Specification of a Stochastic Simulation Model for the Analysis of Monetary and Fiscal Policy

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Department of Finance Working Paper

2001-14

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* The views expressed in this paper are those of the author and do not reflect those of the Department of Finance. The author would like to acknowledge important contributions made by Chris Matier, David Dupuis, Patrick Georges and Nicolas Moreau and also thank Mike Bradnock for valuable research assistance.

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Abstract

This paper documents the specification of a model that was constructed specifically for the purpose of analysing alternative strategies for implementing monetary and fiscal policy in Canada using stochastic simulation methods.

Some key features of the model are as follows. The domestic sector of the model is modelled as a small open economy, which is strongly influenced by economic and financial developments in the foreign sector. Expectations play a central role in determining inflation, interest rates and the exchange rate. Expectations are formed as a combination of forward-looking (model-consistent forecasts) and backward-looking (adaptive) components. Economic developments affect fiscal revenues and expenditures in the model, and vice versa, so that fiscal policy actions have economic repercussions. The public debt is comprised of treasury bills and government bonds with maturities that range from one quarter to twenty years. This is a distinctive feature of the model that results in non-trivial dynamics for debt service costs. Monetary and fiscal policy are determined using simple policy rules. The monetary authority reacts to unanticipated economic developments each quarter in an effort to keep inflation within a target range. The fiscal authority plans a budget each year in an effort to keep the debt-to-GDP ratio on a clear, downward profile. The model is designed to compare alternative monetary policy rules and fiscal planning strategies.

The parameters in the model are specified using an eclectic methodology that combines estimation and calibration procedures. Some parameters are determined on the basis of estimated reduced-form equations; other parameters are calibrated using the method of simulation moments. The structure of the model is designed to allow for a wide range of alternative parameter values to facilitate sensitivity analysis.

1. Introduction

This paper documents the specification of a model that was constructed specifically for the purpose of analysing alternative strategies for implementing monetary and fiscal policy in Canada using stochastic simulation methods.

Ideally, one would like to conduct policy analysis using a model that has a strong theoretical foundation and empirical properties that are supported by the data. Unfortunately, this is beyond the current state of macroeconomics. There is little consensus about several theoretical aspects of the basic structure underlying macro models. Moreover, models with explicit theoretical foundations based on intertemporal dynamic optimization are often be rejected by the data, while models that are constructed to "fit the data" often lack explicit theoretical foundations. Modellers therefore face a trade-off between specifying a macro model with well-defined theoretical versus empirical properties.[1] Moreover, stochastic simulation methods cannot be applied to most existing theoretical and empirical macro models because their complexity makes the computations intractable. In addition, most theoretical models that have few (if any) stochastic elements, which constrains the range of shocks that policy makers are interested in.

The modelling project summarized in this paper is motivated from the perspective of a policy maker faced with uncertainty about future economic and fiscal developments. The monetary authority in the model sets its instrument (the short-term nominal interest rate) in the current period in an effort to keep inflation within a target range over the coming year or two. Similarly, the fiscal authority plans its budget for the coming fiscal year by making discretionary changes to program spending and/or taxes in an effort to keep the debt-to-GDP ratio on a "clear, downward profile". The model is designed to compare how alternative monetary policy rules and fiscal planning strategies can achieve these policy objectives.

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[1] For an overview of recent research on formulating macro models that have well-defined theoretical foundations and good empirical properties, see Fuhrer (1997, 2000), Rotemberg and Woodford (1997), McCallum and Nelson (2000).

The next section of the paper describes the basic structure of the stochastic simulation model (SSM). Section 3 discusses the methodology used to specify the model. Section 4 illustrates some of its key simulation properties.

2. Model Structure

Overview

The model has a foreign and a domestic sector, each with an endogenous monetary policy reaction function. The enables us to examine how economic, financial and monetary policy developments abroad impact on Canada's small open economy. The macroeconomic core of each sector consists of a few reduced-form equations for aggregate demand/supply dynamics and the inflation process. Our preference for a small, aggregate model is largely driven by computational requirements of stochastic simulation methods. The reduced-form aspect of the model enables us to focus on key parameters of interest. This greatly facilitates calibration of the model and sensitivity analysis.

Given our objectives outlined above, our foremost concern is to ensure that the model has "desirable" simulation properties. By this we mean that the dynamic impulse responses should have well-defined economic interpretations and "reasonable" magnitudes. Some equations in the model are specified strictly on theoretical grounds. For example, the exchange rate is determined by the uncovered interest parity condition in one version of the model and the term structure of interest rates is modelled using the expectations hypothesis. These arbitrage conditions are often rejected by empirical tests.[2] It is unclear whether this is due to a rejection of rational expectations, the presence of a time-varying exchange rate risk premium/term premium, imperfect asset substitutability or some combination of all these factors. Despite the empirical shortcomings, uncovered interest parity and the expectations hypothesis of the term structure provide a standard theoretical framework for determining interest rates and exchange rates in macro models.

Output and inflation exhibit considerable persistence in the model. This reflects the view that various frictions, associated with price and wage setting, prevent product

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[2] See Bekaert and Hodrick (2000) and references therein.

markets from clearing instantaneously. The reduced-form equations capture the dynamic adjustment of prices and quantities during the transition to equilibrium. We do not take a strong stance on the precise form of the underlying frictions. We rely instead on the data to determine the average degree of persistence that various reduced-form relationships have exhibited over the historical period. The reduced-form approach to macro modelling is of course subject to the Lucas critique. Sensitivity analysis therefore plays a central role in using the model to draw reliable policy conclusions.

Expectations play a key role in determining inflation, interest rates and the exchange rate. We take an eclectic approach to modelling expectations. Following Buiter and Miller (1985), expectations are formed using a combination of forward- and backward-looking components. This approach provides a high degree of flexibility, which facilitates calibration of the model and sensitivity analysis. For example, the model can be simulated using forward-looking model-consistent (rational) expectations and backward-looking (adaptive) expectations as special cases.

A Few Words About Notation

It is useful to explain our notation before presenting the structure of the model. Lower case letters represent logs of variables scaled by a factor of 100, unless otherwise noted. This enables us to express most variables as percentage deviations from their equilibrium levels. For example, the cyclical component of output y^c_t (the "output gap") is measured as the percentage deviation of real GDP y_t from its potential level y^p_t. The model is specified at the quarterly frequency, but all changes and interest rates are expressed at annual rates. For example, Δp_t refers to the quarterly percentage change in the price level p_t, expressed at an annual rate.

Real output and real interest rates are decomposed into transitory and permanent components. The permanent components have a stochastic trend. The transitory components are interpreted as deviations from equilibrium levels.[3] All variables converge

[3] From an econometric perspective, output and real interest rates are integrated of order one, whereas the cyclical components are integrated of order zero (mean stationary).

to their respective equilibrium levels (the permanent components) in the long run. This ensures that the cyclical components are stationary.

The monetary and fiscal authorities in the model can distinguish between the transitory and permanent components. Hence, there is no uncertainty about the source or persistence of the shocks encountered.[4] The model can be simulated, however, such that policy makers cannot immediately ascertain whether observed changes to output and real interest rates are transitory or permanent.

We will present the foreign and domestic sectors of the model, each in turn.

2.1 The Foreign Sector

The foreign sector of the model in comprised of six stochastic equations along with several identities.

Aggregate supply/demand dynamics

The cyclical component of foreign real output yf_t (the foreign "output gap") is modelled using the following reduced-form equation:

(F.1) $$yf^c_t = (\alpha_1 + \alpha_1)yf^c_{t-1} - \alpha_1\alpha_2 yf^c_{t-2} + \alpha_3 rf^c_t + \varepsilon f_{1t}$$

where rf^c_t is the cyclical component of the foreign real cost of funds and εf_{1t} is a random error term ~ $(0, \sigma_{f1}^2)$. The cyclical component of the real cost of funds is defined as a weighted average of real yields on short- and long-term bonds, given by:

$$rf^c_t = \nu rf^{1c}_t + (1-\nu)rf^{80c}_t$$

where rf^{1c}_t and rf^{80c}_t represent the cyclical components of the real yield on 3-month treasury bills and 20-year discount bonds, respectively. Equation (F.1) attributes cyclical fluctuations in aggregate demand to movements in short- and long-term real interest rates.

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[4] The model can be simulated in such way that policy makers cannot immediately ascertain whether observed changes to output and real interest rates are transitory or permanent.

The Inflation Process

The foreign inflation rate Δpf_t is determined by simultaneous interaction between the reaction of the monetary authority to economic developments and the formation of inflation expectations. This is captured by the following reduced-form equation:

$$\text{(F.2)} \quad \Delta pf_t = \beta_1(yf^c_t + yf^c_{t-1}) + \beta_2\Delta pf_{t-1} + \beta_3\Delta pf^e_{t+4} + (1-\beta_2-\beta_3)\Delta pf^{ess}_t + \beta_6\Delta rpe^s_t + \varepsilon f_{2t}$$

where Δpf_{t-1} is the inflation rate in the previous quarter, Δpf^e_{t+4} is the expected inflation rate over the coming year and Δpf^{ess}_t is the expected steady-state inflation rate, Δrpe^s_t represents "smoothed" changes in the real price of energy and $\varepsilon f_{2t} \sim (0, \sigma_{f2}^2)$ is a random error term.

Changes in the relative price of energy influence inflation with a long lag. The dynamic adjustment process is modelled using a geometric lag structure, specified as follows:

$$\Delta rpe^s_t = \lambda_3\Delta rpe^s_{t-1} + (1-\lambda_3)\Delta rpe_t$$

where the parameter λ_3 determines the speed of adjustment. The geometric lag structure "smoothes" fluctuations in the relative price of energy so that inflation responds gradually to energy price shocks.

Nominal rigidities underlying the inflation equation do not have explicit micro-foundations. Instead, we pursue a reduced-form approach to modelling "persistence" in the inflation process. The parameter β_2 represents the degree to which current inflation is influenced by past inflation Δpf_{t-1}. This captures persistence in inflation that arises from *intrinsic* sources (wage contracts or menu-cost pricing[5]) as well as backward-looking (adaptive) expectations. The parameter β_3 represents the degree to which inflation is influenced by expected inflation over the coming year Δpf^e_{t+4}. We impose the parameter restriction $(1-\beta_2-\beta_3)$ on the expected steady-state inflation rate Δpf^{ess}_t to ensure that inflation and expectations converge to the target rate in the long run.

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[5] Mankiw and Romer (1991) review a variety of models with New-Keynesian rigidities built on costly price adjustment and staggered wage/price setting.

Inflation expectations are determined by equations of the general form:

$$\Delta pf^{e}_{t+i} = \psi(1/i) \sum_{j=1}^{i} E_t \Delta pf_{t+j} + (1 - \psi)\Delta pf_t$$

where $E_t \Delta pf_{t+j}$ represents the model-consistent forecast of inflation in period j. The parameter ψ combines backward-looking and forward-looking elements of inflation expectations.[6] This specification nests *adaptive* expectations ($\psi = 0$) and *rational* (forward-looking model consistent) expectations ($\psi = 1$) as special cases. In the long run, inflation is anchored by the expected steady-state rate, determined by:

$$\Delta pf^{ess}_{t} = \theta \Delta pf^{*} + (1-\theta)\Delta pf^{e}_{t+80} \qquad (0 \leq \theta \leq 1)$$

where Δpf^{*} represents the inflation target and Δpf^{e}_{t+80} is the average expected inflation rate over the coming 20 years.

The parameter θ provides a simple mechanism for varying the credibility aspect of monetary policy. In the case where $\theta = 1$, agents are certain that inflation will eventually converge to the target level. This can be interpreted as a situation in which monetary policy has a high degree of credibility. In the more general case where $0 < \theta < 1$, long-run inflation expectations are influenced by the average expected outcome over a 20 year horizon. This captures the idea that persistent deviations in inflation from the target level may cause agents to doubt whether the monetary authority is able (or willing) to contain fluctuations in inflation. Because inflation expectations have both backward- and forward-looking components, the average expected inflation Δpf^{e}_{t+80} is determined by past performance as well as model-consistent expectations about future developments. Increasing the value of θ can be interpreted as enhanced credibility of monetary policy. This approach to modelling credibility is admittedly simplistic; nonetheless it serves the purpose of exploring the importance of credibility. Ideally, credibility should be determined as an endogenous outcome conditional on inflation expectations and the monetary policy rule in place. We leave this for future research.

[6] This approach to modelling inflation expectations, originally proposed by Buiter and Miller (1985), has been incorporated into several models that have been designed to investigate monetary policy rules.

Interest rates

The Fisher identity is used to decompose nominal interest rates into "real" and "expected inflation" components. This can be represented in the general case as:

$$if^i_t = rf^i_t + \Delta pf^e_{t+i}$$

where if^i_t and rf^i_t represent the nominal and real yield on an "i-period" bond and Δpf^e_{t+i} is the expected inflation rate over the coming "i-periods". Real interest rates are decomposed into cyclical and permanent components:

$$rf^i_t = rf^{1p}_t + \kappa^i + rf^{ic}_t$$

where rf^i_t is the real yield on an "i-period" bond, rf^{ic}_t is the cyclical component, and rf^{1p}_t is the permanent component of the real yield on a one-period bond (3-month treasury bills). Real yields on bonds with different maturities differ by a constant term premium κ^i. For example, the real yield on a 20-year government bond is 100 basis points above that on a one-period bond (κ^{80}=1.0).[7]

The permanent component of the one-period real interest rate evolves stochastically over time according to:

(F.3) $$rf^{1p}_t = rf^{1p}_{t-1} - \rho_3(rf^{1p}_{t-1} - rf^{1p*}) + \varepsilon f_{3t}$$

where rf^{1p*} is the equilibrium short-term foreign real interest rate[8] and $\varepsilon f_{3t} \sim (0, \sigma_{f3}^2)$ is a random error term. Permanent components of real interest rates across the term structure move in tandem with stochastic fluctuations in rf^{1p}_t (the levels differ by constant term premia). Cyclical components are determined by the expectations hypothesis of the term structure discussed below.

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[7] Term premia range from 10 basis points for 6-month treasury bills (κ^2=0.1) to 100 basis points on 20-year government bonds (κ^{80}=1.0), relative to the benchmark yield on 3-month treasury bills (κ^1=0).

[8] The equilibrium foreign short-term real interest rate rf^{1p*} is set to 2.8 per cent in the base case model.

