substituted into the Black–Scholes options-pricing formulae, making it possible to recover options prices for a continuous series of exercise prices. This permitted the use of the Breeden and Litzenberger (1978) approximation to recover the risk-neutral PDF.

A problem arises with smoothing the volatility smile, however. The approximation is applicable only within the range of available exercise prices, since the curve will explode outside that range. Therefore, these methods typically attach ad hoc "tails" to describe the behaviour beyond the range of available exercise prices.

The fourth method for characterizing the risk-neutral PDF follows Aït-Sahalia and Lo (1998), who refer to the state-price density (SPD) rather than the risk-neutral density. They use non-parametric options-pricing formulae, and kernel estimation techniques to construct a smooth options-pricing function, to which the Breeden and Litzenberger result may be applied, and from which the SPD or risk-neutral PDF may be obtained. The SPD can also be called the pricing kernel, since once it has been obtained one can price any asset at the current time, with payoff at date *T*.[14] Though this method can be quite flexible, it also tends to be very data-intensive.

Though many of these approaches differ in the specification and/or estimation of the risk-neutral PDFs, they typically give similar results. For example, Campa, Chang, and Reider (1997) find that smoothing the volatility smile gives results quite similar to the MLN method, while Dumas, Fleming, and Whaley (1998) find that smoothing the volatility smile gives results very similar to deterministic volatility functions.

3 Estimating Risk-Neutral PDFs from Options Prices

The MLN approach and the deterministic local volatility approach are used in this paper to extract risk-neutral PDFs from Canadian dollar futures options prices. The resulting risk-neutral PDFs may also help reveal market sentiment about future Canadian exchange rates, as they allow for non-symmetric distributions. As will be shown, the BSGK model produces symmetric distributions that cannot capture market sentiment.[15]

3.1 Mixtures of lognormal distributions

An MLN is a flexible way to deal with departures from the assumptions underlying the BSGK futures options-pricing model that, according to Bahra (1996), imposes minimal structure on the stochastic process of the price of Canadian dollar futures contracts. For example, the stochastic volatility options-pricing model of Hull and White (1988), and the jump-diffusion options-pricing models, such as Merton (1973), Bates (1991, 1996b), and Malz (1997), can be represented by MLNs.

The empirical analysis will employ a weighted mixture of two lognormal distributions:[16]

$$q[\tilde{S}(T)] = \phi_1 q_1[\tilde{S}(T)] + (1-\phi_1) q_2[\tilde{S}(T)], \tag{9}$$

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14. See Campbell, Lo, and MacKinlay (1997) for more on kernel-pricing methods.

15. This argument applies to the logarithm of the futures price and the logarithm of future exchange rates, rather than their levels.

16. The BSGK model is a special case of the mixture if $\phi_1 = 1$.

where

$$q_i[\tilde{S}(T)] = \frac{1}{\sqrt{2\pi}\sigma_i\tilde{S}(T)}\exp\left\{-\frac{1}{2}\left(\frac{\log(\tilde{S}(T))-\mu_i}{\sigma_i}\right)^2\right\}, \text{ for } i = 1,2.$$

The mixture of two lognormal distributions is used to maintain a consistent specification in the day-to-day analysis of the evolution of the markets' perceptions of the future value of the Canadian dollar. The number of lognormal densities that can be employed day to day depends on the quantity and quality of the available options price data. Unfortunately, the number of options exercise prices traded and the volume of trade in Canadian dollar futures options can be quite low, suggesting that the use of just two lognormal distributions is most appropriate.[17, 18]

The parameters of the risk-neutral PDFs are estimated by minimizing the squared pricing errors associated with the call futures options prices, the put futures options prices, and the currency futures price. The parameter vector, denoted by θ, will depend on whether the BSGK or the MLN model is estimated. Therefore, the same approximation is used for both the American-style BSGK options prices and for the American-style MLN options prices, rather than using the Barone-Adesi and Whaley approximation for the BSGK options prices.[19]

Let the theoretical Canadian dollar futures price derived from the options-pricing model under risk-neutral density $q[\tilde{S}(T)]$ be given by $F_\theta(0, T)$, and the observed Canadian dollar futures price be given by $F(0, T)$. Also, let $C_\theta(0, X)$, and $P_\theta(0, X)$ be the theoretical call and put futures options prices. Let $C(X)$ and $P(X)$ be the observed call and put futures options prices, respectively, with exercise price X. The minimization problem is:

$$\min_\theta\left[\sum_{i=1}^{n}[C(X_i)-C_\theta(0, X_i)]^2 + \sum_{j=1}^{m}[P(X_j)-P_\theta(0, X)]^2 + [F(0, T)-F_\theta(0, T)]^2\right],$$

17. A similar argument has been used by Bahra (1996) to support the use of a mixture of two lognormals.

18. Leahy and Thomas (1996) employed a mixture of three lognormal distributions in their analysis of the Quebec referendum. That was possible because Canadian dollar futures options were actively traded for a broad range of exercise prices during that period.

19. More detail on the estimation technique and further discussion of the pricing American-style options within this framework is included in the Appendix.

where the number of call and put options are allowed to differ. The third term of the minimization ensures that the theoretical Canadian dollar futures price will equal the expected future Canadian dollar exchange rate. That is, $F_{\theta}(0, T) = E_0(\tilde{S}_T)$. Numerical techniques are employed to implement the minimization, obtain estimates of the parameters, and recover the estimated risk-neutral PDF by evaluating (8).

3.2 Deterministic local volatility

There are two methods for modelling deterministic local volatility (DLV). One method follows Rubinstein (1994), Derman and Kani (1994), Dupire (1994), and Jackwerth and Rubinstein (1996) in using binomial or trinomial trees. Unfortunately, implied tree methods can price only to a single maturity, and do not generalize to pricing more complex options. The second method follows Bodurtha and Jermakyan (1996), Lagnado and Osher (1997a, 1997b), and Bouchouev and Isakov (1997), solving the appropriate partial differential equation in what is referred to as solving the inverse problem of options pricing.[20]

The inverse problem begins by extending the diffusion process (3) to allow local volatility to be a deterministic function that depends on the exchange rate and time:

$$\frac{dS}{S} = \{r - r_c\}dt + \sigma(S, t)dz. \tag{10}$$

Using the covered interest rate parity relationship (4), the diffusion process for the futures price is

$$\frac{dF}{F} = \sigma(F, t)dz, \tag{11}$$

and the price of a currency call futures option, $C = C(t, X)$, can be shown to satisfy the extended Black–Scholes partial differential equation

$$\frac{\partial C}{\partial t} + \frac{1}{2}F^2\sigma^2(F,t)\frac{\partial^2 C}{\partial^2 F} - rC = 0. \tag{12}$$

20. The calculation of implied volatility from the BSGK model is an example of solving an inverse problem of options pricing. If all the variables necessary for calculating the BSGK options prices were available, one could easily calculate options prices. This is an example of a forward problem. Instead, the volatility of the underlying asset is missing, and actual options prices are used along with the BSGK formulation to find an estimate of the local volatility that minimizes the difference between the observed options prices and the BSGK options prices. This is the inverse problem.

If the option were European-style, then the terminal condition for (12) is given by equation (1),[21] subject to two conditions: that the value of a call option will equal 0 if the futures price equals 0; and that, as the futures price becomes very large, the value of the call option will equal the futures price.

The standard approach to finding the local volatility surface is to solve

$$\tilde{\sigma}(F, t) = \underset{\sigma}{\operatorname{argmin}}\left(\sum_{i,\,j}[C(X_i) - C_\theta(T_j, X_i)]^2 \right.$$

$$\left. + \sum_{i,\,j}[P(X_i) - P_\theta(T_j, X_i)]^2\right), \tag{13}$$

where T_j allows for the use of multiple maturity dates in the estimation of the local volatility surface; though for comparison to the MLN technique only one maturity date will be considered.

Given a characterization of the local volatility surface $\tilde{\sigma}(F, t)$, and the observed futures options prices, the risk-neutral PDF for the futures price for any future date can be generated from (11) via Monte Carlo simulations.

The above approach for obtaining the implied volatility surface is unstable—this is discussed in the Appendix. That is, arbitrarily small errors in the input data and errors in the numerical solution of the partial differential equation result in significant deviations in the volatility surface. Therefore, we employed regularization methods to make the inverse problem stable and robust.

4 Data and Case Study Methodology

The data consist of end-of-day settlement prices for American-style Canadian dollar futures options that have traded on the Chicago Mercantile Exchange (CME) since June 1986, and Canadian dollar $100,000 futures contracts, which have traded on the CME since May 1972.[22] Through 1996 and 1997, the average daily volume in Canadian dollar futures options was 775 contracts (a C$77 million notional amount) and 1,000 contracts (a C$100 million notional amount) respectively; trading activity was most

21. See the Appendix for a discussion of extending this approach to pricing American options.

22. Although Canadian dollar currency options are listed and traded on the Philadelphia Stock Exchange, the limited number of exercise prices and small trading volumes limit the use of these options for empirical analysis.

concentrated in periods of increased uncertainty. By way of comparison, Miville (1995) presents the results of a triennial Bank of Canada survey showing an average daily turnover for April 1995 of US$580 million in over-the-counter Canadian dollar options.

CME futures and futures options mature on a quarterly cycle of March, June, September, and December and the two nearest serial months. Futures contracts mature on the third Wednesday of the contract month, and futures options mature two Fridays prior to that date.

The nearest-to-maturity futures and futures options maturing on the quarterly cycle of March, June, September, and December are used in the empirical analysis, since these are typically the most actively traded contracts. The options contracts examined will have a term to maturity that declines from just over three months to between one and two weeks.[23]

The risk-neutral PDFs derived from the Canadian dollar futures options will be graphically and numerically analyzed to determine the consistency of the results. The risk-neutral PDFs charted refer to the density over the level of the future exchange rate. The summary statistics are presented with regard to the logarithm of the future exchange rate. If the exchange rate is a lognormal process, the risk-neutral PDF over the level of the future exchange rate will be skewed right, while the risk-neutral PDF over the logarithm will be symmetric. Thus, if the graphs are presented with respect to the logarithm of future exchange rates, the summary statistics will be consistent with the graphs; if the graphs are presented over the level of future exchange rates, they will be "tilted" right with respect to the summary statistics. Hence, if skewness is negative but relatively small the graphs over the level of future exchange rates may appear to be symmetric, but if skewness is zero the graphs may appear to be skewed right.

