Bank of Canada Banque du Canada

Working Paper 2002-32 / Document de travail 2002-32

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2002-32

November 2002

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Acknowledgements

We would like to thank Charles Carlstrom, Walter Engert, Paul Gomme, Hashmat Khan, and seminar participants at the Bank of Canada, Federal Reserve Bank of Cleveland, Ohio State University, and Simon Fraser University for their useful comments. We would also like to thank Guang-Jia Zhang for his help early in this project. This research was partially funded (for Andolfatto) by the Social Sciences and Humanities Research Council.

Abstract

We develop an equilibrium model of the monetary policy transmission mechanism that highlights information frictions in the market for money and search frictions in the market for labour.

A change in monetary policy regime, modelled here as an exogenous reduction in the long-run target for the money-growth rate, results in a large and persistent increase in the interest rate owing to a persistent shortfall in liquidity. This persistent liquidity effect occurs because of the limited information that individuals have concerning the nature of the shock, which implies that individuals optimally update their inflation forecasts using an adaptive expectations rule. The subsequent period of high interest rates curtails job-creation activities in the business sector, making it more difficult for the unemployed to find suitable job matches; employment bottoms out two to three quarters after the shock. In the long run, however, employment rises above its initial level, primarily because of the lower long-run interest rates associated with a tight-money regime.

JEL classification: E4, E5
Bank classification: Transmission of monetary policy; Uncertainty and monetary policy

Résumé

Dans leur étude, les auteurs élaborent un modèle d'équilibre du mécanisme de transmission de la politique monétaire qui fait ressortir les éléments de friction relatifs à l'acquisition d'information sur le marché de la monnaie et à la recherche d'emploi.

Le changement de régime de politique monétaire, qui revêt dans le modèle la forme d'une réduction exogène du taux cible de croissance monétaire à long terme, fait augmenter les taux d'intérêt de façon marquée et durable en raison de la persistance de la pénurie de liquidités. Cette persistance vient du fait que les particuliers disposent d'une information limitée sur la nature du choc survenu, ce qui implique qu'il est optimal pour eux d'actualiser leurs prévisions d'inflation en formant des anticipations adaptatives. Le niveau élevé des taux d'intérêt a pour effet de freiner la création d'emplois dans les entreprises et de diminuer la probabilité que les chômeurs retrouvent un travail qui leur convienne. L'emploi touche un creux de deux à trois trimestres après le changement de régime. En longue période, cependant, il remonte au-dessus de son niveau initial, principalement en raison de la baisse des taux d'intérêt à long terme associée au resserrement monétaire.

Classification JEL : E4, E5
Classification de la Banque : Transmission de la politique monétaire; Incertitude et politique monétaire

1. Introduction

This paper proposes an explanation of the process by which an exogenous shock to monetary policy (a regime shift) affects the economy. An event such as a change in the monetary policy regime may be the closest thing that one might imagine as corresponding to a truly exogenous monetary shock.[1] Romer and Romer (1989, 1994) have attempted to identify several such episodes in U.S. monetary history based on records of Federal Reserve policy meetings. These authors associate regime shifts with episodes in which the Fed appeared to have undertaken a significant shift towards monetary tightening. Each such episode is characterized by two high-profile phenomena: (i) a persistent increase in short-term interest rates, and (ii) a persistent increase (decrease) in unemployment (output).[2] This pattern is evident in other countries as well. For example, a dramatic shift (tightening) in the Bank of Canada's policy in the early 1980s is widely perceived to have contributed to the severity of the 1981–82 recession and the high interest rates prevailing in the early part of that decade (Howitt 1986). In this paper, we take these to be some of the stylized facts associated with changes in monetary policy regimes and ask whether (or to what extent) theory can account for such behaviour.

Until recently, existing dynamic general-equilibrium (DGE) monetary models have been unable to explain why interest rates appear to rise and output to fall in the wake of exogenous monetary tightening. In fact, standard theory predicts that the opposite should happen; see, for example, Greenwood and Huffman (1987). In most environments (including sticky-price models), the behaviour of the nominal interest rate is governed predominantly by a Fisherian anticipated inflation effect, so that an unanticipated (but persistent) monetary tightening leads to a downward revision in inflation forecasts and hence to a decline in the interest rate. Most standard environments also have the implication that output should rise following a monetary contraction, since inflation serves as a tax on market activity.

Lucas (1990) and Fuerst (1992) embed a limited-participation feature into an otherwise standard cash-in-advance model; this allows for the possibility that a monetary shock has a direct impact only on the subset of individuals participating in financial markets at the time of the shock.[3] Households are assumed to be in less frequent contact with financial markets, at least compared with other agents in the business sector, such as firms and financial intermediaries. Should the central bank surprise financial markets by unexpectedly reducing cash reserves in the banking sector, the subsequent shortfall in loanable funds (liquidity) puts upward pressure on the interest

[1] Christiano, Eichenbaum, and Evans (1999) describe some of the problems associated with trying to identify exogenous monetary policy shocks.

[2] This characterization of the economy's response to an exogenous monetary shock is consistent with the evidence provided by Friedman and Schwartz (1963) for earlier periods in U.S. history.

[3] The heterogeneous impact of monetary policy was first modelled by Grossman and Weiss (1983) and Rotemberg (1984).

rate; see Christiano (1991).

The main shortcoming of the limited-participation model is its inability to generate a *persistent* liquidity effect: interest rates and output move in the desired direction only in the period of the shock. One obvious way to generate persistence is to substitute the assumed one-period delay in portfolio substitution with a more general adjustment-cost technology; this is the approach taken by Christiano and Eichenbaum (1992), and more recently by Cooley and Quadrini (1999). Cook (1999) is also able to generate persistence by assuming that financial-intermediation costs depend negatively on some lagged measure of aggregate economic activity. Shi (1998) is able to generate a persistent employment effect in an environment that features search in both the market for output and labour.

In this paper, we depart from the literature in two significant ways. In the first major departure, we move away from modelling a monetary shock as a simple autoregressive process and instead follow Andolfatto and Gomme (1999), who explicitly model shifts in policy regimes.[4] In particular, we think of a monetary policy regime as an operating procedure by the central bank that either implicitly or explicitly manifests itself as a long-run money-growth rate or inflation target. We view a particular regime as reflecting the preferences of central bank governors, or perhaps the political or intellectual forces that serve to influence such preferences. Accordingly, we attribute the following set of characteristics to regimes. First, they tend to persist for long periods of time. They do not, however, persist indefinitely; periodic shifts in regimes can be expected to occur at random intervals, with an expected duration being related to the frequency of preference changes.[5] Second, when regimes do change, they often change in a big way. Third, policy regimes cannot be observed directly by the general public, but must be inferred by relating observable central bank behaviour to the underlying regime; because observable bank behaviour varies for reasons other than regime shifts, inference can be made only imperfectly. Specifically, monetary policy is viewed as having two components: one is determined at each point in time by the prevailing policy regime and manifests itself as a particular stance by the central bank regarding the appropriate long-run

[4]Andolfatto and Gomme (1999) consider only two possible regimes. In contrast, the number of potential regimes in our paper is unrestricted. Modelling regimes in this manner seems more plausible, because in reality private agents usually do not know in advance where the new regime would end up; we avoid the need to interpret monetary history as the outcome of only two alternating policy regimes. In addition, we are able to consider regime changes of varying sizes. Again, we believe that this modification is a step in the direction of greater realism and hence is of some practical interest.

[5]For example, regime shifts could reflect the appointment of a new central bank governor or chairman, the adoption of a policy based on an intellectual argument that suggests that low-cost disinflation is feasible, or a change in policy owing to political pressure caused by the strain of wartime finance or high unemployment. See, for example, Hetzel (1998), who provides an interesting account of U.S. monetary policy during the Vietnam War. Alternatively, DeLong (1997) argues that the high-inflation episodes of the 1970s and their termination at the beginning of the 1980s was a result of shifting views about the shape of the Phillips curve and, more generally, about the nature of the constraints under which monetary policy is conducted.

money-growth rate or inflation target, and the other reflects short-lived interventions. Individuals can observe the historical sequence of money-growth rates, but they cannot distinguish the extent to which a particular money-growth rate represents a shift in the regime or a transitory intervention; i.e., they face a signal-extraction problem (Muth 1960). Consequently, agents' forecasts of future money growth based on past rates naturally display an inertial tendency (much like if adaptive expectations had been assumed), an economic force that will *endogenously* propagate the liquidity effect.

The second major departure concerns the way in which the labour market is modelled. We substitute the conventional Walrasian labour market with a Pissarides (1985)-style labour market search environment. We incorporate this framework primarily because of a growing recognition in the literature that the behaviour of labour-market flows following any type of shock is likely to play a quantitatively important role in any subsequent adjustment dynamics.[6] In addition, because the search model explicitly considers job vacancies and unemployment, we are able to investigate the model's implications for two important empirical relations: the Beveridge curve and the Phillips curve.

Together, the elements described above give rise to the following account of the monetary policy transmission mechanism. Unanticipated monetary tightening (a regime change) leads to a sharp rise in the interest rate, owing to the contemporaneous unexpected shortfall in liquidity. The shortfall in loanable funds (and higher interest rates) means that operating budgets in the business sector must be scaled back (at least, to the extent that operating budgets are financed with cash loans). In particular, firms begin to economize on their recruitment activities, leading to lower job creation and higher unemployment. In addition, to the extent that people do not realize that an actual regime change has occurred, their portfolio adjustment behaviour will display a natural stickiness, and one can expect high interest rates to prevail for some time. That is, with complete information (anticipating a large shortfall in liquidity), households would want to increase their deposit-to-cash ratio significantly to exploit the higher interest rate; but with incomplete information, households attach some weight to the possibility that the current money shock reflects only a transitory shortfall in liquidity, so that the desired increase in deposits is mitigated, with the result that financial markets are once again surprised by a shortfall in liquidity. This latter effect has the additional consequence of reducing investment demand (job-creation activities) for a number of periods, which further contributes to unemployment. Eventually, as people come to

[6] Empirically, worker flows in and out of employment during a short period of time are large relative to the stock (in contrast to physical capital, where the reverse is true). Thus, any change in these flows as a consequence of a change in monetary policy (or any other shock) is likely to have a more significant supply-side effect on output relative to standard models that rely on physical capital for intertemporal propagation. Andolfatto (1996) demonstrates the quantitative improvement that the labour-market search model yields over a conventional environment in the analysis of real shocks.

realize the nature of the monetary policy shock, expectations and interest rates adjust accordingly, leading to lower interest rates and increased rates of job creation. Because of frictions in the labour-market search process, however, this adjustment period takes time.

We find that the mechanisms described above are quantitatively important for generating persistent liquidity effects, although they depend on the precise calibration of the underlying processes for regime and transitory interventions that are employed. In addition, the model's predicted vacancy-unemployment-inflation dynamics appear to be consistent with conventional accounts of the Beveridge and Phillips curves. We conclude that the forces highlighted in the model may have helped shape the pattern of economic activity during some episodes in monetary history (e.g., the inflation episode of the 1970s and the disinflation episodes of the early 1980s and early 1990s). In particular, the model is consistent with empirical evidence that reports that inflation forecasts during the great inflation of the 1970s were consistently below actual inflation rates.[7]

This paper is organized as follows. Section 2 develops the model, which features a standard limited-participation cash-in-advance economy with a signal-extraction problem and labour-market search. The model is calibrated in section 3. A number of impulse-response experiments are performed in section 4. Section 5 concludes.

2. Model

2.1 Household sector

The economy is populated by a fixed mass of households that have identical preferences, defined over stochastic sequences of a cash good, c_{1t}, and a credit good, c_{2t} (Lucas and Stokey 1987); assume that preferences are represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_{1t}) + \gamma \ln(c_{2t})], \tag{1}$$

where $0 < \beta < 1$.

Following Pissarides (1985), employment in this economy is determined by a labour-market search process. Accordingly, let n_t denote the (predetermined) number of households who are employed (matched with a firm) at the beginning of period t, with the mass of households normalized to unity, $1 - n_t$ representing the number of unemployed households.[8] Following Merz (1997), assume that households have access to a complete insurance market, so that the idiosyncratic risk

[7] Andolfatto, Hendry, and Moran (2002) test this result quantitatively with a model that contains information frictions that are similar to those emphasized here. They show that simulated model data on realized and expected inflation replicate some of the available evidence about the rejection of unbiasedness in inflation expectations.

[8] Technically, the unemployed households in the model are more accurately described as non-employed.

associated with stochastic employment opportunities can be fully insured; this allows us to focus on a representative household.

The representative household consists of two members: a shopper and a worker, the latter of which is either employed or unemployed. At the beginning of the period, the shopper and the worker travel to spatially separated marketplaces: a market for output and a market for labour. The shopper uses (predetermined) cash on hand, M_t^c, to purchase the cash good, c_{1t}, and is faced with the following cash-in-advance constraint:

$$M_t^c \geq P_t c_{1t}, \tag{2}$$

where P_t denotes the nominal price of output (all goods sell for the same price).

Assume that the cash-in-advance constraint binds in every period. The shopper also spends $P_t c_{2t}$ on a set of credit goods; this payment is delayed until the end of the period.

Workers have one unit of time, which they supply perfectly inelastically as either work effort or search effort (leisure does not enter into the utility function). If the worker is employed, they travel to their job and earn nominal-wage income, W_t, which, after unemployment insurance deductions, results in take-home pay, $W_t n_t$. These earnings are paid to the worker as an upfront cash payment, but owing to the spatial separation of markets, this cash cannot be used by the shopper to purchase cash goods. If the worker is unemployed, they travel to the labour market in search of work. Moral hazard is not an issue; the unemployed worker also receives an unemployment insurance payment equal to $W_t n_t$, which, like the cash earnings of an employed worker, cannot be used by the shopper until the end of the period.