Monetary Policy

The monetary authority in the foreign sector is modelled using a simple policy rule of the following form:

(F.4) $$rf^{c}_{1t} = \gamma_1[E_t(pf_{t+3} - pf_{t-1}) - \Delta pf^*] + \gamma_2 rf^{c}_{1t-1} + \varepsilon f_{4t}$$

where $E_t(pf_{t+3} - pf_{t-1})$ represents the expected inflation rate over the coming year and Δpf^* is the inflation target rate. Monetary policy is set in a forward-looking manner using model-consistent forecasts of inflation. The autoregressive term is intended to capture the "interest rate smoothing" aspect of setting monetary policy. High (position) values of the parameter γ_2 act to dampen quarterly movements in short-term interest rates, which delays the monetary policy response.[9] This is often motivated by the contention that the monetary policy seeks to preserve orderly financial markets by limiting interest rate and exchange rate volatility.[10] The random error term $\varepsilon_{4t} \sim (0, \sigma_{f4}^2)$ reflects the idea that the monetary authority cannot control short-term interest rates with certainty.

Although the monetary policy reaction function (F.4) is specified with reference to the short-term real interest rate rf^{c}_{1t}, one can nonetheless interpret the short-term nominal interest rate if^{c}_{1t} as the "instrument" of monetary policy. This would entail substituting the Fisher identity into the reaction function (F.4) so that the monetary authority sets the nominal interest rate if^{c}_{1t} to influence real interest rate rf^{c}_{1t}.

It is important to note that the monetary policy rule (F.4) is intended to illustrate the properties of the model.[11] The model has desirable simulation properties under a wide range of alternative monetary policy rules. For example, the model can be used to examine the implications of setting monetary policy in a backward-looking versus a

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[9] See Taylor (1999b) and Sack and Wieland (1999).

[10] Sack and Wieland (1999) show that "interest rate smoothing" can be a feature of an optimal monetary policy rule even if the objective of the monetary authority is defined in terms of minimizing the variability of only output and inflation.

[11] The monetary policy reaction function (F.4) is referred to as an *inflation-forecast* rule in the literature, according to the terminology used by Svensson (1999) and Rudebusch and Svensson (1999).

forward-looking manner.[12] The model can also be used to examine the implications of taking the output gap into account in setting monetary policy (so-called "Taylor rules).[13]

The Term Structure of Interest Rates

The term structure of interest rates is modelled using the expectations hypothesis with constant term premia. This can represented in the general case as follows:

$$\text{(F.5)} \qquad rf_{it} = \varphi[(1/i) \sum_{j=1}^{i} E_t(rf_{1t+j}) + \kappa^i] + (1-\varphi)rf_{it-1} + \kappa^i \varepsilon f_{5t} + (1 - \kappa^i)\varepsilon f_{4t}$$

where rf_{it} is the expected real yield on an "i-period" discount bond, $E_t(rf_{1t+j})$ is the expected real yield on one-period bonds "j-periods" in the future and $\varepsilon f_{5t} \sim (0, \sigma_{f5}^2)$ is a random error term.

The parameter φ combines forward- and backward-looking components of expectations (as in the case of inflation expectations). In the special case where $\varphi = 1$, the expected real yield on an "i-period" bond is determined by model-consistent forecasts (the rational expectation) of future real yields on the one-period bond plus a constant term premium κ^i. In the special case where $\varphi = 0$, expectations of real yields are static. Note that the formation of inflation expectations used to define real interest rates can differ from those used in the inflation equations. This enables us to consider the possibility that inflation expectations in financial markets are more (or less) forward-looking than those undelrying the wage/price setting process.

The random error terms in the term structure equation (F.5) are motivated by the common finding that empirical tests reject the expectations hypothesis of the term structure under rational expectations. For example, Campbell and Shiller (1991) find that

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[12] In the case of "forward-looking" policy rules, monetary policy is set with reference to model-consistent expectations whereas "backward-looking" rules are based on the most recent inflation rate (which is typically reported with a lag). Batini and Haldane (1999) and Levin et al. (1999a) compare the performance of "forward-looking" versus "backward-looking" monetary policy rules.

[13] Hostland (1999) examines how alternative monetary policy rules of this nature influence medium-term fiscal planning. For a more complete discussion of issues relating to monetary policy rules, see Taylor (1999a and 1999b).

the expectations hypothesis can only account for about half of the movements in the spread between long- and short-term interest rates (measured by the yields on 10-year U.S. government bonds versus 3-month U.S. treasury bills). Subsequent empirical work by Fuhrer (1995) shows that allowing for changes in monetary policy greatly improves the explanatory power of the expectations hypothesis.

Most studies find that long-term interest rates fluctuate by more than what is predicted by the expectations hypothesis with rational expectations. Two (mutually-independent) error terms εf_{4t} and εf_{5t} are introduced to make the model more consistent with this empirical finding. The error term εf_{4t} gives rise to stochastic variation in yields at the short-end of the term structure, while the other error term εf_{5t} gives rise to stochastic variation in yields at the long-end. Real yields along the term structure combine the two sources of stochastic variation. For example, stochastic variation in the real yield on a one-year bond is determined as a weighted average of the error terms εf_{4t} and εf_{5t} ($\kappa^4 * \varepsilon f_{5t} + (1-\kappa^4) * \varepsilon f_{4t}$ where $\kappa^4 = 0.2$).

2.2 The Domestic Sector

Aggregate supply/demand dynamics

The domestic sector of the model is essentially an expanded open-economy version of the foreign sector. The cyclical component of output y_t (the "output gap") is modelled using the following reduced-form equation:

$$(1) \qquad y^c_t = (\alpha_1+\alpha_2)y^c_{t-1} + \alpha_1\alpha_2 y^c_{t-2} + \alpha_3 r^c_t + \alpha_4 q^c_t + \alpha_5 rpc^c_t + \alpha_6 pbal^c_t + \alpha_7 yf^c_t + \alpha_8 \Delta yf^c_{t-1} + \varepsilon_{1t}$$

where r^c_t is the cyclical component of the real cost of funds, q^c_t is the cyclical component of the real exchange rate, rpc^c_t is cyclical component of the relative price of (non-energy) commodities, $pbal^c_t$ is the cyclical component of the primary budget balance, yf^c_t is the cyclical component of the foreign output and $\varepsilon_{1t} \sim (0, \sigma_1^2)$ is a random error term.

Equation (1) captures the dynamic response of aggregate demand shocks to cyclical movements in real interest rates, the real exchange rate, relative commodity prices, the stance of fiscal policy and foreign aggregate demand conditions. A transitory increase in the real cost of funds r^c_t causes output to decline relative to its potential level while a transitory increase in the real exchange rate (a depreciation), the relative price of (non-energy) commodities and foreign output cause output to rise relative to its potential level. Fiscal policy influences aggregate demand through liquidity constraints. Output contracts (expands) when the primary balance (as a proportion of potential output) moves toward a surplus (deficit) position.

The trend component of output y^p_t (potential output) evolves over time as a random walk with a time-varying drift component μ_t:

(2) $$\Delta y^p_t = \mu_t + \varepsilon_{2t}$$

where $\varepsilon_{2t} \sim (0, \sigma_2^2)$ is a random error term. The projected profile of the time-varying drift component μ_t declines gradually from a rate of 2.9 per cent in 2000 to 1.5 per cent in 2050. This is intended to capture the expected decline in labour market participation rates associated with the impending demographic trends.

The cyclical component of the real exchange rate is decomposed into trend and cyclical components:

$$q_t = q^t_t + q^c_t$$

where the trend component q^t_t is defined as the steady-state value:

$$q^t_t = E_t q_{t+40}$$

where $E_t q_{t+40}$ represents the model-consistent forecast 10 years in the future.

The relative price of (non-energy) commodities is decomposed into trend and cyclical components:

$$rpc_t = rpc^t_t + rpc^c_t$$

where the trend component rpc^t_t evolves as a geometric lag of observed values:

$$rpc^t_t = \lambda_4 rpc^t_{t-1} + (1-\lambda_4) rpc_t$$

In theory, permanent changes in the real interest rate, the real exchange rate and relative price of (non-energy) commodities affect the level of potential output. These supply-side linkages are not explicit in the model, however. They are presumed to be (unobserved) components of the stochastic process generating potential output.

The Inflation Process

Inflation is captured using the following two reduced-form equations:

$$(3) \qquad \Delta pc_t = \beta_1(y^c_t + y^c_{t-1}) + \beta_2\Delta pc_{t-1} + \beta_3 E_t\Delta pe_{t+4} + (1\text{-}\beta_2\text{-}\beta_3)\Delta p^{ess}_t + \varepsilon_{3t}$$

$$(4) \qquad \Delta pp_t = \beta_1(y^c_t + y^c_{t-1}) + \beta_2\Delta pp_{t-1} + \beta_3 E_t\Delta pe_{t+4} + (1\text{-}\beta_2\text{-}\beta_3)\Delta p^{ess}_t + \beta_4\Delta q^s_t + \beta_5\Delta rpc^s_t + \beta_6\Delta rpe^s_t + \varepsilon_{4t}$$

where Δpc_t is "core" consumer price (CPI excluding food, energy and indirect taxes) inflation, Δpp_t is producer price (GDP deflator) inflation, Δp^{ess}_t is the expected inflation rate in the long run and $\varepsilon f_{3t} \sim (0, \sigma_3^2)$ and $\varepsilon f_{4t} \sim (0, \sigma_4^2)$ are random error terms.

Changes in the real exchange rate and the relative price of commodities influence inflation with long lags. This is captured using geometric lag structures as follows:

$$\Delta q^s_t = \lambda_1\Delta q^s_{t-1} + (1\text{-}\lambda_1)\Delta q_t$$

$$\Delta rpc^s_t = \lambda_2\Delta rpc^s_{t-1} + (1\text{-}\lambda_2)\Delta rpc_t$$

$$\Delta rpe^s_t = \lambda_3\Delta rpe^s_{t-1} + (1\text{-}\lambda_3)\Delta rpe_t$$

where the parameters λ_1, λ_2 and λ_3 determine the speed of adjustment. As mentioned earlier, the geometric lag structures "smooth" fluctuations in relative prices. Hence, Δq^s_t, Δrpc^s_t and Δrpe^s_t represent "smoothed" changes in the real exchange rate, the relative price of (non-energy) commodities, and the relative price of energy, respectively.

Changes in the real exchange rate have a direct effect on producer price inflation Δpp_t, but only affect core inflation Δpc_t indirectly through inflation expectations. This is partly due to the fact that core inflation excludes food and energy, which make up a substantial proportion of imported goods in the CPI. Passthrough from exchange rate changes to the CPI excluding food and energy is much lower than for the total CPI. Exchange rate changes "passthrough" directly into producer price inflation and thereby affect inflation expectations. (Inflation expectations are based on an average of core inflation and producer price inflation.) The magnitude and timing of exchange rate passthrough to core inflation is largely determined by expectations, which are influenced by several factors including the monetary policy rule in place. We view this as a desirable feature of the model.

Inflation expectations are generated along the same lines as in the foreign sector. Expectations over the short- and medium-run are determined by:

$$\Delta p^{e}_{t+i} = \psi(1/i) \sum_{j=1}^{i} E_t\Delta p_{t+j} + (1 - \psi)\, \Delta p_t$$

where Δp_t is an average of core inflation Δpc_t and producer price inflation Δpp_t and $E_t\Delta p_{t+j}$ represents the model-consistent forecast in period j. Expectations over the long run are anchored by the expected steady-state rate inflation rate Δp^{ess}_t, determined by:

$$\Delta p^{ess}_t = \theta\Delta p^* + (1-\theta)\Delta p^e_{t+80} \qquad (0 \leq \theta \leq 1)$$

where Δp^* represents the inflation target and Δp^e_{t+80} is the average expected inflation rate over the next 20 years.

Interest rates

Interest rates are determined along the same lines as in the foreign sector outlined above. The Fisher condition is used to decompose nominal interest rates into “real” and “expected inflation” components:

$$i_{it} = r_{it} + E_t\,(4/i)(p_{t+i} - p)$$

Real interest rates are decomposed into permanent and cyclical components:

$$r^i_t = r^{1p}_t + \kappa^i + r^{ic}_t$$

where r^i_t is the real yield on an “i-period” bond, r^{ic}_t is its cyclical component, r^{1p}_t is the permanent component of the real yield on one-period bonds and κ^i is a constant term premium.

The permanent component of the real yield on one-period bonds r^{1p}_t differs from its foreign counterpart rf^{1p}_t, by a constant exchange rate risk premium κ:[14]

$$r^{1p}_t = rf^{1p}_t + \kappa$$

Cyclical components of real interest rates are determined by the expected real interest rate in the future as implied by the expectations hypothesis of the term structure.

Monetary Policy

As mentioned earlier, the model can be simulated using a wide variety of monetary policy rules. For the purposes of this paper we will illustrate the properties of the model using a simple policy rule of the form:

$$r^c_{1t} = \gamma_1[E_t(pc_{t+3} - pc_{t-1}) - \Delta p^*] + \gamma_2 rf^c_{1t} + \varepsilon_{5t} \tag{5}$$

where $E_t(pc_{t+3} - pc_{t-1})$ represents the model-consistent forecast of *core* inflation over the coming year and Δp^* is the inflation target rate. The autoregressive term captures the "interest rate smoothing" role of monetary policy and ε_{5t} is the random error term that represents the uncertainty surrounding the monetary authority's control over its instrument – the short-term nominal interest rate.

The Term Structure of Interest Rates

The term structure of interest rates is modelled using the expectations hypothesis with constant term premia, which can be represented by:

$$r_{it} = \varphi[(1/i) \sum_{j=1}^{i} E_t(r_{1t+j}) + \kappa^i] + (1-\varphi)r_{it-1} + \kappa^i\varepsilon_{6t} + (1 - \kappa^i)\varepsilon_{5t} \qquad (0 \leq \varphi \leq 1) \tag{6}$$

where r_{it} represents the expected real yield on an "i-period" discount bond, $E_t(r_{1t+j})$ is the model-consistent forecast of the real yield on one-period bonds "j-periods" in the future and $\varepsilon_{5t} \sim (0, \sigma_5^2)$ and $\varepsilon_{6t} \sim (0, \sigma_6^2)$ are random error terms.

