The fair value of the U.S. stock market: A structural VECM approach

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Résumé

Le but de ce travail est d'identifier les composantes permanente et transitoire constituant le prix des titres boursiers américains, à l'aide de la méthodologie SVECM développée par King, Plosser, Stock et Watson (1991). Cette méthodologie utilise l'information contenue dans les vecteurs de cointégration pour identifier les chocs structurels permanents du modèle. Puisque l'impact estimé de ces chocs à long terme sur les prix boursiers constitue la composante permanente du marché boursier, le modèle peut aussi être utilisé pour estimer le « juste » prix des valeurs boursières. Alternativement, la composante transitoire identifiée donne une indication du degré de sur-évaluation (ou de sous-évaluation) du marché boursier. Notre principal résultat est que la hausse du marché boursier américain au cours de la période 1995-1999 s'explique principalement par la hausse des profits des entreprises et la réduction de l'inflation.

Abstract

The goal of this paper is to disentangle permanent and transitory components of U.S stock prices, using the SVECM methodology of King, Plosser, Stock and Watson (1991). This methodology uses the information contained in the cointegration vectors to identify the structural permanent shocks in the model. Since the estimated long run impact of these shocks on stock prices constitutes the stock market permanent component, the model can also be used to estimate the stock market "fair" value. Alternatively, the transitory component of the real stock price gives an indication of the degree of overvaluation (or undervaluation) of the stock market. Our main results are that most of the rise in real stock prices over the 1995-1999 period was associated with the increase in real corporate profits and the reduction in inflation.

1. Introduction

By historical standards, the increase in U.S. stock market prices has been very impressive over the nineties. The S&P500 has quadrupled since the beginning of the nineties and doubled since Alan Greenspan first talked of irrational exuberance, on December 5, 1996. According to many economists, this increase in equity prices has contributed to the increase in consumption and sustained growth in GDP. In this context, an important issue is whether the increase in stock market prices is well justified by "fundamentals" or if it reflects a market "bubble", in which case an abrupt correction may seriously impede consumption and growth.

For some analysts, the current equity market levels are fully justified by fundamentals. They argue that financial deregulation and increased globalisation of good and capital markets since the early eighties, greater monetary and fiscal discipline, and the widespread integration of new technologies, all contribute to increased productivity growth which in turn has boosted corporate profits and asset prices. However, for other analysts, an unsustainable market bubble has developed and a large correction is likely.

Most valuation models rely on unobserved values of expected corporate earnings and equity premium to assess the fair value of equities. Moreover they generally treat all variables, except the stock market price as exogenous, which may lead to biased estimates.

This paper differs from previous studies in two areas. First, it does not rely on proxies of unobserved variables. Instead, it estimates the fair value of stock prices by disentangling its permanent and transitory components, using the methodology developed by King, Plosser, Stock and Watson (KPSW, 1991). The permanent component represents the fair value of the stock market, while the transitory component gives an indication of the degree of overvaluation of the stock market. Second, the methodology addresses the possible endogeneity of all variables.

The second section of this paper presents a basic model of equity valuation and different empirical representations of the model. Our model is presented in more detail in Section 3. The results and conclusions constitute Sections 4 and 5, respectively.

2. A review of equity valuation models

2.1 The basic model

Most models of asset prices in modern finance theory are based on the assumption that households choose the path of their consumption over time and allocate their assets at each point in time so that the risk-adjusted expected rates of return are equated to the risk-free interest rate. Under this assumption, asset prices are determined by the present (risk-adjusted) discounted value of future dividends as showed in equation (1).

$$P_t = \sum_{i=t+1}^{\infty} \frac{D_i}{(1+r_i)^i} \qquad (1)$$

That is, P_t, the price of a firm's share in period t, varies positively with D_i, the expected stream of dividend payments, and negatively with r_i, investors' nominal required rate of return for the security. Under the additional assumptions that the required rate of return is constant and equal to the risk free rate rf plus an equity risk premium ρ,[1] and that dividends grow at the constant rate g, we obtain the *Gordon growth model*:

$$P_t = \frac{D_{t+1}}{rf + \rho - g} \qquad (2)$$

This simple formula can help us to identify the factors that may lead to changes in equity prices. Lower risk-free interest rate and lower equity risk premium would decrease

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[1] An equity security is a claim against an uncertain stream of income. Due to the increased uncertainty about the *ex post* equity return relative to a government bond held until maturity, investors typically require a premium to hold this class of assets relative to a fixed-income claim. The difference between the required rates of return on these two classes of securities is generally called the "equity risk premium". Notice that the equity premium may be a function of various factors, including the rate of inflation.

investor's required nominal rate of return and thus increase security prices. As well, higher expected dividends stream would drive asset prices up.

If we further assume that there are no retained earnings or, equivalently, that all earnings are paid in dividends, then the *Gordon formula* can be written as:

$$P_t = \frac{E_{t+1}}{rf \quad + \rho - g} \qquad (3)$$

One empirical representation of (3) is the Fed Stock Valuation Model (FSVM), in which the fair value for the S&P 500 is derived by dividing expected earnings by the 10-year Treasury bond yield.[2] If the actual index is greater than (less than) this fair value, the market is overpriced (under-priced).[3] The FSVM model implicitly assumes that expected earnings will increase at a rate g equals to the equity premium ρ. Clearly, this is a strong assumption.

In a recent study, Yardeni (1999) proposes to improve the FSVM by including a risk premium and a proxy for the rate of growth of expected earnings. His risk premium measure is the spread between corporate bond yields and Treasury bond yields.[4] The rate of growth of expected earnings beyond the next twelve months is approximated by the spread between the corporate bond yield and the earnings/price ratio (or the inverse of the price/earnings ratio). The rationale is that if this spread is positive, asset holders must anticipate earnings growth in the future to accept a current equity yield lower than the less risky corporate bond yield. However, one can question the usefulness of such a

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[2] A good source of historical data on expected earnings for the S&P 500 is I/B/E/S International Inc. This organisation has been compiling 12-month consensus forecasts for operating earnings since January 1979, based on the projections of Wall Street's industry analysts.

[3] Equivalently, the (expected) earnings/price ratio (E/P) can be calculated as the ratio of 12-months expected earnings to the S&P 500 index. This yield is then compared to the 10-year Treasury bond yield. The stock market is overpriced (underpriced) when the earnings/price ratio is less than (greater than) the Treasury bond yield.

