NAOMI

A New Quarterly Forecasting Model
Part II: A Guide to Canadian NAOMI

Stephen Murchison

Department of Finance Working Paper 2001-25

The author thanks Patrick Georges, Nicolas Moreau and Pierre Siklos for very useful comments and suggestions. The author also thanks Don Coletti , Steven James, Benoît Robidoux and Robert Tetlow for helpful discussions.

Abstract

This paper provides an introduction to the Canadian side of NAOMI (**N**orth **A**merican **O**pen economy **M**acro-econometric **I**ntegrated model), a new economic model developed at the Department of Finance. NAOMI is intended to bridge the gap between pure forecasting models whose forecasts are often difficult to interpret and dynamic general equilibrium models whose predictions often lack precision. NAOMI's intended purpose is to provide quarterly macroeconomic forecasts of the Canadian and U.S. economies along with a measure of the uncertainty associated with each forecast. While ideally suited to this task, NAOMI is also capable of providing sensible answers to a limited set of more general policy questions. Specifically, it may be employed to address both the likelihood and economic implications of a particular risk to the forecast. This paper provides a detailed description of the economic assumptions underlying NAOMI's structure. In addition, a complete set of deterministic shocks is included to illustrate the model's simulation properties. Finally, model validation is provided through an extensive set of out-of-sample forecast statistics.

Résumé

Cet article offre une introduction au bloc canadien de MIOAN (modèle **M**acro-économique **I**ntégré de l'économie **O**uverte de l'**A**mérique du **N**ord), un nouveau modèle économique élaboré au ministère des Finances. Le modèle MIOAN vise à faire le pont entre les modèles prévisionnels purs, dont les prévisions sont souvent difficiles à interpréter, et les modèles d'équilibre général dynamiques, dont les prévisions manquent souvent de précision. Le modèle MIOAN vise à fournir des prévisions macro-économiques trimestrielles relatives aux économies canadienne et américaine, ainsi qu'une mesure de l'incertitude associée à chaque prévision. Bien qu'il ait été expressément conçu à cette fin, le modèle MIOAN permet également d'obtenir des réponses structurées à une série plus limitée de questions générales de politique. Spécifiquement, il permet d'évaluer à la fois la probabilité et les implications économiques d'un risque spécifique associé à la prévision. L'article offre une description détaillée des hypothèses économiques qui sous-tendent la structure du modèle MIOAN. De plus, un ensemble complet de chocs déterministiques est inclus afin d'illustrer les propriétés de simulation du modèle. Enfin, la validation du modèle est illustrée par un ensemble exhaustif de statistiques prévisionnelles hors-échantillon.

1.0 Introduction

This paper provides a detailed description of the Canadian side of NAOMI (**N**orth **A**merican **O**pen economy **M**acro-econometric **I**ntegrated model), a new economic model developed in the Economic Analysis and Forecasting Division at the Department of Finance. NAOMI is a small, estimated, macroeconomic model of the Canadian and U.S. economies. In addition to producing a model-consistent forecast of key Canadian and US variables each quarter, NAOMI also provides a solid foundation for assessing the magnitude and sources of uncertainty over the forecast horizon. Specifically, it can provide valid forecast confidence intervals that take into consideration both shock and parameter uncertainty. This facilitates the analyses of forecast risk in general and risks to the government's budget balance in particular. In addition, by making the U.S. endogenous in the model and carefully considering its influence on Canada, NAOMI will eventually provide an explicit measure of the additional uncertainty associated with the U.S. forecasts. [1] This source of uncertainty is lost in models that treat foreign variables as exogenous.

NAOMI is also capable of providing sensible answers to more general policy-related questions. For instance, it may be employed to provide guidance on the fiscal implications of certain key economic shocks.

This paper details the structure and properties of NAOMI and is organised as follows: section 2.0 outlines the motivation and objectives of this modelling project. Section 3.0 describes the model structure while section 4.0 outlines some of its key simulation properties. Section 5.0 provides model validation including out-of-sample forecast performance and section 6.0 concludes.

2.0 Motivation and Objective

The size and structure of an economic model is intimately related to its intended purpose. The wide spectrum of economic models that currently exist in the academic literature is a reflection of this fact. It is therefore necessary to first define a model's function before one can go about deciding its form. In the case of this project, we require a model suitable to two main tasks;

(1) The capacity to expediently produce accurate macroeconomic forecasts with confidence intervals that reflect both shock and parameter uncertainty

[1] The U.S. side of NAOMI currently remains a work in progress. Thus, all results presented in this paper treat the U.S. as exogenous. A forthcoming paper dealing with fiscal prudence factors will present results that include the effects of U.S. variables uncertainty.

(2) From the standpoint of economic theory, the model should have sensible properties and a meaningful steady state

Satisfying the first requirement places constraints on the size and structure of the model. For instance, measuring parameter uncertainty necessitates estimating the parameters of the model. Non-linear structural models of the sort derived from explicitly optimising behaviour are difficult to estimate using even very sophisticated econometric techniques. Furthermore, they often treat important explanatory variables such as total factor productivity, labour supply and foreign activity as exogenous. In the interest of ensuring the accuracy of the confidence intervals it is important to construct a model that is largely self-contained.

The size of confidence intervals and the time required to calculate them tends to increase quickly with the size of the model. However, making the model too parsimonious will lead to the exclusion of important dynamic channels or explanatory variables. This will tend to compromise the medium-term forecasting ability of the model. Consider, for example, a naïve expectations model of interest rates that always sets future rates equal to their last known value (otherwise known as a unit-root model, $E(R(t+1))=R(t)$). In the very short term this model will perform well compared to more elaborate multivariate models by virtue of the fact that interest rates tend to be quite persistent. However, such a naïve model ignores the impact of changes in expected future inflation on interest rates both through the Fisher equation and through central bank intervention. Consequently, the medium-term forecast uncertainty associated with this model will be large. Indeed, as the forecast horizon increases the confidence interval for interest rates will become infinitely wide. Obviously, the required measure of fiscal prudence predicted by such a model will be extremely high at longer horizons.

These arguments would tend to support the use of a moderate-sized, linear, reduced-form model such as a vector autoregression (VAR) or Bayesian VAR (BVAR). But as Murchison (2001) explains, the vast number of parameters contained in a well-specified, open-economy VAR often leads to considerable parameter uncertainty and wide confidence intervals. In addition, VARs have steady states that are difficult to interpret. For instance, the steady-state inflation rate will be its historical average, which is substantially higher than the midpoint of the Bank of Canada’s official target band. This will call into question the medium-term accuracy of VAR forecasts. Finally, the individual equations of a VAR are often quite difficult to interpret since they contain lags of every variable contained in the model. Thus, for example, short-term interest rates will react to movements in commodity prices holding output, inflation and the exchange rate constant.

By placing exclusion restrictions on the individual equations of the model based on theory, it is often possible to improve the out-of-sample forecasting performance while also making the model's impulse responses easier to interpret. On this basis, we have opted to estimate a small, dynamic model using the model selection strategy outlined in Murchison(2001). This procedure selects lags and includes explanatory variables based on their ability to forecast the dependent variable out of sample. Thus, for instance, commodity prices will only be included in the inflation equation if they improve the equation's forecasting ability out of sample. As can be seen in the following section, this approach tends to choose very parsimonious models.

3.0 Model Description

Canadian NAOMI consists of 6 behavioural equations that determine output growth, CPI excluding food and energy and GDP inflation, the real exchange rate, the slope of the yield curve and long-term nominal interest rates. These equations (with the exception of the yield curve equation) are estimated simultaneously using quarterly data from 1972Q1 to 2000Q1 using maximum likelihood. The monetary policy rule is estimated using generalised method of moments (GMM) separately from 1988Q1 to 1998Q1 (see Appendix 1.0). Exogenous variables in this version of the model include Canadian potential GDP, U.S. GDP, GDP deflator and potential GDP, U.S. short- and long-term interest rates, total and non-energy commodity prices as well the total government's primary structural balance. NAOMI also contains 18 identities that determine, among other things, the nominal exchange rate and nominal short-term interest rates. In terms of model structure, NAOMI can reasonably be thought of as Keynesian in the short run and classical in the long run. Specifically, prices are sticky in the short run, assuring a short-run role for monetary policy, and fully flexible in the long run, guaranteeing long-run monetary-policy neutrality.

3.1 Aggregate Output

Output growth is determined primarily by potential output growth, U.S. output growth and the change in the slope of the yield curve. A somewhat smaller role is played by the (change in the) real exchange rate, real non-energy commodity prices and the government's primary structural balance (as a share of potential output).

$$\Delta ygap = .3\Delta ygap_{-1} + .34\left(\Delta y^{us} - \Delta y^{p}\right) - .3\sum_{i=3}^{4}\frac{\Delta slope_{t-i}}{2} + .06\Delta z_{-3} + .04\Delta z_{-4} + .03\Delta pcne_{-3}$$
$$-.15\Delta gov$$

$\overline{R}^2 = .97 \quad S.E. = 0.005 \quad DW = 1.9$ [2]

or

$$\Delta y = .66\Delta y^{p} + .3\left(\Delta y_{-1} - \Delta y_{-1}{}^{p}\right) + .34\Delta y^{US} - .3\sum_{i=3}^{4}\frac{\Delta slope_{t-i}}{2} + .06\Delta z_{-3} + .04\Delta z_{-4} + .03\Delta pcne_{-3}$$
$$-.15\Delta gov$$

where ygap is the CEFM output gap (at factor cost), y^p is CEFM potential output at factor cost, y and y^{US} are Canadian and U.S. real output, slope is the slope of the yield curve gap (short rate-long rate+term premium), z represents the real exchange rate (using GDP deflators)[3], pcne is the relative price of non-energy commodities (U.S. dollars) and finally gov is the Department of Finance cyclically adjusted total government budget balance as a share of nominal potential GDP.

In the short run, potential output affects aggregate demand but not one-for-one. Stated otherwise, aggregate demand adjusts fully to a shock to potential output only after several quarters. Thus, business cycles are created by both demand and supply shocks in NAOMI. Such sluggish adjustment could arise, for instance, because labour productivity initially falls when a new technology is introduced. Alternatively, it could arise because agents are uncertain as to the duration of the shock. For instance, if permanent income consumers perceive part of a supply shock to be temporary then only a small portion of their increased income will be spent. This in turn may cause output to increase by less than potential in the short-run. Naturally, this behaviour will eventually dissipate as agents learn the true nature of the shock.

An important source of business cycle dynamics in NAOMI is provided by U.S. activity. Note that it is U.S. output rather than the output gap that influences domestic aggregate demand. This reflects the belief that Canadian exports respond to movement in U.S. output irrespective of the

---

[2] Respectively the adjusted R-square, regression standard error and Durbin-Watson statistic.

[3] Defined such that an increase represents a depreciation of the Canadian exchange rate.

underlying source of this movement. Models that include the U.S. output gap only ignore effects of U.S. supply shocks on Canadian activity.[4]

An important source of business cycle dynamics in NAOMI is monetary policy. Monetary policy affects aggregate demand through the slope of the yield curve rather than the unobserved real interest rate. The minimum lag between monetary policy actions and aggregate demand effects is three quarters. A great deal of consideration has been given to the issue of whether it is the *level* or the *change* in the slope that affects output *growth* relative to potential. A brief explanation for the selection of the change in the slope is provided in Appendix 2.0.

Movements in the nominal exchange rate only affect real activity in Canada to the extent that they feed into the real exchange rate. Stated otherwise, a nominal exchange rate depreciation that is offset one-for-one by higher domestic prices will have no effect on real GDP. However, because NAOMI assumes sluggish nominal adjustment in the short run, nominal exchange rate shocks feed through to the real exchange rate initially. Real exchange rate movements are assumed to affect Canada's net export position with respect to the U.S. Thus a weaker exchange rate will tend to increase export demand abroad while reducing domestic import demand. The effect on aggregate demand of a one per cent depreciation of the real exchange rate is approximately one-third that of a 100 basis point decline in the yield curve. Appendix 3.0 describes the dynamics of NAOMI's version of a monetary conditions index and elaborates on the relative importance of the yield curve and the exchange rate in determining aggregate demand.

Real non-energy commodity prices exert a relatively small influence on the output gap. Ceteris paribus, higher commodity prices will tend to boost economic activity. NAOMI implicitly assumes that changes in the relative price of commodities stem from either a shock to world demand or a shock to supply by countries other than Canada. Thus, for instance, the discovery of a large gold mine on Canadian soil that causes an increase in supply and a reduction in price would not be handled well by NAOMI.

Finally, the change in the government's primary structural balance (as a share of potential GDP) exerts a rather modest influence on GDP. With a coefficient only –0.15, NAOMI is substantially Ricardian in its response to tax cuts for instance. In addition to trying the primary balance, we have also investigated the explanatory power of the fiscal conditions index (FCI) suggested by

---

[4] See, for example, Fougère(1999)

James, Robidoux and Wong(2000). The primary differences between measures are that the latter attempts to include the counter cyclical effects of automatic stabilisers in addition to permitting heterogeneous propensities to consume out of tax and transfer changes. However, using this measure introduces the issue of simultaneity between regressor and regressand since this FCI is clearly endogenous to output. Ignoring this complication yields a somewhat higher elasticity of about -0.25.

3.2 Inflation

NAOMI determines the level of both GDP and Consumer Price Index (excluding food and energy) inflation via an expectations-augmented Phillips curve framework. Inflation is determined by a combination of adaptive inflation expectations, the level and change in the output gap, the change in the relative price of total commodities and the change in the real exchange rate.

$$\pi = .67\pi_{-1} + .33\pi_{-2} + .17\text{ygap}_{-1} + .65\Delta\text{ygap}_{-1} + .1\Delta\text{pcom}_{-2} + .07\Delta z_{-1} + .21\Delta z_{-2}$$

$$\bar{R}^2 = .81 \quad \text{S.E.} = 0.014 \quad \text{DW} = 2.2$$

where π is quarterly at annual rates CPI excluding food and energy inflation (CPIXFE) and pcom is the relative price of total commodities (U.S. dollars).

CPIXFE inflation plays a pivotal role in NAOMI as it forms the basis for the determination of short-term and, to a lessor extent, long-term interest rates. It is determined primarily by demand-pull forces (i.e. the aggregate output gap from CEFM, see Robidoux and Wong (1998)) and movements in two relative prices, total commodities relative to the GDP deflator (in U.S. dollars) and the real exchange rate. The influence of the latter two variables reflects Canada’s status as an open economy. The use of total rather than non-energy commodity prices allows for the possibility of second-round effects on core inflation (e.g. through transportation costs) stemming from energy price movements. Noteworthy also is the inclusion of the real rather than the nominal exchange rate. Ceteris paribus, only if the price of imported goods (*expressed in Canadian dollars*) is rising faster than the general price level in Canada will domestic inflation begin to rise. This specification permits ongoing inflation differentials between Canada and U.S. to persist indefinitely (i.e. independent monetary policy) with the differential merely being reflected by movements in the nominal exchange rate (long run relative PPP).

While the level of the output gap is a necessary feature of any output-based Phillips curve, the inclusion of its first difference requires some explanation. So-called ‘speed limit’ effects suggest that when output is approaching potential very quickly production bottlenecks in certain sectors may cause the inflation to rise despite the economy as a whole still being in excess supply (see Gordon 1980). Others, such as Fortin (1990,1991) appeal to the possibility of hysteresis in the labour market as a motivation for including the change term. On the empirical side, Duguay (1984) and Cozier and Wilkinson (1990) have found strong statistical evidence in favour of the inclusion of the change term in the Phillips curve for Canada.

Our research is consistent with these results but is motivated on a somewhat different basis. We argue that because the change in the output gap carries important information regarding the future level of the output gap then it may serve as a proxy for the forward component of inflation expectations in an otherwise purely adaptive specification. Laxton, Shoom and Tetlow(1992) suggest that measurement error associated with using a Hodrick Prescott filter to estimate potential can lead to the erroneous inclusion of the change term. However, it is not clear from their work whether it is measurement error that causes the coefficient on the change term to be biased or the inclusion of lags of inflation.[5] Furthermore, one cannot conclude on the basis of their experiment that the change term does not belong in a Phillips curve. Rather, they can only state that a significant coefficient may or may not be spurious. Finally, the measure of potential used is our estimation work is not derived explicitly from a HP filter.

Inflation expectations, like all expectations in NAOMI with exception of the central bank’s are purely adaptive[6]. Agents place a weight on the first and second lags of inflation in forming their expectations about the future. One could also include the change in the output gap term as a proxy for inflation expectations. Note that there is no explicit distinction made in NAOMI between expectational and intrinsic dynamics. Rather, the coefficients on lagged inflation represent the sum of these two otherwise distinct sources of dynamics. Further, there is no explicit credibility effect in the model. Instead, the pure accelerationist specification is imposed.[7] This ensures

---

[5] OLS is biased but consistent for regressions that include lags of the dependent variable.

[6] Unfortunately, implementing fully rational expectations for all endogenous variables would further complicate estimation and could compromise the model’s forecast accuracy.

[7] This amounts to restricting the coefficients on lagged inflation such that they sum to unity. This restriction is not rejected by the data.

against so called ‘free lunches’ for the central bank that arise when an explicit weight is placed on the inflation target.