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[14] To clarify the notation, κ is a constant *exchange rate premium* that accounts for a permanent differential in real yields on domestic and foreign bonds, whereas κ^i represents a constant *term premium* that accounts

The cyclical component of the real cost of funds r^c_t is defined as:

$$r^c_t = \nu r^{1c}_t + (1-\nu)r^{80c}_t$$

where r^{1c}_t and r^{80c}_t represent the cyclical components of the real yield on one-period and 80-period (20-year) discount bonds, respectively.

The Exchange Rate

There is a great deal of uncertainty about which economic factors influence the exchange rate. This presents a thorny problem for macro modellers. We consider two alternative approaches to exchange rate determination. This enables us to examine the robustness of simulation results in light of the high degree of uncertainty surrounding the determination of the exchange rate. One approach imposes the uncovered interest rate parity condition on the model. The other approach specifies exchange rate movements using an estimated reduced-form equation.

Uncovered Interest Rate Parity

The uncovered interest rate parity condition is applied to the real exchange rate:

$$\text{(7a)} \qquad \Delta q^e_{t+1} = (r^1_t - rf^1_t - \kappa)/4 + \varepsilon_{7t}$$

where Δq^e_{t+1} is the expected change in the real exchange rate,[15] $(r^1_t - rf^1_t - \kappa)$ is the domestic-foreign real interest rate differential that takes into account a constant exchange rate premium κ and $\varepsilon_{7t} \sim (0, \sigma_7^2)$ is a random error term. Expectations are modelled using a combination of backward- and forward-looking components given by:

$$q^e_{t+1} = \varphi E_t q_{t+1} + (1-\varphi)q_{t-1} \qquad (0 \le \varphi \le 1)$$

where $E_t q_{t+1}$ is the model-consistent forecast of the real exchange rate in the coming period and q_{t-1} is the lagged value. Note that the uncovered interest rate parity condition can be imposed without using fully forward-looking model-consistent (rational)

for the differential in real yields on "i-period" bonds relative to the benchmark one-period bond in the domestic and foreign sectors.

[15] To clarify, q_t represents the (log of the) price of foreign exchange (expressed in real terms) so that an increase in q_t implies a real exchange rate depreciation.

expectations (by setting $\varphi < 1$).[16] The expectations formation mechanism used to generate the uncovered parity condition is however consistent with that used to generate the expectation hypothesis of the term structure.[17] This ensures that uncovered interest rate parity holds across the entire term structure of interest rates.

Reduced-Form Equation

Following Amano and van Norden (1993), we model the real exchange rate q_t using a reduced-form equation of the form:

(7b) $$\Delta q_t = \chi_1 \Delta q_{t-1} + \chi_2 \Delta rpc_t + \chi_3 (q_{t-1} - \chi_4\, rpc_{t-1}) + \chi_5 (r^1{}_t - rf^1{}_t - \kappa) + \varepsilon_{7t}$$

where rpc_t is the relative price of (non-energy) commodities and $\varepsilon_{7t} \sim (0, \sigma_7{}^2)$ is a random error term. The error correction term $(q_{t-1} - \chi_4\, rpc_{t-1})$ ensures that changes to the relative price of (non-energy) commodities have a permanent effect on the real exchange rate in the long. The original research by Amano and van Norden (1993), subsequently updated by Lafrance and van Norden (1995), found that the changes in real energy prices also had a permanent effect on the real exchange rate. This finding was not supported by our empirical work and hence, real energy prices were excluded.

Commodity prices

The relative price of (non-energy) commodity prices and energy prices, rpe_t and rpe_t respectively, are each modelled as integrated first-order autoregressive processes:

(8) $$\Delta rpc_t = \rho_1 \Delta rpc_{t-1} + \varepsilon_{8t}$$

(9) $$\Delta rpe_t = \rho_2 \Delta rpe_{t-1} + \varepsilon_{9t}$$

where $\varepsilon_{8t} \sim (0, \sigma_8{}^2)$ and $\varepsilon_{9t} \sim (0, \sigma_9{}^2)$ are random error terms.

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[16] Bryant (1995) provides some empirical support for this approach.

[17] This is implied by our notation. Expectations of future interest rates and exchange rates are formed using a combination of backward- and forward-looking combinations determined by the parameter φ.

2.3 The Fiscal Framework

Fiscal accounting

Our objective is not to produce a detailed account of the various components that make up the fiscal balance sheet. Instead, the model is designed to provide a broad summary of how changes in economic conditions impact on fiscal revenues and expenditures at the aggregate level. For this purpose, we aggregate budgetary revenues from all sources (personal income tax, corporate income tax, employment insurance premiums and excise taxes and duties), except earnings on investments. Earnings on investments are subtracted from (gross) debt service payments to produce a net measure of debt service payments that corresponds to net federal debt (outstanding federal debt less investment assets held by the federal government). On the expenditure side, we focus on total program spending which includes transfers to persons and other levels of government as well as direct program spending.

Net public debt and all fiscal expenditures and revenues are defined on a national accounts basis. Our use of national accounts, rather than public accounts, fiscal measures largely reflects the fact that national accounts fiscal data are readily available on a quarterly basis beginning in 1961. This avoids difficulties entailed in attaining reliable public accounts fiscal data over the historical period. The fiscal accounting underlying the model is specified at both the federal and provincial levels. We will concentrate our discussion on the basic accounting relationships at the federal level only to simplify presentation. Federal expenditures and revenues are defined to exclude contributions and outlays associated with public sector pension plans and the Canada/Québec pension plans.[18] This enables us to abstract from issues pertaining to public pension plans and focus on the medium-term fiscal planning aspect of the federal budget process.

[18] Moreau (2000) illustrates how the measures of the budget balance and net public debt are affected by including contributions and outlays associated with public pension plans.

The basic fiscal accounting relationships used are summarized as follows. The operating budget balance B, defined as total revenues REV less program spending PS and debt service payments DS, is equal to the change in net public debt D:[19]

$$B_t = REV_t - PS_t - DS_t = 4*(D_t - D_{t-1})$$

We will first discuss how debt service payments DS_t are determined in the model before turning to total revenues REV_t and finally, program spending PS_t.

Debt Service Costs

Debt service payments are analyzed with reference to the implicit interest rate on net federal debt and the stock of net federal debt. This can be calculated over the historical period using the following identity:

$$i^d_t = DS_t / D_t$$

where i^d_t is the implicit interest rate on net federal debt, DS_t is the value of debt service payments made in period "t", and D_t is the value of outstanding net federal debt.

The implicit interest rate on net federal debt is endogenous in the simulation model. In order to capture the exact relationship between movements in interest rates, net debt and debt service costs, one would have to keep track of yields on all financial assets and liabilities held by the federal government at each point in time. We make a number of simplifying assumptions to reduce the computational burden.

We abstract from complications associated with having foreign currency-denominated debt, retail debt instruments[20] and investment income earned on financial assets held by the federal government. We also assume that the maturity structure of public debt is constant. The implicit interest rate on net federal debt is calculated as a

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[19] The change in net federal debt is scaled by a factor of 4 because the quarterly budget balance is measured at annual rate to be consistent with the convention used in constructing national accounts data.

[20] Canada Savings Bonds (CSBs) and other retail debt instruments currently make up only about six per cent of outstanding Government of Canada securities. The complicating feature of CSB's is that they can be redeemed before maturity. In order to capture this in our simulation model, we would have to model how unanticipated increases in interest rates lead to CSB redemptions, which is beyond the scope of our analysis.

weighted average of current and lagged interest rates on government securities with different maturity dates. The weights reflect two factors. First, new debt issues are financed by bonds with different maturities. Second, interest rate payments are made on outstanding bonds that were issued at different dates in the past. We approximate the maturity structure of federal debt by focusing on eight maturity categories ranging from one quarter (90-day treasury bills) to 20 years. The implicit interest rate on net federal debt i^d_t is calculated as follows:

$$i^d_t = \omega_1 i^1_t + \omega_2 (1/2)(i^2_t + i^2_{t-1}) + \omega_4 (1/4)(i^4_t + i^4_{t-1} + i^4_{t-2} + i^4_{t-3})$$

$$+ \omega_8 (1/8) \sum_{j=1}^{8} i^8_{t-j-1} + \gamma_{12} (1/12) \sum_{j=1}^{12} i^{12}_{t-j-1} + \omega_{20} (1/20) \sum_{j=1}^{20} i^{20}_{t-j-1}$$

$$+ \omega_{40} (1/40) \sum_{j=1}^{40} i^{40}_{t-j-1} + \omega_{80} (1/80) \sum_{j=1}^{80} i^{80}_{t-j-1}$$

where i_t^k represents the (when issued) yield on a k-period bond issued in period “t” and the weights ω_i represent the proportion of outstanding bonds of each maturity k.

The logic underlying the above equation is as follows. One-period bonds mature each period so that the implicit interest rate on one-period bonds is simply the current short-term interest rate i^1_t. Half of the two-period bonds mature each period so that the implicit interest rate on two-period bonds is given by the average yield on two-period bonds issued in the current period and in the previous period $(1/2)(i^2_t + i^2_{t-1})$. In the case of a 20-year bond, only one of eighty mature each period so that the implicit interest rate is the average yield on those bonds issued in the current period and those issued in each of the previous 79 periods. The parameters ω_i are calibrated such that 35 per cent of the federal debt matures within the forthcoming four quarters, which is consistent with the federal debt management strategy.

Total Revenues and Program Spending

Total revenue as a proportion of GDP moves in a pro-cyclical manner with a stochastic component given by:

(10) $$REV_t / YGDP_t = \upsilon_1 y^c_t + \varepsilon_{10t}$$

where YGDP is nominal GDP and $\varepsilon_{10t} \sim (0, \sigma_{10}^2)$ is a random error term. The pro-cyclical element is consistent with the empirical observation that the average effective tax rate (total tax revenues as a proportion of GDP) tends to rise slightly during economic expansions and decline slightly during contractions. This reflects the progressive aspect of the personal income tax system as well as the fact that corporate profits have a stronger cyclical component than GDP. The stochastic element is intended to capture the uncertainty surrounding tax revenues for given levels of economic activity. This reflects the idea that tax elasticities have a stochastic element and hence, projected tax revenues would involve errors even if economic conditions were known.

Program spending is decomposed into a cyclical and permanent component:

$$PS_t = PSC_t + PSP_t$$

The cyclical component of program spending PSC_t moves in a counter-cyclical manner with a stochastic component:

(11) $$PSC_t = \upsilon_2 y^c_t + \varepsilon_{11t}$$

where $\varepsilon_{11t} \sim (0, \sigma_{11}^2)$ is a random error term. The counter-cyclical element of program spending PSC_t is intended to capture the automatic stabilisation properties of various spending programs (mainly contributions to and benefits drawn from the employment insurance program). The stochastic component represents the uncertainty surrounding fiscal expenditures at given levels of interest rates and economic activity.

The permanent component of program spending PSP_t represents the discretionary component of fiscal policy, which is determined by a fiscal policy rule.

Fiscal Planning Strategies

The model is designed to evaluate alternative strategies for fiscal planning. We illustrate the main features of the model using the case where the fiscal authority aims to keep the debt-to-GDP ratio on a clear, downward profile in the presence of uncertainty about future economic and fiscal developments. The fiscal authority in the model plans a budget at the beginning of each fiscal year. After the budget is set, stochastic shocks impact on program spending, tax revenue and debt service, causing the debt-to-GDP ratio and the budget balance to diverge from projected levels. In the subsequent fiscal year, the fiscal authority reformulates its budget plans taking into account economic and fiscal developments. Under a *flexible debt rule*, the fiscal authority sets discretionary spending (PSP) such that the projected level of the debt-to-GDP ratio gradually reverts to the mid-point of the target range (over a period of about two or three years).[21] This fiscal planning strategy is implemented in the model using a forward-looking policy rule of the following form:

$$(b_t - b^*) = \tau(E_t d_{t+1} - d^*) \tag{12}$$

where $(b_t - b^*)$ represents the deviation of the budget balance from its target level (expressed as proportions of GDP) and $(E_t d_{t+1} - d^*)$ represents the expected deviation of the debt-to-GDP ratio from the mid-point of the target range in the coming fiscal year.[22]

Although the fiscal policy rule (12) is specified in terms of the budget balance-to-GDP ratio b_t, discretionary spending PSP is the instrument of fiscal policy. The fiscal authority in the model sets discretionary spending PSP such that the b_t is consistent with the fiscal policy rule (12). This conveys the idea that discretionary changes to program spending and/or taxes are required in order to offset unanticipated changes in the debt-to-GDP ratio. For instance, in the case where the debt-to-GDP ratio rises unexpectedly, the fiscal authority would have to reduce program spending and/or raise taxes in order to offset the increase in the debt-to-GDP ratio. The forward-looking nature of the policy rule captures the idea that the medium-term fiscal plan is based on economic and fiscal projections over the coming year.

[21] Hostland and Matier (2001) and Georges (2001) examine alternative fiscal planning strategies.

[22] The target level for the budget balance is simply the flow dimension of the mid-point of the debt-to-GDP target range.

3. Model Specification

Calibration Methodology

The forward-looking nature of interest rates, the exchange rate, the monetary policy rule and the fiscal planning strategy in the model prevents us from applying standard econometric methods. Technically, we could apply advanced econometric methods that have been developed for estimating models with rational expectations. These methods would be very difficult to implement in practice, however, because we would need to know the policy rules and planning strategies in place over the historical period. Monetary policy in Canada has not been implemented with reference to a simple policy rule. Explicit inflation targets came into effect in 1992, so we have relatively few observations for estimating a monetary policy rule. Moreover, some monetary policy developments that have occurred since 1992 cannot be captured by a simple rule.[23]

Similar problems arise in modeling the fiscal planning strategy over the historical period. Eliminating the deficit and reducing public debt have been major objectives of fiscal policy in Canada since the mid-1990s. The 1994 federal budget aimed to reduce the deficit/GDP ratio to 3 per cent in 1996-97. *Interim* deficit/GDP targets were lowered in the two subsequent budgets to 2 per cent in 1997-98 and 1 per cent in 1998-99. The 1998 federal budget introduced the *Debt Repayment Plan*, which aims to keep the debt/GDP ratio on a “clear, downward profile”. Aside from these few deficit/debt reduction initiatives, there have been no explicit targets that would enable us to estimate a simple policy rule.