[4] This spread is likely to widen in periods of economic distress, as bond investors tend to worry that profits won't be sufficient to meet the debt-servicing obligations of some companies. As earnings are also likely to be depressed in those periods, this constitutes a good proxy for the equity risk premium.

variable for evaluating the fair stock market value. While long-run optimism about earnings growth helps explain the rise in the P/E ratio, the relevant question is whether this optimism is well founded. In other words**,** to get the fair stock market valuation, we would need the "fair" expected earnings growth.

2.2 The role of inflation

In principle, stock prices should vary proportionately with inflation, as nominal dividends (D) should grow at the rate of inflation, and the nominal rate of return (r) and the growth in nominal dividend (g) increase with the rate of inflation. Real stock prices should not be then correlated with inflation. However, as figure 1 suggests, there has been an evident negative correlation between inflation and real S&P 500 price over time. An interesting question is whether inflation had an economic impact on *real* profits and/or on investors *real* required return, or if the correlation only reflects a statistical (not causal) phenomena.

Sharpe (1999) finds that the economic effect of inflation on required long-run real stock returns can be substantial. In his model, the log earnings-price ratio is expressed as a linear function of expected inflation, expected future returns, expected earnings growth rates, and the log of the current dividend/payout ratio. Investors expectations are drawn from surveys of professional forecasters. According to his results, a one percentage point increase in expected inflation is estimated to raise required real stock returns by about one percentage point and amounts to about a 20 percent decline in stock prices.[5]

There are many reasons why inflation may be so harmfull to the stock market. For example, Danthine and Donaldson (1986) develop a monetary model with rational expectations in which the stock market does provide full insurance against monetary shocks but not against *temporary* inflation induced by real shocks. In their model, real shocks have an effect on consumption goods prices without changing the expected stream

[5] See also Fillion (1992) for an examination of the effect of inflation on the real return on equity.

of dividends (because of dividends stickiness) and thus have an impact on stock market real returns. Another channel, proposed by Fama (1981) and others, suggest that higher inflation induces lower expected real economic activity and/or uncertainty over the conduct of future monetary and fiscal policies, leading investors to demand higher risk premium. Alternatively, Modigliani and Cohn (1979) suggest that investors collectively suffer from money illusion and use a nominal rate to discount real dividends. The negative relation between real returns and inflation may also be related to Lucas (1973) signal extraction problem associated with inflation. In Lucas model, agents have problems assessing whether a price increase is the result of higher demand in a specific market or the result of an overall price inflation. This may result in less efficient resource allocation that translates into higher required real return as inflation goes up, possibly through a higher equity premium.

Sangyong (2000) estimates a VAR system with U.S data, including real stock returns, the growth rate of industrial production, inflation, the Federal Funds rate and the price of oil. He finds that as much as 30% of the negative covariance between real stock returns and inflation is attributable to orthogonal innovations in the Federal Funds rate. This result is consistent with Fama's interpretation of the negative correlation between inflation and real stock returns, but it is not consistent with the implications of Danthine and Donaldson in which only real shocks can account for the negative correlation. Moreover, the possiblility of "money illusion", suggested by Modigliani and Cohn, cannot be ruled out by Sangyong's results.

2.3 Our approach to stock valuation

All the preceding empirical works rely on some market expectations, whether inflation expectations or earnings expectations or both. A relevant question is whether those expectations are well founded. Our approach does not rely on any market expectations to judge if the current valuation is the *fair* valuation. Instead, it disentangles transitory and permanent components of stock prices with the methodology proposed by King, Plosser, Stock and Watson (KPSW, 1991).

Moreover, all the preceding studies (but the paper by Sangyong) adopt a methodology in which all variables but price/earnings ratio are exogenous, which may lead to biased estimates. For example, it is generally admitted that interest rates and stock prices are determined simultaneously. Hence, when it is assumed that only stock prices are endogenous, there is some contemporaneous correlation between the disturbance term and the regressor, which leads to biased estimates. Our methodology allows for dynamic short-run relations between the variables and it explicitly deals with the endogeneity of the variables.

Furthermore, since recent empirical evidence points to a relationship between inflation and real stock returns, our model will also try to identify the estimated effect of inflation on real stock prices.

3. The SVECM model of real stock prices

KPSW's methodology uses the information contained in the cointegration vector, combined with exogeneity assumptions, to identify the structural permanent shocks in the model.[6] The estimated long run impact of these permanent shocks on stock prices constitutes the stock market fair value. We estimate a five-lag VECM model with the following four endogenous variables over the 1959:Q2-1999:Q4 period: the log of the ratio of the S&P 500 to GDP deflator; the log of the ratio of after-tax corporate profits to GDP deflator; the inflation rate measured as the annualised per cent change in the quarterly GDP deflator; and the real long-term interest rate (Moody's Baa nominal corporate yields minus one lag of the inflation rate).[7] The variable names are, respectively, LSP500, LPRO, INFL and RBAA. The data are shown in Figure 1.

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[6] The KPSW's identification methodology is described in detail in Appendix A.

[7] The number of lags is such that we reject the presence of significant autocorrelations in the residuals.

Univariate analysis of the four variables indicates that all the variables can be characterized as non-stationnary I(1) processes. These results are consistent with the large literature on unit-root properties of U.S. macroeconomic time-series.[8] The cointegration tests indicate the presence of one cointegration vector in the model:[9]

Table 1. Estimated cointegration vector (normalized on LSP500)

	RBAA	INFL	LPRO	LSP500
Unconstrained:	9.461	16.267	-1.186	1
Constrained:	8.038	16.372	-1[c]	1

c constrained coefficient

The cointegration vector contains useful information about the long-run relationship between the variables.[10] On the basis of the identification constraints described in the following paragraph, the cointegration vector implies that a one percent change in real profits, ceteris paribus, is expected to be associated with an increase in real stock prices of 1.186 percent.[11] Moreover, the model is unable to reject (p-value = 0.21) that the parameter associated with LPRO in the cointegrating vector is equal to minus one, consistent with the linearizarion of equation (1). It is also worthwhile to note the absence of cointegration in a three-variable model that excludes the role of inflation, which is

[8] As an alternative assumption, recent literature has suggested that the time-series behaviour of inflation and real interest rates might be better described by stochastic changes in regimes than by non-stationary processes. The VECM models are designed for non-stationary variables, but not for variables that are characterised by changes in regimes.

[9] In fact, the Trace test rejects the presence of two cointegration vectors at a level of 90%, although the L-max tests does not reject the presence of two cointegration vectors. However, it is well known that cointegration tests have a tendency to overreject when the null is true [see Godbout, van Norden (1997)].