In addition to their labour-market earnings, households generate interest income on their savings and dividend income from their share of business-sector equity (all households have an identical share). Households enter the period with (predetermined) savings, M_t^d. These savings are in the form of cash deposits with financial intermediaries and earn a nominal net rate of interest, $R_t > 0$; this income accrues at the end of the period. Let Π_t denote the cash-dividend payment accruing to the household at the end of the period. With money income so determined, this period's credit goods consumption and the next period's asset allocations must obey the following restriction:

$$W_t n_t + \Pi_t + (1 + R_t) M_t^d \geq P_t c_{2t} + M_{t+1}^c + M_{t+1}^d. \tag{3}$$

Note that the household's future portfolio allocation (the division of end-of-period money holdings between cash, M_{t+1}^c, and deposits, M_{t+1}^d) must be made during period t, prior to having any information on the state of monetary policy prevailing in period $t+1$.

The household's choice problem is to select a contingency plan:

$$\{c_{1t}, c_{2t}, M_{t+1}^c, M_{t+1}^d \mid t \geq 0\},$$

to maximize (1) subject to (2) and (3), taking as given a stochastic process for:

$$\{n_t, W_t, R_t, P_t, \Pi_t \mid t \geq 0\},$$

with $M_0^c, M_0^d > 0$ given.

The choice problem can be reformulated as a dynamic program:

$$V_t(M_t^c, M_t^d) = \max_{c_{2t}, M_{t+1}^c, M_{t+1}^d} \left\{ \ln(\frac{M_t^c}{P_t}) + \gamma \ln(c_{2t}) + \beta E_t V_{t+1}(M_{t+1}^c, M_{t+1}^d) \right\}, \tag{4}$$

where the maximization is subject to (3) (note that the cash-in-advance constraint (2) has been used to eliminate c_{1t} from the problem). Let λ_t denote the Lagrange multiplier associated with the budget constraint. Intuitively, λ_t represents the utility value of an additional dollar accruing one period in the future (before it has been committed as either cash or deposits). The first-order necessary conditions that characterize optimal behaviour on the part of households are given by:

$$\begin{aligned} \lambda_t &= \beta E_t[\frac{1}{M_{t+1}^c}] = \beta[\frac{1}{M_{t+1}^c}]; \\ \lambda_t &= \beta E_t[(1 + R_{t+1})\lambda_{t+1}]; \\ \gamma &= \lambda_t P_t c_{2t}. \end{aligned}$$

The first restriction above equates λ_t to the expected marginal utility of a dollar allocated as cash; implicitly, this restriction governs the household's portfolio choice.[9] The second restriction governs the optimal intertemporal evolution of money balances. The third restriction characterizes the optimal division of consumption across cash and credit goods. All of these choices must obey constraint (3).

2.2 Business sector

The business sector consists of three distinct agents: goods-producing (active) firms, vacancies, and financial intermediaries. We can assume, without loss of generality, that each active firm consists of a single job-worker pair; consequently, n_t represents both the number of employed workers and the number of active firms. We follow Pissarides (1985) in assuming that idiosyncratic structural disturbances deplete the stock of job-worker capital at rate $0 < s < 1$; at the individual level, s represents the probability of job destruction. At the same time, search and recruiting activities serve to replenish the stock of employment. Assume that the aggregate search intensity of workers is proportional to the stock of unemployed individuals, $1 - n_t$, and that the aggregate recruiting intensity is proportional to the stock of vacancies, v_t. The uncoordinated search and recruiting activities of workers and firms results, in the aggregate, in m_t new matches; this matching process is governed by the following constant returns-to-scale technology:

$$m(v_t, 1 - n_t) = \chi v_t^{\eta}(1 - n_t)^{1-\eta}. \tag{5}$$

[9] Since M_{t+1}^c is chosen at time t, the expectation operator is not necessary.

Consequently, aggregate employment evolves according to:

$$n_{t+1} = (1-s)\left[n_t + m(v_t, 1-n_t)\right], \tag{6}$$

where the above formulation assumes that a newly formed match is subject to the structural shock in the period in which it is formed.

For simplicity, we assume that the productivity of each job-worker match is identical; let y denote the output of an active enterprise.[10] Assume that the period wage bill, W_t, must be financed with a cash loan; a worker-firm match results, for the firm, in a financial flow of $P_t y - (1+R_t)W_t$. With the amount, W_t, being directed to the worker in that pair, the net flow (or surplus) resulting from a match is $P_t y - R_t W_t$. We assume that the firm and the worker split this net surplus according to a fair-share rule $0 < \xi < 1$ (see Howitt 1988), so that:

$$W_t = \xi(P_t y - R_t W_t). \tag{7}$$

Solving for W_t in this last equation leads to:

$$W_t = \frac{\xi}{(1+\xi R_t)} P_t y, \tag{8}$$

which implies a flow profit for the firm equal to:

$$\Pi_t^f = \frac{(1-\xi)}{(1+\xi R_t)} P_t y. \tag{9}$$

Notice that both wages and profits depend negatively on the nominal rate of interest. With the profit flow so determined, the value (measured in utils) of an active firm, J_t, satisfies the following recursion:

$$J_t = \lambda_t \Pi_t^f + \beta(1-s)E_t J_{t+1},$$

where λ_t is the utility value that a household attaches to one extra dollar of cash accruing one period hence.

To open or maintain a job vacancy, a real resource cost equal to $\kappa > 0$ must be spent by the vacant firm. The nominal cost of this expenditure is given by $P_t \kappa$, an amount that must be borrowed from financial intermediaries and paid back at the end of the period at interest rate R_t. Consequently, the cash flow for a vacancy is given by:

$$\Pi_t^v = -P_t(1+R_t)\kappa.$$

This negative cash flow is absorbed by the owners of this vacancy capital (i.e., households pay back this cash loan out of their money income).[11]

[10] The model emphasizes employment capital and abstracts from physical capital. It would be easy to add physical capital, as in Andolfatto (1996). However, doing so would change very little of the quantitative properties of the model, since the investment-capital propagation mechanism is very weak; see Cogley and Nason (1995).

[11] This negative cash flow could also be interpreted as households' purchases of shares during the initial public offerings of start-up firms.

A job-worker match occurs with probability $q_t = q(\phi_t)$, where $\phi_t = v_t/(1 - n_t)$ defines the tightness of the labour market from the viewpoint of a searching firm. The function q is derived from the aggregate matching technology:

$$q(\phi_t) = \frac{m(v_t, 1 - n_t)}{v_t} = \chi \phi_t^{\eta - 1}.$$

Clearly, q is a decreasing function of the labour-market tightness variable, ϕ_t. That is, as the number of vacancies grows relative to the number of available workers, the probability falls that any given vacancy is successful in its recruiting effort.[12]

A successful job match is able to produce output one period hence, conditional on surviving the structural shock. Let Q_t denote the value of a vacant firm (measured in utils). This capital value must obey the following recursive relationship:

$$Q_t = \lambda_t \Pi_t^v + \beta \left[q(\phi_t)(1 - s) E_t J_{t+1} + [1 - q(\phi_t)(1 - s)] E_t Q_{t+1} \right].$$

As is standard, we assume that there is free entry in the creation of job vacancies. Consequently, if $Q_t > 0$, new jobs will continue to appear, with the resulting congestion driving the probability of success low enough until, at the margin, an entrepreneur is simply indifferent between posting a vacancy and not; i.e., $Q_t = 0$ for all t. Imposing this restriction on the equation above yields:

$$\beta q(\phi_t)(1 - s) E_t J_{t+1} = \lambda_t (1 + R_t) P_t \kappa. \tag{10}$$

We next describe the operation of the financial sector. At the beginning of period t, financial intermediaries receive a cash injection, X_t, from the monetary authority; this cash, together with the funds, M_t^d, provided by households, is lent to goods-producing firms and vacancies in the business sector at interest rate R_t. The interest rate paid on loans is the same as that paid on deposits, since financial intermediation is assumed to be costless and competitive. Consequently, the financial sector earns a flow profit at the end of period t equal to:

$$\begin{aligned} \Pi_t^b &= (1 + R_t)(M_t^d + X_t) - (1 + R_t) M_t^d \qquad (11) \\ &= (1 + R_t) X_t. \end{aligned}$$

As all households are imagined to hold an identical share in business sector equity, they are entitled to the aggregate dividend payment: $\Pi_t = \Pi_t^f + \Pi_t^v + \Pi_t^b$. Finally, goods-market and loan-market clearing implies that:

$$c_{1t} + c_{2t} + \kappa v_t = n_t y; \tag{12}$$

$$W_t n_t + P_t \kappa v_t = M_t^d + X_t. \tag{13}$$

[12] The probability of a searching worker finding a match (i.e., finding a job) is equal to $\frac{m(v_t, 1-n_t)}{1-n_t} = \chi \phi_t^{\eta}$.

2.3 Monetary policy

Let M_t denote the total supply of base money at the beginning of period t. During the period, the monetary authority undertakes a cash injection, $X_t = x_t M_t$, where $0 < x_t < 1$ is the (net) growth rate of money supply. We assume that x_t comprises two stochastic components: z_t, which reflects the regime (i.e., the underlying preferences about long-run money expansion), and u_t, which expresses the monetary authority's short-run discretionary policy. We thus have:

$$x_t = \mu + z_t + u_t, \tag{14}$$

where μ is the average rate of growth.

We assume that the regime component of monetary policy, z_t, remains constant for relatively long periods of time and only occasionally shifts to a new regime. Accordingly, the evolution of z_t can be expressed as follows:

$$z_t = \begin{cases} z_{t-1} & \text{with probability } \theta; \\ g_t & \text{with probability } 1-\theta, \quad g_t \sim N(0, \sigma_g^2). \end{cases} \tag{15}$$

The parameter $0 < \theta < 1$ indexes the expected duration of any given regime. As stated in section 1, the regime variable, z_t, can be given a number of interpretations. It could correspond to changes in economic thinking that lead monetary authorities to modify their views about the proper long-term rate of monetary expansion. Alternatively, a change in z_t could reflect the appointment of a new central bank governor, whose preferences over inflation outcomes differ from those of their predecessor. Under either interpretation, however, we would expect the persistence of the regime (expressed by the parameter θ) to be fairly high. Moreover, the size of the parameter σ_g should reflect our understanding that the shifts, when they do occur, are of significant size.

In contrast, the transitory component of money growth, u_t, is assumed to follow a standard AR(1) specification, as follows:

$$u_t = \rho u_{t-1} + e_t, \tag{16}$$

where $0 \leq |\rho| < 1$ and $e_t \sim N(0, \sigma_e^2)$. The variable u_t can be interpreted as instances when monetary authorities wish to intervene in financial markets, for short periods of time and in reaction to factors not modelled here. Alternatively, these shocks can be considered to be errors stemming from the imperfect control exercised by the central bank over monetary aggregates. Again, under either interpretation, our expectation would be that these interventions are very transitory and thus that the value of the parameter ρ would be fairly small.

2.4 Expectations

Assume that individuals in the economy know the parameters $(\mu, \theta, \rho, \sigma_g^2, \sigma_e^2)$. If, in addition, private agents can distinguish the components of money growth (transitory and persistent), then forecasting future money growth is easy:

$$E_t x_{t+1} = \mu + \theta z_t + \rho u_t. \tag{17}$$

In our view, however, an important element of reality is that private agents are unable to determine perfectly which policy regime is actually in place at any given time. In other words, while individuals are able to observe the historical realizations of money-growth rates, $\Omega_t = \{x_t, x_{t-1}, ..., x_0\}$, they are unable to observe the historical realizations of z_t or u_t separately. Consequently, the forecast in (17) is not feasible.

We believe that this information friction characterizes well the operating procedure of the Federal Reserve—which does not release publicly any long-term inflation target—and the Bank of Canada's behaviour prior to the introduction of formal inflation targets in 1991.[13]

Economic agents thus need to solve the signal-extraction problem of separating the two individual components of realized money growth. We assume that they do so by constructing optimal (Bayesian) forecasts of future money growth, x_{t+1}, based on all relevant information, which, in this case, amounts to knowledge of $\Omega_t, \mu, \theta, \rho, \sigma_g^2$, and σ_u^2 (and an initial prior). Given that (z_t, u_t) are distributed joint-normally,[14] Bayes' rule and the Kalman filter (i.e., linear least-squares projections) will coincide; consequently, we employ the latter updating rule. The economic agents featured in this paper are thus rational, in that they use the optimal Bayesian forecasts to overcome the information friction.