Given the difficulties entailed in specifying a monetary policy rule and fiscal planning strategy over the historical period, we do not estimate the model with reference to explicit policy rules. Instead, we follow a model specification procedure that combines estimation and calibration methods. Some of the parameters are calibrated with reference to estimated reduced-form relationships that are reported in the appendix. These estimates

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[23] It would be particularly difficult to specify a simple policy rule that can capture the monetary policy response to international financial developments such as the “Mexican peso” crisis in 1995 and the Asian crisis in 1997.

are summarized in Table 1 below. We report “benchmark” values for each parameter along with a range of “high” and “low” parameter settings. The “benchmark” values correspond to what we refer to as the “benchmark” version of the model. The ranges for the parameters enable us to perform sensitivity analysis of the “benchmark” in a systematic manner. This will be illustrated in the following section of the paper.[24]

[24] The parameter ranges can also be used in stochastic simulation experiments to examine the implications of parameter uncertainty. See Dupuis and Hostland (2000) for research along these lines.

Table 1: Parameters based on Reduced-Form Estimates

The Foreign Sector

Equation	*Parameter*	*Low*	*Benchmark*	*High*
F.1	α_1	0.70	0.75	0.80
F.1	α_2	0.50	0.65	0.80
F.1	α_3	-0.40	-0.30	-0.20
F.2	β_1	0.10	0.15	0.20
F.2	β_2	0.20	0.30	0.40
F.2	β_3	0.30	0.40	0.50
F.2	β_6	0.00	0.01	0.02
F.3	ρ_3	0.00	0.10	0.20
F.4	γ_1	0.50	0.75	1.00
F.4	γ_2	0.70	0.75	0.80
F.5	φ	0.50	0.75	1.00
	ψ	0.50	0.75	1.00
	ν	0.00	0.25	0.50
	θ	0.00	0.25	0.50

The Domestic Sector

Equation	*Parameter*	*Low*	*Benchmark*	*High*
(1)	α_1	0.70	0.75	0.80
(1)	α_2	0.50	0.65	0.80
(1)	α_3	-0.15	-0.1	-0.05
(1)	α_4	0.00	0.05	0.1
(1)	α_5	0.00	0.01	0.02
(1)	α_7	0.025	0.05	0.075
(1)	α_8	-0.20	-0.10	0.00
(3) & (4)	β_1	0.10	0.15	0.20
(3) & (4)	β_2	0.20	0.30	0.40
(3) & (4)	β_3	0.30	0.40	0.50

Table 1 continued

Equation	*Parameter*	*Low*	*Benchmark*	*High*
(4)	β_4	0.00	0.05	0.10
(4)	β_5	0.00	0.05	0.10
(4)	β_6	0.00	0.01	0.02
(5)	γ_1	0.50	0.75	1.00
(5)	γ_2	0.70	0.75	0.80
(6)	φ	0.50	0.75	1.00
(7b)	χ_1	0.30	0.35	0.4
(7b)	χ_2	-0.15	-0.10	-0.05
(7b)	χ_3	-0.30	-0.25	-0.20
(7b)	χ_4	-0.60	-0.50	-0.40
(7b)	χ_5	-0.90	-0.75	-0.60
(8)	ρ_1	0.00	0.10	0.20
(9)	ρ_2	0.00	0.10	0.20
(10)	υ_1	-0.45	-0.35	-0.25
(11)	υ_2	0.00	0.05	0.10
	ψ	0.50	0.75	1.00
	ν	0.25	0.5	0.75
	θ	0.00	0.25	0.50
	λ_1, λ_2, λ_3	0.50	0.60	0.70
	λ_4	0.60	0.70	0.80

The remaining parameters are calibrated using the *method of simulated moments.*[25] The basic intuition underlying this method is to "fit the model to the data" by matching moments of variables simulated by the model to empirical moments observed in the data. The statistical aspect of this methodology can be illustrated with reference to the following general representation of the model:

$$Y_t = f(X_t, \beta) + \varepsilon_t$$

where Y_t is a vector of endogenous variables, X_t is a vector of predetermined variables, $f(X_t, \beta)$ represents the non-linear functional form of the model and ε_t is a vector of random error terms with covariance matrix Ω. Elements of the covariance matrix Ω are calibrated such that second moments (variances) of variables simulated by the model match as close as possible second moments (unconditional variances) of variables calculated over the historical period.[26]

The calibration procedure is implemented as follows. Stochastic simulations are performed using the "benchmark" parameters β reported above in Table 1 along with initial values of the covariance matrix Ω. The benchmark version of the model assumes that most of the underlying "shocks" are mutually uncorrelated.[27] This orthogonality assumption enables us to focus on the diagonal elements of the covariance matrix Ω^{ii} (conditional variances). We can allow for more general covariance structures in the context of sensitivity analysis. The conditional variances Ω^{ii} are jointly set to minimize the following objective function:

$$\sum_{i=1}^{n} [\ \mathrm{Var}(Y_i) - \mathrm{Var}(Y^s{}_i)\ ]$$

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[25] For a more complete discussion of the method of simulated moments, see Gouriéroux and Monfort (1997) and the references therein.

[26] The calibration methodology is not applied to the first moments of variables because the simulated series generated by the model are centred on their equilibrium values regardless of the parameters used.

[27] The error terms in the two domestic inflation equations (3) and (4) are calibrated to have a correlation of 0.25, as implied by the reduced-form estimates reported in the appendix.

where $Var(Y_i)$ is the unconditional variance of selected endogenous variables and $Var(Y^s_i)$ represents the variance of its simulated counterpart.[28] This is implemented using an iterative algorithm that searches across the parameter space to find a set parameters that jointly minimize the distance between simulated and historical moments.

The “moment matching” exercise outlined above can be applied with reference to empirical moments calculated over the historical period. For the purpose of our model, however, we apply the calibration methodology using a fair amount of judgement. To illustrate, we will discuss how we calibrated the domestic and foreign sectors of the model, each in turn.

The Domestic Sector

Tables 2 compares “historical” and “simulated” moments for the key variables in the domestic sector of the model. The “historical” moments are calculated using quarterly data over the period 1953Q1 to 2000Q2 as well as over two sub-periods defined with reference to the change from a fixed to flexible exchange rate regime in June 1970.

First consider how the model can be calibrated to match the amount of historical variation in real commodity prices. Changes in real energy and non-energy commodity prices (Δrpc and Δrpe) are modeled as autoregressive processes and hence, are unaffected by the rest of the model. The absence of feedback makes the calibration procedure straightforward. The standard deviations of the error terms (σ_8 and σ_9) in equations (8) and (9) are equate the variance of simulated commodity prices to those calculated over the historical period.[29] Table 3 reports standard deviations of the error terms (conditional variances Ω^{ii}) that were used to generate the simulated moments reported in Table 2.

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[28] We consider the simple case where the number of calibrated parameters equal the number of moments to match so that the generalized method of moments (GMM) estimator is exactly identified and hence, can be calculated without a “weighting matrix”. See Chapter 14 in Hamilton (1994) and the references therein.

Table 2: "Historical" versus "Simulated" Moments – Domestic Variables

	"Historical" Moments			*"Simulated" Moments*
	1953Q1-1960Q2	*1960Q3-1970Q2*	*1971Q1-2000Q2*	
Δrpc			16.86	*16.86*
Δrpe			34.25	*34.25*
$(pc_t - pc_{t-4})$	1.41	1.51	2.85	*0.75*
y^c	1.25	1.64	2.11	*1.40*
Δq	5.96	3.60	7.67	*7.71*
r^1	1.22	0.82	2.95	*3.16*
i^1	1.25	1.48	3.56	*3.53*
r^{80}	0.65	0.39	1.70	*1.67*
i^{80}	0.73	1.00	2.45	*1.70*

Table 3: Standard deviations of error terms in the Domestic Sector

(Calibrated using the method of simulated moments.)

Equation	*Parameter*	*Calibrated value*
(1)	σ_1	0.6
(2)	σ_2	0.7
(3)	σ_3	0.2
(4)	σ_4	0.28
(5)	σ_5	0.0
(6)	σ_6	1.6
(7)	σ_7	0.42
(8)	σ_8	16.63
(9)	σ_9	33.98
(10)	σ_{10}	0.06
(11)	σ_{11}	0.03

[29] We match the standard deviation of real commodity prices calculated over the post-1970 period because the quality of the data is questionable prior to 1970.

Calibrating the amount of variation in *core* inflation is not as straightforward. The monetary policy rule in the model is specified with reference to a constant inflation target range. However, monetary policy in Canada was not implemented with reference to explicit inflation targets until the early 1990s. The model therefore cannot replicate Canada's experience with inflation over the past several decades (particularly the inflationary episodes that occurred during the 1970s). The variability of *core* inflation relative to the target range is more relevant for calibrating the monetary policy rule used in the model. Hence, we calibrate the model such that core inflation has a standard deviation of 0.75 of a percentage point, which is consistent with the average variation of *core* inflation relative to the mid-point of the target range since the introduction of inflation targets in the early 1990s.[30]

In the case of real output, we abstract from the effects of disinflationary monetary policy, which contributed to the depth and duration of the recessions in the early 1980s and early 1990s. When the monetary policy in the model is specified with reference to an inflation target using a reasonably well-specified policy rule, episodes of disinflation do not occur in stochastic simulation experiments. As a consequence, the average amount of output variation observed during the 1980s and 1990s is judged to be too high for the purpose of calibrating the model. Instead, we calibrate the amount of variation in output to be roughly consistent with the historical data prior to the 1981 recession.[31]

The amount of variation in interest rates also varies substantially across the two sub-periods. The model is calibrated to match the amount of variation during the flexible exchange rate period (1971Q1 to 2000Q2). A similar approach was used to calibrate the average variation in the real exchange rate over the flexible exchange rate period.

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[30] The *Inflation Reduction Guidelines*, announced in February 1991, specified a range for lower inflation from the end of 1992 to the end of 1995. This was followed by the specification of a 1% to 3% target range, which was introduced in February 1993. The standard deviation of core inflation relative to the mid-point of the target range is 0.74 of a percentage point on average over the period 1992Q4 to 2000Q2.

[31] The standard deviation of the measure of the output gap obtained from a state space model has a standard deviation of 1.5 percentage points over the period 1953Q1-1980Q4.

The fiscal authority in the model faces uncertainty about economic and fiscal developments. In other words, there is uncertainty about fiscal expenditures and tax revenues even if economic developments were known. The amount of uncertainty surrounding fiscal developments is largely determined by judgement. The fiscal authority's forecast of program spending in the coming fiscal year has a 90 per cent confidence interval of 0.2 per cent of GDP (currently about $2 billion), conditional on economic developments. In other words, program spending planned for the fiscal year 2000 is expected to be within $1 billion of its projected level (about $133 billion on a national accounts basis) with 90 per cent probability if economic developments turn out exactly as planned. Similarly, tax revenues projected for the coming fiscal year have a 90 per cent confidence interval of 0.4 per cent of GDP (currently about $4 billion), conditional on economic developments. This implies that projected tax revenues for the fiscal year 2000 is expected to be within $2 billion of its projected level (about $176 billion on a national accounts basis) with 90 per cent probability if economic developments turn out exactly as planned.

The Foreign Sector

The foreign sector is calibrated much along the same lines as the domestic sector, with one minor exception. Core inflation in the foreign sector is calibrated to have more variation than in the domestic sector (a standard deviation of 1.0 versus 0.75 of a percentage point). This is intended to capture the fact that our major trading partner – the U.S. – does not have explicit inflation targets and hence, may be willing to tolerate more variation in inflation. Table 4 below compares simulated and historical moments for key variables in the foreign sector. Table 5 reports standard deviations of the error terms that were used to generate the simulated moments reported in Table 4.

It is important to note that the parameter values reported in Tables 2 to 5 represent but one of several ways that the model can be calibrated. The calibration procedure is designed so that it can be applied to alternative scenarios. For instance, one could consider scenarios where variables exhibit more or less variation than observed on average over history. This would enable one to examine policy issues with reference to alternative scenarios that depict tranquil versus turbulent periods.

Table 4: "Historical" versus "Simulated" Moments – Foreign Variables

	"Historical" Moments			***"Simulated" Moments***
	1953Q1-1960Q2	*1960Q3-1970Q2*	*1971Q1-2000Q2*	
$(pf_t - pf_{t-4})$	1.00	1.46	2.45	*1.00*
yf^c	1.71	2.08	1.80	*1.50*
rf^1	0.98	0.41	2.19	*2.26*
if^1	0.97	1.47	2.68	*3.20*
rf^{80}	0.49	0.20	1.72	*1.70*
if^{80}	0.53	0.93	2.20	*1.66*

Table 5: Standard deviations of error terms in the Foreign Sector

(Calibrated using the method of simulated moments.)

Equation	*Parameter*	*Calibrated value*
(F.1)	σ_{f1}	0.46
(F.2)	σ_{f2}	0.25
(F.3)	σ_{f3}	0.18
(F.4)	σ_{f4}	0.0
(F.5)	σ_{f5}	1.68

4. Model Properties

This section of the paper examines a few illustrative shocks to illustrate some of the key properties of the model. The shocks are scaled to correspond to the calibrated values reported in Tables 3 and 5 above. Impulse responses are generated using the ranges of parameter values reported in Table 1 above. This is implemented as follows. Parameter values are drawn from a normal distribution with a mean equal to the “benchmark” values reported in Table 1. The standard deviation of the parameter distribution is set such that the “low” and “high” parameter values reported in Table 1 correspond to a 95 a per cent confidence interval. For each shock, the dynamic response of the model is generated repeatedly using several alternative parameter settings. This enables us to calculate confidence intervals for the impulse responses. The confidence intervals provide a rough measure of the extent to which the simulation properties of the model vary across the range of parameter values.

An Output Shock

Figure 1a shows the dynamic response of selected macro variables in the domestic sector of the model to a one standard deviation – 0.7 percentage point – transitory increase in the output gap. The solid line in each panel represents the mean response across repeated dynamic simulations; the broken lines represent a 90 per cent confidence interval.[32]

The autoregressive roots α_1 and α_2 in the output equations (F.1) and (1) are calibrate to produce a “hump-shaped” response to random shocks. This is believed to be a stylized fact of the business cycle.[33] The output shock persists for between two to four years, which roughly corresponds to the average duration of business cycles. Due to the forward-looking nature of the monetary policy rule, the monetary authority in the model anticipates the inflationary consequences of the rise in aggregate demand and reacts

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[32] Because the model is symmetric, the solid lines in all panels also represent the dynamic response of the model obtained from a single simulation using the “benchmark” parameter values reported in Table 1.

