Forecasting the Price of Crude Oil via Convenience Yield Predictions

Thomas A. Knetsch

Deutsche Bundesbank
Economics Department

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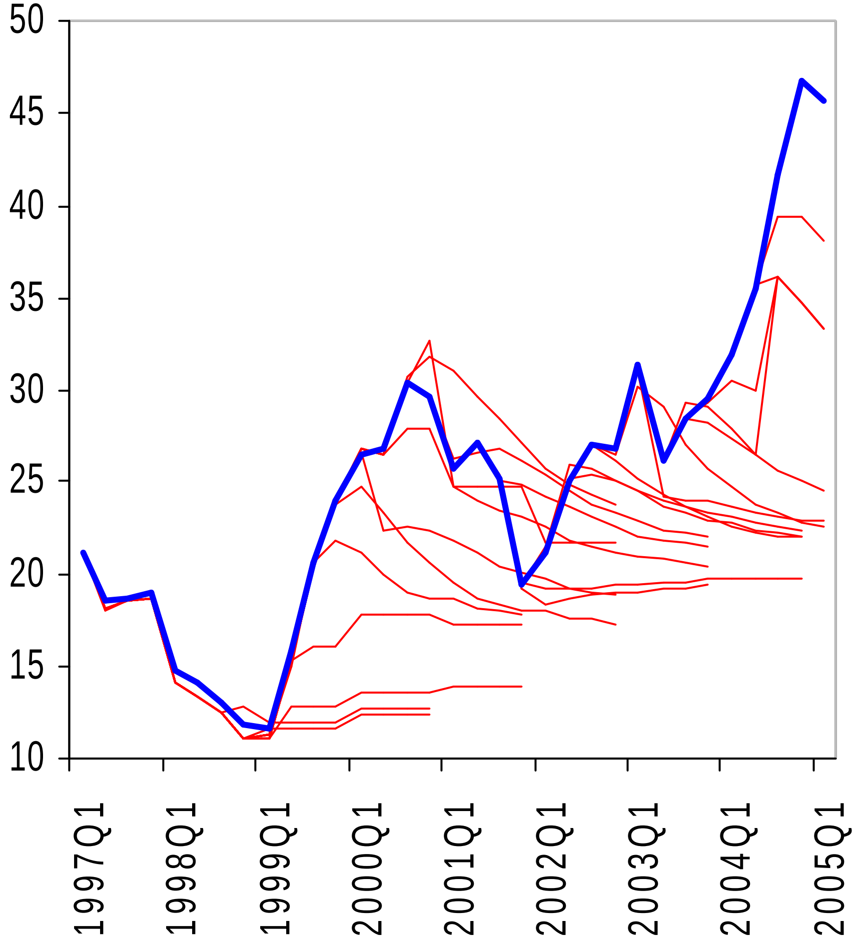
Motivation



Eurosystem's macroeconomic projection exercises use futures prices to prolong the oil price path until the end of the projection horizon.

Poor performance in recent years.

(Ongoing) internal evaluation has mainly focused on two questions:

Was the bias observed in the past projection exercises pure chance or systematic?

Should we work with alternative approaches to oil price forecasting (e.g. random walk assumption)?

Motivation

Practitioners' discussion has been dominated by empirical arguments …

- Chernenko et al. (2004) and Chinn et al. (2005) do not reject the hypothesis that futures prices are unbiased predictors.
- Coimbra and Esteves (2004) find a downward bias, but it is not statistically significant.
- In contrast, Moosa and Al-Loughani (1994) conclude that futures prices are neither unbiased nor efficient.

… but theoretical considerations should not be ignored (see e.g. Pindyck, 1993, 2001).

- The present value model of rational commodity pricing implies that futures prices will only be unbiased predictors if market participants are risk-neutral.
- In oil markets, the cost-of-carry relationship includes a convenience yield which is not neglible empirically.
- Convenience yield is defined as "flow of services which accrues to the owner of a physical inventory but not to the owner of a contract for future delivery".