GDP inflation is determined in a similar fashion to CPIXFE except that the estimated elasticities with respect to exchange rate and commodity prices are larger. The latter reflects the fact that the commodity export component in the GDP deflator is greater than the import component in the CPIXFE index.

$$\pi^{p} = \pi + .3\Delta pcom + .13\Delta pcom_{-1} + .08\Delta z_{-1}$$

$$\overline{R}^{2} = .78 \quad S.E. = 0.019 \quad DW = 1.86$$

where π^{p} represents quarterly at annual rates GDP inflation.

<u>3.3 Slope of the Yield Curve Gap</u>

Consistent with the Bank of Canada’s mandate, the yield curve in NAOMI is set so as to maintain fourth-over-fourth CPIXFE inflation at the midpoint of the official target band of 1 to 3 per cent. Reflecting the lags between policy actions and inflation outcomes, monetary policy is forward looking in NAOMI. In addition, there exists a considerable degree of policy smoothing to reflect, among other factors, the uncertainty faced by the central bank regarding the structure of the economy. Hence the slope of the yield curve is a function of the lagged yield curve and the inflation rate expected to prevail 7 quarters from now. The output gap and exchange rate do not appear explicitly in the policy rule.

$$slope = .85 slope_{-1} + .38\left(E_{t}\hat{\pi}_{t+7} - \pi^{target}\right)$$

$$S.E. = 0.0069$$

where slope is defined to be the 90 Treasury Bill rate minus the over 10 year government of Canada bond yield plus a constant term premium and π^{target} =0.02[8], the midpoint of the Bank of Canada’s current inflation target range.

---

[8] In NAOMI monetary policy targets CPIXFE inflation defined as *log(P(t))-log(P(t-4))* as opposed to quarterly at annual inflation

Monetary policy actions affect the real economy through the yield curve in NAOMI. The peak impact on inflation from a policy innovation in NAOMI occurs about 7 quarters after the shock. Hence, interest rates today move in response to the expected deviation of inflation from the target 7 quarters from now. Consistent with the theory of optimal central bank behaviour under non-additive uncertainty, as well as historical observation, we introduce a smoothing term in the form of a lag of the dependent variable with an estimated coefficient of .85. It is worth noting that the coefficient of 0.38 (2.5×(1-.85)) on expected inflation is estimated from historical data and is in no sense optimal. Appendix 1.0 provides a thorough discussion of NAOMI's policy rule.

<u>3.4 Long-term interest rates (Over 10 year average Government bond yield)</u>

Long-term interest rates are determined by short-term interest rates, U.S. long-term interest rates and to a lesser extent, an equilibrium real rate plus inflation.

$$\mathrm{rgcb} = .8\mathrm{rgcb}_{-1} + .12\mathrm{rgcb}_{-2} + .08(\mathrm{rrgcb}^{*} + \pi) + .12\Delta\mathrm{rtb} + .08\Delta\mathrm{rtb}_{-1} + .08\Delta\mathrm{rtb}_{-2} + .8\Delta\mathrm{rgcb}^{\mathrm{US}}$$

$$\overline{R}^{2} = .99 \quad \mathrm{S.E.} = 0.0028 \quad \mathrm{DW} = 1.94$$

where rgcb is the average yield on over 10 year government bonds, rtb is the 90-day Tbill rate and $\mathrm{rgcb}^{\mathrm{US}}$ is the average market yield on 10 year U.S. bonds. rrgcb* is defined to be the equilibrium real long-term interest rate in Canada and is based on an HP filter.

Modelling long-term interest rates in a backward-looking model is a difficult task. Ideally, one should compute long-term interest rates as a weighted average of future short rates consistent with the expectations theory of the term structure. Estimating these weights presents econometric problems and the resulting equations often forecast poorly. This stems from the fact that observed long-term interest rates display excess sensitivity relative to the predictions of the expectations theory, i.e. they follow too closely movements in short rates. It is also difficult to ignore the high correlation between movements in domestic and U.S. long bonds. Indeed, the historical correlation is even higher than for short-term rates. However, making long rates in Canada a function of those in the U.S. makes interpreting foreign shocks more difficult given the assumption of domestic monetary autonomy. For instance, consider an increase in targeted level of inflation in the U.S. We should expect a permanent increase in short- and long-term bonds abroad consistent with the Fisher identity.[9] By forcing Canadian long rates to follow (in the long run) those in the U.S. we would witness a corresponding increase in domestic short-rates so as to maintain the same steady-state yield slope. Consequently, real interest rates must rise in Canada in response to a permanent increase in inflation abroad.

Blending theory with observation is a particularly difficult task for long-term rates and it represents an area for future research. Currently, in the short run, long-rates are a function of changes in short domestic rates and U.S. long rates. In the long run, however, long-bonds are determined by a (rather preliminary) measure of the equilibrium real rate (HP filter) and inflation.

3.4(b) Short-term Nominal Interest Rate (Tbill)

Given a level for long-term interest rates, the slope determines level of short-term interest rates.

The short-term interest rate in NAOMI (90-day treasury bill) is defined as the slope plus the long rate minus the term premium. We set the term premium equal to the historical average difference between short and long rates. Strictly speaking, this definition is inadequate since the term premium has, in all likelihood, varied considerably through time due to changes in, among other things, the government debt. We leave a more thorough investigation of this issue to future research.

3.5 The Real Exchange Rate

The real exchange rate plays a pivotal role as an equilibrating force in NAOMI. Responding to both internal and external shocks, the real exchange rate works in tandem with monetary policy to restore real (but not nominal) equilibrium in the model. However, a unique feature of NAOMI is the ability of the real exchange rate to restore real equilibrium without the aid of monetary policy.

With two exceptions, long run relative purchasing power parity holds in NAOMI. Thus, for example, higher U.S. inflation does not affect Canadian inflation in the long run since the nominal exchange adjusts one-for-one to offset it. In the short run, PPP is relaxed reflecting, for instance, the influence of sluggish nominal price and/or quantity adjustment. The two previously

[9] Ignoring the initial change in the slope necessary to raise U.S. inflation.

noted exceptions to PPP arise from permanent commodity price shocks and permanent supply shocks.

$$\Delta z = .4\Delta z_{-1} - .25(slope_{-1} - slope_{-1}^{US}) - .1\Delta pcne - .26(ygap - ygap^{US}) - .25(\Delta\pi - \Delta\pi^{US})$$
$$- .08\left(z_{-1} + .41pcne_{-1} + \sum_{i=1}^{\infty}\left(ygap - ygap^{US}\right)_{t-i} + .25\sum_{i=1}^{\infty}\left(slope - slope^{US}\right)_{t-i} \right)$$

$$\overline{R}^2 = .98 \quad S.E. = 0.015 \quad DW = 2.0$$

where π^{US} is U.S. GDP inflation (see previous sections for definitions of other variables).

In the short run, the real exchange rate is determined by the;

(1) Canada/U.S. slope differential

(2) Canada/U.S. output gap differential

(3) change in real non-energy commodity prices

(3) Canada/U.S. inflation differential

Movements in the real interest rate are proxied by the slope of the yield curve.[10] Thus, for instance, an increase in short-term interest rates will increase the slope and cause the real exchange rate to appreciate in the short run.

As previously described, the activity differential between Canada and the U.S. exerts a direct influence on the real exchange rate in NAOMI. For example, an increase in aggregate demand in Canada, holding constant potential output and the U.S. output gap, will lead to an appreciation of the real exchange rate regardless of whether or not monetary policy reacts to the shock. This feature is predicated on the view that a market economy does not require a central bank to restore real equilibrium. Rather monetary policy acts merely as a catalyst in this process by exploiting the presence of short-run nominal rigidities. The theory behind the aforementioned exchange rate response is quite straightforward. Recall that the real exchange rate between country A and B is the price of country A's output relative to country B's expressed in a common currency. An increase in the demand for country A's good, stemming from a shock to preferences for instance,

[10] One lag of the slope differential is used to avoid the obvious issue of simultaneity between interest rates and the exchange rate.

will tend to increase its relative price and hence cause an appreciation of the real exchange rate[11]. This feature has important implications for the model's dynamic properties as demonstrated in section 4.0.

Finally, the Canada/U.S. inflation differential influences the real exchange rate in the short run. Temporary violations of relative PPP are strongly supported by the data and add to the forecasting ability of the equation. So, for example, higher U.S. inflation will cause a temporary depreciation of the real exchange rate that will pass through directly into Canadian inflation (in addition to boosting aggregate demand).

In the long run, the real exchange rate is determined by the relative price of non-energy commodities and the *cumulative* output gap and slope differentials between Canada and the U.S (see Appendix 4.0 for a more detailed explanation). There is now a fairly extensive body of empirical evidence linking the equilibrium value of the real exchange rate with the terms of trade for small open economies such as Canada. We consider it important to acknowledge this link in NAOMI. However, the long run elasticity of the exchange rate with respect to commodity prices is somewhat smaller than the average value reported in the literature.[12]

The importance of the cumulative output gap differential becomes of interest in the presence of supply shocks. Ceteris paribus, an increase in Canadian potential stemming from a TFP shock will create excess supply and thus lead to a depreciation of the exchange rate. This depreciation, combined with the effects of monetary policy, eventually return the output gap to zero. However, the cumulative (or integral) output gap remains negative. Thus a positive potential shock will permanently depreciate the real exchange rate.

4.0 Model Properties

In order facilitate a better understanding of NAOMI's macro properties, we report the results of the model's response to a number of exogenous shocks. Given the linear structure of the model, these responses are invariant to initial conditions.

---

[11] This assumes, among other things, that the majority of the increased demand is for domestically produced goods.

[12] Recent studies on long-run exchange rate determination include Amano and van Norden(1993), Amano, Coletti and Murchison(1998), Charron(1999) and Lafrance and van Norden(1995). Based on these papers, the average value is long-run elasticity is about -0.6.

4.1 Temporary Shocks

4.1.1 Aggregate Demand Shock (Figure 1.0)

In this scenario we consider the impact of a 1 per cent increase in aggregate demand stemming from, for instance, a shock to consumers' rate-of-time preference with no corresponding increase in potential.[13] Historically, aggregate demand shocks have tended to be quite persistent but ultimately temporary in nature. Here we assign a root of 0.9, implying that half of the shock has dissipated after 2 years.

While it takes time for the shock to demand to translate into significantly higher inflation, interest rates rise 25 basis points immediately. This reflects the fact that policy is looking at future rather than present inflation. The peak increase in short-term nominal rates is about 55 basis points which translates into a 40 basis points increase in the yield curve. A combination of higher interest rates and an output gap of about 1 per cent combine to appreciate the real exchange rate by just over 3 per cent. The nominal exchange rate appreciation is somewhat less reflecting a higher domestic price level. A stronger exchange rate and higher slope work to eliminate excess demand within about 2.5 years and ultimately push the economy into excess supply so as to restore inflation to 2 per cent in the 3rd year. Owing to the accelerationist Phillips curve there is no cumulative output gain for this, or any other, temporary shock. The peak increase in quarterly (at annual rates) inflation is almost 0.7 percentage points after 5 quarters.

4.1.2 Unanticipated Relative Price Shock (Figure 2.0)

In this scenario we consider the impact of a 1-percentage point unanticipated, temporary inflation shock that does not become entrenched in inflation expectations. Thus inflation will return to control without the aid of any endogenous model reaction. This is conceptually distinct from a supply shock, which is considered in section 4.3.2. Examples of such shocks are relative price shocks that initially affect the aggregate price level but are price neutral in the long run or a one-time increase in the indirect tax rate. The shock has a root of .95 indicating a high degree of persistence.

Owing to the temporary nature of the shock, monetary policy reacts less aggressively than if it were to base policy on current or lagged inflation, as is the case with a Taylor rule specification.

---

[13] The shock builds to 1 per cent over the course of one year.

Since more than half of the shock has dissipated at the relevant policy horizon of 7 quarters, the monetary response is somewhat muted. Nevertheless, short-term interest rates increase by about 50 basis points in the second year with about half of this increase being reflected in the slope of the yield curve. This is sufficient to reduce output by 0.2 per cent and appreciate the real exchange rate by 0.5 per cent. This combination returns inflation to control at the end of the third year. It is worth noting that while the real exchange rate appreciates the nominal depreciates. This difference arises because of the higher domestic price level. NAOMI illustrates that nominal exchange rate appreciations need not follow tighter monetary policy, even when U.S. policy is unchanged. Long-term rates increase to a peak of 25 basis points above control near the end of the second year.

Owing to the temporary nature of the shock, policy must eventually push the economy into excess demand so as to prevent inflation from undershooting the target. In the long run there is no cumulative output loss.

4.1.2(b) *What if the price shock were fully anticipated*? (Figure 2.1)

To further illustrate the impact of forward looking monetary policy we consider the difference in model behaviour when the aforementioned shock is delayed for 2 years. While shocks are not usually anticipated in this sense, one could imagine an announced future increase in indirect taxes as an example.

When the price increase is anticipated the monetary authority no longer looks through the peak of the shock, as is the case when it is a surprise. Consequently, the peak increase in the slope rises from about 25 to 50 basis points and the real exchange rate appreciates by 9 per cent compared to 5 per cent. Further, because policy reacts before the shock hits, inflation initially declines below control and the peak increase falls from 1 to about 0.8 percentage points. In addition, inflation returns to control in about half the time.

As a result of lower expected inflation, long-term interest rates increase by less than 10 basis points compared to 25 when the shock is unforeseen. The effect of tighter policy just about cancels the impact of lower inflation on short-term rates.

The preceding temporary price shocks illustrate a noteworthy feature of the class of monetary policy rules typically used in models such as NAOMI. Specifically, policy reacts to a positive inflation shock despite the knowledge that it is temporary and will not cause inflation to exceed 3 per cent at any point in time. In a purely deterministic environment where the central bank is truly indifferent to the level of inflation while within the target band, the argument could be made that no policy action is warranted. However, publishing target bands as opposed to a specific level likely reflects the stochastic environment that a central bank operates in rather than indifference toward a range of inflation outcomes. If it is true that the dis-utility to the central bank (and hence the public) is quadratic in the deviation of inflation from the midpoint of the target range, then one can motivate the policy reaction on this basis.

Alternatively, even if the central bank is indifferent regarding inflation outcomes within the bands, a policy response can still be rationalised conditional on the assumption of uncertainty. In a linear, stochastic environment[14] the 'safest' place to be from the policy maker's perspective is the middle of the target range as this uniquely minimises expected loss. For the purpose of these deterministic simulations, we assume that the central bank behaves *as if* it were in a stochastic environment. Thus these impulse responses best reflect the behaviour of the central bank in a true forecast environment.

4.1.3 Nominal Exchange Rate Shock (Figure 3.0)

Here we consider a temporary investor portfolio shock that causes a 3 per cent depreciation of the nominal exchange rate. This is roughly equivalent to a shock that lowers the dollar from 70 to 68 cents U.S. As a result of sticky prices, the real exchange rate depreciates by almost the full amount in the first few quarters following the shock.

A real depreciation raises inflation both via higher import prices and through increased net-export activity. The latter raises demand by about almost 0.3 per cent above potential after 5 quarters. The peak increase to CPIXFE inflation is 0.7 percentage points and occurs after approximately one year. GDP inflation increases marginally more reflecting its higher exchange rate elasticity. Short-term interest rates rise fairly rapidly to a peak of 35 basis points above control. This increase combined with the unwinding of the shock, returns inflation to control after about 2 years. The shock results in a permanently weaker nominal exchange rate because it raises the general price level in Canada visa-vie the U.S.

4.1.4 Monetary Policy Shock (Figure 4.0)

---

[14] With odd moments equal to zero.

We now consider the impact of a 100 basis points increase in the slope of the yield curve that lasts for 2 years[15]. This shock is intended to illustrate further the monetary policy transmission in NAOMI.

The peak decline in output of 0.8 per cent occurs 2.5 years following the onset of the shock. This decline is consistent with that reported by Duguay(1994) (-.8 per cent) and Hendry(1995,1998) (-.9 per cent). However, it is somewhat lower than CEFM (-1.3 per cent) and considerably lower than Gerlach and Smets(1995) and QPM (Coletti, Hunt, Rose and Tetlow(1996)) (-1.5 per cent). There appears to no consensus in the empirical literature as to whether the slope or the change in the slope affects output growth. However, this distinction has a non-trivial effect on the aforementioned elasticity and explains entirely NAOMI's relatively low elasticity[16].

At the end of the third year inflation has fallen 1.4 percentage points below the target and the nominal exchange has appreciated by 3 per cent.

Following the conclusion of the shock, policy aggressively begins working to offset the past increases by lowering short rates more than 150 basis points below control. This facilitates a depreciation of the real exchange rate and moves the output gap from -0.8 to .45 per cent in a little over two years. Inflation has all but returned to the target 3 years following the end of the initial interest rate increase.

4.2 Foreign Shocks

4.2.1 U.S. Aggregate Demand Shock (Figure 5.0)

Although the U.S. side of NAOMI is currently incomplete and therefore exogenous, it remains instructive to provide a flavour of the model's response to foreign shocks. Here we consider the impact of a hypothetical scenario whereby U.S. demand increases above potential and as a result U.S. inflation, commodity prices and interest rates increase. Some caution is warranted in interpreting the results however, since these responses are derived from a separately estimated structural VAR model.

---

[15] A 100 basis point increase in the slope translates into about 120 basis points in short rates.