[33] This is documented by Cogley and Nason (1995) among others using data for the U.S.

immediately by raising the short-term nominal interest rate by between 70 to 110 basis points. Inflation rises to a peak of between 0.4 to 0.7 percentage points two quarters after the increase in aggregate demand. Expected inflation over the coming year rises instantaneously due to the forward-looking expectations component. The short-term real interest rate rises by between 0.4 to 0.7 percentage points on impact, while the real exchange rate appreciates by between 0.6 to 1.4 percentage points.[34] Most of the dynamic adjustment is completed by the end of about two years.

Figure 1b illustrates the response of fiscal variables to the same shock. The increase in output causes the debt-to-GDP ratio to decline by between 0.35 and 0.42 percentage points after two quarters. This partly reflects the unanticipated decline in the denominator – GDP. In addition to the direct effect on the denominator of the debt-to-GDP ratio, the decline in GDP also raises the primary budget balance through automatic stabilisation properties of program spending and tax revenues. The fiscal authority in the model reacts to these fiscal developments by raising discretionary spending. It puts in place a budget plan that aims to increase the debt-to-GDP ratio back to its target level over the coming two to three years.

An Inflation Shock

Figure 2 illustrates the dynamic response of macro variables to a one standard deviation – 0.2 percentage point – transitory increase in inflation. For presentation purposes, we consider the case of an unanticipated increase in both measures (consumer and producer) of inflation.[35] The monetary authority in the model reacts to the rise in inflation by raising short-term nominal interest rate by about 20 basis points. There is persistence in the inflation process, arising from expectational and intrinsic sources, which prevents the monetary authority from bringing inflation back to its target level immediately. Inflation "overshoots" the target level slightly for this particular setting of

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[34] Recall that the exchange rate is defined as the price of foreign exchange so that a decrease (increase) corresponds to an appreciation (depreciation).

[35] The random error terms ε_2 and ε_3 in the inflation equations (2) and (3) have a correlation of 0.25 in stochastic simulation experiments.

the policy parameter γ_1, resulting in some minor secondary cycles in interest rates and the exchange rate. This overshooting can be avoided by using lower values of γ. However, this results in slower convergence to equilibrium and hence, larger fluctuations in inflation.

The fiscal response to an inflation shock is negligible. This is because the fiscal sector of the model is indexed to inflation. An increase in the price level has no direct effect any the fiscal variables in the model because they are expressed as percentages of GDP. Indirect effects arise, however, from the monetary policy response. Higher interest rates lead to higher debt service costs and lower output, which reduces tax revenues and raises program spending (as proportions of GDP). These indirect effects, however, have relatively minor consequences for all of the fiscal variables.

A Real Exchange Rate Shock

Figure 3 illustrates the dynamic response of macro variables to a one standard deviation – 1.2 percentage point – transitory increase (depreciation) in the real exchange rate. The real exchange rate depreciation leads to an increase in output and inflation. The monetary authority responds by raising short-term nominal interest rates by between 5 and 20 basis points.

A Real Commodity Price Shock

Figure 4 illustrates the dynamic response of macro variables to a one standard deviation – 16 percentage point – transitory increase in the real price of commodities. A rise in the relative commodity prices is inflationary. Aggregate demand increases slightly for most, but not all, parameter settings. The ambiguity arises because the direct effect coming through the commodity price term in the output equation can be more than offset by the indirect effect coming from the monetary policy response. The monetary authority reacts to the impending inflationary pressure by raising interest rates (by between 23 and 67 basis points) to reduce output below its potential level. Note that the model exhibits high persistence in response to a commodity price shock – convergence to equilibrium takes between three to five years.

A Foreign Output Shock

Foreign shocks also play an important role in the model. This is illustrated by Figures 5a and 5b for the case of a one standard deviation – 0.7 percentage point – transitory increase in foreign output. Figure 5a shows that the responses of foreign macro variables are quite similar to the corresponding domestic macro variables shown in Figure 1a for the case of a domestic output shock. The major difference is that foreign output exhibits a stronger interest rate response than domestic output. Figure 5b shows that the increase in foreign output stimulates domestic aggregate demand, which raises inflationary pressure. The monetary authority responds by raising short-term nominal interest rates by between 40 and 80 basis points. The response of the real exchange rate is ambiguous – it depends on the domestic-foreign real interest rate differential, which is quite sensitive to alternative parameter values.

Figure 2: Response of Macro Variables to Inflation Shock

(One standard deviation - 0.2 percentage points - transitory increase in inflation)

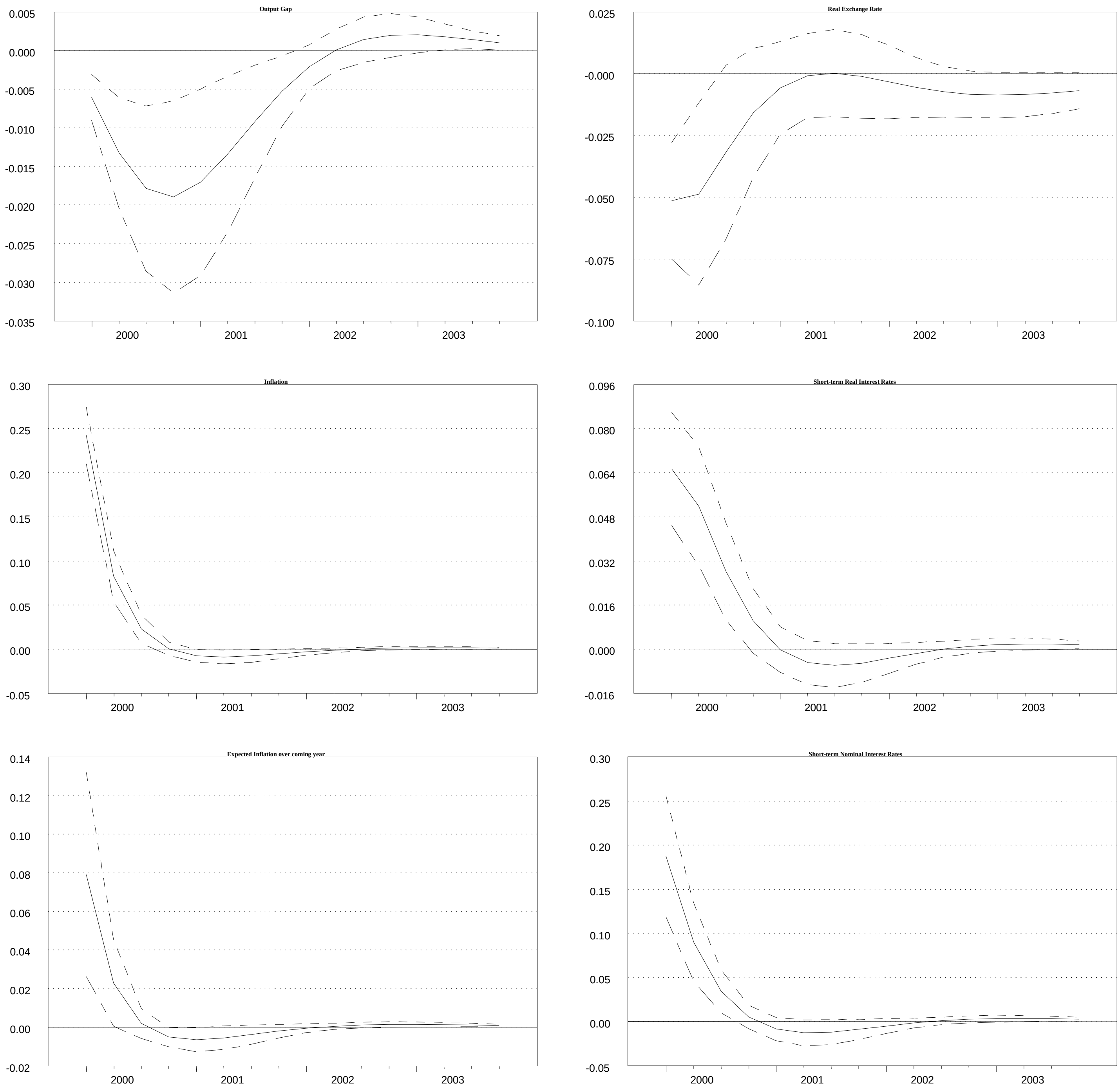


Figure 1a: Response of Macro Variables to Output Shock

(One standard deviation - 0.7 percentage points - transitory increase in aggregate demand)

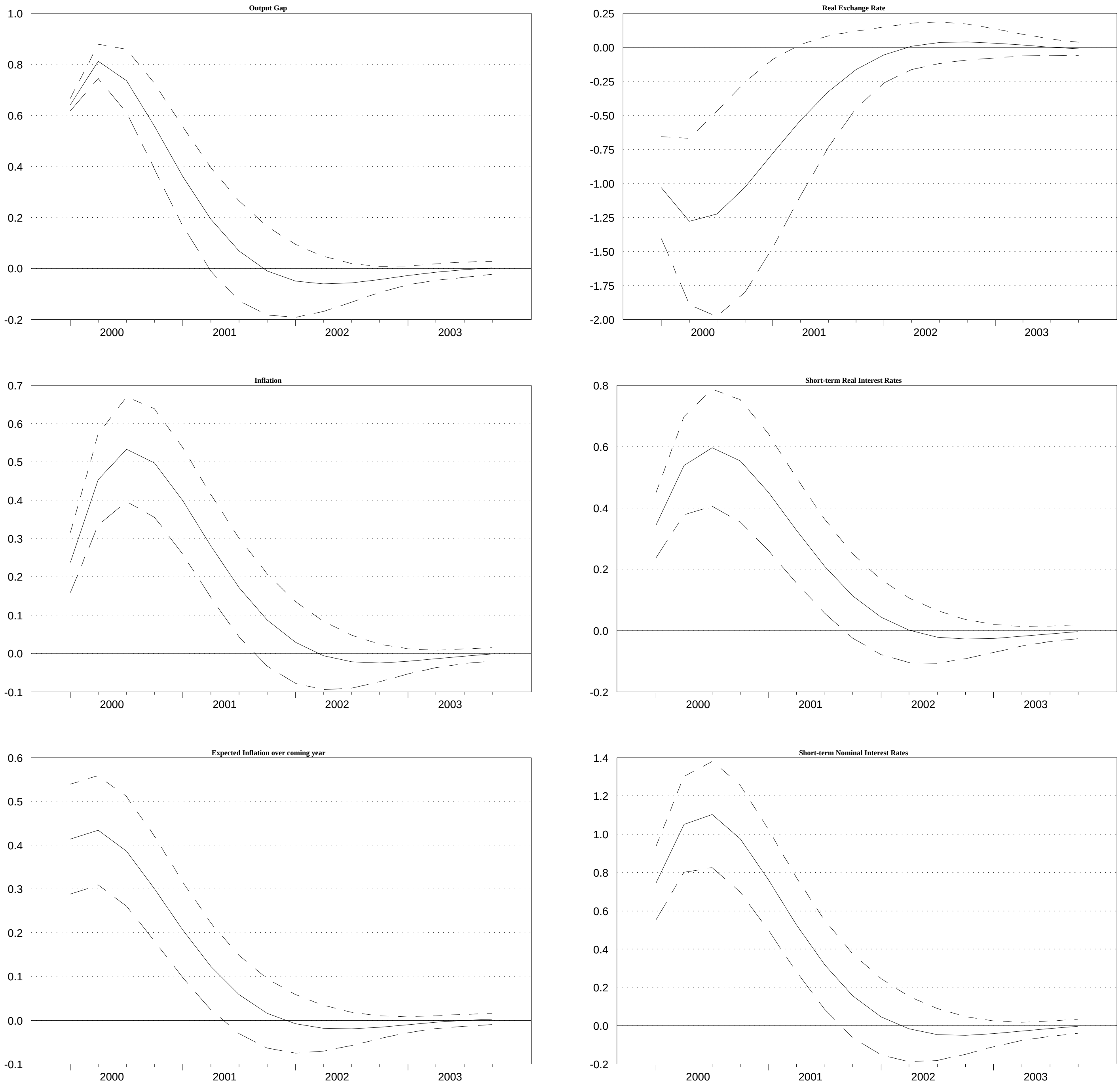


Figure 1b: Response of Fiscal Variables to Output Shock
(Expressed as a percentage of GDP)

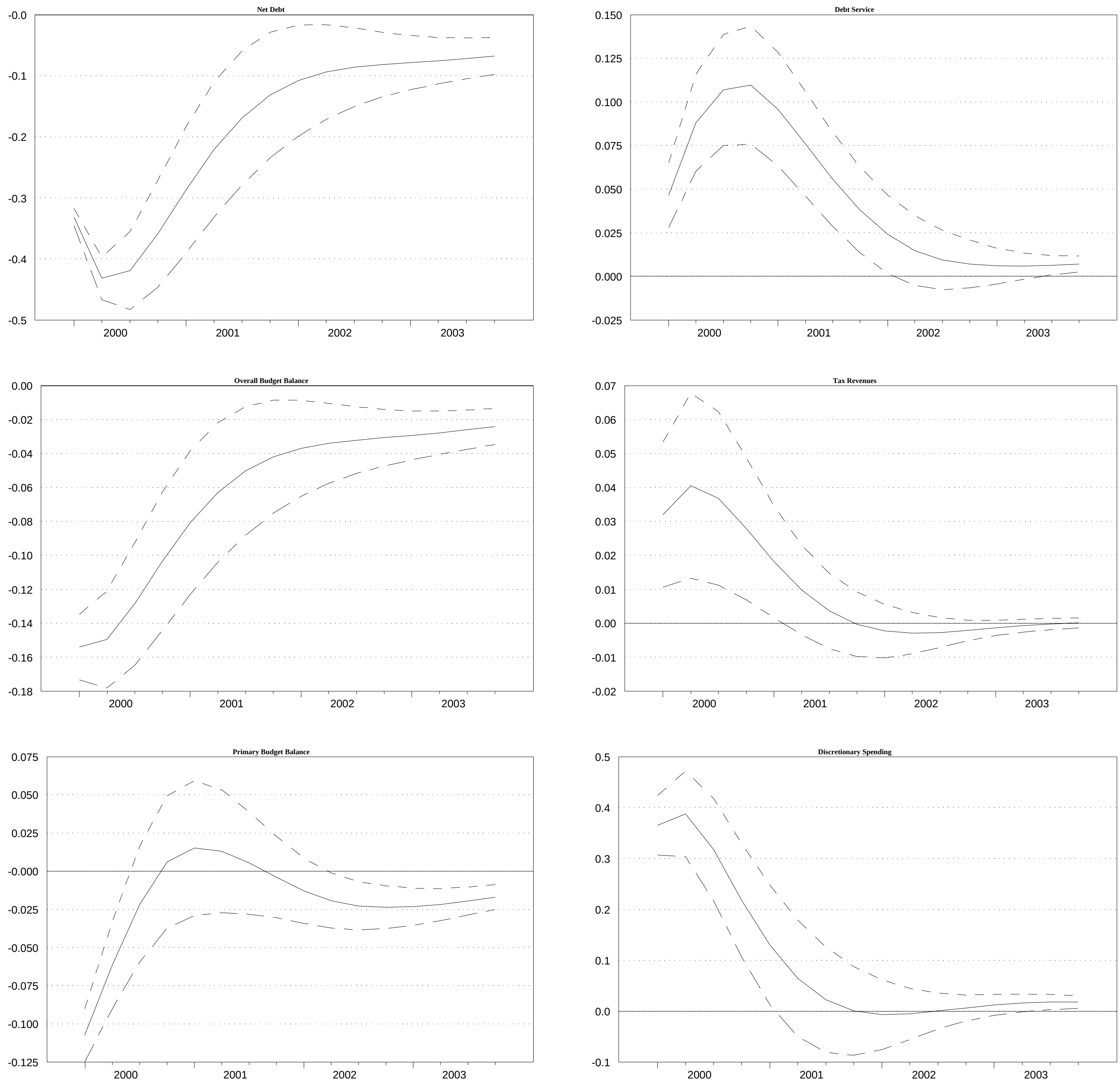


Figure 2: Response of Macro Variables to Inflation Shock

(One standard deviation - 0.2 percentage points - transitory increase in inflation)