Define the projections $z'_{t+1} = P[z_{t+1} \mid \Omega_t]$ and $u'_{t+1} = P[u_{t+1} \mid \Omega_t]$. The *limited-information* forecast will be given by:

$$E\left[x_{t+1} \mid \Omega_t\right] = \mu + z'_{t+1} + u'_{t+1},$$

where these projections satisfy the following recursive formulas:

$$\begin{aligned} z'_{t+1} &= \theta\left[(1-\alpha)z'_t + \alpha(x_t - \mu - u'_t)\right]; \\ u'_{t+1} &= \rho\left[\alpha u'_t + (1-\alpha)(x_t - \mu - z'_t)\right], \end{aligned} \tag{18}$$

where $\alpha = (\Sigma + \Delta)/[(\Sigma + 2\Delta + \Gamma)]$, $\Sigma = E(z_t - z'_t)^2, \Delta = E(z_t - z'_t)(u_t - u'_t)$, and $\Gamma = E(u_t - u'_t)^2$. These quantities, and thus the value of α, are endogenously determined by the Kalman filter

[13] We also believe that it remains relevant for the period since 1991. First, the credibility of the Bank of Canada was initially imperfect and the announcement of the new policy of inflation targets was not fully incorporated in economic agents' expectations. Second, the practical procedures by which the Bank implements its inflation-targeting policy leaves some uncertainty over the length of time inflation would be allowed to drift away from 2 per cent before intervention would occur to correct such a diverging path. Further, sizable uncertainty might remain over the long-term continuation of the inflation-targeting policy. Others have argued that the Federal Reserve has implicitly adopted inflation targets, although it has not publicized them; such a situation matches very well the information friction that we model.

[14] Conditional on the value of z_t, z_{t+1} is not distributed normally. But the unconditional distribution of z_t should be normal, since all changes in that variable arise from the normally distributed g_t; see equation (15).

as a function of the parameters of the underlying processes for money growth $(\mu, \theta, \rho, \sigma_g^2, \sigma_e^2)$. Derivations are supplied in Appendix A.[15]

Observe that α governs the relative weight that individuals attach to a particular money-growth-rate realization; i.e., a large α implies that the deviation $(x_t - \mu)$ is more likely to be perceived as a regime shift, while a small α implies the opposite.

By assuming full knowledge of the parameters that underlie the evolution of the money supply $(\mu, \theta, \rho, \sigma_g^2, \sigma_e^2)$, we are probably understating the information problems faced by real economic agents: the structure of the monetary decision process has most certainly changed over time. Considering shifts in the parameters and learning about those shifts with a least-squares rule could be a further step towards greater realism.

2.5 Equilibrium

Conditional on a stochastic process for monetary policy, and given the nature of available information, an equilibrium for this economy is a collection of stochastic processes such that households and firms are optimizing and markets clear every period. Note that the market for labour does not clear in the conventional sense; rather, wages are determined via bilateral negotiations between the recruit and the firm.

To solve for the equilibrium numerically, we transform a number of variables to induce stationarity. To this end, let $\hat{\lambda}_t = \lambda_t M_{t+1}$ and, similarly, let lower-case nominal variables denote the fact that they have been deflated by M_{t+1} (except for m_t^c, which denotes the ratio M_t^c/M_t). The restrictions that characterize the equilibrium are then written as:

$$\hat{\lambda}_t = \beta[\frac{1}{m_{t+1}^c}] \tag{19}$$

$$\hat{\lambda}_t = \beta E_t[\frac{1+R_{t+1}}{1+x_{t+1}}\hat{\lambda}_{t+1}] \tag{20}$$

$$\gamma = \hat{\lambda}_t p_t c_{2t} \tag{21}$$

$$J_t = \hat{\lambda}_t[1+\xi R_t]^{-1}(1-\xi)p_t y + \beta(1-s)E_t J_{t+1} \tag{22}$$

$$0 = -\hat{\lambda}_t(1+R_t)p_t\kappa + \beta\chi(1-s)\phi_t^{\eta-1}E_t J_{t+1} \tag{23}$$

$$p_t c_{1t} = \frac{m_t^c}{1+x_t} \tag{24}$$

$$\frac{\xi p_t y}{(1+\xi R_t)} = w_t. \tag{25}$$

The law of motion for employment remains unchanged, as does the market-clearing condition

[15] Σ, Δ, and Γ are the elements of the mean-squared-error (MSE) matrix that arises when z_{t+1} and u_{t+1} are forecast based on time-t information. They are written without time subscripts, to indicate that we use their steady-state values in our computations rather than update an initial MSE matrix until convergence. This approach is not essential to the nature of our results, because the convergence to the steady-state MSE matrix is fast. See Appendix A for further details.

in the goods market. The loan-market condition becomes:

$$w_t n_t + p_t \kappa (1-n_t)\phi_t = 1 - \frac{m_t^c}{1+x_t}.$$

The simultaneous solution of our ten equations (the seven above plus (12) and (13)) represents the equilibrium of the economy. A first-order, linear approximation of this solution is computed using the method described and algorithms provided in King and Watson (1998). Details of the solution procedure and how it is adapted to the problem at hand are available from the authors.

3. Calibration

3.1 Computing the steady state

The model's steady state is calibrated to fit various secular measures in the Canadian economy. Evaluating (19) to (25) at their steady-state values leads to the following:

$$\lambda = \beta/m^c \tag{26}$$

$$1+\mu = \beta(1+R) \tag{27}$$

$$\gamma = \lambda p c_2 \tag{28}$$

$$J = \frac{(1-\xi)py\lambda}{(1+\xi R)[1-\beta(1-s)]} \tag{29}$$

$$\lambda(1+R)p\kappa = \beta\chi(1-s)\phi^{\eta-1}J \tag{30}$$

$$pc_1 = \frac{m^c}{1+\mu} \tag{31}$$

$$w = \frac{\xi py}{(1+\xi R)}. \tag{32}$$

To these seven first-order conditions we add the law of motion for employment (6), the goods-market clearing condition, and the loan-market clearing condition, all evaluated at steady-state values:

$$sn = \chi(1-s)(1-n)\phi^\eta \tag{33}$$

$$c_1 + c_2 + k(1-n)\phi = ny \tag{34}$$

$$wn + pk(1-n)\phi = 1 - \frac{m^c}{1+\mu}. \tag{35}$$

These restrictions constitute ten equations in the ten unknowns:

$$(c_1, c_2, \lambda, m^c, J, R, p, w, n, \phi),$$

which can be easily solved for a particular parameterization. Consequently, the model has eight parameters $\{\beta, \gamma, k, \chi, \eta, s, \xi, y\}$ that describe preferences, technologies, and bargaining power, and five parameters $\{\mu, \theta, \rho, \sigma_e, \sigma_g\}$ that describe the evolution of money-growth rates.

3.2 Calibrating preference and technology parameters

Parameter values are chosen as follows. Assuming quarterly time periods, the discount factor is set to a conventional value, $\beta = 0.99$, which implies an average real rate of interest equal to 4 per cent per annum. The productivity parameter serves simply to scale values; it was chosen to be $y = 1$. The parameter γ is related to the importance of the credit good in the overall consumption basket of households. Following the logic in Cooley and Hansen (1995), we fix γ at 0.136 (which corresponds to a value of 0.88 for the parameter α in their model). The quarterly transition probability out of employment was set to $s = 0.15$ based on evidence provided in Jones (1993) for Canada.[16] The parameter η represents the elasticity of new hires with respect to recruiting intensity. Estimates for the United States and the United Kingdom suggest values in the neighbourhood of $\eta = 0.60$.[17]

The parameters κ, ξ, and χ were chosen jointly so that the steady state of the model would exhibit the following properties: (i) a labour share of total income of 80 per cent, (ii) an employment ratio (employment relative to the working-age population) equal to 65 per cent, and (iii) a probability for a searching firm of finding a suitable worker equal to 0.7. The first two properties correspond to averages of observed Canadian data. The third follows Cooley and Quadrini (1999). The parameter values implied by these restrictions are $\kappa = 0.59$, $\xi = 0.82$, and $\chi = 0.52$. Note that the calibration of the separation rate, s, and the steady-state employment, n, is sufficient to determine the steady-state probability that a searching worker finds a job ($\chi\phi^{\eta}$; see equation (33)). Our chosen values for s and n imply that this probability equals 0.33.[18]

3.3 Calibrating the monetary policy process

The parameters that govern the monetary policy process are $\mu, \theta, \rho, \sigma_g$, and σ_e. The first parameter expresses the unconditional mean growth rate of money: it is fixed at 0.015, the average quarterly per-capita growth rate of Canadian base money over the sample period 1955Q2 to 2001Q4.

Recall that θ and σ_g govern the dynamics of the regime component of monetary policy (the variable z_t). These parameters, respectively, express the expected duration of a particular regime and the standard deviation of the distribution from which the value of a regime shift, when one occurs, is drawn. On the other hand, ρ and σ_e denote the autocorrelation and standard deviation

[16] Jones reports that the monthly probability of transition from employment (to either unemployment or out of the labour force) is 4.2 per cent. A quarterly figure would thus be slightly less than 13 per cent.

[17] Blanchard and Diamond (1989) report estimates of η around that value. No Canadian estimate is found among the 25 empirical studies of the matching function referenced by Petrongolo and Pissarides (2000).

[18] This value implies an average unemployment duration of three quarters. Such a value is relatively high compared with the typical estimates of around 20 weeks (or 1.5 quarters) for the average unemployment duration in Canada. However, our concept of unemployment is broader than measured unemployment: it includes both unemployed workers and those out of the labour force. The average unemployment duration of this broader category is likely to be higher than the duration of standard unemployment measures.

of the transitory money-supply shocks around a given regime (the u_t variable).

The interpretations suggested above for the shifts in the variable z_t—changes in economic thinking or the appointment of a new central bank governor—suggest that these shifts occur only infrequently, perhaps once every five or ten years (transposed to the quarterly frequency we use, this corresponds to one shift every 20 to 40 periods). Such an average duration between shifts corresponds to values of θ between 0.95 and 0.975. We thus fix the value of θ at 0.975 (experiments conducted with the lower value, 0.95, do not change the qualitative features of our results).

Calibration of the remaining three parameters (σ_g, ρ, and σ_e) is not straightforward. One possibility, which would be hard to achieve in an uncontroversial manner, would be to identify specific episodes of monetary history as z_t or u_t shocks and study them to uncover the dynamic properties of the shocks. Instead, we simply use the dynamics of observed money growth to achieve this calibration. Specifically, we use the chosen value for θ and the knowledge of the processes in (15) and (16) to match the observed standard deviation (0.013) and the first- (0.47) and second-order (0.40) autocorrelations of base money growth over the sample 1955Q2 to 2001Q4.[19] Table 1 summarizes the calibrated values that emerge from these computations.

Table 1: Calibrated Values of the Parameters

Parameter	Calibrated value
θ	0.975
μ	0.011
ρ	0.119
σ_g	0.0083
σ_e	0.0097
Resulting value for α	0.152

The calibrated value of σ_g implies that when a regime shift does occur, a typical order of magnitude for the change in the long-run inflation target caused by the shift is 0.83 per cent, or 3.3 per cent on an annualized basis. Further, the low value of ρ indicates that, as conjectured earlier, the transitory interventions of monetary policy have only a weak serial correlation.

These parameter values, in turn, determine a value slightly over 0.15 for the parameter α. Recall that a relatively high value of α would have signified that agents tend to consider most shocks to arise from the persistent z-type regime shifts. In contrast, the small value of α in this benchmark calibration means that agents tend to view most monetary disturbances as transitory

[19] Details of the computations are provided in Appendix B. We also provide, in section 4.4, experiments based on M2 data, to check the robustness of our results to the empirical definition of money used.

u-type shocks.[20]

4. Impulse-Response Experiments

4.1 Regime shock

Our first experiment is the implementation of a new (lower) target for the money-growth rate, which may also be considered the implementation of a disinflation policy. Technically, we consider a negative one-standard-deviation money-regime shift; i.e., the variable z_t is lowered by 0.0083 (the value of σ_g). This corresponds to a drop in the (annualized) money-growth rate from 6.3 per cent per annum to just under 3 per cent per annum. Figure 1 shows the economy's dynamic response to this type of shock. Panel A compares the impulse responses when information is complete (solid lines) with those arising when the information friction and learning behaviour emphasized in this paper are activated (the dashed lines). Panel B compares the responses of realized and (one-quarter-ahead) expected inflation, in the case of complete information (bottom left graph), and incomplete information (bottom right graph).

Panel A reveals that, in the presence of information frictions, a strong liquidity effect arises from the shift: the (annualized) nominal rate rises from 10.4 per cent to just over 13 per cent in the impact period of the shock. Further, the interest rate stays above its initial level for five quarters following the shock. In contrast, in the absence of information problems, the interest rate, while increasing substantially in the impact period of the shock, adjusts very quickly. Inflation falls rather quickly in either information environment, although the adjustment appears to be more gradual in the incomplete-information case.[21]

Figure 1 also shows that the liquidity shortage curtails the creation of new job vacancies. Recruiting intensity falls by 2.1 percentage points on impact, and stays below its initial steady state for six quarters following the shock. The slow, upward adjustment in new job-creation reflects in part the slow downward movement in the interest rate. In the long run, the lower steady-state interest rate associated with the new money regime serves to stimulate job-creation activities; relative to the initial steady state, job vacancies are predicted to be over one percentage point higher in two and a half years.

[20] The monetary policy-related shocks on money growth are the only disturbances that affect our artificial economy. The presence of other shocks—which affect money demand, for example—would complicate the learning problem faced by economic agents.

[21] The distinction between cash and credit goods, which serves to make velocity endogenous, helps to mute the response of inflation to the shock. In particular, the interest rate hike induces an increase in velocity, which serves to mute the downward movement in inflation. The reduction in output growth also serves to keep inflation higher than it otherwise might be. Quantitatively, though, both of these forces are not strong enough to prevent a strong initial drop in inflation.

On impact, the higher interest rate reduces the surplus created by a given match, so both wages and profits fall in the initial periods following the regime change. The sustained reduction in job-creation activities leads to a positive and hump-shaped unemployment response. Unemployment peaks in the third period following the shock at 0.6 percentage points above its initial steady-state level and stays above this level for nine quarters. Eventually, lower interest rates and increased job-creation activities imply that unemployment falls in the long run by over 1 percentage point; the full adjustment period takes about five years. A comparison with the complete information case reveals that the information frictions introduce significant persistence in the response of unemployment.

