Can Affine Term Structure Models Help Us to Predict Exchange Rates? - Discussion

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Outline

Forward Premium Puzzle

- Uncovered Interest Rate Parity:

$$\Delta s_{t+1} = \alpha_0 + \alpha_1 \left[r_t - r_t^{\star} \right] + \varepsilon_{t+1},$$

where s_t is in \$/units of foreign currency.

  - UIP condition: $\alpha_0 = 0$ and $\alpha_1 = 1$.
  - In the data: $\alpha_1 < 1$ and mostly $\alpha_1 < 0$.

Exchange Rate Predictability

- Exchange rates are hard to predict.
- Meese and Rogoff (1983): random walk beats macro models for out-of-sample forecasts at short horizons.
  - Clarida and Taylor (1997): VECM beats random walk.
- Longer horizons: Mark (1995).
  - Kilian (1999): results not robust.

This Paper: An Internationally Affine Model

- State vector: $X_t = \begin{bmatrix} r_t \\ r_t^\star \end{bmatrix}$.
- X_t follows an Orstein-Uhlenbeck process.
- Market price of risk Λ_t affine in X_t:

$$\begin{aligned} dX_t &= \Phi(\Theta - X_t) + \Sigma^{1/2} dW_t \\ \Lambda_t &= \Lambda_0 + \Lambda_1 X_t \\ \Lambda_t^\star &= \Lambda_0^\star + \Lambda_1^\star X_t. \end{aligned}$$

Real Exchange Rate in Complete Markets

- For a foreign investor buying a bond from her country, the real return $R^{\star}_{t,t+1}$ satisfies:

$$E_t(M^{\star}_{t,t+1} R^{*}_{t,t+1}) = 1.$$

- But a domestic investor can also buy a foreign bond:

$$E_t(M_{t,t+1} \frac{Q_{t+1}}{Q_t} R^{\star}_{t,t+1}) = 1.$$

- Thus, in complete markets, real exchange rate Q is defined as:

$$\frac{Q_{t+1}}{Q_t} = \frac{M^{\star}_{t,t+1}}{M_{t,t+1}}.$$

Procedure

- No arbitrage in bond and FX markets.
- Interest rates:

$$
\begin{aligned}
r_t^h &= A(h) + B(h)' X_t \\
r_t^{\star,h} &= A^\star(h) + B^\star(h)' X_t, \text{ where } X_t = \begin{bmatrix} r_t \\ r_t^\star \end{bmatrix}.
\end{aligned}
$$

- Exchange rates:

$$\Delta s_{t+1} = C(1) + D(1)' \widetilde{X}_t + v_{t+1}, \text{ where } \widetilde{X}_t = [X_t', vech(X_t X_t')']'.$$

- MLE.

Results

- Forward bias:
  - US-Canada, 3-month horizon:

$$\begin{aligned} \alpha_1 &= -0.5 \text{ in the model} \\ &= -0.8 \text{ in the data.} \end{aligned}$$

  - US-UK, 3-month horizon:

$$\begin{aligned} \alpha_1 &= -1.9 \text{ in the model} \\ &= -1.5 \text{ in the data.} \end{aligned}$$

- Exchange rate predictability out-of-sample:
  - US-Canada, 9% lower RMSE (vs random walk and VAR) at 12-month
  - US-UK: 36% lower.

Two possible mechanisms

- Lustig-Verdelhan (2005).
- Complete markets, no inflation risk.
- Log currency risk premium:

$$std_t m_{t+1} \left[std_t m_{t+1} - \rho_t \left(m_{t+1}, m^\star_{t+1} \right) std_t m^\star_{t+1} \right].$$

- Two possible mechanisms: lower foreign interest rate means
  - Heteroskedasticity: $std_t m^\star_{t+1} \nearrow$
  - Time-varying correlation: $\rho_t \left(m_{t+1}, m^\star_{t+1} \right) \nearrow$
- $\Rightarrow$ Both mechanisms are playing here (see signs of estimated $\lambda_{i,j}$)

US-UK

- $M_{t,t+1} = e^{-r_t} e^{-\lambda_t X_{t+1}} / \phi^P(-\lambda_t, X_t)$.
- $std_t m_{t+1}$ is prop. to $-\lambda_t$.
- As a first approximation, take $dr_t \perp dr_t^\star$.
- Estimation (Table 3 in the paper):

$$\begin{aligned} \Lambda_t &= 5.7 - 2.1 r_t - 0.6 r_t^\star \\ \Lambda_t^\star &= -5.9 + 2.6 r_t + 0.3 r_t^\star \end{aligned}$$

  - $r_t^\star \searrow \Longrightarrow \Lambda_t^\star \searrow \Longleftrightarrow std_t m_{t+1}^\star \nearrow$
  - $r_t^\star \searrow \Longrightarrow cov_t(m_{t+1}, m_{t+1}^\star) \nearrow$

Two issues

- Backus, Foresi and Telmer (2001): Affine models and forward premium.
- Example 2: negative factors $\Rightarrow$ negative interest rates (but low prob.);
- Example 4: interdependent factor model $\Rightarrow$ asymmetry: one shock impacts more the foreign interest rate than the other shock, but impacts less the foreign pricing kernel.

Procedure

- Quasi-MLE, using interest rate and exchange rate data.
- Paper uses two assumptions:
  - Exchange rate innovations homoskedastic: $v_{t+1} \sim N(0, \sigma_v^2)$.
  - Exchange rate innovations v_{t+1} uncorrelated to interest rate residuals.
- Justifications?
  - Heteroskedasticity in FX (Andersen, Bollerslev, 1998)?
  - Currency risk premia only explained by interest rates?

What drives the predictability results?

- Reference point: Clarida, Sarno, Taylor and Valente (2003): MS-VECM beats VECM by 40%.
- Standard errors?
- Comparing to VARs or VECMs:
  - Is this about adding non-linearities $(r_t)^2, (r_t^\star)^2, \ldots$?
  - Or about restrictions on estimated coefficients implied by no-arbitrage?
  - $\Rightarrow$ Compare to VAR or VECM with higher moments?

More moments

- Variance of changes in exchange rates:

$$\frac{Q_{t+1}}{Q_t} = \frac{M^\star_{t,t+1}}{M_{t,t+1}}.$$

$$\sigma^2_{\Delta q} = \sigma^2_{m^\star} + \sigma^2_m - 2\rho_{m^\star,m}\sigma_{m^\star}\sigma_m.$$

- Campbell-Shiller tests of the EH:

$$y^{n-1}_{t+1} - y^n_t = \alpha + \beta_n\left(\frac{y^n_t - y^1_t}{n-1}\right) + \varepsilon_{t+1}.$$

- Does FX data and no-arbitrage condition across countries lead to better yield predictability?

Conclusion

- Combining bond and FX data gives much better FX out-of-sample predictability.
- Very exciting results!