The numerical results will include the summary statistics and risk-neutral probabilities attached to the likelihood that the Canadian dollar will be below specific exchange rate levels on the maturity date of the option. These probabilities may be useful as early warning signals for possible policy responses, or for modifying programs for the management of

23. Options with no trading volume are removed, regardless of the existence of open interest in the contract. Prices for all contracts with open interest are established each day for "marking to market," regardless of trading activity in the contract.

exchange rate risk.[24] For example, if the current futures price of one Canadian dollar is 72 cents U.S., we present the probability that the exchange value of the Canadian dollar on the maturity date of the option will be less than 73 U.S. cents, less than 72 U.S. cents, less than 71 U.S. cents, and so on. Of course, as the futures price moves from day to day, the probability of crossing these benchmarks will change but will provide probabilistic statements with regard to future exchange rates.

Table 1 presents the probabilities for future Canadian exchange rates from the BSGK model and the MLN model for 6 March 1998. These probabilities are broadly similar, though over the range of future exchange rates presented the BSGK model tends to overestimate the probability of lower future exchange rates and underestimate the probability of higher future exchange rates compared with the MLN model.

Figure 1 presents the graphs for the MLN and the DLV models. These graphs demonstrate that, although not identical, the evidence from both sets of graphs is consistent. Thus, as the results from the MLN and the DLV approaches are broadly similar, only the results from the MLN approach will be discussed in depth.[25]

5 Case Study Results

To focus the analysis on the information content of Canadian dollar futures options, two types of events will be discussed. The first type is the release of *Monetary Policy Reports*, through which the Bank of Canada conveys its assessment of the economy and the implications for monetary policy over the medium term. The second type is changes to the operating band for overnight interest rates during the height of impact of the Mexican currency crisis and the Asian financial crisis on the Canadian dollar. These events will therefore represent the announcement effects of Bank of Canada *Monetary Policy Reports* and the effect of Bank of Canada monetary policy actions.

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24. It must be stressed that the Bank of Canada does not attempt to maintain a specific exchange rate level, nor does it attempt to re-establish a level following a period of appreciation or depreciation. If the Canadian dollar has moved significantly and appears likely to remain at those levels for some period of time, the Bank may, depending on the nature of the shock, act to change the operating band so that interest rates move the monetary conditions index (MCI) to a more appropriate level. For more on these issues see Murray, Zelmer, and Mc Manus (1997) and Freedman (1994).

25. The results for the DLV model are available upon request.

Table 1

Comparison of BSKG and MLN Cumulative Probabilities for the Canadian Exchange Rate, in U.S. Cents, on 6 March 1998

January–February 1998	**Prob($S_T \leq X$)**				
	67	**68**	**69**	**70**	**71**
Tuesday, 27 January	9 [10]	25 [28]	56 [55]	82 [79]	93 [93]
Wednesday, 28 January	10 [10]	23 [29]	57 [56]	85 [81]	94 [94]
Thursday, 29 January	16 [20]	42 [44]	73 [70]	90 [88]	96 [97]
Friday, 30 January	9 [11]	27 [31]	57 [58]	83 [81]	95 [94]
Monday, 2 February	8 [8]	23 [26]	54 [54]	82 [79]	93 [94]
Tuesday, 3 February	5 [4]	15 [17]	44 [45]	79 [76]	93 [93]

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 6 March 1998 to be less than the stated value *X*. The dates of changes to the operating band are highlighted.
BSKG probabilities are enclosed in square brackets.

5.1 Monetary Policy Reports

Bank of Canada *Monetary Policy Reports* have been issued semi-annually, each May and November, since 1995. They represent one aspect of the Bank of Canada's commitment to increased transparency in the conduct of monetary policy.[26] The release of the *Report* has become an anticipated event, as it may provide new information about the Bank's future policy intentions. For brevity, we will discuss only the three most recent *Reports*.

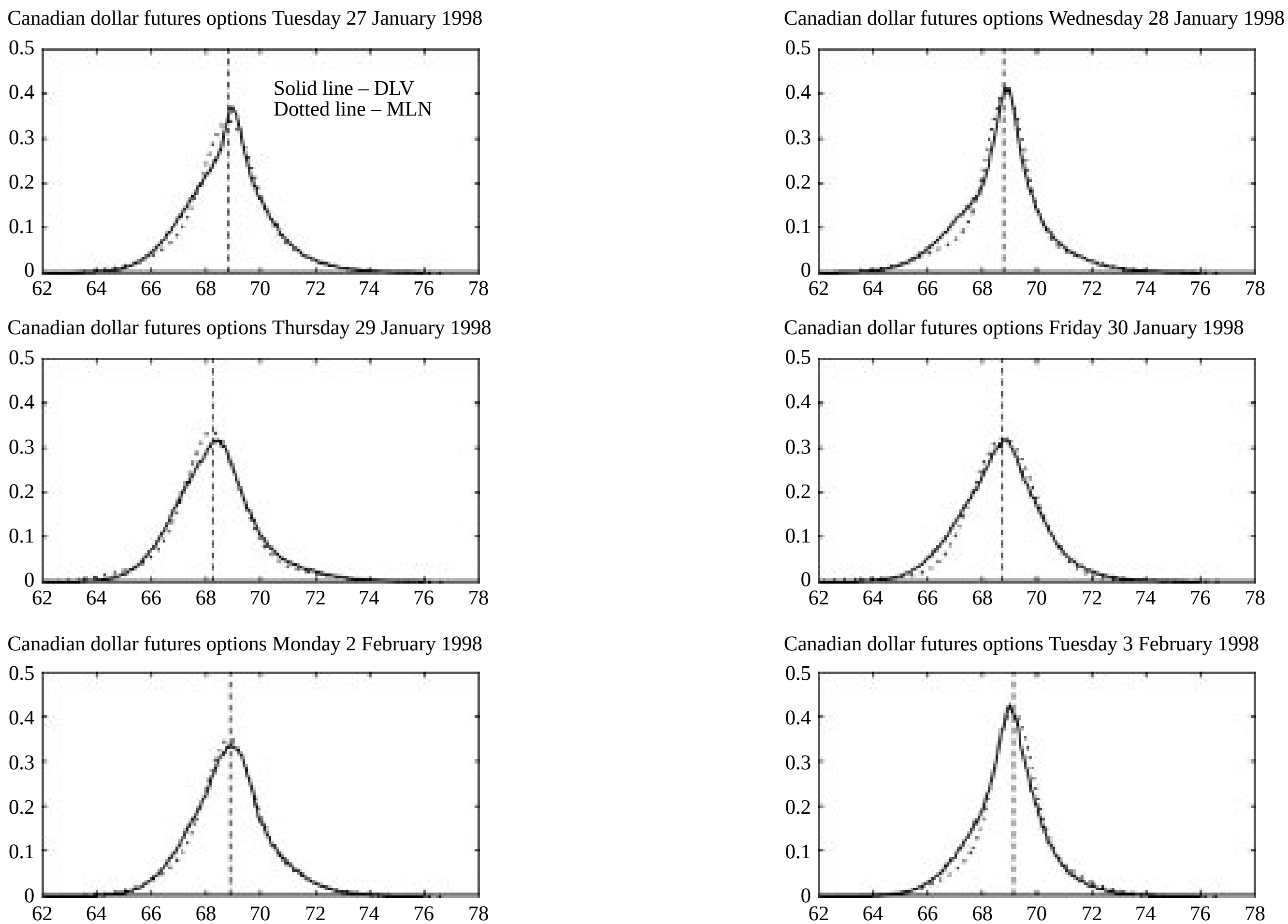
The *Reports* of November 1996 and November 1997 reiterated the medium-term monetary policy stance of the *Reports* immediately before them. The November 1996 *Report* suggested the possible need for further easing in monetary conditions over the medium term. The November 1997 *Report* reiterated the message in the May 1997 *Report* that further action to reduce the amount of monetary stimulus would be appropriate.

Between these two, the *Report* of 15 May 1997 is of particular interest because it signalled an inflection point in the Bank's policy intentions. Specifically, it indicated that a less-stimulative path for monetary

26. See Noël (1995–1996).

Figure 1

Mixture of Lognormal (MLN) and Deterministic Local Volatility (DLV) Probability Distributions



conditions would need to be promoted "…in order to maintain Canada's low trend of inflation, and thereby contribute to a durable expansion in activity consistent with the capacity of the economy to produce goods and services" (Bank of Canada, May 1997). Thus, it is the only *Report* of the three in which significant "new" information on the medium-term course for monetary policy was released.

The impact of the *Reports* on market sentiment are illustrated in Figures 2, 3, and 4, and Tables 2, 3, and 4. One can easily see that it is very difficult to generalize about their impact during this time.

In the days surrounding the November 1996 *Report*, the Canadian dollar weakened as market participants surmised that interest rates were not about to increase. The dollar strengthened prior to the November 1997 *Report*, but ended the period more or less unchanged. In contrast, on the day of release of the May 1997 *Report* the Canadian dollar strengthened significantly and remained higher than before the *Report's* release, as the Bank of Canada signalled a shift over the medium term to a less-stimulative policy stance.

5.1.1 November 1996

The impact of each of the *Reports* on annualized volatility was quite different. There was reduced volatility around the release of the November 1996 *Report*, but by the end of the review period volatility had returned to previous levels.

The *Report* noted a solid increase in economic activity in Canada in the second half of 1996. The market gave it considerable attention, because it closely followed a meeting of the Federal Open Markets Committee of the Federal Reserve Board.