In Panel B of Figure 1, in the complete-information case, realized and expected inflation overlap perfectly immediately after the initial shock. On the other hand, the incomplete-information case is characterized by sticky expectations relative to the actual rates: a significant gap between realized and expected inflation persists for several periods. Even in the long run, inflation expectations remain slightly above the actual inflation rate, because individuals attach at least some probability to a reversion to the long-run mean money-growth rate, μ.[22]

Although the effects reported above exhibit substantial persistence, they may still fall short of matching those uncovered in actual data; for example, Romer and Romer (1989) report effects that last in the order of years. Overall, however, we feel that these responses are broadly consistent with the empirical evidence.

4.2 Transitory shock

We next consider a transitory, negative, one-standard-deviation shock; i.e., u_t is lowered by 0.0097. This corresponds to a fall in (annualized) money growth from 6.3 per cent per annum to about 2.4 per cent per annum. Notice that money growth returns back very quickly to 6.3 per cent, because the persistence parameter of such shocks (the parameter ρ) is low. Figure 2 shows the economy's dynamic response to this type of shock.

In the impact period, inflation and the interest rate behave very much as they would in a regime shock: most notably, the interest rate experiences a significant increase. Unlike in the case of a regime change, however, the interest rate in an incomplete-information environment falls very sharply and undershoots its steady-state level in the periods subsequent to the shock. The interest rate undershoots because individuals have mistakenly attributed some weight in their forecasts to the possibility of a regime change. When money growth turns out to be higher than forecast in the subsequent period, the resulting unexpected increase in liquidity puts additional

[22] Considering the process governing the evolution of the regime variable, z_t, a more appropriate impulse-response experiment would arise by shifting z_t for a given number of periods (say, the expected duration of the shift) and then switching it back to its long-run average. In such a case, all complete- and incomplete-information paths would, at the end of the period, coincide with one another.

downward pressure on the interest rate. It takes individuals close to two years before they become confident that the money-growth shock was actually transitory. In the complete-information case, no undershooting is present and the interest rate is back to its steady-state level in the period immediately following the impact of the shock.

With a transitory shock, the temporary increase in the interest rate curtails vacancy creation in the impact period and this, in turn, leads to a short period of above-average unemployment. In subsequent periods, however, and in the case of incomplete information, the interest rate undershoots its steady-state level, which increases incentives for job creation: unemployment thus soon decreases from its recession levels and also undershoots its steady-state level for several quarters. Incomplete information therefore leads eventually to a persistent (if modest) economic boom; if individuals had complete information concerning the nature of the shock, their inflation forecasts would adjust correctly, leading to higher interest rates, lower job creation, and higher unemployment throughout the transition period (see the solid lines of the graphs in Figure 2).

One interpretation of our experiment is that it measures the efficacy of discretionary monetary policy across environments characterized by a credible (complete-information) and non-credible (incomplete-information) monetary authority.[23] Consider, for example, the case of a transitory *increase* in money growth. The results in Figure 2 suggest that a non-credible monetary authority may be able to engineer a short-lived decrease in unemployment by temporarily increasing money growth and lowering interest rates. The initial reduction in unemployment would, however, be followed by a prolonged period of above-average unemployment. Such a depression in economic activity would result because the public attaches some weight to the possibility that the high money-growth realization was generated by a new loose-money regime (despite the denials that would likely be forthcoming from the authority). Accordingly, individuals would revise their inflation forecasts upward, putting upward pressure on interest rates, and leading to depressed investment (recruiting) activity. In contrast, a credible monetary authority could generate a long-lasting economic boom through what might be termed a credibility dividend.

The bottom panel of Figure 2 illustrates the relative behaviour of expected and actual inflation. In a manner similar to Figure 1, the bottom left graph shows that, in a complete-information environment, actual and expected inflation move in lock-step following the initial shock. In contrast, the bottom right graph indicates that incomplete information gives rise to a persistent wedge between expected inflation and actual levels. Agents overpredict inflation at the onset of the shock, but are slow to recognize the waning of the initial impulse.

[23]These labels are motivated by the fact that a credible monetary authority could reveal the true nature of the shock even if individuals could not observe it.

4.3 A "gradual" disinflation policy

One possible explanation for the persistently high interest rates that seem to accompany some disinflation episodes (e.g., Canada in the early 1980s and early 1990s) is that the regime change actually takes place in steps. For example, the inflation target might be lowered by four percentage points in year one, followed by a further four percentage points in year two. Such an observation is a possible outcome of the stochastic process modelled above.

Figure 3 shows the impulse responses (under incomplete information) following such a two-standard-deviation disinflation policy, starting from an initial steady state where money growth (and thus inflation) is relatively high. The solid line in each graph shows the case in which the disinflation policy is implemented all at once. In the other two cases, the disinflation policy is implemented in two steps: one line shows a shorter implementation period (Gradual #1), and the other shows a longer implementation period (Gradual #2). As expected, the gradual implementation of the disinflation policy leads to smoother movements in the interest rates, relative to those arising from the immediate, benchmark implementation. A longer period of relatively high rates of interest results, as well as more persistence generally in economic outcomes.

4.4 Alternative calibration: using M2

In our model, the liquid funds, the supply of which is controlled by the monetary authorities (M_t), are the only medium of monetary transactions. This assumption holds imperfectly in reality, of course, as the money base, although determined by the actions of the Bank, is only one component of what agents use to conduct monetary transactions. It is therefore important to check the robustness of our results using a different monetary aggregate as the basis for calibrating the parameters that underlie the dynamics of money-supply growth. We expect that the use of a different calibration and an alternative specification of monetary policy would modify the calibrated value of the key parameter in the learning mechanism, α. Recall that this parameter describes the extent to which economic agents will tend to consider that deviations of observed money growth from its mean arise from regime shifts.

Figure 4 shows the results that arise when the monetary aggregate, M2, is used to calibrate the money-growth processes. Such a change in the choice of monetary aggregates, along with our assumption that $\theta = 0.975$, leads us to assign a value of 0.28 to the parameter α.[24] A relatively broad aggregate such as M2 is often used to build monetary DGE models, because it internalizes most intra-aggregate substitutions that contaminate the movements of narrower aggregates (such

[24] The other parameter values that result from this alternative calibration are $\rho = 0.69$, $\sigma_g = 0.011$, and $\sigma_e = 0.0068$. The high value of ρ that results from this calibration choice is less coherent with our priors concerning the nature of the u_t shocks that affect the money supply.

as M1). In Canada, these substitutions are related mostly to institutional and regulatory features, and hence it is difficult to interpret them within the context of the standard monetary DGE model.

This experiment has a large influence on the persistence of all responses. As Figure 4 shows, the liquidity effect on the nominal interest rate indicates little, if any, persistence. Vacancies stay below their steady-state level for only two quarters and unemployment falls much quicker than in Figure 1. These important differences between the two calibrations underscore the importance of parameter α, a relatively high value of which implies that agents are faster to recognize a regime shift than a transitory intervention. Moreover, the fact that parameter ρ is, in this case, quite high means that the two types of monetary disturbances are not very different from each other, as they were in the calibration that used base-money data. This lessens the importance of the information frictions that we emphasize in this paper.[25]

Panel B of Figure 4 is similar to Panel B of Figure 1: in the complete-information case, expected and actual inflation merge together immediately after the initial period of the shift. In the incomplete-information case, expected inflation overpredicts actual levels for a few quarters and, although these errors appear less pronounced and persistent than those features in Figure 1, they remain economically significant.

4.5 Slowly evolving regime changes

It might be possible that monetary policy regimes cannot be changed instantly, as we have assumed in this paper. Considerations of caution and gradualism might encourage central banks to phase in desired regime changes over several quarters. Alternatively, inside-money creation might respond only gradually to outside-money shocks, and therefore monetary aggregates might not increase as quickly as process (14) implies. Consequently, it may be useful to model the evolution of money-growth rates as follows:

$$x_t = \tau x_{t-1} + (1-\tau)(\mu + z_t + u_t),$$

where the parameter τ measures the speed at which a desired regime change is (or can be) implemented. Assuming that such a process exists (with $\tau = 0.5$), alongside our benchmark calibration of the other parameters, gives rise to Figure 5.[26]

The gradual decrease in money-growth rates is shown in the upper left corner of the figure. Because the magnitude of the initial innovation to money growth is lower, the responses of the other variables are also lower. The interest rate, for example, increases from its steady-state value

[25] A calibration strategy that would allow relatively high values of α while retaining the small values of ρ would likely improve our exposition of the information-friction framework's essential features.

[26] The calibration procedure and the derivation of the filter must be modified to take this new process for money growth into account. In particular, assigning another parameter value requires an additional moment restriction: we rely on the third-order autocorrelation for base money. Details are available from the authors.

of 10.4 per cent to a rate of 11.4 per cent initially. Vacancies decrease by only 0.9 percentage points, whereas in Figure 1 they were shown to decrease by 2.3 percentage points. Because of the reduced availability of money (relative to our previous experiments), inflation initially responds less strongly and its adjustment towards its new steady-state value is noticeably smoother than it was in the experiments plotted in Figure 1.

Panel B of Figure 5 indicates that, as in Figure 1, expected inflation stays above actual levels for several quarters an incomplete-information environment. The learning mechanism of the model is thus capable of generating persistent episodes of what appear to be expectations errors.

4.6 The Beveridge and Phillips curves

Empirically, the Beveridge curve shows the negative correlation typically observed between measures of aggregate recruiting intensity (e.g., vacancy counts or the help-wanted index) and the unemployment rate (Blanchard and Diamond 1989). Andolfatto (1996) demonstrates how this negative correlation can be generated in a real-business-cycle model modified to incorporate labour-market search. In the model developed here, the same negative relation is apparent for the incomplete-information case.

Figure 6 plots the vacancy-unemployment dynamics in an incomplete-information environment, following a *positive* one-standard-deviation regime shift and transitory shock, respectively.[27] The dynamics in each case feature the classic counter-clockwise loops associated with the Beveridge curve. In both cases, the surprise cash injection stimulates vacancy creation (owing to the temporarily lower interest rate). Following the regime change, job creation remains relatively high for a few periods (owing to the relatively low interest rates), and this leads to decreases in the unemployment rate for several periods. However, the long-term effects of the shift eventually take hold: along with rising interest rates comes lower levels of vacancies and, eventually, higher unemployment rates. Following the transitory shock, vacancies soon decline from their short-lived spike and continue to decrease (and unemployment to increase), because of the overshooting in interest rates the shock has brought into play. Once this overshooting is over, vacancies and unemployment converge back to their steady-state values.

Figure 7 plots the inflation-unemployment dynamics following the same regime shift and transitory shock, respectively. As emphasized by Cooley and Quadrini (1999), many economists consider the Phillips curve to be an essential tool for the conduct of monetary policy. Because the Phillips curve does appear to be a stylized fact, it is presumably important that a model used to organize thought on monetary policy be able to replicate at least the qualitative nature of the Phillips relationship. As Figure 7 reveals, our model generates a negatively sloped Phillips curve when

[27] These experiments are the exact opposite of those presented in the preceding figures.

monetary disturbances are transitory. When the monetary disturbance takes the form of a regime change, the Phillips curve maintains its negative slope only in the short run; in the long run, the Phillips curve is positively sloped. The pattern described here is consistent with conventional wisdom.

It should be emphasized that the mechanism that generates the negative Phillips curve relation is somewhat different from many popular explanations; e.g., explanations based on the notion of low unemployment somehow causing upward wage pressure. In our set-up (as in Cooley and Quadrini 1999), low unemployment is caused by increased job-creation activities created by lower interest rates.

5. Conclusion

In this paper, we have explored the properties of a monetary model that featured information frictions in the market for money and search frictions in the labour market. The information friction created sticky expectations, whereas the search frictions served to propagate a monetary shock forward in time even after the initial liquidity effect had dissipated. We feel that inflation expectations are sticky because monetary policy is characterized by both persistent and transitory components that cannot be separately observed; i.e., individuals are compelled to form an adaptive-expectations rule.

We found that the implementation of a disinflation policy (modelled here as a switch to a lower long-term inflation regime) generated a persistent increase in the short-term interest rate, owing to a persistent unexpected shortfall in liquidity, and a persistent increase in unemployment. In the longer run, both the interest rate and unemployment fell below their initial levels. This persistent liquidity effect arose because of a sticky portfolio response, which itself was the manifestation of the sticky-expectations mechanism. The subsequent periods of higher-than-normal interest rates curtailed job-creation activities in the business sector, making it more difficult for the unemployed to find suitable job matches. This, in turn, created a persistent decrease in the unemployment rate. The vacancy-unemployment-inflation dynamics created by the model were consistent with conventional accounts of the Beveridge and Phillips curves.

We conclude that the forces highlighted in our model may have played a role in shaping the pattern of economic activity witnessed during some episodes of monetary history (e.g., the inflation episode of the 1970s and the disinflation episodes of the early 1980s and early 1990s). In particular, the model is consistent with empirical evidence that inflation forecasts during the 1970s were consistently below actual inflation rates. Consider, for example, the implementation of a loose-money regime. Figure 8 plots the responses of variables to a gradual, two-standard-deviation regime shift.[28] The bottom right graph of the figure indicates that inflation expectations remain below actual inflation for all of the period plotted. Andolfatto, Hendry, and Moran (2002), building on this result in a related model, show that these differences between the paths of realized and expected inflation can replicate some of the sylized facts about empirical measures of inflation expectations.