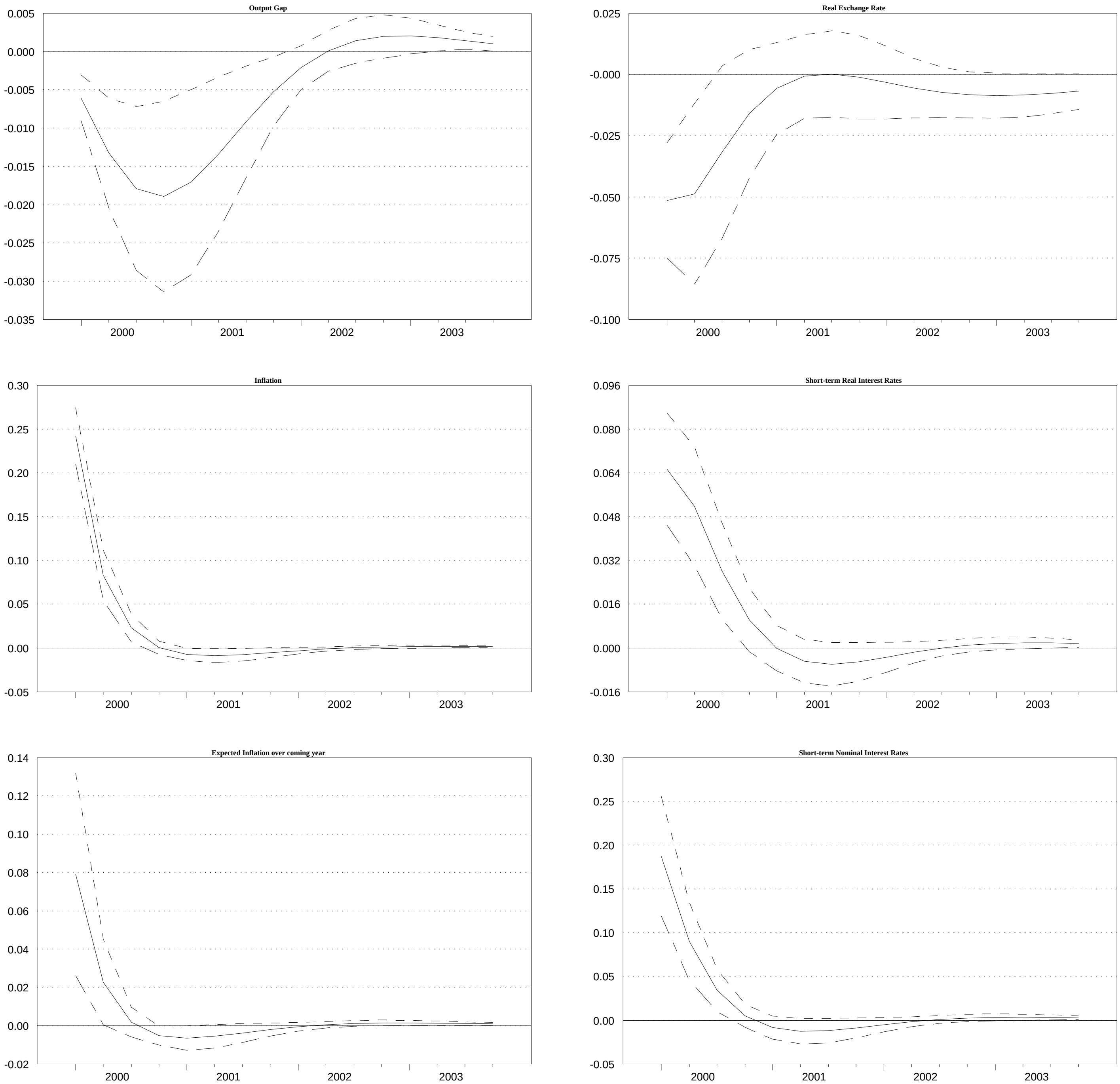


Figure 3: Response of Macro Variables to Real Exchange Shock

(One standard deviation - 0.8 percentage points - transitory increase in real exchange rate)

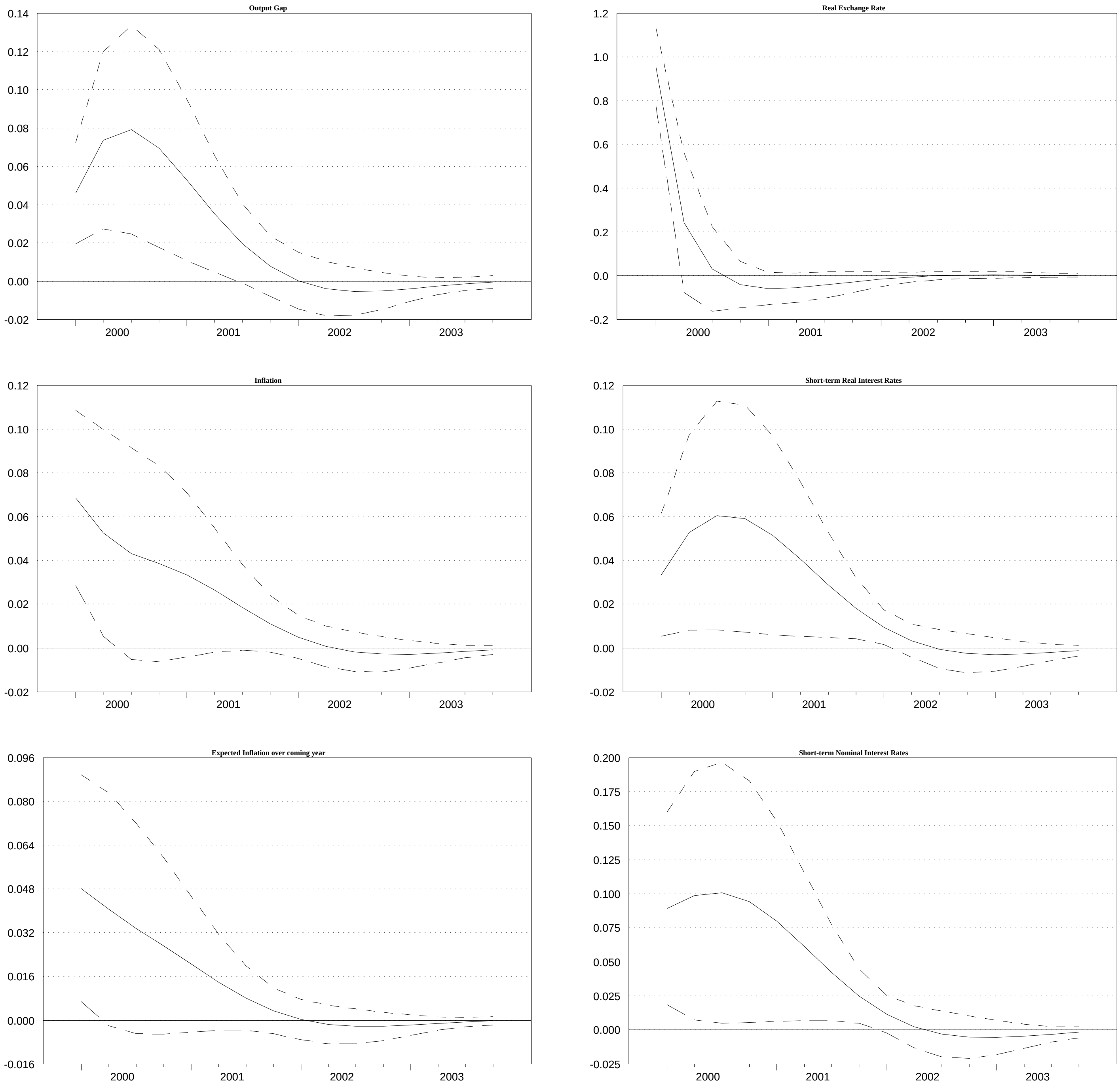


Figure 4: Response of Macro Variables to Commodity Price Shock

(One standard deviation - 16 percentage points - transitory increase in real commodity price)

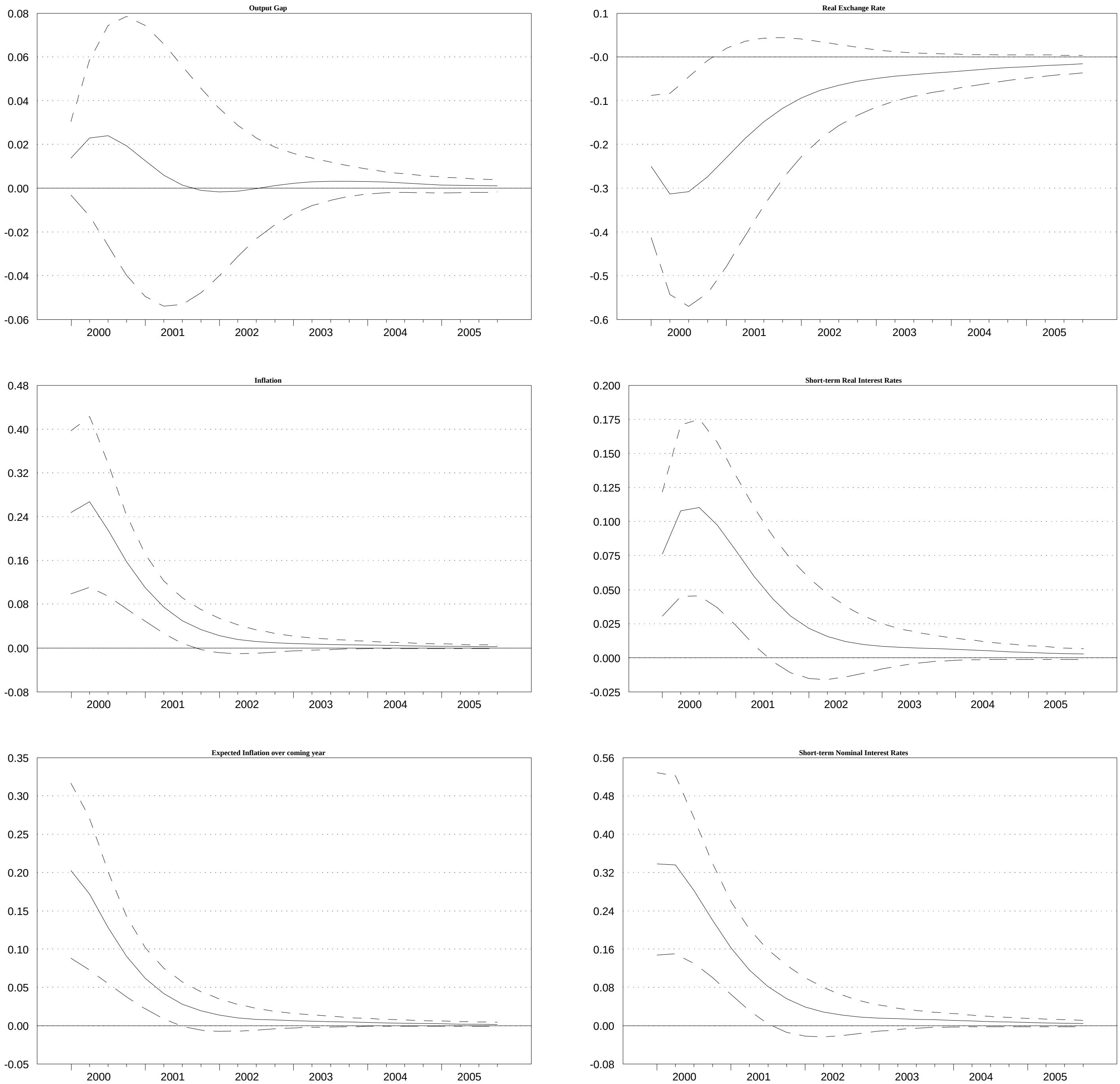


Figure 5a: Response of Foreign Macro Variables to Foreign Output Shock

(One standard deviation - 0.7 percentage points - transitory increase in foreign aggregate demand)