Except for the day of the *Report's* release, the market was mildly optimistic about the Canadian exchange rate over the next month. The *Report's* suggestion that further reductions in interest rates may be forthcoming, with continued stimulus to economic activity, did lead to optimism about the Canadian economy. The negative tone on the day of the *Report* might be a reflection of the market's temporary reaction to the Bank of Canada's suggestion that interest rates were not about to rise, which led to increased risk-neutral weight being placed on lower future exchange rates.[27]

As the Canadian dollar and the futures price fell before and after the November 1996 *Report*, the probability of the Canadian exchange rate being below 75 U.S. cents on 5 December 1996 rose from 36 per cent to about

27. It is difficult to analyze the results for Tuesday, 12 November 1996, because of the low trading volume.

Figure 2

***Monetary Policy Report*, November 1996**

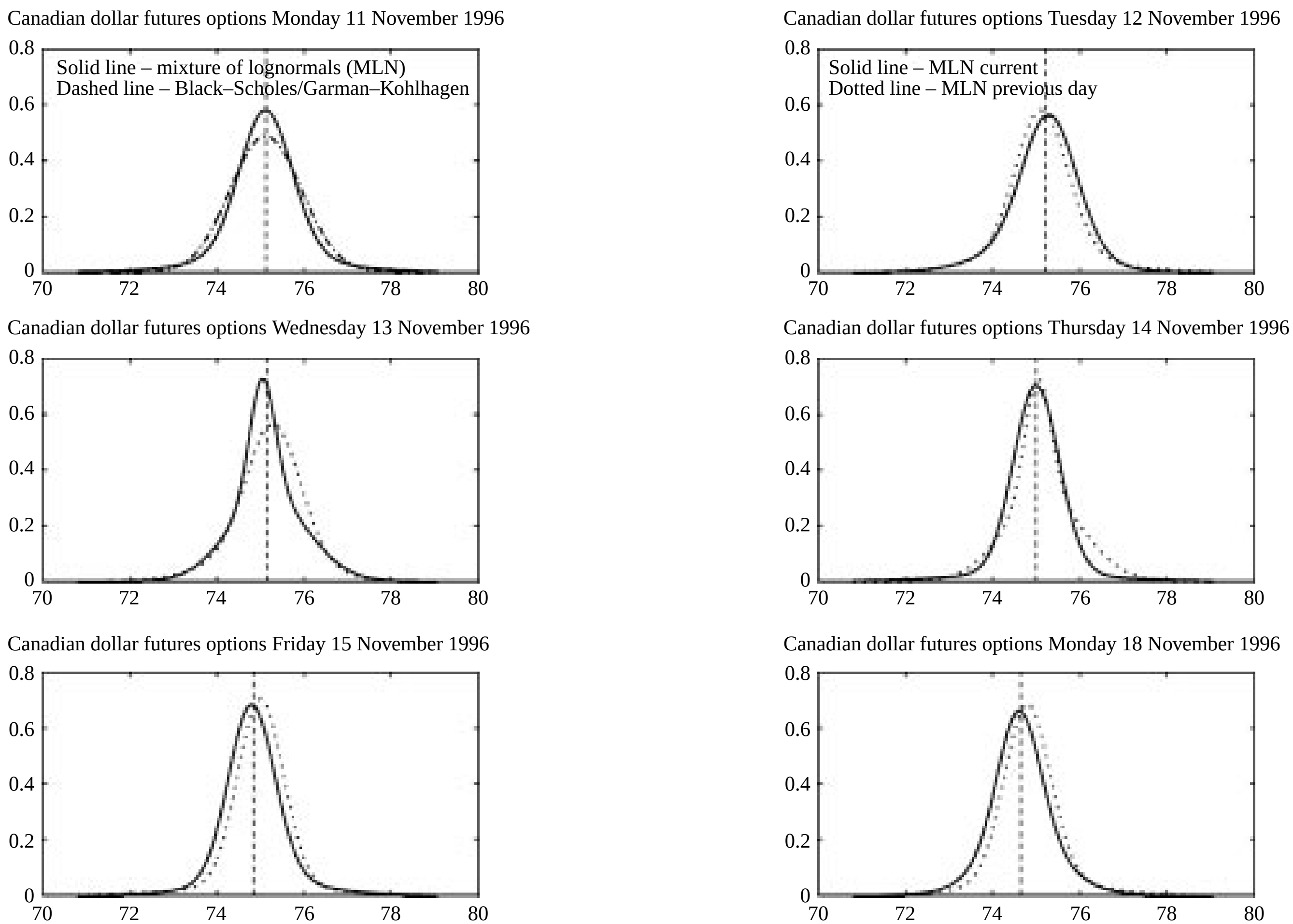


Figure 3

***Monetary Policy Report*, May 1997**

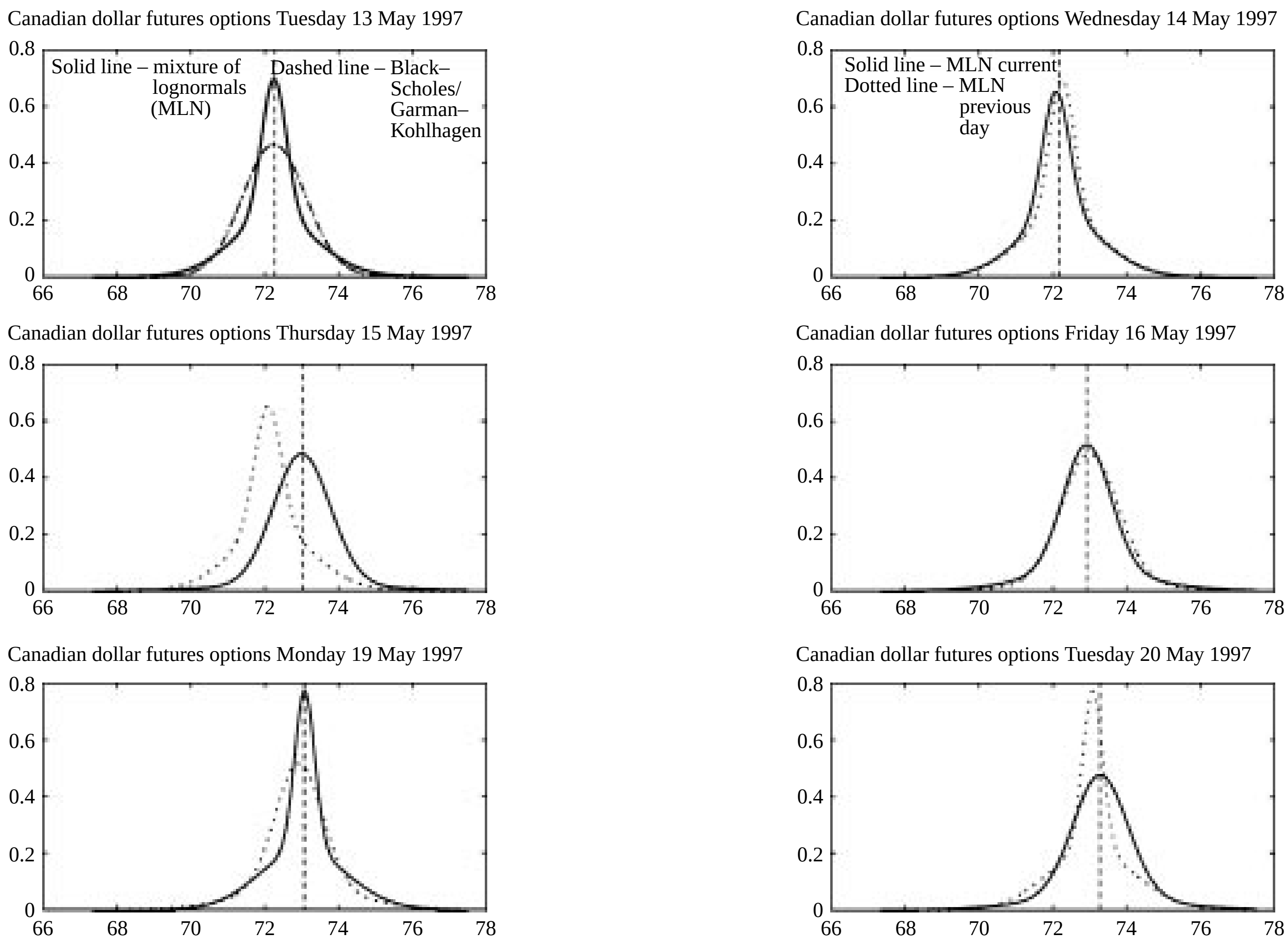


Figure 4

***Monetary Policy Report*, November 1997**

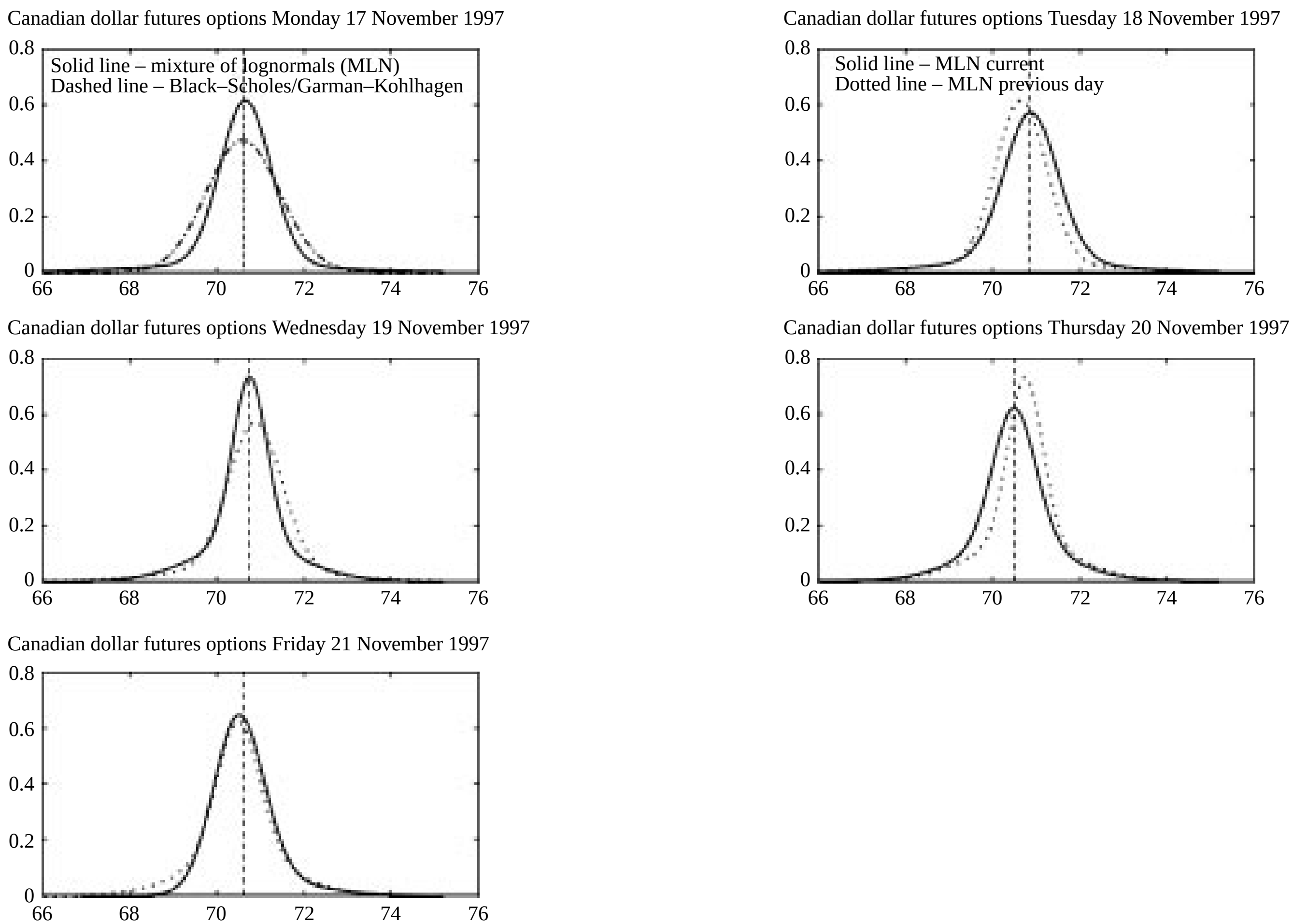


Table 2

Monetary Policy Report, 14 November 1996

November 1996	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Tuesday, 12 November	75.02	75.12	4.08	4.20	−0.431	4.530	215
Wednesday, 13 November	74.98	75.14	3.91	4.06	0.140	3.975	905
Thursday, 14 November	74.83	74.99	3.70	4.16	−0.286	9.613	767
Friday, 15 November	74.69	74.84	3.77	3.99	0.353	6.186	354
Monday, 18 November	74.47	74.65	4.10	4.21	0.121	4.216	581

Notes: Volatility is expressed as an annualized percentage. BSGK volatility corresponds to the at-the-money implied volatility. Skewness and kurtosis refer to the MLN model, as skewness and kurtosis for the BSGK model are 0 and 3 respectively. The release date of the *Monetary Policy Report* is highlighted.

Table 2a

Monetary Policy Report, 14 November 1996
Probabilities for the Canadian Exchange Rate on 5 December 1996

November 1996	Prob($S_T \leq X$)			
	73	74	75	76
Tuesday, 12 November	1	7	36	86
Wednesday, 13 November	1	7	43	87
Thursday, 14 November	2	6	50	94
Friday, 15 November	1	9	62	95
Monday, 18 November	1	15	72	96

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 5 December 1996 to be less than the stated value *X*. The release date of the *Monetary Policy Report* is highlighted.

Table 3

Monetary Policy Report, 15 May 1997

May 1997	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Tuesday, 13 May	72.16	72.27	4.59	4.87	0.026	4.550	500
Wednesday, 14 May	71.94	72.17	4.80	5.01	0.140	4.423	391
Thursday, 15 May	72.97	73.01	5.26	5.66	−0.058	8.024	1,588
Friday, 16 May	72.82	72.93	5.46	5.80	−0.027	5.905	1,486
Monday, 19 May	na	73.07	5.49	5.79	−0.033	4.559	303
Tuesday, 20 May	73.19	73.28	6.19	6.43	−0.131	6.142	601

Notes: See Table 1. Monday, 19 May 1997 was a holiday in Canada.

Table 3a

Monetary Policy Report, 15 May 1997
Probabilities for the Canadian Exchange Rate on 6 June 1997

May 1997	Prob$(S_T \leq X)$				
	70	71	72	73	74
Tuesday, 13 May	1	8	33	84	96
Wednesday, 14 May	1	8	41	85	96
Thursday, 15 May	1	2	12	50	88
Friday, 16 May	1	3	13	54	89
Monday, 19 May	0	2	11	44	87
Tuesday, 20 May	1	2	8	37	80

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 6 June 1997 to be less than the stated value X. The release date of the *Monetary Policy Report* is highlighted.

Table 4

***Monetary Policy Report*, 19 November 1997**

November 1997	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Monday, 17 November	70.59	70.62	5.34	6.26	−0.051	10.365	429
Tuesday, 18 November	70.71	70.84	5.59	6.26	−0.389	7.544	275
Wednesday, 19 November	70.62	70.73	5.15	5.62	−0.122	5.620	228
Thursday, 20 November	70.39	70.50	5.56	5.91	−0.018	5.025	230
Friday, 21 November	70.52	70.61	4.94	5.03	0.744	4.901	679

Notes: See Table 1.

Table 4a

***Monetary Policy Report*, 19 November 1997**
Probabilities for the Canadian Exchange Rate on 5 December 1997

November 1997	Prob$(S_T \leq X)$				
	68	**69**	**70**	**71**	**72**
Monday, 17 November	2	4	17	70	96
Tuesday, 18 November	1	3	13	58	93
Wednesday, 19 November	1	3	13	68	94
Thursday, 20 November	1	4	23	77	96
Friday, 21 November	0	0	18	75	96

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 5 December 1997 to be less than the stated value X. The release date of the *Monetary Policy Report* is highlighted.

