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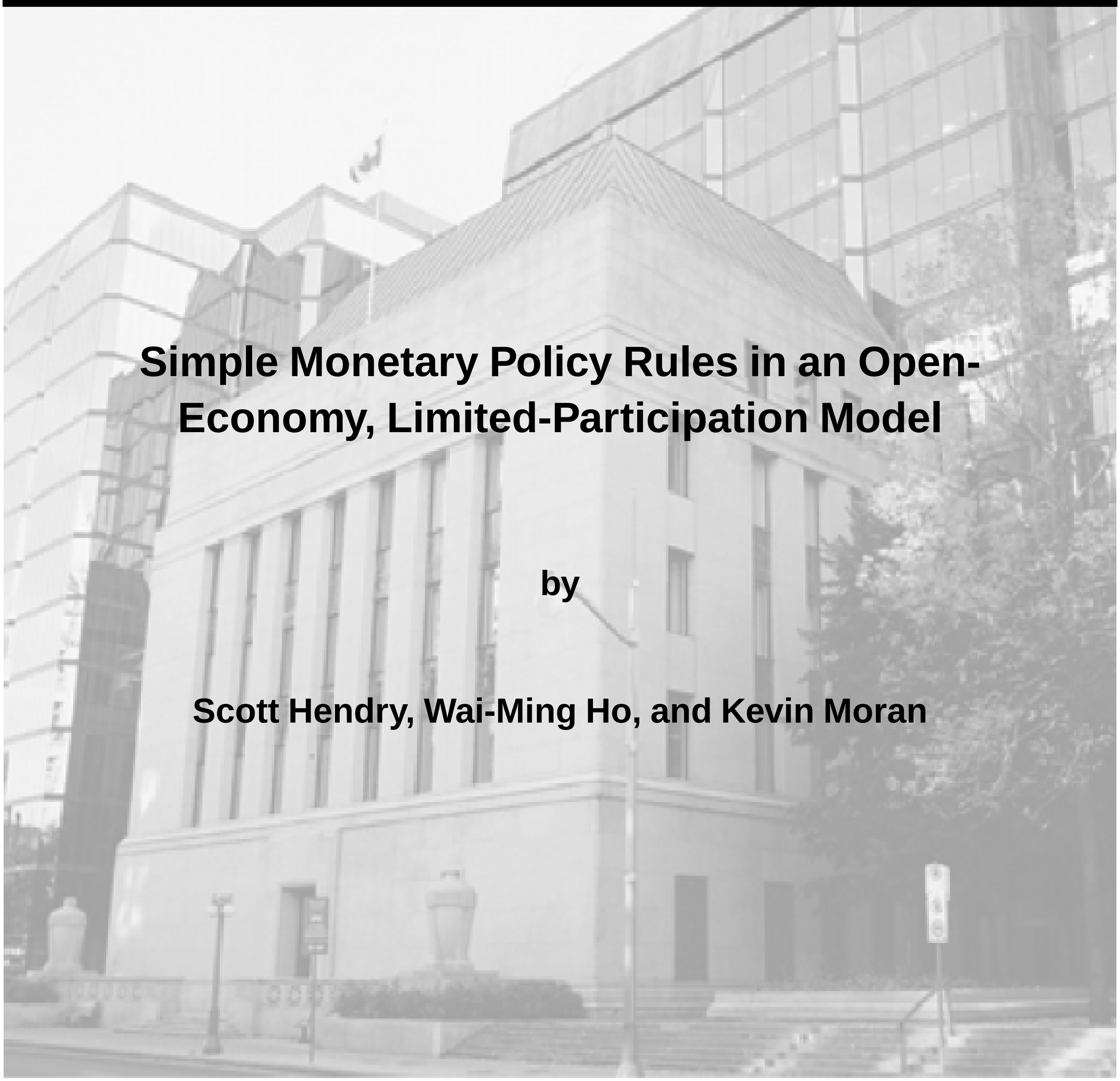

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Abstract

The authors assess the stabilization properties of simple monetary policy rules within the context of a small open-economy model constructed around the limited-participation assumption and calibrated to salient features of the Canadian economy. By relying on limited participation as the main nominal friction that affects the artificial economy, the authors provide an important check of the robustness of the results obtained using alternative environments in the literature on monetary policy rules, most notably the now-standard “New Keynesian” paradigm that emphasizes rigidities in the price-setting mechanism.

The authors’ analysis identifies general principles to which a rule should adhere to possess favourable stabilization properties. The rule should direct monetary authorities to increase nominal interest rates significantly when lagged interest rates are already high. By contrast, upward pressures on output (and perhaps also on inflation) should lead to decreases in interest rates. Further, monetary policy should be essentially unconcerned with exchange rate movements. In addition, responding to future inflation, rather than the current rate, does not generate significant welfare improvements.

While some of these principles are similar to those obtained using alternative environments, the recommendation that monetary policy should lower rates when output or inflation is pushing upward is more specific to limited-participation models. This recommendation is linked to the fact that, in such models, expected rises in inflation lead the financial system to contract aggregate loanable funds and push economic activity downward, thus embedding a negative correlation between inflationary and output pressures in the model economy. In that sense, the authors’ analysis might be interpreted as the study of an economy in which financial markets have limited flexibility to react to shocks and how this limited flexibility impinges on the choice of a “good” monetary policy rule.

JEL classification: E52, E44, E58, F31
Bank classification: Monetary policy framework; Transmission of monetary policy

Résumé

Les auteurs étudient les propriétés stabilisatrices de règles de politique monétaire simples dans le cadre d'un modèle de petite économie ouverte à participation limitée qui est étalonné en fonction des principales caractéristiques de l'économie canadienne. En faisant de la participation limitée la friction centrale dans la sphère nominale de l'économie modélisée, les auteurs testent la robustesse des résultats obtenus pour d'autres paradigmes dans les travaux consacrés à ces règles, en particulier le paradigme des nouveaux économistes keynésiens, maintenant répandu, qui met l'accent sur les rigidités entravant le processus d'établissement des prix.

Au terme de leur analyse, les auteurs cernent les principes généraux qu'une règle doit respecter pour posséder des propriétés stabilisatrices. Ainsi, les autorités monétaires devraient relever fortement les taux d'intérêt nominaux si les taux retardés sont déjà élevés. Par contre, l'existence de pressions à la hausse sur la production (et peut-être aussi sur l'inflation) devrait les amener à réduire les taux d'intérêt. En outre, elles ne devraient pas se soucier des fluctuations du taux de change. Enfin, la prise en compte du taux d'inflation anticipé, plutôt que du taux observé, dans la fonction de réaction des autorités ne génère pas de gains de bien-être notables.

Si certains des principes énoncés ressemblent à ceux que font ressortir d'autres paradigmes, il n'en reste pas moins que la recommandation d'abaisser les taux d'intérêt en cas de hausse de la production ou de l'inflation est plus caractéristique des modèles à participation limitée. En effet, dans ces modèles, l'augmentation attendue de l'inflation pousse le système financier à comprimer le volume total du financement et entraîne un ralentissement de l'activité, établissant de la sorte dans l'économie modélisée une corrélation négative entre les pressions inflationnistes et les pressions s'exerçant sur la production. Les auteurs se trouvent en un sens à analyser le cas d'une économie dans laquelle les marchés financiers disposent de peu de latitude pour réagir aux chocs, ainsi que l'incidence de ce manque de flexibilité sur le choix d'une « bonne » règle de politique monétaire.

Classification JEL : E52, E44, E58, F31
Classification de la Banque : Cadre de la politique monétaire; Transmission de la politique monétaire

1. Introduction

This paper assesses the stabilization properties of simple monetary policy rules in the context of a small open-economy model calibrated to salient features of the Canadian economy. The model is constructed around the limited-participation assumption, which postulates that a temporary segmentation in financial markets may prevent, sometimes for extended periods, liquid funds from travelling between the goods market and the financial market. The monetary non-neutrality that this friction introduces into the artificial economy potentially gives monetary policy (defined here as a rule that links the nominal interest rate to the value of various economic variables) a role in stabilizing economic fluctuations. We consider several types of such monetary policy rules, with the type of a rule defined as the list of variables to which monetary authorities respond (the deviations of current inflation and output from their steady-state values, for example), whereas the exact magnitude of the responses identify a specific rule within a type.

The paper shares with several existing studies the general objective of identifying rules with good stabilization properties in open-economy environments.[1] The other studies, however, are built around the "New Keynesian" paradigm, in which the central friction that affects the artificial economy is the assumption of sticky prices or wages. Our paper is the first evaluation of monetary policy rules in an open-economy setting using a model that places the limited-participation assumption at the heart of the analysis.[2]

In addition to the hypothesis of limited participation, the model economy we use features two traded goods and one non-traded good, as well as the opportunity for domestic financial intermediaries to access foreign financial markets. The economy is assumed to be small relative to the rest of the world, so that the foreign prices of the two traded goods are exogenous to the domestic economy. The artificial economy is affected by shocks to technology, preferences, and foreign-determined prices and interest rates. The absence of frictions on the price-setting decisions implies that all domestic prices are perfectly flexible at all times.

The simulations we undertake first separate rules that lead to stable and unique equilibria from those that lead to indeterminate or explosive equilibria.[3] The latter cases are interpreted as

1. Among many others, see Devereux (2000), Ravenna (2000), Ghironi (2000), and Batini, Harrison, and Millard (2003).
2. Christiano and Gust (1999) use a limited-participation model to discuss monetary policy rules, but restrict their analysis to a closed-economy setting, and do not identify specific loss-minimizing rules.
3. In stable unique equilibria, episodes of self-fulfilling expectations (sunspots) are not possible, whereas they may occur when an equilibrium is indeterminate. Further, the economy always converges back to its initial state, whereas it may permanently diverge from it when the equilibrium is explosive.

instances where monetary policy has a destabilizing, rather than a stabilizing, effect on the economy. Second, specific cases among the rules that lead to stable and unique equilibria are identified as minimizing economic loss, which is computed under three alternative measures.

Our analysis identifies general principles a rule should follow to achieve favourable outcomes. The rule should lead monetary authorities to increase nominal interest rates significantly when inflation is pushing upwards or when lagged interest rates are already high. By contrast, monetary policy should respond to output by increasing rates when output is under decreasing pressure. Further, monetary policy should not react directly to movements in the exchange rate. Finally, it is important to react to current inflationary pressures, instead of only to expected future inflationary pressures. Although these principles are robust across alternative definitions of the loss function, the precise numerical values of the coefficients are not: monetary policy should thus follow these principles without relying mechanically on the magnitude of the responses drawn from the use of one particular definition of economic loss.

The appropriateness of a rule that features strong responses to inflation and lagged interest rates but no response to exchange rates has been suggested before by researchers who have used models derived from the “New Keynesian” paradigm. Along those dimensions, our analysis supports the validity of these principles in a model built on an alternative source of non-neutrality. Note that to instruct monetary policy not to react directly to exchange rate movements does not imply that the open-economy environment of the analysis is itself irrelevant. Rather, it suggests, as Taylor (2001) points out, that the transmission channel from exchange rates to inflation is successfully internalized by a rule that responds to consumer price index (CPI) inflation. Further, most of the welfare improvements that arise from choosing the better rules that we identify result from reductions in the variability of the consumption of imported goods. As such, the inclusion of open-economy features has important consequences for our assessment of what a “good” monetary policy rule is.

On the other hand, the recommendation that monetary policy should increase rates when output is already low is more specific to limited-participation models. In those models, an expected rise in inflation leads depositors to withdraw funds from financial markets, which restricts the supply of loans and increases their price, causing output reductions. Were monetary authorities to lower rates in response to these output decreases, the stimulating impact of the loosening would further exacerbate already-increasing inflation, which in itself would require interest rate increases, thereby deteriorating output further when the intent was to limit its decrease. In our simulations, these second-round effects are quantitatively significant, to the extent that an increase in rates when output falls reduces the pressure on financial markets and inflation enough to actually alleviate the negative pressures on output.

The recommendation to refrain from alleviating output decreases by reducing rates resembles similar discussions among policy-makers about the course monetary policy should take when an economy is affected by supply shocks. This similarity stems from the fact that, within our limited-participation environment, financial markets have limited flexibility to modify their lending supply in response to shocks. Our analysis can thus be interpreted as an attempt to determine which type of monetary policy rule may be adequate for an economy that is affected by recurring episodes of severe rigidity in the supply of credit.

This paper is organized as follows. Section 2 discusses the related literature on limited participation and simple monetary policy rules. Section 3 describes the details of our model, and section 4 the manner in which the model is calibrated and solved. Section 5 assesses the model's properties, to develop intuition about its mechanisms and provide a basis upon which the normative results that follow can be evaluated. Section 6 reports detailed results on the stabilization properties of various rules; section 7 discusses and synthesizes these results, highlighting the dimensions along which the introduction of limited participation impinges on the analysis. Section 8 offers some conclusions.

2. Limited Participation and Monetary Policy Rules

2.1 Limited participation and the liquidity effect

Vector autoregressions (VARs), introduced in applied macroeconomic analysis by Sims (1980), have been used extensively to identify and quantify the effects of monetary policy shocks. On balance, the literature has shown that contractionary monetary policy shocks cause the following responses of economic variables: (i) short-term interest rates rise on impact and remain above their initial level for a few quarters; (ii) narrow money (the liquidity of financial markets) declines on impact and its return to pre-shock levels approximately mirrors that of short-term rates; (iii) output suffers a significant decline shortly after the impact period and remains low for several periods; (iv) prices do not respond at first (and may actually increase), but they eventually experience significant declines that fade away only gradually; (v) these responses are staggered, so that little overlap exists between the movements of the variables that move first (interest rates) and those that move last (prices); and, (vi) nominal and real exchange rates undergo persistent appreciation.[4]

4. Papers that establish and discuss the closed-economy subset of these facts include Christiano, Eichenbaum, and Evans (1996, 1999), Leeper, Sims, and Zha (1996), and Bernanke and Mihov (1998). Exchange rates are analyzed in Eichenbaum and Evans (1995) and Grilli and Roubini (1996). Papers that verify the general results apply to Canadian data include Fung and Gupta (1997), Cushman and Zha (1997), and Fung and Kasumovich (1998).

Quantitative models that emphasize the New Keynesian paradigm (price or wage rigidities embedded in an optimizing framework) can generate responses of output and prices that are mostly in line with this evidence.[5] Further, open-economy extensions of these models can replicate, at least in a qualitative fashion, the movements in exchange rates (real and nominal) that accompany monetary policy shocks.[6] The negative correlation between short-term interest rates and liquidity measures has, however, been harder to replicate with models that stem from that paradigm, because the Fisher relation (which equates the nominal rate to the sum of the real rate and of the expected rate of inflation) always holds in these models. Typically, new injections of liquidity are associated with increases in expected inflation; when the Fisher relation holds, this positive correlation extends to the nominal interest rate.

A negative correlation between short-term interest rates and measures of liquidity (narrow money) arises more naturally in models that rely on limited participation to generate monetary non-neutralities. The core assumption of these models is that there exists, possibly for an extended period, a segmentation between the goods sector and the financial sector that makes it difficult for liquidity to flow from one sector to the other. When the central bank unexpectedly injects liquidity in the financial sector, it creates a financial imbalance between that sector (where liquidity is relatively abundant) and the goods sector (where it is relatively scarce): the relative scarcity of liquidity in financial markets puts downward pressure on nominal interest rates, resulting in the negative correlation identified in the empirical literature.[7] Households, which could eliminate the imbalance by transferring some of their financial assets between sectors, are barred from doing so by assumption. In effect, it is assumed that households access or modify their financial portfolios at a lower frequency than financial intermediaries and firms do, and so households enjoy only "limited" participation in financial markets.[8]

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5. The New Keynesian paradigm is described by Clarida, Galí, and Gertler (1999) and King (2000). Among numerous recent contributions to this literature, see Erceg, Henderson, and Levin (2000) and Christiano, Eichenbaum, and Evans (2001).
6. The literature that analyzes open-economy extensions to the New Keynesian framework includes Obstfeld and Rogoff (1995), Betts and Devereux (2000), Kollmann (2001), and Chari, Kehoe, and McGrattan (2002).
7. In contrast to models of the New Keynesian paradigm, the Fisher relation holds only in expectation in limited-participation models: the realized nominal rate will deviate from its Fisherian fundamentals, sometimes for extended periods of time, because of the imbalance of funds between the two sectors. Note also that limited-participation models can more easily rationalize the decrease in corporate profits that accompanies contractionary monetary shocks; see Christiano, Eichenbaum, and Evans (1997) for a discussion.
8. The limited-participation paradigm originates in work by Grossman and Weiss (1983), Rotemberg (1984), and Lucas (1990). The structure of production and the timing of financial flows that we use are closer, however, to the contributions of Christiano (1991) and Fuerst (1992). Recent examples of the use of limited participation in closed-economy, quantitative settings include Christiano and Gust (1999), Cooley and Quadrini (1999), and Dhar and Millard (2000). Open-economy extensions of the limited-participation paradigm include Grilli and Roubini (1992), Ho (1993), Schlagenhauf and Wrase (1995a,b), and Sill and Wrase (1999).

2.2 Simple monetary policy rules

Concurrent with the recent development of optimizing models that emphasize the price rigidities of the New Keynesian paradigm (the bulk of the literature) or limited participation, there has been a great increase in the number of papers that study monetary policy rules.

The starting point of this literature (Taylor 1993a) is itself a synthesis of work already completed that assessed the properties of various types of simple rules to guide monetary policy, within the previous generation of macroeconomic models. These models were, in general, larger than the more recent ones in the New Keynesian literature, and contained fewer references to explicit optimizing behaviour. They did appeal, however, to rational-expectations concepts for their numerical solutions (see Taylor 1993b for an illustration).

In turn, the review essay in Taylor (1993a) generated an extensive body of work that analyzes the properties of simple monetary policy rules within smaller scale, fully optimizing, rational-expectations models. This literature, the progress of which is summarized in the volume edited by Taylor (1999a), continues to expand. Among the issues studied within that literature, the extent to which forward-looking rules, rather than backward-looking ones, deliver better monetary policy outcomes,[9] the likelihood that the rules employed by monetary authorities exacerbate economic fluctuations rather than help contain them, and the properties of rules that react directly to exchange rates have generated some of the liveliest debates.[10]

In parallel with these quantitative-theoretic advances, econometric estimations have recently established that these rules can fit, to a significant degree, the course of actual monetary policy actions across several episodes of monetary policy history.[11]

These results suggest that there might be conclusions about the appropriateness of simple rules that are robust to various sources of uncertainty, or views, about the exact way in which modern economies work. Although considerable energy has been expended in verifying this robustness

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9. Generically, backward-looking rules would direct monetary authorities to respond to deviations of *current* or *past* inflation from target, while a forward-looking rule would suggest they should respond to deviations of *expected future* inflation from target.
10. On the benefits of forward-looking rules, see Batini and Haldane (1999). On the properties of rules that contain explicit references to exchange rates, or other variables that represent world linkages, see Ball (1999), Devereux (2000), and Batini, Harrison, and Millard (2003). For a discussion of the link between the response of monetary authorities to deviations of inflation from its target, on the one hand, and the global stability of the economy, on the other, see Rotemberg and Woodford (1999).
11. Some of this evidence is obtained using casual econometric analysis (Taylor 1993a, 1999b). Other researchers (Clarida, Galí, and Gertler 1998, 2000) base their results on formal econometric methodology.

proposition across different specifications of the New Keynesian paradigm, no exhaustive assessment exists of the properties of simple rules within limited-participation models.