[10] However, we have to be careful with the economic interpretation of the cointegration vector. It is well known that structural assumptions are necessary to interpret cointegration vectors (see for example Wickens [1996]).

[11] As discussed in the Appendix, the same interpretation of the cointegration vector can not be made for the other parameters.

consistent with our assumption that inflation plays an important role in the long-run determination of real stock prices.

In the context of our four-variable model, the KPSW's methodology helps to identify three permanent shocks and one transitory shock.[12] A permanent shock is one which has a long run (or permanent) impact on the *level* of the variables. Without further assumptions, the stochastic trend in each I(1) variable is a linear combination of those three permanent shocks. We have normalised the cointegration vector on the stock price variable and have ordered the variables in the following way: RBAA, INFL, LPRO, LSP500. With KPSW's methodology, the ordering assumes that the first identified permanent shock has a permanent effect on RBAA, the second has a permanent effect on INFL, and the third has a permanent effect on LPRO. Moreover, a variable may be affected by a permanent shock to a higher variable in the system, but not the reverse. Thus, the ordering implies three long run constraints. The first one is a long run Fisher effect, that is, a permanent change in inflation is constrained not to have an effect on the real interest rate in the long run.[13] The second and third constraints are that a permanent change in the level of real profits is assumed to be neutral in the long run on RBAA and INFL. According to the modified Golden rule, higher productivity growth implies higher real profit growth and a higher real interest rate. However, since productivity growth is stationary, a permanent shock in the level of productivity will have a permanent effect on real profits, but only a temporary effect on interest rates. A similar argument can be offered for inflation. Higher temporary productivity growth will permanently increase the *level* of profits and potential output. This will temporarily reduce inflation until the output gap is closed again.

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[12] In the absence of cointegration, all four variables would have their own stochastic trend or, equivalently, would be affected by their own permanent shock. One cointegration relation means that one variable's stochastic trend is a linear combination of the stochastic trends in the other three variables or, equivalently, that the model is affected by three permanent shocks.

[13] Given that some recent studies [see for example Koustas and Serletis (1999) and Chouinard and Gauthier (2000)] tend to reject the Fisher effect in the long run, a model without this constraint has been estimated and is discussed later in the paper.

Let the structural shocks be partitioned as permanent and transitory, $\eta_t = (\eta_t^p, \eta_t^t)$, and let the notation for the “interest rate”, “inflation rate” and “profit” shocks be $\eta_t^p = (\eta_t^r, \eta_t^i, \eta_t^{pr})$. The structural model, as described in Appendix A, can then be written as:

$$\Delta X_t = \mu + \Gamma^r(L)\eta_t^r + \Gamma^i(L)\eta_t^i + \Gamma^{pr}(L)\eta_t^{pr} + \Gamma^t(L)\eta_t^t$$

Since X_t can be approximated by

$$X_t \approx \mu t + X_0 + \sum_{j=1}^{t} \Delta X_j$$

we can decompose each variable into an interest rate component, an inflation component, a profit component and a transitory component; each historical component being the cumulative contribution of each structural shock over time.

The estimated model also allows for linear trends. The estimated linear trends in the level are:

RBAA	INFL	LPRO	LSP500
0.0303	-0.0112	0.9934	0.9340

As shown, the trends in the real interest rate and the inflation rate are only 3 and 1 basis points per quarter. The trends in real profits and stock prices are close to 1 percent per quarter.

4. Analysis of the structural shocks[14]

In this section, we first examine the impulse reponse functions of the four structural shocks (three permanent and one temporary) on each variable. We also discuss the results of the variance decomposition and the historical decomposition of each variable based on the four structural shocks.

Impulse response functions

As shown in Figure 6 (page 29), a persistent shock of about 80 basis points in the real interest rate, reduces inflation by about 40 basis points in the long run. Furthermore, a higher interest rate depresses both real profits and stock prices. The negative long-term effect of interest rate shocks is slightly less important on profits than on stock prices (minus 1.6% vs minus 2.2%), illustrating the double negative impact of higher interest rates on stock prices via lower profits and portfolio adjustments toward bonds.[15]

Given the assumption that the real interest rate is exogenous in the long run, inflation has no permanent impact on it. Moreover, the results show that inflation is neutral to real profits in the long-run but has very important depressing effects on real stock prices. A persistent increase of 40 basis points in inflation is associated with a decrease in stock prices of around 6 percent.[16] As discussed in Section 2.2, there are many reasons why inflation may have such an important impact on real stock prices. However, one could argue that our model exacerbates the role of inflation on stock prices. To examine this, we estimated a second model in which the first two variables are inverted. That is, INFL is allowed to have a long run impact on the real interest rate, but a permanent shock in RBAA cannot have a permanent effect on inflation. As expected, the contribution of

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[14] I have modified the catsmis.src procedure in CATS to implement the KPSW's identification methodology. The programs are available on request.

[15] More precisely, as can be seen in the long run multiplier matrix in the Appendix, a permanent real interest rate increase of 83 basis points decreases inflation by 37 basis points, real profits by 164 basis points, and real stock prices by 222 basis points in the long-run.

[16] This is qualitatively in line with Sharpe (1999), although the real impact of inflation on stock prices is more tenuous in our model.

inflation to real stock prices becomes much less important since it is counterbalanced by the effect of inflation on RBAA. But this translates into a strong rejection of the Fisher effect as the long run matrix indicates that a one percent permanent increase in inflation depresses the *real* interest rate by more than one percent. This would imply that higher inflation decreases the *nominal* rate of interest, which is very difficult to justify.[17]

Our permanent real profit shock mimics a supply shock. It pushes the real interest rate higher by about 40 basis points after a year and decreases inflation by 30 basis points after 6 quarters. Our results give support to Danthine and Donaldson's study since a positive real profits shock, which may be interpreted as a positive supply shock, has a negative short-run impact on inflation and a permanent positive impact on real stock prices. Furthermore, consistent with the linearisation of (1), a change in long run real profits is associated with a proportional change in real stock prices.

Transitory shocks are difficult to interpret since they typically consist of a combination of many shocks from different sources, all having transitory impacts on possibly many variables. In this study, these shocks show very important effects on the stock market, increasing them by more than 3 percentage points in the short-run, and decreasing profits by more than one percent over the same period. The opposite impacts on profits and stock prices suggest that transitory shocks might be caused by speculators whose decisions are not based on fundamentals like profits.