Overview

Central idea of the forecasting approach:

- **Transpose Pindyck's present value model of rational commodity pricing into a forecasting tool.**
- **Use market expectations and autoregressive forecasts of the marginal convenience yield to prolong the oil price path into the future.**

Structure of the presentation:

1. **The convenience yield forecasting model**
   a. Model setup
   b. Parameter estimation
2. **Predictability: oil price percentage change vs. marginal convenience yield**
3. **Modelling exercises and forecast evaluations**
   a. Recursive model selection and parameter estimation
   b. Recursive out-of-sample forecast evaluation

Convenience yield forecasting model

<u>Crude oil pricing equation:</u> $$p(t) = d \sum_{i=0}^{\infty} d^{\,i}\, \mathrm{E}y\,(t+i)$$

is solution of difference equation $$\mathrm{E}p(t+1) = (1+m)\,p(t) - \mathrm{E}y\,(t)$$

with $$d = (1+m)^{-1}$$

$m = r + r$ **oil-specific 1-period discount rate (i.e. expected rate of return required for holding a unit of crude oil)**

[= risk-free rate + oil-specific risk premium]

$y\,(t)$ **1-period (net) marginal convenience yield**

Convenience yield forecasting model

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<u>Cost-of-carry relationship:</u> $\mathrm{E_M} y\,(t,T) = (1 + r_T)\,p(t) - f\,(t,T)$

r_T	T-period risk-free interest rate
$y\,(t,T)$	T-period convenience yield
$f\,(t,T)$	futures price for delivery in t+T
$y\,(t) \equiv \mathrm{E_M} y\,(t,1)$	marginal convenience yield

<u>Hypothesis:</u> Marginal convenience yield is mean-reverting.

$$y\,(t) = a_0 + a\,(L) y\,(t-1) + e_t$$

where $a_0 > 0$ **and** $a(L)$ **stable AR polynomial of finite order.**

Convenience yield forecasting model

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Convenience yield predictions:

1. **Market expectations:** $$\mathrm{E}_{\mathrm{M}} y(t+h) = \mathrm{E}_{\mathrm{M}} y(t, h+1) - (1+m)\mathrm{E}_{\mathrm{M}} y(t, h)$$

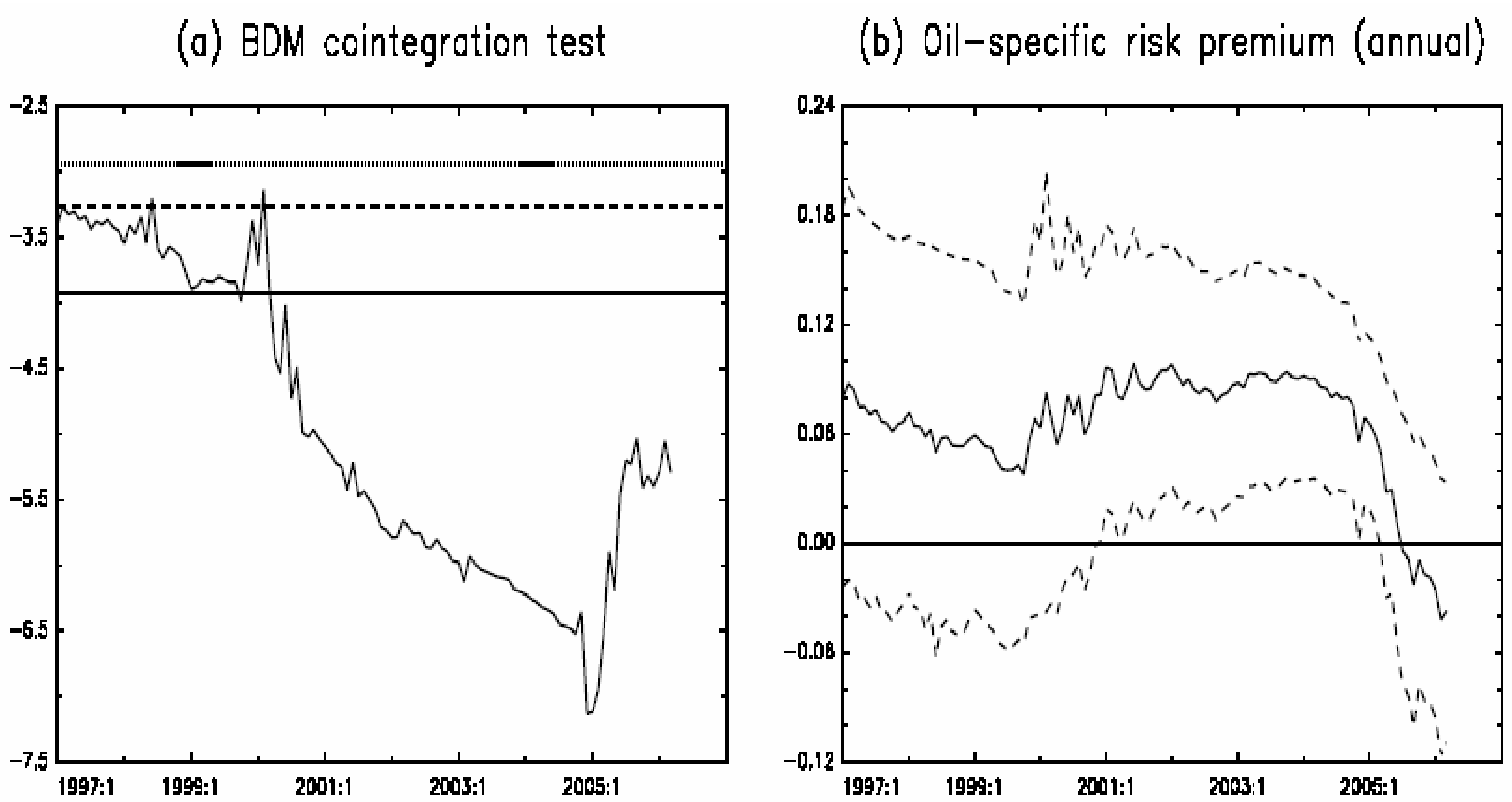
2. **Autoregressive forecasts:** $$\mathrm{E}_{\mathrm{A}} y(t+h) = [1 - a^{h+1}(1)] y_0 + a^{h+1}(L) y(t-1)$$