Monetary policy rules, which link the nominal interest rate decisions by monetary authorities to the value of various other economic variables, might affect the economy differently within a limited-participation environment. First, recall that nominal interest rates may deviate significantly and for extended periods of time from their Fisherian fundamentals. Since a rule links nominal rates to other economic variables, the departure from Fisherian fundamentals introduces a wedge that may affect the stabilization properties of various rules. As stated earlier, the limited-participation framework may be interpreted as an analysis of an economy where the aggregate supply of loanable funds reacts only in a limited manner to most shocks, which introduces an important constraint on the conduct of monetary policy and thus on the appropriateness of various policy rules. Such factors suggest that any examination of the robustness of rules should include simulations conducted using limited-participation models. This paper's main contribution is to provide such a robustness check with a quantitative, calibrated open-economy model.

Finally, the high degree of openness of the Canadian economy requires that monetary policy rules be studied within open-economy environments calibrated to the specifics of the Canadian economy before recommendations for such rules are introduced into the Bank of Canada's decision-making process. A second contribution of this paper is thus to help provide policy prescriptions that are of practical relevance to Canadian monetary policy.[12]

3. The Model

The artificial economy we consider is first characterized by the segmentation between the goods and the financial sectors. Second, the economy features extensive links with the rest of the world, through the presence of two distinct traded goods and the opportunity to access a foreign bond market. The economy we describe is, however, small relative to foreign markets. The prices of the two traded goods, as well as the interest rate of foreign bonds, are therefore taken as given by the agents in the economy. Third, all markets are perfectly competitive.[13]

12. Studies of monetary policy rules couched within the specific policy environment of Canada include Armour, Fung, and Maclean (2002) and Côté et al. (2003), which document the stabilization properties of various rules using larger-scale models with incomplete optimizing and rational behaviour, and Ravenna (2000) and Ghironi (2000), whose small-scale, fully optimizing, and rational models are used to assess the inflation-targeting experience of Canada since the introduction of explicit targets in 1991.

13. The literature usually uses monopolistic competition structures to introduce market power and pricing decisions that are consistent with price or wage rigidities. Because the main source of monetary non-neutrality in this model is the segmentation in financial markets, we can retain the convenience of perfect competition.

The first traded good (good 1) is produced domestically and either consumed, invested, or exported; the other traded good (good 2) is imported. The domestic economy also produces another, non-traded good (good 3). Domestic production requires the use of physical capital and labour. Importers require no production inputs: they act as intermediaries, buying goods in foreign markets and transporting them back to the domestic economy. By bringing together several such features of actual open economies into one environment, the model extends the reach of the existing open-economy, limited-participation literature.[14]

Money is introduced into the model by imposing a cash-in-advance constraint on certain household purchases, a standard strategy in the limited-participation literature. This contrasts with the introduction of real money balances in the utility function, the modelling strategy most often followed in the New Keynesian literature. When open-economy models are analyzed, the use of cash-in-advance constraints provides an intuitive determination of the nominal exchange rate, as the relative price that will set foreign exchange markets in equilibrium.[15]

The presence of physical capital in the model, as well as the inclusion of adjustment costs that restrict its flow from one sector to the next, is another distinguishing feature of our modelling strategy. The literature that assesses the properties of monetary policy rules, either in closed-economy or open-economy settings, usually abstracts from physical capital.

3.1 Households

The representative household seeks to maximize lifetime expected utility, subject to a number of constraints. The optimization problem it must solve is expressed as follows:

$$\underset{\substack{c_{1t+k},\, c_{2t+k},\, c_{3t+k} \\ n_{t+k},\, M^c_{t+k+1},\, M^d_{t+k+1} \\ I_{1t+k},\, I_{3t+k}\big|_{k=0}^{\infty}}}{MAX} \quad E_t \sum_{k=0}^{\infty} \beta^{t+k} \cdot u\left(c_{1t+k}, c_{2t+k}, c_{3t+k}, n_{t+k}, \frac{M^c_{t+k+1}}{M^c_{t+k}}\right), \qquad (1)$$

14. The models in Schlagenhauf and Wrase (1995a,b) do not allow domestic agents to borrow on foreign financial markets. The model in Ho (1993) allows firms to borrow abroad, but within a very stylized model with no capital or choice of labour. None of the existing open-economy, limited-participation models allows for the existence of a non-traded good.
15. In open-economy models with money in the utility function, the nominal exchange rate is determined by assuming that the law of one price holds on a subset of goods (as in the so-called “producer pricing” settings, see Obstfeld and Rogoff 1995), or by appealing to an intertemporal balance of payments equilibrium (as in the “pricing to market” settings, see Chari, Kehoe, and McGrattan 2002).

subject to the following sequences of constraints:

$$P_{1t}c_{1t} + P_{2t}c_{2t} + P_{3t}c_{3t} + P_{1t}I_{1t} + P_{1t}I_{3t} \le M_t^c + W_t n_t; \qquad (\lambda_{1t}) \tag{2}$$

$$M_{t+1}^c + M_{t+1}^d \le R^d{}_t M^d{}_t + \Pi^F{}_t + \Pi^B{}_t + R_{1t}k_{1t} + R_{3t}k_{3t} + \\ (M_t^c + W_t n_t - P_{1t}c_{1t} - P_{2t}c_{2t} - P_{3t}c_{3t} - P_{1t}I_{1t} - P_{1t}I_{3t}); \qquad (\lambda_{2t}) \tag{3}$$

$$k_{1,\,t+1} = (1-\delta_1)\cdot k_{1t} + I_{1t} - \frac{\phi_{I1}}{2}\left(\frac{I_{1t}}{k_{1t}} - \delta_1\right)^2 k_{1t}; \qquad (\eta_{1t}) \tag{4}$$

$$k_{3,\,t+1} = (1-\delta_1)\cdot k_{3t} + I_{3t} - \frac{\phi_{I3}}{2}\left(\frac{I_{3t}}{k_{3t}} - \delta_3\right)^2 k_{3t}. \qquad (\eta_{3t}) \tag{5}$$

The constraints have the following interpretation. Equation (2) is the cash-in-advance constraint. It states that the liquid funds that households hold in the goods market at the beginning of the period, M_t^c, plus their wage payments, $W_t n_t$, must be sufficient to cover the nominal value of their consumption of the three goods ($P_{1t}c_{1t} + P_{2t}c_{2t} + P_{3t}c_{3t}$) and their planned investment in new capital in the two (domestic) productive sectors ($P_{1t}I_{1t} + P_{1t}I_{3t}$).[16] Equation (3) is the households' end-of-period wealth constraint: their available financial wealth is composed of the return on their non-liquid funds ($R^d{}_t M^d{}_t$), the dividends that arise from their ownership of all firms and banks ($\Pi^F{}_t + \Pi^B{}_t$), the rental income derived from renting the capital they own to domestic firms ($R_{1t}k_{1t} + R_{3t}k_{3t}$), and any liquid funds left over from the purchases described in equation (2). They allocate this financial wealth between beginning-of-next-period liquid balances (M_{t+1}^c) and balances deposited at the financial intermediaries (M_{t+1}^d). Equations (4) and (5) state that tomorrow's stock of installed physical capital in each production sector will consist of undepreciated capital already in place (the depreciation rates of capital are represented by δ_1 and δ_3, respectively), plus the new investment directed towards that sector, minus adjustment costs that depend on how important planned investment is relative to already-installed capital. The presence of these adjustment costs lessens the facility with which capital can move from one sector to the other, and thus prevents excessive volatility in investment from occurring in the simulations.

The functional form we employ to describe current utility is the following:

$$u\left(c_{1t}, c_{2t}, c_{3t}, n_t, \frac{M_{t+1}^c}{M_t^c}\right) = \gamma_1 \ln(c_{1t} + \iota_t) + \gamma_2 \ln(c_{2t}) + \gamma_3 \ln(c_{3t} - \iota_t) + \psi(1 - n_t - AC_t) ,$$

16. Investment targeted towards sector 3 (the sector that produces the non-traded good) is priced at P_{1t}, because the investment good is produced in the traded sector.

where

$$AC_t = \frac{\phi_{PC}}{2}\left(\frac{M^c_{t+1}}{M^c_t} - \mu\right)^2.$$

Utility is separable in its three consumption and single leisure arguments. Moreover, leisure enters linearly, as originally proposed by Hansen (1985): this results in a high aggregate elasticity of labour supply with respect to the real wage, even though this elasticity can be thought of as being very small at the individual level. Second, the utility flows from consumption of given levels of type 1 and type 3 goods depend on the preference shock, ι_t.[17] Third, leisure is defined as one minus hours worked (n_t), minus portfolio adjustment costs (AC_t). This last variable expresses the costs involved when modifying households' financial portfolios: since μ is the steady-state rate of monetary expansion, any increase in the level of liquid funds held in excess of this rate entails costs that increase with the square of the deviation. The presence of these costs ensures that monetary policy shocks will have effects that persist for several periods.[18] Finally, the aggregate price index that is implied by this specification of utility is the following[19]:

$$P_t = \frac{(P_{1t})^{\gamma_1}(P_{2t})^{\gamma_2}(P_{3t})^{\gamma_3}}{\gamma_1^{\gamma_1}\gamma_2^{\gamma_2}\gamma_3^{\gamma_3}}. \tag{6}$$

3.2 Production structure and firms

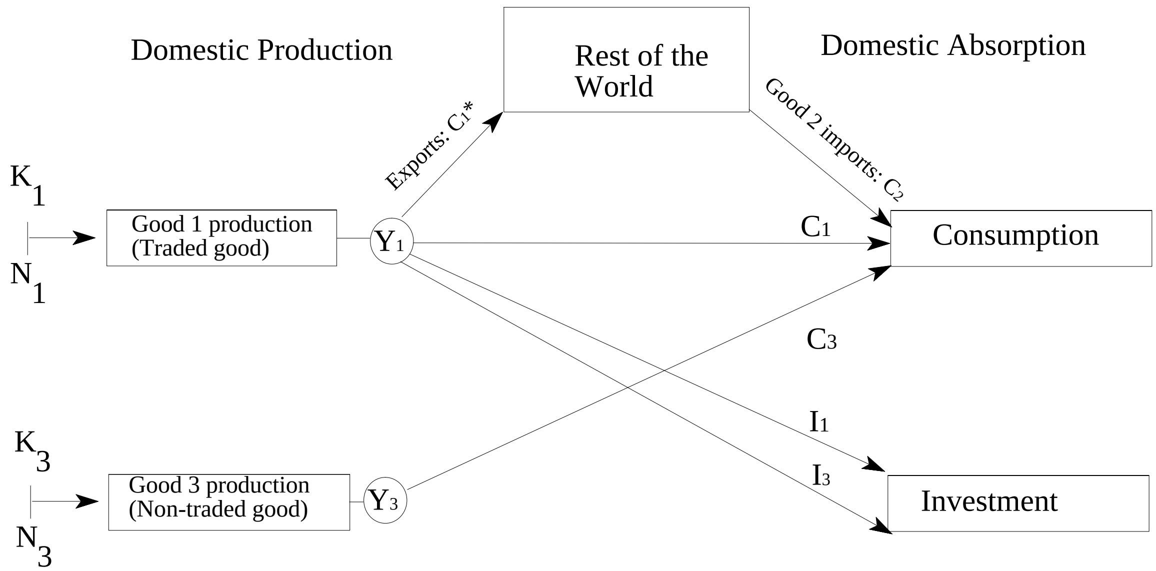
3.2.1 Production structure

Chart 1 depicts the structure of production and the flow of goods across the economy. First, (traded) good 1 is produced domestically: the inputs that enter into this production are drawn from the (domestic) stocks of physical capital and labour. Production of the good is then allocated to

17. Shocks similar to this one are also used by Erceg, Henderson, and Levin (2000) to analyze optimal monetary policy in a sticky price–sticky wage model. These shocks are meant to act as "demand" disturbances and induce a positive correlation between output and inflation. The fixed aggregate supply of liquid funds in our model (absent monetary policy actions) makes positive co-movements of prices and output difficult to obtain as increases in demand are met by upward pressure on interest rates and thus declines in output.
18. These costs can be interpreted as the time costs of deciding upon, and then implementing, the optimal change in households' holdings of liquid balances. Modelling them in terms of goods—rather than time—costs would not modify the results. In the absence of such costs, the imbalance of funds between the financial and goods sectors that a monetary policy shock initiates would last only one period. Such portfolio adjustment costs are also used in recent papers that use the limited-participation assumption (Christiano and Gust 1999; Cooley and Quadrini 1999).
19. This aggregate price index represents the minimum cost of purchasing one unit of an aggregate consumption index, C_t, where $C_t = (c_{1t})^{\gamma_1}(c_{2t})^{\gamma_2}(c_{3t})^{\gamma_3}$.

exports, consumption, or investment in the stock of capital in the two production sectors. Second, good 2 is imported from foreign markets and is allocated entirely to domestic consumption. Third, good 3 is produced domestically but is non-traded: production is thus allocated to domestic consumption.

Chart 1: Production Structure of the Domestic Economy



Only good 1 can contribute to investment in physical capital. It can thus be interpreted as the generic "good" of this economy; (non-traded) good 3 can be understood to be its generic "service," and imports (good 2) are assumed to not include any investment goods.[20]

3.2.2 Producers of goods 1 and 3

Firms that produce good 1 rent the necessary capital and labour inputs from households and sell their products in a competitive market. We assume that these firms must pay a portion of their wage bill before they receive the proceeds from their sales and, therefore, must borrow from financial intermediaries to cover those expenses. There are no intertemporal dimensions to the optimization problems of these firms, so they choose labour and capital inputs to maximize per-period real profits, as follows:

$$\underset{\{N_{1t}, K_{1t}\}}{MAX} \quad \frac{P_{1t}Y_{1t} - R_{1t}K_{1t} - J_t R^l_t W_t N_{1t} - (1-J_t)W_t N_{1t}}{P_t}, \tag{7}$$

20. To improve the mapping between the artificial economy's structure and the actual Canadian economy, it would be interesting to modify the model and allow the imported good to contribute to the economy's capital stock.

with respect to the following production function:

$$Y_{1t} = F_1(K_{1t}, N_{1t}) = A_{1t} \cdot K_{1t}^{\alpha_1} \cdot N_{1t}^{1-\alpha_1}. \tag{8}$$

In these expressions, R_{1t} represents the rental rate for capital allocated to the production of type-1 goods, R^l_t is the lending rate on bank loans, W_t is the economy-wide nominal wage rate and, as before, P_t is the aggregate price level and P_{1t} the price of type-1 goods. Moreover, A_{1t} represents an exogenous productivity shock that affects the production capabilities of all firms in this sector; J_t is the fraction of the wage bill that must be paid in advance and thus necessitates the borrowing of liquid funds. The remaining fraction of the wage bill $(1 - J_t)$ is not subject to borrowing costs, because it can be paid out of the revenues from sales. An increase in J_t means that firms must borrow more liquidity from financial intermediaries to operate at a given scale, and it can thus be interpreted as a shock to the demand for credit (or money) from production firms. The calibration of the process by which J_t evolves is discussed in section 4.2.[21]

The optimization problem of firms that produce good 3 is very similar to those of good 1 producers (the only difference being that, since capital goods are uniquely drawn from type-1 goods, the capital employed by these firms is valued at the price P_{1t}). The following optimization problem thus emerges (these firms are also affected by the money-demand shock, J_t):

$$\underset{\{N_{3t}, K_{3t}\}}{MAX} \frac{P_{3t}Y_{3t} - Rk_{3t}K_{3t} - J_tR^l_tW_tN_{3t} - (1-J_t)W_tN_{3t}}{P_t}, \tag{9}$$

with respect to:

$$Y_{3t} = F_3(K_{3t}, N_{3t}) = A_{3t} \cdot K_{3t}^{\alpha_3} \cdot N_{3t}^{1-\alpha_3}. \tag{10}$$

The shocks that affect the productive capabilities in each sector, A_{1t} and A_{3t}, are exogenous and are assumed to evolve according to the following bivariate AR(1) process:

$$\begin{bmatrix} \ln(A_{1t}) \\ \ln(A_{3t}) \end{bmatrix} = \mathbf{A} + \mathbf{H} \begin{bmatrix} \ln(A_{1,t-1}) \\ \ln(A_{3,t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon^{z1}_t \\ \varepsilon^{z3}_t \end{bmatrix}; \qquad \begin{bmatrix} \varepsilon^{z1}_t \\ \varepsilon^{z3}_t \end{bmatrix} \sim (0, \Omega_z). \tag{11}$$

In (11), the (2 by 1) vector **A** contains the long-run mean of the shocks and the (2 by 2) matrix **H** contains the feedback components. Section 4.2 discusses the calibration of this process.

21. This J-shock to money demand is also used in Christiano and Gust (1999).

The price of good 3 is determined endogenously, as part of the general equilibrium of the economy. Because good 1 is traded internationally and the domestic economy is small relative to foreign markets, its price is assumed to be determined on foreign markets and taken to be exogenous to our model. This (foreign currency) price is denoted as P^*_{1t}. We consider that standard arbitrage mechanisms ensure that the law of one price holds for that good. The domestic currency price of good 1 is therefore:

$$P_{1t} = e_t \cdot P^*_{1t}, \tag{12}$$

where e_t is the nominal exchange rate (the domestic currency price of foreign currency).[22]

3.2.3 Importers of good 2

The importers of good 2 buy the good on foreign markets (paying with foreign currency) and transport it back to the domestic economy, where they sell it to consumers in a competitive market. We assume that these firms must borrow a fraction, J_t, of the funds necessary for their purchases from banks. The optimization problems of these firms are as follows:

$$\underset{\{Y_{2t}\}}{MAX} \quad P_{2t}Y_{2t} - R^l{}_t \cdot J_t(e_tP^*_{2t}Y_{2t}) - (1-J_t)(e_tP^*_{2t}Y_{2t}), \tag{13}$$

where P^*_{2t} is the (foreign currency) price of the good. The total (foreign currency) cost to importers is thus $P^*_{2t}Y_{2t}$, which, when multiplied by the nominal exchange rate, e_t, gives total domestic currency costs. Since importers borrow a fraction of this amount, the gross nominal lending rate enters the determination of the total costs. Again, the evolution of P^*_{2t} is taken to be exogenous to the model and calibrated in section 4.2[23] This maximization problem is trivial and results in the following arbitrage condition:

$$P_{2t} = [J_tR^l{}_t + (1-J_t)]e_tP^*_{2t}. \tag{14}$$

The quantity of good 2 supplied (Y_{2t}) is determined by households' demand for that good. The presence of $R^l{}_t$ (which results from our assumption that importers of good 2 must borrow some of their funds before travelling to foreign markets) introduces a wedge between P_{2t} and its determinants under the law of one price ($e_tP^*_{2t}$). This may imply that changes in lending rates

22. An increase in e_t thus represents a depreciation of the domestic currency.
23. We actually calibrate processes for the *relative* world price P^*_{1t}/P^*_t and P^*_{2t}/P^*_t, where P^*_t represents the total, world price level.

will modify the relative (domestic) price of goods 1 and 2. Further, for given values of the exchange rate, it creates an immediate pass-through from interest rate increases to the price of good 2, and from there to the aggregate price level: a tightening of monetary policy could thus lead to increases in prices. Although in equilibrium the effect from the exchange rate dominates, some of the empirical papers on the effects of monetary policy shocks report such increases in price following a tightening in monetary policy, which are only gradually transformed into the expected declines.[24]

3.3 Financial intermediaries

Financial intermediaries (banks) collect funds from households and lend them to firms. They discount future profits at the same rate at which households discount future streams of income, so that the optimization problem of banks is the following:

$$\underset{L_{1t+k}, L_{2t+k}, L_{3t+k}, Q_{t+k+1}\big|_{k=0}^{\infty}}{MAX} \quad E_t \sum_{k=0}^{\infty} \beta^{t+k} \cdot \lambda_{t+k} \cdot \Pi^B{}_{t+k}, \tag{15}$$

where λ_{t+k} is the weight that a firm attaches to future profits (in equilibrium, it will be equal to the households' marginal utility of income), and bank profits, $\Pi^B{}_t$, are defined as:

$$\Pi^B{}_t = R^l{}_t L_{1t} + R^l{}_t L_{2t} + R^l{}_t L_{3t} - R^d{}_t (M^d{}_t + X_t); \tag{16}$$

while the maximization is done with respect to the following constraint:

$$L_{1t} + L_{2t} + L_{3t} \le M^d{}_t + X_t - e_t(q_t Q_{t+1} - Q_t). \tag{17}$$

In those expressions, L_{1t}, L_{2t}, and L_{3t} represent bank lending to the three types of domestic firms: the first three terms on the right-hand side of (16) are thus revenues from lending activities, with $R^l{}_t$ being the lending rate. Costs arise from the need to remunerate household funds (at rate $R^d{}_t$), both those deposited by households themselves at the end of the preceding period ($M^d{}_t$) and the current injection of liquidity (X_t), which the central bank deposits in households' accounts.