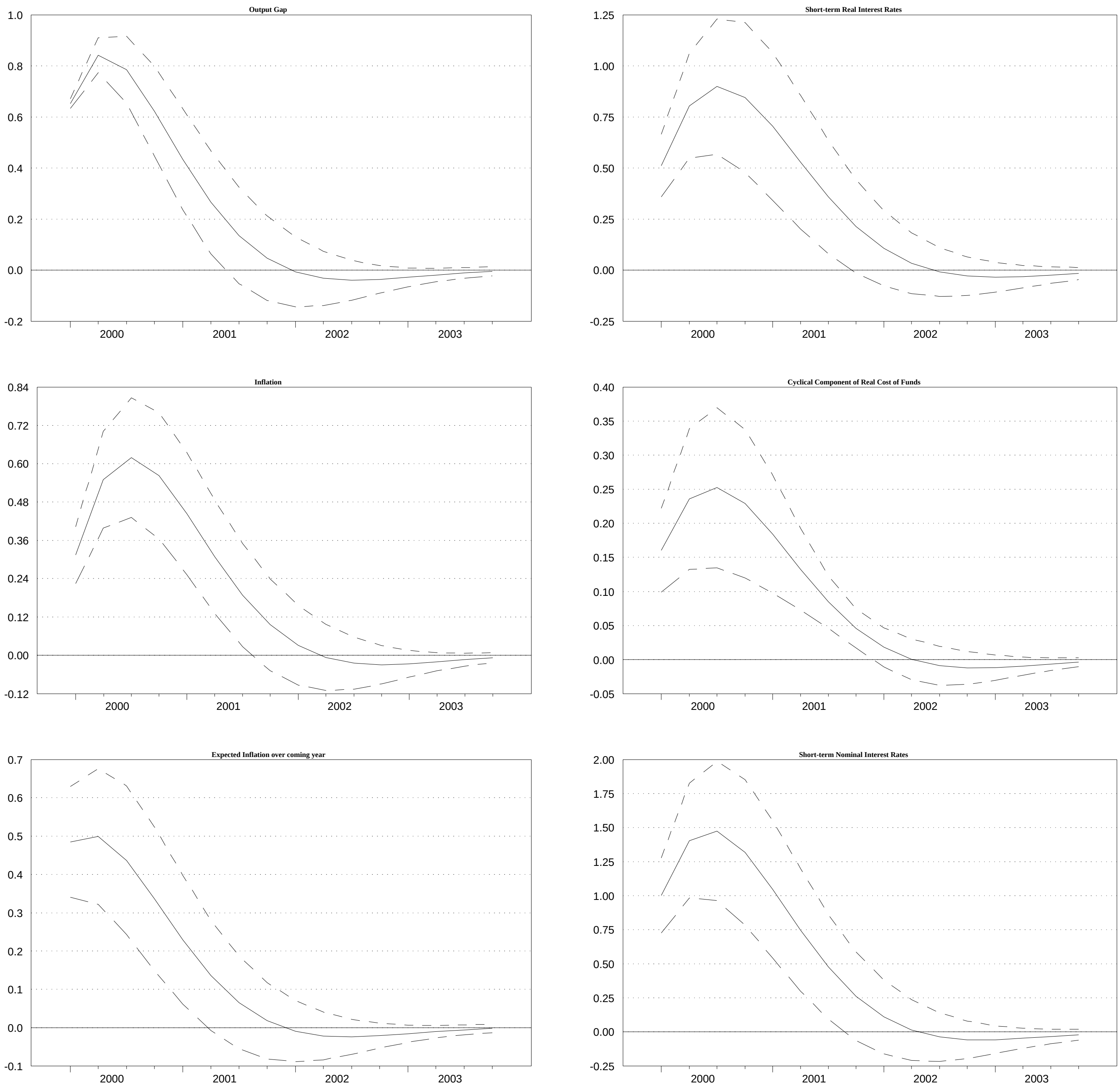
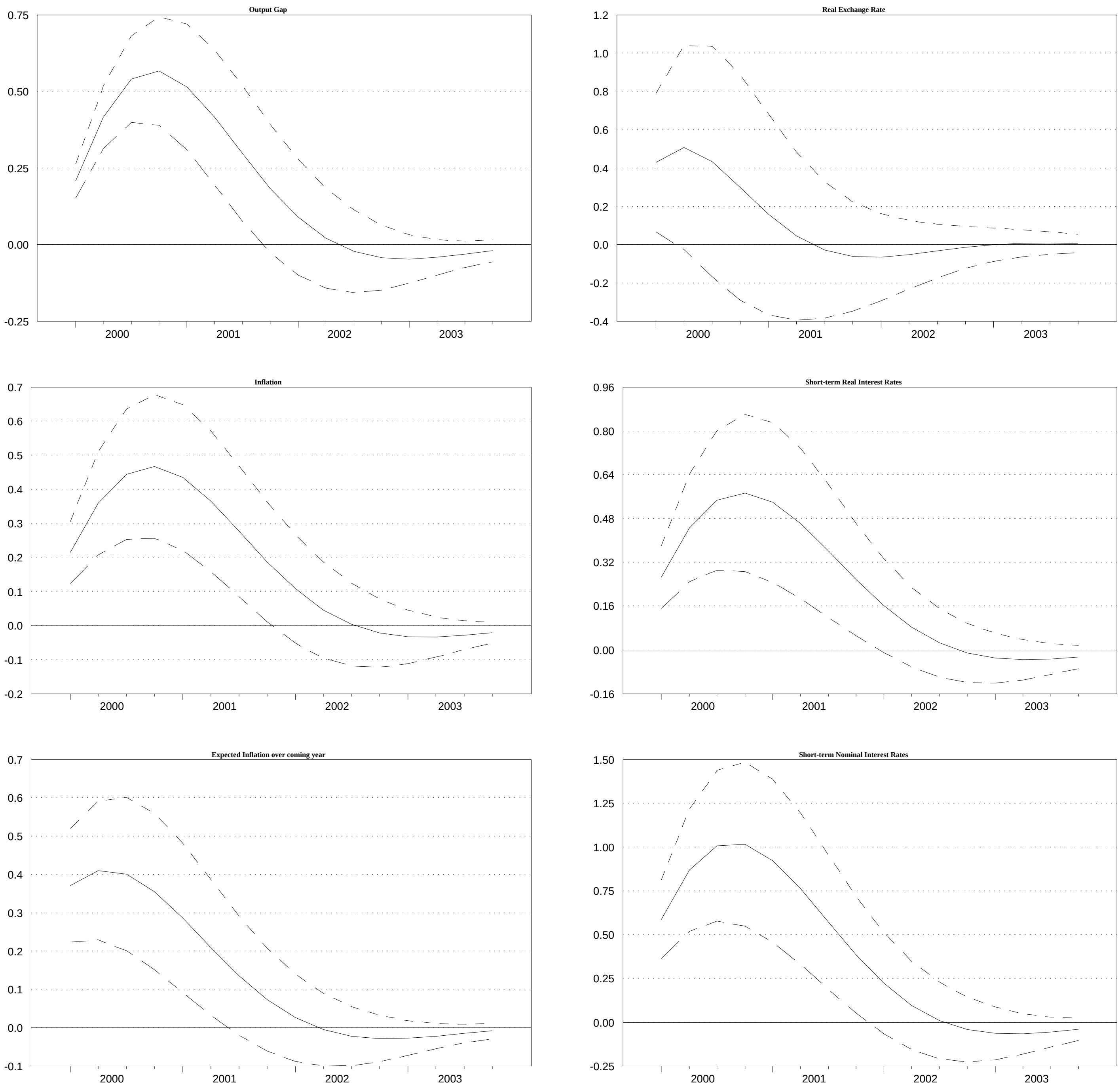


Figure 5b: Response of Domestic Macro Variables to Foreign Output Shock

(One standard deviation - 0.7 percentage points - transitory increase in foreign aggregate demand)



Appendix: Reduced-Form Estimates

Reduced-Form Equations

The Foreign Sector

Our empirical work on the foreign sector focuses on the following three reduced-form equations for output and inflation:[36]

(F.1) $$yf^c_t = \alpha_1 yf^c_{t-1} + \alpha_2 yf^c_{t-2} + \alpha_3\, rf^c_t + \varepsilon f_{1t}$$

(F.2a) $$\Delta pcf_t = \beta_{11} yf^c_t + \beta_{12} yf^c_{t-1} + \beta_2 \Delta pcf_{t-1} + \beta_3 \Delta pf^e_{t+4} + (1-\beta_2-\beta_3)\Delta pf^{ess}_t + \beta_6 \Delta rpo^s_t + \varepsilon_{f2at}$$

(F.2b) $$\Delta ppf_t = \beta_{11} yf^c_t + \beta_{12} yf^c_{t-1} + \beta_2 \Delta ppf_{t-1} + \beta_3 \Delta pf^e_{t+4} + (1-\beta_2-\beta_3)\Delta pf^{ess}_t + \beta_6 \Delta rpo^s_t + \varepsilon_{f2bt}$$

where $\varepsilon_{f1t} \sim (0, \sigma^2_{f1})$, $\varepsilon_{f2at} \sim (0, \sigma^2_{f2a})$ and $\varepsilon_{f2bt} \sim (0, \sigma^2_{f2b})$. These equations are quite similar to those discussed earlier. There are, however, a few differences worth noting. The parameters of the inflation equation (F.2) are estimated using two alternative measures of inflation (changes in consumer prices Δpcf_t and producer prices Δppf_t). This enables us to gauge the average response of foreign inflation Δpf_t to changes in aggregate demand conditions, inflation expectations and changes in real energy prices Δrpe^s_t. This is in contrast to the foreign sector of the model, which is specified using just one measure of inflation as a simplification.

It is also important to note that the inflation equations (F.2a) and (F.2b) are estimated using various surveys of inflation forecasts to proxy inflation expectations. For example, consensus forecasts of inflation over the one-year horizon are used to proxy Δpf^e_{t+4}. This allows us to avoid the econometric complications associated with estimating models with model-consistent forward-looking (rational) expectations. Moreover, we take into account the endogenous nature of the short-term interest rate in equation (F.1) by using instrument variables.[37]

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[36] A detailed description of the data follows later in this appendix.

[37] All the predetermined variables in the model are included in the instrument list.

The Domestic Sector

Our empirical work on the domestic sector focuses on the following four reduced-form equations for output, inflation and the real exchange rate:

$$y^c_t = \alpha_1 y^c_{t-1} + \alpha_2 y^c_{t-1} + \alpha_{31} r^{1c}_t + \alpha_{32} r^{80c}_t + \alpha_4 \Delta q^c_t + \alpha_5 \Delta rpc^c_t + \alpha_6 \Delta rpe^c_t + \alpha_7 yf^c_t + \alpha_8 \Delta yf^c_t + \varepsilon_{1t} \quad \text{(A.1)}$$

$$\Delta pc_t = \beta_1 (y^c_t + y^c_{t-1}) + \beta_2 \Delta pc_{t-1} + \beta_3 \Delta p^e_{t+4} + (1-\beta_2-\beta_3)\Delta p^{ess}_t + \beta_{41}\Delta q^s_t + \beta_{51}\Delta rpc^s_t + \beta_{61}\Delta rpe^s_t + \varepsilon_{2t} \quad \text{(A.2)}$$

$$\Delta pp_t = \beta_1 (y^c_t + y^c_{t-1}) + \beta_2 \Delta pp_{t-1} + \beta_3 \Delta p^e_{t+4} + (1-\beta_2-\beta_3)\Delta p^{ess}_t + \beta_{42}\Delta q^s_t + \beta_{52}\Delta rpc^s_t + \beta_{62}\Delta rpe^s_t + \varepsilon_{3t} \quad \text{(A.3)}$$

$$\Delta q_t = \chi_1 \Delta q_{t-1} + \chi_2 \Delta rpc_t + \chi_3 (q_{t-1} - \chi_4\, rpc_{t-1}) + \chi_5 (r^1_t - rf^1_t) + \varepsilon_{4t} \quad \text{(A.4)}$$

where $\varepsilon_{1t} \sim (0, \sigma^2{}_1)$, $\varepsilon_{2t} \sim (0, \sigma^2{}_2)$, $\varepsilon_{3t} \sim (0, \sigma^2{}_3)$ and $\varepsilon_{4t} \sim (0, \sigma^2{}_4)$.

The reduced-form equations outlined above were estimated using different procedures. We first estimate the equations individually using instrumental variables (IV).[38] We then estimate the equations jointly as a system using maximum likelihood (ML). IV estimates are reported below in Tables A1, A2 and A3; ML estimates are reported in Tables A4 and A5.

We provide some insight into the robustness of the estimates by estimating the parameters over the period 1953-1999 as well as over the flexible exchange rate period 1971-1999. In addition, we consider two alternative measures of potential output. One measure was generated by applying the Hodrick-Prescott (H-P) filter to real GDP.[39] Estimates obtained using the H-P measure of trend output are reported in Tables A1 to A4. The other measure was obtained by estimating potential output as an unobserved variable in the model.[40] These estimates are reported in Table A5.

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[38] The inflation equations were estimated as seemingly related regressions (SUR) to allow for a non-zero covariance between the error terms.

[39] We apply the H-P filter using a "smoothing parameter" of 50,000. This results in a measure of trend output that closely approximates the potential output series in the econometric model CEFM.

[40] This involves estimating the state-space representation of the model where potential output is a state variable. For examples of applications along these lines, see Kichian (1999) and the references therein.

Discussion of the Empirical Results

We will not go over all the empirical results in detail due to space limitations. We will instead focus our discussion on the empirical findings that have important implications for simulation properties of the model.

The dynamic specification of the reduced-form equations was largely determined by the data using a general-to-specific testing procedure. For example, we found that the dynamics of the output equations (F.1) and (A.1) based on the H-P measure of trend output could be well approximated by a second-order autoregressive process. Our estimates of the state-space representation of the model led to a similar result. In the case of the inflation equations, a first-order autoregressive process was found to be sufficient when inflation expectations were proxied by consensus forecasts of inflation.[41]

The response of output to interest rate changes is another key element of the monetary policy transmission mechanism. The magnitude of this linkage is largely determined by two parameters α_3 and ν. The parameter α_3 determines the response of output to changes in the real cost of funds r^c_t, which is measured as a weighted average of short-term and long-term real interest rates: $r^c_t = \nu r^{1c}_t + (1-\nu) r^{80c}_t$. Estimates of the foreign sector indicated that ν is close to zero, and statistically insignificant, implying that foreign aggregate demand is influenced primarily by changes in long-term real interest rates. In contrast, estimates of ν obtained for the domestic sector vary widely across the 0.1 to 0.5 range, indicating that short-term interest rates have a significant effect on output in Canada. We also found considerable differences in estimates of the parameter α_3 across the domestic and foreign sectors. Our estimates indicate that a 100 basis point increase in the real cost of funds would reduce output by 0.2 to 0.4 of a percentage point on impact in the foreign sector but only a 0.1 of a percentage point in the domestic sector. These empirical findings are reflected in the differences in the calibration of the domestic and foreign sector of the model reported in Table 1.

