Factor-Market Structure, Shifting Inflation Targets, and the New Keynesian Phillips Curve

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Introduction

Understanding the economic forces that drive inflation dynamics is important for a monetary authority, especially for those practising inflation targeting. The recent economics literature has used versions of the so-called New Keynesian Phillips curve (NKPC) in an effort to improve our understanding of inflation dynamics. The standard NKPC links current inflation to real marginal cost and the expectation of future inflation. The main advantage of this approach over more traditional reduced-form approaches is that it has a theoretical foundation and, therefore, a clear structural interpretation. While the ability of the standard NKPC to deliver a structural interpretation of inflation dynamics is important, its empirical support has been weak (for an example based on Canadian data, see Guay, Luger, and Zhu 2003). Countless variations of the NKPC have been proposed in an effort to improve its congruence with the data. The most notable variations are, perhaps, the ones proposed by Galí and Gertler (1999) and Christiano, Eichenbaum, Evans (2005), which effectively add lagged inflation to create a "hybrid NKPC." Galí and Gertler motivate the lagged inflation term by the presence of firms that use rule-of-thumb pricing strategies, whereas Christiano, Eichenbaum, and Evans appeal to dynamic price indexation.

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Notwithstanding the way lagged inflation is introduced into these models, the main aim of the lagged dependent variable is to address previously noted empirical shortcomings of the standard NKPC. The empirical evidence based on Canadian data, however, has been mixed. Gagnon and Khan (2005), for instance, find evidence in favour of the hybrid NKPC, whereas Nason and Smith (2004) statistically reject the model.

In this paper, we take another look at the ability of the NKPC with partial dynamic price indexation to capture key features of Canadian inflation data. Our study, however, differs from previous research along three important dimensions. In particular, we relax three assumptions often made in the NKPC literature: (i) constant inflation target; (ii) labour's share of income as a measure of marginal cost; and (iii) rental market for capital. We believe the empirical performance of the NKPC is an especially important question, since the behaviour of inflation dynamics has important implications for monetary policy, and in particular for how central banks should react to real events while maintaining their inflation target. For instance, the degree to which inflation is a predetermined variable is critical to the question of how forward looking monetary policy should be (see Batini and Nelson 2001).

Why do we choose these three particular assumptions and not others? First, there is strong empirical evidence against these restrictions and, second, we are able to readily address them in a reasonable manner.

An important maintained assumption of previous empirical NKPC studies is that the monetary policy regime has been constant over the sample period of estimation. We relax the assumption of a constant historical inflation target, since empirical evidence suggests that it is an unrealistic assumption, at least for the Bank of Canada.

Next, we replace the usual proxy for marginal cost, labour's share of income, with a definition that allows for non-Cobb-Douglas production, adjustment costs to labour, and an explicit role for imported intermediate goods. Empirical evidence reported in Amano and Wirjanto (1997) for Canada indicates that capital and labour are less substitutable than the Cobb-Douglas production function admits, suggesting that a more general production function may be a more appropriate description of short-run production. With respect to labour adjustment costs, there is an extensive literature documenting the presence of statistically significant costs of labour adjustment. As well, McCallum and Nelson (2000) find an important role for imported intermediate goods for the ability of a small open economy model to replicate data-based impulse-response functions.

We also abstract from the standard assumption of a rental market for capital and assume the presence of firm-specific capital. In our work, we treat the capital stock of each firm as invariant to their relative price. Sbordone (2002) and Galí, Gertler, and López-Salido (2001) also treat firm-specific capital in this manner.

Finally, our version of the NKPC is estimated using a simulated method of moments (SMM) estimator proposed by Smith (1993). This procedure compares the properties of the reduced-form vector autoregression (VAR) representation of the structural model to an unconstrained VAR. One notable advantage of this estimation method is that it allows us to avoid identification problems that often plague instrumental variables (that is, generalized method of moments—GMM) estimation of NKPC models (see, for example, Nason and Smith 2005).

After presenting the structure of the price-setting model in section 1, we describe and use two approaches for estimating time-varying inflation targets in section 2. In section 3, we review the estimation methodology, and in section 4, we report the empirical results. In particular, we examine the implications of the parameter estimates for average price contract duration and the sensitivity of inflation to movements in marginal cost. In the final section, we offer concluding remarks and suggestions for future work.

1 A Small Open Economy Model

In this section, we formulate a price-setting framework incorporating a non-constant inflation target, firm-specific capital, and a constant elasticity of substitution (CES) production technology that includes imported intermediate goods and labour adjustment costs. Within this framework, prices are determined according to Calvo (1983) with partial dynamic price indexation.

1.1 Final goods production

Final goods in our economy, Z_t, are produced by a representative, perfectly competitive firm combining a continuum of intermediate finished goods, Z_{it} $i \in [0, 1]$, using the technology

$$Z_t = \left[\int_0^1 Z_{it}^{\frac{c-1}{c}} di\right]^{\frac{c}{c-1}}, \tag{1}$$

and charging the price, P_t, according to

$$P_t = \left[\int_0^1 P_{it}^{1-c} di\right]^{\frac{1}{1-c}}. \tag{2}$$

Final goods can be thought of as being consumed or invested in our model economy. Thus, the consumption and investment deflators are the same and are equal to P_t. However, given that the focus of this paper is on the consumer price index (CPI), P_t will correspond to the CPI excluding the eight most volatile items[1] (hereafter the CPIX) for all empirical work. Profit maximization implies the following demand function by the aggregator for the i^{th} firm's output

$$Z_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon_t} Z_t. \tag{3}$$

1.2 Production technology and marginal cost

We assume a continuum of monopolistically competitive firms, indexed by i, $i \in [0, 1]$, that each produce a differentiated final good using a constant elasticity of substitution (CES) production technology in labour, L_{it}, capital, K_{it}, and imported inputs, M_{it}:

