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Interpreting Money-Supply and Interest-Rate Shocks as Monetary-Policy Shocks

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Abstract

In this paper two shocks are analysed using Canadian data: a money-supply shock (“M-shock”) and an interest-rate shock (“R-shock”). Money-supply shocks are derived using long-run restrictions based on long-run propositions of monetary theory. Thus, an M-shock is represented by an orthogonalized innovation in the trend shared by money and prices. An R-shock is represented by the orthogonalized innovation in the overnight interest rate. Either type of shock might be interpreted as a monetary-policy shock.

A permanent increase in the nominal stock of M1 generates: a temporary fall in the interest rate, consistent with the liquidity effect; a temporary rise in real output; a permanent increase in the price level; and a permanent depreciation of the nominal exchange rate. Although the behaviour of M1 is not directly controlled by the central bank, the identifying assumption that the central bank controls the long-run trend in money and prices and has no long-run effect on real output appears to be quite reasonable. A temporary positive real-interest-rate shock generates a temporary fall in money and output, but prices rise initially (a “price puzzle”) before eventually declining. Both the M-shock and R-shock models are consistent with an active role for money in the transmission of monetary policy.

Résumé

Dans le présent document, l’auteur analyse deux types de choc au moyen de données canadiennes : un choc d’offre de monnaie (le choc M) et un choc de taux d’intérêt (le choc R). Pour identifier le premier type de choc, il recourt aux restrictions de long terme découlant de certaines propositions avancées par la théorie monétaire. Le choc M est représenté par une innovation orthogonale dans la tendance commune qu’affichent la monnaie et les prix, et le choc R, par une innovation orthogonale dans le taux d’intérêt à un jour. L’un ou l’autre de ces chocs peut être interprété comme un choc de politique monétaire.

Une augmentation permanente du stock nominal de monnaie, M1, entraîne les conséquences suivantes: une chute temporaire du taux d’intérêt, conforme à l’effet de liquidité; une hausse passagère de la production exprimée en termes réels; une augmentation permanente du niveau des prix; et une dépréciation permanente du taux de change nominal. Bien que le comportement de M1 ne soit pas contrôlé directement par la banque centrale, l’hypothèse que cette dernière détermine la tendance de la monnaie et des prix à long terme mais qu’elle n’exerce aucune influence sur la production en termes réels en longue période semble fondée. Une hausse temporaire du taux d’intérêt réel provoque une baisse temporaire de la masse monétaire et de la production, mais les prix affichent un comportement plutôt déconcertant, puisqu’ils s’élèvent d’abord avant de redescendre. Les résultats obtenus à l’aide des modèles relatifs aux deux types de choc cadrent avec la thèse voulant que la monnaie joue un rôle actif dans la transmission de la politique monétaire.

Contents

1. Introduction

Central to the analysis of a monetary economy is the concept of the demand for money (Friedman 1956). Temporal stability in the behaviour of the demand for money is of particular interest for monetary policy. Given a stable demand-for-money function, a stable path of monetary expansion will lead to a stable path for prices. However, the empirical evidence of the stability of demand-for-money functions is mixed. Most recently, Stock and Watson (1993) estimate M1 as a function of prices, real income and a short-term interest rate using postwar U.S. data and find that the parameters of the function are not stable over time. In contrast, restricting income elasticity to unity, Hoffman, Rasche and Tieslau (1995) argue that the hypothesis of parameter stability cannot be rejected for a similar function across five industrialized countries, including the United States.

Even if the demand-for-money function is stable, there remains the issue of the dynamic relationship between monetary policy actions and economic fluctuations. Following the work of Lucas (1973) and Sargent and Wallace (1975), empirical research on this issue has been attentive to unanticipated movements in monetary policy. The application of vector autoregression (VAR) models and the identification of monetary-policy shocks have recently dominated this research (for example, Sims 1980a, 1986; Bernanke 1986; Keating 1992; Christiano and Eichenbaum 1992; Leeper and Gordon 1992; Lastrapes and Selgin 1995).

Monetary-policy shocks have been modelled as money-supply shocks, interest-rate shocks and combinations of the two (for example, Cochrane 1994). Sims (1986) argues that, since movements in M1 are the combination of private and central banking behaviour, an M1 shock may not be an appropriate monetary-policy shock. Instead, Sims argues that a Treasury-bill-rate shock is a more reasonable measure of a monetary-policy shock. Similar to Sims (1986), Christiano and Eichenbaum (1992) use contemporaneous restrictions to identify monetary-policy shocks. Using monetary base and M1 as the measures of the money supply for quarterly and monthly U.S. data, they argue that money-supply shocks lead to implausible interest rate and output movements. However, the orthogonalized innovation in non-borrowed reserves, the central bank's instrument of monetary policy, could be interpreted as a monetary-policy shock.

Fung and Gupta (1994) identify a Canadian monetary-policy shock focussing on the instruments of monetary policy. Of the two instruments of monetary policy — settlement-balance management and open-market operations — settlement-balance management is the primary instrument of monetary policy in Canada.[1] Accordingly, Fung and Gupta represent the orthogonalized innovation in excess settlement balances (excess cash) as a monetary-policy shock. However, excess cash is difficult to motivate empirically as a source of macroeconomic fluctuations, since it has properties very different from typical monetary aggregates and has no discernible correlation with other macro-variables. In addition, Fung and Gupta found that an unanticipated increase in excess cash — an easing of monetary policy — was followed by a short-run fall in prices (a "price puzzle").[2]

Lastrapes and Selgin (1995) use long-run (Blanchard-Quah) restrictions to identify money-supply shocks. They measure the money supply in the United States as monetary base, M1 and M2. The authors find that a permanent money-supply shock generates a temporary fall in interest rates, consistent with a monetary-policy shock. In contrast to the conclusions when using contemporaneous restrictions, this result is common across the different measures of the money supply. Thus, the dynamic effects of a U.S. monetary-policy shock are sensitive to the identification strategy.

In this paper, we use the information from the central proposition of monetary theory, the long-run demand-for-money function, as well as other restrictions to identify two monetary-policy shocks: a money-supply shock and an interest-rate shock.[3] Then,

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[1] In Canada the central bank controls the supply of deposits at the central bank held by financial institutions (settlement balances). Excess cash reserves are chartered bank deposits at the Bank of Canada in excess of the statutory minimum. (Reserve requirements were phased out between June 1992 and July 1994.)

[2] However, in subsequent unpublished work, this price puzzle was not found when the overnight interest rate was used as the measure of monetary policy and the U.S. interest-rate instrument (the federal funds rate) was included in the central bank's reaction function. (On this point, see Armour, Engert and Fung 1996.)

[3] Fisher, Fackler and Orden (1995) identify a "monetary" shock using long-run cointegration restrictions. However, the authors do not examine the dynamics of interest rates or the exchange rate, which are central to the conduct of monetary policy in a small open economy.

interpreting the supply of money in excess of its long-run demand as an indicator of the stance of monetary policy, the dynamics of the transmission mechanism in the Canadian economy are analysed. The long-run identification strategy is preferred since monetary theory is built on long-run propositions. In this sense, relying on long-run restrictions is considered to be less ad hoc than its contemporaneous counterpart.

The intuition of the money-supply shock is straightforward. Given a stable long-run demand-for-money function, an orthogonalized innovation in the trend shared by money and prices is interpreted as a money-supply shock (M-shock). That is, a monetary-policy shock is interpreted as an exogenous disturbance to the common trend between money and prices. This interpretation follows from the common-trends model developed by King, Plosser, Stock and Watson (1987, 1991). In that model, a permanent productivity shock is identified as the common stochastic trend in output, consumption, and investment.

An M-shock under our representation does not mean that the central bank exogenously creates or destroys M1, for example. Instead, it represents a central bank action that disturbs the evolution of the trend shared by money and prices and that is not accounted for by any other influences. This central bank action leads to a permanent change in the nominal money supply and generates a new nominal equilibrium path in the economy, with no long-run real economic consequences.