[16] Experimentation with the level of the slope in the aggregate demand function suggests a peak impact of about –1.6 per cent for the same 8 quarter shock to interest rates.

The shock we consider is 0.75 per cent increase in U.S.aggregate demand that causes inflation to peak at about 0.35 percentage points above control after about 1.5 years. Both the U.S. slope and long-term interest rates rise by a peak of about 35 basis points in the first year of the shock. This implies a 70 basis point increase to short rates. Finally, level of real commodity prices rises by about 0.5 per cent above control but eventually returns to control (as it is a relative price). The path of the U.S. output gap has been modified slightly to ensure that the cumulative output gain is zero (consistent with an accelerationist Phillips curve)

We now turn to the impact of these shocks on the Canadian economy. An increase in U.S. output (and output gap) will put upward pressure on Canadian export activity and cause the real exchange rate to depreciate slightly (the increase in U.S. spending falls predominantly on U.S. produced goods[17]). The latter effect is strengthened by an increase in the U.S. slope which makes U.S. denominated debt instruments relatively more attractive and by the increase in U.S. inflation. Recall that PPP does not hold completely in the short run implying that the nominal exchange rate fails to adjust sufficiently to leave import prices unchanged. Finally, the effect of these three factors on the dollar is mitigated somewhat by higher real commodity prices.

The combination of higher U.S. demand, a weaker real exchange rate and stronger real commodity prices all work to push the Canadian economy into excess demand. The initial increase is about 35 per cent of the U.S. demand shock, reflecting the share of Canadian output that is exported to the United States. Output then rises to peak of about 0.4 per cent above control as the exchange rate and commodity effects come into play.

Higher demand in Canada, again combined with a weaker exchange rate and higher commodity prices, causes inflation to rise by about the same amount as in the U.S. (.35 percentage points) albeit somewhat faster. When we compare the inflation response here to that of a domestic aggregate demand shock of the same magnitude we see that inflation is more responsive when the shock originates abroad. This is mainly due to the short run violation of relative PPP whereby there is some direct passthrough into Canadian prices stemming from higher U.S. prices.

Monetary policy reacts to this increase in inflation by increasing the slope of the yield curve by a peak of about 35 basis points. However, because long-term interest rates respond directly to

---

[17] Recall that this exchange rate response would be the opposite if the source of higher U.S. activity were supply rather than demand.

higher U.S. rates, short-term rates must increase by more than if the same demand shock arose in Canada only. The peak increase at the short end is 60 basis points early in the second year.

In the long run all real variables return to there control levels and there is no net output gain arising from the shock. However, owing to the fact that the U.S. price level increases by more than the Canadian, the nominal exchange is about 2 per cent stronger than before the shock. This is roughly equivalent to a 1.5 cent increase from a starting point of 70 cents/U.S.

4.2.2 U.S. Inflation Shock (that is fully accommodated by the Fed) (Figure 6.0)

While somewhat contrived, this shock is intended to show the impact of higher U.S. inflation and nominal interest rates on the Canadian economy in the short and long run.

Recall that relative PPP is violated in the short run in NAOMI but holds in the long run for general price level inflation (but not for relative price changes or supply shocks). Moreover, while the level of U.S. long rates influences Canadian long rates in the short run, interest rates are determined entirely by the domestic inflation rate and an exogenously determined equilibrium real rate in the long run.

With this in mind we can now analyse the impact of higher U.S. inflation and interest rates on Canada. Recall that higher foreign inflation temporarily depreciates the real exchange rate putting upward pressure on domestic inflation as import prices begin rising faster than the general price level. Slightly less than one-tenth of the U.S. inflation shock feeds into GDP inflation after one year. The exchange rate depreciation places very modest upward pressure on aggregate demand (less than .03 per cent).

The combination of higher domestic inflation (expectations) and foreign long rates pushes domestic long rates up by about 85 basis points. Short-term rates increase by almost the same amount indicating almost no change in the slope. As the nominal exchange rate eventually adjusts to the inflation differential the real exchange rate appreciates back toward its starting point and inflation returns to the target. Note that inflation will return to its pre-shock level regardless of whether or not monetary policy intervenes. The slight tightening of the slope merely speeds the adjustment back to equilibrium.

Finally, as inflation returns to 2 per cent both short and long rates return to control. The nominal exchange forever appreciates at 1 per cent per year consistent with the higher Canada/U.S. inflation differential.

4.3 Permanent Shocks

4.3.1 Permanent Real Commodity Price Shock (Figure 7.0)

This shock is similar to that experienced in Canada during the Asian crisis of 1997-98. Specifically, commodity prices dropped significantly while U.S. growth remained robust. Thus, the fall in commodity prices was not caused by a negative U.S. demand shock. Here we examine the impact of a permanent negative shock to real non-energy commodity prices that builds over the course of one year to a peak of 15 per cent. All U.S. variables are held fixed in the scenario.

Initially, lower commodity prices should induce a negative supply response by commodity exporters as they are pushed down their supply/marginal cost curves. Stated otherwise, high cost producers no longer find it profitable to supply the initial quantity at the new lower price and hence exports fall.

The response of profit maximising producers to relative price shocks like this are not well articulated by a model such as NAOMI. Rather, the model simply predicts a fall in output for about the first 1.5 years that is presumably driven by lower exports following the shock. The magnitude of the reduction underscores an interesting property of NAOMI. Specifically, commodity prices seem to exert a very modest influence on aggregate demand in Canada. Indeed, finding a strong link between these variables, even controlling for monetary policy and exchange rate movement, has thus far proved elusive. The low elasticity may stem from endogenous labour movements between the commodity and manufacturing sector in the presence of commodity price shocks. Alternatively, there may be other endogenous relative price movements not captured by the real exchange rate that are excluded from NAOMI's demand function. Finally, commodity price shocks of this sort may affect potential output rather than output relative to potential.

The combination of modest excess supply and the negative shock to a relative price work to lower CPIXFE inflation to a peak of about 0.5 percentage points below control. The magnitude of the decline in GDP inflation is more than twice as large. This reflects the higher commodity share in the GDP deflator relative to the CPIXFE.

The monetary authority begins working immediately to offset the shock by easing policy. This results in a 40 basis points reduction to the treasury bill rate at the end of the first year. This intervention, combined with a substantial depreciation of the real exchange rate, acts to push the economy into very mild excess demand near the end of the second year. Allowing the real exchange rate to fully adjust to the shock clearly minimises the negative impact on output and inflation stemming from the shock.[18] If policy were to instead target the exchange rate at its pre-shock level, the impact on output would be considerably greater. Indeed, ultimately the central bank would have to abandon this strategy or be faced with an ever-falling inflation rate.

4.3.2 A Reduction to the Midpoint of the Inflation Target (Figure 8.0)

Here we consider the impact of a reduction to the midpoint of the Bank of Canada's inflation target bands equal to one percentage point.[19]

Short-term interest rates rise by a peak of about 85 basis points, generating a one per cent appreciation of the real exchange rate and a 0.45 per cent peak loss in output in the beginning of the third year. Given the form of the monetary reaction function, inflation will only approach the new target asymptotically. However, the vast majority of the disinflation is completed within 5 years.

The cumulative output loss is 2 per cent, implying a sacrifice/benefit ratio of **2 to 1**.[20] This ratio lies exactly between QPM's sacrifice ratio of 3:1 and benefit ratio of 1:1. The difference between sacrifice and benefit ratios in the Bank staff's model reflects a (kink) non-linearity in the Phillips curve. No such non-linearity exists in NAOMI.

Consistent with long run relative PPP, the nominal exchange rate appreciates 1 per cent per year forever and the real exchange rate returns to control.

4.3.3 Is NAOMI's Policy Rule Credible? (Figure 8.1)

Monetary policy is only credible in the eyes of the public if it is both willing and able to fulfil its stated mandate. Consequently, it is worth investigating whether the estimated policy rule in NAOMI could in fact generate a path for inflation that lies within the Bank's inflation target range. In early 1991 Bank Governor John Crow committed to a moving target that would allow

---

[18] Recall that non-commodity exporters benefit from the exchange rate depreciation

[19] This can otherwise be thought of as a reduction in the bands from 1 to 3 per cent down to 0 to 2 per cent beginning with inflation at 2 per cent.

[20] A sacrifice (benefit) ratio is cumulative loss (gain) in output associated with a permanent 1 percentage point reduction (increase) to the rate of inflation. Owing to a linear Phillips curve, NAOMI scarifice and benefit ratios are the same.

inflation to fall gradually from 6 per cent down to 3 per cent in about 2 years (end 1992) and then to 2 per cent by the end of 1995, plus or minus one percentage point.

Given the structure of NAOMI, including the estimated policy rule, we can get a rough idea of what such a disinflation might have looked like. Of course we would not expect it to look identical to the actual disinflation that occurred even if NAOMI were a good approximation of the actual economy (including the policy rule). This stems from the fact that the economy was hit by shocks over this period that are not accounted for in the experiment. Furthermore, we have made no attempt to include movements in exogenous variables. Rather this represents an experiment to answer the question*; a priori could the Bank achieve what it set out to achieve?* In this sense, the experiment seeks to generate an ex-ante expected outcome rather than an ex-post counterfactual.

As indicated in Figure 10.0, the Bank is able to maintain inflation within the target range, but just barely. The most difficult period is at the beginning of the experiment where the targeted level of inflation is falling fastest. This is partly due to the inherent lag between monetary policy actions and inflation outcomes in the model. An added complication in the experiment is the fact that in 1991Q1 the output gap was already –3 per cent (CEFM definition of potential). Thus we should have expected inflation to be falling even before the first policy action was taken. Putting in a –3 per cent output shock into this experiment would surely push inflation closer to the midpoint of the target range. Instead however, we have assumed that the Bank began the disinflation 6 months before the actual announcement of the targets. At this point the economy was close to equilibrium (output gap was almost zero). This amounts to allowing the model two extra quarters to achieve the initial 3 percentage point reduction to inflation.

Based on these results one might conclude the policy rule in NAOMI is a little on the timid side. Recall however that it has been estimated over the entire 1990’s, not just the disinflation period. Over much of this period the Bank has largely accommodated an average core inflation rate of less than 2 per cent. Explanations for this seeming inconsistency in behaviour are varied but suffice to say that parameterising a policy rule based solely on the Banks behaviour during disinflations may generate overly aggressive policy responses in the current forecast environment.

Using the estimated policy rule, we see that output falls to a peak of –1.4 per cent below control at the end of 1992 and then begins to move back toward potential. This output decline stems from a combination of a 250 basis points increase in short-term interest rates and a 3.6% appreciation of the real exchange rate. The cumulative output loss stemming from the disinflation is about 8 per cent.

4.3.4 Permanent Supply Shock (Figure 9.0)

Because supply shocks generate business cycle dynamics in NAOMI, it is worth considering the short run impact of, for example, a 1 per cent increase to potential stemming from a permanent increase in total factor productivity.

As discussed in Section 3.0, positive (negative) supply shocks initially create excess supply (demand). Thus in the shock considered here, the output gap initially falls to –0.35 per cent which initiates a fall in inflation and a real depreciation of the exchange rate. The peak decline in inflation of –0.35 percentage points occurs at the beginning of the second year. This generates a modest easing of the slope equal to about 30 basis points. Short-term interest rates peak at about 45 basis points below control in the second year.

Output takes almost 5 years to adjust fully to the increase in potential. Finally, owing the permanence of the shock, the nominal (and real) exchange rate is now forever weaker by about 2.5 (3.5) per cent or about 1.7 cents lower (from 70 cents). To the extent that a real exchange rate reflects a country's terms-of-trade (ignoring non-traded goods) we can conclude from this that a relative supply shock in Canada reduces the relative price of Canada's exportables. Stated otherwise, domestic supply conditions exert some influence over the world price of our exports. Thus NAOMI violates the strict assumption that Canada is a *small open economy* on world markets. It is worth noting, however, that the depreciation of both the real and nominal exchange rate is closer to 1 per cent after two years which is perhaps a more relevant time horizon for forecasters. Owing to this relatively small short-run elasticity, NAOMI reflects the behaviour of an *almost* small open economy.[21]

4.3.4(b) What if Monetary Policy Misinterprets the Shock (Figure 9.1)

The analysis above indicates that in the presence of a positive shock to potential output, the appropriate response by the central bank is a modest easing of policy. However, this assumes that the nature of the shock is known with certainty. In practice, however, potential output and

[21] The assumption that Canada exerts no influence over the price of capital (the real interest rate) regardless of how much it borrows is retained in the model.

therefore the output gap are not directly observed. Rather what the monetary authority initially sees is a 0.65 per cent increase in output[22]. We now consider the implications for inflation and interest rates of the policy maker initially interpreting the increase in output as a persistent positive demand shock. Only after 6 quarters does the central banks begin to learn the nature of the shock. The choice of 6 quarters is arbitrary but it represents a sufficient amount of time in NAOMI for the monetary authority to observe that its policy actions are not consistent with returning inflation to the target.

Consistent with a demand-shock interpretation, interest rates initially rise, eventually reaching about 50 basis points above control at the end of the first year. Despite this tightening of policy, however, the nominal and real exchange rates still depreciate. This stems from the fact that the economy is being pushed ever further into excess supply. Thus while the economy is becoming more productive on the supply side, higher interest rates are choking off domestic consumption of the extra output. A depreciating exchange rate helps to create an outlet for the excess goods and services by making them cheaper on foreign markets. Thus we should expect to witness a pronounced substitution from consumption and investment to net exports. Nevertheless, output continues to fall for the first year and a half indicating that the fall in domestic demand is greater than the rise in net exports.

Consistent with the combination of excess supply stemming from the potential shock and the erroneous tightening of policy, core inflation falls to a trough of about 0.75 percentage points below the target at the end of year two. This trough occurs about 6 months after the monetary authority begins to learn the true nature of the shock. Within a year, interest rates fall from 50 basis points above control to 100 basis points below. This facilitates an even faster decline in the value of the dollar. By the end of the fourth year output has reached its new equilibrium one per cent above control and the output gap is again zero.

In summary, this mistake is costly from a policy perspective since inflation falls by more than *twice* as much as when the shock is correctly interpreted. Moreover, the mistake introduces extra volatility into both the output and interest rate markets.

5.0 Model Validation

[22] Recall that a 1 per cent increase to potential translates to a .65 per cent increase in output in NAOMI at the time of the shock.

Having described NAOMI's structure and simulation properties, we now turn the focus to the model's ability to replicate history. In order to get a realistic picture of its behaviour through the business cycle we have elected to focus on the 1988Q1-1993Q1 period. Given the CEFM definition of potential, this period includes both a business-cycle peak and trough. We begin by analysing how each equation performs over this period, given actual outcomes for the independent variables. However, the simulations remain dynamic in the sense that past errors get carried forward through lags of the dependent variable. In the next section we move to a full model dynamic simulation with the exception of the slope of the yield curve. Finally, we provide the average error (AE), average absolute error (AAE), root mean-squared-error (RMSE) and Theil's U statistic at various horizons over the last decade.

5.1 Single Equation Dynamic Out-of-Sample Forecast Analysis

Single equation analysis can be quite useful for determining if one or more of the estimated parameters have changed considerably through time or if there are important omitted variables. However, it is critical to omit the evaluation sample (1988Q1-1993Q1) from the estimation period since doing so provides a more realistic picture of how the equation is likely to perform in a true forecast environment. Here we have gone one step further and omitted the last two years prior to the evaluation period. Thus each equation is estimated from 1972Q1 to 1985Q4.

Table 1.0 provides the single-equation dynamic out-of-sample forecast statistics. The average error (AE) provides a measure forecast bias while the corresponding T-stat provides a test of the statistical significance of the bias where the null is no bias. The average absolute error (AAE) provides an indication of how far, on average, a forecast is likely to be from the true value while the standard deviation penalises large errors disproportionately more than small ones but ignores bias. Root mean squared error (RMSE) provides disproportionate weighting and includes the effect of bias. Hence an unbiased forecast will have the same standard deviation and root-mean-squared error.

Table 1.0: Single Eq. Dynamic Out-of-Sample Forecast Statistics

	Ave. Error	T-stat	Ave. Abs. Error	Std.Dev Error	RMSE
CPIXFE Inf (Q@A)	-0.0008	-0.36	0.008	0.010	0.010
CPIXFE Inf (4th/4th)	-0.0013	-1.00	0.005	0.006	0.006
Output Gap	0.0050	1.30	0.014	0.018	0.019
Real Exch. Rate	-0.0050	-1.50	0.012	0.015	0.016
Long-term Int. Rate	0.0018	1.70	0.004	0.005	0.005

All errors are expressed in percentage points except the exchange rate which is in cents

Beginning with quarterly at annual (Q@A) CPIXFE inflation (see Figure 11.0) we note that the equation does a reasonably good job of tracking the broad movements in inflation over this period. Specifically, it picks up the rise in inflation beginning from about 4 to 6 per cent beginning in 1988. However, it does underpredict inflation in 1989Q1, which then becomes nested in inflation expectations over the next couple of quarters. It also tracks fairly well the decline associated with the disinflation of the early 1990s[23]. It is worth reiterating at this point that there is no implicit or explicit inflation target in the equation. The AE over this period is -0.08 percentage points while the AAE and RMSE are 0.8 and 1 percentage point, respectively. Note that on the basis of the t-stat we conclude the forecast is unbiased. On a fourth-over-fourth basis, the RMSE is 0.6 percentage points, reflecting the lower volatility associated with this measure of underlying inflation. Overall the equation performs reasonably well over this rather volatile period of history.