72 per cent. Further, the probability of the Canadian exchange rate being between 74 and 75 U.S. cents rose from 29 per cent to about 57 per cent. It was therefore considered likely that the Canadian dollar exchange rate would stay below 75 U.S. cents and that the exchange rate would be between 74 and 75 U.S. cents on 5 December 1996.

5.1.2 May 1997

The May 1997 *Report* was associated with an increase in volatility over the review period. Specifically, volatility, kurtosis, trading volume, the spot exchange rate, and the futures price increased significantly on the day of its release.

The *Report* noted broad-based strength in the economy, and that over the medium term, as slack in the economy is absorbed, less-stimulative economic conditions would be required. Since there was no indication of the timing of the move to less-stimulative monetary conditions, it would appear that the uncertainty over future values of the Canadian dollar rose, as the spot Canadian exchange rate rallied.

The *Report* led to a market tone best described as relatively "neutral" to weakly "bearish." This may have been because of the significant increase in the Canadian dollar exchange rate and the futures price, and the uncertainty over the timing of any increase in interest rates. The market's tone following the *Report* might reflect the announcement effect that less-stimulative monetary conditions would be forthcoming, even though interest rates remained unchanged as the Canadian dollar significantly strengthened.

The *Report* led to an increased probability that the exchange rate would end up higher on 6 June 1997. The probability that the exchange rate would be below 72 U.S. cents fell from 33 per cent to about 8 per cent, while the probability that the exchange rate would be below 73 U.S. cents fell from about 84 per cent to about 37 per cent. Further, the probability that the exchange rate would be between 72 and 73 U.S. cents fell from 51 per cent to 29 per cent, while the probability that the exchange rate would be between 73 and 74 U.S. cents rose from 12 per cent to 43 per cent.

5.1.3 November 1997

The volume of trade in Canadian dollar futures options around the November 1997 *Report* was quite low, making interpretation of the data problematic. Over the period as a whole, very little changed though volatility appears to have decreased. The Canadian dollar and the futures price were more or less unchanged, as were the probabilities associated with future Canadian exchange rates.

The *Report* was released at a time of soft economic data on growth in Canada and of increasing uncertainty in financial markets because of the emerging crisis in Asia. Though the *Report* noted a possible need for less-stimulative monetary conditions, there was no sense of immediacy, and the market apparently concluded that the Bank of Canada was not about to increase rates.

The *Report* had little impact on the probability of the Canadian exchange rate being below 70 U.S. cents, though there was a slight increase in the probability of the exchange rate being below 71 U.S. cents on 5 December 1997, possibly associated with the decrease in volatility.

5.1.4 General observations

In summary, *Monetary Policy Reports* apparently have their biggest impact on the distribution of future Canadian exchange rates when they reveal fundamentally “new” information, such as an inflection point in monetary policy intentions. It would also appear that the tone of the market cannot necessarily be interpreted independently of the level of the futures price, which is the mean of the risk-neutral PDF. On a day-to-day basis, the changes in the probabilities associated with a particular level of the exchange rate are mainly associated with changes in the futures price. Marginal contributions also come from changes in the higher moments of the risk-neutral PDF.

5.2 International financial crises and market participants’ views on the Canadian dollar

Both the Mexican currency crisis and the Asian financial crisis led to a flight to quality of capital into the U.S. market, and to a subsequent appreciation of the U.S. dollar relative to most other currencies, including the Canadian dollar. At the times of the two crises, there were some similarities in the Canadian macroeconomic environments, such as lower inflation and economic growth relative to the United States, but there were also several important macroeconomic differences. It would therefore be interesting to compare the market’s perceptions of future Canadian exchange rates and their reaction to Bank of Canada policy actions during these two crises.