$$Z_{it} = \left(\delta_1^{\frac{1}{\sigma}}(A_t L_{it})^{\frac{\sigma-1}{\sigma}} + \delta_2^{\frac{1}{\sigma}}(K_{it})^{\frac{\sigma-1}{\sigma}} + (1-\delta_1-\delta_2)^{\frac{1}{\sigma}}(M_{it})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} - \Omega_{i,t}, \tag{4}$$

and charge a price, P_{it}, for their good that maximizes present and expected future discounted profits. A_t is labour-augmenting technology. Imports are included as a factor of production, since approximately 20 per cent of Canadian consumption goods are imported. The parameters δ_1 and δ_2 are increasing functions of the shares of labour and capital in production, and the elasticity of substitution among the three factors is assumed equal and constant at $\sigma, \sigma \neq 1$. Empirical implementations of the NKPC have often used labour's share of output as a proxy for marginal cost (see, for example, Gagnon and Khan 2005 for Canadian data) when modelling the GDP deflator. While labour share is a convenient proxy for marginal cost, there is

1. The eight items are: fruit, vegetables, gasoline, fuel oil, natural gas, mortgage interest, intercity transportation, and tobacco products.

good reason to believe that it understates fluctuations in true marginal cost. Studies using Canadian data (see, for example, Amano and Wirjanto 1997) place this elasticity between 0.3 to 0.6, suggesting a greater degree of complementarity between capital and labour than admitted by the Cobb-Douglas production function. Thus, in an effort to capture such complementarity, we use a CES production function to describe short-run Canadian production.

When purchasing labour and imports, firms are assumed to take nominal wage, W_t, and price of imports, P_{mt}, as given. However, we assume that varying the level of employment is costly to the firm, and the costs, governed by the parameter χ, take the form of lost labour productivity. Specifically, we assume a quadratic penalty function in the growth rate of employment,

$$\Omega_{i,t} = \frac{Z_t \chi}{2}\left(\frac{L_{it}}{L_{i,t-1}} - 1\right)^2, \tag{5}$$

so in steady state, $\Omega = 0$. Profit maximization on the part of the firm, subject to equations (3, 4, and 5), implies the following first-order conditions for imported inputs and labour:

$$0 = \left(\frac{(1-\delta_1-\delta_2)Z_{i,t}}{M_{i,t}}\right)^{\frac{1}{\sigma}} - \frac{P_{mt}}{\lambda_{i,t}}, \tag{6}$$

$$0 = \left\{ \begin{array}{l} \lambda_{i,t}\dfrac{(\delta_1 Z_{i,t})^{\frac{1}{\sigma}}}{L_{i,t}} A_t^{\frac{\sigma-1}{\sigma}} - \dfrac{Z_{i,t}\lambda_{i,t}\chi}{L_{i,t-1}}\left(\dfrac{L_{i,t}}{L_{i,t-1}} - 1\right) \\ + \mathbf{E}_t \dfrac{\lambda_{i,t+1} Z_{i,t+1}\chi}{(1+R_{i,t})L_{i,t}^2} L_{i,t+1}\left(\dfrac{L_{i,t+1}}{L_{i,t}} - 1\right) - W_t \end{array} \right\}. \tag{7}$$

Expressed in terms of *real* marginal cost and labour's share of output, $s_{i,t}$, we have

$$\frac{\lambda_{i,t}}{P_{i,t}} = \left\{ \frac{s_{i,t}}{\delta_1^{\frac{1}{\sigma}}\left(\dfrac{A_t L_{it}}{Z_{i,t}}\right)^{\frac{\sigma-1}{\sigma}} - \chi\left(\dfrac{L_{i,t}}{L_{i,t-1}}\left(\dfrac{L_{i,t}}{L_{i,t-1}} - 1\right) + \kappa_{t+1}\left(\dfrac{L_{i,t+1}}{L_{i,t}} - 1\right)\right)} \right\},$$

where the denominator is the elasticity of current and future production with respect to time-*t* labour and

$$\kappa_{t+1} \equiv \frac{\lambda_{i,t+1} L_{i,t+1} Z_{i,t+1}}{\lambda_{i,t} L_{i,t} Z_{i,t}(1+R_t)}.$$

Linearizing around a steady state characterized by $L_{i,t} = \bar{L}$ and $\bar{\kappa} = 1$ (the real interest rate equals the real growth rate of the economy), we obtain

$$\hat{\lambda}_{i,t} = \Theta\left(\hat{s}_t - \frac{\chi}{s_L}(\Delta^2 \mathbf{E}_t \hat{L}_{i,t+1})\right) + \Delta\hat{z}_{k,i,t} + (1-\Theta)\hat{p}_{m,t}, \tag{8}$$

where $\hat{z}_{k,t} = \hat{Z}_{i,t} - \hat{K}_t$, and $\hat{\lambda}_{i,t}$ captures the percentage deviation of *real* marginal cost from its steady state and

$$\Theta = \frac{s_L}{s_L + (1-\sigma)(1-s_L-s_K)},$$

$$\Lambda = \frac{1-\sigma}{\sigma}\frac{s_K}{s_L + (1-\sigma)(1-s_L-s_K)},$$

where s_L and s_κ are labour and capital share parameters, respectively. Now consider a simple autoregressive model for computing $\mathbf{E}_t \Delta\hat{L}_{i,t+1}$:

$$\Delta\hat{L}_{t+1} = \rho_1 \Delta\hat{L}_t + \rho_2 \Delta\hat{L}_{t-1} + u_t,$$

which we estimate by ordinary least squares using Canadian employment data from 1980 to 2004.[2] Equation (8) can now be rewritten in terms of variables observed at time *t*:

$$\hat{\lambda}_{i,t} = \Theta\left(\hat{s}_{i,t} - \frac{(\rho_1-1)\chi}{s_L}\left(\Delta\hat{L}_{i,t} + \frac{\rho_2}{\rho_1-1}\Delta\hat{L}_{i,t-1}\right)\right)$$
$$+ \Lambda\hat{z}_{k,i,t} + (1-\Theta)\hat{p}_{m,t}. \tag{9}$$

Note that if $\sigma = 1$ and $\chi = 0$, we obtain $\hat{\lambda}_{i,t} = \hat{s}_{i,t}$ and with no imports in production, $\hat{s}_{i,t}$ corresponds to labour's share of nominal GDP, the typical proxy for real marginal cost.