We next consider the performance of the real exchange rate equation. Given the important role played by the exchange rate as an equilibrating force in the model it is critical that the equation be able to track the data out–of-sample. Furthermore, empirical validation is of special interest when the choice of included variables is derived largely from theory, as is the case with this equation. In particular, the inclusion of output gap and inflation differentials in the short run dynamics and the cumulative slope and output gap differential in the cointegrating vector is somewhat unique to this model if not unprecedented.

Over this particular episode we witness the real exchange rate appreciating from 84 to 96 cents U.S. followed by a dramatic slide back down to 85 cents. Similar to the Phillips curve, the

[23] The equation's ability to pick up the GST shock stems from the inclusion of a dummy variable.

exchange rate equation captures trends and turning points quite well. The AE, AAE and RMSE are respectively -0.5, 1.2 and 1.6 cents. Considering the amount of volatility over this period, the performance of the real exchange equation is impressive. Furthermore, an important role is played by the output gap differential in generating such accurate forecasts.

With regard to long-term interest rates, the late 1980s are a particularly illustrative period as they exemplify the main shortcoming associated with backward-looking expectations equations. During this episode, long rates were declining, albeit slightly, while short rates were increasing rapidly. This episode was later followed by declines at the short end throughout the early 90's. Thus long rates behaved broadly consistent with the expectations theory over this period. The equation, however, places too large a weight on recent short-rate behaviour and no weight on future behaviour. Thus they over predict long rate levels by an average of 60 basis points from 1988Q3-90Q3. After this episode, however, the equation does a reasonable job tracking the decline in rates. Over the 5-year period the AE, AAE and RMSE are 18, 40 and 50 basis points, respectively. With a t-value of 1.7 we are tempted to reject no bias in favour of an upward bias over this particular episode.

Finally, we consider the forecasting performance of what is arguably the most important equation in the system, the aggregate demand function. While the equation does quite a good job of tracking the rather pronounced slowdown in the Canadian economy beginning in early 1988 stemming from higher interest rates, it misses both the timing and magnitude of the trough in the early 1990s. Specifically, the equation predicts that the economy should not have plummeted into a recession of the depth actually witnessed. Moreover, it suggests that the duration of the recession should have been shorter. This prediction failure is nothing new for estimated Canadian demand functions simulated over this period. For instance, Freedman and Macklem(1998) and Robidoux and Wong (1998) arrive at a similar conclusion using similarly specified functions.[24]Indeed, both NAOMI and the Freedman and Macklem equations begin to over-predict output growth at about the point in time. Freedman and Macklem use this result to argue that the severity of the recession cannot be explained on the basis of monetary policy alone (real interest rates and the real exchange rate) since they are included in the equation. Likewise, the demand function in NAOMI does a good job of tracking the 1981-82 recession as well as the first half of

[24] Freedman and Macklem model output growth as a function of U.S. growth, the change in the ex-post real interest rate, the change in the real exchange rate, commodity price growth and the change in the cyclically-adjusted total government balance.

this recession. Thus one is tempted to conclude that there is something special about the 1990 recession.

Turning to the forecast statistics for the output gap equation we see that the AE, AAE and RMSE are 0.5, 1.4 and 1.9 percent, respectively. Thus on average the equation over-predicted the level of output by 0.5 per cent. The large difference between the AAE and the RMSE indicates that the equation made at least a few very large errors.

We have omitted the GDP deflator because of problems created by the increase in CPIXFE inflation stemming from the GST. Because this shock caused CPIXFE inflation to rise dramatically but had little effect on the GDP deflator, the equation (that models GDP inflation as a function of CPIXFE inflation) breaks down over this period. However, after this episode until 1993Q1, the equation has a RMSE of 1.2 percentage points (quarterly at annual rates), slightly higher than the CPIXFE equation.

5.2 Full Model Dynamic Out-of-Sample Forecast Analysis

We now consider NAOMI's ability to forecast the late 1980's and early 1990's when simulated rather than actual outcomes are used as explanatory variables (with the exception of the slope of the yield curve). As indicated by Table 2.0 and Figure 11.1, differences between the single equation and full-model simulation are fairly small with the exception of the real exchange rate. For this variable, RMSE increases from 0.016 to 0.024 mainly because it under predicts the extent of the depreciation that begins near the end of 1991. This stems from the fact that the simulated value of the output gap over this period is much higher than the actual (as previously) discussed. In turn, NAOMI predicts a stronger real exchange rate then actually prevailed.

Table 2.0: Full Model Dynamic Out-of-Sample Forecast Statistics

	Ave. Error	T-stat	Ave. Abs. Error	Std.Dev Error	RMSE
CPIXFE Inf (Q@A)	-0.001	-0.4	0.01	0.013	0.013
CPIXFE Inf (4th/4th)	-0.002	-1.1	0.007	0.009	0.009
Output Gap	0.007	1.8	0.014	0.018	0.02
Real Exch. Rate	-0.002	-0.5	0.016	0.024	0.024
Long-term Int. Rate	0.0005	0.3	0.006	0.008	0.008

5.3 Out-of-Sample Forecast Statistics over the 1990's

The most informative forecast statistics are those generated when the entire model is simulated dynamically out-of-sample over a period of time that is believed to be reflective of the near future. For this reason, evaluating the model over the 1990s is likely more informative than evaluating it over 1970s largely because the Bank of Canada was not explicitly targeting inflation in the 1970s. By making every variable endogenous, including the yield slope, we receive the most accurate indication of how the model is likely to perform in a true forecast setting at different horizons. Indeed, this type of experiment will capture shock, parameter and *model* uncertainty. Unfortunately, this type of information is often omitted, particularly when the model is forward looking. This stems from the fact that is it more difficult to control for the actual information set available at the time the conditional forecast is made. For instance, forward-looking DGE models are often 'grounded' by a calibrated (exogenous) steady-state. Further, the dynamic forecast is heavily dependent on these calibrated constants (e.g. capital-to-output ratio, relative prices). Thus when a conditional forecast is made at time *t*, one must choose values for these exogenous parameters without being influenced by what has actually happened since time *t*.

Since NAOMI's steady state consists of having inflation equal to the target, which requires an output gap of zero and constant relative prices, the only steady state parameter required is the inflation target. However, since the target was announced in 1991, this value is known a priori for the full forecast horizon when each conditional expectation is made.

While this experiment will provide the most accurate assessment of NAOMI's forecast accuracy, it is not perfect. When each forecast is made, actual outcomes from the exogenous variables are used. This will tend to bias the true forecast uncertainty downward because actual exogenous outcomes are unknown in a forecast setting and are consequently subject to error. When the U.S. side of NAOMI is completed we will be able to get a more accurate indication of the true level of forecast uncertainty.[25]

Table 3.0 provides forecast accuracy statistics for each variable at horizons ranging from 1 to 8 quarters starting 1991Q1 (when the inflation target was announced) and ending in 1999Q4 (30 observations at each horizon). Thus for each quarter the model is solved up to 8 quarters ahead and then compared to the actual outcomes for each variable. Again we compute the average error, the standard deviation of the error and the root-mean-squared error.

---

[25] However, even with this version of the model potential output will remain exogenous.

In addition, the Theils U statistic and *content horizon* is provided. Theils U provides an indication of the forecast accuracy of the model relative to a naïve no-change forecast. Specifically, it is the model's RMSE divided by the RMSE of a no-change forecast. Depending on the historical evaluation sample chosen, a U value of less than one may or may not be easy to achieve. For instance, if one chooses a sample for which there is a noticeable trend in the data then almost any model will produce a low U statistic. Conversely, if markets are efficient, arbitrage is possible and every variable in the model is endogenous then achieving a U statistic of less than one should be impossible (e.g. exchange rates)[26].

The content horizon of a forecast refers to the number of periods for which the forecast contains useful information beyond that contained in the unconditional mean of the series. Thus a content horizon is only defined for stationary processes. It can easily be shown that the content horizon is the horizon for which the forecast error variance just equals the variance of the series over that period. Defining Z(h) as the standard deviation of the forecast error at horizon h divided by the standard deviation of the series, the content horizon is the value of h for which Z(h)=1. For all horizons less than the content horizon, Z(h) should be less than one. Content horizon is a particularly stringent test because it assumes that the unconditional mean is known. Hence more information is being provided to the naïve forecast than is available to the model being evaluated. This is particularly true in this instance for the exchange rate. Since the exchange rate possesses a definite trend over the 1990's we evaluate the first difference instead. Of course the first difference of the (log of the) exchange rate is the average depreciation over that period. Thus the naïve forecast knows what the trend is but NAOMI does not. This information asymmetry will bias down considerably the content horizon[27].

So while the Theils U provides a good indication of the models ability to forecast at short horizons, the content horizon provides a very reliable metric by which one can judge medium term forecast accuracy[28]. Whereas the U statistic should fall as the forecast horizon increases, the content horizons provided in Table 4.0 have been computed based on no-change in the exogenous variables. This modification is critical since a forecast conditioned on the actual outcome of an

---

[26] The fact that exchange rate equations are sometimes able to achieve U values of less than one reflects the inclusion of exogenous contemporary regressors or a rather judicious choice of evaluation sample.

[27] A solution to this is to use the conditional mean. However this introduces other problems.

[28] Content horizon is particularly useful for distributed lag models such as NAOMI.

exogenous variable will *always* contain information beyond that contained in the unconditional mean. Thus the content horizon will be infinite unless every variable in the model is endogenous. Assuming that the U.S. side of NAOMI will be able to forecast better than the naïve no-change model, the content horizons will be greater than those reported here. Finally, we have included the uncentred 10^{th} and 90^{th} percentiles of the forecast errors at each horizon.

Table 3.0: Forecast Statistics (Endogenous Interest Rates, 1991Q1-1999Q4)

Horizon	Ave. Error	Std. Dev.	RMSE	10 Perc.	90 Perc.	Theil U	Z-stat	Content H.
CPIXFE Inflation (Q@A)								
1	-0.001	0.010	0.010	-0.016	0.011	0.597	0.512	
2	-0.002	0.011	0.012	-0.016	0.011	0.498	0.589	
4	-0.004	0.013	0.013	-0.022	0.013	0.510	0.712	
8	-0.004	0.014	0.014	-0.023	0.013	0.572	0.914	**10 quarters**
CPIXFE Inflation (t/t-4)								
1	0.000	0.003	0.003	-0.004	0.003	0.413	0.173	
2	-0.001	0.005	0.005	-0.006	0.005	0.435	0.314	
4	-0.003	0.009	0.009	-0.012	0.008	0.544	0.658	
8	-0.004	0.011	0.012	-0.018	0.007	0.677	1.06	**8 quarters**
Output Gap								
1	0.001	0.004	0.004	-0.004	0.006	0.936	0.256	
2	0.003	0.006	0.007	-0.005	0.009	0.908	0.477	
4	0.005	0.010	0.011	-0.008	0.019	0.894	0.821	
8	0.006	0.011	0.013	-0.007	0.024	0.777	1	**8 quarters**
Output Growth (Q@A)								
1	0.004	0.015	0.015	-0.014	0.025	0.774	0.632	
2	0.006	0.016	0.017	-0.019	0.027	0.794	0.716	
4	0.004	0.016	0.016	-0.018	0.024	0.562	0.737	
8	0.001	0.016	0.016	-0.019	0.021	0.503	0.801	**10 quarters**
Real Exchange Rate								
1	-0.004	0.017	0.017	-0.030	0.015	0.763	0.897*****	
2	-0.003	0.021	0.021	-0.030	0.023	0.594	0.897	
4	-0.003	0.027	0.028	-0.029	0.039	0.457	0.907	
8	0.008	0.047	0.048	-0.061	0.066	0.472	0.98	**9 quarters**
Nominal Exchange Rate								
1	-0.003	0.017	0.017	-0.028	0.015	0.869	0.72*****	
2	-0.003	0.022	0.022	-0.027	0.029	0.730	0.82	
4	-0.004	0.031	0.031	-0.038	0.039	0.609	0.941	
8	0.005	0.049	0.049	-0.046	0.066	0.579	1	**8 quarters**
Short-term Interest Rate (Tbill)								
1	0.000	0.008	0.008	-0.009	0.008	0.950	0.469	
2	0.000	0.011	0.011	-0.017	0.013	0.923	0.693	
4	-0.001	0.017	0.017	-0.022	0.022	0.894	1.04	
8	-0.003	0.023	0.023	-0.029	0.029	0.900		**4 quarters**
Long-term Interest Rate (Over 10 year)								
1	0.000	0.002	0.002	-0.003	0.003	0.520	0.155	
2	0.001	0.003	0.003	-0.002	0.005	0.469	0.211	
4	0.001	0.005	0.005	-0.005	0.008	0.521	0.331	
8	0.002	0.008	0.008	-0.008	0.012	0.540	0.452	**14 quarters**
GDP Inflation (Q@A)								
1	0.002	0.011	0.011	-0.011	0.015	0.601	0.635	
2	0.000	0.015	0.015	-0.015	0.013	0.708	0.842	
4	0.003	0.014	0.014	-0.014	0.018	0.522	0.780	
8	0.007	0.015	0.016	-0.010	0.028	0.542	0.844	**12 quarters**

*-Z-stat and content horizon for the change in the exchange rate (see text)

To summarise Table 3.0, NAOMI can beat a no-change forecast for all variables at all horizons, in most cases by a considerable margin. For instance, a Theil's U statistic of .77 (.56) for quarterly output growth at a one quarter (year) horizon indicates that the model can beat a naïve forecast by almost 25 (50) percent. Furthermore, NAOMI can do better than simply forecasting the average growth rate of output over 1990's (assuming this was known) at horizons up to about 2.5 years. The performance for CPIXFE inflation is quite similar. Both the nominal and real exchange rates have U statistics considerably lower than one at all horizons. Furthermore, the content horizon for the *change* in the exchange rate is just over 2 years.

Whereas the U and Z statistics are very impressive for long-term interest rates, short-term rates do not fare so well. Part of the reason for the precision of the long rate in terms of the U statistics is the fact that it is partially determined by exogenous U.S. rates[29]. The single biggest contributing factor to the mediocre performance of short rates is their close link to monetary policy. For the last 2 years (1998-99) NAOMI consistently predicts looser monetary conditions (in the form of lower interest rates) than actually witnessed over this period. This is due to the fact that inflation has consistently been below the midpoint of the target. Omitting this period improves the forecast performance non-trivially. Nevertheless, the ability of forward-looking policy rules to track closely actual interest-rate movements appears limited. This may be true even if the model does a good job of forecasting inflation. This is because monetary policy depends not on what actual inflation will be 7 quarters from now but rather what the central bank thinks it will be. NAOMI may not do a good job at replicating the bank's conditional expectations.

Overall, NAOMI appears to provide significant forecast information up to about 2 years when naïve forecasts for exogenous variables are employed.

---

[29] This is not true, however, for the Z statistic where U.S. rates are forecasted using a naïve model.

6.0 Conclusion

This paper describes the structure, simulation properties and forecast performance of the Canadian side of NAOMI, a new econometric model developed in the Economic Analysis and Forecasting Division at the Department of Finance. The results highlight that NAOMI represents a reasonable compromise between pure forecasting devices such as reduced-form VARs and structural models based explicitly on optimal decision rules.

NAOMI exhibits short-run (long-run) properties that are largely consistent with a wide class of Keynesian (classical) models. Specifically, prices are rigid in the short run and fully flexible in the long run. Particular attention has been given to the role of the real exchange rate in Canadian NAOMI, reflecting Canada's status as an open economy that relies heavily on trade. The real exchange rate is influenced by both short- and long-term factors and serves as an important equilibrating force, guaranteeing real equilibrium in the model. Monetary policy is represented by an inflation-forecast rule that includes a smoothing parameter that reflects, among other factors, the uncertainty faced by the Bank of Canada in setting interest rates.

NAOMI is useful for producing sensible quarterly forecasts of key macroeconomic variables on a timely basis. In addition, analyses of the uncertainty associated with each forecast can easily be performed. Finally, NAOMI can be used to perform risk scenarios and address a limited set of policy related questions. Future work will include using NAOMI to assess risks to the government's budget balance.

Appendix 1.0 Monetary Policy Rule[30]

In NAOMI we characterize the stance of monetary policy in terms of the slope of the term structure of interest rates (s_t : the short rate less the long rate). The use of s_t rather than the short-term interest rate reflects several considerations. Most importantly, we think that the slope of the term structure is helpful for the identification of monetary and non-monetary shocks in the historical data (Cozier and Tkacz 1994).

Disentangling monetary and non-monetary shocks from the short-term interest rate is particularly difficult. Let R_t be the 1-quarter nominal interest rate at time t which can be decomposed into a 1-period real interest rate, r_t , and expected 1-period-ahead inflation, $E_t\pi_{t+1}$:

$$R_t = r_t + E_t\pi_{t+1}. \tag{1.1}$$

The real interest rate, r_t , is also assumed to consist of two elements r_t^*, a time-varying equilibrium world real interest rate determined by non-monetary factors and mp_t a monetary policy factor reflecting changes in the supply of settlement balances engineered by the monetary authority

$$r_t = r_r^* + mp_t . \tag{1.2}$$

We can see from this simple decomposition that movements in the nominal short-term interest rate can come about due to several factors. First, common shocks across the world influence expected inflation, $E_t\pi_{t+1}$ in a similar manner. Second, movements in world real interest rates, r_t^*, as determined by global investment and savings conditions, influence domestic interest rates in all countries in a common fashion. Barro and Sala i Martin (1990) find (for 10 OECD countries) that each country's real interest rate depends primarily on world factors, rather than own-country factors, thereby suggesting a good deal of integration of world capital markets.