Armour, Engert and Fung (1996) argue that the orthogonalized innovation in the overnight interest rate provides a good operational measure of monetary-policy shocks in Canada. Thus, in addition to money-supply shocks, we also represent a monetary-policy shock by the orthogonalized innovation in the overnight interest rate (R-shock).

To preview the results, we find that the M-shock models conform to a monetary-policy shock. A permanent increase in the nominal stock of M1 generates: a temporary fall in the interest rate, consistent with the liquidity effect; a temporary rise in real output; a permanent increase in the price level; and a permanent depreciation of the nominal exchange rate. The response of output is, however, negative for one quarter. The simple M-shock models do not display short-run price and exchange-rate puzzles; a positive money-supply shock leads to an increase in prices and a depreciation of the nominal exchange rate. Thus although the behaviour of M1 is not directly controlled by the central bank, the identifying assumption that the central bank controls the long-run trend in money and prices and has no long-run effect on real output appears to be quite reasonable.

The R-shock models yield results that are broadly consistent with previous literature: a temporary real-interest-rate shock generates a temporary fall in money and output, but prices rise initially (a "price puzzle") before eventually declining. Both the M-shock and R-shock models are consistent with an active role for money in the transmission of monetary policy.

Overall, the experiments, which focus on monetary-policy shocks defined in two different ways and in a variety of model specifications, suggest the following four principal conclusions:

- A long-run demand-for-M1 function is a robust feature of the data.
- A monetary-policy shock disturbs the relationship between money and its long-run demand so as to create a long-lasting monetary disequilibrium. Consistent with Hendry (1995), such money gaps are eliminated over time as prices gradually adjust.
- Monetary-policy shocks clearly affect prices with a long lag (and the lag is variable across the models considered here).
- A monetary-policy shock has a transitory effect on output but the effect may be long-lasting.

This paper is organized as follows. Section 2 reviews the role for money in the transmission of monetary policy, as well as two long-run propositions of economic theory. The empirical methodology, an adaptation of King, Plosser, Stock and Watson (1991), is summarized in Section 4. Briefly, consistent with the propositions of economic theory, cointegration relationships condition the matrix of long-run multipliers and define the long-run restrictions in the VAR. The data and its properties are considered in Section 5. The empirical results of the M-shock models and R-shock models are presented in Sections 6 and 7. Finally, Section 8 summarizes the principal conclusions of the paper.

2. Money in the Transmission Mechanism

2.1 Alternative views of money

According to one view of the monetary transmission mechanism, the monetary authority controls a short-term interest rate and the nominal quantity of money evolves endogenously and passively according to its demand. In this case, money is a passive channel with no meaningful causal role in the transmission of monetary policy.

In an alternative approach, discrepancies between the nominal quantity of money demanded and the nominal quantity of money supplied are pivotal to the analysis of the transmission mechanism (Friedman 1970). According to an active-money view, while the quantity of money may be endogenous, it is also subject to the independent influence of the central bank. This influence, among other things, can lead to a real quantity of money holdings that is larger (smaller) than desired. In contrast to the passive-money view, the attempt to eliminate these excess balances (restore deficient balances) is considered to have an important role in the transmission of monetary policy.

The interpretation of a nominal "monetary shock" highlights the distinction between the two views. According to the passive-money view, a monetary shock is the consequence of a change in the demand for money (caused by an output shock, for example) that is accommodated by the central bank as it targets short-term interest rates.

In contrast, the active-money view interprets a monetary shock as the consequence of a change in the supply of money induced by the central bank that is unanticipated by agents. Consider a positive shock: initially, agents have to hold the additional nominal balances.[4] Over time, individuals perceive that the nominal quantity of money they hold

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[4] As an example of an active-money view, proponents of the "buffer-stock view" argue that money balances are used by agents to absorb unanticipated variations in income flows. For example, expansionary monetary shocks, generating a short-run fall in interest rates, can lead to changes in the expected returns of a planned portfolio. In the time that it takes to choose an alternative portfolio, average money holdings increase. (A similar example can be constructed for consumption decisions.) While this increase may seem trivial for the individual, it is argued that, in the aggregate, the increase in money holdings is important. For a further examination of the buffer-stock view, see Johnson (1962), Carr and Darby (1981) and Laidler (1990, 1994).

corresponds to a real quantity that is larger than desired, at current prices, and that this is not a temporary condition. That is, individuals are “off” their long-run demand-for-money function. However, all individuals cannot collectively dispose of the *aggregate* excess nominal balances. Nonetheless, the attempt to do so has economic effects. The increase in expenditure leads to an increase in nominal spending, an increase in economic activity, and ultimately an increase in prices. This transmission continues until the factors affecting the supply and the demand for money adjust to restore monetary equilibrium.

These alternative views of money in the transmission mechanism — the passive-money and the active-money views — can be distinguished by dynamic empirical analysis. There are, for example, two distinguishing features of these two views. First, the active-money view suggests that there is a tendency for discrepancies between the nominal quantity of money demanded and the nominal quantity of money supplied to persist. According to the passive-money view, instantaneous interest-rate adjustments eliminate any excess money balances. A second distinguishing feature is that the active-money view argues that individuals’ attempts to eliminate such monetary disequilibriums have aggregate short-run economic consequences. The passive-money view makes no similar claims.

Given the brief discussion of the role of money in the transmission mechanism, the following subsection reviews some simple long-run propositions of economic theory.

2.2 Long-run propositions of economic theory

Demand for money

Views about money in the transmission mechanism presume the existence of a long-run demand-for-money function. In the aggregate, the demand for real money balances is thought to increase with real economic activity. The opportunity cost of holding real balances — the foregone investment income (real rate of interest) and the lost purchasing power over the holding period (expected rate of inflation) — is summarized by the nominal rate of interest. Thus, the long-run demand for money can be expressed as

$$m^d - p = \beta_1 y - \beta_2 R \tag{2.1}$$

where m is the log of the nominal stock of money, p is the log of the price level, y is the log of real output and R is the nominal rate of interest.

The demand-for-money function has an empirical interpretation. It is well documented that the variables described in (2.1) are non-stationary. If, however, relation (2.1) is stationary, then the long-run demand for money can be interpreted as a cointegration relationship.

Open-economy propositions

For the dynamic analysis of a small open-economy like Canada, consideration of open-economy equilibrium propositions is necessary. Since the demand for money is thought of as (virtually) independent of the openness of the economy, a respecification of this function is, in general, unnecessary.[5] Moreover, with flexible exchange rates, domestic nominal variables are determined by domestic policy, assuming aggregate supply and real interest rates are determined on the real side of the economy, including international developments. Consequently, the role of the exchange rate is the primary focus of the open-economy propositions.

Open-economy propositions rely on the competitiveness of domestic goods and the mobility of domestic capital in world markets. By the “law of one price,” competition in goods markets and capital mobility in capital markets imply that, at least in the long run, no arbitrage opportunities can exist by trading domestic goods (capital) for identical foreign goods (capital).

Purchasing power parity (PPP) summarizes the law of one price for goods markets: the domestic price and foreign price of the same good will be equal, adjusted for the

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[5] McKinnon (1982), Poloz (1984, 1986) and Filosa (1995) admit the possibility of money holders shifting among currencies. In these “currency substitution” models, the expected depreciation of domestic currency is included in the demand-for-money function.

exchange rate between the two currencies. Thus, one role of the exchange rate is to reconcile movements of foreign and domestic prices. PPP can be expressed as

$$PFX = p - p^f \tag{2.2}$$

where p^f is the log of the foreign price level and PFX is the log of the price of domestic currency relative to foreign currency. The PPP relationship has an empirical interpretation similar to that of the demand-for-money function. Although the relationship may not hold at any given point in time, if (2.2) is stationary, then in the long run PPP describes the equilibrium real exchange rate.

Interest-rate parity (IRP) summarizes the law of one price for capital markets: the domestic nominal rate of interest will be equal to the foreign nominal rate of interest, adjusted for the expected rate of change in the exchange rate between the two currencies, abstracting from risk premiums. IRP can be expressed as

$$E(\Delta PFX) = R - R^f \tag{2.3}$$

where R^f is the foreign nominal rate of interest and $E(\Delta PFX)$ is the expected rate of change in the exchange rate. (The empirical interpretation of IRP is parallel to the demand for money and PPP cases.)