2. The choice of two lags is based on Akaike's information criterion.

1.3 New Keynesian Phillips curve equation

In this section, we describe a log-linearized inflation equation based on Calvo (1983) price setting, augmented with firm-specific capital and partial dynamic price indexation. The latter assumption implies that a firm that cannot reoptimize its price follows the rule

$$P_{it} = P_{i,t-1}(1+\pi_{t-1})^{\gamma}. \tag{10}$$

A number of researchers have argued that this kind of modification to the standard NKPC results in a more realistic specification. Christiano, Eichenbaum, and Evans (2005) and Giannoni and Woodford (2005) argue that a model with $\gamma = 1$ improves its ability to reproduce key moments in the data. Smets and Wouters (2003) treat γ as a free parameter and, in contrast, conclude that the best-fitting value of γ is around 0.6. Smets and Wouters show that the Calvo model with partial dynamic indexation, indexed by γ with $\gamma \in (0, 1)$, may be written as

$$\hat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}\hat{\pi}_{t+1} + \phi\hat{\lambda}_t, \tag{11}$$

where $\hat{\pi} = \pi_t - \pi_t^T, \pi_t = \ln(P_t/P_{t-1})$, and P_t is steady-state inflation or the Bank of Canada's inflation target. Equation (11) says that the deviation of inflation from its target (i.e., the inflation gap, $\hat{\pi}_t$) depends on past and expected future inflation deviations and on current real marginal cost. When γ, the parameter governing the magnitude of indexation, is zero, the equation reverts to its standard form. Conversely, when γ is positive, the degree of indexation to lagged inflation provides a measure of the degree of persistence in Canadian inflation dynamics after accounting for shifts in the inflation target and the persistence in real marginal cost. In the empirical section, we conduct formal tests to determine whether γ is statistically different from zero.

An important feature to note is that the only difference between the log-linearized inflation equation in the homogeneous versus firm-specific capital case pertains to the structural relationship between inflation and marginal cost. While the form of the equation in both cases is identical, the difference lies in the mapping between the reduced-form parameter governing the effect of marginal cost on inflation and the structural parameters. Under the capital rental market assumption, the elasticity of inflation with respect to changes in marginal cost depends primarily on γ and the fraction of firms that reoptimize prices within a period, $1-\theta$, or more specifically,

$$\phi = \frac{(1-\theta)(1-\beta\theta)}{(1+\beta\gamma)\theta}.$$

Under the assumption of firm-specific capital, ϕ is a function of a broader set of structural parameters:

$$\phi = \eta \cdot \frac{(1-\theta)(1-\beta\theta)}{(1+\beta\gamma)\theta}; \quad \eta < 1, \tag{12}$$

where η captures the difference between average and firm-specific marginal cost. Furthermore, if we assume that firms are unable (exogenous capital) or unwilling (owing to adjustment costs on investment that approach infinity) to change their capital stock in response to changes in economic conditions, then

$$\eta = \left\{\frac{\sigma\mu s_L}{\sigma\mu s_L + \varepsilon(1-\mu s_L)}\right\}, \tag{13}$$

where μ is steady-state markup of price over marginal cost, and ε represents the demand elasticity by the aggregator. Effectively, η steepens the marginal cost curve at the firm level, dampening the effect of marginal cost movements on inflation. We will discuss this effect and its implications in greater detail.

2 Estimating the Inflation Target

Many observers have noted changes in Canadian monetary policy over the post-Bretton Woods period. Nelson (2005) provides a lucid, quantitative, and graphical overview of Canadian monetary policy in the 1970s and 1980s, based on newspaper articles and statements of policy-makers. Nelson finds, *inter alia*, evidence suggesting changes in monetary policy over time. Perhaps the most convincing piece of evidence is a 2000 lecture given by then Bank of Canada Governor Gordon Thiessen that described the evolution of Canadian monetary policy. In the lecture, Governor Thiessen identifies three monetary policy regimes since 1971: (i) stagflation and monetarism, 1971–81; (ii) the search for a new nominal anchor, 1982–90; and (iii) inflation targets, 1991–present. Taken together, there is much evidence suggesting that the Bank of Canada's implicit inflation target has shifted over time.

The evidence suggests that the assumption of a constant inflation target is untenable, and so it is replaced with a target that varies over time. It should be noted that relaxing the assumption of a constant inflation target has important implications for inflation dynamics. Researchers have found recently that the lagged inflation term needed in standard NKPC models to help explain key features of aggregate inflation data may reflect shifts in the monetary policy regime rather than “structural” backward-looking behaviour. Indeed, Coenen and Levin (2004), Cogley and Sbordone (2005), and Kozicki and Tinsley (2003) argue that it is essential to account for shifts in monetary policy to avoid finding spurious evidence of inflation persistence. Coenen and Levin find that a standard NKPC is able to account for the persistence of German inflation once shifts in monetary policy are taken into account. As well, Kozicki and Tinsley (2003) report, *inter alia*, empirical evidence suggesting that shifts in monetary policy regime and less than full credible policy have contributed importantly to observed persistence of US and Canadian inflation.