That said, under certain conditions the slope of the term structure, s_t^k, can reflect the stance of monetary policy better than a short-term interest rate

---

[30] This appendix was prepared by Jian-Guo Cao, Don Coletti and Stephen Murchison

$$s_t^k = R_t - R_t^k \quad (1.3)$$

where R_t^k denotes the k-period (or long-term) nominal interest rate (Cozier and Tkacz 1994). According to the expectations theory of the term structure R_t^k can be written as the average of expected future short rates plus a term premium, ψ_t^k

$$R_t^k = \frac{1}{k}(R_t + E_t R_{t+1} + ... + E_t R_{t+k-1}) + \psi_t^k . \quad (1.4)$$

Substituting equations (1.1), (1.2) and (1.4) into (1.3) yields the following equation for the slope of the yield curve:

$$s_t^k = (\frac{k-1}{k})r_t^* - \frac{1}{k}E_t\sum_{i=1}^{k-1} r_{t+i}^* - (\frac{k-1}{k})mp_t + \frac{1}{k}E_t\sum_{i=1}^{k-1} mp_{t+1} + (\frac{k-1}{k})E_t\pi_{t+1} - \frac{1}{k}E_t\sum_{i=1}^{k-1}\pi_{t+1+i} + \psi_t^k . \quad (1.5)$$

Abstracting from the term premium, equation (1.5) implies that the slope of the yield curve is related positively to the gap between the short-run equilibrium real rate and its expected future level, negatively to current monetary policy actions relative to its expected future course, and positively to short-run expected inflation relative to its longer-run future expected level. Therefore, if inflation expectations are highly persistent and if movements in equilibrium world real interest rates are also persistent the slope of the yield curve isolates monetary policy[31].

1.1 Methodology

The technique we use to estimate the monetary reaction function of NAOMI comes from Clarida, Gali and Gertler (1998)[32]. The baseline specification for our analysis is the inflation forecast-targeting rule:

$$s_t^* = \bar{s} + \beta(E[\pi_{t+7} | \Omega_t] - \pi^*) \quad (2.1)$$

where $\bar{s}$ is the sample average term premium, Ω_t is the information set available to the central bank at time t and π^* is the central bank's inflation target.[33]

[31] Whether these conditions are satisfied is an empirical question beyond the scope of this paper.

To hit the inflation target, the monetary authority acts to move the intermediate target variable, s, raising (lowering) the slope of the term structure when inflation is expected to be above (below) the target rate seven quarters ahead. In NAOMI, the short nominal rate instrument is adjusted taking into account any movements in long rates in order to achieve a particular desired setting for s, and ultimately for the expected path of inflation.

To allow for the high degree of persistence in interest rate movements we follow Clarida *et al.* (1998) and assume that the slope of the term structure partially adjusts to the target as follows:

$$s_t = (1-\rho)s_t^* + \rho s_{t-1} + \nu_t \tag{2.2}$$

where the parameter $0 \le \rho \le 1$ captures the degree of interest rate smoothing and ν_t is the exogenous random shock to the interest rates assumed to be i.i.d.

Combining equation (2.1) and (2.2) and setting $\alpha = \bar{s} - \beta\pi^*$ yields:

$$s_t = (1-\rho)(\alpha + \beta E[\pi_{t+7} | \Omega_t]) + \rho s_{t-1} + \upsilon_t \,. \tag{2.3}$$

Replacing the unobserved variables with realized variables yields:

$$s_t = (1-\rho)(\alpha + \beta\pi_{t+7}) + \rho s_{t-1} + \varepsilon_t \tag{2.4}$$

where $\varepsilon_t \equiv -(1-\rho)\{\beta(\pi_{t+7} - E[\pi_{t+7} | \Omega_t])\} + \upsilon_t \,.$ (2.5)

Using actual values of future inflation however introduces endogenity bias into the regression estimates. To circumvent this problem the authors suggest Hansen's (1982) generalized method of moments (GMM) estimator. Potential instruments must be available to the central Bank at time t, useful for forecasting future inflation, orthogonal to ε_t and, for our purposes, an integral part of inflation determination in NAOMI.

---

[32] The following discussion essentially follows Clarida, *et al* (1998)

[33] We assume that the monetary authority does not know the true level of inflation in period t when choosing the interest rate.

The estimated coefficient on expected inflation seven quarters ahead, β , equals 2.5, with a standard error of 0.33. This means that a rise in the expected inflation of one percentage point induces the Bank of Canada to eventually raise the short-term interest rate relative to the long-term rate by 250 basis points. [34] The first quarter reaction to such as shock is about 40 basis points. Note the high degree of persistence in the slope of the term structure as the estimated coefficient on the lagged slope of the term structure equals 0.84 with a standard error of 0.02.

The J-statistic = 6.46, chi-squared(18) with p-value = 0.994 implies that we cannot reject the over-identifying restrictions of the base-case model.[35] Sensitivity tests that vary the number of lags of the instruments indicate that the results are robust to the choice of lag length.

[34] Note that the shock to the level of short-term interest rates is indeterminate without an equation for long-term interest rates.

[35] This means that we cannot reject the proposition that the slope of the term structure reacts to movements in the instruments that are correlated with inflation seven quarters ahead and not to independent movements in these variables.

Appendix 2.1 Output and the Instrument of Monetary Policy

Considerable time has effort has gone into the choice of the relevant measure of monetary policy in NAOMI. It is now widely accepted by the economics profession that monetary is able to exert an influence over aggregate demand because of the existence of so-called sticky prices[36]. For whatever reason, the aggregate price level is slow in its adjustment to shocks and as a result changes to growth rate of the money supply can affect the real interest rate (and real exchange rate) in the short run. Consequently it is able to temporarily affect the level of real output in the economy.

It would seem reasonable then to use the real interest rate as the measure of monetary policy stance in NAOMI. However, as explained in Cao, Coletti and Murchison (2000) there are good reasons to consider the slope of the yield curve adjusted for the term premium instead. Among these is the fact that the ex-ante real interest rate is an unobserved variable since inflation expectations are unobserved. Furthermore attempts to use the ex-post real interest rate often produce poor results in aggregate demand functions. Often one must resort to the use of arbitrary moving averages of lagged inflation as a proxy for inflation expectations so as to achieve a significant role for the real interest rate. The approach used Duguay (1994) and Freedman and Macklem (1998) is to use one-period lagged inflation rate to define the real interest rate and then use a moving average of this with an arbitrary number of lags.

The argument can also be made that the slope of the yield curve does a better job of isolating those movements in the real interest rates that are due to monetary policy alone.

One the basis of these factors we have elected to use the slope of the yield curve. However, the issue remains whether or not it is the level of the adjusted slope or the change that affects output *growth*. Among the factors to consider are the following;

(a) The change in the slope will eventually produce a constant MCI weight ratio. Using the level will produce an MCI weight ratio that goes to infinity unless the level of the real exchange rate is also used in the demand function. Empirically, using the level of the slope will tend to produce a larger MCI weight ratio at all horizons beyond the first couple of quarters.

[36] Although numerous other channels have been explored.

(b) If it is true that $dy_t = f(r_t)$ then it follows that $y_t = \int_{-\infty}^{t} f(r_t)dr$. Hence *temporary policy shocks have permanent effects on real output*. So output today is a function of every policy shock that has occurred over history despite the fact that these shocks are necessarily temporary in nature. So in the absence of inflation targeting the monetary authority can permanently raise output.

(c) Studies that look at relationship between real interest rates and growth focus on the change in real rates. Whereas studies that analyse the predictive power of the slope tend to regress growth on the level of the slope. Using the level can approximate the change and vice versa quite well over the first few periods of a shock so it can be difficult to test which is appropriate using actual data. However the specification will give profoundly different results in a full model beyond the first few periods.

(d) Using the level slope introduces considerably more secondary cycling into the model. However, by using the change in the slope, we are most likely underestimating somewhat the impact of monetary policy on output. Indeed, the impact of monetary policy on output in NAOMI is as low as in any other study we have seen.

Predicated mainly on the belief that temporary policy shocks should not have permanent effects on output, we have elected to use the change in the slope. However, we hope to gain greater insight on the issue through future research.

Appendix 3.0 Monetary Conditions in NAOMI

In this section we briefly describe the relative importance of the yield slope and the real exchange rate in the determination of real GDP. Typically this relationship is captured by the weights used in the construction of a monetary conditions index (MCI). However, the real (nominal) MCI as defined by Freedman (1994,1995) is the relationship between real (nominal) short-term interest rates and the real (nominal) exchange rate. Since real and nominal short-term interest rates play no explicit role in NAOMI such a direct comparison is difficult. Nevertheless we can provide a measure of the relative importance of the slope and real exchange rate through time.

Table 4.0: Quasi MCI Weighting

Period	Weight
1	-
2	-
3	2.5
4	3
5	3
Infinity	3

As shown in Table 4.0 it take 3 quarters for the slope and real exchange rate to affect output. In the 3rd quarter a 100 basis point increase in the slope has the same effect on output as a 2.5 per cent appreciation of the real exchange rate. In the 4th quarter and thereafter this ratio increases to 3.

Appendix 4.0 1 Long run Determinants of the Real Exchange Rate in NAOMI

The cointegrating relationship for the real exchange rate is given as;

(2) $$z = -.41\text{pcne} - \sum_{i=1}^{\infty}\left(\text{ygap} - \text{ygap}^{\text{US}}\right) - .25\sum_{i=1}^{\infty}\left(\text{slope} - \text{slope}^{\text{US}}\right)$$

In Section 3.5 we described the role of real exchange as an important equilibrating relative price in NAOMI. For instance, a permanent increase to government spending relative to output holding taxes constant will result in a permanent appreciation of the real exchange rate. This appreciation will eventually reduce net exports such that output returns to control. In steady state we will run current account deficits consistent with the government's budget deficit[37]. NAOMI is structured such that the real exchange rate will continue to appreciate until real equilibrium is again restored. This in turn implies that its long-run level is related to the cumulative sum of all past output gaps (relative to the same sum for the U.S.). The same argument can be made for the cumulative slope differential since the change in the exchange rate is currently a function of the level of the slope differential.

Since the output gap in the U.S. and Canada are both I(0) by construction, their differential is necessarily I(0). Furthermore, term-structure theory states that even if short- and long-term interest rates are individually I(1) their difference must be I(0). If a series is truly I(0) then its integral (or cumulative sum in discrete time) will be I(1)[38]. Given that both the slope and output gap differential display a fair degree of inertia we should expect their integrals to be I(1). ADF unit-root tests fail to reject a unit root at even 10% for either series.

Table 5.0 ADF Unit Root Tests

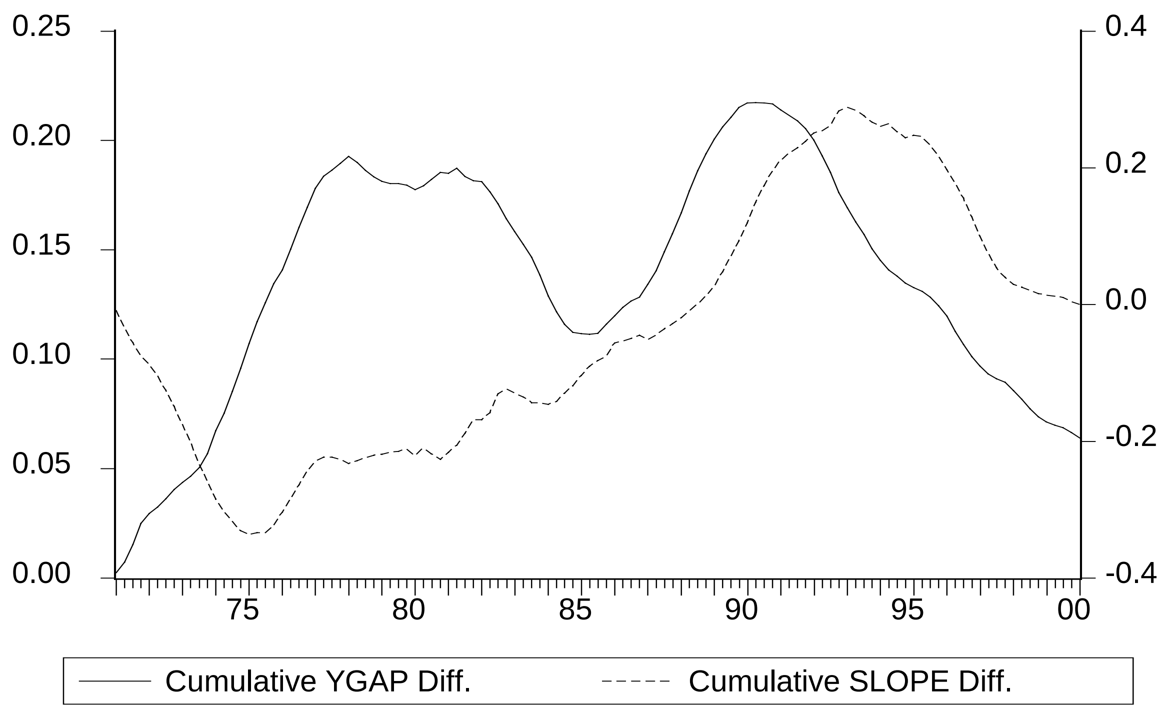
Variable	ADF T-value	10% C.V.	5% C.V.	1% C.V.
Z	-1.35	-2.58	-2.89	-3.49
LRBCNE	-2.56	-2.58	-2.89	-3.49
CUM_SDIFF	-1.6	-2.58	-2.89	-3.49
CUM_YGAP-CUMYGAPUS	-2.3	-2.58	-2.89	-3.49

[37] Since NAOMI does not acknowledge stock conditions, the steady-state depreciation that would be required to finance a higher domestic debt is not generated by the model.

Since all series are assumed to contain a single unit root we may test for the possibility of a cointegrating relationship. Here we have employed the Johansen and Juselius (1988,1990) FIML test to reflect the endogeneity of the cumulative differentials in the estimation of the coitegrating vector as well as the possibility of multiple cointegrating vectors. Table 5.0 presents the results of the J&J cointegration tests. The hypothesis of 0 vectors (no cointegration) can easily be rejected at the 1% level but the hypothesis of at most one vector cannot be rejected at 5%. On this basis we conclude that there is one unique long-run relationship. It is worth noting that the actual estimated coefficients are based on the standard OLS static regression which is valid under the maintained hypothesis of cointegration.

Table 6.0 J&J Cointegration Test

Eigenvalue	Likelihood Ratio	5% Critical Value	1% Critical Value	# of C.E.'s
0.28	62.58	47.21	54.46	0
0.13	27.50	29.68	35.65	at most 1
0.09	12.11	15.41	20.04	at most 2
0.02	2.21	3.76	6.65	at most 3



[38] Assuming that we denote the nth difference of an I(0) series as I(-d)

References

Barro, Robert J. and Xavier Sala-I-Martin. (1990): “World Real Interest Rates” NBER Working paper 3317.

Clarida, Richard, Jordi Gali, and Mark Gertler. (1998): “Monetary Policy Rules in Practice: Some International Evidence,” *European Economic Review*, vol. 42, pp. 1033-67.

Coletti, Don, Benjamin Hunt, David Rose and Robert Tetlow (1996): The Bank of Canada’s New Quarterly Projection Model Part 3: The Dynamic Model: QPM’. Bank of Canada Technical Report No. 75

Cozier, Barry V. and Gordon Wilkinson (1990): ‘How Large are the Costs of Disinflation in Canada?’ Bank of Canada Working Paper 90-6.

_____________ and Greg Tkacz. (1994): “The Term Structure and Real Activity in Canada” Bank of Canada Working Paper 94-3.

Delage, Paul and Jean-François Fillion (1999): L’écart à terme comme indicateur de l’activité économique, Department of Finance, Mimeo

Duguay, Pierre (1984): ‘Empirical Evidence on Price Determination in Canada: An Aggregate Approach’ Working paper prepared for *Price Flexibility and Business Cycle Fluctuations in Canada- a survey* (Royal Commission on the Economic Union and Development Prospects for Canada

______________ (1994): ‘Empirical Evidence on the Strength of the Monetary Transmission Mechanism in Canada: An Aggregate Approach’. *Journal of Monetary Economics* (February, 1994)

______________ (1994): ‘The Use of Indicators and of the Monetary Conditions Index in Canada’. *Frameworks for Monetary Stability*, International Monetary Fund

_______________ (1995): ‘The Role of Monetary Conditions and of the Monetary Conditions Index in the Conduct of Policy’. Bank of Canada Review, Autumn 1995.