The model we have described, however, suffers from a number of quantitative shortcomings. While the mechanisms that we have highlighted are clearly capable of contributing to the persistence of the liquidity effect, the speed of that learning seems to be fairly sensitive to the calibration of the underlying processes for regime shifts and transitory interventions. A more robust calibration strategy, perhaps one relying on actual episodes in monetary history, would likely improve the exposition of the potential contribution of information frictions to macroeconomic modelling. Another apparent shortcoming in this class of models concerns the behaviour of actual inflation, which, in the data, appears to respond much more sluggishly to a monetary disturbance. Accounting for these discrepancies continues to prove a challenge.

Finally, actual monetary policy practices may be better characterized by an interest-rate-targeting rule, which, through the demand of private agents for transactions assets and a (possibly variable) outside/inside money multiplier, gives rise to a required course of action for the total supply of base money. A complete and satisfactory modelling of these operational aspects of monetary policy would likely improve our understanding of the relevance of information frictions for the conduct of monetary policy.

[28] There are no asymmetries in the model: the paths displayed in Figure 8 are therefore closely related to those in Figure 3.

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Figure 1: Responses Following a One-Standard-Deviation (Negative) Regime Shift

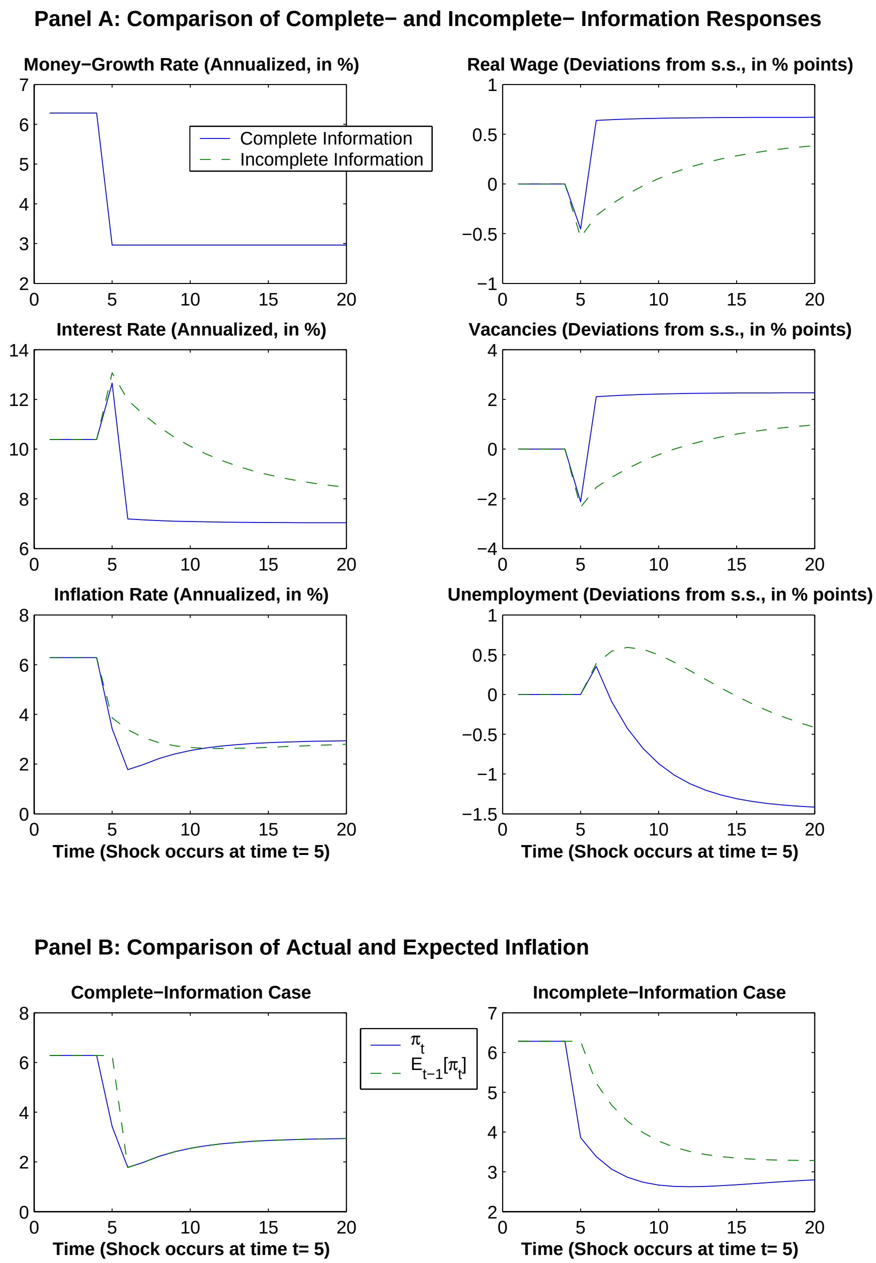
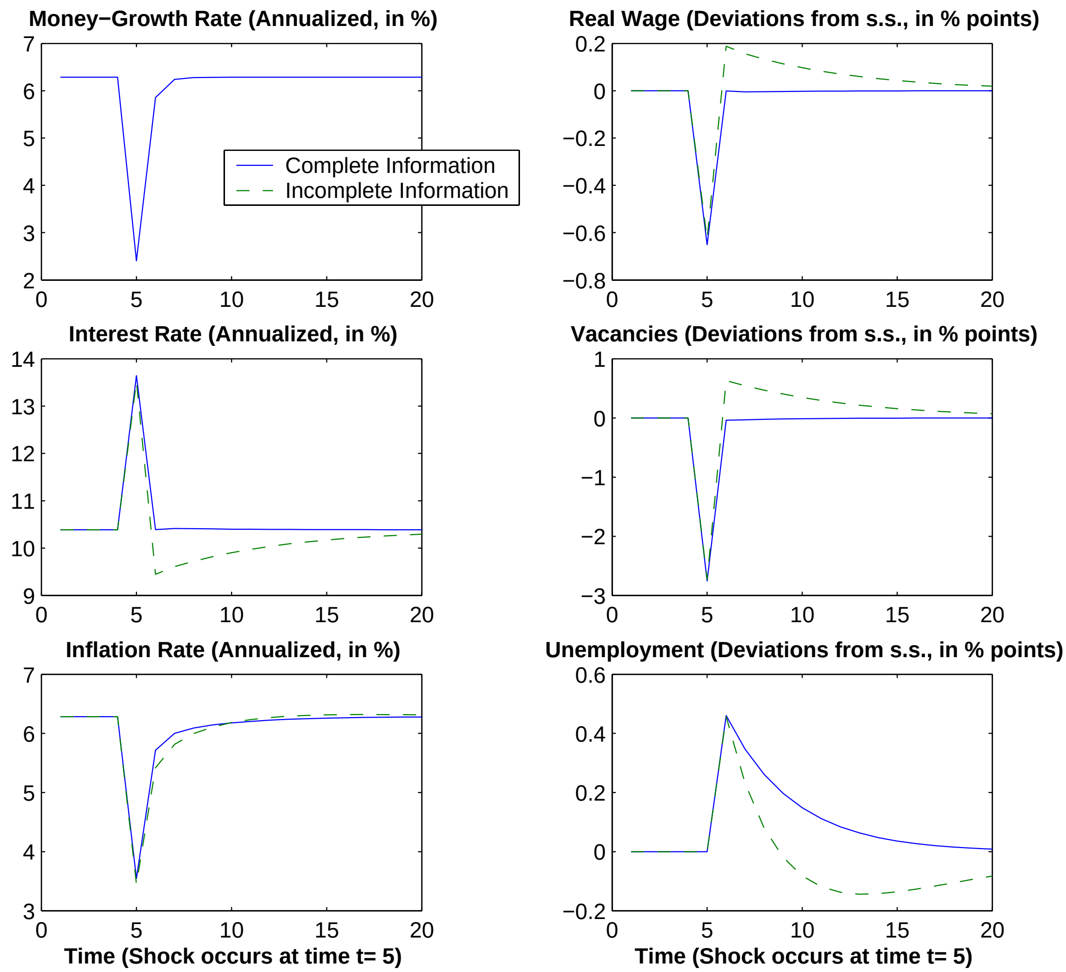


Figure 2: Responses Following a One-Standard-Deviation (Negative) Transitory Shock