3. **Combined forecasts:** $$\mathrm{E}_{C} y(t+h) = p_{0,h} + p_h(L) y(t-1) + w_h \mathrm{E}_{\mathrm{M}} y(t+h)$$

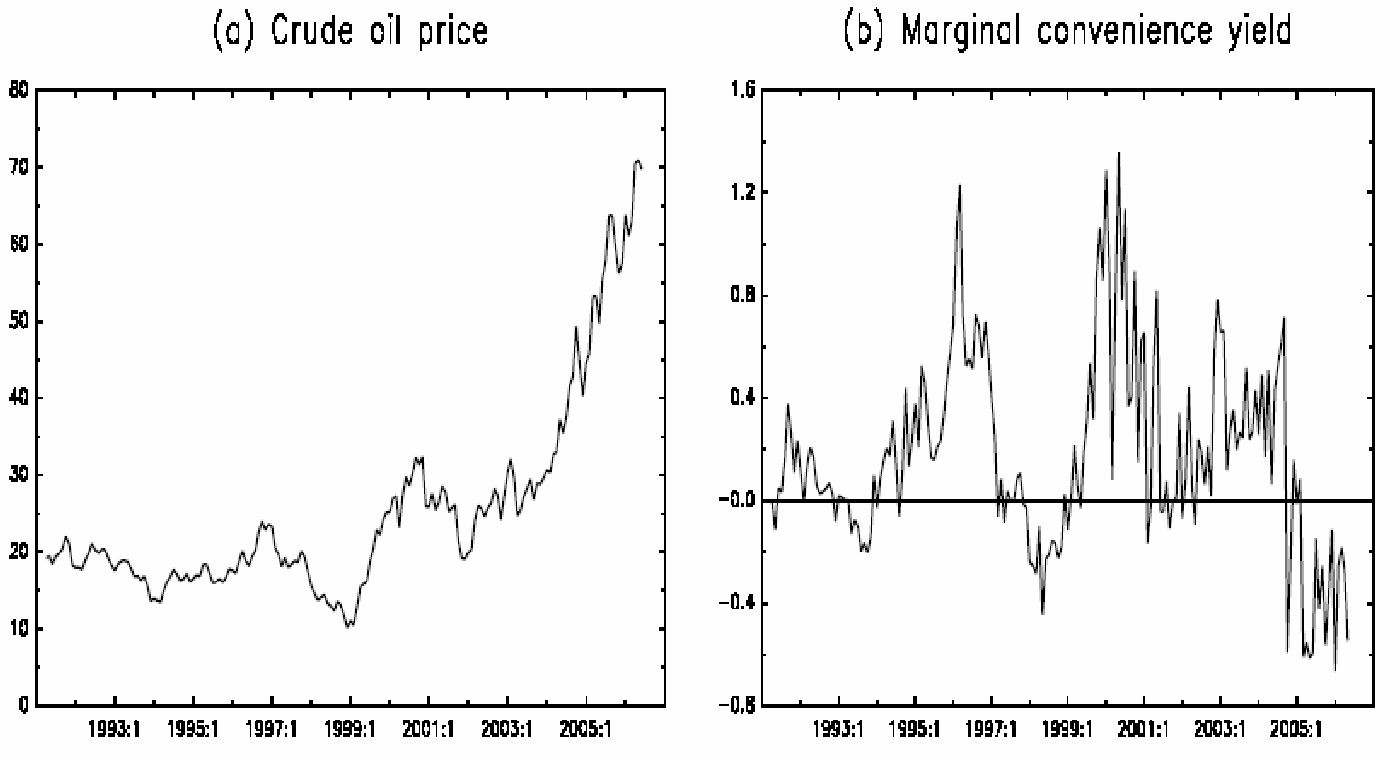
Estimation of the oil-specific risk premium:

- **Spot and nearest futures price are cointegrated with the latter being weakly exogenous. → single equation error correction approach**
- **Risk premium can be derived from the freely estimated coefficient of the cointegrating relation.**
- **Critical assumption: risk premium constant over the estimation sample.**

Recursive risk premium estimates

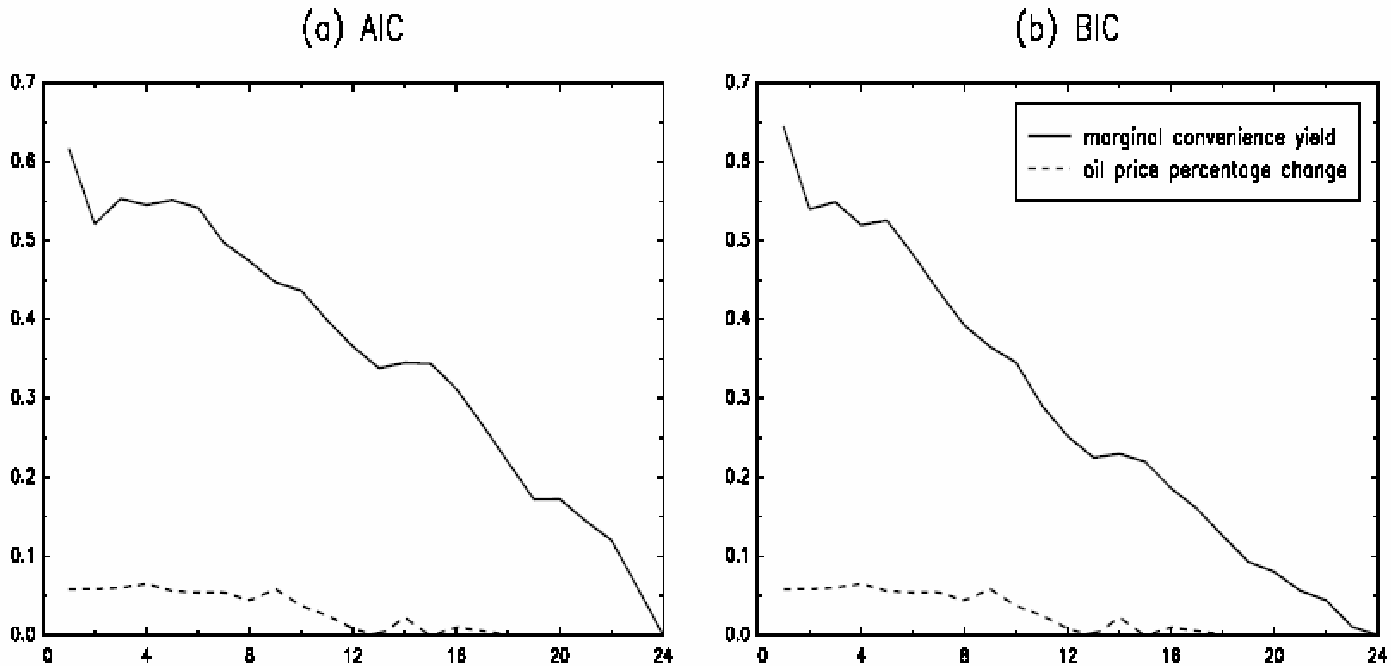


Data: Futures prices for Brent oil



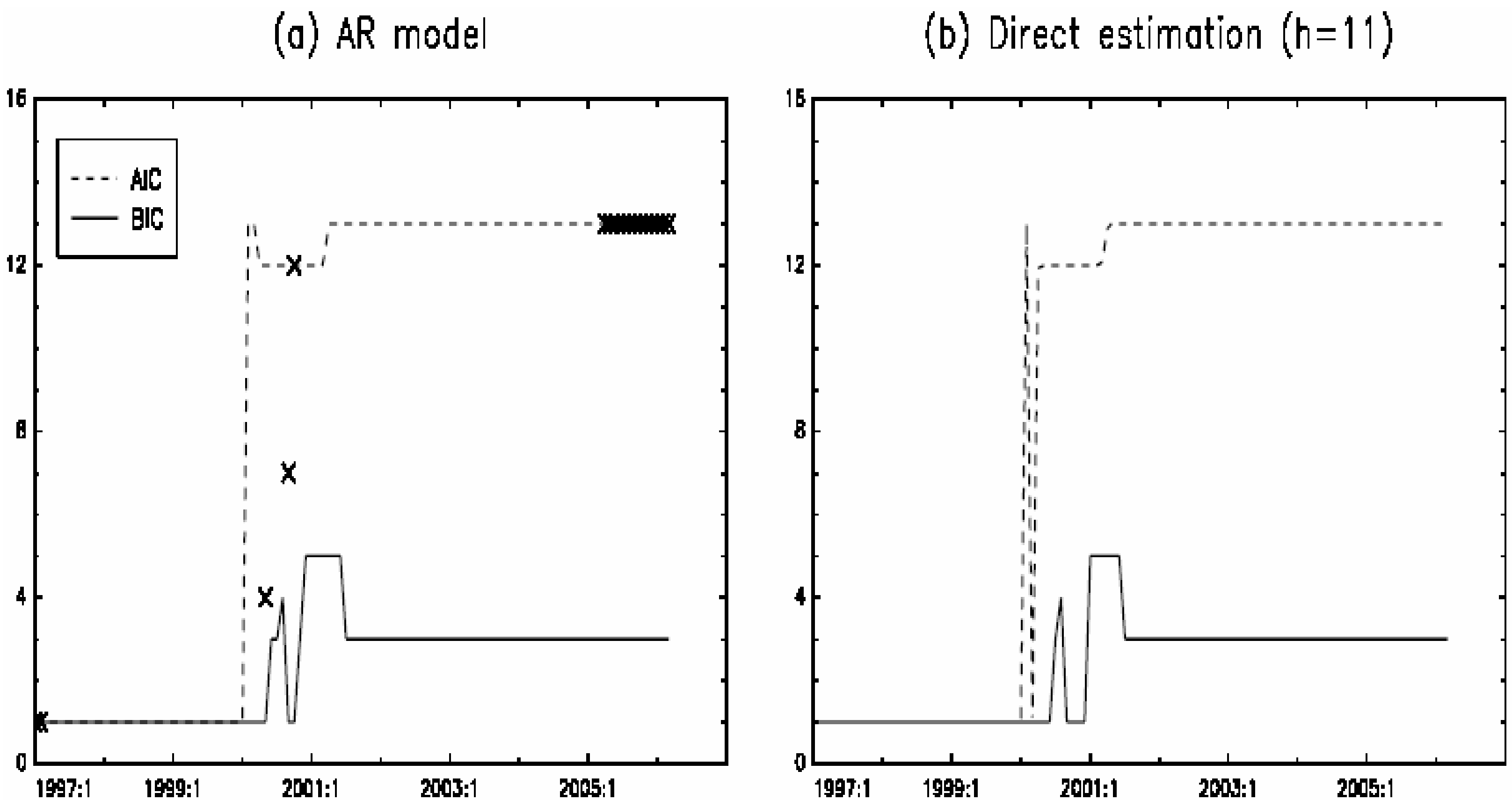
The left-hand graph plots the time series of the price for a barrel of Brent oil. The right-hand graph depicts the time series of the monthly marginal convenience yield. Both entities are measured in U.S. dollars.