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[17] Among others, Koustas and Serletis (1999) and Chouinard et Gauthier (2000) also reject the Fisher effect on nominal interest rates, but not as strongly as suggested by the alternative ordering assumption discussed above.

Variance decomposition[18]

The results of the variance decomposition exercise are presented in Table 2. In the long run only the interest rate shocks affect real interest rates since we have assumed they were exogenous to the other shocks. In the short run, profits shocks explain about 20% of the variance of the interest rate forecast error within the first year, possibly reflecting the temporary positive effect of aggregate supply shocks on productivity growth and real interest rates.[19] The Fisher effect is rejected in the short run, since almost 20% of RBAA forecast-error variance is explained by the inflation shocks after one year, and still 10% after 2 years. Moreover, more than 15% of the interest rate forecast-error variance is explained by transitory shocks after one year.

Almost 20% of the variations in inflation is due to transitory shocks, and about 25% is explained by permanent shocks in real profits in the short run. In the long run, inflation is explained at almost 60% by its own permanent shock, the other 40% being attributed to interest rate shocks, possibly reflecting the ultimate effect of monetary policy on inflation.

Real profits are mainly affected by their own permanent shocks. To a lesser extent, profits are also affected by interest rate shocks. However, inflation is neutral to real profits in the long-run, even though the methodology did not impose any money neutrality constraint.

Finally, about 50% of the real stock price fluctuations are attributed to inflation shocks at the long horizon forecast. Since inflation has no long run impact on real interest rates and real profits, investors real required return must be affected by inflation through the equity premium. Real profits shocks explain about 40% of LSP500 variations in the

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[18] This section discusses the variance decomposition of the variables in level.

[19] The positive short-run contribution of the profit shocks to real interest rates might also reflect the reaction of monetary policy following a persistent aggregate demand shock.

Table 2. Forecast-Error Variance Decomposition

A. Fraction of real interest rate (RBAA) forecast-error variance attributed to the four structural shocks

Horizon	η^r	η^i	η^{pr}	η^t
1	.77	.15	0	.07
4	.47	.18	.19	.16
8	.70	.09	.11	.09
20	.9	.03	.04	.03
120	.99	0	.0	0

B. Fraction of inflation (INFL) forecast-error variance attributed to the four structural shocks

Horizon	η^r	η^i	η^{pr}	η^t
1	.01	.64	.20	.15
4	.02	.57	.25	.17
8	.04	.60	.23	.13
20	.21	.60	.12	.07
120	.42	.55	.02	.01

C. Fraction of profit (LPRO) forecast-error variance attributed to the four structural shocks

Horizon	η^r	η^i	η^{pr}	η^t
1	.04	.07	.84	.05
4	.07	.02	.89	.02
8	.09	.01	.88	.01
20	.10	0	.89	0
120	.08	0	.91	0

D. Fraction of real stock price (LSP500) forecast-error variance attributed to the four structural shocks

Horizon	η^r	η^i	η^{pr}	η^t
1	0	.08	.32	.59
4	.02	.43	.29	.26
8	.01	.57	.30	.12
20	.04	.54	.38	.04
120	.06	.52	.41	0

long run and about 30% in the short run. Finally, almost 60% of one quarter ahead stock prices forecast error is explained by transitory shocks, possibly driven by noisy traders (or chartists as opposed to fundamentalists). This still represents 25% after a year, and 10% after 2 years, indicating that the periods of potentially large under/over valuation may last more than a few quarters in our model.

Historical decomposition of stochastic components

First, looking at the historical decomposition of real interest rates in Figure 2 (page 24), we observe that most of the large increase in the real interest rate at the beginning of the eighties is identified as a "permanent" shock. However, most of this increase was reversed by negative shocks in the second-half of the 1980's and the early 1990's.

As shown in Figure 3 (page 26), the inflation permanent shock captures well the upward trend in inflation of the 1970's and the decreasing trend of the 1980's and 1990's. The contribution of interest rate shocks to inflation is also important, decreasing inflation by more than 3 percentage points at the beginning of the 1980's, but increasing it by about the same magnitude in the second half of the 1980's and the beginning of the 1990's.

Figure 4 (page 27) shows the large contribution of the profit shocks to the variance of real profits. In particular, an important increase in real profits of about 40 percent is identified since 1994. This surge in profits has contributed to a real stock price increase of the same importance. As expected in theory, the identified long run impact of a one percent increase in real after-tax profits is to increase real stock prices by one percent. Interest rate shocks also have a large quantitative impact on profits (Figure 4) and stock prices (Figure 5).

As shown in Figure 5 (page 28), the results also confirm that an important part of the real stock market tendency over the last three decades is explained by changes in the inflation stochastic trend. Actually, over the 1967-1980 period, which includes the two oil price shocks, the increase in inflation contributed to a decrease in *real* stock prices of about 80%.[20] The subsequent decrease in inflation explains an increase of the real stock market prices of more than 80% over the 1981-1999 period. During the 1994-99 period, the decrease in inflation trend contributed to a 30% increase in real stock prices. As shown in the variance decomposition analysis, inflation has no long-run impact on real interest rates and a negligeable effect on real profits. Thus, investors real required return must be

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[20] That decrease is relative to an estimated deterministic trend.

affected by inflation possibly through the equity premium. A simple calculation of the impact of inflation on the earnings/price ratio, or equivalently of the implicit required real equity return, shows that a one percent increase in inflation increases the required real return by 1.5% when evaluated at the average earnings/price ratio over the sample. Inflation's impact on the required return decreases to 1 percent for the lowest earnings/price ratios (when inflation was low) and may be as high as 2.8% for the periods of higher earnings/price ratios (when inflation was high).

Finally, as shown by the contribution of the transitory shock to stock prices in Figure 5, the 1981-82 bear market has been associated with a correction of close to 30%, while the 1987 stock market correction has been estimated at about 15%. The stock market was around 10% undervalued at the beginning of 1994 and around 10% overvalued at the end of 1999Q4. This is in line with the S&P500's correction we have observed since the beginning of the new millenium. It is also interesting to notice the market tendency to overreact, an overevaluation being usually followed by an undervaluation.

6. Conclusions

In this paper, we have developed an econometric model of stock price valuation, with a role for the impact of inflation on the equity premium. The model relies neither on market expectations nor on "subjective" fair equity premium, but estimates the relationship between stock prices, after-tax profits, real long-term interest rates and inflation, using the SVECM methodology of King, Plosser, Stock and Watson (1991). It then decomposes the individual variables into their permanent and transitory components. The sum of the estimated permanent components of stock prices is assumed to represent the long-term fair value of the S&P500, while the transitory component gives an indication of the degree of overvaluation.