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24. Authors interpret such findings as evidence that monetary policy has not been properly identified; they refer to the phenomenon as the "price puzzle." Christiano, Eichenbaum, and Evans (2001) describe a structural model in which such a "price puzzle" arises as an equilibrium phenomena, and suggest that it might be a genuine feature of the data, rather than a sign of a misspecified model.

In a closed-economy environment, the constraint limiting what banks can lend to domestic firms would be that total lending cannot exceed domestic saving balances. Here, however, we assume that banks can participate in foreign financial markets. Specifically, we assume that they participate in a market that trades a discount bond representing a promise to pay one unit of foreign currency in the next period. The (foreign currency) price of such a bond is q_t.

This market enables banks to gather additional liquidity when lending opportunities outnumber the balances that are available to them domestically or, inversely, it provides an outlet for banks to dispose of excess liquidity when domestic lending opportunities are slim. Equation (17) describes how the presence of this foreign financial market modifies the constraint faced by the banks.[25] In that expression, Q_{t+1} expresses a bank's net purchases of (foreign currency) discount bonds. A negative value of Q_{t+1} thus expresses a situation where a domestic bank is borrowing on international markets. Suppose, for example, that a bank starts the period with a zero balance of international bonds ($Q_t = 0$); the equation thus states that any excess of domestic lending over savings balances available domestically will be covered by borrowing on international markets.[26]

The (foreign currency) gross return on holding this discount bond (or the interest cost on international borrowing, if holdings are negative) is the following:

$$R^{eff}{}_t = \frac{1}{q_t}. \tag{18}$$

We assume that $R^{eff}{}_t$ is first composed of an exogenous component that follows the evolution of short-term, risk-free rates on world markets and, second, of an endogenous component that responds to the domestic banking sector's level of foreign indebtedness. Supposing that deviations from steady state (x^{ss}) are denoted by a hat (so that $\hat{x}_t = (x_t - x^{ss})/x^{ss}$ for any variable x_t), we assume that the following describes the evolution of $R^{eff}{}_t$:

$$\hat{R^{eff}}_t = \hat{R^*}_t - \eta N\hat{F}A_t, \tag{19}$$

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25. See Ho (1993) for an earlier example of a limited-participation model in which lending can be obtained from foreign channels in addition to the domestic ones. Another mechanism by which to loosen the connection between total amount lent and saving balances available domestically is to model reserves, inside-money creation, and the existence of a money multiplier. See Chari, Christiano, and Eichenbaum (1995) for such an analysis within the limited-participation environment.
26. This assumption reduces the extent to which the limited-participation assumption "bites." In the standard model, a surprise injection leaves a bank with too much liquidity: to lend all of it, banks are forced to push nominal lending rates lower than their Fisherian fundamentals. Banks can, to a certain extent, lend out excess liquidity on international markets without having to lower interest rates too much to place this liquidity in domestic markets.

where R^*_t is the world risk-free rate and $N\hat{F}A_t$ is the change in the quantity of foreign bonds held by domestic banks. Lending rates on foreign borrowing can thus increase, because the world rate, R^*_t, has increased, or because the net indebtedness of domestic banks, $N\hat{F}A_t$, has worsened. The elasticity parameter, η, describes the sensitivity of effective rates to that indebtedness.[27] We discuss the calibration of the process for R^*_t and the numerical value of η in section 4.2.

Chart 2: Financial Flows in the Domestic Economy

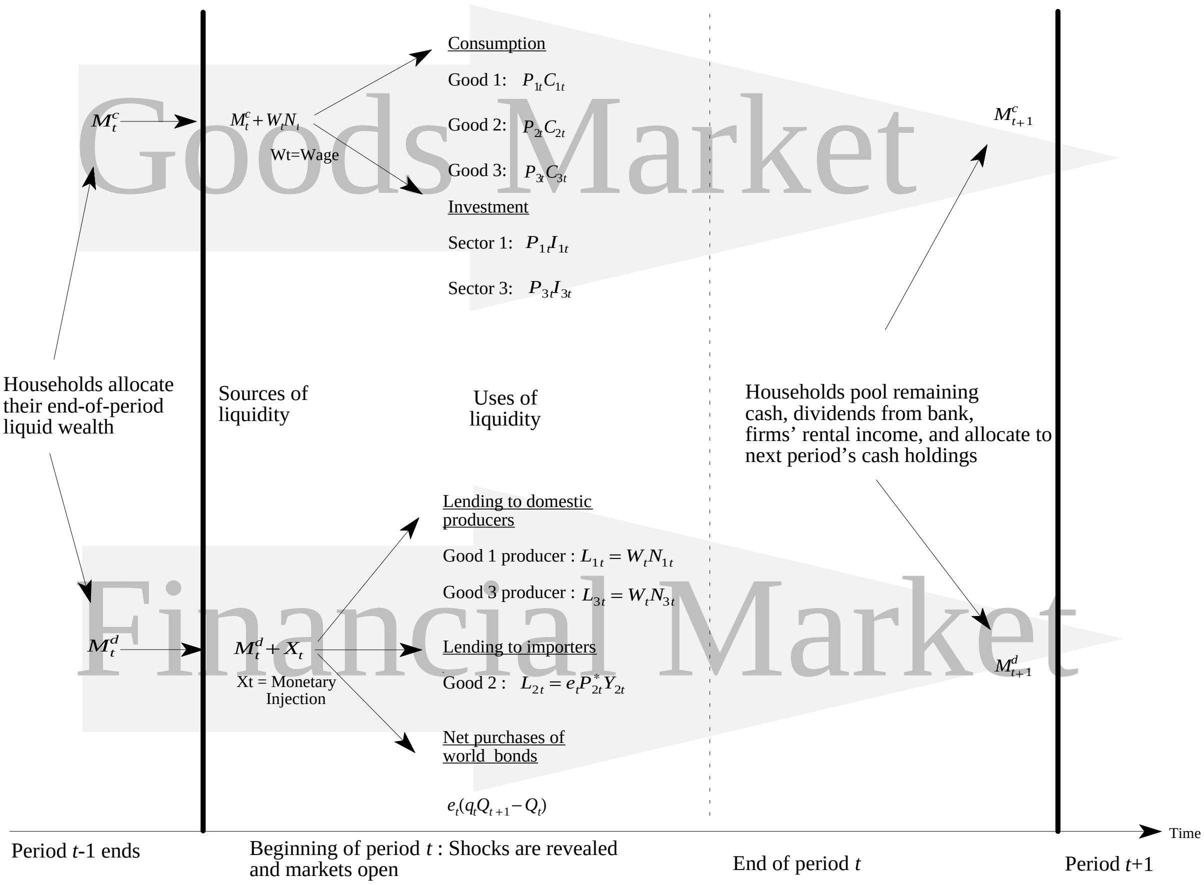

Chart 2 summarizes the financial flows within the economy. Households allocate their end-of-period financial wealth to either the goods sector or the financial sector by choosing M_t^c and M_t^d at the end of period *t*-1, before the value of the shocks that affect the economy in period *t* is known. From that moment, a segmentation exists between the two sectors that prevents liquidity (and, particularly, any new liquidity injection, X_t, that results from monetary policy actions) from

27. In addition to its intuitive appeal, this mechanism has the advantage of ensuring that the model has a stationary steady state. Similar mechanisms for the evolution of effective rates on foreign borrowing are used by Devereux (2000) and Ravenna (2000). Correia, Neves, and Rebelo (1995) and Schmitt-Grohe and Uribe (Forthcoming) discuss stationarity in small open-economy models. We could have assumed that households are the economic agents that participate on international lending markets. This would not have affected the stationarity issue, but would have implied a different interpretation of net foreign indebtedness in our economy.

flowing from one sector to the other. Even at the end of the period, when households pool all their liquid funds and choose to allocate them between M^c_{t+1} and M^d_{t+1}, the adjustment costs continue to reduce the flexibility of their decisions.

3.4 The central bank and government

The instrument of monetary policy is the supply of liquid funds (money). Injections of new money during the current period are denoted by $X_t = M_{t+1} - M_t$ (M_t is the total stock of money at the beginning of period *t*). This notation implies that the gross rate of monetary expansion, μ_t, is:

$$\mu_t = \frac{M_{t+1}}{M_t} = \frac{X_t}{M_t} + 1. \tag{20}$$

We assume that the central bank manipulates μ_t in such a way that its desired level for the nominal interest rate is achieved. More precisely, a desired level for interest rates, along with the state variables of the economy and the optimizing behaviour of economic agents, implies a specific value for the demand for real money balances. When monetary authorities set money supply equal to this demand, the desired interest rate is achieved.

While the underlying instrument of monetary policy remains the rate of monetary expansion, we describe monetary policy as a rule that links the desired level of nominal interest rates to a deterministic function of variables known at time *t* and a stochastic shock, as in the following:

$$\hat{R}_t = \Omega(I_t) + \varepsilon^{MP}{}_t, \tag{21}$$

where $\hat{R}_t$ is the deviation of nominal rates from their steady-state levels, $\Omega(I_t)$ is a (linear) function of variables known at time *t*, and $\varepsilon^{MP}{}_t$ is an exogenous disturbance to the rule, or a monetary policy shock.[28] The general form in (21) can accommodate a number of specific rules analyzed in the literature. For example, the rule originally described in Taylor (1993a) would consist, in the present framework, of the following:

$$\hat{R}_t = 1.5\hat{\pi}_t + 0.5\hat{y}_t + \varepsilon^{MP}{}_t. \tag{22}$$

28. The interpretation of $\varepsilon^{MP}{}_t$ is thus similar to those proposed in the empirical literature on identified VARs. See Christiano, Eichenbaum, and Evans (1999) for a discussion.

This rule directs monetary authorities to increase nominal rates, in response to any positive deviation of inflation from steady state ($\hat{\pi}_t$) by more than one for one. Further, the rule calls for nominal rate increases whenever output increases above its trend or steady-state level.[29]

The general expression in (21) can accommodate a variety of rules. For example, a forward-looking rule, where the monetary authorities react to deviations of expected future inflation from target, would be:

$$\hat{R}_t = \alpha E_t[\hat{\pi}_{t+k}] + \beta \hat{y}_t + \varepsilon^{MP}_t, \quad (23)$$

where we have replaced specific numerical values of the responses by the generic parameters α and β.[30] Further, a smoothing behaviour, where monetary authorities wish to only gradually achieve their desired level of interest rates, can be expressed with the following form[31]:

$$\hat{R}_t = \alpha E_t[\hat{\pi}_{t+k}] + \beta \hat{y}_t + \gamma \hat{R}_{t-1} + \varepsilon^{MP}_t. \quad (24)$$

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29. Throughout the analysis, we assume that monetary policy responds to output deviations from steady state. Alternatively, we could identify potential output at any point in time as the level that would have been obtained if the portfolio rigidities were not present, define the output gap as the difference between the actual and this measure of potential, and direct monetary policy to react to the gap. We do not pursue this route for two reasons. First, in a limited-participation environment, the output gap may not be a reliable forecast of inflationary pressures, as it is in models based on price rigidities and containing a Phillips curve; responding to the gap may not necessarily allow monetary policy to achieve better outcomes. Second, the "flexible portfolio" definition of potential may not correspond even roughly to the measures of potential available to policy-makers.

30. In the literature, the interest and inflation rate variables are often measured on an annualized basis. There is thus a scaling problem if one wishes to compare the parameter values in (23) (we use quarterly rates throughout) to those commonly discussed in the literature. To enable such a comparison, we use the value $\beta/4$ as a response to output deviations in rules like (23), while continuing to frame the discussion in terms of the parameter β.

31. This form of rule is the one estimated by Clarida, Galí, and Gertler (1998, 2000), and is found to fit recent monetary policy history well. These authors write the rule in a slightly different manner, which involves the following two steps. First, the target rate for interest rates is modelled as responding to inflation and output deviations from steady-state values:

$$R_t^{\text{target}} = \alpha E_t[\hat{\pi}_{t+k}] + \beta \hat{y}_t.$$

Next, they assume that monetary authorities only gradually converge towards this targeted rate, so that actual rates are a weighted sum of their own lagged values and of the target, as in:

$$\hat{R}_t = (1-\gamma)R_t^{\text{target}} + \gamma \hat{R}_{t-1} = (1-\gamma)(\alpha E_t[\hat{\pi}_{t+k}] + \beta \hat{y}_t) + \gamma \hat{R}_{t-1} + \varepsilon^{MP}_t.$$

Conditional on a slight rewriting of the parameters α and β, this form is the same as the one in (24).

Finally, a general rule, where monetary authorities potentially react to all state variables and shock processes that affect the economy, can be represented in the following vector notation:

$$\hat{R}_t = \mu_s'\hat{\boldsymbol{s}}_{\boldsymbol{t}} + \mu_x'\hat{\boldsymbol{x}}_{\boldsymbol{t}} + \varepsilon^{MP}{}_t, \tag{25}$$

where $\hat{\boldsymbol{s}}_{\boldsymbol{t}}$ and $\hat{\boldsymbol{x}}_{\boldsymbol{t}}$, respectively, denote the state variables and the exogenous shocks that affect the economy, and $\mu_{\boldsymbol{s}}$ and $\mu_{\boldsymbol{x}}$ denote vectors of monetary policy responses to those variables.

In most of our quantitative work using these rules, we assume full commitment when computing approximate solutions to the equilibrium. That is, the particular form of (21) under study is entered directly into the rational-expectations solution, imposing that economic agents assign a probability of zero to an event where monetary authorities would deviate from that rule at any time in the future. We do provide, however, some results arrived at under the assumption of discretion (or period-by-period optimization).[32]

3.5 Market clearing and definition of the equilibrium

3.5.1 Foreign exchange

The market that exchanges the foreign currency for the domestic currency determines the value of the nominal exchange rate. The only participants in this market are domestic agents. The supply of foreign currency is provided by the exporters: having sold a quantity, $c^*{}_{1t}$, of good 1 to foreigners at a (foreign currency) price, $P^*{}_{1t}$, they hold $c^*{}_{1t} \cdot P^*{}_{1t}$ in foreign currency that they want to convert to domestic currency. Importers, on the other hand, want to buy a quantity, Y_{2t}, of (foreign-made) good 2, which carries a purchase price of $P^*{}_{2t}$ in foreign currency. Finally, banks demand foreign currency to purchase their (net) investment of $q_tQ_{t+1} - Q_t$. The following equilibrium condition in the foreign currency market therefore arises and implicitly determines the nominal exchange rate:

$$c^*{}_{1t} \cdot P^*{}_{1t} = Y_{2t} \cdot P^*{}_{2t} + (q_tQ_{t+1} - Q_t) . \tag{26}$$

32. For a detailed discussion of the steps required to solve full-commitment and discretionary solutions (or time-inconsistent and time-consistent solutions, respectively), see Cooley and Quadrini (2002). To compute discretionary equilibria in our setting, we follow the spirit of the algorithms described in Dennis (2003).

3.5.2 Goods markets

Equilibrium in the market for good 1 requires that domestic production be sufficient to cover domestic consumption of that good, investment in both production sectors, and exports:

$$A_{1t} \cdot K_{1t}^{\alpha_1} \cdot N_{1t}^{1-\alpha_1} = c_{1t} + I_{1t} + I_{3t} + c^*_{1t}. \tag{27}$$

Equilibrium in the market for good 3 states that production equals domestic consumption:

$$A_{3t} \cdot K_{3t}^{\alpha_3} \cdot N_{3t}^{1-\alpha_3} = c_{3t}. \tag{28}$$

Finally, market good 2 is in equilibrium when the quantity that importers purchase in foreign markets is equal to households' consumption of the good:

$$Y_{2t} = c_{2t}. \tag{29}$$

3.5.3 Savings and loan markets

The loans extended by the banks must be sufficient to cover the borrowing needs of the three types of firms (the fraction of the wage bills of good 1 and good 3 producers and of the total input costs of importers that are subject to borrowing):

$$L_{1t} = J_t W_t N_{1t}; \qquad L_{3t} = J_t W_t N_{3t}; \qquad L_{2t} = J_t e_t P^*_{2t} Y_{2t}. \tag{30}$$

Banks do not hold any excess liquidity, so that the constraint (17) holds with equality:

$$e_t(q_t Q_{t+1} - Q_t) + L_{1t} + L_{2t} + L_{3t} = M^d{}_t + X_t\ . \tag{31}$$

Perfect competition in the savings and loan markets (and the fact that intermediation is costless) ensures that financial intermediaries equate the lending and the savings rate. Further, that rate is also the one targeted by monetary authorities:

$$R^L{}_t = R^d{}_t = R_t. \tag{32}$$

3.5.4 Labour and capital rental markets

Total labour supply is equal to total demand for labour that arises from the activities of the domestic firms that produce goods 1 and 3:

$$n_t = N_{1t} + N_{3t} . \quad (33)$$

Finally, the installed capital in each of the domestic production sectors (which is owned by the households) must be equal to the quantity of capital that the firms producing goods 1 and 3 plan to use:

$$k_{1t} = K_{1t}; \qquad k_{3t} = K_{3t}. \quad (34)$$

3.5.5 Definition of the equilibrium

Denote the value at time *t* of the exogenous shocks that affect the economy as s_t. Next, let $s^t = (s_0, s_1, \ldots, s_t)$ define the history of all shocks up to and including period *t*. An equilibrium for this economy consists of sequences of allocation functions for households $\{c_{1t}(s^t), c_{2t}(s^t), c_{3t}(s^t), n_t(s^t), M^c_{t+1}(s^t), M^d_{t+1}(s^t), I_{1t}(s^t), I_{3t}(s^t), k_{1t+1}(s^t), k_{3t+1}(s^t)\}$, firms $\{N_{1t}(s^t), N_{3t}(s^t), K_{1t}(s^t), K_{3t}(s^t), Y_{2t}(s^t)\}$, and banks $\{L_{1t}(s^t), L_{2t}(s^t), L_{3t}(s^t), Q_{t+1}(s^t)\}$; sequences of pricing functions $\{P_{1t}(s^t), P_{2t}(s^t), P_{3t}(s^t), R_{1t}(s^t), R_{3t}(s^t), W_t(s^t), R^d{}_t(s^t), R^l{}_t(s^t), P_t(s^t)\}$; a monetary policy rule that describes monetary authorities' actions (equation (21)); starting values for the state variables $(k_{1, t=0}, k_{3, t=0}, M_0^c, M_0^d, Q_0\}$; and, finally, data-generating processes (DGPs) for the exogenous shock variables $(\iota_{1t}, A_{1t}, A_{3t}, R^*{}_t, \pi^*{}_t, P^*{}_{1t}, P^*{}_{2t}, J_t)$.[33] Allocations, pricing functions, the policy rule, starting values, and exogenous processes are such that (i) taking prices as given, the allocations solve the optimization problem of households described in (1) to (5) and the profit maximization problems of the three types of firms and the banks, and (ii) the market-clearing equations in (26) to (34) are respected.[34]

A numerical representation of this equilibrium is obtained by first computing a non-stochastic steady state for the economy and then constructing a first-order approximation of the solution around that steady state. Appendix A provides details of the solution method.