While assuming that a non-constant inflation target adds a degree of potential realism to our model, we must construct a measure of an implied inflation target. We use two approaches to construct such a variable and describe each method.

2.1 Moving endpoints method

The first method is the VAR with a moving endpoints (MEP) approach developed in Kozicki and Tinsley (1998). The method entails estimating a VAR with variables in deviations from steady-state form, so that any non-stationarity arising in the VAR is attributed to shifts in the steady state. Following Kozicki and Tinsley (2003), we assume that only the steady state of inflation displays non-stationary behaviour. The reduced-form model assumes that the dynamics of the variables under consideration are well described by a j-lag VAR. In each quarter, variation in the inflation target is assumed to be an independent normal innovation. The reduced form is given by

$$\begin{bmatrix}\pi_t\\ \lambda_t\\ R_t\end{bmatrix} = \sum_{i=1}^{j}\mathbf{A}_i\begin{bmatrix}\pi_{t-i}\\ \lambda_{t-i}\\ R_{t-i}\end{bmatrix} + \left(\mathbf{I} - \sum_{i=1}^{j}\mathbf{A}_i\right)\begin{bmatrix}\bar{\pi}_t\\ \bar{\lambda}\\ \bar{r}+\bar{\pi}_t\end{bmatrix} + \mathbf{u}_t. \tag{14}$$

$\bar{r}$ is interpreted as the steady-state real interest rate and the inflation target follows a random-walk process:

$$\bar{\pi}_t = \bar{\pi}_{t-1} + v_t, \tag{15}$$

with $\mathbf{E}(v_t u_{it}) = 0 \quad i = 1, 2, 3$. The VAR and inflation-target innovations are assumed to be serially uncorrelated and uncorrelated with each other. Owing to the unobserved state variable π_t^T, we use Kalman filtering methods to estimate the model. Figure 1 shows actual inflation and the implied inflation target from the VAR with the MEP approach. The estimated inflation target is the unsmoothed estimate of the state variable from the Kalman filter. The estimated inflation target follows the path of actual inflation reasonably well. Interestingly, the implied inflation target is higher than actual inflation during the disinflation of 1981–82, suggesting that the Bank of Canada did not have full credibility in its efforts to reduce inflation. The estimated inflation target also appears to capture the announced downward inflation-target path from 3 per cent to 2 per cent (1992 to 1995), as well as the current 1 to 3 per cent inflation-targeting range.

Figure 2 shows a plot of real marginal cost and the MEP-based inflation gap. We see here that both series broadly move together, particularly over the early and late 1980s and early 1990s. The full-sample correlation is 0.45.

2.2 Staff economic projection method

Our second method follows the approach developed in Amano and Murchison (2005), which exploits access to Bank of Canada staff inflation projection data.[3] We begin by positing that the Bank of Canada has set policy in a manner generally consistent with a simple rule of the form

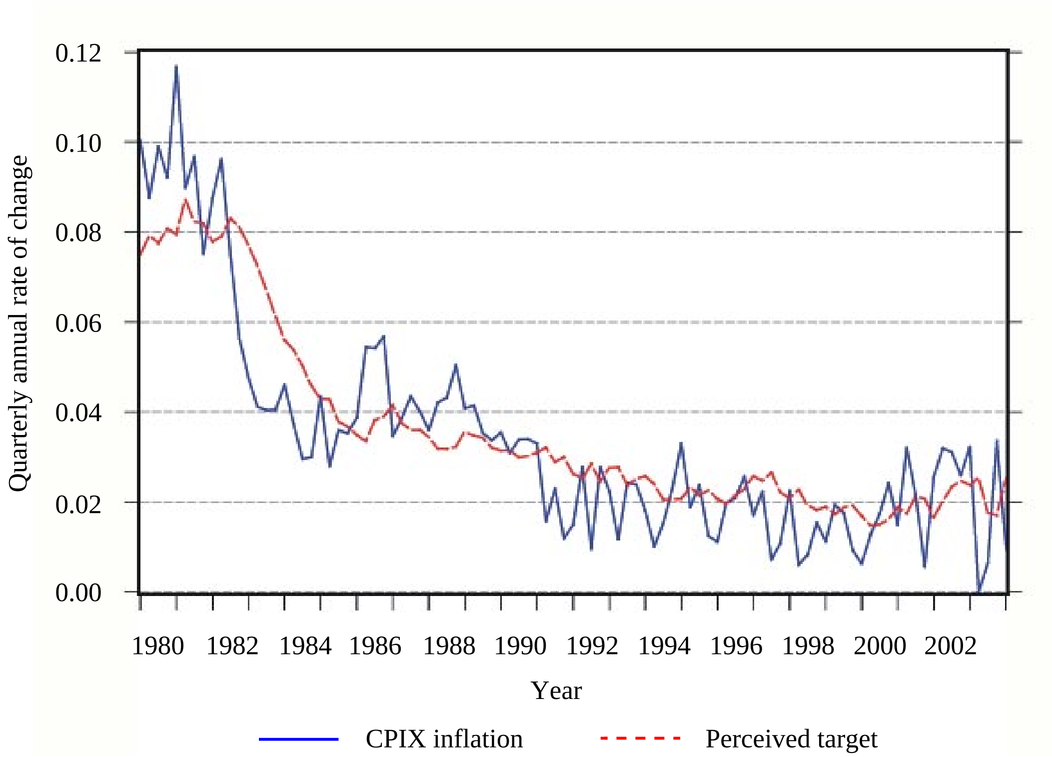
$$\begin{aligned} R_t = {} & \zeta_t R_{t-1} + \mathbf{E}_{t-1}(1-\zeta_t)(\bar{r} + \pi_t \\ & + w_{1,t}(\pi_t + \bar{\pi}_t) + w_{2,t}\tilde{y}_t + w_{3,t}\Delta z_t) + \varepsilon_t, \end{aligned} \tag{16}$$