Engert, Walter and Scott Hendry (1998): ‘Forecasting Inflation from the M1-VECM: Part 2’. Bank of Canada Working Paper 98-6

Freedman, Charles and Tiff Macklem (1998): 'A Comment on the Great Canadian Slump'. *Canadian Journal of Economics* Vol. 31 No. 3

Fortin, Pierre (1990): 'Can Canadian Anti-Inflation Policy be Made More Cost Effective?' Centre de reserche sur les politiques economiques, Université du Québes à Montréal

_____________ (1991): 'The Phillips Curve, Macroeconomic Policy, and the Welfare of Canadians' Innes Lecture, 1991 Meetings of the Canadian Economics Association

Gerlach, Stefan and Frank Smets (1995): 'The Monetary Transmission Mechanism: Evidence from the G-7 Countries' Working Paper No. 26. Bank for International Settlements, Basel.

Gordon, Robert J. (1980): 'A Consistent Characterization of a Near-Century of Price Behaviour' *American Economic Review, Papers and Proceedings* 70 (p.p 243-249)

Hamilton, James D. (1994) Time Series Analysis, Princeton University Press

Hendry, Scott (1995): 'Long-Run Demand for M1' Bank of Canada Working Paper 95-11

Johansen, S. (1988): 'Statistical Analysis of Cointegrating Vectors'. *Journal of Economic Dynamics and Control*, 12 No. 2

____________, and K. Juselius (1990): 'Maximum Likelihood Estimation and Inference on Cdointegration: with Applications to the Demand for Money'. *Oxford Bulletin of Economics and Statistics*, 52

Laxton, David, Kevin Shoom and Robert Tetlow (1992): 'Should the Change in the Gap Appear in the Phillips Curve? Some Consequences of Mismeasuring Potential Output'. Bank of Canada Working Paper 92-1

Murchison, Stephen (2001): 'NAOMI Part I: Proposed Model Selection Strategy ' Department of Finance Working Paper 2001-19

Robidoux, Benoît and Bing-Sun Wong (1998):'The Canadian Economic and Fiscal Model 1996 – Part I' Department of Finance Working Paper 1998-05

Vincent, Nicholas (2001): 'NAOMI Part III: A Guide to U.S. NAOMI' ' Department of Finance Working Paper (Forthcoming)