In order to interpret shocks with causal inference (an economic interpretation), dynamic empirical analysis requires a set of identification assumptions. The above propositions, central to the monetary analysis of an open economy, are straightforward, although questionable. The question of how to use this long-run information in a set of identification restrictions is a methodological issue addressed in the following section.

3. Empirical Methodology

The purpose of this section is to briefly discuss alternative identification strategies and then show how the long-run propositions of monetary theory are used to identify economic shocks through long-run restrictions.[6]

3.1 Alternative identification strategies

Define X_t as the $n \times 1$ vector of economic variables and ε_t as the $n \times 1$ vector of serially uncorrelated disturbances with covariance matrix Σ_{ε}. A linear characterization of the economy can be described by the following structural autoregressive model:

$$D_0 X_t = D_1 X_{t-1} + \ldots + D_l X_{t-l} + \varepsilon_t \tag{3.1}$$

where D_i is an unknown $n \times n$ matrix of parameters and the number of autoregressive lags is truncated to l. It is assumed that the researcher knows the following reduced-form model

$$X_t = H_1 X_{t-1} + \ldots + H_l X_{t-l} + e_t \tag{3.2}$$

where $H_i = D_0^{-1} D_i$ and $e_t = D_0^{-1} \varepsilon_t$. Since model (3.2) is estimated, we know the H_i's and e_t. The distinguishing feature of models (3.1) and (3.2) is the matrix of contemporaneous relationships D_0. In terms of the classical identification problem, additional assumptions are required to recover model (3.1) from model (3.2).

Common to the literature on VAR models, a recursive causal chain can be assumed (Sims 1980b). Ordering the variables in terms of their causal importance implies that the matrix of contemporaneous relationships is lower triangular (a Wold ordering). In a two-variable system of prices and money, for instance, a prices-money ordering assumes that,

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[6] This section provides an intuitive discussion of the identification strategy used in this paper. Readers interested only in the results can skip to Section 4. The algebraic detail of the empirical methodology is provided in Appendix 2.

contemporaneously, money responds to price shocks but prices do not respond to money shocks.

A fundamental criticism of the Wold ordering is the inability to interpret the shocks (Cooley and Leroy 1985). Relying on well-defined economic theory as the constraining structure in a VAR model addresses the Cooley-Leroy criticism. In this regard, several empirical studies impose structure implied by the contemporaneous predictions of economic theory. In a four-variable model of money, prices, output and interest rates, the identification strategy used by Keating (1992), for example, includes a short-run demand-for-money function. The long-run predictions of economic theory have also been used in identification strategies. For example, in a bivariate model of output and unemployment, Blanchard and Quah (1989) assumed that fluctuations in GDP are characterized by two types of shocks: those that have a (long-run) permanent effect on output and those that do not.

A long-run identification strategy can be an appropriate device for analysing the dynamics of the monetary transmission mechanism for two primary reasons. First, as a theoretical matter, competing views of the role of money in the transmission mechanism rely on long-run propositions. In the short run, however, economic theory does not necessarily predict these propositions will hold. In this sense, relying on long-run structure is considered to be less ad hoc than its contemporaneous counterpart.[7] Second, as an empirical matter, it is well-documented that several macro-variables can be characterized as unit-root processes; that is, the variables are subject to a stochastic trend (for example, Nelson and Plosser 1982). This implies that shocks to these variables have *permanent* effects. Many of these same variables share stochastic trends, implying the existence of a stationary linear combination of the variables; that is, the variables are cointegrated (Engle and Granger 1987). This mean-reversion (or trend-reversion) property implies that shocks to the combination of the variables have only *temporary* effects. We argue, therefore, that a long-run identification strategy can best exploit the theoretical and empirical properties of the macro-variables.

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[7] Long-run restrictions do, however, make the strong assumption that, contemporaneously, the central bank observes all the variables included in the VAR model (see Faust and Leeper 1994).

As a result of these considerations, this paper uses the cointegration structure, which is consistent with monetary theory, as the primary vehicle of identification.

3.2 Estimation of the VAR and the long-run cointegration restrictions

By simple algebra, model (3.2) can be rewritten in error-correction form as

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{l-1} \Delta X_{t-l+1} + \pi X_{t-l} + e_t \tag{3.3}$$

where Δ is the first difference operator, $\Gamma_i = -I_n + \sum_{j=1}^{i} H_j$ and $\Gamma_l = \pi$. This representation is convenient for a system of equations where the components of X are difference-stationary and cointegrated. In that case, π can be decomposed into two full column rank matrices such that $\pi = \alpha\beta'$, where α and β are $n \times r$ matrices and $0 < r < n$. The stationary combinations of the non-stationary variables are represented by $\beta' X_{t-l}$ and describe the low-frequency relationships in the system (long-run equilibrium). That is, the r columns of β represent the cointegration vectors. The elements of the matrix α are the adjustment parameters. Any deviation from long-run equilibrium results in a change in X that is consistent with the system returning to equilibrium. The path to equilibrium is described by the short-run dynamics of the model, the parameters of the lagged endogenous variables. Following Johansen and Juselius (1990), tests of the cointegration rank (rank of β) are performed and the parameters of the cointegration vectors are estimated. Then, the remaining parameters of model (3.3) are estimated by ordinary least squares (OLS).

After estimating model (3.3), we can generate the following reduced-form moving-average representation (MAR):

$$\Delta X_t = G(L) e_t \tag{3.4}$$

where G is a known $n \times n$ polynomial matrix and L is the lag operator.[8] For a starting value of zero, the cumulation of model (3.4) is the moving-average representation of model (3.2). The structural moving-average representation (SMAR) can be expressed as

$$\Delta X_t = \Phi(L)\varepsilon_t \tag{3.5}$$

where Φ is an unknown $n \times n$ polynomial matrix. For a starting value of zero, the cumulation of model (3.5) is the moving-average representation of model (3.1).

The MAR and SMAR are related by

$$\Phi(L)\Phi_0^{-1} = G(L) \tag{3.6}$$

$$\Phi_0\varepsilon_t = e_t \tag{3.7}$$

where Φ_0 is the matrix of contemporaneous relationships (impact matrix). Parallel to the case of the autoregressive representations, the matrix of contemporaneous relationships Φ_0 is the distinguishing feature of models (3.4) and (3.5). In the long-run ($L = 1$), there are kn unique unknown elements in $\Phi(1)$ (by the orthogonality condition that $\beta'\Phi(1) = 0$, where the number of permanent shocks is $k = n - r$), n^2 independent unknown elements in Φ_0 and $n(n+1)/2$ unique unknown elements in Σ_ε. Similarly, there are kn unique known (reduced-form) elements in $G(1)$ and $n(n+1)/2$ unique known elements in Σ_e. Thus, an additional n^2 restrictions are necessary to identify the structural model. Below, we discuss the strategy used to identify the dynamic multipliers of the permanent shocks in the structural model.

The first component of the identification strategy assumes that permanent and temporary shocks originate from independent sources, so the covariance matrix of the structural shocks is

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[8] In principle, the order of the moving average is infinite. In the empirical results, an arbitrarily large truncation is used (300 quarters).

$$\Sigma_\varepsilon = E(\varepsilon_t \varepsilon_t') = \begin{bmatrix} \Sigma_{\varepsilon^P} & 0 \\ 0 & \Sigma_{\varepsilon^T} \end{bmatrix} \quad (3.8)$$

and Σ_ε is partitioned conformably with $\varepsilon_t = (\varepsilon_t^{P}{}' \;\; \varepsilon_t^{T}{}')'$ where ε_t^P is the $k \times 1$ vector of permanent shocks and ε_t^T is the $r \times 1$ vector of temporary shocks. This assumption generates $n(n-1)/2$ unique restrictions.