Predictability

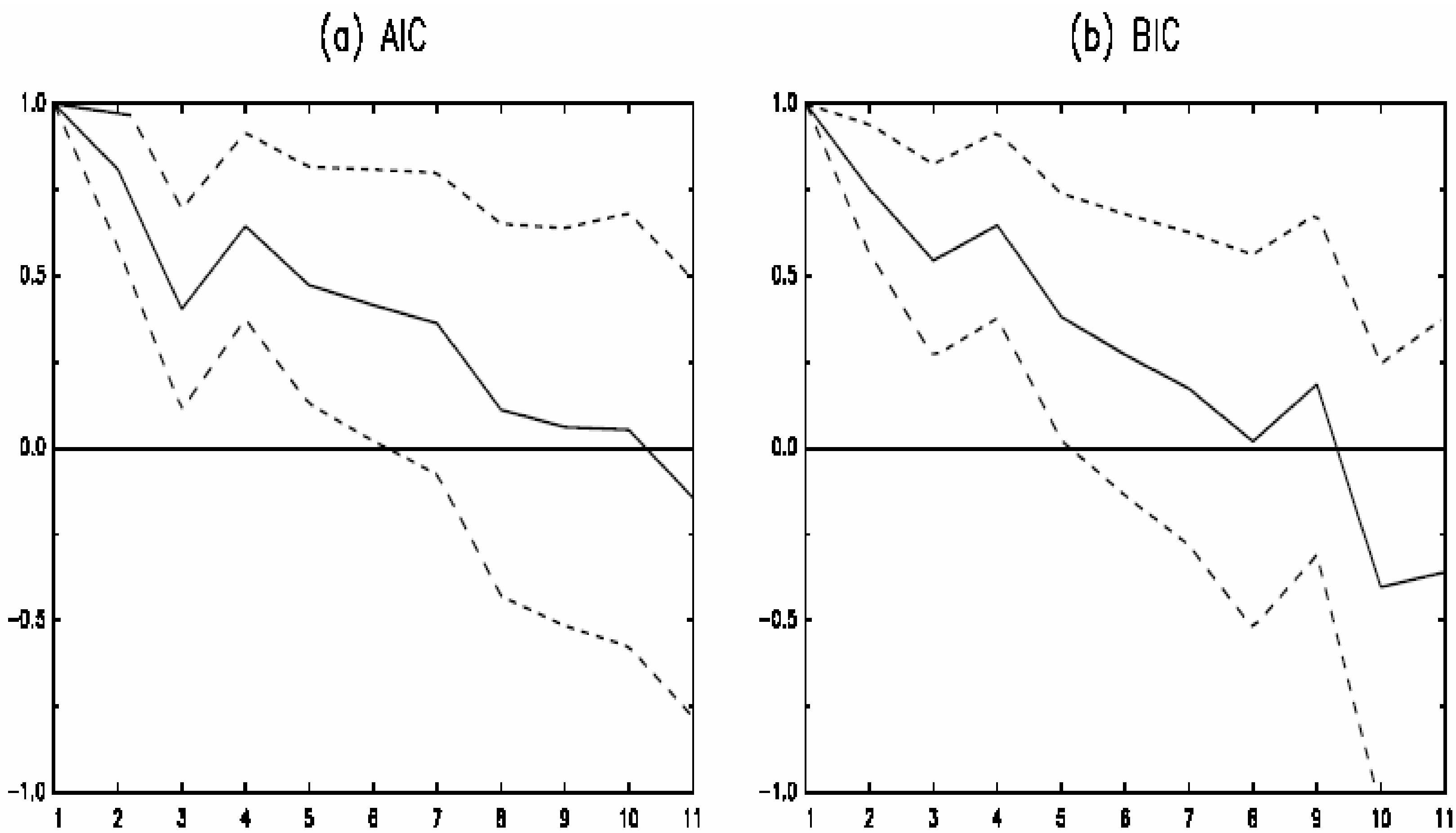


The graphs depict the Diebold and Kilian's (2001) predictability measure for the marginal convenience yield and the oil price percentage change. Results for the AIC and the BIC approximations of the "true" data generating processes are presented. The scale of the horizontal axis is the prediction horizon in months.

Modelling of convenience yield forecast equations



Weigths of futures information in combined model



Modelling exercise: Summary

<u>Main results of the model selection and parameter estimation analysis:</u>

- **In most periods, estimate of the oil-specific risk premium is between 6 and 10 percentage points, but noteworthy collapse since early 2005;**
- **AR model of the marginal convenience yield (plug-in technique):**
  - AIC: long lag order 13;
  - BIC: lag order more parsimonious, but sometimes autocorrelated residuals;
  - HQ: long lag order 13, but less significant regressors in AR polynomial.
- **Direct estimation method (either pure AR or combined model):**
  - Lag order selection yields uniform pattern across forecast horizons.
  - AIC and BIC/HQ choices are the same as above.
  - Weight of futures market information decreases in the forecast horizon.

Forecast evaluation exercise

Aim: Evaluation of the convenience yield forecasting models

Forecast horizons (in months): h = 1, 2, …, 11

Benchmarks:

- random walk assumption (RWA),
- futures market hypothesis (FMH);

Recursive out-of-sample forecast exercises:

- samples of forecast origins: Jan 97 / Jul 00 – Mar 06 (about 100 / 60 forecasts),
- model selection and parameter estimation anew at each forecast origin;

Evaluation criteria:

- root mean squared error (RMSE),
- mean error (ME),
- frequency of correct direction-of-change predictions (DC);

Forecast accuacy tests:

- Diebold-Mariano (1995) test of equal predictive ability against the benchmarks;
- Pesaran-Timmermann (1992) test of "valuable" direction-of-change predictions.