Figure 1.0: 1% Temporary Aggregate Demand Shock

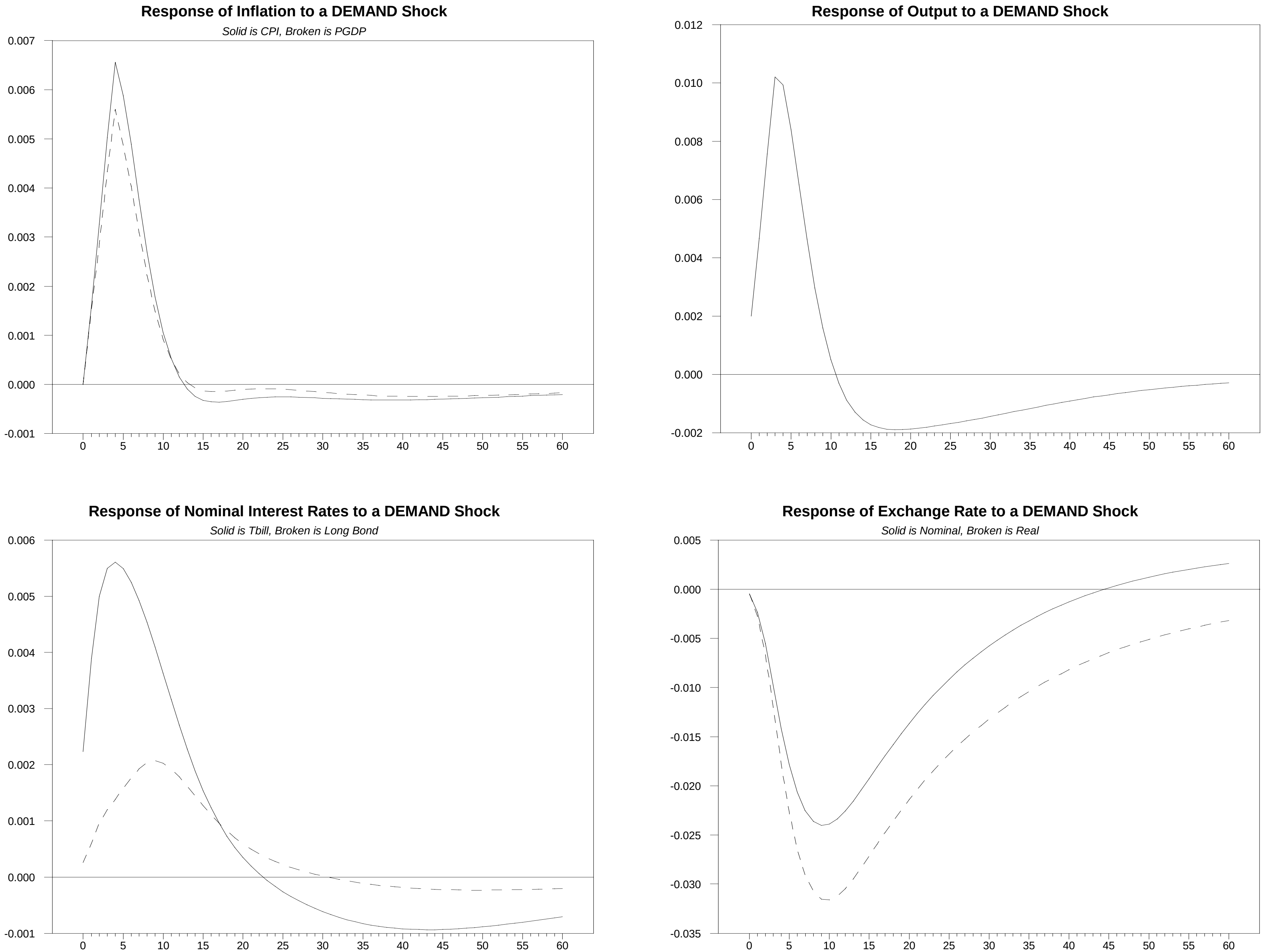


Figure 2.0: 1p.p. Unanticipated Price Shock

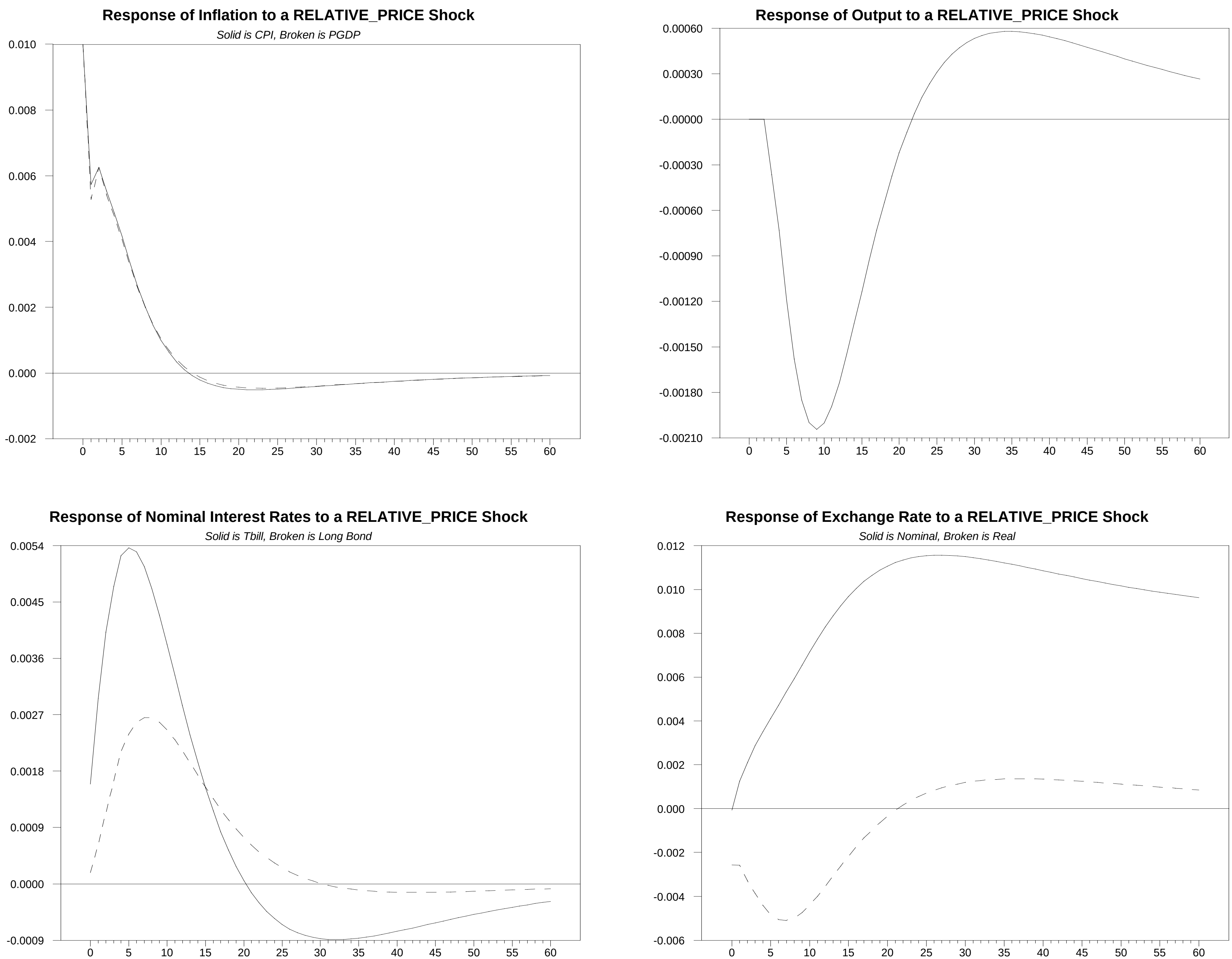


Figure 2.1: 1p.p. Anticipated Price Shock

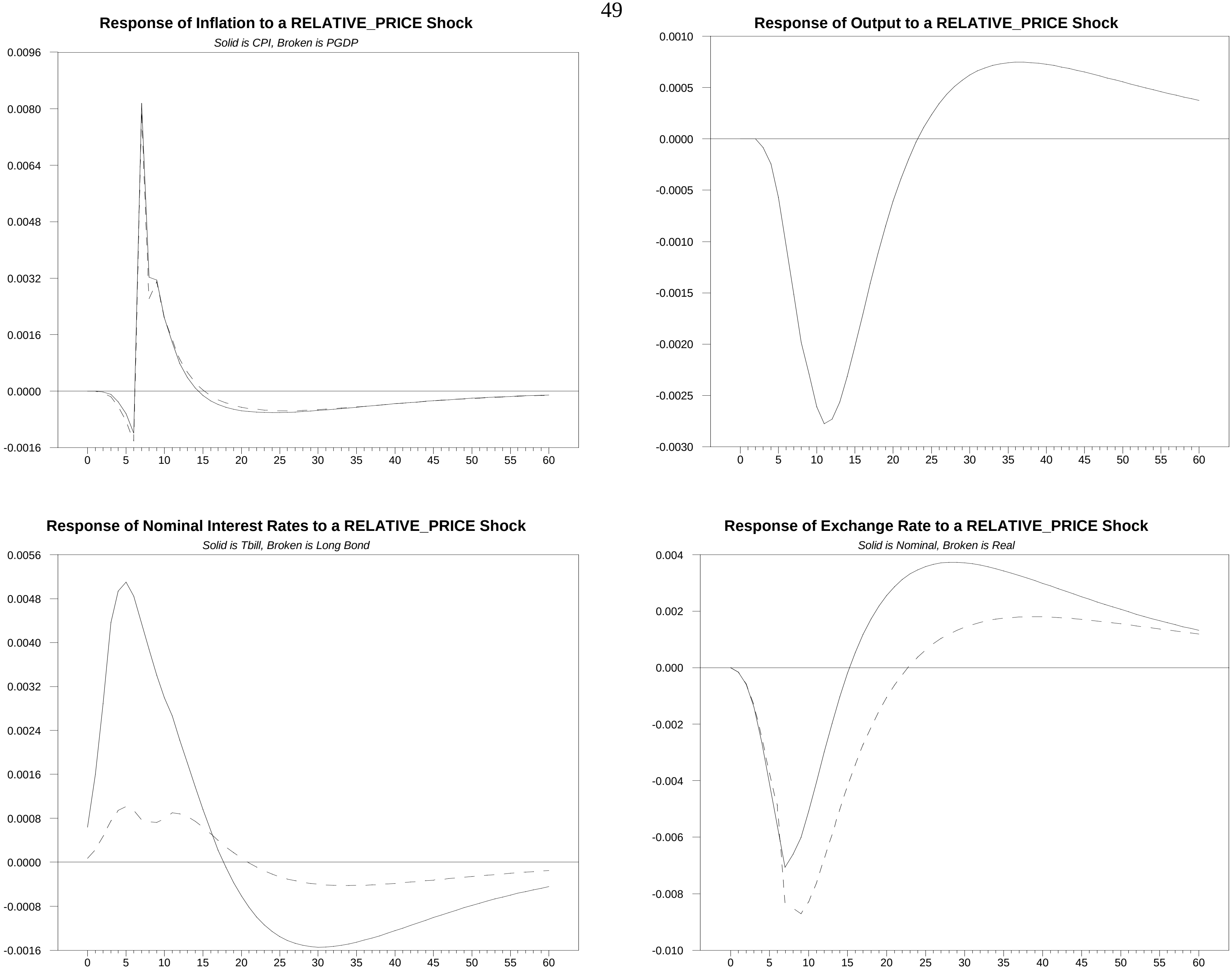


Figure 3.0: -3% Nominal Exchange Rate Shock

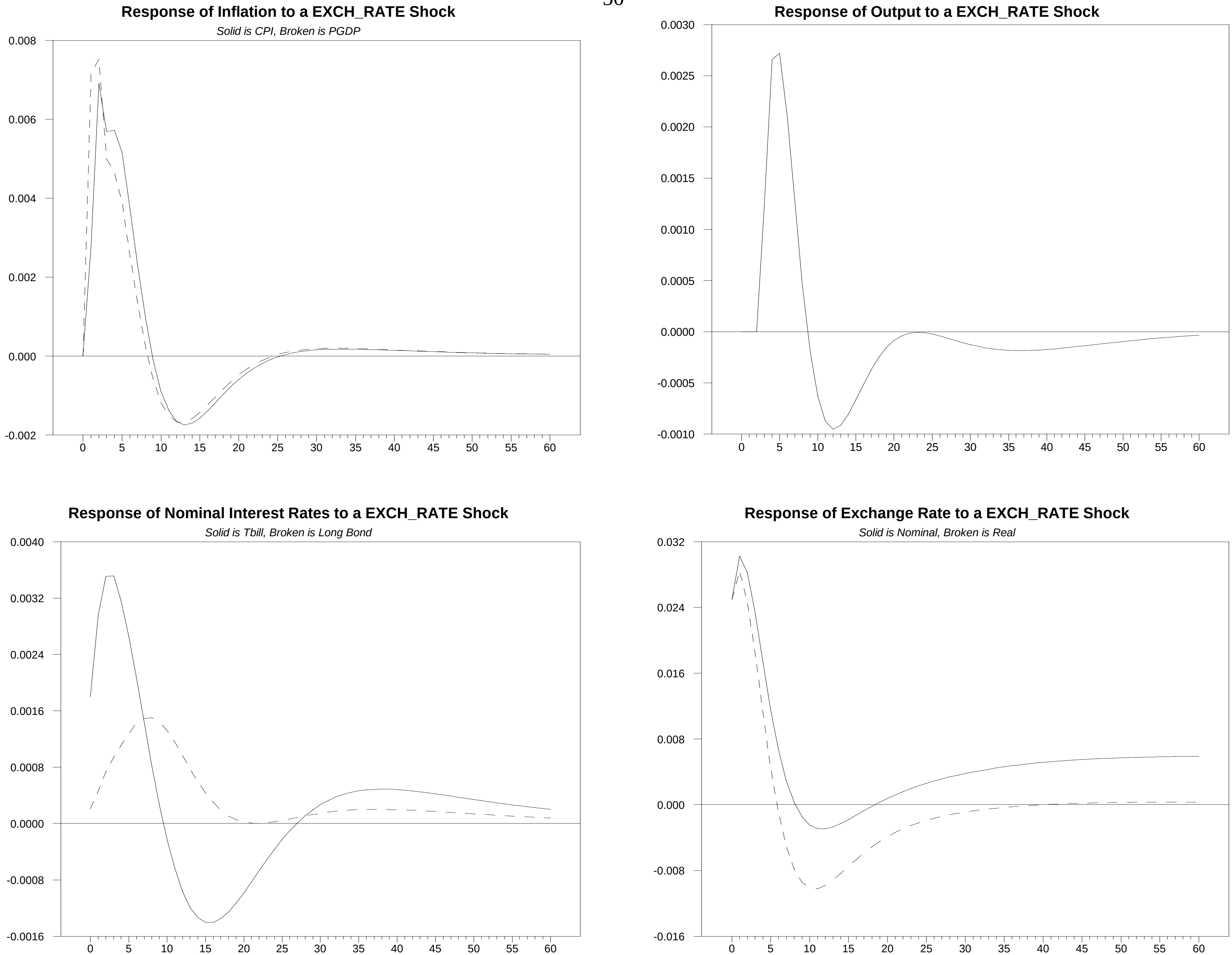


Figure 4.0 100 b.p. Yield Curve Shock

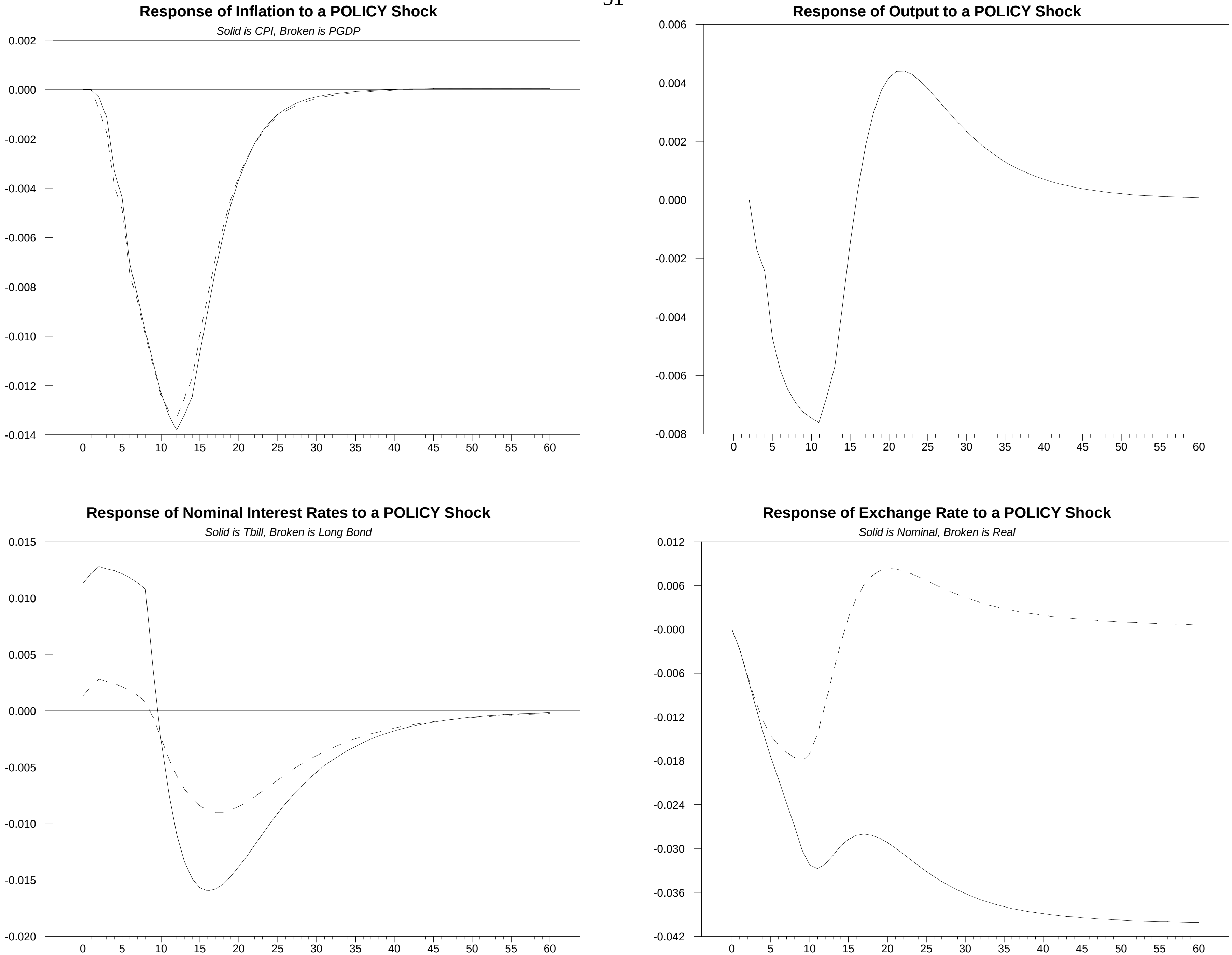


Figure 5.0: 0.75% U.S. Aggregate Demand Shock

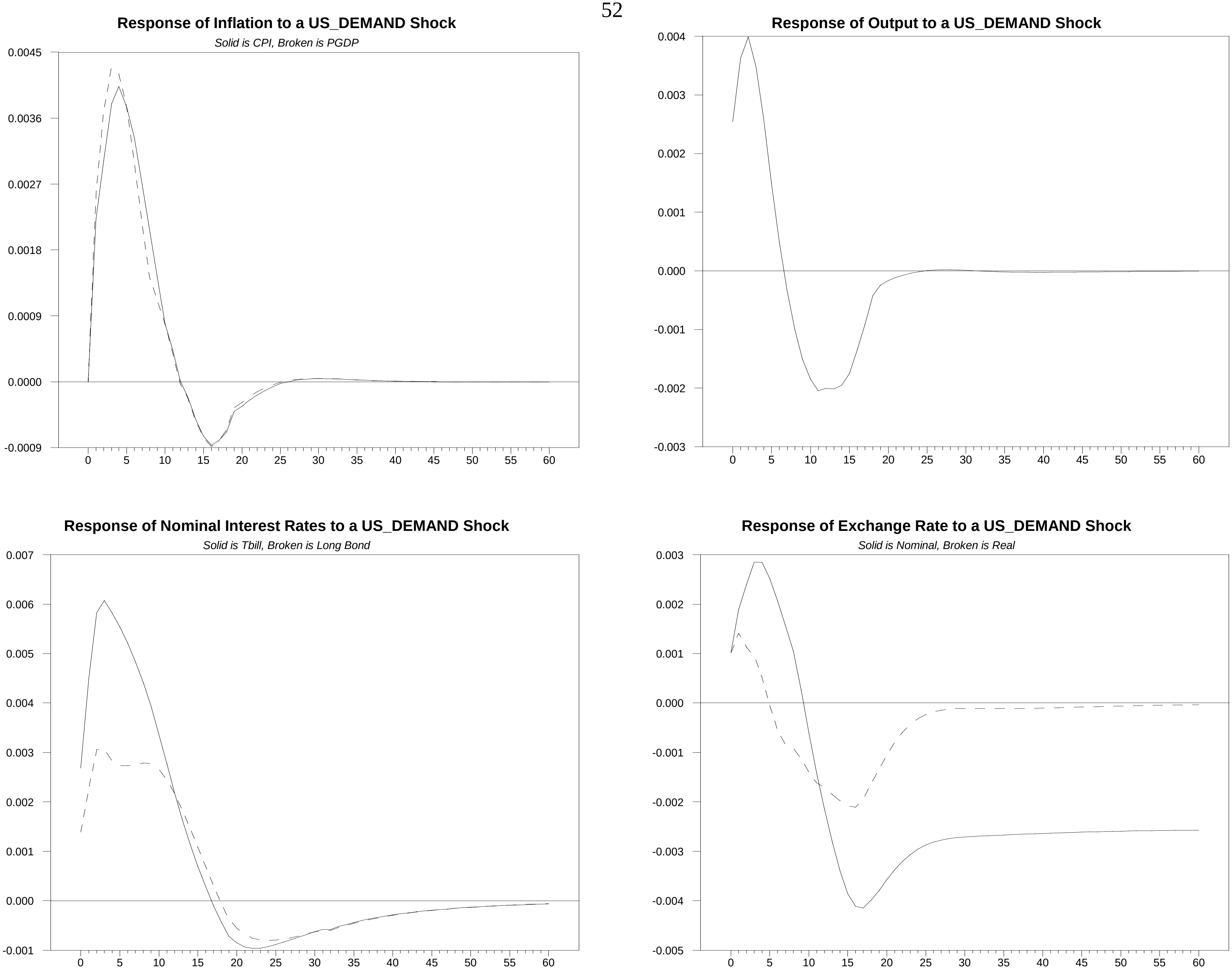


Figure 6.0 1p.p. U.S. Inflation Shock (Accommodated by the Fed)