where $\bar{r}$ is the steady-state real interest rate (assumed to be constant), $\tilde{y}_t$ is the output gap, and Δz_t is the growth rate of the real exchange rate. We close the system with an unrestricted VAR for the relevant variables needed in order to set the interest rate and forecast future inflation. That is, we assume that the staff forecasts can be well captured by a small-dimension VAR(p) in the variables of interest $\mathbf{X}'_t = \{1, \pi_t, \tilde{y}_t, \Delta z_t\}$. At this point, we have said nothing about how to identify the historical target. Our methodology for doing so is as follows. Suppose that we wish to compute the central bank's target for period s. We would first estimate the five-variable reduced-form VAR(p) on data up to and including period $s-1$. Next, we define the vector

$$\mathbf{X}_t = \sum_{i=1}^{p} \mathbf{A}_{i,t}\mathbf{X}_{t-i} + \mathbf{u}_t, \tag{17}$$

3. We thank Jean Boivin for suggesting this idea.

Figure 1
MEP-based perceived inflation target



Note:
MEP—moving endpoints.

where $\mathbf{E}_{s-1}\pi_{s+4}$ is the five-quarter-ahead forecast generated by the VAR, and π^*_{s+4} is the one-year-ahead staff economic projection (SEP) of inflation, produced in period *s*, conditional on information up to $s-1$. Finally, the constant in the reduced-form interest rate equation is chosen to minimize the quadratic $\mathbf{Q}'_{s-1}\mathbf{W}\mathbf{Q}_{s-1}$, where $\mathbf{W}$ is a matrix that weights the different forecast horizons in the loss function.[4] We can then recover the target based on our choice of this constant. This process is subsequently repeated for all observations in the sample from 1980 up to the official

4. We give a weight of 1 to forecast errors at all horizons except 20 quarters, which has a weight of 5. This reflects the idea that longer-horizon forecasts should reveal more about the Bank's underlying inflation target.

adoption of an inflation target by the Bank of Canada in 1991. Thereafter, we use the actual stated target for the SEP target.[5]

$$\bar{\pi}_s = \left\{ \frac{(\zeta_{s-1} - 1)^{-1} \xi_{1,\,s-1} - \bar{r}}{(1 - w_{1,\,s-1})} \right\}, \tag{18}$$

$$w_{i,\,s-1} = (\zeta_{s-1} - 1)^{-1} \xi_{i+1,\,s-1} \qquad i = 1, 2, 3, 4\,. \tag{19}$$

The results for the SEP approach are reported in Figure 3. The deviations of inflation from the estimated inflation target are very similar to those from the Kozicki and Tinsley approach, at least qualitatively. The same gap develops during the 1981–82 disinflation, and the correlation between the two inflation target estimates is 0.79 over the full sample.

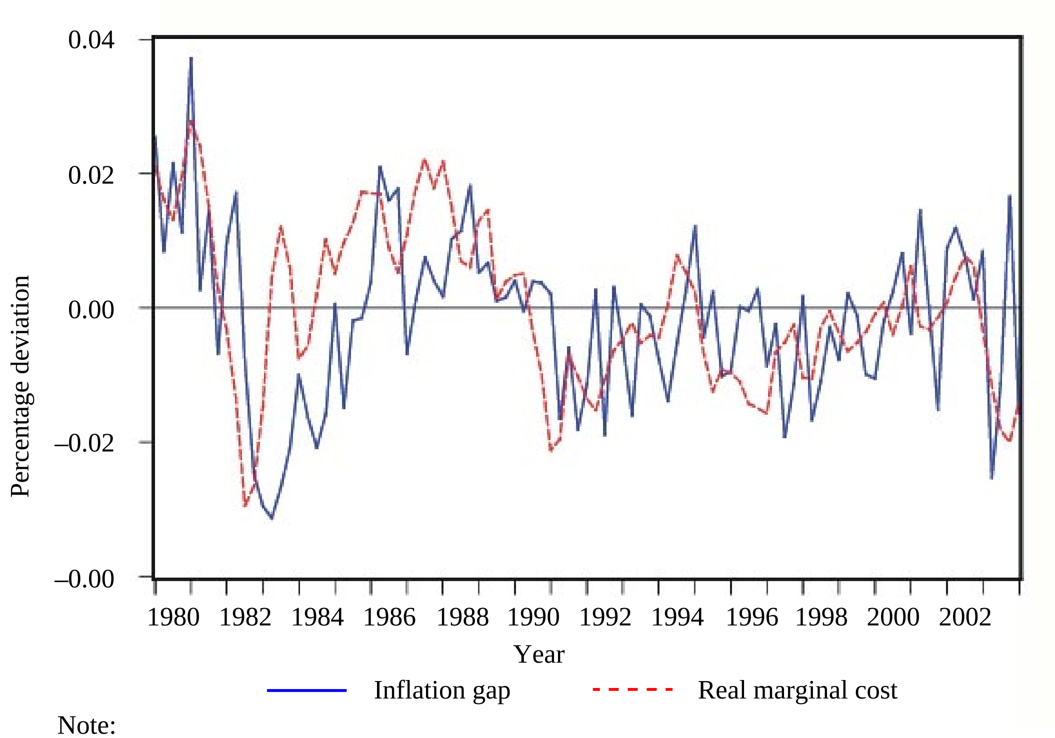
Table 1 provides the standard deviations and two measures of persistence for raw CPIX inflation, the two inflation-gap measures, and our measure of real marginal cost. In terms of the persistence measures, AR(1) refers to the first-order autocorrelation coefficient, and ρ refers to the largest estimated root in the series.