4. Calibration

4.1 Calibrating the parameters

The model is calibrated using several Canadian data counterparts to the steady-state properties of the model. First, the discount rate, β, is set to 0.99 so that the steady-state real annual rate of

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33. The variable π_t^* represents foreign inflation. Even though we have not used it when describing the model, it appears in the deflated, detrended, and linearized system the solution is based on.

34. The concept behind this solution is from Chari, Kehoe, and McGrattan (2002).

interest will be close to 4 per cent. Next, using the Canadian national accounts data, consumption of good 1 is identified with personal consumption of goods, consumption of good 2 is matched with imports, and consumption of good 3 is identified with personal expenditure on services. The consumption parameters in the utility function are set at $\gamma_1 = 0.43$, $\gamma_2 = 0.26$, and $\gamma_3 = 0.31$, which ensures that the relative size of the consumption categories is approximately as in the data: the ratio of consumption of good 1 to good 3 is 1.41, and that of good 2 to good 3 is 0.84. The values of the production function parameters, α_1 and α_3, the labour utility parameter, ψ, and the depreciation rate, δ, are set such that the following are approximately as in the data: the ratio of investment to output of good 1 (0.21); the ratio of production of good 1 to good 3 (2.91); the total labour supply (0.18 of available time); and the ratio of wages to GDP (0.68).

According to data on GDP at factor cost, the goods-producing sector is characterized by a capital-output ratio that is 55 per cent larger than the one in the service-producing sector.[35] Consequently, the scale parameter in the good 1 production function is set at 1.55.

The ratio of net foreign asset holdings to total capital is found to be -0.012, on average, from 1975 to 2000 using Canadian data on chartered bank assets. Therefore, we set the (appropriately scaled down) measure of the steady-state value of Q to -0.012.

Finally, we set $\eta = 0.05$, $\phi_{PC} = 0.6$, and $\phi_{I1} = \phi_{I3} = 5.0$, so that net foreign assets are much more volatile than output; inflation and output have a positive contemporaneous correlation of about 0.2, as in the data; and investment is about four times as volatile as output.

4.2 Calibrating the exogenous shocks

4.2.1 Productivity shocks

Taking logs of the production functions in (8) and (10) and rearranging to isolate $\ln(A_{it})$ yields:

$$\ln(A_{it}) = \ln(Y_{it}) - \alpha_i \cdot \ln(K_{it}) - (1-\alpha_i) \cdot \ln(N_{it}),\ i = 1,3.$$

We identify Canadian quarterly GDP at factor cost in the goods-producing sector and in the service-producing sectors as Y_{1t} and Y_{3t}, respectively. Next, the amount of capital and labour

35. Data on GDP at factor cost (on a value-added basis), labour input, and capital input all show that the service sector is much larger than the goods-producing sector. However, the expenditure-based GDP numbers (on a final expenditure basis) imply that the goods-producing sector is larger. Obviously, a large part of the final value of the goods production comes from inputs of services. Since our model does not have input goods, we are not able to replicate this pattern. We use the expenditure-based GDP numbers to calibrate the relative size of the sectors, but use the factor cost and capital data to calibrate the relative capital intensity of the two sectors.

employed in these two sectors provides us with data series for K_{1t}, K_{3t}, N_{1t}, and N_{3t}. Finally, the values of α_1 and α_3 established above allow us to compute time series for $\ln(A_{1t})$ and $\ln(A_{3t})$. The series are then detrended using the Hodrick-Prescott (HP) filter and the cyclical components are used to estimate the process given in equation (11) over the sample 1987Q1–2000Q4, with the following results[36]:

$$\begin{bmatrix} \ln(\hat{A}_{1t}) \\ \ln(\hat{A}_{3t}) \end{bmatrix} = \begin{bmatrix} 0.70 & 0.0 \\ 0.0 & 0.51 \end{bmatrix} \begin{bmatrix} \ln(\hat{A}_{1t-1}) \\ \ln(\hat{A}_{3t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon^{z1}{}_t \\ \varepsilon^{z3}{}_t \end{bmatrix}, \qquad \Omega_z = \begin{bmatrix} 0.00752^2 & 0.00253^2 \\ 0.00253^2 & 0.00502^2 \end{bmatrix}. \tag{35}$$

The estimated process implies that there is very little diffusion from the technology shocks that affect one sector to those that affect the other (the off-diagonal elements in the matrix of coefficients are essentially zero). There still remains, however, some relationship between the two technology shocks, because the innovations $\varepsilon^{z1}{}_t$ and $\varepsilon^{z3}{}_t$ have a contemporaneous correlation of 0.17.[37]

4.2.2 Consumption preference shocks

We assume that a standard AR(1) process governs the evolution of the consumption shock, ι_{1t}. Recall that this shock affects the relative contribution of consumption of goods 1 and 3 towards overall utility. Because there are no available data on these shocks, the parameter values of the process were chosen to approximately replicate the observed volatility of consumption relative to output in Canadian data. The process is as follows:

$$\iota_t = 0.7 \cdot \iota_{t-1} + \varepsilon^{\text{cons}}{}_t; \qquad \varepsilon^{\text{cons}}{}_t \sim (0, 0.01^2). \tag{36}$$

4.2.3 Foreign shocks

There are four foreign shocks in the model: $R^*{}_t$, $\pi^*{}_t$, $P^*{}_{1t}$, and $P^*{}_{2t}$. To calibrate the process that describes their evolution, we first identify data counterparts. The world short-term rate is the one featured in the projections conducted with the Quarterly Projection Model (QPM), the Bank of Canada's main policy model.[38] Inflation is the net annual rate of growth in the foreign GDP

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36. The labour data are available only since 1987 on a sector basis. Recall that we denote a variable with a hat as the deviation of this variable from its steady state or trend.
37. Batini, Harrison, and Millard (2003) also find modest diffusion when they calibrate their model to the U.K. economy.
38. This rate, as well as the measure of the foreign deflator, $P^*{}_t$, is a trade-weighted average. For Canadian data, these averages will naturally be heavily dominated by American data.

deflator.[39] The relative foreign price of good 1, P^*_{1t}, is constructed as follows. Recall that good 1 is the traded good that the domestic economy exports. To compute a foreign currency price series for that good, we multiply the nominal exchange rate by the deflator for Canadian exports, $P_t^{\exp}$. Dividing by the world price deflator then gives the relative price:

$$P^*_{1t} = \frac{e_t \cdot P_t^{\exp}}{P^*_t}. \tag{37}$$

Similarly, we identify the deflator for Canadian imports as the series that underlies the process for the price of good 2, so that we have:

$$P^*_{2t} = \frac{e_t \cdot P_t^{imp}}{P^*_t}. \tag{38}$$

The four series are detrended using the HP filter and the following VAR(1) is estimated[40]:

$$\begin{bmatrix} \hat{R}^*_t \\ \hat{\pi}^*_t \\ \hat{P}^*_{1t} \\ \hat{P}^*_{2t} \end{bmatrix} = \begin{bmatrix} 0.66 & 0.06 & 0.03 & -0.02 \\ 0.19 & 0.37 & -0.002 & 0.004 \\ -0.60 & 1.97 & 0.60 & 0.24 \\ -0.53 & 1.61 & -0.35 & 1.12 \end{bmatrix} \begin{bmatrix} \hat{R}^*_{t-1} \\ \hat{\pi}^*_{t-1} \\ \hat{P}^*_{1t-1} \\ \hat{P}^*_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon^{R^*}_t \\ \varepsilon^{\pi^*}_t \\ \varepsilon^{P_1^*}_t \\ \varepsilon^{P_2^*}_t \end{bmatrix}, \tag{39}$$

with the estimated variance-covariance matrix of the vector $[\varepsilon^{R^*}_t, \varepsilon^{\pi^*}_t, \varepsilon^{P_1^*}_t, \varepsilon^{P_2^*}_t]'$ as follows:

$$\Omega_w = \begin{bmatrix} 4.95e-6 & 1.34e-6 & 1.34e-5 & 1.50e-5 \\ 1.34e-6 & 3.54e-6 & 6.09e-6 & 4.65e-7 \\ 1.34e-5 & 6.09e-6 & 4.55e-4 & 5.43e-4 \\ 1.50e-5 & 4.65e-7 & 5.43e-4 & 7.62e-4 \end{bmatrix}. \tag{40}$$

Note that these results imply that there is some connection between the foreign interest rate and the inflation rate: the diffusion parameters are 0.06 and 0.19, respectively, and the contemporaneous correlation in the innovations is 0.32. Further, there are strong links between the two relative price series; most notably, the innovations to P^*_{1t} and P^*_{2t} display a contemporaneous

39. As a result, we have $\pi^*_t = P^*_t / P^*_{t-1} - 1$, where P^*_t is the world price deflator.
40. Estimation using non-detrended data does not fundamentally modify the results shown in (39).

correlation of 0.92. By contrast, the relationship between the interest rate and inflation shocks, on the one hand, and the relative price shocks, on the other, is more modest.

4.2.4 Shocks to monetary policy and money demand

To verify the capacity of our model to replicate the macroeconomic history measured by Canadian data, we need to employ a specific form for the monetary policy rule. Ravenna (2000) estimates the following forward-looking rule for the Canadian experience, using the methodology described in Clarida, Galí, and Gertler (1998, 2000):

$$\hat{R}_t = 0.333 E_t[\hat{\pi}_{t+k}] + 0.0078\hat{y}_t + 0.84\hat{R}_{t-1} + \varepsilon^{MP}{}_t. \tag{41}$$

We use this policy rule with a variance of 0.001 for the monetary policy shocks, $\varepsilon^{MP}{}_t$. The steady-state inflation rate is not determined by this rule: we fix it at 4.5 per cent, the average rate of inflation in Canada over the sample used by Ravenna to estimate (41).

Finally, following Christiano and Gust (1999), we assume that the J-demand for money shock evolves according to a standard AR(1) process. We set the numerical values of this process to the following (which imply that money growth is more volatile than inflation)[41]:

$$\log(J_t) = 0.5 \cdot \log(J_{t-1}) + \varepsilon^{\mathrm{MD}}{}_t; \qquad \varepsilon^{MD}{}_t \sim (0, 0.01^2). \tag{42}$$

5. Assessing the Model

5.1 Impulse responses

The first type of diagnostic we perform is to compute the impulse responses of the model following a particular shock. For the first type of shock—an innovation in monetary policy—there exist empirical counterparts to the responses we report, so that we can assess their validity. For the other two shocks we report (an adverse technology innovation in sector 1 and a positive money-demand shock), empirical counterparts are harder to find. We nevertheless can assess the extent to which the responses are in accord with our expectations, and provide some assessment of the model's performance.

41. Money growth experiences 4 to 5 times the volatility of inflation in the actual data, but the model is not able to replicate this even for very large money-demand shocks.

5.1.1 Responses following a monetary policy shock

Figure 1 shows the impulse response of the economy following a monetary policy shock; i.e., a negative deviation of the rule in (41) with $\varepsilon^{MP}{}_t = -0.001$. This corresponds to a decrease of around 60 basis points in the annualized nominal interest rate. This decline in interest rates is achieved by a strong, transitory increase in the money-growth rate, the underlying instrument of monetary policy, which is expected to undershoot its steady-state value in the following periods.

The responses of interest rates, exchange rates, GDP, and inflation are roughly in line with the empirical responses identified by Cushman and Zha (1997) and Fung and Kasumovich (1998) using Canadian data. Interest rates decline and continue to be below steady state for a few quarters. Both the nominal and the real exchange rates depreciate on impact; the real exchange rate returns gradually to its steady-state value. The response of inflation is immediate and short-lived, in contrast with the responses in Cushman and Zha (1997) (as well as with conventional wisdom about the behaviour of inflation), which suggests that inflation reacts only very gradually to economic shocks. Finally, the response of output is also rapid and transitory, when the empirical evidence would suggest that its response is characterized by a smooth hump-shaped path. The model thus succeeds in generating responses following monetary policy shocks that are roughly of the correct sign and persistence, except for the case of inflation (and, to a lesser extent, output), where the model's responses should be more gradual.[42]

5.1.2 Responses following a negative technology shock in the traded-goods sector (sector 1)

Figure 2 reports the response of the economy following a negative technology shock to the production function of sector 1; i.e., $\varepsilon^{z1}{}_t$ in equation (35) takes a value of -0.01. Were the monetary authorities not to react (and leave money growth unchanged), domestic production and consumption of good 1 would fall sharply (recall that the perfect flexibility of prices means that the relative price of good 1 would increase immediately). Further, the reduced demand of funds stemming from the reduction in the production of good 1 would entail a decrease in the nominal interest rate; that decrease, in turn, would lead to a depreciation of the domestic currency. The reduction in nominal rates would stimulate the production and consumption of the non-traded good (good 3). Because of the adverse-supply shock, inflation would increase sharply but would decrease slightly in the following periods.

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42. The persistence in the responses of output and inflation following monetary policy shocks has been the subject of intensive research in the last several years. It has been shown that the addition of some non-standard features to limited-participation environments (monopolistic competition, non-separability of utility in consumption and leisure, etc.) helps the models generate persistence. See Christiano and Gust (1999).

According to the rule implemented in this benchmark version of the model (the rule in (41)), monetary authorities do not consider the immediate increase in the inflation rate a problem, but wish to limit the decrease in inflation that will occur in the following periods if they do not intervene. Their reaction is to reduce money-growth rates today but signal a big increase in the following period. They thus exacerbate the negative impact of the shock in the first period; interest rates do not decrease and may actually increase, at the same time that the adverse technology shock affects the economy. This results in worse outcomes in sector 1, while the lack of decrease in the interest rate means that the boom in sector 3 cannot occur. (Note, however, that the real interest rate declines sharply.) The loose monetary policy in the future, however, means that the spike in inflation is very short-lived, and inflation is essentially back to its steady-state level in the following period.

5.1.3 Responses following a positive money-demand shock

Figure 3 reports the response of the economy when it is affected by a positive shock to money demand; i.e., $\varepsilon^{MP}{}_t = 0.01$. Were the monetary authorities not to react, the increased demand for money would increase its scarcity and thus the nominal interest rate. Within our limited-participation environment, this increase in nominal rates would have a detrimental effect on all market activities and consumption would fall, along with the production of good 1. Further, the increase in the nominal interest rate would lead to nominal and real depreciations of the currency.

The monetary authorities do react to the shock, however, and increase the supply of money. This insulates almost perfectly the economy from the effects of the shock; the scale of all graphs in Figure 3 is very small. Accommodating the increase in money demand results in a flat profile for nominal interest rates, which limits drastically the response of other variables.

6. Searching for a "Good" Rule

6.1 Uniqueness, indeterminacy, and explosiveness

One way to determine the relative desirability of different monetary policy rules is to assess their consequences for the overall stability of the economic model. Some rules imply a uniquely determined, stable equilibrium where no episodes of self-fulfilling shocks are possible, others will lead to a multiplicity of equilibria (i.e., an indeterminate equilibrium) where such episodes can occur, and yet others yield unstable equilibria that never converge back to their initial state after the onset of a shock. We interpret the indeterminate or explosive cases as situations where the rule followed by monetary authorities actually exacerbates fluctuations rather than stabilizes them, and therefore is welfare-reducing.[43] An exploration of a rule's consequences for the overall stability of the model economy is therefore a natural starting point for an investigation into what a "good" rule consists of.[44]

Figure 4 presents such an exploration. It depicts the stability characteristics of the equilibria implied by a monetary policy rule that responds to deviations of inflation and output from their steady-state values as well as to lagged values of the interest rate, so that we have the following:

$$\hat{R}_t = \alpha\hat{\pi}_t + \beta\hat{y}_t + \gamma\hat{R}_{t-1} + \varepsilon^{MP}{}_t. \tag{43}$$

The figure explores various specifications of (43); the three panels of the figure correspond to values of $\gamma = 0, 0.5, \text{and } 1.5$, respectively, and each panel examines a wide range of values for α and β. For each combination of parameters, the figure shows whether the equilibrium is unique and stable (light grey), indeterminate (white), or explosive (dark grey). In addition, we indicate in black the rare cases where we could not find any solution.

The first lesson one can draw from the graphs in Figure 4 is that a strong, positive response of interest rates to inflation deviations increases the likelihood of obtaining a unique, stable equilibrium. Second, for given values of the inflation response, a stronger response to output decreases the possibility of uniqueness and makes (welfare-reducing) indeterminacy more likely. Finally, a significant response to lagged interest rates makes it more likely that the equilibrium will be unique and stable.

The stabilizing effect of a strong response to inflation, as well as the undermining of this stabilization that a strong response to output causes, is also present in the stability assessment presented in Christiano and Gust (1999), within their closed-economy, limited-participation model.[45] This occurs because, in limited-participation models, an expected rise in inflation leads agents to increase the cash holdings they keep for consumption good purchases and to reduce the funds they send to financial markets. Banks cannot create substitutes for these deposits and must therefore reduce the total supply of loanable funds, increasing lending rates and thus creating

43. In the indeterminacy case, the excess volatility arises because sunspot shocks have the ability to make the economy switch from one equilibrium to another and thus affect real allocations. Further, indeterminacy implies that the economy may overreact to fundamental shocks.

44. Christiano and Gust (1999, footnote 8) argue that "first-order welfare gains are to be had by avoiding these bad outcomes" and that "once these outcomes have been avoided, there is relatively less to be gained from moving to the globally optimal specification."

45. We explore the possibility that α and β take on negative as well as positive values, rather than study only the positive quadrant, as Christiano and Gust (1999) do. Figure 4 reveals that negative responses to output or inflation deviations can also lead to unique stable equilibria.

downward pressure on output.[46] A rule that directs monetary authorities to dampen this rise in interest rates by injecting liquidity in the financial markets (i.e., a low value of α or a high value of β) may provide the support that enables a shock to expected inflation to become self-fulfilling. In contrast, a rule that responds strongly to inflation but only in a limited manner to output variability (a high value of α and a low value of β) will reduce liquidity following a rise in expected inflation, and the chain that may have linked expected increases in inflation to increases in the actual rate is cut: no self-fulfilling episodes can exist (see Christiano and Gust 1999 for further discussion).