The second component of the identification strategy imposes cointegration constraints on the matrix of long-run multipliers $\Phi(1)$. The following subsection outlines how this is done for two monetary models.

Demand for money: Benchmark

The benchmark model employs the proposed demand-for-money cointegration relationship $m_t^d = \mu_{0,1} + p_t + \beta_1 y_t - \beta_2 R_t + \varepsilon_{1,t}^T$ where $\varepsilon_{1,t}^T$ is a stationary money-demand shock and $\mu_{0,1}$ is a constant. As shown below, in the benchmark model, M-shocks are permanent.

The matrix of long-run multipliers, $\Phi(1)$, is partitioned by the number of permanent shocks in the model. In this four-variable model, since there is one cointegration vector ($r = 1$), there are three permanent shocks ($k = 3$). The first three columns of the matrix of long-run multipliers represent the long-run responses of the change in X_t to the permanent innovations. The long-run response of the change in X_t to the temporary innovation is represented by the last column of the matrix of long-run multipliers and is equal to zero by definition. Specifically, for $L = 1$, model (3.5) is rewritten as

$$\Delta X_t = [A \;\; 0]\varepsilon_t \quad (3.9)$$

where $\Phi(1) = [A \;\; 0]$, the 4×3 matrix A is equal to $\tilde{A}\Pi$, the 4×3 $\tilde{A}$ is a known matrix that describes the cointegration "structure" of the model, and 0 is a 4×1 matrix of zeros. The matrix of long-run multipliers is determined by the condition that its columns are orthogonal to the cointegration relations so that $\beta'\Phi(1) = 0$ (Engle and Granger 1987). The partition of the matrix of long-run multipliers, combined with the orthogonality condition, generates kr (unique) identification restrictions. The matrix Π is a $k \times k$ lower triangular matrix with full column rank and diagonal elements normalized to one. Π contains $k(k-1)/2$ unknown elements and $k(k+1)/2$ identification restrictions (a long-run recursive Wold ordering). This matrix is the normalization used to distinguish permanent shocks; the restrictions associated with Π exactly identify the permanent components of the model.[9] From the assumption that the permanent shocks are mutually uncorrelated, the unknown parameters of Π can be determined (see Appendix 2).[10]

Defining $X = \begin{bmatrix} R & y & m & p \end{bmatrix}'$, the matrix A can be expressed as

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\beta_2 & \beta_1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ \pi_{21} & 1 & 0 \\ \pi_{31} & \pi_{32} & 1 \end{bmatrix}. \tag{3.10}$$

According to the first column of $\tilde{A}$, a 1 per cent permanent real-interest-rate shock has a negative effect on the demand for money of β_2. This is interpreted as either a foreign interest-rate shock or a risk-premium shock. The second column of $\tilde{A}$ represents a 1 per cent permanent output shock. This is interpreted as a productivity shock. In words, a 1 per cent increase in output has a positive effect on the demand for money of β_1. The third column says that a 1 per cent permanent change in the level of money leads to a proportionate change in the price level. In other words, the four-variable system has three stochastic trends, as represented by the demand-for-money function. Given the output and interest-rate stochastic trends, M-shocks are defined as the innovation in the common trend of money and prices.

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[9] In the case where there is one permanent shock ($k = 1$), Π is a scalar and therefore redundant. However, when more than one permanent shock is present ($k > 1$), the permanent shocks are non-unique. In other words, for any non-singular matrix P, $(AP)(P^{-1}\varepsilon_t) = A\varepsilon_t$.

[10] Notice that to identify the temporary components of the model, an additional $r(r+1)/2$ restrictions are necessary. In this regard, we use contemporaneous restrictions similar to Armour, Engert and Fung (1996).

Combined with Π, the matrix of long-run multipliers implies that a real-interest-rate shock can have a permanent effect on all other variables, that productivity shocks have no long-run interest-rate effects but can effect (real) money and prices in the long-run, and that money shocks have a long-run impact on prices only.

Demand for money: Open-economy extension

The open-economy extension of the benchmark model includes the purchasing-power-parity relationship $PFX_t = \mu_{0,2} + p_t - p_t^f + \varepsilon_{2,t}^T$ where $\varepsilon_{2,t}^T$ is a stationary real exchange-rate shock and $\mu_{0,2}$ is a constant. Defining $X = \left[p^f \; PFX \; R \; y \; m \; p\right]'$, where p^f is restricted to be strictly exogenous, the matrix of long-run multipliers for the five endogenous variables can be summarized as

$$A = \begin{bmatrix} -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -\beta_2 & \beta_1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ \pi_{21} & 1 & 0 & 0 \\ \pi_{31} & \pi_{32} & 1 & 0 \\ \pi_{41} & \pi_{41} & \pi_{43} & 1 \end{bmatrix}. \qquad (3.11)$$

The first column of $\tilde{A}$ represents a 1 per cent foreign price-level shock; this leads to a 1 per cent appreciation of domestic currency (a decrease in the exchange rate). The next two columns of the matrix $\tilde{A}$ correspond to the interest-rate and productivity shocks in the benchmark model. The fourth column describes a domestic money shock but differs from the benchmark model in that it leads to a depreciation of domestic currency (an increase in the exchange rate). As in the benchmark model, money-supply shocks have no long-run real economic consequences.

4. The Data

In general, for classical statistical inference, stationarity is a necessary property. Therefore, after determining the variables' order of integration, the highest order of integration in the variable set is transformed to one (that is, I(1)). The cointegrating combinations of the non-stationary variables will then necessarily be I(0). In terms of an error-correction model, since all of the components are stationary, we are able to test the proposed theoretical cointegration relations using standard distribution theory.

The data used in this study are quarterly observations. The domestic data set is an overnight rate of interest (R); gross domestic product (in logarithms, y); the consumer price index (in logarithms, p); net nominal M1 balances, defined as currency plus chartered bank net demand deposits (in logarithms, m); and the Canadian dollar price of the U.S. dollar (in logarithms, PFX). The foreign data set is the federal funds rate (R^f) and the U.S. gross domestic product deflator (in logarithms, p^f).[11] The sample period for the regressions is 1954Q3 to 1994Q4 for systems defined in terms of the price level, and 1954Q4 to 1994Q4 for systems specified in terms of inflation.

Univariate analysis indicated that the data set was I(1), with two exceptions: in the case of prices, there was evidence supporting either an I(1) or an I(2) process; and the ex post real interest rate could be either I(1) or I(0). In examining a role for money, in particular a demand-for-money relationship, Stock and Watson (1993) found similar results for U.S. data (both the net national product price deflator and U.S. nominal money supply (M1) were either I(1) or I(2)). Given the indeterminate results, two systems are analysed depending on the order of integration of the price level (and the money supply).

The first set of models is based on prices being I(1); the second set of models is based on prices being I(2), so that the inflation rate (which is I(1)) is the relevant variable for the VAR.

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[11] From 1954Q3 to 1971Q1 R is the average of the day-to-day loan rate. From 1971Q2 to 1995Q1 R is the average overnight financing rate. GDP is expenditure-based, in millions of 1986 Canadian dollars. The nominal money supply is the average of the last Wednesday of the month, in millions of dollars. The nominal exchange rate is the average of the end-of-month closing price. The U.S. interest rate is the average of the end-of-month observations.

5. M-shock Models: Estimation Results

In this section, we estimate the error-correction model and generate impulse response functions. This is done in three steps. First, we test for the number of cointegration vectors and estimate their parameters (using the Johansen and Juselius (1990) methodology combined with the finite sample critical values generated in Appendix 3). Second, the proposed theoretical cointegration relationships are tested. In the third step, the parameter estimates of the cointegration vectors (as well as other restrictions) are used to identify the economic shocks and the impulse response functions are then examined.

5.1 Demand for money: Benchmark

The benchmark model employs the demand-for-money cointegration relationship $m_t^d = \mu_{0,1} + p_t + \beta_1 y_t - \beta_2 R_t + \gamma D81 + \varepsilon_{1,t}^T$ and defines $X = \begin{bmatrix} R & y & m & p \end{bmatrix}'$ where $D81$ is a linear deterministic shift parameter.[12] For the purposes of cointegration analysis, we first assume that the price level is integrated of order one (which implies that the ex post real interest rate is also integrated of order one, since the nominal interest rate is I(1)). The following subsection summarizes the tests for the number of permanent innovations in the system and estimates the parameters of the demand-for-money function.