The Bank of Canada policy actions are focused on changes in the operating band for overnight interest rates,[28] the primary day-to-day policy tool of the Bank of Canada.[29] Our empirical analysis considers increases in the operating band that were perceived as being in reaction to persistent weakness in the Canadian dollar, which are identified by newspaper accounts prior to 22 February 1996 and by Bank of Canada press releases after 22 February 1996.[30]

5.2.1 The Mexican currency crisis

The Mexican financial crisis was particularly harsh on countries that were perceived as having debt problems, such as Italy, Sweden, and Canada. Other factors also weighed on the Canadian dollar, such as a sharp narrowing of interest rate spreads with the United States and heightened concerns over the fiscal situation, including the federal budget that would be presented on 27 February 1995 and concerns over the political situation.[31] There were other signs of a “bearish” tone toward Canada: the downward movement of the currency; a negative tone in the Canadian bond market with the selling of Canadian dollar denominated assets in global markets; and rising yields on Government of Canada bonds (Stinson 1995a).

The Mexican currency crisis had its greatest impact on the Canadian dollar from 9 January 1995 to 20 January 1995. Over this period, the operating band for overnight interest rates was raised three times (10, 12, and 17 January) by 50 basis points each time, and the Bank Rate rose twice (10 and 17 January). The interest rate increases were interpreted as being in defence of the weakening Canadian dollar, even though the Bank of Canada made no public pronouncements (Stinson 1995b). The increases in the overnight interest rates also followed prior movement in short-term interest rates. Therefore, the Bank was ratifying increases in interest rates that had already been made by the market.

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28. Note that the options-pricing models discussed were developed under the assumption that interest rates are constant. It is therefore inconsistent to attribute the evolution of the risk-neutral PDFs solely to the markets’ changing perceptions of future exchange rates, assuming everything else is unchanged.

29. See the *Bank of Canada Review* (1996) for discussions of the introduction and use of the operating band for overnight interest rates.

30. On that date, the Bank Rate was redefined as the upper rate of the operating band for overnight interest rates, and changes to the Bank Rate were thereafter accompanied by press releases. The press releases that have accompanied changes in the Bank Rate are available at the Bank of Canada’s Web site: < http://www.bank-banque-canada.ca>.

31. See Zelmer (1996) for a more detailed analysis of the Canadian economic situation around this time.

The analysis of the impact of the Mexican currency crisis can be found in Figure 5 and Table 5. Over several days in the midst of the Mexican currency crisis, the risk-neutral PDFs were skewed left, suggesting an underlying “bearish” tone with regard to the future value of the Canadian dollar.

Since Canadian interest rates were above U.S. rates at that time, the Canadian dollar was already expected to depreciate, according to the covered interest parity relationship. Together with the skewed-left distributions, an even greater depreciation than that already expected was possible, despite the increases in the operating band. In fact, during the crisis the price of Canadian dollar futures contracts fell almost one full U.S. cent, the risk-neutral PDFs maintained the negative skew, and the degree of uncertainty surrounding future exchange rates appeared to increase.

Figure 5 and Table 5 show that the Bank of Canada’s changes to the operating band for overnight interest rates did not measurably change the market’s view of future Canadian exchange rates at the height of the impact of the Mexican currency crisis on the Canadian dollar. However, the Bank’s intervention might have prevented the currency from depreciating even more. The continuing fall in the price of Canadian dollar futures contracts over the period led to an increase in the probability that the exchange rate would be below 71 U.S. cents on 3 March 1995, from 51 per cent to 71 per cent. The probability that the Canadian exchange rate would be below 70 U.S. cents rose from 26 per cent to 47 per cent, and the probability that the Canadian exchange rate would be below 69 U.S. cents rose from 11 per cent to 24 per cent.

By mid-February, overnight interest rates had risen by 200 basis points, and the market tone towards future Canadian exchange rates was no longer “bearish.” However, volatility and the probability of a large move in the Canadian exchange rate by 3 March remained fairly high. The tone in the market turned pessimistic, and the Canadian dollar weakened somewhat following the release of a report by Moody’s Investors Service on 16 February stating that Canada’s credit rating was under review. The resulting sell-off of Canadian dollar denominated assets quickly led the Bank to effect a 50-basis-point increase in the operating band for overnight interest rates that afternoon (Stinson 1995d). Short-term interest rates moved concurrently with the increase in the operating band. Therefore, this move was not merely a ratification of market interest rates, although the selling of Canadian dollar assets was building pressure for higher rates.

Figure 6 and Tables 6 and 6a present the analysis of the risk-neutral PDFs around 16 February 1995. The figures and tables show that the market was apparently “neutral” toward future Canadian exchange rates—except for the day of the Moody’s report, when a “bearish” tone was evident.

Figure 5

Mexican Currency Crisis

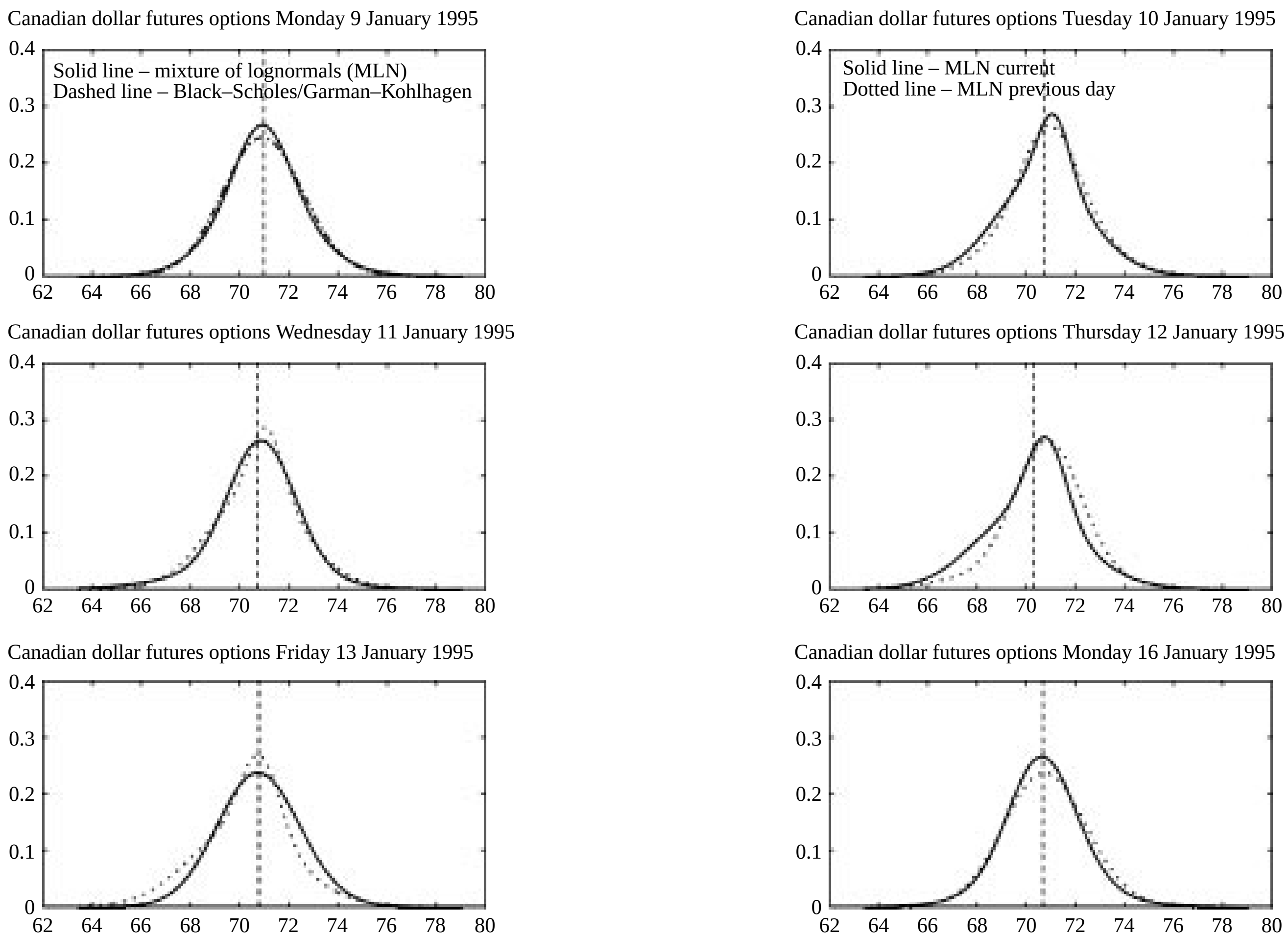
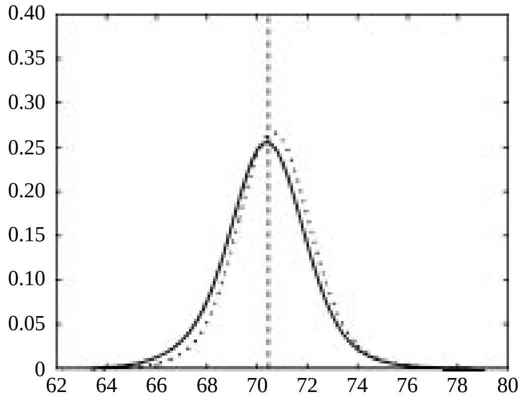


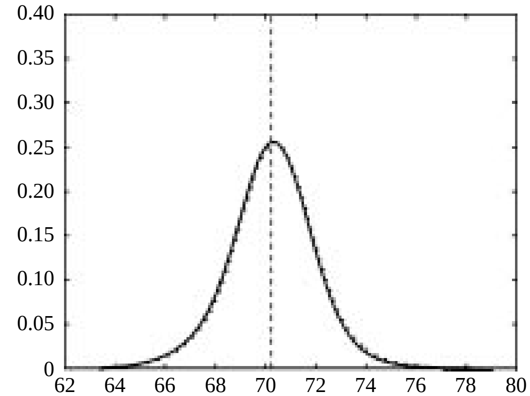
Figure 5 (cont'd)