First, we see that the two gap measures are less volatile and less persistent than raw inflation from 1980 to 1992, as we would expect, since the low-frequency component of inflation has, in principle, been removed. Second, we note that real marginal cost is more than twice as volatile than either inflation gap, suggesting that an important role will be played by the assumption of firm-specific capital in producing a reasonable value for θ.

Finally, we see that for the subsample 1993Q1–2004Q1, the volatility and persistence of the two gap series decline relative to the full sample, whereas only the volatility of real marginal cost falls (the AR(1) coefficient falls modestly from 0.86 to 0.78). Thus, while the coincident decline in volatilities is reassuring, the fact that inflation is now essentially white noise while real marginal cost remains persistent, represents an outstanding issue and a useful area for future research.

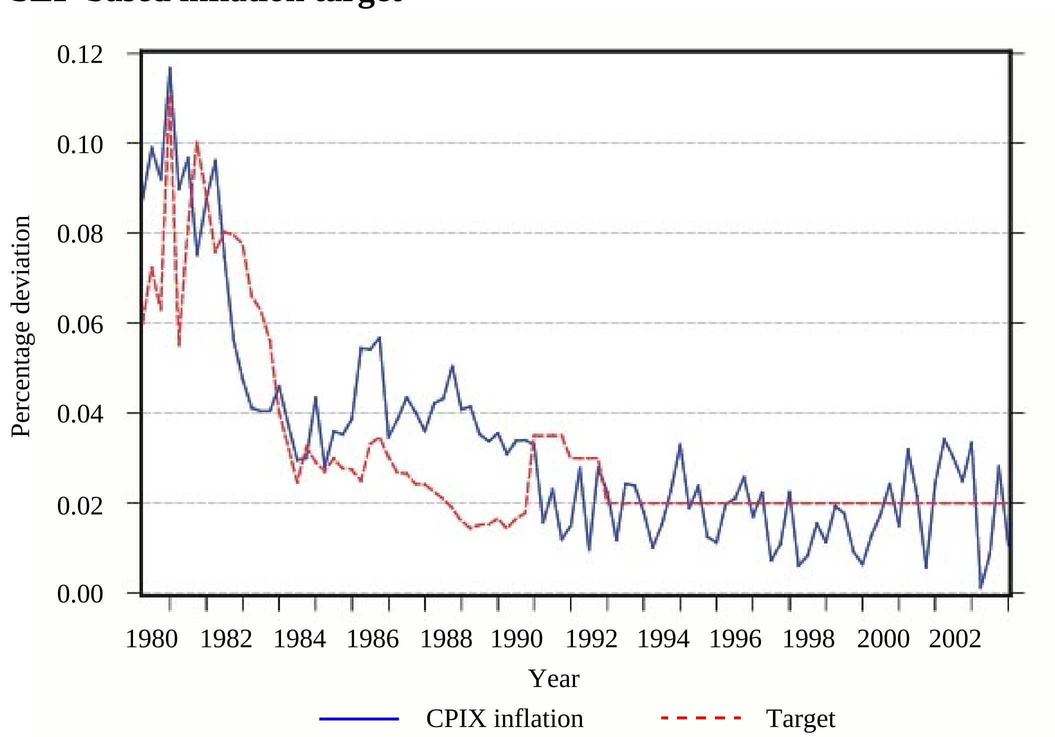
5. A potential issue arises as to whether the target identified here can be interpreted as agents' perceived target, as in the case of the MEP approach, given that the staff projection data are not available to the public. To investigate this issue, we considered applying the signal extraction approach advocated by Erceg and Levin (2003), whereby agents must infer the target based on interest rate changes. Apart from making the inflation gap slightly more persistent, this modification does not change the results presented in the next section. Also, given the fact that inflation falls faster than the SEP target in the early 1980s disinflation, we believe that this variable can also be interpreted as a perceived target.

Figure 2
MEP-based inflation gap and normalized marginal cost



Note:
MEP—moving endpoints.

Figure 3
SEP-based inflation target



Note:
SEP—staff economic projection.

Table 1
Summary statistics for main variables of interest

Variable	1980Q1–1992Q4			1993Q1–2004Q1		
	Std. dev.	Persistence		Std. dev.	Persistence	
	p.p.	ρ	AR(1)	p.p.	ρ	AR(1)
Raw CPIX	2.4	0.93*	0.87*	0.8	0.14	0.14
MEP gap	1.3	0.69*	0.45*	0.9	0.02	0.02
SEP gap	1.5	0.63*	0.63*	0.8	0.14	0.14
Marginal cost	3.0	0.79*	0.86*	1.7	0.92*	0.78*

Notes:
Std. dev.—standard deviation.
p.p.—percentage point.

2.3 Estimation methods

As mentioned in the introduction, we use the SMM approach developed in Smith (1993) to estimate the parameters of the model.[6] Generally speaking, SMM provides a method of comparing the key properties generated by a structural model to those from the data. The data-based moments are generated from an approximating statistical model that should fit the data reasonably well but that need not necessarily nest the structural model. It is noteworthy that the SMM approach using an unconstrained VAR as the approximating model has at least two useful features. First, the inflation equation within the VAR provides a useful and natural metric for the degree of inflation persistence that should be captured by the structural model. Second, an unconstrained VAR does not require controversial identifying assumptions.