Forecast evaluation: RMSE

Table 1: Root mean squared error

Evaluation	h	CY(M)	CY(Da)	CY(Db)	CY(Pa)	CY(Pb)	RWA	FMH
starting in Jan 97	1	2.479 [0.44 ; 0.13]	2.479 [0.44 ; 0.13]	2.479 [0.44 ; 0.13]	2.479 [0.44 ; 0.13]	2.479 [0.44 ; 0.13]	2.513 [0.89]	2.506
	2	3.508 [0.25 ; 0.09]	3.544 [0.40 ; 0.46]	3.546 [0.39 ; 0.47]	3.557 [0.53 ; 0.59]	3.537 [0.40 ; 0.35]	3.618 [0.78]	3.587
	3	4.178 [0.41 ; 0.04]	4.250 [0.74 ; 0.39]	4.234 [0.62 ; 0.25]	4.264 [0.85 ; 0.47]	4.238 [0.71 ; 0.31]	4.293 [0.74]	4.345
	4	4.683 [0.53 ; 0.05]	4.727 [0.73 ; 0.24]	4.704 [0.61 ; 0.13]	4.738 [0.79 ; 0.28]	4.682 [0.56 ; 0.14]	4.790 [0.41]	4.952
	5	5.150 [0.94 ; 0.06]	5.101 [0.87 ; 0.14]	5.060 [0.70 ; 0.07]	5.155 [0.94 ; 0.24]	5.070 [0.79 ; 0.10]	5.134 [0.16]	5.557
	6	5.961 [0.79 ; 0.08]	5.834 [0.79 ; 0.11]	5.784 [0.58 ; 0.06]	5.971 [0.81 ; 0.27]	5.857 [0.91 ; 0.13]	5.892 [0.15]	6.481
	7	6.745 [0.81 ; 0.10]	6.545 [0.59 ; 0.10]	6.486 [0.36 ; 0.06]	6.724 [0.90 ; 0.29]	6.596 [0.82 ; 0.15]	6.678 [0.16]	7.372
	8	7.457 [0.75 ; 0.12]	7.184 [0.54 ; 0.10]	7.080 [0.24 ; 0.06]	7.367 [0.98 ; 0.28]	7.258 [0.82 ; 0.17]	7.356 [0.17]	8.202
	9	8.060 [0.63 ; 0.14]	7.660 [0.46 ; 0.09]	7.540 [0.17 ; 0.06]	7.922 [0.97 ; 0.30]	7.807 [0.83 ; 0.19]	7.905 [0.17]	8.935
	10	8.630 [0.50 ; 0.16]	8.089 [0.38 ; 0.09]	7.980 [0.12 ; 0.07]	8.486 [0.88 ; 0.33]	8.357 [0.89 ; 0.21]	8.413 [0.18]	9.650
	11	9.277 [0.53 ; 0.18]	8.646 [0.28 ; 0.09]	8.555 [0.07 ; 0.09]	9.216 [0.73 ; 0.37]	9.032 [0.92 ; 0.23]	9.064 [0.22]	10.420

Forecast evaluation: ME

Table 2: Mean error

Evaluation	h	CY(M)	CY(Da)	CY(Db)	CY(Pa)	CY(Pb)	RWA	FMH
starting in Jan 97	1	0.310 [0.15 ; 0.00]	0.310 [0.15 ; 0.00]	0.310 [0.15 ; 0.00]	0.310 [0.15 ; 0.00]	0.310 [0.15 ; 0.00]	0.361 [0.04]	0.447
	2	0.653 [0.41 ; 0.00]	0.656 [0.39 ; 0.00]	0.662 [0.40 ; 0.00]	0.651 [0.39 ; 0.00]	0.655 [0.41 ; 0.00]	0.737 [0.09]	0.934
	3	1.087 [0.70 ; 0.00]	1.043 [0.50 ; 0.00]	1.049 [0.50 ; 0.00]	1.024 [0.48 ; 0.00]	1.044 [0.53 ; 0.00]	1.153 [0.08]	1.518
	4	1.514 [0.94 ; 0.00]	1.408 [0.61 ; 0.00]	1.414 [0.60 ; 0.00]	1.366 [0.54 ; 0.00]	1.405 [0.62 ; 0.00]	1.533 [0.06]	2.098
	5	1.932 [0.91 ; 0.00]	1.782 [0.72 ; 0.00]	1.787 [0.70 ; 0.00]	1.692 [0.58 ; 0.00]	1.754 [0.68 ; 0.00]	1.896 [0.04]	2.675
	6	2.398 [0.82 ; 0.00]	2.224 [0.83 ; 0.00]	2.220 [0.79 ; 0.00]	2.062 [0.60 ; 0.00]	2.153 [0.72 ; 0.00]	2.309 [0.04]	3.304
	7	2.899 [0.78 ; 0.00]	2.702 [0.89 ; 0.00]	2.693 [0.85 ; 0.00]	2.492 [0.63 ; 0.00]	2.618 [0.77 ; 0.00]	2.768 [0.03]	3.978
	8	3.391 [0.77 ; 0.00]	3.189 [0.94 ; 0.00]	3.162 [0.87 ; 0.00]	2.930 [0.65 ; 0.00]	3.099 [0.82 ; 0.00]	3.231 [0.03]	4.645
	9	3.835 [0.76 ; 0.00]	3.621 [0.97 ; 0.00]	3.601 [0.92 ; 0.00]	3.330 [0.66 ; 0.00]	3.558 [0.89 ; 0.00]	3.648 [0.03]	5.267
	10	4.237 [0.76 ; 0.00]	4.040 [0.99 ; 0.00]	4.008 [0.96 ; 0.00]	3.701 [0.67 ; 0.00]	3.995 [0.95 ; 0.00]	4.034 [0.04]	5.844
	11	4.592 [0.77 ; 0.00]	4.421 [0.96 ; 0.01]	4.374 [0.99 ; 0.00]	4.025 [0.67 ; 0.00]	4.389 [0.99 ; 0.00]	4.378 [0.04]	6.371