Mexican Currency Crisis

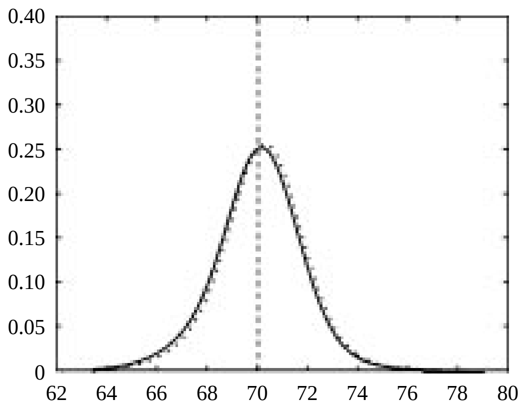
Canadian dollar futures options Tuesday 17 January 1995



Canadian dollar futures options Wednesday 18 January 1995



Canadian dollar futures options Thursday 19 January 1995



Canadian dollar futures options Friday 20 January 1995

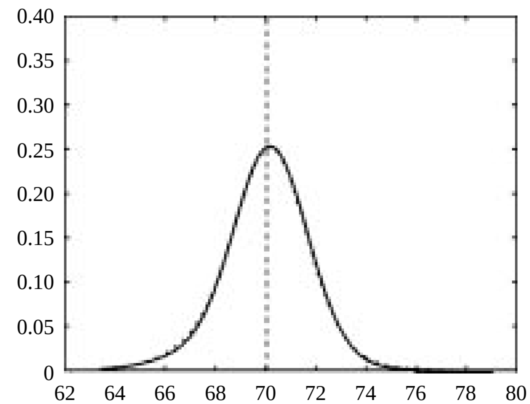


Table 5

Mexican Currency Crisis, January 1995

January 1995	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Monday, 9 January	70.99	70.98	5.98	6.11	0.002	3.583	1,918
Tuesday, 10 January	70.92	70.78	6.19	6.34	−0.125	3.457	1,956
Wednesday, 11 January	70.88	70.74	6.31	6.65	−0.498	4.865	1,083
Thursday, 12 January	70.55	70.32	6.86	7.14	−0.257	3.620	1,040
Friday, 13 January	70.87	70.79	6.44	6.45	−0.025	3.028	1,292
Monday, 16 January	70.93	70.69	6.31	6.45	0.000	3.882	150
Tuesday, 17 January	70.49	70.36	6.83	7.01	−0.129	3.845	917
Wednesday, 18 January	70.36	70.23	6.85	7.06	−0.220	3.848	1,072
Thursday, 19 January	70.20	70.04	7.08	7.31	−0.289	3.868	1,269
Friday, 20 January	70.29	70.05	7.00	7.23	−0.433	4.057	618

Notes: See Table 1. The dates of changes to the operating band are highlighted.

The Moody’s report had a very short-lived impact on the market’s views about future Canadian exchange rates. This is illustrated in Table 6a: The probability that the Canadian exchange rate would fall below 71 U.S. cents on 3 March 1995 was 40 per cent on the Tuesday before the report, and rose to 58 per cent on the day of the report before falling back to 36 per cent two business days after the report. The Bank of Canada’s pre-emptive action appears to have helped contain the market reaction. Considering that the Canadian dollar futures option contract was only 18 days away from maturity, these movements are quite large. However, it must be remembered that the asset underlying the futures option contract is a Canadian dollar futures contract, not the Canadian dollar itself.

Table 5a

Mexican Currency Crisis, January, 1995
Probabilities for the Canadian Exchange Rate on 3 March 1995

January 1995	Prob($S_T \le X$)					
	68	**69**	**70**	**71**	**72**	**73**
Monday, 9 January	4	11	26	51	75	89
Tuesday, 10 January	6	15	30	54	79	91
Wednesday, 11 January	6	13	30	55	78	92
Thursday, 12 January	11	22	39	64	85	94
Friday, 13 January	5	14	32	55	76	90
Monday, 16 January	5	13	33	59	81	93
Tuesday, 17 January	8	20	41	65	84	94
Wednesday, 18 January	9	22	43	68	86	95
Thursday, 19 January	11	25	47	72	88	96
Friday, 20 January	11	24	47	71	88	96

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 3 March 1995 to be less than the stated value X. The dates of changes to the operating band are highlighted.

5.2.2 The Asian financial crisis

In contrast to the time of the Mexican currency crisis, the Canadian economic situation during the Asian financial crisis was much stronger, characterized by low inflation, steady growth, a strong fiscal performance, and a relatively stable political situation. These factors contributed to Canadian short-term interest rates being roughly 100 basis points below U.S. short-term rates. In fact, given Canada's relatively strong fundamentals, many analysts considered the Canadian dollar to be undervalued, not weak.[32]

32. See Milner and Drohan (1998).

Figure 6

Moody's Debt Report

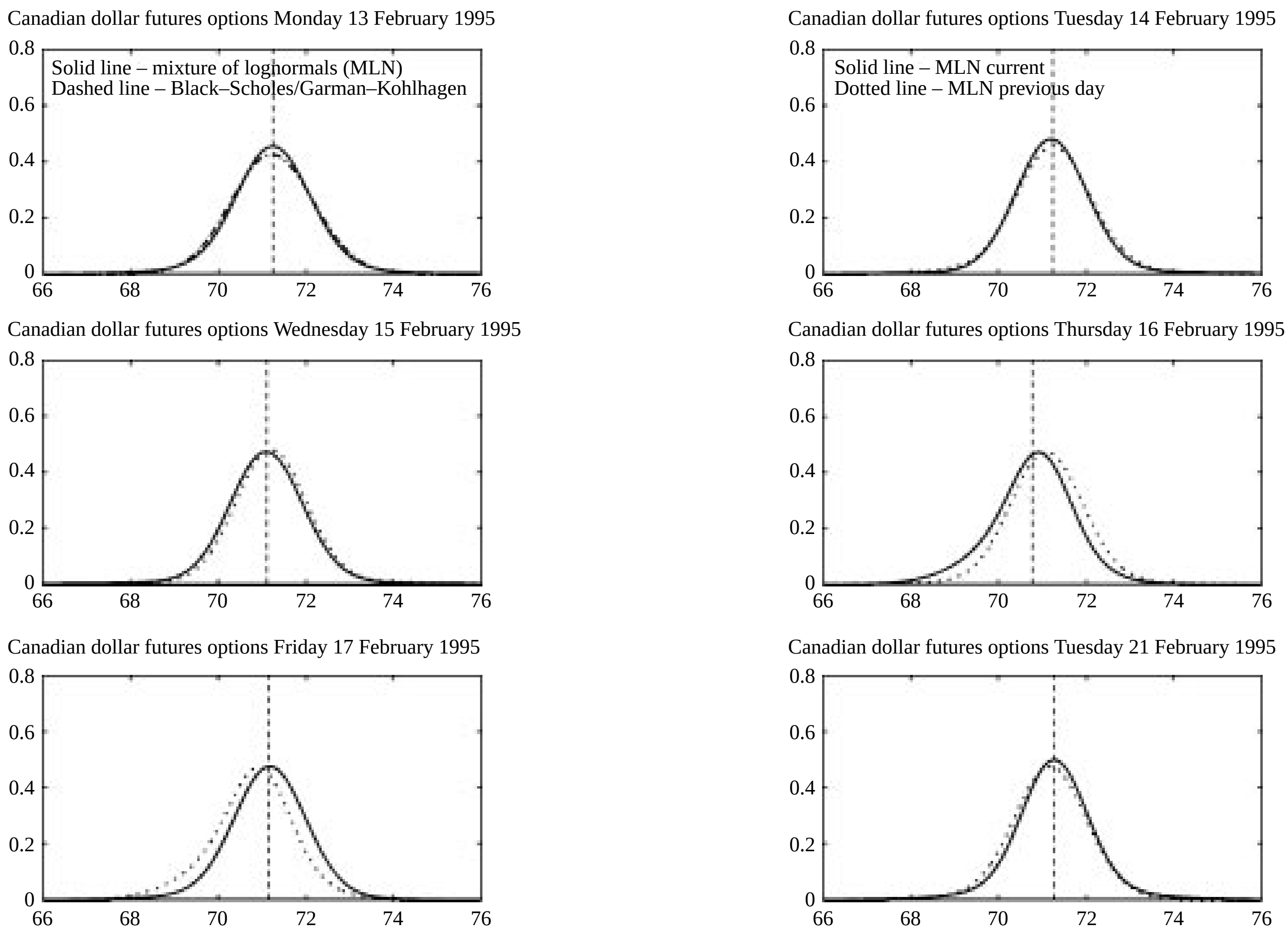


Table 6

Moody's Debt Review, 16 February 1995

February 1995	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Tuesday, 14 February	71.34	71.24	5.94	6.39	0.365	8.516	907
Wednesday, 15 February	71.27	71.10	6.00	6.29	−0.130	6.061	651
Thursday, 16 February	70.80	70.78	6.44	6.61	−0.333	3.733	371
Friday, 17 February	71.29	71.15	6.26	6.40	−0.286	4.143	1,205
Tuesday, 21 February	71.35	71.28	7.53	7.91	−0.036	5.402	1,451

Notes: See Table 1. The date of the change to the operating band is highlighted.

Table 6a

Moody's Debt Review, 16 February 1995
Probabilities for the Canadian Exchange Rate on 3 March 1995

February 1995	Prob$(S_T \leq X)$				
	69	70	71	72	73
Tuesday, 14 February	1	7	40	82	97
Wednesday, 15 February	1	10	45	85	98
Thursday, 16 February	4	19	58	92	99
Friday, 17 February	1	9	42	84	98
Tuesday, 21 February	1	7	36	81	97

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 3 March 1995 to be less than the stated value *X*. The date of the change to the operating band is highlighted.

Also unlike the Mexican currency crisis, yields on the Canadian bond market did not generally rise during the Asian crisis. This would appear to suggest that, even though the Canadian dollar was depreciating, market participants were not very pessimistic about Canada's economic prospects. The situation may have changed over the few days prior to the 30 January change in the operating band, since bond yields in Canada ticked up slightly,[33] but there was no sustained weakness in the bond market, nor was there a sense of crisis.