Cointegration results

The cointegration results are summarized in Table 1 (Appendix 1). Following Johansen and Juselius (1990), the maximum eigenvalue and trace tests are used to determine the number of stochastic trends in model (3.3) (the number of permanent innovations).[13] Based on finite-sample critical values, the system is concluded to have at least two

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[12] Hendry (1995) illustrated that the relationship between money, prices, output and interest rates is stable with the inclusion of a linear deterministic shift parameter that accounts for the financial innovation from 1980 to 1983.

[13] An adaptation of the likelihood-ratio test was used to determine the optimal lag structure of the models. DeSerres and Guay (1995) show that, for the purposes of imposing long-run restrictions, this sequential test is preferred to information criteria (such as the Akaike criteria). The optimal lag structure (*l*) for the benchmark model is six. The optimal *l* for the open-economy extension is four.

stochastic trends (at most two cointegration vectors).[14] In the benchmark model, the demand-for-money function is considered as the unique cointegration vector. Thus, we restrict the cointegration space to one vector. This vector can easily be interpreted as a demand-for-money function. The hypothesis of unitary-price elasticity cannot be rejected (p-value 0.22), the income elasticity of 0.70 is (significantly) less than one and the interest-rate semi-elasticity is -0.04. According to the statistically significant adjustment parameters, changes in money and prices eliminate a deviation of money from its desired level.

Given the parameter estimates of the demand-for-money function, the restrictions to the matrix of long-run multipliers allow for the dynamic analysis of the permanent innovations in the system. With respect to a monetary-policy shock, the following experiment is considered: an unanticipated contemporaneous movement in monetary policy that generates a 1 per cent permanent change in the nominal equilibrium path of the economy. The following subsection summarizes the impulse response functions.

Impulse responses to the M-shock

In response to a positive M-shock, money increases gradually to the new nominal equilibrium (Figure 1, column 1). The intuition of the response function is clear. Since the endogenous properties of the money stock are known by the central bank (estimated by model (3.3)), in order to induce a 1 per cent permanent change in the money stock, the central bank need only induce a contemporaneous change of 0.36 per cent. This can be interpreted as a multiplier effect.

Consistent with a liquidity effect, the overnight interest rate responds to the M-shock with a significant decrease for three quarters.[15] Output responds positively and significantly after four quarters. At the sixth quarter, the output response peaks (0.2 per

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[14] 14. Appendix 3 summarizes the methodology for generating asymptotic and finite-sample critical values based on a system with the intercept term and the deterministic variable *D81* restricted to the cointegration space. The finite-sample values lead to (at times) vastly different conclusions with respect to the dimensionality of the cointegration space.

[15] 15. For the analysis of the impulse response functions, “significant” will mean “statistically significantly different from zero.” This corresponds to the case where the one-standard-deviation confidence bands of the response function lie on one side of the x-axis (which represents a zero response).

cent) and then decreases back to steady state. The impact response of the price level is quite sluggish (0.06 per cent); the response of the price level increases gradually towards steady state. Figure 1 (column 2) plots the deviation of money from its long-run demand. This “money gap” is long-lasting. Following an M-shock, monetary disequilibrium is above 0.4 per cent for seven quarters. One interpretation of the positive deviation of money from its long-run demand is that the supply of money is moving independently of, rather than in response to, the demand for money. The restoration of monetary equilibrium appears most closely related to the equilibrium adjustment of the price level.

The majority of previous empirical work has represented a money-supply shock as the orthogonalized component of the innovation in a monetary aggregate (for example, Christiano and Eichenbaum 1992). In contrast, in this paper, an unanticipated contemporaneous increase in the money supply leads to a permanent change in the nominal equilibrium path of the economy; however, a money-demand shock, which is transitory, leads to only a temporary deviation from equilibrium (by virtue of the cointegration restrictions). This may explain why, in previous empirical work, temporary money shocks behave like money-demand innovations.

Several VAR-based studies of monetary-policy shocks find a “price puzzle.” For example, Sims (1992) finds that a positive interest-rate shock leads to an increase in prices in several industrialized countries. He argues that this puzzle may be the endogenous policy response to inflationary pressures that are observed by the central bank but that are not included in the model. Fung and Gupta (1994) find a counterintuitive price response in Canada. However, the simple M-shock model does not give rise to this puzzle. This suggests that the price puzzle may be the result of the contemporaneous identification strategy and the measure of monetary policy used in previous studies.

5.2 Demand for money: Open-economy extension

The open-economy model extends the benchmark model with the inclusion a purchasing-power-parity cointegration relationship $PFX_t = \mu_{0,2} + p_t - p_t^f + \varepsilon_{2,t}^T$. In this model $X = \begin{bmatrix} p^f & PFX & R & y & m & p \end{bmatrix}'$ and p^f is strictly exogenous. Thus domestic shocks are restricted to have no impact on the foreign price level. In addition, following a domestic shock, purchasing power parity is restored through the reaction of the domestic price level and the nominal exchange rate.

Cointegration results

The cointegration results are summarized in Table 2 (Appendix 1). The maximum eigenvalue and trace tests support three to five stochastic trends in the system (one to three cointegration vectors). The open-economy model is considered to have two cointegration vectors. Thus, we restrict the cointegration space to two vectors. The first vector can easily be interpreted as a demand-for-money function with an income elasticity of 0.73 and an interest-rate semi-elasticity of -0.08. The joint restrictions associated with the money-demand and purchasing-power-parity relationships, as well as the zero restriction on the speed of adjustment parameters of foreign prices, are rejected by the data (p-value 0.05). Maintaining the assumption of output-neutrality from the benchmark model, we assume that monetary-policy shocks do not have a long-run effect on the real exchange rate. That is, we assume that monetary-policy shocks are not the source of non-stationarity in the real exchange rate.

The experiment conducted is the same as in the benchmark model: a monetary-policy shock that generates a 1 per cent permanent change in the nominal equilibrium path of the economy. The following subsection summarizes the impulse response functions in comparison with the benchmark model.

Impulse responses to the M-shock

In response to a positive M-shock, money adjusts to the new nominal equilibrium after four quarters, much faster than in the benchmark model (Figure 2, column 1). This may explain one role of the exchange rate in the transmission of monetary policy: exchange-rate fluctuations allow the stock of money to reach equilibrium faster than would otherwise be the case.

The overnight interest rate initially responds with a significant decrease (25 basis points). This liquidity effect lasts for about four quarters and is very similar to that of the benchmark model. Output responds positively after two quarters; the effect on output is transitory but long-lasting. The nominal exchange rate depreciates contemporaneously by 0.29 per cent and then continues to depreciate towards the new nominal equilibrium. As expected, the money-supply shock affects the exchange rate more quickly than it does prices in the goods market, as measured by the consumer price index; the contemporaneous increase in the price level is significant but small (0.07 per cent). Figure 2 (column 2) plots the response of the two equilibrium conditions of the model: the demand-for-money and the purchasing-power-parity relationships. As in the benchmark model, the equilibrium adjustment of the price level appears most closely related to the monetary disequilibrium. The real-exchange-rate deviation from parity is above 20 basis points for over 9 quarters. This overshooting, which occurs because the equilibrium adjustment of the nominal exchange rate is faster than the price level, may be the source of the long-lasting output effect following the money-supply shock.

In addition to the short-run response of the price level, a second puzzle for VAR-based studies of monetary-policy shocks is the “exchange-rate puzzle.” Grilli and Roubini (1995), for example, find that a positive interest-rate shock leads to a depreciation of the exchange rate for all G-7 countries other than the United States. However, the M-shock model generates an intuitive nominal exchange-rate response. This suggests that the exchange-rate puzzle also may be the result of the contemporaneous identification strategy and the measure of monetary policy used in previous studies.