For the current exercise, we follow Coenen and Levin (2004) and use an unconstrained VAR in the inflation gap and our measure of marginal cost as the approximating statistical model. In effect, the method estimates the parameters of the structural model by matching its reduced-form (constrained) VAR representation as closely as possible to its unconstrained data-based counterpart. More specifically, we begin by estimating a bivariate VAR(p) from 1980 to 2004 by ordinary least squares, which we will refer to as the "auxiliary model," and then proceed to construct the vector Γ, which

6. Our application differs slightly from that of Smith (1993) in that our model is linearized prior to estimation and there are no unobserved variables. Therefore, there is no need to generate artificial data to compute the model's reduced-form VAR representation.

contains the estimated parameters of the inflation equation.[7] Next, for a given parameterization, we combine the structural inflation equation (given by equation (NKPC)) with the VAR equation for real marginal cost and then solve the resulting system, which we refer to as the "structural model," using a QZ decomposition (as advocated by Sims 2001). This resulting system is a restricted VAR(*p*). We again extract the parameters from the reduced-form inflation equation of this system and form the vector $\Gamma(\gamma, \phi)$. Finally, the estimates of γ and ϕ from equation (NKPC) are chosen to solve

$$\min_{\{\gamma, \phi\}} (\Gamma - \Gamma(\gamma, \phi))'\mathbf{W}(\Gamma - \Gamma(\gamma, \phi)). \tag{20}$$

Then, conditional on the estimate of ϕ and our assumptions about the parameters that determine η, we can recover θ.

3 Empirical Results

3.1 Parameter estimates

In this section, we discuss the ability of the NKPC with dynamic indexation to match important features of Canadian inflation data. Before proceeding to the estimation results, however, we outline the calibration of some model parameters (see Table 2). The discount rate, β, is set to 0.99, implying an annual real interest rate of 4 per cent. Following Gagnon and Khan (2005), the elasticity of substitution among the factors of production, σ, is set equal to 0.5 and, ε, the demand elasticity, is calibrated to 11 and implies a steady-state markup of 10 per cent. The labour adjustment cost parameter, χ, is 6.0, consistent with Rotemberg and Woodford (1999). The share parameters (s_L and s_K) are calibrated to their historical averages. Finally, given our assumptions regarding σ, ε, and s_L, the degree of real rigidity stemming from the assumption of firm-specific capital, η, is equal to 0.045. In other words, inflation in our model is more than 20 times less sensitive to movements in marginal cost than a model that assumes a rental market for capital. This value, while implicitly calibrated, is consistent with those values estimated by Coenen and Levin (2004), who use German data.

The SMM estimation results for the two measures of inflation (MEP and SEP inflation gaps) are presented in Table 2. The estimates of θ imply that firms, on average, reoptimize their price about once every eight months, a number well in line with survey evidence for Canada (see Amirault, Kwan,

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7. In terms of determining the lag length of the auxiliary VAR, we restrict the lag length to be the same as that of the structural model's reduced-form representation, which, in the case of the Calvo model with indexation, is one lag. In this case, the choice of weighting matrix, *W*, in equation (15) is inconsequential since the model is just identified.

Table 2
Calibrated parameters

Parameter	Value
σ	0.50
β	0.99
ε	11
χ	6.00
s_L	0.46
s_K	0.37
Functions of calibrated parameters	
μ	1.10
η	0.045
Θ	0.84
Λ	0.66

and Wilkinson 2004–2005). Furthermore, the estimated duration is robust to the choice of the methodology for calculating the historical target. Under the MEP (SEP) methodology, we obtain a point estimate for the duration between reoptimizations of 2.8 (2.6) quarters, with a 90 per cent confidence interval of two to four quarters. A question that may arise from these results is: How does the NKPC model admit aggregate inflation that is moderately inertial despite the fact that firms change prices frequently? The answer lies in the result that when firms do change prices, they do so by only a "small" amount. This dampened price response is due to the fact that under the firm-specific capital assumption, each firm's short-run marginal cost is increasing in its own output. To better understand this result, consider a firm contemplating a price increase. The firm understands that a higher price implies less demand and less output. A lower level of output reduces marginal cost and induces the firm to post a lower price. Thus, the dependence of marginal cost on firm-level output reduces the firm's incentive to raise its price. This dampening influence explains why aggregate inflation responds less to a given aggregate cost shock, even though firms reoptimize their price relatively frequently.

The degree of dynamic indexation is estimated to be a very moderate 0.37, regardless of the measure of the inflation gap. The 5th and 95th percentiles for the point estimate (using the MEP-based gap) are 0.1 and 0.65; we can therefore rule out both zero and full indexation. The values for the forward- and backward-looking components are consistent with the results reported in Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2001) and show that, even if the standard NKPC is rejected in favour of an equation allowing for additional inertia coming from lagged inflation, the weight on the forward-looking component is quantitatively more relevant (0.72 versus

0.27 on lagged inflation, according to our estimates), an especially important point from the perspective of a monetary authority.

Turning to the overall fit of the model, we see (Table 3) that the structural model explains a slightly higher (lower) proportion of the overall variation in the SEP (MEP) inflation gap relative to the unrestricted VAR(2) inflation equation.[8] This would suggest that nothing is lost by working with the structural model (with the SEP gap), at least in terms of in-sample fit. Given this result, it is not surprising that the restrictions imposed by the Calvo model with indexation are not rejected by the data using the SEP gap. For the MEP gap, the difference in $\bar{R}^2$ is just 0.02, 0.36 versus 0.38 VAR(2). Nevertheless, this difference is sufficiently large to produce a probability value of 0.051 using the LR test under the null that the restrictions imposed by the structural model are true.

Comparisons with the VAR(1) can be easily summarized once we recognize that the structural model does not impose any binding restrictions relative to a VAR(1). With the indexation parameter free to vary on the [0,1] interval and ϕ only restricted to be positive, the reduced form of the estimated structural model corresponds exactly to that of the auxiliary model.