[41] Consensus forecasts of inflation obtained from various surveys tend to be highly correlated with current and past inflation, suggesting that expectations are formed in an adaptive manner (Hostland 1997).

The slope of the Phillips curve is another key element of the monetary policy transmission mechanism. Our empirical analysis indicated that it is difficult to determine the extent to which inflation responds to current versus lagged aggregate demand/supply conditions. The current and lagged value of the output gap together fit well in most specifications.[42] Taken together, our estimates imply that a one percentage point increase in output above its potential level would increase inflation by about 0.1 to 0.2 percentage points after two quarters.

We also experienced difficulty in obtaining robust estimates of other parameters in the inflation equations. For example, we found it very difficult to disentangle how inflation responds to past inflation versus expectations. This is implied by the wide range in estimates of the parameters β_2 and β_3. The calibrated values for these parameters reported in Table 1 (β_2 = 0.3 and β_3 = 0.4) largely reflect our judgement based on the simulation properties of the model.

Changes in relative prices were found to affect output and inflation with long lags. We model the dynamic adjustment process in a parsimonious manner using a geometric lag structure. For example, changes in the relative price of oil are filtered as follows:

$$\Delta rpo^s_t = \lambda_3 \Delta rpo^s_{t-1} + (1-\lambda_3)\Delta rpo_{t-1}$$

where Δrpo^s_t is the variable specified in the foreign inflation equations (F.2a) and (F.2b). We initially searched for values of the parameter λ that maximize the “fit” of the regression equations. This resulted in values in the 0.7 to 0.9 range, which implies that the dynamic adjustment process takes several years. This was judged to be implausible and instead λ was set to be the 0.5 to 0.7 range, which results in a more plausible rate of dynamic adjustment to relative price changes.

In the domestic sector, changes in the real exchange rate and relative commodity and energy prices generally were found to have a significant effect on producer price inflation Δpp_t, but little effect on “core” consumer price inflation Δpc_t. The estimated

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[42] We found no empirical support for the hypothesis that inflation responds to the change in the output gap (in addition to, or in place of the level of the output gap).

response of producer price inflation, however, varies substantially across estimation procedures and sample periods. These empirical results are taken into account in calibrating the model by allowing these parameters to vary within a fairly wide range.

Preliminary estimates of a reduced-form exchange rate equation similar to (A.4) indicated that the relative price of energy has an insignificant effect on the real exchange rate, contrary to the results obtained by Amano and van Norden (1993) and Lafrance and van Norden (1995). These conflicting results can largely be explained by the fact that we have extended the sample period from 1972Q2-1994Q3 (used by Lafrance and van Norden) to 1971Q1-1999Q4. We find strong evidence of a cointegrating relationship between non-energy commodity prices and the real exchange rate, however.

Table A1: IV Estimates of output equations

(Based on measure of the output gap obtained from the H-P filter.)

Foreign sector: equation (F.1)

Sample:	***1953Q2-99Q4***		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	1.275	0.071	1.202	0.089
α_2	-0.393	0.070	-0.313	0.089
α_3	-0.403	0.141	-0.296	0.130
σ_{f1}	0.895		0.826	

Domestic sector: equation (A.1)

	1954Q1-99Q4		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	0.925	0.072	0.997	0.090
α_2	-0.016	0.074	-0.167	0.086
α_{31}	-0.032	0.038	-0.049	0.035
α_{32}	-0.075	0.114	-0.073	0.103
α_4	0.050	0.055	-0.017	0.054
α_5	0.013	0.009	-0.002	0.010
α_6	0.007	0.012	0.004	0.010
α_7	0.057	0.045	0.222	0.080
α_8	0.141	0.024	0.105	0.029
σ_1	0.798		0.690	

Table A2: IV Estimates of inflation equations

(Based on measure of the output gap obtained from the H-P filter.)

Foreign sector: equations (F.2a) and (F.2b)

Sample:	*1953Q2-99Q4*		*1971Q2-99Q4*	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
β_{11}	0.065	0.072	0.006	0.094
β_{12}	0.087	0.072	0.153	0.098
β_2	0.418	0.046	0.315	0.062
β_3	0.106	0.121	0.219	0.203
β_6	0.021	0.006	0.018	0.006
σ_{f2a}	1.19		1.25	
σ_{f2b}	1.04		0.99	
Corr(ε_{f2at}, ε_{f2bt})	0.275		0.211	

Domestic sector: equations (A.2) and (A.3)

	1954Q1-99Q4		*1971Q1-99Q4*	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
β_1	0.199	0.052	0.180	0.050
β_2	0.163	0.112	0.425	0.134
β_3	0.763	0.119	0.588	0.144
β_{41}	0.005	0.038	0.028	0.031
β_{51}	0.002	0.016	0.009	0.013
β_{61}	0.009	0.010	0.003	0.006
β_{42}	0.095	0.053	0.083	0.044
β_{52}	0.112	0.024	0.089	0.021
β_{62}	0.034	0.014	0.009	0.010
σ_2	1.79		1.34	
σ_3	2.51		1.99	
Corr(ε_{1t}, ε_{2t})	-0.030		0.054	

Table A3: IV Estimates of exchange rate equation

Domestic sector: equation (A.4)

Sample:	***1954Q1-99Q4***		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
χ_1	0.299	0.073	0.351	0.085
χ_2	-0.009	0.035	-0.108	0.038
χ_3	-0.256	0.072	-0.264	0.085
χ_4	-0.569	0.185	-0.524	0.214
χ_5	-0.471	0.322	-0.635	0.373
σ_4	6.38		6.82	

Note:

The parameter χ_4 and its standard error are estimated using the lead-lag procedure proposed by Phillips and Loretan (1991). The null hypothesis of no cointegration can be rejected at the 0.002 level of significance based on the estimates obtained from this procedure,.

Table A4a: ML Estimates

(Based on measure of the output gap obtained from the H-P filter.)

Foreign sector: equations (F.1), (F.2a) and (F.2b)

Sample:	***1953Q2-99Q4***		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	1.264	0.069	1.205	0.086
α_2	-0.383	0.068	-0.322	0.087
α_3	-0.395	0.137	-0.303	0.127
$\beta_{11} = \beta_{12}$	0.066	0.015	0.048	0.022
β_2	0.494	0.045	0.464	0.060
β_3	0.215	0.119	0.533	0.209
β_6	0.019	0.006	0.019	0.007
σ_{f1}	0.88		0.81	
σ_{f2a}	1.19		1.28	
σ_{f2b}	1.06		1.02	

Domestic sector: equations (A.1), (A.2), (A.3) and (A.4)

Sample:	***1954Q1-99Q4***		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	0.968	0.067	1.012	0.081
α_2	-0.058	0.068	-0.178	0.077
α_{31}	-0.036	0.035	-0.065	0.031
α_{32}	-0.056	0.108	-0.061	0.095
α_4	0.011	0.018	-0.013	0.017
α_5	0.009	0.008	-0.009	0.008
α_6	0.005	0.005	0.002	0.004
α_7	0.057	0.039	0.226	0.065
α_8	0.136	0.022	0.113	0.026

Table A4a continued: ML Estimates

Domestic sector: equations (A.1), (A.2), (A.3) and (A.4)

Sample:	***1953Q2-99Q4***		***1971Q1-99Q4***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
β_1	0.100	0.026	0.096	0.024
β_2	0.336	0.106	0.445	0.128
β_3	0.590	0.112	0.562	0.136
β_{41}	0.006	0.041	0.044	0.031
β_{51}	0.006	0.017	0.011	0.013
β_{61}	0.008	0.010	0.006	0.007
β_{42}	0.168	0.053	0.143	0.044
β_{52}	0.095	0.023	0.092	0.021
β_{62}	0.028	0.013	0.018	0.011
χ_1	0.367	0.067	0.377	0.082
χ_2	-0.089	0.030	-0.109	0.035
χ_3	-0.248	0.074	-0.243	0.089
χ_4	-0.442	0.116	-0.480	0.142
χ_5	-0.955	0.296	-0.904	0.344
σ_1	0.78		0.65	
σ_2	1.88		1.31	
σ_3	2.65		1.98	
σ_4	6.25		6.76	

Table A4b: ML Estimates of Correlation between Residuals

Domestic Sector

	1953Q2-99Q4			*1971Q1-99Q4*		
	ε_{2t}	ε_{3t}	ε_{4t}	ε_{2t}	ε_{3t}	ε_{4t}
ε_{1t}	-0.02	-0.33	0.00	0.02	-0.32	-0.01
ε_{2t}		-0.07	0.03		0.02	-0.16
ε_{3t}			-0.49			-0.43

Foreign Sector

	1953Q2-99Q4		*1971Q1-99Q4*	
	ε_{2t}	ε_{3t}	ε_{2t}	ε_{3t}
ε_{1t}	0.04	-0.13	0.17	0.04
ε_{2t}		0.28		0.25

Table A5: ML Estimates

(Based on measure of the output gap obtained from state-space model.)

Sample period: 1955Q3 to 2000Q2

Foreign sector: equations (F.1), (F.2a) and (F.2b)

	Unrestricted		***Restricted***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	0.750	0.053	0.750	–
α_2	0.750	0.053	0.650	–
α_3	-0.188	0.164	-0.256	0.129
$\beta_{11} = \beta_{12}$	0.101	0.023	0.103	0.024
β_2	0.390	0.068	0.389	0.069
β_3	0.274	0.160	0.275	0.161
β_6	0.168	0.078	0.168	0.078
σ_{f1}	0.505	0.062	0.505	0.062
σ_{f2a}	1.122	0.013	1.176	0.114
σ_{f2b}	1.177	0.115	1.051	0.066

Domestic sector: equations (A.1), (A.2) and (A.3)

Dynamics:	***Unrestricted***		***Restricted***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_1	0.825	0.107	0.750	–
α_2	0.529	0.151	0.650	–
α_{31}	-0.004	0.091	-0.006	0.042
α_{32}	-0.115	0.090	-0.112	0.066
α_4	0.021	0.030	0.016	0.016
α_5	0.032	0.014	0.029	0.011
α_6	0.000	–	0.000	–

Table A5 continued: ML Estimates

Domestic sector: equations (A.1), (A.2) and (A.3)

	Unrestricted		***Restricted***	
Parameter	*Estimate*	*Standard Error*	*Estimate*	*Standard Error*
α_7	0.034	0.047	0.033	0.026
α_8	0.054	0.019	0.048	0.014
β_1	0.140	0.058	0.142	0.053
β_2	0.000	–	0.000	–
β_3	0.253	0.333	0.248	0.323
β_{41}	0.008	0.100	0.010	0.045
β_{51}	0.042	0.073	0.041	0.038
β_{61}	0.012	0.027	0.013	0.019
β_{42}	0.084	0.079	0.085	0.083
β_{52}	0.323	0.053	0.322	0.054
β_{62}	0.038	0.032	0.039	0.023
σ_1	0.318	0.044	0.315	0.037
σ_2	1.656	0.116	1.655	0.118
σ_3	2.240	0.202	2.239	0.205

Description of the Data[43]

Foreign Variables

Foreign output:

yf^c = the deviation of foreign real output yf from its potential level yf^p

yf = U.S. real GDP

yf^p = potential GDP in the U.S. measured using the following two approaches:

1. trend obtained from applying H-P filter to yf
2. estimate obtained from state-space representation of the foreign sector

Foreign real interest rates:

rf^c_t the real cost of funds in the U.S. = $\nu rf^{1c}_t + (1-\nu)rf^{80c}_t$

rf^{1c} = the deviation of the foreign short-term real interest rate rf^1 from its trend rf^{1p}

rf^1 = the yield on 3-month U.S. treasury bills less expected inflation over coming quarter obtained from surveys of inflation forecasts

rf^{1p} = trend component of rf^1 measured using the following two approaches:

1. trend obtained from applying H-P filter to rf^1
2. estimate obtained from state-space representation of the foreign sector

rf^{80c} = the deviation of the foreign long-term real interest rate rf^{80} from its trend rf^{80p}

rf^{80} = the yield on 20-year U.S. government bonds less expected inflation over 10-year horizon obtained from surveys of inflation forecasts

rf^{80p} = trend component of rf^{80} measured using the following two approaches:

1. trend obtained from applying H-P filter to rf^{80}
2. estimate obtained from state-space representation of the foreign sector

Foreign prices:

Δpcf = quarterly change in U.S. CPI excluding price and energy

Δppf = quarterly change in U.S. GDP price deflator

Δpf^e_{t+4} = expected inflation over coming year obtained from surveys of inflation forecasts

Δpf^{ess} = expected inflation over 10-year horizon obtained from surveys of inflation forecasts

Δrpo = quarterly change in the price of crude petroleum (in $US) relative to the price of finished goods

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[43] The data series listed below are available upon request.

Domestic Variables

Domestic output:

y^c = the deviation of real output y from its potential level y^p

y = real GDP

y^p = potential GDP measured using the following two approaches:

1. trend obtained from applying H-P filter to y
2. estimate obtained from state-space representation of the domestic sector

Domestic real interest rates:

rf^c_t the real cost of funds = $\nu r^{1c}_t + (1-\nu) r^{80c}_t$

r^{1c} = the deviation of the short-term real interest rate r^1 from its trend r^{1p}

r^1 = the yield on 3-month treasury bills less expected inflation over coming quarter constructed using surveys of inflation forecasts

r^{1p} = trend component of r^1 measured using the following two approaches:

1. trend obtained from applying H-P filter to r^1
2. estimate obtained from state-space representation of the domestic sector

r^{80c} = the deviation of the long-term real interest rate r^{80} from its trend r^{80p}

r^{80} = the yield on 20-year government bonds less expected inflation over 10-year horizon constructed using surveys of inflation forecasts

r^{80p} = trend component of r^{80} measured using the following two approaches:

1. trend obtained from applying H-P filter to r^{80}
2. estimate obtained from state-space representation of the domestic sector

Domestic prices:

Δpc = quarterly change in CPI excluding price, energy and indirect taxes

Δpp = quarterly change in GDP price deflator

Δp^e_{t+4} = expected inflation over coming year constructed using surveys of inflation forecasts

Δp^{ess} = expected inflation over 10-year horizon constructed using surveys of inflation forecasts

rpc = the Bank of Canada (non-energy) commodity price index (in $US) relative to the U.S. GDP price deflator

rpe = the Bank of Canada energy price index (in $US) relative to the U.S. GDP price deflator

q_t = the Canada-U.S. real effective exchange rate ($CN price of $U.S.)

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