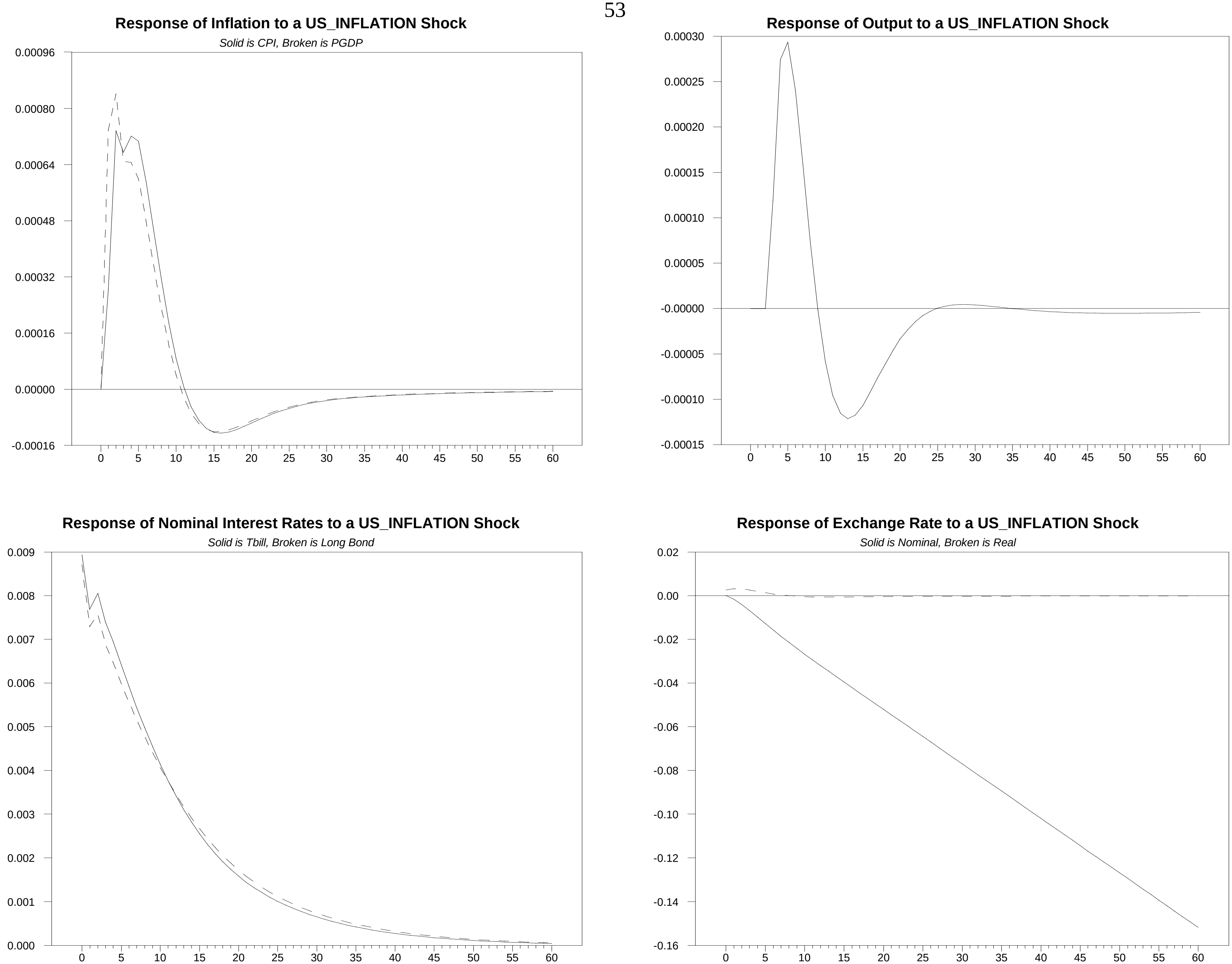


Figure 7.0: Permanent -15% (non-en.) Commodity Price Shock

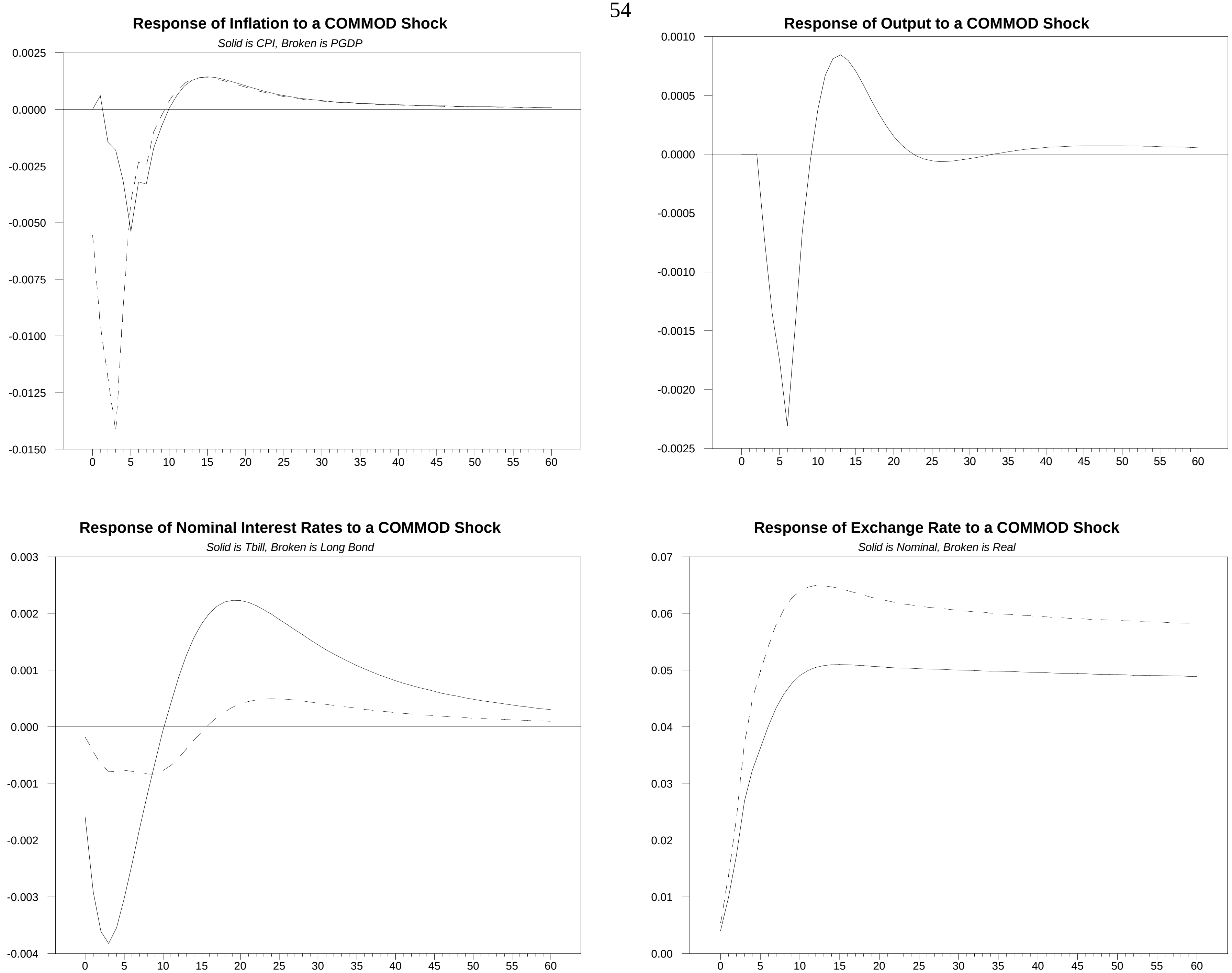


Figure 8.0: 1p.p. Reduction to the Midpoint of the Inflation Target

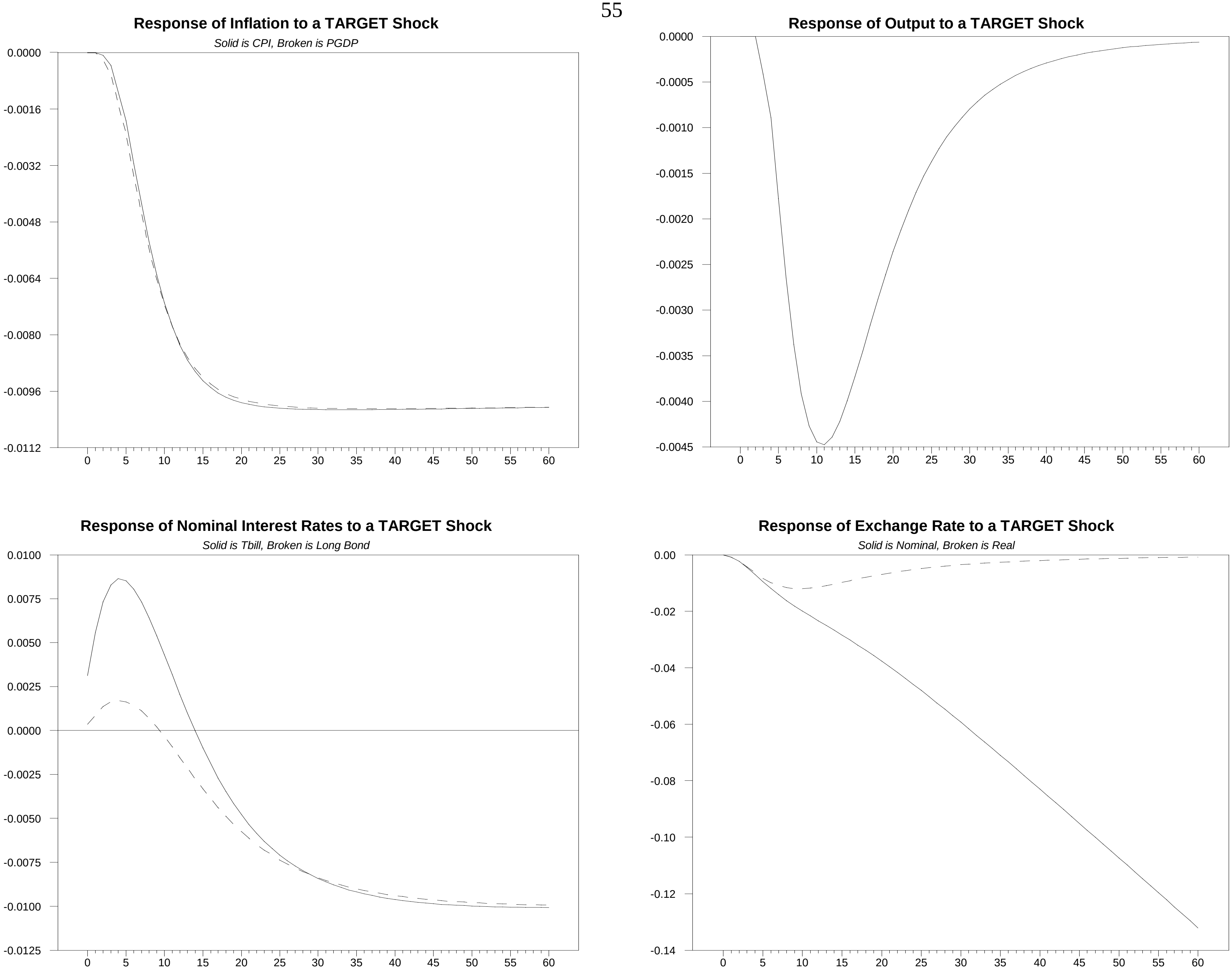


Figure 8.1: Counterfactual Simulation - BOC Inflation Targets

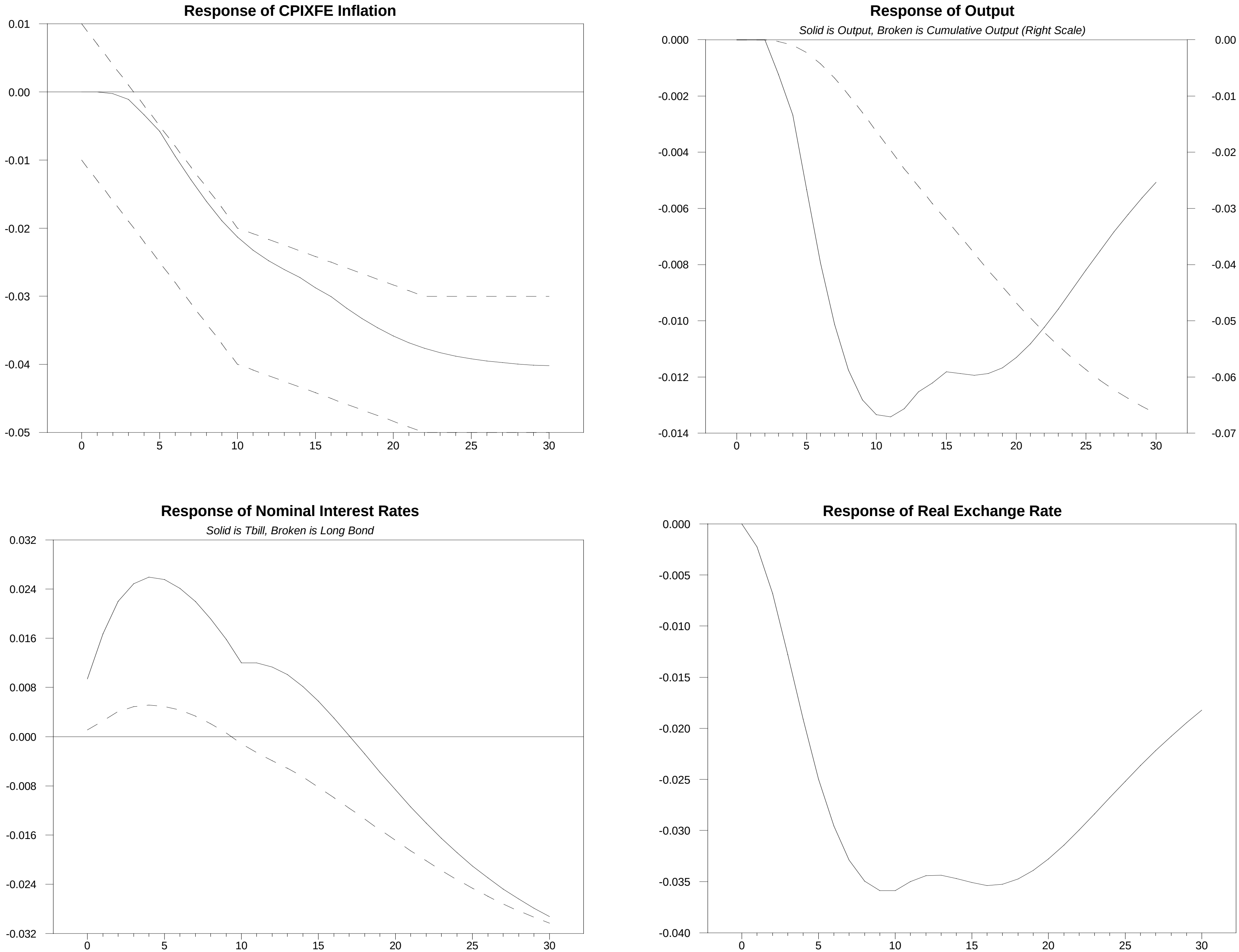


Figure 9.0: 1% Permanent Supply Shock

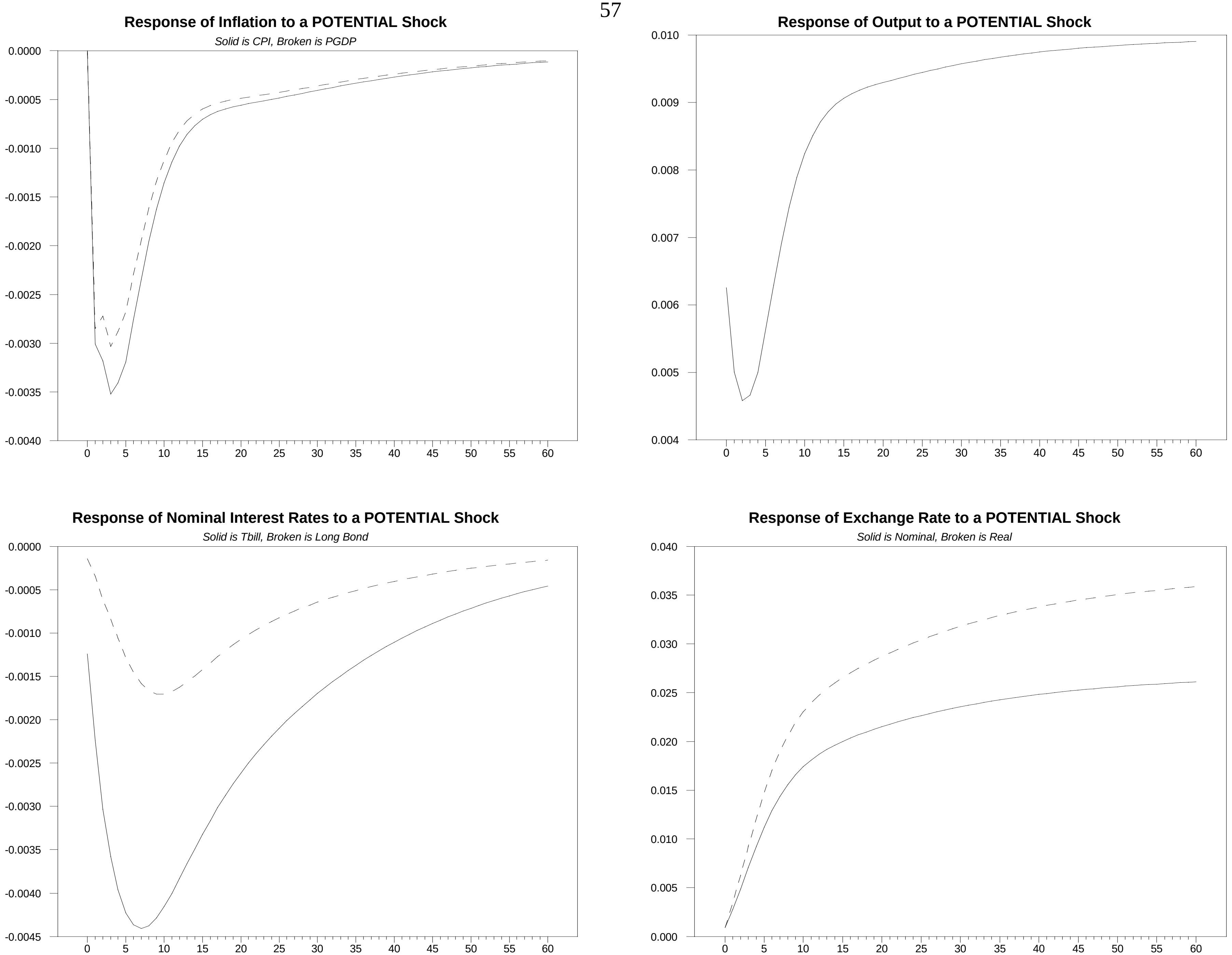


Figure 9.1: 1% Permanent Supply Shock Perceived as Demand Shock

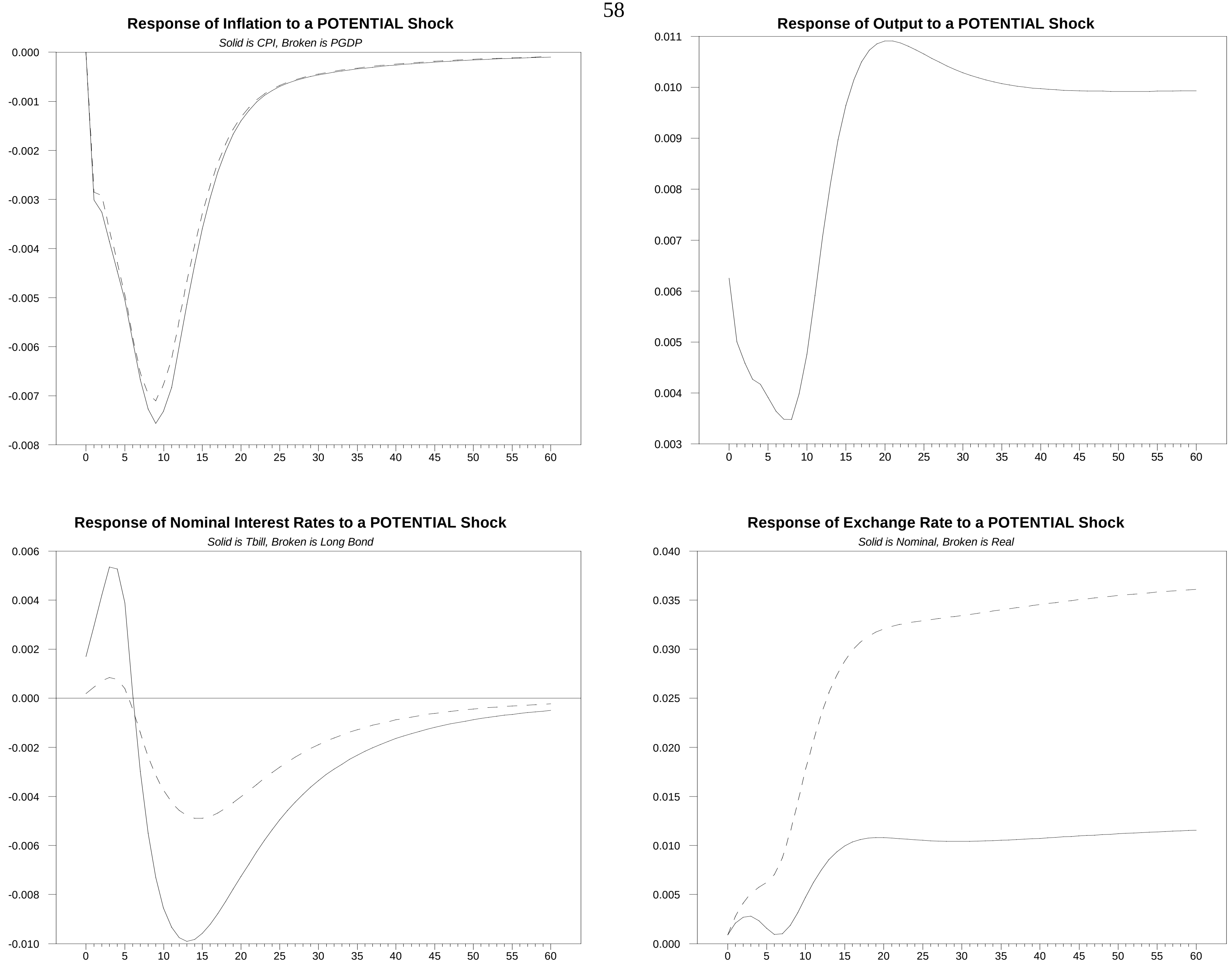


Figure 11.0: Quasi-dynamic Out-of-Sample Simulation (1988-1993)

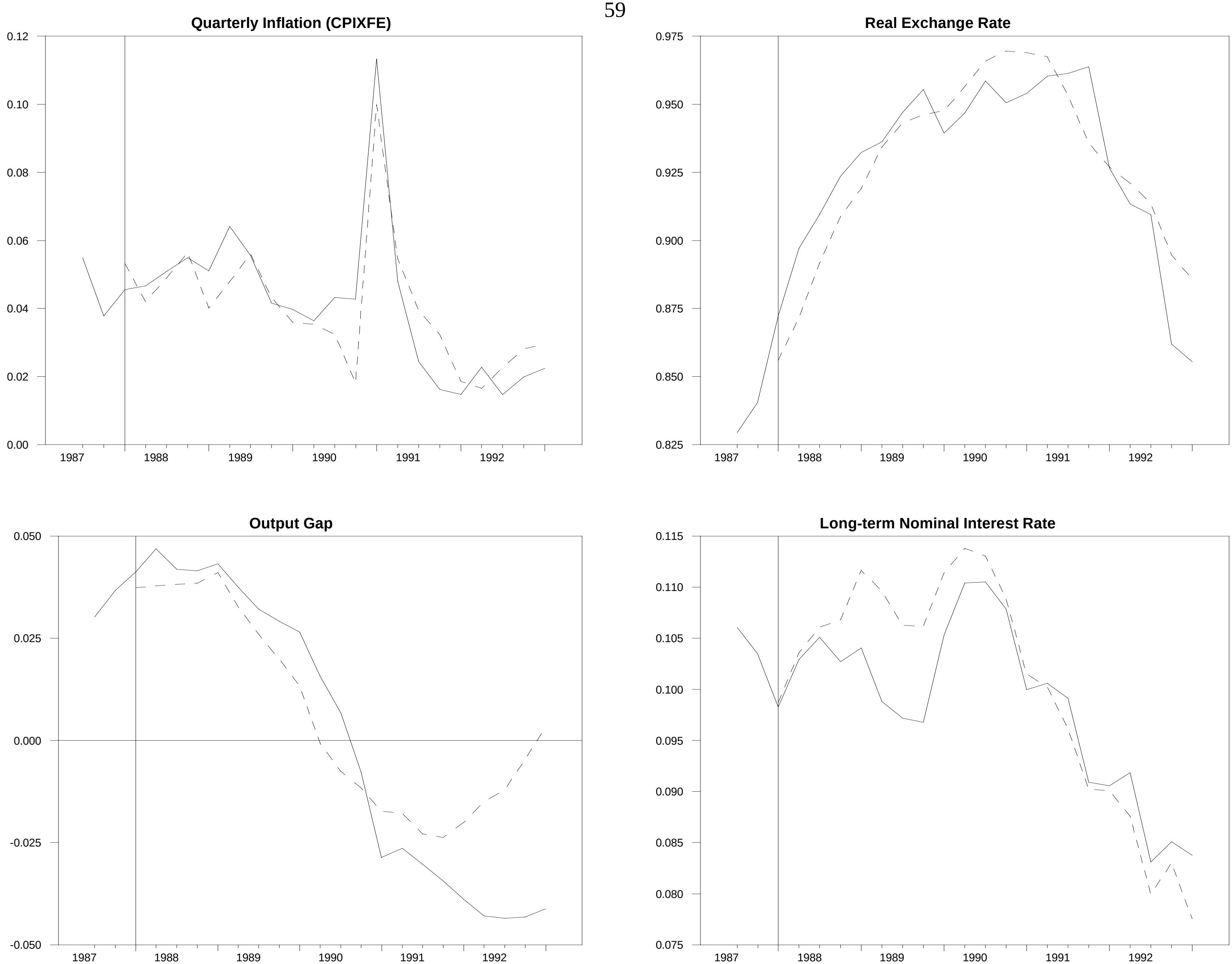


Figure 11.1: Fully-dynamic Out-of-Sample Simulation (1988-1993)