Our results differ significantly from those of Christiano and Gust (1999) along one dimension, however: relative to what they report, the inclusion of open-economy aspects shrinks the size of the region of explosiveness, with the indeterminacy and uniqueness regions expanding to cover the gap. Notably, this implies that, in our analysis, the original Taylor rule with $\alpha = 1.5, \beta = 0.5,$ and $\gamma = 0$ yields a stable equilibrium, while it does not in Christiano and Gust's analysis. Although an expected rise in inflation still reduces the domestic supply of liquidity to financial markets, banks have, in our framework, access to international financial markets as an alternative source of financing. Therefore, the reduction in the supply of funds will be lessened, and the upward pressure on the interest rate and the downward pressure on output will be diminished. A strong response to output deviations (a high value of β) may not lead to the injection of new liquidity (which would help make the rise in expected inflation self-fulfilling), because the rise had a smaller impact on interest rates and output to begin with.

Figure 5 shows the stability characteristics of a similar rule for which expected future inflation replaces actual inflation:

$$\hat{R}_t = \alpha E_t[\hat{\pi}_{t+1}] + \beta \hat{y}_t + \gamma \hat{R}_{t-1} + \varepsilon^{MP}{}_t. \tag{44}$$

46. Agents increase their cash holdings because an expected burst of inflation makes holding deposits in financial markets less attractive; recall that the return from holding such deposits can be used only in the next period, by which time inflation will have devalued its purchasing power. The inability of financial markets to create any substitutes for household deposits (i.e., to create inside money) can be interpreted as a situation where the supply of loanable funds is severely constrained for reasons exogenous to the model. Chari, Christiano, and Eichenbaum (1995) present a model where financial intermediaries have the ability to create inside money. An exploration of the stabilization properties of simple monetary policy rules within their environment would make an interesting contribution to the literature.

The results shown in Figure 5 suggest that, under such a specification, unique and stable equilibria are much less common, unless the coefficient on lagged interest rates (γ) is high.[47] As evidenced above, inflation reacts quickly and in a transitory manner to monetary policy shocks. A rule that directs monetary authorities to respond to future inflationary pressures rather than current ones may be inefficient, because it affects current inflation without modifying the future rate very much. In responses to technology shocks, for example, the inflation reaction may be concentrated in the period of the shock, in which case a rule like (44) in essence bars monetary authorities from responding to the shock. Compared with Christiano and Gust (1999), the open-economy features of our paper seem to further reduce an already small region of unique and stable equilibria.

In summary, a rule that does not contain a strong positive response to output tends to be associated with unique, stable equilibria, as does a rule that includes a strong weight on lagged interest rates; this latter fact is particularly relevant when a rule reacts to expected future inflation, as in (44). Section 6.2 describes the ranking of rules according to their stabilization properties. We explore only the subset of rules that imply stable, unique equilibria. We therefore implicitly assume that rules that lead to indeterminate or explosive equilibria reduce welfare to such an extent as to render them uninteresting for further study.[48]

6.2 Stabilization properties

6.2.1 The experiments

The first measure we use to rank policy rules is a welfare-based loss function. This function is obtained by computing a second-order Taylor expansion of households' lifetime utility around the non-stochastic steady state of the model. This approximation yields the following:

$$W \approx u^{ss} - \frac{\gamma_1}{2}\sigma^2(\hat{c}_{1t}) - \frac{\gamma_2}{2}\sigma^2(\hat{c}_{2t}) - \frac{\gamma_3}{2}\sigma^2(\hat{c}_{3t}), \tag{45}$$

where W is lifetime utility, u^{ss} is the utility obtained in the non-stochastic steady state, and $\sigma^2(\hat{c}_{1t})$, $\sigma^2(\hat{c}_{2t})$, and $\sigma^2(\hat{c}_{3t})$ are the variance in each type of consumption (see Appendix B for

47. The interpretation of the coefficients in (44) is slightly different than if the rule had been written in the form commonly employed in the empirical literature (e.g., Clarida, Galí, and Gertler 1998):

$$\hat{R}_t = (1-\gamma)[\alpha E_t \hat{\pi}_{t+1} + \beta \hat{y}_t] + \gamma[\hat{R}_{t-1}] + \varepsilon^{MP}{}_t .$$

In particular, when considering values of γ bigger than one and positive values of α, this form would call for a decrease in interest rates if inflation threatened to increase above its target.

48. It is common in the literature to disregard rules that do not lead to stable equilibria. Some authors, however, do take into account such rules in their quantitative work: Lubik and Schorfheide (2003) allow for the possibility of indeterminate equilibria in their estimation of a New Keynesian model.

the derivation of (45)). Since the value of u^{ss} is common across rules (they all lead to the same steady state), good stabilization properties can be defined as keeping the weighted sum of variances in (45) to a minimum. We thus have the following welfare-based loss function:

$$L^1 = \gamma_1\sigma^2(\hat{c}_{1t}) + \gamma_2\sigma^2(\hat{c}_{2t}) + \gamma_3\sigma^2(\hat{c}_{3t}) . \quad (46)$$

When we discuss the welfare implications of a rule, we discuss both the welfare according to (46) and the lifetime utility computed from (45).

To provide a robustness check on our results, we also report results based on a loss function that penalizes variability in output and inflation equally, so that the loss is:

$$L^2 = \sigma^2(\hat{\pi}_t) + \sigma^2(\hat{y}_t) . \quad (47)$$

One justification central banks use when they introduce an inflation-targeting strategy is that it limits the variability and uncertainty in inflation, which in turn increases the prospects for stable and high economic growth. The inclusion of inflation variability in the loss function, as in (47), may serve as a proxy for such concerns. Further, such a loss function is often used in the literature that examines policy rules with models of the New Keynesian paradigm.[49]

We also examine a loss function that penalizes interest rate variability in addition to inflation and output variability. This follows Batini, Harrison, and Millard (2003), among others, and is interpreted as a dislike, on the part of monetary authorities, of big movements in interest rates, perhaps to avoid situations where the zero lower bound on nominal interest rates becomes binding.[50] We use a weight of 0.25 on the interest rate variability, so that the loss is:

$$L^3 = \sigma^2(\hat{\pi}_t) + \sigma^2(\hat{y}_t) + 0.25\sigma^2(\hat{R}_t) . \quad (48)$$

In Tables 1 to 3, we compute the best specification within each type of rule, according to the loss L^1 (Table 1), L^2 (Table 2), and L^3 (Table 3).[51] Recall that we refer to the type of a rule as the list of feedback variables to which interest rates respond; the best specification within a type consists

49. In such models, however, a formal correspondence between inflation variability and welfare, arising from the price rigidities, can be established.

50. A dislike for drastic movements in interest rates can be formally rationalized by positing that monetary authorities, having limited information about the current economic situation, can only gradually learn about it; such learning is what appears as interest rate smoothing (see Aoki 2003). Batini, Harrison, and Millard (2003) introduce interest rate volatility in the loss of monetary authorities by including the variance of the *change* in interest rates, in contrast to (48).

51. The best specification is identified using Matlab's simplex algorithm (*fmins*).

of the coefficient values that achieve the lowest loss possible.[52] The first panel of Tables 1 to 3 reports the optimized values of the coefficients. The second panel shows the loss associated with each rule, as well as the percentage deviation of this loss with respect to the one that the economy would attain were the optimal policy rule to be followed. The latter is a function of all of the state variables in the model as well as of a set of lagged Lagrangean multipliers associated with the optimal decision rules of the agents in the model.[53] We use the summary statistics from this rule (shown in the last column of each table) as a benchmark to compare with the other rules.

In the third panel of Tables 1 to 3, we report the standard deviation of economic variables when the rule is applied to the economy. In the last panel, we show the loss associated with the rule according to the two alternative loss definitions, as well as the likelihood of negative nominal interest rates occurring in the simulations (the linear method we use to compute approximate solutions to the model does not preclude such an occurrence). Further analysis should explicitly account for the non-linear behaviour of the economy at the zero bound as well as the possible changes in the implementation of monetary policy (through direct purchases of financial assets, for example).[54]

The results in Tables 1 to 3 are computed under the assumption that monetary authorities are fully committed to the rule being studied; economic agents put zero weight on the possibility of monetary policy deviating from the systematic part of the rule. We discuss this assumption in section 6.2.4 and also report some results of the hypothesis that monetary authorities reoptimize every period.

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52. For example, Rule 2 in Table 1 is of the type where interest rates respond only to deviations in output and inflation from steady state. The type of the rule is written generically as

$$\hat{R}_t = \alpha\hat{\pi}_t + \beta\hat{y}_t + \varepsilon^{MP}{}_t,$$

while the optimized rule is described by the numerical values for α and β that minimize L^1.

53. The rule is involved and thus its exact form is not reported. Soderlind (1999) and Dennis (2003) discuss globally optimal rules. We use the latter's notation and methodology to compute the optimal rules. The lagged Lagrangean multipliers in the rule ensure that policy is set in the current period in a manner consistent with how private sector expectations were formed in the past.

54. The tables indicate that the zero bound on interest rates is not violated frequently by the rules, except for the optimal rule in Table 1. This frequency would increase significantly if the steady-state rate of inflation was lower than 4.5 per cent. Rotemberg and Woodford (1999) increase the steady-state inflation associated with a rule when it implies high interest rate volatility, to avoid the zero-bound problem on nominal rates.

6.2.2 The results: welfare-based loss function

Table 1 reports the implications of nine rules, ranking them according to the welfare-based loss function in (46). One noticeable feature is that the welfare losses associated with the rules are very small (second panel of the table). Stated in another way, the lifetime utility of the representative agent is in all cases very close to the steady-state utility, so that the welfare losses resulting from economic fluctuations are small and the stabilization effects that monetary policy can provide carry only modest welfare benefits. For example, the original Taylor rule, with (non-optimized) coefficients of 0.5 on output and 1.5 on inflation deviations from target, yields a loss of 0.000314 and the lifetime utility attained under this rule is only 0.07 *per cent* lower than the steady-state utility.[55] Nevertheless, the simple Taylor rule is not the best achievable outcome for monetary authorities: its loss is 25 per cent above that which would occur under the optimal policy (last column). The remainder of this subsection describes the extent to which this gap can be bridged by alternative rules.

Retaining the same feedback variables (current inflation and output deviations), but searching for the optimal values of the responses to these variables, leads to Rule 2, the implications of which are shown in the second column of Table 1. The loss is reduced by approximately 10 per cent relative to that of the original Taylor rule, and is only 13 per cent above the loss implied by the optimal rule. The welfare improvements from following Rule 2 rather than Rule 1 result mostly from a reduction in the variability of c_2 (the consumption of imported goods). Further, the coefficient on inflation is significantly higher in Rule 2 and the coefficient on output is negative. These results are consistent with the analysis in the preceding section: within limited-participation environments, monetary policy rules require high coefficients on inflation and low coefficients on output to perform well. The simulations reported in Table 1 show that pushing the response to output down into negative values is the best policy; it would lead monetary authorities to react to upward pressure on output by reducing interest rates, seemingly providing further stimulus to the economy.

Rule 3 contains a response to lagged interest rates. Optimizing over the three coefficients leads to an outcome where the loss, relative to the original Taylor rule, is reduced by 15 per cent and is

55. Very small welfare gains from the stabilization effect of monetary policy are common in the literature (e.g., Devereux 2000). Such results echo the general finding of Lucas (1987), who observes that fluctuations of an order similar to that observed in modern developed economies may simply not be very costly in terms of lifetime utility, at least as measured by standard utility functions. These small welfare gains do not mean that the rewards of good monetary policy are insignificant. Recall that we have already identified and discarded rules that did not lead to stable and unique equilibria. In doing so, it may be argued that we have already achieved first-order welfare gains and are minimizing second-order losses.

only 9 per cent above the loss implied by the optimal rule. This welfare improvement originates from modest reductions in the variability of all three types of consumption. The coefficients on both output and inflation are negative, while the coefficient on lagged interest rates is positive and higher than 1. In relation to the graphs shown in Figure 4, the optimal rule is thus located in the bottom left quadrant of panel C. The negative coefficient on inflation is not a standard feature of quantitative analyses of monetary policy rules: it implies that monetary authorities should react to inflationary pressures by reducing interest rates. Further, the high value of the coefficient on lagged interest rates would lead to explosive economic outcomes in models that do not have the strong forward-looking behaviour of our model (we discuss these results in section 7).[56] Rule 3 leads to variable inflation (a feature common to all remaining rules reported in Table 1). The second- and third-last rows of Table1 show that Rules 3 to 9 perform badly if inflation variability appears in the loss (as in L^2 and L^3).

Rules 4, 5, and 6 explore the consequences of adding the real exchange rate, s_t, to the list of feedback variables to which monetary authorities should respond. The respective rules differ in that the current value, the current and lagged values, or the growth rate of s_t enters the rule. Rule 4 shows that reacting only to the current level of the real exchange rate has a limited impact: the coefficient on the real exchange rate is close to zero and the welfare results, as well as the coefficients on inflation, output, and lagged interest rates, are virtually unchanged from those reported under Rule 3. Batini, Harrison, and Millard (2003), using a small open-economy model with price rigidities, and Côté et al. (2003), analyzing large models of the Canadian economy, also report little benefit from reacting directly to exchange rate movements. On the other hand, noticeable improvements in the welfare loss do result from reacting to both current and lagged values of the real exchange rate, as in Rules 5 and 6. The signs of the coefficients indicate that positive changes in s_t relative to s_{t-1} (real depreciations) should lead, somewhat counterintuitively, to decreases in the interest rate set by monetary authorities. We do not focus on these rules, because this is the only instance in which we find that reacting to exchange rate-based variables affects the analysis.

Rule 7 responds to the same variables as Rule 3: inflation and lagged interest rates. The difference is that expected future deviations of inflation from steady state enter the rule, rather than the current deviations. Although this modification of the rule's structure does not significantly change the coefficients on inflation and lagged interest rates, the coefficient on output is positive. The welfare loss is also slightly lower than it was under Rule 3, consistent with the results in Batini,

56. Inflation enters the rule with a positive coefficient when the two alternative measures of loss are used; see Tables 2 and 3.

Harrison, and Millard (2003), where such an inflation-forecast-based (IFB) rule is found to minimize loss. The type of rule is the same as Ravenna (2000) estimates—his results are reproduced above in (41)—using observed monetary policy outcomes in Canada over the last decade. Although the numerical coefficients of Rule 7 are fairly different from those estimated by Ravenna, the high coefficient on lagged interest rates is common to the two rules.

Rule 8 explores the consequence of reacting to lagged output and inflation deviations from steady state, rather than their current values, which might not be known with certainty when monetary authorities set interest rates. Table 1 reports that this rule leads to a slightly higher loss than Rule 3. Further, the response to interest rates is lower, and responses to both inflation and output are positive.[57] The deterioration in welfare is caused by an increase in the variance of c_3, the consumption of the non-traded goods. These results suggest that there is a slight but noticeable difference in both the welfare attained and the correct responses to inflation and output when there are considerable lags between the monetary policy actions and the release of the data.

Rule 9 explores the appropriateness of including responses to both current and future inflation in the rule. The resulting coefficients are similar to those of Rule 7, which react only to expected future inflation; this suggests that the key response is the one that governs the reaction to future inflation. Nevertheless, Rule 9 does produce small reductions in the loss, relative to the already low level achieved under Rule 7. Interestingly, the welfare improvements arise from a reduction in the variability of the non-traded good consumption, c_3. The last column of Table 1 shows that further reductions in the volatility of imported good and non-traded good consumption are possible with the optimal rule, which reacts to all shocks and state variables of the economy. Relative to the Taylor rule, the variance of good 2’s consumption is reduced by 50 per cent.

In summary, we find that a rule that responds to lagged interest rates as well as to current output and inflation deviations exhibits good stabilization properties. The signs of these responses would lead monetary authorities, however, to take decisions that might appear at first counterintuitive: lowering rates when output or inflation is pushing upwards, and reacting to lagged rates with seemingly explosive force.[58] In addition, there was little robust evidence to suggest that directly reacting to exchange rates might significantly improve welfare. Moreover, responding to current inflation, rather than expected future inflationary pressures or past values of inflation, appears to

57. Rotemberg and Woodford (1999) also report slight increases in loss when the lagged values of the feedback variables are entered in a rule that responds to inflation, output, and interest rates.

58. We repeated the experiments reported in Table 1 under the constraint that the response to inflation should be positive. The welfare results were noticeably worse and the coefficients on lagged interest rates were mostly negative. The negativity of the inflation response thus seemed to have been transferred to the response to lagged interest rates (results available upon request).

be important to lower economic loss. Finally, most of the welfare improvements that arise from choosing the better rules result from a reduction in the variability of the imported good consumption, and at the expense of inflation variability. As such, the inclusion of open-economy features in the analysis is found to have important consequences for our assessment of what a "good" monetary policy rule is, even though the inclusion of the real exchange rate in the rules does not produce significant changes to their stabilization properties.[59] Section 7 will discuss these results further; first, section 6.2.3 will explore the extent to which these results depend on the loss function used to rank the rules.

6.2.3 The results: alternative loss functions

Table 2 reports simulation results for the case when the ranking is governed by loss function L^2, the sum of output and inflation deviations from steady state. One result that is repeated from Table 1 is that the original Taylor rule, with coefficients of 0.5 and 1.5 on output and inflation, can be significantly improved by using the optimized coefficients (Rule 2): this reduces the loss from 33.8 to 4.5, a decline of over 85 per cent.[60] The predominant source of this reduction is a substantial decline in the volatility of inflation. Further, as in Table 1, the optimized coefficient on inflation is positive and the response to output is negative. The inflation responses remain positive across all the rules analyzed. The objective of limiting the variability of inflation, which was not a concern in the ranking presented in Table 1, leads monetary authorities to increase rates in response to upward pressures on inflation, the standard response discussed in the literature.[61]

The addition of the lagged value of interest rates to the list of feedback variables slightly reduces the loss (see Rule 3). The signs of the output and inflation coefficients do not change, and the lagged interest rate coefficient continues to be above one, as was the case in Table 1. A reduction in output variability leads this rule to a superior loss result: compared with Rule 2, the volatility of inflation actually increases slightly.

Including the real exchange rate in the rule, the implications of which are illustrated by Rules 4, 5, and 6, does not modify results significantly. The responses to the exchange rate are close to zero, and the coefficients on output, inflation, and lagged interest rates, as well as the values attained by

59. Taylor (2001) expands on this idea, arguing that a "closed-economy" rule already reacts (indirectly) to the exchange rate through its effects on inflation and output, even though the exchange rate does not enter directly into the rule.

60. The absolute magnitudes of the losses are not comparable across the different tables (i.e., across the different specifications of the loss function).

61. Stated another way, in Table 1, all the rules that contain a negative reaction of monetary policy to inflation, which performed well under loss L^1, did much worse under L^2 or L^3 (see the second-to-last and third-to-last rows of Table 1).

the loss, are virtually unchanged from Rule 3. Reacting explicitly to exchange rates does not have much of an effect on the ability of monetary policy rules to minimize loss when measured by the sum of output and inflation variability. This is in contrast with some of the results in Table 1, which suggest that reacting to real depreciations might lead to some decreases in the loss.

Rule 7, which reacts to expected future inflation rather than current inflation, significantly worsens the loss; the results are the lowest of all rules analyzed. This is in sharp contrast with Table 1 and also with the results in Batini, Harrison, and Millard (2003), where such a rule has a very good outcome. This result illustrates that, within our limited-participation model with flexible prices (recall from Figures 1 to 3 that the reaction of inflation to shocks is immediate and short-lived), reacting only to expected future inflation is not an efficient way to limit inflation's variability. Again, were inflation volatility to cease being a concern, such a rule would start performing better: when the rules in Table 2 are ranked according to the welfare-based loss, L^1, Rule 7 is the best.