Our results indicate that about 20% of the increase of the stock market in the 1994-1999 period is attributable to the deterministic trend in real economic activity and about 40% is due to the large "stochastic" increase in after-tax profits during that period. Another 30%

of the increase in stock prices increase is associated with the decline in inflation, and about 10% seems to be transitory . This portion is assumed to represent the degree of overvaluation of the S&P 500 at the end of 1999.

In line with Sharpe (1999), our model indicates the change in the inflation stochastic trend has been a very important driver of *real* stock prices over the last decades. Our results suggest that investors ask for a higher risk premium as inflation increases. We generalised Sharpe’s 1979-1998 result over a longer period of three decades, with a methodology that does not rely on market expectations.

Appendix A

A. KPSW's identification methodology

Let X_t be a $n\,x1$ vector of cointegrated I(1) variables written in the error correction form:

$$\Delta X_t = \mu + \sum_{j=1}^{l} A_j \Delta X_{t-j} + \alpha\beta' X_{t-1} + \varepsilon_t \qquad (A1)$$

where α and β are $n\ x\ r$ matrices of full rank, and r is the number of cointegrating vectors. We can invert (A1) to obtain the following MA representation:

$$\Delta X_t = \mu + \varepsilon_t + C_1\varepsilon_{t-1} + ... = \mu + \sum_{j=0}^{\infty} C_j \varepsilon_{t-j} \equiv \mu + C(L)\varepsilon_t \qquad (A2)$$

where ε_t is a $(n\times1)$ vector of innovations and C_i's are $(n\times n)$ matrices of estimated parameters. From the reduced-form model (A2), we need to identify the following structural model:

$$\Delta X_t = \mu + \Gamma_0\eta_t + \Gamma_1\eta_{t-1} + ... = \mu + \sum_{j=0}^{\infty} \Gamma_j \eta_{t-j} \equiv \mu + \Gamma(L)\eta_t \qquad (A3)$$

where η_t is a $(n\times1)$ vector of unknown structural shocks, $Var(\eta_t) = \Sigma_\eta$, and Γ_j's are (n x n) matrices to be identified, where the τ_{kl} element of Γ_j measures the impact on the k^{th} variable of the l^{th} structural shock, after *j* periods.

The first identification restriction is that Σ_η is block-diagonal, the two blocks corresponding to the partition $\eta_t = (\eta_t^p, \eta_t^t)$ where η_t^p is a vector $(k\times1)$ of permanent shocks, and η_t^t is a vector $((n-k)\times1)$ of transitory shocks. The Σ_{η^p} matrix is also assumed to be diagonal, that is, each permanent shock is uncorrelated with the others.

The other identification restrictions are:

$$\Gamma(1) = [\tilde{A}\Pi \quad 0] \qquad (A4)$$

where $\tilde{A}$ is a known full rank $(n \times k)$ matrix, whose columns are orthogonal to the cointegration vectors, i.e. $\beta'\tilde{A} = 0$ (see Engle et Granger [1987]). Π is a "to be identified" $(k \times k)$ lower triangular matrix whose role is to allow some shocks to have a long run impact on more than one variable, and 0 is a $(n \times (n-k))$ matrix of 0's. Given that Π is usually not diagonal, the order of the variables become important since the lower is a variable in the system, the bigger the number of permanent shocks which will have an impact on it in the long run (see section B below).

We will now show that these restrictions are sufficient to identify the structural model. Equations (A2) and (A3) are related by:

$$\Gamma_0 \eta_t = \varepsilon_t \qquad (A5)$$

$$C(L) = \Gamma(L)\Gamma_0^{-1} \qquad (A6)$$

$$C(1) = \Gamma(1)\Gamma_0^{-1} \qquad (A7)$$

Let D be any solution of $C(1) = \tilde{A}D$. Since $C(1)\varepsilon_t = \Gamma(1)\eta_t = \tilde{A}\Pi\eta_t^p$, we can write:

$$\tilde{A}D\varepsilon_t = \tilde{A}\Pi\eta_t^p \qquad (A8)$$

$$D\Sigma_\varepsilon D' = \Pi\Sigma_{\eta^p}\Pi' \qquad (A9)$$

Let $\Pi^* = chol(D\Sigma_\varepsilon D') = \Pi\Sigma_{\eta^p}^{1/2}$. Since Π is triangular and Σ_{η^p} is diagonal, we obtain a unique solution for Π et Σ_{η^p}. By (A8), we thus can identify the permanent shocks:

$$\eta_t^p = \Pi^{-1}D\varepsilon_t = G\varepsilon_t$$

It is easy to show (see KPSW) that the dynamic multipliers of η_t^p are identified by:

$$\Gamma(L) = C(L)\Sigma_{\varepsilon} G' \Sigma_{\eta^p}^{-1}$$

B. Our estimated matrix of long run multipliers

For our model, the estimated matrix of long run multipliers for the three permanent shocks, $\tilde{A}\Pi$, is the product of the two following matrices:

$$\tilde{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8.04 & -16.4 & 1 \end{bmatrix} \qquad \Pi = \begin{bmatrix} .83 & 0 & 0 \\ -0.37 & 0.4 & 0 \\ -1.64 & 0.23 & 5.59 \end{bmatrix}$$

The $\tilde{A}$ matrix is such that $\beta'\tilde{A} = 0$ and is taken directly from the cointegration vector. The first column says that a one percent increase in real interest rate, ceteris paribus, results in a 8.04% lower stock price level. The second column says that a one percent permanent increase in inflation depresses the real stock market by 16.4%, and, finally, the third column says that a one percent increase in real profits increases real stock prices by 1 percent. Should $\Gamma(1) = [\tilde{A} \;\; 0]$, the model would assume that LSP500 is the only endogenous variable in the long run. The Π matrix is a result of KPSW's identification and allows a more realistic long term dynamic structure. The product $\tilde{A}\Pi$ is the estimated structural long run impact when we let each permanent shock to have a long run impact on each variable (except for the three zero's constraints). The long term impacts are quite different than with $\tilde{A}$ alone:

$$\tilde{A}\Pi = \begin{bmatrix} .83 & 0 & 0 \\ -.37 & .398 & 0 \\ -1.64 & .23 & 5.59 \\ -2.22 & -6.28 & 5.59 \end{bmatrix}$$

For example, when a permanent increase in interest rates is allowed to have an impact on all variables, its depressing effects on the stock market is much less important, but it also decreases profits and inflation.