Panel A: Comparison of Complete− and Incomplete− Information Responses



Panel B: Comparison of Actual and Expected Inflation

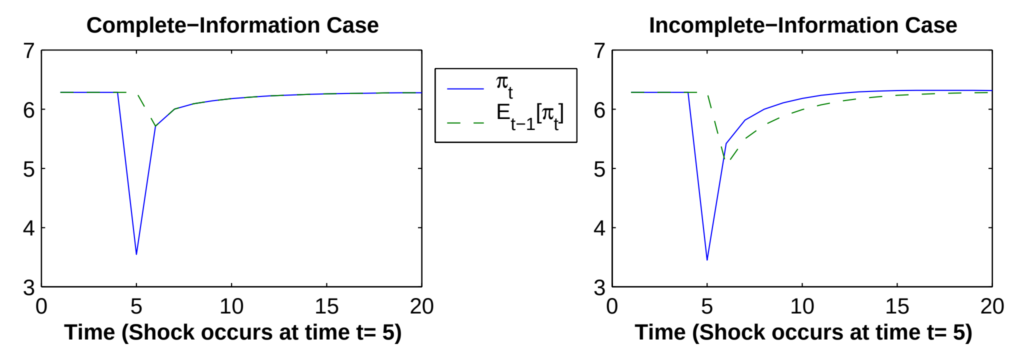


Figure 3: Two-Standard-Deviation Regime Shift: Immediate and Gradual

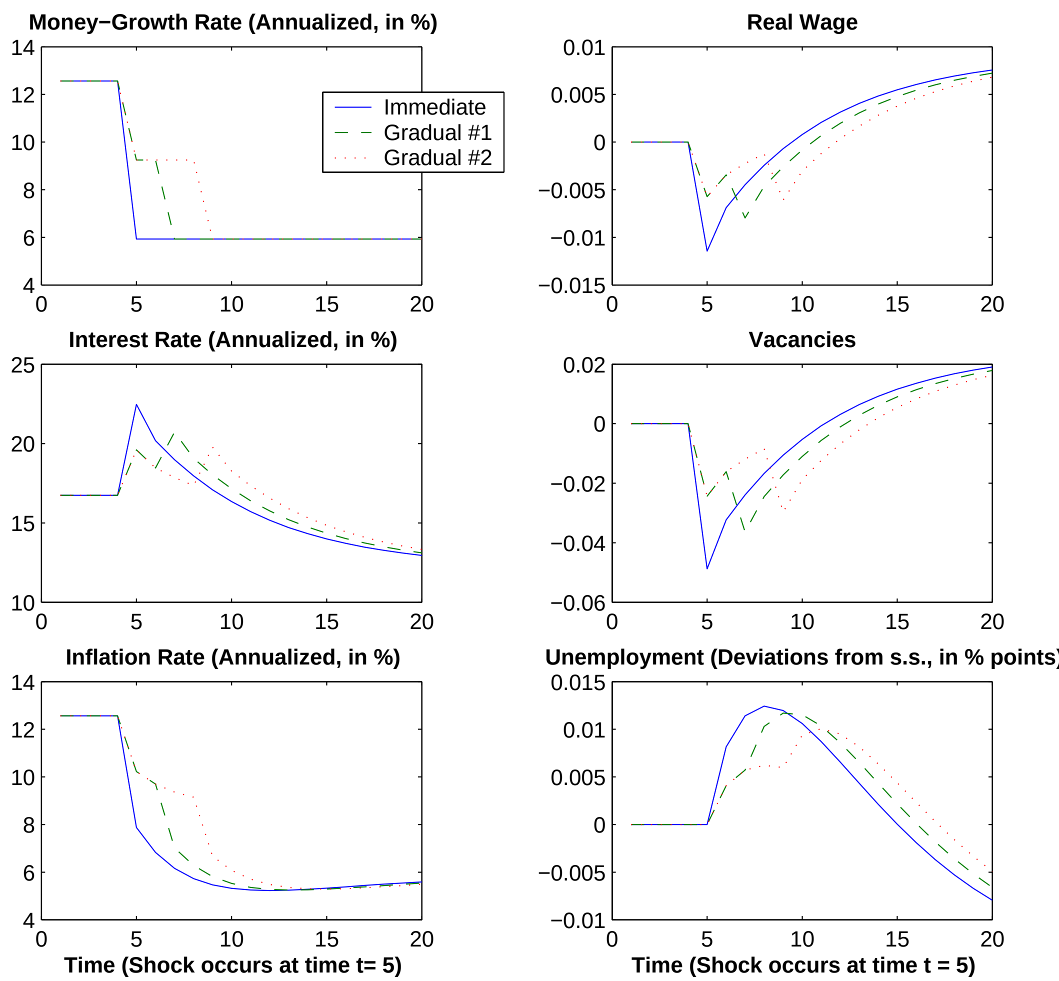


Figure 4: One-Standard-Deviation (Negative) Regime Shift: Calibration using M2

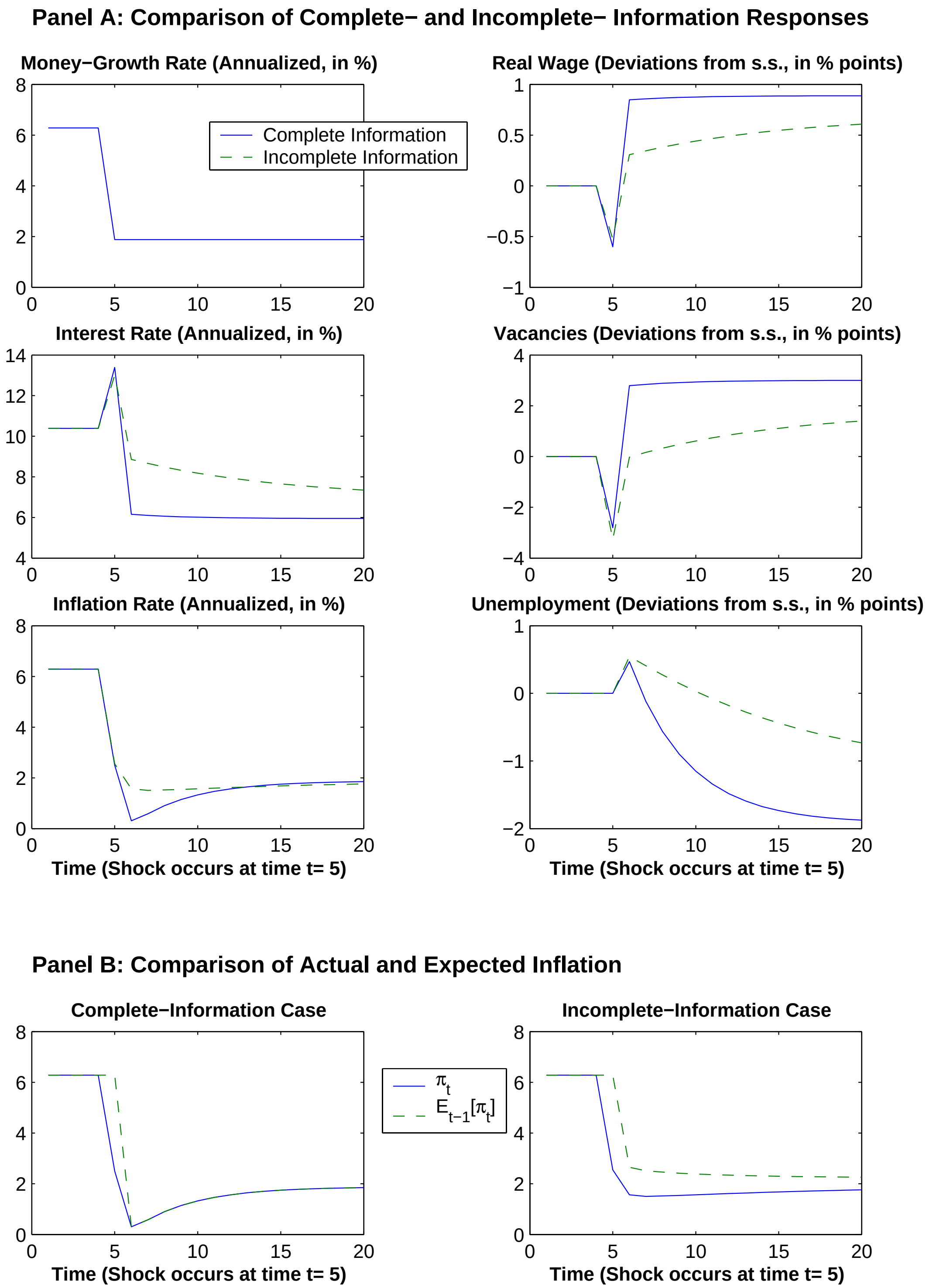
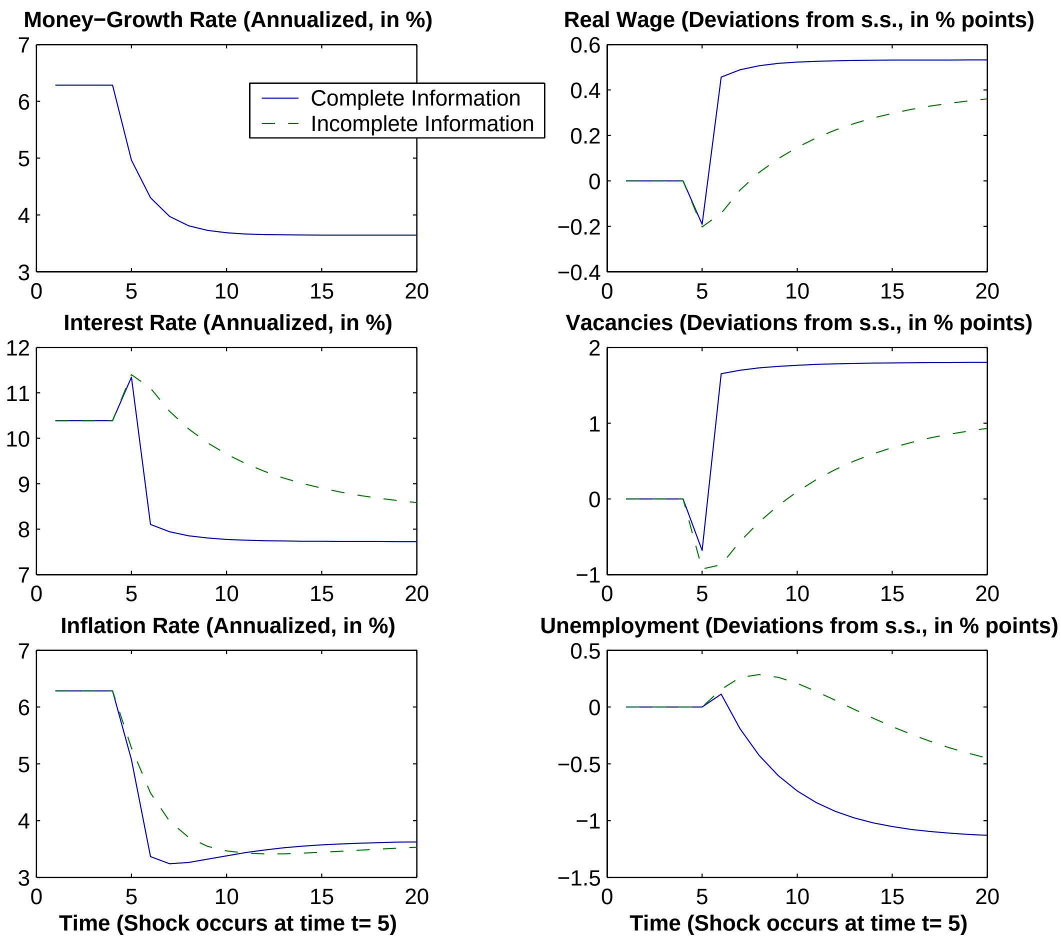


Figure 5: One-Standard-Deviation (Negative) Regime Shift: Alternative Process of Money Growth ($\tau = 0.5$)

Panel A: Comparison of Complete− and Incomplete− Information Responses



Panel B: Comparison of Actual and Expected Inflation

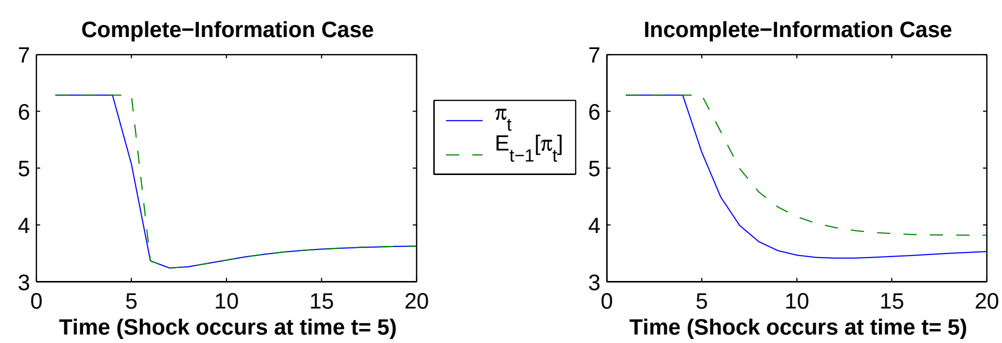


Figure 6: The Beveridge Curve Arising from a (Positive) Regime Shift or Transitory Shock

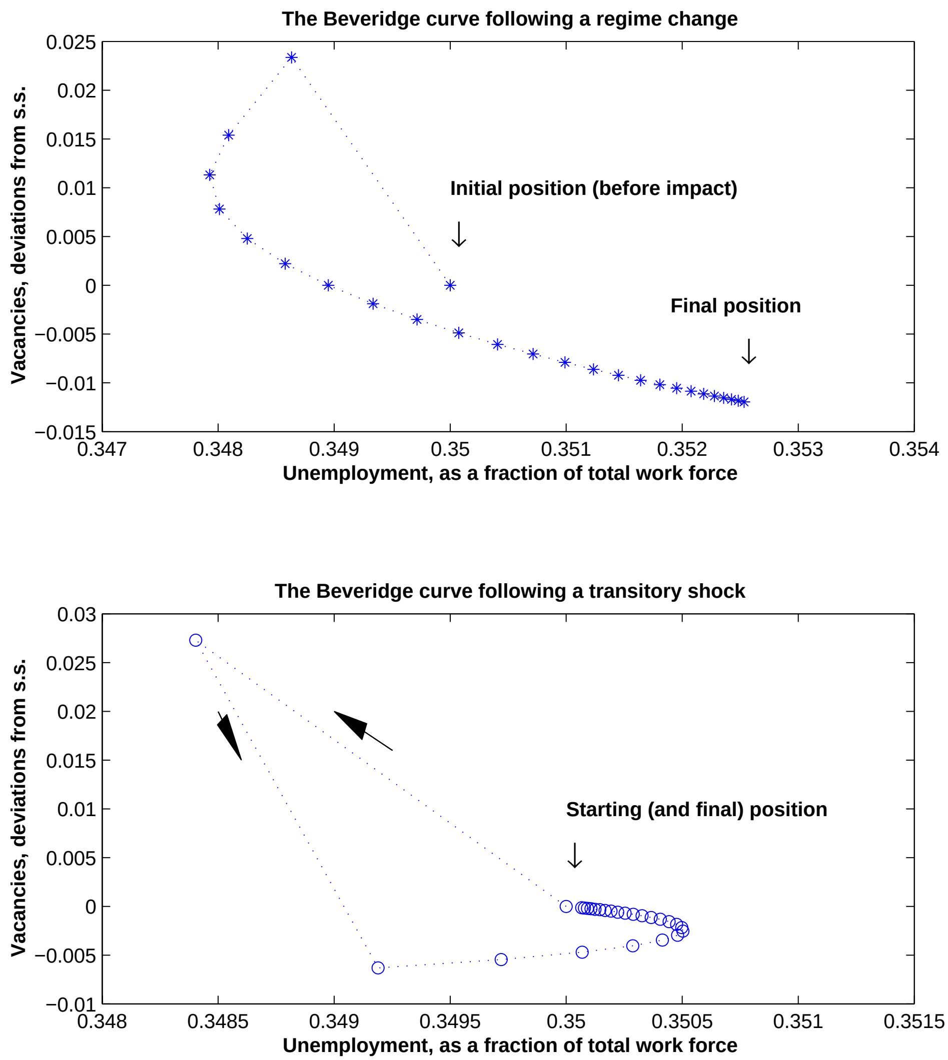


Figure 7: The Phillips Curve Arising from (Positive) Regime Shift or Transitory Shock