Though many fundamental indicators were supportive of the Canadian dollar, a growing current account deficit and declining commodity prices indicated that the Canadian dollar also faced some definite risks. The declining commodity prices had a particularly significant impact on what were perceived to be commodity-based currencies such as the Canadian and Australian dollars. However, the weaknesses in these currencies were considered to be temporary, but possibly persistent.[34]

The Asian financial crisis had its greatest impact on Canada through mid- to late January 1998, when the Canadian dollar fell to its lowest historic levels relative to the U.S. dollar. Therefore, we will analyze the events leading up to and following the increase in operating band on 30 January 1998. As with the analysis of the Moody's report, the increase in the operating band on 30 January was not a ratification of market interest rates, and may therefore be said to have surprised the market.

The results for the Asian crisis are presented in Figure 7 and Table 7. They indicate that the tone in the market turned "bearish" early in the week. Following a half-cent decline in the Canadian futures price between Wednesday and Thursday, the risk-neutral PDF became essentially symmetric, which may suggest that market participants were essentially "neutral" with regard to future Canadian exchange rates from that point onward.

Volatility apparently rose over the week prior to the increase in the operating band but declined following the increase. The spot exchange rate and the futures price also rose by almost 1 cent from the day before the Bank Rate increase to a few days after. Following the increase in the futures price and the exchange rate, the market's tone over future exchange rates was basically "neutral" and associated with a decline in volatility.

Figure 7 presents the graphs associated with the Asian crisis. One can definitely notice a decline in volatility on 3 February relative to 30 January.

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33. See Bloomberg 1998.
34. See Little 1998.

Figure 7

Asian Financial Crisis

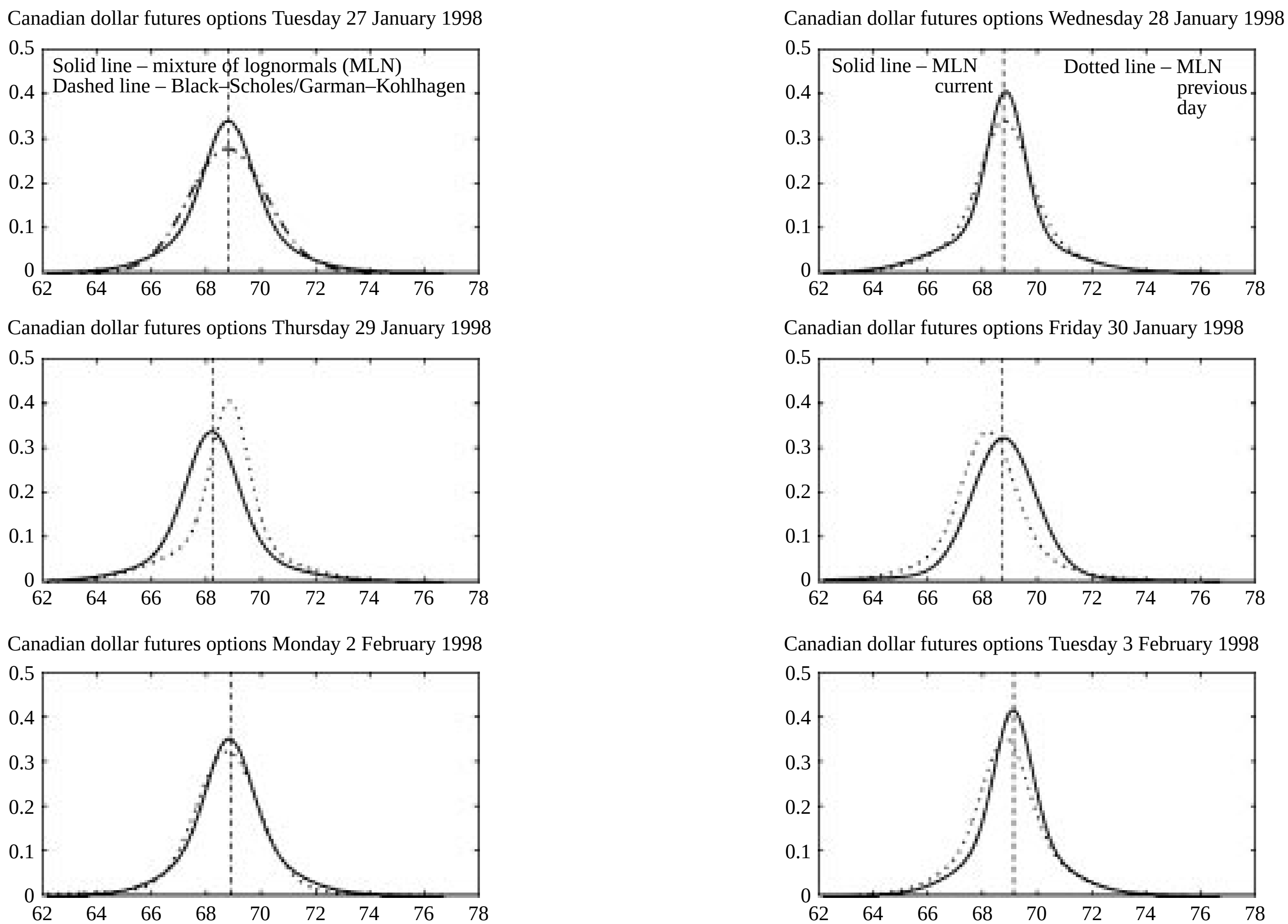

Table 7

Asian Financial Crisis, January 1998

January–February 1998	Spot C$	Futures price	BSGK volatility	MLN volatility	MLN skewness	MLN kurtosis	Trading volume
Tuesday, 27 January	68.70	68.83	6.44	6.76	−0.034	4.484	807
Wednesday, 28 January	68.57	68.78	6.38	6.96	−0.197	5.178	482
Thursday, 29 January	68.25	68.25	6.81	7.23	0.016	5.103	539
Friday, 30 January	68.70	68.74	6.72	7.52	−0.580	8.787	356
Monday, 2 February	68.82	68.89	6.64	6.95	0.022	4.261	130
Tuesday, 3 February	69.08	69.15	6.07	6.40	−0.006	4.655	1,269

Notes: See Table 1. The date of the change to the operating band is highlighted.

The volatility on 30 January disguises to some extent the skewness, although a negative skew is evident.

The dynamics in the days around 30 January were also reflected in the risk-neutral probabilities attached to the Canadian exchange rate for 6 March 1998. During the days preceding 30 January, the probability of the future exchange rate falling below 68 U.S. cents rose from 25 per cent to 42 per cent, while the probability of falling below 67 U.S. cents went from 9 per cent to 16 per cent. Following the change in the operating band, the probabilities fell to 15 per cent that the exchange rate would fall below 68 U.S. cents, and to 5 per cent that the exchange rate would fall below 67 U.S. cents.

5.2.3 General observations

There was a significant difference of opinion around the time of the Asian crisis that was not apparent around the time of the Mexican crisis. The macroeconomic fundamentals underlying the Canadian dollar at the time of the Asian crisis were for the most part sound. Also, given the negative interest rate differential vis-à-vis the United States, the Canadian dollar was expected to appreciate toward the futures price, even though the currency was depreciating over the period. Therefore, some analysts considered the weakness in the Canadian dollar to be temporary and eventually self-

Table 7a

Asian Financial Crisis, January 1998
Probabilities for the Canadian Exchange Rate on 6 March 1998

January–February 1998	Prob$(S_T \leq X)$				
	67	**68**	**69**	**70**	**71**
Tuesday, 27 January	9	25	56	82	93
Wednesday, 28 January	10	23	57	85	94
Thursday, 29 January	16	42	73	90	96
Friday, 30 January	9	27	57	83	95
Monday, 2 February	8	23	54	82	93
Tuesday, 3 February	5	15	44	79	93

Notes: The probabilities are the risk-neutral probabilities that the market assigns on the given date for the exchange rate on 6 March 1998 to be less than the stated value X. The date of the change to the operating band is highlighted.

correcting. The other opinion in the market suggested that an increase in the operating band was necessary in order to restore the Bank of Canada's credibility at a time of historic weakness in the Canadian dollar. Unfortunately, Canadian dollar futures options do not support one view or the other.

In contrast to the changes in the operating band during the Mexican crisis, the change in the Bank Rate on 30 January 1998 did apparently change the market's views on future Canadian exchange rates. This may demonstrate that Bank of Canada actions that are a ratification of previous market movements in interest rates do not have a significant impact on market sentiment (except perhaps to prevent a further deterioration in the situation). Conversely, actions that do surprise the market convey new information and thus can have significant effects.

Conclusions

The results demonstrate several interesting issues in the analysis of risk-neutral PDFs for future Canadian exchange rates. Monetary policy statements or actions that suggest a change in policy intentions, or which were not anticipated by the market, convey new information that was not previously embedded in the prices for Canadian dollar futures options. Hence, these statements or actions tend to have a greater impact on the risk-neutral PDF for future Canadian exchange rates than do statements or actions that reiterate the policy stance or that are anticipated by the market.

Given the analysis above, we must also mention that the options data used have some limitations. For example, the market is often quite thin; on several occasions only a few strike prices were traded. With a 50-basis-point step between each strike, the information that can be extracted from futures options is limited, as we are unable to obtain a great deal of information about the shape of the density function over the range of available data. In these situations, it might be useful to employ methods that are better suited to situations where there is limited data availability, such as the method of maximum entropy used by Buchen and Kelly (1996).

On other occasions, Canadian dollar futures options will be actively traded one day and virtually inactive the next day. This complicates the analysis of market participants' views of future Canadian exchange rates, as the distributions that result on thinly traded days are typically very close to symmetric. Therefore, one could observe a "bearish" tone one day and a "neutral" tone the next, before returning to a "bearish" tone, mostly because the options on the second day were just not very informative. This suggests that estimates of the higher moments of the distribution can be sensitive to the amount of data available.

The results demonstrate that certain preconditions are necessary in order for futures options to provide information over and above that contained in futures prices and forward rates. One precondition is the need for a wide number of strike prices in an actively traded market. Another precondition is that the futures option not be too close to maturity, since as the option nears maturity its implied volatility will fall. These preconditions tend to limit the applicability of analyzing Canadian dollar currency futures options on a day-to-day basis, since this market tends to be active sporadically. However, when this market is active, it is usually at times that are significant to the market as a whole.