The M-shock models are consistent with the view that money plays an active role in the transmission of monetary policy. The slow equilibrium adjustment of the models suggests that monetary policy can have real short-run economic effects. The models also help explain the role of the interest rate and the exchange rate in the monetary transmission mechanism. The similarity between the interest-rate dynamics in the two M-shock models suggests that the central bank manipulates a quantity instrument (such as settlement balances) in order to attain a particular interest-rate path. Nominal exchange-rate fluctuations allow the stock of money to adjust more quickly than would otherwise be the case. However, since the equilibrium adjustment of the nominal exchange rate is faster than the price level, the real exchange rate overshoots. Thus, a monetary-policy shock can have long-lasting output effects.

6. An Alternative Specification of Monetary-Policy Shocks: R-Shock Model Estimation Results

An interest-rate shock (R-shock) can also be interpreted as a monetary-policy shock, as in several VAR-based studies. The overnight interest rate makes a relatively attractive monetary-policy variable because it is subject to considerable influence by the central bank. (However, it is not a variable that is under the central bank's control.) In the identification strategy for the R-shock model, we maintain the assumption that a monetary-policy shock has no long-run real economic consequences. Thus the appropriate R-shock is considered to be a *temporary* real-interest-rate shock which has only *temporary* economic effects. In the remainder of this section, we analyse such a shock in an open-economy model.

6.1 Demand for money and R-shocks

In Section 5, the real interest rate was assumed to be integrated of order one. However, if inflation is I(1), then cointegration between the nominal interest rate and the rate of inflation would imply that the real interest rate is I(0). The evidence of the order of integration of the real interest rate was mixed. Univariate unit-root tests could not reject a non-stationary real rate. In a multivariate model of real money, output, interest rates and inflation, however, there was evidence of cointegration between the nominal interest rate and the rate of inflation.

We consider an R-shock in the context of an open economy. In particular, relying on the parity conditions PPP and IRP, an R-shock is represented as a monetary-policy action that generates a temporary deviation from the equilibrium real interest rate, where the equilibrium real interest rate is the cointegration relationship between domestic and foreign real interest rates. This interpretation of an R-shock can accommodate either an I(0) or I(1) domestic/foreign real interest rate. For the I(0) case, cointegration between domestic and foreign real interest rates is redundant; a temporary R-shock is adequately represented as an innovation in the domestic real interest rate. For the I(1) case, a real-interest-rate shock is non-stationary and may therefore have permanent economic effects. However, cointegration between the domestic and foreign real interest rates implies that domestic R-shocks are stationary, representing a temporary deviation from the non-stationary foreign real interest rate.

In this model, there are two cointegration relationships to consider. First, the demand-for-money function is expressed as

$$m_t^d - p_t = \mu_{0,1} + \beta_1 y_t - \beta_2 R_t + \gamma D81 + \varepsilon_{1,t}^T \tag{7.1}$$

where $\varepsilon_{1,t}^T$ is a stationary money-demand shock (temporary shock). The second relationship follows from PPP and IRP. Applying the first-difference operator to the PPP relationship (equation (2.2)) and combining it with the IRP relationship (equation (2.3)), the (long-run) equilibrium real interest rate can be expressed as

$$(R_t - E_t \Delta p_{t+1}) = \mu_{0,2} + (R_t^f - E_t \Delta p_{t+1}^f) + \varepsilon_{2,t}^T \tag{7.2}$$

where $\varepsilon_{2,t}^T$ is a stationary real-interest-rate shock (temporary shock). An R-shock is a monetary-policy action that generates a temporary deviation from the equilibrium real interest rate.

As in the M-shock models, the structure of the R-shock model is summarized by the two cointegration relationships, (7.1) and (7.2). In this model, $X = \left[R^f \ \Delta p^f \ y \ \Delta p \ R \ (m-p)\right]'$ and the foreign variables, R^f and Δp^f, are restricted to be strictly exogenous. Thus, domestic shocks are restricted to have no impact on the foreign inflation rate and the foreign nominal rate of interest. In addition, following a domestic shock, the equilibrium real rate of interest is restored through the reaction of the domestic inflation rate and the domestic nominal interest rate.

The matrix of long-run multipliers is partitioned by the number of permanent shocks in the model; in this six-variable system, there are two hypothesized temporary shocks and four hypothesized permanent shocks. As in the M-shock model, the cointegration relationships impose restrictions on the matrix of long-run multipliers. These restrictions are used to interpret the permanent shocks in the model. In this system, the four hypothesized permanent shocks are: an output shock, a neutral domestic inflation shock, a neutral foreign inflation shock and a foreign real-interest-rate shock. There are two hypothesized temporary shocks: a money-demand shock and a real-interest-rate shock.

However, as discussed in Section 3, the long-run restrictions are not sufficient to identify the temporary shocks in the model. Thus, additional restrictions are required. In this regard, an interest-rate shock is assumed to have a contemporaneous impact on only real money balances.

Cointegration results

The cointegration results are summarized in Table 3 (Appendix 1). The system has three to five permanent innovations (one to three cointegration vectors). The restrictions corresponding to equations (7.1) and (7.2) could not be rejected (not shown, p-value 0.40). However, the joint test of the restrictions associated with equations (7.1) and (7.2) and the zero restrictions on the speed of adjustment parameters of the foreign variables were rejected by the data (p-value 0.004). Nonetheless, we restrict the foreign variables to be (strictly) exogenous when generating the impulse response functions.

The estimates of the demand-for-money function are similar to previous cases. From the significant adjustment parameters, deviations of money from its desired level and deviations of the real interest rate from parity are eliminated by changes in inflation and money. The following subsection summarizes the impulse response functions for the experiment of a one-standard deviation contemporaneous real-interest-rate shock.

Impulse responses to the R-shock

In response to a positive temporary R-shock (106 basis points), the response of the overnight interest rate is above zero for about seven quarters (Figure 3, column 1). The impact response of money is negative and significant. The fall in output is long-lasting: it takes over 25 quarters for output to return to its pre-shock level (zero). The deviation of domestic real interest rates from equilibrium remains above 20 basis points for over 3 quarters (Figure 3, column 2). This is the source of persistent negative response of output.

In contrast to the M-shock model, the initial response of inflation is positive and significant. This “price puzzle” is found in several VAR-based studies when the interest rate is used as the instrument of monetary policy (for example, Sims 1992); it is generally viewed as a curious result.[16] Similar to the M-shock model, the correspondence between inflation and the money gap remains strong. The long-run demand for money falls initially with the increase in nominal (and real) interest rates (Figure 3, column 2). This leads to a positive money gap initially. In order to accommodate the fall in demand, a contraction of the money supply is required. However, after the fourth quarter, the supply of money overshoots the long-run demand and the money gap becomes negative. As in the M-shock models, this long-lasting contraction leads to a fall in the rate of inflation after six quarters. Overall, the response functions for the R-shock model are consistent with an active-money view.

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16. In comparison, Armour, Engert and Fung (1996) found no evidence of a price puzzle following an overnight interest-rate shock. The R-shock model used in this paper differs from that of Armour, Engert and Fung primarily in the measure of the price level: the latter paper measured the price level as the GDP deflator instead of the CPI, and this may help account for the different results.

7. Conclusions

The empirical results of the M-shock models conform to a monetary-policy shock. A permanent increase in the nominal stock of M1 generates: a temporary fall in interest rates, consistent with the liquidity effect; a temporary rise in real output; a permanent increase in the price level; and a permanent depreciation of the nominal exchange rate. Previous literature, such as Sims (1986) and Christiano and Eichenbaum (1992), argue that M1 shocks are poor measures of monetary-policy shocks. This paper suggests that the conclusions of previous research may be attributable to the identification strategy. In addition, using a quantity measure of monetary policy and long-run cointegration restrictions to identify monetary-policy shocks, the M-shock models do not display the price and exchange-rate puzzles found in several previous VAR-based studies. The R-shock models yield results that are broadly consistent with previous literature: a temporary real-interest-rate shock generates a temporary fall in money and output, but prices rise initially (a “price puzzle”) before eventually declining.