Rule 8, in which the lagged (rather than the current) values of inflation and output enter the rule, worsens the loss, though not as drastically as the drop associated with Rule 7. This result (and a similar one reported in Table 1) suggests that the ability to observe and react to inflation quickly is important in our framework; the quick and transient response of inflation to economic shocks plays a role in generating this result. Finally, Rule 9, which responds to both current and future inflation, performs the best of all the rules examined.

In summary, most of the general results first described in Table 1 are robust to ranking the rules according to L^2. Responding to lagged interest rates in addition to inflation and output shows the best potential for loss minimization; the response to lagged interest rates should be strong, while the coefficient on output should be weak, or even negative and, on balance, the response to the exchange should be zero. The results are less robust with respect to the coefficient on inflation, which is negative most of the time when ranking rules according to L^1, but positive when using L^2. Further, responding to current values of inflation is important, as the loss is significantly affected if monetary authorities react only to future or only to lagged inflation. It is important to note that the numerical values of the coefficients in the rules with good stabilization properties are quite different across loss functions, even within the same rule type. Our analysis should therefore be interpreted as identifying principles to guide monetary policy practice across a wide variety of situations, rather than finding a precise numerical rule.

Table 3 confirms that these general principles are robust to yet another measure of loss. The table illustrates the consequence of modifying loss L^2 by including the variability in interest rates in the loss, which is written as in (48). The table indicates that loss is significantly reduced by first optimizing over the coefficients on current inflation and output (Rule 2), and then adding lagged interest rates to the rule (Rule 3). The presence of interest rate volatility in the loss significantly increases the coefficient on lagged interest rates in the rule, relative to Table 2. Again, the inclusion of an exchange rate variable has no significant impact on the analysis. Finally, the absence of current inflation from the list of variables that monetary policy responds to does not lead to good outcomes. The precise numerical values of the coefficients in the rules have again changed from Table 2 to Table 3.

6.2.4 Responding to the GDP deflator

To this point, the inflation variable that has been included in the rule was the model equivalent to the growth in the CPI index (recall the definition of the aggregate price index in (6)). Table 4 reports the ranking of a subset of the rules when the growth in the model equivalent to the GDP deflator is used as the measure of inflation. A majority of the rules perform substantially worse than when CPI inflation was used. Only the rule that minimizes the welfare-based loss function using domestic inflation, output, and the lagged interest rate is close to its counterpart with CPI inflation reported in Table 1. While the inflation response is negative for all but one rule, the output coefficient is negative only for the rules operating under the welfare-based loss. Interestingly, including the change in the real exchange rate in the rule (Rule 11) provides improvements to the losses, even under L^2 and L^3. This is logical, because by restraining monetary policy to react only to domestic prices, the price of the imported good, c_2, has been excluded from the analysis: reacting to the exchange rate is one way to reintroduce a concern for the variability in c_2. In summary, reacting only to the domestic portion of inflation would not be recommended by this model. If a central bank decided to follow such a strategy, however, including the change in the real exchange rate variable would improve the performance of the monetary policy rule.

6.2.5 Full-commitment rules and discretionary rules

The rules discussed to this point have been chosen under the assumption that a commitment device exists that ensures monetary authorities will follow the specified rule from the present until the infinite future, with zero probability that they will deviate from it.

Christiano and Gust (1999) argue that this assumption might be unrealistic. Faced with a very substantial adverse-supply shock, monetary authorities will need to increase interest rates significantly (and in doing so damage an already fragile economy) in order to control inflation. Alternatively, exploding inflation expectations can be brought under control only by significant increases to interest rates. There might therefore be a strong incentive for monetary authorities to deviate from their announced rules in these situations, to lessen the impact of shocks to the real economy, making the full-commitment assumption less tenable.

Consequently, we compute optimal discretionary rules, following the strategy described in Dennis (2003). These rules are self-enforcing, because they allow monetary authorities to re-evaluate the actions they should take every period. Imagine that a Nash game is played between the central bank and the private agents of the economy. The central bank chooses its rule given decision rules made by private agents, while agents set their decision rules based on what they believe the central bank is doing at the time and with the understanding that it may reoptimize in the future. The (Nash equilibrium) solution to this game is what we refer to as the discretionary equilibrium.[62]

Table 5 shows the results for the globally optimal rules under both commitment and discretionary assumptions (the commitment results repeat those already shown in Tables 1, 2, and 3). The table shows that there are important benefits from the commitment device: the loss functions decline substantially when the commitment assumption is used, relative to the discretionary equilibria. The commitment device, if it exists, is quite beneficial to the economy.

7. Discussion

In this section, we discuss our results, some of which might appear at first to be puzzling, and relate them to others in the literature on monetary policy rules, emphasizing the areas where our limited-participation-based analysis affects the results.

7.1 Negative responses to output

Most of the rules shown in Tables 1 to 4 contain a negative response to output. A priori, raising interest rates when output is already low would seem to exacerbate fluctuations and lead monetary policy to have a destabilizing effect on economic activity. In this model, however, this logic does not hold: the second-round effects of these negative responses facilitate rather than undermine stabilization.

Recall, however, that our limited-participation model contains a channel by which shocks affect the supply of loanable funds. Expected rises in inflation lead households to withdraw funds from financial markets and, since financial institutions do not have the ability to create money

62. See Cooley and Quadrini (2002) for a discussion of full-commitment and discretionary equilibria.

substitutes, this leads them to restrict credit supply, which increases lending rates and depresses economic activity. This “liquidity supply” channel thus associates inflationary pressures with declines in output, a negative correlation that does not occur in models from the New Keynesian paradigm.[63]

In this context, consider first a policy rule that reacts to inflation only, where the response to inflation is denoted by the coefficient $\alpha > 1$. Abstracting from monetary policy shocks, we have:

$$\hat{R}_t = \alpha\hat{\pi}_t. \tag{49}$$

Under this rule, inflationary pressures lead monetary authorities to increase nominal rates. In a limited-participation model, this requires that decreases be generated in money-growth rates, which serve to limit the increase in inflation and thus help stabilize the economy.

Consider a rule that reacts to output in addition to inflation, with a coefficient $\beta > 0$:

$$\hat{R}_t = \alpha\hat{\pi}_t + \beta\hat{y}_t. \tag{50}$$

In our economy, most shocks produce negatively correlated pressures on inflation and output because of the limited-participation hypothesis. In such a context, the responses described by α and β undermine each other. An adverse technology shock, for example, leads to decreases in output and increases in inflation. The coefficient $\beta > 0$ directs monetary authorities to lower rates, to attenuate the decline in output. Such rate decreases require an acceleration to the money-growth rates, thus exacerbating inflationary pressures. Because of the response in α, these exacerbated pressures lead to increases in rates and thus, if the effect is strong enough, to further output declines, whereas the original intent was to attenuate these declines.

By contrast, when the coefficient β is negative, the two responses work together to stabilize the economy. On the one hand, the increase in inflation requires monetary authorities, through parameter α, to decrease money growth in order to increase nominal rates. On the other hand, the original decline in output leads monetary authorities to *increase* nominal rates, seemingly at the expense of output. But this action reduces the upward pressures on inflation, which reduces the extent to which monetary authorities need to fight inflationary pressures in the first place. Again, this second-round effect is very significant in our simulations, so that monetary authorities need to increase interest rates by less, which stabilizes output fluctuations.

63. This correlation is similar to the one often associated with supply shocks; in our model, these supply effects arise from the inability of banks to create substitutes to households’ savings when lending.

Figure 6 illustrates the situation. It depicts the response of economic variables following a shock to sector 1 productivity, under three possible rules: an exogenous money-growth process (so that monetary authorities do not react to shocks), a rule that reacts only to inflation, and a rule that reacts to inflation and output with the coefficients of Rule 2 in Table 1 (which features a negative response to output deviations).

If monetary authorities did not react to the shock (exogenous money growth), inflation would increase and output would fall; the fall in output would reduce the demand for funds, which would contribute to keeping interest rates low. In contrast, under the rule that reacts to inflation but not to output, monetary authorities counteract the strong increase in inflation by reducing money-growth rates; through the liquidity effect, this keeps interest rates relatively high, and further exacerbates the reduction in output.

Consider the rule with a negative response to output. The output decline initiated by the shock calls for an interest rate increase and thus further decreases in money-growth rates. Because such decreases lead to similar decreases in inflation, and because of the positive coefficient on inflation in the rule, second-round *decreases* in interest rates result, contrary to the original intent. This actually helps alleviate the initial decline in output. In a model where the contemporaneous correlation between output and inflation was not as strongly negative as it is in the present case, these second-round effects would probably not be as pronounced and the usual intuition of a strong positive coefficient on output being able to control output fluctuations might continue to apply.

In summary, under the assumption that most shocks affecting an economy are supply-type shocks, or that it is likely and common for the financial markets to lose their ability (or willingness) to find substitutes to private funds in order to fund firm borrowing, the correct response to negative output pressures might be to increase rates, to preserve the inflation stabilization objective.

7.2 Explosive responses to lagged interest rates

Tables 1 to 4 illustrate that, when a response to lagged interest rates is added to the rule, the optimized value of that coefficient is most often larger than one. This appears at first to lead to explosive paths and thus to be inconsistent with a stable equilibrium. In our model, however, such a potential for explosiveness helps to preserve rather than undermine stability. Consider a rule that responds to inflation one-for-one and to lagged interest rates with a coefficient of, say, 1.5:

$$\hat{R}_t = \hat{\pi}_t + 1.5\hat{R}_{t-1} . \tag{51}$$

A shock of 1 per cent to inflation thus leads to an increase of 1 per cent in interest rates, starting from where rates are at their steady-state value. Abstracting from future values of inflation, this would lead to explosive responses of interest rates in the future to 1.5 per cent in the next period, 2.25 per cent in the next, etc. But within the limited-participation environment in our model, these ever-increasing interest rates would be associated with substantial reductions in money-growth rates, themselves causing future *decreases* in inflation and, thus, because of the presence of inflation in the rule in (51), decreases in the interest rates. Stability is preserved if the negative pressures on nominal rates emanating from those falls in future inflation overweigh the explosive nature of the coefficient on lagged interest rates.

One interpretation of this rule is that, in response to positive inflation shocks, monetary authorities are (credibly) committing themselves to embark on a series of ever-increasing tightenings, until subsequent declines in inflation become substantial enough to undermine the explosive path. Economic agents, with full knowledge of the rule, thus expect declines in inflation, which puts in place the conditions for those very declines to occur, or even for the initial inflation increase not to materialize. The credibility of the rule and the forward-looking behaviour of agents thus enables monetary policy to affect inflation pressures without actually reacting strongly to them, by simply threatening to react strongly in the future (the coefficient on inflation could have been small in the rule in (51)).[64] An alternative way to understand high coefficients on lagged interest rates is that they allow monetary authorities to influence long-term as well as short-term interest rates. Recall that, using the expectations hypothesis, long-term interest rates consist of the discounted, weighted sum of future short rates; a rule such as (51) commits monetary policy to increase short-term rates for the foreseeable future, thus leading to immediate increases in long-term rates. In turn, an increase in the long-term rates might have a strong influence on current economic activity and help stabilize inflation quickly. For these forward-looking effects to be operative, the assumption that monetary authorities are fully committed to the rule is key.

7.3 Negative responses to inflation

The rules evaluated using the welfare-based measure (Table 1) that include responses to lagged interest rates contain a negative response to inflation ($\alpha < 0$).[65] Again, this would appear to promote rather than mitigate instability: a burst of inflation would lead monetary authorities to

64. The mechanism by which high coefficients on lagged interest rates impart stability on an economy is discussed further in Rotemberg and Woodford (1999). See also Feldstein (1999).

65. We do encounter local minima with $\alpha > 0$, but they are dominated by the global minimum with $\alpha < 0$. For low or zero values of response to lagged interest rates, the local minimum with $\alpha > 0$ dominates, as evidenced by the results for Rule 2. Rotemberg and Woodford (1999) also report negative responses to inflation for some rule specifications.

lower nominal interest rates (generating increases in money-growth rates), thereby creating more inflationary pressures and further declines in interest rates, in a seemingly explosive chain.

The explosive chain of events may be broken, however, considering that the negative correlation between money-growth and nominal interest rates is not a built-in feature of the limited-participation environment and may actually disappear in some situations. Although monetary injections, by saturating the financial market with liquidity, do tend to push interest rates lower (the liquidity effect), these injections, if they are expected to persist for long periods of time, may be accompanied by increases in expected future inflation, pushing the nominal interest rate higher (the anticipated-inflation effect). Were this latter effect to dominate, the correlation between money growth and nominal interest rates may become positive.

Consider a persistent, negative technology shock that increases inflation and is expected to continue to do so for a long time. To counteract this increase in inflation, monetary authorities need to generate substantial and persistent decreases in money-growth rates. The anticipated-inflation effect is thus likely to dominate under this scenario, which indicates that this long period of reduced money-growth rates will be associated with reduced nominal interest rates, generating a negative correlation between interest rates and inflation ($\alpha < 0$). Stated another way, monetary authorities react to persistent upward inflationary pressure by persistently lowering the inflation tax; when their objective is to reduce fluctuations in consumption (Table 1)—which may be particularly affected by this tax—such a policy turns out to be optimal.

The results described in sections 7.1 to 7.3 lead to Rule 3 in Table 1. One the one hand, the rule raises rates when output is already low, because lowering rates would only create more inflationary pressures, and through the liquidity-supply channel exacerbate further the original declines in output. On the other hand, the rule leads monetary authorities to (slightly) increase rates when inflation is already low, because, faced with very persistent shocks, the best economic stabilizer they can provide is to manipulate the inflation tax. The slight increase in rates is propagated through time by the high coefficient on lagged interest rates, and is eventually stabilized by subsequent inflation increases.

8. Conclusion

This paper has quantitatively analyzed of the stabilization properties of several types of monetary policy rules. The analysis was conducted using a small open-economy model with limited participation that is affected by several sources of shocks and calibrated to salient features of the Canadian economy.

Similar to many other recent studies, we find that a strong response of monetary policy to lagged interest rates is likely to stabilize economic fluctuations and reduce the policy-makers' loss. We also find little evidence that reacting to exchange rates generates significant loss reductions.

Our results do differ from those of other studies on important aspects, however. First, we find that negative responses to output and sometimes to inflation may be the best course for monetary policy. The negative response to output can be rationalized by the presence of limited participation and a "liquidity-supply channel" of monetary policy, which affects banks' credit supply and produces negative correlations between output and inflation. The negative response to inflation may be interpreted as situations where monetary authorities react to long-lasting shocks by modifying their behaviour for long periods of time, so that the correlation between nominal rates and inflation is the positive one prevailing over the long term in our model. To some extent, because the response of inflation in the model to most shocks is immediate and short-lived, the long-term and short-term behaviour of inflation collapse into one, leading the rules we study to deviate from the standard forms they take in other models, in which short-term stabilization dominates the behaviour of monetary policy. This is further evidenced by our finding that little welfare benefit is to be gained by reacting to future expected inflation rather than the current rate.

In this respect, the policy implications of our results may be interpreted as complementing those that arise from similar analyses that use small models with price rigidities (Batini, Harrison, and Millard 2003), or larger empirical models of the Canadian economy (Côté et al. 2003). Policy-makers should perhaps study a battery of different monetary rules (along with the alternative policy prescriptions derived from those rules) and weigh different interest rate scenarios according to their best knowledge of the shocks or frictions most likely to affect the economy at the time of the decisions.[66] In such a context, the principles we have identified might serve to guide monetary policy during episodes when it is perceived that financial markets have lost their capability or willingness to modify credit supply to match credit demand.

Our model covers only one aspect of a complex problem. It may be interesting to nest our model environment within a more general one. For example, adding price rigidities to the existing portfolio rigidities might improve the model's ability to replicate the short-run dynamics of inflation, whereas allowing financial intermediaries to create a substitute for households deposits in order to lend more flexibly would perhaps limit the extent to which the "liquidity-supply channel" affects the transmission of monetary policy in the model. Such extensions would allow us to verify whether the main principles we identify remain to construct a "good rule."

66. See also Feldstein (1999) for such an argument.

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Table 1: Policy Rules Ranked According to $L^1 = \gamma_1\sigma^2(\hat{c}_{1t}) + \gamma_2\sigma^2(\hat{c}_{2t}) + \gamma_3\sigma^2(\hat{c}_{3t})$

Coefficient on	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	Rule 9	Optimal
$\hat{\pi}_t$	1.5	2.86	-0.23	-0.23	-0.16	-1.32			0.06	
$\hat{y}_t$	0.5	-5.35	-7.47	-7.18	-0.64	4.94	1.19		0.50	
$\hat{R}_{t-1}$			1.14	1.08	0.86	2.20	2.04	0.42	1.43	
$\hat{s}_t$				-0.05	-0.26					
$\hat{s}_{t-1}$					0.20					
$\Delta\hat{s}_t$						-2.02				
$\hat{\pi}_{t-1}$								0.22		
$\hat{y}_{t-1}$								4.90		
$E_t\hat{\pi}_{t+1}$							-0.38		-0.23	
Loss assessment										
Loss	0.00314	0.00282	0.00272	0.00272	0.00263	0.00268	0.00271	0.00277	0.00269	0.00250
% above optimal policy	25.11	12.70	8.56	8.57	4.89	7.18	8.09	10.70	7.56	0.00
Lifetime utility	2.0177	2.0178	2.0179	2.0179	2.0179	2.0179	2.0179	2.0178	2.0179	2.018
% below steady state	0.074	0.069	0.064	0.064	0.064	0.064	0.064	0.069	0.064	0.059
Second moments										
$\sigma(\hat{\pi}_t)$	5.05	3.09	18.49	18.37	27.75	15.16	20.83	15.34	27.50	22.10
$\sigma(\hat{y}_t)$	2.87	1.58	0.25	0.22	1.98	1.66	2.30	1.16	2.50	2.20
$\sigma(\hat{R}_t)$	6.53	4.49	5.39	5.50	7.19	6.31	4.95	4.04	5.38	8.77
$\sigma(\hat{c}_{1t})$	5.60	5.62	5.58	5.59	5.61	5.65	5.57	5.49	5.64	5.93
$\sigma(\hat{c}_{2t})$	3.80	2.70	2.56	2.57	1.32	2.29	2.16	2.57	2.22	1.90
$\sigma(\hat{c}_{3t})$	6.77	6.43	6.25	6.24	6.30	6.16	6.36	6.48	6.22	5.38
Other										
Loss according to L^2	33.75	12.03	342.11	337.53	774.37	232.64	439.49	236.85	762.94	493.29
Loss according to L^3	44.42	17.08	349.37	345.1201	787.30	242.62	445.63	240.94	770.20	512.53
% obs where $R_t < 0$	9.38	2.77	5.51	5.91	11.56	8.65	4.11	1.66	5.50	16.31

Table 2: Policy Rules Ranked According to $L^2 = \sigma^2(\hat{\pi}_t) + \sigma^2(\hat{y}_t)$