3.2 Comparisons to the standard model

In this section, we investigate the effects of relaxing several assumptions implicit in our preferred specification. More specifically, we begin by estimating the standard NKPC, the Calvo model without indexation and firm-specific capital (see column 1 of Table 4). Here we see that, consistent with our priors and past research using Canadian GDP deflator inflation, the standard model fails to adequately capture the dynamics of inflation along several margins. First, we can easily reject the null hypothesis of no serial correlation in the residuals using the Q-statistic. Second, the model produces a $\bar{R}^2$ of less than 0.1, compared to 0.83 for the unrestricted VAR(2) equation for inflation. Not surprisingly, we easily reject the restrictions imposed by the structural model using a LR (likelihood ratio) test. Finally, the model suggests that firms reoptimize prices on average about once every two years, which seems unreasonably long.

When partial indexation is added to this basic model, the overall fit improves significantly, but we continue to reject the restrictions imposed by the model. Specifically, the unrestricted model prefers two lags of inflation, whereas the indexation model admits just one. Furthermore, it implies an

8. The restricted model is able to explain a higher proportion, because our measure of fit is the adjusted *R*-square, which adjusts for degrees of freedom.

Table 3
Estimation results—preferred model

Variable	**SMM (1980Q1–2004Q1)**			
	MEP inflation gap		**SEP inflation gap**	
NKPC	**VAR(1)**	**VAR(2)**	**VAR(1)**	**VAR(2)**
γ	0.37		0.37	
	[0.10	0.65]	[0.23	0.70]
Average duration	**2.8** quarters		**2.6** quarters	
	[2.00	4.0]	[1.80	3.50]
$\bar{R}^2$	0.36	0.36	0.54	0.54
LB Q-statistic	2.18	2.18	1.99	1.99
VAR(2)				
$\bar{R}^2$	0.35	0.38	0.53	0.52
LB Q-statistic	2.18	2.04	0.53	2.00
H_0: $NKPC = VAR$	**1.00**	**0.051**	**1.00**	**0.30**

Notes:
SMM—simulated method of moments.
LB—Ljung-Box Q-statistic.

Table 4
Estimation results—variations on the preferred model

Variable	**Raw CPIX inflation**		**SEP inflation gap**		
NKPC	$\eta = 1, \gamma = 0$	$\eta = 1$	$\eta = 1$	$\eta < 1$	$\hat{\lambda}_t = \hat{s}_t$
γ	0	0.97	0.37	0.37	0.53
Av. duration	8.9 quarters	16.10	10.80	2.60	∞
$\bar{R}^2$	0.06	0.80	0.54	0.54	0.38
$LB\ Q - stat$	0.00	0.00	0.40	0.40	0.28
VAR(2)					
$\bar{R}^2$	0.83	0.83	0.52	0.52	0.52
$NKPC = VAR$	**0.00**	**0.00**	**0.30**	**0.30**	**0.00**

Table 5
Forecast results (1985Q1–2004Q1)

Model	**Forecast RMSE tests**	
	MEP inflation gap	**SEP inflation gap**
NKPC	**0.23**	**0.21**
YGAP phil. curve	0.26	0.25
AR(2)	0.25	0.24
Diebold-Mariano test	Prob. value under null of equal RMSE (σ)	
$\sigma_{NKPC} < \sigma_{YGAP}$	0.00	0.00
$\sigma_{NKPC} > \sigma_{YGAP}$	1.00	1.00
$\sigma_{NKPC} < \sigma_{AR(2)}$	0.01	0.02
$\sigma_{NKPC} > \sigma_{AR(2)}$	0.99	0.98

average duration between price reoptimizations of four years. Thus, it appears that neither of these two models is consistent with raw CPIX inflation. Next, we attempt to model the SEP-based inflation gap, allowing for partial indexation but continue to maintain the assumption of a rental market for capital $\eta = 1$. In this case, the model matches the preferred model in every respect except that it predicts that firms reoptimize prices, on average, about once every 11 quarters.

Finally, we estimate the preferred specification (indexation with firm-specific capital) but replace our measure of marginal cost with labour's share of final good income, which is equivalent to setting adjustment costs on employment to zero and the elasticity of substitution between production inputs to unity; that is, $\chi = 0$ and $\sigma = 1$. Interestingly, we see that in this instance, the optimization algorithm drives $\phi = 0$, implying an average contract duration that is infinite. This stems from the fact that this measure of marginal cost is unrelated to inflation. Owing to the inclusion of indexation, the model succeeds in explaining about 38 per cent of the historical movements in the inflation gap, compared to 54 per cent for the unrestricted model that uses the preferred marginal cost measure ($\chi = 6$ and $\sigma = 0.5$).[9] Finally, labour's share is not significant in the inflation equation of the unrestricted auxiliary model.

Conclusion

In this paper, we examined the ability of the New Keynesian Phillips curve with partial dynamic price indexation to capture key features of Canadian inflation data. Our study, however, differs from earlier research along three important dimensions. In particular, we relax three assumptions often made in the NKPC literature: (i) constant (and observable and credible) inflation target; (ii) labour's share of income as a measure of marginal cost; and (iii) rental market for capital. Overall, we find that the NKPC with partial dynamic indexation appears capable of reproducing important moments of Canadian inflation data. Indeed, the estimated model replicates both the inflation persistence found in macroeconomic data in addition to durations between price reoptimizations that we view as very reasonable. We view this as support for microfounded models of pricing behaviour such as the Calvo (1983) model.

It would be interesting to further explore the reasons behind the apparent decline in the persistence of both of our inflation-gap measures. As discussed, it is difficult to understand why such a decline in persistence

9. We do not conduct a likelihood ratio test in this instance since the models are non-nested.

should not be accompanied by a corresponding decline in the persistence of real marginal cost. This would seem to point to some form of misspecification to either our marginal cost series or our inflation-gap variables. It would also be interesting to explore whether the SEP approach to identifying the historical inflation objective of the central bank as discussed in this paper could also be usefully applied to other countries.

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