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Figure 1. The Data

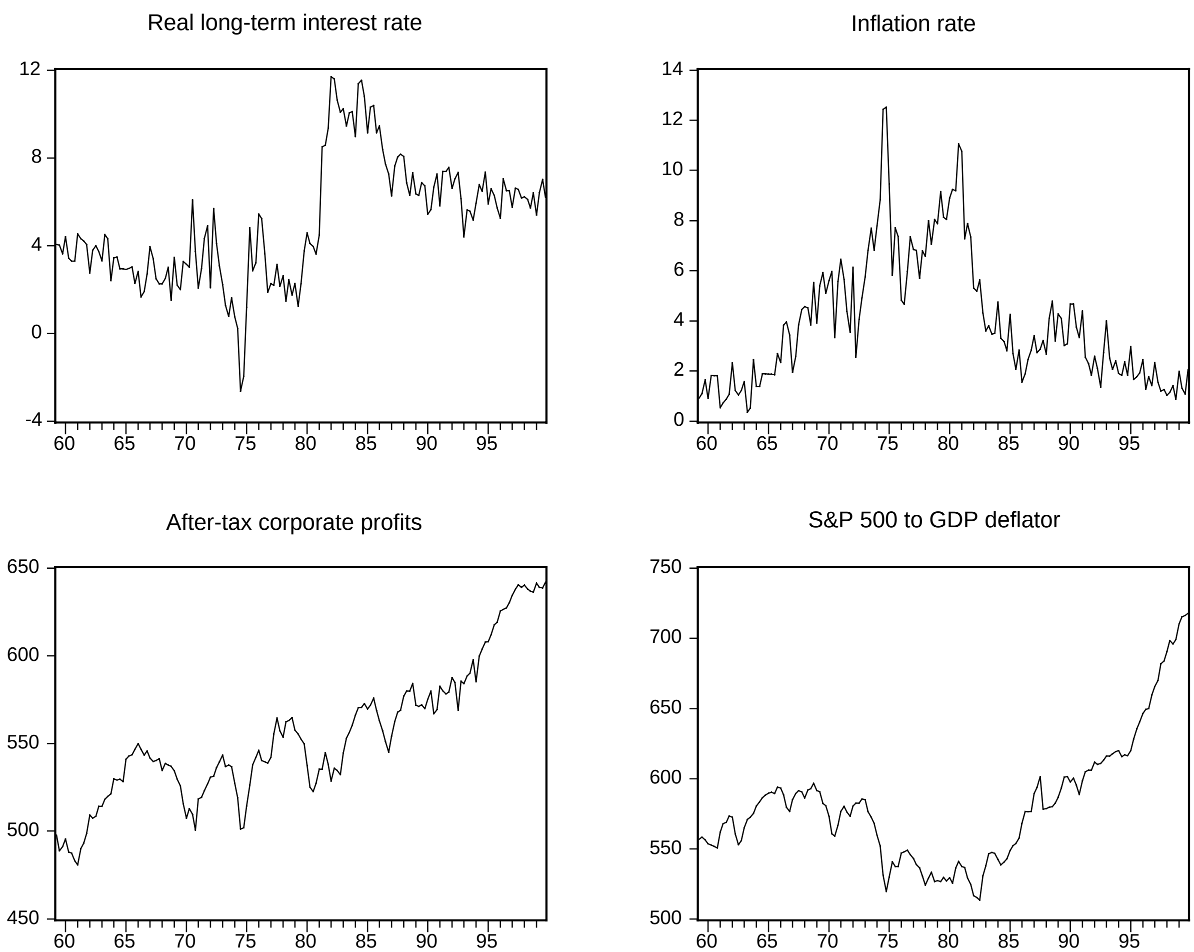

Figure 2

Stochastic components of real long-term interest rates (RBAA)