Both the M-shock and R-shock models are consistent with the view that money plays an active role in the transmission of monetary policy. The slow equilibrium adjustment of the models implies that monetary policy can have short-run real economic effects. The similarity between interest-rate dynamics in the two M-shock models suggests that the central bank manipulates a quantity instrument (such as settlement balances) in order to attain a particular interest-rate path. Also in the M-shock model, since the equilibrium adjustment of the nominal exchange rate is faster than the price level, the real exchange rate overshoots. In the R-shock model, the real interest rate also overshoots its equilibrium. These overshooting properties suggest that a monetary-policy shock may have long-lasting output effects.

Although there are considerable differences in the institutional setting and the implementation of monetary policy across industrialized countries, there is no reason to believe that the fundamental effects of unanticipated monetary policy are very different. Future research will examine whether the dynamics of M-shock models are robust across industrialized countries.

Appendix 1. Appendix of Results

Table 1. Cointegration test results for the benchmark M-shock model ($X = \left[R\ y\ m\ p\right]'$)

Cointegration rank: Tests for the number of permanent innovations in the system

$n-r$	r	Test Statistics		90% Confidence[a]		95% Confidence[a]		Conclusion (# vectors)
		λ Max[b]	Trace[c]	λ Max	Trace	λ Max	Trace	
4	0	31.68	78.44	R (R)	R (R)	A (R)	R (R)	at most 2 vectors
3	1	24.79	46.75	A (R)	R (R)	A (R)	R (R)	
2	2	21.96	21.96	A (A)	A (A)	A (R)	A (R)	

Cointegration space: Estimated vector space based on one cointegration relationship

	m	p	y	R	constant	D81
β matrix	1.00[d]	-0.834	-0.87	0.04	0.55	0.11
α matrix	-0.10*	0.03*	0.01	3.96		

Restricted cointegration space: Testing unitary price elasticity

	m	p	y	R	constant	D81	p-value
β matrix	1.00[d]	-1.00	-0.70	0.04	-1.66	0.18	0.22
α matrix	-0.09*	0.03*	0.03	1.67			

Cointegration structure: The cointegration vector used as the structure of the VAR

$$m_t^d = 1.66 + p_t + 0.70y_t - 0.04R_t - 0.18D81$$

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a. R and A are the cases when the null hypothesis can be rejected and cannot be rejected, respectively (based on the finite-sample critical values generated in Appendix 3). R or A in parentheses are the conclusions corresponding to the asymptotic critical values of Osterwald-Lenum (Case 1*).

b. The null hypothesis is r cointegration vectors and the alternative is $r + 1$.

c. The null hypothesis is (less than or equal to) r cointegration vectors and the alternative is $r = n$.

d. Normalized to one

* Significantly different from zero at the 5 per cent level of significance

Table 2. Cointegration test results for the open-economy M-shock model ($X = \left[p^f \; PFX \; R \; y \; m \; p\right]'$)

Cointegration rank: Tests for the number of permanent innovations in the system

$n-r$	r	Test Statistics		90% Confidence[a]		95% Confidence[a]		Conclusion (# vectors)
		λ Max[b]	Trace[c]	λ Max	Trace	λ Max	Trace	
6	0	51.69	151.95	R (R)	R (R)	R (R)	R (R)	1 to 3 vectors
5	1	32.97	100.27	A (R)	R (R)	A (A)	R (R)	
4	2	31.95	67.29	R (R)	R (R)	A (A)	R (R)	

Cointegration space: Estimated vector space based on two cointegration relations

	m	p	y	R	PFX	p^f	Constant	D81
β matrix	1.00[d]	1.08	-0.60	0.05	-0.42	-2.37	-3.19	0.22
	-0.20	1.00[d]	-0.29	-0.12	2.27	-1.10	5.18	0.63
α matrix	-0.099*	0.006	-0.006	0.574	0.008	-0.007		
	-0.013*	-0.002	0.002	1.890	-0.009	-0.003		

Restricted cointegration space: Testing unitary price elasticity

	m	p	y	R	PFX	p^f	Constant	D81	p-value
β matrix	1.00[d]	-1.00	-0.73	0.08	0	0	6.23	0.15	0.05
	0	1.00[d]	0	0	-1.00	-1.00	-7.48	0	
α matrix	-0.048*	0.007*	-0.006	-1.759	0.019	0			
	-0.050*	0.007*	-0.006	-1.662	0.020	0			

Cointegration structure: The cointegration vectors used as the structure of the VAR

$$m_t^d = -6.23 + p_t + 0.73y_t - 0.08R_t - 0.15D81_t$$

$$PFX = 7.48 + p - p^f$$

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a. R and A are the cases when the null hypothesis can be rejected and cannot be rejected, respectively (based on the finite-sample critical values generated in Appendix 3). R or A in parentheses represent the conclusions corresponding to the asymptotic critical values of Osterwald-Lenum (Case 1*).

b. The null hypothesis is r cointegration vectors and the alternative is $r + 1$.

c. The null hypothesis is (less than or equal to) r cointegration vectors and the alternative is $r = n$.

d. Normalized to one

* Significantly different from zero at the 5 per cent level of significance

Table 3. Cointegration test results for the open-economy R-shock model ($X = \left[\Delta p^f \; R^f \; y \; \Delta p \; R \; (m-p)\right]'$)

Cointegration rank: Tests for the number of permanent innovations in the system

$n-r$	r	Test Statistics		90% Confidence[a]		95% Confidence[a]		Conclusion (# vectors)
		λ Max[b]	Trace[c]	λ Max	Trace	λ Max	Trace	
6	0	54.18	169.72	R (R)	R (R)	R (R)	R (R)	1 to 3 vectors
5	1	37.27	115.54	A (R)	R (R)	A (R)	R (R)	
4	2	32.76	78.27	R (R)	R (R)	A (R)	R (R)	

Cointegration space: Estimated vector space based on two cointegration relations

	$(m-p)$	Δp	y	R	Δp^f	R^f	Constant	D81
β matrix	1.00[d]	-0.04	-0.85	0.06	0.05	-0.03	0.07	0.32
	-8.93	0.45	4.98	1.00[d]	-0.75	-0.73	25.94	-3.35
α matrix	-0.072*	8.859*	0.002	0.556	5.657*	2.827*		
	-0.001	-0.117	-0.002*	-0.089*	0.094	0.069		

Restricted cointegration space: Testing for unitary price elasticity and real interest rate relationship

	$(m-p)$	Δp	y	R	Δp^f	R^f	Constant	D81	p-value
β matrix	1.00[d]	0	-0.87	0.03	0	0	1.04	0.31	0.004
	0	-1.00	0	1.00[d]	1.00	-1.00	5.43	0	
α matrix	-0.081*	5.027*	0.017	-0.902	0	0			
	-0.002*	0.247*	-0.001	-0.041	0	0			

Cointegration structure: The cointegration vectors used as the structure of the VAR

$$m_t^d - p_t = -1.04 + 0.87y_t - 0.03R_t - 0.31D81$$

$$(R - \Delta p) = -5.43 + (R^f - \Delta p^f)$$

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a. R and A are the cases when the null hypothesis can be rejected and cannot be rejected, respectively (based on the finite sample critical values generated in Appendix 3). R or A in parentheses represent the conclusions corresponding to the asymptotic critical values of Osterwald-Lenum (Case 1*).

b. The null hypothesis is r cointegration vectors and the alternative is $r + 1$.

c. The null hypothesis is (less than or equal to) r cointegration vectors and the alternative is $r = n$.

d. Normalized to one

* Significantly different from zero at the 5 per cent level of significance

FIGURE 1.

Dynamic responses to a monetary-policy shock in the benchmark M-shock model (25 quarters)

M-Shock

Money gap

m

R

y

p

The interest-rate response functions are in basis points; all other responses are at a quarterly per cent.
The response functions are illustrated with one standard deviation confidence intervals (based on a Monte Carlo simulation).

FIGURE 2.