Coefficient on	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	Rule 9	Optimal
$\hat{\pi}_t$	1.5	188.00	27.31	37.28	34.89	33.39			11.17	
$\hat{y}_t$	0.5	-40.22	-5.55	-7.53	-7.05	-7.21	0.07		-0.48	
$\hat{R}_{t-1}$			2.49	3.53	3.39	3.01	1.17	5.57	0.89	
$\hat{s}_t$				0.13	0.08					
$\hat{s}_{t-1}$					0.04					
$\Delta\hat{s}_t$						0.02				
$\hat{\pi}_{t-1}$								12.39		
$\hat{y}_{t-1}$								-0.97		
$E_t\hat{\pi}_{t+1}$							-0.13		-11.30	
Loss assessment										
Loss	33.75	4.48	4.39	4.38	4.38	4.39	93.64	6.71	4.38	4.04
% above optimal	733.91	10.89	8.51	8.43	8.43	8.51	2213.52	65.94	8.23	0.00
Second moments										
$\sigma(\hat{\pi}_t)$	5.05	0.44	0.50	0.50	0.50	0.53	9.49	1.54	0.51	0.72
$\sigma(\hat{y}_t)$	2.87	2.07	2.03	2.03	2.03	2.02	1.88	2.08	2.02	1.87
$\sigma(\hat{R}_t)$	6.53	3.83	3.46	3.45	3.45	3.44	1.08	3.18	3.46	3.44
$\sigma(\hat{c}_{1t})$	5.60	2.84	5.55	5.55	5.55	5.55	5.48	5.50	5.54	5.56
$\sigma(\hat{c}_{2t})$	3.80	5.56	2.62	2.62	2.62	2.61	2.38	2.59	2.59	2.55
$\sigma(\hat{c}_{3t})$	6.77	2.66	6.59	6.59	6.59	6.59	6.65	6.67	6.59	6.54
Other										
Loss according to L^1	31.40	28.60	28.52	28.53	28.53	28.52	28.16	28.62	28.47	28.30
Loss according to L^3	44.42	8.15	7.38	7.36	7.36	7.36	93.94	9.24	7.38	7.02
% obs where $R_t < 0$	9.38	1.23	0.63	0.63	0.63	0.62	1.01 e-13	0.34	0.65	0.62

Table 3: Policy Rules Ranked According to $L^3 = \sigma^2(\hat{\pi}_t) + \sigma^2(\hat{y}_t) + 0.25\sigma^2(\hat{R}_t)$

Coefficient on	Rule 1	Rule 2	Rule 3	Rule 4	Rule 5	Rule 6	Rule 7	Rule 8	Rule 9	Optimal
$\hat{\pi}_t$	1.5	-9.07	23.70	70.26	42.38	24.79			-2.99	
$\hat{y}_t$	0.5	1.63	-3.50	-10.15	-6.10	-3.67	0.06		0.73	
$\hat{R}_{t-1}$			9.93	30.09	18.77	10.34	1.14	7.21	-1.25	
$\hat{s}_t$				0.53	0.16					
$\hat{s}_{t-1}$					0.20					
$\Delta\hat{s}_t$						0.01				
$\hat{\pi}_{t-1}$								8.40		
$\hat{y}_{t-1}$								-0.35		
$E_t\hat{\pi}_{t+1}$									-2.86	
Loss assessment										
Loss	44.41	7.98	6.55	6.54	6.53	6.55	93.90	8.37	6.55	6.14
% above optimal	622.46	29.84	6.56	6.37	6.35	6.56	1427.35	36.15	6.56	0.00
Second moments										
$\sigma(\hat{\pi}_t)$	5.05	0.64	1.06	1.07	1.06	1.06	9.49	1.82	1.06	1.11
$\sigma(\hat{y}_t)$	2.87	2.05	1.95	1.95	1.95	1.95	1.87	1.97	1.95	1.83
$\sigma(\hat{R}_t)$	6.53	3.65	2.51	2.50	2.50	2.51	0.93	2.14	2.51	2.47
$\sigma(\hat{c}_{1t})$	5.60	5.55	5.52	5.52	5.52	5.52	5.48	5.49	5.52	5.53
$\sigma(\hat{c}_{2t})$	3.80	2.65	2.52	2.54	2.54	2.53	2.41	2.50	2.52	2.47
$\sigma(\hat{c}_{3t})$	6.77	6.59	6.62	6.62	6.27	6.62	6.66	6.68	6.62	6.58
Other										
Loss according to L^1	31.39	28.58	28.41	28.42	28.42	28.41	28.21	28.45	28.40	28.20
Loss according to L^2	33.75	4.64	4.97	4.96	4.96	4.97	93.68	7.22	4.97	4.61
% obs where $R_t < 0$	9.37	0.92	0.03	0.03	0.03	0.03	2.59e-18	0.002	0.03	0.02

Table 4: Policy Rules Ranking: Domestic Inflation

Coefficient on	Rule 10	Rule 11	Rule 12	Rule 10	Rule 11	Rule 12	Rule 10	Rule 11	Rule 12	Optimal
$\hat{\pi}^{dom}_t$	-0.21	-1.00	0.90	-3.03	-6.30	-4.31	-2.99	-20.60	-5.73	
$\hat{y}_t$	-7.04	-4.25	-6.27	0.55	1.11	0.73	0.33	2.14	0.57	
$\hat{R}_{t-1}$	1.14	2.07	2.54	0.70	0.47	0.63	-0.27	-7.26	-1.24	
$\Delta\hat{s}_t$		-1.45			0.49			1.74		
$\Delta\hat{e}_t$			-2.55			0.21			0.29	
Loss assessment										
Loss according to L^1	0.00272	0.00270	0.00269							0.00250
% above optimal policy	8.57	7.73	7.49							0.00
Loss according to L^2				6.32	5.90	6.15				4.04
% above optimal policy				56.18	45.77	51.98				0.00
Loss according to L^3							8.45	7.99	8.27	6.14
% above optimal policy							37.56	30.04	34.52	0.00

Table 5: Globally Optimal Commitment vs Discretionary Rules

	Loss according to L^1		**Loss according to L^2**		**Loss according to L^3**	
	Commitment	Discretionary	Commitment	Discretionary	Commitment	Discretionary
Loss	0.00250		4.04	4.8	6.14	8.82
% above commitment	0.00		0.00	18.81	0.00	43.64
Lifetime utility	2.018					
% above commitment	0.00					
Second moments						
$\sigma(\hat{\pi}_t)$	22.10		0.72	0.10	1.11	0.27
$\sigma(\hat{y}_t)$	2.20		1.87	2.18	1.83	2.21
$\sigma(\hat{R}_t)$	8.77		3.44	3.90	2.47	3.92
$\sigma(\hat{c}_{1t})$	5.93		5.56	5.55	5.53	5.55
$\sigma(\hat{c}_{2t})$	1.90		2.55	2.69	2.47	2.73
$\sigma(\hat{c}_{3t})$	5.38		6.54	6.62	6.58	6.62
Loss comparisons						
Loss according to L^1	0.00250		28.30	28.74	28.20	28.81
Loss according to L^2	493.29		4.04	4.80	4.61	4.97
Loss according to L^3	512.53		7.02	8.61	6.14	8.82

Figure 1: Responses of the Economy to a Monetary Policy Easing

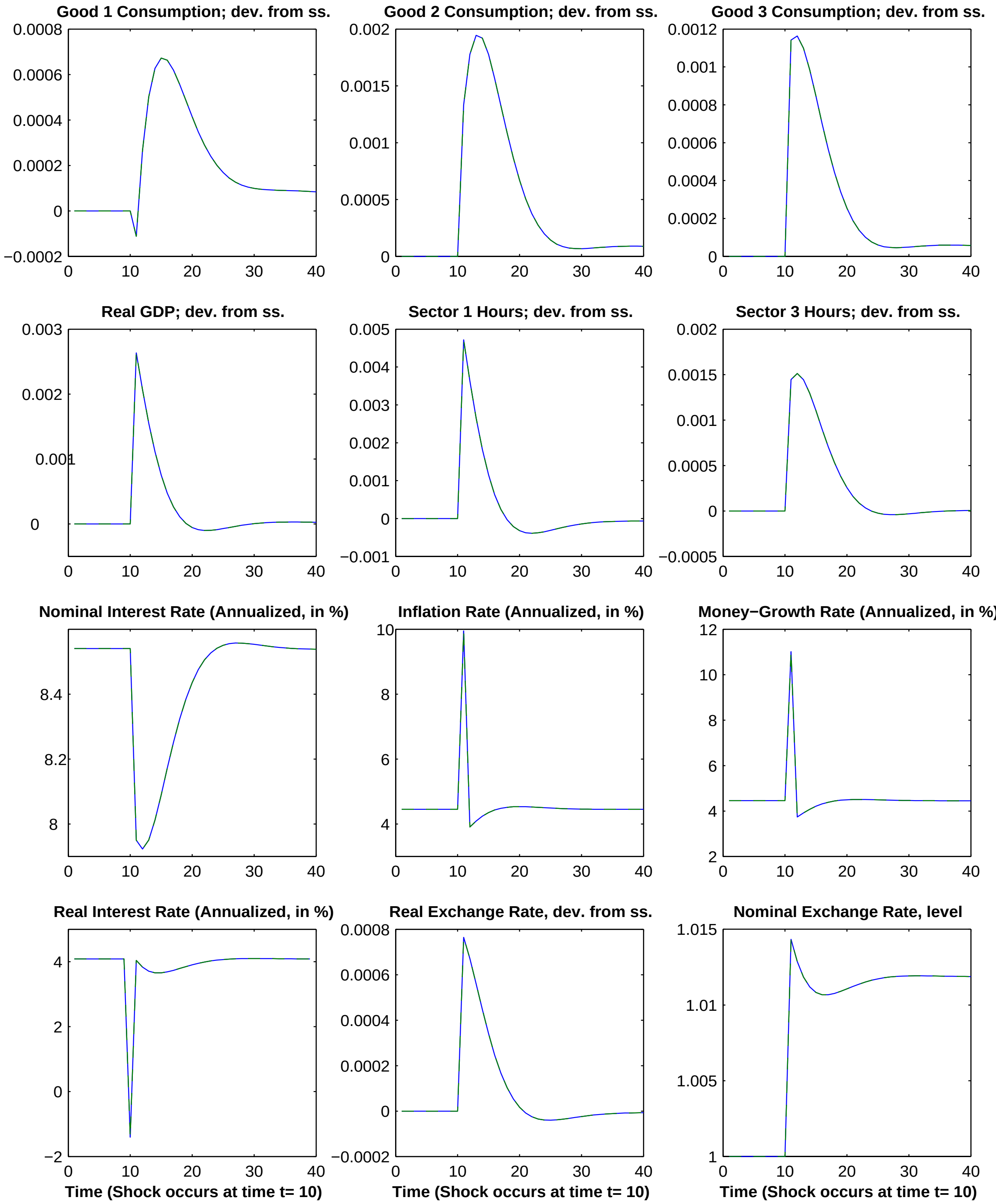


Figure 2: Response of the Economy to a Negative Technology Shock (Sector 1)

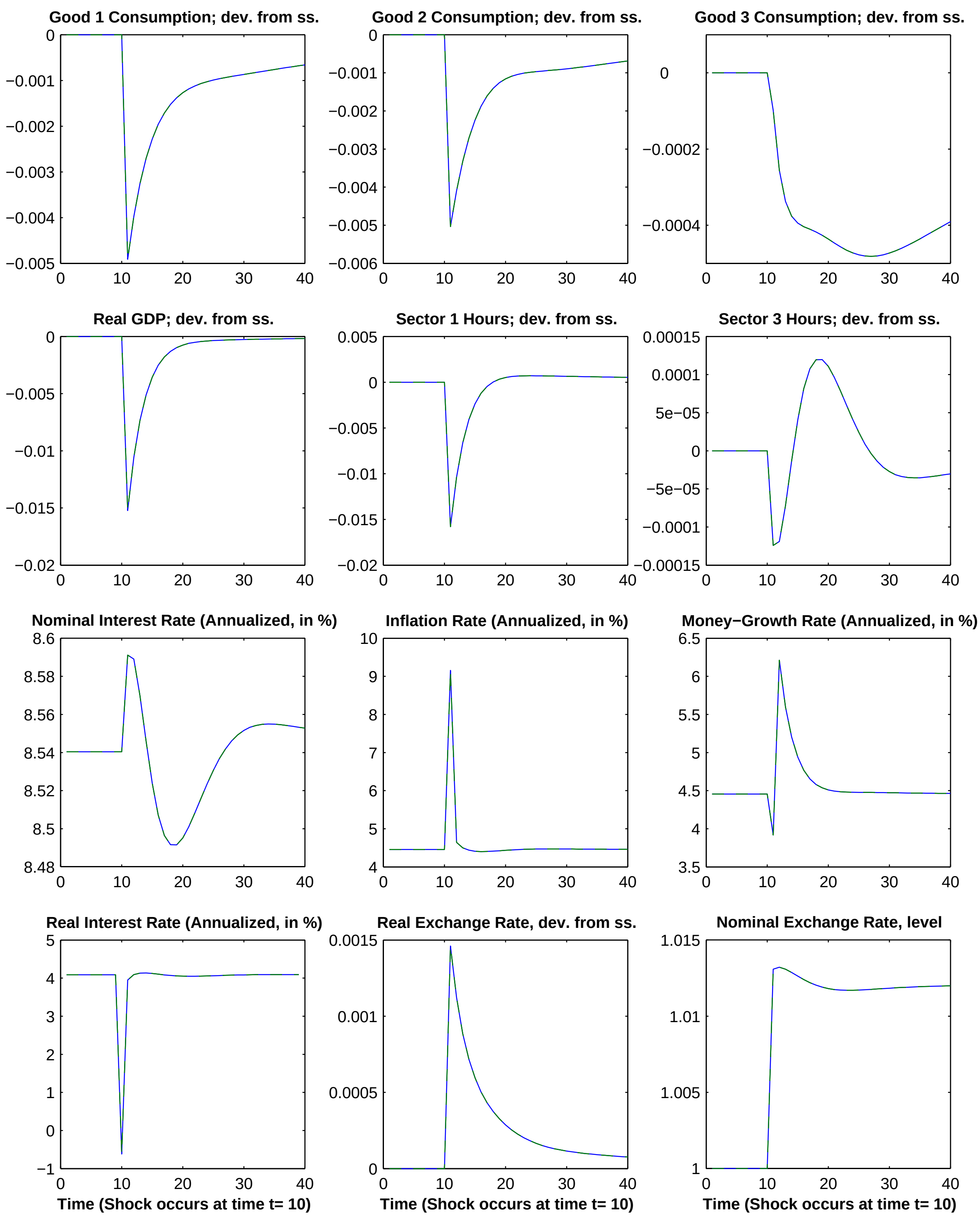


Figure 3: Response of the Economy to a Positive Money-Demand Shock

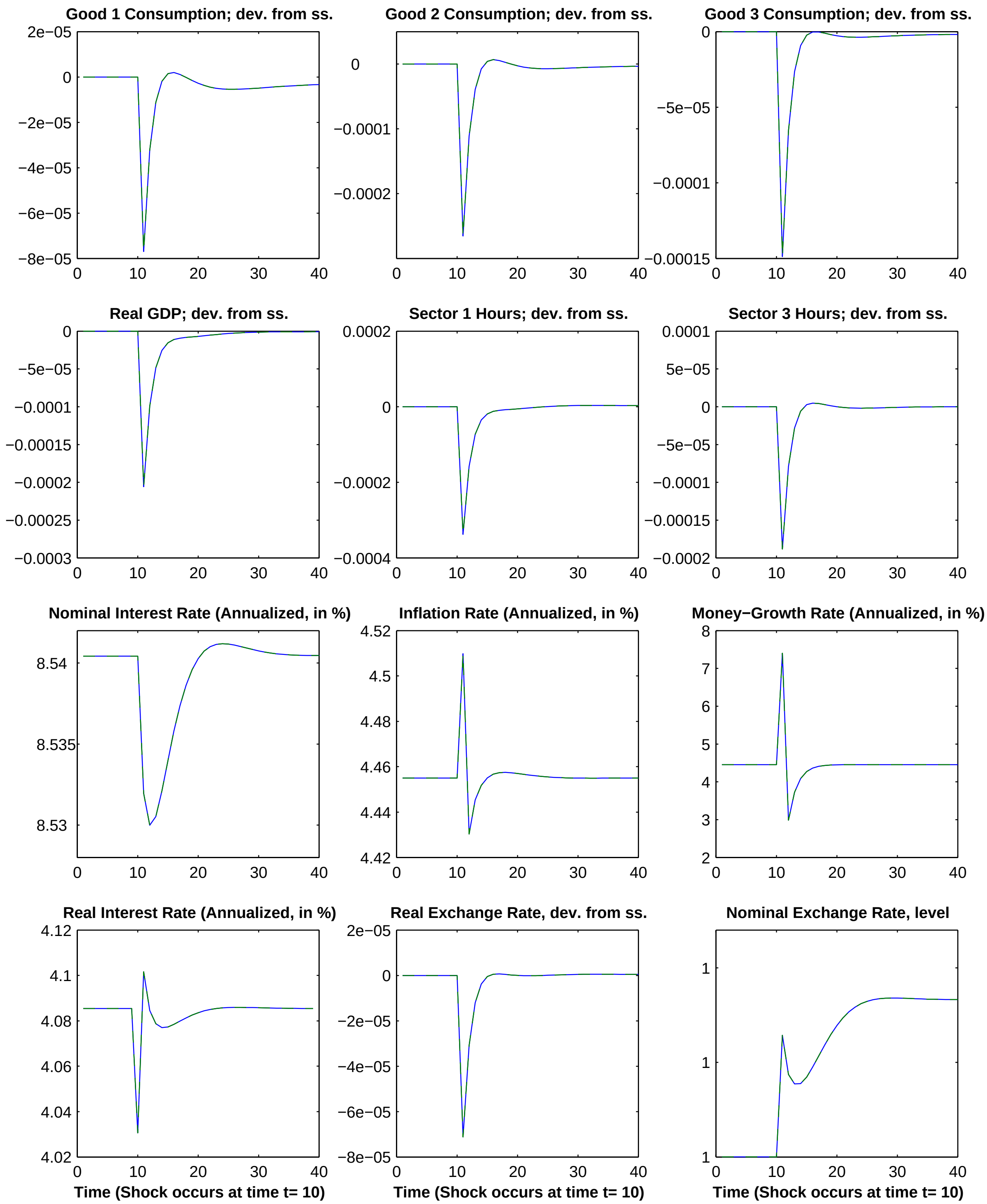


Figure 4: Stability Characteristics of the Rule $\hat{R}_t = \alpha\hat{\pi}_t + \beta\hat{y}_t + \gamma\hat{R}_{t-1} + \varepsilon^{MP}_t$