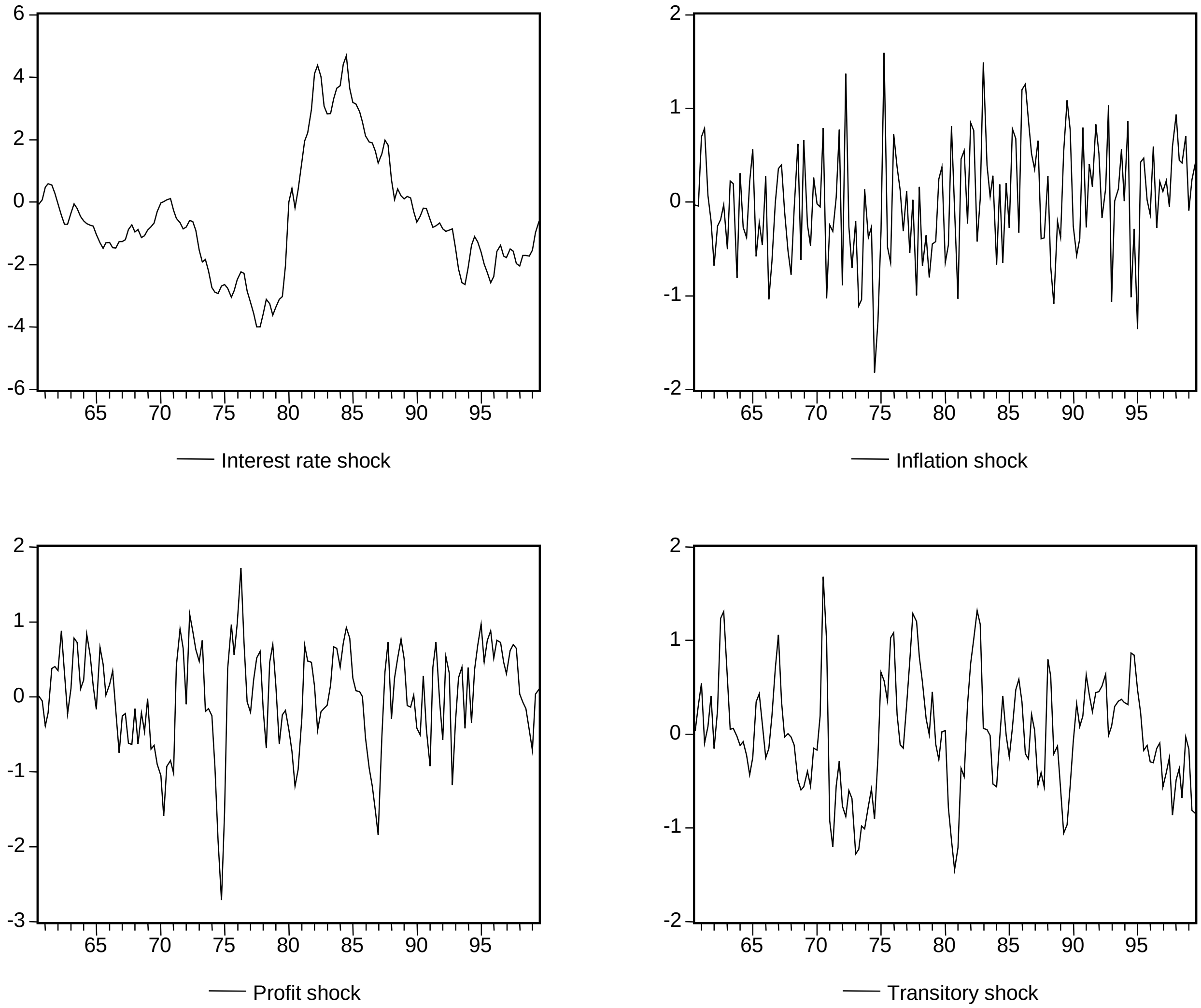

Figure 3. Stochastic components of inflation (INFL)

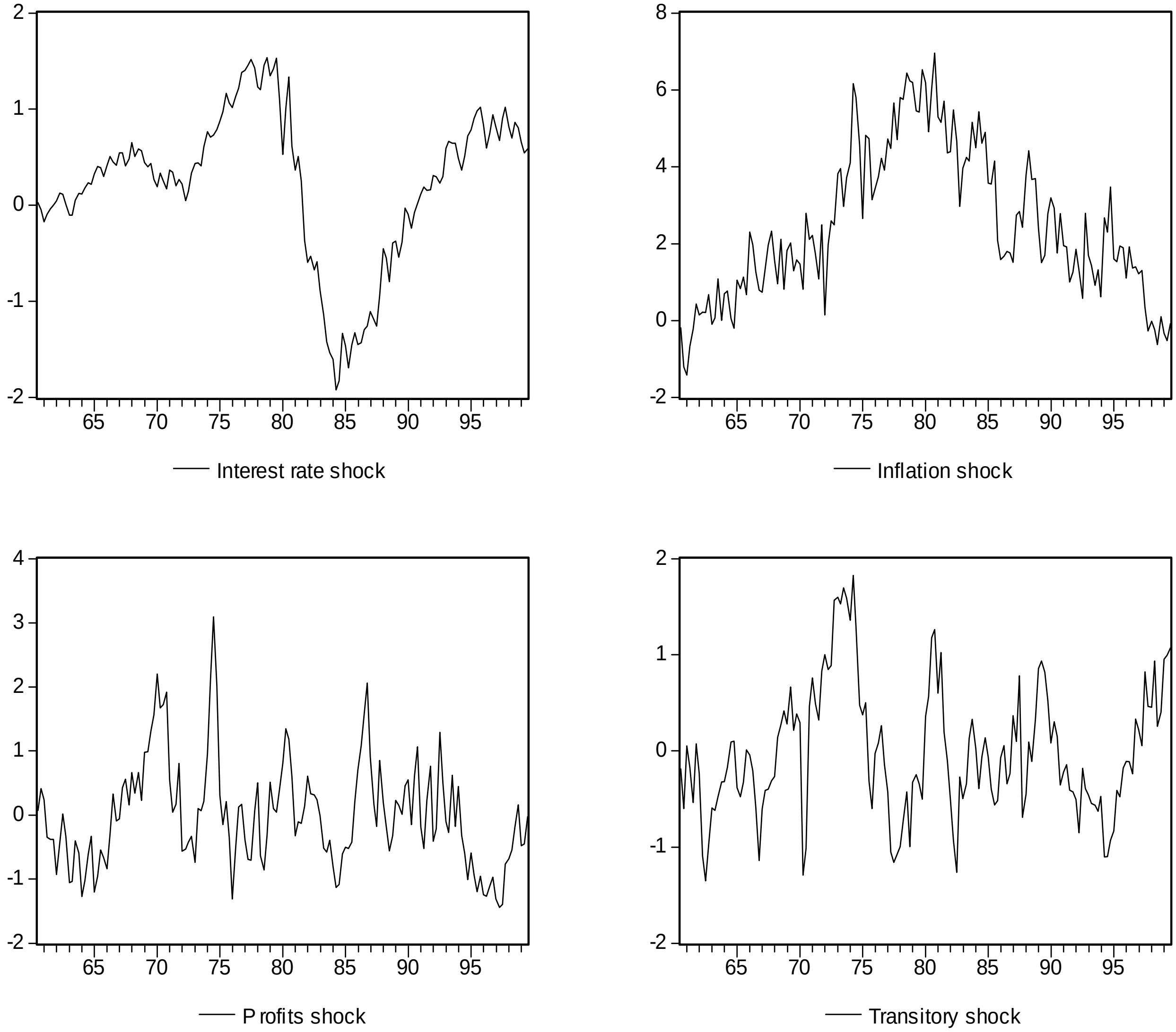

Figure 4.

Stochastic components of profits (LPRO)