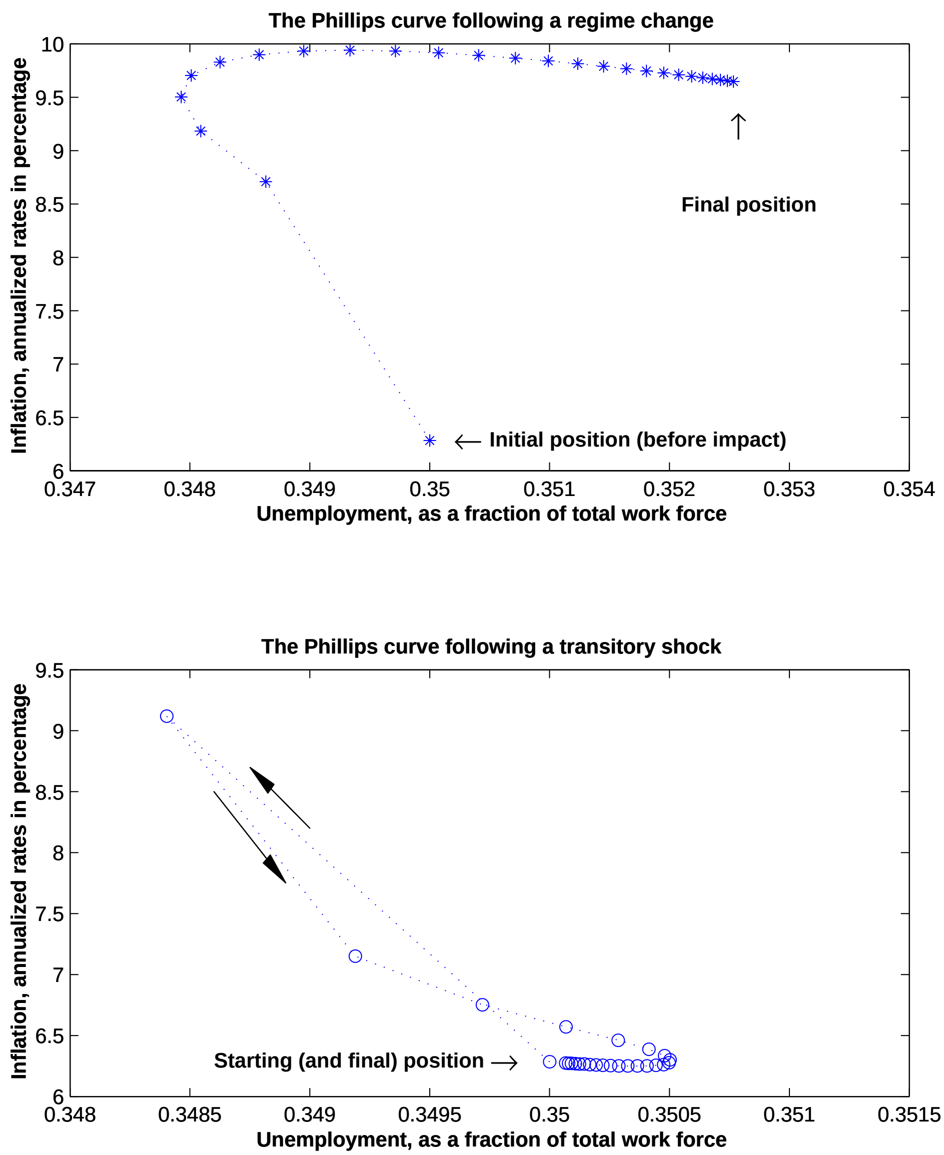
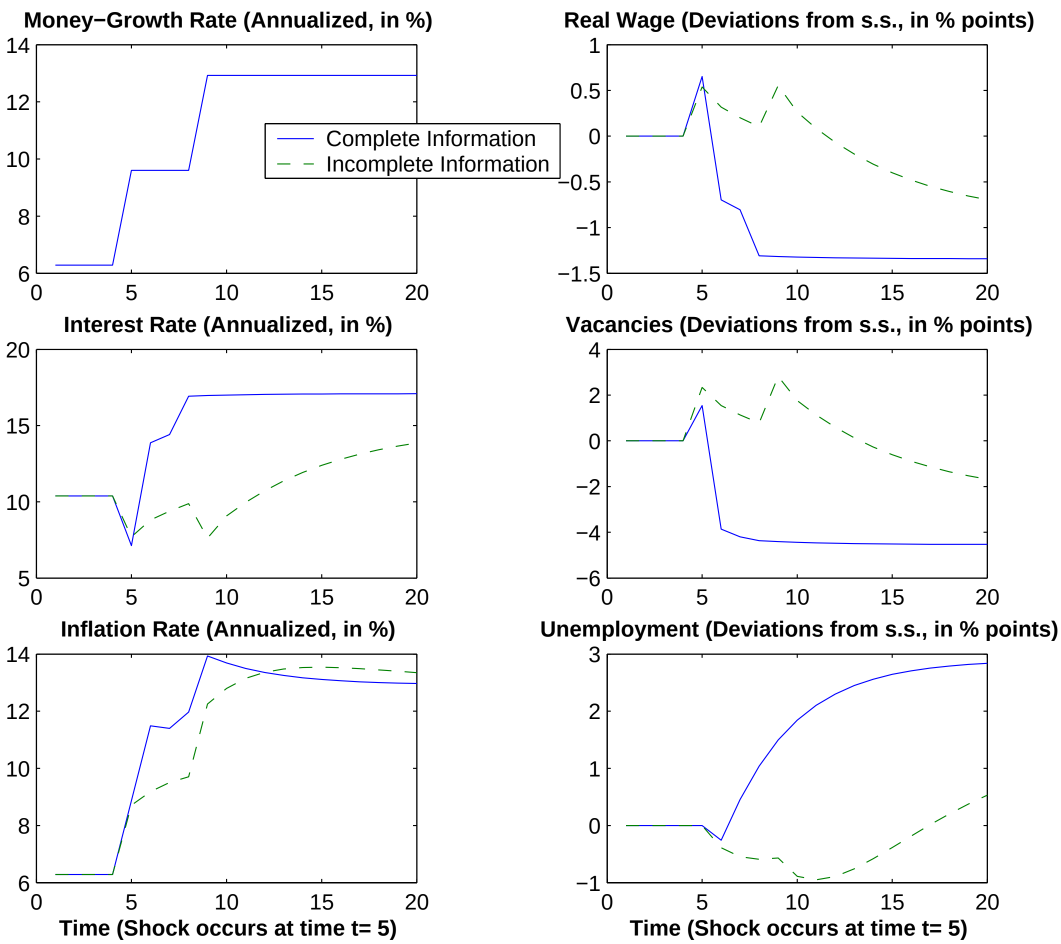
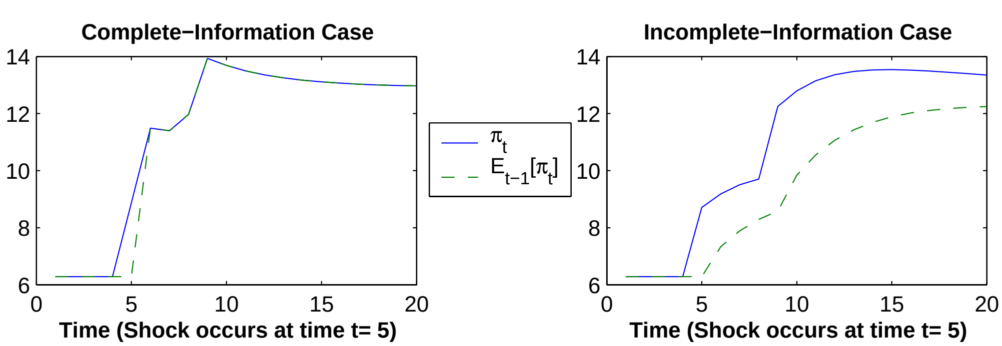


Figure 8: Responses Following a Gradual, Two-Standard-Deviation (Positive) Regime Shift

Panel A: Comparison of Complete− and Incomplete− Information Responses



Panel B: Comparison of Actual and Expected Inflation



Appendix A: Kalman Filter

A.1 The steady-state filter

This appendix provides the derivation for the Kalman filter (18). Let z'_{t+1} and u'_{t+1} denote the projections of z_{t+1} and u_{t+1} on $\Omega_t = \{x_t, x_{t-1}, ..., x_0\}$, respectively. We denote these projections by:

$$\begin{aligned} z'_{t+1} &\equiv P\left[z_{t+1} \mid \Omega_{t-1}, x_t\right]; \\ u'_{t+1} &\equiv P\left[u_{t+1} \mid \Omega_{t-1}, x_t\right]. \end{aligned}$$

These projections can be shown to satisfy the following recursive formulae (Sargent 1987):

$$P\left[z_{t+1} \mid \Omega_{t-1}, x_t\right] = P\left[z_{t+1} \mid \Omega_{t-1}\right] + P\left[z_{t+1} - P[z_{t+1} \mid \Omega_{t-1}] \mid x_t - P[x_t \mid \Omega_{t-1}]\right]; \tag{A.1}$$

$$P\left[u_{t+1} \mid \Omega_{t-1}, x_t\right] = P\left[u_{t+1} \mid \Omega_{t-1}\right] + P\left[u_{t+1} - P[u_{t+1} \mid \Omega_{t-1}] \mid x_t - P[x_t \mid \Omega_{t-1}]\right]. \tag{A.2}$$

Let us work through these expressions, element by element.

Consider the first terms on the right-hand side of (A.1) and (A.2): $P\left[z_{t+1} \mid \Omega_{t-1}\right]$ and $P\left[u_{t+1} \mid \Omega_{t-1}\right]$. Using (15) and (16), we see that:

$$\begin{aligned} P\left[z_{t+1} \mid \Omega_{t-1}\right] &= \theta P\left[z_t \mid \Omega_{t-1}\right] + (1-\theta) P\left[v_{t+1} \mid \Omega_{t-1}\right] \\ &= \theta z'_t; \end{aligned}$$

$$\begin{aligned} P\left[u_{t+1} \mid \Omega_{t-1}\right] &= \rho P\left[u_t \mid \Omega_{t-1}\right] + P\left[e_{t+1} \mid \Omega_{t-1}\right] \\ &= \rho u'_t, \end{aligned}$$

where the last terms drop out because they are orthogonal to Ω_{t-1}. We can now rewrite (A.1) and (A.2) as:

$$\begin{aligned} z'_{t+1} &= \theta z'_t + P\left[z_{t+1} - \theta z'_t \mid x_t - P[x_t \mid \Omega_{t-1}]\right]; \\ u'_{t+1} &= \rho u'_t + P\left[u_{t+1} - \rho u'_t \mid x_t - P[x_t \mid \Omega_{t-1}]\right]. \end{aligned} \tag{A.3}$$

Consider the term $P[x_t \mid \Omega_{t-1}]$, which can be written as:

$$\begin{aligned} P\left[\mu + z_t + u_t \mid \Omega_{t-1}\right] &= P\left[\mu \mid \Omega_{t-1}\right] + P\left[z_t \mid \Omega_{t-1}\right] + P\left[u_t \mid \Omega_{t-1}\right] \\ &= \mu + z'_t + u'_t. \end{aligned}$$

Then, from (14), we have $x_t = \mu + z_t + u_t$, so that

$$x_t - P[x_t \mid \Omega_{t-1}] = (z_t - z'_t) + (u_t - u'_t),$$

which allows us to rewrite (A.3) as:

$$\begin{aligned} z'_{t+1} &= \theta z'_t + P\left[z_{t+1} - \theta z'_t \mid (z_t - z'_t) + (u_t - u'_t)\right]; \qquad \text{(A.4)} \\ u'_{t+1} &= \rho u'_t + P\left[u_{t+1} - \rho u'_t \mid (z_t - z'_t) + (u_t - u'_t)\right]. \end{aligned}$$

Let us now consider the term $P\,[.]$ in the expressions above:

$$\begin{aligned} & P\left[z_{t+1} - \theta z'_t \mid (z_t - z'_t) + (u_t - u'_t)\right] \\ = \; & P\left[z_{t+1} \mid (z_t - z'_t) + (u_t - u'_t)\right] - P[\theta z'_t \mid (z_t - z'_t) + (u_t - u'_t)] \\ = \; & \theta P\left[z_t \mid (z_t - z'_t) + (u_t - u'_t)\right] - \theta P[z'_t \mid (z_t - z'_t) + (u_t - u'_t)] \\ = \; & \theta P\left[(z_t - z'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right]; \end{aligned}$$

$$\begin{aligned} & P\left[u_{t+1} - \rho u'_t \mid (z_t - z'_t) + (u_t - u'_t)\right] \\ = \; & P\left[u_{t+1} \mid (z_t - z'_t) + (u_t - u'_t)\right] - P[\rho u'_t \mid (z_t - z'_t) + (u_t - u'_t)] \\ = \; & \rho P\left[u_t \mid (z_t - z'_t) + (u_t - u'_t)\right] - \rho P[u'_t \mid (z_t - z'_t) + (u_t - u'_t)] \\ = \; & \rho P\left[(u_t - u'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right]. \end{aligned}$$

Therefore, we can again simplify (A.4) to read:

$$\begin{aligned} z'_{t+1} &= \theta z'_t + \theta P\left[(z_t - z'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right]; \qquad \text{(A.5)} \\ u'_{t+1} &= \rho u'_t + \rho P\left[(u_t - u'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right]. \end{aligned}$$

Then, by the definition of a projection,

$$P\left[(z_t - z'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right] = \alpha_t\left[(z_t - z'_t) + (u_t - u'_t)\right],$$

for some value α_t. This value must satisfy:

$$\alpha_t = \arg\min_{\alpha} E\{(z_t - z'_t) - \alpha\left[(z_t - z'_t) + (u_t - u'_t)\right]\}^2.$$

The solution is characterized by the first-order condition:

$$E\left\{(z_t - z'_t) - \alpha_t\left[(z_t - z'_t) + (u_t - u'_t)\right]\right\}\left[(z_t - z'_t) + (u_t - u'_t)\right] = 0,$$

which can be solved for:

$$\alpha_t = \frac{\Sigma_t + \Delta_t}{\Sigma_t + 2\Delta_t + \Gamma_t},$$

where $\Sigma_t \equiv E(z_t - z'_t)^2$, $\Delta_t \equiv E(z_t - z'_t)(u_t - u'_t)$, and $\Gamma_t \equiv E(u_t - u'_t)^2$. Similarly, we know that

$$P\left[(u_t - u'_t) \mid (z_t - z'_t) + (u_t - u'_t)\right] = \beta_t\left[(z_t - z'_t) + (u_t - u'_t)\right],$$

for some value β_t, where:

$$\beta_t = \arg\min_{\beta} E\{(u_t - u_t') - \beta\left[(z_t - z_t') + (u_t - u_t')\right]\}^2,$$

the solution to which is given by:

$$\beta_t = \frac{\Gamma_t + \Delta_t}{\Sigma_t + 2\Delta_t + \Gamma_t} = (1 - \alpha_t).$$

Substituting these results into (A.5), we derive:

$$\begin{aligned} z_{t+1}' &= \theta z_t' + \theta\alpha_t\left[(z_t - z_t') + (u_t - u_t')\right]; \\ u_{t+1}' &= \rho u_t' + \rho(1-\alpha_t)\left[(z_t - z_t') + (u_t - u_t')\right]. \end{aligned} \tag{A.6}$$

Using $x_t = \mu + z_t + u_t$, we can rewrite these expressions as:

$$\begin{aligned} z_{t+1}' &= \theta\left[(1-\alpha_t)z_t' + \alpha_t(x_t - \mu - u_t')\right]; \\ u_{t+1}' &= \rho\left[\alpha_t u_t' + (1-\alpha_t)(x_t - \mu - z_t')\right]. \end{aligned} \tag{A.7}$$

At the beginning of date t, the values for (z_t', u_t') and $(\Sigma_t, \Delta_t, \Gamma_t)$ are predetermined; (z_0', u_0') and $(\Sigma_0, \Delta_0, \Gamma_0)$ are given arbitrarily. As the period unfolds, the current money-growth rate, x_t, is realized, and the individual updates their forecast of z_{t+1} and u_{t+1} according to (A.7). Note that, because the individual is unsure as to whether the current money-growth realization was caused by a persistent or transitory shock (or both), they generally attach some weight to both events.