Forecast evaluation: Direction of change

Table 3: Direction of change

Evaluation	h	CY(M)	CY(Da)	CY(Db)	CY(Pa)	CY(Pb)	FMH
starting in Jan 97	1	0.518 [0.47 ; 0.16]	0.518 [0.47 ; 0.16]	0.518 [0.47 ; 0.16]	0.518 [0.47 ; 0.16]	0.518 [0.47 ; 0.16]	0.500
	2	0.523 [0.52 ; 0.16]	0.541 [0.36 ; 0.15]	0.514 [0.58 ; 0.16]	0.514 [0.58 ; 0.16]	0.532 [0.46 ; 0.16]	0.450
	3	0.528 [0.41 ; 0.15]	0.537 [0.36 ; 0.15]	0.500	0.500	0.509 [0.57 ; 0.16]	0.481
	4	0.458	0.523 [0.48 ; 0.16]	0.505 [0.53 ; 0.16]	0.495	0.477	0.393
	5	0.472	0.509 [0.63 ; 0.15]	0.500	0.491	0.491	0.387
	6	0.476	0.533 [0.37 ; 0.15]	0.505 [0.56 ; 0.16]	0.486	0.486	0.381
	7	0.529 [0.27 ; 0.13]	0.567 [0.14 ; 0.09]	0.577 [0.08 ; 0.06]	0.442	0.490	0.423
	8	0.495	0.583 [0.09 ; 0.07]	0.583 [0.08 ; 0.06]	0.485	0.466	0.417
	9	0.500	0.608 [0.04 ; 0.03]	0.588 [0.06 ; 0.05]	0.480	0.471	0.431
	10	0.495	0.653 [0.00 ; 0.00]	0.693 [0.00 ; 0.00]	0.465	0.505 [0.55 ; 0.16]	0.416
	11	0.470	0.680 [0.00 ; 0.00]	0.680 [0.00 ; 0.00]	0.460	0.490	0.420

Forecast evaluation: Summary

Root mean squared error:

- CY(M) performs best at short horizons (h<5) and CY(Db) at longer horizons.
- Differences in forecast ability against RWA not statistically significant; weak significance against FMH is reported for best CY performers.

Mean error:

- FMH has the strongest bias, RWA and CY models perform significantly better.
- At all horizons, CY(Pa) ist best performer among CY models, also better than RWA.

Direction of change:

- FMH fails more frequently than it succeeds to predict the direction of change.
- At short horizons, all CY models hit the direction of change in more than 50 percent.
- At longer horizons, only CY(Da) and CY(Db) come up with frequencies above 0.5, but those are statistically significant.

Combined forecasting models:

- never best performer within the class of CY models.

Conclusions

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Specific results:

- The futures price for delivery in h months should not be used as predictor of the spot price h months ahead because this practice leads to strong biases. The proposed forecasting technique reduces the bias significantly.
- In terms of RMSE and ME, the random-walk model appears to be an even competitor. Vis-à-vis this benchmark, however, the proposed forecasting device is advantageous because it provides valuable direction-of-change predictions.

Some limitations:

- Convenience yield may be modelled structurally (Pindyck, 1994; French, 2005).
- Forecast improvement is envisaged by letting the risk premium be time-variant.
- Evaluate longer horizons and take further oil price forecasting tools into account.

General assessment:

- Although improvement is marked in relative terms, the oil price is and remains a highly unpredictable quantity.
- Compared with the RWA, the proposed forecasting device provides an informative direction-of-change forecast and, compared with the FMH, a less biased forecast.