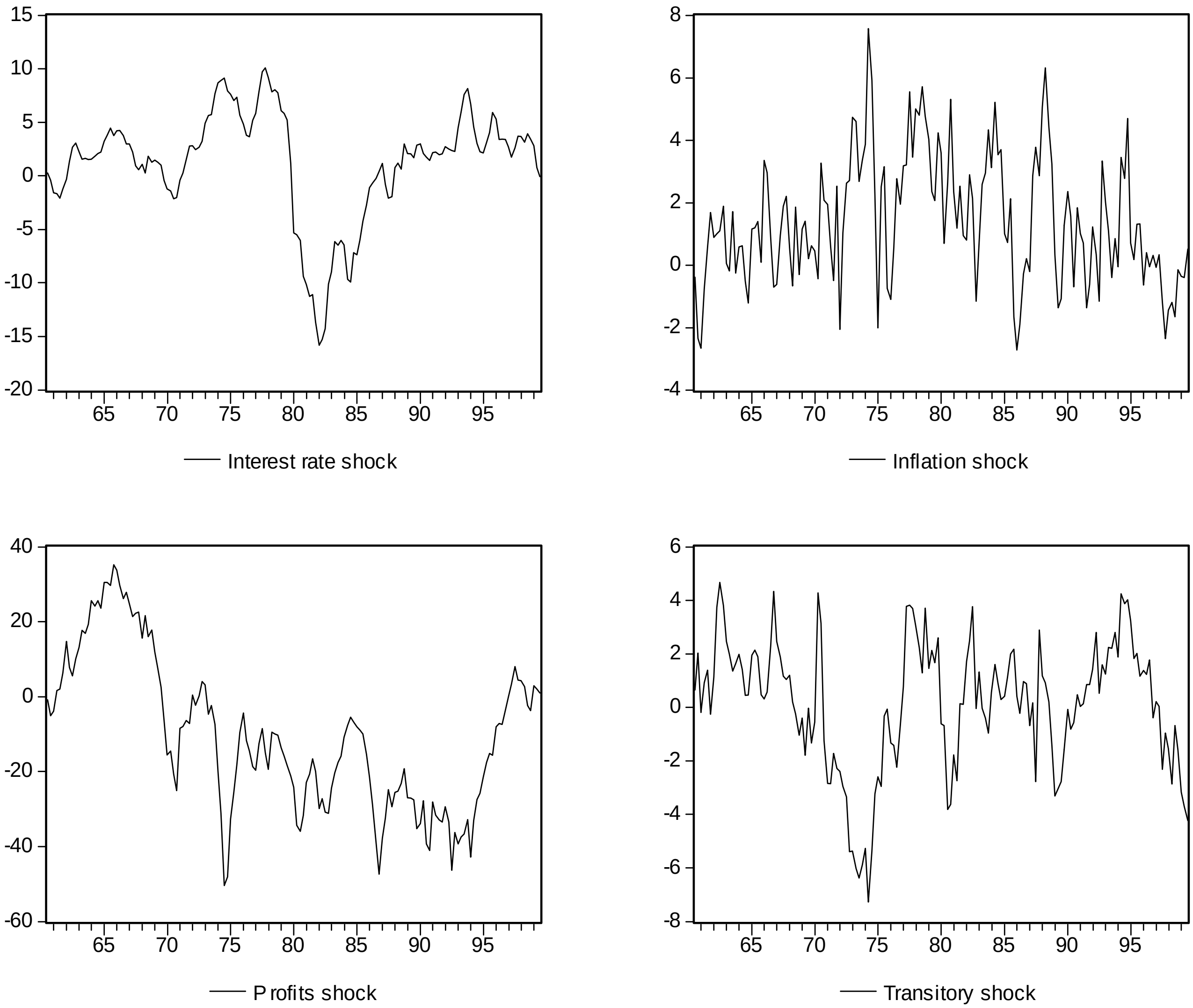

Figure 5.

Stochastic components of real stock prices (LSP500)

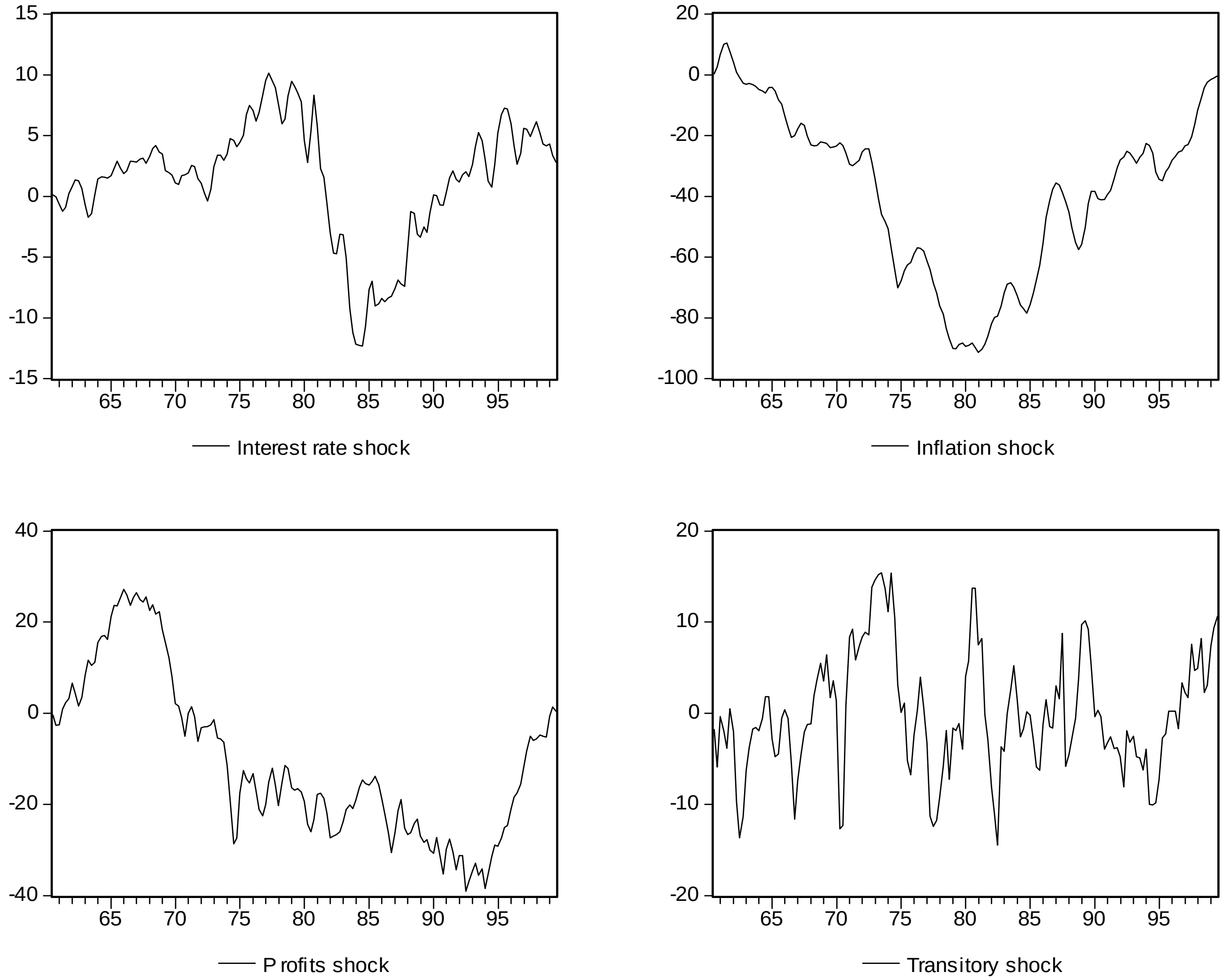

Figure 6. Impulse response functions