Dynamic responses to a monetary-policy shock in the open-economy M-shock model (25 quarters)

M-shock

Money gap

m

R

PFX

The real exchange rate (deviation from PPP)

y

p

The interest-rate response functions are in basis points; all other responses are at a quarterly per cent.
The response functions are illustrated with one standard deviation confidence intervals (based on a Monte Carlo simulation).

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FIGURE 3.

Dynamic responses to a monetary-policy shock in the open economy R-shock model (25 quarters)

Temporary R-shock

Money gap

R

(m-p)

Domestic Real Interest Rate

y

p(t)-p(t-1)

The interest-rate response functions are in basis points; all other responses are at a quarterly per cent.
The response functions are illustrated with one standard deviation confidence intervals (based on a Monte Carlo simulation).

Appendix 2. Econometric Methodology Econometric Methodology

Algebraic details of the identification strategy [1]

Recall from Section 3 the MAR $\Delta X_t = G(L)e_t$ and the SMAR $\Delta X_t = \Phi(L)\varepsilon_t$ which are related by the following two relations:

$$\Phi(L)\Phi_0^{-1} = G(L) \tag{3.6}$$

$$\Phi_0\varepsilon_t = e_t. \tag{3.7}$$

From equation (3.6), setting $L = 1$ gives

$$\Phi(1) = G(1)\Phi_0 \ . \tag{A.1}$$

By multiplying both sides of equation (A.1) by ε_t and combining it with equation (3.7), we obtain

$$\Phi(1)\varepsilon_t = G(1)e_t \ . \tag{A.2}$$

By definition of the restricted matrix of long-run multipliers (equation (3.9)) equation (A.2) can be rewritten as

$$\Phi(1)\varepsilon_t = [\tilde{A}\Pi \quad 0]\varepsilon_t = \tilde{A}\Pi\varepsilon_t^P \ . \tag{A.3}$$

Combining equations (A.2) and (A.3) gives

$$\tilde{A}\Pi\varepsilon_t^P = G(1)e_t \ . \tag{A.4}$$

By multiplying both sides of equation (A.4) by their respective transposes and taking the expectation, we obtain

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[1] This section is an adaptation of the appendix of King et al. (1991).

$$\tilde{A}\Pi\Sigma_{\varepsilon^P}\Pi'\tilde{A}' = G(1)\Sigma_\varepsilon G(1)' \tag{A.5}$$

where the LHS is spectral density evaluated at zero. Pre-multiplying both sides of equation (A.5) by $(\tilde{A}'\tilde{A})^{-1}\tilde{A}'$ and $\tilde{A}(\tilde{A}'\tilde{A})^{-1}$ yields

$$\Pi\Sigma_{\varepsilon^P}\Pi' = (\tilde{A}'\tilde{A})^{-1}\tilde{A}'G(1)\Sigma_\varepsilon G(1)'\tilde{A}(\tilde{A}'\tilde{A})^{-1} \ . \tag{A.6}$$

To simplify notation, define the $k \times n$ matrix $\theta = (\tilde{A}'\tilde{A})^{-1}\tilde{A}'G(1)$, which implies $\tilde{A}\theta = G(1)$. Thus equation (A.6) can be rewritten as

$$\Pi\Sigma_{\varepsilon^P}\Pi' = \theta\Sigma_\varepsilon\theta'. \tag{A.7}$$

Since the LHS of equation (A.7) is symmetric and positive definite, it can be decomposed into a unique lower triangular matrix Π^* by the Choleski Factorization Theorem. Thus

$$\Pi^*\Pi^{*\prime} = \Pi\Sigma_{\varepsilon^P}\Pi' \tag{A.8}$$

which also implies that

$$\Pi^* = \Pi\Sigma_{\varepsilon^P}^{1/2}. \tag{A.9}$$

Thus unique values of Π and Σ_{ε^P} are obtained.

Next consider the dynamic multipliers. Combining $\tilde{A}\theta = G(1)$ with equations (3.9) and (A.2) gives

$$\tilde{A}\theta = [\tilde{A}\Pi \ \ 0]\Phi_0^{-1}. \tag{A.10}$$

Thus $\tilde{A}\theta = \tilde{A}\Pi\Phi_0^{-1}$. By construction, the first k rows of Φ_0^{-1} are $\Pi^{-1}\theta$. Finally, $\varepsilon_t = \Phi_0^{-1}e_t$ implies that $\varepsilon_t^P = \Pi^{-1}\theta e_t$.

The dynamic multipliers associated with ε_t^P are given by the first k columns of $\Phi(L)$. First, consider the components of the impact matrix Φ_0. Similar to the matrix of long-run multipliers, the matrix Φ_0 is partitioned by the number of permanent innovations in the model. Define the partition of Φ_0 as

$$\Phi_0 = [\Phi_{k0} \ \ \Phi_{r0}] \tag{A.11}$$

where Φ_{k0} is $n \times k$, Φ_{r0} is $n \times r$ and $k + r = n$. Since $\Phi(L) = G(L)\Phi_0$, the first k columns of $\Phi(L)$ are given by

$$G(L)\Phi_{k0} \, . \tag{A.12}$$

Consider now the recovery of Φ_{k0}. Since $\Phi_0\varepsilon_t = e_t$ we have

$$\Sigma_\varepsilon\Phi_0' = \Phi_0^{-1}\Sigma_e \, . \tag{A.13}$$

Expanding (A.13) and solving for Φ_{k0} gives $\Phi_{k0}' = \Sigma_\varepsilon^{-1}\Pi^{-1}\theta\Sigma_e$. Therefore the dynamic multipliers for the permanent shocks, ε_t^P, are

$$\Phi(L)\Sigma_\varepsilon^{-1}\Pi^{-1}\theta\Sigma_e \, . \tag{A.14}$$

Appendix 3. Finite-Sample Critical Values

This appendix briefly summarizes the methodology employed in generating the finite-sample critical values. The non-deterministic parts of the data generated process (DGP) are drawn from a mean zero 400-period Gaussian distribution with variance one. The constant term is drawn from a mean ten 400-period Gaussian distribution with variance one. The analytic asymptotic distribution of the test statistics is approximated by the Gaussian 400-period random walk (Osterwald-Lenum 1992). For each row, the null hypothesis of no cointegration is tested ($n-r$ stochastic trends). By varying n, the number of endogenous variables in the system, critical values are generated row by row. After replicating this procedure 6,000 times, the asymptotic quantiles are calculated. There are two differences between the way the finite-sample and asymptotic critical values are derived. First, and most obvious, the non-deterministic parts of the DGP are drawn from an *f* period Gaussian distribution (where *f* corresponds to the number of observations in the estimated models). Second, the quantiles are calculated from 12,000 replications. The finite-sample critical values of the case with no constant in the DGP but a constant and a linear shift parameter spanned by the cointegration space (restricted constant) are presented below.

Table 4. Finite-sample Distribution of the Cointegration Test Statistics: Case 1* [a]

DGP & SM: $\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{k-1} \Delta X_{t-k+1} + \alpha(\beta', \mu_0, \delta)(X'_{t-1}, 1, D81)' + \Psi D_t + \varepsilon_t$

$\varepsilon_t \sim N(0, I_n)$

	λ -max			Trace		
$n-r$	90%	95%	99%	90%	95%	99%
1	11.19	13.08	17.34	11.19	13.08	17.34
2	17.94	20.19	24.69	24.45	27.04	32.67
3	24.49	27.02	32.27	41.63	44.93	51.99
4	30.84	33.41	39.19	62.71	66.31	75.05
5	37.37	40.17	45.82	88.39	92.95	102.67
6	43.60	46.66	52.95	117.97	123.51	134.30
7	49.79	52.83	59.93	152.52	158.20	169.79
8	56.23	59.60	66.21	190.56	197.74	210.81
9	62.86	66.23	73.30	233.87	241.75	256.10
10	69.22	72.98	80.57	281.52	289.75	305.55

a. Aside from the inclusion of *D81*, this table corresponds to Case 1* in Osterwald-Lenum (1992).

Note: *D* represents three centered seasonal dummies.

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