

Discussion

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The Melick and Thomas paper highlights the importance of the two main issues associated with the construction of probability density functions (PDFs) using options prices: the adjustment of risk-neutral PDFs for maturity dependence (or rather contract-switching); and the introduction of confidence intervals. The issue of how to construct confidence intervals is particularly interesting.

Risk-neutral PDFs can be used in practice to infer the probability of:

- central bank movements;
- European Monetary Union convergence;
- market crashes, trading ranges, breakouts, etc. (particularly for equity markets).

However, it is important to bear in mind that, since they are risk-neutral, one is never completely certain that a change in the PDFs reflects a change in expectations or in risk preferences.

The remainder of my comments can be organized into three cautions with respect to the data that the author should consider when modelling risk-neutral PDFs. The points are summarized as follows.

The Pitfalls of Low-Vega and High-Gamma Options

Vega is the first partial derivative of the option price with respect to the option's volatility parameter (σ) or, less formally, the rate of change in the value of the option with respect to the volatility of the underlying asset. Low-vega options are short-dated, deep-out-of-the-money options. As a

result, these options typically trade at a very small premium relative to the bid–offer spread. A small change in the price of the underlying asset can lead to large changes in implied volatility. This explains why short-dated options often show large implied volatilities. When using these low-vega options to estimate a risk-neutral PDF, one may observe large swings in the PDF, which are an artifact of small premiums relative to the bid–offer spread.

Gamma is the second partial derivative of the option's price with respect to the price of the underlying asset. Less formally, one can consider it the rate of change in the option's price with respect to the delta of the option—the first partial derivative of the option price with respect to the price of the underlying asset. This measure is analogous to the convexity measure in the fixed-income world. High-gamma options are short-dated and very close-to-the-money.

The issue is that gamma hedging is notoriously difficult to perform. As a result, these options are more expensive in the market (they carry a risk premium), which relates only to the hedging cost, not to market expectations. Thus, using high-gamma options may be misleading because prices might not reflect probabilities of certain outcomes but rather the hedging expense. This leads me to question the ability of the technique Melick and Thomas use to extract bimodal densities from the data.

Volatility Associated with Economic Events

The timing of economic announcements tends to follow a deterministic pattern. This pattern may introduce a bias into the development of a constant-maturity series from exchange-traded options. Realized volatility (as opposed to implied volatility) appears to depend greatly on the timing of economic events; as a result, this volatility pattern might be an interesting way to correct for time patterns.

Volatility Smiles and Skews

There are many explanations for the volatility smile (or smirk), which means that in a volatility curve short-dated and long-dated options tend to exhibit higher implied volatility than do intermediate-dated options. Note that implied volatility is essentially another way to express the premium of the option (with only two unknowns, price and volatility, at least in the Black–Scholes formula, one solves for the implied volatility given the option premium). As a result, the volatility smile suggests that short-dated

and long-dated options are relatively more expensive. Some explanations for this are:

- extreme volatility;
- gapping markets;
- leptokurtotic, or fat-tailed PDFs; the leptokurtosis may represent unhedgeable risks. This suggests to me that there may be risk premiums in the options premiums, implying that there is no riskless hedge for these instruments. This would explain their higher implied volatility—or higher prices.