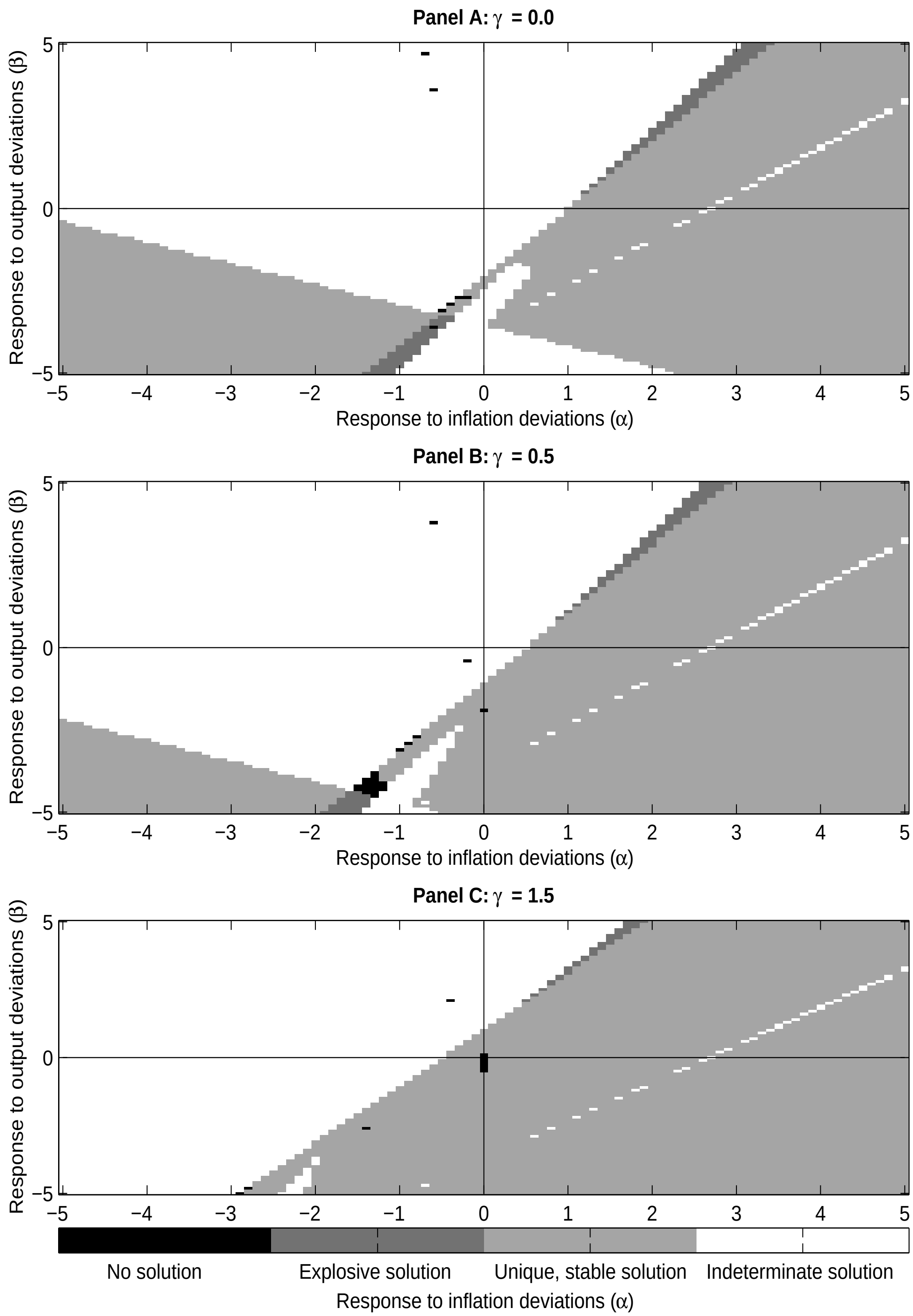


Figure 5: Stability Characteristics of the Rule $\hat{R}_t = \alpha E_t[\hat{\pi}_{t+1}] + \beta \hat{y}_t + \gamma \hat{R}_{t-1} + \varepsilon^{MP}_t$

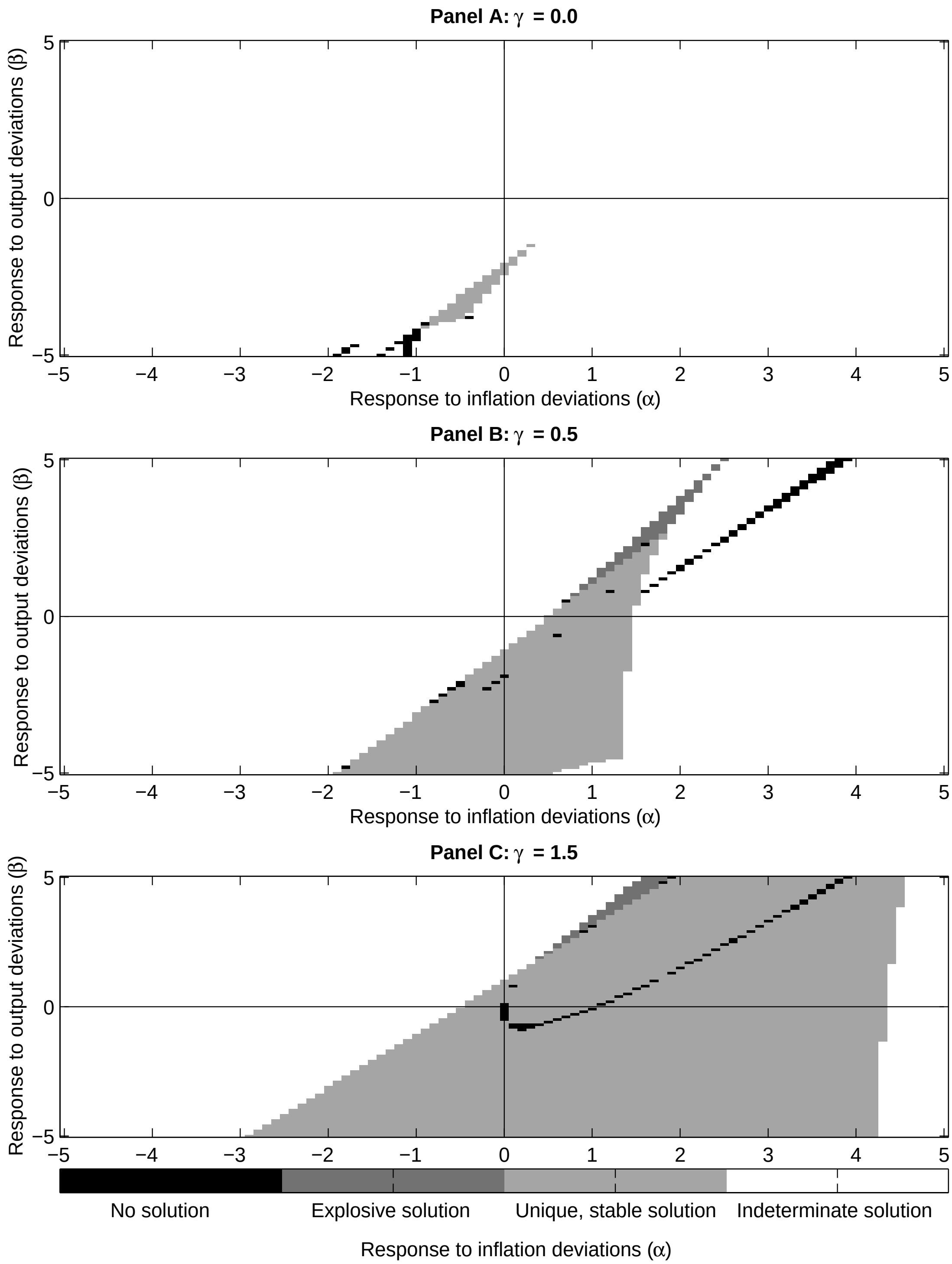
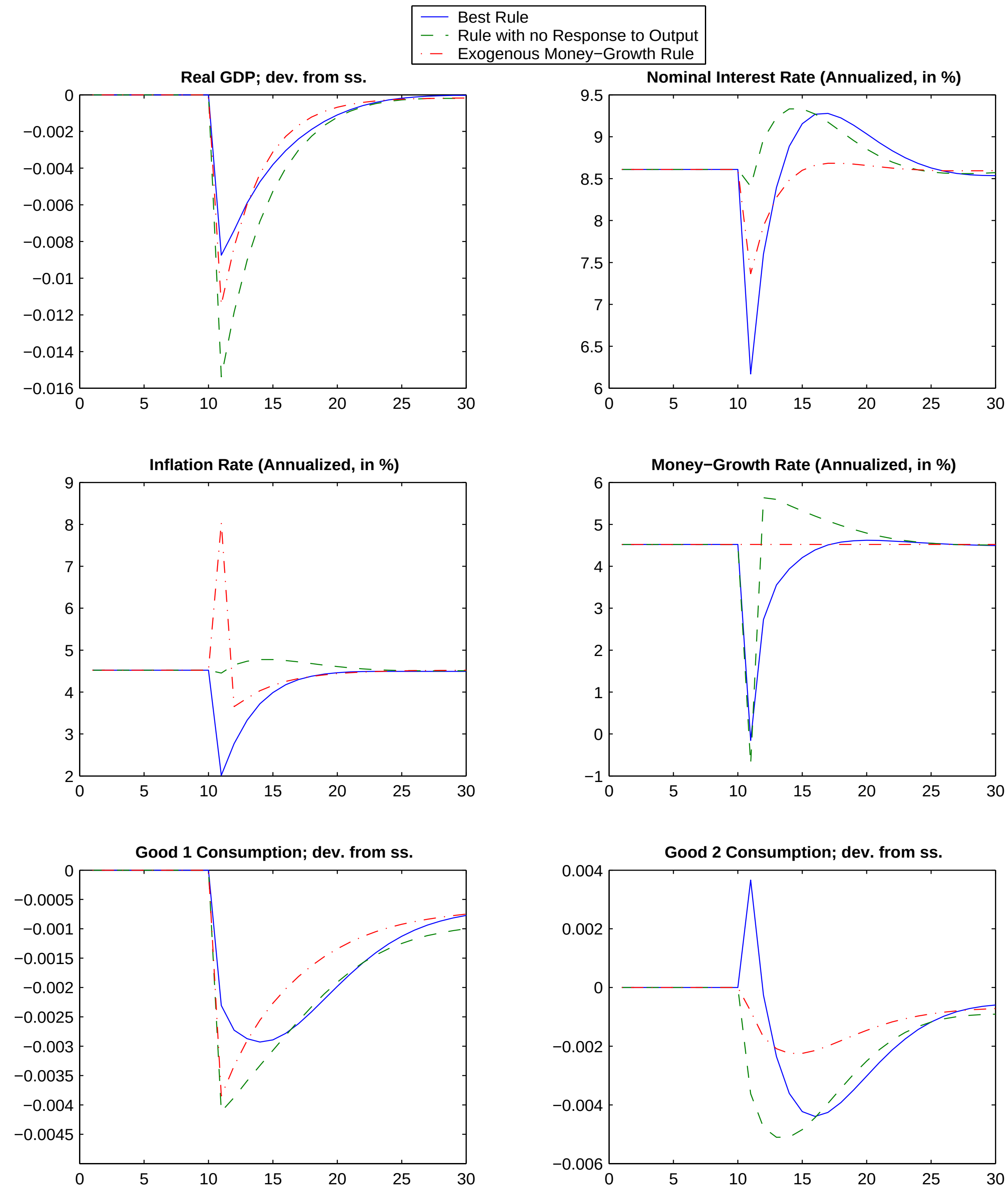


Figure 6: Responses to a Negative Technology Shock to Sector 1 Under Three Types of Rules



Appendix A: Solution of the Model

A first-order approximation to the solution of the model is computed using the algorithms in King and Watson (1998). First, collect all the equations that describe the equilibrium of the economy: the first-order conditions of the optimizing problems of the households (equations (1) to (5)); of each of the three domestic firms (equations (7) and (8), (9) and (10), and (13)); and of the domestic banks (equations (17) and (16)); the equations that determine the effective rate on international borrowing (19); the arbitrage conditions (11) and (14); the monetary policy rule (21); the market-clearing conditions (26) to (34); and the processes that describe the evolution of the exogenous processes in section 4.2. All nominal variables (the average growth of which is the rate of steady-state inflation) are detrended.

Second, a non-stochastic steady state is computed for this system of equations, by fixing all exogenous variables to their expected values and removing all time subscripts from the endogenous variables. Next, a first-order, Taylor approximation of this system of equations around the steady state is computed. The resulting expressions are arranged and stacked according to the notation in King and Watson (1998),

$$\boldsymbol{A}E_t[\hat{\boldsymbol{y}}_{t+1}] = \boldsymbol{B}\hat{\boldsymbol{y}}_t + \boldsymbol{C}(F)E_t[\hat{\boldsymbol{x}}_t]. \tag{A.1}$$

In this expression, $\hat{\boldsymbol{y}}_t$ is a (n by 1) vector containing all the endogenous variables of the system, $\hat{x}_t$ is a (x by 1) vector containing the exogenous variables, and $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}(F)$ are matrices of coefficients. The possibility of including future values of the vector of exogenous variables is made possible by the forward operator, F.

The solution delivered by King and Watson's methodology takes the following form:

$$\begin{bmatrix} \hat{\boldsymbol{y}}_t \\ \hat{k}_{t+1} \end{bmatrix} = \begin{bmatrix} \Pi \\ \boldsymbol{M} \end{bmatrix} \hat{k}_t, \tag{A.2}$$

where $\hat{k}_t$ contains both the subset of the endogenous variables that are predetermined as of the beginning of period t, as well as all the exogenous variables contained in $\hat{x}_t$; Π and $\boldsymbol{M}$ are matrices of coefficients. This solution can be used to conduct various types of economic analysis, like tracing out the impulse responses of the economy following a given innovation in one of the exogenous variables in $\hat{x}_t$, or computing unconditional moments (variance, covariances) of the economy. Full details about the linearized equations are available from the authors. Moran (2003) gives step-by-step descriptions of the implementation of King and Watson's methodology, within much simpler models.

Appendix B: Derivation of the Welfare-Based Loss Function

Ranking monetary policy rules according to a welfare criterion requires that we link the lifetime utility of the representative household to the economic outcomes implied by any policy rule. To do so, we follow Rotemberg and Woodford (1997, 1999) and Erceg, Henderson, and Levin (2000) and compute a second-order approximation to the expected lifetime utility around the steady state of the economy.

Recall that households maximize the following expectation of their lifetime discounted utility:

$$W_t = E_t \sum_{k=0}^{\infty} \beta^{t+k} u\left(c_{1t+k}, c_{2t+k}, c_{3t+k}, n_{t+k}, \frac{M^c{}_{t+k+1}}{M^c{}_{t+k}}\right), \tag{B.1}$$

where the following functional form is assumed:

$$u\left(c_{1t}, c_{2t}, c_{3t}, n_t, \frac{M^c{}_{t+1}}{M^c{}_t}\right) = \gamma_1 \ln\left(\frac{c_{1t}}{\iota_t}\right) + \gamma_2 \ln(c_{2t}) + \gamma_3 \ln\left(\frac{c_{3t}}{-\iota_t}\right) + \psi(1 - n_t - AC_t), \tag{B.2}$$

with the last term, AC_t, defined as:

$$AC_t = \frac{\phi^{PC}}{2}\left(\frac{M^c{}_{t+1}}{M^c{}_t} - \mu\right)^2. \tag{B.3}$$

To compare the unconditional expectation of W_t across economic outcomes, write $E[W_t]$ as the following infinite sum of unconditional expectations of period utility:

$$E[W_t] = \sum_{k=0}^{\infty} \beta^{t+k} E\left[u\left(c_{1t+k}, c_{2t+k}, c_{3t+k}, n_{t+k}, \frac{M^c{}_{t+k+1}}{M^c{}_{t+k}}\right)\right]. \tag{B.4}$$

As these unconditional expectations do not depend on *t*, we can simplify this last expression to:

$$E[W_t] = \frac{1}{1-\beta} E\left[u\left(c_{1t}, c_{2t}, c_{3t}, n_t, \frac{M^c{}_{t+1}}{M^c{}_t}\right)\right] \propto E\left[u\left(c_{1t}, c_{2t}, c_{3t}, n_t, \frac{M^c{}_{t+1}}{M^c{}_t}\right)\right]. \tag{B.5}$$

Next, we compute a second-order Taylor expansion of $u(c_{1t}, c_{2t}, c_{3t}, n_t)$ around its steady-state value, $u^{ss}(c_1{}^{ss}, c_2{}^{ss}, c_3{}^{ss}, n^{ss})$. Such an expansion delivers the following[1]:

1. We disregard the impact of the preference shock, ι_t, and of the portfolio variables, $M^c{}_{t+1}/M^c{}_t$, in the second-order expansion. This strategy is common in the literature.

$$u_t \approx u^{ss} + \begin{bmatrix} \frac{\gamma_1}{c_1^{ss}} & \frac{\gamma_2}{c_2^{ss}} & \frac{\gamma_3}{c_3^{ss}} & -\psi \end{bmatrix} \begin{bmatrix} c_{1t} - c_1^{ss} \\ c_{2t} - c_2^{ss} \\ c_{3t} - c_3^{ss} \\ n_t - n^{ss} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} c_{1t} - c_1^{ss} \\ c_{2t} - c_2^{ss} \\ c_{3t} - c_3^{ss} \\ n_t - n^{ss} \end{bmatrix}^T \begin{bmatrix} -\frac{\gamma_1}{\left(c_1^{ss}\right)^2} & 0 & 0 & 0 \\ 0 & -\frac{\gamma_2}{\left(c_2^{ss}\right)^2} & 0 & 0 \\ 0 & 0 & -\frac{\gamma_3}{\left(c_3^{ss}\right)^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{1t} - c_1^{ss} \\ c_{2t} - c_2^{ss} \\ c_{3t} - c_3^{ss} \\ n_t - n^{ss} \end{bmatrix}. \quad \text{(B.6)}$$

Zeros are in the cross products of the matrix of second derivatives of u_t and at its (4,4) position, because the utility is separable in the three consumption types and linear in leisure. Recalling that $\hat{x}_t = (x_t - x^{ss})/x^{ss}$ for all x_t, the second-order approximation can be written:

$$u_t \approx u^{ss} + \gamma_1 \hat{c}_{1t} + \gamma_2 \hat{c}_{2t} + \gamma_3 \hat{c}_{3t} - \psi n^{ss} \hat{n}_t - \frac{\gamma_1}{2} \hat{c}_{1t}^{\,2} - \frac{\gamma_2}{2} \hat{c}_{2t}^{\,2} - \frac{\gamma_3}{2} \hat{c}_{3t}^{\,2}. \quad \text{(B.7)}$$

We are interested in the unconditional expectation of u_t. We thus have:

$$E[u_t] \approx u^{ss} + \gamma_1 E[\hat{c}_{1t}] + \gamma_2 E[\hat{c}_{2t}] + \gamma_3 E[\hat{c}_{3t}] - \psi n^{ss} E[\hat{n}_t] - \frac{\gamma_1}{2} E\left[\hat{c}_{1t}^{\,2}\right] - \frac{\gamma_2}{2} E\left[\hat{c}_{2t}^{\,2}\right] - \frac{\gamma_3}{2} E\left[\hat{c}_{3t}^{\,2}\right]. \quad \text{(B.8)}$$

The linearity of our solution method implies $E[\hat{x}_t] = 0$ and therefore $E[\hat{x}_t^2] = \sigma^2(\hat{x}_t)$ for all x_t:

$$E[u_t] \approx u^{ss} - \frac{\gamma_1}{2}\sigma^2(\hat{c}_{1t}) - \frac{\gamma_2}{2}\sigma^2(\hat{c}_{2t}) - \frac{\gamma_3}{2}\sigma^2(\hat{c}_{3t}). \quad \text{(B.9)}$$

The unconditional expectation of lifetime utility is thus approximately equal to the utility achieved in a non-stochastic steady state with all shocks shut down, minus a weighted sum of the variability in consumption imposed by those shocks. Since u^{ss} is constant across all rules examined, one policy rule will dominate another when the weighted sum of variances in equation (B.9) is smaller. Stated another way, we seek policy rules to minimize the following welfare-based loss function:

$$L^W = \gamma_1 \sigma^2(\hat{c}_{1t}) + \gamma_2 \sigma^2(\hat{c}_{2t}) + \gamma_3 \sigma^2(\hat{c}_{3t}). \quad \text{(B.10)}$$

The loss function for our (flexible price) model does not include terms that represent deviations of inflation from its target. Such terms appear in the loss functions derived by Rotemberg and Woodford (1997, 1999) because the nominal price rigidities of their models produce welfare-reducing dispersions in relative prices that are related to the overall volatility of inflation.

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