Knowledge of Σ_t, Δ_t, and Γ_t insures that we can solve for α_t. We concentrate on solving for the stationary Σ, Δ, Γ, and α (but see the end of this appendix). To that end, drop the expectations operator, with the understanding that raw moments such as Ez_t^2 will be displayed as z^2. We need to solve for nine raw moments:

$$\{z^2, zz', (z')^2, u^2, uu', (u')^2, uz', zu', u'z'\},$$

and a tenth moment that we know a priori to be $zu = 0$.

Tedious algebra reveals that the nine restrictions that characterize the stationary moments above are given by:[29]

$$z^2 = \sigma_g^2; \tag{A.8}$$

$$zz' = \theta\left\{(1-\alpha)zz' + \alpha z^2 - \alpha zu'\right\} \tag{A.9}$$

$$\begin{aligned} (z')^2 &= \theta^2\{(1-\alpha)^2(z')^2 + 2(1-\alpha)\alpha[zz' + uz' - u'z'] \\ &\quad + \alpha^2[z^2 - 2zu' + u^2 - 2uu' + (u')^2]\}; \end{aligned} \tag{A.10}$$

[29]The MATLAB code to solve this system is available from the authors upon request.

$$
\begin{aligned}
u'z' &= \rho\theta\{\alpha(1-\alpha)u'z' + (1-\alpha)^2[zz' + uz' - (z')^2] \\
&\quad +\alpha^2[zu' + uu' - (u')^2] \\
&\quad +(1-\alpha)\alpha[z^2 + u^2 - zz' - uz' - zu' - uu' + u'z']\};
\end{aligned} \tag{A.11}
$$

$$
uz' = \rho\theta\{(1-\alpha)uz' + \alpha u^2 - \alpha uu'\}; \tag{A.12}
$$

$$
zu' = \rho\theta\{\alpha zu' + (1-\alpha)z^2 - (1-\alpha)zz'\}; \tag{A.13}
$$

$$
u^2 = (1-\rho^2)^{-1}\sigma_e^2; \tag{A.14}
$$

$$
uu' = \rho^2\{\alpha uu' + (1-\alpha)u^2 - (1-\alpha)uz'\}; \tag{A.15}
$$

$$
\begin{aligned}
(u')^2 &= \rho^2\{\alpha^2(u')^2 + 2\alpha(1-\alpha)[zu' + uu' - u'z'] \\
&\quad +(1-\alpha)^2[z^2 + u^2 - 2zz' - 2uz' + (z')^2]\};
\end{aligned} \tag{A.16}
$$

where $\alpha = (\Sigma + \Delta)/(\Sigma + 2\Delta + \Gamma)$, and

$$
\begin{aligned}
\Sigma &= z^2 - 2zz' + (z')^2; \\
\Delta &= u'z' - uz' - u'z; \\
\Gamma &= u^2 - 2uu' + (u')^2.
\end{aligned}
$$

A.2 The transition to the steady-state filter

Using the steady-state value of α implicitly assumes that agents have had, prior to the simulations depicted in our impulse-response experiments, sufficient time and data for their projections to converge to those steady-state values. It is therefore of interest to check the speed at which this convergence occurs. Were the speed found to be excessively slow, the assumption that agents use the steady-state value of α in their updating would be less tenable.

To check the speed at which convergence occurs, we solve the updating problem in an alternative way. Recall equations (14), (15), and (16), which describe the evolution of observed money-growth rates from their averages. These three equations can be rewritten in the following system:

$$
\begin{bmatrix} z_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} \theta & 0 \\ 0 & \rho \end{bmatrix} \cdot \begin{bmatrix} z_t \\ u_t \end{bmatrix} + \begin{bmatrix} N_{t+1} \\ e_{t+1} \end{bmatrix}; \tag{A.17}
$$

$$
x_t - \mu = \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} z_t \\ u_t \end{bmatrix}; \tag{A.18}
$$

with N_{t+1} defined as follows:

$$
N_{t+1} = \begin{cases} (1-\theta)\, z_t & \text{with probability } \theta; \\ g_{t+1} - \theta z_t & \text{with probability } 1-\theta; \end{cases} \tag{A.19}
$$

and where, again, it is assumed that $e_{t+1} \sim N(0, \sigma_e^2)$ and $g_{t+1} \sim N(0, \sigma_g^2)$.

Compare this system to the one described in Hamilton's (1994 chapter 13) discussion of state-space models and the Kalman filter:

$$\begin{aligned} y_t &= \mathbf{A}' \cdot \mathbf{x}_t + \mathbf{H}' \cdot \xi_t + \mathbf{w}_t; \\ \xi_{t+1} &= \mathbf{F} \cdot \xi_t + \mathbf{v}_{t+1}; \\ E(\mathbf{v}_t \mathbf{v}_t') &= \mathbf{Q}; \\ E(\mathbf{w}_t \mathbf{w}_t') &= \mathbf{R}. \end{aligned}$$

The equivalence between the two systems is established by defining $y_t = x_t - \mu$, $x_t = 0$, $\xi_t = [z_t \; u_t]'$, $\mathbf{w}_t = 0$, $\mathbf{v}_t = [N_t \; e_t]'$, and the following matrices:

$$\mathbf{A} = 0; \mathbf{H} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \mathbf{F} = \begin{bmatrix} \theta & 0 \\ 0 & \rho \end{bmatrix}; \mathbf{Q} = \begin{bmatrix} \sigma_N^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}; \mathbf{R} = 0.$$

Note that one can show that $\sigma_N^2 = (1-\theta)(1+\theta)\sigma_g^2$.

Denote the MSE of the one-step-ahead forecasts of the unobserved states, conditional on time-t information, as $\mathbf{P}_{t+1}|_t$.[30] Conditional on starting values $\hat{\xi}_1|_0$ and $P_1|_0$,[31] the following recursive structure that describes the evolution of $\hat{\xi}_{t+1}|_t$ and $P_{t+1}|_t$ emerges:

$$\mathbf{K}_t = \mathbf{F}\mathbf{P}_t|_{t-1}\mathbf{H}(\mathbf{H}'\mathbf{P}_t|_{t-1}\mathbf{H})^{-1}; \quad (A.20)$$

$$\widehat{\xi}_{t+1}|_t = \mathbf{F}\hat{\xi}_t|_{t-1} + \mathbf{K}_t(y_t - \mathbf{H}'\hat{\xi}_t|_{t-1}); \quad (A.21)$$

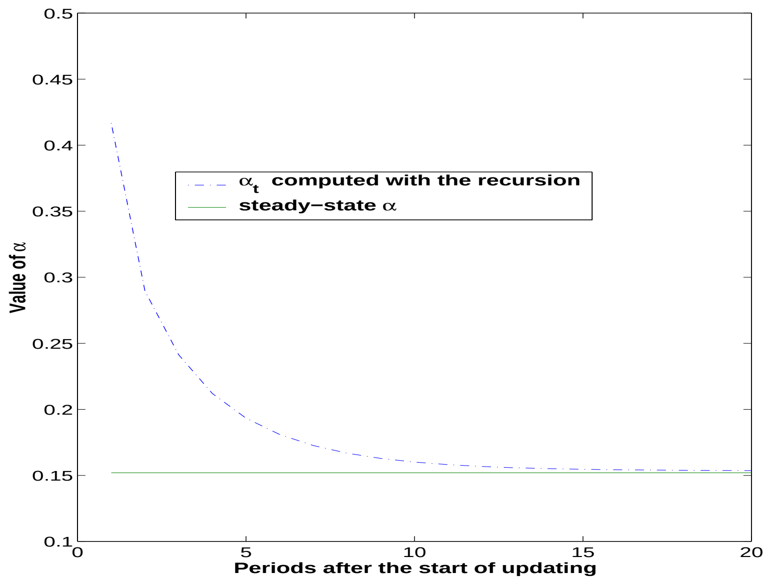
$$\mathbf{P}_{t+1}|_t = (\mathbf{F} - \mathbf{K}_t\mathbf{H}')\mathbf{P}_t|_{t-1}(\mathbf{F}' - \mathbf{H}\mathbf{K}_t') + \mathbf{Q}. \quad (A.22)$$

The $(1,1)$ element of the matrix $\mathbf{K}_t$ is the weight that the updating puts on observed deviations of money-growth rates towards the first element of $\widehat{\xi}_{t+1}|_t$, and is thus comparable to the parameter α_t. Figure A.1. shows that, starting from an unconditional mean just above 0.4, the value of α_t converges rapidly to its steady-state value of 0.152, which was used in the impulse-response experiments documented. The use of the steady-state value was thus fairly innocuous.

[30] So that $\mathbf{P}_{t+1}|_t = E_t\left[(\xi_{t+1} - \widehat{\xi}_{t+1}|_t)(\xi_{t+1} - \widehat{\xi}_{t+1}|_t)'.\right]$

[31] We use the unconditional expectations.

Figure A.1: Convergence of the Updating Parameter α



Appendix B: Calibrating the Money-Growth Process

By construction, z_t and u_t are orthogonal, so that

$$cov(x_t, x_{t-1}) = Ez_t z_{t-1} + Eu_t u_{t-1}, \tag{B.1}$$

where we have used $Ez_t = Eu_t = 0$. Clearly, we can rewrite the right-hand side of the above expression as:

$$\begin{aligned} cov(x_t, x_{t-1}) &= \frac{Ez_t z_{t-1}}{std(z_t)std(z_{t-1})} std(z_t)std(z_{t-1}) + \frac{Eu_t u_{t-1}}{std(u_t)std(u_{t-1})} std(u_t)std(u_{t-1}) \\ &= cor(z_t, z_{t-1})std(z_t)std(z_{t-1}) + cor(u_t, u_{t-1})std(u_t)std(u_{t-1}) \\ &= cor(z_t, z_{t-1})var(z) + cor(u_t, u_{t-1})var(u). \end{aligned}$$

Dividing both sides of the above expression by $var(x)$, we derive

$$cor(x_t, x_{t-1}) = cor(z_t, z_{t-1})\frac{var(z)}{var(x)} + cor(u_t, u_{t-1})\frac{var(u)}{var(x)}. \tag{B.2}$$

Define $weight = var(z)/var(x)$. Using $var(x) = var(z) + var(u)$, $cor(z_t, z_{t-1}) = \theta$, $cor(u_t, u_{t-1}) = \rho$, we can rewrite the above expression as:

$$cor(x_t, x_{t-1}) = \theta\, weight + \rho\,(1 - weight). \tag{B.3}$$

Recall that the parameter θ governs the expected duration of a particular regime, ρ expresses the persistence of the transitory monetary policy shocks around a given regime, and *weight* corresponds to the relative importance of regime shocks in the overall variation of money growth. We need to assign values to these three parameters, but so far we have only one equation that links them to observed data (B.3). Because we have stronger a priori views about the duration of a given regime than about the weight of regime shocks in the overall variance of money growth, we decided to fix the value of θ but to use an additional moment restriction to identify the value of the *weight*.

Using the same logic that led from (B.1) to (B.2), we can write, in the case of the second-order autocorrelation of x_t:

$$cor(x_t, x_{t-2}) = \theta^2\, weight + \rho^2\,(1 - weight). \tag{B.4}$$

By assuming a value for θ and using (B.3) and (B.4) it is possible to extract values of ρ and *weight*. The values are as follows:

$$weight = \frac{cor(x_t, x_{t-2}) - cor(x_t, x_{t-1})^2}{cor(x_t, x_{t-2}) + \theta^2 - 2\theta cor(x_t, x_{t-1})},$$

and

$$\rho = \frac{cor(x_t, x_{t-1}) - \theta\, weight}{(1 - weight)}.$$

As stated in the text, data on the Canadian monetary base reveals that $cor(x_t, x_{t-1}) = 0.53$ and $cor(x_t, x_{t-2}) = 0.47$. The assumption that there is a relatively high value for θ (0.95) implies values of *weight* and ρ equal to 0.52 and 0.08. A value for *weight* equal to 0.5 would have been the natural first choice if we had not imposed the second autocorrelation restriction.

We can identify values for the variance in the innovations to the two monetary policy components. First, note that the unconditional variance of u_t is given by:

$$var(u) = \frac{\sigma_e^2}{1-\rho^2}.$$

Because $var(x) = var(z) + var(u)$, we also know that:

$$var(u) = (1 - weight) \cdot var(x).$$

From the data, we have $var(x) = 0.013^2$. Combining this information allows us to compute:

$$\sigma_e = [(1 - weight) \cdot 0.013^2 \cdot (1-\rho^2)]^{1/2}.$$

Furthermore, the unconditional variance of z_t is simply σ_g^2, so that

$$\sigma_g = [weight \cdot 0.013^2]^{1/2}.$$

Finally, we set $\mu = 0.0118$ to replicate the average quarterly money-growth rate in our sample.

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2002

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