Appendix 1

Deterministic local volatility

The BSGK diffusion model (3) is extended by allowing the U.S. and Canadian interest rates and the local volatility to be deterministic functions:

$$\frac{dS}{S} = \{r(t) - r_c(t)\}dt + \sigma(S, t)dz. \qquad \text{(A1.1)}$$

The price of a currency call option, $C = C(t, X)$, can be shown to satisfy the extended Black–Scholes partial differential equation,

$$\frac{\partial C}{\partial t} + \frac{1}{2}S^2\sigma^2(S, t)\frac{\partial^2 C}{\partial^2 S} + \{r(t) - r_c(t)\}S\frac{\partial C}{\partial S} - r(t)C = 0. \qquad \text{(A1.2)}$$

If the option is an American-style option, then its price C satisfies well-known boundary and final conditions.[1] If the option is a European-style option, then the final condition for the backwards-in-time parabolic equation (15) is given by equation (1). In this case the boundary conditions are given by

$$C(T, X)|_{S=0} \text{ and } C(T, X) \sim S\exp\left(-\int_0^T r_c(\tau)d\tau\right)$$

in the limit that S tends to infinity.

Black's model can be used to describe futures prices, F:

$$\frac{dF}{F} = \sigma(F, t)dz. \qquad \text{(A1.3)}$$

The price of a call option on a futures contract satisfies equation (A1.2), where S and r_f are replaced by F and r, respectively. Furthermore, given the local volatility surface $\sigma(F, t)$, the risk-neutral PDF for F can be generated from (A1.3) via Monte Carlo simulations.

A realistic estimate of the local volatility surface is found by solving the inverse problem for futures options. The inverse problem for futures options and currency options consists of finding the local volatility that minimizes the pricing errors between the theoretical options prices and the market options prices (see Section 3). Unfortunately, the process of calculating American options prices for many different strikes and maturities is data-intensive. Hence, an approximate procedure is employed for adjusting market prices of American futures options. The BSGK prices for the corresponding European futures options are calculated using

1. See Wilmott, Dewynne, and Howison (1993).

historical volatility. The Bjerksund–Stensland (1993) analytic approximation for the prices of American futures options are also calculated using the same historical volatility. The market prices of American futures options are adjusted by subtracting off the difference between the American and European options prices that were calculated using the constant historical volatility.[2]

As a further aid to reducing the computation time the adjoint problem is considered. The adjoint problem converts the BSGK partial differential equation to an adjoint differential equation, and enables the simultaneous finding of vectors of theoretical prices for European calls and puts, given a vector of strike prices

$$\{X_i\}_{i = 1 \ldots N}$$

with the same maturity. (The method increases the performance of calibration N-fold.) The adjoint partial differential equation for (A1.2) can be reduced[3] to

$$\frac{\partial C}{\partial t} + \frac{1}{2} X^2 \sigma^2(S, t) \frac{\partial^2 C}{\partial^2 X} + \{r_c(t) - r(t)\} X \frac{\partial C}{\partial X} - r_c(t) C = 0, \quad \text{(A1.4)}$$

with the final condition $C(T, X)|_{S = S_0} = \max(S_0 - X, 0)$ and the boundary conditions,

$$C(t, 0) = S \exp\left(-\int_0^t r_c(\tau) d\tau\right) \text{ and } C(t, \infty) = 0.$$

The standard approach to finding the local volatility surface is to solve

$$\hat{\sigma} = \underset{\sigma}{\operatorname{argmin}}\left(\sum_{i, j} [C(X_i) - C_\theta(T_j, X_i)]^2 + \sum_{i, j} [P(X_i) - P_\theta(T_j, X_i)]^2\right). \quad \text{(A1.5)}$$

However, the above inverse problem is unstable with respect to the local volatility. In other words, arbitrarily small errors in the input data and the numerical solution of the partial differential equation result in significant deviations in the volatility surface. Special regularization methods are employed to make the inverse problem stable and robust. In particular, an

2. This adjustment is similar to the control variate technique used in lattice methods for American options pricing—see Hull and White (1988).

3. See Levin (1998) and Bouchouev and Isakov (1997).

extension of Tikhonov's regularization method [Tikhonov and Arsenin (1977), Levin (1988, 1989, 1998), and Levin and Filatova (1995)] is used. The regularization method consists in minimizing the functional

$$\begin{aligned} M_\alpha(\sigma) = & \sum_{i,j} \omega_{ij}[C(X_i) - C_\theta(T_j, X_i)]^2 \\ & + \sum_{i,j} \eta_{ij}[P(X_i) - P_\theta(T_j, X_i)]^2 + \alpha_1 \|\nabla\sigma\|^2 \\ & + \alpha_2 \|\sigma - \sigma_{hist}\|^2 + \lambda \max(\sigma - \sigma_{min}, 0)^2, \end{aligned} \tag{A1.6}$$

where ω_{ij} and η_{ij} are positive weights, σ_{hist} is the historical volatility, σ_{min} is a small minimal level of local volatility, λ is a large penalty parameter that ensures that local volatility is positive, α_1 is a regularization parameter that defines the "smoothness" of the local volatility surface, and α_2 is a regularization parameter that controls the deviation of the local volatility from the historical volatility. The weights ω_{ij} and η_{ij} are assigned large values for strikes and maturities that correspond to more-liquid options. The ∇ operator is the gradient operator, and $\|\bullet\|$ denotes the L^2 norm.

Mixtures of lognormal distributions

To price American-style options using the mixture of lognormal distributions, we calculate the price of the American-style options as a weighted average of the upper and lower bounds from (7). The American-style options prices used in the minimization problem are given by:

$$C_\theta(0, X) = \omega_{it}\bar{C}_A(0, X) + (1 - \omega_{it})\underline{C}_A(0, X)$$

$$P_\theta(0, X) = \omega_{it}\bar{P}_A(0, X) + (1 - \omega_{it})\underline{P}_A(0, X) \text{ where } i = 1,2. \tag{A1.7}$$

Following Melick and Thomas (1997), the weights applied will depend on whether the particular option is in-the-money or out-of-the-money. That is, $i = 1$ for an in-the-money call or put option, while $i = 2$ for out-of-the-money call or put options. This increases the number of parameters that must be estimated by two for both the Black–Scholes model and for the MLN model.

The skewness and kurtosis statistics tend to be sensitive to how far into the tails one goes in their calculation. To that end, analytic expressions for skewness and kurtosis are utilized for the mixture of lognormal distributions.

To find the analytic expressions for skewness and kurtosis, suppose that the PDF of the random variable x is given by

$$q[\tilde{x}] = \sum_{i=1}^{k} \phi_i q_i[\tilde{x}], \tag{A1.8}$$

where $\sum_{i=1}^{k} \phi_i = 1$ and $0 \le \phi_i \le 1$ for all i. Then the mean of x is given by

$$\mu = \sum_{i=1}^{k} \phi_i q \mathrm{E}_i[\tilde{x}], \tag{A1.9}$$

where $\mathrm{E}_i[x](\equiv \mu_i)$ is the expectation with respect to the PDF $q_i[\tilde{x}]$. The central moments of x are given by:

$$\begin{aligned} \mathrm{E}[(\tilde{x}-\mu)^n] &= \sum_{i=1}^{k} \phi_i \mathrm{E}_i[(\tilde{x}-\mu)^n] \\ &= \sum_{i=1}^{k} \phi_i \sum_{j=0}^{n} \binom{n}{j} (\mu_i - \mu)^j \mathrm{E}_i\left[(\tilde{x}-\mu_i)^{n-j}\right]. \end{aligned} \tag{A1.10}$$

If the PDF $q_i[\tilde{x}]$ is normally distributed with mean μ_i and variance σ_i^2, i.e.,

$$q_i[\tilde{x}] \sim N(\mu_i, \sigma_i^2),$$

then:

$$\begin{aligned} \mathrm{E}_i\left[(\tilde{x}-\mu_i)^n\right] &= \frac{n!}{2^{n/2}(n/2)!}\sigma_i^2 && \text{if } n \text{ is even} \\ &= 0 && \text{if } n \text{ is odd.} \end{aligned} \tag{A1.11}$$

In the MLN model, the PDF for the exchange rate was assumed to be a mixture of two lognormal distributions. Thus, the logarithm of the exchange rate is distributed as a mixture of two normals. Hence, equations (20) through (23) imply that the mean, variance, skewness, and kurtosis of the logarithm of the exchange rate process[4] are given by:

4. The variance, skewness, and kurtosis of the logarithm of the exchange rate are given by: $\sigma^2 = \mathrm{E}[(\log(\tilde{S}(T)-\mu)^2]$, Skew = $(\mathrm{E}[(\log(\tilde{S}(T)-\mu)^3])/\sigma^3$, and Kurt = $(\mathrm{E}[(\log(\tilde{S}(T)-\mu)^4])/\sigma^4$, respectively.

$$\mu = \phi_1\mu_1 + \phi_2\mu_2$$

$$\sigma^2 = \phi_1\sigma_1^2 + \phi_2\sigma_2^2 + \phi_1\phi_2(\mu_1 - \mu_2)^2$$

$$\sigma^3\mathrm{Skew} = \phi_1\phi_2(\mu_1 - \mu_2)\left[3\left(\sigma_1^2 - \sigma_2^2\right) + (\phi_2 - \phi_1)(\mu_1 - \mu_2)^2\right]$$

$$\sigma^4\mathrm{Kurt} = 3\left(\phi_1\sigma_1^4 + \phi_2\sigma_2^4\right) + 6\phi_1\phi_2(\mu_1 - \mu_2)^2$$

$$\left[\phi_2\sigma_1^2 + \phi_1\sigma_2^2\right] + \phi_1\phi_2(\mu_1 - \mu_2)^4\left(\phi_1^3 + \phi_2^3\right).$$

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